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A Theory of Safe Asset Creation, Systemic Risk, and Aggregate Demand

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Abstract

This paper presents a theory of safe asset creation and the interactions between systemic risk and aggregate demand. The creation of private safe assets by financial intermediaries requires them to take leverage, which generates a risk of future crisis (systemic risk) in which intermediaries liquidate assets to service their debt. In contrast, the creation of public safe assets by the government does not generate systemic risk as the government's power to tax allows it to better absorb losses. The level of systemic risk determines the neutral rate of interest through households' precautionary saving and aggregate demand. The model features a two-way interaction between systemic risk and aggregate demand. Monetary and fiscal policy can stabilize aggregate demand and reduce systemic risk by altering the mix of private and public safe assets held by savers. When monetary policy is constrained, the economy can enter a *risk-driven stagnation trap* in which economic stagnation arises due to excessive systemic risk. Macroprudential policies which reduce systemic risk can stimulate aggregate demand.

*Levent Altinoglu: Federal Reserve Board of Governors. The analysis and the conclusions set forth are those of the author and do not indicate concurrence by other members of the research staff or the Board of Governors of the Federal Reserve. Ben Roscoe provided excellent research assistance.

1 Introduction

The relationship between risks in the financial system and macroeconomic fluctuations has been at the forefront of academic and policy debates over the last two decades, which have featured periods of booms in real activity accompanied by a buildup of vulnerabilities in the financial system.[1] Moreover, persistent economic slumps have often followed financial crises and often coincide with widespread deleveraging among financial intermediaries.[2]

One mechanism which may link macroeconomic and financial vulnerabilities is the creation of safe assets, in which financial intermediaries provide insurance to savers by producing relatively safe liabilities out of the risky assets which back them. The literature has highlighted the role of safe assets in creating financial instability (Bocola and Lorenzoni (2023), Caramp (2023)) and low aggregate demand (Caballero and Farhi (2018), Caballero and Simsek (2020), Caballero and Simsek (2021)). However, much of the literature either focuses on the determination of aggregate demand, in which case the supply of safe assets is pinned down by an exogenous constraint, or uses a real model in which aggregate demand does not play a meaningful role in determining output.[3] A deeper understanding of the interactions between financial instability and aggregate demand may improve our understanding of macroeconomic fluctuations.

In this paper, I introduce a theory of safe asset creation, systemic risk, and aggregate demand which yields insights into the nature of macroeconomic booms, financial crises, and persistent slumps. The paper makes three contributions. First, it builds a model in which the creation of safe assets generates a two-way interaction between aggregate demand and systemic risk (i.e., the severity of future crises). In the model, the neutral rate of interest depends on the level of systemic risk, which in turn depends on the relative share of private versus public safe assets held by savers. Second, the paper shows the possibility of risk-driven stagnation traps in which economic growth is low because systemic risk is high, which may yield insight into the nature of persistent slumps. Third, the paper shows how monetary, fiscal, and macroprudential policies may operate through new channels once one accounts for the interactions between systemic risk and aggregate demand.

I develop these arguments using a model in which the production of safe assets generates meaningful interactions between systemic risk and aggregate demand. The model combines elements from literature on financial leverage (such as Bocola and Lorenzoni (2023), Caramp (2023), Acharya, Dogra and Singh (2022), and Segura and Villacorta (2023)), in which the leverage of fi-

[1] For example, the economic boom of the early 2000s in the United States was accompanied by a high appetite for risk and leverage among financial intermediaries.

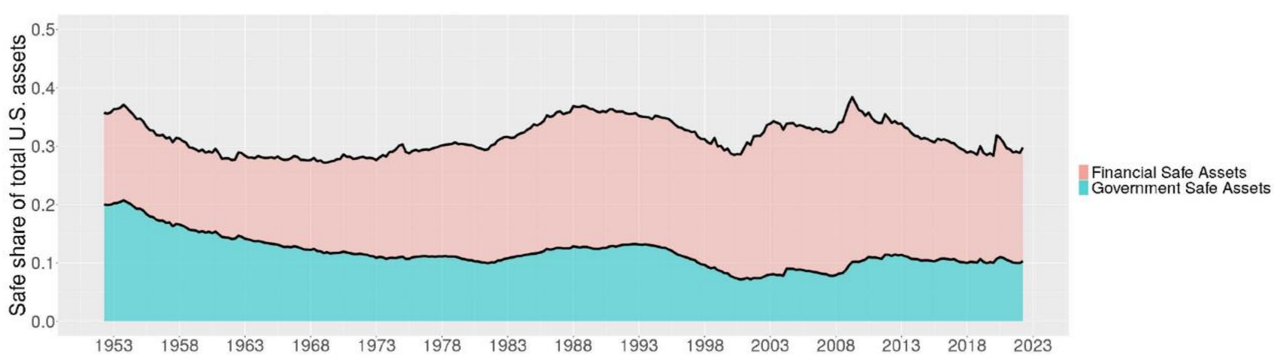
[2] Notable such episodes include the Great Depression, the slow recoveries of the US and eurozone economies following the Global Financial Crisis, and the slump in Japan from the early 1990s until 2020.

[3] Some exceptions are the series of papers following Caballero and Simsek (2020), which use a model without financial frictions in which fluctuations in aggregate demand are driven by asset prices and heterogeneous beliefs, and Boissay et al. (2023), which examines in a New Keynesian model with endogenous financial crises.

nancial intermediaries can trigger socially costly crises, and macroeconomic models of risk sharing and nominal rigidities (such as Caballero and Farhi (2018), Caballero and Simsek (2020), Korinek and Simsek (2016), and Farhi and Werning (2016)) in which inefficient risk sharing arrangements can depress aggregate demand and output. As such, the model presents a unified framework in which both aggregate demand and the quantity of aggregate risk in the economy are determined jointly.

The model is in part motivated by an important distinction, often overlooked in the literature, between safe assets issued by private financial institutions (such as bank deposits, money market mutual fund liabilities, and asset backed-securities) and those issued by the consolidated government (such as government bonds and central bank reserves).[4] As shown in Figure 1, the relative share of privately issued safe assets in the United States, for example, tends to increase in the run-up to a financial crisis, and subsides afterword, while the share of public safe assets tends to increase following crises.[5] Moreover, the large government interventions during the periods following the Global Financial Crisis appear to have substituted private safe assets for public ones.

Figure 1: Safe asset share in the U.S. (1952-2022)



Data source: Federal Reserve Flow of Funds.

To the extent that the public and private sectors have a different capacity to bear aggregate risk, the macroeconomic consequences of the creation of public versus private safe assets may differ. In issuing safe liabilities backed by risky assets, financial intermediaries take leverage and bear

[4]By private safe assets, I have in mind claims issued by financial institutions which are relatively stable in value compared to the risky investments which back them, such as bank deposits, mortgage and asset backed securities, liabilities issued by money market mutual funds, and commercial paper. For simplicity, I abstract from government guarantees of the liabilities of private institutions, such as deposit guarantees or expected bailouts, which could also be thought of as generating public safe assets.

[5]Figure 1 plots the total value of safe liabilities in the U.S. relative to total U.S. assets using data from the Federal Reserve Flow of Funds, and decomposes safe liabilities into those issue by financial institutions and the government, as defined by Gorton, Lewellen and Metrick (2012). The Online Appendix lists the data series used. See also and Ferreira and Shousha (2022).

the risk associated with their assets. Such risk-taking can lead to socially-costly financial crises in which institutions are forced to liquidate their assets in order to service their debt. By contrast, a government which has the power to tax may be able to service or rollover its debt without systemic consequences. To understand the macroeconomic implications of private versus public safe asset creation requires a model which captures the distinct means by which the private and public sectors bear risk.

The model features three periods (dates 0, 1, and 2) and four representative agents: risk-averse households, risk-neutral banks, good-producing firms, and a government. Households supply labor inelastically in all periods and, at date 0, can save in two types of safe bonds: private bonds issued by banks and public bonds issued by the government. Each safe bond pays an uncontingent rate of return in all states of the world at date 1, and agents cannot default. Bank use the proceeds of their debt issuance at date 0 to invest in capital which has a risky return at date 1. The government can use the proceeds of its debt issuance to buy capital at date 0 and can issue lump-sum taxes and transfers at any date.

Firms combine capital and labor to produce a consumption good each period, they can vary their utilization of capital, and they have fully rigid prices. Monetary policy targets the neutral rate of interest (that is, the interest rate at which output would be at potential) each period, but is subject to an effective lower bound. A focus of the paper is the role of aggregate demand at date 0. Therefore, while date 0 output may be demand-determined, I ensure that monetary policy is unconstrained by the effective lower bound in all states at dates 1 and 2, which ensures that output is at potential at those dates.

At date 1, the economy is subject to an aggregate TFP shock to firms' production (taking a high or low value) which affects banks' return on capital. Safe asset creation at date 0 thus plays two roles: It finances risky investment in capital and it insures households against this risk.

The model's core mechanism is built on two key premises. First, the creation of private safe assets by banks has different macroeconomic consequences than does the creation of public safe assets by the government. In order to issue private safe assets at date 0, banks take leverage which generates a risk of future crisis (systemic risk) in which banks must liquidate capital at a loss to service their debt in the bad state at date 1.[6] In contrast, the creation of public safe assets by the government does not generate systemic risk, as the government's power to tax allows it to smooth losses over time and over agents. These assumptions imply that the level of systemic risk (that is, the severity of a future crisis) is endogenously determined by the composition of safe assets (public versus private) held by households at date 0.

[6]One can think of systemic risk as the endogenous component of aggregate risk which stems from the liquidation of capital in the bad state of the world, as opposed to the fundamental component of aggregate risk, which is TFP shock itself.

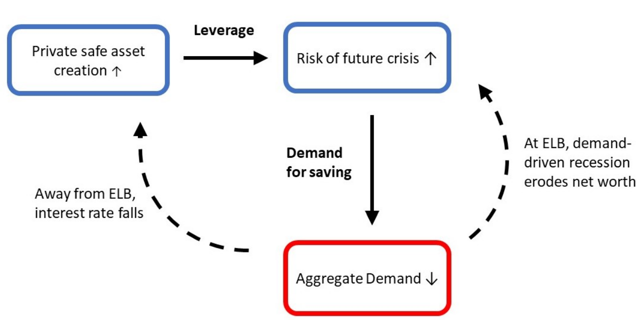
Second, crises reduce the household's future labor income due to a macroeconomic spillover, similar to that in Bocola and Lorenzoni (2023). In particular, when banks liquidate capital to repay their debt in bad states of the world at date 1, they reduce the future stock of capital at date 2. The lower future stock of capital, in turn, reduces the future wage to be earned by households at date 2, owing to complementarities between capital and labor in the production of goods. As a result, the cost of crises is shared in general equilibrium by households in the form of lower future labor income.

Thus, the creation of private safe assets by banks entails a kind of risk transformation in which the risk to banks' earnings (due to the TFP shock at date 1) is transformed into labor income risk for the household (due to the effect of liquidation at date 1 on the wage at date 2). This risk transformation highlights a contradictory aspect of safe asset creation: While private safe assets fully insure individual savers against the TFP shock at date 1, it also forces them to bear labor income risk in general equilibrium. However, households take their future wage as given and therefore do not internalize the systemic consequences of private safe asset creation when choosing their portfolios of private and public safe assets at date 0.

In response to the risk of a future crisis, households increase their precautionary saving at date 0 in order to smooth consumption across future dates and states by increasing their holdings of safe assets. In this manner, the risk of a future crisis depresses current aggregate demand at date 0. As a result, the neutral rate of interest at date 0 depends on the level of systemic risk, and by extension, is determined by the composition of safe assets at date 0.

The central mechanism which emerges from this environment is that safe asset creation gives rise to a two-way interaction between systemic risk and aggregate demand, stylistically illustrated in Figure 1. The creation of private safe assets by banks insures savers against adverse shocks, but generates a risk of future crises as a side-effect. The risk of future crises increases the household's precautionary saving which depresses current aggregate demand. When monetary policy is unconstrained by the effective lower bound (ELB), the monetary authority can reduce the nominal interest rate to ensure output remains at potential.

Figure 2: Stylistic depiction of the core mechanism



In the model, monetary policy stimulates aggregate demand not only directly through the household's Euler equation, but also through a macroprudential channel in reducing systemic risk: A lower interest rate at date 0 reduces the debt burden of banks, which reduces the amount of capital they must liquidate in a future crisis state to service their debt at date 1. This lower level of liquidation increases the household's labor income in the future crisis state at date 2, reducing the household's precautionary saving at date 0.

Moreover, the model yields insight on the debate about the role monetary policy should play in tamping down on vulnerabilities in the financial system. The literature on "leaning against the wind" (e.g., Boissay et al. (2023), Goldberg and Lopez-Salido (2023)) has shown that a cost of using tight monetary policy to reduce financial vulnerabilities is that it directly reduces economic activity in normal times. There is an additional channel in my model: A higher policy rate increases banks' cost of debt service, and therefore exacerbates the severity of a future crisis. This higher systemic risk further depresses aggregate demand and economic activity ex ante.

When monetary policy is constrained by the effective lower bound, the decline in aggregate demand leads to a demand-driven recession at date 0 (similar to Caballero and Farhi (2018)). Such a recession reduces bank earnings and erodes their net worth at date 0, further increasing the risk of future crises. The model thus features a two-way interaction between systemic risk and aggregate demand at the effective lower bound: High systemic risk lowers aggregate demand by increasing households' precautionary saving. This leads to a demand-driven recession which erodes the net worth of banks, further increasing systemic risk. Hence, the economy is characterized by a *paradox of safety* at the effective lower bound, similar to Acharya, Dogra and Singh (2022) and Segura and Villacorta (2023), in which the demand for insurance against systemic risk generates further systemic risk through the creation of private safe assets.

If the two-way interaction between systemic risk and aggregate demand is sufficiently strong, the equilibrium may feature a *risk-driven stagnation trap*, in which stagnant output growth emerges as the result of an excessively high level of systemic risk. A high level of systemic risk depresses aggregate demand and forces the neutral rate of interest to fall below the effective lower bound. The resulting demand-driven recession lowers date 0 output and the resources available for investment. Therefore, even though a higher share of output goes to investment, the level of investment falls. As a result, the future capital stock is lower at date 1, and so is future output in all states of the world.[7] Thus, the risk-driven stagnation trap arises because both the creation of safe assets and investment in capital requires that banks take more leverage, which only serves to increase systemic risk and worsen the demand-driven recession ex ante.

The risk-driven stagnation trap is similar in spirit to the stagnation trap first identified in Be-

[7]Moreover, the lower expected future output reduces banks' expected future earnings, thereby increasing the banks' burden of debt and further increasing systemic risk ex ante.

nigno and Fornaro (2018). However, the trap in my paper derives from high systemic risk rather than self-fulfilling expectations of low future growth. Therefore, in contrast to that paper, policies designed to stimulate investment may be counterproductive to the extent they further incentivize bank leverage. The trap in this model is also similar to the safety trap first identified in Caballero and Farhi (2018) in which a demand-driven recession arises due to a shortage of safe assets. However, the trap here derives in part from an *oversupply* of private safe assets. Therefore, policies designed to increase the supply of private safe assets would only worsen the demand-driven recession.

At the effective lower bound, macroprudential policies that reduce bank leverage can serve as a substitute for monetary policy and stimulate aggregate demand ex ante by reducing systemic risk. Moreover, by stimulating aggregate demand, such macroprudential policies can increase the neutral rate of interest and thereby alleviate the burden on monetary policy.

Fiscal policy can stabilize aggregate demand and reduce systemic risk by altering the mix of private and public safe assets held by savers. For instance, when the effective lower bound is binding, deficit spending financed by public safe asset issuance at date 0 can both stimulate aggregate demand directly, and indirectly by crowding-out private safe asset issuance and reducing systemic risk. These macroprudential effects increase the size of fiscal multipliers. Quantitative easing can stimulate aggregate demand at date 0 through macroprudential effects. By purchasing capital from banks at date 0 in exchange for public debt, the government can shift the composition of safe assets held by households toward public safe assets, effectively shifting the risk associated with investment from the private sector to the government's balance sheet. This reduces systemic risk and stimulates aggregate demand, even if the total supply of safe assets is held constant. Government bailouts of banks in bad states of the world at date 1 can stimulate aggregate demand ex ante by reducing systemic risk.[8]

1.1 Literature review

The seminal paper of Caballero and Farhi (2018) established the notion of the safety trap in which demand-driven recessions arise due to a general shortage of safe assets. The main innovation in my model is the dynamic interplay between aggregate demand and systemic risk, which stems chiefly from how I model the model the supply of safe assets. In the baseline case of Caballero and Farhi (2018), the supply of (private) safe assets is pinned down by an exogenous collateral constraint, and as a result, the supply of safe assets is not affected by macroeconomic conditions. While there is a social benefit to issuing safe assets, there is no social cost and therefore the supply

[8] I abstract here from moral hazard considerations whereby the anticipation of government bailouts leads to perverse risk-taking incentives at date 1.

of safe assets is an aggregate demand shifter: increase in the supply of safe asset only ever boosts aggregate demand.[9]

In this paper, by contrast, the dynamic interaction between aggregate demand and systemic risk gives rise to a social cost of issuing private safe assets. This is because (private) safe asset creation endogenously generates the risk of future crisis. In turn, crisis risk affects the demand for safe assets and aggregate demand ex ante due to a macroeconomic spillover from crises to future labor income. However this social cost is not internalized by agents ex ante because the macroeconomic spillover materializes only in general equilibrium. As a result of these interactions, the response of economy to shocks and to various policies are qualitatively different in this model. This paper also implies a sharp distinction between public and private safe assets with regard to their macroeconomic consequences. Therefore, in this model, aggregate demand (and the level of systemic risk) is determined not only by the total supply of safe assets, but also by the *composition* of safe assets between private and public.[10] Indeed, safety traps can arise from an *oversupply* of private safe assets, which can drastically alter the policy implications.

The macroeconomic spillover first modeled in Bocola and Lorenzoni (2023) plays a similar role in my model. However, their focus is on the risk-sharing problem between consumers and banks, and so these agents can trade full set of state-contingent claims. By contrast, I take as given market incompleteness by assuming that households and banks trade fixed rate bonds, and as a result, inefficient risk-sharing manifests as excessive private safe asset creation. Moreover, while Bocola and Lorenzoni (2023) use a real model whereas my focus is on aggregate demand and its interaction with systemic risk. Finally, the contrast between how the public and private sectors absorb risk is central to my results.

This paper also relates to the series of papers Caballero and Simsek (2020) and Caballero and Simsek (2021) in which the ability of the economy to absorb the risk associated with investment interacts with aggregate demand and output. While financial frictions are essential to give rise to the notion of systemic risk in my model, these papers abstract from financial frictions and focus on how asset prices affect the distribution of wealth across agents who vary in their beliefs or risk tolerance. Using a similar framework to these papers, Goldberg and Lopez-Salido (2023) identify new channels through which monetary policy may affect the severity of speculative booms and demand-driven recessions, which informs the debate surrounding the macroprudential use of monetary policy (see Ajello et al. (2019)). Boissay et al. (2023) and Collard et al. (2017) also

[9] In an extension of their baseline model, Caballero and Farhi (2018) allow for agents to relax the collateral constraint and increase the safe asset supply subject to a convex cost. However, this cost is private while the benefit of supplying safe assets in a safety trap is social. Hence, the extension still features a general under-provision of safe assets.

[10] While the supply of public safe assets acts as an aggregate demand shifter, as in Caballero and Farhi (2018), this is not generally the case for private safe assets: The risk sharing externality in my paper implies that private safe asset creation may be excessively high in a safety trap.

analyze optimal monetary and macroprudential policies when monetary policy affects financial stability.

Benigno and Fornaro (2018) is the first paper to formalize the notion of a stagnation trap in which deficient demand results in persistently low economic growth. While in that paper, the stagnation trap arises due to an endogenous fall in investment which reduces innovation and future productivity, in my paper, productivity is exogenous; the fall in expected future output arises due to a fall in investment and the future stock of capital. In addition, while self-fulfilling expectations of low growth and multiplcity of equilibria are central to the stagnation trap of Benigno and Fornaro (2018), in my model, the two-way interaction between systemic risk and aggregate demand is what sustains low investment and growth in equilibrium. Therefore, policies designed to incentivize investment may be counterproductive to the extent that they lead to higher financial leverage. Other papers which study secular stagnation include **?**, Cuba-Borda and Singh (2021), and Xavier (2023).

Korinek and Simsek (2016) and Farhi and Werning (2016) have a similar role for macroprudential policy to address an aggregate demand externality. While in Korinek and Simsek (2016), output is always supply-determined ex ante and demand-determined in bad states ex post, the reverse is true in my model. Therefore, in contrast to that paper, macroprudential policy can be used to stimulate current aggregate demand, while fiscal or monetary policy can be beneficial ex ante both by stimulating demand and by reducing systemic risk.

This paper is related to Acharya, Dogra and Singh (2022), who use a real model without aggregate risk to show that the supply of private safe assets may create its own demand. While they abstract from uncertainty to focus on the multiplicity of equilibria, I instead abstract from multiplicity to focus on how safe asset creation amplifies uncertainty through macroeconomic spillovers, and its interactions with aggregate demand. Benigno and Robatto (2019) and Infante and Ordonez (2021) examine the optimal supply of private and public liquidity. The latter paper focuses comparing the ability of public and private safe assets to facilitate the sharing of idiosyncratic liquidity risk through their use as collateral in light of their different exposure to aggregate risk. Angeletos, Collard and Dellas (Forthcoming) also examine the optimal supply of public debt in the presence of tradeoff between reducing the severity of financial frictions and reducing fiscal space. Relative to these papers, I abstract from liquidity benefits of public safe assets to focus on how public versus private safe assets generate systemic risk. Since agents in my model do not internalize that private safe assets generate more aggregate risk in general equilibrium, the convenience yield on public debt is inefficiently low.

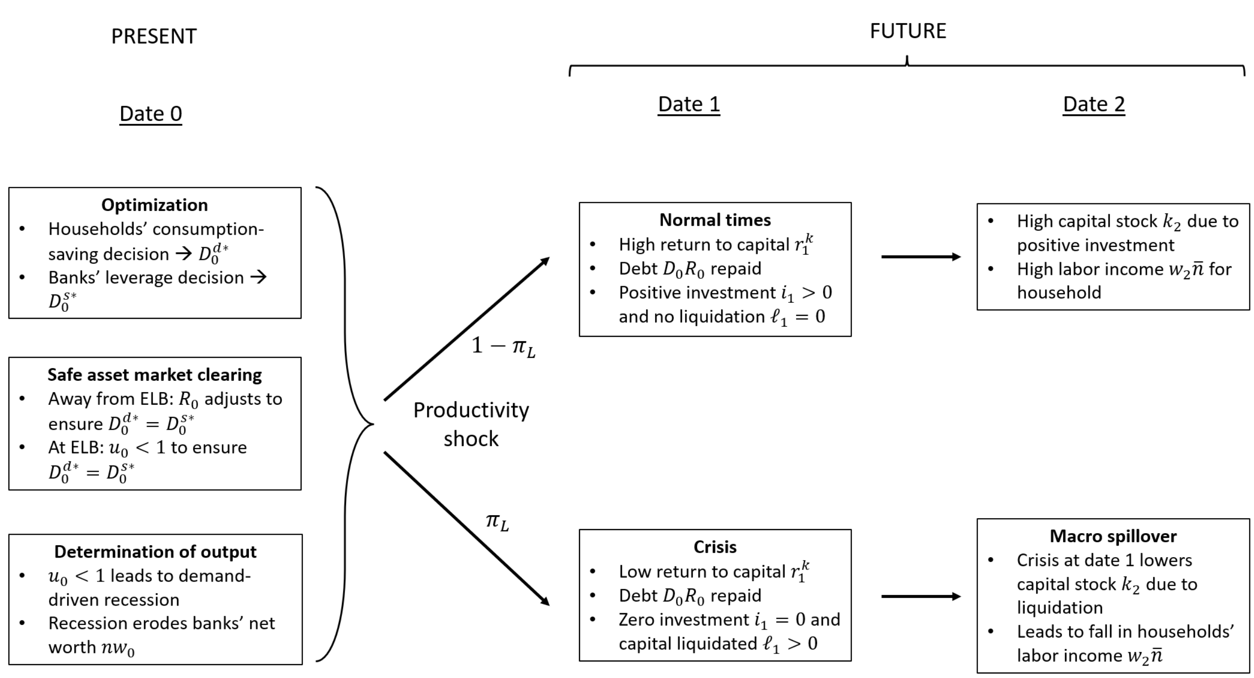
This paper is also related to several recent papers on safe assets and crises, such as Diamond (2020), Lenel (2023), Luck and Schempp (2023), Ross (2023), and Segura and Villacorta (2023) which imply that the production of safe assets generates financial externalities. However, these papers abstract from macroeconomic dynamics such as the role of aggregate demand, or are in partial equilibrium rather than general equilibrium, both of which are central to my mechanism. Finally, Azzimonti and Yared (2019) study optimal provision of public versus private safe assets in a model without aggregate risk, and Benigno and Nistico (2017) study optimal monetary policy in a model with a kind of cash-in-advance constraint for private and public safe assets.

2 Model

2.1 Overview of setup

There are three periods: dates 0, 1, and 2. There are two types of goods: capital and the consumption good. There are two types of agents which consume: a measure one of identical, risk-averse households, and a measure one of identical, risk-neutral banks. In addition, there are good-producing firms who are owned by the household and have rigid prices. I also assume there is a government whose behavior I assume is determined exogenously, for now. (In the normative section, I will suppose the government is an optimizing agent and characterize its optimal behavior.) Figure 3 summarizes the sequence of events and key features of the equilibrium to be discussed in section 3.

Figure 3: Summary of model environment



On the supply side of the economy, the consumption good is produced each period by a representative final goods producer who rents capital from banks and hire labor in competitive markets.

These firms have pre-set nominal prices which are fixed within and across periods, and normalized to 1, implying the inflation rate is always 0.[11] Firms meet demand at these prices by costlessly varying the utilization of capital. Thus, output can be below its potential level due to a shortage of aggregate demand. Any profits are distributed as dividends back to the household at the end of each period.

Since prices are fully sticky, the real interest rate is equal to the nominal interest rate, which is controlled by the monetary authority. In equilibrium at date 0, the policy rate set by the monetary authority will be equal to the nominal rate on government bonds $R_0^{MP} = R_0^B$. Therefore, I henceforth ignore the notation R_0^{MP} and instead refer to R_0^B as both the rate on government bonds and the monetary policy rate. Moreover, since inflation is always zero, R_0^B is also the real rate of interest at date 0. (I normalize the price level to $P_t = 1$ for each period t.)

I assume that the monetary authority attempts to replicate the supply-determined output level. However, there is a lower bound constraint on the gross nominal interest rate, implying $R_t^B \geq 1$. Thus, the monetary authority sets the nominal interest rate according to $R_t^B = \max\{R_t^*, 1\}$, where R_t^* is the gross natural rate of interest rate which ensures output is at its potential.

For simplicity, I assume that while output may fall below potential at date 0, output is equal to its potential at dates 1 and 2 – that is, I assume the gross natural interest rate weakly exceeds 1 in all states at date 1 and 2.[12] Since the monetary authority targets the natural rate, the (real and nominal) rate of interest at date 1 will always be given by $R_1^B = 1$.

The source of aggregate uncertainty is a shock to the total factor productivity z_1 of final goods producers at date 1. There are two aggregate states at date 1: $z_1 \in \{z_1^H, z_1^L\}$ where $z_1^H > z_1^L$.

The representative bank is endowed with capital at date 0 and rents capital to intermediate producers in a competitive market within each period. between periods, the representative bank can store capital freely and can create new capital by investing consumption good into an investment technology with constant returns-to-scale. In equilibrium, the TFP shock to final good firms affects the banks rental income from capital. Hence, the bank's return to date 0 investment in capital is subject to aggregate risk.

To finance investment in new capital at date 0, the bank can issue nominally safe, one-period debt D_0 to the household in a competitive market at date 0. This private bond pays a nominal gross rate of return denoted R_0^D at date 1 which is not contingent on the state of the world. [13] The bank cannot issue new debt at date 1. Therefore, it must pay its date 1 debt out of its date 1 rental income

[11] The nominal rigidity can be microfounded with the standard assumption of monopolistically competitive firms who face some degree of rigidity in price-setting. See Caballero and Simsek (2021).

[12] This is reminiscent of the simplifying assumptions employed in Caballero and Simsek (2021) and Caballero and Farhi (2018).

[13] I abstract from default and assume that borrowers have full commitment to repay debt, but relaxing this assumption would not alter central insights of the model.

or by converting a portion of its capital holdings into the consumption good. However, capital is partially illiquid at date 1 and is subject to a liquidation cost. In particular, if ℓ_1 denotes the fraction of its capital stock that the bank liquidates at date 1, then its date 2 capital stock is given by

$$k_2(s) = i_1 + (1 - \ell_1 - \phi(\ell_1(s)))\, k_1(s) \tag{1}$$

Here, $\phi(\ell_1(s))$ denotes the liquidation cost, indexed by the state of the world s, and is a strictly convex function of ℓ_1: $\phi(\ell_1) = \ell_1^\eta$ where $\eta > 1$. Then $\phi(0) = 0$, $\phi' \geq 0$, $\phi'(0) = 0$, and $\phi'' > 0$.[14]

Thus, at date 1, the bank is potentially liquidity constrained for three reasons: it cannot issue state-contingent debt ex ante, it cannot raise new debt to finance repayment ex post, and capital is partially illiquid at date 1. As a result, in bad states of the world (that is, when productivity is low), the bank may be forced to resort to costly liquidation of capital to repay its debt. Therefore, while private debt insures the household against aggregate shocks in partial equilibrium, this insurance may lead to a lower future capital stock in general equilibrium (and therefore lower future consumption). Finally, capital fully depreciates after date 2.

On the demand side of the economy, the risk-averse representative household is endowed with the consumption good at date 0. Each period, the household supplies labor to intermediate goods producers inelastically, and solves a consumption-saving decision. At date 0, the household has access to two financial assets: the private bond and a public bond issued by the government. In equilibrium, the private bond allows the household and the bank to share aggregate risk to some extent. At date 1, the household has access to an exogenous storage technology with constant returns-to-scale between dates 1 and 2. The quantity of consumption goods that the household stores from date 1 to date 2 denoted B_1, where the real gross rate of return on this technology is denote $R_1^B = 1$.[15]

At date 0, agents may receive an unanticipated news shock about future TFP z_1 (i.e., an MIT news shock). Agents can re-optimize their date 0 decisions in response to this news, but prices are fully rigid. Thus, the MIT shock may cause output to deviate from its potential at date 0. I consider the effects of such a shock in section 6.

In each period, the government can implement taxes transfers on agents, both lump-sum and distortionary. At date 0, the government can purchase capital from the banks in a competitive spot market – a policy similar in spirit to quantitative easing. Moreover, the government can issue a one-

[14] This liquidation cost is a reduced form way to capture endogenous fire sales similar to Lorenzoni (2008). While this reduced-form representation yields very similar dynamics, it makes the model considerably more tractable.

[15] The storage technology is not critical for the qualitative results but improves the tractability of the model and also ensures that the natural rate of interest is equal to 1 in all states at dates 1 and 2 so that the effective lower bound never binds at those dates. If I instead assumed that the household could invest in government bonds at date 1, the results would be similar – I would simply restrict my analysis to the cases in which the natural rate is weakly greater than 1 at dates 1 and 2, ensuring that output is at its potential at these dates.

period, nominally risk-free bond B_0 at date 0 that pays a nominal gross rate of return R_0^B, which is not contingent on the state of the world. The government's power to tax gives it a comparative advantage over the private sector in issuing safe debt, which will be internalized by a social planner but not necessarily by private agents.

2.2 Household

The representative household has log utility and is endowed with e_0 units of the consumption good at date 0. Each period, the household supplies labor $\bar{n}$ in inelastically, and solves consumption-saving decision each period. At date 0 the household has two assets to save in: private bonds (in endogenous supply) or public bonds (in exogenous supply). Households cannot hold capital directly. At date 1, the household has access to a riskless storage technology with constant returns-to-scale, where B_1 and $R_1^B = 1$ respectively denote the quantity of the consumption good that the households investment in the date 1 storage technology and the real gross rate of return on the technology.

At dates 0 and 1, the household effectively faces a consumption-saving decision and a portfolio choice: how to allocate savings between the public and private bond at date 0, and between the public bond and traditional firms at date 1. The household's optimization problem at date 0 is therefore to choose its consumption each period c_0, c_1,c_2, its date 0 nominal holdings of the private and public bonds D_0, B_0, and its date 1 saving decision B_1 to maximize its expected utility, taking as given the wage w_t, nominal rates of return, and price level P_t at each date t.

$$\max_{c_0,c_1,c_2,D_0,B_0,B_1} \log c_0 + E_0\left[\log c_1 + \log c_2\right]$$

Each date t, the household's available funds in real terms is given by the sum of its endowment, labor income $w_t\bar{n}$, dividends d_t^F, and real return on assets, net of lump-sum taxes T_t. It can allocate these funds between consumption and investment in each type of asset. Therefore, the household's budget constraints for dates 0, 1, and 2, respectively, are

$$c_0 + \frac{D_0}{P_0} + \frac{B_0}{P_0} \leq e_0 - T_0 + d_0^F + w_0\bar{n} \tag{2}$$

$$c_1(s) + B_1(s) \leq \frac{R_0^D D_0}{P_1(s)} + \frac{R_0^B B_0}{P_1(s)} + d_1^F(s) - T_1 + w_1\bar{n} \tag{3}$$

$$c_2(s) \leq d_2^F(s) + R_1^B(s)B_1(s) - T_2 + w_2\bar{n} \tag{4}$$

All endogenous date 0 variables are conditional on the date 0 MIT shock α, while all endogenous variables at dates 1 and 2 are conditional on α and the aggregate state of the world s.

The household's optimality conditions, derived in Online Appendix 1, imply that the nominal rates of the return on the private and public bonds must be equalized in equilibrium, $R_0^D = R_0^B$, as the private and public bond are equivalent assets from the perspective of private agents as they both offer a nominally risk-free, state-uncontingent return in date 1 (although the two assets are not equivalent from a social perspective, as I show later). I henceforth use $R_0 \equiv R_0^D = R_0^B$ to denote the nominal interest rate. The household's date 0 consumption-saving decision is governed by the Euler equation $\frac{1}{c_0} = R_0 E_0 \left[\frac{1}{c_1(s)}\right]$, which pins down the household's demand for total saving (i.e., its demand for both types of bonds together).

The household's demand function for private bonds, derived from its date 0 Euler equation and budget constraint, is downward-sloping in the interest rate R_0 and depends on the level of utilization u_0 in general equilibrium.

$$D_0^d(R_0, B_0; u_0) = e_0 - T_0 + d_0^F + w_0 \bar{n} - B_0 - \frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-1} \tag{5}$$

The negative relationship between the household's demand for private bonds D_0^d and the government's supply of public bonds B_0 reflects that a higher supply of public bonds crowds out the household's demand for private bonds, since the bonds are perfect substitutes from the perspective of the household. Because only the household consumes at date 0, D_0^d inversely reflects aggregate consumption demand at date 0. As a result, the quantity of public bonds supplied by the government acts as a consumption-demand shifter. Moreover, the optimality condition for the storage technology B_1 implies that the household perfectly smooths its consumption between dates 1 and 2 so that $c_2(s) = c_1(s)$.[16]

Equation (5) is one of they key equations which governs the demand for safe assets and will play an important role in the model's mechanism to be discussed later. Since the demand for saving D_0^d is directly related to aggregate (consumption) demand at date 0, I refer to this equation as the *aggregate demand (AD) curve*. The AD curve reflects the dependence of aggregate demand on systemic risk $\ell_1(s_L)$ via future consumption $E_0\left[\frac{1}{c_1(s)}\right]$. Moreover, the partial derivative of D_0^d with respect to $\ell_1(s_L)$, $\frac{\partial D_0^d}{\partial \ell_1(s_L)}$, an elasticity I will refer to as *Channel 2*, reflects the sensitivity of aggregate demand to systemic risk and constitutes part of the dynamic interaction between

[16] The storage technology could be interpreted as as riskless international bond with which households in a small open economy can borrow or save between dates 1 and 2. This assumption is not necessary for my qualitative results: All that is required is that the household have some means of smoothing consumption between dates 1 and 2, to some extent. For example, I could alternatively allow the government to issue bonds at date 1 through which the household could smooth consumption between dates 1 and 2. Nevertheless, this assumption improves the model's tractability in two ways: 1) Due to perfect consumption smoothing between dates 1 and 2, I need to keep track only of total future consumption (dates 1 and 2) rather than consumption at each date separately; 2) It ensures that output is supplied-determined at date 1 by implying that the gross natural rate of interest at date 1 is 1.

systemic risk and aggregate demand at the heart of the model. This will be discussed in more detail in sections 3 and 4.

2.3 Banks

The representative, risk-neutral bank as linear utility $v(c_2^E)$ and consumes only at date 2, where c_2^E denotes the bank's date 2 consumption. The bank is endowed with k_0 units of capital at date 0. Within each period, the bank rents its capital holdings to intermediate goods producers in a competitive market and receives a real gross rental rate of capital $r^k(s)$ within the same period. Capital does not depreciate from use in production.

Banks also have access to an investment technology at dates 0 and 1 which converts one unit of the consumption good into one unit of new capital the next period and has constant returns-to-scale. While this investment technology is itself risk-free, the bank's date 0 investment ultimately involves aggregate risk since the date 1 return on capital $r_1^k(s)$ will reflect the aggregate shock at date 1.

To finance investment, the bank issues debt to the household in a competitive market at date 0 with nominal face value D_0 and gross nominal rate of return R_0, subject to a natural borrowing limit which never binds in equilibrium. At date 0, the bank can participate in a competitive spot market for capital together with the government, where k_1^{QE} denotes the quantity of capital that the bank sells to the government at the spot price q_0, which equals $q_0 = 1$ in equilibrium.

The bank's date 0 budget constraint on intertemporal choices at the end of date 0 ensures that investment i_0 in new capital in the second stage of date 0 can be financed by issuing private debt, rental income $r_0^k k_0$, proceeds from sales of capital to the government $q_0 k_1^{QE}$, or government transfers T_0^E.

$$\left(1 - \tau_0^D\right) \frac{D_0}{P_0} + r_0^k k_0 + q_0 k_1^{QE} + T_0^E \geq i_0 \tag{6}$$

τ_0^R is a distortionary tax which increases the cost to the bank of issuing debt at date 0. Any proceeds from asset sales to the government can immediately be reinvested in the creation of new capital. The bank's stock of capital evolves between dates 0 and 1 according to

$$k_1 = i_0 + k_0 - k_1^{QE} \tag{7}$$

At date 1, banks have to meet their obligations $\tau_0^R \frac{D_0 R_0}{P_1(s)}$ and can invest in new capital at date 1, subject to a non-negativity constraint on investment $i_1(s) \geq 0$. They can finance these expenditures out of their earnings from renting capital at date 1 $r_1^k(s) k_1$, from liquidating a fraction $\ell_1(s)$ of their capital holdings at date 1, or out of any lump-sum government transfers T_1^E. The bank's date 1 budget constraint is therefore

$$r_1^k(s)k_1 + \ell_1(s)k_1 + T_1^E \geq i_1(s) + \tau_0^R \frac{D_0 R_0}{P_1(s)} \tag{8}$$

τ_0^R is a distortionary tax which increases the effective interest the bank must pay on its debt: The bank pays total of $\tau_0^R R_0 D_0$ in interest at date 1, of which $R_0 D_0$ goes to the households who hold the bonds and the remaining $\left(\tau_0^R - 1\right) R_0 D_0$ goes to the government. Liquidation is costly and involves a loss of capital given by the strictly convex function $\phi\left(\ell_1\right) = \ell_1^\eta$, where $\eta > 1$. Therefore, the bank's date 2 capital holdings evolves according to

$$k_2(s) = i_1 + \left(1 - \ell_1 - \phi\left(\ell_1(s)\right)\right) k_1(s) \tag{9}$$

At date 2, the bank rents its capital stock to intermediate goods firms and it consumes, and capital fully depreciates at date 2 after being used in production. The bank's date 2 budget constraint is

$$r_2^k k_2 + T_2^E \leq c_2^E \tag{10}$$

where T_2^E are date 2 transfers.

Bank's optimality conditions The problem of the bank is to choose in each period and each state how much to invest in new capital i_0, i_1, how much of its holdings of capital to sell at date 0 k_1^{QE} and liquidate in date 1 ℓ_1, and how much date 0 debt to issue D_0 in order to maximize its expected date 2 consumption $E_0\left[v(c_2^E)\right]$, subject to its budget constraints, the natural borrowing limit, the non-negativity constraint on date 1 investment, and the law of motions for capital. The full problem and the derivation of the optimality conditions is given in Online Appendix 2.

In the bad state at date 1, the bank forgoes investment, $i_1(s_L) = 0$, and liquidates just enough capital to repay its debt, so that $\ell_1(s_L)$ is pinned down by the binding non-negativity constraint on date 1 investment,

$$\ell_1(s_L) = Lev_0 - r_1^k(s_L) \tag{11}$$

where I have defined the leverage of the bank at date 0 as $Lev_0 := \frac{\tau_0^R D_0 R_0 - T_1^E}{k_1}$, the ratio of the bank's effective liabilities net of lump-sum transfers to its assets. Equation (11), which I refer to as the *systemic risk (SR) curve*, is one of the key equations which will determine the demand for safe assets.[17] In Appendix 4, I show that the severity of crises $\ell_1(s_L)$, and hence systemic risk, is strictly increasing in the bank's date 0 debt, $\frac{d\ell_1(s_L)}{dD_0^s} > 0$. I refer to this positive relationship between date

[17] As will become clear in section 4, the severity of crises $\ell_1(s_L)$ is directly related to the household's expected future

0 debt D_0 and the severity of crises $\ell_1(s_L)$ through the SR curve as *Channel 1*, which constitutes one of the key channels making up the dynamic interaction. This will be discussed in more detail in sections 3 and 4.

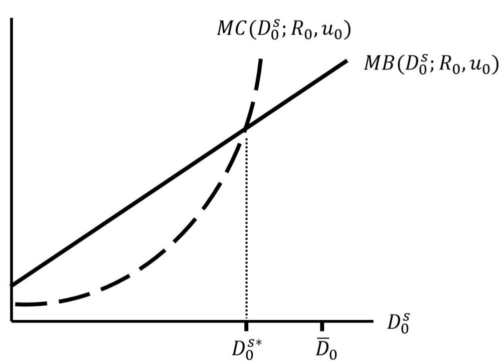
Because the bank's leverage at date 0 is determined by its choice of D_0, the bank's desired leverage ratio is pinned down by its optimal choice of date 0 debt D_0, which balances the marginal benefit of debt with the marginal cost.

$$\underbrace{\left(1-\tau_0^D\right)\left\{E\left[r_2^k\left(r_1^k(s)+1\right)\right]+\lambda_1(s_L)\left[r_1^k(s_L)+\ell_1(s_L)\right]\right\}}_{marginal\ benefit}=\underbrace{E\left[r_2^k\left(\tau_0^R R_0+\phi\left(\ell_1(s)\right)\left(1-\tau_0^D\right)\right)\right]+\lambda_1(s_L)\tau_0^R R_0}_{marginal\ cost} \tag{12}$$

The marginal benefit of date 0 debt, net of the tax rate τ_0^D, is given by the expected return to capital across dates 1 and 2 (the first term on the left-hand side of (12)) and the value of relaxing the non-negativity constraint on investment in the bad state (the second term on the left), where $\lambda_1(s_L)$ reflects the shadow price of funds in the bad state. The marginal cost of debt is given by the effective interest rate $\tau_0^R R_0$ and the expected cost of liquidating extra capital in the bad state (the first term on the right-hand side). Moreover, each unit of debt issued at date 0 tightens the bank's non-negativity constraint on date 1 investment in the bad state by the effective interest rate $\tau_0^R R_0$, which has a cost given by the shadow value of liquid funds in that state, $\lambda_1(s_L)$ (the second term on the right-hand side).

The convex liquidation cost $\phi(\ell_1)$ introduces a concavity in the bank's date 1 payoff function (as a function of D_0) which makes the bank behave at date 0 as if it is risk averse, which I show in Appendix 1. This is illustrated in Figure 4, where $MB(D_0;R_0)$ and $MC(D_0;R_0)$ denote the marginal benefit and marginal cost, respectively, of date 0 private debt D_0 at a given interest rate R_0.

Figure 4: Bank's leverage choice



labor income and consumption $E_0\left[\frac{1}{c_1(s)}\right]$ via the macroeconomic spillover from liquidation to the household's labor income.

Intuitively, the strict convexity of the liquidation cost implies that the bank's net marginal benefit is strictly decreasing in D_0 for a sufficiently high D_0. I show in Appendix 1 that, as long as the liquidation cost function $\phi(\cdot)$ is sufficiently convex, the bank is at an interior optimum for his choice of D_0 and the natural borrowing limit $\overline{D}_0$ is non-binding in equilibrium. The balance between the bank's marginal cost and benefit of debt issuance defines the bank's supply curve $D_0^{s*}(R_0; u_0)$ for private safe assets as a function of R_0 for a given level of utilization u_0.

Risk premium A corollary of the bank's risk-averse behavior is that, in equilibrium, there is a strictly positive risk premium at date 0, which is defined as the difference in the expected (gross) rate of return to capital, $1 + \frac{E\left[r_2^k(s) r_1^k(s)\right]}{E\left[r_2^k(s)\right]}$, and the effective risk-free rate of return, $\tau_0^R R_0$.

$$RP_0 := \left(1 - \tau_0^D\right)\left(1 + \frac{E\left[r_2^k(s) r_1^k(s)\right]}{E\left[r_2^k(s)\right]}\right) - \tau_0^R R_0 > 0 \tag{13}$$

I discuss the intuition behind this expression in Appendix 2. In this setting, the positive risk premium results from the costly liquidation of capital which occurs in the bad state at date 1, $\phi\left(\ell_1(s_L)\right) > 0$. In the absence of a liquidation cost, the bank would want to increase its invest in capital up to the point that the return on capital and the cost of borrowing is zero – i.e. until the risk premium is 0. However, the positive expected cost of liquidation reduces the attractiveness of investing in capital, resulting in a positive equilibrium risk premium. Indeed, in Appendix 2, I show that this risk premium is strictly positive in equilibrium if and only if $\ell_1(s) > 0$ for some state s.[18]

2.4 Nominal rigidity

There is a representative firm who rents capital from bank and the government and hires labor from the household in competitive markets to produce the consumption good within each period according to the production function below, where $\alpha \in (0,1)$ and where $\tilde{k}_t = k_t + k_t^G$ is the total capital stock at date t.

$$y_t = z_t \left(u_t \tilde{k}_t\right)^{\alpha} \bar{n}^{1-\alpha} \tag{14}$$

[18]Thus, the liquidation cost in this model lays a similar role that the collateral constraint plays in Caballero and Farhi (2018). In that model, a binding collateral constraint results in a positive risk premium, as borrowers are unable to scale investment up to the point that the expected rate of return on risky assets and the safe rate of return are equalized.

The price of the consumption good is predetermined at date 0 and is fully rigid across all periods.[19] Hence, there is no inflation. The price level P is fixed and normalized to 1.

In each period t, the firm chooses how much labor to hire, how much capital to rent, and how much capital to utilize to maximize its profits, taking prices and the demand y_t^D it faces as given and subject to a standard budget constraint $d_t^F = y_t - w_t\bar{n} - r_t^k \tilde{k}_t$. The first order conditions, derived in Online Appendix 3, equate the rental rate of capital and the wage to the factor shares of income: $r_t^k = \alpha \frac{y_t}{\tilde{k}_t}$ and $w_t = (1-\alpha)\frac{y_t}{n_t}$. Since firms cannot adjust prices in response to the demand they face, capital utilization $u_t \in (0,1]$ is chosen optimally to ensure that their output is exactly equal to the demand they face, $y_t = y_t^D$.

Potential output in date t is defined as the level output given full utilization of resources, $y_t = z_t k_t^\alpha \bar{n}^{1-\alpha}$. Because prices are rigid, output can possibly be below potential and determined by the level of aggregate demand which prevails given prices and the interest rate. I assume that monetary policy ensures that output is at potential when possible. That is, away from the effective lower bound, the policy rate is set to the natural rate of interest – the rate such the level of aggregate demand is equal to potential output. As discussed in section 2.1, the natural rate is at date 1 is $R_1^B = 1$ implying that output is always at potential at date 1.

2.5 Government

At date 0, the government can issue to the household one-period debt on a competitive market, where B_0 denotes the nominal face value of the debt, which yields a state-uncontingent nominal gross rate of return R_0^B. The government can also levy lump-sum and distortionary taxes on agents, and can purchase capital from the bank in a competitive spot market, where k_1^G denotes the quantity of capital that the government purchases.[20] Therefore, the government's date 0 budget constraint is

$$\frac{B_0}{P_0} + T_0 + \tau_0 - T_0^E = q_0 k_1^G \tag{15}$$

where $\tau_0 := \tau_0^D D_0$ are the distortionary taxes levied on the bank at date 0. At date 1, the government earns rental income on its capital holdings, and can also levy lump-sum and distortionary taxes to

[19]The nominal rigidity can be microfounded with the standard assumption of monoplistically competitive firms who face some degree of rigidity in price-setting. See Caballero and Simsek (2021).This extreme form of nominal rigidity is also used in Caballero and Farhi (2018), Caballero and Simsek (2020), Caballero and Simsek (2021), and Korinek and Simsek (2016). This assumption is not critical to my results but significantly improves the model's tractability while delivering one of the key frictions (nominal rigidities) necessary for output to potentially be demand-determined in equilibrium.

[20]Rather than buying capital directly, I could equivalently assume that the government buys at date 0 a claim on the date 1 rental rate of capital, and issues a lump-sum transfer to the bank in the bad state at date 1 to prevent it from liquidating its capital holdings. This alternative would yield the same allocation.

finance the repayment of its date 0 debt.

$$T_1 + \tau_1 - T_1^E + r_1^k(s)k_1^G = R_0^B \frac{B_0}{P_0}\frac{P_0}{P_1} \tag{16}$$

where $\tau_1 := \left(\tau_0^R - 1\right) R_0 D_0$ are the distortionary taxes levied on the bank at date 1. The government stores its date 1 capital holdings through date 2 without depreciation. Therefore, the law of motion for the government's capital holdings at date 2 is

$$k_2^G = k_1^G$$

At date 2, the government earns rental income on its holdings of capital, can levy lump-sum and distortionary taxation on both agents, keeps a balanced budget.

$$T_2 - T_2^E + r_2^k k_2^G = 0 \tag{17}$$

For now, I take the government's behavior as given.

The government's power to tax gives it a comparative advantage over the private sector in issuing safe debt. Hence, unlike the bank, the government does never needs to liquidate its capital holdings to service its debt. [21]

3 Equilibrium

The equilibrium is a set of processes for allocations, prices, and returns such that households and banks maximize expected utility, firms maximize profits, capital evolves according to its laws of motion, the nominal interest rate follows the rule described in section 2.1, and the markets for labor, capital, and private and public bonds clear. Recall that supply of public bonds B_0 is taken as exogenous until the normative section. I solve for the equilibrium recursively. Before solving for the equilibrium I impose the following assumptions.

Assumption 1:
A) $z_1(s_L) \in \left(\frac{\tau_0^D R_0 D_0 - k_1}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}}, \frac{\tau_0^R D_0 R_0 - T_1^E}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}} \right)$ and $z_1(s_H) \geq \frac{\tau_0^R D_0 R_0 - T_1^E}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}}$
B) $T_1^E < k_1$

Appendix 12 shows that these assumptions ensure that the bank's date 0 natural borrowing limit on borrowing D_0 is never binding in equilibrium, and that liquidation occurs only in the bad

[21] While the government could liquidate capital holdings, a benevolent government would never find it optimal to do so, as liquidation entails a higher deadweight loss compared to lump-sum taxation due to the liquidation cost $\phi(\ell_1)$.

state, $\ell_1(s_L) > 0$, $\ell_1(s_H) = 0$. The appendix also shows that these restrictions are satisfied by a non-empty set of parameters.

3.1 Equilibrium at dates 1 and 2

Output is at potential at dates 1 and 2 At date 2, monetary policy ensures output is at its potential, which is determined by the household's inelastic labor supply and the total capital stock which the bank and government bring into date 2 from date 1. I solve for the date 2 equilibrium in Online Appendix 4.

At date 1, the return on the storage technology $R_1^B = 1$ pins down the gross natural rate of interest rate at $R_1^* = R_1^B = 1$ at date 1, ensuring that output is at potential. The household's quantity of saving at date 1 simply adjusts to satisfy its Euler equation at this interest rate.

Two regimes at date 1 The equilibrium at date 1 features two regimes, depending on whether the bank liquidates capital or not. This is shown in Online Appendix 5. In the *normal regime*, the bank's non-negativity constraint on date 1 investment is non-binding and the bank does not liquidate any of its capital holdings so that $\ell_1(s) = 0$, $\lambda_1(s) = 0$. In this regime, the bank's date 1 return from rental income is sufficient to meet its debt obligations $\tau_0^R \frac{D_0 R_0}{P_1}$. In the *crisis regime*, on the other hand, the banks' date 1 income is insufficient to cover its debt obligations, forcing the bank to liquidate some portion $\ell_1(s) > 0$ of its capital holdings, which is pinned down by its binding non-negativity constraint on date 1 investment.

$$\ell_1(s) = \frac{\tau_0^R D_0 R_0}{k_1} - r_1^k(s) - \frac{T_1^E}{k_1} \tag{18}$$

The lower the realization of date 1 TFP z_1, the more severe the crisis.[22] Assumption 1 about TFP in each state $z_1(s)$ imply that the normal regime obtains in when TFP is high and the crisis regime obtains when TFP is low , i.e. $\ell_1(s_H) = 0$ and $\ell_1(s_L) > 0$.

3.2 Date 0 equilibrium

The determination of the date 0 equilibrium can be understood in two stages: the determination of the demand for and supply of safe assets D_0^{d*}, D_0^{s*} for a given interest rate R_0 and utilization rate u_0 (partial equilibrium); and the determination of the interest rate R_0 and the utilization rate u_0 to clear the market for safe assets (general equilibrium). I first begin with the partial equilibrium determination of the supply and demand for safe assets.

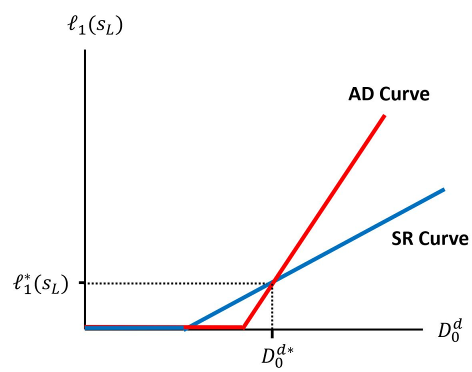
Supply and demand for safe assets in partial equilibrium As discussed in section 2.3, the bank's supply curve of safe assets is the outcome of the balance between the bank's marginal

[22]Formally, because k_1 is predetermined at date 1 and labor is in fixed supply, I have $\frac{d\ell_1}{dz_1} = -\frac{dr_1^k}{dz_1} = -\frac{\alpha}{\tilde{k}_t} \frac{y_1}{z_1} < 0$.

benefit and marginal cost of leverage at date 0 summarized in equation (12), which respectively reflect the expected return to investment and the expected future liquidation costs. The bank's optimal choice of date 0 debt D_0^s for a given R_0 and u_0 yields the supply curve of safe assets as a function of the interest rate and utilization rate.

The determination of the household's demand curve for safe assets is the result of a balance between aggregate demand and systemic risk, respectively given by the key equations for the AD curve (5) and the SR curve (11), for a given R_0 and u_0. (Recall that the AD curve (5) summarizes the household's consumption-saving decision conditional on a level of systemic risk through $\ell_1(s_L)$, while the SR curve summarizes how the household's demand for saving (and therefore aggregate demand) affects systemic risk through $\ell_1(s_L)$, ceteris paribus.) This balance, or fixed point, can be illustrated graphically in Figure 5, which plots in a stylized manner the AD curve and SR curve, with the household's date 0 demand for safe assets (holding all else fixed) D_0^d on the x-axis and the severity of crisis $\ell_1(s_L)$ on the y-axis.[23]

Figure 5: Demand for safe assets from the balance between AD and SR



The slope of the SR curve is given by Channel 1 of the dynamic interaction between aggregate demand and systemic risk, $\frac{d\ell_1(s_L)}{dD_0^s}$, which captures the (positive) relationship between date 0 debt D_0 and the severity of crises at date 1 $\ell_1(s_L)$.[24] The slope of the AD curve, in turn, is given by Channel 2 of the dynamic interaction, $\frac{\partial D_0^d}{\partial \ell_1(s_L)}$, which captures how systemic risk affects aggregate demand at date 0.[25] Appendices 3 and 4 show that both channels strictly positive $\frac{d\ell_1(s_L)}{dD_0^s}, \frac{\partial D_0^d}{\partial \ell_1(s_L)} > 0$

[23] Note that in the figure, I are plotting the inverse function of the AD curve (5) by showing $\ell_1(s_L)$ as a function of D_0^d. Moreover, for each curve I are plotting the relation between D_0^d and $\ell_1(s_L)$ to a first order approximation for ease of exposition.

[24] The x-intercept of the SR curve is given by $D_0 = \frac{r_1^k(s_L)k_1 + T_1^E}{\tau_0^R R_0}$, which reflects how much private debt at date 0 it takes for there to be no liquidation in the bad state at date 1. The x-intercept of the AD curve is given by a complex expression not reproduced here and reflects the household's demand for safe assets when $\ell_1(s_L) = 0$, that is, in the absence of systemic risk.

[25] Technically, since I define Channel 2 as $\frac{\partial D_0^d}{\partial \ell_1(s_L)}$, the slope of the (inverse) AD curve plotted in Figure 5 is given by the inverse of Channel 2.

under a mild assumption, and hence both curves are upward sloping.[26]

The fixed point $\left(D_0^{d*}, \ell_1^*(s_L)\right)$ between the two curves is the result of the dynamic interplay between aggregate demand and systemic risk (Channels 1 and 2), and reflects that the equilibrium values of systemic risk implied by ℓ_1 and aggregate demand implied by D^d must be consistent with both the household's consumption-saving decision (captured by the AD curve equation) and the relationship between the bank's leverage and systemic risk (captured by the SR curve equation). [27]The fixed point in turn determines the household's demand curve $D_0^{d*}(R_0; u_0)$ for safe assets for given levels of R_0 and u_0.[28]

How does this figure compare relate to the literature? In much of the related literature, systemic risk is either non-existent, because the supply of safe assets is pinned down by a binding borrowing constraint which prevents default or liquidation, or systemic risk is fixed exogenously. Therefore, comparable SR curve in the literature would either be absent or would be horizontal and unresponsive to changes in other variables. As a result, relative to the literature, the determination of the equilibrium differs because the levels of systemic risk and aggregate demand are determined jointly, and the response of the equilibrium to shocks and changes in monetary, fiscal, or macroprudential policy differs as Ill, as I will later show.

General equilibrium in the market for safe assets Given the household's demand curve D_0^{d*} for safe assets and the bank's supply curve for safe assets D_0^{s*}, the interest rate and/or utilization rate $R_0; u_0$ are determined in general equilibrium to clear the market for safe assets $D_0^{d*}(R_0; u_0) = D_0^{s*}(R_0; u_0)$.[29] Whether the interest rate R_0 or utilization u_0 adjusts to clear this market depends on whether the effective lower bound on monetary policy is binding or not. Therefore, the date 0 equilibrium features two regimes depending on whether this constraint binds, as is shown in the following lemma. The full derivation of the date 0 equilibrium is given in Online Appendix 6.

Lemma 1: There are two regimes at date 0: an aggregate supply (AS) regime in which the

[26]For ease of exposition, in Figure 5 I plot the case in which the AD curve is steeper than the SR curve and its x-intercept larger, though this need not be the case in general.

[27]To understand convergence to the fixed point, note that, for all $D_0^d > D_0^{d*}$, the level of $\ell_1(s_L)$ implied by the AD curve is strictly greater than that implied by the SR curve. Therefore, for D_0^d to be consistent with the household's consumption-saving decision, $\ell_1(s_L)$ (and therefore systemic risk) must by higher than it actually is given D_0^d. So given the low level of systemic risk, the household's demand for safe assets D_0^d must be lower. Similarly, for all $D_0^d < D_0^{d*}$, the level of $\ell_1(s_L)$ implied by the AD curve is strictly less than that implied by the SR curve. Therefore, for D_0^d to be consistent with the household's consumption-saving decision, $\ell_1(s_L)$ must be lower than it actually is for that D_0^d. So given that systemic risk is relatively high, the household's demand for safe assets D_0^d must be higher. D_0^{d*} is the point at which the level of systemic risk implied by $\ell_1(s_L)$ and the level of aggregate demand implied by D_0^d are consistent with both relations (5) and (11).

[28]While D_0^d denotes the household's demand for private safe assets, recall that household is indifferent between holding private versus public safe assets. Since the supply of public safe assets B_0 is thus far taken as exogenous, B_0 simply crowds out demand for private safe assets. This is reflected in the AD curve equation (5) by the negative relationship between B_0 and D_0^d.

[29]Of course, these variables are determined together with the the monetary policy rule described in section 2.1 and the final good producing firm's optimal choice of capital utilization described in section 2.4.

effective lower bound on the nominal interest is non-binding $R_0^B > 1$ and capital is fully utilized $u_0 = 1$; and an aggregate demand (AD) regime in which the effective lower bound is binding $R_0^B = 1$ and capital is under-utilized $u_0 < 1$.

Proof: See Online Appendix 6.

Essentially, which regime prevails at date 0 depends on how the economy adjusts to clear an excess demand for saving, denoted by $D_0^d(R_0, B_0; u_0) - D_0^s(R_0)$.[30] [31]

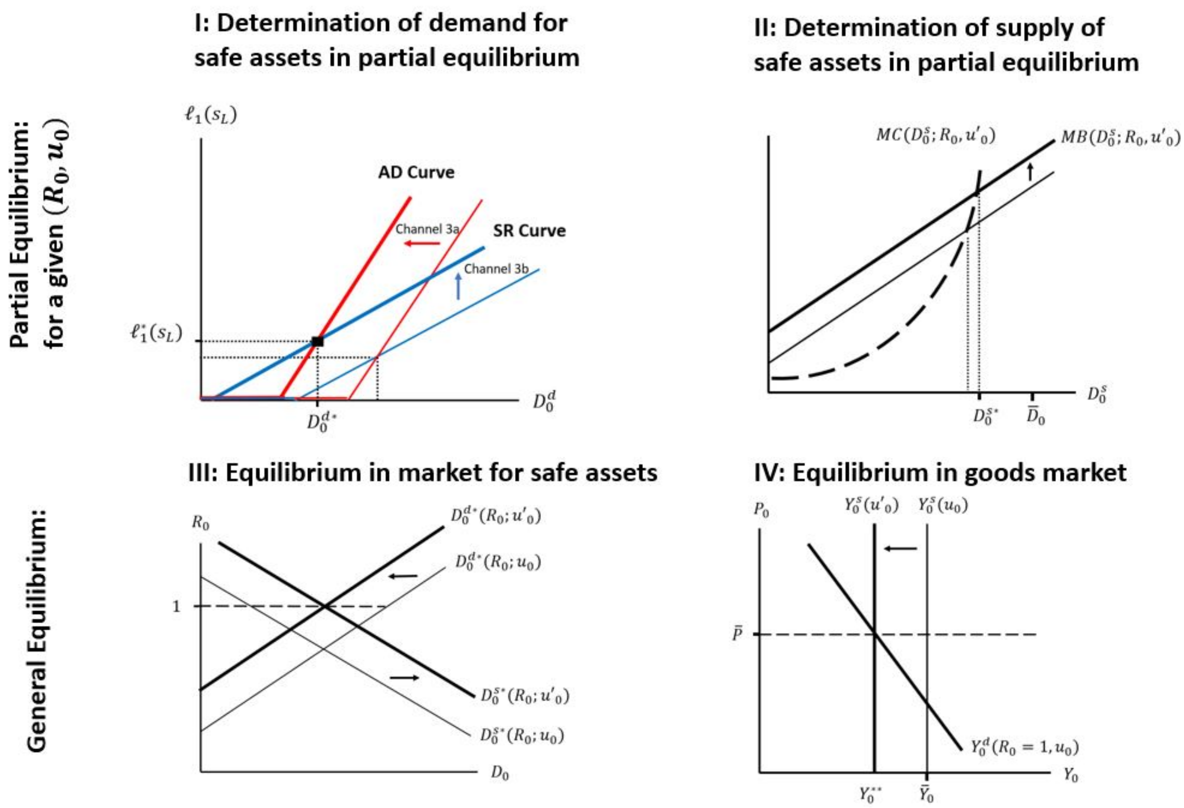
Supply-determined regime In the aggregate *supply-determined regime*, the effective lower bound is not binding and output is at potential. The nominal interest rate R_0 is able to adjust to equilibrate the demand for saving at date 0 and is determined such that the bank's optimality condition for borrowing holds. In Online Appendix 7, I show how the adjustment in R_0 equilibrates the market for safe assets while leaving output at potential.

Demand-determined regime In the aggregate *demand-determined regime*, the effective lower bound is binding $R_0 = 1$ and aggregate output at date 0 is demand-determined. At the effective lower bound $R_0 = 1$, there is an excess demand for private debt $D_0^d(R_0, B_0; u_0) - D_0^s(R_0)$. To reach equilibrium, the utilization of capital must fall to eliminate any excess demand for saving $D_0^d(R_0, B_0; u_0) - D_0^s(R_0)$, depressing output below its potential. In Online Appendix 7, I show how a fall in utilization u_0 equilibrates the economy by reducing the household's incentive to save. Figure 6 illustrates how the adjustment in utilization equilibrates the economy at the effective lower bound, resulting in a demand-driven recession – that is, a safety trap.

[30]Recall that I had showed above that D_0^d reflects aggregate consumption demand while D_0^s reflects aggregate investment demand at date 0. Thus, aggregate demand relative to potential output at date 0 is captured by the excess demand for saving at date 0, $D_0^d(R_0, B_0; u_0) - D_0^s(R_0)$. That is, if there is an excess demand for safe assets, given by $D_0^d - D_0^s > 0$, aggregate demand is below total output. In the supply-determined regime, when the lower bound on the nominal interest is not binding, monetary policy can fall to be equal to the natural rate of interest ensuring that aggregate demand for output equals potential (that, there is no excess demand for saving $D_0^d - D_0^s = 0$).

[31]Note that the expression for excess demand for private debt already embeds the market clearing condition in the market for public bonds, and reflects that the government's exogenous supply of public bonds affects household's demand for private bonds through $D_0^d(R_0, B_0; u_0)$.

Figure 6: Safety trap in the demand-determined regime



At the effective lower bound $R_0 = 1$, there exists an excess demand for safe assets, shown in panel III. This also results in deficient aggregate demand in the market for goods (panel IV), given the fixed price level. In order to clear these markets, utilization falls, shifting the demand curve for safe assets in and the supply curve for safe assets out. This also results in a fall in the aggregate supply of goods.

In addition, the fall in utilization has feedback effects to the SR, AD, MC, and MB curves in panels I and II. In particular, the demand-driven recession affects systemic risk and the demand of safe assets through two general equilibrium feedback channels, Channels 3a and 3b, which I discuss in detail in section 4. Together with Channels 1 and 2, these channels constitute a two-way feedback loop between systemic risk and aggregate demand, depicted stylistically in Figure 1. In addition, the fall in u_0 also affects the bank's incentive to issue debt, which shifts the bank's supply curve of safe assets.[32] In order to reach an equilibrium, the fall in u_0 must reduce the excess demand for debt on net when taking into account all effects on the demand and supply of debt.

General equilibrium is the fixed point between these curves, and hence through the dynamic interaction between systemic risk and aggregate demand which arises in general equilibrium.

[32]In Online Appendix 7, I trace out these effects. In panel II of Figure 6, I depict only one of these effects, which is that the demand-recession at date 0 increases the expected return to capital, shifting the MB curve up.

3.3 Central features of the model

It is worth summarizing the key features of the model which will generate the main mechanisms and insights of the paper.

1) Crises at date 1 result in a macroeconomic spillover to the household's date 2 labor income A *crisis* at date 1 occurs when the bank is forced to liquidate some of its capital, $\ell_1(s) > 0$, in order to service its debt. In reducing the capital stock at date 2 and therefore the marginal product of labor at date 2, a crisis lowers the household's date 2 labor income $w_2\bar{n}$ in general equilibrium. As a result of this macroeconomic spillover, the household's labor income is low in the same states of the world that the bank's income is low.[33]

2) Systemic risk affects the supply of and demand for safe assets ex ante On the demand side, the macroeconomic spillover implies that an increase in *systemic risk* – defined as expected future liquidation at date 1, $\ell_1(s)$ – increases the household's date 0 demand for safe asset due to both lower expected future labor income at date 2 and through a precautionary saving motive. Therefore, an increase in systemic risk will increase the demand for safe assets and reduce aggregate demand at date 0.

On the supply side, the willingness of banks to create safe assets depends on their capacity to absorb risk. In order to insure bondholders against the date 1 productivity shocks, banks must themselves bear this risk either through their net worth or by liquidating capital at date 1 to raise funds when their funds are insufficient to service debt.[34] Therefore, fluctuations in the bank's net worth at date 0 will affect the supply of safe assets ex ante at date 0.

3) Private safe asset creation generates systemic risk while public safe asset creation does not Private safe asset creation generates systemic risk at date 0, as it forces banks to bear aggregate risk in part through liquidation of capital in the bad state at date 1. Though insured against the productivity shock z at date 1, bondholders are not insured against this systemic risk – that is, households bear the risk of low labor income in the bad state at date 2 owing to the macroeconomic spillover. Nevertheless, in making their portfolio choice at date 0, households do not internalize that private safe asset creation generates systemic risk, as the macroeconomic spillover materializes only in general equilibrium.

Unlike private safe assets, however, the production of public safe assets does not create systemic risk as a side-effect. This is because, due to the government's power to tax, at date 1 the government can meet its obligations without liquidating capital.

4) Safety traps emerge as result of dynamic interaction between aggregate demand and systemic

[33] Since the bank's decision to liquidate capital at date 1 depends in part on its leverage, the household's decision to invest in private debt has an effect on $w_2\bar{n}$ in general equilibrium through this spillover. However, households do not internalize this spillover when making their date 0 decision to invest in private debt. This has important positive and normative implications

[34] This decision of how to absorb shocks is summarized in the bank's first-order condition for $\ell_1(s)$.

risk The effective lower bound on nominal interest rates generates the possibility of demand-driven recessions due to a shortage of safe assets, known in the literature as a *safety trap*. However, unlike in the literature, safe asset shortages are entirely endogenous here, resulting from a dynamic interaction between systemic risk and aggregate demand which causes supply of and demand for safe assets to be out of balance at the effective lower bound. These insights are explored in more detail in section 5.3.

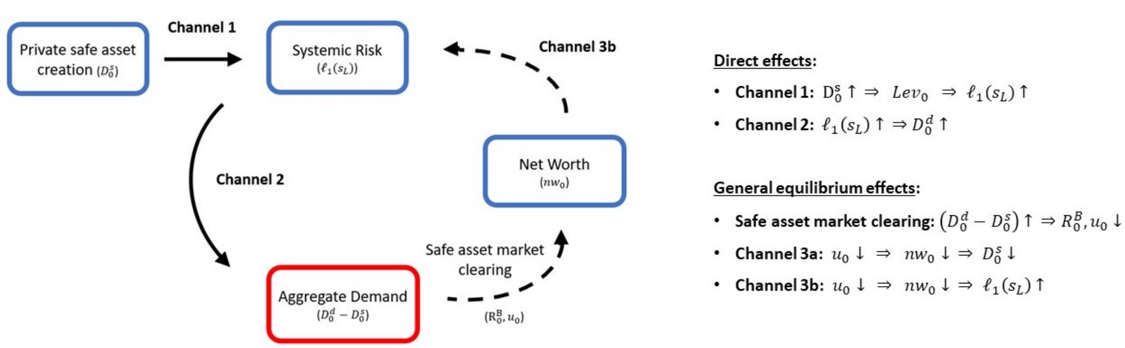
4 Interactions between debt, systemic risk, and aggregate demand

In this section, I study how public and private debt affect systemic risk and aggregate demand. To do so, I characterize the marginal effect of an increase in the supply of either type of debt on the equilibrium allocation, and a provide decomposition of these effects into three separate channels which together constitute a dynamic interaction between systemic risk and aggregate demand. For simplicity, I leave aside distortionary taxation for my positive characterizations, $\tau_0^D = 0, \tau_1^R = 1$.

4.1 Dynamic interaction between systemic risk and aggregate demand

To understand how the quantity of private (public) debt affects the equilibrium allocation at the margin, one can evaluate the total derivative of each endogenous variable with respect to D_0^s (B_0).[35] The total effect can be described by three channels, illustrated in the figure below for private debt.[36]

Figure 7: Dynamic interplay between aggregate demand and systemic risk



[35] Effectively, I examine the first order effect of on the equilibrium of an outward shift in the supply curve of debt of one type, holding constant the supply of the the other type of debt. While the total effect on the equilibrium allocation is determined in general equilibrium by the confluence of many channels, separating the effects of a change in the supply versus demand of one type of debt allows us to trace out these channels more clearly.

[36] This figure is special case of Figure 1 in that it applies when monetary policy is constrained in the demand-determined regime.

Channels 1 and 2 capture how the supply of safe assets affects aggregate demand by changing the excess demand for saving $D_0^d(R_0, B_0; u_0) - D_0^s(R_0)$, at a given interest rate R_0 and utilization rate u_0. These channels emerge by deriving the household's demand for saving (7) with respect to the supply of private safe assets.

$$\frac{dD_0^d}{dD_0^s} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial D_0^s} + \underbrace{\overbrace{\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}}^{Channel\ 2}}_{dynamic\ interaction\ from\ SR\ to\ AD} \overbrace{\frac{d\ell_1(s_L)}{dD_0^s}}^{Channel\ 1} \tag{19}$$

The first term on the right-hand side captures the effect of the supply of private debt on consumption demand, *excluding* the effects via systemic risk, i.e. expected losses due to liquidation in the bad state $\ell_1(s_L)$. The second term captures the role of systemic risk in shaping the household's saving demand and is comprised of the two key channels of the model. Channel 1 reflects the effect of issuance of private debt on systemic risk (i.e., the severity of crises); while Channel 2 reflects the effect of systemic risk on the household's consumption demand through both the effect of liquidation on expected future consumption on consumption risk (i.e. consumption volatility).[37]

In Appendices 3, 4, and 5, I show that both Channels 1 and 2 are strictly positive under mild assumptions: $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)} > 0$ and $\frac{d\ell_1(s_L)}{dD_0^s} > 0$.[38] Therefore, an increase in the supply of private debt unambiguously increases systemic risk, ceteris paribus, by increasing the bank's leverage and therefore the severity of crises at date 1 (Channel 1). In turn, higher systemic risk lowers aggregate demand at date 0 by increasing the household's desire to save (Channel 2). [39]

Channel 3 captures a general equilibrium feedback effect. The excess demand for saving $D_0^d(R_0, B_0; u_0) - D_0^s(R_0)$ must be eliminated in general equilibrium through an adjustment in the date 0 interest rate R_0 or the utilization rate u_0 (depending on whether the supply-determined or demand-determined regime prevails at date 0). In general equilibrium, the adjustment of R_0 or u_0 feeds back to the date 0 supply of debt D_0^s and aggregate demand. This general equilibrium feedback effect is captured by Channel 3, represented by the dashed lines in Figure 7. For now, I leave aside this general equilibrium feedback 3 by holding both R_0 and u_0 constant, and return to

[37]Channel 2 thus captures one direction of a dynamic interaction between systemic risk and aggregate demand – the effect of systemic risk on aggregate demand at date 0, through the severity of crises at date 1. I show later that, in general equilibrium, there is also an effect in the reverse direction. Together these two channels will generate a feedback loop between aggregate demand and systemic risk.

[38]A sufficient condition for the former result is that $\frac{k_2^G}{\bar{k}_2} < \frac{1}{\bar{n}}$, while a sufficient condition for the latter is that lump-sum transfers are sufficiently small for latter. These assumptions are interpreted in Appendix 3 and 4, respectively.

[39]Note that the effect of higher systemic risk on aggregate demand at date 0 reflects both a desire to smooth consumption across time due to lower expected future consumption, and for precautionary motives due to higher consumption risk.

it in section 4.3.

4.2 Public safe asset creation does not directly generate systemic risk

In order to compare the role of public versus private debt, I now study the effect of an increase in the supply of public debt B_0 on aggregate demand, which requires that I make assumptions about what the government does with the proceeds of debt issuance at the margin. In this section, I assume that the government invests all proceeds from debt issuance in capital purchased on the spot market at date 0, which it holds through date 2, and rebates the rental income earned on these capital holdings to the bank in lump-sum fashion. These assumptions minimize the distortions associated with issuance of public debt by ensuring that the distribution of wealth across agents is not affected directly, allowing us to focus on the effects of public debt issuance per se.[40]

Deriving the household's demand for saving $D_0^d(R_0, B_0; u_0)$ with respect to B_0, shown in Appendix 6, I obtain a similar expression to (37).

$$\frac{dD_0^d}{dB_0} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0} \tag{20}$$

Comparing (37) and (38), I can see that, unlike with private safe assets, the supply of public safe assets does not affect consumption demand through systemic risk – Channels 1 and 2 are not active for public safe asset production. The government's special power to tax frees it from the burden of having to liquidate capital in order to service its debt in the bad state at date 1. Therefore, the government can produce safe assets, and thus absorb aggregate risk, without directly exacerbating the severity of crises at date 1.

As a result, the dynamic interaction between aggregate demand and systemic risk applies only for private safe asset production, but not for the production of public safe assets. I show in Online Appendix 8 that, as a result, I have $\frac{dD_0^d}{dD_0^s} > \frac{dD_0^d}{dB_0}$; that is, an increase in the supply of private debt unambiguously reduces consumption demand (i.e. increases demand for saving) at the margin by more than an increase in the supply of public debt does.[41]

[40] These assumptions are made explicit in Appendix 6. In the normative section, I relax these assumptions about the government's behavior.

[41] Public safe asset creation may have indirect effect on systemic risk by crowding-in or -out private safe asset creation through the term $\frac{dD_0^s}{dB_0}$. I take this issue up in the normative section of the paper.

4.3 General equilibrium feedback effects from aggregate demand to systemic risk

I now characterize Channel 3 in Figure 7, the general equilibrium feedback effects from the adjustment in R_0 or u_0 following a change in the supply of safe assets. The precise nature of Channel 3 depends on the regime that prevails at date 0 – that is, whether R_0 or u_0 is adjusting to reach equilibrium.

General equilibrium feedback effects in the supply-determined regime Recall that systemic risk (i.e. $\ell_1(s_L)$) depends on $R_0 D_0$ and k_1 through the bank's leverage ratio at date 0, Lev_0. Therefore, depending on the net effect of the shock to $R_0 D_0$, the bank's date 0 leverage ratio, and therefore systemic risk, may decrease or increase in general equilibrium.[42]

General equilibrium feedback effects in the demand-determined regime In the demand-determined regime, utilization u_0 falls in response to the increase in excess demand. The consequent fall in output lowers the rental income of banks and thus erodes their date 0 net worth. Ceteris paribus, the fall in the bank's net worth increases its date 0 leverage ratio and hence the expected future liquidation costs. Depending on how the bank manages this rise in liquidation risk, there are two general equilibrium feedback channels that may emerge through which the fall in net worth affects the equilibrium: through a fall in the supply of private safe assets or through a rise in the demand for safe assets.

Channel 3a: Demand recession further reduces supply of private safe assets Channel 3a captures the effect of the recession on the supply of private safe assets, $\frac{dD_0^s}{du_0}$. If, in a bid to manage the liquidation risks associated with its lower net worth, the bank tries to delever by reducing its issuance of debt, then $\frac{dD_0^s}{du_0} > 0$. While this channel does not directly exacerbate systemic risk $\ell_1(s_L)$, it exacerbates the shortage of safe assets by contracting the supply and hence worsens the demand-driven recession.

Channel 3b: Demand recession further increases systemic risk Channel 3b captures the effect of the recession on the severity of crises, $\frac{d\ell_1(s_L)}{du_0}$. If, in response to the erosion in its net worth, the bank allows its leverage ratio to rise rather than delevering, then the bank's leverage ratio rises so that liquidation risk rises, $\frac{d\ell_1(s_L)}{du_0} < 0$. This constitutes a feedback channel from aggregate demand to systemic risk, illustrated in Figure 7.

The total marginal effect of general equilibrium feedback Channels 3a and 3b on the severity of the demand-recession can be expressed as the sum of these two channels, which are controlled

[42] In particular, as R_0 falls in to clear an excess demand for saving, the bank's date 1 interest payments fall, lowering liquidation in the bad state. However, R_0 also induces the bank to increasing date 0 borrowing D_0 at the margin, which has a positive effect on liquidation. The response of systemic risk to the the fall in R_0 depends on the net of these two effects. While I could layout the conditions under which one effect dominates the other, the net change in D_0 in the supply-determined regime is not central to the results I wish to characterize.

by the elasticities $\frac{dD_0^s}{du_0}, \frac{d\ell_1(s_L)}{du_0}$ which respectively govern the general equilibrium response of the supply of private debt and systemic risk to the demand driven recession.[43]

$$\frac{d\left(D_0^d - D_0^s\right)}{dD_0^s}\left[\overbrace{\frac{dD_0^s}{du_0}}^{Channel\ 3a} + \overbrace{\frac{d\ell_1(s_L)}{du_0}}^{Channel\ 3b}\right] \tag{21}$$

Feedback loop may amplify the demand recession Taking into account Channel 3a and 3b, the model features a dynamic feedback loop between systemic risk and aggregate demand in general equilibrium, illustrated in Figure 7: Systemic risk affects aggregate demand at date 0 in partial equilibrium (Channel 2) and, in general equilibrium, aggregate demand in turn affects systemic risk (Channels 1 and 3).

Appendix 7 shows that, under relatively weak conditions, I have $\frac{\partial D_0^s}{\partial u_0} > 0$ and $\frac{d\ell_1(s_L)}{du_0} < 0$ so that both general equilibrium Channels 3a and 3b imply that date 0 output in the demand-determined regime is lower than it would otherwise be. In the case of Channel 3a, the fall in the supply of private safe assets further increases the excess demand for private safe assets at date 0, while in the case of Channel 3b, the higher systemic risk associated with larger liquidation costs causes the household to demand more safe assets at date 0. In this manner both channels may cause a higher demand for safe assets, requiring a deeper demand-driven recession to equilibrate the economy at date 0.[44]

5 The safety trap and systemic risk

In this section, I show that the demand-determined regime features a safety trap, which I define as a demand-driven recession which arises due to a shortage of either type of safe asset.[45] I also show that the dynamic interaction between aggregate demand and systemic risk at the core of this model may fundamentally alter the nature of safety traps relative to the literature, and implies that there are different types of safety traps.

Remark on safety traps versus liquidity traps As discussed in Caballero and Farhi (2018), safety traps are a special type of liquidity trap in which there is a shortage of safe assets as opposed to a shortage of assets more broadly. For systemic risk (which is the focus of this paper) to play a role in my model, I must consider an environment of positive risk premia and hence a shortage of

[43] For simplicity, I omit from the illustration the effect of utilization on D_0^d through the wage w_0.

[44] In section 6, I show that, under weak conditions, Channels 3a and 3b also amplify the severity of demand recessions in response to a shock at date 0.

[45] Nevertheless, I show in section 5 that, in this setting, a safety trap must feature a shortage of public safe assets, but not necessarily a shortage of private safe assets.

safe assets.[46]

5.1 Conditions defining a safe asset shortage

Conceptually, a shortage of one type of safe asset implies that, in equilibrium, a marginal increase in the supply of that asset reduces the severity of the demand recession at date 0. Therefore, to determine whether there is a shortage of public (private) safe assets in the demand-determined regime, I ask whether, at the margin, an increase in the supply of public (private) safe assets increases or decreases the excess demand for saving. The explicit conditions are given in Appendix 8.

Intuitively, a marginal increase in the supply of a safe asset has broadly two effects on the excess demand for saving at date 0: an effect on the supply of private bonds $\frac{dD_0^s}{dB_0}$, and an effect on the demand for private bonds $D_0^d\left(R_0, B_0; u_0\right)$ through the household's Euler equation $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0}$. If the net effect is a fall in the excess demand for saving, then utilization must rise at the margin to equilibrate the economy, resulting in a less severe demand recession.

5.2 A taxonomy of safety traps

In this section, I show how the dynamic interaction between systemic risk and aggregate demand alters the nature of safety traps. I first show that the safety trap in the demand-determined regime must feature a shortage of public safe assets, as long as $\frac{dD_0^s}{dB_0}$ is not too negative – that is, if an increase in the supply of public safe assets crowds-in private safe asset creation, or at least does not crowd it at too much.

Lemma 2: *Safety trap features shortage of public safe assets*

Suppose that $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0} - 1 < \frac{dD_0^s}{dB_0}$ is satisfied in equilibrium in the demand-determined regime. Then, in the demand-determined regime, I have $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0} \leq ?1 + \frac{dD_0^s}{dB_0}$, implying that the demand-determined regime features a shortage of public safe assets.

Proof: See Appendix 9.

The condition spelled out in the lemma is a sufficient condition for the demand-determined regime to always feature a shortage of public safe assets (and therefore, a safety trap).[47] Hence-

[46] Central to safety traps are positive risk premiums. As I discussed in section 2.3, a positive risk premium arises in this model due to the liquidation cost of capital. Thus, by construction, the safety trap is the only type of liquidity trap that occurs in equilibrium in this model.

[47] Since I showed previously that $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0} < 0$, a sufficient condition for this to hold would be $\frac{dD_0^s}{dB_0} \geq 0$.

forth, I restrict the analysis to regions of the parameter space in which this condition is satisfied in equilibrium in the demand-determined regime.

In this model, not all safety traps are alike. Paradoxically, the demand-determined regime can feature an *oversupply* of private safe assets at the same time that it features a shortage of public safe assets. Indeed, this model produces a taxonomy of safety traps depending on whether the equilibrium features a shortage or an oversupply of private safe assets.

Proposition 1: *A taxonomy of safety traps*

A) The demand-determined regime at date 0 features a safety trap which is one of two types:

In a *conventional safety trap*, the economy features a shortage of both public and private safe assets in equilibrium: At the margin, an increase in the supply of either public or private safe assets would increase aggregate output at date 0.

In a *risk-intensive safety trap*, the economy features a shortage of public safe assets and an oversupply of private safe assets in equilibrium: At the margin, an increase in the supply of public or a decrease in the supply of private safe assets would increase aggregate output at date 0.

B) This case obtains if and only if the product of Channels 1 and 2 is sufficiently large.

$$\overbrace{\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}}^{Channel\ 2}\overbrace{\frac{d\ell_1(s_L)}{dD_0^s}}^{Channel\ 1} > 1 - \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial D_0^s}$$

where the left-hand side is strictly positive.

Proof: See Appendix 10.

While both types of safety trap feature a shortage of public safe assets, what distinguishes them is whether there is a shortage or an oversupply of *private* safe assets. In a conventional safety trap, the demand recession at date 0 is driven by a shortage of both public and private safe assets, and reflects a general insufficiency in the means of smoothing household consumption across time and states. This type of generalized shortage of safe assets is what characterizes the literature on safety traps, such as Caballero and Farhi (2018).

In a risk-intensive safety trap, by contrast, the demand recession is driven is by a shortage of public safe assets together with an *oversupply* of private safe assets.[48] [49] Importantly, in a risk-intensive safety trap, the economy is characterized by a *paradox of safety*, in which the desire of households to hold safe assets as insurance against systemic risk ultimately increases systemic risk in general equilibrium, through the higher leverage of banks. In this case, the cause of the safe asset shortage is financial instability (i.e. high systemic risk), which leads to a deficiency of aggregate demand due to precautionary saving. In turn, this endogenously leads to a safety trap because low output erodes banks' date 0 net worth and thus reduces their ability to produce private safe assets without further increasing systemic risk.

The proposition also shows that the nature of safety traps is determined by the strength of dynamic interaction between systemic risk and aggregate demand: Risk-intensive safety traps occur if and only if the dynamic interaction between systemic risk and aggregate demand, given by the product of Channels 1 and 2, is sufficiently strong.[50] To understand why, recall that in this setting, the supply of private safe assets has an effect not only on the aggregate supply of safe assets but also on the demand for safe assets due to its effect on systemic risk. A safety trap features an oversupply of private safe assets if a marginal increase in the supply of private safe assets increases the demand for safe assets by more than the supply, worsening a shortage of safe assets. In this case, a marginal rise in the supply of private safe assets would increase bank leverage and therefore increase systemic risk (Channel 1). In turn, the rise in systemic risk would, ceteris paribus, raise the household's demand for saving (Channel 2), as it would have more incentive to transfer resources to the bad state of the world at dates 1 and 2. In such a case, the safety trap would, counterintuitively, be ameliorated through a reduction in the supply of private safe assets.

As I show in the normative section, the nature of the safety trap will have important implications for the direction of externalities and the design of optimal policy.[51]

[48] In a risk-intensive safety trap, a higher supply of private safe assets could increase future output, y_1, y_2, even if it reduces current output, y_0. For example, a higher supply of private safe assets allows the bank to increase date 0 investment, and therefore could cause future output y_1, y_2 to rise due to a higher stock of capital. At the same time, the higher supply of private safe assets increases bank's leverage and therefore systemic risk. If the dynamic interaction between systemic risk and aggregate demand (Channels 1 and 2) is sufficiently strong, the latter effect dominates, meansing that the higher supply of private safe assets reduces current output at the margin, even if it increases future output.

[49] Key to this taxonomy is the differential manner in which the production of public and private safe assets affects systemic risk.

[50] Recall also that the strength of each channel is controlled by the elasticities of crisis risk to the leverage of banks and of households' saving demand to crisis risk, respectively.

[51] As I will show later, this also equivalently means the direction of an aggregate demand externality is such that planner would reduce the bank's leverage relative to the competitive equilibrium.

5.3 Discussion

Two remarks about the above results are worth emphasizing. First, in this model, safety traps are truly endogenous in that the supply of private safe assets may be low precisely because the demand for safe assets is high. In particular, the results above show that high demand for safe assets can itself reduce the capacity of the private sector to produce safe assets: At the effective lower bound, high demand for safe assets leads to a demand-recession, which can erode bank net worth and thus limit the ability of banks to create safe assets by issuing debt. Moreover, increasing the supply of private safe assets may, under some conditions, only serve to further increase demand and thereby worsen the safe asset shortage (the paradox of safety): Issuing more private safe assets may increase systemic risk by forcing banks to liquidate more capital in bad future states, thereby increasing the household's demand for safe means of storing consumption across time.

Second, the interaction between systemic risk and aggregate demand at the heart of this model is crucial to deriving this taxonomy and understanding the nature of safety traps. To show this, in Online Appendix 9, I consider a counterfactual version of this economy in which $\ell_1 = 0$ in all states. If there is a a shortage of public safe assets in equilibrium at date 0, then there must also be a shortage of private safe assets in equilibrium. In such an economy, there can be only conventional safety traps – that is, cannot have an oversupply of private safe assets in a safety trap and the paradox of safety never emerges.

6 Effects of a shock to systemic risk

In this section, I analyze how this economy responds to a shock which increases systemic risk at date 0, and show that the dynamic feedback loop between systemic risk and aggregate demand (the confluence of Channels 1, 2, and 3) generate an amplification mechanism. To do so, I trace out the channels of transmission of an unanticipated (MIT) shock to future TFP in the bad state. In particular, agents learn at date 0 that TFP in the bad state at date 1, $z_1(\underline{s})$, is lower than initially thought – essentially an adverse news shock. This shock has the effect of increasing the severity of crises in the bad state at date 1, and therefore can be interpreted as an exogenous increase the systemic risk faced by agents at date 0.[52] The effects of the shock are stylistically illustrated in Figure 8, and the effects are traced out in analytical detail in Online Appendix 10.

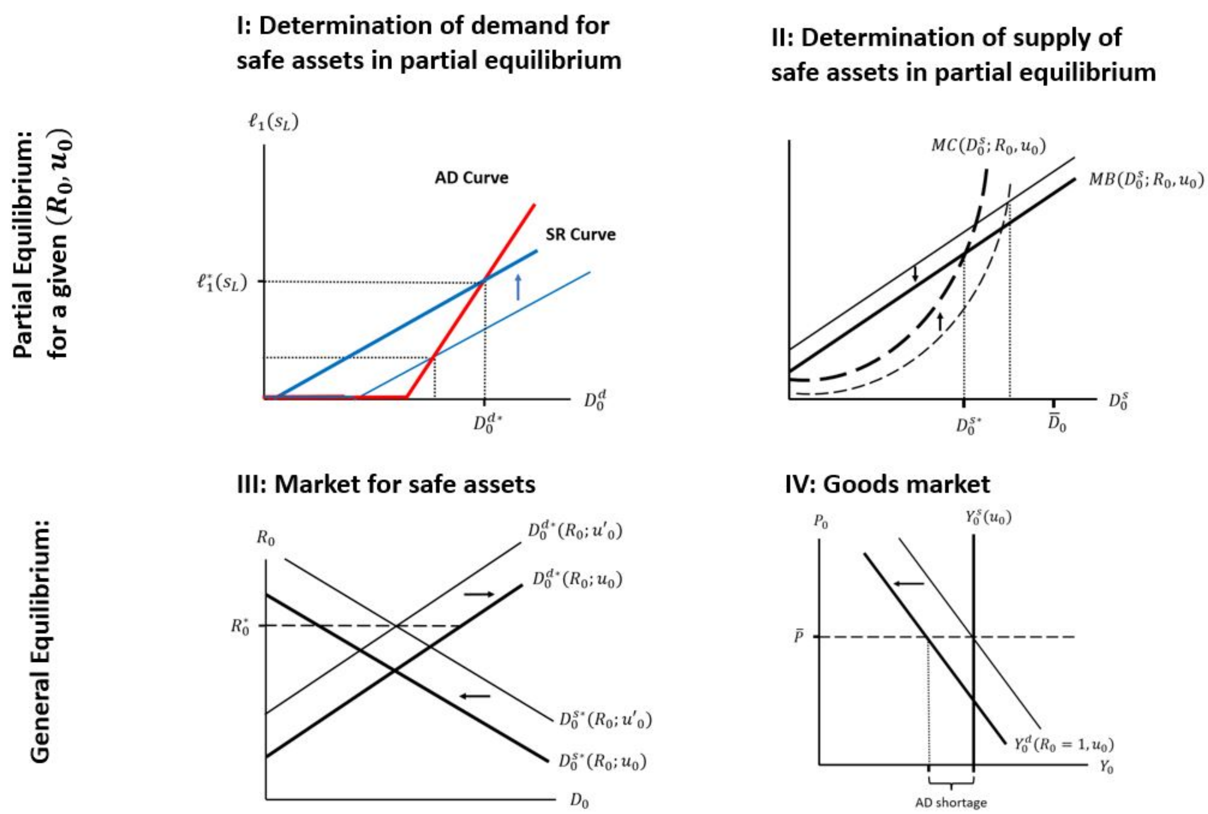
Since adverse shock to date 1 TFP reduces the bank's rental income in the bad state at date 1, it increases the level of liquidation in that state, $\ell_1(s_L)$. Therefore, by increasing the level of liquidation, the anticipated shock to date 1 TFP effectively shifts the SR curve in panel I up,

[52]The response of the economy, and hence the channels I outline below, will be similar across any shock which causes an excess demand for safe assets at date 0.

resulting in a higher demand for safe assets D_0^* at any interest rate and level of utilization, and therefore shifting the demand curve for safe assets in panel III to the right.

Moreover, the higher expected liquidation costs to the bank, and the lower expected return to capital at date 1, causes the MC curve in panel II to shift up and the MB curve to shift down. Both of these effects serve to reduce the bank's supply of private safe assets D_0^{s*} at any interest rate and utilization rate, shifting left the supply curve for safe assets in panel III. As a result, the shock causes an excess demand for safe asset at date 0. Depending on whether monetary policy is constrained by the effective lower bound, this safe asset shortage must be cleared through a fall in the interest rate or through a demand-driven recession (see section 3).

Figure 8: Effect of shock to $\ell_1(s_L)$



General equilibrium feedback from date 0 to dates 1 and 2: The excess demand for safe assets requires either the date 0 interest rate or utilization to adjust to equilibrate the economy, depending on whether the economy is in the supply-determined or demand-determined regime at date 0. In turn, the adjustment in R_0 and u_0 may feed back to the bank's leverage and therefore affect the allocation at dates 1 and 2 through Channels 3a and 3b (not shown in Figure 8).

Supply-determined regime In the supply-determined regime, monetary policy facilitates an adjustment in the composition of output going to saving versus consumption through a lower interest rate, but a recession at date 0 is avoided as monetary policy is not constrained.[53]

Demand-determined regime As I showed in section 3.2, in the demand-determined regime, the excess demand is cleared through fall in u_0 which erodes the bank's net worth at date 0 and increases its leverage ratio and expected future liquidation costs, ceteris paribus. Depending on how the bank manages this rise in liquidation risk, Channels 3a and 3b shock worsens the decline in aggregate output at date 0 through the general equilibrium Channels 3a and 3b. The fall in the bank's net worth causes the supply of private safe assets to fall further through general equilibrium Channel 3a, while through general equilibrium Channel 3b, it causes systemic risk to rise, increasing the household's date 0 demand for safe assets.

The proposition below outlines the conditions under which these general equilibrium channels amplify the demand-driven recession at date 0 in response to the shock.

Proposition 2: Amplification mechanism If the demand recession at date 0 causes the bank to reduce its debt issuance ($\frac{dD_0^s}{du_0} > 0$) and/or if it causes systemic risk to rise ($\frac{d\ell_1(s_L)}{du_0} < 0$), then Channels 3a and 3b together may amplify fall in date 0 output in response to an unanticipated fall in $z_1(s_L)$. Furthermore, if both channels are active, then the condition for amplification is weaker than if only one channel holds.

Proof: See Appendix 11.

Thus, the dynamic, two-way interaction between systemic risk and aggregate demand that arises in general equilibrium can give rise to an amplification mechanism which exacerbates demand-driven recessions in the presence of safe asset shortages.

7 Normative Implications

In this section, I examine the normative and policy implications of the model. I begin by tracing out the transmission channels of various different policies. I then solve a constrained social planner's problem, identify the externalities at play, and solve for optimal policy. In the presence of this dynamic interplay, insufficient coordination between policies designed to stimulate aggregate demand at the effective lower bound and those to mitigate systemic risk can result in an inefficiently low level of aggregate demand and output and an excessively high level of systemic risk at date 0.

[53] In the supply-determined regime, output is supply-determined, and the shock has no effect on the supply side of the economy at date 0, as k_0 is a fixed endowment, labor is inelastically supplied, and utilization is optimally $u_0 = 1$ in the supply-determined regime. Of course, if the shock to date 1 TFP is sufficiently severe, the economy may enter the demand-determined regime at date 0. However, my analysis focuses on the marginal effects of shocks.

7.1 Transmission channels of policy instruments

I begin by tracing out the transmission channels of various different policies and show that the dynamic interplay between aggregate demand and systemic risk at the heart of the model implies that policies in general may have effects on both aggregate demand and systemic risk.

7.1.1 Monetary Policy

The dynamic interplay between aggregate demand and systemic risk introduces a new, macroprudential channel through which monetary policy can affect output at date 0: through affect the risk of a future crisis.

At date 0, monetary policy affects both the demand and supply side of the economy. Suppose I are in the supply-determined regime at date 0 so that monetary policy is unconstrained, and suppose that, off-equilibrium, there is an excess demand for safe assets. (Equivalently, output is below potential at date 0.) A lower nominal interest rate $R_0^{MP} = R_0^B$ lowers demand for safe assets D_0^d through the household's Euler equation, and at the same time increases demand for goods. This is the conventional effect of monetary policy the demand side of the economy.

However, there is an additional indirect effect on aggregate demand through its effect on future crisis risk. By boosting aggregate demand and keeping output at the natural rate, monetary policy boosts the bank's date 0 net worth, allowing banks to produce private safe assets without significantly increasing the risk of a future crisis. By keeping systemic risk low, this further boosts aggregate demand at date 0.

On the supply side, monetary policy has opposing effects on systemic risk: A lower policy rate incentivizes the bank to issue more private safe assets. This both increases returns (to both banks and households, in the form of labor income) in the good state at date 1, but magnifies the severity of crises in the bad state.

In particular, a lower nominal interest rate increases the bank's issuance of private debt D_0, in turn, boosting the bank's investment in capital i_0. The higher leverage of the bank Lev_0 has opposing effects on the labor income risk borne by the household. In the good state s_H, the higher capital stock of the bank k_1 implies that output is higher y_1, and therefore the household's date 1 labor income is higher $w_1\bar{n} = (1-\alpha)y_1$. Moreover, the higher return r_1^k earned by the bank due to the higher capital stock k_1 implies that the bank's date 1 investment i_1 in capital is higher, meaning the date 2 capital stock k_2 is higher. As a result, date 2 output and labor income are both higher. Both of these effects imply that higher date 0 bank leverage unambiguously increases the total future consumption of the household in the good state $c_1(s_H) + c_2(s_H)$. This effect lowers household's demand for consumption smoothing D_0^d, reinforcing the decline in date 0 demand for private debt mentioned before.

In contrast, the effect of higher date 0 leverage Lev_0 on the household's total future consumption in the bad state is ambiguous. As before, the higher date 0 leverage Lev_0 means that the bank's date 1 capital stock k_1 is higher. As a result, date 1 output y_1 is higher than it would otherwise be, and hence the household's date 1 labor income $w_1\bar{n} = (1-\alpha)y_1$ is higher. However, in the bad state, the higher leverage implies that the bank is forced to liquidate a higher fraction ℓ_1 of its date 1 capital holdings to repay its debt. As a result, the date 2 capital stock k_2 is lower in the bad state. This means that date 2 output y_2 and labor income $w_2\bar{n} = (1-\alpha)y_2$ are lower as a result of the higher level of leverage. Thus, higher date 0 leverage increases date 1 labor income $w_1(s_L)\bar{n}$ but reduces date 2 labor income $w_2(s_L)\bar{n}$ in the bad state. The net effect on the household's date 1 and 2 total consumption in the bad state $c_1(s_L) + c_2(s_L)$ is ambiguous.

Taking stock, higher bank leverage increases the household's total future consumption in the good state, but has an ambiguous effect on total future consumption in the bad state. Depending on the net effect of on the household's date 0 saving motive, this supply-side effect may boost or dampen aggregate demand at date 0. If the net effect is to increase the household's expected discounted total future consumption, then the reduction in the nominal interest rate reduces the household's demand for private debt D_0^d through the household's date 0 Euler equation, which reinforces the direct effect on the demand for saving. (If, on the other hand, the net effect of the lower nominal interest rate is to lower the households total expected future consumption, then a reduction in the interest rate could have a contractionary effect at date 0 by incentivizing the household to save more on net. I will not focus on this case since it is not empirically relevant.) Either way, monetary policy works in part through the interplay between systemic risk and aggregate demand.

7.1.2 Fiscal policies

I consider two classes of fiscal policies: ex post government bailouts and ex ante intervention in which the government finances spending by issuing public safe assets at date 0.

Government bailouts Consider first an ex post bailout policy in which the government commits to issuing transfers to banks in the bad state at date 1, which are just sufficient such that the banks can repay their debt without liquidating capital, so that $\ell_1(s_L) = 0$. Suppose that the government finances these transfers by exacting lump-sum taxes on the household at date 1. Effectively, this transfers the bank's losses to the households contemporaneously.[54] By forcing the households to bear some of the losses associated with investment, this policy eliminates liquidations from materializing ex post and thus prevents the associated deadweight loss.

Suppose the government can commit to such a policy ex ante at date 0. Effectively, private

[54] Alternatively, the government could finance the bailout by issuing debt at date 1 to be repaid using taxes ate date 2. This would smooth losses across time. However, recall that the household perfectly smooths consumption across dates 1 and 2 using the storage technology anyway.

safe assets are then implicitly backed by government guarantee, which (partially) transforms them into public safe assets. By transferring sufficient amount of investment risk from the banks to the household via the government's balance sheet, this policy eliminates systemic risk at date 0.[55] Thus, when monetary policy is constrained by the effective lower bound, an ex ante commitment to bailing out banks ex post can improve safety traps by mitigating systemic risk and thus stimulating aggregate demand.

Deficit spending Ex ante fiscal interventions, such as deficit spending at date 0 in which the government finances transfers through the issuance of public safe assets, affect aggregate demand both directly and indirectly through macroprudential effects which reduce systemic risk. The conventional, direct effect is to stimulate aggregate demand, which boosts output when monetary policy is constrained.

In addition, there are two macroprudential effects of deficit spending at date 0. First, by stimulating aggregate demand and output directly, spending boosts net worth of banks at date 0, allowing them to bear losses with less risk of a future crisis. Second, to the extent that it crowds out private safe asset issuance, deficit spending effectively changes the composition of the safe assets held by households at date 0. By reducing the share of private safe assets in their holdings, fiscal policy transfers some of the risk associated with investment to the government's balance sheet, and therefore reduces systemic risk. Both of these effects reduce systemic risk, and therefore boost aggregate demand ex ante.

Fiscal multiplier These macroprudential effects imply that fiscal multipliers are larger than they otherwise would be. Thus, the dynamic interplay between aggregate demand and systemic risk constitutes an additional channel through which fiscal spending may affect output. This model thus introduces another component of fiscal multipliers. The size of the fiscal multiplier at date 0 depends in part on the strength of the dynamic interplay between aggregate demand and systemic risk, which is controlled by the elasticities identified in section 4.

Importantly, the model suggests that, in a safety trap characterized by a high level of systemic risk, public expenditure ought to be financed through deficit spending rather than contemporaneous taxes. Note that Ricardian equivalence does not generally hold in this environment because public safe asset production (financed by future taxes) entails higher future output because, to the extent that it crowds out private safe asset production, fiscal spending reduces crisis risk. Therefore, by simultaneously spending and providing public safe assets as a way for the households to smooth consumption without generating systemic risk, fiscal policy can take full advantage of the fiscal multiplier stemming from the dynamic interplay between aggregate demand and systemic risk.

[55] I abstract here from moral hazard considerations whereby the anticipation of government bailouts leads to perverse risk-taking incentives at date 1.

7.1.3 Quantitative easing

Quasi-fiscal monetary policy, such as quantitative easing, whereby the government purchases capital from banks at date 0 by issuing public safe assets, can stimulate aggregate demand at date 0 through macroprudential effects.

In order to implement this policy, I assume that at date 0, the government issues public safe assets B_0 to the household and uses the proceeds to buy capital from on the date 0 spot market at date 0, taking the competitive price q_0 as given.[56] The government holds this capital through date 2 and rents out this capital to firms at dates 1 and, earning the competitive rental rate r_t^k. I repays its debt at date 1 (and possibly date 2) using the rental income and lump-sum taxes on the household.

Quantitative easing affects the allocation in part through macroprudential effects on systemic risk. In particular, there are two broad channels through which it works. First, this policy alters the composition of safe assets held by households toward public safe assets. Effectively, this shifts the risk of investment from the banks' balance sheets to that of the government. And since the government finances any losses at date 1 with lump-sum taxes on the household, the government effectively forces the household to bear some of the losses associated with low productivity in the bad state instead of having the banks bear the entirety of these losses. Ultimately, this reduces the fraction of total capital stock that is liquidated in the bad state and reduces the associated deadweight losses.[57] And since the deadweight loss associated with liquidation is lower, the household may be better off in general equilibrium as its future labor income may be higher in the bad state at date 2 than it would otherwise be.

Thus, quantitative easing can reduce systemic risk by potentially improving risk sharing between the household and banks. Effectively, quantitative easing changes the composition of the assets which implicitly back safe assets, as a higher fraction is backed by the government's tax power. I discuss the optimality of this policy in the normative section.

A second, indirect effect is that higher aggregate demand may crowd in private safe asset creation without a significant rise in systemic risk, as higher output boosts bank's date 0 net worth. Effectively, the policy can thus create a virtuous cycle between high aggregate demand and low

[56] In practice, supposed to work by directly affecting these prices. I abstract from these considerations here.

[57] The key for quantitative easing to work in the model is that, by holding private debt on its balance sheet, the government prevents the banks from liquidating their capital holdings to service their debt in the bad state. How Ill does this reflect reality? In practice, liquidations often occur when, for example, a drop in the price of the assets held by financial intermediaries lead to margin calls or runs by their private creditors, forcing the intermediaries to inefficiently liquidate these assets. When a central bank purchases the debt of these intermediaries, it becomes the creditor. If the intermediaries' assets backing the debt decline in value, the central bank typically does not issue a margin call, unlike private creditors. Hence, quantitative easing effectively reduces the incidence of inefficient liquidations, as the public sector does not issue margin calls in bad states of the world to the extent that the private sector does. Quantitative easing When the consolidated government becomes the creditor of these intermediaries through asset purchases, ass does not margin call. Therefore, my assumption in the model that banks do not liquidate capital to service debt held by the government can thus be thought of a reduced form way to capture this phenomenon.

systemic risk at date 0. (Moreover, the policy can allow the bank to expand investment in capital at date 0.)

Note that because quantitative easing affects the level of overall investment at both dates 0 and 1, in general, there is a social tradeoff to quantitative easing. In particular, at date 0, quantitative easing my crowd-in or -out public safe asset issuance and date 0 investment in capital. Since there are decreasing returns to investment, this may lead to suboptimal investment levels at date 0. (That said, the bank may be compensated for this via the price of date 0 capital, which would allow it to increase investment at date 0.)

Moreover, by holding capital, the government denies banks some rental income at date 1. This reduces the bank's capacity to invest in new capital at date 1 and pay off debt at date 1. By contrast, the government does not have investment technology at date 1; the government's rental income at date 1 is used to pay off debt or transfers, but cannot directly be used to fund investment in new capital at date 1. So in principle, absent other interventions or transfers, quantitative easing may result in less total investment at date 1. (Of course, the government could offset some of these effects by transferring funds to the bank at date 1 to expand investment.)

This social tradeoff to quantitative easing implies that, in general, it is not socially optimal for the government to fully intermediate between the household and banks by issuing public debt and transferring all proceeds to the bank, nor is it in general socially optimal for public safe assets to be the only type of debt held by households. I revisit this issue in section on optimal policy.

7.1.4 Macroprudential policy

The model's mechanism also introduces a new role for macroprudential policy – one of aggregate demand management. Consider the Pigouvian taxes τ_0 which disincentivize the bank from issuing debt at date 0 by increasing its marginal cost, as can be seen in the bank's optimality condition for leverage (18). One can interpret these taxes as macroprudential regulations, such as bank capital requirements, which limit the leverage of financial intermediaries.

By reducing the leverage of banks at the margin, such taxes reduce the severity of crises in the bad state at date 0. By reducing systemic risk, this policy can raise the households' labor income in the bad state at date 2, and thereby potentially reduce aggregate demand at date 0. Thus, the dynamic interplay between aggregate demand and systemic risk implies that, at the margin, macroprudential policy at date 0 has an effect on aggregate demand at date 0.

In general, tight macroprudential policy has two opposing effects on date 0 aggregate demand when monetary policy is constrained. On the one hand, by reducing the supply of private safe assets, it worsens a safe asset shortage. On the other hand, by reducing systemic risk, it lowers the demand for safe assets at date 0, mitigating the shortage. The net effect on the severity of the safe asset shortage, and aggregate demand, depends on the nature of the safety trap. In a conventional safety trap, which features a shortage of both public and private safe assets, the net effect of tight macroprudential policy is to worsen the safe asset shortage, and amplify the demand recession at date 0.

By contrast, in a risk-intensive safety trap, which feature an oversupply of private safe assets, the net effect of macroprudential policy is to boost aggregate demand by restricting the supply of private safe assets, even though this results in a net decrease in the total safe asset supply. Hence, policies which restrict the supply of private safe assets, such as bank capital regulation, may counterintuitively mitigate safety traps under these conditions. Thus, this paper highlights that, when monetary policy is constrained by the effective lower bound, macroprudential policy can substitute for monetary policy and stimulate aggregate demand and boost output at date 0.[58]

7.2 Normative implications

7.2.1 Social planner's problem

To evaluate the allocative efficiency of the economy, I solve the problem of a social planner who solves for allocation which maximizes welfare of household, subject to some level of welfare for the bank, taking as given the constraints to which agents are subject, including incomplete markets, the non-negativity constraint on date 1 investment, the deadweight loss associated with liquidation, and the effective lower bound on monetary policy. The solution to this problem constitutes the constrained efficient allocation. I first solve the social planner's problem taking as given the government's behavior, to elucidate the externalities at play and understand how different policy interventions can improve the allocation at the margin. Then I solve the Ramsey problem to characterize optimal policy.

7.2.2 Externalities

Risk sharing externality: The competitive equilibrium is in general constrained inefficient due to the presence of two externalities. The first is a risk-sharing externality, similar to that in Bocola Lorenzoni (2023), in which households do not internalize how their demand for safe assets at date 0 lowers their date 2 labor income in the bad state of the world due to the macroeconomic spillover

[58] This result is similar in spirit to that of Korinek and Simsek (2016) in which macroprudential regulation can boost future output due to an aggregate demand externality. However, the nature of the aggregate demand externality here is different and therefore so are the policy implications. In Korinek and Simsek (2016), leverage today causes demand-driven a recession in bad states tomorrow. Therefore, macroprudential regulation today reduces the likelihood of demand-driven recessions tomorrow. But in my model, macroprudential policies today (at date 0) stimulate aggregate demand today (also at date 0). This is due to the dynamic interplay between aggregate demand at date 0 and the risk of crises at date 1. Hence, my paper indicates that in an environment of low current aggregate demand, macroprudential policy may substitute for monetary policy and boost current output.

in which a crisis at date 1 lowers date 2 labor income.[59]

Relative to the social optimum, banks bears too much of the risk associated with investment at date 0 while the households bear too little. In order to insure households against investment risk at date 0, banks preserve the safety of their liabilities by liquidating capital in the bad state at date 1. However, due to the deadweight loss and macroeconomic spillover associated with liquidation, this forces the household to bear losses at date 2 in the form of labor income. Thus, by preventing the household from bearing investment risk at date 1, private safe asset creation forces the household to bear labor income risk at date 2 in general equilibrium, which is not internalized by the household.

The inefficient risk-sharing associated with private safe assets does not obtain for public safe assets. From the point-of-view of individual savers, these are equivalent instruments to smooth consumption as both the private and public bonds promise the same payoff profile at date 1. However, these are not equivalent means of smoothing consumption from a social perspective. Public safe assets are implicitly backed in part by the state-contingent stream of future tax revenue, which the government can generate without liquidating capital inefficiently. Indeed, I later show that the government can optimal use public safe assets and lump-sum taxation to force the household to bear some of the risk associated with investment, thus reducing systemic risk.

Because of the two-way interaction between systemic risk and aggregate demand, agents' date 0 borrowing and saving s are also associated with an aggregate demand externality, as effect on crisis and distribution of future income affects current aggregate demand, which affects current output when monetary policy is constrained by the effective lower bound.

Aggregate demand externality: The second externality, which obtains only in the demand-determined regime, is an aggregate demand externality in which households do not internalize how, at the effective lower bound, their date 0 spending affects boosts date 0 and therefore other households' and banks' income/net worth.[60] Moreover, because of the two-way interaction between aggregate demand and systemic risk, agents' date 0 spending decisions (aggregate demand) are also associated with the risk-sharing externality as date 0 output affects banks' net worth, and hence the severity of crises and the households' date 2 labor income.

Margins of inefficiency These two externalities leads to two margins of constrained inefficiency when monetary policy is constrained by the effective lower bound. The first margin is that aggregate demand is inefficiently low at date 0. (Relative to the social optimum, the share of private safe assets held by the household is high as they do not internalize how their holdings of

[59]Note that the effect of debt issuance on the severity of crises (the size of $\ell_1(s_L)$) is priced in the interest rate (the banks' cost of borrowing). However, the effect that liquidation has on the household's date 2 labor income in general equilibrium is not priced in, and hence there is an externality.

[60]While in similar spirit to that in Korinek and Simsek (2016), in that private leverage creates risk of demand-driven recessions, here the nature of the aggregate demand externality is meaningfully different. I discuss this further in the literature review.

private safe assets generate systemic risk in general equilibrium, resulting in depressed output and excessive systemic risk.) This affects not only date 0 output, but also lowers the bank's net worth at date 0, potentially increasing the risk of future crises and therefore lowering future output in the bad state.

The second margin of inefficiency is that the household holds too much private safe assets and too little public safe assets at date 0. Put differently, households are bearing too little of the risk associated with investment while banks are bearing too much. At a given level of saving, this results in inefficiently high systemic risk, and therefore lower aggregate demand at date 0. Thus, in this setting, inefficient demand-driven recessions and excessive systemic risk arises not only due to a broad shortage of safe assets at date 0, but also due to the composition of safe assets held by the households being inefficiently skewed toward private safe assets.

7.2.3 Elasticities controlling the externalities

The externalities outlined above are intimately linked with the channels which makeup the dynamic interplay between, and therefore also the nature of the safety trap and optimal policy response.

What is the relationship between the externalities and channels making up the dynamic interaction between aggregate demand and systemic risk? To be precise, while the channels are not same thing as the externalities, they are affected by the externalities. For example, the risk sharing externality is controlled by the product of two elasticities which determine the marginal effect of private safe asset creation on the household's date 1 and 2 labor income in the bad state through date 1 liquidation.

$$\frac{\partial\left(\frac{1}{c_1(s_L)}\right)}{\partial \ell_1(s_L)}\frac{d\ell_1(s_L)}{dD_0^s}$$

This is a subcomponent of the product of Channels 1 and 2 in (19).

$$\underbrace{\overbrace{\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}}^{Channel\ 2}}_{dynamic\ interaction\ from\ SR\ to\ AD}\overbrace{\frac{d\ell_1(s_L)}{dD_0^s}}^{Channel\ 1}$$

Thus, the product of Channels 1 and 2, which control one direction of the dynamic interplay from systemic risk to aggregate demand, embeds the risk-sharing externality.

Similarly, the general equilibrium feedback Channels 3a and 3b relate closely to the aggregate demand externality. The aggregate demand externality, which captures how the household's date

0 consumption-saving decision affects output at date 0, can be expressed as the product of the following elasticities.

$$\frac{\partial y_0}{\partial u_0}\frac{du_0}{d\left(D_0^d-D_0^s\right)}\frac{d\left(D_0^d-D_0^s\right)}{dD_0^d}$$

Recall that Channels 3a and 3b are respectively controlled by the following elasticities, $\frac{dD_0^s}{du_0}$ and $\frac{d\ell_1(s_L)}{du_0}$, both of which depend in part on $\frac{\partial y_0}{\partial u_0}$, which in turn partially reflects the aggregate demand externality.

What is the relationship between the strength of the externalities and the type of safety trap? Recall that the nature of safety trap (and its associated policy responses) depends on whether a marginal increase in the supply of private safe assets increases or decreases the date 0 excess demand for safe assets. As established in section 5, this depends in part on the confluence of the three channels (the dynamic interplay), and also on the (potentially) countervailing direct effect of changing the supply of safe assets on the safe asset shortage whereby higher D^s reduces $D_0^d - D_0^s$ directly (in case of private safe asset production) or indirectly by crowding-in private safe assets via B_0. The net effect of these forces depends in part on strength of externalities, and determines marginal effect of safe asset creation on excess demand for safe assets – hence, the nature of the safety trap.

For example, consider a marginal increase in the supply of private safe assets at date 0 D_0^s. What is the effect on the safe asset shortage at date 0? If the risk sharing externality is strong, then this is more likely to cause a large increase in systemic risk through the macroeconomic spillover. This would depress aggregate ex ante, making it more likely that this would exacerbate a safe asset shortage. Moreover, the depressed aggregate demand reduces date 0 output when at the effective lower bound. If the aggregate demand externality is strong, then the fall in output is more likely to have a strong adverse effect on banks' net worth, further increasing systemic risk and reducing aggregate demand. Thus, if the two externalities are stronger (and hence the dynamic interplay between systemic risk and aggregate demand is stronger), then the economy is more likely to be in a risk-intensive safety trap at date 0, which implies very different policy prescriptions from a conventional safety trap.

7.2.4 Allocative distortions

To understand how these externalities distort the equilibrium allocation, consider the problem of a constrained social planner who (in part) aims to allocate consumption optimally across the three dates 1, 2, and 3. Recall that the household can perfectly smooth consumption between dates 1 and 2 due to presence of the storage technology and the absence of uncertainty after date 0. Therefore, the allocation of consumption between dates 1 and 2 will always be efficient (that is, the planner cannot improve on that margin). Therefore, I can agglomerate the households' consumption at dates 1 and 2 into a 'future' period (1,2), and thereby reduce the planner's problem to allocating consumption between date 0 and date (1,2). This problem is affected by two margins: how to the total amount of resources available at date 0 and the allocation of these resources between date 0 and date (1,2).

The risk-sharing externality (i.e., private safe asset production) has two effects on the allocation. First, due to the deadweight loss associated with liquidation in the bad state, it leads to a reduction in the total amount of resources available for consumption across both dates 1 and 2. Second, because of the macroeconomic spillover from liquidation to the household's date 2 labor income, it reallocates the household's losses from the bad state in date 1 (in the form of losses on the investment of capital) to the bad state of date 2 (in the form of lower labor income). The second effect is irrelevant, as the household can perfectly smooth losses between dates 1 and 2 using the storage technology. Therefore, the risk sharing externality ultimately reduces total resources available for consumption at dates (1,2) due to the deadweight loss. Now consider how the aggregate demand externality affects the allocation. The fall in aggregate demand lowers output (when monetary policy is constrained) and therefore lowers the total resources at date 0 available for date 0 consumption or future consumption at date (1,2).

Thus, because of these two externalities, the desire of the household to push consumption into the future (1,2) (that is, the demand for private safe assets) results in lower output at date 0 (aggregate demand externality), and also lower wealth in the bad state of date (1,2) due to the deadweight loss associated with liquidation (excessive systemic risk due to risk sharing externality). Optimal policy will therefore involve distorting private choices – the household's date 0 consumption-saving decision (aggregate demand), and the composition of the household's saving portfolio (risk sharing) – to increase aggregate demand at date 0 and reduce the deadweight loss incurred in the bad state at date 1.

7.2.5 Policy coordination in a safety trap

How should these policies be jointly set? When monetary policy is constrained by the effective lower bound, the equilibrium features two sources of inefficiency: depressed aggregate demand and high systemic risk. But because of the dynamic interplay between these two , policies can only be imperfectly targeted to address each inefficiency – that is, policies which reduce systemic risk have spillover effects to aggregate demand, and vice versa. Therefore, there is, in general, scope to coordinate policies designed to mitigate systemic risk and policies designed for aggregate demand management in order to jointly achieve the desired level of aggregate demand and systemic risk. The extent to which different policies imply a tradeoff depends on nature of the safety trap.

Policy tradeoffs in conventional safety traps First consider the conventional safety trap, which features a shortage of both public and private safe assets. The safety trap features excessive systemic risk and insufficient aggregate demand. Moreover, in a conventional safety trap, private safe asset creation involves a social tradeoff between aggregate demand and systemic risk. (There is no such tradeoff for public safe asset creation, except to the extent that it may crowd-in private safe asset creation further.) In particular, as discussed in the previous section, while macroprudential policy can mitigate systemic risk by restricting the supply of private safe assets, this worsens the safe asset shortage in a conventional safety trap, resulting in, ceteris paribus, even lower aggregate demand and output. Hence, there is a tradeoff to which reduce systemic risk.

On the other hand, fiscal policies designed to stimulate aggregate date 0 demand increase output at date 0, thereby increasing the total resources available for current and future consumption, but, to the extent that they crowd-in private safe asset creation, they may also increase systemic risk, reducing the future wealth of the household due to the deadweight loss and macroeconomic spillover.

Quantitative easing, by contrast, can in principle both stimulate aggregate demand and mitigate systemic risk, as it both reduces the share of private safe assets held by the household and substitutes this with a higher supply of public safe assets. However, recall from the previous section that, there is also a social tradeoff to quantitative easing associated with over-investment. Therefore, in general it is not optimal to use only quantitative easing to address the allocative inefficiencies. As a result, there may be scope for coordination between quantitative easing and macroprudential or fiscal policies: one could use both policies to stimulate aggregate demand, reduce systemic risk, and achieve the optimal level of investment at the same time.

Therefore, due to the social tradeoff between systemic risk and aggregate demand that obtains in a conventional safety trap, when some policies restrict the supply of private safe assets, such as through macroprudential policy, one should should compensate for this with an increase in the supply of public safe assets, either through quantitative easing or fiscal policy. Moreover, insufficient coordination between macroprudential regulation and quantitative easing can lead to demand-driven recessions characterized by excessively high systemic risk.

Policy tradeoffs in risk-intensive safety traps In a risk-intensive safety trap, coordination may not be necessary. In particular, using macroprudential policy may suffice to achieve the constrained social optimum as it can both reduce systemic risk and stimulate aggregate demand, creating a virtuous cycle between the two, facilitating an exit from the trap. By contrast, fiscal policy could be useful only to the extent that they, on net, crowd-out private safe asset creation. Otherwise, these policies would stimulate demand but potentially increase systemic risk. Similarly to conventional safety traps, quantitative easing could improve both aggregate demand and systemic risk, but may be lead to excessive investment at date 0.

8 Conclusion

This paper introduces a theory in which the creation of safe assets generates a two-way interaction between aggregate demand and the risk of future crises. The model highlights the role played by the composition of safe assets between public and private safe assets in the determination of economic activity, systemic risk, and growth. The paper also showed that policy interventions, including monetary, macroprudential, and fiscal policies, can operate through additional channels once one accounts for the interactions between systemic risk and aggregate demand. The paper thus sheds light on the nature of macroeconomic booms and busts, safe asset shortages, and persistent slumps, and provides insight on how a range of policy interventions can influence these episodes.

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Appendices

APPENDIX 1: Proof of Lemma 2: bank behaves as if it is risk-averse.

In this appendix, I show that under Condition 1, although the bank is risk neutral, the convex liquidation cost $\phi(\ell_1)$ introduces a convexity in the bank's payoff function (as a function of D_0). This convexity makes the bank behave at date 0 as if it's risk averse. First, let us define the marginal cost MC and marginal benefit MB of borrowing to the bank from its optimality condition for D_0.

$$MC \equiv E\left[v' r_2^k \left(\phi\left(\ell_1(s)\right)\left(1-\tau_0^D\right)+\tau_0^R R_0\right)\right]+\lambda_1(s_L)\tau_0^R R_0$$

$$MB \equiv \left(1-\tau_0^D\right) E\left[v' r_2^k \left(r_1^k(s)+1\right)\right]+\left(1-\tau_0^D\right)\lambda_1(s_L)\left[r_1^k(s_L)+\ell_1(s_L)\right]$$

To show this, I will show that, although E's marginal benefit MB and marginal cost MC from borrowing are increasing in D_0, at the equilibrium value of D_0 (such that $MB = MC$), the marginal cost is increasing at a faster rate than the marginal benefit. That is, $\frac{\partial MC}{\partial D_0}|_{D_0^*} > \frac{\partial MB}{\partial D_0}|_{D_0^*}$, in partial equilibrium (i.e. taking prices as given). This is because of our assumption that the cost of liquidating capital in the bad state at date 1 is convex, $\phi(\ell_1) \geq 0$, $\phi'(\ell_1) \geq 0$, and $\phi''(\ell_1) \geq 0$, where the inequalities hold strictly for all $\ell_1 > 0$.For this reason, in partial equilibrium (i.e. taking prices as given), the bank can reach an interior optimum for his choice of D_0 – that is, he doesn't necessarily try to maximize borrowing, and so I can have a situation in which his

To show this, first recall from the bank's non-negativity constraint on date 1 investment, and the expression for ℓ_1 in the low state that, I have

$$\ell_1(s) = \frac{\tau_0^R D_0 R_0 - T_1^E}{k_1} - r_1^k(s) = Lev_0 - r_1^k(s) \tag{22}$$

where $Lev_0 := \frac{\tau_0^R D_0 R_0 - T_1^E}{k_1}$. Moreover, from the expression for the Lagrange multiplier, the extent to which the constraint binds depends also on ℓ_1, and hence leverage.

$$\lambda_1(s) = v' r_2^k(s)\phi'\left(\ell_1(s)\right) \tag{23}$$

$$\frac{\partial \lambda_1(\underline{s})}{\partial D_0} = v' r_2^k(\underline{s})\phi''\left(\ell_1(\underline{s})\right)\frac{\partial \ell_1(\underline{s})}{\partial D_0}$$

Under what conditions do I have $\frac{\partial MC}{\partial D_0} > \frac{\partial MB}{\partial D_0}$? First note that, since the bank takes prices as given in making its leverage decision, I are interested in the derivatives $\frac{\partial MC}{\partial D_0}, \frac{\partial MB}{\partial D_0}$ in partial equilibrium, i.e. leaving prices fixed. (While the bank's leverage decision will affect prices in general equilibrium, these effects are not internalized at the margin by the bank. Since here I are interested in characterizing the bank's marginal decisions in partial equilibrium, I hold prices fixed.) Therefore, from the definitions of MC and MB above, I have

$$\frac{\partial MC}{\partial D_0} = \left[\pi(\underline{s})\left(1-\tau_0^D\right)\phi'\left(\ell_1(\underline{s})\right) + \tau_0^R R_0 \phi''\left(\ell_1(\underline{s})\right)\right] v' r_2^k(\underline{s}) \frac{\partial \ell_1(\underline{s})}{\partial D_0}$$

And

$$\frac{\partial MB}{\partial D_0} = \left(1-\tau_0^D\right)\left[r_1^k(s_L) + \ell_1(s_L)\right] v' r_2^k(\underline{s}) \phi''\left(\ell_1(\underline{s})\right)\frac{\partial \ell_1(\underline{s})}{\partial D_0} + \left(1-\tau_0^D\right)\lambda_1(s_L)\frac{\partial \ell_1(\underline{s})}{\partial D_0}$$

Imposing the equilibrium result that $\underline{s} = s_L$.

$$= \left(1-\tau_0^D\right)\left[\left(r_1^k(s_L) + \ell_1(s_L)\right)\phi''\left(\ell_1(s_L)\right) + \phi'\left(\ell_1(s_L)\right)\right] v' r_2^k(s_L) \frac{\partial \ell_1(\underline{s})}{\partial D_0}$$

Therefore, $\frac{\partial MC}{\partial D_0} > \frac{\partial MB}{\partial D_0}$ iff

$$\left[\pi(\underline{s})\left(1-\tau_0^D\right)\phi'\left(\ell_1(\underline{s})\right) + \tau_0^R R_0 \phi''\left(\ell_1(s_L)\right)\right] v' r_2^k(s_L) \frac{\partial \ell_1(s_L)}{\partial D_0} > \left[\left(r_1^k(s_L) + \ell_1(s_L)\right)\phi''\left(\ell_1(s_L)\right) + \phi'\left(\ell_1(s_L)\right)\right]\left(1-\right.$$

$$0 > \left[\left(1-\tau_0^D\right)\left(r_1^k(s_L) + \ell_1(s_L)\right) - \tau_0^R R_0\right]\phi''\left(\ell_1(s_L)\right) + \left(1-\pi(s_L)\right)\left(1-\tau_0^D\right)\phi'\left(\ell_1(s_L)\right) \tag{24}$$

What is the sign of $\left(1-\tau_0^D\right)\left(r_1^k(s_L) + \ell_1(s_L)\right) - \tau_0^R R_0$ when evaluated at the equilibrium D_0^*? Recall that the non-negativity constraint on date 1 investment binds in the bad state, so that

$$r_1^k(s_L) + \ell_1(s_L) = \tau_0^R R_0 \frac{D_0}{k_1} - \frac{T_1^E}{k_1}$$

So I can write $\left(1-\tau_0^D\right)\left(r_1^k(s_L) + \ell_1(s_L)\right) - \tau_0^R R_0$ as

$$\left(1-\tau_0^D\right)\left(r_1^k(s_L) + \ell_1(s_L)\right) - \tau_0^R R_0 = \left(1-\tau_0^D\right)\left(\tau_0^R R_0 \frac{D_0}{k_1} - \frac{T_1^E}{k_1}\right) - \tau_0^R R_0$$

As long as T_0^E and T_1^E are not significantly negative (indeed, I will assume they are both weakly positive), I have that $\left(1-\tau_0^D\right)\left(\tau_0^R R_0 \frac{D_0}{k_1} - \frac{T_1^E}{k_1}\right) - \tau_0^R R_0 < 0$. To see this

$$\left(1-\tau_0^D\right)\left(\tau_0^R R_0 \frac{D_0}{k_1} - \frac{T_1^E}{k_1}\right) <? \tau_0^R R_0$$

Note that $\left(1-\tau_0^D\right) D_0 - k_1$ can be expressed as

$$\left(1-\tau_0^D\right) D_0 - k_1 = -\left(r_0^k + 1\right) k_0 - T_0^E$$

Replace this in the above inequality

$$-\left[\left(r_0^k+1\right) k_0 + T_0^E\right] \tau_0^R R_0 <? \left(1-\tau_0^D\right) T_1^E$$

This holds as long as as T_0^E and T_1^E are not significantly negative. (A sufficient condition is that $T_0^E, T_1^E \geq 0$.) Thus the sign of $\left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right) - \tau_0^R R_0$ is $\left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right) - \tau_0^R R_0 < 0$.

Therefore, I can write condition (10) as

$$\left[\tau_0^R R_0 - \left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right)\right] \phi''\left(\ell_1(s_L)\right) > \left(1-\pi(s_L)\right)\left(1-\tau_0^D\right)\phi'\left(\ell_1(s_L)\right) \tag{25}$$

where both the right-hand and left-hand sides are strictly positive. I can rewrite this condition as

$$\frac{\phi''\left(\ell_1(s_L)\right)}{\phi'\left(\ell_1(s_L)\right)} > \left(1-\pi(s_L)\right) \frac{\left(1-\tau_0^D\right)}{\left[\tau_0^R R_0 - \left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right)\right]} \tag{26}$$

Thus, I have that the banks behaves as if it is risk averse ($\frac{\partial MC}{\partial D_0} > \frac{\partial MB}{\partial D_0}$) if and only if this condition holds. To interpret this condition, there are three terms which affect it. First, if the liquidation cost function is sufficiently convex (so that ϕ'' is sufficiently large relative to ϕ') then this condition is more likely to hold. This is is because then, at the margin, higher leverage will be associated with a higher liquidation cost. Second, if the bank's losses $\tau_0^R R_0 - r_1^k(s_L) - \ell_1(s_L)$ (i.e. the difference between its repayment and its revenue plus liquidation value) is sufficiently high, then this condition is more likely to hold. This is again because higher losses in the bad state make the cost of borrowing larger at the margin. And third, if the probability of the bad state $\pi(\underline{s})$ is sufficiently high, then this condition is more likely to hold. This is because the bank incurs losses in the bad state, making borrowing more costly.

Let us break down this condition further. Since I have $\phi\left(\ell_1\right) = \ell_1^\eta$ and $\phi'\left(\ell_1\right) = \eta \ell_1^{\eta-1}$ and $\phi''\left(\ell_1\right) = \eta\left(\eta-1\right)\ell_1^{\eta-2}$, where $\eta > 1$, this can condition can be written as

$$\phi''\left(\ell_1(s_L)\right)\left[\tau_0^R R_0 - \left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right)\right] > \left(1-\pi(s_L)\right)\left(1-\tau_0^D\right)\phi'\left(\ell_1(s_L)\right) \tag{27}$$

$$\ell_1(s_L) < \left[\tau_0^R R_0 - \left(1-\tau_0^D\right) r_1^k(s_L)\right] \frac{(\eta-1)}{\left(\eta-\pi(s_L)\right)\left(1-\tau_0^D\right)} \tag{28}$$

And recall that $\ell_1(s_L) = Lev_0 - r_1^k(s)$. So this condition is

$$Lev_0 < \left[\tau_0^R R_0 - \left(1-\tau_0^D\right) r_1^k(s_L)\right] \frac{(\eta-1)}{(\eta-\pi(s_L))\left(1-\tau_0^D\right)} + r_1^k(s) \tag{29}$$

where the right-hand side is strictly positive. Thus, as long as, in equilibrium, Lev_0 is sufficiently small then I have $\frac{\partial MC}{\partial D_0} > \frac{\partial MB}{\partial D_0}$. In that case, a marginally higher D_0 will make MC higher than MB.

Therefore, I henceforth assume the following condition holds in equilibrium.

Condition 1:
A) $\frac{\phi''(\ell_1(s_L))}{\phi'(\ell_1(s_L))} > (1-\pi(s_L)) \frac{\left(1-\tau_0^D\right)}{\left[\tau_0^R R_0 - \left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right)\right]}$
B) $T_0^E, T_1^E \geq 0$

As I show in Appendix 3, this condition ensures that the bank's expected date 1 losses from borrowing at date 0 are sufficiently high that the bank behaves at date 0 as if it is risk-averse. The condition is benign and amounts to saying that the liquidation cost function $\phi(\cdot)$ is sufficiently convex, the probability of the bad state is sufficiently high, and the bank's losses in bad state are sufficiently high. Moreover, note that $\frac{\phi''(\ell_1(s_L))}{\phi'(\ell_1(s_L))} = \frac{(\eta-1)\eta\ell_1^{\eta-2}}{\eta\ell_1^{\eta-1}} = \frac{(\eta-1)\ell_1}{\ell_1^2} = \frac{(\eta-1)}{\ell_1}$. Therefore, since the only restriction on η is that $\eta > 1$, it is otherwise a free parameter which can always make sufficiently large that this condition holds.

APPENDIX 2: Risk premium

Define the risk premium at date 0 RP_0 as

$$RP_0 := \left(1-\tau_0^D\right)\left(1 + \frac{E\left[r_2^k(s) r_1^k(s)\right]}{E\left[r_2^k(s)\right]}\right) - \tau_0^R R_0 > 0$$

Note that, since the bank consumes only at date 2, the net expected discounted rate of return on capital is defined as the consumption units it is expected to provide at date 2, $E\left[r_2^k(s) r_1^k(s)\right]$, and is then discounted by the bank's date 1 discount factor $E\left[r_2^k(s)\right]$ to reflect the date 1 value of this expected consumption, yielding the net expected discounted return $\frac{E\left[r_2^k(s) r_1^k(s)\right]}{E\left[r_2^k(s)\right]}$. This is the rate of return the bank takes into account when is making its date 0 investment decision.

(At date 1, the rental rate of capital is $r_1^k(s)$. This return can be reinvested in capital at date 1, yielding $r_2^k(s)$ units of consumption for the bank at date 2. Hence, the date 2 consumption units that the bank can expect to consume for each unit of capital is $E\left[r_2^k(s) r_1^k(s)\right]$. At date 0, the bank's expected discount rate at date 1 is $E\left[r_2^k(s)\right]$. Note that the net expected discounted rate of return on capital can alternatively be expressed as $E\left[r_1^k(s)\right] + \frac{Cov\left(r_2^k, r_1^k\right)}{E\left[r_2^k(s)\right]}$. Note also that if the rental rates of capital at dates 1 and 2 Ire orthogonal (i.e. if $Cov\left(r_2^k(s), r_1^k(s)\right) = 0$), and if there were no distortionary taxes, then our expression for the risk premium would reduce to $1+E\left[r_1^k(s)\right] - R_0$. But the rental rates are not orthogonal due to the endogenous accumulation of capital between dates 1 and 2. That is, if $r_1^k(s)$ is low in state s due to a high date 1 capital stock $k_1(s)$, then the date 2 capital stock $k_2(s)$ will likely also be high, which means that $r_2^k(s)$ will likely be low as Ill.)

The risk premium can identically be expressed as a function of the bank's expected liquidation cost due to the binding non-negativity constraint on date 1 investment. To see this, simply rearrange the bank's FOC for D_0.

$$\underbrace{\left(1-\tau_0^D\right)\left\{E\left[v' r_2^k\left(r_1^k(s)+1\right)\right] + \lambda_1(s_L)\left[r_1^k(s_L)+\ell_1(s_L)\right]\right\}}_{marginal\ benefit} = \underbrace{E\left[v' r_2^k \phi\left(\ell_1(s)\right)\left(1-\tau_0^D\right)\right] + E\left[v' r_2^k \tau_0^R R_0\right] + \lambda_1(s_L)}_{marginal\ cost} \tag{30}$$

$$\left(1-\tau_0^D\right)\left(\frac{E\left[r_2^k r_1^k\right]}{E\left[r_2^k\right]}+1\right) - \tau_0^R R_0 = \left(1-\tau_0^D\right)\frac{E\left[r_2^k \phi\left(\ell_1(s)\right)\right]}{E\left[r_2^k\right]} + \lambda_1(s_L)\frac{\left[\tau_0^R R_0 - \left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right)\right]}{E\left[r_2^k\right]} \tag{31}$$

The left-hand side of the equation is the definition of the risk premium, while the right-hand side is a function of the bank's expected liquidation cost.

$$RP_0 = \left(1-\tau_0^D\right)\frac{E\left[r_2^k \phi\left(\ell_1(s)\right)\right]}{E\left[r_2^k\right]} + \lambda_1(s_L)\frac{\left[\tau_0^R R_0 - \left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right)\right]}{E\left[r_2^k\right]} \tag{32}$$

Adjusted risk premium I can define a notion of the risk premium adjusted for the distortionary taxes τ_0^D, τ_0^R as follows. Define the adjusted risk-free rate as

$$R_0^{D,adj} := \left(\frac{\tau_0^R}{1-\tau_0^D}\right) R_0$$

From the bank's FOC for D_0, this implies

$$R_0^{D,adj} = \frac{\overbrace{E\left[v' r_2^k\left(r_1^k(s)+1-\phi\left(\ell_1(s)\right)\right)\right]}^{E's\ return\ from\ capital} + \overbrace{\lambda_1(s_L)\left[r_1^k(s_L)+\ell_1(s_L)\right]}^{shadow\ price\ in\ crisis\ state}}{\lambda_1(s_L)+E\left[v' r_2^k\right]}$$

And define the adjusted risk premium as

$$RP_0^{adj} := \frac{RP_0}{1-\tau_0^D}$$

Then it follows from the definition of RP_0 that

$$RP_0 := \left(1-\tau_0^D\right)\left(1+\frac{E\left[r_2^k(s) r_1^k(s)\right]}{E\left[r_2^k(s)\right]}\right)-\tau_0^R R_0$$

$$RP_0 = \left(1-\tau_0^D\right)\left[1+\frac{E\left[r_2^k r_1^k\right]}{E\left[r_2^k\right]}\right]-\left(1-\tau_0^D\right) R_0^{D,adj}$$

$$RP_0 = \left(1-\tau_0^D\right)\left[1+\frac{E\left[r_2^k r_1^k\right]}{E\left[r_2^k\right]}-R_0^{D,adj}\right]$$

And so

$$RP_0^{adj} = 1+\frac{E\left[r_2^k r_1^k\right]}{E\left[r_2^k\right]}-R_0^{D,adj}$$

I can then rewrite the bank's FOC for D_0 to express RP_0^{adj} as a function of the bank's expected liquidation cost.

$$RP_0 = \left(1-\tau_0^D\right)\frac{E\left[r_2^k \phi\left(\ell_1(s)\right)\right]}{E\left[r_2^k\right]}+\lambda_1(s_L)\frac{\left[\tau_0^R R_0-\left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right)\right]}{E\left[r_2^k\right]} \tag{33}$$

$$RP_0^{adj} = \frac{E\left[r_2^k \phi\left(\ell_1(s)\right)\right]}{E\left[r_2^k\right]}+\lambda_1(s_L)\frac{\left[\tau_0^R R_0-\left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right)\right]}{\left(1-\tau_0^D\right)E\left[r_2^k\right]} \tag{34}$$

Risk premium when the non-negativity constraint on date 1 investment never binds Suppose the bank's non-negativity constraint on date 1 investment never binds at date 1. Then the bank's FOC for D_0 would be

$$E\left[v' r_2^k \frac{dk_2(s)}{dD_0}\right]=0 \tag{35}$$

where $\frac{dk_2(s)}{dD_0}=\left[r_1^k(s)+1-\phi\left(\ell_1(s)\right)\right]\left(1-\tau_0^D\right)-\tau_0^R R_0=\left(r_1^k(s)+1\right)\left(1-\tau_0^D\right)-\tau_0^R R_0$. This would reduce to

$$E\left[r_2^k\left(\left(r_1^k(s)+1\right)\left(1-\tau_0^D\right)-\tau_0^R R_0\right)\right]=0 \tag{36}$$

$$\left(1-\tau_0^D\right)E\left[r_2^k\left(r_1^k(s)+1\right)\right]=\tau_0^R R_0 E\left[r_2^k\right] \tag{37}$$

i.e.

$$\left(1-\tau_0^D\right)\left\{E\left[r_2^k r_1^k\right]+E\left[r_2^k\right]\right\}=\tau_0^R R_0 E\left[r_2^k\right] \tag{38}$$

$$\left(1-\tau_0^D\right)\left\{\frac{E\left[r_2^k(s) r_1^k(s)\right]}{E\left[r_2^k(s)\right]}+1\right\}=\tau_0^R R_0 \tag{39}$$

Thus, the risk premium RP_0 would be 0 if and only if $\ell_1(s) = 0$ for all s. Moreover, since $\ell_1(s), \lambda_1(s) = 0$, the adjusted risk premium satisfies $RP_0^{adj} = 0$ if and only if $\ell_1(s) =$ for all s.

APPENDIX 3: Proof that Channel 2 is strictly positive

Proof that show both Channels 1 and 2 >0: $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)} > 0$ and $\frac{d\ell_1(s_L)}{dD_0^s} > 0$, under assumptions that lump-sum transfers are sufficiently small, and assumption that $\frac{k_2^G}{\tilde{k}_2} < \frac{1}{\bar{n}}$. Start with Channel 2:

CLAIM: $\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} > 0$

PROOF:

$$\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})}$$

I proceed by showing first that $\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} > 0$, and then that $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} > 0$.

Claim: $\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} > 0$

Proof:

Note that

$$\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} = -\frac{\pi(\underline{s})}{(c_1(\underline{s}))^2} \frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})}$$

where

$$c_1(s) = c_2(s) = \frac{1}{2}\left[(w_1 + w_2)\bar{n} + R_0(D_0 + B_0) - T_1 - T_2\right]$$

So

$$\frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{\bar{n}}{2}\left(\frac{\partial w_1(\underline{s})}{\partial \ell_1(\underline{s})} + \frac{\partial w_2(\underline{s})}{\partial \ell_1(\underline{s})} - \frac{\partial T_1}{\partial \ell_1(\underline{s})} - \frac{\partial T_2}{\partial \ell_1(\underline{s})}\right)$$

Note that

$$w_1 = (1-\alpha)\frac{y_1}{\bar{n}} \tag{40}$$

$$w_2 = (1-\alpha)\frac{y_2}{\bar{n}} \tag{41}$$

So

$$\frac{\partial w_1(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{(1-\alpha)}{\bar{n}} \frac{\partial y_1(\underline{s})}{\partial \ell_1(\underline{s})} = 0$$

and

$$\frac{\partial w_2(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{(1-\alpha)}{\bar{n}} \frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})}$$

And since

$$y_2 = z_2 \left(\tilde{k}_2\right)^{\alpha} \bar{n}^{1-\alpha} \tag{42}$$

where $\tilde{k}_2 = k_2 + k_2^G$, then

$$\frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{\partial y_2(\underline{s})}{\partial \tilde{k}_2(\underline{s})} \frac{\partial \tilde{k}_2(\underline{s})}{\partial k_2(\underline{s})} \frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})} = \alpha \frac{y_2}{\tilde{k}_2} \frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})}$$

and $k_2(s) = \left[1 + r_1^k(s) - \phi\left(\ell_1(s)\right)\right] k_1(D_0) - \tau_0^R D_0 R_0 + T_1^E$ so

$$\frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})} = -k_1(\underline{s})\phi' = -\eta k_1(\underline{s}) \ell_1^{\eta-1}(\underline{s}) < 0$$

The last inequality follows from the fact that $\ell_1(\underline{s}) > 0$ in the bad state.

Note also that

$$T_1 = R_0^B B_0 - \tau_1 + T_1^E - r_1^k(s) k_1^G \tag{43}$$

$$T_2 = T_2^E - \tau_2 - r_2^k k_2^G \tag{44}$$

So

$$\frac{\partial T_1}{\partial \ell_1(\underline{s})} = -k_1^G \frac{\partial r_1^k(\underline{s})}{\partial \ell_1(\underline{s})} = 0$$

where the last equality holds since $r_1^k = \alpha \frac{y_1}{\tilde{k}_1}$ and $\frac{\partial \tilde{k}_1}{\partial \ell_1(\underline{s})} = 0$ since $\tilde{k}_1 = k_1^G + k_1$ and $k_1 = D_0 + T_0^E + \left(r_0^k + 1\right) k_0$. Moreover

$$\frac{\partial T_1}{\partial \ell_1(\underline{s})} = -k_2^G \frac{\partial r_2^k(\underline{s})}{\partial \ell_1(\underline{s})}$$

Note that, since $r_2^k = \alpha \frac{y_2}{k_2 + k_2^G}$,

$$\frac{\partial r_2^k}{\partial \ell_1(\underline{s})} = \frac{\alpha}{k_2 + k_2^G} \frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})} - \alpha \frac{y_2}{\left(k_2 + k_2^G\right)^2} \frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})}$$

And since $k_2(s) = \left[1 + r_1^k(s) - \phi\left(\ell_1(s)\right)\right] k_1(D_0) - \tau_0^R D_0 R_0 + T_1^E$, I have

$$\frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})} = -k_1 \phi'\left(\ell_1(s)\right)$$

So

$$\frac{\partial r_2^k}{\partial \ell_1(\underline{s})} = \frac{\alpha}{k_2 + k_2^G} \frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})} + \alpha \frac{y_2}{\left(k_2 + k_2^G\right)^2} k_1 \phi'\left(\ell_1(s)\right)$$

Plugging these expressions into the equation of $\frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})}$.

$$\frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{\bar{n}}{2}\left(\frac{(1-\alpha)}{\bar{n}} \frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})} + k_2^G \frac{\alpha}{k_2 + k_2^G} \frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})} + k_2^G \alpha \frac{y_2}{\left(k_2 + k_2^G\right)^2} k_1 \phi'\left(\ell_1(s)\right)\right)$$

and since I showed that $\frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})} = \alpha \frac{y_2}{\tilde{k}_2} \frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})}$ and $\frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})} = -k_1(\underline{s})\phi' < 0$, this is

$$= \frac{\bar{n}}{2}\left(-\left[\frac{(1-\alpha)}{\bar{n}} + k_2^G \frac{\alpha}{k_2 + k_2^G}\right] \alpha \frac{y_2}{\tilde{k}_2} k_1(\underline{s})\phi' + k_2^G \alpha \frac{y_2}{\left(\tilde{k}_2\right)^2} k_1(\underline{s})\phi'\right)$$

Thus, a rise in liquidation in the bad state causes future consumption to fall through a decrease in the wage, but also pushes up consumption through a possible rise in the rental rate of capital. In what follows, I find a condition which ensures that this latter effect does not dominate. I have $\frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})} < 0$ if and only if

$$-\left[\frac{(1-\alpha)}{\bar{n}} + k_2^G \frac{\alpha}{k_2 + k_2^G}\right] \alpha \frac{y_2}{\tilde{k}_2} k_1(\underline{s})\phi' + k_2^G \alpha \frac{y_2}{\left(\tilde{k}_2\right)^2} k_1(\underline{s})\phi' < 0$$

$$\frac{k_2^G}{\tilde{k}_2} < \frac{1}{\bar{n}}$$

which holds if labor supply is sufficiently small or government's share of date 2 capital stock is sufficiently small.

Suppose then that $\frac{k_2^G}{\tilde{k}_2} < \frac{1}{\bar{n}}$. Then it follows that $\frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})} < 0$. Therefore,

$$\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} = -\frac{\pi(\underline{s})}{\left(c_1(\underline{s})\right)^2} \frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})} > 0$$

Q.E.D.

Claim: $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} > 0$. Recall the household's demand function $D_0^d(R_0, B_0) = e_0 - T_0 + d_0^F + w_0\bar{n} - B_0 - \frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-1}$. This implies that $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} = \frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-2} > 0$

Thus, since both $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} > 0$ and $\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} > 0$, I have

$$\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} > 0$$

$$\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} = \frac{1}{R_0\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^2} \frac{\pi(\underline{s})}{(c_1(\underline{s}))^2} \frac{\bar{n}}{2} \frac{(1-\alpha)}{\bar{n}} \alpha \frac{y_2}{\tilde{k}_2} \eta k_1(\underline{s}) \ell_1^{\eta-1}(\underline{s}) > 0$$

Thus Channel 2: $\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} > 0$. Q.E.D.

To summarize, I have

$$\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} = -\frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-2} \frac{\pi(\underline{s})}{(c_1(\underline{s}))^2} \frac{\bar{n}}{2}\left(-\left[\frac{(1-\alpha)}{\bar{n}} + k_2^G \frac{\alpha}{k_2 + k_2^G}\right] \alpha \frac{y_2}{\tilde{k}_2} k_1(\underline{s}) \phi' + k_2^G \alpha \frac{y_2}{\left(\tilde{k}_2\right)^2} k_1(\underline{s}) \phi'\right) > 0$$

APPENDIX 4: Proof that Channel 1 is strictly positive

Now consider Channel 1, the effect of date 0 borrowing on the severity of crises $\frac{d\ell_1(s_L)}{dD_0^s}$. To a first order approximation, the partial equilibrium effect of date 0 saving on the severity of a crisis, conditional on a crisis occurring at date 1, can be summarized by the derivative of liquidation in the bad state with respect to date 0 private debt, $\frac{d\ell_1(s_L)}{dD_0}$. Here, I show that in equilibrium I have $\frac{d\ell_1(s)}{dD_0} > 0$ if and only if $\ell_1(s) > 0$, and $\frac{d\ell_1(s)}{dD_0} = 0$ otherwise – that is, higher date 0 borrowing increases the severity of a crises, conditional on a crisis occurring.

CLAIM: Channel 1: As long as lump-sum transfers are sufficiently small, I have $\frac{d\ell_1(s_L)}{dD_0} > 0$ and $\frac{d\ell_1(s_H)}{dD_0} = 0$.

PROOF: I already showed that, in the good state $s = s_H$, I are in the normal regime so that $\frac{d\ell_1(s_H)}{dD_0} = 0$. In the bad state $s = s_L$, the variable $\ell_1(s)$ is pinned down by the binding non-negativity constraint on date 1 investment.

$$\ell_1(s) = \frac{\tau_0^R D_0 R_0}{k_1} - r_1^k(s) - \frac{T_1^E}{k_1}$$

Recall the law of motion for k_1.

$$k_1 = i_0 + k_0 - k_1^{QE} \tag{45}$$

Replacing i_0 with the bank's binding date 0 budget constraint yields

$$k_1 = \frac{D_0}{P_0} + \left(r_0^k + 1\right) k_0 + T_0^E \tag{46}$$

So

$$\frac{dk_1}{dD_0} = \frac{1}{P_0} = 1$$

Therefore, I can express the derivative $\frac{d\ell_1(s)}{dD_0}$ as

$$\frac{d\ell_1(s)}{dD_0} = \frac{\tau_0^R R_0}{k_1} - \frac{dr_1^k(s)}{dD_0} + \frac{T_1^E}{(k_1)^2} - \frac{\tau_0^R D_0 R_0}{(k_1)^2}$$

Recall

$$r_1^k = \alpha \frac{y_1}{\tilde{k}_1} = \alpha z_1 \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha} \tag{47}$$

So

$$\frac{dr_1^k(s)}{dD_0} = \alpha(\alpha-1) z_1 \left(\tilde{k}_1\right)^{\alpha-2} \bar{n}^{1-\alpha} < 0$$

Since $\frac{dr_1^k(s)}{dD_0} < 0$, a sufficient condition for $\frac{d\ell_1(s)}{dD_0} > 0$ is

$$\frac{\tau_0^R R_0}{k_1} + \frac{T_1^E}{(k_1)^2} - \frac{\tau_0^R D_0 R_0}{(k_1)^2} > 0$$

i.e.

$$\tau_0^R R_0 (k_1 - D_0) > T_1^E$$

Henceforth, I assume that T_1^E, which for now I take as exogenous, satisfies this condition. (Note that if I set transfers to $T_1^E = 0$, then this condition would reduce to

$$k_1 > D_0$$

which holds because of net worth.)

$$k_1 = \frac{D_0}{P_0} + \left(r_0^k + 1\right) k_0 + T_0^E > D_0 \tag{48}$$

Thus, as long as T_1^E is sufficiently small, I have $\frac{d\ell_1(s)}{dD_0} > 0$ in the bad state.

Thus, date 0 saving increases the severity of crises when they occur. I show in Appendix 4 that, as long as lump-sum transfers are sufficiently small, I have $\frac{d\ell_1(s_L)}{dD_0} > 0$ and $\frac{d\ell_1(s_H)}{dD_0} = 0$. That is, the fraction of the bank's capital which is liquidated in the bad state is increasing (to a first order approximation) in initial saving D_0. This is because higher D_0 (holding all else constant(?)) increases the bank's date 0 leverage Lev_0, which implies that the bank's losses in the bad state (when its rental income is low) are larger. As a result, the bank is forced to liquidate a higher fraction of its capital holdings in order to repay its debt.

Thus, higher date 0 saving increases systemic risk (in partial equilibrium) by increasing the bank's leverage, thereby exacerbating the severity of crises in the bad state at date 1.

APPENDIX 5: Effect of crisis risk on precautionary saving

Below, I show that $\frac{\partial D_0^d}{\partial \ell_1(\mathrm{s})} > 0$.

First, let's reduce the dynamic system of equations a bit. Note that I can get rid of both c_1 and B_1. The first order condition for B_1 implies $c_1 = c_2$. And from the expressions for c_1 and c_2, I can compute the sum of c_1 and c_2 as

$$c_1(s) + c_2(s) = B_1(s) - T_2 + w_2\bar{n} + R_0 D_0 + R_0^B B_0 - T_1 + w_1\bar{n} - B_1(s)$$

$$= (w_1 + w_2)\,\bar{n} + R_0\,(D_0 + B_0) - T_1 - T_2$$

And since $c_1 = c_2$, this means I can ignore both c_1 and B_1, and capture how shocks affect total consumption after date 0:

$$c_2(s) = \frac{1}{2}\left[(w_1 + w_2)\,\bar{n} + R_0\,(D_0 + B_0) - T_1 - T_2\right]$$

Intuitively, the household uses the storage technology $B_1(s)$ (with $R_1^B = 1$) to fully smooth consumption between dates 1 and 2. So I only need to keep track of the household's total income at dates 1 and 2 - its divided optimally via $B_1(s)$.

Recall the household's FOC for D_0^d:

$$\frac{1}{c_0} = R_0 E_0 \left[\frac{1}{c_1(s)}\right] \tag{49}$$

Plug this into the household's date 0 budget constraint:

$$c_0 + D_0^d + B_0 = e_0 - T_0 + d_0^F + w_0 \bar{n} \tag{50}$$

$$D_0^d = e_0 - T_0 + w_0 \bar{n} - B_0 - \frac{1}{R_0 E_0 \left[\frac{1}{c_1(s)}\right]} \tag{51}$$

So

$$\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} = \frac{1}{R_0 \left(E_0 \left[\frac{1}{c_1(s)}\right]\right)^2} \frac{\partial E_0 \left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})}$$

where

$$\frac{\partial E_0 \left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} = -\frac{\pi(\underline{s})}{(c_1(\underline{s}))^2} \frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})}$$

where

$$c_1(s) = c_2(s) = \frac{1}{2}\left[(w_1 + w_2)\bar{n} + R_0 (D_0 + B_0) - T_1 - T_2\right]$$

So

$$\frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{\bar{n}}{2}\left(\frac{\partial w_1(\underline{s})}{\partial \ell_1(\underline{s})} + \frac{\partial w_2(\underline{s})}{\partial \ell_1(\underline{s})}\right)$$

Note that

$$w_1 = (1-\alpha)\frac{y_1}{\bar{n}} \tag{52}$$

$$w_2 = (1-\alpha)\frac{y_2}{\bar{n}} \tag{53}$$

So

$$\frac{\partial w_1(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{(1-\alpha)}{\bar{n}} \frac{\partial y_1(\underline{s})}{\partial \ell_1(\underline{s})} = 0$$

and

$$\frac{\partial w_2(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{(1-\alpha)}{\bar{n}} \frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})}$$

And since

$$y_2 = z_2 \left(\tilde{k}_2\right)^{\alpha} \bar{n}^{1-\alpha} \tag{54}$$

where $\tilde{k}_2 = k_2 + k_2^G$, then

$$\frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{\partial y_2(\underline{s})}{\partial \tilde{k}_2(\underline{s})} \frac{\partial \tilde{k}_2(\underline{s})}{\partial k_2(\underline{s})} \frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})} = \alpha \frac{y_2}{\tilde{k}_2} \frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})}$$

and $k_2(s) = i_1 + (1 - \phi(\ell_1(s)))\, k_1(s)$, where $i_1 = 0$ in the bad state, so

$$\frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})} = -k_1(\underline{s})\phi' = -\eta k_1(\underline{s}) \ell_1^{\eta-1}(\underline{s}) < 0$$

The last inequality follows from the fact that $\ell_1(\underline{s}) > 0$ in the bad state. Plugging this into our expression for $\frac{\partial D_0^d}{\partial \ell_1(\underline{s})}$ yields

$$\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} = \frac{1}{R_0\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^2} \frac{\pi(\underline{s})}{(c_1(\underline{s}))^2} \frac{\bar{n}}{2} \frac{(1-\alpha)}{\bar{n}} \alpha \frac{y_2}{\tilde{k}_2} \eta k_1(\underline{s}) \ell_1^{\eta-1}(\underline{s}) > 0$$

APPENDIX 6: Effect of public debt supply on the competitive equilibrium

In order to compare the role of public versus private debt, I now study the effect of an increase in the supply of public debt B_0 on aggregate demand. A characterization of this effect reveals that, in contrast to private debt, public debt has no direct effect on systemic risk (i.e. the severity of crises).

To evaluate the effects of public debt issuance requires that I make assumptions about what the government does with the proceeds. In what follows, I make the following assumptions about the government's behavior.

Assumptions about government's behavior

i. The government invests any additional proceeds from borrowing in its capital holdings k_1^G (via quantitative easing), so that $\frac{\partial k_1^G}{\partial B_0} = 1$.

ii. The government keeps its holdings of capital constant across time, so that $k_2^G = k_1^G$.

iii. At date 1, any government revenue net of interest payments is transferred in lump-sum fashion to the bank, so that $T_1^E = r_1^k(s) k_1^G - R_0^B B_0$. Therefore, the government's date 1 budget constraint implies $T_1 = -\tau_1$.

iv. At date 2, the government's proceeds from renting its capital holdings $r_2^k k_2^G$ are paid in lump-sum fashion to the bank, so that $T_2^E = r_2^k(s)k_2^G$. The government's date 2 budget constraint then implies $T_2 = 0$.

As with private debt, I focus on the effects of public debt supply on aggregate demand, and for now abstract from general feedback effects from the change in aggregate demand reflected in Channel 3 via $\frac{dD_0^s}{dD_0^d}$ and changes in R_0 and u_0. To characterize the effect of D_0^s on aggregate demand, I can take the derivative of the household's demand function $D_0^d(R_0, B_0; u_0)$, given in equation (7), with respect to B_0.

$$\frac{dD_0^d}{dB_0} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{dE_0\left[\frac{1}{c_1(s)}\right]}{dB_0}$$

As with private debt, this can be decomposed into two terms, one capturing the role of systemic risk and the other excluding it.

$$\frac{dD_0^d}{dB_0} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \left[\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0} + \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)} \frac{d\ell_1(s_L)}{dB_0}\right]$$

However, public debt B_0 has no effect on aggregate demand through systemic risk (that is $\frac{d\ell_1(s_L)}{dB_0} = 0$), except through general equilibrium feedback effects. Since I set aside such effects here, this implies that the second term is 0.

To see this, recall the expression (11) for $\ell_1(s)$

$$\ell_1(s) = \frac{\tau_0^R D_0 R_0}{k_1} - r_1^k(s) - \frac{T_1^E}{k_1}$$

Since, abstracting from general equilibrium effects through u_0 and R_0, I have $\frac{dk_1}{dB_0}, \frac{dT_1^E}{dB_0}, \frac{dr_1^k(s_L)}{dB_0} = 0$. Recall that, although in general ℓ_1 may respond to B_0 through the equilibrium value of D_0, this effect is a general equilibrium effect which is determined only by the general equilibrium response of u_0 or R_0. As discussed above, I abstract from these GE effects (Channel 3) here, and return to them later.

Therefore $\frac{d\ell_1(s_L)}{dB_0} = 0$ when holding u_0 constant (that its, when ignoring GE feedback channel). Hence Channel 1 is not active for public debt. So

$$\frac{dD_0^d}{dB_0} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0}$$

Since Channel 1 not active, neither is Channel 2 (that is, public debt has no effect on aggregate demand through either Channels 1 or 2). Hence, systemic risk doesn't play a role, and so neither does dynamic interaction, in the case of public debt.

Thus, I can now see how public and private debt differ. Compare the effect of the supply of either type of debt on aggregate demand:

$$\frac{dD_0^d}{dD_0^s} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial D_0^s} + \overbrace{\underbrace{\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}}_{dynamic\ interaction\ from\ SR\ to\ AD}}^{Channel\ 2}\overbrace{\frac{d\ell_1(s_L)}{dD_0^s}}^{Channel\ 1}$$

versus

$$\frac{dD_0^d}{dB_0} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0}$$

In Appendix 6, I show that these expressions imply that $\frac{dD_0^d}{dD_0^s} > \frac{dD_0^d}{dB_0}$; that is, an increase in the supply of private debt unambiguously reduces aggregate demand (i.e. increases demand for saving) by more than an increase in the supply of public debt does.

Hence, $\frac{dD_0^d}{dD_0^s}$ and $\frac{dD_0^d}{dB_0}$ differ only in the Channel 1: role played by systemic risk. This indicates that supply of public debt does not affect equilibrium via systemic risk. Only affects via other effect on $E\left[\frac{1}{c_1(s)}\right]$ (investment, consumption), which in turn has effect on demand for saving.

APPENDIX 7: General equilibrium feedback channels

First, in order to establish the relationship between the two GE channels, $\frac{\partial \ell_1(\mathrm{s})}{\partial u_0}$ and $\frac{\partial D_0}{\partial u_0}$, let's derive an expression for $\frac{\partial \ell_1(\mathrm{s})}{\partial u_0}$.

Recall that $\ell_1(s) = Lev_0 - r_1^k(s)$, where $Lev_0 := \frac{\tau_0^R D_0 R_0 - T_1^E}{k_1}$. Also recall from Appendix 8 (Effect of utilization on the bank's supply of debt), that

$$\frac{\partial r_1^k}{\partial u_0} = -\alpha(1-\alpha)z_1(k_1)^{\alpha-2}\bar{n}^{1-\alpha}\frac{\partial k_1}{\partial u_0} \tag{55}$$

$$\frac{\partial Lev_0}{\partial u_0} = \frac{\tau_0^R R_0}{k_1}\frac{\partial D_0}{\partial u_0} - \left(\frac{\tau_0^R D_0 R_0 - T_1^E}{(k_1)^2}\right)\frac{\partial k_1}{\partial u_0} \tag{56}$$

Recall that $\frac{\partial k_1}{\partial u_0} = \frac{\partial D_0}{\partial u_0} + k_0\frac{\partial r_0^k}{\partial u_0}$, and $k_1 = D_0 + r_0^k k_0 + T_0^E + k_0$ and $r_t^k = \alpha\frac{y_t}{\tilde{k}_t}$, and also $\frac{\partial r_1^k}{\partial u_0} =$

$-\alpha(1-\alpha)z_1(k_1)^{\alpha-2}\bar{n}^{1-\alpha}\frac{\partial k_1}{\partial u_0}$, and $\frac{dr_0^k}{du_0}=-\alpha(1-\alpha)z_0(u_0)^{\alpha}(k_0)^{\alpha-2}\bar{n}^{1-\alpha}<0$. Using these expressions I can write $\frac{\partial \ell_1(\underline{s})}{\partial u_0}$ as

$$\frac{\partial \ell_1(\underline{s})}{\partial u_0}=A\frac{\partial D_0}{\partial u_0}+F$$

where I have used the following definitions.

$$A\equiv\frac{\partial Lev_0}{\partial D_0}+\frac{\partial Lev_0}{\partial k_1}-\frac{\partial r_1^k}{\partial k_1}$$

$$=\underbrace{\frac{\tau_0^R R_0}{k_1}}_{\frac{\partial Lev_0}{\partial D_0}}-\underbrace{\frac{\tau_0^R D_0 R_0-T_1^E}{(k_1)^2}}_{-\frac{\partial Lev_0}{\partial k_1}}+\underbrace{\alpha(1-\alpha)z_1(k_1)^{\alpha-2}\bar{n}^{1-\alpha}}_{-\frac{\partial r_1^k}{\partial k_1}}$$

$$F\equiv-\left[-\frac{\partial Lev_0}{\partial k_1}+\frac{\partial r_1^k}{\partial k_1}\right]k_0\frac{\partial r_0^k}{\partial u_0}$$

$$=\left(\underbrace{\frac{\tau_0^R D_0 R_0-T_1^E}{(k_1)^2}}_{-\frac{\partial Lev_0}{\partial k_1}}-\underbrace{\alpha(1-\alpha)z_1(k_1)^{\alpha-2}\bar{n}^{1-\alpha}}_{-\frac{\partial r_1^k}{\partial k_1}}\right)k_0\underbrace{\alpha(1-\alpha)z_0(u_0)^{\alpha}(k_0)^{\alpha-2}\bar{n}^{1-\alpha}}_{-\frac{\partial r_0^k}{\partial u_0}}$$

The above expresses $\frac{\partial \ell_1(\underline{s})}{\partial u_0}$ (which controls general equilibrium Channel 3b) as a function of $\frac{\partial D_0}{\partial u_0}$. Needless to say, in the demand-determined regime in which $R_0=1$, the behavior of the quantity of private debt D_0 in response to a change in utilization reflects both the response of the supply of debt D_0^s and demand D_0^d. I would, however, like to express it as a function of $\frac{\partial D_0}{\partial u_0}$ (which controls general equilibrium Channel 3a). Therefore, to a first order approximation, I can express $\frac{\partial D_0}{\partial u_0}$ as a linear function of $\frac{\partial D_0^s}{\partial u_0}$ (Channel 3a) and $\frac{\partial D_0^d}{\partial u_0}$: $\frac{\partial D_0}{\partial u_0}=a\frac{\partial D_0^s}{\partial u_0}+b\frac{\partial D_0^b}{\partial u_0}$, where a,b are constants. Note also that, since, holding all else constant (to a first-order approximation), a rise in either the supply or demand of debt would increase the quantity of debt, I have $a,b>0$.

Furthermore, I can derive the expression for $\frac{\partial D_0^b}{\partial u_0}$ from equation (5) for the household's demand for private debt.

$$D_0^d(R_0, B_0; u_0) = e_0 - T_0 + d_0^F + w_0\bar{n} - B_0 - \frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-1} \tag{57}$$

In the demand-determined regime, $R_0 = 1$. Taking the derivative of this function with respect to u_0 shows that utilization has an effect on D_0^d through two channels: through the effect on systemic risk (Channel 3b), and through the households labor income $w_0\bar{n}$: $\frac{dD_0^d}{du_0} = \frac{dD_0^d}{d\ell_1(s_L)}\frac{d\ell_1(s_L)}{du_0} + \frac{\partial w_0\bar{n}}{\partial u_0}$. Moreover, I have already shown in Appendix 15 that $\frac{dD_0^d}{d\ell_1(s_L)} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)} > 0$. And recall from equation (289) that $\frac{\partial w_0\bar{n}}{\partial u_0} = \alpha(1-\alpha)\frac{y_0}{u_0} > 0$. Thus, I have

$$\frac{dD_0^d}{du_0} = \underbrace{\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}}_{>0}\frac{d\ell_1(s_L)}{du_0} + \underbrace{\frac{\partial w_0\bar{n}}{\partial u_0}}_{>0}$$

Using the expression $\frac{\partial D_0}{\partial u_0} = a\frac{\partial D_0^s}{\partial u_0} + b\frac{\partial D_0^b}{\partial u_0}$ and the above expression for $\frac{dD_0^d}{du_0}$, I can insert this into our expression for $\frac{\partial \ell_1(\underline{s})}{\partial u_0}$.

$$\frac{\partial \ell_1(\underline{s})}{\partial u_0} = A\left(a\frac{\partial D_0^s}{\partial u_0} + b\frac{\partial D_0^b}{\partial u_0}\right) + F$$

$$\frac{\partial \ell_1(\underline{s})}{\partial u_0}\left[1 - Ab\underbrace{\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}}_{>0}\right] = Aa\frac{\partial D_0^s}{\partial u_0} + Ab\underbrace{\frac{\partial w_0\bar{n}}{\partial u_0}}_{>0} + F$$

Thus, I have expressed Channel 3b $\frac{\partial \ell_1(\underline{s})}{\partial u_0}$ as a function of Channel 3a $\frac{\partial D_0^s}{\partial u_0}$ and some constants, where recall that A, F are functions of some elasticities as defined above.

$$A = \underbrace{\frac{\partial Lev_0}{\partial D_0}}_{>0} + \left[\underbrace{\frac{\partial Lev_0}{\partial k_1}}_{<0} - \underbrace{\frac{\partial r_1^k}{\partial k_1}}_{<0}\right]$$

$$F=\left[\underbrace{\frac{\partial Lev_0}{\partial k_1}}_{<0}-\underbrace{\frac{\partial r_1^k}{\partial k_1}}_{<0}\right]k_0\underbrace{\frac{\partial r_0^k}{\partial u_0}}_{<0}$$

Note that, the above expression for $\frac{\partial \ell_1(\mathrm{s})}{\partial u_0}$ shows that it is possible that neither general equilibrium channels holds in equilibrium – that is, it is possible that both $\frac{\partial D_0^s}{\partial u_0}\leq 0$ and $\frac{d\ell_1(s_L)}{du_0}\geq 0$ in equilibrium. Indeed, the above expressions show that, given $\frac{\partial D_0^s}{\partial u_0}\leq 0$, it can also be the case that $\frac{d\ell_1(s_L)}{du_0}\geq 0$ depending on size of $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}=\frac{\partial D_0^d}{\partial \ell_1(s_L)}$ relative to A,b, and also depending on the size of $\frac{\partial w_0\bar{n}}{\partial u_0}$, B, etc.

Intuitively, one might expect that if the bank's supply of private safe assets D_0^s rises in response to the fall in utilization, that systemic risk $\ell_1(s_L)$ would rise as a result of the higher leverage. That is, one might expect that even if Channel 3a does not hold, 3b would hold. How can both 3a and 3b not hold simultaneously?

Note that, even if lower utilization increases the supply of private debt D^s (so that 3a is not active, $\frac{\partial D_0^s}{\partial u_0}\leq 0$), one could still have a fall in systemic risk $\ell_1(s_L)$, so that 3b is also not active $\frac{d\ell_1(s_L)}{du_0}\geq 0$. This could happen if, in equilibrium, the fall in demand for private debt D_0^d falls by more than the rise in the supply, such that D_0 falls on net, and hence lowers systemic risk $\ell_1(s_L)$. As I showed above, the response of D_0^d to u_0, $\frac{\partial D_0^d}{\partial u_0}$, depends on how u_0 affects the wage through term $\frac{\partial w_0\bar{n}}{\partial u_0}$ and on how the equilibrium change in systemic risk $\ell_1(s_L)$ feeds into the household demand for private debt D_0^d through the terms $\frac{\partial D_0^d}{\partial \ell_1(s_L)}=\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}$. Indeed, these are precisely the relevant elasticities which appear in the above expression for Channel 3b $\frac{\partial \ell_1(\mathrm{s})}{\partial u_0}$.

I now derive some sufficient conditions for both to be active ($\frac{\partial D_0^s}{\partial u_0}>0$ and $\frac{d\ell_1(s_L)}{du_0}<0$) in equilibrium. Looking at the above expression for $\frac{\partial \ell_1(\mathrm{s})}{\partial u_0}$, if 3a is active $\frac{\partial D_0^s}{\partial u_0}>0$, then a sufficient condition for 3b to also be active $\frac{d\ell_1(s_L)}{du_0}<0$ is that the expression $A>0$ (i.e. $\underbrace{\frac{\partial Lev_0}{\partial k_1}}_{<0}>\underbrace{\frac{\partial r_1^k}{\partial k_1}}_{<0}$), and that $F<0$ is not too negative. Thus, for both 3a and 3b to be active in equilibrium, a sufficient condition is that the following three conditions all hold:

i) Channel 3a is active, i.e. $\frac{\partial D_0^s}{\partial u_0}>0$. I could derive explicit necessary and sufficient conditions for $\frac{dD_0}{du_0}>0$ using our expression for $\frac{dD_0}{du_0}$ derived in Online Appendix 7. In any case, this condition already implicitly embeds how u_0 affects D_0 through the bank's net worth. Intuitively, the demand-driven recession induced by the fall in u_0 should cause the bank's supply of private debt D_0^s to fall. This is natural, given that firms typically seek to reduce debt issuance during demand-driven downturns.

ii) $\underbrace{\frac{\partial Lev_0}{\partial k_1}}_{<0} > \underbrace{\frac{\partial r_1^k}{\partial k_1}}_{<0}$, so that $A > 0$. To understand this condition, recall that $\ell_1(s_L) = Lev_0 - r_1^k(s_L)$. Thus, this condition implies that, at the margin, a lower capital stock reduces systemic risk $\ell_1(s_L)$. That is, even though, a lower capital stock increases leverage (holding other variables constant) and thus increases $\ell_1(s_L)$, it also increases the rental rate of capital at date 1, which reduces $\ell_1(s_L)$. This condition is that the latter effect dominates.

iii) $|F|$ is not too large, to ensure that the condition for $\frac{d\ell_1(s_L)}{du_0} < 0$ holds. Given our definition of F above, this amounts to a condition that k_0 is small and/or $\frac{\partial r_0^k}{\partial u_0} < 0$ is small in absolute value. The latter implies that, at the margin, the recession does not cause the rental rate of capital at date 0 to increase too much.

Thus, these conditions are relatively weak.

APPENDIX 8: Conditions defining a safety trap

Defining a safety trap in general

To determine whether there is a shortage of bonds, I ask whether, at the margin, would an increase in the supply of private safe assets D_0^s increase or decrease the demand-driven recession, that is, utilization. Therefore, the key elasticity which defines the nature of the safety trap is the derivative of utilization with respect to the supply of private safe assets, evaluated at the equilibrium

$$\frac{du_0}{dD_0^s}|_{u_0=u_0^*, D_0^d - D_0^s = 0, R_0 = 1}$$

In particular, the sign of this elasticity, evaluated at the date 0 equilibrium, determines whether I are in a conventional safety trap (in which there's a shortage of both public and private safe assets) or a risky safety trap (in which there's a shortage of public safe assets, and an oversupply of private ones): if the derivative is strictly negative, then I have a risky safety trap, otherwise I have a conventional safety trap.

Note that, in Appendix 8, I already showed that that u_0 is determined to ensure that there is no excess demand $D^d(R_0, u_0, B_0) - D_0^s = 0$ when $R_0 = 1$. That is, at $R_0 = 1$, equilibrium utilization $u_0$can be expressed as a function only of excess demand for debt, $D^d(R_0, u_0, B_0) - D_0^s$. Therefore, utilization falls if and only if excess demand increases (holding utilization fixed). That is, $\frac{du_0}{dD_0^s}|_{u_0=u_0^*, D_0^d - D_0^s = 0, R_0 = 1} < 0$ if and only if $\frac{d}{dD_0^s}\left(D_0^d - D_0^s\right)|_{u_0=u_0^*, D_0^d - D_0^s = 0, R_0 = 1} > 0$. Thus, the response of utilization ultimately boils down to the sign of this elasticity. Therefore, I can equivalently define the nature of the safety trap based on the sign of the following derivative (evaluated at the equilibrium, and holding u_0 fixed).

$$\frac{d}{dD_0^s}\left(D_0^d - D_0^s\right)|_{u_0=u_0^*, D_0^d-D_0^s=0, R_0=1}$$

Therefore, I can reformulate the definition as follows:

To determine the nature of the safety trap, I ask whether, at the margin, would an increase in the supply of private safe assets D_0^s increase or decrease the excess demand for safe assets, holding u_0 fixed at its equilibrium value (i.e. not allowing u_0 to adjust to the change in excess demand). (Think of model relationship between excess demand for safe assets and safe asset supply, that is $D_0^d - D_0^s$ as a function of D_0^s. Equilibrium condition defined as $D_0^d - D_0^s = 0$. Can take derivative of the function with respect to D_0^s evaluated at the equilibrium, and holding u_0 fixed. This derivative can be positive or negative in principle.)

That is, the key elasticity which defines the nature of the safety trap is

$$\frac{d}{dD_0^s}\left(D_0^d - D_0^s\right)$$

where $\frac{dD_0^s}{dD_0^s} = 1$ and $\frac{dD_0^d}{dD_0^s} = \frac{dD_0^d}{d\ell_1(s_L)}\frac{d\ell_1(s_L)}{dD_0^s}$, and I have suppressed the notation that these derivatives are eventuated at the equilibrium (including $u_0 = u_0^*$).

Note on General Equilibrium Feedback Effects In general, a marginal increase in the supply of debt (private or public) has an effect on utilization u_0 via Channels 1 and 2 outlined earlier. Moreover, the change in u_0 may have a general equilibrium feedback effect on the supply of private debt in turn, through what I termed Channel 3. (For example, suppose that , at the margin, a higher supply of private debt D_0^s causes utilization u_0 to fall via Channels 1 and 2. Depending on the direction of each of the channels, the fall in u_0 could cause D_0 either to rise or fall as a feedback effect. The net effect on u_0 and D_0 would depend on the direction and strength of these feedback effects relative to the first round effect. That is, the general equilibrium relationship between the supply of debt and u_0 can vary depending on the feedback effects from u_0 to other variables in general equilibrium, via Channel 3. In either case, the feedback effects must be diminishing in intensity in order for there to be an equilibrium.)

Our definition of the safety trap above can accommodate such general equilibrium interactions. However, in deriving conditions on the elasticities which delineate the nature of the safety trap, I abstract from general equilibrium feedback effects in two ways: First, I do not account for general equilibrium feedback effects from u_0 onto other variables (that is, Channel 3). That is, while u_0 will respond to a change in the supply of either type of debt, I do not account here for how this change in u_0 in turn affects other variables. Second, I do not account for how a change in the supply of either type of debt would affect the value of D_0 traded in equilibrium. The response of

D_0 necessarily depends on the balance of supply D_0^s and demand D_0^d for debt, and the response of utilization u_0. In deriving conditions on elasticities, I do not account for these effects. In that sense, the sufficient statistics that I derive below abstract from general equilibrium feedback effects through u_0. Nevertheless, I take up these general equilibrium feedback effects in section 4.3, and characterize the amplification mechanism to which it gives rise.

Aggregate demand, (i.e. the household's demand function for private debt) is given by

$$D_0^d(R_0, u_0, B_0) = e_0 - T_0 + d_0^F + w_0\bar{n} - B_0 - \frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-1} \tag{58}$$

while the bank's supply curve for private debt is given by the optimality condition:

$$\underbrace{E\left[v' r_2^k\left(r_1^k(s)+1\right)\right] + \lambda_1(s_L)\left[r_1^k(s_L)+\ell_1(s_L)\right]}_{marginal\ benefit} = \underbrace{E\left[v' r_2^k\left(\phi\left(\ell_1(s)\right)+\tau_0^R R_0\right)\right] + \lambda_1(s_L)\tau_0^R R_0}_{marginal\ cost} \tag{59}$$

$\ell_1(\underline{s}) = \frac{\tau_0^R D_0 R_0 - T_1^E}{k_1} - r_1^k(\underline{s})$

Shortage of Public Safe Assets

There is a shortage of public safe assets if and only if

$$\frac{d}{dB_0}\left(D_0^d - D_0^s\right) < 0$$

where I have suppressed the notation that these derivatives are eventuated at the equilibrium (including $u_0 = u_0^*$).

I make the same assumptions about government behavior as listed in Appendix 12. I now show that, in the demand-determined regime, I always have a shortage of public safe assets under certain conditions. I have

$$\frac{dD_0^d}{dB_0} = \frac{\partial D_0^d}{\partial B_0}\frac{dB_0}{dB_0} - \frac{\partial D_0^d}{\partial T_0}\frac{dT_0}{dB_0} + \frac{\partial D_0^d}{\partial d_0^F}\frac{dd_0^F}{dB_0} + \frac{\partial D_0^d}{\partial w_0}\frac{dw_0}{dB_0} + \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{dE_0\left[\frac{1}{c_1(s)}\right]}{dB_0}$$

$$= -1 - \frac{dT_0}{dB_0} + \frac{dd_0^F}{dB_0} + \bar{n}\frac{dw_0}{dB_0} + \frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-2}\frac{dE_0\left[\frac{1}{c_1(s)}\right]}{dB_0}$$

Since our assumptions about the government's behavior implies that higher issuance of public debt is not rebated as a transfer at date 0, I have $\frac{dT_0}{dB_0} = 0$.

Terms depend on $\frac{du_0}{dB_0}$, and last term also depends on $\frac{d\ell_1(s_L)}{dB_0}$. (Recall that so far I are holding u_0 constant because I are focusing on the effect of B_0 on excess demand for (private) safe assets.) I have in the demand-determined regime that $d_0^F = 0$, $w_0 = (1-\alpha)\frac{y_0}{\bar{n}}$, where $y_0 = z_0 (u_0 k_0)^{\alpha} \bar{n}^{1-\alpha}$, which are affected by B_0 only via u_0. So if I hold u_0 constant, then $\frac{dd_0^F}{dB_0}, \frac{dw_0}{dB_0} = 0$. Then $\frac{dD_0^d}{dB_0}$ simplifies to

$$\frac{dD_0^d}{dB_0} = -1 + \frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-2} \frac{dE_0\left[\frac{1}{c_1(s)}\right]}{dB_0}$$

$$= -1 + \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{dE_0\left[\frac{1}{c_1(s)}\right]}{dB_0}$$

where the second term is the effect of B_0 on the household's consumption risk, and the effect of consumption risk on D_0^d (precautionary saving demand). Note that part of the effect of $\frac{dE_0\left[\frac{1}{c_1(s)}\right]}{dB_0}$ will capture the effect of higher B_0 on lump-sum taxes at date 1 T_1 (or, isomorphically, date 2, where this isomorphism arises because of perfect consumption smoothing between dates 1 and 2).

Note that so far, the condition for a shortage of public safe assets is: I have a shortage of public safe assets if and only if:

$$\frac{d}{dB_0}\left(D_0^d - D_0^s\right) \leq 0$$

$$\frac{dD_0^d}{dB_0} - \frac{dD_0^s}{dB_0} \leq 0$$

$$\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{dE_0\left[\frac{1}{c_1(s)}\right]}{dB_0} \leq 1 + \frac{dD_0^s}{dB_0}$$

where the first term is the effect of B_0 on H's expected future marginal utility of consumption $E_0\left[u'\left(c_1(s)\right)\right]$ (which includes consumption risk), and the effect of consumption risk on D_0^d (saving demand), and the second effect is the effect of B_0 on E's supply schedule of private debt.

Recall that I already showed that, when ignoring general equilibrium feedback effects (via u_0 and R_0), I have $\frac{d\ell_1(s_L)}{dB_0} = 0$ when holding u_0 constant. Thus, our condition for a shortage of public safe assets reduces to: I have a shortage of public safe assets iff:

$$\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0} \leq 1+\frac{dD_0^s}{dB_0}$$

where this reflects the fact that the effect of B_0 on systemic risk ℓ_1 is 0 in partial equilibrium (i.e. holding u_0 and R_0 fixed).

Effect of B_0 on the Supply of Private Debt

How about effect of B_0 on the bank's willingness to issue private debt, $\frac{dD_0^s}{dB_0}$? Since $c_1(s) = c_2(s)$, I have $c_1(s) = \frac{1}{2}\left[(w_1+w_2)\bar{n}+R_0(D_0+B_0)-T_1-T_2\right]$, and so a rise in B_0 increases the household's date 1 and 2 consumption. D_0^s is determined by the bank's first order condition, which balances the marginal benefit and cost of borrowing.

$$\underbrace{E\left[v'r_2^k\left(r_1^k(s)+1\right)\right]+\lambda_1(s_L)\left[r_1^k(s_L)+\ell_1(s_L)\right]}_{marginal\ benefit} = \underbrace{E\left[v'r_2^k\left(\phi\left(\ell_1(s)\right)+\tau_0^R R_0\right)\right]+\lambda_1(s_L)\tau_0^R R_0}_{marginal\ cost} \quad (60)$$

Therefore, a change in the supply of B_0 affects the supply of private debt if and only if it alters this tradeoff in some way. I show in Online Appendix 11 (Characterizing the sign of $\frac{dD_0^s}{dB_0}$) that a sufficient condition for $\frac{dD_0^s}{dB_0} < 0$ in the demand-determined regime (when holding u_0 and R_0 constant, i.e. ignoring general equilibrium feedback effects) is that expected liquidation $\pi_L\ell_1(s_L)$ to be sufficiently small in equilibrium that the following condition holds:

$$\pi_L\ell_1(s_L) \leq \left(\frac{k_1-D_0+T_1^E}{k_1}\right)\eta$$

I will later impose an assumption that $\frac{\partial D_0^s}{\partial B_0} < 0$ is not too negative, in order to ensure that the demand-determined regime always features a shortage of public safe assets (see Appendix 10).

Private Safe Assets: Shortage or Oversupply?

This can be either a shortage of public safe assets (which I will take up in the normative part) or private safe assets, or both. Our model reveals that not all safety traps are alike. There can be a general shortage of safe assets (i.e. a shortage of both public and private safe assets), similar to that in Caballero and Farhi (2017) and other papers in the literature, which I term conventional safety trap; or there can be a shortage of public safe assets and an oversupply of private safe assets. I refer to the latter as risk-intensive safety traps.

To determine the nature of the safety trap, I ask whether, at the margin, would an increase in the supply of private safe assets D_0^s increase or decrease the excess demand for safe assets, holding

u_0 fixed at its equilibrium value (i.e. not allowing u_0 to adjust to the change in excess demand). (I can think of the relationship between excess demand for safe assets and safe asset supply, that is $D_0^d - D_0^s$, as a function of D_0^s. The equilibrium condition is defined as $D_0^d - D_0^s = 0$. I can take the derivative of the function with respect to D_0^s evaluated at the equilibrium, and holding u_0 fixed. This derivative can be positive or negative in principle.)

That is, the key elasticity which defines the nature of the safety trap is

$$\frac{d}{dD_0^s}\left(D_0^d - D_0^s\right)$$

where $\frac{dD_0^s}{dD_0^s} = 1$ and $\frac{dD_0^d}{dD_0^s} = \frac{dD_0^d}{d\ell_1(s_L)}\frac{d\ell_1(s_L)}{dD_0^s}$, and I have suppressed the notation that these derivatives are eventuated at the equilibrium (including $u_0 = u_0^*$).

A sufficient statistic (ignoring general equilibrium feedback effects from changes in u_0) for determining the nature of the safety trap is then $\frac{d}{dD_0^s}\left(D_0^d - D_0^s\right)$ with respect to 0, i.e.

$$\frac{dD_0^d}{dD_0^s} - 1$$

with respect to 0.

$$\frac{dD_0^d}{dD_0^s} = \frac{\partial D_0^d}{\partial d_0^F}\frac{dd_0^F}{dD_0^s} + \frac{\partial D_0^d}{\partial w_0}\frac{dw_0}{dD_0^s} + \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{dE_0\left[\frac{1}{c_1(s)}\right]}{dD_0^s}$$

Again, I have in the demand-determined regime that $d_0^F = 0$, $w_0 = (1-\alpha)\frac{y_0}{\bar{n}}$, where $y_0 = z_0\left(u_0 k_0\right)^{\alpha}\bar{n}^{1-\alpha}$, which are affected by D_0^s only via u_0. So given that I are holding u_0 constant, then $\frac{dd_0^F}{dD_0^s}, \frac{dw_0}{dD_0^s} = 0$. Then $\frac{dD_0^d}{dD_0^s}$ simplifies to

$$\frac{dD_0^d}{dD_0^s} = \frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-2}\frac{dE_0\left[\frac{1}{c_1(s)}\right]}{dD_0^s}$$

Therefore, the condition defining whether there is a shortage of private safe assets (a sufficient statistic) is: there is a shortage of private safe assets (conventional safety trap) if and only if:

$$\frac{d}{dD_0^s}\left(D_0^d - D_0^s\right) \leq 0$$

$$\frac{dD_0^d}{dD_0^s} \leq 1$$

where, at the effective lower bound, this is

$$\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{dE_0\left[\frac{1}{c_1(s)}\right]}{dD_0^s} \leq 1$$

This is the effect of D_0^s on the household's consumption risk, and the effect of consumption risk on D_0^d (precautionary saving demand). Moreover, recall these elasticities can be decomposed into a channel reflecting the effect of systemic risk. Thus, there's a shortage of private safe assets if and only if

$$\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \left(\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial D_0^s} + \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)} \frac{d\ell_1(s_L)}{dD_0^s} \right) \leq 1$$

where the second term captures the effect of debt on expected future marginal utility of consumption via systemic risk, while the first term captures other effects such as through the household's interest income on holdings of bonds in both states.

APPENDIX 9: Proof of Lemma 2: *Safety trap features shortage of public safe assets*

I now show that, under regularity conditions, the demand-determined regime always features a safety trap (that is, a shortage of public safe assets). In particular, I will show that, in the demand-determined regime, I have

$$\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0} \leq 1 + \frac{dD_0^s}{dB_0}$$

I begin by showing that the left-hand side is weakly negative in the demand-determined regime.

I begin by showing that $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} > 0$. Note from the household's demand function D_0^d that

$$D_0^d(R_0, B_0) = e_0 - T_0 + d_0^F + w_0\bar{n} - B_0 - \frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-1} \tag{61}$$

So $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} = \frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-2} > 0$.

I now show that $\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0} < 0$. It suffices to show that, for each state s, $\frac{\partial c_1(s)}{\partial B_0} > 0$, abstracting from general equilibrium feedback effects (that is, holding u_0 and R_0 fixed). Recall that, in equilibrium,

$$c_1(s) = c_2(s) = \frac{1}{2}\left[(w_1+w_2)\bar{n} + R_0(D_0+B_0) - T_1 - T_2\right]$$

First note that $\frac{\partial c_1(s)}{\partial B_0} = \frac{R_0}{2} > 0$. Leaving ℓ_1 constant (since this is grouped in the term reflecting Channel 1), I must also account for how each type of debt affects $c_1(s)$ through the other terms in the above expression.

$$\frac{\partial c_1(s)}{\partial B_0} = \frac{R_0}{2} + \bar{n}\frac{\partial w_2(s)}{\partial B_0} + \bar{n}\frac{\partial w_1(s)}{\partial B_0} - \frac{\partial T_1}{\partial B_0} - \frac{\partial T_2}{\partial B_0}.$$

Recall that under the assumptions I've made about the government's behavior, I have $T_1 = -\tau_1$ and $T_2 = 0$ in equilibrium. Therefore, $\frac{\partial T_1}{\partial B_0}, \frac{\partial T_2}{\partial B_0} = 0$. Moreover, I already showed that $\frac{\partial \ell_1(s)}{\partial B_0} = 0$. Note, then, that

$$\frac{\partial w_1(s)}{\partial B_0} = \frac{(1-\alpha)}{\bar{n}}\frac{\partial y_1(s)}{\partial B_0} = \frac{(1-\alpha)}{\bar{n}}\frac{\partial y_1(s)}{\partial \tilde{k}_1(s)}\frac{\partial \tilde{k}_1(s)}{\partial k_1^G(s)}\frac{\partial k_1^G(s)}{\partial B_0} = \frac{(1-\alpha)}{\bar{n}}\frac{\partial y_1(s)}{\partial \tilde{k}_1(s)} = \frac{(1-\alpha)}{\bar{n}}\alpha\frac{y_1(s)}{\tilde{k}_1(s)} > 0$$

Hence, $\frac{\partial w_1(s)}{\partial B_0} > 0$.

Now turn to $\frac{\partial w_2(s)}{\partial B_0}$.

$$\frac{\partial w_2(s)}{\partial B_0} = \frac{(1-\alpha)}{\bar{n}}\frac{\partial y_2(s)}{\partial B_0} = \frac{(1-\alpha)}{\bar{n}}\frac{\partial y_2(s)}{\partial \tilde{k}_2(s)}\left[\frac{\partial \tilde{k}_2(s)}{\partial k_2^G(s)}\frac{\partial k_2^G(s)}{\partial B_0} + \frac{\partial \tilde{k}_2(s)}{\partial k_2(s)}\frac{\partial k_2(s)}{\partial B_0}\right]$$

$$= \frac{(1-\alpha)}{\bar{n}}\frac{\partial y_2(s)}{\partial \tilde{k}_2(s)}\left[1 + \frac{\partial k_2(s)}{\partial B_0}\right]$$

Recall that, in equilibrium, I have $k_2(s) = \left[1 + r_1^k(s) - \phi\left(\ell_1(s)\right)\right]k_1(D_0) - \tau_0^R D_0 R_0 + T_1^E$, and that, based on the assumptions above about the government's behavior, I have $T_1^E = r_1^k(s)k_1^G - R_0^B B_0$. Therefore,

$$k_2(s) = \left[1 + r_1^k(s) - \phi\left(\ell_1(s)\right)\right]k_1 - \tau_0^R D_0 R_0 + r_1^k(s)k_1^G - R_0^B B_0$$

and so

$$\frac{\partial k_2(s)}{\partial B_0} = \left[1 + r_1^k(s) - \phi\left(\ell_1(s)\right)\right]\frac{\partial k_1}{\partial B_0} - \tau_0^R R_0\frac{\partial D_0}{\partial B_0} + k_1\frac{\partial r_1^k(s)}{\partial B_0} + r_1^k(s)\frac{\partial k_1^G}{\partial B_0} + k_1^G\frac{\partial r_1^k(s)}{\partial B_0} - R_0^B$$

Recall that $k_1 = D_0 + T_0^E + \left(r_0^k + 1\right) k_0$, $\frac{\partial k_1^G}{\partial B_0} = 1$, and, as I showed above, B_0 doesn't affect the supply of debt, save for GE feedback effects from which I are abstracting. Therefore,

$$\frac{\partial k_2(s)}{\partial B_0} = \left[k_1 + k_1^G\right] \frac{\partial r_1^k(s)}{\partial B_0} + r_1^k(s) - R_0^B$$

where $r_1^k = \alpha \frac{y_1}{\tilde{k}_1}$, so $\frac{\partial r_1^k(s)}{\partial B_0} = \alpha \frac{1}{\tilde{k}_1} \frac{\partial y_1}{\partial B_0} - \alpha \frac{y_1}{\left(\tilde{k}_1\right)^2} \frac{\partial \tilde{k}_1}{\partial B_0} = \alpha \frac{1}{\tilde{k}_1} \frac{\partial y_1}{\partial B_0} - \frac{r_1^k}{\tilde{k}_1} \frac{\partial \tilde{k}_1}{\partial B_0}$. And since $\frac{\partial \tilde{k}_1}{\partial B_0} = \frac{\partial k_1}{\partial B_0} + \frac{\partial \tilde{k}_1^G}{\partial B_0} = 1$ and $\frac{\partial y_1}{\partial B_0} = \frac{\partial y_1}{\partial \tilde{k}_1} \frac{\partial \tilde{k}_1}{\partial B_0} = \frac{\partial y_1}{\partial \tilde{k}_1} = \alpha \frac{y_1}{\tilde{k}_1} = r_1^k$, I have $\frac{\partial r_1^k(s)}{\partial B_0} = \alpha \frac{r_1^k}{\tilde{k}_1} - \frac{r_1^k}{\tilde{k}_1} = -\frac{r_1^k}{\tilde{k}_1}(1-\alpha) < 0$. Putting these derivatives together, I can see that $\frac{\partial c_1(s)}{\partial B_0} > 0$ for both states of the world.

$$\frac{\partial c_1(s)}{\partial B_0} = \frac{R_0}{2} + \bar{n} \frac{\partial w_2(s)}{\partial B_0} + \bar{n} \frac{(1-\alpha)}{\bar{n}} \alpha \frac{y_1(s)}{\tilde{k}_1(s)}$$

$$= \frac{R_0}{2} + (1-\alpha)\, r_2^k(s) \left[1 + \alpha r_1^k(s) - R_0^B\right] + (1-\alpha)\, \alpha \frac{y_1(s)}{\tilde{k}_1(s)} > 0$$

Thus, it follows that $\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0} < 0$. Hence, thus far I have

$$\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0} < 0$$

I now assume that the following condition holds in equilibrium, which amounts to assuming that $\frac{dD_0^s}{dB_0}$ is not too negative:

$$\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0} - 1 < \frac{dD_0^s}{dB_0}$$

where $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0} < 0$. (Note that it would suffice to assume that $\frac{dD_0^s}{dB_0} \geq 0$.) Given this assumption, it follows that

$$\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial B_0} < 0 < 1 + \frac{dD_0^s}{dB_0}$$

and so the demand-determined regime always features a shortage of public safe assets.

APPENDIX 10: Proof of Proposition 1: *Oversupply of private assets*

I claim that there is an oversupply of private safe assets if and only if the dynamic interaction between systemic risk (ℓ_1) and AD is sufficiently large. To prove this, first recall that there's an over supply of private safe assets if and only if

$$\frac{dD_0^d}{dD_0^s} > 1$$

From our decomposition of channel 1 $\frac{dD_0^d}{dD_0^s}$, I know that the above elasticity can be decomposed into two terms.

$$\underbrace{\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}\frac{d\ell_1(s_L)}{dD_0^s}}_{>0} > 1 - \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial D_0^s}$$

Thus, there is an oversupply of private safe assets if and only if the dynamic interaction between aggregate demand and systemic risk (more precisely the product of Channels 1 and 2) is sufficiently strong. Q.E.D.

So the condition defining nature of safety trap indicates that I have a risk-intensive safety trap if and only if the product of the two channels is sufficiently strong - that is, if the aggregate demand and systemic risk is sufficiently strong. If this dynamic interaction is sufficiently strong, then at the margin, an increase in the supply of private debt will cause systemic risk to increase and the recession to become worse, implying that there's an oversupply of private debt. (As I will show later, this also equivalently means the direction of the aggregate demand externality is such that planner would reduce the bank's leverage relative to the competitive equilibrium.) Alternatively, if it is not sufficiently strong, then a marginal increase in the supply of private debt may increase systemic risk and precautionary demand, but this will be outweighed by the effect of more private debt on reducing excess demand for saving, reducing the severity of the demand recession. In that case, there's a shortage of private safe assets (i.e. a conventional safety trap).

APPENDIX 11: Proof of Proposition 2: Amplification mechanism

Here, I show that under weak conditions, the confluence of the general equilibrium Channels 3a and 3b, outlined in section 4.3, amplifies the demand-recession in response to an adverse, unanticipated fall in $z_1(s_L)$. Recall that each channel amplifies the downturn if active, and dampens it if not (that is, if the effects go the opposite direction). If, in equilibrium, only one of the channels is active, then whether the general equilibrium effects, in total, amplify or dampen the shock depends on the net effect of the two.

The net effect of both 3a and 3b on date 0 excess demand for saving, and therefore the severity of the date 0 recession, is given by the marginal total effect of both channels on excess demand for saving at date 0: $\frac{dD_0^d}{du_0} - \frac{dD_0^s}{du_0}$. In particular, the net effect of 3a and 3b is to *dampen* the severity of the recession if excess demand for saving at date 0 falls at the margin as u_0 falls, that is if $\frac{d}{du_0}\left(D_0^d - D_0^s\right) > 0$.

$$\frac{dD_0^d}{du_0} - \frac{dD_0^s}{du_0} > 0$$

Note that $\frac{dD_0^s}{du_0}$ is Channel 3a in itself, while Channel 3b is embedded in the term $\frac{dD_0^d}{du_0}$. In particular, recall equation (5) for the household's demand for private safe assets.

$$D_0^d\left(R_0, B_0; u_0\right) = e_0 - T_0 + d_0^F + w_0\bar{n} - B_0 - \frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-1} \tag{62}$$

In the demand-determined regime, $R_0 = 1$. Taking the derivative of this function with respect to u_0 shows that utilization has an effect on D_0^d through two channels: through the effect on systemic risk (Channel 3b), and through the households labor income $w_0\bar{n}$: $\frac{dD_0^d}{du_0} = \frac{dD_0^d}{d\ell_1(s_L)}\frac{d\ell_1(s_L)}{du_0} + \frac{\partial w_0\bar{n}}{\partial u_0}$. Moreover, I have already shown in Appendix 15 that $\frac{dD_0^d}{d\ell_1(s_L)} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)} > 0$. And recall from equation (289) that $\frac{\partial w_0\bar{n}}{\partial u_0} = \alpha\left(1-\alpha\right)\frac{y_0}{u_0} > 0$. Therefore, I can rewrite this condition for dampening as

$$\underbrace{\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}}_{>0}\underbrace{\frac{d\ell_1(s_L)}{du_0}}_{3b} + \underbrace{\frac{\partial w_0\bar{n}}{\partial u_0}}_{>0} > \underbrace{\frac{dD_0^s}{du_0}}_{3a}$$

Proof of Part A:

Claim: If neither general equilibrium channel is active ($\frac{dD_0^s}{du_0} < 0$ and $\frac{d\ell_1(s_L)}{du_0} > 0$), then the general equilibrium Channels 3a and 3b together dampen the downturn. To see this, revisit the condition for dampening:

$$\underbrace{\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}}_{>0}\underbrace{\frac{d\ell_1(s_L)}{du_0}}_{>0} + \underbrace{\frac{\partial w_0\bar{n}}{\partial u_0}}_{>0} > \underbrace{\frac{dD_0^s}{du_0}}_{<0}$$

This condition holds by the fact that, when $\frac{dD_0^s}{du_0} < 0$ and $\frac{d\ell_1(s_L)}{du_0} > 0$, the left-hand side is strictly positive while the right-hand side is unambiguously strictly negative. In this case, the general equilibrium channels cannot amplify the downturn. Therefore, the confluence of the general equilibrium feedback channels (together with the effect on $\frac{\partial w_0 \bar{n}}{\partial u_0}$) is to dampen the downturn. Q.E.D.

Proof of Part B:

Claim: If one or both channels is active, amplification may hold.

To see this, when 3a holds ($\frac{dD_0^s}{du_0} > 0$) but 3b does not ($\frac{d\ell_1(s_L)}{du_0} \geq 0$), the condition for amplification is

$$\underbrace{\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}}_{>0} \underbrace{\frac{d\ell_1(s_L)}{du_0}}_{>0} + \underbrace{\frac{\partial w_0 \bar{n}}{\partial u_0}}_{>0} < \underbrace{\frac{dD_0^s}{du_0}}_{>0}$$

which can hold. When 3a does not hold ($\frac{dD_0^s}{du_0} \leq 0$) but 3b does ($\frac{d\ell_1(s_L)}{du_0} < 0$), the condition for amplification is

$$\underbrace{\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}}_{>0} \underbrace{\frac{d\ell_1(s_L)}{du_0}}_{<0} + \underbrace{\frac{\partial w_0 \bar{n}}{\partial u_0}}_{>0} < \underbrace{\frac{dD_0^s}{du_0}}_{<0}$$

which can also hold.

If both channels are active, then the condition for amplification is weaker than if only one channel holds. E.g. if both channels are active the condition for amplification is

$$\underbrace{\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}}_{>0} \underbrace{\frac{d\ell_1(s_L)}{du_0}}_{<0} + \underbrace{\frac{\partial w_0 \bar{n}}{\partial u_0}}_{>0} < \underbrace{\frac{dD_0^s}{du_0}}_{>0}$$

Due to the signs of the terms in the condition, this is a weaker condition than both of the previous conditions. Q.E.D.

APPENDIX 12: Proofs regarding Assumption 1

In this appendix, I show that Assumption 1 ensures that the bank's date 0 natural borrowing limit on borrowing D_0 is never binding in equilibrium, and that liquidation occurs only in the bad state, $\ell_1(s_L) > 0$, $\ell_1(s_H) = 0$. I also show that these restrictions are satisfied by a non-empty set of parameters satisfies these restrictions.

Claim 1: $z_1(s_L) < \frac{\tau_0^D R_0 D_0 - k_1}{k_1 \alpha (\tilde{k}_1)^{\alpha-1} \bar{n}^{1-\alpha}}$ implies that $\ell_1(s_L) > 0$.

Proof: I have $\ell_1(s_L) > 0$ if and only if

$$\ell_1(s_L) = \frac{\tau_0^R D_0 R_0}{k_1} - r_1^k(s_L) - \frac{T_1^E}{k_1} > 0 \tag{63}$$

i.e.

$$\frac{\tau_0^R D_0 R_0 - T_1^E}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}} > z_1(s_L) \tag{64}$$

Q.E.D.

Claim 2: $z_1(s_H) \geq \frac{\tau_0^R D_0 R_0 - T_1^E}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}}$ implies that $\ell_1(s_H) = 0$.

Proof: From the expression for ℓ_1, it follows that $\ell_1(s_H) = 0$ if and only if

$$\frac{\tau_0^R D_0 R_0 - T_1^E}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}} \leq z_1(s_H) \tag{65}$$

I can set $z_1(s_H)$ arbitrarily high to ensure this is satisfied. Q.E.D.

Claim 3: The natural borrowing limit is non-binding.

Proof: Recall that natural borrowing limit is $\tau_0^D R_0 D_0 \leq P_1(\underline{s}) \left(r_1^k(\underline{s}) k_1 + \bar{\ell}_1 k_1\right)$ where $\bar{\ell}_1$, the maximum fraction of its capital that the bank can liquidate without violating the non-negativity constraint on k_2, solves $k_2(s) = 0$ when $i_1 = 0$:

$$k_2(s) = i_1 + (1 - \ell_1 - \phi(\ell_1(s))) k_1(s) \tag{66}$$

$$1 = \ell_1 + \ell_1^\eta \tag{67}$$

So the natural borrowing limit is non-binding if and only if

$$\tau_0^D R_0 D_0 < P_1(\underline{s}) \left(r_1^k(\underline{s}) k_1 + \bar{\ell}_1 k_1\right)$$

$$\frac{\tau_0^D R_0 D_0}{k_1} - r_1^k(s_L) < \bar{\ell}_1$$

Note that since $\bar{\ell}_1$ solves $1 = \ell_1 + \ell_1^\eta$, as $\eta > 1$ approaches infinity, $\bar{\ell}_1 > 0$ approaches 1. Therefore, making η arbitrarily large would by itself not suffice to ensure the natural borrowing limit is always non-binding. But it would suffice if it is also the case that

$$\frac{\tau_0^D R_0 D_0}{k_1} - r_1^k(s_L) < 1$$

i.e.

$$\frac{\tau_0^D R_0 D_0 - k_1}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}} < z_1(s_L)$$

Thus, I can ensure that the natural borrowing limit is non-binding by simultaneously making η arbitrarily large and $z_1(s_L)$ is not too small, so that it satisfies the above inequality. Q.E.D.

Claim 4: There is a non-empty set of parameters which satisfy these assumptions.

Proof: First recall that I can make $z_1(s_H)$ arbitrarily large to ensure that $\frac{\tau_0^R D_0 R_0 - T_1^E}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}} \leq z_1(s_H)$.

To simultaneously ensure that both the natural borrowing limit is non-binding and that $\ell_1(s_L) > 0$, I must jointly assume that $z_1(s_L)$ satisfies

$$z_1(s_L) \in \left(\frac{\tau_0^D R_0 D_0 - k_1}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}}, \frac{\tau_0^R D_0 R_0 - T_1^E}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}} \right)$$

Note that such a $z_1(s_L)$ exists as long as

$$\frac{\tau_0^D R_0 D_0 - k_1}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}} < \frac{\tau_0^R D_0 R_0 - T_1^E}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}}$$

i.e.

$$T_1^E < k_1$$

Thus, part (B) of Assumption (1) ensures such a $z_1(s_L)$ exists. Q.E.D.