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Labor Market Discrimination and the Racial Unemployment Gap: Can Monetary Policy Make a Difference?*

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Abstract

Black workers experience a higher unemployment rate, as well as more volatile employment dynamics, than white workers, and the racial unemployment rate gap is largely unexplained by observable characteristics. We develop a New Keynesian model with search and matching frictions in the labor market, endogenous separations, and employer discrimination against Black workers to explain these outcomes. The model is consistent with key features of the aggregate economy and is able to explain key labor market disparities across racial groups. We then use this model to assess the effects of the Federal Reserve's new monetary policy framework—interest rates respond to *shortfalls* of employment from its maximum level rather than *deviations*—on racial inequality in the labor market. We find that shifting from a Deviations interest rate rule to a Shortfalls rule reduces the racial unemployment rate gap and the model-based measures of labor market discrimination but increases the average inflation rate. From a welfare perspective, we find that the Shortfalls approach does not do much to reduce racial inequality in our model economy.

JEL Classification: E24, E52, J15, J70
Keywords: Unemployment, Monetary Policy, Racial Inequality, Discrimination

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1 Introduction

Unemployment rates differ systematically across racial groups—and, in particular, between Black and white workers. In the United States, the average jobless rate of Black workers is more than 6 percentage points larger than that of white workers over the past four decades. In addition, Black workers have more volatile employment dynamics than white workers, which results in a highly countercyclical racial unemployment rate gap. That is, the Black unemployment rate rises faster in downturns than the white unemployment rate but also falls faster during periods of economic expansions. Importantly, the racial unemployment rate gap between Black and white workers has been persistent since data have been available and is largely unexplained by differences in observable characteristics such as age, education, marital status, and state of residence (Cajner et al., 2017).

The first objective of this paper is to develop a search and matching model that is able to explain the heterogeneous labor market outcomes between Black and white workers. Given the compelling empirical evidence consistent with the presence of racial discrimination in the labor market, our model features employer discrimination against Black workers.

The second objective of this paper is to use our model to evaluate the macroeconomic effects, both at the aggregate level and by racial group, of the new monetary policy framework adopted by the Federal Reserve. In August 2020, the Federal Open Market Committee (FOMC) updated its Statement on Longer-Run Goals and Monetary Policy Strategy and, in particular, its understanding of maximum employment as a "broad-based and inclusive goal." At the same time, the updated statement assesses that in setting monetary policy, the FOMC seeks over time to mitigate *shortfalls* of employment from its maximum level, whereas the previous statement instead mentioned the FOMC would seek over time to mitigate *deviations* of employment from its maximum level. This change from mitigating *deviations* to mitigating *shortfalls* can be read as introducing an asymmetry in the monetary policy strategy, if monetary policy will not tighten as a consequence of a strong labor market unless inflationary pressures are present. A key contribution of this paper is to quantify the macroeconomic effects of switching from a Deviations interest rate rule to a Shortfalls rule at the aggregate level and by racial group, with a special focus on labor market outcomes.

This paper builds a search and matching model with endogenous separations à la Mortensen and Pissarides (1994) and adds worker heterogeneity by allowing for two types of representative households that differ in terms of a nonproductive attribute (i.e., race). In addition, the model features employer taste-based discrimination against Black workers.[1] In particular, following Becker (1971), we assume that employers have prejudice and incur a per-period perceived cost of employing a certain type of worker (i.e., Black workers). This cost is assumed to be time invariant and independent of monetary policy in the sense that monetary policy cannot directly affect discrimination—only indirectly by changing business cycle dynamics. Our model also incorporates nominal price rigidities and an effective lower bound (ELB) constraint on nominal interest rates. Our baseline model spec-

[1] This paper focuses on employer taste-based discrimination. Other ways of modeling discrimination in the labor market (e.g. customer taste-based discrimination, statistical discrimination) have been used in the existing literature. See Lang and Lehmann (2012) for a survey of the economic literature on racial discrimination.

ification features a monetary authority that sets the nominal interest rate by means of a standard Taylor-type interest rate rule, the "Deviations rule," that responds symmetrically to deviations of inflation and unemployment from their respective steady-state values. We calibrate our baseline model to be consistent with key aggregate moments of the U.S. economy and show that a small degree of discrimination against Black workers is enough to generate the mean racial unemployment rate gap between Black and white workers observed in the data.

Our baseline model endogenously generates four key results regarding labor market dynamics across racial groups, which, importantly, are all in line with the empirical evidence. First, the model endogenously generates higher mean separation rates and lower mean job-finding rates for Black workers relative to white workers. Second, the model is able to explain the higher volatility of the unemployment rate for Blacks than for whites. As a result, even though the discriminatory parameter is time invariant, our model generates a racial unemployment rate gap that is highly volatile and strongly countercyclical. In recessions (expansions), the unemployment rate for Blacks increases (decreases) by more than the corresponding one for whites, widening (shrinking) the racial unemployment rate gap. Third, our model also finds that the separation rate margin is key in accounting for the mean and volatility of the racial unemployment rate gap. This result is in line with the empirical evidence that the separation rate margin is more important in explaining the dynamics of the racial unemployment rate gap than the job-finding rate margin. Finally, the model generates positively skewed unemployment rates for both racial groups—and also for the racial unemployment rate gap. We demonstrate that the ELB constraint on the nominal interest rate plays a key role in generating distributions of endogenous variables skewed to the downside.

Our baseline model delivers two additional results regarding the presence of discrimination in the labor market. First, we develop novel model-based measures of racial discrimination in the labor market. In our model, employers exhibit taste-based discrimination against Black workers, which manifests itself in discrimination in two labor market margins: hiring and separations. Specifically, we can quantify the mass of workers who are not hired and the mass of workers who are fired for no other reason than for being Black. In theory, such workers could file a charge of racial discrimination. We then compare our model-based measures of discrimination with data on race-based charges of discrimination coming from the U.S. Equal Employment Opportunity Commission (EEOC). We find that, consistent with the data, our model-based discrimination measures are strongly countercyclical, meaning that labor market discrimination by race is highly dependent on labor market conditions, with more discrimination during periods of slack labor markets. Second, we find that inequality in labor market outcomes between white and Black workers leads to a meaningful difference in terms of welfare for the two types of households.

We then use our model to study the macroeconomic implications, at the aggregate level and by racial group, of switching from a Deviations rule to a Shortfalls rule, which continues to respond symmetrically to inflation deviations from its target but responds asymmetrically to the deviations of unemployment from its steady-state value. In particular, the Shortfalls rule responds to deviations of the unemployment rate from its steady-state value only when the unemployment rate is *above* its steady-state level. The Shortfalls rule is meant to capture, in a reduced-form way, one reading of the updated monetary strategy unveiled by the Federal Reserve in August 2020. We find that, switching from a Deviations rule to a Shortfalls rule strengthens economic expansions by keeping interest rates lower than they otherwise would be under a symmetric interest rate rule. As a result, the aggregate unemployment rate in the model falls by about 0.7 percentage point, from 6.4 percent to 5.6 percent. Importantly, given that the Black unemployment rate is more cyclical than its white counterpart in the model (consistent with the data, as discussed earlier), the unemployment rate falls more for Black workers than for white workers. Thus, even though the Shortfalls rule still targets the aggregate unemployment rate gap, it has disproportionately larger benefits for Blacks than for whites in terms of unemployment rates. As a result, switching to a Shortfalls rule narrows the racial unemployment rate gap by 0.5 percentage point, from 6.5 percent to 6.0 percent. Consistent with the lower stabilization properties of the Shortfalls rule relative to the Deviations rule, we find that switching to the Shortfalls rule increases the volatility of all labor market variables. However, all labor market variables are much less skewed to the downside under the Shortfalls rule. For example, the negative skewness of the racial unemployment rate gap is basically eliminated. As a result, under the Shortfalls rule, the distribution of the racial unemployment rate gap shifts toward being more symmetric around a lower mean , with a lower probability of experiencing very high values over the business cycle. Importantly, we see declines in the model-based discrimination measures when switching to a Shortfalls rule, both at the hiring margin and at the separation margin.

In contrast to all the benefits that the adoption of a Shortfalls rule entails for the labor market, we find that switching to a Shortfalls rule increases the average inflation rate by 0.5 percentage point, from 1.9 percent to 2.4 percent. We then assess the welfare effects of switching from a Deviations rule to a Shortfalls rule, contrasting the benefits in terms of lower unemployment rates for all workers versus the costs in terms of higher average inflation rates, which represent a loss of efficiency. Overall, we find that, on average, both types of households experience welfare gains when switching from a Deviations rule to a Shortfalls rule, though the welfare gains in consumption-equivalent terms are quantitatively very small. We conclude that, while switching to a Shortfalls rule can have a small but meaningful effect on the racial unemployment rate gap and on the model-based measures of labor market discrimination, it does not do much to reduce racial inequality in terms of welfare in our model economy. While we believe that our model represents an important step toward understanding the welfare implications of employment inequality between Black and white workers, our analysis might represent a lower bound on the welfare differences between the two racial groups, given some model assumptions like the presence of within-household consumption insurance and the absence of liquidity constraints. Relaxing some of these assumptions can be a fruitful area of future research.

Related Literature This paper contributes to four strands of literature. First, our paper relates to the literature that documents labor market disparities across racial groups and studies the role of racial discrimination in generating those outcomes. A key feature of our model is that there are no productive differences between Black and white workers, and thus all differences in labor market outcomes result from discrimination. In the context of the empirical literature, our model attributes all disparities in labor market outcomes between Black and white workers to unobserved or unexplained features, which is consistent with the existing empirical evidence. In particular, Stratton (1993), Ritter and Taylor (2011), and Cajner et al. (2017) show that most of the racial unemployment rate gap between Black and white workers is unexplained by observable characteristics. Lang and Lehmann (2012) and Daly et al. (2017) find that a substantial portion of the wage gap between Black and white workers is also unexplained by differences in easily measured characteristics. Relatedly, Charles and Guryan (2008) test the predictions of Becker's original model about employer prejudice and find that one-fourth of the racial wage gap is due to prejudice, with nontrivial consequences for Black lifetime earnings. In addition, there is empirical evidence consistent with hiring discrimination, in line with our model economy. In particular, Bertrand and Mullainathan (2004) conduct a field experiment with fictitious resumes and find direct evidence of labor market discrimination in the hiring process. Also, Forsythe and Wu (2021) use the American Time Use Survey and find that Black workers see a particularly low interview response rate per time spent actively searching for a job. Our model assumes the existence of a representative firm with taste-based discrimination, which contrasts with Becker's (1971) classic argument that discrimination should disappear in the long run. However, in their survey of the economic literature on racial discrimination in the labor market, Lang and Lehmann (2012) point to several theories that justify the existence of discriminating firms in the long run.[2]

Second, our paper contributes to the theoretical literature on random search and matching models by incorporating taste-based discrimination that can successfully account for differential labor market outcomes by race. Contributions to this literature include Bartel (1995), Bowlus and Eckstein (2002), Rosén (2003), Flabbi (2010), and Borowczyk-Martins et al. (2017). Our model differs from these papers in two key dimensions. On the one hand, in most of these papers, the worker's exit to unemployment is assumed to be exogenous, while our model includes endogenous separations. We view the addition of endogenous separations as a key contribution of our paper to the literature, as, in the data, the separation rate margin explains most of the mean and business cycle volatility of the racial unemployment rate gap between Blacks and whites. On the other hand, none of the aforementioned papers include nominal rigidities and thus cannot be used to study the macroeconomic consequences of changes in monetary policy frameworks. Also, our paper considers both demand and supply shocks, while most of the papers in this literature focus on supply shocks.

Third, our paper is also related to the literature that studies the implications for monetary policy of introducing worker heterogeneity in the context of New Keynesian search and matching models. Our modeling approach is most similar to Ravenna and Walsh (2012) and Bergman et al. (2022),

[2]For example, Black (1995) presents a model where discriminating high-ability entrepreneurs sacrifice some of the returns to their high ability but can still compete in the market with nondiscriminating low-ability entrepreneurs. Rosén (1997) demonstrates that in the presence of information asymmetry, even if only a few firms discriminate against Black workers, it is rational for all firms to do so. Sasaki (1998) argues that if one group of workers dislikes working with another group of workers, discriminating firms will arise to accommodate the discriminating workers.

which also include endogenous separations.[3] The primary distinction between these papers and our work is that these papers focus on differences between high-productivity and low-productivity workers. In contrast, because we are interested in labor market disparities between Black and white workers and the racial unemployment rate gap is largely unexplained by observables, our paper models the heterogeneous labor market outcomes between Black and white workers as a result of discrimination instead of differences in productivity. Our paper also differs in terms of the types of monetary policy strategies under analysis. In particular, Ravenna and Walsh (2012) explore the differences between a pure price-stability rule and a Taylor-type rule that responds to both inflation and unemployment, while Bergman et al. (2022) study the implications of switching from strict inflation targeting to average inflation targeting. In contrast, our paper studies the effects of switching from a Deviations rule, which responds symmetrically to both the inflation gap and the unemployment rate gap, to a Shortfalls rule, which responds asymmetrically to the unemployment rate gap (while continuing to respond symmetrically to the inflation rate gap), a defining feature of the Federal Reserve's updated monetary policy framework. Another difference from the previously mentioned papers is that we include the ELB constraint on nominal interest rates in our analysis. This addition allows us to analyze the differential effects of the ELB constraint on labor market outcomes by racial group, as well as study whether the macroeconomic and distributional effects of switching to a Shortfalls rule are affected by the presence of the ELB constraint.

Other recent papers that use theoretical models to study the heterogeneous effects of monetary policy on workers of different races include Lee et al. (2022), Nakajima (2023), and Ait Lahcen et al. (2023). Lee et al. (2022) study the tradeoff between unemployment and inflation stabilization in a stylized macro model where the monetary authority targets the Black unemployment rate rather than the aggregate unemployment rate. Nakajima (2023) builds a heterogeneous-agent New Keynesian model with racial inequality in income and wealth and studies how monetary policy affects different racial groups (taking the observed racial disparities in the labor market as given). Ait Lahcen et al. (2023) study the implications of different inflation regimes on the racial unemployment gap, using a New Monetarist approach, and assume exogenous differences in the job-finding and separation rates between Black and white workers. Our contribution to this literature is to build a New Keynesian search and matching model with endogenous separations, where the observed labor market disparities between Black and white workers arise as an endogenous result of the presence of discrimination, and to use that model to study the differential effects on racial groups of a Shortfalls strategy that can be interpreted as consistent with the FOMC's updated Statement on Longer-Run Goals and Monetary Policy Strategy.

Finally, our paper is also related to the recent literature that studies the macroeconomic implications of the Federal Reserve's updated monetary policy framework. Contributions to this literature include Bergman et al. (2022), which was already discussed and studies the effects of switching from a strict inflation-targeting regime to an average-inflation-targeting regime, and Bundick and

[3] Other papers featuring endogenous separations in New Keynesian models include Walsh (2005), Krause and Lubik (2007), Thomas and Zanetti (2009), Trigari (2009), Campolmi and Faia (2011), and Zanetti (2011).

Petrosky-Nadeau (2021), which studies the macroeconomic effects on the aggregate economy of switching from a Deviations rule to a Shortfalls rule. Our paper studies the same change in monetary policy strategy as in the latter paper but, instead, focuses on the heterogeneous effects on labor market outcomes for Black and white workers.

The rest of this paper proceeds as follows. Section 2 establishes the key empirical regularities about racial differences in labor market outcomes that we wish to highlight. Section 3 develops our model to explain these regularities. Section 4 discusses our solution method and calibration strategy. Section 5 presents the results of our baseline economy under the Deviations rule. Section 6 presents the aggregate and distributional effects from switching from a Deviations rule to a Shortfalls rule. Finally, Section 7 concludes with a discussion of possible avenues for future research.

2 Empirical Evidence

2.1 Labor Market Outcomes by Race

It is well documented in the existing literature that unemployment rates differ between racial groups—and, in particular, between Black and white workers. In the U.S., the jobless rate of Black workers is, on average, more than 6 percentage points greater than that of white workers (see Panel A of Table 1).[4] Moreover, this racial unemployment rate gap has been persistent since data have been available. As documented by Cajner et al. (2017), the racial unemployment rate gap between Black and white workers is largely unexplained by differences in observable characteristics like age, education, marital status, state of residence, etc. In addition, Black unemployed workers not only experience a higher unemployment risk than white workers, but also suffer a more volatile unemployment rate than white unemployed workers over the business cycle (see Panel B of Table 1, which provides standard deviations of the detrended unemployment rates by race).[5] Importantly, a higher cyclical volatility for Blacks means that the racial unemployment rate gap is highly countercyclical, as the Black unemployment rate falls faster than the white unemployment rate in economic expansions but rises faster in downturns. In particular, the correlation between the cyclical components of the racial unemployment rate gap and the aggregate unemployment rate stands at 0.77 in the U.S. data.

In order to understand whether a higher and more volatile unemployment rate for Black workers is due to a higher probability of becoming unemployed or a lower probability of finding a job, we compute unemployment flows by racial group, following the methodology in Elsby et al. (2009) and Shimer (2012). Table 1 reports the means and cyclical volatilities of unemployment inflow rates (separation rates) and outflow rates (job-finding rates) by racial group. The results show that both

[4] Unless otherwise noted, the labor market data presented in this paper are constructed from the monthly Current Population Survey (CPS) microdata from January 1976 to December 2019. Figure A.1 in Appendix A.1 plots the unemployment rates by race over time.

[5] To compute cyclical volatility, we first detrend the data using the HP filter with a smoothing parameter of 10^5. We follow Cairó and Cajner (2018) and focus on absolute volatilities and thus do not express the variables in natural logarithms when calculating volatilities. The reason to focus on absolute volatilities is to avoid the distorting effect due to different mean unemployment rates between the two racial groups. See Appendix A.1 for additional details.

Table 1: Labor Market Outcomes by Race (in percent)

	Aggregate	Blacks	Whites
Panel A: Means			
Unemployment rate	6.3	12.0	5.5
Separation rate	3.3	5.8	3.0
Job-finding rate	51.3	43.5	54.1
Panel B: Cyclical volatility			
Unemployment rate	1.3	2.1	1.2
Separation rate	0.3	0.8	0.3
Job-finding rate	9.7	7.9	10.7

Note: Series are computed as quarterly averages of monthly data from the CPS. Cyclical volatility is defined as the standard deviation of the data expressed in deviations from an HP trend with a smoothing parameter of 10^5.

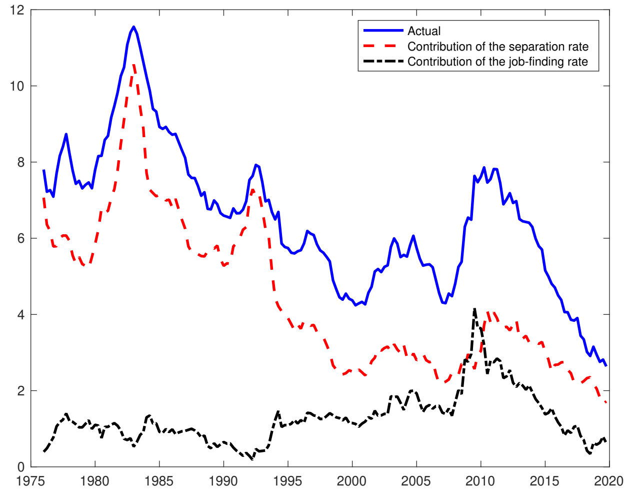
margins seem to contribute to the average unemployment rate differential across racial groups. In particular, Black workers have a much higher probability of becoming unemployed and a lower probability of finding a job than white workers. In terms of volatility, the higher volatility of the separation rate for Blacks is key in explaining their higher unemployment volatility.

2.1.1 Assessing the Contribution of Unemployment Flow Rates to the Dynamics of the Racial Unemployment Rate Gap

In order to assess the contribution of each unemployment flow rate in generating the observed racial unemployment rate gap, we follow Shimer (2012) and Cajner et al. (2017) by exploiting the steady-state unemployment approximation $U_t^{i*} \approx \lambda_t^i/(\lambda_t^i + f_t^i)$, which replicates well the actual unemployment rates (λ_t and f_t stand for the separation rate and job-finding rate, respectively) for group i (Black or white). To determine the relative importance of the separation rate margin, we first construct a counterfactual white unemployment rate, $U_t^\lambda = \lambda_t^b/(\lambda_t^b + f_t^w)$, that would prevail if white workers faced the same separation rate as Black workers. We then assess the contribution of the separation rate margin by computing a counterfactual unemployment rate gap, $ugap_t^\lambda = U_t^\lambda - U_t^{w*}$, which we compare with the actual racial unemployment rate gap (approximated by its steady-state representation), $ugap_t = U_t^{b*} - U_t^{w*}$. Similarly, to assess the relative importance of the job-finding rate margin, we first construct the counterfactual white unemployment rate that would prevail if white workers faced the Black job-finding rate, $U_t^f = \lambda_t^w/(\lambda_t^w + f_t^b)$. We then assess the contribution of the job-finding rate margin by computing a counterfactual unemployment rate gap, $ugap_t^f = U_t^f - U_t^{w*}$, which again is to be compared with the actual racial gap, $ugap_t$.

Figure 1 plots the steady-state approximation of the actual racial unemployment rate gap (solid blue line), together with the counterfactual unemployment rates that assess the contribution of the separation rate (dashed red line) and the job-finding rate (dashed-dotted black line). As can be seen, most of the variation in the racial unemployment rate gap is explained by the separation rate, and it is only in the later part of the sample period that the job-finding rate takes a larger role.

Figure 1: Racial Unemployment Rate Gap: Actual versus Counterfactuals



Note: The separation rate and job-finding rate are computed as quarterly averages of monthly data. Data are expressed in terms of 4-quarter moving averages.

Table 2 computes the contribution of the separation rate and job-finding rate to the mean and variance of the racial unemployment rate gap, extending the work done by Shimer (2012) for the case of the aggregate unemployment rate. The contribution of the separation rate to the mean of the racial gap is computed as the ratio of the average counterfactual racial unemployment rate gap to the average of the actual gap, i.e., $\frac{E[ugap_{cst}]}{E[ugap_t]}$. The contribution of the separation rate to the variance of the racial gap is computed as the fraction of the variance of the cyclical component of the racial unemployment rate gap that is attributable to the covariance between the cyclical components of the actual racial gap and the counterfactual racial gap, i.e., $\frac{\mathrm{Cov}(ugap_t, ugap_{cst})}{\mathrm{Var}(ugap_t)}$—or, in other words, to the coefficient on $ugap_t$ in a regression of $ugap_{cst}$ on $ugap_t$ and a constant. The contributions of the job-finding rate are computed analogously, using $ugap_{cft}$ instead of $ugap_{cst}$.

The results of Table 2 show that, even though both the separation rate and the job-finding rate matter for explaining the mean and volatility of the racial unemployment rate gap over the 1976–2019 period, the separation rate is the most important one.[6] In particular, the separation rate margin explains about two-thirds of the average racial unemployment rate gap. In turn, the covariance of the cyclical components of the actual racial gap and the counterfactual racial gap generated by the separation rate accounts for 60 percent of the variance of the cyclical component of the actual racial gap. Section 3 builds a theoretical framework that is consistent with this empirical regularity.

2.2 Aggregate Labor Market Conditions and Racial Discrimination

This section documents a strong countercyclical pattern for racial discrimination in the U.S. Following Boulware and Kuttner (2019), we use data on race-based charges of discrimination coming from the EEOC charge statistics, but at the national level instead of at the state level. A charge

[6] Appendix A.2 shows that this result is robust to the inclusion of transitions in and out of the labor force.

Table 2: Contributions of Unemployment Flow Rates to the Racial Unemployment Rate Gap in the Data

	Separation rate	Job-finding rate
Mean	0.67	0.20
Cyclical variance	0.60	0.27

Note: The contribution of the separation rate to the mean of the racial gap is computed as $\frac{E[ugap_{cst}]}{E[ugap_t]}$. The contribution of the separation rate to the variance of the racial gap is computed as $\frac{\text{Cov}(ugap_t, ugap_{cst})}{\text{Var}(ugap_t)}$. The contributions of the job-finding rate are computed analogously, using $ugap_{cft}$ instead of $ugap_{cst}$. The rows do not sum to 100 percent, as the decomposition is not exact. Variances and covariances are computed using data in deviations from an HP trend with a smoothing parameter of 10^5.

of discrimination is defined as "a signed statement filed with the EEOC asserting that an employer, union, or labor organization engaged in employment discrimination in the workplace and requests that the EEOC take remedial action" (Boulware and Kuttner, 2019, p.166). Employees, job applicants, part-time employees, and former employees can all file charges. The national data are available starting in 1997. We construct a "race charges" variable, defined as the number of race-based charges of discrimination per nonwhite member of the labor force. Figure 2 plots the cyclical component of the race charges variable from 1997 to 2019, together with the cyclical component of the aggregate unemployment rate.[7] For context in interpreting these numbers, the state with the most discrimination is Arkansas, with an average race charges rate of 26 basis points, and the state with the least discrimination is Maine, with an average race charges rate of 2 basis points.[8] So a 1 basis point change in the race charges rate is commensurate with 4 percent of the difference between Arkansas and Maine. As can be seen, there is a very strong correlation between the cyclical components of the race charges and the cyclical components of the unemployment rates, with a contemporaneous correlation of 0.65. The data show that discrimination is highly dependent on labor market conditions, with racial discrimination being more prevalent in a slack labor market, when the unemployment rate is high. Our model economy, which is described next, features model-based discrimination measures that are consistent with this empirical regularity.

3 Model

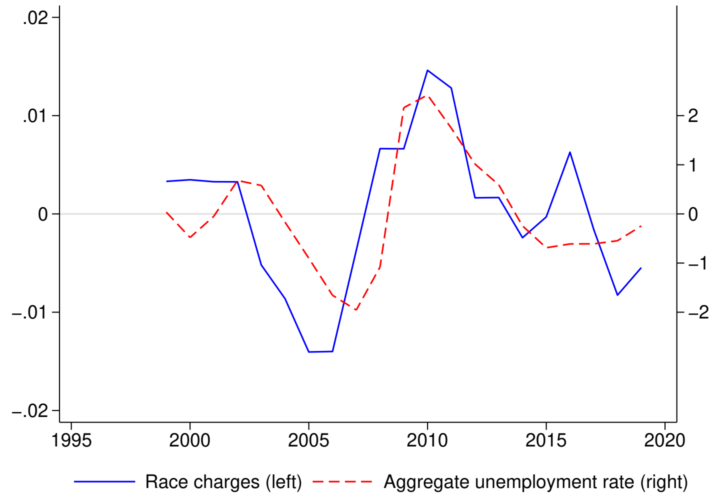
This section presents a New Keynesian model with search and matching frictions and endogenous separations à la Mortensen and Pissarides (1994). We include employer taste-based discrimination in order to study the heterogeneous labor market outcomes between Black and white workers over the business cycle. Our model economy is populated by five types of agents: households, an intermediate goods producer, retailers, a final good producer, and a monetary authority.[9]

[7] To compute the cyclical volatility we first detrend the data using the HP filter. We use a smoothing parameter of 100, given that race-based charges are available at an annual frequency.

[8] The state level data are available at an annual frequency for the 2009–2019 period.

[9] Appendix C provides additional model details, the definition of equilibrium, the full set of model equations, and details on how to compute the steady state.

Figure 2: Aggregate Labor Market Conditions and Racial Discrimination



Note: Data are annual and series are cyclical components from an HP filter with a smoothing parameter of 100.

3.1 Environment

There are two types of representative households, type 1 and type 2, who differ in terms of a nonproductive attribute (i.e., race). There is a continuum of members within each household. We normalize the total labor force to 1, and δ is the fixed share of workers of type 1 in the labor force. We assume that employers exhibit taste-based discrimination against type 1 workers. In particular, following Becker (1971), we assume that employers have prejudice and incur a per-period perceived cost κ_1 of employing a type 1 worker.

Workers can be either employed or unemployed. Employed workers of type i, with idiosyncratic productivity z, receive a real wage $w_t^i(z)$ in period t, whereas unemployed workers receive unemployment benefits h. We abstract from labor force participation decisions and thus assume that all unemployed workers are searching for jobs. We also abstract from modeling the intensive margin of labor and assume all workers supply a constant indivisible unit of labor.

A continuum of perfectly competitive intermediate goods producers have access to a production technology that uses labor as the only input of production. Intermediate goods producers hire workers by posting vacancies. Search is random, i.e., a firm cannot direct its search to a particular type of worker.[10] After a match between a firm and a worker of type i is formed, the match draws an independently and identically distributed idiosyncratic productivity z from a time-invariant distribution with cumulative distribution function, or c.d.f., $G(z)$ and density $g(z)$. If the draw is above a certain threshold, described in more detail later, a firm-worker match is formed and production starts. Each period, every firm-worker match draws a new idiosyncratic productivity, which again needs to be above the reservation threshold to avoid an endogenous separation. We

[10]This specification is consistent with federal laws that make it illegal to discriminate against a job applicant or an employee because of the person's race.

also assume that matches are subject to exogenous separations, independent of productivity, and we allow those exogenously separated workers to search for a job within the same period. In a given period, the timing of labor market events is as follows: First, a fraction λ^x of workers are exogenously separated from their jobs. Second, vacancies are posted, and all those workers who were not employed the previous period search for jobs along with the newly separated workers. Third, firm-worker matches are formed based on a matching technology taking searchers and vacancies as inputs. Fourth, existing matched workers and newly matched workers draw a new idiosyncratic productivity, and all matches with idiosyncratic productivity below the reservation threshold are endogenously separated. Finally, production takes place, and wages are paid.

3.2 Labor Market

The matching process between workers and firms is formally depicted by the existence of a Cobb-Douglas matching function:

$$m(s_t, v_t) = \varsigma s_t^{1-\varepsilon} v_t^{\varepsilon},$$

where s_t is the measure of total searchers; v_t, aggregate vacancies; ς, matching efficiency, and ε, matching function elasticity. The meeting probabilities for unemployed workers and for vacancies are p_t and q_t, respectively, defined as

$$\begin{aligned} p_t \equiv p(\theta_t) &= \frac{m(s_t, v_t)}{s_t} = \varsigma \theta_t^{\varepsilon}, \\ q_t \equiv q(\theta_t) &= \frac{m(s_t, v_t)}{v_t} = \varsigma \theta_t^{\varepsilon - 1}, \end{aligned}$$

where $\theta_t = \frac{v_t}{s_t}$ is labor market tightness. Note that a match is successfully formed between a firm and a worker of type i if the idiosyncratic productivity draw is above the corresponding reservation productivity threshold, z_t^{Ri}. As a result, the job-finding rate for a worker of type i in period t is $f_{it} = p_t[1 - G(z_t^{Ri})]$. The evolution of the searching population is given by

$$\begin{aligned} s_t &= s_{1t} + s_{2t}, \\ s_{1t} &= \delta - n_{1t-1} + \lambda^x n_{1t-1}, \\ s_{2t} &= (1 - \delta) - n_{2t-1} + \lambda^x n_{2t-1}, \end{aligned}$$

with subscripts 1 and 2 referring to the type of worker. In turn, employment evolves according to

$$\begin{aligned} n_t &= n_{1t} + n_{2t}, \\ n_{it} &= [1 - G(z_t^{Ri})][(1 - \lambda^x) n_{it-1} + q_{it} v_t], \end{aligned}$$

where q_{it} is the probability of a vacancy meeting a type i worker, defined as

$$q_{it} \equiv \frac{s_{it}}{s_t} q_t. \tag{1}$$

Equation (1) results from the random search assumption and the fact that both types of workers participate in a common labor market. As a result, new matches are distributed between the two types of workers in proportion to the number of searchers of each type. Total new matches can be represented as $q_t v_t$, so a proportion $\frac{s_{it}}{s_t}$ of them are type i searchers. Separations can occur for both exogenous and endogenous reasons, and the total separation rate for type i workers is defined as $\lambda_{it} \equiv \lambda^x(1-p_t) + [(1-\lambda^x) + p_t\lambda^x]G(z_t^{Ri})$. The unemployment rates at the end of the period for the aggregate economy and for each type of worker are given by, respectively,

$$
\begin{aligned}
U_t &= 1 - n_t, \\
U_{1t} &= \frac{\delta - n_{1t}}{\delta}, \\
U_{2t} &= \frac{(1-\delta) - n_{2t}}{1-\delta}.
\end{aligned}
$$

The racial unemployment rate gap in period t is defined as $U_{1t} - U_{2t}$.

3.3 Intermediate Goods Producer

The intermediate goods producer has access to the following production function:

$$
x_t = A_t \left(n_{1t} \int_{z_t^{R1}} \frac{zg(z)}{1-G(z_t^{R1})} dz + n_{2t} \int_{z_t^{R2}} \frac{zg(z)}{1-G(z_t^{R2})} dz \right),
$$

where A_t is aggregate productivity, which evolves stochastically according to a first-order autoregressive, or AR(1), process in logs:

$$
\log A_t = \rho_A \log A_{t-1} + \sigma_A \epsilon_{A,t}, \quad \epsilon_{A,t} \sim N(0,1).
$$

The intermediate goods producer posts vacancies, v_t, in order to hire labor, n_t, out of the searching population, s_t. When posting vacancies and setting the reservation productivities, the intermediate goods producer takes wages, the vacancy-filling rates, and technology as given, and solves

$$
\begin{aligned}
\max_{x_t, n_{1t}, n_{2t}, v_t, z_t^{R1}, z_t^{R2}} & E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} [p_t^m x_t - n_{1t}\bar{w}_{1t} - n_{2t}\bar{w}_{2t} - \chi v_t - \kappa_1 n_{1t} - \kappa_2 n_{2t}] \\
s.t \quad & x_t = A_t \left(n_{1t}\bar{z}_{1t} + n_{2t}\bar{z}_{2t} \right), \\
& n_{it} = [1 - G(z_t^{Ri})][(1-\lambda^x)n_{it-1} + q_{it}v_t],
\end{aligned}
$$

where p_t^m is the real price of the intermediate good; $\Lambda_{t,t+i}$ is the stochastic discount factor; $\bar{w}_{it} = \int_{z_t^{Ri}} \frac{w_{it}(z)g(z)}{1-G(z_t^{Ri})} dz$ is the average wage across employed type i workers; $\bar{z}_{it} = \int_{z_t^{Ri}} \frac{zg(z)}{1-G(z_t^{Ri})} dz$ is average productivity across employed type i workers; χ is the per-period vacancy posting cost; and κ_i is a perceived cost due to employing type i workers, which captures the level of discrimination against type i workers.[11]

[11] By assumption, $\kappa_2 = 0$, but we include κ_2 for generality.

The solution to the intermediate goods producer's problem yields the job creation condition

$$\chi = q_{1t}(1 - G(z_t^{R1}))\phi_{1t} + q_{2t}(1 - G(z_t^{R2}))\phi_{2t} \tag{2}$$

and the job destruction condition for each type of worker i:

$$p_t^m A_t z_t^{Ri} - w_t(z_t^{Ri}) - \kappa_i + (1 - \lambda^x) E_t \Lambda_{t,t+1}(1 - G(z_{t+1}^{Ri}))\phi_{it+1} = 0, \tag{3}$$

where ϕ_{it} is the expected value to the firm from hiring a type i worker and is given by the Lagrange multiplier on the employment evolution constraint:

$$\phi_{it} = \int_{z_t^{Ri}} (\varphi_t A_t z - w_t(z)) \frac{g(z)}{1 - G(z_t^{Ri})} dz - \kappa_i + (1 - \lambda^x) E_t \Lambda_{t,t+1} \phi_{it+1}(1 - G(z_{t+1}^{Ri}).$$

The job creation condition (2) equates the cost of posting a vacancy with the firm's expected benefit from filling that vacancy. The job destruction condition (3) states that at the reservation productivity, the value of the match for the firm is exactly zero for each type of worker i.

3.4 Households

There are two types of representative households, type 1 and type 2, who differ in terms of a nonproductive attribute (i.e., race). There is a continuum of members within each household. There is perfect consumption insurance within each household but not across households, and we assume that the two types of households cannot trade bonds with each other. A household of type i maximizes expected lifetime utility subject to the per capita period-by-period budget constraint, solving

$$\max_{c_{it}, B_{it}} E_0 \sum_{t=0}^{\infty} \beta^t \log(c_{it})$$
$$st. \quad c_{it} + \frac{B_{it}}{P_t} = \xi_t (1 + i_{t-1}) \frac{B_{it-1}}{P_t} + (1 - U_{it})\bar{w}_{it} + U_{it} h + \Pi_t - T_t,$$

where c_{it} is per capita consumption, B_{it} are per capita holdings of one-period nominal bonds, P_t is the price of the final good, i_t is the nominal interest rate, Π_t are firm profits reverted to households in a lump-sum fashion, and T_t is a lump-sum tax that finances unemployment benefits. Note that we assume that profits and the tax burden are shared evenly by all members of the economy—to focus on inequality arising from differential labor market outcomes between the two types of households. Thus, given that we normalize the aggregate population to 1, per capita profits and taxes are equal to aggregate profits and taxes in the model. Following Smets and Wouters (2007), we introduce a risk premium shock, ξ_t, which is equivalent to a wedge between the interest rate controlled by the central bank and the return on assets held by the household. The risk premium shock represents an aggregate demand disturbance and evolves stochastically according to an AR(1) process in logs:

$$\log \xi_t = \rho_A \log \xi_{t-1} + \sigma_\xi \epsilon_{\xi,t}, \quad \epsilon_{\xi,t} \sim N(0, 1).$$

The solution to the household problem yields the standard Euler equation:

$$\frac{1}{c_{it}} = \beta\xi_t(1+i_t)E_t\frac{1}{\pi_{t+1}}\frac{1}{c_{it+1}},$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

3.5 Final Good Producer

There is a representative final good producer that combines the intermediate goods packaged by retailers into the final good with a standard Dixit-Stiglitz aggregator technology, solving

$$\max_{y_{jt},y_t} P_t y_t - \int_0^1 P_{jt}y_{jt}dj \quad st. \quad \int_0^1 \left(\frac{y_{jt}}{y_t}\right)^{\frac{\gamma-1}{\gamma}} dj = 1,$$

where y_t is final output, y_{jt} is the intermediate good packaged by retailer j, and P_{jt} is the price charged by retailer j. The solution to this problem yields the well-known demand schedule faced by a given retailer and the price aggregation equation:

$$y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\gamma} y_t,$$

$$P_t = \left(\int_0^1 P_{jt}^{1-\gamma}dj\right)^{\frac{1}{1-\gamma}}.$$

3.6 Retailers

There is a mass 1 continuum of monopolistically competitive price-setting retailers indexed by $j \in [0,1]$, who get to optimally reset their price with probability $(1-\lambda_p)$ each period. The retailers who are not able to adjust their price, index their price to inflation—that is, $P_{jt+1} = \pi_t^{\gamma_p}\pi^{1-\gamma_p}P_{jt}$, where π is the steady-state inflation and γ_p is the degree of indexation. The retailers who are able to reset their price in period t do so to maximize the preset value of real expected profits at time t, solving

$$\max_{P_{jt}^*} E_t\left[\sum_{s=0}^{\infty}\lambda_p^s\Lambda_{t,t+s}\left(\frac{P_{jt}^*}{P_{t+s}}\Pi_{k=1}^s\pi_{t+k-1}^{\gamma_p}\pi^{1-\gamma_p} - \frac{P_{t+s}^m}{P_{t+s}}\right)y_{t+s}y_{jt,t+s}^*\right],$$

where

$$y_{jt,t+s}^* = \left(\frac{\Pi_{k=1}^s\pi_{t+k-1}^{\gamma_p}\pi^{1-\gamma_p}P_{jt}^*}{P_{t+i}}\right)^{-\gamma}.$$

The solution to this problem, along with the price aggregation equation, yields the standard nonlinear New Keynesian Phillips curve equations:

$$
\begin{aligned}
p_t^* &= \frac{\gamma}{\gamma-1}\frac{\mathcal{P}_t^N}{\mathcal{P}_t^D}, \\
\mathcal{P}_t^N &= p_t^m y_t + E_t \lambda_p \Lambda_{t,t+1}\left(\frac{\pi_t^{\gamma_p}\pi^{1-\gamma_p}}{\pi_{t+1}}\right)^{-\gamma}\mathcal{P}_{t+1}^N, \\
\mathcal{P}_t^D &= y_t + E_t \lambda_p \Lambda_{t,t+1}\left(\frac{\pi_t^{\gamma_p}\pi^{1-\gamma_p}}{\pi_{t+1}}\right)^{1-\gamma}\mathcal{P}_{t+1}^D, \\
1 &= (1-\lambda_p){p_t^*}^{1-\gamma} + \lambda_p\left(\frac{\pi_{t-1}^{\gamma_p}\pi^{1-\gamma_p}}{\pi_t}\right)^{1-\gamma},
\end{aligned}
$$

where p_t^* is the real price set by all retailers who get to optimally choose their prices.

3.7 Wage Bargaining

The value of employment in period t to a worker of type i with idiosyncratic productivity z can be written recursively as the sum of the current wage and the continuation value:

$$
\begin{aligned}
H_{it}(z) = w_{it}(z) &+ \lambda^x E_t\Lambda_{t,t+1}\left[p_{t+1}\left(\int_{z_{t+1}^{Ri}} H_{it+1}(y)g_i(y)dy + G(z_{t+1}^{Ri})U_{it+1}\right) + (1-p_{t+1})U_{it+1}\right] \\
&+ (1-\lambda^x)E_t\Lambda_{t,t+1}\left[\int_{z_{t+1}^{Ri}} H_{it+1}(y)g(y)dy + G(z_{t+1}^{Ri})U_{it+1}\right].
\end{aligned}
$$

An employed worker receives the wage $w_{it}(z)$ in period t. Next period, there is a λ^x probability that the worker is exogenously separated. If the worker is exogenously separated, there is a p_{t+1} probability they can still be matched to a new job in the same period (provided that the idiosyncratic productivity draw is above its corresponding reservation threshold), and there is a $1-p_{t+1}$ probability that they become unemployed and receive the value of unemployment, U_{it+1}. The return for all those matches with an idiosyncratic productivity $z < z_{t+1}^{R_i}$ is simply the value of unemployment as the match endogenously separates. Note that if the worker is not exogenously separated, the worker continues employment if the idiosyncratic productivity draw is above its corresponding reservation threshold; otherwise, the worker would become unemployed.

The value of unemployment is similarly defined recursively as the sum of unemployment benefits and the continuation value:

$$
U_{it} = h + E_t\Lambda_{t,t+1}\left[p_{t+1}\left(\int_{z_{t+1}^{Ri}} H_{it+1}(y)g(y)dy + G(z_{t+1}^{Ri})U_{it+1}\right) + (1-p_{t+1})U_{it+1}\right].
$$

The surplus from a job in period t to a worker of type i with productivity z is defined as

$$
V_{it}(z) \equiv H_{it}(z) - U_{it}.
$$

The value of an individual worker of type i with productivity z to the firm in period t is defined recursively as

$$J_{it}(z) = p_t^m A_t z - w_{it}(z) - \kappa_i + (1-\lambda^x) E_t \Lambda_{t,t+1} \int_{z_{t+1}^{Ri}} J_{it+1}(y) g(y) dy.$$

In the current period, the value to the firm is the real return to the worker's production less the wage and the perceived cost κ_i paid to employ that worker. There is then a λ^x probability that the worker is exogenously separated in the next period and the firm receives no benefit, and there is a $1-\lambda^x$ probability that the worker is not exogenously separated, in which case the firm's expected return is the integral of its returns across the idiosyncratic productivity distribution. The return for all $z < z_{t+1}^{R_i}$ is 0, as the match is endogenously destroyed.

We assume that wages are set by Nash bargaining, which requires that the wage for each worker type and at each productivity level is set to maximize the Nash product $V_{it}(z)^{1-\zeta} J_{it}(z)^{\zeta}$, where $(1-\zeta)$ is the workers' bargaining power. The first-order condition of this problem is

$$(1-\zeta) J_{it}(z) = \zeta V_{it}(z).$$

As shown in Appendix C, the resulting equilibrium wage is given by

$$w_{it}(z) = (1-\zeta)(A_t p_t^m z - \kappa_i + (1-\lambda^x) E_t \Lambda_{t,t+1} p_{t+1} (1 - G(z_{t+1}^{Ri})) \phi_{it+1}) + \zeta h.$$

We can see that discrimination will adversely affect the wages of the group that is discriminated against but that the relative importance of this effect will diminish as productivity increases. Integrating over z, we find that the average wage for type i workers is

$$\bar{w}_{it} = (1-\zeta)(A_t p_t^m \bar{z}_{it} - \kappa_i + (1-\lambda^x) E_t \Lambda_{t,t+1} p_{t+1} (1 - G(z_{t+1}^{Ri})) \phi_{it+1}) + \zeta h. \tag{4}$$

3.8 Monetary Policy

In the model, the monetary authority sets the nominal interest rate i_t according to an inertial Taylor-type interest rate rule. Under the baseline calibration, the central bank follows a "*Deviations* rule" given by equation (5), which treats deviations of inflation and unemployment from their steady-state values symmetrically and is subject to the ELB:

$$i_t = \max\{0, \phi_i i_{t-1} + (1-\phi_i)[i + \phi_\pi(\log(\pi_t) - \log(\pi) + \phi_u(U_t - U)]\}, \tag{5}$$

where i, π, and U are steady-state values for the nominal interest rate, inflation, and aggregate unemployment rate, respectively. The parameter ϕ_i is the inertial component, while ϕ_π and ϕ_u determine the policy reactions to deviations of inflation from steady state and to deviations of the aggregate unemployment rate from steady state, respectively.

In Section 6, we analyze an alternative monetary policy rule, denoted as the "*Shortfalls* rule" and given by equation (6), that does not react to the deviations of the aggregate unemployment rate from its target when the unemployment rate is below its steady-state value:

$$i_t = \left\{ \begin{array}{ll} \max\{0, \phi_i i_{t-1} + (1-\phi_i)[i + \phi_\pi(\log(\pi_t) - \log(\pi)) + \phi_u(U_t - U)]\} & \text{if } U_t > U \\ \max\{0, \phi_i i_{t-1} + (1-\phi_i)[i + \phi_\pi(\log(\pi_t) - \log(\pi))]\} & \text{if } U_t < U \end{array} \right\}. \tag{6}$$

The Shortfalls rule is meant to capture, in a reduced-form way, the reinterpretation of the maximum employment goal by the FOMC. In particular, the updated Statement on Longer-Run Goals and Monetary Policy Strategy assesses that "in setting monetary policy, the Committee seeks over time to mitigate shortfalls of employment from the Committee's assessment of its maximum level and deviations of inflation from its longer-run goal." The previous statement instead mentioned that "in setting monetary policy, the Committee seeks to mitigate deviations of inflation from its longer-run goal and deviations of employment from the Committee's assessments of its maximum level."

4 Solution Method and Calibration

We solve our model economy using a nonlinear solution method that allows us to satisfy the ELB constraint. We use the extended path method (also known as the perfect foresight solution or deterministic solution) implemented in Dynare to solve our nonlinear model of equilibrium equations. This method uses a variant of the Fair and Taylor (1983) algorithm, and Juillard (1996) provides details about its implementation.[12]

The model is calibrated at a quarterly frequency to match key empirical moments for the U.S. economy over the 1976–2019 period.[13] Table 3 presents the parameter values for the baseline calibration of the model. Table 4 shows that the calibrated model does a good job in matching the targeted moments. While most of the parameters do not have a one-to-one relationship to a moment, it is informative to describe the calibration in steps and highlight the key parameter that informs each moment. Importantly, all parameters are assumed to be the same for both types of households in the model, except for the κ_1, which is assumed to be positive and the κ_2, which is normalized to zero. In order to bring the model to the data, we will explicitly regard type 1 workers as Black workers and type 2 workers as white workers in the data.

The elasticity of substitution between goods is set to $\gamma = 6$, consistent with a 20 percent markup. We draw from the evidence reported in Petrongolo and Pissarides (2001) to calibrate the elasticity of the Cobb-Douglas matching function to $\varepsilon = 0.5$. We follow most of the literature and set the firm's bargaining power to $\zeta = 0.5$. We set the vacancy cost equal to 0.11, equivalent to 11 percent of average idiosyncratic productivity across employed workers, consistent with Hagedorn and Manovskii (2008) and very similar to other values used in the literature. The unemployment benefits parameter is set to $h = 0.71$, as in Hall and Milgrom (2008) and Pissarides (2009).

The efficiency parameter ς in the matching function targets a quarterly aggregate job-finding

[12] This method has been recently used by Lindé and Trabandt (2018).

[13] To calibrate the volatility of inflation and the correlation of unemployment and inflation, we consider data from 1984 to 2019, after the Great Moderation. For mean inflation, we consider data from 1995 to 2019 so the high inflation of the 1980s does not mask the deflationary bias of the ELB on nominal interest rates.

Table 3: Parameter Values

Parameter	Description	Value	Rationale
γ	Elasticity of subs. between goods	6	Literature
ε	Matching function elasticity	0.5	Petrongolo and Pissarides (2001)
ζ	Firm's bargaining power	0.5	Literature
χ	Vacancy posting cost	0.11	Hagedorn and Manovskii (2008)
h	Unemployment benefits	0.71	Hall and Milgrom (2008)
ς	Matching efficiency	0.966	Job-finding rate
λ^x	Exogenous separation rate	0.15	JOLTS data
μ_z	Mean log idiosyncratic prod.	-0.0236	Normalization
σ_z	SD log idiosyncratic prod.	0.157	Separation rate
δ	Type 1 labor share	0.15	Labor share of Blacks
κ_1	Discrimination against type 1	0.0292	Racial unemployment rate gap
κ_2	Discrimination against type 2	0	Normalization
ϕ_i	Monetary policy: Inertia	0.85	English et al. (2015)
ϕ_π	Monetary policy rule: Inflation gap	1.5	Taylor (1999)
ϕ_u	Monetary policy rule: Unemployment gap	-0.5	Taylor (1999)
π	Inflation target/steady state	0.005	2% annual inflation rate
λ_p	Price stickiness	0.84	Inflation volatility
ρ_A	Autocorr., productivity shock	0.93	Autocorr. of labor productivity
σ_A	SD, productivity shock	0.003	Unemp. volatility
ρ_ξ	Autocorr., risk premium shock	0.93	Corr. of output and unemp.
σ_ξ	SD, risk premium shock	0.00145	Corr. of inflation and unemp.
β	Discount factor	$1/(1.0001^4)$	Probability of binding ELB

rate of about 85 percent, consistent with the CPS microevidence for individuals 16 years of age and over from 1976 to 2019.[14] Regarding the idiosyncratic productivity process, we follow the standard practice in the literature by assuming that idiosyncratic shocks are independent draws from a log-normal distribution with parameters μ_z and σ_z. The parameter μ_z is set to -0.0236, which normalizes average worker's productivity to 1. To calibrate the standard deviation σ_z and the exogenous separation rate λ^x, we match the empirical evidence on the aggregate separation rate. The mean quarterly inflow rate to unemployment is 5.5 percent in the CPS.[15] Following Fujita and Ramey (2012), we assume that two-thirds of aggregate separations are endogenous.[16] Consequently, we set $\lambda^x = 0.15$ and $\sigma_z = 0.157$. Note that we assume that both types of households have the same exogenous separation rate. This assumption allows us to focus on the role of discrimination in generating differences across both types of households, but it is also consistent with the fact that job-to-job transition rates from the CPS are very similar between Black and white workers (see

[14]The aggregate job-finding rate in the model is given by $f_t = f_{1t}\frac{\delta - n_{1t-1}}{1-n_{t-1}} + f_{2t}\frac{1-\delta-n_{2t-1}}{1-n_{t-1}}$, while the aggregate separation rate is equal to $\lambda_t = \lambda_{1t}\frac{n_{1t-1}}{n_{t-1}} + \lambda_{2t}\frac{n_{2t-1}}{n_{t-1}}$. A quarterly job-finding rate of 85 percent is equivalent to a monthly job-finding rate of 50 percent. Note that the quarterly job-finding rate is the probability that a worker who is unemployed at the beginning of the quarter is employed at the end of the quarter. Thus, if f_m is the monthly job-finding rate and λ_m is the monthly separation rate, the quarterly job-finding rate is given by $f = f_m(1-\lambda_m)^2 + (1-f_m)f_m(1-\lambda_m) + (1-f)^2 f_m + f_m^2\lambda_m$. Similarly, the quarterly job separation rate is given by $\lambda = \lambda_m(1-f_m)^2 + (1-\lambda_m)\lambda_m(1-f_m) + (1-\lambda_m)^2\lambda_m + \lambda_m^2 f_m$.

[15]This value is consistent with a monthly rate of 3.3 percent.

[16]This assumption is also consistent with the Job Openings and Labor Turnover Survey (JOLTS) data, available from December 2000 onward. In particular, the mean monthly layoff rate is about 1.5 percent, computed from December 2000 to December 2019. This result compares with the roughly 2.5 percent mean monthly inflow rate into unemployment from the CPS over the same period.

Table 4: Targeted Moments

Moments	Data	Model
Aggregate job-finding rate in steady state	83	85
Aggregate separation rate in steady state	5.5	5.5
Racial unemployment rate gap in steady state	6.4	6.4
Mean inflation rate	1.8	1.9
Volatility unemployment rate	1.3	1.1
Volatility inflation rate	0.8	0.9
Corr. unemployment rate and inflation rate	-0.30	-0.32
Corr. unemployment rate and output	-0.81	-0.85
Autocorrelation of labor productivity	0.9	0.8
Probability of binding ELB	16	11

Note: Data are at a quarterly frequency for the 1976–2019 period unless otherwise specified. Output and labor productivity data are logged. In the "Data" column, "steady state" refers to the mean. To compute cyclical volatilities (standard deviation) and correlations, we first detrend the series using the HP filter with a smoothing parameter of 10^5.

Appendix A.3 for further details).

The labor share of Blacks is set to $\delta = 0.15$, consistent with the data. The discriminatory parameter κ_1 is set to 0.0292 (and $\kappa_2 = 0$) to match an average unemployment rate gap between Blacks and whites of 6.4 percentage points, consistent with the CPS evidence. This value for κ_1 is equivalent to 3.6 percent of the steady-state average wage.

The monetary policy rule in our baseline economy is symmetric, so it responds equally to positive and negative deviations of the actual unemployment rate (and inflation) with respect to its steady-state value. In particular, we follow Taylor (1999) and set the inflation coefficient to $\phi_\pi = 1.5$ and the unemployment gap coefficient to $\phi_u = -2/4$.[17] Following the empirical evidence in English et al. (2015), the inertial component of the monetary policy rule, ϕ_i, is set to 0.85. We consider a steady-state inflation rate of 2 percent, consistent with the FOMC inflation objective. Regarding the degree of nominal rigidities, we set the probability of resetting the price to 0.84 so that the cyclical component of inflation in the baseline economy is as volatile as the cyclical component of total PCE (personal consumption expenditures) price inflation in the data. We do not allow for price indexation and set $\gamma_p = 0$.

Finally, we consider two types of aggregate shocks: a demand shock implemented via a risk premium shock and a supply shock implemented via an aggregate productivity shock.[18] We assume that both shock processes have the same persistence, and we calibrate that persistence, together with the standard deviation of both shocks, to target the following cyclical targets: the autocorrelation of labor productivity, the correlation of output and the aggregate unemployment rate, the

[17] The value for ϕ_u corresponds to a coefficient of 1 on the output gap as in Taylor (1999), which we then transform to an unemployment rate gap by using the empirical Okun's law relationship. This transformation roughly delivers a coefficient of -2 when regressing the output gap on the unemployment rate gap. The adjustment of dividing by 4 is justified by Taylor's use of annual inflation in the rule, while we use quarterly inflation in equations (5) and (6).

[18] Bundick and Petrosky-Nadeau (2021) also consider a combination of demand and supply shocks in their analysis.

standard deviation of the aggregate unemployment rate, and the correlation between inflation and the unemployment rate.[19] The resulting values are $\rho_A = \rho_\xi = 0.93$, $\sigma_A = 0.003$, and $\sigma_\xi = 0.00145$. These values, along with a discount factor consistent with an annual interest rate of effectively 0 percent, allow the model to deliver a probability of a binding ELB on the nominal interest rate of 11 percent, which is not far from the 16 percent observed in the data.[20]

Table 4 shows that the calibrated model is able to match the targeted key moments relatively well. Consistent with the empirical evidence, the model delivers a slight deflationary bias due to the ELB, with inflation averaging 1.9 percent, below its 2 percent steady-state value. The ELB also biases unemployment upward, leading to a mean aggregate unemployment rate of 6.37 percent, slightly above its steady-state value. We discuss all model results in the next section.

5 Simulation Results under the Deviations Rule

This section presents the simulation results of our baseline model economy that uses the Deviations monetary policy rule. First, we present results for the aggregate economy. Second, we compare labor market dynamics across racial groups in our model and quantify the role played by the separation rate in explaining the dynamics of the racial unemployment rate gap. Third, we define a model-based measure of discrimination and compare with its empirical counterpart. Fourth, we discuss the model's mechanisms in generating all the results. Finally, we show that inequality in labor market outcomes between Black and white workers leads to meaningful differences in terms of welfare between both types of households.

5.1 Aggregate Outcomes

Table 5 reports simulation results for the unemployment rate and flows at the aggregate level for the baseline model with the ELB constraint imposed (Panel B), together with the actual U.S. data moments over the 1976–2019 period (Panel A).[21] In order to better understand the role played by the ELB constraint, Panel C in Table 5 shows the simulation results without imposing the ELB constraint on nominal interest rates.

Starting with Panel B, the simulation results show that the model performs reasonably well at the aggregate level. It basically hits the empirical means of the job-finding rate and the separation rate, and mostly matches the volatility of the unemployment rate and inflation, by construction of the exercise. The model is able to endogenously replicate the observed volatility of the aggregate separation rate, though it underpredicts the volatility of the job-finding rate. Also, the baseline model is able to account for the positive skewness of the aggregate unemployment rate observed

[19] To compute cyclical components, we detrend the series using the HP filter with a smoothing parameter of 10^5.

[20] The target federal funds rate set by the FOMC reached the ELB in December 2008, and it was not above its ELB until December 2015. Thus the economy was at the ELB for 28 quarters. which, dividing by the total number of quarters from 1976 to 2019, yield a binding ELB for 16 percent of the time.

[21] The reported model values are means of statistics computed from 1,000 simulations. In each simulation, 276 quarterly observations for all variables are obtained. The first 100 quarters are discarded, and the last 176 quarters, corresponding to the 1976–2019 period, are used to compute the statistics in the same way as we do for the data.

Table 5: Aggregate Outcomes: Deviations Rule

	U	λ	f	π
Panel A: Data				
Mean	6.29	3.30	51.27	1.81
Volatility	1.31	0.33	9.68	0.85
Skewness	0.94	0.41	0.02	-0.45
Panel B: Model, Deviations rule with ELB				
Mean	6.37	3.36	50.14	1.88
Volatility	1.11	0.37	3.49	0.86
Skewness	1.25	1.10	-0.21	-0.54
Panel C: Model, Deviations rule without ELB				
Mean	6.12	3.28	50.73	1.98
Volatility	0.72	0.26	2.85	0.71
Skewness	-0.03	-0.09	0.53	-0.10

Note: Statistics for the model are means across 1,000 simulations. Means and volatilities are in percentage points. The cyclical volatility and skewness are computed for the variables expressed in deviations from an HP trend with a smoothing parameter of 10^5. U refers to unemployment rate, f to job-finding rate, λ to separation rate, and π to annual inflation. λ and f are the implied monthly rates from quarterly rates.

in the data, which is mainly coming from the positive skewness on the separation rate, consistent with the data. As shown in Panel C, the ELB constraint is key in explaining the skewness result, as, without imposing that constraint, the model would basically exhibit no skewness in the aggregate unemployment rate. The binding ELB constraint also increases the average aggregate unemployment rate by $\frac{1}{4}$ percentage point, as it increases the mean separation rate and lowers the mean job-finding rate, and amplifies labor market volatilities. Regarding inflation, the presence of the ELB constraint results in a slight degree of deflation bias, with the inflation rate averaging 0.1 percentage point below its 2.0 percent steady state. The ELB constraint is also responsible for having a more volatile and more negatively skewed inflation rate.

5.2 Labor Market Dynamics across Racial Groups

Table 6 reports simulation results for labor market variables by racial group for the baseline economy under the Deviations rule (Panel B), together with actual U.S. data moments during the 1976–2019 period (Panel A). Panel C reports simulation results without imposing the ELB. To start, first note that the only model result in Table 6 that is a target of our calibration strategy is the mean of the racial unemployment rate gap, which stands at 6.5 percentage points in the model, in line with the data. The rest of the model outcomes in Table 6 are untargeted moments.

Our model economy delivers four key results regarding labor market dynamics across racial groups, which are all in line with the empirical evidence. First, the model *endogenously* generates a higher mean separation rate and a lower mean job-finding rate for Black workers relative to white workers, as in the data. In particular, the ratio of separation rates between Black and white

Table 6: Labor Market Outcomes by Race: Deviations Rule

	Unemployment rate			Separation rate		Job-finding rate	
	B	*W*	Racial gap	*B*	*W*	*B*	*W*
Panel A: Data							
Means	11.96	5.51	6.46	5.84	2.99	43.46	54.06
Volatility	2.12	1.21	1.08	0.84	0.29	7.90	10.68
Skewness	0.88	0.99	0.78	0.92	0.35	0.02	0.05
Panel B: Model, with ELB							
Means	11.89	5.39	6.50	6.53	2.90	49.01	51.67
Volatility	1.70	1.01	0.68	0.54	0.35	3.64	3.63
Skewness	1.23	1.26	1.18	1.28	1.09	-0.18	-0.15
Panel C: Model, without ELB							
Means	11.52	5.17	6.35	6.42	2.83	49.63	52.28
Volatility	1.10	0.65	0.45	0.36	0.24	2.98	2.99
Skewness	-0.03	-0.02	-0.04	0.01	-0.09	0.56	0.58

Note: See note to Table 5. *B* refers to Blacks, which correspond to group 1 in the model, and *W* to whites, or group 2 in the model.

workers is 1.9 in the data and 2.3 in the model, while the ratio of job-finding rates between Black and white workers is 0.8 in the data and 0.9 in the model. Second, regarding cyclical volatilities, the model is able to endogenously generate higher volatility of the unemployment rate for Blacks than for whites, because of higher volatility of the separation rate margin, as is the case in the data. Third, our baseline model generates a racial unemployment rate gap that is highly volatile and strongly countercyclical, consistent with the data. In particular, the correlation between the racial unemployment rate gap and the aggregate unemployment rate stands at 0.77 in the U.S. data, and it is almost equal to 1 in our model economy. Fourth, the baseline model generates positively skewed unemployment rates for both racial groups, mainly because of skewness in the respective separation rates, as observed in the data. The model also reproduces the observed positively skewed racial unemployment rate gap. Interestingly, the model is able to explain these key results about labor market dynamics across racial groups with a relatively small value of κ_1, about 3.6 percent of the steady-state average wage.

Panel C of Table 6 illustrates the role played by the ELB constraint in generating the model results just described. The presence of the ELB constraint generates higher separation rates and lower job-finding rates—and thus higher unemployment rates—for both racial groups, as well as higher volatility for all labor market variables. Importantly, however, our model is able to generate similar differences between the two racial groups in terms of means and volatilities for the unemployment rate and means and volatilities for the job-finding rate and separation rate, regardless of the ELB constraint. Also, the ability of the model to generate a strongly countercyclical racial unemployment rate gap is robust to the exclusion of the ELB constraint. As was the case for the aggregate labor market outcomes, the ELB constraint is key in generating skewness in labor market variables by race in the model.

Table 7: Contributions of Unemployment Flow Rates to the
Racial Unemployment Rate Gap in the Model

	Separation rate	Job-finding rate
Mean	0.92	0.05
Cyclical variance	0.81	0.14

Note: See note to Table 2.

5.2.1 The Role of the Separation Rate for the Dynamics of the Racial Unemployment Rate Gap

An important result of our theoretical model is that the separation rate margin is of crucial importance in explaining the differences in the unemployment rate between Blacks and whites in the model, in terms of both means and volatilities, in line with the empirical evidence. This section quantifies the role of the separation rate margin in explaining the dynamics of the racial unemployment rate gap in the model using the same methodology as the empirical analysis in Section 2.1. We construct the same counterfactual racial unemployment rate gaps using stochastic simulations from our baseline model economy and report the contributions of each unemployment flow rate in Table 7, which is to be compared with the empirical findings in Table 2. Importantly, the separation rate margin is key in the model to explaining the mean and variance of the racial unemployment rate gap, consistent with the empirical evidence, although the model overstates the importance of the separation rate relative to the job-finding rate when compared with the data. In particular, the separation rate margin explains about 90 percent of the average unemployment rate gap in the model (67 percent in the data). In turn, the covariance of the cyclical components of the racial gap and the counterfactual racial gap generated by the separation rate accounts for about 80 percent of the variance of the cyclical component of the racial gap (60 percent in the data). The reason why the model understates the role played by the job-finding rate in explaining the differences in the unemployment rate between Blacks and whites is that the bulk of the variation in the job-finding rate comes from the job-meeting rate component, which is common to both types of workers.

5.3 Model-Based Measures of Labor Market Discrimination

In the model, employers exhibit discrimination against type 1 workers, which manifests itself in two labor market margins: hiring and separations. The first type of discrimination occurs during the hiring process. Recall that the hiring process in the model involves two stages. During the first stage, an unemployed worker and a vacancy meet with probability p_t. During the second stage, the match draws an idiosyncratic productivity. The firm-worker match will form and start producing only if the idiosyncratic productivity is above the reservation productivity threshold. Otherwise, the firm-worker match will be dissolved. Our baseline calibration features a positive κ_1 discriminatory parameter against type 1 workers and normalizes $\kappa_2 = 0$. This feature results in a higher reservation threshold for type 1 workers than for type 2 workers—and thus a lower job-finding rate and higher job separation rate for type 1 workers than for type 2 workers. We define the labor market discrimination associated with the hiring margin, labeled $\mathcal{D}_t^f$, as those type 1 unemployed workers who are not hired for no other reason than for being type 1, as they would otherwise be hired if they were instead type 2. We can formally define $\mathcal{D}_t^f$ as a fraction of the type 1 labor force:

$$\mathcal{D}_t^f \equiv \frac{u_{1t-1}}{\delta} p_t \left[G(z_t^{R1}) - G(z_t^{R2}) \right].$$

In other words, those type 1 unemployed workers whose idiosyncratic productivity draw is above the reservation threshold for type 2 workers, but below the reservation threshold for type 1 workers, will remain unemployed solely due to the presence of discrimination during the hiring process. In theory, such an unemployed worker could file a charge of discrimination against the employer.

The second type of discrimination occurs during the separation process. In the model, each matched worker-firm draws a new idiosyncratic productivity each period, which needs to be above the reservation threshold to avoid an endogenous separation. The labor market discrimination associated with the separation margin, labeled $\mathcal{D}_t^\lambda$, occurs when a type 1 worker-firm match draws an idiosyncratic productivity that is above the reservation threshold for type 2 workers but below the corresponding one for type 1 workers. As a result, this type 1 employed worker is endogenously separated for no other reason than for being type 1, as they would otherwise remain employed if they were instead type 2. In theory, such an employed worker could file a charge of discrimination against the employer. We can formally define $\mathcal{D}_t^\lambda$ as a fraction of the type 1 labor force:

$$\mathcal{D}_t^\lambda \equiv \frac{n_{1t-1}}{\delta} \left[(1-\lambda^x) + p_t \lambda^x \right] \left[G(z_t^{R1}) - G(z_t^{R2}) \right].$$

The data on race-based charges of discrimination do not differentiate between the hiring and separation margins, so to compare discrimination in the model with its empirical counterpart, we sum the two measures of discrimination, which are already expressed as a fraction of the type 1 labor force:

$$\mathcal{D}_t = \mathcal{D}_t^f + \mathcal{D}_t^\lambda. \tag{7}$$

Table 8 presents simulation results for the model-based measures of labor market discrimination with and without imposing the ELB constraint. Notably, our baseline model generates strongly countercyclical measures of labor market discrimination, as shown in the last row of Panel A, at both the hiring and firing margins. This result means that labor market discrimination is more prevalent during recessions than during expansions, as observed in the data. In particular, the correlation between the aggregate rate of labor market discrimination and the aggregate unemployment rate stands at 0.99 in our model economy, while it is 0.64 in the data. The strong countercyclical pattern of labor market discrimination measures in the model is independent of the presence of the ELB constraint. The ELB constraint is, though, responsible for the positive skewness in discrimination measures as well as for slightly higher means and volatilities of those measures.

Table 8: Model-Based Labor Market Discrimination Measures: Deviations Rule

	Hiring margin $\mathcal{D}_t^f$	Separation margin $\mathcal{D}_t^\lambda$	Aggregate $\mathcal{D}_t$
Panel A: Model, with ELB			
Means	0.64	5.22	5.86
Volatility	0.12	0.40	0.48
Skewness	1.35	1.12	1.13
Corr. with aggregate U	0.80	0.95	0.99
Panel B: Model, without ELB			
Means	0.61	5.15	5.76
Volatility	0.07	0.28	0.33
Skewness	0.07	-0.05	-0.06
Corr. with aggregate U	0.78	0.97	1

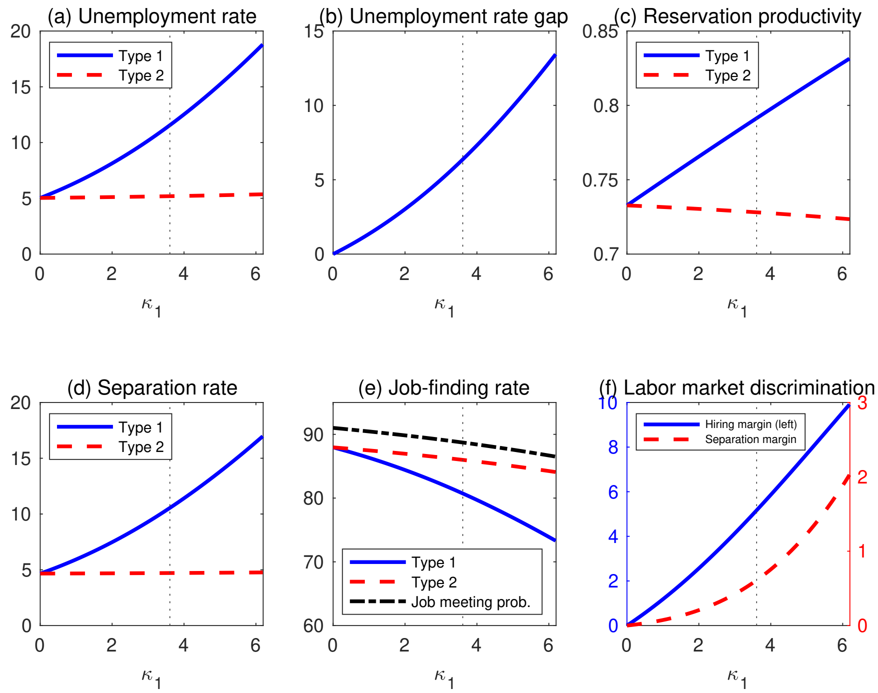
Note: See note to Table 5.

5.4 Discussion of the Model's Mechanism

In order to understand the mechanism at work in our model to generate different unemployment dynamics between Black and white workers, we analyze the role played by the discriminatory parameter κ_1 by means of two figures. We first solve the steady state of the model for different values of κ_1 while keeping the rest of the parameters fixed at their calibrated values from Table 3, including $\kappa_2 = 0$. Figure 3 presents the results of this exercise, where black dotted vertical lines are used to mark the baseline calibration for κ_1 (expressed as a percentage of the aggregate wage). The first thing to notice is that, when $\kappa_1 = \kappa_2 = 0$, there are no differences between the two types of workers, and both types share the same labor market outcomes, with a zero racial unemployment rate gap and no labor market discrimination.

As κ_1 increases (while keeping $\kappa_2 = 0$), Panels (c) and (d) in Figure 3 show that the reservation productivity threshold for type 1 workers also rises, leading to a higher separation rate for those workers. In contrast, the reservation threshold and the separation rate for type 2 workers remain almost unchanged when κ_1 rises. The reason for the increase in the reservation threshold for type 1 workers is that firms need to compensate for a higher perceived cost of employing a Black worker. Panel (e) shows that the job-meeting probability that both types of workers face when being unemployed slightly falls as κ_1 increases. This decrease is the result of the random search assumption, as the expected benefit of filling a vacancy falls when κ_1 rises, reducing firms' incentives to post vacancies. Importantly, though, as κ_1 increases, the job-finding rate for type 1 workers falls more than for type 2 workers, as there is a higher probability after meeting a vacancy that the idiosyncratic productivity draw for type 1 workers falls below their reservation threshold than is the case for type 2 workers. As a result, as κ_1 increases, Panel (a) shows that the unemployment rate of type 1 workers increases, while the unemployment rate of type 2 workers remains almost unchanged. Thus, the racial unemployment rate gap increases (see Panel (b)). Finally, as the discriminatory parameter κ_1 increases, model-based measures of labor market discrimination widen both at the hiring margin and at the separation margin, as shown in Panel (f).

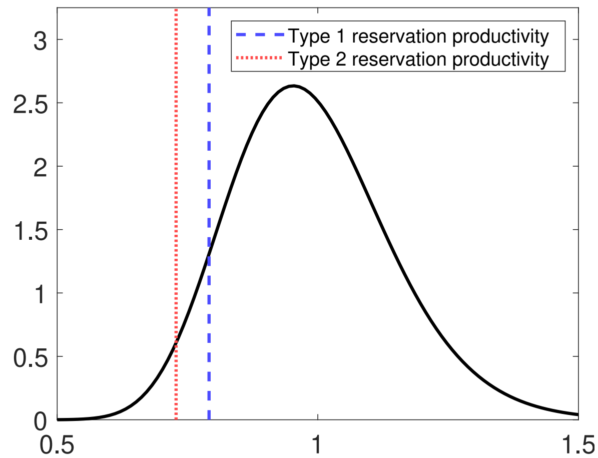
Figure 3: Labor Market Effects of Higher Discrimination against Type 1



Note: We plot steady-state results when all parameters are kept at their calibrated values in Table 3 and κ_1 is allowed to vary from 0 to 0.05. On the horizontal axis, we express κ_1 as a percentage of the aggregate wage. The vertical dotted lines correspond to the baseline calibration of the model.

We turn next to explain why type 1 workers have higher unemployment rate volatility than type 2 workers. Figure 4 plots the distribution of idiosyncratic productivity under our baseline calibration, which is the same for both types of workers and invariant to aggregate shocks. We also add to the figure two vertical lines that mark the positions of the steady-state reservation productivities: a dashed blue line for type 1 workers and a dotted red line for type 2 workers. As has been shown previously, given that $\kappa_1 > \kappa_2$, type 1 workers have a higher reservation threshold than type 2 workers. As the model economy is subject to aggregate shocks (both supply and demand shocks), both reservation thresholds move. For example, if an aggregate shock results in a higher (lower) separation rate, then both reservation thresholds move to the right (left). Importantly, because of the assumed log-normal distribution for idiosyncratic productivity, there is a greater density mass around the reservation threshold for type 1 workers than for type 2 workers. Therefore, an aggregate shock that buffets the economy will imply larger volatility for the separation rate and job-finding rate, and thus for the unemployment rate, of type 1 workers than for type 2 workers. This outcome explains why type 1 workers encounter larger volatility of the separation rate, job-finding rate, and unemployment rate than type 2 workers. It also explains why the racial unemployment rate gap and labor market discrimination measures are countercyclical, as there is a larger mass of Black workers around the reservation threshold than of white workers when the economy is hit by aggregate shocks.

Figure 4: Idiosyncratic Productivity Distribution and Reservation Thresholds



5.5 Welfare

This section shows that inequality in labor market outcomes between white and Black workers leads to a meaningful difference in terms of welfare for the two types of households. As explained later, in our model economy, inequality in labor market outcomes results in consumption inequality, and thus welfare differences, between the two types of households. In addition, in our New Keynesian model, there is a loss of efficiency in production, as there is price dispersion across firms due to Calvo price rigidities. Our welfare measure takes into account the welfare loss due to price dispersion. However, in our model, we assume that both types of households face the same inflation rate, and we thus abstract from allowing different households to consume different baskets of goods and services resulting in different inflation rates.[22]

We define welfare for the type i household in period t, $\mathcal{W}_{it}$, as the discounted sum of utility generated from the per-person stream of consumption in period t and every future period:[23]

$$\mathcal{W}_{it} \equiv E_t \sum_{s=0}^{\infty} \beta^s \log(c_{it+s}).$$

We can then compute the consumption-equivalent welfare wedge between type 1 and type 2 households in period t, Ψ_t, as the percentage increase in per-person consumption in the type 1 household necessary in period t and every future period to equalize welfare between the two households.

[22]Appendix A.4 provides empirical support for that assumption, showing that average inflation rates between Black and white households have been very similar. These results are consistent with McGranahan and Paulson (2006) and Hobijn et al. (2009), for example, which find that the inflation experiences of the different demographic groups are highly correlated with, and similar in magnitude to, the inflation experiences of the overall urban population. We leave the model extension of allowing for different inflation volatility across racial groups for future research. Lee et al. (2022) study, in a reduced-form Phillips curve framework, how monetary policy affects real incomes by racial group, when inflation rates and unemployment rates differ by race.

[23]See Appendix C.7 for additional details.

Formally, we implicitly define Ψ_t as

$$E_t \sum_{s=0}^{\infty} \beta^s \log((1+\Psi_t)c_{1t+s}) = E_t \sum_{s=0}^{\infty} \beta^s \log(c_{2t+s}). \tag{8}$$

A positive value of Ψ_t implies that type 2 households have higher welfare than type 1 households.

In our baseline model economy, we find that the consumption-equivalent welfare wedge between Black and white workers averages 2.4 percent. That is, Black households would need to increase their consumption by 2.4 percent every period to achieve the same welfare as white households, which is a meaningful value. Note that in our model economy, labor market inequality leads to consumption inequality, and thus welfare differences, through two margins. First, the presence of discrimination against type 1 workers lowers their average wage (see the wage equation (4)), which enters as income into the budget constraint for the household:[24]

$$c_{it} = (1-U_{it})\bar{w}_{it} + U_{it}h + \Pi_t - T_t.$$

In our baseline economy, the mean wage for Black workers is, on average, 2.2 percent below the mean wage for white workers.[25] The second margin leading to consumption inequality in our model is that the Black unemployment rate is, on average, 6.5 percentage points higher than its white counterpart, and unemployment benefits are lower than the average wage for both types of workers.

Despite the large welfare wedge between Black and white workers in the model, we find that this welfare wedge hardly varies over the business cycle, with a standard deviation quantitatively indistinct from zero. The reasons for this result are that both households can smooth their consumption, and that the welfare measure is a forward-looking variable and the central bank's monetary policy rule stabilizes the economy in the long run.

Finally, while we believe that our model represents an important step toward understanding the welfare implications of employment inequality between white and Black workers, it is important to note that the welfare analysis in this paper represents a lower bound on the welfare differences between racial groups. In particular, our model certainly underestimates the welfare loss due to unemployment, mainly because it assumes that there is within-group consumption insurance and that individuals are not subject to liquidity constraints—and also because we assume that all unemployed workers receive unemployment benefits that do not expire regardless of the duration of the unemployment spell.

[24] The presence of κ_1 has a direct effect on the wage of type 1 workers, as well as an indirect effect through the continuation value of employing a type 1 worker, which also affects the wage. Note that we have zeroed out bond holdings in the budget constraint as bonds are in zero-net supply in equilibrium.

[25] Note that this gap is smaller than κ_1, in part because the average productivity of Black employed workers is larger than the one for whites, given that Blacks have a larger reservation productivity threshold than white workers.

6 Simulation Results: Shortfalls Rule versus Deviations Rule

This section studies the macroeconomic consequences of switching from a Deviations rule to a Shortfalls rule. Recall that the Shortfalls rule continues to respond symmetrically to inflation deviations from its target but responds to deviations of unemployment from its target only when the unemployment rate is above its target. In order to do so, we solve the model by keeping all parameters at their baseline values of Table 3, and we use the Shortfalls rule (6), instead of the Deviations rule (5), to govern the nominal interest rate. We first show results via impulse-response functions (IRFs) and then present summary statistics from stochastic simulations for the aggregate economy and for each racial group. Finally, we discuss the welfare implications of the change in the monetary policy rule.[26]

6.1 Impulse-Response Functions

This section presents the nonlinear dynamic responses to the risk premium shock (demand shock) in the model under two alternative monetary policy rules: the baseline Deviations rule and the Shortfalls rule. For illustrative purposes, we select the sign and the size of the shock to target a maximum increase of output of 1 percent under the baseline rule.

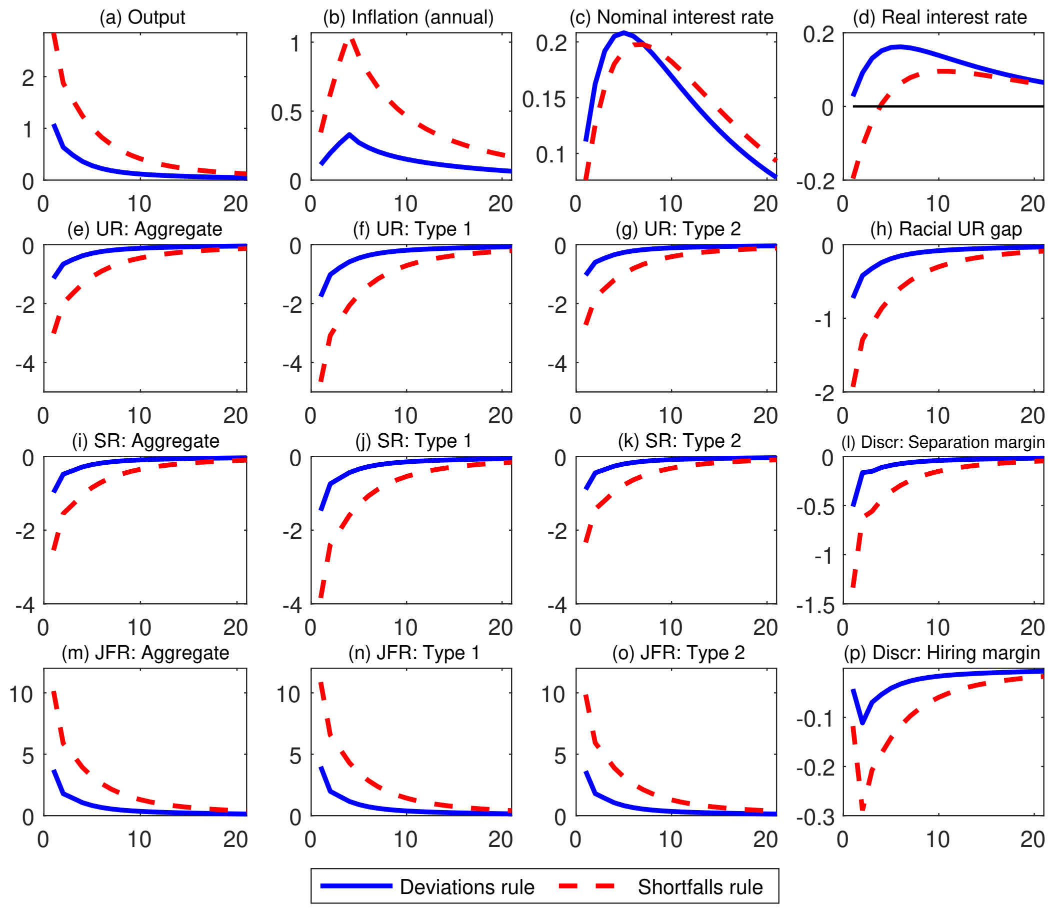
Figure 5 displays the impulse responses to an expansionary risk premium shock, which lowers the effective rate of return from the risk-free bond, increasing consumption and output. Under the Deviations rule, higher aggregate demand increases the surplus of a match for both types of workers. Consequently, firms post more vacancies, increasing the aggregate job-meeting rate, and the job separation rate declines for both types of workers because of lower reservation productivity thresholds. As a result, the unemployment rate falls for both types of workers. Higher aggregate demand also generates upward pressure on inflation. Given the increase in inflation and the fall in the aggregate unemployment rate, the Deviations rule prescribes raising the nominal interest rate, although not sufficiently to perfectly offset the shock.

Importantly, the response to the aggregate demand shock is not the same for the two types of workers. In particular, the separation rate for Black workers falls more than that for whites. Thus, the decline in the unemployment rate for Blacks is relatively more pronounced than the one for whites, and the racial unemployment rate gap falls. In turn, the model-based aggregate labor market discrimination measure falls. This countercyclical behavior of the racial unemployment rate gap and the labor market discrimination measure is consistent with the data.

When comparing the results under the Shortfalls rule, we see that the expansionary effects of the aggregate demand shock are larger. Specifically, given that the Shortfalls rule does not respond to deviations of the aggregate unemployment rate when it is below the target, the response of the policy rate to the risk premium shock is more muted than under the Deviations rule. As a result, the real rate falls below its steady-state value on impact, and the response of it becomes positive

[26] In all the exercises, we assume that the ELB constraint on nominal interest rates is binding. However, the results regarding the effects of switching from a Deviations rule to a Shortfalls rule are robust to excluding the ELB constraint. See Appendix B.2.2 for additional details.

Figure 5: Impulse-Response Functions to an Expansionary Demand Shock



Note: All panels plot percentage absolute deviations from steady state, except for Panel (a), which plots relative deviations instead. *UR* stands for unemployment rate, *SR* for separation rate, and *JFR* for job-finding rate.

only one year after the shock. Consequently, the positive responses for output and inflation and the decline in the unemployment rate are larger than under the Deviations rule. Given that Black workers benefit relatively more than white workers during longer expansionary periods, the racial unemployment rate gap falls by more under the Shortfalls rule than under the Deviations rule, as is the case for both measures of labor market discrimination. Thus, under a Shortfalls rule, the effects of expansionary demand shocks benefit relatively more the disadvantaged group in the labor market. Similar results are obtained for the aggregate productivity shock, and we relegate that discussion to Appendix B.1.

6.2 Aggregate Outcomes

Table 9 reports simulation results for the aggregate economy under the Deviations rule (in Panel A) and under the Shortfalls rule (in Panel B), both in the presence of the ELB constraint on nominal interest rates. Moving from a Deviations rule to a Shortfalls rule has quantitatively significant effects on both the average outcomes of the economy and their business cycle properties. Consistent with

Table 9: Aggregate Outcomes: Deviations Rule versus Shortfalls Rule

	U	λ	f	π
Panel A: Deviations rule				
Mean	6.37	3.36	50.14	1.88
Volatility	1.11	0.37	3.49	0.86
Skewness	1.25	1.10	-0.21	-0.54
Panel B: Shortfalls rule				
Mean	5.65	3.10	56.99	2.40
Volatility	1.68	0.64	16.32	0.90
Skewness	-0.09	-0.83	2.90	-0.28

Note: See note to Table 5.

the more accommodative response of the Shortfalls rule to a low aggregate unemployment rate, the economy under the Shortfalls rule features an average unemployment rate 0.7 percentage point below the one governed by the Deviations rule. This lower unemployment rate is the result of both a lower aggregate separation rate and a higher aggregate job-finding rate. In turn, the Shortfalls rule raises average inflation by 0.5 percentage point, from 1.9 percent to 2.4 percent. In terms of business cycle properties, the Shortfalls rule generates slightly larger labor market volatility and significantly changes the skewness of labor market variables. In particular, the Shortfalls rule is able to eliminate the positive skewness in the aggregate unemployment rate and reduces by half the negative skewness in the inflation rate. Relatedly, the Shortfalls rule also reduces the probability of a binding ELB by almost 2 percentage points with respect to the Deviations rule, and it strengthens the negative correlation between the inflation rate and the unemployment rate, and the negative correlation between output and the unemployment rate, when compared with an economy governed by the Deviations rule (see Table A.5 in Appendix B.2).

6.3 Labor Market Dynamics across Racial Groups

This section studies how moving from a Deviations rule to a Shortfalls rule differently affects Black and white workers in our model economy. Table 10 presents simulation results under the Shortfalls rule (in Panel B), to be compared with the results under the Deviations rule (in Panel A). Consistent with the more accommodative response of the Shortfalls rule to a below-steady-state aggregate unemployment rate than the Deviations rule, the unemployment rate of both types of workers is lower on average. Importantly, the adoption of the Shortfalls rule has distributional effects in the economy, as it benefits more the disadvantaged group in the labor market. In particular, under the Shortfalls rule, the decline in the unemployment rate for Black workers is larger than the decline in the unemployment rate for white workers. As a result, the average racial unemployment rate gap falls by 0.5 percentage point. In terms of unemployment flows, the adoption of the Shortfalls rule results in lower separation rates and higher job-finding rates for both types of workers. As was the case for the aggregate economy, all labor market variables are more volatile, as the Shortfalls rule

Table 10: Labor Market Outcomes by Race: Deviations Rule versus Shortfalls Rule

	Unemployment rate			Separation rate		Job-finding rate		Aggregate
	B	W	Racial gap	B	W	B	W	Discr.
Panel A: Model, Deviations rule								
Means	11.89	5.39	6.50	6.53	2.90	49.01	51.67	5.86
Volatility	1.70	1.01	0.68	0.54	0.35	3.64	3.63	0.48
Skewness	1.23	1.26	1.18	1.28	1.09	-0.18	-0.15	1.13
Panel B: Model, Shortfalls rule								
Means	10.78	4.74	6.03	6.29	2.65	56.56	59.37	5.52
Volatility	2.58	1.53	1.06	0.79	0.62	16.93	17.33	0.78
Skewness	-0.10	-0.09	-0.13	-0.40	-1.02	2.64	2.67	-0.26

Note: See note to Table 5. B refers to Blacks, which correspond to group 1 in the model, and W to whites, or group 2 in the model.

has lower stabilization properties than the Deviations rule. However, all labor market variables are much less skewed to the downside under the Shortfalls rule. For example, the negative skewness of the racial unemployment rate gap is basically eliminated. As a result, the distribution of the racial unemployment rate gap shifts toward being more symmetric around a lower mean under the Shortfalls rule, with a lower probability of experiencing very high values over the business cycle. This outcome can also be seen in Table A.6 in Appendix B.2, where we compare the distributions of unemployment rates at the aggregate level and by racial group, as well as the distribution of the racial unemployment rate gap, under both monetary policy rules.

Finally, the adoption of a Shortfalls rule results in a lower degree of discrimination in the labor market, as our model-based measure of aggregate discrimination is, on average, lower under a Shortfalls rule.[27] Some of the business cycle properties of the model-based measures of discrimination are also affected by the adoption of a Shortfalls rule. In particular, despite both measures remaining highly countercyclical, their volatility is slightly increased but their degree of skewness is significantly reduced, similar to the racial unemployment rate gap.

6.4 Welfare

This section assesses the welfare effects of switching from a Deviations rule to a Shortfalls rule. We first discuss the welfare changes for each type of household and then discuss the changes on the welfare wedge between the two types of households. To facilitate interpretation, we express welfare for both types of households in consumption-equivalent terms. To do so, we define the fraction ψ_{it} that we must raise type i consumption by each period in order to reach its corresponding steady-state welfare. Concretely, we define ψ_{it} implicitly as

$$E_t \sum_{s=0}^{\infty} \beta^s \log((1+\psi_{it})c_{it+s}) = \mathcal{W}_i,$$

[27] We observe a decline in discrimination both in the hiring margin and in the separation margin when moving from a Deviations rule to a Shortfalls rule (see Table A.7 in Appendix B.2).

where $\mathcal{W}_i$ is the steady-state welfare of the type i household.[28]

We find that, under both types of monetary policy rules, welfare for both types of households remains extremely close to its respective steady-state value throughout the business cycle. The most noticeable difference is that welfare for both types of households averages just slightly *below* steady-state welfare under the Deviations rule, while welfare for both groups averages just *above* steady-state welfare under the Shortfalls rule. Taken at face value, the increase in average inflation is more than compensated for by the higher consumption stream that allows a lower unemployment rate. We find that the unemployment rate falls by 1.1 percentage points for Black workers and 0.7 percentage point for white workers, on average, when switching from a Deviations rule to a Shortfalls rule. In turn, the average wage for Black workers increases by 0.43 percent, and the average wage for white workers increases by only 0.48 percent. Overall, on average, consumption for Black households increases by 0.62 percent, and consumption for white households increases by 0.64 percent. However, the differences in consumption-equivalent terms are quantitatively very small and almost indistinct from zero.

Regarding the consumption-equivalent welfare wedge between the two types of households, Ψ_t, defined in equation (8), we find that it slightly *increases* when moving from the Deviations rule to the Shortfalls rule. The reason for this result is that while the racial unemployment rate gap narrows by 0.5 percentage point, the wage gap between type 1 workers and type 2 workers actually slightly widens. On average, mean wages for both types of households slightly increase as match surplus increases on average; however, the mean wage for white workers increases by more than the mean wage for Black workers. The reason for that result is related to the changes in the productivity reservation thresholds for both types of households when adopting the Shortfalls rule. In particular, when switching to the Shortfalls rule, average productivity falls by more for Black workers than for white workers, contributing to increase the gap in average wages between Black and white households. Importantly, though, the change in the welfare wedge is effectively zero in quantitative terms. So while switching to a Shortfalls rule can have a small but meaningful effect on the racial unemployment rate gap and on the model-based measures of labor market discrimination, it does not help much to reduce racial inequality in terms of welfare in our model economy.

A note of caution regarding our welfare results is warranted again. As discussed in Section 5.5, our model certainly underestimates the welfare loss due to unemployment, as it features consumption insurance within each type of household and no liquidity constraints, among other simplifying assumptions. While we believe that our model represents an important step toward understanding the welfare implications of adopting a Shortfalls rule, extending our model economy to include some of those features could be a valuable avenue for future work to better assess the welfare consequences of the change in the conduct of monetary policy.

[28] See further details in Appendix C.7.

7 Conclusion

In this paper, we build a theoretical search and matching model that is able to explain the heterogeneous labor market outcomes between Black and white workers over the business cycle. In particular, we add worker heterogeneity and employer discrimination to a New Keynesian model with search and matching frictions in the labor market and endogenous separations. The model is consistent with key features of the aggregate economy and is able to explain key labor market facts across racial groups. We then apply this model to explore the implications of different monetary policy rules on these labor market outcomes—specifically, the effect of switching from an interest rate rule that responds to inflation and *deviations* of unemployment from its steady-state value, to one that responds to inflation and *shortfalls* in unemployment from its steady state. The latter policy rule is meant to capture in a reduced-form way one reading of the updated Statement on Longer-Run Goals and Monetary Policy Strategy by the FOMC. We find that the adoption of the Shortfalls rule reduces the average racial unemployment rate gap and the rate of labor market discrimination against Black workers in the model, while it increases average inflation in the model. From a welfare perspective, we find that while both Black and white households experience welfare gains when switching from a Deviations rule to a Shortfalls rule, the welfare gains in consumption-equivalent terms are quantitatively very small.

This paper focuses on the unemployment rate gap between Blacks and whites, which results from the fact that Black workers have a lower probability of finding a job when unemployed, and a greater probability of separating from employment, than white workers. However, labor market disparities between these two racial groups are also present in terms of other labor market margins. For example, both the labor force participation rate gap between Blacks and whites and the racial gap in terms of involuntary part-time employment are remarkably large. Importantly, both racial gaps are mostly unexplained by observables. We believe that our model could be a good starting point to incorporate additional margins of labor supply and reassess the welfare benefits of the newly adopted monetary policy framework. We leave these extensions for future research.

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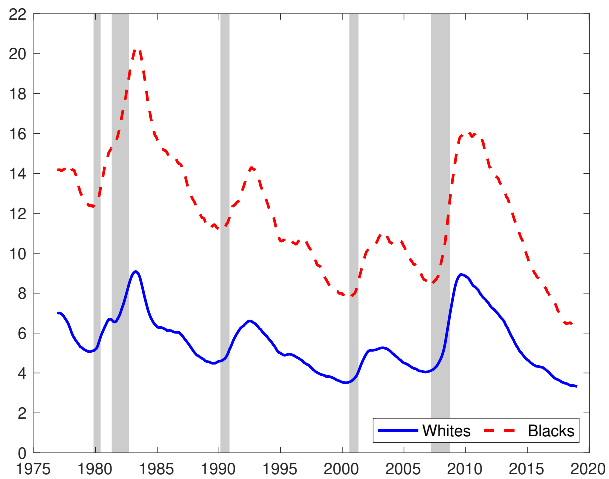
Online Appendix - Not Intended for Publication

A Additional Empirical Results

A.1 Unemployment Rates and Labor Market Volatility by Race

Figure A.1 plots the unemployment rate by race over the sample period analyzed in this paper.

Figure A.1: Unemployment Rates by Race



Note: We plot 12-month moving averages. The shaded areas represent NBER recessions.

Table A.1 reports two measures of volatility for the main labor market variables of interest. Absolute volatilities are defined as standard deviations of the data expressed in deviations from an HP trend with a smoothing parameter of 10^5. Relative volatilities are defined analogously, except that all variables are initially expressed in natural logarithms. Relative volatilities have been used in the recent search and matching literature to assess the quantitative properties of search and matching models. Given the interest of this paper in studying the unemployment rate gap between Blacks and whites, our preferred measure of volatilities are absolute volatilities. This preference avoids the distorting effect of different unemployment rate means between Blacks and whites on relative volatility measures.

Table A.1: Labor Market Volatility by Race

	Absolute volatilities			Relative volatilities		
	Whites	Blacks	Ratio	Whites	Blacks	Ratio
Employment rate	1.2	2.1	1.8	1.3	2.4	1.9
Unemployment rate	1.2	2.1	1.8	20.7	17.6	0.9
Job-finding rate	10.7	7.9	0.7	21.1	20.8	1.0
Separation rate	0.3	0.8	2.9	9.6	13.0	1.4

Note: Absolute volatilities are defined as standard deviations of the data expressed in deviations from an HP trend with a smoothing parameter of 10^5. Relative volatilities are defined analogously, except that all variables are initially expressed in natural logarithms.

As shown in Table A.1, Black workers experience larger employment volatility than white workers, regardless of which volatility measure is used. The same conclusion is reached in terms of unemployment volatility when looking at absolute volatilities. However, the opposite is reached when looking at the relative volatility, as the volatility of the white unemployment rate is actually *higher* than that of the Black unemployment rate. The reason why white workers have a more volatile unemployment rate in terms of log deviations is only due to their lower mean unemployment rate.

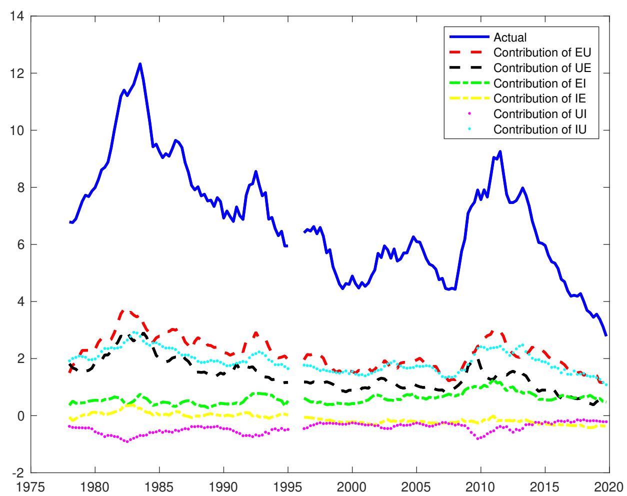
A.2 Unemployment Gross Flow Rates

We now assess the contribution of each unemployment flow in generating the observed racial unemployment rate gap by considering a three-state system where individuals can be employed (E), unemployed (U), or out of the labor force (I). We follow the same approach as in Section 2.1 to determine the relative importance of each margin, but with the steady-state unemployment approximation now

$$u_t^{i*} \approx \frac{EI_t^i IU_t^i + IE_t^i EU_t^i + IU_t^i EU_t^i}{EI_t^i IU_t^i + IE_t^i EU_t^i + IU_t^i EU_t^i + UI_t^i IE_t^i + IU_t^i UE_t^i + IE_t^i UE_t^i}.$$

Figure A.2 is the analog of Figure 1, and Table A.2 is the analog of Table 2, for the three-state system. The results show that the separation rate (EU transition rate) is the most important contributor in explaining the mean and the variance of the racial unemployment rate gap even when considering the transitions in and out of the labor force.

Figure A.2: Racial Unemployment Rate Gap:
Actual versus Counterfactuals (Three-State System)



Note: Series are quarterly averages of monthly data spanning from 1978:Q1 to 2019:Q4. Data are expressed in terms of 4-quarter moving averages.

Table A.2: Contributions of Unemployment Gross Flow Rates to the Racial Unemployment Rate Gap in the Data (Three-State System)

	EU	UE	EI	IE	UI	IU
Mean	0.32	0.20	0.09	-0.02	-0.06	0.28
Cyclical variance	0.29	0.20	0.07	0.05	-0.05	0.17

Note: The contribution of the separation rate to the mean of the racial gap is computed as $\frac{E[ugap_{cst}]}{E[ugap_t]}$. The contribution of the separation rate to the variance of the racial gap is computed as $\frac{\text{Cov}(ugap_t, ugap_{cst})}{\text{Var}(ugap_t)}$. The contributions of the job-finding rate are computed analogously, using $ugap_{cft}$ instead of $ugap_{cst}$. The rows do not sum to 100 percent, as the decomposition is not exact. Variances and covariances are computed using data in deviations from an HP trend with a smoothing parameter of 10^5.

A.3 Employer-to-Employer Transition Rates by Race

Our model assumes that exogenous separation rates are the same between Black and white workers. In the literature, exogenous separation rates are related to quits. Given that the JOLTS data do not allow the computation of quits by demographic characteristics, we compute employer-to-employer transition rates as an alternative measure of quits by race from the CPS data. In particular, we compute employer-to-employer transition rates by race, following the approach in Fallick and Fleischman (2004). We focus, though, on self-reported responses in the CPS in order to avoid the selection bias from the changes in the interviewing process in 2007 documented by Fujita et al. (forthcoming). Table A.3 shows that employer-to-employer transition rates differ very little by demographic groups, supporting the assumption that exogenous separation rates in the model do not differ by race.

Table A.3: Employer-to-Employer Transition Rates by Race

	Aggregate	Whites	Blacks
Average 2004–2019 (in percent)	2.03	1.96	2.20

A.4 Inflation Rates by Race

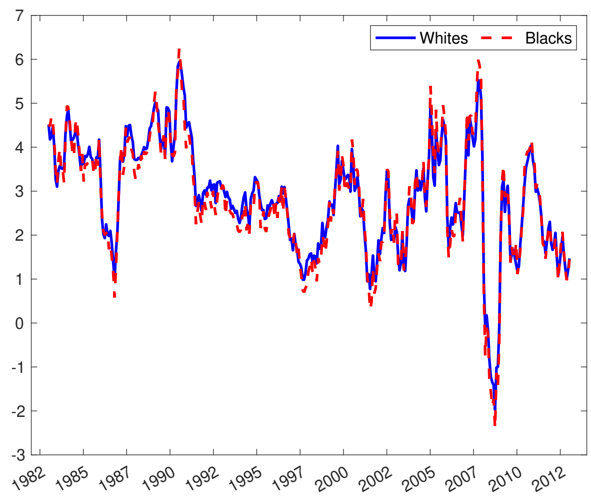
We use the Chicago Fed Income Based Economic Index (IBEX) 12 Month Inflation Rates, which are monthly inflation measures designed to capture the inflation experiences of specific socioeconomic and demographic groups available from 1983 to 2013. Each monthly IBEX inflation rate measures the percentage change in prices over the past 12 months for a particular group. Details on the methodology to compute IBEX inflation rates are described in McGranahan and Paulson (2006). Table A.4 and Figure A.3 show that, on average, the inflation rates faced by Black and white individuals are not very different over the available period. In particular, the average inflation rates over the 1983–2013 period are 2.87 percent for whites and 2.80 percent for Blacks. The cumulative gap over the 30-year period between whites and Blacks is equal to 5.27 percentage points, or roughly 18 basis points per year on average. Overall, the results presented in this section support the assumption of equal inflation rates across Black and white households in our model.

Table A.4: IBEX Inflation Rates by Race: 1983–2013 (in percent)

	All	White	Black
Average	2.86	2.87	2.80

Note: Chicago Fed Income Based Economic Index (IBEX) 12 Month Inflation Rates. “All” refers to all urban consumer units, “White” means reference person and spouse are white, and “Black” means reference person or spouse is Black or of Afro-American origin.

Figure A.3: IBEX Inflation Rates by Race (in percent)



B Additional Model Results

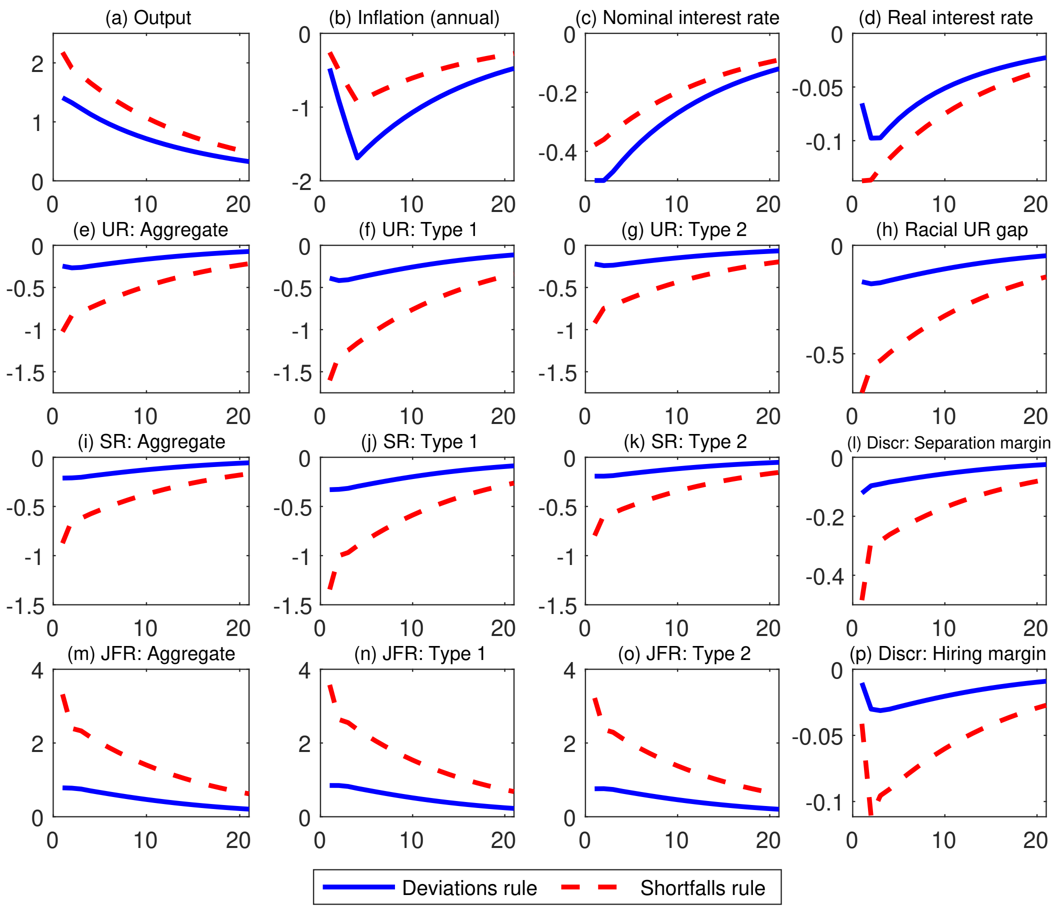
B.1 Impulse-Response Functions

This section presents the IRFs to a productivity shock that increases output by a maximum of 1 percent. In order to build intuition, we first present, in Figure A.4, the IRFs when the inertial component of the monetary policy rule is equal to zero (i.e., $\phi_i = 0$). We then show the IRFs under the baseline calibration in Figure A.5. Solid blue lines show the responses under the baseline Deviations rule, while red dashed lines show the responses under the alternative Shortfalls rule.

With respect to the Deviations rule, Figure A.4 shows that higher aggregate productivity has an expansionary effect on aggregate demand, increasing both consumption and output. This effect results in a lower aggregate unemployment rate. Inflation, however, runs lower, as higher productivity lowers marginal costs. Under the Deviations rule (baseline policy), the monetary policy faces a tradeoff between closing the inflation gap and closing the unemployment rate gap. Given the baseline calibration of the model, the monetary authority responds to the productivity shock by lowering the nominal interest rate, which allows the real interest rate to fall. In terms of disaggregated labor market outcomes, higher aggregate productivity lowers the unemployment rate gap between type 1 and type 2 workers, as the unemployment rate of type 1 workers falls more than the unemployment rate of type 2 workers. Again, the separation margin is key in explaining the decline of the unemployment rate gap, as the separation rate of type 1 workers falls more than the separation rate of type 2 workers, while the job-finding rate for both types of workers increases in similar amounts. As a result, the unemployment share of type 1 workers falls, and the labor market discrimination measure also declines.

If the central bank were to follow a Shortfalls rule instead, the central bank would then not

Figure A.4: Impulse-Response Functions to a Productivity Shock with No Inertia in the Monetary Policy Rule

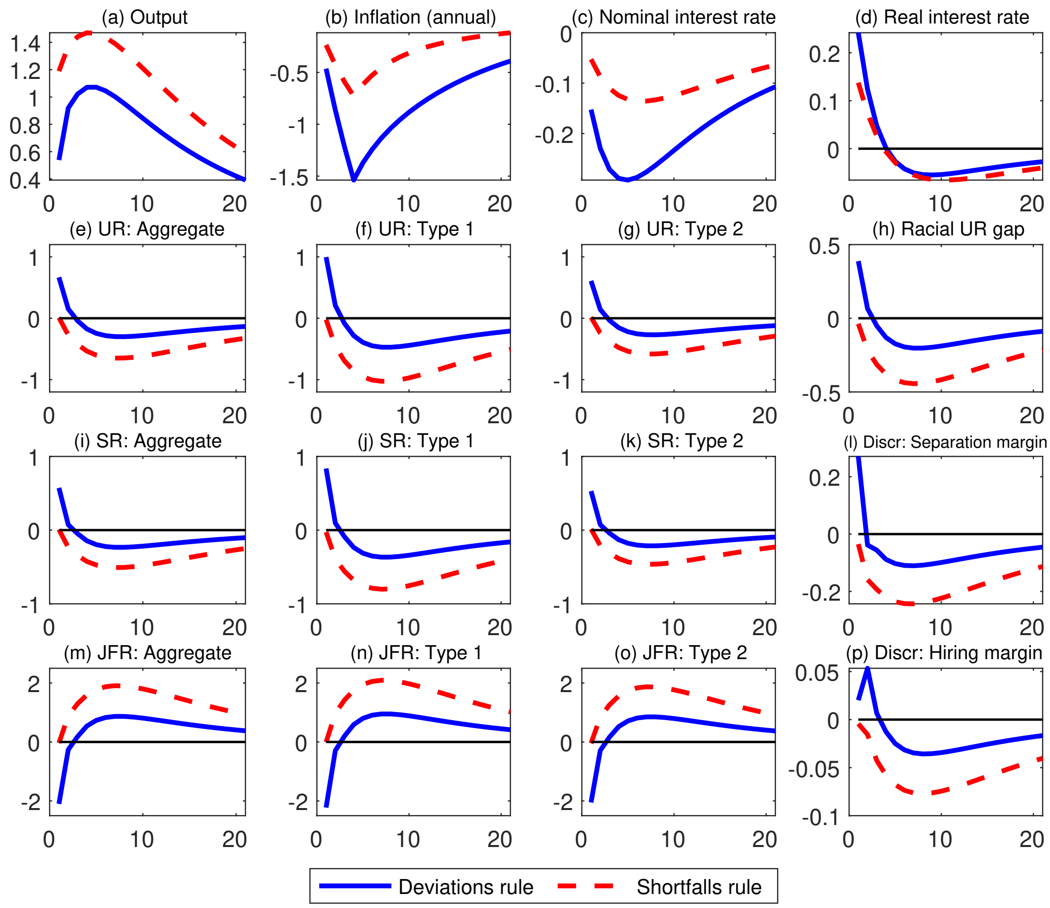


Note: All panels plot percentage absolute deviations from steady state except for Panel (a), which plots relative deviations instead. *UR* stands for unemployment rate, *SR* for separation rate, and *JFR* for job-finding rate.

face a tradeoff between inflation and unemployment rate stabilization upon a positive aggregate productivity shock. Thus, the central bank would focus just on the decline in inflation and lower the nominal interest rate accordingly, allowing the real interest rate to fall more than under the Deviations rule. As a result, the expansionary macroeconomic effects of a higher productivity shock (higher output and lower aggregate unemployment rate) would be larger than under the baseline Deviations rule. Importantly, adopting a Shortfalls rule would also improve distributional outcomes. In particular, the unemployment rate gap between the two types of workers, as well as the labor market discrimination measures, would fall by more under the Shortfalls rule than under the Deviations rule. The separation margin is again key for this result.

Figure A.5 plots the IRFs to the same expansionary shock as in Figure A.4, but under the assumption that $\phi_i = 0.85$, as in the baseline calibration. Under the baseline Deviations rule, and with a relatively high degree of inertia, the central bank is less aggressive in responding to inflation and unemployment deviations from their targets upon a positive productivity shock. As a result, the real interest rate increases on impact. The positive productivity shock allows aggregate output and consumption to expand, despite the temporary increase in the unemployment rate. After the first few quarters, though, the central bank is able to reduce the nominal interest rate, putting downward pressure on the real interest rate, and the aggregate unemployment rate eventually falls.

Figure A.5: Impulse-Response Functions to a Productivity Shock



Note: See note to Figure A.4.

Note that at that point, the central bank faces a tradeoff, as both aggregate unemployment and the inflation rate are below their targets under the Deviations rule, which would not be the case if the central bank were to follow a Shortfalls rule. After the first few quarters, the qualitative responses of all variables are similar to those in the case without inertia described earlier.

B.2 Stochastic Simulations

B.2.1 Shortfalls Rule versus Deviations Rule, with the ELB Constraint

This section presents three additional sets of simulation results. First, Table A.5 shows that the Shortfalls rule reduces the probability of a binding ELB by almost 2 percentage points with respect to the Deviations rule, and it also strengthens the negative correlation between the inflation rate and the unemployment rate, and the negative correlation between output and the unemployment rate, when compared with an economy governed by the Deviations rule.

Table A.5: Additional Business Cycle Moments: Data versus Model

	ELB probability	Corr(u, π)	Corr(u, y)
Data	15.9	-0.27	-0.81
Model: Deviations rule	10.6	-0.32	-0.85
Model: Shortfalls rule	8.5	-0.51	-0.94

Note: "Corr" refers to the cyclical correlation, and it is computed for the variables expressed in deviations from an HP trend with a smoothing parameter of 10^5.

Second, Table A.6 summarizes the distributions of unemployment rates at the aggregate level and by racial group, as well as the distribution of the racial unemployment rate gap, under both monetary policy rules. We can see that the distributions are shifted toward lower unemployment levels when shifting from a Deviations rule to a Shortfalls rule.

Table A.6: Distributions of Unemployment Rates under Alternative Monetary Policy Rules

	Deviations rule				Shortfalls rule			
	Aggregate	B	W	Racial Gap	Aggregate	B	W	Racial Gap
< 3	0.0	0.0	0.2	0.0	9.8	1.3	15.8	1.7
$[3, 5)$	8.6	0.0	39.0	0.6	22.1	2.9	33.2	17.6
$[5, 6)$	33.6	0.0	42.6	23.8	18.6	2.9	34.2	22.6
$[6, 7)$	38.6	0.0	12.3	59.7	31.7	4.2	11.7	43.6
$[7, 9)$	15.3	2.5	3.6	13.7	14.4	13.4	3.1	12.6
$[9, 11)$	2.1	30.2	1.3	1.7	1.8	21.1	1.1	1.4
$>= 11$	1.8	67.3	1.0	0.6	1.6	54.1	0.9	0.5

Note: Statistics for the model are means across 1,000 simulations. Means and volatilities are in percentage points. The cyclical volatility and skewness are computed for the variables expressed in deviations from an HP trend with a smoothing parameter of 10^5. B refers to Blacks which correspond to group 1 in the model, and W to whites, or group 2 in the model.

Finally, Table A.7 compares the model-based measures of discrimination between the Deviations rule and the Shortfalls rule. For all measures, discrimination levels are lower, slightly more volatile, and less skewed toward higher levels of discrimination when monetary policy is governed by the Shortfalls rule instead of the Deviations rule.

Table A.7: Model-Based Labor Market Discrimination Measures

	Hiring margin $\mathcal{D}_t^f$	Separation margin $\mathcal{D}_t^\lambda$	Aggregate $\mathcal{D}_t$
Panel A: Model, Deviations rule			
Means	0.64	5.22	5.86
Volatility	0.13	0.41	0.48
Skewness	1.35	1.11	1.13
Corr. with aggregate U	0.80	0.95	0.99
Panel B: Model, Shortfalls rule			
Means	0.56	4.95	5.52
Volatility	0.18	0.67	0.78
Skewness	0.28	-0.30	-0.26
Corr. with aggregate U	0.81	0.97	1

Note: See note to Table A.6.

B.2.2 Shortfalls Rule versus Deviations Rule, without the ELB Constraint

This section confirms that the main results of switching from a Deviations rule to a Shortfalls rule presented in the main text are robust to a nonbinding ELB constraint on nominal interest rates. In particular, we simulate the economy under the same parameter values as in our baseline case but without imposing the ELB constraint. Table A.8 shows that the aggregate effects are similar to those in the case with the binding ELB constraint, with a 0.7 percentage point decline in the aggregate unemployment rate; an increase in labor market volatility, with labor market variables much less skewed to the downside; and a 0.5 percentage point higher inflation rate.

Table A.8: Aggregate Outcomes without the ELB Constraint

	U	λ	f	π
Panel A: Model, Deviations rule, without ELB				
Mean	6.12	3.28	50.73	1.98
Volatility	0.72	0.26	2.85	0.71
Skewness	-0.03	-0.09	0.53	-0.10
Panel B: Model, Shortfalls rule, without ELB				
Mean	5.43	3.03	57.61	2.49
Volatility	1.43	0.58	16.32	0.78
Skewness	-0.84	-1.51	3.04	0.12

Note: See note to Table A.6. U refers to unemployment rate, f to job-finding rate, λ to separation rate, and π to annual inflation. λ and f are the implied monthly rates from quarterly rates.

Table A.9 shows that the effects across household types of switching from a Deviations rule to a Shortfalls rule are similar to those in the case with the binding ELB constraint. In particular, we still find a 0.5 percentage point decline in the racial unemployment rate gap, a 1.1 percentage points decline in the unemployment rate for Blacks, and a 0.7 percentage point decline in the unemployment rate for whites. We also find comparable declines in the measures of labor market discrimination. Therefore, we conclude that the main results of the paper regarding the effects of switching from a Deviations rule to a Shortfalls rule are robust to including the ELB constraint on nominal interest rates.

Table A.9: Labor Market Outcomes by Race, without the ELB Constraint

	Unemployment rate			Separation rate		Job-finding rate		Aggregate
	B	*W*	Racial gap	*B*	*W*	*B*	*W*	Discr.
Panel A: Model, Deviations rule, without ELB								
Means	11.52	5.17	6.35	6.42	2.83	49.63	52.28	5.76
Volatility	1.10	0.65	0.45	0.36	0.24	2.98	2.99	0.33
Skewness	-0.03	-0.02	-0.04	0.01	-0.09	0.56	0.58	-0.06
Panel B: Model, Shortfalls rule, without ELB								
Means	10.45	4.55	5.90	6.19	2.59	57.21	60.03	5.43
Volatility	2.21	1.30	0.91	0.67	0.56	16.94	17.34	0.69
Skewness	-0.83	-0.84	-0.81	-1.28	-1.70	2.76	2.78	-0.87

Note: See note to Table A.6.

C Additional Model Details

C.1 Intermediate Goods Producer Details

The first-order conditions with respect to employment, vacancies, and output are

$$\begin{aligned}
v_t &: \chi = q_{1t}(1-G(z_t^{R1}))\phi_{1t} + q_{2t}(1-G(z_t^{R2}))\phi_{2t},\\
n_{1t} &: \phi_{1t} = \int_{z_t^{R1}} (\varphi_t A_t z - w_t(z)) \frac{g(z)}{1-G(z_t^{R1})} dz - \kappa_1 + (1-\lambda^x) E_t \Lambda_{t,t+1} \phi_{1t+1}(1-G(z_{t+1}^{R1}),\\
n_{2t} &: \phi_{2t} = \int_{z_t^{R2}} (\varphi_t A_t z - w_t(z)) \frac{g(z)}{1-G(z_t^{R2})} dz - \kappa_2 + (1-\lambda^x) E_t \Lambda_{t,t+1} \phi_{2t+1}(1-G(z_{t+1}^{R2})),\\
x_t &: p_t^m = \varphi_t,
\end{aligned}$$

where ϕ_{it} and φ_t are Lagrange multipliers. The first-order condition for the reservation productivities is slightly more involved. Start with

$$\begin{aligned}
&- n_{it}\left(\frac{-1}{1-G(z_t^{Ri})} w_{it}(z_t^{Ri}) g(z_t^{Ri}) + \int_{z_t^{Ri}} w_{it}(z) g_i(z) \frac{g(z_t^{Ri})}{(1-G(z_t^{Ri}))^2}\right)\\
&- \phi_{it}[(1-\lambda^x) n_{t-1} + q_{it} v_t] g(z_t^{Ri}) - \varphi_t A_t n_{it}\left(\frac{1}{1-G(z_t^{Ri})} z_t^{Ri} g(z_t^{Ri}) - \int_{z_t^{Ri}} z g_i(z) dz \frac{g(z_t^{Ri})}{(1-G(z_t^{Ri})^2}\right) = 0,
\end{aligned}$$

and factor out $\frac{-n_{it} g_i(z_t^{Ri})}{1-G(z_t^{Ri})}$ to get

$$\left(-w_{it}(z_t^{Ri}) + \int_{z_t^{Ri}} \frac{w_{it}(z) g(z)}{1-G(z_t^{Ri})}\right) + \phi_{it} + \varphi_t A_t \left(z_t^{Ri} - \int_{z_t^{Ri}} \frac{z g(z) dz}{1-G(z_t^{Ri})}\right) = 0.$$

Then substitute for ϕ_{it} from the first-order condition with respect to n_{it} to get

$$\begin{aligned}
&- w_{it}(z_t^{Ri}) + \varphi_t A_t z_t^{Ri} - \kappa_i + (1-\lambda^x) E_t \Lambda_{t,t+1} \phi_{it+1}(1-G(z_{t+1}^{Ri})\\
&+ \int_{z_t^{Ri}} \frac{(w_{it}(z) - \varphi_t A_t z) g(z)}{1-G(z_t^{Ri})} dz + \int_{z_t^{Ri}} \frac{(\varphi_t A_t z - w_{it}(z)) g(z)}{1-G(z_t^{Ri})} dz = 0 \implies\\
&\varphi_t A_t z_t^{Ri} - w_{it}(z_t^{Ri}) - \kappa_i + (1-\lambda^x) E_t \Lambda_{t,t+1} \phi_{it+1}(1-G(z_{t+1}^{Ri}) = 0.
\end{aligned}$$

Combining and rearranging the first-order conditions yield

$$\begin{aligned}
(1-G(z_t^{Ri}))\phi_{it} &= \int_{z_t^{Ri}} p_t^m A_t (z - z_t^{Ri}) - (w_t(z) - w_t(z_t^{Ri})) g(z) dz\\
p_t^m A_t z_t^{Ri} + (1-\lambda^x) E_t \Lambda_{t,t+1}(1-G(z_{t+1}^{Ri}))\phi_{it+1} &= w_t(z_t^{Ri}) + \kappa_i.
\end{aligned}$$

Realized per-period profits are

$$\Pi_t^{int} = p_t^m A_t (n_{1t}\bar{z}_{1t} + n_{2t}\bar{z}_{2t}) - \bar{w}_{1t} n_{1t} - \bar{w}_{2t} n_{2t} - \chi v_t.$$

C.2 Household Details

The first-order conditions for the household problem are

$$\begin{aligned}
c_{it}: &\quad \frac{1}{c_{it}} = \lambda_t,\\
B_{it}: &\quad \lambda_{it} = \beta \xi_t (1+i_t) E_t \frac{P_t}{P_{t+1}} \lambda_{it+1},
\end{aligned}$$

where λ_{it} is the Lagrange multiplier associated with the budget constraint. Combining these two first-order conditions leads to the Euler equation. The stochastic discount factor is

$$\Lambda_{t,t+\tau} = \beta^\tau \frac{c_{1t}}{c_{1t+\tau}} = \beta^\tau \frac{c_{2t}}{c_{2t+\tau}},$$

where the complete-market trading of aggregate state-contingent claims to consumption equalizes these ratios of marginal utilities.

C.3 Final Good Producer Details

The first-order conditions for the final good producer's problem are

$$\begin{aligned} y_{jt}: \quad & P_{jt} = \nu_t \frac{\gamma-1}{\gamma}\left(\frac{y_{jt}}{y_t}\right)^{\frac{-1}{\gamma}} \frac{1}{y_t}, \\ y_t: \quad & P_t = \nu_t \frac{\gamma-1}{\gamma}\left(\frac{1}{y_t}\right)^{\frac{-1}{\gamma}} \frac{1}{y_t^2} \int_0^1 y_{jt}^{\frac{\gamma-1}{\gamma}} dj, \end{aligned}$$

where ν_t is the Lagrange multiplier on the aggregator constraint. Dividing these two equations yields

$$\frac{P_{jt}}{P_t} = \left(\frac{y_{jt}}{y_t}\right)^{\frac{-1}{\gamma}} \int_0^1 \left(\frac{y_{jt}}{y_t}\right)^{\frac{\gamma-1}{\gamma}} dj \implies y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\gamma} y_t.$$

Thus, $\frac{y_{jt}}{y_t} = \left(\frac{P_{jt}}{P_t}\right)^{-\gamma}$, which, if we plug into the aggregator, results in

$$\int_0^1 \left(\frac{P_{jt}}{P_t}\right)^{1-\gamma} dj = 1 \implies P_t = \left(\int_0^1 P_{jt}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}.$$

We will also note that final good producer profits are zero in equilibrium, as they behave competitively and their production technology exhibits constant returns to scale.

C.4 Retailer Details

The first-order condition for the retailers' problem can be written as

$$\sum_{s=0}^{\infty} E_t \left[\lambda_p^s \Lambda_{t,t+s} y_{t+s} y^*_{jt,t+s} \left(\frac{X_{t,t+s}}{P_{t+s}}\left(1 + \frac{P^*_{jt}}{y^*_{jt,t+s}} \frac{\partial y^*_{jt,t+s}}{\partial P^*_{jt}}\right) + p^m_{t+i} \frac{-1}{y^*_{jt,t+s}} \frac{\partial y^*_{jt,t+s}}{\partial P^*_{jt}}\right)\right] = 0,$$

where

$$X_{t,t+s} = \begin{cases} 1 & i = 0 \\ \Pi_{k=1}^s \pi_{t+k-1}^{\gamma_p} \pi^{1-\gamma_p} & i = 1, ..., \infty. \end{cases}$$

Then, defining

$$\begin{aligned} X^p_{t,t+s} &\equiv \frac{P_{t+s}}{P_t} = \begin{cases} 1 & i = 0 \\ \Pi_{k=1}^s \pi_{t+k} & i = 1, ..., \infty, \end{cases} \\ \Theta^*_{jt,t+s} &\equiv -\frac{P^*_{jt}}{y^*_{jt,t+s}} \frac{\partial y^*_{jt,t+s}}{\partial P^*_{jt}} = \gamma, \end{aligned}$$

and multiplying through by P_{jt}^* yield

$$E_t \sum_{s=0}^{\infty} \lambda_p^s \Lambda_{t,t+s} \left[\left(1 - \Theta_{jt,t+s}^*\right) \frac{P_{jt}^*}{P_{t+s}} X_{t,t+s} + \Theta_{jt,t+s}^* p_{t+s}^m \right] y_{jt,t+s}^* y_{t+s} = 0.$$

Noting that all retailers have the same solution to the optimization problem, we can rewrite the first-order condition as

$$\frac{P_t^*}{P_t} = \frac{E_t \sum_{i=0}^{\infty} \lambda_p^i \Lambda_{t,t+i} \Theta_{t,t+i}^* p_{t+i}^m y_{t+i} y_{t,t+i}^*}{E_t \sum_{i=0}^{\infty} \lambda_p^i \Lambda_{t,t+i} (\Theta_{t,t+i}^* - 1) \frac{X_{t,t+i}}{X_{t,t+i}^p} y_{t+i} y_{t,t+i}^*} \implies$$

$$p_t^* = \frac{E_t \sum_{i=0}^{\infty} \lambda_p^s \Lambda_{t,t+s} \gamma p_{t+s}^m y_{t+s} \left(\frac{X_{t,t+s} P_t^*}{P_{t+s}}\right)^{-\gamma}}{E_t \sum_{i=0}^{\infty} \lambda_p^s \Lambda_{t,t+s} (\gamma - 1) \frac{X_{t,t+s}}{X_{t,t+s}^p} y_{t+s} \left(\frac{X_{t,t+s} P_t^*}{P_{t+s}}\right)^{-\gamma}} = \frac{\gamma}{\gamma - 1} \frac{E_t \sum_{s=0}^{\infty} \lambda_p^i \Lambda_{t,t+s} p_{t+s}^m y_{t+i} \left(\frac{X_{t,t+s}}{X_{t,t+s}^P}\right)^{-\gamma}}{E_t \sum_{s=0}^{\infty} \lambda_p^s \Lambda_{t,t+s} \frac{X_{t,t+s}}{X_{t,t+s}^p} y_{t+i} \left(\frac{X_{t,t+s}}{X_{t,t+s}^P}\right)^{-\gamma}}.$$

Then define the equations

$$\begin{aligned}
p_t^* &= \frac{\gamma}{\gamma - 1} \frac{p_t^N}{p_t^D}, \\
p_t^N &= E_t \sum_{s=0}^{\infty} \lambda_p^s \Lambda_{t,t+s} p_{t+i}^m y_{t+s} \left(\frac{X_{t,t+s}}{X_{t,t+s}^P}\right)^{-\gamma}, \\
p_t^D &= E_t \sum_{s=0}^{\infty} \lambda_p^s \Lambda_{t,t+s} \frac{X_{t,t+s}}{X_{t,t+s}^p} y_{t+s} \left(\frac{X_{t,t+s}}{X_{t,t+s}^P}\right)^{-\gamma},
\end{aligned}$$

and write p_t^N and p_t^D recursively as

$$\begin{aligned}
p_t^N &= p_t^m y_t + E_t \lambda_p \Lambda_{t,t+1} \left(\frac{\pi_t^{\gamma_p} \pi^{1-\gamma_p}}{\pi_{t+1}}\right)^{-\gamma} p_{t+1}^N, \\
p_t^D &= y_t + E_t \lambda_p \Lambda_{t,t+1} \left(\frac{\pi_t^{\gamma_p} \pi^{1-\gamma_p}}{\pi_{t+1}}\right)^{1-\gamma} p_{t+1}^D.
\end{aligned}$$

Now note that, due to the law of large numbers, the aggregate price index is

$$\begin{aligned}
P_t &= \left((1 - \lambda_p) P_t^{*1-\gamma} + \lambda_p (\pi_{t-1}^{\gamma_p} \pi^{1-\gamma_p} P_{t-1})^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \implies \\
1 &= (1 - \lambda_p) {p_t^*}^{1-\gamma} + \lambda_p \left(\frac{\pi_{t-1}^{\gamma_p} \pi^{1-\gamma_p}}{\pi_t}\right)^{1-\gamma}.
\end{aligned}$$

Retailer j earns per-period real profits

$$\left(\frac{P_{jt}}{P_t} - p_t^m\right) y_{jt} = \left(\frac{P_{jt}}{P_t} - p_t^m\right) \left(\frac{P_{jt}}{P_t}\right)^{-\gamma} y_t = \left(\frac{P_{jt}}{P_t}\right)^{1-\gamma} y_t - \left(\frac{P_{jt}}{P_t}\right)^{-\gamma} p_t^m y_t.$$

Now, integrating across all retailers yields

$$\Pi_t^{ret} = y_t - p_t^m y_t \Delta_t.$$

where $\Delta_t = \int_0^1 \left(\frac{P_{jt}}{P_t}\right)^{-\gamma} dj$ measures price dispersion and we recall that

$$\int_0^1 \left(\frac{P_{jt}}{P_t}\right)^{1-\gamma} dj = 1.$$

Aggregate profits are given by

$$\Pi_t = \Pi_t^{int} + \Pi_t^{ret}.$$

C.5 Wage Bargaining Details

Combining the equations for H_{it} and U_{it} and splitting the continuation value of U_{it} between the λ^x and $(1-\lambda^x)$ terms in $H_{it}(z)$ yield

$$\begin{aligned}
V_{it}(z) = w_{it}(z) - h + (1-\lambda^x)E_t\Lambda_{t,t+1}\left[\int_{z_{t+1}^{Ri}} H_{it+1}(y)g(y)dy + G(z_{t+1}^{Ri})U_{it+1}\right] \\
- (1-\lambda^x)E_t\Lambda_{t,t+1}\left[p_{t+1}\left(\int_{z_{t+1}^{Ri}} H_{it+1}(y)g(y)dz + G(z_{t+1}^{Ri})U_{it+1}\right) + (1-p_{t+1})U_{it+1}\right] \\
= w_{it}(z) - h + (1-\lambda^x)E_t\Lambda_{t,t+1}\left[(1-p_{t+1})\left(\int_{z_{t+1}^{Ri}} H_{it+1}(y)g(y)dy + G(z_{t+1}^{Ri})U_{it+1}\right) - (1-p_{t+1})U_{it+1}\right] \\
= w_{it}(z) - h + (1-\lambda^x)E_t\Lambda_{t,t+1}\int_{z_{t+1}^{Ri}} V_{it+1}(y)g(y)dy - (1-\lambda^x)E_t\Lambda_{t,t+1}p_{t+1}\int_{z_{t+1}^{Ri}} V_{it+1}(y)g(y)dy.
\end{aligned}$$

Using the first-order condition from the Nash bargaining problem, we can rewrite

$$\int_{z_{t+1}^{Ri}} V_{it+1}(y)g(y)dy = \frac{1-\zeta}{\zeta}\int_{z_{t+1}^{Ri}} J_{it+1}(z)g(y)dy.$$

Next, we integrate $J_{it}(z)$ against $g(z)$ to get

$$\int_{z_t^{Ri}} J_{it}(z)g(z)dz = \int_{z_t^{Ri}} (p_t^m A_t z - w_{it}(z)g(z)dz - (1-G(z_t^{Ri}))\kappa_i + (1-G(z_t^{Ri}))(1-\lambda^x)E_t\Lambda_{t,t+1}\int_{z_{t+1}^{Ri}} J_{it+1}(y)g(y)dy$$

$$\implies \int_{z_t^{Ri}} \frac{J_{it}(z)g(z)}{1-G(z_t^{Ri})}dz = \int_{z_t^{Ri}} \frac{(p_t^m A_t z - w_{it}(z)g(z)}{1-G(z_t^{Ri})}dz - \kappa_i + (1-\lambda^x)E_t\Lambda_{t,t+1}(1-G(z_{t+1}^{Ri}))\int_{z_{t+1}^{Ri}} \frac{J_{it+1}(y)g(y)}{1-G(z_{t+1}^{Ri})}dy.$$

Thus, from the first-order condition of the firm's problem with respect to n_{it}, we can see that the average value across all employed workers of type i to the firm, $\int_{z_t^{Ri}} \frac{J_{it}(z)g_i(z)}{1-G_i(z_t^{Ri})}dz$, evolves according to the same equation as ϕ_{it}. If we employ this substitution into the equation for J, we see that

$$J_{it}(z) = p_t^m A_t z - w_{it}(z) - \kappa_i + (1-\lambda^x)E_t\Lambda_{t,t+1}\phi_{it+1}(1-G(z_{t+1}^{Ri})),$$

which, by the job destruction condition, we can see is equal to 0 when $z = z_t^{Ri}$. In other words, the surplus to the firm (and thus to the worker via the surplus sharing equation) from a worker of type i with the reservation productivity is exactly 0. Now we can write

$$V_{it}(z) = w_{it}(z) - h - (1-\lambda^x)E_t\Lambda_{t,t+1}p_{t+1}\frac{1-\zeta}{\zeta}(1-G(z_{t+1}^{Ri}))\phi_{it+1} + (1-\lambda^x)E_t\Lambda_{t,t+1}\frac{1-\zeta}{\zeta}(1-G(z_{t+1}^{Ri}))\phi_{it+1}.$$

Plugging this value into the first-order condition of the Nash bargaining problem along with the equation for $J_t(z)$ results in

$$(1-\zeta)A_t p_t^m z - (1-\zeta)w_{it}(z) - (1-\zeta)\kappa_i + (1-\zeta)(1-\lambda^x)E_t\Lambda_{t,t+1}\phi_{it+1}(1-G(z_t^{Ri})) =$$
$$\zeta w_t(z) - \zeta h - (1-\zeta)(1-\lambda^x)E_t\Lambda_{t,t+1}p_{t+1}(1-G(z_{t+1}^{Ri}))\phi_{it+1} + (1-\zeta)(1-\lambda^x)E_t\Lambda_{t,t+1}(1-G(z_{t+1}^{Ri}))\phi_{it+1}.$$

Rearranging, we obtain

$$w_{it}(z) = (1-\zeta)A_t p_t^m z + \zeta h - (1-\zeta)\kappa_i + (1-\zeta)(1-\lambda^x)E_t\Lambda_{t,t+1}p_{t+1}(1-G(z_{t+1}^{Ri}))\phi_{it+1},$$
$$w_{it}(z) = (1-\zeta)(A_t p_t^m z - \kappa_i + (1-\lambda^x)E_t\Lambda_{t,t+1}p_{t+1}(1-G(z_{t+1}^{Ri}))\phi_{it+1}) + \zeta h.$$

C.6 Equilibrium

Using the wage equation to substitute for $w_{it}(z) - w_{it}(z_t^{Ri})$ in

$$(1-G(z_t^{Ri}))\phi_{it} = \int_{z_t^{Ri}} p_t^m A_t(z - z_t^{Ri}) - (w_t(z) - w_t(z_t^{Ri}))g(z)dz$$

yields

$$\phi_{1t} = \frac{1}{1-G(z_t^{R1})}\zeta p_t^m A_t \int_{z_t^{R1}} (z - z_t^{R1})g(z)dz = \zeta p_t^m A_t(\bar{z}_{1t} - z_t^{R1}),$$
$$\phi_{2t} = \frac{1}{1-G(z_t^{R2})}\zeta p_t^m A_t \int_{z_t^{R2}} (z - z_t^{R2})g(z)dz = \zeta p_t^m A_t(\bar{z}_{2t} - z_t^{R2}).$$

Then, evaluating the wage equation at z_t^{Ri} and substituting into

$$p_t^m A_t z_t^{Ri} = w_t(z_t^{Ri}) + \kappa_i - (1-\lambda^x)E_t\Lambda_{t,t+1}(1-G(z_{t+1}^{Ri}))\phi_{it+1}$$

result in

$$\zeta p_t^m A_t z_t^{R1} = (1-\lambda^x)E_t\Lambda_{t,t+1}\left[(1-\zeta)p_{t+1} - 1\right](1-G(z_{t+1}^{R1}))\phi_{1t+1} + \zeta(h+\kappa_1),$$
$$\zeta p_t^m A_t z_t^{R2} = (1-\lambda^x)E_t\Lambda_{t,t+1}\left[(1-\zeta)p_{t+1} - 1\right](1-G(z_{t+1}^{R2}))\phi_{2t+1} + \zeta(h+\kappa_2).$$

The aggregate resource constraint given pecuniary hiring costs is

$$y_t = \delta c_{1t} + (1-\delta)c_{2t} + \chi v_t.$$

Given $y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\gamma} y_t$, we also have that

$$A_t\left(n_{1t}\bar{z}_{1t} + n_{2t}\bar{z}_{2t}\right) = x_t = \int_0^1 y_{jt}dj = y_t\Delta_t,$$

where

$$\Delta_t = \int_0^1 \left(\frac{P_{jt}}{P_t}\right)^{-\gamma} dj = (1-\lambda_p)p_t^{*-\gamma} + \lambda_p\left(\frac{\pi_{t-1}^{\gamma_p}\pi^{1-\gamma_p}}{\pi_t}\right)^{-\gamma}\Delta_{t-1}.$$

Finally,

$$G_{1t} \equiv G(z_t^{R1}) = logncdf(z_t^{R1}, \mu_z, \sigma_z),$$
$$G_{2t} \equiv G(z_t^{R2}) = logncdf(z_t^{R2}, \mu_z, \sigma_z)$$

show that worker productivity is governed by a log-normal distribution.

C.7 Welfare

Given an equilibrium consumption stream c_{it}, welfare from the perspective of period t for a household of type i is given by

$$\mathcal{W}_{it} \equiv E_t \sum_{s=0}^{\infty} \beta^s \log(c_{it+s}) = \log(c_{it}) + E_t E_{t+1} \sum_{s=0} \beta^{s+1} \log(c_{it+s+1}) = \log(c_t) + \beta E_t E_{t+1} \sum_{s=0} \beta^s c_{it+1+s},$$

which we can rewrite recursively as

$$\mathcal{W}_{it} = \log(c_{it}) + \beta E_t \mathcal{W}_{it+1}.$$

We define the consumption-equivalent welfare wedge between type 1 and 2 households as the value Ψ_t such that

$$E_t \sum_{s=0}^{\infty} \beta^s \log((1+\Psi_t)c_{1t+s}) = E_t \sum_{s=0}^{\infty} \beta^s \log(c_{2t+s}),$$

which implies that

$$\Psi_t = e^{(1-\beta)(\mathcal{W}_{2t} - \mathcal{W}_{1t})} - 1.$$

A positive value of Ψ_t implies that type 2 households have higher welfare than type 1 households.

Now, for interpretability, we want to express welfare in consumption-equivalent terms. To do so, we will compute the fraction ψ_{it} that we must raise consumption by each period in order to reach steady-state welfare. Concretely, we solve

$$E_t \sum_{s=0}^{\infty} \beta^s \log((1+\psi_{it})c_{it+s}) = \mathcal{W}_i$$

for ψ_{it}. Note that

$$E_t \sum_{s=0}^{\infty} \beta^s \log((1+\psi)c_{it+s}) = \mathcal{W}_{it} + \sum_{s=0}^{\infty} \beta^s \log(1+\psi_{it}) = \mathcal{W}_{it} + \frac{\log(1+\psi_{it})}{1-\beta}.$$

So we solve

$$\mathcal{W}_{it} + \frac{\log(1+\psi_{it})}{1-\beta} = \mathcal{W}_i,$$

which implies

$$\psi_{it} = e^{(1-\beta)(\mathcal{W}_i - \mathcal{W}_{it})} - 1.$$

A positive value for ψ_{it} represents a welfare loss with respect to steady state. Now, treating utility as cardinal (which is often done in these types of models), we can define aggregate welfare as

$$\mathcal{W}_t = \delta \mathcal{W}_{1t} + (1-\delta)\mathcal{W}_{2t}.$$

We can define consumption-equivalent aggregate welfare in the same way to get

$$\psi_t = e^{(1-\beta)(\mathcal{W} - \mathcal{W}_t)} - 1.$$

C.8 Definition of Equilibrium and Full Set of Model Equations

Equilibrium in our baseline economy is defined as the path of

$$\{p_t^m, c_{1t}, c_{2t}, \Lambda_{t,t+1}, p_t^*, \pi_t, n_{1t}, n_{2t}, n_t, s_{1t}, s_{2t}, s_t, v_t, p_t, q_t, q_{1t}, q_{2t}, z_t^{R1}, z_t^{R2}, \theta_t, G_{1t}, G_{2t}, T_t,$$

$$U_t, U_{1t}, U_{2t}, \phi_{1t}, \phi_{2t}, i_t, y_t, \Delta_t, \Pi_t^{int}, \Pi_t^{ret}, \Pi_t, \bar{z}_{1t}, \bar{z}_{2t}, \mathcal{P}_t^N, \mathcal{P}_t^D, r_t, \bar{z}_t, \bar{w}_{1t}, \bar{w}_{2t}, \bar{w}_t, \lambda_{1t}, \lambda_{2t}, \lambda_t,$$

$$f_{1t}, f_{2t}, f_t, \lambda_{1t}^m, \lambda_{2t}^m, f_{1t}^m, f_{2t}^m, \lambda_t^m, f_t^m, lp_t, ugap_t, \mathcal{D}_t^f, \mathcal{D}_t^\lambda, \mathcal{D}_t, \lambda_{xt}, \mathcal{W}_{1t}, \mathcal{W}_{2t}, \psi_{1t}, \psi_{2t}, \Psi_t, \mathcal{W}_t, \psi_t\}$$

that satisfies equations (C.1) to (C.68) for all $t \geq 0$, given the evolution of the exogenous shocks $\{\epsilon_{A,t}, \epsilon_{\xi,t}\}_{t=0}^{\infty}$, the laws of motion for $\{A_t, \xi_t\}$ given by equations (C.69) and (C.70), and the initial values of the endogenous state variables $\{n_{1s}, n_{2s}, i_s, \Delta_s, \pi_s\}$ for $s = -1$.

Labor market tightness:

$$\theta_t = \frac{v_t}{s_t} \tag{C.1}$$

Job-meeting probability:

$$p_t = \varsigma \theta_t^{\varepsilon} \tag{C.2}$$

Vacancy-filling probability:

$$q_t = \varsigma \theta_t^{\varepsilon - 1} \tag{C.3}$$

Job searchers:

$$s_t = s_{1t} + s_{2t} \tag{C.4}$$

Type 1 job searchers:

$$s_{1t} = \delta - n_{1t} + \lambda^x n_{1t-1} \tag{C.5}$$

Type 2 job searchers:

$$s_{2t} = (1 - \delta) - n_{2t} + \lambda^x n_{2t-1} \tag{C.6}$$

Employment:

$$n_t = n_{1t} + n_{2t} \tag{C.7}$$

Evolution of type 1 employment:

$$n_{1t} = [1 - G_{1t}][(1 - \lambda^x) n_{1t-1} + q_{1t} v_t] \tag{C.8}$$

Evolution of type 2 employment:

$$n_{2t} = [1 - G_{2t}][(1 - \lambda^x) n_{2t-1} + q_{2t} v_t] \tag{C.9}$$

Type 1 vacancy-meeting probability:

$$q_{1t} = \frac{s_{1t}}{s_t} q_t \tag{C.10}$$

Type 2 vacancy-meeting probability:

$$q_{2t} = \frac{s_{2t}}{s_t} q_t \tag{C.11}$$

Unemployment rate:

$$U_t = 1 - n_t \tag{C.12}$$

Type 1 unemployment rate:

$$U_{1t} = \frac{\delta - n_{1t}}{\delta} \tag{C.13}$$

Type 2 unemployment rate:

$$U_{2t}\frac{(1-\delta)-n_{2t}}{1-\delta} \tag{C.14}$$

Unemployment rate gap:

$$ugap_t = U_{1t} - U_{2t} \tag{C.15}$$

Separation rate:

$$\lambda_t = \frac{\lambda_{1t}n_{1t-1} + \lambda_{2t}n_{2t-1}}{n_{t-1}} \tag{C.16}$$

Job-finding rate:

$$f_t = \frac{f_{1t-1}(\delta - n_{1t-1}) + f_{2t-1}(1-\delta-n_{2t-1})}{1-n_{t-1}} \tag{C.17}$$

Type 1 separation rate:

$$\lambda_{1t} = \lambda^x(1-p_t) + [(1-\lambda^x) + p_t\lambda^x]G_{1t} \tag{C.18}$$

Type 2 separation rate:

$$\lambda_{2t} = \lambda^x(1-p_t) + [(1-\lambda^x) + p_t\lambda^x]G_{2t} \tag{C.19}$$

Type 1 job-finding rate:

$$f_{1t} = p_t[1-G_{1t}] \tag{C.20}$$

Type 2 job-finding rate:

$$f_{2t} = p_t[1-G_{2t}] \tag{C.21}$$

Implied monthly separation rate:

$$\lambda_t = \lambda_t^m(1-f_t^m)^2 + (1-\lambda_t^m)\lambda_t^m(1-f_t^m) + (1-\lambda_t^m)^2\lambda_t^m + (\lambda_t^m)^2 f_t^m \tag{C.22}$$

Implied monthly job-finding rate:

$$f_t = f_t^m(1-\lambda_t^m)^2 + (1-f_t^m)f_t^m(1-\lambda_t^m) + (1-f_t^m)^2 f_t^m + (f_t^m)^2\lambda_t^m \tag{C.23}$$

Implied type 1 monthly separation rate:

$$\lambda_{1t} = \lambda_{1t}^m(1-f_{1t}^m)^2 + (1-\lambda_{1t}^m)\lambda_{1t}^m(1-f_{1t}^m) + (1-\lambda_{1t}^m)^2\lambda_{1t}^m + (\lambda_{1t}^m)^2 f_{1t}^m \tag{C.24}$$

Implied type 1 monthly job-finding rate:

$$f_{1t} = f_{1t}^m(1-\lambda_{1t}^m)^2 + (1-f_{1t}^m)f_{1t}^m(1-\lambda_{1t}^m) + (1-f_{1t}^m)^2 f_{1t}^m + (f_{1t}^m)^2\lambda_{1t}^m \tag{C.25}$$

Implied type 2 monthly separation rate:

$$\lambda_{2t} = \lambda_{2t}^m(1-f_{2t}^m)^2 + (1-\lambda_{2t}^m)\lambda_{2t}^m(1-f_{2t}^m) + (1-\lambda_{2t}^m)^2\lambda_{2t}^m + (\lambda_{2t}^m)^2 f_{2t}^m \tag{C.26}$$

Implied type 2 monthly job-finding rate:

$$f_{2t} = f_{2t}^m(1-\lambda_{2t}^m)^2 + (1-f_{2t}^m)f_{2t}^m(1-\lambda_{2t}^m) + (1-f_{2t}^m)^2 f_{2t}^m + (f_{2t}^m)^2\lambda_{2t}^m \tag{C.27}$$

Hiring margin discrimination:

$$\mathcal{D}_t^f = \frac{\delta - n_{1t-1}}{\delta}p_t[G_{1t} - G_{2t}] \tag{C.28}$$

Firing margin discrimination:

$$\mathcal{D}_t^{\lambda} = \frac{n_{1t-1}}{\delta}[(1-\lambda^x) + p_t\lambda^x][G_{1t} - G_{2t}] \tag{C.29}$$

Aggregate discrimination:

$$\mathcal{D}_t = \mathcal{D}_t^f + \mathcal{D}_t^{\lambda} \tag{C.30}$$

Job creation condition:

$$\chi = q_{1t}(1-G_{1t})\phi_{1t} + q_{2t}(1-G_{2t})\phi_{2t} \tag{C.31}$$

Type 1 job separation condition:

$$\zeta p_t^m A_t z_t^{R1} = (1-\lambda^x)E_t\Lambda_{t,t+1}[(1-\zeta)p_{t+1} - 1](1-G_{1t+1})\phi_{1t+1} + \zeta(h+\kappa_1) \tag{C.32}$$

Type 2 job separation condition:

$$\zeta p_t^m A_t z_t^{R2} = (1-\lambda^x)E_t\Lambda_{t,t+1}[(1-\zeta)p_{t+1} - 1](1-G_{2t+1})\phi_{2t+1} + \zeta(h+\kappa_2) \tag{C.33}$$

Type 1 value of employment:

$$\phi_{1t} = \zeta p_t^m A_t(\bar{z}_{1t} - z_t^{R1}) \tag{C.34}$$

Type 2 value of employment:

$$\phi_{2t} = \zeta p_t^m A_t(\bar{z}_{2t} - z_t^{R2}) \tag{C.35}$$

Average type 1 employment:

$$\bar{z}_{1t} = \int_{z_t^{R1}} \frac{z}{1-G_{1t}} g(z)dz \tag{C.36}$$

Average type 2 employment:

$$\bar{z}_{2t} = \int_{z_t^{R2}} \frac{z}{1-G_{2t}} g(z)dz \tag{C.37}$$

Average type 1 wage:

$$\bar{w}_{1t} = (1-\zeta)(A_t p_t^m \bar{z}_{1t} - \kappa_1(1-\lambda^x)E_t\Lambda_{t,t+1}p_{t+1}(1-G(z_{t+1}^{R1}))\phi_{1t+1}) + \zeta h \tag{C.38}$$

Average type 2 wage:

$$\bar{w}_{2t} = (1-\zeta)(A_t p_t^m \bar{z}_{2t} - \kappa_2(1-\lambda^x)E_t\Lambda_{t,t+1}p_{t+1}(1-G(z_{t+1}^{R2}))\phi_{2t+1}) + \zeta h \tag{C.39}$$

Aggregate average wage:

$$\bar{w}_t = \frac{\bar{w}_{1t}n_{1t} + \bar{w}_{2t}n_{2t}}{n_t} \tag{C.40}$$

Stochastic discount factor:

$$\Lambda_{t,t+1} = \beta\frac{c_{1t}}{c_{1t+1}} \tag{C.41}$$

Euler equation:

$$1 = \beta\xi_t(1+i_t)E_t\frac{1}{\pi_{t+1}}\Lambda_{t,t+1} \tag{C.42}$$

Nonlinear Phillips curve equation 1:

$$p_t^* = \frac{\gamma}{\gamma-1}\frac{\mathcal{P}_t^N}{\mathcal{P}_t^D} \tag{C.43}$$

Nonlinear Phillips curve equation 2:

$$\mathcal{P}_t^N = p_t^m y_t + E_t \lambda_p \Lambda_{t,t+1} \left(\frac{\pi_t^{\gamma_p} \pi^{1-\gamma_p}}{\pi_{t+1}} \right)^{-\gamma} \mathcal{P}_{t+1}^N \tag{C.44}$$

Nonlinear Phillips curve equation 3:

$$\mathcal{P}_t^D = y_t + E_t \lambda_p \Lambda_{t,t+1} \left(\frac{\pi_t^{\gamma_p} \pi^{1-\gamma_p}}{\pi_{t+1}} \right)^{1-\gamma} \mathcal{P}_{t+1}^D \tag{C.45}$$

Nonlinear Phillips curve equation 4:

$$1 = (1-\lambda_p) {p_t^*}^{1-\gamma} + \lambda_p \left(\frac{\pi_{t-1}^{\gamma_p} \pi^{1-\gamma_p}}{\pi_t} \right)^{1-\gamma} \tag{C.46}$$

Deviations interest rate rule:

$$i_t = \max\{0, \phi_i i_{t-1} + (1-\phi_i)[i + \phi_\pi(\log(\pi_t) - \log(\pi)) + \phi_u(U_t - U)]\} \tag{C.47}$$

Aggregate resource constraint:

$$y_t = \delta c_{1t} + (1-\delta) c_{2t} + \chi v_t \tag{C.48}$$

Output:

$$A_t(n_{1t}\bar{z}_{1t} + n_{2t}\bar{z}_{2t}) = y_t \Delta_t \tag{C.49}$$

Price dispersion:

$$\Delta_t = (1-\lambda_p) p_t^{*-\gamma} + \lambda_p \left(\frac{\pi_{t-1}^{\gamma_p} \pi^{1-\gamma_p}}{\pi_t} \right)^{-\gamma} \Delta_{t-1} \tag{C.50}$$

Intermediate goods producer profits:

$$\Pi_t^{int} = p_t^m A_t(n_{1t}\bar{z}_{1t} + n_{2t}\bar{z}_{2t}) - \bar{w}_{1t} n_{1t} - \bar{w}_{2t} n_{2t} - \chi v_t \tag{C.51}$$

Retailer profits:

$$\Pi_t^{ret} = y_t \tilde{\Delta}_t - p_t^m y_t \Delta_t \tag{C.52}$$

Aggregate profits:

$$\Pi_t = \Pi_t^{int} + \Pi_t^{ret} \tag{C.53}$$

Type 1 budget constraint:

$$c_{1t} = (1 - U_{1t}) \bar{w}_{1t} + U_{1t} h + \Pi_t - T_t \tag{C.54}$$

Taxes:

$$T_t = (1 - n_t) h \tag{C.55}$$

Productivity c.d.f. at type 1 reservation productivity threshold:

$$G_{1t} = logncdf(z_t^{R1}, \mu_z, \sigma_z) \tag{C.56}$$

Productivity c.d.f. at type 2 reservation productivity threshold:

$$G_{2t} = logncdf(z_t^{R2}, \mu_z, \sigma_z) \tag{C.57}$$

Ex ante net real interest rate:

$$r_t = \frac{1+i_t}{E_t\pi_{t+1}} - 1 \tag{C.58}$$

Average idiosyncratic productivity:

$$\bar{z}_t = \frac{n_{1t}}{n_t}\bar{z}_{1t} + \frac{n_{2t}}{n_t}\bar{z}_{2t} \tag{C.59}$$

Labor productivity:

$$lp_t = \frac{y_t}{n_t} \tag{C.60}$$

Exogenous separation rate:

$$\lambda_{xt} = (1-p_t)\lambda^x \tag{C.61}$$

Type 1 welfare:

$$\mathcal{W}_{1t} = \log(c_{1t}) + \beta E_t \mathcal{W}_{1t+1} \tag{C.62}$$

Type 2 welfare:

$$\mathcal{W}_{2t} = \log(c_{2t}) + \beta E_t \mathcal{W}_{2t+1} \tag{C.63}$$

Type 1 consumption-equivalent welfare:

$$\psi_{1t} = e^{(1-\beta)(\mathcal{W}_1 - \mathcal{W}_{1t})} - 1 \tag{C.64}$$

Type 2 consumption-equivalent welfare:

$$\psi_{2t} = e^{(1-\beta)(\mathcal{W}_2 - \mathcal{W}_{2t})} - 1 \tag{C.65}$$

Welfare wedge:

$$\Psi_t = e^{(1-\beta)(\mathcal{W}_{2t} - \mathcal{W}_{1t})} - 1 \tag{C.66}$$

Aggregate welfare:

$$\mathcal{W}_t = \delta\mathcal{W}_{1t} + (1-\delta)\mathcal{W}_{2t}. \tag{C.67}$$

Aggregate consumption-equivalent welfare:

$$\psi_t = e^{(1-\beta)(\mathcal{W} - \mathcal{W}_t)} - 1 \tag{C.68}$$

Aggregate productivity:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_A \epsilon_{A,t} \tag{C.69}$$

Risk premium:

$$\log \xi_t = \rho_\xi \log \xi_{t-1} + \sigma_\xi \epsilon_{\xi,t} \tag{C.70}$$

C.9 Calibration and Steady State

The exogenous parameters in the model are

$$\delta, \beta, \varsigma, \lambda^x, \mu_{1z}, \sigma_{1z}, \mu_{2z}, \sigma_{2z}, \varepsilon, \zeta, \gamma, h, \chi, \lambda_p, \kappa_1, \kappa_2, \sigma_g, \sigma_m, \sigma_A, \rho_g, \rho_A, \phi_\pi, \phi_u, \phi_i.$$

The one targeted steady-state value is π.
From the risk premium equation,

$$\xi = 1.$$

From the aggregate productivity equation,

$$A = 1.$$

From nonlinear Phillips curve equation 4,

$$p^* = 1.$$

From nonlinear Phillips curve equations 1–3,

$$p^N = \frac{p^m y}{1 - \lambda_p \beta},$$

$$p^D = \frac{y}{1 - \lambda_p \beta},$$

$$p^* = \frac{\gamma}{\gamma - 1} p^m \implies p^m = \frac{\gamma - 1}{\gamma} p^*.$$

From the price dispersion equation,

$$\Delta = {p^*}^{-\gamma}.$$

From the stochastic discount factor equation,

$$\Lambda = \beta.$$

From the Euler equation,

$$i = \frac{\pi}{\beta} - 1.$$

Now observe the following system of 19 equations in the 19 variables

$$s_1, s_2, s, v, p, q, \theta, q_1, q_2, n_1, n_2, \phi_1, \phi_2, \bar{z}_1, \bar{z}_2, z^{R1}, z^{R2}, G_1, G_2.$$

Type 1 searchers:

$$s_1 = \delta + (\lambda^x - 1) n_1$$

Type 2 searchers:

$$s_2 = 1 - \delta + (\lambda^x - 1) n_2$$

Aggregate searchers:

$$s = s_1 + s_1$$

Job-meeting probability :

$$p = \varsigma \theta^{\varepsilon}$$

Vacancy-meeting probability:

$$q = \varsigma \theta^{\varepsilon - 1}$$

Labor market tightness:

$$\theta = \frac{v}{u}$$

Type 1 vacancy-filling probability:

$$q_1 = \frac{s_1}{s} q$$

Type 2 vacancy-filling probability:

$$q_2 = \frac{s_2}{s} q$$

Type 1 labor force evolution:

$$n_1 = (1 - G_1)[(1 - \lambda^x)n_1 + q_1 v]$$

Type 2 labor force evolution:

$$n_2 = (1 - G_2)[(1 - \lambda^x)n_2 + q_2 v]$$

Hiring condition:

$$\chi = q_1(1 - G_1)\phi_1 + q_2(1 - G_2)\phi_2$$

Type 1 value of employment:

$$\phi_1 = \zeta p^m A(\bar{z}_1 - z^{R1})$$

Type 2 value of employment:

$$\phi_2 = \zeta p^m A(\bar{z}_2 - z^{R2})$$

Type 1 average productivity:

$$\bar{z}_1 = \int_{z^{R1}} \frac{zg(z)dz}{1 - G_1}$$

Type 2 average productivity:

$$\bar{z}_2 = \int_{z^{R2}} \frac{zg(z)dz}{1 - G_2}$$

Type 1 job separation condition:

$$\zeta p^m A z^{R1} = (1 - \lambda^x)\beta[(1 - \zeta)p - 1](1 - G_1)\phi_1 + \zeta(h + \kappa_1)$$

Type 2 job separation condition:

$$\zeta p^m A z^{R2} = (1 - \lambda^x)\beta[(1 - \zeta)p - 1](1 - G_2)\phi_2 + \zeta(h + \kappa_2)$$

Productivity c.d.f. at type 1 reservation productivity threshold:

$$G_1 = logncdf(z^{R1}, \mu_z, \sigma_z)$$

Productivity c.d.f. at type 2 reservation productivity threshold:

$$G_2 = logncdf(z^{R2}, \mu_z, \sigma_z)$$

We can then collapse this system into a system of 5 equations in the 5 variables

$$n_1, n_2, \theta, G_1, G_2.$$

$$n_1 = \frac{\delta(1 - G_1)p(\theta)}{1 - (1 - G_1)(1 - \lambda^x)(1 - p(\theta))},$$

$$n_2 = \frac{1 - \delta(1 - G_2)p(\theta)}{1 - (1 - G_2)(1 - \lambda^x)(1 - p(\theta))},$$

$$\zeta p^m A \cdot logninv(G_1) = (1-\lambda^x)\beta[(1-\zeta)p(\theta)-1](1-G_1)\zeta p^m A\left(\int_{logninv(G_1)} \frac{zg(z)dz}{1 - G_1} - logninv(G_1)\right) + \zeta(h+\kappa_1),$$

$$\zeta p^m A \cdot loginv(G_2) = (1-\lambda^x)\beta[(1-\zeta)p(\theta)-1](1-G_2)\zeta p^m A\left(\int_{logninv(G_2)} \frac{zg(z)dz}{1 - G_2} - logninv(G_2)\right) + \zeta(h+\kappa_2),$$

$$\chi = \frac{\delta + (\lambda^x - 1)n_1}{1 + (\lambda^x - 1)n_1 + (\lambda^x - 1)n_2} q(\theta)(1 - G_1)\zeta p^m A \left(\int_{logninv(G_1)} \frac{zg(z)dz}{1 - G_1} - logninv(G_1) \right),$$

$$+ \frac{1 - \delta + (\lambda^x - 1)n_2}{1 + (\lambda^x - 1)n_1 + (\lambda^x - 1)n_2} q(\theta)(1 - G_1)\zeta p^m A \left(\int_{logninv(G_2)} \frac{zg(z)dz}{1 - G_2} - logninv(G_2) \right),$$

which we solve with a nonlinear solver. We can then substitute appropriately into the other 14 equations to get the steady states of the other 14 variables.
From the type 1 unemployment equation,

$$U_1 = \frac{\delta - n_1}{\delta}.$$

From type 2 unemployment,

$$U_2 = \frac{(1 - \delta) - n_2}{1 - \delta}.$$

From the aggregate labor force equation,

$$n = n_1 + n_2.$$

From the aggregate unemployment equation,

$$U = 1 - n.$$

From the output equation,

$$y = \frac{A(n_1 \bar{z}_1 + n_2 \bar{z}_2)}{\Delta}.$$

From Phillips curve equations 2 and 3,

$$p^N = \frac{p^m y}{1 - \lambda_p \beta},$$
$$p^D = \frac{y}{1 - \lambda_p \beta}.$$

From the unemployment rate gap equation,

$$ugap = U_1 - U_2.$$

From the hiring margin discrimination equation,

$$\mathcal{D}^f = \frac{\delta - n_1}{\delta} p[G_1 - G_2].$$

From the firing margin discrimination equation,

$$\mathcal{D}^\lambda = \frac{n_1}{\delta}[(1 - \lambda^x) + p\lambda^x][G_1 - G_2].$$

From the aggregate discrimination equation,

$$\mathcal{D} = \mathcal{D}^f + \mathcal{D}^\lambda.$$

From the type 1 separation rate equation,

$$\lambda_1 = (1 - p)\lambda^x + G_1[(1 - \lambda^x) + p\lambda^x].$$

From the type 2 separation rate equation,

$$\lambda_2 = (1-p)\lambda^x + G_2[(1-\lambda^x) + p\lambda^x].$$

From the separation rate equation,

$$\lambda = \lambda_1 \frac{n_1}{n} + \lambda_2 \frac{n_2}{n}.$$

From the exogenous separation rate equation,

$$\lambda_x = (1-p)\lambda^x.$$

From the type 1 job-finding rate equation,

$$f_1 = (1-G_1)p.$$

From the type 2 job-finding rate equation,

$$f_2 = (1-G_2)p.$$

From the aggregate job-finding rate equation,

$$f = f_1 \frac{\delta - n_1}{1-n} + f_2 \frac{1-\delta-n_{2-1}}{1-n}.$$

From the labor productivity equation,

$$lp = \frac{y}{n}.$$

From the real interest rate equation,

$$r = \frac{1+i}{\pi} - 1.$$

From the average productivity equation,

$$\bar{z} = \frac{n_1}{n}\bar{z}_1 + \frac{n_2}{n}\bar{z}_2.$$

From the average wage equations,

$$\begin{aligned}
\bar{w}_1 =& (1-\zeta)(Ap^m \bar{z}_1 - \kappa_1(1-\lambda^x)\Lambda p(1-G(z^{R1}))\phi_1 + \zeta h, \\
\bar{w}_2 =& (1-\zeta)(Ap^m \bar{z}_2 - \kappa_1(1-\lambda^x)\Lambda p(1-G(z^{R2}))\phi_2 + \zeta h, \\
\bar{w} =& \frac{n_1\bar{w}_1 + n_2\bar{w}_2}{n}.
\end{aligned}$$

From the intermediate goods producer profits equation,

$$\Pi^{int} = p^m A(n_1\bar{z}_1 + n_2\bar{z}_2) - \bar{w}_1 n_1 - \bar{w}_2 n_2 - \chi v.$$

From the retailer profits equation,

$$\Pi^{ret} = y\tilde{\Delta} - p^m y\Delta.$$

From the aggregate profit equation,

$$\Pi = \Pi^{int} + \Pi^{ret}.$$

From the tax equation,

$$T = (1-n)h.$$

From the type 1 budget constraint,

$$c_1 = (1 - U_1)\bar{w}_1 + U_1 h + \Pi - T.$$

From the aggregate resource constraint,

$$c_2 = \frac{y - \chi v - \delta c_1}{1 - \delta}.$$

From the welfare equations,

$$\begin{aligned}
\mathcal{W}_1 =& \frac{\log(c_1)}{1-\beta}, \\
\mathcal{W}_2 =& \frac{\log(c_2)}{1-\beta}, \\
\mathcal{W} =& \delta \mathcal{W}_1 + (1-\delta)\mathcal{W}_2, \\
\psi_1 =& \psi_2 = \psi = 0, \\
\Psi =& e^{(1-\beta)(\mathcal{W}_2 - \mathcal{W}_1)} - 1.
\end{aligned}$$

We then use a nonlinear solver to derive the implied monthly job finding and separation rates from their quarterly counterparts using the following equations:

$$\lambda = \lambda^m(1-f^m)^2 + (1-\lambda^m)\lambda^m(1-f^m) + (1-\lambda^m)^2\lambda^m + (\lambda^m)^2 f^m,$$

$$f = f^m(1-\lambda^m)^2 + (1-f^m)f^m(1-\lambda^m) + (1-f^m)^2 f^m + (f^m)^2\lambda^m,$$

$$\lambda_1 = \lambda_1^m(1-f_1^m)^2 + (1-\lambda_1^m)\lambda_1^m(1-f_1^m) + (1-\lambda_1^m)^2\lambda_1^m + (\lambda_1^m)^2 f_1^m,$$

$$f_1 = f_1^m(1-\lambda^m)^2 + (1-f_1^m)f_1^m(1-\lambda_1^m) + (1-f_1^m)^2 f_1^m + (f_1^m)^2\lambda_1^m,$$

$$\lambda_2 = \lambda_2^m(1-f_2^m)^2 + (1-\lambda_2^m)\lambda_2^m(1-f_2^m) + (1-\lambda_2^m)^2\lambda_2^m + (\lambda_2^m)^2 f_2^m,$$

$$f_2 = f_2^m(1-\lambda_2^m)^2 + (1-f_2^m)f_2^m(1-\lambda_2^m) + (1-f_2^m)^2 f_2^m + (f_2^m)^2\lambda_2^m.$$