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Sticky Leverage: Comment*

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Abstract

We revisit the role of long-term nominal corporate debt for the transmission of inflation shocks in the general equilibrium model of Gomes et al. (2016, henceforth GJS). We show that inaccuracies in the model solution and calibration strategy lead GJS to a model equilibrium in which nominal long-term debt is systematically mispriced. As a result, the quantitative importance of corporate leverage in the transmission of inflation shocks to real activity in their framework is 6 times larger than what arises under the rational expectations equilibrium.

JEL classification: E12, E31, E44, E52, G01, G32, G35

Key words: Corporate leverage, Nominal long-term debt, Debt overhang, Generalized Euler equation

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1 Introduction

In advanced economies, the share of nonfinancial corporate long-term debt over GDP has been rising since the Great Financial Crisis and reached all-time highs after the COVID-19 pandemic (IMF, 2023; FRB, 2022). Moreover, positive inflation surprises hit the global economy as it recovered from the 2020 recession, calling for aggressive monetary policy responses from central banks. Against this backdrop, understanding and quantifying the role of long-term nominal corporate debt in the propagation of inflation and monetary policy shocks is of paramount importance.

GJS build, solve, and calibrate a quantitative general equilibrium model in which firms issue long-term nominal debt. Nominal debt creates a link between inflation and the real economy that they report to be a quantitatively important source of monetary nonneutrality even when prices are fully flexible. They also emphasize that the long-term maturity of corporate debt makes leverage sticky and enhances the amplification and propagation of inflation and monetary policy shocks on real activity.

In revisiting GJS's sticky leverage framework, we find that the quantitative importance of long-term corporate debt in the transmission of inflation and monetary policy shocks is overstated. We show that this is due to consequential inaccuracies in their model solution and calibration strategies that introduce systematic underpricing of the leverage ratchet effect (Admati et al., 2018) from the side of corporate bondholders—an unintended source of leverage stickiness. Once corrected, we find that long-term nominal debt in their model provides limited amplification of surprise inflation on real activity.

GJS compute the steady state of a simplified version of their model to inform bondholders' generalized Euler equation in the perturbation solution of their baseline model. In doing so, they disregard dynamic restrictions that are crucial for holders of long-term debt claims to correctly price the effect of firms' shareholders' leverage decisions on expected future defaults. Two simplifying assumptions are consequential in delivering an inaccurate steady-state solution. First, GJS assume that investment is independent of leverage.

However, in the presence of long-term debt, higher leverage increases the expected default rate, reduces shareholders' returns from investing, and has a negative effect on investment. Second, they assume that the functional form of the leverage policy function is a smooth polynomial. In contrast, we find that in the baseline model (i) the leverage policy function has a kink, and (ii) that the economy reaches a steady state at that kink. In such a steady state, equity holders choose the highest level of leverage that is compatible with zero default risk—thereby defying GJS's calibration strategy targeting a positive steady-state default rate.

As a result of these inaccuracies, GJS introduce an unintended source of leverage stickiness in their model: model-inconsistent beliefs. Bondholders believe that the price of debt is less sensitive to changes in leverage than it would be in an equilibrium with rational bondholders. Intuitively, when leverage is elevated, bondholders underestimate the expected cost of defaults, as they wrongly forecast that the firms' shareholders will bring leverage down swiftly. As a result, borrowing costs underprice default risk and shareholders will find it optimal to keep leverage higher for longer than they would if bondholders' expectations were rational. This feature of their solution algorithm overamplifies the response of leverage and macroeconomic outcomes to aggregate disturbances that push the corporate leverage ratio up, such as the disinflationary shocks and restrictive monetary policy shocks showcased in the paper.

We uncover an additional error in how GJS deal with a missing dynamic equation for the first derivative of the leverage policy function with respect to leverage. While solving their model with perturbation methods in the presence of a generalized Euler equation, GJS add an extra equilibrium condition obtained by differentiating the firms' leverage first-order condition (FOC) with respect to leverage. This addition results in the violation of the second-order condition (SOC) for optimality of the firm's problem with respect to leverage.

To reassess the quantitative implications of the model, we adopt a different solution strategy and revisit the model calibration. We adopt a perturbation-based solution method that can account for the presence of a generalized Euler equation without relying on GJS's simplifying assumptions. We deploy an algorithm based on Dennis (2022) and grounded in Klein et al. (2008) that relies on model-consistent dynamic restrictions from higher-order perturbations to obtain an accurate first derivative of the leverage policy function. To the extent possible, we also verify our claims in a model solution obtained using global methods.

We show that GJS's moment targets for the steady-state market value of leverage, default rates, and leverage volatility do not uniquely pin down the degree of amplification and persistence of leverage in the dynamic equilibrium. In particular, under the rational expectations solution we identify two regions of the parameter space that deliver the same steady-state moment targets with starkly different implications for leverage stickiness and the volatility of borrowing costs and real activity. Within the set of possible parameter combinations, the ones associated with an empirically plausible tradeoff between tax benefits and expected default costs deliver a significantly lower degree of leverage persistence than the one reported in GJS. Moreover, any attempt to increase leverage persistence by adjusting the calibration comes at the cost of substantially reducing the amplification of inflation shocks.

We show that the unintended departure from rational expectations magnifies the effect of their baseline disinflationary shock on real activity by a factor of 6 relative to the model solved under rational expectations and purposefully calibrated to achieve a high degree of leverage persistence. We also show that long-term nominal debt provides no amplification of monetary policy shocks on aggregate output relative to a model with short-term nominal debt, once their baseline model is extended to include sticky prices.

The comment is organized as follows. In Section 2, we describe the unintended source of leverage stickiness and shock amplification introduced by the inaccuracies in GJS's solution strategy. In Section 2.2, we revisit GJS's calibration strategy and show that it is not well suited to uniquely pin down the degree of leverage stickiness in the model. In Section 3, we evaluate the extent to which bondholders' model-inconsistent beliefs under their calibration and solution overemphasize the quantitative importance of long-term debt in the transmission of inflation shocks relative to our rational expectations solution in a set of plausible model calibrations. Section 4 concludes.

2 Inaccuracies in GJS's Model Solution and Calibration

In this section, we argue that GJS's proposed solution method generates a systematic wedge between the firm's leverage policy function and bondholders' perception of it—i.e., the leverage policy is not time-consistent. We provide an alternative model solution. As we recalibrate the model, we show that GJS's calibration strategy to match three steady-state moment targets is insufficient to uniquely pin down the equilibrium degree of leverage stickiness.

2.1 An Unintended Source of Leverage Stickiness: Model-Inconsistent Beliefs

In the presence of a generalized Euler equation to price long-term debt, any solution method that perturbs equilibrium conditions around the deterministic steady state faces the difficulty that in order to determine equilibrium variables at a particular point of the state space, one needs to know how the leverage policy function, h, behaves in the neighborhood of that point. As a result, the deterministic steady state cannot be computed without knowing the equilibrium dynamics of leverage around that steady state. We can see this by studying the FOC for leverage, ω':

$$Q(\omega') + \frac{\partial Q}{\partial \omega'}\left(\omega' - \frac{(1-\lambda)\omega}{g(i)\mu}\right) = -(1-\tau)\mathbb{E}\left[M'\Phi(z^{*\prime})\left(\frac{\partial z^{*\prime}}{\partial \omega'}\right)\right], \qquad (1)$$

and noting that the partial derivative of the price of debt with respect to next-period leverage, $\frac{\partial Q}{\partial \omega'}$ can be derived by differentiating the demand function for long-term corporate debt:

$$Q(\omega') := \mathbb{E}\left[M'\left(\Phi(z^{*'})\left(\frac{\mathsf{c}+\lambda+(1-\lambda)q'}{\mu'}\right) + \left(1-\Phi(z^{*'})\right)\mathfrak{p}(\omega')\right)\right]. \qquad (2)$$

with respect to ω':

$$\frac{\partial Q}{\partial \omega'} = \mathbb{E}\left[M'\left\{(1-\lambda)\left(\frac{\partial Q}{\partial \omega''}\right)\frac{h'_\omega}{\mu'} + \phi(z^{*\prime})\left(\frac{\partial z^{*\prime}}{\partial \omega'}\right)\left(\frac{\tau \mathsf{c}}{\mu'} + \frac{\xi}{\omega'}\right) - (1-\Phi(z^{*\prime}))\frac{\mathfrak{p}(\omega')}{\omega'}\right\}\right], \tag{3}$$

where $\mathfrak{p}(\omega')$ denotes creditors' real payoff per unit of outstanding debt in the event of default. The assumption of rational expectations requires that the equilibrium bond price equals the demand function evaluated at the equilibrium leverage policy function, i.e., $q = Q \circ h$.[1]

For consistency with GJS's notation, we denote the partial derivative of leverage policy h with respect to predetermined leverage as $h_\omega := \frac{\partial h}{\partial \omega}$. The appearance of h_ω in (3) poses a nontrivial problem for the computation of the deterministic steady state: one needs to "guess" the steady-state value of h_ω.[2] GJS frame this issue as follows: "Essentially, there is one additional variable to solve for, namely h_ω, without an additional equation." (p3812). However, this statement is only true if one ignores the fact that the equilibrium conditions provide cross-equation restrictions at each point of the state space, *not just at the steady state*. While global solution techniques automatically take these extra restrictions into account, they tend to be computationally intensive.[3]

GJS propose a compromise and devise a two-step perturbation-based solution method to approximate a rational expectations equilibrium of their model with corporate nominal long-term debt. In the first step, they use global techniques to solve for the steady-state of a *simplified* version of the model. In the second step, they use the steady-state equilibrium from the simplified model to pin down h_ω in the generalized Euler equation of the *baseline* model. In other words, GJS conjecture that the steady state of the simplified model provides a good guess of the unknown value of h_ω at the steady state of the baseline

[1] See appendix A for a detailed description of the financial block of GJS's model and its notation.

[2] In other words, the combination of (1) and (3) gives rise to a "generalized Euler equation." Generalized Euler equations are different from standard Euler equations in that they contain derivatives of variables with respect to endogenous state variables.

[3] Two recent examples that use global methods are Jungherr and Schott (2021) and Jungherr and Schott (2022). In both cases, their analysis abstracts from aggregate uncertainty.

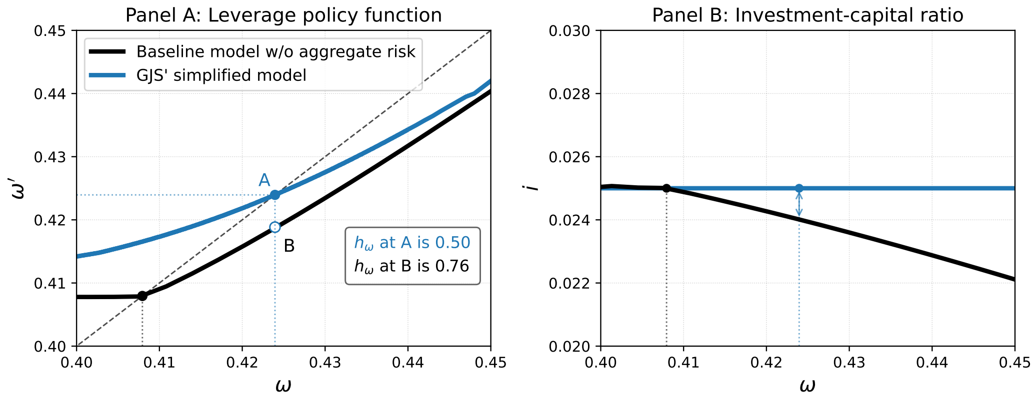


Figure 1: Policy functions from GJS's simplified model vs baseline model without aggregate risk

Note: Comparison of the results from GJS's value function iteration code (blue) and a global solution of the baseline model without aggregate risk (black). In the baseline model, there are two states: capital k and leverage ω. For the plots we fixed capital at its steady-state level. The left plot shows the leverage policy function $h(\omega, k_{ss})$ as a function of predetermined leverage ω. The right plot shows the investment-to-capital ratio $i(\omega, k_{ss})$ as a function of predetermined leverage ω. All parameters are as in Tables 1 of Gomes et al. (2016).

model.

We show that this conjecture is incorrect. The public code that implements their solution strategy reveals that a wedge exists between the value of h_ω derived from GJS's steady-state solution (≈ 0.53) and the equilibrium value from the first-order approximation of the model dynamics (≈ 0.93). Hence, GJS report a solution in which the creditors' perceived h_ω underestimates the leverage ratchet effect and thus the actual h exhibits an erroneously high level and persistence of leverage.[4]

We find that the wedge emerges from the model simplifications that they adopt to solve for the deterministic steady state. Two of such simplifications warrant special attention:

1. **Assuming that the steady-state investment-to-capital ratio i is**

[4]For an intuitive explanation of how bondholders' model-inconsistent beliefs can affect leverage persistence relative to rational expectations, see Appendix B.

constant rather than downward-sloping in leverage lowers h_ω as perceived by bondholders. To simplify the firm's problem and make it univariate, GJS assume that (a) the investment-to-capital ratio, i, is fixed and equal to the depreciation rate, $i = \delta$; (b) creditors' stochastic discount factor is $M = \beta$; and (c) there is no aggregate risk. Assumption (a) proves consequential in introducing model-inconsistent beliefs, as it fails to account for the fact that investment and leverage decisions are intertwined in a model with long-term debt. As leverage increases, expected defaults also increase. Hence, the rate of return on capital for shareholders drops, as does investment—that is, the steady-state investment-to-capital ratio is downward sloping in the leverage ratio. The right panel of Figure 1 shows the steady-state relationship between the investment-to-capital ratio (on the y axis) and the leverage ratio (on the x axis) computed from the global solution of GJS's baseline model without aggregate risk (in black) and compares it with GJS's simplified steady-state solution with a constant $i = \delta$ (in blue). The plot confirms that under GJS's calibration, the partial derivative of the investment-to-capital ratio with respect to leverage is negative, $\frac{\partial i}{\partial \omega} \leq 0$, and that the equilibrium investment-to-capital ratio declines progressively below δ as steady-state leverage increases.

How does $\frac{\partial i}{\partial \omega}$ affect the persistence of the leverage policy function? This mechanism comes into direct focus in the FOC (1) of the firm's problem with respect to ω' that we rearrange as:

$$\omega' \left(1 + \frac{Q(\omega')}{\frac{\partial Q(\omega')}{\partial \omega'}\omega'}\right) - \frac{\mathbb{E}_t\left[M'\Phi(z^{*\prime})\Big((1-\tau)\mathsf{c} + \lambda + (1-\lambda)q'\Big)\frac{1}{\mu'}\right]}{\frac{\partial Q(\omega')}{\partial \omega'}} = \frac{1}{g(i)}\frac{(1-\lambda)\omega}{\mu}$$

to reveal that next period leverage ω' is increasing in current leverage ω and that the investment-to-capital ratio, i, inversely affects the slope of such a relationship—and hence conditions the first derivative of the leverage policy function in equilibrium, h_ω. The left panel of Figure 1 shows that the slope of the policy function derived from GJS's simplified model computed under the assumption $i = \delta$ (in blue) is indeed lower than the slope of the time-consistent policy function derived from the baseline model (in black) for any value of $\omega : i < \delta$.[5]

2. **A bounded distribution for firms' idiosyncratic risk introduces a kink in the leverage policy function, while GJS approximate such a function with a smooth polynomial**. In the baseline model, the idiosyncratic shock distribution $\Phi(z)$ has a bounded support implying that firms always have the option to make their debt risk-free by choosing the highest ω' compatible with zero default probability, $\Phi(z^{*\prime}) = 1$. This specification introduces a kink in the demand function for corporate debt Q. GJS implicitly disregard this kink by restricting the functional form of Q to be a smooth, higher-order polynomial in the value function iteration algorithm that solves for the steady state.

The quantitative implications of these assumptions are illustrated in Figure 1. The figure compares the policy functions from GJS's simplified model (blue) with the policy functions from the baseline model without aggregate risk (black) using the parameter values in Table 1 of GJS. The two policy functions intersect with the 45-degree line in the left quadrant at the model's steady-state leverage. Both the steady-state level for leverage and the first derivative—i.e., the persistence—of the leverage policy function are very different in the two models. In fact, the steady state of the baseline model (the black dot) lies at the kink with zero default risk: firms choose an ω' that maximally exploits the tax advantage of debt without having to bear any future costs of default. Moreover, at the steady state of the simplified model, investment $i = \delta$ is constrained to be higher than it would be in the baseline model. Hence, the simplified model downplays the costs of the leverage ratchet effect. As a result, the slope of the leverage policy at the steady state (point A) is lower than the slope of the rational expectations leverage policy at the same leverage value (point B).

[5]In addition, assumptions (b)-(c) ignore the fact that higher-order terms associated with risk premia affect both the deterministic steady state and the first-order approximation of the model solution around that steady state through their influence on h_ω. Admittedly, the assumption of logarithmic utility makes the quantitative effect of (b)-(c) relatively small.

Once GJS compute the steady state and the slope of the policy function h_ω from the incorrect simplified model solution, they use this information to write down the generalized Euler equation of the baseline model and to perturb the FOC of the dynamic system. In doing so, GJS de facto assume that bondholders' beliefs downplay the leverage ratchet effect relative to the firms' optimal policy. As detailed in Appendix B, the emergence of such a wedge in beliefs implies that both the level and the degree of persistence of leverage in the dynamic equilibrium are higher than they would be in a model solution in which bondholders' beliefs were model-consistent.[6]

Moreover, in the second step of their solution strategy, GJS resolve the missing-equation problem described earlier by producing an additional equation that, we argue, imposes undue restrictions on the second-order conditions of the firms' problem. They propose to differentiate the firm's FOC (1) with respect to the firm's leverage choice ω'. GJS adopt this strategy in the spirit of Klein et al. (2008), who solve the time-consistent steady state of an optimal policy problem without commitment. Crucially, in Klein et al. (2008), differentiating the private sector's FOCs with respect to the social planner's own choice variables is valid, because the private sector's FOCs must hold for all attainable values of the planner's choice variables. We argue, however, that GJS's approach—which they highlight as an important additional novelty of their paper—is conceptually wrong in their competitive equilibrium problem. In their set-up, the firm's FOC with respect to leverage (1) does not have to hold for all values of leverage, ω'. Including this restriction as an additional model equation amounts to imposing that the second derivative of the objective function with respect to the leverage choice is equal to zero and, therefore, that the firm's SOC for local optimality is violated at the steady state.[7] In sum, GJS's proposed solution method effectively restricts the curvature of the shareholder value function in the direction of leverage and casts doubt on the local optimality of firms' policies under their solution method.

[6]Incidentally, the qualitative implication of this error is similar in spirit to Miao and Wang (2010), who *assume* $\bar{h}_\omega = 0$.

[7]Using our global solution, we verify that the value of the SOC under the time-consistent solution is always strictly negative.

In what follows, we overcome GJS's two-step solution by relying on the solution method of Dennis (2022), which elicits the required information about the model dynamics from higher-order perturbations in the neighborhood of the steady state. We postulate a first-order approximation of h_ω around the steady state and pin down the coefficients of this policy function by iterating on the second-order approximation of the full model, which includes a second-order approximation of h. This algorithm ensures that the solution is accurate to the first order. In Appendix C, we describe this method in more detail, including how it can be used to obtain an exact solution to any desired order of approximation.

2.2 GJS's Calibration Strategy Cannot Uniquely Pin Down Leverage Stickiness

In this section, we deploy our rational expectations solution of the model to reassess GJS's model calibration strategy. As we adopt the same steps detailed in their paper, we find that there exist at least two calibrations that can match their moment targets and have very different implications for leverage stickiness.

After setting the coupon rate c so that the price of default-free corporate debt is 1 and debt maturity λ so that the average corporate debt maturity is 5 years, GJS identify three parameters as crucial determinants of the persistence of firm leverage: default cost ξ, corporate tax rate τ, and the parameter η_1 that is inversely related to the dispersion of the idiosyncratic shock distribution. In order to calibrate the triplet (ξ, η_1, τ) they propose three moment targets:

(I) average quarterly default rate of 0.26 percent, i.e., $\Phi(\bar{z}^*) = 1 - 0.0026$,

(II) market value of leverage of 42 percent, i.e., $\bar{q}\bar{\omega} = 0.42$,

(III) unconditional volatility of the leverage ratio of 1.7, i.e., $\sigma_\omega = 1.7$[8],

[8] The model-implied unconditional volatility of ω is computed as the standard deviation of the log difference between a simulated ω-path and its HP-filtered trend (with $\lambda = 1600$).

where the bars $\bar{\cdot}$ denote values of model variables at the deterministic steady state.

GJS acknowledge that their model is unable to match all three moment targets—in particular, the unconditional leverage volatility of 1.7 appears much higher than what a model with two aggregate shocks can attain. We address this difficulty by calibrating (ξ, η_1, τ) in two steps. First, we identify the set of (ξ, τ) pairs that lead to steady states that satisfy moment targets (I) and (II) *exactly.* The red line on the left panel of Figure 2 shows the set of such (ξ, τ) pairs. The blue dot represents the parameter values of GJS. The fact that the blue dot is not on the red line is a manifestation of the fact that the parameter values in Table 1 of GJS are calibrated with inconsistent bond holders' beliefs that cannot satisfy their moment targets under the rational expectations solution.[9] We then note that conditional on targets (I) and (II) being satisfied, the steady-state version of the firm's FOC for investment

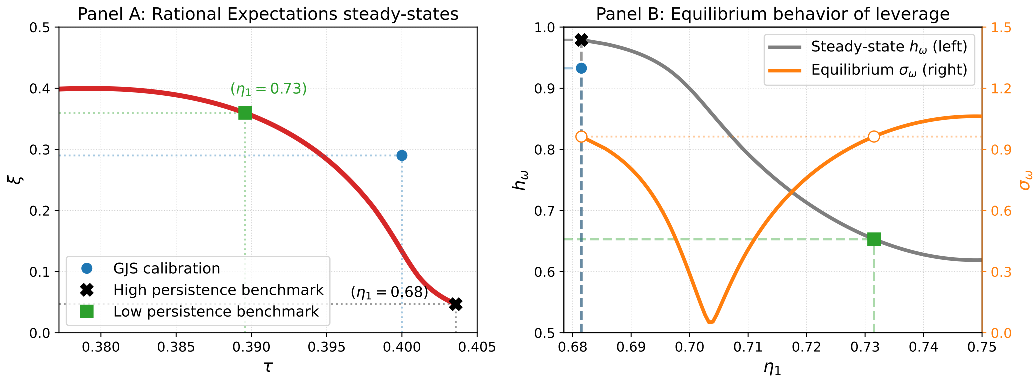
$$1 - \bar{q}\bar{\omega} = (1-\tau)\beta \left(\bar{z}^* \Phi(\bar{z}^*) - \int_{\underline{z}}^{\bar{z}^*} z d\Phi \right),$$

provides a one-to-one mapping between τ and η_1 via the dependence of the distribution Φ on η_1. This means that the three-dimensional problem reduces to a two-dimensional search in terms of (ξ, τ), because each (ξ, τ) automatically determines a specific η_1 value. In the second step of our calibration strategy, we choose the pair (ξ, τ)—and thereby η_1—so that the implied equilibrium σ_ω minimizes the distance from target (III).

The orange line on the right panel of Figure 2 depicts the equilibrium leverage volatility σ_ω as a function of η_1 (each value corresponding to a different point on the red line of the left panel). The nonmonotonic relationship arises from the fact that the partial derivative of the leverage policy with respect to inflation, $h_\mu := \frac{\partial h}{\partial \mu}$, switches signs along the red line, from being positive for relatively low values of η_1 to being negative for relatively high values of η_1 reaching zero around $\eta_1 \approx 0.703$. This implies that we can generate relatively

[9] In fact, as we saw in Section 2.1, the corresponding steady-state default rate under the time-consistent solution is zero, rather than the desired 0.26 percent.

Figure 2: Steady states consistent with moment targets (I) and (II)



Note: The red line on the left panel shows the set of (ξ, τ) pairs that lead to steady states with model-consistent beliefs and a quarterly default rate of 0.26 percent and a market value of leverage of 42 percent. Conditional on these two moment targets, there is a one-to-one relationship between τ and η_1. Parameter η_1 runs from 0.6815 (GJS's value) to 0.75 which is the theoretical upper bound for values that generate strictly positive density for the idiosyncratic shock distribution. The gray line on the right panel shows the steady state h_ω (a proxy of leverage persistence) as a function of η_1 (left y-axis). The orange line on the right panel shows the unconditional leverage volatility σ_ω as a function of η_1 (right y-axis). The blue dots represent GJS's values. The black cross represents our "high persistence benchmark" of $(\xi, \tau, \eta_1) = (0.047, 0.404, 0.6815)$ with $(h_\omega, \sigma_\omega) = (0.98, 0.96)$. The green square represents our "low persistence benchmark" of $(\xi, \tau, \eta_1) = (0.359, 0.389, 0.7315)$ with $(h_\omega, \sigma_\omega) = (0.65, 0.96)$. All other parameters as in Table 1 of Gomes et al. (2016).

high levels of leverage volatility σ_ω in two ways: either with a combination of high idiosyncratic risk volatility (low η_1), low default costs (ξ), and a high corporate tax rate (τ) or with a combination of low idiosyncratic risk volatility (high η_1), high default costs (ξ) and low corporate tax rate (τ). In other words, there are at least two regions of the parameter space (represented by the two humps of the orange line) that can get us close to GJS's three moment targets. The gray line on the right panel of Figure 2—showing the steady-state value of h_ω that we view as a proxy for leverage stickiness—demonstrates that the two regions exhibit starkly different leverage persistence. This different leverage persistence means that GJS's calibration strategy is not suitable to uniquely pin down a plausible degree of leverage stickiness.

To get a sense of the *range* of leverage stickiness the model can generate, we will use two economies that are meant to represent the two relevant regions of the parameter space:

- **high-persistence benchmark** (denoted by the black cross), which keeps GJS's idiosyncratic shock distribution fixed and calibrates (ξ, τ) to match moment targets (I) and (II)

- **low-persistence benchmark** (denoted by the green square), which we choose so that it exhibits the same $(\Phi(\bar{z}^*),\ \bar{q}\bar{\omega},\ \sigma_\omega)$ values as the high-persistence benchmark, but we require $\eta_1 > 0.71$

Additional consideration can be devoted to determining which combination of calibrated parameters is more in line with microeconomic evidence. Along these lines we note that the low-persistence benchmark calibration appears more empirically plausible, with a relatively low capital tax τ and a relatively high default cost ξ compared with the high-persistence benchmark.[10] In contrast, the location of the blue dot on the right panel of Figure 2 suggests that GJS's calibration is much closer to the high-persistence benchmark, so the latter might be viewed as the counterpart of GJS with model-consistent beliefs.

3 Leverage Stickiness Provides Limited Amplification of Inflation and Monetary Policy Shocks

In this section, we reexamine the main finding of GJS—namely, that the presence of nominal long-term defaultable debt significantly amplifies the effect and propagation of inflation shocks to the real economy—and find that the calibration and solution method implemented in GJS overemphasize the importance of this transmission mechanism. We wish to understand how much

[10] For example, Elenev et al. (2021) argue that ξ should be around 0.5, while He and Milbradt (2014) find that bankruptcy costs are, on average, about 40% of the value of firms' bonds.

leverage stickiness one can expect from the leverage ratchet effect alone without the amplifying forces of model-inconsistent beliefs that we described in Section 2.1.

To assess how the existence of nominal long-term debt affects the propagation of inflation shocks, Figure 2 in GJS reports the responses of aggregate variables to a negative inflation shock. These impulse response functions (IRFs) are replicated in Figure 3 by the blue dotted lines. They show how a negative shock to inflation increases the real burden of outstanding corporate debt and expected default rates in a very persistent way, even though inflation returns to its steady-state value relatively quickly. This persistent response of corporate leverage and default translates into a persistent response of investment and output.

We contrast these IRFs that combine the leverage ratchet effect and model-inconsistent beliefs with responses shaped by the leverage ratchet effect alone. In particular, the dashed black and solid green lines in Figure 3 represent IRFs from our high-persistence benchmark and low-persistence benchmark calibrations, respectively. As discussed previously, these economies were calibrated so that they satisfy moment targets (I) and (II) exactly and get as close as possible to target (III). By and large, these IRFs are representative of the dynamics that we can expect from the model with a very low default cost (high persistence) and an empirically more plausible level of default cost (low persistence). In addition, the shaded areas in the panels of Figure 3 show the min-max ranges of attainable IRFs, assuming that one uses only targets (I) and (II) and disregards target (III). In other words, they summarize the IRFs that can be obtained by any of the calibrations compatible with the red line in the left panel of Figure 2.

The responses of aggregate real variables to the inflation shock under model-consistent beliefs are markedly different from those in GJS. In particular, inflation shocks no longer have strong and persistent effects on investment and output. Under calibrations for which the initial response is somewhat strong, persistence tends to be very low. Under calibrations for which persistence is higher, the initial response is substantially weaker than reported by GJS. Moreover, as we discussed earlier, high persistence requires unrealistically low bankruptcy costs ξ. The systematic misperception by creditors implicit in GJS, in combination with the other inaccuracies we have pointed out, overemphasizes the degree of amplification of output arising from leverage stickiness by a factor between 1.2 and 6 on impact depending on the triplet of parameter values (ξ, τ, η_1) under consideration. Under the high-persistence benchmark, a disinflationary shock causes output to drop by 0.09% on impact compared to 0.52%. Under the low-persistence benchmark, the same shock causes output to drop by 0.42%, although the effects are short-lived. While the half-life of the inflation shock to output reported by GJS is 4 years, it is 2 quarters in the low-persistence benchmark and more than 10 years in the high-persistence benchmark.

Finally, we apply the above solution and calibration methodology to the New Keynesian version of GJS's model that includes sticky prices (described in Section IV of the original paper). Figure 4 shows a comparison between GJS's published impulse responses for the one-period and long-term debt version of their New Keynesian model—the orange dashed-dotted and the blue dotted lines respectively—and the corresponding responses computed under the low-persistence and high-persistence benchmarks under the rational expectations equilibrium of the model with long-term debt—the green and black dashed lines. The impulse responses suggest that under rational expectations and a range of plausible model calibrations, long-term nominal debt provides no amplification of the effects of monetary policy shocks on aggregate output relative to a model with a New-Keynesian model with one-period nominal debt.[11]

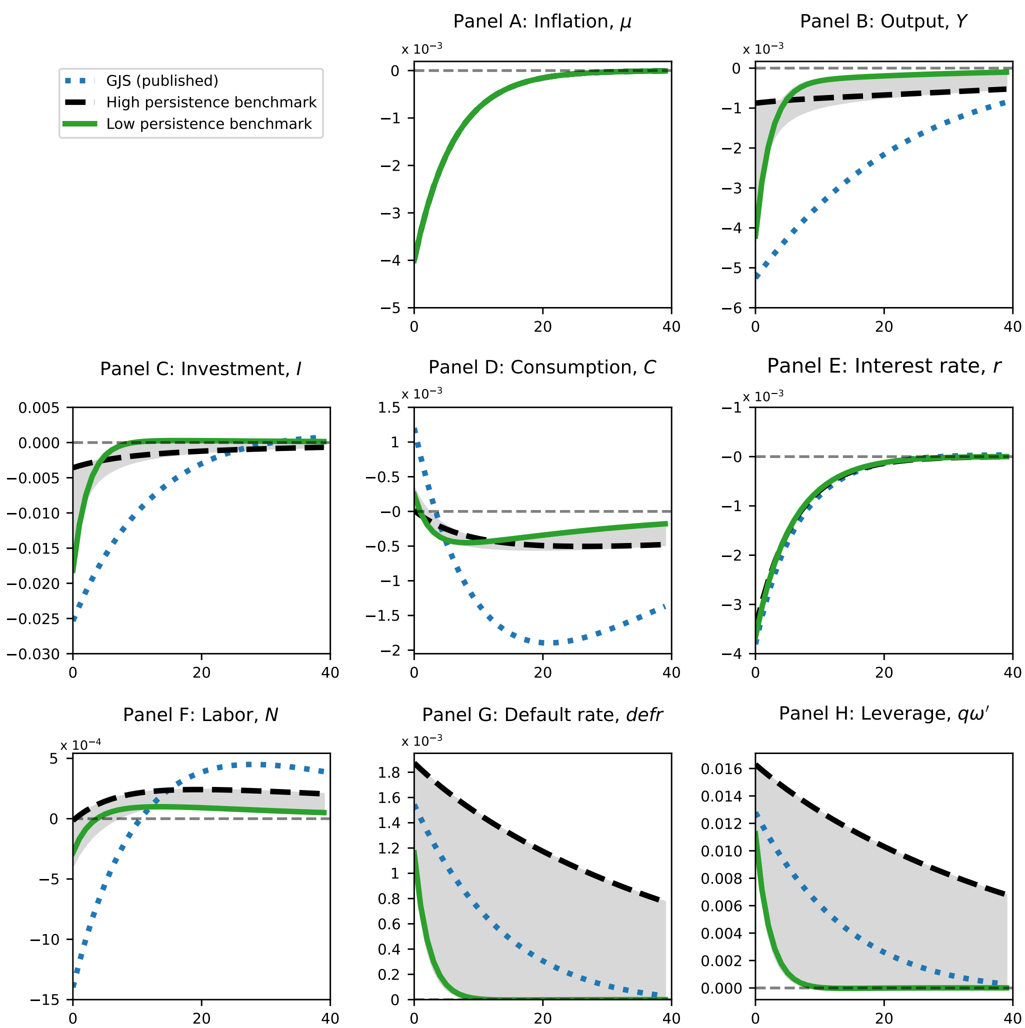
4 Concluding Remarks

In this paper, we revisit the role of long-term corporate debt in the transmission of inflation shocks in Gomes et al. (2016). We show that inaccuracies

[11] In Appendix D, we highlight the important quantitative consequence of GJS's calibration of a 100% deadweight loss from corporate default. We find that the majority of the output response in the top-right panel of Figure 3 is attributable to this direct effect.

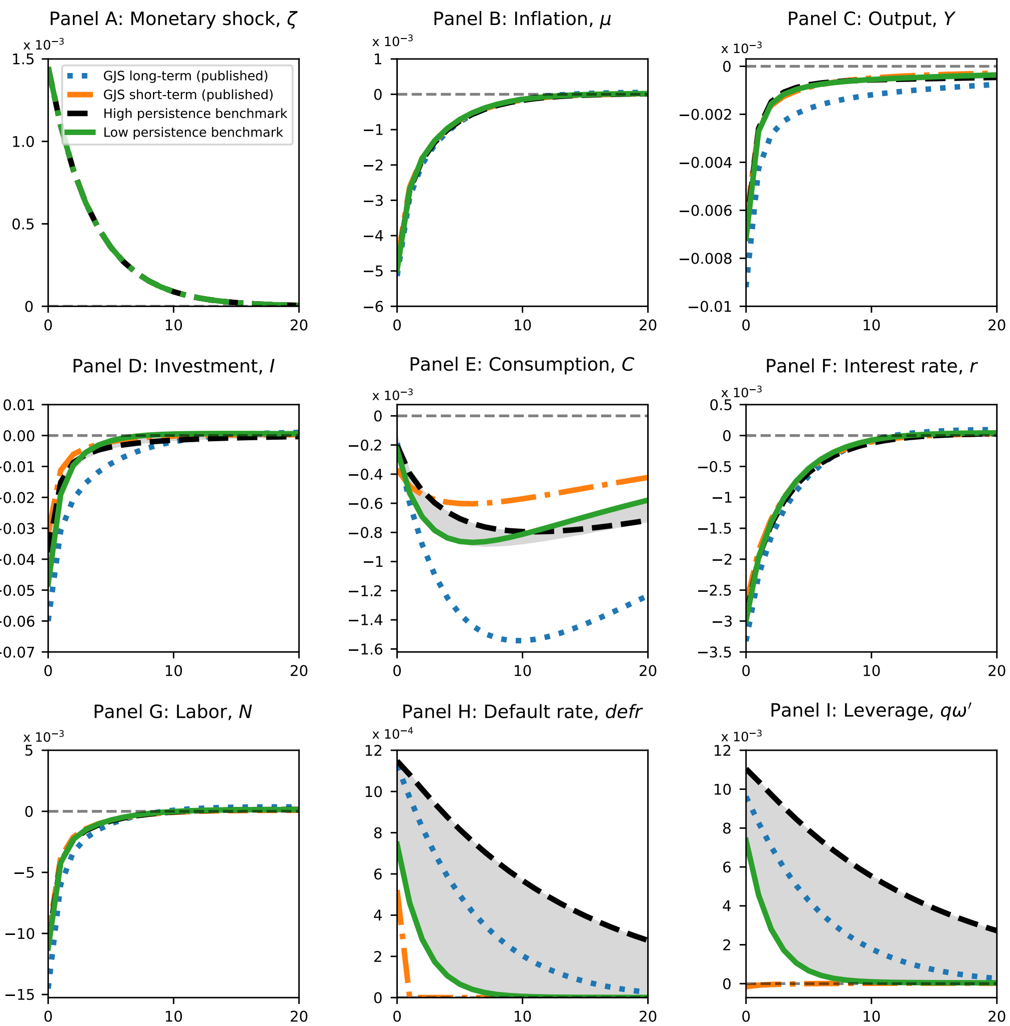
in the model solution and calibration strategy result in an overstatement of the quantitative importance of long-term corporate debt in the transmission of inflation shocks. The excessive amplification and propagation of shocks stem from GJS's solution method reaching an equilibrium in which creditors display beliefs on firms' leverage decisions that are incompatible with rational expectations. A systematic misperception dampens the response of borrowing costs to changes in expected default risk and exaggerates the implications of leverage stickiness for real activity of inflation shocks by up to a factor of six. Any attempt to recalibrate the model to increase leverage stickiness comes at the cost of lower amplification of the effects of inflation shocks on real activity.

Figure 3: Impulse Responses to a Disinflation Shock in Model with Flexible Prices



Note: This figure reproduces the impulse responses from the solution of their flexible-price model with model-inconsistent beliefs displayed in Figure 2 of Gomes et al. (2016) (blue dotted lines) and compares them to a range of possible impulse responses coming from rational expectations equilibria with different (ξ, η_1, τ) tuples that satisfy GJS's moment targets. The gray areas show the min-max range of attainable impulse responses for each horizon. The black dashed lines represent IRFs from our "high persistence benchmark" with $(\xi, \eta_1, \tau) = (0.047, 0.404, 0.6815)$. The green solid lines represent IRFs from our "low persistence benchmark" with $(\xi, \eta_1, \tau) = (0.359, 0.389, 0.7315)$. All other parameters are as in Table 1 of Gomes et al. (2016).

Figure 4: Impulse Responses to a Monetary Policy Tightening Shock in Model with Sticky Prices



Note: This figure shows the impulse responses from GJS's solution of their New Keynesian sticky-prices model with model-inconsistent beliefs (blue dotted lines) and compares them to a range of possible impulse responses coming from rational expectations equilibria with different (ξ, η_1, τ) tuples that satisfy GJS's moment targets. The gray areas show the min-max range of attainable impulse responses for each horizon.

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A The Intended Source of Leverage Stickiness: the Leverage Ratchet Effect

GJS introduce *nominal defaultable long-term debt* in a quantitative general equilibrium model. In their framework, corporate debt financing has a tax advantage relative to equity, and limited liability makes voluntary default possible. Costly default gives rise to a tradeoff between debt and equity, so that firm leverage—defined as the ratio $\omega := b/k$ between debt b and capital k—is a well-defined choice variable. When debt is *long-term*, firm leverage becomes an endogenous state variable as a result of the leverage ratchet effect of Admati et al. (2018). Finally, when long-term debt contracts are set in *nominal* terms, surprise inflation or deflation has a direct effect on the real burden of debt and leverage, leading to potentially persistent real effects. GJS articulate this mechanism in a flexible-price model that they later extend by adding sticky goods prices and a monetary policy rule. In this comment, we focus on GJS's flexible-price model, but all of our findings carry through in their extended New Keynesian model.

There is a continuum of measure one of firms. At the beginning of the period, each firm j is hit by an idiosyncratic additive revenue shock z_t^j. The shock z_t^j is i.i.d. across firms and time, has mean zero, and cumulative distribution function Φ. Firms finance capital investment by issuing equity and a defaultable *nominal* debt instrument b_t^j with price q_t, coupon rate c, and (fixed) average maturity $1/\lambda$. GJS set up the environment so that the firm's problem is linearly homogeneous in capital and debt, so, given predetermined leverage ω, firms' choices are identical, and the j superscript can be dropped. The shareholder value per unit of capital at the end of the period (excluding period t earnings) is denoted by $v(\omega)$.

Limited liability implies that shareholders will choose to default on their debt obligations whenever after-tax earnings plus shareholder value $v(\omega)$ fall below the debt service cost. This occurs whenever the idiosyncratic revenue shock is larger than a threshold value, $z \geq z^*$, with the cutoff z^* being determined by

$$(1-\tau)z^* = (1-\tau)R + \delta\tau + \upsilon(\omega) - ((1-\tau)\mathsf{c} + \lambda)\,\frac{\omega}{\mu}, \tag{4}$$

where μ is gross inflation, R denotes the pretax rental rate on capital, τ is the linear tax rate on earnings, and $\delta\tau$ is the tax shield accrued from capital depreciation at rate δ. The last term on the right-hand side is the debt service cost, which includes a tax shield term, $\tau\mathsf{c}$, because coupon payments are tax deductible.

At the end of each period, firms choose their investment-to-capital ratio, i, and next-period leverage, ω', subject to capital accumulation $k'/k = 1-\delta+i =: g(i)$ and the representative investor's demand function for risky corporate debt $Q(\omega')$, which expresses the required price for any level of ω'. The demand function is ω' dependent, because leverage affects the default threshold z^* and, hence, the *probability of default* (see (4)). As a result, when purchasing corporate debt, the representative investor must take the issuing firm's leverage policy into account. Let h denote the leverage policy of a firm with current leverage ω and aggregate states s so that $\omega' = h(\omega, \mathsf{s})$.

In the event of default, creditors' real payoff per unit of outstanding debt is given by

$$\mathfrak{p}(\omega, \mathsf{s}) = \frac{(1-\tau)(R - \mathbb{E}^z[z|z > z^*]) + \delta\tau - \xi + \upsilon(\omega) + (1-\lambda)q\frac{\omega}{\mu}}{\omega},$$

where ξ represents an additive default cost. The demand function for corporate debt is then determined by the Euler equation of the investor with one-period-ahead stochastic discount factor M':

$$Q(\omega') := \mathbb{E}\left[M'\left(\Phi(z^{*'})\left(\frac{\mathsf{c} + \lambda + (1-\lambda)q'}{\mu'}\right) + \Big(1 - \Phi(z^{*'})\Big)\mathfrak{p}(\omega')\right)\right]. \tag{5}$$

Taking the default threshold z^*, stochastic discount factor M', and demand function Q as given, firms maximize their shareholders' value by solving the following Bellman equation:[12]

$$v(\omega) = \max_{i,\omega'} \quad Q(\omega')\Big(\omega' g(i) - (1-\lambda)\frac{\omega}{\mu}\Big) - i + g(i)\mathbb{E}\left[M' \int_{\underline{z}}^{z^{*\prime}} (1-\tau)\left(z^{*\prime} - z'\right) d\Phi\right]. \quad (6)$$

The demand function $Q(\omega')$ is a downward-sloping function of leverage: higher leverage implies higher expected default rates and lowers bond prices. Low bond prices hurt the firm, because issuing new debt becomes more costly. When debt is long term ($\lambda < 1$), however, increasing leverage benefits shareholders because it dilutes old bondholders' claims. As a result, long-term debt introduces a potential conflict between firm value maximization and shareholder value maximization, which, in equilibrium, resolves in favor of shareholders resisting buying back outstanding long-term debt—what Admati et al. (2018) call the "leverage ratchet effect." Rational forward-looking creditors understand the shareholders' incentives toward higher leverage and will consistently adjust bond prices to be compensated for the additional default risk that they will have to bear. In a Markov Perfect Equilibrium with *time-consistent* policies, the interaction of shareholders' incentives and creditors' best response makes leverage a persistent state variable and delivers a policy function h that is upward sloping and convex.

B The Unintended Source of Leverage Stickiness: Model-Inconsistent Beliefs vs. Rational Expectations

The demand function $Q(\omega')$ and the equilibrium bond price q are distinct objects, but the assumption of rational expectations links them together by requiring that the equilibrium bond price equals the demand function evaluated at the equilibrium leverage policy function, $q = Q \circ h$. Suppose, however, that the bondholders' expectation of what leverage policy firms will choose in the future is not rational: for unspecified reasons, instead of h, they incor-

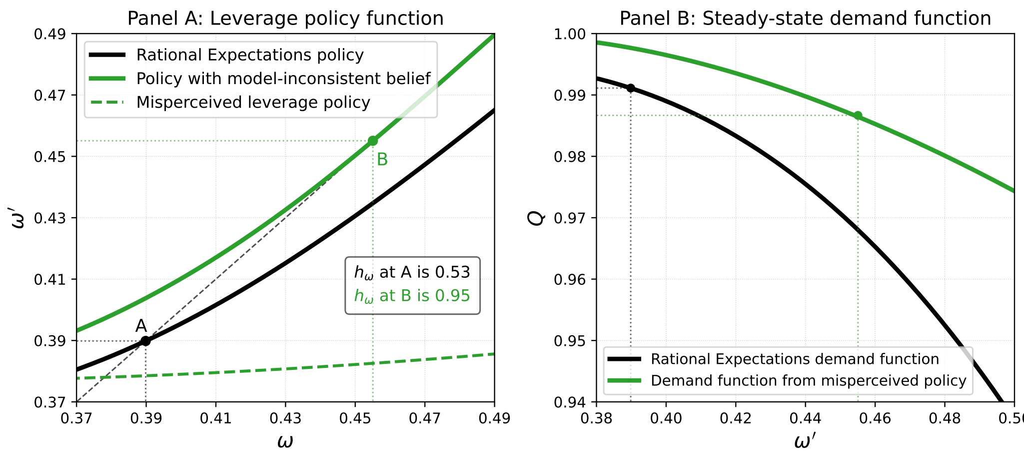
[12] The assumption that firms take the demand function Q as given is a stronger notion than rational expectations, because it requires that firms also know the prices of assets that are not traded in equilibrium. This approach follows Makowski (1983) and—as explained by Gale and Gottardi (2020)—delivers the same results as assuming complete markets for all debt and equity contracts, as in Hart (1979) and Allen and Gale (1991).

rectly and systematically perceive that firms' leverage policy will be governed by some other function $\bar{h}$. As a result, when bondholders price long-term corporate debt, they will input $q' = Q \circ \bar{h}'$ instead of $q' = Q \circ h'$ in the pricing equation (5). The demand function will depend on model-inconsistent beliefs over the leverage policy function, $\bar{h}$. Firms take this $\bar{h}$-dependent demand function as given when choosing their optimal leverage policy h to maximize shareholders' value, as a solution to (6). Therefore, in an equilibrium with model-inconsistent beliefs, a wedge arises between the perceived leverage policy $\bar{h}$, which shapes the demand for corporate debt, and the actual leverage policy h, which is the firms' best response to that demand.

To build intuition on the implications of model-inconsistent beliefs on the degree of leverage stickiness, Figure 5 depicts leverage policies (left) and steady-state demand functions (right) from a version of the model without aggregate uncertainty, solved with global methods. The figure compares the time-consistent equilibrium (black solid lines) with an equilibrium with model-inconsistent beliefs (green solid lines) under the assumption that bondholders' perceived $\bar{h}_\omega$ (green dashed line) is lower than the equilibrium h_ω. The steady-state demand functions in the right panel are downward sloping and concave in ω', showing how creditors—anticipating the leverage ratchet effect—try to discourage shareholders from leveraging up by making new debt increasingly costly. This conflict of interest explains why firms with a high level of leverage tend to maintain next period's leverage elevated, illustrating that a large stock of outstanding old debt makes shareholders less averse to bearing the increasing cost of new debt.

Figure 5 shows that the demand function from the equilibrium with model-inconsistent beliefs (green) is flatter than the rational expectations counterpart (black) for any value of ω'. The policy functions intersect with the 45 degree line in the left quadrant at the model's steady-state leverage. What makes the green demand curve flat is the fact that it incorporates the bondholder's model-inconsistent leverage policy $\bar{h}$ (green dashed line in the left panel) rather than the equilibrium firm behavior h (green solid line in the left panel) and that the

Figure 5: Leverage policy h and demand function Q in a model without aggregate risk



Note: Results from a global solution of the model without aggregate risk. There are two states: capital k and leverage ω. For the plots we fixed capital at its steady-state level. The left plot shows leverage policy functions $h(\omega, k_{ss})$ as a function of predetermined leverage. The right plot shows demand functions $Q(\omega'; \omega_{ss}, k_{ss})$ at the respective steady state (colored dotted lines) as a function of next period leverage ω'. Black solid lines are from the time-consistent equilibrium. Green solid lines are from an equilibrium with model-inconsistent beliefs. The green dashed line on the left panel represents the bondholders' misperceived policy $\bar{h}(\omega) = h_{ss} + h_\omega(\omega - h_{ss}) + \frac{h_{\omega\omega}}{2}(\omega - h_{ss})^2$ with $(h_{ss}, h_\omega, h_{\omega\omega}) = (0.378, 0.04, 0.5)$. In addition, we used $(\eta_1, \xi, \tau, \lambda) = (0.7192, 0.3089, 0.37, 0.09)$. The rest of the parameters are as in Tables 1 of Gomes et al. (2016).

slope—and hence the persistence—of the misperceived leverage policy is much lower than that of the actual policy. As a result, the equilibrium policy with model-inconsistent beliefs (the green solid line in the left panel) shows that next period leverage ω' is optimally higher for any present level of leverage ω relative to the rational expectations policy function. Moreover, at the steady state, the slope of the policy function under model-inconsistent beliefs is higher relative to its rational expectations counterpart. Intuitively, the flat demand curve of bondholders fails to provide sufficient incentives for the shareholders to reduce firm's leverage, effectively amplifying the leverage stickiness via the leverage ratchet effect.[13] Consequently, while leverage is persistent under the rational expectations policy, Figure 5 shows how the magnitude of its persistence can be amplified when the creditors' perceived leverage policy $\bar{h}$ is at odds with actual firm behavior h. In Section 2 we detail how inaccuracies in GJS's solution algorithm introduce model-inconsistent beliefs in their model.

C A Perturbation-Based Solution Algorithm to Compute the Rational-Expectations Equilibrium

The algorithm below describes how to obtain the n^{th}-order accurate perturbation of the model solution around the steady state. The starting point is the formulation of an initial guess of the n^{th}-order policy rule for h_ω, followed by the implementation of this iterative algorithm:

- Step 1: Use the model equilibrium conditions augmented with the n^{th}-order guess of h_ω to pin down the model steady state and compute the $(n+1)^{th}$-order perturbation solution around that steady state.
- Step 2: Differentiate the h policy function obtained in the $(n+1)^{th}$-order solution of Step 1 with respect to ω to obtain the n^{th}-order equilibrium policy rule for h_ω.
- Step 3: Update the n^{th}-order guess of h_ω using the equilibrium value obtained in Step 2 and return to Step 1. Iterate until convergence of the guess of h_ω used in Step 1 and the equilibrium policy rule for h_ω extracted in Step 2.

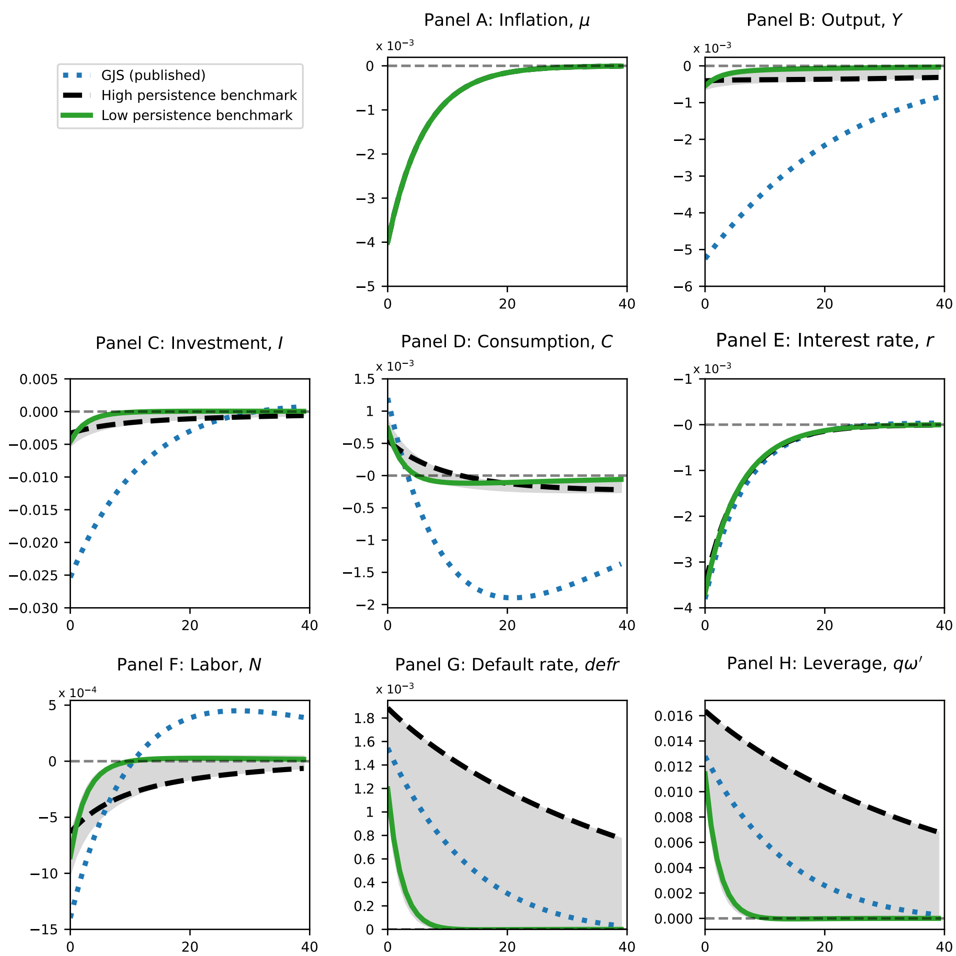
D The Dynamic Effect of Default Losses on Aggregate Output

We point out an important quantitative consequence of an arguably arbitrary and extreme calibration decision in GJS; the assumption that 100% of the

[13] The same force works the other way as well (not shown): if the perceived $\bar{h}$ exhibits higher persistence than the rational expectations policy, the best response h will have a relatively low persistence relative to the rational expectations policy.

bankruptcy losses represent real deadweight losses ($\xi^r = 1$), which leads to a direct link between the responses of default rate and output. We find that the majority of the output response in the top-right panel of Figure 3 is attributable to this direct effect. To illustrate this point, Figure 6 relaxes that assumption and assumes, instead, that bankruptcy costs are fully rebated to households ($\xi^r = 0$). As a result, the amplification and propagation of inflation shocks become much weaker. Assuming 100% deadweight losses increases the degree of amplification of inflation shocks on output by a factor in excess of about 1.5 to 8 on impact, compared with a calibration in which 0% of bankruptcy costs are deadweight losses.

Figure 6: Impulse Responses to a Disinflation Shock in the Flexible-Price Model with Deadweight Losses $\xi^r = 0$



Note: This figure shows the impulse responses from GJS's flexible-price model solution with model-inconsistent beliefs (blue dotted lines) and compares them to a range of possible impulse responses coming from rational expectations equilibria with different (ξ, η_1, τ) tuples that satisfy GJS's moment targets. The gray areas show the min-max range of attainable impulse responses for each horizon.