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On the Negatives of Negative Interest Rates*

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Abstract

Major central banks remunerate reserves at negative rates (NIR). To study the long-run effects of NIR, we focus on the role of reserves as intertemporal stores of value that are used to settle interbank liabilities. We construct a dynamic general equilibrium model with commercial banks holding reserves and funding investments with retail deposits. In the long run, NIR distorts investment decisions, lowers welfare, depresses output, and reduces bank profitability. The type of distortion depends on the transmission of NIR to retail deposits. The availability of cash explains the asymmetric effects of policy-rate changes in negative vs positive territory.

JEL Classification: E40, E42, E43, E50, E58.
Keywords: negative interest rate, money market, monetary policy, interest rates.

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"From a micro perspective, low rates undermine financial intermediaries' health by reducing their profitability, impede the efficient allocation of capital by enabling even the weakest firms to meet debt-service obligations, and may also inhibit competition by favoring incumbent firms. There is something unhealthy about an economy in which corporations can profitably borrow and invest even if the project in question pays a zero return. These considerations suggest that reducing interest rates may not be merely insufficient, but actually counterproductive, as a response to secular stagnation."

Anna Stansbury and Lawrence H. Summers (2019).
Whither Central Banking? Project Syndicate Commentary.

1 Introduction

Following the 2008 global financial crisis and subsequent developments, the Bank of Japan (in 2016), the Danmarks Nationalbank (in 2014), the European Central Bank (in 2014), the Swedish Riksbank (in 2015), and the Swiss National Bank (in 2015) have implemented negative interest rates by decreasing the remuneration of central bank reserves into negative territory (NIR). Initially, the expectation was that NIR would be needed for a short period of time only, until the economic conditions would allow to normalize rates to their positive long-term averages. However, except for the Swedish case, NIR was implemented until mid-2022. NIR has thus been in place for almost a decade, justifying a theoretical study of its long-run implications.[1] The Bank of Japan continues to apply negative interest rates to reserves that exceed a particular threshold.

In this paper, we develop a dynamic general equilibrium model to study the long-run effects of NIR on investment decisions, welfare, bank profitability and output. Banks fund productive investment with retail deposits, hold central bank reserves, and borrow or lend reserves in the money market. The central bank implements monetary policy by setting the interest rate on reserves. It further controls the stock of reserves by means of open-market operations. Interbank liabilities arise due to banks' investment activity—an individual bank finances investment by creating deposits, but depositors will transfer some of their deposits to other banks, generating liabilities vis-à-vis the other banks. These liabilities must be settled with reserves, giving rise to a mechanism in which reserve holdings affect the equilibrium allocation. We find that NIR distorts

[1] Steady-state analysis is well suited to reflect the monetary conditions between 2015 and 2022, during which negative rates were implemented over a long time horizon. Switzerland, for example, introduced a negative interest rate on reserves of -0.75 percent in mid-January 2015. Only in mid-June 2022, with inflation rising, Switzerland increased the policy rate to -0.25 percent and they moved out of negative territory in mid-September 2022 by increasing the rate to 0.5 percent.

investment decisions, lowers welfare, depresses output, and reduces bank profitability in the long run. The reason is that NIR reduces banks' willingness to hold reserves, which restricts aggregate investment because of limited means to settle interbank liabilities. The type of long-run distortion that we uncover depends on the transmission of NIR to retail deposit rates and the availability of zero-interest cash to households.

With a perfectly competitive market for retail deposits, the rate on central bank reserves perfectly transmits to retail deposit rates. However, with negative rates households replace a fraction of their deposits with cash, which prevents banks from fully passing on the NIR to depositors. This affects investment as follows. Banks with investment projects of a small efficient scale invest too much to avoid the NIR on idle reserves. Banks with investment projects of a large efficient scale invest too little because NIR decreases the value of the collateral that these banks use to borrow additional reserves in the money market. Overinvestment only occurs under NIR and this is one of the reasons why reducing interest rates into negative territory is different compared to regular reductions and, according to our mechanism, *counterproductive* in the long run (see the quote by Summers and Stansbury (2019)).

With a perfectly competitive market for retail deposits, the overinvestment distortion can be eliminated by abolishing cash as proposed by, for example, Rogoff (2017). The intuition is that in the absence of cash, banks can fully pass on the NIR to depositors.[2] Nevertheless, even without cash large-scale projects remain underfunded because of binding collateral constraints. Underfunding is made worse when a central bank lowers the interest rate on reserves.

There is little evidence for a perfect transmission of the reserve rate to retail deposit rates. In contrast, empirical evidence for imperfect transmission of monetary policy to retail deposit rates is widely documented.[3] In particular, it has a strong support in the empirical NIR-literature and is true across the different NIR-currencies. Overall, the literature agrees that retail deposits (the main funding source for many banks) are largely insulated from NIR.[4] Furthermore, the data also strongly suggest that the transmission of the policy rate to retail deposit rates is especially weak in NIR periods.

For an imperfect transmission of the reserve rate to the deposit rate, we also find that banks with small-scale projects invest too much and banks with large-scale projects invest too little under NIR. In contrast to the perfect transmission case, these inefficiencies

[2] In practice, eliminating cash will not be sufficient because households have other payment instruments available to avoid negative deposit rates such as cryptocurrencies (See Schär and Berentsen 2020).

[3] See for example Drechsler et al. (2017), Dell'Ariccia et al. (2017), Basten and Mariathasan (2020), Heider et al. (2019), Eisenschmidt and Smets (2019), Demiralp et al. (2021), or Eggertsson et al. (2019).

[4] Stettler (2020) and Brunnermeier and Koby (2019) provide a microfoundation of the limited pass-through of NIR to retail deposit rates and Heider et al. (2020) discuss additional reasons for a zero lower bound on retail deposits.

are independent of the availability of cash. Our mechanism thus highlights that imperfect transmission leads to a misallocation of capital during NIR episodes that would not arise during positive interest rate episodes.

With perfect and with imperfect transmission of NIR, lowering the interest rate on reserves into negative territory unambiguously lowers welfare in the long run. The intuition for this result are the distortions discussed above. This result is independent of the availability of cash to households.

NIR also undermines commercial banks' financial health by reducing their profitability.[5] To address this concern, most central banks grant exemptions from NIR. That is, a fraction of reserves is remunerated at a rate of zero or even a positive interest rate, while the remaining part is remunerated at the NIR.[6] We study the effects of exempting a part of banks' reserve holdings from NIR. We find that exemptions improve bank profitability without affecting the central bank's ability to control the money market rate. However, the investment distortions created by NIR, and the resulting long-run negative welfare effects of NIR, are not mitigated.

Our results have important policy implications that differ substantially from most of the existing literature that focuses on the short-run effects of NIR: First, our model provides a microfoundation for the asymmetric transmission of policy rates to the real economy during NIR periods, since the overinvestment distortion only manifests itself under NIR. This demonstrates that negative and positive interest rates are asymmetric with respect to their effects on the economy.[7] Second, the overinvestment distortion can be mitigated by abolishing cash in an economy with a perfectly competitive market for retail deposits. However, empirical evidence clearly indicates that this market is not perfectly competitive. In this case all the long-run investment distortions identified in the paper are present even in the absence of cash. Thus, the suggestion to abolish cash to make NIR more effective should be taken with several grains of salt. Third, in our model NIR depress aggregate output in the long run. Although overinvestment can occur for small-scale investment projects, the damaging effect on large-scale projects dominates. In that sense, our paper is related to Brunnermeier and Koby (2019), Eggertsson et al. (2019) and Arce et al. (2021), who also show that NIR can be contractionary under

[5] While the NIR is the relevant remuneration of banks' asset side (reserves and money market lending), it does not (fully) transmit to the banks' liability side (deposits). This decreases the banks' interest rate margins and thus profits.

[6] All of the above-mentioned central banks except for the Swedish Riksbank exempt a fraction of reserves from NIR, remunerating it at zero or even a positive interest rate. Monetary policy with a tiered remuneration of reserves schedule is also studied in Boutros and Witmer (2020) and Fuhrer et al. (2021). Boutros and Witmer (2020) focus on the implication of exemptions on the demand for physical cash, whereas Fuhrer et al. (2021) study the effect of exemption thresholds on interbank markets.

[7] For example, a reduction of the policy rate in positive territory, say from 2 percent to 1 percent, has different effects than a reduction in negative territories, say from -1 percent to -2 percent.

certain conditions. These results are also closely aligned with Balloch and Koby (2022) and Wang (2020). Balloch and Koby (2022) study the long-run impact of low interest rates on banks in Japan and find that prolonged low interest rates negatively impact bank profitability and loan issuance. Also Wang (2020) studies the effects of low interest rates and finds a negative impact on loan growth in the long-run.[8] Fourth, exemptions are an effective remedy against declining bank profits, while at the same time leaving the transmission of NIR to money market interest rates unaffected. Finally, NIR reduces the value of a country's currency, indicating that it can be used as a tool to dampen the appreciation of a currency. The Swiss National Bank and the Danmarks Nationalbank explicitly introduced NIR to make their respective currencies less attractive and thus to dampen the appreciation pressure. Although the real value of the unit of account decreases, we also find, in line with observations from NIR-countries, that the aggregate value of reserves increases when NIR goes along with quantitative easing.

Finally, it is interesting to compare our results with those of, among others, Agarwal and Kimball (2019), who take a stimulating effect of NIR in the short run as given. The mechanism we have in mind shows that NIR is counterproductive if implemented over a long time horizon. However, we also consider transitional dynamics and find that NIR can have positive effects on welfare and aggregate output in the short run. That being said, NIR might also be justified for reasons outside of our model, such as to dampen an excessive appreciation of a currency.

The remainder of this paper is organized as follows. Section 2 discusses the implementation of NIR across the above-mentioned central banks. Section 3 describes the theoretical model. Section 4 discusses the equilibrium in the baseline economy and the key insights of the model. Section 5 introduces cash and a perfectly competitive market for deposits into the baseline model. Section 7 reviews the literature on NIR and Section 8 concludes. Appendix B discusses an extension with a haircut on reserves used as collateral and all proofs are in Appendix C.

2 Implementation of NIR

In what follows, we describe how NIR have been implemented in the past across different central banks, provide evidence how NIR have transmitted to the economy and discuss the reasons why NIR were implemented by the respective central bank. We focus on the implementation of NIR-policies by the Bank of Japan (BOJ), the Danmarks Nationalbank (DN), the European Central Bank (ECB), the Swedish Riksbank (Riksbank) and

[8]Another paper that studies NIR in steady-state is Dong and Wen (2017). However, they do not focus on the long-run effects of NIR. Instead, they study why NIR are possible and also the effects of liquidity injections during NIR periods.

the Swiss National Bank (SNB). Most central banks have increased their policy rates back into the positive territory in the fall of 2022 due to rising inflation. As of March 2023, only the BOJ continues to apply negative rates to reserves above a particular threshold.

Before introducing NIR, the five central banks under consideration operated in an environment with excess reserves. Due to excess reserves, monetary policy is characterized by a so-called floor system (see Berentsen et al. (2014)). They all implemented NIR by decreasing the remuneration of reserves or the interest rate applied in reserve-absorbing operations into negative territory.

Some central banks, namely the ECB, the DN and the SNB implemented a two-tiered system, where part of the reserves are exempt from NIR and earn an interest rate of 0% (SNB (2015), Jørgensen and Risbjerg (2012)). The Riksbank on the other hand did not implement reserve tiering. Excess reserves were absorbed in fine-tuning operations and deposited at the deposit facility are remunerated at -0.1%. Lastly, the BOJ adopted a three-tiered system, where a fraction of reserves earns a positive interest rate, a second fraction earns a zero interest rate and a third fraction is negatively remunerated (Bank of Japan (2016)).[9]

The primary rationale for exemptions was to address profitability considerations. Ceteris paribus, NIR decrease banks' interest rate margins if they cannot pass-on NIR to all their liabilities. Heider et al. (2019), Dell'Ariccia et al. (2017), Eisenschmidt and Smets (2019), Eggertsson et al. (2019), Zurbrügg (2016) and Basten and Mariathasan (2020) provide ample evidence that NIR are indeed not passed on to all liabilities. Moreover, the fact that all central banks excluded at least minimum reserve requirements from NIR suggests that central banks are reluctant to charge NIR on required reserves holdings, possibly due to legal considerations.[10] Further, exemptions may have been introduced due to central banks' mandate to ensure the functioning of cashless payments.

Consensus in the literature is that NIR were transmitted to money market interest rates and fixed-income markets. This was despite the fact that NIR were introduced with exemptions for all central banks except the Riksbank. Wholesale lending and deposit

[9]Banks' exemption threshold calculation was linked to the minimum reserve requirements in cases of the SNB, the ECB and to some extent in case of the BOJ (ECB (2019), SNB (2015), Bank of Japan (2016)). In case of the BOJ, the exemption threshold is based on average reserves holdings, reserve requirements and borrowing in BOJ's loan supporting programs. The latter two are considered in the so-called "Macro-Add on Balance", which is remunerated at 0% and the former represents the so-called "Basic Balance", which is remunerated at +0.1%. Reserve holdings exceeding the basic balances and the macro-add on balances are called the "Policy-Rate Balance" and are remunerated at -0.1% (Bank of Japan (2016)). The DN based its calculation of the exemption threshold on banks' activity in the payment system. Reserves exceeding the threshold are automatically converted into negatively remunerated certificates of deposits with a one-week tenor (Abildgren et al. (2015)).

[10]There are no minimum reserve requirements at the Riksbank.

rates were only partly affected and retail deposits were largely exempt from NIR. This was true for all five central banks discussed (see Bech and Malkhozov (2016), Dell'Ariccia et al. (2017) and Section 6 that discusses the related literature).

The reason for the introduction of NIR differed across central banks. In case of the ECB and the Riksbank, the introduction of NIR was part of a larger program including quantitative easing measures, with the goal to increase inflation. Also the BOJ introduced NIR to increase inflation and economic activity together with quantitative easing measures (Bank of Japan (2016)). The Danmarks Nationalbank and the SNB introduced NIR to dampen the appreciation pressure on their respective currencies. In Appendix A, we discuss the implementation of NIR and the environment in which NIR were introduced by the respective central banks in more detail.

3 The Model

Our theoretical model is motivated by the implementation of NIR discussed above. Time is discrete and continues forever: $t = 0, 1, ..., \infty$. There are two types of infinitely-lived agents: a unit mass bankers and a unit mass households. The focus of our attention will be on bankers, who we model to represent commercial banks. The government sector consists of a central bank in charge of monetary policy and a treasury department in charge of fiscal policy.[11]

In each period, two perfectly competitive markets open sequentially (see Figure 1).[12] First, an *investment-money (IM) market*, where production and investment of a capital good takes place, and where bankers can borrow and lend reserves. Second, a *settlement market*, where liabilities are settled and a general good is produced and consumed. All goods are perfectly divisible and nonstorable, which means that they cannot be carried from one market to the next. There are three perfectly divisible financial assets: bonds, reserves, and deposits. Bonds are one-period lived nominal liabilities of the treasury, which can be traded in the settlement market and can be used as collateral by bankers in the money market. Reserves are issued by the central bank and can only be held by bankers. Deposits are liabilities of bankers that can only be held by households.[13]

[11]The way in which we model the government and its policies is based on Williamson (2012).

[12]The theoretical model presented in Section 3 shares elements of Berentsen et al. (2014) and Berentsen et al. (2018). Our theoretical framework allows for a more realistic modeling of the effects of NIR on the economy. Furthermore, effects of NIR were not studied in these papers. Other theoretical papers on money markets include Orr and Mellon (1961), Poole (1968), Furfine (2000), Whitesell (2006), Berentsen and Monnet (2008), Afonso and Lagos (2015), Bech and Monnet (2016), Rocheteau et al. (2018).

[13]We assume that bankers can commit to households, but that households lack commitment. We make these assumptions so that bankers in our model resemble real-world commercial banks. Because we are interested in NIR, we take bankers and how they operative as primitives rather than outcomes. For the latter approach, see, for instance, Gu et al. (2013).

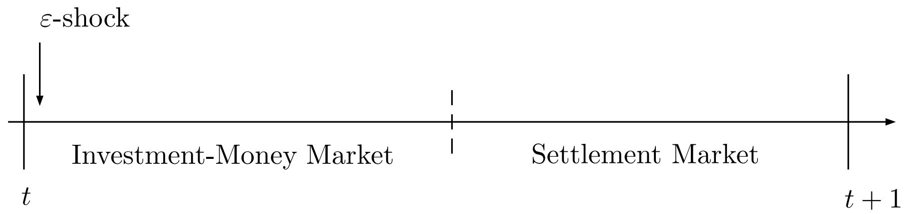

Figure 1: Environment

Bankers are willing to accept liabilities of other bankers at par. Deposits held across periods earn an exogenous nominal rate i_d in the settlement market. In Section 5, we endogenize the deposit rate and introduce cash as an outside option for households.

Preferences of households are given by the flow utility function

$$\mathcal{U}(k, x, h) = -k + x - h.$$

Here, x and h are consumption and production, respectively, of the general good by the household, and k is the households' production of capital goods.

Preferences of bankers are given by the flow utility function

$$\mathcal{V}_\varepsilon(k, x, h) = \varepsilon^{1/\alpha} f(k) + x - h,$$

which can be interpreted as a banker's net income. Specifically, bankers receive returns $\varepsilon^{1/\alpha} f(k)$ from investing k units of capital. The production function satisfies $f(k) = \frac{k^{1-1/\alpha}}{1-1/\alpha}$, with $\alpha > 1$, and ε is an idiosyncratic investment shock. This shock has a continuous distribution $G(\varepsilon)$ with support $[0, \infty]$, and the shock is i.i.d. across bankers and is serially uncorrelated. The main purpose of the investment shock is to generate a distribution of reserve holdings across bankers and to study how NIR affect investment decisions for different values of ε. Also, x and h are consumption and production, respectively, of the general good by the banker. Production (consumption) in this context can be interpreted as increasing (resp. decreasing) a bankers' equity.

Finally, bankers and households discount future utility at the common rate $\beta = (1 + r)^{-1} < 1$, where r is the time rate of discount.

First-best allocations. Due to the quasi-linear preference structure, production and consumption of general goods are irrelevant for aggregate welfare. First-best allocations however require that the marginal return from an investment project equals the marginal cost of capital. That is,

$$\varepsilon^{1/\alpha} f'(k) = 1.$$

Solving for k_ε yields

$$k_\varepsilon^* = \varepsilon.$$

Thus, the first-best investment quantities satisfy $k_\varepsilon^* = \varepsilon$ for all ε. Note that $\varepsilon^{1/\alpha} f(k_\varepsilon^*) - k_\varepsilon^* \geq 0$ for all ε. The implication is that from a societal point of view, all projects should be implemented.

In what follows, we consider matters in a market economy. We first discuss the specific structure of the settlement market and the IM market, and the associated decisions of our agents in these two markets. Then, we define equilibrium and characterize welfare.

3.1 Settlement Market

In the settlement market, the general good trades at nominal price P_t^x. We define $\phi_t \equiv 1/P_t^x$ and focus on steady states in which all nominal prices grow at a constant rate between periods.

3.1.1 Treasury Department

In the settlement market, the treasury controls the supply of nominal bonds $B_{TR,t+1}$. These bonds mature in the time $t+1$ settlement market and pay an endogenous interest rate $i_{b,t}$, which is constant in steady state. The treasury also receives the profits from the central bank's operations and it can provide lump-sum subsidies to (or levy lump-sum taxes on) households and bankers. We focus on steady-state policies and let γ denote the gross rate at which bond supply grows between periods. We write $b_{TR} = \phi_t B_{TR,t}$ for the real value of maturing bonds, which is constant in steady state. In the steady state all nominal prices in the economy therefore grow at a gross rate γ, in particular $\gamma\phi_{t+1} = \phi_t$. We denote with τ_H (τ_B) a real lump-sum transfer received by all households (resp. bankers) during the settlement market, and we denote with π_{CB} the central bank's profits in real terms. The treasury's budget constraint implies

$$\tau_H + \tau_B = b_{TR}[\gamma - (1 + i_b)] + \pi_{CB}. \tag{1}$$

3.1.2 Central Bank

In the settlement market, the central bank controls the nominal stock of reserves M_{t+1} carried into the next period through purchasing treasury bonds.[14] We write $B_{t+1} = B_{TR,t+1} - M_{t+1}$ for the amount of bonds supplied to the private sector, i.e., net of central bank purchases, and we write $\eta_{t+1} = M_{t+1}/B_{t+1}$ for the reserves-to-bonds ratio.

[14]This setup implies that the entire stock of reserves is backed by treasury bonds.

This ratio is controlled by the central bank by means of open-market operations. The ratio can take any positive value, but has to be constant in the steady state.

The central bank sets an exemption threshold $\overline{M}_t$, and pays nominal interest rate $i_{p,t}$ on reserves below the exemption threshold and $i_{n,t}$ on reserves above the exemption threshold. A banker that carries $\widehat{M}_{\varepsilon,t}$ units of reserves into the settlement market, where we index bankers according to the shocks drawn in the IM market, therefore receives a net interest payment

$$\mathcal{P}_{\varepsilon,t} = \begin{cases} i_{p,t}\widehat{M}_{\varepsilon,t} & \text{if } \widehat{M}_{\varepsilon,t} \leq \overline{M}_t \\ i_{p,t}\overline{M}_t + i_{n,t}\left(\widehat{M}_{\varepsilon,t} - \overline{M}_t\right) & \text{if } \widehat{M}_{\varepsilon,t} > \overline{M}_t \end{cases}$$

from the central bank. For most parts of our policy analysis, we think of $i_{n,t}$ to be negative (the NIR) and $i_{p,t}$ to be equal to zero to capture the current interest rate policies in NIR countries. However, the equations that follow allow for all cases in which $i_{n,t} \leq i_{p,t}$.

Focusing on a steady state, η is constant over time so that the real value of reserves is also constant. We assume that there is a positive initial stock of assets such that $M_0 = \eta B_0$, and that the exemption threshold grows at the same rate as the stock of reserves.[15] It follows that the remaining policy variables of the central bank are time-invariant interest rates i_n and i_p, and an initial exemption threshold $\overline{M}_0$.

Let $m = \phi_t M_t$ denote the aggregate value of reserves carried into the settlement market, let $\bar{m} = \phi_t \overline{M}_t$ denote the exemption threshold expressed in real terms, and let $\hat{m}_\varepsilon = \phi_t \widehat{M}_{\varepsilon,t}$ denote the real value of reserves carried into the settlement market by a banker that faced investment shock ε. These quantities are all constant in steady state. Given its policies and the distribution of reserves across bankers, the central bank's real profits during the settlement market, denoted with π_{CB}, are given by:

$$\pi_{CB} = i_b m - \int_0^\infty [i_p \min\{\hat{m}_\varepsilon, \bar{m}\} + i_n \max\{\hat{m}_\varepsilon - \bar{m}, 0\}] \mathrm{d}G. \tag{2}$$

That means, profits of the central bank equal seignorage revenues minus aggregate interest payments to bankers.

3.1.3 Households

To ensure that households' problem is recursive, we characterize value functions for real quantities. Consider a household that enters the settlement market with $\hat{d}$ deposits,

[15] Since the assets are nominal objects, the government and the central bank can start the economy off with one-time injections of reserves M_0 and bonds $B_{TR,0}$.

expressed in terms of general goods. Let d denote the real amount of deposits that the household carries out of the settlement market, evaluated at the next period's price of general goods. Also, let $W_{IM}(d)$ denote the utility value of entering the next period's IM market with deposits worth d general goods. Without loss, we assume that households do not hold bonds.[16] Defining $\rho_d = 1/(1+i_d)$ and assuming that households cannot short-sell deposits, we obtain the following indirect utility function:

$$\begin{aligned} &W_S(\hat{d}) = \max_{x,h,d} \{x - h + \beta W_{IM}(d)\} \\ \text{s.t.} \quad &x + \gamma d \leq h + \hat{d}/\rho_d + \tau_H, \quad x \geq 0, \quad h \geq 0, \quad \text{and} \quad d \geq 0. \end{aligned}$$

Because the household's budget constraint will always hold with equality, we can eliminate x and h to obtain a value function that is linear in $\hat{d}$:

$$W_S(\hat{d}) = \max_{d \geq 0} \{-\gamma d + \beta W_{IM}(d)\} + \hat{d}/\rho_d + \tau_H. \tag{3}$$

Conjecture that $W_{IM}(d)$ is continuously differentiable and concave, and let $W_{IM}^d = \partial W_{IM}/\partial d$. We obtain the following necessary and sufficient condition for optimality of deposits carried out of the settlement market:

$$0 \geq -\gamma + \beta W_{IM}^d(d), \quad \text{with equality if } d > 0. \tag{4}$$

3.1.4 Bankers

To ensure that bankers' problem is recursive, we again characterize value functions for real quantities. Consider a banker that enters the settlement market with bonds worth $\hat{b}$ general goods, reserves worth $\hat{m}$ general goods, reserves borrowed from other bankers during the preceding IM market worth $\hat{z}$ general goods, and deposits worth $\hat{d}$ general goods. Define $\rho_b = 1/(1+i_b)$, $\rho_n = 1/(1+i_n)$, $\rho_p = 1/(1+i_p)$, and $\rho_m = 1/(1+i_m)$, where i_m is the nominal interest rate for borrowing reserves from other bankers in the IM market. Let m and b denote the real amount of reserves and bonds, respectively, that the banker carries out of the settlement market, evaluated at the next period's price of general goods. We impose a non-negativity constraint on m and b to capture that bankers cannot short these assets. Also, let d be the real amount of deposits on the banker's balance sheet at the end of the settlement market, again evaluated at the next period's price of general goods. Finally, let $V_{IM}(m, b, d|\varepsilon)$ denote the banker's indirect utility function of entering the next IM market with reserves, bonds, and deposits worth

[16] Bankers can use bonds as collateral whereas households can only use bonds as a savings instrument. Bankers are therefore willing to incur a lower interest rate on bonds compared to households.

m, b, and, respectively, d general goods, conditional on receiving investment shock ε. For the banker, we obtain the following indirect utility function associated with entering the settlement market:

$$V_S(\hat{m}, \hat{z}, \hat{b}, \hat{d}) = \max_{x,h,m,b} \left\{ x - h + \beta \int_0^\infty V_{IM}(m, b, d|\varepsilon) \mathrm{d}G \right\}$$
$$\text{s.t.} \qquad x + \frac{\hat{d}}{\rho_d} + \gamma(m + b - d) \leq h - \frac{\hat{z}}{\rho_m} + \frac{\min\{\hat{m}, \bar{m}\}}{\rho_p} + \frac{\max\{\hat{m} - \bar{m}, 0\}}{\rho_n} + \frac{\hat{b}}{\rho_b} + \tau_B,$$
$$x \geq 0, \quad h \geq 0, \quad m \geq 0, \quad \text{and} \quad b \geq 0.$$

Because the banker's budget constraint will always hold with equality, we can eliminate x and h to obtain a value function that is linear in $\hat{m}$, $\hat{z}$, $\hat{b}$, and $\hat{d}$:

$$V_S(\hat{m}, \hat{z}, \hat{b}, \hat{d}) = \max_{m \geq 0, b \geq 0} \left\{ -\gamma(m + b - d) + \beta \int_0^\infty V_{IM}(m, b, d|\varepsilon) \mathrm{d}G \right\} - \hat{d}/\rho_d - \hat{z}/\rho_m$$
$$+ \min\{\hat{m}, \bar{m}\}/\rho_p + \max\{\hat{m} - \bar{m}, 0\}/\rho_n + \hat{b}/\rho_b + \tau_B. \quad (5)$$

Conjecture that $V_{IM}(m, b, d|\varepsilon)$ is continuously differentiable and concave in m and b, and let $V_{IM}^m = \partial V_{IM}/\partial m$ and $V_{IM}^b = \partial V_{IM}/\partial b$. We then obtain the following necessary and sufficient conditions for optimality of assets carried out of the settlement market:

$$m: \quad 0 \geq \beta \int_0^\infty V_{IM}^m(m, b, d|\varepsilon) \mathrm{d}G - \gamma, \qquad \text{with equality if } m > 0, \quad (6)$$
$$b: \quad 0 \geq \beta \int_0^\infty V_{IM}^b(m, b, d|\varepsilon) \mathrm{d}G - \gamma, \qquad \text{with equality if } b > 0. \quad (7)$$

3.2 Investment-Money Market

During the IM market, bankers and households can trade capital goods at nominal price P_t^k. Let $p = P_t^k / P_t^x$ denote the relative price of capital goods in terms of general goods, which is constant in steady state. We assume that each household produces the same amount of capital k_s and that each banker has the same customer base. By producing capital, households therefore acquire a portfolio of deposits that is evenly distributed across bankers. In Section 5, we shall allow households to convert these deposits into cash. Besides acquiring capital, bankers can lend or borrow reserves in a money market.

The demand for reserves in our model arises from bankers' acquisition of capital goods. Particularly, a banker purchases capital goods by creating deposits, but because households prefer an even distribution of deposits across bankers, households transfer the created deposits to other bankers. This creates liabilities vis-à-vis the other bankers, which have to be settled with reserves.[17]

3.2.1 Households

The linearity of $W_S(d)$ implies that $W_S(d) = d/\rho_d + W_S(0)$. Using this property, the household's indirect utility function of entering the IM market with deposits worth d general goods, is given by:

$$W_{IM}(d) = \max_{k_s \geq 0} \{-k_s + [pk_s + d]/\rho_d\} + W_S(0). \tag{8}$$

It follows that households are indifferent with respect to the amount of capital that they want to supply if and only if

$$p = \rho_d. \tag{9}$$

Moreover, $W_{IM}(d)$ is continuously differentiable and concave, and $W^d_{IM}(d) = 1/\rho_d$.

3.2.2 Bankers

During the IM market, all bankers face an inflow pk_s of deposits from households, which the bankers take as given. Because these deposits are made by households that produced capital for other bankers, the inflow of deposits generates an associated inflow pk_s of reserves. We assume that this inflow takes place just after the IM market has convened.[18]

Consider a banker that has drawn investment shock ε (henceforth the ε-banker). Let m_ε denote the real value of reserves carried out of the IM market by this banker. Note that this quantity is subject to a non-negativity constraint, as reserves cannot be shorted:

$$m_\varepsilon \geq 0. \tag{10}$$

The ε-banker's real reserve holdings at the beginning of the settlement market are then given by $m_\varepsilon + pk_s$. In the money market, the ε-banker can borrow reserves from (or lend reserves to) other bankers at a competitive nominal rate i_m. Let z_ε denote the real value of net reserves borrowed in the money market by the ε-banker. We assume, in line with the operation of interbank markets in many NIR countries, that bankers must pledge collateral to borrow reserves.[19] A banker can pledge a fraction $\chi \in [0, 1]$ of its bond holdings and a fraction σ of its reserves carried out the IM market as collateral. Also, a fraction $\theta \in [0, 1]$ of the reserves acquired just after the IM has convened can

[17] This assumption serves to mimic that commercial banks use reserves to settle interbank liabilities.

[18] We find that the timing of this inflow is irrelevant for our results.

[19] Berentsen and Monnet (2008) and Berentsen et al. (2018) briefly describe the operation of money markets in the Euro Area and, respectively, Switzerland. Like us, they assume collateralized lending.

be pledged as collateral.[20] The interpretation of $\chi, \sigma, \theta < 1$ is a haircut on the value of assets pledged as collateral, for example because the banker can abscond with some of these assets in case it chooses to default on its obligations. With bond holdings worth b general goods, we obtain

$$z_\varepsilon \leq \chi b + \sigma m_\varepsilon + \theta p k_s. \tag{11}$$

Suppose that the ε-banker enters the IM market with m real reserve holdings and d real deposits. When k_ε denotes the amount of capital goods acquired by the ε-banker, we obtain as budget constraint

$$m_\varepsilon + p k_\varepsilon \leq m + z_\varepsilon,$$

reflecting that the banker has to cover the acquisition of capital goods, which generates liabilities vis-à-vis the other bankers, with reserves. Because profit maximizing bankers do not leave resources on the table, the budget constraint must hold with equality. Combining with the borrowing constraint, we find

$$p k_\varepsilon + m_\varepsilon(1-\sigma) \leq \chi b + \theta p k_s + m. \tag{12}$$

Exploiting the linearity of $V_S(\hat{m}, \hat{z}, \hat{b}, \hat{d})$ in Equation (6), the ε-banker faces the following indirect utility function:

$$V_{IM}(m, b, d|\varepsilon) = \max_{k_\varepsilon, m_\varepsilon} \left\{ \begin{array}{l} \varepsilon^{1/\alpha} \dfrac{k_\varepsilon^{1-1/\alpha}}{1-1/\alpha} + V_S(0) + \dfrac{b}{\rho_b} - \dfrac{d + p k_s}{\rho_d} - \dfrac{m_\varepsilon + p k_\varepsilon - m}{\rho_m} \\ + \dfrac{\min\{m_\varepsilon + p k_s, \bar{m}\}}{\rho_p} + \dfrac{\max\{m_\varepsilon + p k_s - \bar{m}, 0\}}{\rho_n} \end{array} \right\}, \tag{13}$$

subject to (10) and (12). We can ignore a non-negativity constraint for k_ε, as this constraint will never bind.

Let μ_ε and λ_ε be the Lagrange multipliers associated with (10) and (12), respectively. First-order conditions for the ε-banker are then given by

$$k_\varepsilon: \quad 0 = (\varepsilon/k_\varepsilon)^{1/\alpha} - p(1/\rho_m + \lambda_\varepsilon) \tag{14}$$

$$m_\varepsilon: \quad 0 \geq -1/\rho_m + \mathcal{I}_+/\rho_n + (1-\mathcal{I}_+)/\rho_p - (1-\sigma)\lambda_\varepsilon + \mu_\varepsilon \tag{15}$$

$$0 \leq -1/\rho_m + \mathcal{I}_-/\rho_n + (1-\mathcal{I}_-)/\rho_p - (1-\sigma)\lambda_\varepsilon + \mu_\varepsilon \tag{16}$$

[20] Because we did not introduce cash as an outside option for households yet, the idea here is that all bankers rationally expect an inflow pk_s of reserves after the IM market has convened. Bankers are therefore willing to lend reserves to other bankers in the IM market if they are able to seize (part of) the anticipated inflow pk_s in case their counter party defaults.

where

$$\mathcal{I}_+ = \begin{cases} 1 & \text{if } m_\varepsilon + pk_s - \bar{m} \geq 0 \\ 0 & \text{if } m_\varepsilon + pk_s - \bar{m} < 0 \end{cases} \quad \text{and} \quad \mathcal{I}_- = \begin{cases} 1 & \text{if } m_\varepsilon + pk_s - \bar{m} > 0 \\ 0 & \text{if } m_\varepsilon + pk_s - \bar{m} \leq 0 \end{cases}.$$

In Equations (15) and (16), we take into account that a small increase in m_ε may imply that a banker becomes subject to NIR. Equation (15) imposes that a marginal increase in m_ε should not make the bank better off, with $\mathcal{I}_+ = 0$ if the banker remains exempted from NIR for a marginal increase m_ε and $\mathcal{I}_+ = 1$ otherwise. Analogously, Equation (16) imposes that a marginal decrease in m_ε should not make the banker better off, with $\mathcal{I}_- = 1$ if a marginal decrease in m_ε leaves the banker subject to NIR and $\mathcal{I}_- = 0$ otherwise. It can be verified that $V_{IM}(m, b, d|\varepsilon)$ is concave in m and b and continuously differentiable in m, b, d, and ε. Also,

$$V_{IM}^m(m, b, d|\varepsilon) = \frac{1}{p}\left(\frac{\varepsilon}{k_\varepsilon}\right)^{1/\alpha} \quad \text{and} \quad V_{IM}^b(m, b, d|\varepsilon) = \chi\left[\frac{1}{p}\left(\frac{\varepsilon}{k_\varepsilon}\right)^{1/\alpha} - \frac{1}{\rho_m}\right] + \frac{1}{\rho_b}. \tag{17}$$

3.3 Equilibrium and Welfare

Having derived agents' optimal decisions given prices, we can now define what constitutes an equilibrium in our decentralized economy. We focus on steady-state equilibria with a strictly positive demand for reserves. In equilibrium, the aggregate demand for capital goods by bankers should equal the production of capital goods by households. Moreover, the aggregated net amount of reserves borrowed by bankers in the IM market should equal zero. Together with agents' optimal decisions, we find:

Definition 1 *Given policy $(\gamma, \eta, \rho_p, \rho_n, \overline{M}_0)$ and an initial supply of reserves and bonds satisfying $M_0 = \eta B_0$, equilibrium is a tuple of real quantities $\{m, b_{TR}, b, d, k_s, k_\varepsilon, m_\varepsilon\}$ and a pair of prices $\{p, \rho_m, \rho_b\}$ such that:*

1. *Markets clear: $m = \eta b$, $m = b_{TR} - b$, $k_s = \int_0^\infty k_\varepsilon \mathrm{d}G$, and $0 < m = \int_0^\infty m_\varepsilon \mathrm{d}G + pk_s$.*
2. *Bankers maximize profits: with $\overline{m}M_0 = m\overline{M}_0$, m and b solve (6) and (7), and $\{m_\varepsilon, k_\varepsilon\}$ solves (13) subject to (10) and (12).*
3. *Households maximize utility: d solves (4) and k_s solves (8).*

Proposition 2 *Steady-state welfare $\mathcal{W}$ satisfies*

$$(1 - \beta)\mathcal{W} = \int_0^\infty \left[\varepsilon^{1/\alpha} f(k_\varepsilon) - k_\varepsilon\right] \mathrm{d}G. \tag{18}$$

Because households' and bankers' flow utility function is linear in production and consumption of general goods, only capital investment matters for welfare.

4 Equilibrium in a Baseline Economy

We now solve for a baseline model characterized by $\sigma = 1$. We see this as a reasonable case, since reserves are record-keeping entries at the central bank and therefore good collateral. That means, there is no haircut imposed on them so that bankers can redistribute reserves among each other in order to minimize NIR payments. In appendix B, we show that our results hold true for the case with $\sigma < 1$.

We start the analysis by noting that households can only hold one asset, i.e., deposits, and that they only use deposits as a store of value. Recall, that the deposit rate i_d is exogenous in the baseline economy (we will introduce cash and endogenize the deposit rate in Section 5). Combining the first-order condition for deposits carried out of the settlement market (4) with $W_{IM}^d(d) = 1/\rho_d$, we find that households are willing to carry deposits out of the settlement market only if $\gamma\rho_d \leq \beta$. When $\gamma\rho_d < \beta$, households want to carry infinitely many deposits out of the settlement market, as the real return earned by deposits exceeds households' rate of time preference. We therefore need $\gamma\rho_d \geq \beta$ for equilibrium existence. Without loss, we can then focus on equilibria in which households acquire deposits only when they produce capital during the IM market.

Investment Market. For an equilibrium in the investment market, we need that demand for capital goods equals supply of capital goods. This requires that Equation (9) holds, as otherwise households do not want to supply any capital goods (i.e., when $\rho_d > p$) or want to supply infinitely many capital goods (i.e., when $\rho_d < p$). Using $p = \rho_d$ in bankers' first-order condition for capital investment (14) and noting that $b = m/\eta$, we obtain the following:

Lemma 3 *There exist a critical value ε' which uniquely solves*

$$\varepsilon' = \left(\frac{\rho_d}{\rho_m}\right)^{\alpha} \frac{m}{\rho_d}\frac{\chi+\eta}{\eta} + \theta\left[\int_0^{\varepsilon'} \varepsilon \mathrm{d}G + \int_{\varepsilon'}^{\infty} \varepsilon' \mathrm{d}G\right], \tag{19}$$

such that the quantities of capital invested satisfy

$$k_\varepsilon = \begin{cases} \varepsilon\left(\rho_m/\rho_d\right)^{\alpha} & \text{if } \varepsilon \leq \varepsilon' \\ \varepsilon'\left(\rho_m/\rho_d\right)^{\alpha} & \text{if } \varepsilon > \varepsilon' \end{cases}. \tag{20}$$

The intuition for Lemma 3 is as follows. Bankers determine how much of their disposable reserves to use for acquiring capital at relative price ρ_d and how much reserves to lend or borrow in the money market at relative price $1/\rho_m$. Ideally, bankers equate the marginal product of capital investment $(\varepsilon/k_\varepsilon)^{1/\alpha}$ to the opportunity cost of capital investment ρ_d/ρ_m. Bankers with productivity $\varepsilon \leq \varepsilon'$ are able to do so. These bankers' unconstrained amount of investment, $k_\varepsilon = \varepsilon(\rho_m/\rho_d)^\alpha$, is sufficiently small so that it can be financed with $m + \chi b + \theta\rho_d k_s$ reserves. This is not true for bankers with $\varepsilon > \varepsilon'$. These bankers exhaust their borrowing capacity to finance capital investment, as the marginal product of investment then still exceeds the opportunity cost.

Money Market. We first note that the money market rate cannot fall short of NIR. Otherwise, bankers want to borrow an infinitely large amount of reserves in the money market to earn at least $i_n > i_m$ on these reserves.

Next, we note that clearance of the money market requires $\int_0^\infty z_\varepsilon \mathrm{d}G = \int_0^\infty [m_\varepsilon + \rho_d k_\varepsilon - m]\mathrm{d}G = 0$, so that gross borrowing equals gross lending. Because of the non-negativity constraint on reserves carried out of the IM market (10), in an equilibrium we have

$$\int_0^\infty k_\varepsilon \mathrm{d}G \leq \frac{m}{\rho_d}. \tag{21}$$

Using Equations (19) and (20) from Lemma 3, we can rewrite Equation (21) as:

$$\varepsilon' \geq \underline{\varepsilon}, \text{ where } \underline{\varepsilon} = \left(\frac{\chi + \eta}{\eta} + \theta\right)\left[\int_0^{\underline{\varepsilon}} \varepsilon \mathrm{d}G + \int_{\underline{\varepsilon}}^\infty \underline{\varepsilon} \mathrm{d}G\right] \text{ and } \underline{\varepsilon} > 0 \text{ if } \chi > 0 \text{ or } \theta > 0. \tag{22}$$

Here, $\underline{\varepsilon}$ is strictly increasing in θ and χ, and strictly decreasing in η. Intuitively, the lower the haircuts, the less likely it is that the borrowing constraint binds for a given ε. Consequently, the lower bound for ε' is increasing in the pledgability parameters θ and χ. Moreover, we have that $\chi = \theta = 0$ implies $\underline{\varepsilon} = 0$ as well as $\lim_{\chi\to 0, \theta\to 0} \underline{\varepsilon} = 0$.

Equation (22) imposes a lower bound on ε'. Recall that bankers with a binding borrowing constraint have $\varepsilon \geq \varepsilon'$. Equation (22) can therefore be interpreted as an upper bound on the measure of constrained bankers. Keeping the money market rate constant, a large measure of constrained bankers (so that ε' is small) implies a large demand for reserves in the money market. In turn, because there are few bankers facing a slack borrowing constraint, the amount of reserves that can be supplied to the money market is small. When $\varepsilon' < \underline{\varepsilon}$, demand inevitably exceeds supply. To attain an equilibrium in the money market, the money market rate then has to increase until $\varepsilon' = \underline{\varepsilon}$. This follows from Equation (19), which imposes a positive relationship between the money market rate and ε'. The reason is that a higher money market rate implies a higher opportunity cost of capital investment, so that bankers' optimal amount of unconstrained investment declines. In turn, keeping the value of bankers' asset fixed, this implies a smaller measure of bankers facing tight borrowing constraints and thus a decline in the demand for reserves.

On the other hand, with a sufficiently small measure of constrained bankers (so that $\varepsilon' > \underline{\varepsilon}$), the amount of reserves that can be supplied to the money market exceeds the demand for reserves. To clear the money market, either the money market rate should decrease or the unconstrained bankers should hold excess reserves at accounts with the central bank, i.e., carry reserves out of the money market. As we shall uncover next, excess reserves have important implications for the transmission of policy rates to the money market rate.

Define $\bar{m}' = \max\{\bar{m} - \rho_d k_s, 0\}$, which is the minimal amount of reserves that a banker needs to carry out of the IM market to become subject to NIR. Using our first-order conditions for m_ε (15 and 16), we find that:

Lemma 4 *Reserves carried out of the IM market by the ε-banker satisfy:*

$$m_\varepsilon \begin{cases} \leq \bar{m}' & \text{if } i_n < i_m \leq i_p \\ = 0 & \text{if } i_m > i_p \geq i_n \end{cases} \quad \text{and} \quad m_\varepsilon \geq \begin{cases} \bar{m}' & \text{if } i_n \leq i_m < i_p \\ 0 & \text{if } i_m \geq i_p \geq i_n \end{cases}. \tag{23}$$

Lemma 4 is based on the fact that if the money market rate falls short of the interest rate on exempted reserves, bankers want to carry at least $\bar{m}'$ reserves out of the IM market. Otherwise, when $\bar{m}' > 0$ bankers can make a profit by borrowing more reserves in the IM market at rate i_m and holding them at accounts with the central bank that earn $i_p > i_m$. Additionally, only when the money market rate equals NIR, bankers are willing to carry more than $\bar{m}'$ reserves out of the IM market. Similarly, only when the money market rate falls short of or equals the interest rate on exempted reserves, bankers are willing to carry a strictly positive amount of reserves out of the IM market.

Finally, using that clearance of the investment market implies $\int_0^\infty k_\varepsilon \mathrm{d}G = k_s$, we find:

Proposition 5 *With a competitive money market, there is full pass through of the policy rate i_n to the money market rate i_m if $\bar{m} < m$ and*

$$\frac{m}{\rho_d} \geq \left(\frac{\rho_n}{\rho_d}\right)^\alpha \left[\int_0^{\underline{\varepsilon}} \varepsilon \mathrm{d}G + \int_{\underline{\varepsilon}}^\infty \underline{\varepsilon} \mathrm{d}G\right]. \tag{24}$$

In the proof of Proposition 5, we first show that there is a floor on the equilibrium money market rate:

$$\rho_m \leq \begin{cases} \rho_n & \text{if } \bar{m} \leq m \\ \rho_p & \text{if } \bar{m} > m \end{cases}. \tag{25}$$

What determines the floor in Equation (25), is whether bankers can avoid the NIR by redistributing reserves in the IM market. This is the case when the exemption threshold exceeds the aggregate supply of reserves, as bankers can then distribute reserves in the money market so that no banker is subject to NIR. Should the money market rate then fall short of i_p, all bankers want to enter the settlement market with reserves at or beyond the exemption threshold (see Lemma 4) while this cannot be the case in equilibrium.

Second, we show that whether the floor in Equation (25) is attained, depends on the borrowing constraint and the value of reserves carried into the IM market:

$$\frac{m}{\rho_d} \geq \left(\frac{\rho_m}{\rho_d}\right)^{\alpha} \left[\int_0^{\underline{\varepsilon}} \varepsilon \mathrm{d}G + \int_{\underline{\varepsilon}}^{\infty} \underline{\varepsilon} \mathrm{d}G\right], \text{ with } = \text{ if } \rho_m < \begin{cases} \rho_n & \text{if } \bar{m} < m \\ \rho_p & \text{if } \bar{m} \geq m \end{cases}. \tag{26}$$

Specifically, when the money market rate exceeds the floor, no banker carries reserves out of the IM market. In that case, $\varepsilon' = \underline{\varepsilon}$ and the money market rate is determined by Equation (26), which must hold with equality to ensure that supply of reserves equals demand for reserves. However, when the value of reserves carried into the IM market and the money-to-bonds ratio are large and the pledgeability parameters χ and θ are small (meaning that $\underline{\varepsilon}$ is small), Equation (26) can only hold with equality if the money market rate drops below the floor derived in Equation (25). Recall that small values for parameters θ and χ imply a tighter borrowing constraint. Additionally, if the money-to-bonds ratio is large, bonds are scarce, which also contributes to a tighter borrowing constraint. In that case, the money market can only clear if some bankers carry reserves out of the IM market and we thus obtain an environment with excess reserves. In turn, this requires the money market rate to be at the floor in Equation (25).

Proposition 5 has important policy implications: The exemption threshold can be chosen arbitrarily close to the stock of reserves without affecting the transmission of NIR to the money market rate. After all, as long as $\bar{m} < m$, the money market rate is determined uniquely by the equilibrium value of reserves carried into the IM market. Moreover, an increase of $\bar{m}$ obviously reduces the interest rate payments of bankers to the central bank. In the limit as $\bar{m} \to m$, these payments become arbitrarily small and hence, negatively remunerated reserves do not affect bankers' profitability.[21] We discuss

[21] It is important to note that here, we focus on bankers' reserves holdings. In reality, reserves (and money market lending) only represents a sub-set of banks' investment opportunities and, depending on the transmission of NIR, they are also subject to the NIR and thus matter for profitability.

the implications of NIR and exemption thresholds for the equilibrium value of reserves and profitability further below.

Equilibrium. We focus on symmetric stationary equilibria with a strictly positive demand for reserves, full pass-through of NIR to the money market rate, an exogenous deposit rate, and at least some bankers that are subject to the NIR.[22] Such equilibria meet the requirements in Definition 1 and exist only if $\bar{m} < m$. Combining Lemma 3 with (17) and (6), we find:

Proposition 6 *For $\bar{m} < m$, a symmetric stationary equilibrium with a positive demand for reserves and full pass-through of the NIR to the money market rate is sufficiently described by an $\varepsilon' \geq \underline{\varepsilon}$ that solves:*

$$\frac{\gamma \rho_n}{\beta} = \int_0^{\varepsilon'} \mathrm{d}G + \int_{\varepsilon'}^{\infty} \left(\varepsilon/\varepsilon'\right)^{1/\alpha} \mathrm{d}G. \tag{27}$$

The LHS of Equation (27) captures bankers' ex-ante cost of financing capital investment, taking into account that this requires holding reserves. The RHS of Equation (27) governs bankers' expected marginal return on capital investment. To equate the costs of carrying reserves to the expected marginal benefits, ε' is determined endogenously. All equilibrium quantities and prices can then be derived as follows. First, Proposition 5 yields $\rho_m = \rho_n$. Second, Equation (7) and (17) imply $1/\rho_b = \chi/\rho_m + (1-\chi)\gamma/\beta$; the money market rate is partially passed through to bonds, with the degree of pass-through increasing in the pledgeability parameter χ. Third, the relative price of capital p is given by Equation (9); $p = \rho_d$. Fourth, Lemma 3 yields the real value of reserves m and capital investment k_ε for each ε-banker. Fifth, supply of capital goods satisfies $k_s = \int_0^\infty k_\varepsilon \mathrm{d}G$. Sixth, reserve holdings at the end of the IM market m_ε are given by Lemma 4. Seventh, the value of bonds supplied by the treasury and the value of bonds held by bankers follow from $b_{TR} = m + b$ and $b = m/\eta$.

Because the RHS of Equation (27) is strictly decreasing in ε', an equilibrium with full pass-through of NIR to the money market rate is characterized by a unique ε' and exists if and only if

$$1 \leq \frac{\gamma \rho_n}{\beta} \leq \int_0^{\underline{\varepsilon}} \mathrm{d}G + \int_{\underline{\varepsilon}}^{\infty} (\varepsilon/\underline{\varepsilon})^{1/\alpha} \mathrm{d}G. \tag{28}$$

Equation (28) imposes a threshold on i_n, below which the NIR no longer passes through to the money market rate. The reason is that the equilibrium value of reserves carried into the IM market then becomes sufficiently low so that the money market rate cannot

[22] We endogenize the deposit rate in Section 5.

be at the floor in Equation (25). The threshold for i_n is decreasing in the money-to-bonds ratio, which indicates that the central bank can lower this threshold by means of quantitative easing—purchasing treasury bonds to drive up the money-to-bonds ratio. The reason is that by purchasing bonds, the central bank floods the banking system with liquidity, in turn making it more likely that the floor on the money market rate is reached. The importance of quantitative easing for the implementation of low interest rates is in line with observations from NIR countries. For instance, the implementation of NIR has gone along with large expansions in the nominal supply reserves, often due to asset purchasing programs like quantitative easing (see Section 2).[23,24]

When away from the floor, i.e., when the money-to-bonds ratio is sufficiently small, the equilibrium money market rate satisfies

$$\frac{\gamma\rho_m}{\beta} = \int_0^{\underline{\varepsilon}} \varepsilon \mathrm{d}G + \int_{\underline{\varepsilon}}^{\infty} (\varepsilon/\underline{\varepsilon})^{1/\alpha}\, dG, \tag{29}$$

which together with $\varepsilon' = \underline{\varepsilon}$ pins down allocations. Also, from (6)-(7) and (17) it follows that we again have $1/\rho_b = \chi/\rho_m + (1-\chi)\gamma/\beta$. Away from the floor, small changes in NIR do not affect allocations as NIR is not passed through to the money market rate. However, an increase in the money-to-bonds ratio (quantitative easing) reduces the money market rate, and therefore also the nominal interest rate earned by bonds. Once the money market rate is a the NIR, a further increase in the money-to-bonds ratio is irrelevant for real economic activity; quantitative easing only raises the aggregate real value of reserves and reduces the aggregate real value of bonds held by bankers. When $\chi < 1$, an increase in η also reduces b_{TR}; the aggregate real value of bonds supplied by the treasury. All other real variables remain unaffected by a change in η.

4.1 Effects of NIR in Baseline Economy

In this section, we discuss the long-run effects of NIR for investment decisions and welfare. A key insight of the model is that the effects of NIR depend on the transmission of the money market rate i_m to deposit rate i_d. Evidence of the relevance for the deposit channel of monetary policy can be found in Drechsler et al. (2017). Here, we do not provide a theory about how the deposit rate is determined since empirically we observe a wide range of deposit rate behavior.[25] Rather, we assume that the deposit rate i_d is

[23] Though our model only includes government bonds, it can be established that purchasing any type of asset used as collateral by bankers, helps in driving down the effective lower bound on i_n.

[24] These results are related to Williamson (2012), where open-market operations have no effect on prices in a liquidity trap with excess money and Williamson (2016) where quantitative easing increases the value of collateralizable debt and therefore loosens borrowing constraints.

[25] The transmission of NIR is widely discussed in the literature (see Section 6).

exogenous and provide results for two competing assumptions: perfect transmission and imperfect transmission. Under perfect transmission, we assume that the deposit rate adjusts immediately to a change in i_m so that at any point of time $i_d = i_m$. Under imperfect transmission, we assume that the deposit rate does not adjust to a change in i_m. Furthermore, we distinguish between two cases of imperfect transmission. In the first case $i_d > i_m$. We refer to this case as the NIR case because we observe empirically in all NIR countries that the deposit rates are above the money market rates. In the second case $i_d < i_m$. We refer to this case as the US case because we observe for the US that the federal funds rate is positive while the deposit rates are at zero.

In Section 5, we provide an extension that endogenizes the deposit rate i_d and that introduces cash as an outside option for depositors. The transmission of policy rates to the real economy in the extended model is equivalent to the perfect transmission case in the model presented here when $i_n \geq 0$ and to the imperfect transmission NIR case when $i_n < 0$. In this sense, the extension supports a view of NIR policies having qualitatively different effects compared to regular reductions in policy rates. Note, the extension in Section 5 cannot provide a micro-foundation for the US case discussed here. We nevertheless find the discussion of the US case useful, as there exist broad empirical evidence of a limited transmission of money market rates to retail deposits in positive interest rate periods, such that $i_m > i_d$.

To guide our discussion, first observe that NIR only affects allocations when there is pass-through of NIR to money market rates. Otherwise, the money market rate is determined independently of NIR by Equation (29). Hence, to study effects of NIR we can restrict attention to an equilibrium described by Proposition 6. Lemma 7 characterizes two additional critical values.

Lemma 7 *There exist critical values $\tilde{\varepsilon}$ and $\hat{\varepsilon}$, which satisfy*

$$\tilde{\varepsilon} = \left[\varepsilon' - \theta\left(\int_0^{\varepsilon'} \varepsilon \mathrm{d}G + \int_{\varepsilon'}^{\infty} \varepsilon' \mathrm{d}G\right)\right] \frac{\eta}{\chi+\eta}$$
$$- \max\left\{\varepsilon' \frac{\bar{m}}{m} - \left(\frac{\chi+\eta}{\eta} + \theta\frac{\bar{m}}{m}\right)\left[\int_0^{\varepsilon'} \varepsilon dG + \int_{\varepsilon'}^{\infty} \varepsilon' dG\right], 0\right\} \frac{\eta}{\chi+\eta}, \tag{30}$$

$$\hat{\varepsilon} = \varepsilon'(\rho_n/\rho_d)^{\alpha}. \tag{31}$$

If and only if $\tilde{\varepsilon} < \varepsilon'$ there are bankers that borrow in the money market, and if $\tilde{\varepsilon} < \varepsilon'$ then $z_\varepsilon > 0$ for all $\varepsilon > \tilde{\varepsilon}$. If and only if $\hat{\varepsilon} > \varepsilon'$, there exist ε such that $k_\varepsilon > k_\varepsilon^*$ and if $\hat{\varepsilon} > \varepsilon'$, then $k_\varepsilon > k_\varepsilon^*$ if and only if $\varepsilon < \hat{\varepsilon}$.

The implication of Lemma 7 is twofold. First, when $\bar{m} - \rho_d k > 0$ (meaning that bankers which leave the IM market with zero reserves are not subject to NIR) and/or $\chi, \theta > 0$, lending and borrowing must take place in the money market. Then, $\tilde{\varepsilon} < \varepsilon'$ and bankers with $\varepsilon \leq \tilde{\varepsilon}$ lend reserves and bankers with $\varepsilon > \tilde{\varepsilon}$ borrow reserves until they reach the exemption threshold. Second, when $\hat{\varepsilon} > \varepsilon'$ all bankers with $\varepsilon < \hat{\varepsilon}$ invest more than the first-best.

We now discuss each of the different transmission cases in detail.

Perfect transmission: Assume that NIR perfectly transmits to the deposit rate. In this case, $i_d = i_m = i_n$. Figure 2 shows investment quantities under perfect transmission. The 45° line in Figure 2 represents the first-best allocation.

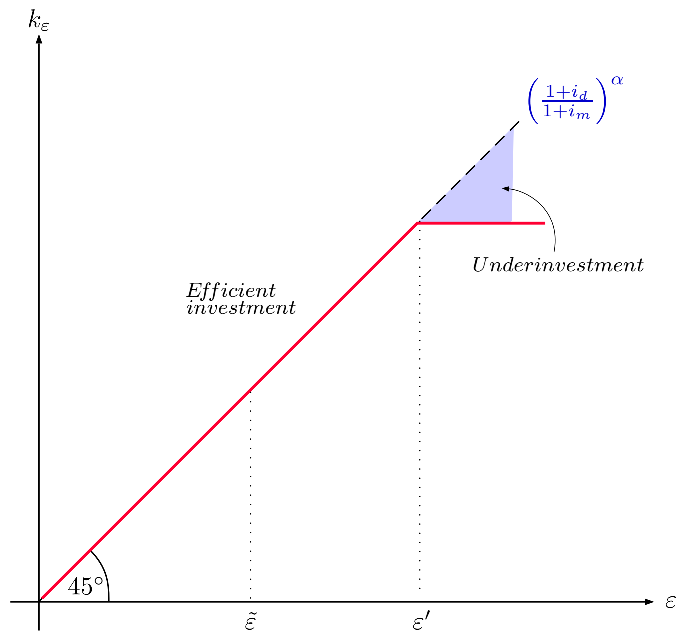

Figure 2: Perfect transmission

The first-best quantities are achieved for all $\varepsilon \leq \varepsilon'$. Bankers with $\varepsilon > \varepsilon'$ are constrained and invest less than the first-best quantities. In what follows, we refer to such an investment behavior as underinvestment.

Imperfect transmission (NIR case): Imperfect transmission for the NIR case involves $i_d > i_m = i_n$. As shown in Figure 3, in this case bankers with $\varepsilon < \hat{\varepsilon}$ invest more than the first-best quantities. In what follows, we refer to such an investment behavior as overinvestment. Bankers with $0 \leq \varepsilon \leq \tilde{\varepsilon}$ overinvest, because they cannot pass on the NIR to their depositors. Bankers with $\tilde{\varepsilon} \leq \varepsilon \leq \hat{\varepsilon}$ overinvest because they can borrow reserves at a lower rate i_m and deposit them at the central bank at interest rate i_p.

Among the bankers that overinvest, those with $\varepsilon \leq \varepsilon'$ are unconstrained and those with $\varepsilon' < \varepsilon < \hat{\varepsilon}$ are constrained by their reserve holdings. Finally, bankers with $\varepsilon > \hat{\varepsilon}$ are constrained and underinvest. A banker with investment shock $\varepsilon = \hat{\varepsilon}$ is constrained, but nevertheless invests the first-best quantity (see Figure 3).

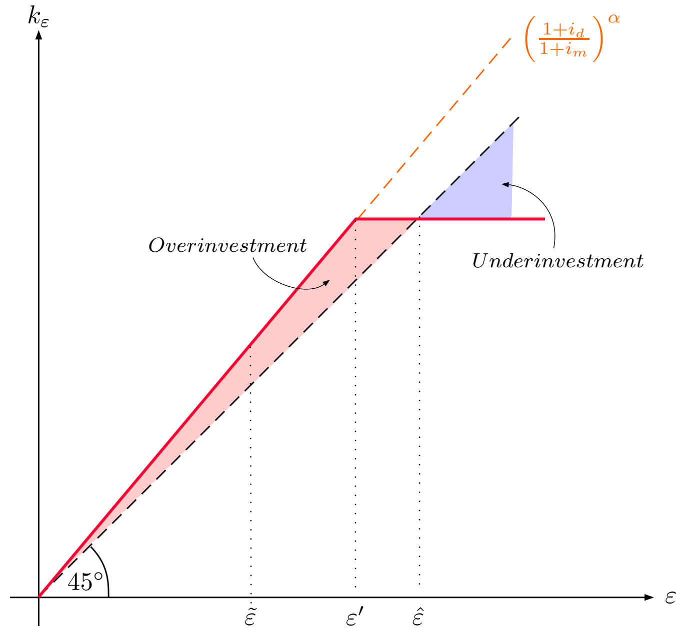

Figure 3: Imperfect transmission: NIR

Empirical evidence for imperfect transmission of NIR to retail deposit rates is widely documented in the literature (see for example Dell'Ariccia et al. 2017, Basten and Mariathasan 2020, Heider et al. 2019, Eisenschmidt and Smets 2019, Demiralp et al. 2021, Eggertsson et al. 2019 or Heider et al. 2020). Stettler (2020) and Brunnermeier and Koby (2019) provide a microfoundation of the limited pass-through of NIR to retail deposit rates and Heider et al. (2020) discuss additional reasons for a zero lower bound on retail deposits. Empirics and theory therefore suggest that the transmission of money market rates to retail deposit rates is distinctly different in NIR periods, compared to positive interest rate periods. Our model predicts that the limited transmission to retail deposit rates observed in NIR periods leads to a misallocation of capital, that would not arise in positive or zero interest rate periods when $i_m \geq i_d$. Furthermore, the model predicts overinvestment by unconstrained bankers in NIR periods, shedding light upon the quote by Summers and Stansbury (2019) (see the introduction).

Imperfect transmission (US case): Imperfect transmission for the US case involves $i_d < i_m = i_n$. As shown in Figure 4, in this case all bankers underinvest because they receive a higher rate on reserves than the interest rate on deposits. Here, bankers with $0 \leq \varepsilon \leq \varepsilon'$ are unconstrained and those with $\varepsilon > \varepsilon'$ are constrained by their reserves.

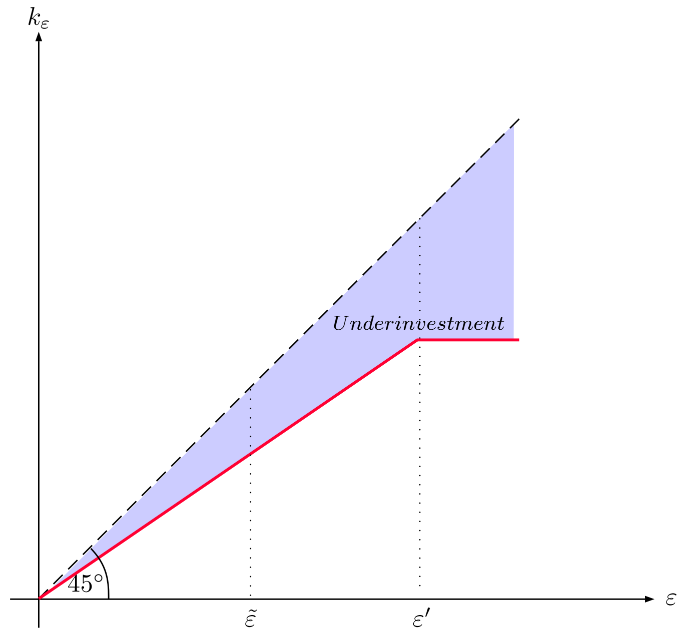

Figure 4: Imperfect transmission: US

4.2 Welfare effects from lowering NIR in Baseline Economy

We now turn towards discussing the long-run welfare effects of a reduction in i_n for the different transmission regimes. Recall that if NIR is fully passed-through to the money market rate, any change in i_n is directly reflected in a change in i_m. In this section, we focus on this case.[26] In order to establish the welfare effects of NIR, we first show how lowering i_n affects all thresholds and investment quantities with pass-through of NIR to money market rates.

Proposition 8 *A decrease in i_n decreases thresholds $\tilde{\varepsilon}$, ε', and $\hat{\varepsilon}$. Furthermore, for all $\varepsilon < \varepsilon'$ with perfect transmission $\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} = 0$ and with imperfect transmission $\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} > 0$. Finally, for all $\varepsilon > \varepsilon'$ with both perfect and imperfect transmission $\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} < 0$.*

A decrease in i_n increases the measure of bankers that underinvest since all bankers with $\varepsilon > \hat{\varepsilon}$ underinvest and $\hat{\varepsilon}$ is increasing in i_n. Similarly, lowering i_n increases the measure of constrained bankers and implies that all constrained bankers invest less. The

[26] If the NIR does not pass-through to the money market rate, then small changes in NIR have no effect on equilibrium allocations.

reason is that lower i_n makes holding reserves less attractive, so that, keeping everything else constant, the real value of reserves declines through an increase in nominal prices. In turn, this reduces ε'. Further, a decrease in i_n decreases $\tilde{\varepsilon}$. When $\tilde{\varepsilon} < \varepsilon'$ this leads to more borrowing in the money market, since the measure of bankers with $\varepsilon > \tilde{\varepsilon}$ increases.

Proposition 9 *Using Proposition 2, we show that the derivative* $\frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n}$ *can be broken into three terms as follows:*

$$\begin{aligned} \frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n} &= A + B + C, \text{ where} \\ & A \equiv \rho_d \int_0^{\varepsilon'} (i_n - i_d) \frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} \mathrm{d}G, \\ & B \equiv \int_{\varepsilon'}^{\hat{\varepsilon}} \left[(\varepsilon/\hat{\varepsilon})^{1/\alpha} - 1 \right] \frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} \mathrm{d}G, \\ & C \equiv \int_{\hat{\varepsilon}}^{\infty} \left[(\varepsilon/\hat{\varepsilon})^{1/\alpha} - 1 \right] \frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} \mathrm{d}G, \end{aligned} \tag{32}$$

Furthermore, we can show that the welfare effect of a decrease in the NIR with perfect or imperfect transmission is always negative, $\frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n} < 0$.

The term A captures the welfare changes of those bankers that are unconstrained. The term B captures the welfare changes of those bankers that are constrained and overinvest. Finally, the term C captures the welfare changes of those bankers that are constrained and underinvest. The aggregate effect of a decrease in i_n on welfare is always negative.[27] Below, we provide an intuition for this result depending on transmission of money market rates to deposits.

Perfect transmission: Assume that NIR perfectly transmits to the deposit rate. In this case, $i_d = i_m = i_n$. Figure 5 shows the change in welfare under perfect transmission.

The first-best quantities are achieved for all $\varepsilon \leq \varepsilon'$. After a decrease in i_n, the new equilibrium value is ε''. Consequently, more bankers are constrained and invest less than the first-best quantities. This is clearly welfare decreasing. This result is also confirmed by inspecting Equation (32). The term A is zero because with perfect transmission $\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} = 0$. The term B is also zero because with perfect transmission $\varepsilon' = \hat{\varepsilon}$. Finally, the term C is negative because $\varepsilon > \hat{\varepsilon}$ and $\frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} < 0$.

[27] Note, we treat the growth rate γ as fixed. If γ would adjust one-for-one with ρ_n, it follows immediately from Equation (27) that ε' remains unchanged. In turn, from Equation (20) it follows that with perfect transmission to deposit rates, also bankers' investment behavior remains unchanged. In case of imperfect transmission, all bankers will increase their investment following a reduction in ρ_n. Depending on whether over- or underinvestment dominates, this can have negative or a positive effect on welfare.

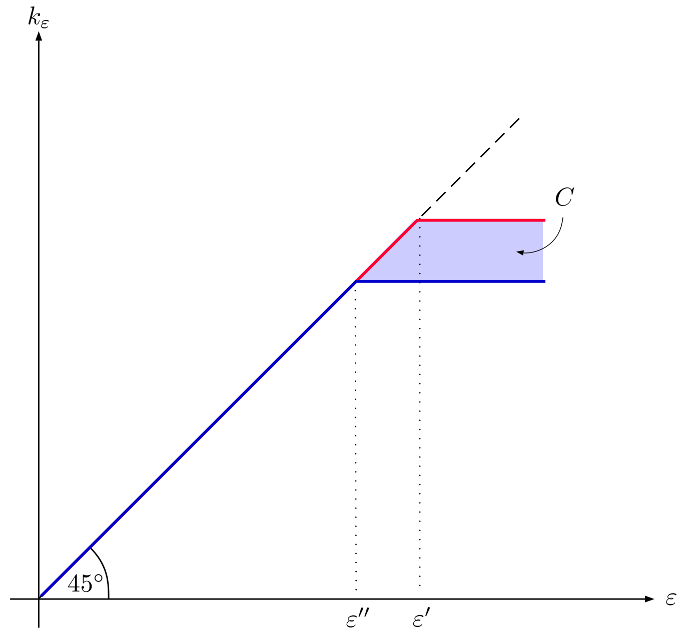

Figure 5: Perfect transmission

Imperfect transmission (NIR case): Imperfect transmission for the NIR case involves $i_n = i_m < i_d$, with i_d fixed while $i_n = i_m$ as there is pass-through of NIR to the money market rate. Here we distinguish between two cases. First, we consider the effect of a decrease in i_n when $i_d = i_m = i_n$ is the initial condition. Second, we consider the effect of a decrease in i_n with $i_d > i_m = i_n$ as initial condition. We summarize the effects for the NIR case in Table 1.

The first case with the initial condition $i_d = i_m = i_n$, is depicted in Figure 6. Unconstrained bankers with $0 \leq \varepsilon \leq \varepsilon'$ initially invest the efficient quantity. The decrease in i_n generates a wedge, causing unconstrained bankers to overinvest. This effect is clearly negative. Constrained bankers with $\varepsilon \geq \varepsilon'$ can invest less after a decrease in i_n because the real value of reserves (expressed in terms of capital goods, i.e., m/ρ_d) decreases. Furthermore, the decrease in i_n also causes a decrease in the critical value ε'. Hence, there are more bankers that are constrained and each constrained banker can invest less. This effect is also clearly negative.

Figure 6 suggests that the welfare effect is always negative. This result is confirmed by inspecting Equation (32). The term A is negative because the decrease in i_n implies a negative wedge $i_n - i_d$ and $\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} > 0$. The term B is zero because with the initial condition $i_d = i_m = i_n$, $\varepsilon' = \hat{\varepsilon}$. Finally, the term C is negative because $\varepsilon > \hat{\varepsilon}$ and $\frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} < 0$. These effects are summarized in Table 1 in row A' and C'.

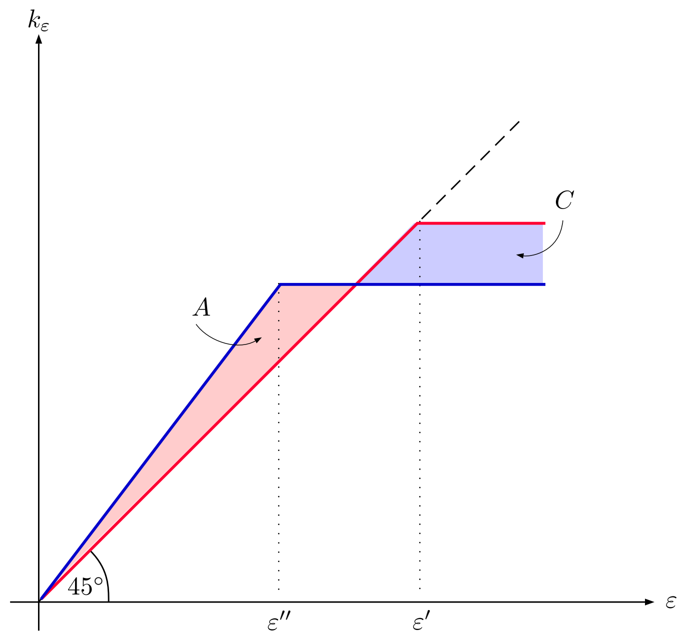

Figure 6: Imperfect transmission: Initial condition $i_d = i_n$

The second case with the initial condition $i_d > i_m = i_n$, is shown in Figure 7. Here, bankers with $0 \leq \varepsilon \leq \varepsilon'$ initially overinvest. A decrease in i_n lowers the critical values ε' and $\hat{\varepsilon}$ to ε'' and $\hat{\varepsilon}'$, respectively. A decrease in i_n further increases the wedge between i_n and i_d, leading to more overinvestment by unconstrained bankers. This effect is clearly negative. Further, the decrease in i_n is also decreasing the real value of reserves. Hence constrained bankers with $\varepsilon > \hat{\varepsilon}$ invest less. This effect is also clearly negative. Here, there is also a third effect for bankers with $\varepsilon' \leq \varepsilon \leq \hat{\varepsilon}$. As shown in the graph, these bankers are constrained and since the decrease in i_n lowers the real value of reserves, these bankers can now invest less. Note further, these bankers overinvest before and after the decrease in the NIR. The decrease in the real value of reserves causes these bankers to invest a quantity that is lower and therefore closer to the first-best allocation. This has a positive effect on welfare.

The effect on unconstrained bankers that overinvest corresponds to term A in Equation (32) and to row A in Table 1. The effect is negative because with imperfect transmission $\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} < 0$ and $i_d > i_m = i_n$. The effect on constrained bankers that overinvest corresponds to term B in Equation (32) and to row B in Table 1. This effect is positive because with imperfect transmission $\varepsilon' < \hat{\varepsilon}$. Finally, the effect on unconstrained bankers that underinvest corresponds to term C in Equation (32) and to row C in Table 1. This term is negative because of $\varepsilon > \hat{\varepsilon}$ and $\frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} < 0$.

In Proposition 9, we show that a decrease in i_n is always welfare decreasing for

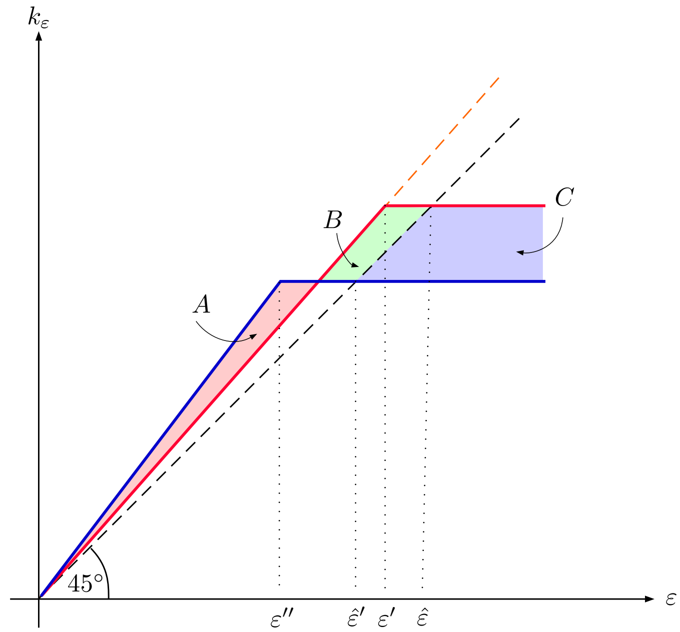

Figure 7: Imperfect transmission: Initial condition $i_d > i_n$

	$i_m < i_d$	overinvest	underinvest	decreasing i_n	$\mathcal{W}$
A	$\varepsilon < \varepsilon'$	$k_\varepsilon > k^*_\varepsilon$		$\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} > 0$	negative
B	$\varepsilon' < \varepsilon \leq \hat{\varepsilon}$	$k_{\varepsilon'} > k^*_\varepsilon$		$\frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} < 0$	positive
C	$\varepsilon > \hat{\varepsilon}$		$k_{\varepsilon'} < k^*_\varepsilon$	$\frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} < 0$	negative
	$i_n = i_d$				
A'	$\varepsilon < \varepsilon' = \hat{\varepsilon}$			$\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} > 0$	negative
C'	$\varepsilon > \varepsilon' = \hat{\varepsilon}$		$k_{\varepsilon'} < k^*_\varepsilon$	$\frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} < 0$	negative

Table 1: Effects of a decrease in i_n in the NIR case

imperfect transmission. Intuitively, the positive welfare effect of reduced overinvestment in region B is larger when the interest rate on deposits is high. In the proof of Proposition 9, we confirm our intuition by showing that in an imperfect transmission regime $\mathrm{d}\mathcal{W}(1-\beta)/\mathrm{d}\rho_{\mathrm{n}}$ is increasing in the interest rate earned by deposits. However, because deposits can be used as a store of value, there is an upper bound on i_d to have equilibrium existence. This upper bound implies that the negative effects of terms A and C always outweigh the positive effect of term B, leading to an overall negative effect on welfare.

Imperfect transmission (US case): Imperfect transmission for the US case involves $i_n = i_m > i_d$. As shown in Figure 8, all bankers underinvest. A decrease in i_n decreases the real value of reserves and therefore decreases ε' to ε''. All bankers with $\varepsilon \geq \varepsilon''$ can

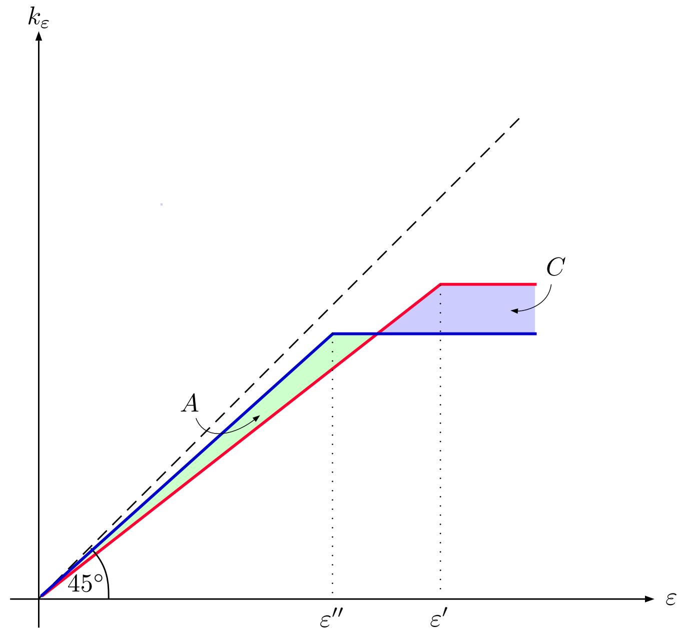

Figure 8: Imperfect transmission: US

invest even less with the decrease in the real value of reserves. This effect is clearly negative. However, unconstrained bankers with $0 \leq \varepsilon \leq \varepsilon''$ invest more with a decrease in i_n. A decrease in i_n in this scenario, means decreasing the wedge between i_n and i_d and therefore holding reserves becomes relatively less attractive. As a result, unconstrained bankers invest more, which has a positive effect on welfare. Proposition 9 shows that a decrease in i_n is always welfare decreasing, implying that the negative effect of term C always outweighs the positive effect of term A. The intuition is again that the positive welfare effect of decreasing i_n depends positively on the interest rate earned by deposits i_d, which is bounded from above to have equilibrium existence.

4.3 Effects from lowering NIR on aggregate output and the real value of reserves in steady state.

In this section, we discuss the long-run effects from lowering i_n on aggregate output and the real value of reserves. Consider first aggregate output, denoted $Q = \int_0^\infty \varepsilon^{1/\alpha} f(k_\varepsilon) \mathrm{d}G$.

Proposition 10 *A decrease in i_n reduces aggregate output.*

Though NIR can lead to more investment, this effect only applies to investment projects with a small efficient scale ($\varepsilon < \varepsilon'$). At the same time, because NIR reduces the amount of investment that constrained bankers can undertake, NIR leads to less investment in projects with a large efficient scale ($\varepsilon > \varepsilon'$). Proposition 10 shows that the latter effect dominates the former, so that NIR leads to less output.

Proposition 11 *A decrease in i_n reduces the real amount of reserves, m, and the real supply of treasury bonds, b_{TR}. This effect is the dominating force in how i_n affects output with imperfect transmission; keeping the real amount of reserves and bonds fixed, with imperfect transmission a decrease in i_n would cause output to expand.*

Low policy rates are typically perceived as expansionary, at least in the short run. In the current environment with excess reserves, a lower value for i_n however reduces the real return on reserves and bonds. Proposition 11 implies that in steady state, this results in a lower equilibrium value of reserves and bonds, or equivalently a permanently higher nominal price for general goods and capital goods. These price adjustments can be thought of as long-run effects that are contractionary. At the same time, in an imperfect transmission regime, keeping the real amount of reserves m fixed implies a negative relationship between policy rate i_n and aggregate output. This relationship arises because a lower money market rate reduces the opportunity cost of investment, suggesting that lower interest rates can be expansionary in the short run. With perfect transmission, the effect of lower rates for given m is however always contractionary, as the relative price of capital goods ($p = \rho_d$) increases and the opportunity cost of capital investment remain unchanged. In Section 5, where we will show that the imperfect transmission case arises endogenously in case of NIR, we further explore the short-run effects of NIR when policy smoothens the transition towards the new steady state.

Note that Proposition 11 only holds under the assumption that the money-to-bonds ratio remains constant. In particular, Proposition 11 shows that NIR reduces the real value of a unit of reserves. During NIR episodes in various NIR countries, the data indicates that the real value of aggregate reserves has increased. This observation can be reconciled with our model if the implementation of NIR goes along with an increase in the money-to-bonds ratio, for instance due to quantitative easing. The reason is that raising the money-to-bonds ratio increases the real value of aggregate reserves, as the central bank increases the supply of reserves by purchasing bonds. Furthermore, in a floor system, a change in the money-to-bonds ratio leaves investment and output unchanged. Our insights of how NIR affects real allocations thus remain valid when NIR is combined with quantitative easing.

Because NIR reduces the real value of a unit of reserves, Proposition 11 also indicates that introducing NIR can be used to dampen the appreciation of a currency, as the SNB and DN did. Again, this result holds true when the implementation of NIR is combined with quantitative easing. In fact, increasing the money-to-bonds ratio in a floor system will actually lead to a further decline in the real value of a unit of reserves when bond holdings have a reduced pledgeability, that is $\chi < 1$. We show this formally in the proof of Proposition 11.

4.4 Effects from lowering NIR on bankers' profitability

We discuss profitability by looking at $\mathcal{P} = \int_0^\infty V_{IM}(m, b, d|\varepsilon)\mathrm{d}G$, i.e., bankers' aggregate utility or equivalently, the present value of their income. In appendix C, we show that

$$(1-\beta)\mathcal{P} = \int_0^\infty \left[\varepsilon^{1/\alpha} f(k_\varepsilon) - k_\varepsilon\right] \mathrm{d}G + b\left(1/\rho_b - \gamma\right) + \bar{m}(1/\rho_p - \gamma) + (m - \bar{m})(1/\rho_n - \gamma) + \tau_B.$$

Keeping the transfer τ_B fixed, an decrease in i_n (an increase in ρ_n) has a direct negative effect, as the return on non-exempted reserves $(m - \bar{m})$ decreases. There are also two indirectly negative effects. First, the profitability from investment activity, i.e., $\int_0^\infty \left[\varepsilon^{1/\alpha} f(k_\varepsilon) - k_\varepsilon\right] \mathrm{d}G$, decreases, which follows from the welfare result in Proposition 9. Second, when $\chi > 0$, the interest rate on bonds, $1/\rho_b = \chi/\rho_n + (1-\chi)\gamma/\beta$, decreases. Finally, there is an ambiguous indirect effect coming from the reduction in the value of reserves m, which also reduces the real exemption threshold $\bar{m} = m(\overline{M}_0/M_0)$ and the real value of bankers' bond holdings $b = m/\eta$. When the average nominal return on bankers' reserve and bonds exceeds inflation, i.e.,

$$\frac{1}{1+\eta}\frac{1}{\rho_b} + \frac{\eta}{1+\eta}\left(\frac{\overline{M}_0}{M_0}\frac{1}{\rho_p} + \frac{M_0 - \overline{M}_0}{M_0}\frac{1}{\rho_n}\right) > \gamma,$$

this indirect effect is negative, and otherwise it is positive. In the likely case that the overall effect is negative, the reduction in profitability can be dampened by increasing the exemption threshold, which leaves the equilibrium allocation unchanged.

4.5 Optimal Monetary Policy in the Baseline Economy

In what follows, we briefly discuss the optimal monetary policy in the baseline economy. From Equation (27), the Friedman rule implies setting $\rho_n = \beta/\gamma$.

Proposition 12 *The Friedman rule is optimal and implements the first-best allocation for $\rho_n = \rho_d$. For $\rho_d > \rho_n$, the Friedman rule does not implement the first-best allocation.*

This shows that the Friedman rule is the optimal monetary policy and implements the first-best allocation under $\rho_n = \rho_d$. Further, if the central bank implements the Friedman rule, the economy cannot be in the NIR case since equilibrium existence requires $\gamma\rho_d \geq \beta$

and the NIR case would require $\rho_n = \beta/\gamma > \rho_d$. Lastly, for $\rho_n < \rho_d$, the Friedman rule is optimal, but it does not implement the first-best—all bankers underinvest.

5 On the Asymmetry of NIR When Cash is Available

In this section we provide a microfoundation for the qualitatively different transmission of policy rates to the real economy during NIR periods. Up to this point we have been agnostic about how the deposit rates are determined in equilibrium and furthermore, deposits were the only means of exchange that households could hold. We now amend the model as follows: first, we assume that households and bankers participate in a perfectly competitive market for deposits; second, we introduce cash. Cash allows households to avoid negative deposit rates and perfect competition provides a microfoundation for the determination of deposit rates.

The purpose of these changes is to demonstrate in a realistic set-up that negative and positive interest rates differ with respect to their effects on the economy. That means, a reduction of the policy rate in positive territory, say from 2 percent to 1 percent, has different effects than a reduction in negative territories, say from -1 percent to -2 percent.

As mentioned above, we modify the baseline model by introducing zero-interest cash as a new payment option for households. Without loss in generality, households are willing to exchange at most a fraction $x \in [0, 1]$ of deposits into cash if it is profitable to do so.[28] The upper bound x reflects the fact that in reality not all payments can be done with cash and so the households will keep some of their earning as deposits even under NIR. We abstract from costs of holding cash.[29]

It is straightforward to show that with perfect competition for deposits, the deposit rate is always equal to the money market rate, that is $i_d = i_m$. Furthermore, recall that in the baseline model with excess reserves the money market rate is always equal to the policy rate, that is $i_m = i_n$. Accordingly, with perfect competition and excess reserves in equilibrium the rates satisfy $i_d = i_m = i_n$.

In what follows we determine the equilibrium quantities of deposits and production of capital goods. After the IM market has closed, households hold pk_s units of deposits

[28] In particular, we assume that the earnings of households from capital production are deposited in their bank account. Households can then choose to withdraw a fraction $\tilde{x} \leq x$ of their deposits as cash. In order to meet the demand for cash, bankers can exchange their reserves for cash at the central bank. We furthermore assume that bankers cannot use cash to avoid the NIR. Therefore, bankers only exchange reserves for cash if their depositors choose to withdraw parts of their deposits. The central bank simply meets the demand for cash.

[29] In reality, it is reasonable to assume that holding large amounts of cash is costly. Introducing a linear cost of holding cash would simply shift the level of i_n at which the transmission of policy rates to the real economy changes.

(in real terms) in their respective bank accounts. Households' production of capital goods k_s and deposit holdings d (after withdrawals) are therefore determined by

$$W_{IM} = \max_{k_s \geq 0, \tilde{x} \in [0,x]} \{-k_s + (1+i_n)(1-\tilde{x})pk_s + \tilde{x}pk_s\} + W_S(0),$$

where $\tilde{x}$ is the fraction of deposits, accumulated by capital production, that the households withdraw. In equilibrium, as in the baseline model, households are indifferent with respect to their production of capital goods and this production equals aggregate demand for capital goods by bankers. The price of capital goods and the real value of money deposited with bankers (after withdrawals) therefore satisfy

$$p = \frac{1}{1 + i_n - x \min\{i_n, 0\}} \quad \text{and} \quad d = (1-\tilde{x})pk_s = \begin{cases} (1-x)pk_s & \text{if } i_n < 0 \\ pk_s & \text{if } i_n \geq 0 \end{cases}. \tag{33}$$

Recall that in the baseline model, θ is the fraction of reserves that the bankers can pledge as collateral in the money market. Because households can now withdraw cash, the borrowing constraint for bankers in the money market becomes tighter during NIR periods. Specifically, the constraint on the amount of reserves z_ε borrowed by the ε-banker (see Equation (11)) becomes dependent on the fraction of deposits $\tilde{x}$ (which in equilibrium is determined by Equation (33)) withdrawn by the households:

$$z_\varepsilon < m_\varepsilon + \chi b + \tilde{\theta} p k_s, \quad \text{where} \quad \tilde{\theta} = (1-\tilde{x})\theta. \tag{34}$$

Finally, to ensure that NIR still pass-trough to the money market rate in the modified environment, we need to take into account that some reserves are turned into cash to meet households' withdrawals. As in the baseline environment, the exemption threshold should be smaller than the aggregate amount of reserves carried into the settlement market. The condition for the exemption threshold in Proposition 5 therefore becomes dependent on the fraction $\tilde{x}$ of deposits withdrawn by the households:

$$\bar{m} < m - \tilde{x} p k_s. \tag{35}$$

Implications. Equation (33) demonstrates that the policy rate i_n is perfectly transmitted to the price of capital when positive, but imperfectly when negative. In turn, this results in asymmetric effects of positive and negative interest rates on the real economy. To see this, define $\tilde{\rho}_d = p$ with p given by Equation (33). Equilibrium investment, welfare, and output in our modified environment are then the same as in the baseline environment with $\rho_d = \tilde{\rho_d}$.[30] Setting $x = 1$, we obtain $\tilde{\rho_d} = \min\{\rho_n, 1\}$. In the baseline environment, this resembles perfect transmission of policy rates to deposit rates when $i_n \geq 0$ and no transmission when $i_n < 0$. Most prominently, with a perfectly competitive market for deposits, the presence of cash as an outside option for depositors implies that NIR policies lead to overinvestment by the low ε-bankers.[31]

Another implication of Equation (33) is that when the ability of households to withdraw cash disappears, that is $x \to 0$, the asymmetric effects of NIR policies vanish. Abolishing cash, as proposed by Rogoff (2017), would therefore mitigate some of the distortions caused by NIR policies. Nevertheless, this beneficial effect of abolishing cash relies on the perfect transmission of policy rates to deposit rates, while there is substantial evidence for imperfect transmission during NIR periods.

The implication of Equation (34) is that in the modified environment, the equilibrium relationship between the real value of reserves m and the policy rate i_n changes compared to the baseline economy (see Proposition 11). The reason is that when policy rates turn negative, there are now two opposing forces affecting the equilibrium value of reserves. First, as in the baseline model, holding reserves becomes less attractive for bankers, which reduces the equilibrium value of reserves. Second, with $\theta > 0$ the borrowing constraint of bankers becomes tighter ($\tilde{\theta}$ jumps down) when policy rates become negative. In turn, this increases the equilibrium value of reserves. Which of the two effects dominates, depends on the pledgeability parameter θ.

5.1 Smoothed effects of NIR

Proposition 10 continues to hold when the retail deposit rate is endogenously determined in a model with cash as outside option, i.e., NIR is contractionary in the long run. Lower interest rates are however, at least in the short run, perceived as expansionary.[32] Below, we show that when policy rates are moved into the negative domain, this perception can be reconciled with our model featuring an endogenous deposit rate. Specifically, introducing NIR can generate a short-run expansionary effect when the effects of NIR

[30] The fact that bankers' borrowing constraints depend on withdrawals made by households, as captured by Equation (34), is irrelevant for this insight. The reason is that in the equilibrium of our baseline model, investment, welfare, and output are independent of both θ and bankers' inflow of deposits d.

[31] Note, in Section 4 we discuss the effects on investment based on the transmission of i_n to the deposit rate i_d. As discussed above in a competitive deposit market, the interest rate i_n always perfectly transmits to the deposit rate i_d. We nevertheless get asymmetric effects in NIR periods, because when $i_n < 0$, the price of capital decouples from the deposit rate i_d. Intuitively, in NIR periods households can avoid the negative interest rate on deposits by holding cash. Producing capital goods therefore becomes more attractive compared to a cashless economy, which is reflected in a lower price of capital. This leads to the asymmetric effects of NIR.

[32] For example, Wang (2020) finds an expansionary effect in the short-run and a contractionary effect in the long-run.

on the money market are smoothened over time.

Focusing on a floor system, it is easy to verify that in the spirit of Equation (27) inflation at time t determines the time t critical threshold ε_t' according to:

$$\frac{\rho_{n,t}\phi_{t-1}}{\beta\phi_t} = \int_0^{\varepsilon_t'} \mathrm{d}G + \int_{\varepsilon_t'}^{\infty} \left(\varepsilon/\varepsilon_t'\right)^{1/\alpha} \mathrm{d}G. \tag{36}$$

In turn, the associated value of reserves follows from Equation (19), and the value of treasury bonds can be calculated from the relationships $b_t = m_t/\eta_t$ and $b_{TR,t} = m_t + b_t$:

$$\varepsilon_t' = \left(\frac{\tilde{\rho}_{d,t}}{\rho_{n,t}}\right)^{\alpha} \frac{m_t}{\tilde{\rho}_{d,t}} \frac{\chi+\eta_t}{\eta_t} + \tilde{\theta}_t \left[\int_0^{\varepsilon_t'} \varepsilon \mathrm{d}G + \int_{\varepsilon_t'}^{\infty} \varepsilon_t' \mathrm{d}G\right] \quad \text{and} \quad b_{TR,t} = \frac{1+\eta_t}{\eta_t} m_t, \tag{37}$$

where $\tilde{\theta}_t = (1-\tilde{x}_t)\theta$. Finally, the relationship between the supply of treasury bonds and the real demand for treasury bonds implies $\phi_t = b_{TR,t}/B_{TR,t}$, so that inflation satisfies

$$\frac{\phi_{t-1}}{\phi_t} = \frac{B_{TR,t}}{B_{TR,t-1}} \frac{b_{TR,t-1}}{b_{TR,t}}.$$

In the long run, the nominal supply of treasury bonds grows at rate γ, which pins down the rate of inflation in the economy and in turn, through (36), the new steady state for ε'. In the short run, as the equations above demonstrate, the treasury and the central bank can smooth the effect of a change in policy rates on borrowing conditions in the money market. Of course, this requires appropriate changes in the money-to-bonds ratio and/or the growth rate of the supply of treasury bonds. As a numerical exercise we calculate the effects of a decrease of i_n on aggregate output and welfare assuming that interest rates adjust immediately and ε_t' moves to the new steady state ε_{ss}' in a gradual fashion:

$$\varepsilon_t' = \delta\varepsilon_{t-1}' + (1-\delta)\varepsilon_{ss}'.$$

From Equation (37), it follows that this is very similar to letting the aggregate real value of reserves move gradually towards the new steady state. In particular, when $\theta = 0$ and the money-to-bonds ratio η_t remains constant, the value of reserves develops proportionally with ε_t'. In our numerical exercise, we set $\theta = 0$ and assume that η_t remains constant, and we use $\beta = 0.99, \gamma = 1.01$ and $\alpha = 1.5$. The specification of i_p, η, χ and the exemption threshold are irrelevant for the numerical exercise. Furthermore, the distribution of ε is assumed to follow a log-normal distribution with mean 1 and a standard deviation of 0.9 and we assume that all households withdraw all their deposits as soon as the retail deposit rate becomes negative, that is $x = 1$. As shown above, this implies $\tilde{\rho}_{d,t} = \min\{\rho_{n,t}, 1\}$. Additionally, we assume that the economy only transitions slowly to the new steady state with $\delta = 0.9$. Lastly, we assume that the central bank changes the interest rate on reserves from $i_n = -0.02$ to $i_n = -0.03$ in period $t = 1$.

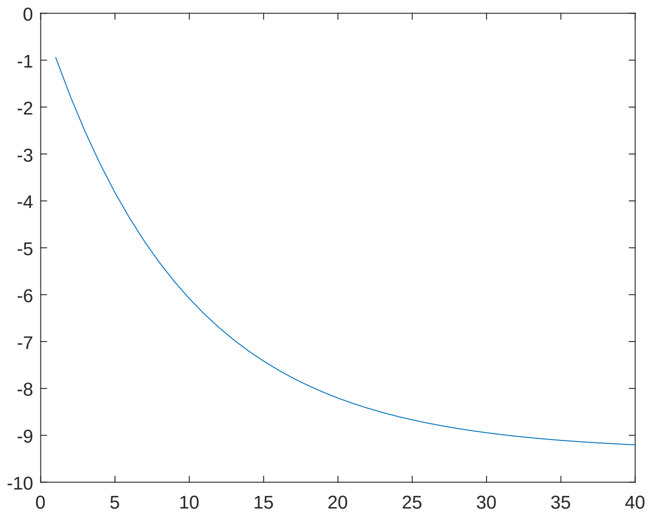

Figure 9: Adjustment of ε'_t, expressed as a percentage deviation from ε'_0.

Figure 9 depicts the relative adjustment of ε'_t, and thus also that of m_t when η_t remains constant, from period $t = 1$ to period $t = 40$ as it slowly moves to the new steady state.

Figure 10 depicts the short-run effects on welfare and aggregate output. In the spirit of Equation (18), flow welfare is calculated as $\int_0^\infty [\varepsilon^{1/\alpha} f(k_{\varepsilon,t}) - k_{\varepsilon,t}]\mathrm{d}G$. Aggregate flow output is calculated as $\int_0^\infty \varepsilon^{1/\alpha} f(k_{\varepsilon,t})\mathrm{d}G$. Both welfare and aggregate output are initially increasing, before declining. This initially expansionary effect is a result of the increase in the level of investment by unconstrained bankers, which arises because i_n is moved into the negative domain while i_d cannot follow this move due to the availability of cash. Despite increasing overinvestment, which hurts welfare, the very slow adjustment shown in Figure 10 implies that initially the effect of constrained bankers being able to invest more (as ε'_t is initially very close to the initial steady-state when δ is low) outweighs the negative impact of overinvestment by unconstrained bankers.[33]

[33]For higher values of δ, the economy moves faster to the new steady state. Therefore, it is possible to have cases in which ε'_1 is already much lower compared to the old steady state at $t = 0$. Hence, at $t = 1$, though the constrained bankers face a relatively low opportunity cost of financing capital, the drop in ε'_1 dominates and implies less investment by constrained bankers, most of which underinvest.

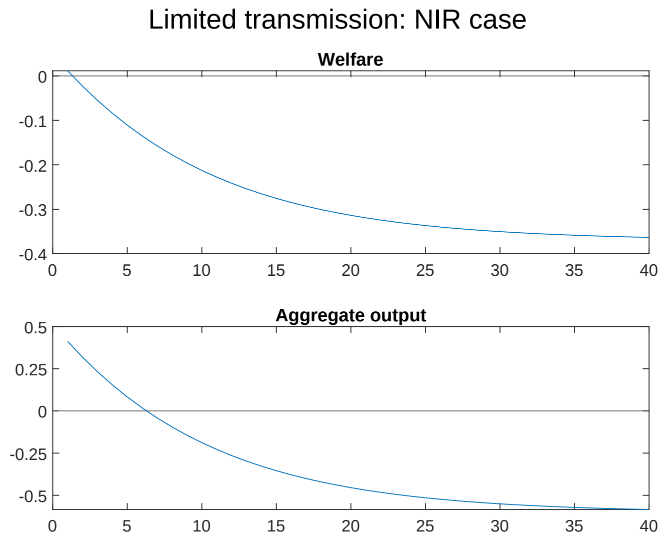

Figure 10: Adjustment of flow welfare and aggregate flow output, expressed as percentage deviations from their respective values at $t = 0$.

6 Related Literature on Negative Interest Rates

The introduction of NIR by various central banks led to a growing literature that studies NIR as a monetary policy tool. Here, we provide an overview of that literature.[34] The NIR literature mainly focuses on the transmission of NIR. Consensus among the literature is that NIR have transmitted to money market interest rates and fixed-income markets. Wholesale lending and deposit rates are only partly affected, and retail deposits are largely exempt from NIR. The literature identifies potential risks for bank profitability and financial stability stemming from NIR. These are particularly relevant when NIR persist over a long period.

Overview articles on NIR include Bech and Malkhozov (2016), Bernhardsen and Lund (2015), Jobst and Lin (2016), Dell'Ariccia et al. (2017), Demiralp et al. (2021), and Jackson (2015). They all discuss the context in which NIR were introduced and how NIR were implemented by respective central banks. Moreover, they discuss the transmission to the money market as well as potential side effects of NIR.

The theoretical NIR literature focuses on the optimality of NIR. Our paper is most closely related to Dong and Wen (2017), Rognlie (2016), Brunnermeier and Koby (2019), Eggertsson et al. (2019), Ulate (2019), Porcellacchia (2019), Bittner et al. (2022), Balloch and Koby (2022) and Wang (2020). Dong and Wen (2017) analyze optimal monetary

[34]See also Heider et al. (2021) for an overview of the existing literature

policy under NIR in a theoretical framework and find that NIR can be welfare improving because they relax borrowing constraints. Similarly, Rognlie (2016) finds that NIR can be optimal when average output is below potential output. Brunnermeier and Koby (2019) introduce the concept of a reversal interest rate. While an interest rate cut decreases the remuneration on safe assets, it simultaneously increases the valuation of banks' asset holdings. If the first effect dominates the second, there exists a so-called reversal rate, a rate at which monetary policy becomes contractionary. Similarly, Eggertsson et al. (2019) show that NIR need not be expansionary as NIR decrease interest rate margins, which in turn decreases bank profitability. If this effect translates to higher intermediation costs, NIR are contractionary. In contrast, Ulate (2019) studies the effects of NIR in a New-Keynesian framework and finds that NIR are an expansionary monetary policy tool. Also, Porcellacchia (2019) finds that in a Diamond and Dybvig (1983) framework, NIR lead to increased aggregate demand as the incentive to save decreases. Bittner et al. (2022) develop a model where bankers monitor projects, and can finance themselves with their own funds and outside funding, such as deposits. Negative policy rates increase bankers' funding costs due to imperfect transmission to the deposit rate. This directly leads to tighter credit constraint for bankers, and also indirectly: higher cost of funding reduces bankers' incentive to monitor and this leads to less outside funding. Balloch and Koby (2022) develop a model to study the long-run effects of low nominal interest rates. In their model, low interest rates decrease banks' market power, leading to declines in bank profits and loan issuance. Also, Wang (2020) develops a model to study the impact of low interest rates and finds that low interest rates can have a negative impact in the long-run on bank profits, equity, and loan issuance, and also increases loan spreads even if interest rates do not hit the ZLB.

Boutros and Witmer (2020) discuss NIR as a means to overcome the ZLB and specifically address exemption thresholds in their theoretical framework. They show that once the exemption threshold depends on the amount of cash withdrawals, NIR in combination with such an exemption threshold can discourage cash withdrawals and therefore effectively lower the ELB.

The empirical literature focuses on the effects of NIR on bank profitability, bank lending and lending rates. Further topics include the transmission to longer term interest rates, the ELB and investment decisions by firms. Altavilla et al. (2022), who use data on firms in the EU, suggest that NIR are an expansionary monetary policy tool as they find that firms that are subject to negative corporate deposit rates, increase investment.

The effect of NIR on bank profitability is a key topic in the NIR literature. Given that banks earn NIR on (part) of their assets, while only passing it on to part of their liabilities, the banks' interest rate margins deteriorate.[35] This in turn is expected to contribute negatively to the profitability. Turk (2016) however shows that bank profitability in Sweden and Denmark was not negatively affected by NIR. The reason is that banking service fees were increased and wholesale funding costs could be decreased to compensate for the lower interest rate margins from other banking activities. Similar results were found by Basten and Mariathasan (2020) for Swiss banks. Also, Scheiber et al. (2016) find no adverse effects on bank profitability in Denmark, Sweden and Switzerland. For European and Japanese banks, Lopez et al. (2020) find that losses in net interest income have been offset by lower funding costs and higher non-interest income. Arseneau (2017) analyzes the effect of NIR on banks in the US by using stress test data. He finds that while all banks would anticipate lower profits in a NIR environment, only some would expect lower profits due to net interest rate margins while others would expect higher profits through this channel. For the Eurozone, Demiralp et al. (2021) show that banks holding excess reserves try to circumvent NIR by increasing bank lending and by decreasing wholesale funding.

Evidence that at least in expectation, NIR contributes negatively to bank profitability is provided by Ampudia and Van den Heuvel (2022). They find that during NIR episodes and especially for banks with a high deposit-to-asset ratio, a lower policy rate has a negative effect on banks' equity value. Also, Nucera et al. (2017) find that for smaller banks with a high deposit-to-asset ratio, NIR increases the propensity to become undercapitalized in a crisis.

Consensus in the literature is that NIR fully transmitted to money market rates as documented by Bech and Malkhozov (2016), Bernhardsen and Lund (2015), Dell'Ariccia et al. (2017), Turk (2016), Jackson (2015), Jensen and Spange (2015) and Bräuning and Wu (2017). Also, the literature largely agrees that NIR transmitted to fixed-income markets. Grisse and Schumacher (2017) investigate the effects of short-term interest rate changes on long-term interest rates. Although theory would predict a weakening effect as interest rates approach the effective lower bound, the authors find a stronger effect during the NIR period than during a zero lower bound period. Similarly, Bräuning and Wu (2017) find that expansionary monetary policy measures had a stronger effect on long-term interest rates during a NIR period than with positive rates. Furthermore, Altavilla et al. (2022) show that NIR transmitted to corporate deposit rates as NIR continued and were lowered further into the negative territory.

The transmission of NIR on bank lending rates is somewhat less clear. Using data

[35] Evidence that that retail deposit rates exhibit a zero lower bound as many banks are reluctant to charge NIR to their retail customers is documented in Eggertsson et al. (2019), Eisenschmidt and Smets (2019), Basten and Mariathasan (2020), Demiralp et al. (2021), Heider et al. (2019) and Heider et al. (2020).

on German banks, Eisenschmidt and Smets (2019) find no evidence that banks with a higher dependence on retail deposits price their loans differently than banks with a lower dependence on retail deposits. Bräuning and Wu (2017) however find that short-term lending rates decrease in both pre-NIR periods and NIR periods, as a consequence of a decrease in the central bank policy rate. During the NIR period, the authors even find a more distinct reaction on long-term lending rates. For the Swiss case, Schelling and Towbin (2020) find that banks which rely more on deposits offer more generous lending terms. In contrast, for the Swedish case Eggertsson et al. (2019) find a decrease of average lending rates. Similarly Amzallag et al. (2019) find that banks who are more exposed to NIR increased rates on fixed-rate mortgages in Italy. Also, Arce et al. (2021) find that in Spain, banks who are more exposed to NIR and less well capitalized increased their lending rates.

There is also mixed evidence on the effect of NIR on bank lending volumes in the literature. Bräuning and Wu (2017) find an increase in loan volumes due to NIR. Boeckx et al. (2020) analyze the effect of the ECB's credit-easing policies, which also include NIR, and find increased bank lending volumes due to these policies. However, they do not disentangle the effect arising from NIR and other measures. Ulate (2019) discusses two effects of NIR on bank lending. First, a lower policy rate reduces bank lending rates and thus increases bank lending. Second, a decrease in the net interest rate margin reduces bank profitability and thus lowers the ability of banks to issue loans. Using data on banks in both (i) countries where central banks adopted NIR and (ii) where NIR have not been adopted, he finds that the first effect dominates the second effect, suggesting that NIR lead to more bank lending. Also, Bottero et al. (2022) find that NIR lead to increases in bank lending in Italy and similar results are found by Grandi and Guille (2022) for France. These findings stand in contrast to Heider et al. (2019) and Eggertsson et al. (2019), who find that lowering interest rates into negative territory did not increase bank lending in the Euro area and in Sweden. Finally, Heider et al. (2019) find that bank lending shifted towards riskier borrowers in the Euro area. This is especially true for banks that rely more on deposit funding.

There are a few papers that study the effective lower bound (ELB) with respect to NIR. Lemke and Vladu (2017) use a shadow-rate term structure model to estimate the lower bound, showing that the lower bound became negative after the ECB's second interest rate cut in the NIR territory, but they find the ELB to be above the ECB's negative policy rate at that time. Grisse et al. (2017) analyze how changes in the believed lower bound has affected long-term interest rates. They find that a decrease in the believed lower bound decreases long-term interest rates without the need for a policy rate cut. Related to the ELB discussion, flight to cash issues or the effects on financial stability were discussed by McAndrews (2015). Jensen and Spange (2015) study demand for cash under NIR in Denmark and find no strong increase since the introduction of NIR.

7 Conclusion

Around the year 2015, major central banks have introduced NIR with the aim to increase inflation, stimulate the economy or to dampen the appreciation pressure on the local currency. The hope was that NIR would only be needed for a short period of time. Most central banks however implemented NIR for almost a decade, moving rates back into positive territory only in 2022 to fight inflation.

Our paper serves as an analysis of the long-effects of NIR by focusing on the role of central bank reserves as a means to settle interbank liabilities. When banks in our model cannot pass on the NIR to their depositors, we identify two distortions. First, overinvestment occurs for small investment projects where banks invest more than the first-best quantity in an attempt to avoid the NIR. Second, investment quantities of large investment projects are too small because collateral constraints bind and NIR decreases the real value of collateral (reserves and treasury bonds). If the transmission is perfect, we find the same distortions if cash is available because cash allows depositors to avoid negative rates on deposits. Abolishing cash will eliminate the overinvestment distortion because it allows banks to pass on negative rates on reserves to depositors. For this result, we implicitly assume that households have no other payment instrument available such as cryptocurrencies (Schär and Berentsen 2020).

There is a consensus in the literature that NIR is not or only rarely passed on to retail depositors which suggest that the imperfect transmission case is the more realistic case. In this case the investment inefficiencies are present with or without cash. In all cases that we study, NIR unambiguously decreases welfare. The reason is simply because of the distortions just described.

We also study exemptions from NIR, which are motivated by the fact that most central banks exclude part of the reserves holdings from the NIR, remunerating it at zero or a positive interest rate. We show that exemptions can mitigate the bank-profitability concerns. At the same time, the negative welfare results continue to hold. Finally, we find that NIR lowers the real value of the local currency and therefore is a tool to dampen the appreciation pressure of a currency. The Swiss National Bank and the Danmarks Nationalbank introduced NIR against this background.

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Appendix A

Bank of Japan

In early 2016, the BOJ introduced the so-called "Quantitative and Qualitative Monetary Easing with a Negative Interest Rate" program to increase inflation to it's target of 2%. This program entailed the decrease of the interest rate applied to excess reserves to -0.1% and announced a set of loan support programs.[36] At the same time, the BOJ also decided to increase the monetary base and implement an asset purchase program of Japanese government bonds and other financial assets (Bank of Japan 2016).

The BOJ implemented a three-tiered program for the remuneration of reserves. Average reserve holdings are allocated to the first tier and are referred to as the "Basic Balance", remunerated at a positive interest rate of 0.1%. The second tier called "Macro-Add-on-Balance" is remunerated at 0% and consists of minimum reserve requirements and any amount outstanding from the so-called "Loan Support Program" and the "Funds-Supplying Operation to Support Financial Institutions in Disaster Areas affected by the Great East Japan Earthquake". The third tier is the "Policy-Rate Balance", which is remunerated at a negative rate of -0.1% and contains all reserves not covered in the first two tiers (Bank of Japan 2016). The first and second tier each have an upper bound.[37][38]

By-and-large, NIR transmitted to Japanese Yen overnight money market interest rates. Turnover in the money market has changed significantly in reaction to the introduction with NIR.

[36] The loan support programs included the "Loan Support Program", a "Funds-Supplying Operations to Support Financial Institutions in Disaster Areas affected by the Great East Japan Earthquake" and a "Funds-Supplying Operations against Pooled Collateral at zero interest rate" program.

[37] For the first tier the upper bound consist of the difference of the average current account balance or benchmark balance and the required reserves. If the amount computed in the first tier exceeds the upper bound for a financial institution, only the amount covered in the upper bound is remunerated at the positive interest rates. The rest will then count towards the second tier. The upper bound of the second tier is the sum of amount that count towards the loan programs and funds supplying programs as well as some amount related to the benchmark balances. Again, if the amount computed in the second tier exceeds the upper bound, only the amount of the upper bound is remunerated at zero interest rate. The rest will count towards the third tier and thus will be subject to NIR (from `https://www.boj.or.jp/en/statistics/outline/notice_2016/not160216a.pdf`

[38] The BOJ furthermore also monitors cash holdings of financial institutions to avoid a flight into cash. If the BOJ observes banks with an increase in cash holdings, this amount will be deducted from the second tier and if necessary from the first tier (Bank of Japan 2016).

European Central Bank

The ECB introduced NIR by setting the interest rate on the deposit facility (the lower bound of the channel system) to -0.1% in June 2014. Before that, the deposit facility has been at 0% since 2012. NIR were accompanied by an announcement of targeted longer-term refinancing operations (TLTRO) and a preparation of an expanded purchase program of asset-backed securities. All of these measures were taken to increase inflation and to support bank lending.

This first interest rate cut into the negative territory was followed by four additional interest rate cuts. The last cut occured in September 2019 to -0.5%. The NIR policy ended in June 2022, when the ECB increased it's policy rate to zero.

During the period of NIR policy, NIR applied to reserves deposited at the ECB deposit facility as well as all average reserve holdings that exceed the six-fold of minimum reserve requirements held at the current account. Deposits at the deposit facility of the ECB were not subject to exemptions. NIR also applied to government deposits held at the Eurosystem that exceed a certain threshold and to account balances in TARGET2 (the payment and settlement system of the Euro area), non-Eurosystem overnight deposits held in TARGET2, to Eurosystem reserve management services that were not subject to any other interest rate and other accounts held with Eurosystem central banks that were not subject to other interest rates (ECB 2014).

The two tiered system was implemented in September 2019, when the threshold for exempt reserves was increased from the minimum reserve requirements to the six-fold of minimum reserve requirements. The reasons for implementing a two-tiered system was to support the transmission of monetary policy trough the banking system (ECB 2019).

In the Euro area, NIR transmitted to the money market rates. Turnover in the money market did not markedly change in reaction to the introduction of NIR (Bech and Malkhozov 2016).

Danmarks Nationalbank

The DN has introduced NIR twice in the last few years by lowering the one-week deposit certificate rate into the negative territory. Similar to the SNB, NIR were primarily introduced to alleviate the appreciation pressure on the exchange rate.[39] The first reduction of the one-week certificates of deposit rate was in 2012 and was accompanied by interventions in the foreign exchange market to stabilize the exchange rate of the Danish kroner against the Euro (Jørgensen and Risbjerg 2012). The second reduction of interest rates into the negative territory was in September 2014. After several changes

[39] The DN aims at keeping a nearly fixed exchange rate between the Danish kroner and the Euro, following closely the European Exchange Rate Mechanism II (ERM II).

to the level of the NIR, the DN has increased the one-week deposit certificate rate to 0.65% in September 2022 from previous negative levels.

The DN also largely implemented NIR with a two-tiered system. Reserves held at the central bank are remunerated at the current account rate, which was zero until March 2021, when it turned negative. With the increase to positive interest rate levels in September 2022, the current account rate is now equal to the one-week deposit certificate rate. During the NIR period, there was a limit on the amount in the current account, determined by the transaction volume in the payment system. Reserve holdings that exceeded this limit were subject to the one-week deposit rate (Bech and Malkhozov 2016, Dell'Ariccia et al. 2017, Jørgensen and Risbjerg 2012). The DN adjusted the NIR as well as the distribution of current account limits, while keeping the latter constant in the aggregate (Danmarks Nationalbank 2016).

By-and-large, NIR transmitted to money market rates in Denmark as well. Bech and Malkhozov (2016) reported some decreases in the unsecured money market turnover in Denmark.

Sveriges Riksbank

Already in 2009, the Swedish Riksbank introduced NIR, by decreasing the interest rate on the deposit facility to -0.25%. Yet, as the Riksbank conducts daily reserve-absorbing repo fine-tuning operations, the amount of reserves subject to NIR was very small and the NIR did not transmit to money market rates (Jørgensen and Risbjerg 2012, Bech and Malkhozov 2016).

In 2014, the Riksbank entered again the negative territory and introduced NIR by decreasing the repo rate and the deposit facility rate to the negative territory. The introduction of NIR was accompanied by a quantitative easing program. These measures were introduced to increase inflation and to safeguard the role of inflation as nominal anchor (Bech and Malkhozov 2016, Dell'Ariccia et al. 2017, Sveriges Riksbank 2015). The period of NIR ended, when the Riksbank increased it's policy rate to zero in Januay 2020.

Given the Riksbank's monetary policy implementation framework, the relevant NIR is the repo rate. The Riksbank issues one-week debt certificates, which are remunerated at the repo rate. Moreover, to drain reserves prior to the close of business, fine-tuning operations are conducted. The fine-tuning operations are remunerated at 0.10% below the repo rate. Any residual reserve holdings can be deposited at the deposit facility of the central bank which is currently 10 basis points below the repo rate.

In contrast to all other central banks, the Riksbank effectively operated NIR without exemptions.[40]

During the NIR period in Sweden, NIR have also transmitted to the money market.

Swiss National Bank

The SNB announced the introduction of NIR of -0.25% on December 18th 2014 taking effect as of January 21st 2015 and further decreased the NIR to -0.75% on January 15th when the SNB discontinued the minimum exchange rate of the Swiss Franc against the Euro. Note, that the announcement on January 15th also took effect as of January 21th 2015. NIR were introduced to make investments in Swiss assets relatively less attractive, alleviating the appreciation of the Swiss Franc against the Euro.

The interest rate on excess reserves has been set to -0.75% until June 2022, when the SNB increased the policy rate to -0.25%, followed by another increase to 0.50% in September 2022.

The SNB implemented NIR with a two-tiered system. Reserve holdings above the exemption threshold were remunerated at -0.75% and reserve holdings below the exemption threshold at 0%. Exemptions are calculated according to two methods and were at least CHF 10 millions. The first method was based on a basis component and a cash component. The basis component consisted of the moving average of minimum reserve requirements over 36 reference periods multiplied by a threshold factor, which was last set at 30. The cash component consisted of cash holdings during the last reference period and was subtracted from the basis component. As a consequence, banks could not effectively use cash to avoid NIR. Lastly, the exemption threshold had to be at least as high as minimum reserve requirements in the last reference period. The second method, applying to all financial institutions that were not subject to the first method, set a fixed exemption threshold (SNB 2019).

In Switzerland, NIR have transmitted to money market rates. The introduction of NIR with exemptions caused furthermore a sharp increase of money market turnover in Switzerland as banks reallocated reserves from financial institutions facing NIR to those, which held reserves below their exemptions.

Other Central Banks

Aside from the five central banks discussed above, Norges Bank, Hungary and the Bulgarian National Bank have also adopted some sort of NIR for some time. These central banks are not in the focus of our analysis because the NIR did not transmit to money markets and beyond.

[40] Swedish banks are not subject to minimum reserve requirements.

Appendix B

In the baseline model, we have assumed that bankers are able to pledge reserves held with the central bank as collateral for reserves borrowed in the money market. Specifically, we have imposed $\sigma = 1$ in Equation (11), so that there is no haircut on reserves. This setup allowed the bankers to redistribute reserves among each other in the money market, with the aim of avoiding the NIR as much as possible.

Here, we investigate the effects of NIR policies when the ability to avoid the NIR through borrowing and lending in the money market is impaired. We do so by characterizing equilibria for arbitrary $\sigma \in [0, 1)$, and reconsider our environment with an exogenous deposit rate i_d and without cash. An extreme example is the parameter specification $\sigma = \chi = \theta = 0$, implying that the money market shuts down.

We start the analysis by verifying that there is a floor on the money market rate given by the NIR.

Lemma 13 *Clearance of the IM market implies $\rho_m \leq \rho_n$.*

We then focus on the relevant case in which there must be at least an ε-banker that ends up with reserves beyond the exemption threshold. To ensure this is indeed the case, following Proposition 5, we assume $\bar{m} < m$. Given this assumption, if the money market rate exceeds the NIR, no banker is willing to carry more than $\bar{m}' = \max\{\bar{m} - pk_s, 0\}$ reserves out of the IM market. Since IM market clearance implies that $pk_s + \int_0^\infty m_\varepsilon \mathrm{d}G = m$, we then have $pk_s \geq m - \max\{\bar{m} - pk_s, 0\}$. In turn, this implies $\bar{m} < pk_s$, so all bankers leave the IM market without reserves if $i_m > i_n$. Qualitatively, the economy is therefore in one of the following three cases.

No pass-through of NIR. In this case, the money market rate exceeds the NIR so all bankers leave the IM market without any reserves. It immediately follows that parameter σ is irrelevant. Hence, bankers' optimal investment behavior is given by Lemma 3. Because the money market rate exceeds the floor implied by NIR, the equilibrium money market rate is given by Equation (29). Together with $\varepsilon' = \underline{\varepsilon}$ this pins down allocations. Because we need $\rho_m < \rho_n$, existence of the current case requires

$$\frac{\gamma \rho_n}{\beta} > \int_0^{\underline{\varepsilon}} \mathrm{d}G + \int_{\underline{\varepsilon}}^{\infty} (\varepsilon/\underline{\varepsilon})^{1/\alpha} \, \mathrm{d}G.$$

Pass-through of NIR with all bankers subject to NIR. In this case, the money market rate equals the NIR and all bankers enter the settlement market with reserves greater than the exemption threshold. Evaluating bankers' first-order conditions for

$\sigma < 1$, the latter requires $\bar{m} - pk_s \leq 0$ so that $m_\varepsilon = 0$ is optimal for all bankers. It again follows that parameter σ is irrelevant. Bankers' optimal investment behavior is therefore given by Lemma 3 and because the money market rate is now at the floor implied by NIR, ε' is pinned down by Equation (27). Using investment market clearance and Equations (9), (19), and (20), the condition $\bar{m} - pk_s \leq 0$ can be written as:

$$\varepsilon' \leq \left(\frac{m}{\bar{m}} \frac{\chi + \eta}{\eta} + \theta \right) \left[\int_0^{\varepsilon'} \varepsilon \mathrm{d}G + \int_{\varepsilon'}^{\infty} \varepsilon' \mathrm{d}G \right]. \tag{38}$$

With $\bar{m} \leq m$, Equation (38) is satisfied for $\varepsilon' \leq \bar{\varepsilon}$, where $\bar{\varepsilon}$ depends on $m/\bar{m}$, χ, η, θ, and G. Moreover, $\bar{\varepsilon}$ is strictly increasing in $m/\bar{m}$, $\lim_{m/\bar{m} \to 1} \bar{\varepsilon} = \underline{\varepsilon}$, and $\lim_{m/\bar{m} \to \infty} \bar{\varepsilon} = \infty$. Since Equation (22) also needs to be satisfied, existence of the current case requires

$$\int_0^{\bar{\varepsilon}} \mathrm{d}G + \int_{\bar{\varepsilon}}^{\infty} (\varepsilon/\bar{\varepsilon})^{1/\alpha} \mathrm{d}G \leq \frac{\gamma \rho_n}{\beta} \leq \int_0^{\underline{\varepsilon}} \mathrm{d}G + \int_{\underline{\varepsilon}}^{\infty} (\varepsilon/\underline{\varepsilon})^{1/\alpha}.$$

Pass-through of NIR with some bankers subject to NIR. In this case, the money market rate rate equals the NIR, and some bankers enter the settlement market with reserves strictly greater than the exemption threshold and some with reserves strictly smaller than the exemption threshold.

From bankers' optimization problems and Equation (9), we obtain

$$k_\varepsilon = \begin{cases} \varepsilon \left(\frac{\rho_n}{\rho_d} \right)^\alpha & \text{if } \varepsilon \leq \varepsilon' \\ \varepsilon' \left(\frac{\rho_n}{\rho_d} \right)^\alpha & \text{if } \varepsilon' < \varepsilon \leq \varepsilon'', \\ \varepsilon \left(\frac{\rho_n}{\rho_d} \frac{1-\sigma}{\rho_n/\rho_p - \sigma} \right)^\alpha & \text{if } \varepsilon'' < \varepsilon \leq \varepsilon''' \\ \varepsilon''' \left(\frac{\rho_n}{\rho_d} \frac{1-\sigma}{\rho_n/\rho_p - \sigma} \right)^\alpha & \text{if } \varepsilon''' < \varepsilon \end{cases}, \tag{39}$$

$$m_\varepsilon \geq \underline{m}_\varepsilon = \min \left\{ \bar{m} - \rho_d k_s, \frac{m \frac{\chi+\eta}{\eta} + \theta \rho_d k_s - \rho_d k_\varepsilon}{1 - \sigma} \right\} \quad \text{with} = \text{if } \varepsilon > \varepsilon', \tag{40}$$

$$\varepsilon' = \left[\frac{m}{\rho_d} \frac{\eta + \chi}{\eta} - (1 - \sigma) \frac{\bar{m}}{\rho_d} + (1 - \sigma + \theta) k_s \right] \left(\frac{\rho_d}{\rho_n} \right)^\alpha, \tag{41}$$

$$\varepsilon'' = \varepsilon' \left(\frac{\rho_n/\rho_p - \sigma}{1 - \sigma} \right)^\alpha, \tag{42}$$

$$\varepsilon''' = \left[\frac{m}{\rho_d} \frac{\chi + \eta}{\eta} + \theta k_s \right] \left(\frac{\rho_d}{\rho_n} \frac{\rho_n/\rho_p - \sigma}{1 - \sigma} \right)^\alpha. \tag{43}$$

Most importantly, we now have three critical values for ε. Like before, all bankers with $\varepsilon \leq \varepsilon'$ face a slack borrowing constraint. For all bankers with a slack borrowing constraint, the opportunity cost of capital investment is given by $\frac{\rho_d}{\rho_n}$ as additional reserves carried out of the IM market are subject to NIR for these bankers. New is that bankers with tight borrowing constraints face a trade-off between carrying reserves out of the IM market and investing in productive capital. Specifically, if ε increases beyond ε', bankers continue to invest $k_\varepsilon = k_{\varepsilon'}$ but only until the marginal return from capital investment equals the opportunity cost of capital investment when not subject to NIR, which is given by $\frac{\rho_d}{\rho_n} \frac{\rho_n/\rho_p - \sigma}{1-\sigma}$.[41] That means, until $\varepsilon = \varepsilon''$. Then, as ε increases beyond ε'', it becomes attractive to carry less reserves out of the IM market and to invest more in productive capital. When ε increases beyond ε''', the banker has devoted all available resources to capital investment and invests $k_\varepsilon = k_{\varepsilon'''}$.

Combining Equations (39)-(43) with investment market clearance, money market clearance, and Equation (6), we obtain:

Proposition 14 *Equilibrium with pass-through of NIR and only some bankers subject to NIR, is sufficiently described by a tuple* $(\varepsilon', \varepsilon'', \varepsilon''')$ *that satisfies* $0 < \varepsilon' < \varepsilon''' \left(\frac{1-\sigma}{\rho_n/\rho_p - \sigma}\right)^\alpha$, $\varepsilon'' = \varepsilon' \left(\frac{\rho_n/\rho_p - \sigma}{1-\sigma}\right)^\alpha$, *and the system of equations*

$$\frac{\gamma \rho_n}{\beta} = \int_0^{\varepsilon'} \mathrm{d}G + \int_{\varepsilon'}^{\varepsilon''} \left(\frac{\varepsilon}{\varepsilon'}\right)^{\frac{1}{\alpha}} \mathrm{d}G + \frac{\rho_n/\rho_p - \sigma}{1-\sigma} \left[\int_{\varepsilon''}^{\varepsilon'''} \mathrm{d}G + \int_{\varepsilon'''}^{\infty} \left(\frac{\varepsilon}{\varepsilon'''}\right)^{\frac{1}{\alpha}} \mathrm{d}G\right],$$

$$\frac{\varepsilon' - \left[1 - (1-\sigma)\frac{\eta}{\chi+\eta}\frac{\bar{m}}{m}\right] \varepsilon''' \left(\frac{1-\sigma}{\rho_n/\rho_p - \sigma}\right)^\alpha}{(1-\sigma)\left(1 + \theta \frac{\eta}{\chi+\eta}\frac{\bar{m}}{m}\right)} = \int_0^{\varepsilon'} \varepsilon \mathrm{d}G + \int_{\varepsilon'}^{\varepsilon''} \varepsilon' \mathrm{d}G + \left(\frac{1-\sigma}{\rho_n/\rho_p - \sigma}\right)^\alpha \left[\int_{\varepsilon''}^{\varepsilon'''} \varepsilon \mathrm{d}G + \int_{\varepsilon'''}^{\infty} \varepsilon''' \mathrm{d}G\right].$$

In the proof of Proposition 14, we show that an equilibrium with pass-through of NIR and only some bankers subject to NIR is characterized by a unique tuple $(\varepsilon', \varepsilon'', \varepsilon''')$ and exists if and only if

$$1 \leq \frac{\gamma \rho_n}{\beta} \leq \int_0^{\bar{\varepsilon}} \mathrm{d}G + \int_{\bar{\varepsilon}}^{\infty} (\varepsilon/\bar{\varepsilon})^{1/\alpha} \mathrm{d}G. \tag{44}$$

[41] Bankers with $\varepsilon'' < \varepsilon < \varepsilon'''$ carry some reserves out of the IM but are not subject to NIR. In Equations (15) and (16), this implies $\mathcal{I}_+ = \mathcal{I}_- = \mu_\varepsilon = 0$. Therefore, $\lambda_\varepsilon(1-\sigma) = 1/\rho_p - 1/\rho_m$. Using the latter, $p = \rho_d$, and $\rho_m = \rho_n$ in $p(1/\rho_m + \lambda_\varepsilon)$, i.e., the second part of Equation (14), gives the cost of capital for bankers with $\varepsilon'' < \varepsilon < \varepsilon'''$.

7.1 Discussion

For low values of θ, and a relatively high value for i_n and the exemption threshold, we find that allocations depend on σ. The reason is that bankers which do not carry reserves out of the IM market, are then not subject to NIR in the settlement market. Borrowing reserves in the money market at $i_m = i_n$ to earn $i_p > i_n$ on reserves held with the central bank, then becomes a feasible and profitable investment strategy. With $\sigma < 1$, doing so affects the amount of reserves that a banker can borrow to finance capital investment. As a result, a trade-off between two investment opportunities arises: investment in productive capital and investment in reserves held with the central bank. This trade-off yields different investment behavior than in the baseline economy, as implied by comparing Equation (20) with Equation (39).

Most importantly, even with the trade-off highlighted above present, low ε-bankers ($\varepsilon < \varepsilon'$) do not exhaust their borrowing constraints and leave the IM market with reserves that are subject to NIR. These bankers therefore face an opportunity cost of capital investment governed by the NIR. We therefore find that bankers with ε sufficiently low, still overinvest in an imperfect transmission regime with a NIR case ($i_n = i_m < i_d$). Figure 11 illustrates that when i_n is relatively close to i_d, only bankers that are subject to NIR overinvest. When i_n is relatively far away from i_d, there are also some bankers that are not subject to NIR but that still overinvest, as illustrated in Figure 12.

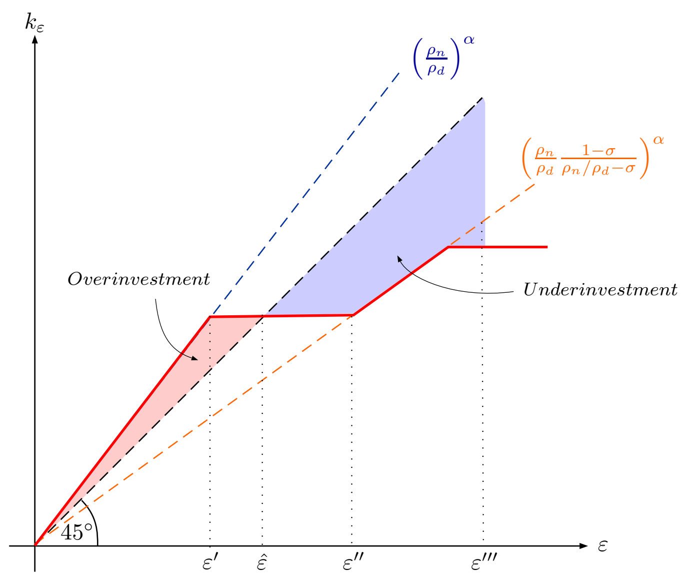

Figure 11: Imperfect transmission: NIR case and $\sigma < 1$ with i_n relatively close to i_d.

For the baseline economy with $\sigma = 1$, both with perfect and imperfect transmission of the money market rate to deposits, we know that welfare, output, and the value

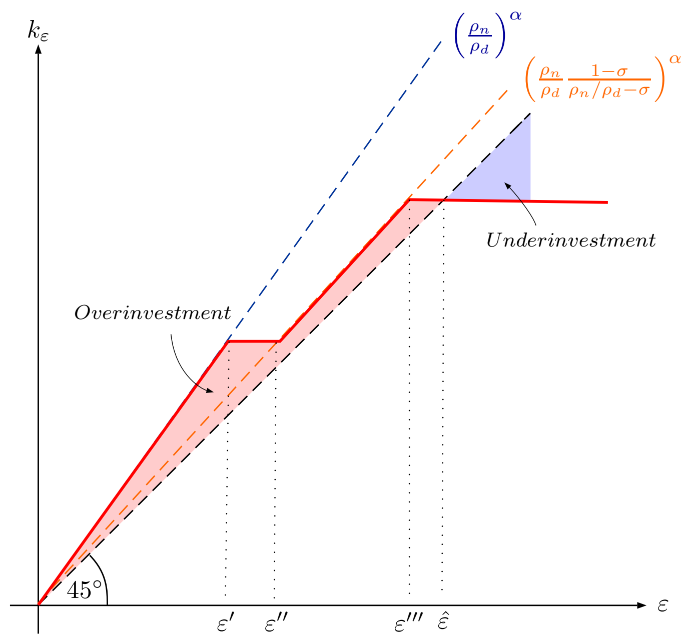

Figure 12: Imperfect transmission: NIR case and $\sigma < 1$ with i_n relatively far from i_d

of reserves are monotonically decreasing in ρ_n when the money-to-bonds ratio remains constant. For the economy with $\sigma < 1$, when ρ_n approaches the Friedman rule, the measure of constrained bankers in the IM market approaches zero. Therefore, allocations and welfare become equivalent to that in the baseline model with $\sigma = 1$. Also, when ρ_n exceeds a critical threshold implied by Equation (44), allocations become equivalent to that in the baseline economy. Hence, at least globally, in the economy with $\sigma < 1$ welfare, output, and the real value of reserves are also decreasing in ρ_n. Our most important qualitative results therefore remain unchanged.

An important difference compared to the baseline model is that the exemption threshold can now affect real allocations. Specifically, a higher exemption threshold pushes allocations away from those in the baseline economy because of two reasons. First, it becomes more likely that bankers face a trade-off between investment in productive capital and investment in reserves held with the central bank. Second, with a higher exemption threshold that trade-off becomes stronger.

Appendix C

Proof of Proposition 2. In steady-state equilibrium, all bankers and households enter the IM with reserves worth m general goods and deposits worth d general goods, respectively. By construction, $\mathcal{W} = \int_0^\infty V_{IM}(m, b, d|\varepsilon)\mathrm{d}G + W_{IM}(d)$. By Definition 1,

$\{m_\varepsilon, k_\varepsilon\}$ solves (13) subject to (10) and (12), and k_s solves (8). Hence, we can substitute out $V_{IM}(m, b, d|\varepsilon)$ and $W_{IM}(d)$ to obtain

$$\mathcal{W} = \int_0^\infty \begin{bmatrix} \varepsilon^{1/\alpha} f(k_\varepsilon) + (m - m_\varepsilon - pk_\varepsilon)/\rho_m + b/\rho_b \\ \quad + \min\{m_\varepsilon + pk_s, \bar{m}\}/\rho_p + \max\{m_\varepsilon + pk_s - \bar{m}, 0\}/\rho_n \end{bmatrix} \mathrm{d}G$$
$$- k_s + V_S(0) + W_S(0).$$

Observe that by construction, reserves carried into the settlement market by a ε-banker satisfy $\hat{m}_\varepsilon = m_\varepsilon + pk_s$. Using IM market clearance conditions $\int_0^\infty k_\varepsilon \mathrm{d}G = k_s$ and $m = \int_0^\infty m_\varepsilon \mathrm{d}G + pk_s$, we may rewrite the above as

$$\mathcal{W} = \int_0^\infty \left[\varepsilon^{1/\alpha} f(k_\varepsilon) - k_\varepsilon + i_p \min\{\hat{m}_\varepsilon, \bar{m}\} + i_n \max\{\hat{m}_\varepsilon - \bar{m}, 0\}\right] \mathrm{d}G$$
$$+ m + b/\rho_b + V_S(0) + W_S(0).$$

From Definition 1, it follows that m and b solve (5) and that d solves (3). Hence, using (5) and (3), we can substitute out $V_S(0)$ and $W_S(0)$ to obtain:

$$\mathcal{W} = \int_0^\infty \left[\varepsilon^{1/\alpha} f(k_\varepsilon) - k_\varepsilon + i_p \min\{\hat{m}_\varepsilon, \bar{m}\} + i_n \max\{\hat{m}_\varepsilon - \bar{m}, 0\}\right] \mathrm{d}G$$
$$+ m(1-\gamma) + b(1/\rho_b - \gamma) + \tau_B + \tau_H + \beta \left[\int_0^\infty V_{IM}(m, d|\varepsilon)\mathrm{d}G + W_{IM}(d)\right].$$

Combining Equation (1) and (2) together with the fact that $m = b_{TR} - b$, we find

$$\tau_H + \tau_B = b(\gamma - 1/\rho_b) + (\gamma - 1)m - \int_0^\infty [i_p \min\{\hat{m}_\varepsilon, \bar{m}\} + i_n \max\{\hat{m}_\varepsilon - \bar{m}, 0\}]\mathrm{d}G. \quad (45)$$

Using this together with the fact that $\mathcal{W} = \int_0^\infty V_{IM}(m, d|\varepsilon)\mathrm{d}G + W_{IM}(d)$, we arrive at:

$$(1-\beta)\mathcal{W} = \int_0^\infty \left[\varepsilon^{1/\alpha} f(k_\varepsilon) - k_\varepsilon\right] \mathrm{d}G.$$

■

Proof of Lemma 3. We first derive the investment schedule for bankers. Consider a banker for which the borrowing constraint (12) is slack, so that $\lambda_\varepsilon = 0$. From first-order condition (14) and Equation (9), we find that for this banker

$$k_\varepsilon = \varepsilon \left(\frac{\rho_m}{\rho_d}\right)^\alpha. \quad (46)$$

To ensure that the borrowing constraint (12) is indeed slack, we need $pk_\varepsilon \leq m + \chi b + \theta p k_s$.

Again using Equation (9), it follows that bankers face a slack borrowing constraint if and only if

$$\varepsilon \leq \left(\frac{\rho_d}{\rho_m}\right)^{\alpha} \left(\frac{m+\chi b}{\rho_d} + \theta k_s\right). \tag{47}$$

Investment by bankers for which borrowing constraint (12) is tight, is given by $pk_\varepsilon = m + \chi b + \theta p k_s$. Again using Equation (9), it follows that for these bankers

$$k_\varepsilon = \frac{m+\chi b}{\rho_d} + \theta k_s \tag{48}$$

Combing Equations (46), (47), and (48), it follows that bankers' investment schedule is given by

$$k_\varepsilon = \begin{cases} \varepsilon \left(\rho_m/\rho_d\right)^{\alpha} & \text{if } \varepsilon \leq \varepsilon' \\ \varepsilon' \left(\rho_m/\rho_d\right)^{\alpha} & \text{if } \varepsilon > \varepsilon' \end{cases}, \quad \text{where} \quad \varepsilon' = \left(\frac{\rho_d}{\rho_m}\right)^{\alpha} \left(\frac{m+\chi b}{\rho_d} + \theta k_s\right). \tag{49}$$

Then, using Equation (49) together with $\int_0^\infty k_\varepsilon \mathrm{d}G = k_s$ and $b = m/\eta$, gives

$$0 = \varepsilon' - \left(\frac{\rho_d}{\rho_m}\right)^{\alpha} \frac{m}{\rho_d} \frac{\chi+\eta}{\eta} - \theta \left[\int_0^{\varepsilon'} \varepsilon \mathrm{d}G + \int_{\varepsilon'}^{\infty} \varepsilon' \mathrm{d}G\right]. \tag{50}$$

The partial derivative of the RHS of Equation (50) w.r.t. ε' is given by $1 - \theta[1 - G(\varepsilon')]$. With $\theta \in [0,1]$, the RHS is therefore monotonically increasing in ε'. Moreover, the RHS is negative when $\varepsilon' = 0$ and approaches ∞ when ε' approaches ∞. Hence, Equation (50) uniquely pins down ε'. ■

Proof of Lemma 4. Equation (15) cannot hold if $i_m < i_n \leq i_p$. Hence, we can focus on cases in which $i_m \geq i_n$ as otherwise demand for reserves carried out of the IM becomes infinitely large.

When $i_n \leq i_m < i_p$, Equation (15) holds if and only if $m_\varepsilon \geq \max\{\bar{m} - pk_s, 0\}$. When $i_n \leq i_m = i_p$, (15) holds if and only if $m_\varepsilon \geq 0$. Finally when $i_n \leq i_p < i_m$, (15) is trivially satisfied.

Similarly, when $i_m = i_n \leq i_p$, Equation (16) is trivially satisfied. When $i_n < i_m \leq i_p$, (16) holds if and only if $0 \leq m_\varepsilon \leq \max\{\bar{m} - pk_s, 0\}$. Finally, when $i_n \leq i_p < i_m$, (16) holds if and only if $m_\varepsilon = 0$.

Combining the insights above, we obtain that m_ε solves (15) and (16) if and only if:

$$m_\varepsilon \leq \begin{cases} \max\{\bar{m} - pk_s, 0\} & \text{if } i_n < i_m \leq i_p \\ 0 & \text{if } i_m > i_p \geq i_n \end{cases}, \; m_\varepsilon \geq \begin{cases} \max\{\bar{m} - pk_s, 0\} & \text{if } i_n \leq i_m < i_p \\ 0 & \text{if } i_m \geq i_p \geq i_n \end{cases}.$$

Using Equation (9) and defining $\bar{m}' = \max\{\bar{m} - \rho_d k_s, 0\}$, gives Equation (23). ■

Proof of Proposition 5. We distinguish between money market equilibria without and with excess reserves: $\int_0^\infty m_\varepsilon \mathrm{d}G = 0$ and $\int_0^\infty m_\varepsilon \mathrm{d}G > 0$, respectively.

In an equilibrium without excess reserves, clearance of the money market implies $m = \int_0^\infty pk_\varepsilon \mathrm{d}G$. Using Equations (9) and (20), the latter becomes

$$\frac{m}{\rho_d} = \left(\frac{\rho_m}{\rho_d}\right)^\alpha \left[\int_0^{\varepsilon'} \varepsilon \mathrm{d}G + \int_{\varepsilon'}^\infty \varepsilon' \mathrm{d}G\right]. \tag{51}$$

Using Equation (19) in Equation (51), we find that

$$\varepsilon' = \left(\frac{\chi + \eta}{\eta} + \theta\right) \left[\int_0^{\varepsilon'} \varepsilon \mathrm{d}G + \int_{\varepsilon'}^\infty \varepsilon' \mathrm{d}G\right]. \tag{52}$$

With χ and $\theta = 0$, Equation (52) holds only if $\varepsilon' = 0$. According to Equation (51), that must however imply $m = 0$, contradicting the notion of an equilibrium in Definition 1. With χ and/or $\theta > 0$, there is a unique strictly positive value for ε' that satisfies Equation (52), which we have defined as $\underline{\varepsilon}$. It follows that the money market rate satisfies

$$\frac{m}{\rho_d} = \left(\frac{\rho_m}{\rho_d}\right)^\alpha \left[\int_0^{\underline{\varepsilon}} \varepsilon \mathrm{d}G + \int_{\underline{\varepsilon}}^\infty \underline{\varepsilon} \mathrm{d}G\right]. \tag{53}$$

It remains to check whether $\int_0^\infty m_\varepsilon \mathrm{d}G = 0$, which must imply $m_\varepsilon = 0$ for all ε, is in line with Equation (23). By construction $\bar{m}' = \min\{\bar{m} - m, 0\}$. Clearly, with $\bar{m} \leq m$, $m_\varepsilon = 0$ requires $\rho_m \leq \rho_n$ according to Equation (23). Also, with $\bar{m} > m$, $m_\varepsilon = 0$ requires $\rho_m \leq \rho_p$ according to Equation (23). Hence, in an equilibrium without excess reserves

$$\rho_m \leq \begin{cases} \rho_n & \text{if } \bar{m} \leq m \\ \rho_p & \text{if } \bar{m} > m \end{cases}. \tag{54}$$

In an equilibrium with excess reserves, $m_\varepsilon > 0$ for at least some ε and $pk_s < m$. When $\bar{m} < m$, then $m_\varepsilon > \bar{m}'$ for at least some ε. Otherwise $m = pk_s + \int_0^\infty m_\varepsilon \mathrm{d}G \leq pk_s + \max\{\bar{m} - pk_s, 0\}$, which implies either $pk_s \geq m$ or $\bar{m} \geq m$; a contradiction. Hence, Equation (23) holds if and only if $\rho_m = \rho_n$. Similarly, when $\bar{m} > m$, then $m_\varepsilon < \bar{m}'$ for at least some ε. First, because $pk_s < m$ and $\bar{m} > m$, we have $\bar{m}' > 0$. Second, if $m_\varepsilon \geq \bar{m}'$ for all ε, then money market clearance implies that $m = \int_0^\infty [pk_\varepsilon + m_\varepsilon]\mathrm{d}G \geq pk_s + \bar{m}' = \bar{m}$; a contradiction. Hence, because $m_\varepsilon > 0$ for at least some ε and $m_\varepsilon < \bar{m}'$ for some ε, Equation (23) requires that $\rho_m = \rho_p$. Finally, with $m = \bar{m}$, we must have either $\rho_p = \rho_n$ or $m_\varepsilon = \bar{m}'$. Clearly, with $\rho_p < \rho_n$ there cannot be bankers with $m_\varepsilon > \bar{m}'$ as well as bankers with $m_\varepsilon < \bar{m}'$. Next, because $pk_s < m$ and $\bar{m} = m$, we again have $\bar{m}' > 0$. Moreover, when $m_\varepsilon < \bar{m}'$ for all ε then $m = \int_0^\infty [pk_\varepsilon + m_\varepsilon]\mathrm{d}G < pk_s + \bar{m}' = \bar{m}$ and when $m_\varepsilon > \bar{m}'$ for all ε then $m = \int_0^\infty [pk_\varepsilon + m_\varepsilon]\mathrm{d}G > pk_s + \bar{m}' = \bar{m}$; both contradictions. With $m_\varepsilon > 0$ for at least some ε and either $\rho_p = \rho_n$ or $m_\varepsilon = \bar{m}' > 0$, it follows that Equation (23) is satisfied for all $\rho_m \in [\rho_p, \rho_n]$. In an equilibrium with excess reserves we therefore find

$$\rho_m \in \begin{cases} \{\rho_n\} & \text{if } \bar{m} < m \\ [\rho_p, \rho_n] & \text{if } \bar{m} = m \\ \{\rho_p\} & \text{if } \bar{m} > m \end{cases}. \tag{55}$$

To ensure that the money market clears with $\int_0^\infty m_\varepsilon \mathrm{d}G > 0$, we need $m > \int_0^\infty pk_\varepsilon \mathrm{d}G$. Using Equation (20), the latter becomes

$$\frac{m}{\rho_d} > \left(\frac{\rho_m}{\rho_d}\right)^\alpha \left[\int_0^{\varepsilon'} \varepsilon \mathrm{d}G + \int_{\varepsilon'}^\infty \varepsilon' \mathrm{d}G\right]. \tag{56}$$

With $\chi = 0$ and $\theta = 0$, Equation (19) implies that Equation (56) is satisfied when $m > 0$ With $\chi > 0$ and/or $\theta > 0$, Equation (19) implies that Equation (56) is satisfied when $\varepsilon' > \underline{\varepsilon}$, which in turn is satisfied whenever

$$\frac{m}{\rho_d} > \left(\frac{\rho_m}{\rho_d}\right)^\alpha \left[\int_0^{\underline{\varepsilon}} \varepsilon \mathrm{d}G + \int_{\underline{\varepsilon}}^\infty \underline{\varepsilon} \mathrm{d}G\right].$$

Because $\lim_{\chi \to 0, \theta \to 0} \underline{\varepsilon} = 0$, the model exhibits no discontinuity at $\chi, \theta = 0$. Without loss, matters in the money market are therefore characterized by a floor on the money market rate given by Equation (54) and the following equilibrium condition

$$\frac{m}{\rho_d} \geq \left(\frac{\rho_m}{\rho_d}\right)^\alpha \left[\int_0^{\underline{\varepsilon}} \varepsilon \mathrm{d}G + \int_{\underline{\varepsilon}}^\infty \underline{\varepsilon} \mathrm{d}G\right], \quad \text{with = if} \quad \rho_m < \begin{cases} \rho_n & \text{if } \bar{m} < m \\ \rho_p & \text{if } \bar{m} \geq m \end{cases}.$$

■

Proof of Proposition 6. To derive Equation (27), differentiate $V_{IM}(m, b, d|\varepsilon)$ with respect to m to get

$$V_{IM}^m(m, b, d|\varepsilon) = 1/\rho_m + \lambda_\varepsilon.$$

Using Equation (14) to replace λ_ε and using Equation (9) to replace p, yields

$$V_{IM}^m(m, b, d|\varepsilon) = \frac{\varepsilon^{1/\alpha} f'(k_\varepsilon)}{\rho_d}. \tag{57}$$

Then, use Equation (57) in Equation (6), which should hold with equality when $m > 0$, to get

$$\frac{\gamma}{\beta} = \int_0^\infty \frac{\varepsilon^{1/\alpha} f'(k_\varepsilon)}{\rho_d} \mathrm{d}G.$$

Finally, recall that $f'(k) = k^{-1/\alpha}$ and replace the quantities k_ε using Lemma 3 to get

$$\frac{\rho_m \gamma}{\beta} = \int_0^{\varepsilon'} \mathrm{d}G + \int_{\varepsilon'}^\infty (\varepsilon/\varepsilon')^{1/\alpha} \mathrm{d}G, \tag{58}$$

which is the same as Equation (27), because $\rho_m = \rho_n$ in an equilibrium with full pass-through of NIR to money market rates. With $\bar{m} < m$, combining Equation (24) and our definition of ε' in Equation (19), Proposition 5 implies that $\rho_m = \rho_n$ if $\varepsilon' \geq \underline{\varepsilon}$. The RHS of (27) is strictly decreasing in ε', approaches 1 if $\varepsilon' \to \infty$, and approaches ∞ when $\varepsilon' \to 0$. It follows that an ε' that solves Equation (27) exists if and only if $\gamma\rho_n \geq \beta$ and satisfies $\varepsilon' \geq \underline{\varepsilon}$ if and only if $\gamma\rho_n/\beta \leq \int_0^{\underline{\varepsilon}} \mathrm{d}G + \int_{\underline{\varepsilon}}^\infty (\varepsilon/\underline{\varepsilon})^{1/\alpha} \mathrm{d}G$. ■

Proof of Lemma 7. To derive the critical value $\tilde{\varepsilon}$, we consider the meaningful case in which $i_n < i_p$. All bankers that are constrained, i.e., $\varepsilon > \varepsilon'$, have $\rho_d k_\varepsilon = m + \chi b + \theta \rho_d k_s$ and $m_\varepsilon \geq \max\{\bar{m} - \rho_d k_s, 0\}$. Hence, net borrowing in the money market for these bankers satisfies

$$z_\varepsilon \geq \chi b + \theta \rho_d k_s + \max\{\bar{m} - \rho_d k_s, 0\}, \quad \text{for all } \varepsilon > \varepsilon'.$$

Therefore, $z_\varepsilon > 0$ for all $\varepsilon > \varepsilon'$ if $\chi > 0$ and/or $\theta > 0$. If $\chi = 0$ and $\theta = 0$, we have $z_\varepsilon > 0$ for all $\varepsilon > \varepsilon'$ if $\max\{\bar{m} - \rho_d k_s, 0\} > 0$.

Bankers that are unconstrained, i.e., $\varepsilon \leq \varepsilon'$, have $k_\varepsilon = \varepsilon (\rho_m/\rho_d)^\alpha$. Since k_ε is increasing in ε, there exists a critical value $\tilde{\varepsilon}$ such that

$$\rho_d k_{\tilde{\varepsilon}} + \max\{\bar{m} - \rho_d k_s, 0\} = m. \tag{59}$$

For bankers with $\varepsilon < \tilde{\varepsilon}$, $z_\varepsilon < 0$ is feasible. We can rewrite Equation (59) as follows

$$k_{\tilde{\varepsilon}} = \frac{m}{\rho_d} - \max\left\{\frac{\bar{m}}{m}\frac{m}{\rho_d} - k_s, 0\right\}.$$

Then, use $k_{\tilde{\varepsilon}} = \tilde{\varepsilon}\,(\rho_m/\rho_d)^{\alpha}$ and Equation (19) to get

$$\tilde{\varepsilon} = \left[\varepsilon' - \theta\left(\int_0^{\varepsilon'} \varepsilon \mathrm{d}G + \int_{\varepsilon'}^{\infty} \varepsilon' \mathrm{d}G\right)\right]\frac{\eta}{\chi+\eta}$$
$$- \max\left\{\frac{\bar{m}}{m}\left[\varepsilon' - \theta\left(\int_0^{\varepsilon'} \varepsilon \mathrm{d}G + \int_{\varepsilon'}^{\infty} \varepsilon' \mathrm{d}G\right)\right]\frac{\eta}{\chi+\eta} - (\rho_d/\rho_m)^{\alpha} k_s, 0\right\}. \quad (60)$$

Using that $k_s = \int_0^{\infty} k_{\varepsilon} \mathrm{d}G$ in combination with Equation (20), implies that Equation (60) becomes

$$\tilde{\varepsilon} = \left[\varepsilon' - \theta\left(\int_0^{\varepsilon'} \varepsilon \mathrm{d}G + \int_{\varepsilon'}^{\infty} \varepsilon' \mathrm{d}G\right)\right]\frac{\eta}{\chi+\eta}$$
$$- \max\left\{\varepsilon'\frac{\bar{m}}{m} - \left(\frac{\chi+\eta}{\eta} + \theta\frac{\bar{m}}{m}\right)\left[\int_0^{\varepsilon'} \varepsilon \mathrm{d}G + \int_{\varepsilon'}^{\infty} \varepsilon' \mathrm{d}G\right], 0\right\}\frac{\eta}{\chi+\eta},$$

where $\varepsilon' \leq \tilde{\varepsilon}$ is automatically satisfied. Also, $\varepsilon' < \tilde{\varepsilon}$ when $\chi > 0$ and/or $\theta > 0$.

Clearly, with χ and/or $\theta > 0$ it follows that all bankers with $\varepsilon > \tilde{\varepsilon}$ borrow in the money market. Without loss we can assume that all bankers with $\varepsilon < \tilde{\varepsilon}$ then lend in the money market.

With χ and $\theta = 0$, if $\max\{\bar{m} - \rho_d k_s, 0\} > 0$ all bankers with $\varepsilon > \tilde{\varepsilon}$ borrow in the money market. Without loss we can assume again that all bankers with $\varepsilon < \tilde{\varepsilon}$ then lend in the money market. If $\max\{\bar{m} - \rho_d k_s, 0\} = 0$, which with $\chi = 0$ and $\theta = 0$ holds if and only if $\tilde{\varepsilon} = \varepsilon'$, we can assume without loss that bankers are not active in the money market; $z_{\varepsilon} = 0$ for all ε.

To derive the critical value $\hat{\varepsilon}$, note that $k^*_{\varepsilon} = \varepsilon$ and that $k_{\varepsilon} \leq \varepsilon(\rho_m/\rho_d)^{\alpha}$. Thus, ε for which $k_{\varepsilon} > \varepsilon$ exist only if $1 < (\rho_m/\rho_d)^{\alpha}$. When $1 < (\rho_m/\rho_d)^{\alpha}$, it follows from Equation (20) that $k_{\varepsilon} > \varepsilon$ if and only if $\varepsilon < \varepsilon'(\rho_m/\rho_d)^{\alpha}$. It follows that $\hat{\varepsilon} = \varepsilon'(\rho_m/\rho_d)^{\alpha}$. Using that $\rho_m = \rho_n$ with full pass-through of NIR to money market rates, gives Equation (31). ■

Proof of Proposition 8. To prove the Proposition, define $\Phi_0 \equiv \rho_n\gamma/\beta$, $\Phi_1 \equiv \int_0^{\varepsilon'} \mathrm{d}G$, and $\Phi_2 \equiv \int_{\varepsilon'}^{\infty} (\varepsilon/\varepsilon')^{1/\alpha} \mathrm{d}G$. First, we derive $\frac{\mathrm{d}\varepsilon'}{\mathrm{d}\rho_n}$. From Equation (27), ε' satisfies

$$\frac{\rho_n\gamma}{\beta} = \int_0^{\varepsilon'} \mathrm{d}G + \int_{\varepsilon'}^{\infty} (\varepsilon/\varepsilon')^{1/\alpha} \mathrm{d}G.$$

The derivative satisfies

$$\frac{\mathrm{d}\varepsilon'}{\mathrm{d}\rho_n} = -\alpha\varepsilon' \left(\gamma/\beta\right) \Phi_2^{-1} = -\alpha\varepsilon' \Phi_0 \Phi_2^{-1} \rho_n^{-1}.$$

Rearranging yields

$$\frac{\mathrm{d}\varepsilon'}{\mathrm{d}\rho_n} \frac{\rho_n}{\varepsilon'} = -\alpha\Phi_0\Phi_2^{-1} < 0, \tag{61}$$

indicating that there are more constrained bankers.

Next, from Equation (30)

$$\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}\rho_n} = \left\{1 - \theta[1 - G(\varepsilon')] - \mathcal{I}\left[\frac{\bar{m}}{m} - \left(\frac{\chi+\eta}{\eta} + \theta\frac{\bar{m}}{m}\right)[1 - G(\varepsilon')]\right]\right\} \frac{\eta}{\chi+\eta} \frac{\mathrm{d}\varepsilon'}{\mathrm{d}\rho_n}, \tag{62}$$

where

$$\mathcal{I} = \begin{cases} 0 & \text{if } \varepsilon\bar{m}/m - [(\chi+\eta)/\eta + \theta\bar{m}/m][\int_0^{\varepsilon'} \varepsilon \mathrm{d}G + \int_{\varepsilon'}^{\infty} \varepsilon' \mathrm{d}G] < 0 \\ 1 & \text{if } \varepsilon\bar{m}/m - [(\chi+\eta)/\eta + \theta\bar{m}/m][\int_0^{\varepsilon'} \varepsilon \mathrm{d}G + \int_{\varepsilon'}^{\infty} \varepsilon' \mathrm{d}G] \geq 0 \end{cases}.$$

Rearrange Equation (62) as

$$\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}\rho_n} = \left\{\left(1 - \mathcal{I}\frac{\bar{m}}{m}\right)\left(1 - \theta[1 - G(\varepsilon')]\right) + \mathcal{I}\frac{\chi+\eta}{\eta}[1 - G(\varepsilon')]\right\} \frac{\eta}{\chi+\eta} \frac{\mathrm{d}\varepsilon'}{\mathrm{d}\rho_n}.$$

With $\frac{\mathrm{d}\varepsilon'}{\mathrm{d}\rho_n} < 0$, $\theta \in [0, 1]$, and $\bar{m} < m$ we clearly have $\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}\rho_n} < 0$, indicating that there are more bankers that borrow in the money market.

Then, from Equation (31) and using that $\frac{\mathrm{d}\varepsilon'}{\mathrm{d}\rho_n}\frac{\rho_n}{\varepsilon'} = -\alpha\Phi_0\Phi_2^{-1}$, we obtain

$$\frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} \frac{\rho_n}{\hat{\varepsilon}} = \alpha\left(1 - \frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d} - \Phi_0\Phi_2^{-1}\right).$$

With perfect transmission, $\rho_n = \rho_d$ and $\mathrm{d}\rho_n = \mathrm{d}\rho_p$ so $\frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} < 0$. With imperfect transmission, $\mathrm{d}\rho_d = 0$. Using that $\Phi_0 = \Phi_1 + \Phi_2$, it also follows that $\frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} < 0$. So, when $\varepsilon' < \hat{\varepsilon}$ less bankers overinvest.

Regarding investment quantities, using Lemma (3) and $\frac{\mathrm{d}\varepsilon'}{\mathrm{d}\rho_n}\frac{\rho_n}{\varepsilon'} = -\alpha\Phi_0\Phi_2^{-1}$, we have

$$\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n}\frac{\rho_n}{k_\varepsilon} = \begin{cases} \alpha\left(1 - \frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}\right) & \text{if } \varepsilon < \varepsilon' \\ \alpha\left(1 - \frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d} - \Phi_0\Phi_2^{-1}\right) & \text{if } \varepsilon > \varepsilon' \end{cases}. \tag{63}$$

Clearly, for $\varepsilon < \varepsilon'$ with perfect transmission $\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} = 0$ and with imperfect transmission $\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} > 0$. Moreover, for $\varepsilon > \varepsilon'$ we have $\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n}\frac{\rho_n}{k_\varepsilon} = \frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n}\frac{\rho_n}{\hat{\varepsilon}}$ so $\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} < 0$ with both perfect and imperfect transmission. Hence, constrained bankers invest less. Unconstrained bankers invest more with imperfect transmission and their investment remains unaffected with perfect transmission. ■

Proof of Proposition 9. First, we derive the expression for $\frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n}$. Take the derivative of Equation (2) with respect to ρ_n to get

$$\frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n} = \int_0^{\varepsilon'} \left[\varepsilon^{(1/\alpha)} f'(k_\varepsilon) - 1\right] \frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} dG + \int_{\varepsilon'}^{\infty} \left[\varepsilon^{(1/\alpha)} f'(k_{\varepsilon'}) - 1\right] \frac{\mathrm{d}k_{\varepsilon'}}{\mathrm{d}\rho_n} dG.$$

Note that ε' depends on ρ_n but the two changes in the integral bounds cancel each other out. We can use Proposition 8 to write this expression as follows

$$\frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n} = \int_0^{\varepsilon'} \left[\varepsilon^{(1/\alpha)} (k_\varepsilon)^{-1/\alpha} - 1\right] \frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} dG + \int_{\varepsilon'}^{\infty} \left[\varepsilon^{(1/\alpha)} (k_{\varepsilon'})^{-1/\alpha} - 1\right] \frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} dG.$$

Now use Lemma 3 to replace $k_\varepsilon = \varepsilon (\rho_n/\rho_d)^\alpha$ and $k_{\varepsilon'} = \varepsilon' (\rho_n/\rho_d)^\alpha$ to get

$$\frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n} = \int_0^{\varepsilon'} [\rho_d/\rho_n - 1] \frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} dG + \int_{\varepsilon'}^{\infty} \left[(\rho_d/\rho_n) (\varepsilon/\varepsilon')^{1/\alpha} - 1\right] \frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} dG.$$

Thus, the derivative $\frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n}$ can be broken into three terms as follows:

$$\begin{aligned}
\frac{d(1-\beta)\mathcal{W}}{d\rho_n} &= A + B + C, \text{ where} \\
& A \equiv \int_0^{\varepsilon'} [\rho_d/\rho_n - 1] \frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} dG, \\
& B \equiv \int_{\varepsilon'}^{\hat{\varepsilon}} \left[(\rho_d/\rho_n) (\varepsilon/\varepsilon')^{1/\alpha} - 1\right] \frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} dG, \\
& C \equiv \int_{\hat{\varepsilon}}^{\infty} \left[(\rho_d/\rho_n) (\varepsilon/\varepsilon')^{1/\alpha} - 1\right] \frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} dG.
\end{aligned}$$

Using $(k_{\varepsilon'})^{-1/\alpha} = (1/\varepsilon')^{1/\alpha} (\rho_d/\rho_n)$ and rearranging $\left[(\rho_d/\rho_n) (\varepsilon/\varepsilon')^{1/\alpha} - 1\right] = \left[(\varepsilon/k_{\varepsilon'})^{1/\alpha} - 1\right]$ we obtain

$$\begin{aligned}
\frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n} &= A + B + C, \text{ where} \\
& A \equiv \rho_d \int_0^{\varepsilon'} (i_n - i_d) \frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n} dG, \\
& B \equiv \int_{\varepsilon'}^{\hat{\varepsilon}} \left[(\varepsilon/\hat{\varepsilon})^{1/\alpha} - 1\right] \frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} dG \\
& C \equiv \int_{\hat{\varepsilon}}^{\infty} \left[(\varepsilon/\hat{\varepsilon})^{1/\alpha} - 1\right] \frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} dG.
\end{aligned}$$

Now, we prove $\frac{d(1-\beta)\mathcal{W}}{d\rho_n} < 0$. We restrict attention to a case in which $\gamma\rho_n > \beta$, as $\varepsilon' = \infty$ otherwise. First, we suppose that $\rho_n > \rho_d$ (NIR case). Here, because $A \leq 0$ and $\frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n} < 0$, it suffices to show that $\int_{\varepsilon'}^{\infty}[(\varepsilon/\hat{\varepsilon})^{1/\alpha} - 1]\mathrm{d}G > 0$. Use that $(\hat{\varepsilon}/\varepsilon')^{1/\alpha} = \rho_n/\rho_d$ and that Equation (27) implies $(\gamma\rho_n - \beta)/\beta = \int_{\varepsilon'}^{\infty}[(\varepsilon/\varepsilon')^{1/\alpha} - 1]\mathrm{d}G$, to find

$$\begin{aligned}\int_{\varepsilon'}^{\infty}[(\varepsilon/\hat{\varepsilon})^{1/\alpha} - 1]\mathrm{d}G &= \frac{\rho_d}{\rho_n}\left[\frac{\gamma\rho_n - \beta}{\beta} + \frac{\rho_d - \rho_n}{\rho_d}\left[1 - G(\varepsilon')\right]\right] \\ &\geq \frac{\rho_d}{\rho_n}\left[\frac{\beta\rho_n/\rho_d - \beta}{\beta} + \frac{\rho_d - \rho_n}{\rho_d}\left[1 - G(\varepsilon')\right]\right] \\ &= \frac{\rho_n - \rho_d}{\rho_n}G(\varepsilon') > 0,\end{aligned}$$

where the second step uses $\gamma\rho_d \geq \beta$ and the third step uses $\rho_n > \rho_d$.

Next, we drop our supposition $\rho_n > \rho_d$ and note that $\frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n} = \Omega_1 - \Omega_2$, with

$$\Omega_1 = \int_0^{\infty}\left(\frac{\varepsilon}{k_\varepsilon}\right)^{1/\alpha}\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n}\mathrm{d}G \quad \text{and} \quad \Omega_2 = \int_0^{\infty}\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n}\mathrm{d}G.$$

Using Lemma (3) together with insights from the proof of Proposition 8, with imperfect transmission we find for Ω_1

$$\begin{aligned}\Omega_1 &= \alpha\frac{1}{\rho_n}\left(\frac{\rho_n}{\rho_d}\right)^{\alpha-1}\left[\left(1 - \frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}\right)\int_0^{\varepsilon'}\varepsilon\mathrm{d}G + \varepsilon'\left(1 - \frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d} - \Phi_0\Phi_2^{-1}\right)\int_{\varepsilon'}^{\infty}\left(\frac{\varepsilon}{\varepsilon'}\right)^{1/\alpha}\mathrm{d}G\right] \\ &= \alpha\frac{1}{\rho_n}\left(\frac{\rho_n}{\rho_d}\right)^{\alpha-1}\left[\left(1 - \frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}\right)\int_0^{\varepsilon'}(\varepsilon - \varepsilon')\mathrm{d}G - \varepsilon'\Phi_0\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}\right],\end{aligned}$$

where the second step uses how Φ_0, Φ_1, and Φ_2 are defined. For Ω_2 we find

$$\Omega_2 = \alpha\frac{1}{\rho_n}\left(\frac{\rho_n}{\rho_d}\right)^{\alpha}\left[\left(1 - \frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}\right)\int_0^{\varepsilon'}\varepsilon\mathrm{d}G + \left(1 - \frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d} - \Phi_0\Phi_2^{-1}\right)\int_{\varepsilon'}^{\infty}\varepsilon'\mathrm{d}G\right].$$

With perfect transmission, meaning that $\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d} = 1$, we find

$$\begin{aligned}\frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n} &\propto -\int_{\varepsilon'}^{\infty}\left[\frac{\rho_d}{\rho_n}\left(\frac{\varepsilon}{\varepsilon'}\right)^{1/\alpha} - 1\right]\mathrm{d}G \\ &= -\int_{\varepsilon'}^{\infty}\left[\left(\frac{\varepsilon}{\hat{\varepsilon}}\right)^{1/\alpha} - 1\right]\mathrm{d}G\end{aligned}$$

With $\rho_n > \rho_d$, we have already established that $\int_{\varepsilon'}^{\infty}\left[(\varepsilon/\hat{\varepsilon})^{1/\alpha} - 1\right] > 0$. With $\rho_n \leq \rho_d$, we have $\varepsilon' \geq \hat{\varepsilon}$. It follows that with perfect transmission, $\frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n} < 0$.

With imperfect transmission, meaning that $\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}=0$, we find

$$\frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n} \propto \frac{\rho_d}{\rho_n}\int_0^{\varepsilon'}(\varepsilon-\varepsilon')\mathrm{d}G - \left[\int_0^{\varepsilon'}\varepsilon \mathrm{d}G + \varepsilon'(1-\Phi_0\Phi_2^{-1})\int_{\varepsilon'}^{\infty}\mathrm{d}G\right]$$

Note that with imperfect transmission, ρ_d does not depend on ρ_n. Moreover ε' depends only on $\gamma\rho_n/\beta$ and G, and not on ρ_d. As $\int_0^{\varepsilon'}(\varepsilon-\varepsilon')\mathrm{d}G<0$, the RHS of the equation above must be decreasing in ρ_d. To conclude, evaluate matters when $\rho_d=\rho_n$:

$$\left.\frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n}\right|_{\rho_d=\rho_n} \propto \varepsilon'\left(\Phi_0\Phi_2^{-1}\int_{\varepsilon'}^{\infty}\mathrm{d}G-1\right)=\varepsilon'\left(\frac{\Phi_0[1-G(\varepsilon')]}{\Phi_0-G(\varepsilon')}-1\right)<0,$$

where we use that $\Phi_2=\Phi_0-G(\varepsilon')$ and $\Phi_0>1$ when $\gamma\rho_n>\beta$. It follows that $\frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n}<0$ when $\rho_n\leq\rho_d$. ■

Proof of Proposition 10. In equilibrium, aggregate output satisfies

$$Q=\int_0^{\varepsilon'}\varepsilon^{1/\alpha}f(k_\varepsilon)\mathrm{d}G+\int_{\varepsilon'}^{\infty}\varepsilon^{1/\alpha}f(k_{\varepsilon'})\mathrm{d}G.$$

The total derivative of Q satisfies

$$\frac{\mathrm{d}Q}{\mathrm{d}\rho_n}=\int_0^{\varepsilon'}\varepsilon^{1/\alpha}f'(k_\varepsilon)\frac{\mathrm{d}k_\varepsilon}{\mathrm{d}\rho_n}dG+\int_{\varepsilon'}^{\infty}\varepsilon^{1/\alpha}f'(k_{\varepsilon'})\frac{\mathrm{d}\hat{\varepsilon}}{\mathrm{d}\rho_n}dG. \tag{64}$$

Using the results of the proof of Proposition 8, this equation can be rearranged to

$$\frac{\mathrm{d}Q}{\mathrm{d}\rho_n}=\alpha\frac{1}{\rho_n}\left(\frac{\rho_n}{\rho_d}\right)^{\alpha-1}\left[\left(1-\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}\right)\int_0^{\varepsilon'}(\varepsilon-\varepsilon')\mathrm{d}G-\varepsilon'\Phi_0\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}\right].$$

For imperfect transmission $\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}=0$ and for perfect transmission $\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}=1$. Clearly, $\frac{\mathrm{d}Q}{\mathrm{d}\rho_n}<0$ for both cases. Thus, a decrease in i_n has a negative effect on aggregate output. ■

Proof of Proposition 11. To derive the effect of an increase in ρ_n on the real value of reserves and bonds, and to also understand how this effect can change when there is a simultaneous change in η, we take the total derivative of Equation (19):

$$\begin{aligned}\mathrm{d}\varepsilon'=&\left[\frac{\mathrm{d}m}{\rho_d}\left(\frac{\rho_d}{\rho_n}\right)^{\alpha}-\frac{m}{\rho_d}\left(\frac{\rho_d}{\rho_n}\right)^{\alpha}\frac{\mathrm{d}\rho_d}{\rho_d}+\alpha\frac{m}{\rho_d}\left(\frac{\rho_d}{\rho_n}\right)^{\alpha}\frac{\mathrm{d}\rho_d}{\rho_d}-\alpha\frac{m}{\rho_d}\left(\frac{\rho_d}{\rho_n}\right)^{\alpha}\frac{\mathrm{d}\rho_n}{\rho_n}\right]\frac{\chi+\eta}{\eta}\\&+\frac{m}{\rho_d}\left(\frac{\rho_d}{\rho_n}\right)^{\alpha}\left(\frac{\mathrm{d}\eta}{\eta}-\frac{\chi+\eta}{\eta^2}\mathrm{d}\eta\right)+\theta\left[\int_{\varepsilon'}^{\infty}\mathrm{d}\varepsilon' dG\right].\end{aligned}$$

Rearranging yields

$$\frac{\mathrm{d}m}{\mathrm{d}\rho_n}\frac{\rho_n}{m} = \frac{\mathrm{d}\varepsilon'}{\mathrm{d}\rho_n}\rho_n\frac{\rho_d}{m}\left(\frac{\rho_n}{\rho_d}\right)^{\alpha}\left[1-\theta\int_{\varepsilon'}^{\infty} dG\right]\frac{\eta}{\chi+\eta} + \alpha + \frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}(1-\alpha) + \frac{\chi}{\chi+\eta}\frac{\mathrm{d}\eta}{\mathrm{d}\rho_n}\frac{\rho_n}{\eta} \tag{65}$$

We now assume $\mathrm{d}\eta = 0$, i.e., when ρ_n changes, there is no simultaneous change in η. The term $\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}(1-\alpha)$ is zero for imperfect transmission and negative for perfect transmission since $\alpha > 1$. To show that $\left.\frac{\mathrm{d}m}{\mathrm{d}\rho_n}\frac{\rho_n}{m}\right|_{\mathrm{d}\eta=0} < 0$ for both perfect and imperfect transmission, it therefore suffices to show that

$$\frac{\mathrm{d}\varepsilon'}{\mathrm{d}\rho_n}\rho_n\frac{\rho_d}{m}\left(\frac{\rho_n}{\rho_d}\right)^{\alpha}\left[1-\theta\int_{\varepsilon'}^{\infty} dG\right]\frac{\eta}{\chi+\eta} + \alpha < 0. \tag{66}$$

From the proof of Proposition 8,

$$\frac{\mathrm{d}\varepsilon'}{\mathrm{d}\rho_n} = -\alpha\varepsilon'\Phi_0\Phi_2^{-1}\rho_n^{-1}.$$

Using this in Equation (66) and then rearranging terms yields

$$\alpha\left[1-\varepsilon'\frac{\Phi_0}{\Phi_2}\frac{\rho_d}{m}\left(\frac{\rho_n}{\rho_d}\right)^{\alpha}\left[1-\theta\int_{\varepsilon'}^{\infty} dG\right]\frac{\eta}{\chi+\eta}\right] < 0.$$

Thus, it suffices to show that

$$\varepsilon'\frac{\Phi_0}{\Phi_2}\frac{\rho_d}{m}\left(\frac{\rho_n}{\rho_d}\right)^{\alpha}\left[1-\theta\int_{\varepsilon'}^{\infty} dG\right]\frac{\eta}{\chi+\eta} > 1.$$

Rearranging yields

$$\frac{\Phi_0}{\Phi_2}\frac{\varepsilon' - \theta\int_{\varepsilon'}^{\infty}\varepsilon' dG}{\frac{m}{\rho_d}\left(\frac{\rho_d}{\rho_n}\right)^{\alpha}\frac{\chi+\eta}{\eta}} > 1$$

Using Equation (19) in the denumerator yields

$$\frac{\Phi_0}{\Phi_2}\frac{\varepsilon' - \theta\int_{\varepsilon'}^{\infty}\varepsilon' dG}{\varepsilon' - \theta\left[\int_0^{\varepsilon'}\varepsilon dG + \int_{\varepsilon'}^{\infty}\varepsilon' dG\right]} \geq \frac{\Phi_0}{\Phi_2} \geq 1,$$

with strict inequalities if ε' is bounded, i.e., $\gamma\rho_n > \beta$.

To understand the effect of $\mathrm{d}\eta$, observe that Equation (65) implies that $\frac{\mathrm{d}m}{\mathrm{d}\rho_n}\frac{\rho_n}{m}$ is increasing in $\mathrm{d}\eta$. Therefore, if an increase in ρ_n goes along with an increasing in η, the effect on m becomes less negative and if $\mathrm{d}\eta$ is sufficiently large, the effect becomes positive. Regarding the effect on b_{TR}, we note first that $b_{TR} = \frac{1+\eta}{\eta} m$. Totally differentiating this equation and combining the result with (65) yields

$$\frac{\mathrm{d}b_{TR}}{\mathrm{d}\rho_n}\frac{\rho_n}{b_{TR}} = \frac{\mathrm{d}\varepsilon'}{\mathrm{d}\rho_n}\rho_n\frac{\rho_d}{m}\left(\frac{\rho_n}{\rho_d}\right)^{\alpha}\left[1-\theta\int_{\varepsilon'}^{\infty} dG\right]\frac{\eta}{\chi+\eta} + \alpha + \frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}(1-\alpha) - \frac{\eta}{1+\eta}\frac{1-\chi}{\chi+\eta}\frac{\mathrm{d}\eta}{\mathrm{d}\rho_n}\frac{\rho_n}{\eta}$$

It follows immediately that we have $\frac{\mathrm{d}b_{TR}}{\mathrm{d}\rho_n}\frac{\rho_n}{b_{TR}}\Big|_{\mathrm{d}\eta=0} < 0$. When $\chi < 1$, we also have that $\frac{\mathrm{d}b_{TR}}{\mathrm{d}\rho_n}\frac{\rho_n}{b_{TR}}$ is decreasing in $\mathrm{d}\eta$. If we therefore have $\frac{\mathrm{d}\eta}{\mathrm{d}\rho_n} \geq 0$, it must be that $\frac{\mathrm{d}b_{TR}}{\mathrm{d}\rho_n}\frac{\rho_n}{b_{TR}} < 0$. Because $B_{TR,t+1} = \gamma B_{TR,t}$ and $B_{TR,0}$ is given exogenously, $\frac{\mathrm{d}b_{TR}}{\mathrm{d}\rho_n}\frac{\rho_n}{b_{TR}} < 0$ suggests that the real value of a unit of reserves, i.e., ϕ_t, drops.

To show the second part of the proof, use that $\mathrm{d}m = 0$ and $\mathrm{d}\eta = 0$ in Equation (65). Rearrange the result and then use Equation (19) to obtain

$$\frac{\mathrm{d}\varepsilon'}{\mathrm{d}\rho_n}\frac{\rho_n}{\varepsilon'}\Big|_{\substack{\mathrm{d}m=0\\ \mathrm{d}\eta=0}} = \left[(\alpha-1)\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d} - \alpha\right]\frac{\varepsilon' - \theta\left[\int_0^{\varepsilon'}\varepsilon\mathrm{d}G + \int_{\varepsilon'}^{\infty}\varepsilon'\mathrm{d}G\right]}{\varepsilon' - \theta\int_{\varepsilon'}^{\infty}\varepsilon'\mathrm{d}G} \tag{67}$$

Then use Equation (64) in combination with Equations (61) and (63) to find:

$$\frac{\mathrm{d}Q}{\mathrm{d}\rho_n}\Big|_{\substack{\mathrm{d}m=0\\ \mathrm{d}\eta=0}} = \frac{\alpha}{\rho_n}\left(\frac{\rho_n}{\rho_d}\right)^{1-\alpha}\left[\begin{array}{l}\left(1-\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}\right)\int_0^{\varepsilon'}\varepsilon\mathrm{d}G \\ +\quad \varepsilon'\left(1-\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}+\frac{1}{\alpha}\frac{\mathrm{d}\varepsilon'}{\mathrm{d}\rho_n}\frac{\rho_n}{\varepsilon'}\right)\int_{\varepsilon'}^{\infty}\left(\frac{\varepsilon}{\varepsilon'}\right)^{1/\alpha}\mathrm{d}G\end{array}\right]. \tag{68}$$

Using Equation (67) in Equation (68) implies that

$$\begin{aligned}\frac{\mathrm{d}Q}{\mathrm{d}\rho_n}\Big|_{\substack{\mathrm{d}m=0\\ \mathrm{d}\eta=0}} \propto & \left[1-\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}\right]\int_0^{\varepsilon'}\varepsilon\mathrm{d}G \\ & + \varepsilon'\left[1-\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}+\left(\frac{\alpha-1}{\alpha}\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d}-1\right)\frac{\varepsilon' - \theta\left[\int_0^{\varepsilon'}\varepsilon\mathrm{d}G + \int_{\varepsilon'}^{\infty}\varepsilon'\mathrm{d}G\right]}{\varepsilon' - \theta\int_{\varepsilon'}^{\infty}\varepsilon'\mathrm{d}G}\right] \\ & \times\int_{\varepsilon'}^{\infty}\left(\frac{\varepsilon}{\varepsilon'}\right)^{1/\alpha}\mathrm{d}G.\end{aligned}$$

With imperfect transmission, $\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d} = 0$ and it follows that $\frac{\mathrm{d}Q}{\mathrm{d}\rho_n} > 0$. With perfect transmission $\frac{\mathrm{d}\rho_d}{\mathrm{d}\rho_n}\frac{\rho_n}{\rho_d} = 1$ and it follows that $\frac{\mathrm{d}Q}{\mathrm{d}\rho_n} < 0$. ■

Proposition 15 $\mathcal{P}$ *satisfies*

$$(1-\beta)\mathcal{P} = \int_0^\infty \left[\varepsilon^{1/\alpha} f(k_\varepsilon) - k_\varepsilon\right] \mathrm{d}G + b(1/\rho_d - 1) + \bar{m}(1/\rho_p - \gamma) + (m - \bar{m})(1/\rho_n - \gamma)$$

Proof of Proposition 15. In equilibrium, households leave the settlement market without deposits, so that $\mathcal{P} = \int_0^\infty V_{IM}(m, b, 0|\varepsilon)\mathrm{d}G$. From (13), we have

$$\int_0^\infty V_{IM}(m, b, 0|\varepsilon)\mathrm{d}G = \int_0^\infty \begin{bmatrix} \varepsilon^{1/\alpha} f(k_\varepsilon) + V_S(0) + b/\rho_d - pk_s/\rho_d - (m_\varepsilon + pk_\varepsilon - m)/\rho_m \\ + \min\{m_\varepsilon + pk_s, \bar{m}\}/\rho_p + \max\{m_\varepsilon + pk_s - \overline{m}, 0\}/\rho_n \end{bmatrix} \mathrm{d}G.$$

Because $m_\varepsilon + pk_s \geq \bar{m}\ \forall\varepsilon$, $m = \int_0^\infty [m_\varepsilon + pk_\varepsilon]\mathrm{d}G$, $k_s = \int_0^\infty k_\varepsilon$, $\rho_d = p$, and $\rho_m = \rho_n$, (see earlier proofs) we obtain

$$\mathcal{P} = \int_0^\infty \left[\varepsilon^{1/\alpha} f(k_\varepsilon) - k_\varepsilon\right] \mathrm{d}G + b/\rho_d + \bar{m}/\rho_p + (m - \bar{m})/\rho_n + V_S(0).$$

Also, since household leave the settlement market without deposits, from (5) we have that

$$V_S(0) = \tau_B - \gamma(m + b) + \beta \int_0^\infty V_{IM}(m, b, 0|\varepsilon)\mathrm{d}G.$$

With $\mathcal{P} = \int_0^\infty V_{IM}(m, b, 0|\varepsilon)\mathrm{d}G$, combining the last two equations completes the proof. ■

Proof of Proposition 12. From the proof of Proposition 8, we know $\frac{\mathrm{d}(1-\beta)\mathcal{W}}{\mathrm{d}\rho_n} < 0$ for $\rho_n\gamma > \beta$. It follows from continuity that the Friedman rule is optimal. Under the Friedman rule, the LHS of Equation (27) equals 1. The RHS of Equation (27) can only equal 1 if $\varepsilon' \to \infty$. Thus, the Friedman rule implies $\varepsilon' \to \infty$.

Note, for $\rho_n = \rho_d$, the Friedman rule implements the first-best allocation, as all bankers invest the first-best quantity. After all, $k_\varepsilon = \varepsilon$ for all $\varepsilon < \varepsilon'$ and $\varepsilon' = \infty$. For $\rho_d > \rho_n$, the Friedman rule is optimal, but does not implement the first-best allocation. In this case, all bankers underinvest since $k_\varepsilon = \varepsilon(\rho_n/\rho_d)^\alpha < k_\varepsilon^*$. Lastly, if the central bank runs the Friedman rule, the economy cannot be in the NIR case since the Friedman rule would require $\rho_n = \beta/\gamma > \rho_d$, for which an equilibrium does not exist. ■

Proof of Lemma 13. Suppose that $\rho_n < \rho_m$ in a steady state equilibrium. Equation (13) then implies that bankers do not lend out reserves in the money market; $m_\varepsilon + pk_\varepsilon - m \geq 0$ for all ε. Moreover, Definition 1 requires $\int_0^\infty [m - m_\varepsilon - pk_\varepsilon]\mathrm{d}G(\varepsilon) = 0$. It follows that $m - m_\varepsilon - pk_\varepsilon = 0$ for all ε.

Without loss, we can focus on a case in which $\chi+\sigma+\theta>0$, as otherwise borrowing reserves in during the IM is infeasible and the money market can be ignored. With $\chi+\sigma+\theta>0$ and $m-m_\varepsilon-pk_\varepsilon=0$ for all ε, the borrowing constraint must be slack for all ε-bankers; $\lambda_\varepsilon=0$ for all ε. But with $\rho_p\le\rho_n<\rho_m$, Equation (15) then requires $\mu_\varepsilon<0$ for all ε, which cannot be the case. That means, if bankers' borrowing constraints are slack but $\rho_p\le\rho_n<\rho_m$, bankers have an incentive to borrow (additional) reserves in the money market and deposit them at the central bank. ■

Proof of Proposition 14. First, we need that some bankers enter the settlement market with reserves strictly smaller than the exemption threshold. This is the case if and only if $\bar{m}-\rho_d k_s>0$. Using the latter in Equation (41) and comparing the result with Equation (43), it follows that $\bar{m}-\rho_d k_s>0$ if $\varepsilon'<\varepsilon'''\left(\frac{1-\sigma}{\rho_n/\rho_p-\sigma}\right)^\alpha$.

Second, combining Equations (6), (39), and (9) with the fact that $V_{IM}^m(m,b,d|\varepsilon)=(\varepsilon/k_\varepsilon)^{1/\alpha}/p$ yields

$$\frac{\gamma\rho_n}{\beta}=\int_0^{\varepsilon'}\mathrm{d}G+\int_{\varepsilon'}^{\varepsilon''}\left(\frac{\varepsilon}{\varepsilon'}\right)^{1/\alpha}\mathrm{d}G+\frac{\rho_n/\rho_p-\sigma}{1-\sigma}\left[\int_{\varepsilon''}^{\varepsilon'''}\mathrm{d}G+\int_{\varepsilon'''}^{\infty}\left(\frac{\varepsilon}{\varepsilon'''}\right)^{1/\alpha}\mathrm{d}G\right]. \quad (69)$$

Third, combining Equations (42) and (43) with $k_s=\int_0^\infty k_\varepsilon \mathrm{d}G$ yields

$$\frac{\varepsilon'-\left[1-(1-\sigma)\frac{\eta}{\chi+\eta}\frac{\bar{m}}{m}\right]\varepsilon'''\left(\frac{1-\sigma}{\rho_n/\rho_p-\sigma}\right)^\alpha}{(1-\sigma)\left(1+\theta\frac{\eta}{\chi+\eta}\frac{\bar{m}}{m}\right)}$$
$$=\int_0^{\varepsilon'}\varepsilon\mathrm{d}G+\int_{\varepsilon'}^{\varepsilon''}\varepsilon'\mathrm{d}G+\left(\frac{1-\sigma}{\rho_n/\rho_p-\sigma}\right)^\alpha\left[\int_{\varepsilon''}^{\varepsilon'''}\varepsilon\mathrm{d}G+\int_{\varepsilon'''}^{\infty}\varepsilon'''\mathrm{d}G\right]. \quad (70)$$

Fourth, we show that $\varepsilon'<\varepsilon'''\left(\frac{1-\sigma}{\rho_n/\rho_p-\sigma}\right)^\alpha$ if and only if $\varepsilon'\ge\bar{\varepsilon}$. Let

$$\mathcal{H}=\frac{\varepsilon'-\left[1-(1-\sigma)\frac{\eta}{\chi+\eta}\frac{\bar{m}}{m}\right]\varepsilon'''\left(\frac{1-\sigma}{\rho_n/\rho_p-\sigma}\right)^\alpha}{(1-\sigma)\left(1+\theta\frac{\eta}{\chi+\eta}\frac{\bar{m}}{m}\right)}$$
$$-\int_0^{\varepsilon'}\varepsilon\mathrm{d}G-\int_{\varepsilon'}^{\varepsilon''}\varepsilon'\mathrm{d}G-\left(\frac{1-\sigma}{\rho_n/\rho_p-\sigma}\right)^\alpha\left[\int_{\varepsilon''}^{\varepsilon'''}\varepsilon\mathrm{d}G+\int_{\varepsilon'''}^{\infty}\varepsilon'''\mathrm{d}G\right].$$

Observe that $\varepsilon''=\varepsilon'(\frac{\rho_n/\rho_p-\sigma}{1-\sigma})^\alpha$ pins down ε'' as a function of ε'. Also, $\partial\mathcal{H}/\partial\varepsilon'''<0$ when $\bar{m}<m$ and $\partial\mathcal{H}/\partial\varepsilon''\Big|_{\varepsilon''=\varepsilon'(\frac{\rho_n/\rho_p-\sigma}{1-\sigma})^\alpha}=0$. Therefore, $\varepsilon'<\varepsilon'''\left(\frac{1-\sigma}{\rho_n/\rho_p-\sigma}\right)^\alpha$ holds if and only if

$$\mathcal{H}\Big|_{\varepsilon'''=\varepsilon'\left(\frac{\rho_n/\rho_d-\sigma}{1-\sigma}\right)^\alpha} > 0 \Rightarrow \varepsilon' \frac{\frac{\eta}{\chi+\eta}\frac{\bar{m}}{m}}{1+\theta\frac{\eta}{\chi+\eta}\frac{\bar{m}}{m}} - \int_0^{\varepsilon'} \varepsilon \mathrm{d}G - \int_{\varepsilon'}^{\infty} \varepsilon' \mathrm{d}G > 0, \tag{71}$$

where we use that $\varepsilon'' = \varepsilon'''$ if $\varepsilon''' = \varepsilon'\left(\frac{\rho_n/\rho_d-\sigma}{1-\sigma}\right)^\alpha$. In turn, Equation (71) holds if and only if $\varepsilon' > \bar{\varepsilon}$.

Fifth, we show that a pair $(\varepsilon', \varepsilon''')$ that solves both Equations (69) and (70) with $\varepsilon'' = \varepsilon'(\frac{\rho_n/\rho_p-\sigma}{1-\sigma})^\alpha$, and satisfies $\varepsilon' > \bar{\varepsilon}$, is unique. Because $\varepsilon' > \bar{\varepsilon}$, we have that $1 - \left(\frac{m}{\bar{m}}\frac{\chi+\eta}{\eta} + \theta\right)[1 - G(\varepsilon')] > 0$. With $\frac{m}{\bar{m}}\frac{\chi+\eta}{\eta} + \theta \le 1$ this is obvious. With $\frac{m}{\bar{m}}\frac{\chi+\eta}{\eta} + \theta > 1$, note that

$$\mathcal{G} = \varepsilon' - \left(\frac{m}{\bar{m}}\frac{\chi+\eta}{\eta} + \theta\right)\left[\int_0^{\varepsilon'} \varepsilon \mathrm{d}G + \int_{\varepsilon'}^{\infty} \varepsilon'\right] \tag{72}$$

is first decreasing in ε' (i.e., until ε' is such that $1-\left(\frac{m}{\bar{m}}\frac{\chi+\eta}{\eta} + \theta\right)[1-G(\varepsilon')] = 0$) and then increasing in ε'. Since, $\mathcal{G} = 0$ when $\varepsilon' = 0$, it follows that $1-\left(\frac{m}{\bar{m}}\frac{\chi+\eta}{\eta} + \theta\right)[1-G(\varepsilon')] > 0$ when $\mathcal{G} > 0$, i.e., when $\varepsilon' > \bar{\varepsilon}$.

With this result, we can perform the following manipulations regarding $\partial\mathcal{H}/\partial\varepsilon'$:

$$\begin{aligned}
\frac{\partial\mathcal{H}}{\partial\varepsilon'} &= \frac{1}{(1-\sigma)\left(1+\theta\frac{\eta}{\chi+\eta}\frac{\bar{m}}{m}\right)} - \int_{\varepsilon'}^{\varepsilon''} \mathrm{d}G \\
&= \frac{1-(1-\sigma)\left(1+\theta\frac{\eta}{\chi+\eta}\frac{\bar{m}}{m}\right)[G(\varepsilon'') - G(\varepsilon')]}{(1-\sigma)\left(1+\theta\frac{\eta}{\chi+\eta}\frac{\bar{m}}{m}\right)} \\
&> \frac{1-(1-\sigma)\left(1+\theta\frac{\eta}{\chi+\eta}\frac{\bar{m}}{m}\right)[1 - G(\varepsilon')]}{(1-\sigma)\left(1+\theta\frac{\eta}{\chi+\eta}\frac{\bar{m}}{m}\right)} \\
&> \frac{1-(1-\sigma)\frac{\eta}{\chi+\eta}\frac{\bar{m}}{m}}{(1-\sigma)\left(1+\theta\frac{\eta}{\chi+\eta}\frac{\bar{m}}{m}\right)} \\
&> 0.
\end{aligned}$$

In equilibrium $\mathcal{H} = 0$, so Equation (70) pins down ε''' as a strictly increasing function of ε'. Because the RHS of Equation (69) is strictly decreasing in both ε' and ε''', and does not respond to small changes in ε'' when $\varepsilon'' = \varepsilon'(\frac{\rho_n/\rho_p-\sigma}{1-\sigma})^\alpha$, it follows that an equilibrium is unique.

Sixth, to have existence of a pair $(\varepsilon', \varepsilon''')$ that solves both Equations (69) and (70), and satisfies $\varepsilon' > \bar{\varepsilon}$, it follows that we need

$$1 \leq \frac{\gamma \rho_n}{\beta} < \int_0^{\bar{\varepsilon}} \mathrm{d}G + \int_{\bar{\varepsilon}}^{\infty} (\varepsilon/\bar{\varepsilon})^{1/\alpha} \, \mathrm{d}G.$$

Finally, to ensure that we indeed have an equilibrium, it remains to verify that the money market clears: $m \geq \rho_d k_s + \int_0^{\infty} \underline{m}_{\varepsilon} \mathrm{d}G$. Because $\underline{m}_{\varepsilon} \leq \bar{m} - \rho_d k_s$ by Equation (40), we have that $m \geq \rho_d k_s + \int_0^{\infty} \underline{m}_{\varepsilon} \mathrm{d}G$ holds for sure if $m \geq \bar{m}$. ∎

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