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The Effects of CBDC on the Federal Reserve’s Balance Sheet

Christopher Gust\textsuperscript{1}, Kyungmin Kim\textsuperscript{1}, and Romina Ruprecht\textsuperscript{1}

\textsuperscript{1}Federal Reserve Board

September 14, 2023

Abstract

We propose a parsimonious framework to understand how the issuance of central bank digital currency (CBDC) might affect the financial system, the Federal Reserve’s balance sheet, and the implementation of monetary policy. We show that there is a wide range of outcomes on the financial system and the Federal Reserve’s balance sheet that could reasonably occur following CBDC issuance. Our analysis highlights that the potential effects on the financial sector depend critically on how the Fed manages its balance sheet. In particular, CBDC could in principle put substantial upward pressure on the spread of the federal funds rate and other wholesale funding rates over the interest rate on reserves unless the Fed expanded its balance sheet to accommodate CBDC issuance.

Keywords: Central bank digital currency, monetary policy implementation, bank disintermediation, central bank balance sheet

JEL Classifications: E50, E51, E52, E58

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1 Introduction

Academic researchers and central banks around the world have begun to study the monetary policy implications of introducing a publicly available digital currency that can be held by a broad set of counterparties and used to make payments.\footnote{For an overview of the literature, see Carapella and Flemming (2020), Ahnert et al. (2022), Auer et al. (2022) and Infante et al. (2022). Auer et al. (2020) discuss real-world CBDC experiments by central banks.} Like cash and reserves, a central bank digital currency (CBDC) would be a liability of a central bank. However, CBDC is different from cash in that CBDC is digital—existing virtually on some ledger—while cash is physical—held in the form of paper bills or coins. CBDC is also different from reserves, because reserves can be held only by banks in addition to a very narrow group of selected institutions, while CBDC is typically intended for a much broader set of counterparties, including possibly individual consumers.

Because CBDC represents a new liability of a central bank that could be widely held, it could potentially have far-reaching consequences for monetary policy and the economy. Research on the potential effects of CBDC has covered a broad range of topics including the provision of private and public money and how the introduction of a CBDC might impact the transmission of monetary policy to the financial system and broader economy.

In this paper, we focus on how the introduction of a CBDC in the United States might affect the implementation of monetary policy and the Fed’s balance sheet. To do so, we develop a parsimonious analytical framework that incorporates linkages between a banking sector, a nonbanking sector comprised of households and firms, and a central bank managing the supply of reserves.

The analysis shows that there is a wide range of possible outcomes for how the introduction of a CBDC might affect the Federal Reserve’s balance sheet and the implementation of monetary policy. These outcomes depend on the extent to which CBDC is substituted for cash and bank deposits, how banks are affected by the introduction of CBDC and respond to its introduction, and how the Fed chooses to manage the supply of reserves and its balance sheet. The model suggests that the impact of a CBDC on the size and composition of the Fed’s balance sheet can differ significantly depending on these factors.

If the demand for CBDC was relatively modest or primarily reflected the public exchanging cash rather than bank deposits for CBDC, then the model implies that the impact of CBDC on the Fed’s balance sheet would be limited. However, if the demand for CBDC was high and primarily reflected the public exchanging bank deposits for CBDC, then the impact on the Fed’s balance sheet could be significant. In that case, the effect on the size of the balance sheet would depend on how the Fed managed the supply of reserves in response to the introduction of CBDC. If the Fed did not make any reserve management (asset)
purchases, then the size of the balance sheet would remain unchanged. But there would be a substantial change in the composition of liabilities with a large decline in reserves and an offsetting large increase in CBDC. In the model, a large fall in reserves could in principle put substantial upward pressure on the level of wholesale funding rates relative to the interest paid on reserve balances (IORB). In that case, a central bank operating in an ample reserve regime might need to make reserve management purchases and expand the size of its balance sheet to maintain a sufficient reserve buffer and moderate the effects of CBDC on the banking sector.

Building on this analysis, we use the model to quantify the potential effects of CBDC on the Fed’s balance sheet, using estimates from the banking literature. Because there is considerable uncertainty about the potential demand for CBDC, we consider scenarios in which CBDC demand is low and ones in which it is high. In low-CBDC-demand scenarios, the effects on the balance sheet are modest with reserves declining less than $100 billion—about 0.4 percent of U.S. GDP in 2022—if the Fed does not engage in reserve management purchases. In this case, the upward pressure on the wholesale funding rate (relative to the IORB rate) in the model is low, and there is little need to conduct reserve management purchases to maintain effective interest rate control. In a scenario where CBDC demand is high, on the order of $1 trillion—about 4 percent of U.S. GDP in 2022—and involves significant outflows from bank deposits to CBDC, the model suggests that reserve management purchases and a significant expansion of the balance sheet may be necessary to alleviate the upward pressure on the wholesale funding rate. Under the assumption that reserve management purchases are undertaken to maintain the level of reserves at their level prior to the introduction of CBDC, the balance sheet will expand by nearly $1 trillion.

While the model is useful for quantifying the potential effects of CBDC on the size and composition of the Fed’s balance sheet, any quantitative exercise regarding the effects of CBDC on the balance sheet should be viewed as preliminary and is subject to considerable uncertainty. In particular, there is little information about potential design features of CBDC, the demand for CBDC, and how banks might respond to the introduction of a CBDC. Also, it is important to note that the model does not incorporate some important channels through which CBDC might impact the balance sheet such as foreign demand for a U.S. CBDC or precautionary demand that might arise from flight-to-safety considerations. These considerations suggest that the demand for CBDC could be even larger than in the scenarios we model and that there could be an even larger expansion in the size of the Federal Reserve’s balance sheet. Alternatively, the impact on the size of the balance sheet could be small, as demand for CBDC could be lower than in the scenarios that we consider.

In stating that there is little need for reserve management purchases, we are abstracting from reserve management purchases that may be needed regardless of the effect of CBDC due to the general growth of the economy and the associated increases in the size of the banking sector and non-reserve liabilities of the Federal Reserve.
This paper is closely related to the literature on CBDC and its effects on monetary policy implementation. Our contribution is to introduce a stylized model that explicitly characterizes the behavior of various sectors in a closed economy that are directly relevant for a central bank’s balance sheet management, while embedding practical considerations regarding policy implementation and a central bank’s management of its balance sheet. Our paper is most closely related to Malloy et al. (2022) and Infante et al. (2022), who study the effects of CBDC on the Fed’s balance sheet using a stylized, scenario-based balance sheet analysis and discuss how the introduction of a CBDC may affect the level of reserves that is consistent with an ample reserve regime. The stylized balance sheets of various sectors in the economy underlying our model are very close to those that appear in Malloy et al. (2022) and Infante et al. (2022). However, unlike these papers, we explicitly model agents’ problems to study the effect of introducing a CBDC on equilibrium quantities and prices.

One important concern about introducing CBDC is its potentially detrimental effect on the banking sector such as bank disintermediation. Previous studies—such as Andolfatto (2021), Assenmacher et al. (2021), Chiu et al. (2023) and Whited et al. (2022)—have generally found that the banking sector would not necessarily be disintermediated due to CBDC, a conclusion shared by our paper. In our model, introducing CBDC also affects the banking sector through its effect on deposit rates, the magnitude of which depends on a number of factors. We describe the effects on deposit rates across various scenarios using our model, but do not explore their implications for how changes in the monetary policy stance affect the real economy. Rather, we focus on the implications of introducing CBDC on the balance sheets of the central bank and other sectors in the economy as well as its implications for interest-rate control.

Our framework is built upon changes in asset allocations by the private sector and their effect on the central bank’s balance sheet, and it does not account for the production of consumption goods or the welfare of households. For analysis of welfare effects from introducing CBDC, see Keister and Sanches (2022), Williamson (2022a) and Williamson (2022b).

The paper is structured as follows. Section 2 presents the model and characterizes the equilibrium. Section 3 discusses qualitative results and Section 4 presents some numerical exercises. Finally Section 5 concludes.

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3Bindseil (2020), Kumhof and Noone (2021) and Meaning et al. (2021) also have scenario- or example-based balance sheet analyses.
4The potential implications of CBDC issuance on monetary policy pass-through is explored by Garratt et al. (2022).
2 Model

The model consists of a banking sector, a nonbanking sector comprised of domestic households and firms, and a central bank (the Fed) that supplies reserves to the banking sector.\textsuperscript{5} Households and firms in the nonbanking sector use currency and interest-bearing bank deposits to purchase goods and services, and the introduction of a non-interest bearing CBDC provides households and firms with a new medium to conduct transactions.\textsuperscript{6} We study the long-run implications of introducing CBDC on the Fed’s balance sheet by comparing the equilibrium of the model without CBDC to the equilibrium after CBDC has been introduced; the model is static and we interpret the equilibrium as representing a “steady state” of the economy. In doing so, we assume CBDC to be a direct liability of the central bank that is intermediated by banks and potentially other financial institutions.\textsuperscript{7}

To simplify the presentation of the model, we formulate most variables in terms of changes (denoted by $\Delta$) in equilibrium quantities and prices between a pre-CBDC and post-CBDC economy, rather than explicitly describing the two equilibria separately. This is especially convenient in describing balance sheet identities and allows us to abstract from quantities and prices that are unaffected by the introduction of CBDC.

2.1 Banking sector

The banks in the model obtain funding in deposit and non-deposit wholesale markets, make interest-bearing loans and hold reserves, which earn interest. Banks maximize their profits by choosing the amount of deposit and non-deposit wholesale funding they wish to obtain, as well as the quantity of loans to issue and the quantity of reserves to hold. Banks are competitive in each of the markets where they operate and take as given the loan rate, the interest on reserve balances (IORB) rate, the deposit rate, and the wholesale funding rate. In the aggregate, banks’ demand for funds is an important determinant of the deposit and wholesale funding rates, while their supply of loans helps determine the loan interest rate in the economy.

The model incorporates two key frictions in the banking sector. The first is that each bank faces (marginal) balance sheet costs that are increasing in its total assets. As emphasized in Martin et al. (2016) or Afonso et al. (2019), balance sheet costs could reflect

\textsuperscript{5}We interpret the firms in the nonbanking sector to include financial non-depository institutions such as insurance companies, pension funds, and mutual funds.

\textsuperscript{6}We analyze the effects of an unremunerated, domestically held CBDC, which we see as an important first step to understanding the impact that CBDC may have and leave the extension of the model to a remunerated CBDC that can be held abroad to future work.

\textsuperscript{7}We interpret these intermediaries as the custodians and maintainers of end-user CBDC accounts, while the central bank maintains and processes accounts at the intermediary level.
regulations or internal limits designed to reduce a bank’s exposure to risk and raise the cost of a bank becoming too large. In the model, a bank’s balance sheet costs are an important determinant of its demand for wholesale funds and the wholesale funding rate. Because of these costs, banks will be more willing to pay up for wholesale funds when their balance sheets shrink with a decline in the private sector’s demand for deposits, putting upward pressure on the wholesale funding rate. The second key friction in the banking sector is that there are synergies between a bank accepting deposits and lending money, making it attractive for a bank to bundle these services together. As emphasized in Mester et al. (2007), bundling loans and deposits helps banks build relationships with their customers and monitor the customers’ credit risk. In the model, the synergy between loans and deposits is a crucial channel through which the outflow of deposits to CBDC affects bank lending and causes bank disintermediation.

There are $N_B$ banks in the economy, which are indexed by $i$.\footnote{Because of the balance sheet cost being a function of a bank’s size, a bank has the incentive to divide itself into smaller entities and reduce these costs. We rule this out \textit{a priori} by fixing the number of banks. Alternatively, we could have assumed instead that banks have monopoly power and there are fixed entry costs though this would considerably complicate the exposition of the model.} Each bank earns profit $P_i$ based on its assets and liabilities:

$$P_i = i_R R_i + (i_L + \alpha N_B D_i - c_L) L_i - i_D D_i - i_E E_i - \frac{1}{2} N_B c_B (R_i + L_i)^2. \tag{1}$$

$R_i, D_i, L_i$ and $E_i$ denote reserves, deposits, loans and non-deposit funding of bank $i$. The variables $i_R, i_L, i_D$ and $i_E$ denote the interest rates on reserves (IORB), loans, deposits, and non-deposit funding, respectively; $c_B$ represents the magnitude of the marginal balance sheet cost; and $\alpha$ captures loan-deposit synergy. The coefficients $c_B$ and $\alpha$ are multiplied by $N_B$ for convenience.\footnote{Specifically, such normalization of $c_B$ and $\alpha$ lets us interpret them as coefficients characterizing the relationship between equilibrium prices and aggregate, not individual banks’ assets and liabilities.} Finally, the parameter $c_L$ reflects that banks incur costs to maintaining and servicing loans.

Each bank maximizes its profit $P_i$, taking interest rates as given while being subject to its balance sheet constraint:

$$R_i + L_i = D_i + E_i. \tag{2}$$
Profit maximization by a bank implies:  

\[ i_L + \alpha N_B D_i - c_L - i_R = 0. \]  

(3)  

\[ -i_D + \alpha N_B L_i + i_R - N_B c_B (R_i + L_i) = 0. \]  

(4)  

\[ -i_E + i_R - N_B c_B (R_i + L_i) = 0. \]  

(5)  

Since these three equations are linear and hold for all banks, we can aggregate them across all banks to characterize the change in the banking sector between pre- and post-CBDC equilibria:  

\[ \Delta i_L + \alpha \Delta (Deposits) = 0. \]  

(6)  

\[ -\Delta i_D - (c_B - \alpha) \Delta (Loans) - c_B \Delta (Reserves) = 0. \]  

(7)  

\[ -\Delta i_E - c_B (\Delta (Loans) + \Delta (Reserves)) = 0. \]  

(8)  

The changes in aggregate quantities are sums of changes in individual quantities: \( \Delta (Deposits) = \sum_i \Delta D_i \) and so on. Note that we are assuming \( \Delta i_R = 0 \); the central bank does not change the interest on reserve balances following the issuance of CBDC. The unchanged IORB rate reflects that we assume that the central bank adjusts the IORB rate to achieve its macroeconomic objectives, and because CBDC is assumed not to have a material impact on macroeconomic variables such as real GDP, the unemployment rate or inflation, the IORB rate is held constant in response to the introduction of CBDC. The individual banks’ balance sheet constraints can be aggregated and expressed as:  

\[ \Delta (Reserves) + \Delta (Loans) = \Delta (Deposits) + \Delta (Wholesale funding). \]  

(9)  

Equations (6), (7), (8) and (9) characterize the response of the banking system to the introduction of CBDC. Figure 1 illustrates the aggregate balance sheet of banks and those of other sectors in the model.

2.2 Central Bank

The central bank manages the supply of reserves, and we consider two alternative assumptions for how the central bank may adjust the level of reserves in response to the introduction of a CBDC. Under the first assumption, there are no reserve management purchases of Treasury securities by the central bank and the size of its balance sheet remains unchanged. In that case, the substitution of CBDC for deposits by households affects

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10 These first-order conditions can be obtained by taking partial derivatives of the associated Lagrangian with respect to \( R_i, L_i, D_i \) and \( E_i \) and eliminating the shadow cost of the constraint from the resulting four equations. Alternatively, we can replace \( R_i \) by \( D_i + E_i - L_i \) in the expression for profit and compute its partial derivatives with respect to \( L_i, D_i \) and \( E_i \).
<table>
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<td>Reserves</td>
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<th>Government (excluding the central bank)</th>
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<tr>
<td>National debt (Δ=0)</td>
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Figure 1: Sectoral Balance Sheets in the Model

The figure illustrates the stylized balance sheets of the financial sectors included in the model. The top-left portion illustrates the aggregate balance sheet of banks, consistent with Equation (9). The top-right portion illustrates the balance sheet of the central bank (Federal Reserve), consistent with Equation (11) in section 2.2; the bottom-left that of the private sector, consistent with Equation (16) in Section 2.3; and the bottom-right that of the government, consistent with Equation (17) in Section 2.4. “Net worth” for the private sector and “national debt” for the government are residual terms that are included only for descriptive purposes, which remain unchanged (“Δ = 0”) and thus do not appear in any of the equations.
the composition of a central bank’s liabilities. When households and businesses transfer funds in their bank accounts to receive CBDC in exchange, their bank needs to purchase CBDC from the central bank and provide the CBDC to the households and firms. To do so, the bank pays the central bank the desired amount in reserves, thus reducing reserve liabilities of the central bank. While the total value of the central bank’s liabilities remains unchanged if there are no reserve management purchases, the composition of liabilities changes, with the supply of reserves declining and the new CBDC liability increasing by the same amount.

With the supply of reserves declining, there could be upward pressure on the spread between the wholesale funding rate (the model’s proxy for the federal funds rate) and the IORB rate, which could in principle force a central bank to inject reserves into the banking system to maintain effective interest rate control and keep reserves ample.\textsuperscript{11} Although not explicitly recognized in the model, a central bank might also decide to inject reserves without any material upward pressure on the wholesale funding rate to maintain a larger desired buffer stock of reserves in an ample reserve regime.\textsuperscript{12} These considerations motivate the second assumption that we make about how a central bank manages reserves in response to the introduction of CBDC: As an alternative to the first assumption in which there are no reserve management purchases, we assume that the central bank engages in purchases of Treasury securities so that the supply of reserves remains unchanged at its level prior to the introduction of CBDC.\textsuperscript{13} In the scenarios that we consider, such purchases help alleviate the upward pressure on the wholesale funding rate and maintain rate control.

Generalizing these two assumptions, we assume that the central bank manages reserve supply as follows:

\[
\Delta(Reserves) = \beta_1 \Delta(CBDC) + \beta_2 \Delta(Deposits). \tag{10}
\]

\(\beta_1\) and \(\beta_2\) are exogenous policy parameters that determine how the central bank responds

\textsuperscript{11}Because of banks’ balance sheet costs, the model generates a negative relationship between banks’ aggregate reserve holdings and the level of the wholesale funding rate relative to the IORB rate. This is consistent with models in the literature such as Afonso et al. (2019) and Kim et al. (2020), which link banks’ marginal balance sheet costs to the level of the federal funds rate relative to the IORB rate. Also, models in the literature generally assume a scarcity value of reserves, at least for low enough levels of reserve demand, which is absent in our model. In our model, banks’ balance sheet costs also imply that the aggregate demand for reserves varies inversely with the volume of bank loans.

\textsuperscript{12}The introduction of CBDC could in principle lead to larger, unpredictable fluctuations in reserves. In that case, a central bank may want to increase the level of reserves relative to its level prior to the introduction of CBDC in order to maintain a larger buffer of reserves to absorb the additional variability. Afonso et al. (2023) present a model in which a central bank chooses the optimal supply of reserves based on such considerations.

\textsuperscript{13}In the model, it is assumed that the central bank purchases Treasury securities exclusively from the nonbanking sector. In reality, however, some of the Treasury securities may reflect sales of these securities by banks. Extending the model to allow for this possibility would have little impact on our results for the Fed’s balance sheet if banks’ demand for Treasury securities is not affected much by the issuance of CBDC.
to changes in CBDC and bank deposits. Since there is no CBDC in the pre-CBDC equilibrium, $\Delta(CBDC)$ represents the amount of CBDC issued to or purchased by the private sector. Note that this expression encompasses the two alternative assumptions discussed earlier: The assumption of no reserve management purchase implies $\beta_1 = 0$ and $\beta_2 = 1$, and the assumption of purchasing Treasury securities to hold reserve supply constant implies $\beta_1 = \beta_2 = 0$. Note that the assumption of no reserve management purchase can be more generally expressed as that of keeping the central bank’s total liabilities fixed, $\Delta(Reserves) = -\Delta(CBDC) - \Delta(Cash)$. Due to the way we characterize the private sector in the next section, this is equivalent to assuming $\beta_1 = 0$ and $\beta_2 = 1$ in our paper, but this need not be the case in general.\footnote{For example, if we assumed $r_D \neq 1$ in equation (14), the expressions would not be equivalent and we would need to specifically assume $\Delta(Reserves) = -\Delta(CBDC) - \Delta(Cash)$.}

The central bank must balance its assets and liabilities. In addition to CBDC, the central bank issues reserves and cash, and we assume that both the private sector and the government hold cash. Government cash can be thought of as funds deposited at a special account at the central bank, similar to the Treasury General Account (TGA) at the Fed. On the asset side, the central bank holds Treasury securities, which satisfy:

$$\Delta(Treasury_{CB}) = \Delta(Reserves) + \Delta(Gov\ cash) + \Delta(Cash) + \Delta(CBDC). \quad (11)$$

### 2.3 Private sector (excluding banks)

When CBDC is introduced into the economy, it is assumed that some households and firms will find it an attractive medium for conducting retail transactions and that they will use CBDC in place of cash or bank deposits. The extent of substitution of CBDC for cash and deposits depends on the convenience and utility of using CBDC in purchasing goods and services relative to cash and deposits.\footnote{CBDC may prove attractive due to its perceived high level of safety, particularly relative to uninsured deposits, as well as due to its ease of use relative to physical currency.} The substitution of CBDC for bank deposits will also depend on the interest rate banks pay on deposits, which can rise in the model as banks respond to the introduction of CBDC. This increase in the deposit rate, all else equal, increases the public’s demand for deposits, and as discussed further below, helps offset some of the deposit outflows that occur in response to the introduction of CBDC.\footnote{Because the banking sector is competitive in the model, the introduction of CBDC tends to push up the deposit rate and reduce the quantity of bank deposits. However, it is theoretically possible for the quantity of bank deposits to increase in response to the introduction of CBDC due to imperfect competition in the banking sector. For a discussion on how the competitiveness of the banking sector affects deposits and deposit rates when CBDC is introduced, see Infante et al. (2022).}

We model the private sector’s optimal choice for holding alternative monetary assets,
which leads to the following characterization of private-sector demand for these assets:

$$\Delta(Cash) = -C_0.$$  (12)
$$\Delta(\text{Deposits}) = -D_0 + g_D \Delta i_D.$$  (13)
$$\Delta(CBDC) = -r_C \Delta(Cash) - r_D \Delta(\text{Deposits}).$$  (14)

An extensive discussion and derivation of these relationships is provided in Appendix A. The terms $C_0 \geq 0$ and $D_0 \geq 0$ in Equations (12) and (13) represent the substitution of CBDC for cash and deposits, respectively, based on either efficiency or convenience considerations that are unrelated to changes in demand that occur in response to movements of interest rates. The second term on the right-hand side of Equation (13), $g_D \Delta i_D$, captures the response to a change in the deposit rate: Bank deposits pay interest $i_D$ while CBDC is unremunerated, and a change in the interest rate spread between deposits and CBDC, $\Delta i_D$, affects the degree of substitution between CBDC and deposits. If the deposit rate increases in response to the introduction of CBDC, $\Delta i_D > 0$, then this increase, all else equal, can partially offset the deposit outflow that occurs based on efficiency or convenience considerations. The parameter $g_D \geq 0$ captures the interest rate sensitivity of the demand for deposits to the deposit rate. In Equation (14), $r_C$ and $r_D$ are what may be called conversion rates. If the cash-to-CBDC conversion rate, $r_C$, were equal to one, one-dollar decline in cash would lead to one-dollar increase in CBDC. In the remainder of the paper, we always set these conversions rates to one, which implies that the private sector simply chooses over different forms of money with overall quantity of money demand fixed. However, these conversion rates could in principle differ from one depending on, for example, the efficiency of using CBDC relative to cash or deposits.

The substitution of CBDC for currency and bank deposits are the two channels through which CBDC affects the central bank’s balance sheet in the model. Because we focus on a non-interest bearing CBDC, we do not consider the substitution of CBDC for short-duration liquid investments such as shares of money market mutual funds.\(^\text{17}\) It is also assumed that any aggregate income or wealth effects associated with CBDC increasing the efficiency of the payment system are small and can be ignored. In addition, by focusing on only domestic households and firms, the model abstracts from the demand for a U.S. CBDC that might arise abroad, which would likely magnify the effects discussed here.\(^\text{18}\)

\(^\text{17}\)We focus on steady states in which interest rates are well away from their effective lower bounds. In principle, the substitutability between CBDC and short-duration liquid investments could become important in an environment where short-term interest rates on these investments were near their effective lower bounds.

\(^\text{18}\)Besides amplifying the effects on the size and composition of the Fed’s balance sheet that we discuss, demand for CBDC from abroad would likely lead to an appreciation of the dollar and an associated reduction in the value of the Fed’s holdings of foreign currency. However, at the end of 2022, foreign currency holdings only amounted to 0.22 percent of the Fed’s total assets and thus these valuation effects would likely be very small.
In addition to currency, deposits, and CBDC, households and firms in the nonbanking sector also hold Treasury securities and lend wholesale or non-deposit funds to the banks in the economy. Broadly interpreted, these wholesale funds supplied by households and firms to banks in the model can be thought of as including loans made in the federal funds market, and the interest rate paid on these wholesale funds serves as the model’s proxy for the effective federal funds rate (EFFR).\textsuperscript{19} Finally, households and firms borrow from the banks and pay interest on these loans.

The private sector’s demand for loans decreases in the interest rate on loans, $i_L$:

$$\Delta(Loans) = -s_L \Delta i_L. \quad (15)$$

In Equation (15), $s_L$ is the slope of the loan demand curve and $\Delta i_L$ is the change in the loan interest rate induced by the introduction of CBDC. In addition to its borrowing from banks, the private sector demands Treasury securities and supplies wholesale funding to banks, while respecting its resource constraint:

$$\Delta(\text{Deposits}) + \Delta(\text{Cash}) + \Delta(\text{Wholesale funding}) + \Delta(\text{Treasury}_{Private}) + \Delta(\text{CBDC}) = \Delta(\text{Loans}). \quad (16)$$

As described earlier, there is no net creation of private sector wealth in response to CBDC issuance, and the changes in assets are balanced by the changes in liabilities. We do not need to specify the functional forms of the private sector’s demand for Treasury securities and supply of wholesale funding to determine equilibrium allocation; assuming that the private sector’s choice clears these respective markets is sufficient. In the model, the supply of Treasury securities by the government and the demand for Treasuries by the central bank do not depend on their price or yield, thus the residual supply is held by the private sector in equilibrium, regardless of their price or yield. Once the private sector’s holdings of Treasury securities are determined, the resource constraint determines its supply of wholesale funding, which in equilibrium will coincide with the banking sector’s demand for wholesale funding.

### 2.4 Government (excluding the central bank)

The government holds cash in its central bank account and issues Treasury securities that can be held by the central bank and the private sector. We assume that the government does not make any net fiscal transfers to any agent in the economy in response to the

\textsuperscript{19}In the model, the wholesale funding rate increases one-to-one with decreases in marginal balance sheet cost faced by banks. The same can be said of EFFR if reserve supply is large enough so that reserve demand curve is essentially flat in the relevant domain and there is no other factor affecting pricing; see Kim et al. (2020).
introduction of CBDC. Accordingly, the changes in its assets are identical to the changes in its liabilities:

\[ \Delta(\text{Gov cash}) = \Delta(\text{Treasury}_{CB}) + \Delta(\text{Treasury}_{Private}) \].

We assume that the government does not change its cash position in response to CBDC issuance, and it follows that \( \Delta(\text{Gov cash}) = 0 \). Therefore, any change in a central bank’s holdings of Treasury securities will be absorbed by the private sector.

### 2.5 Equilibrium

The model describes steady-state changes between pre- and post-CBDC economies, and the equilibrium is characterized by the set of changes in quantities—\( \Delta(\text{Cash}) \), \( \Delta(\text{Deposits}) \) and so on—and interest rates—\( \Delta i_D \), \( \Delta i_L \) and \( \Delta i_E \)—consistent with individual agents’ choices. These equilibrium changes can be characterized as the solution to a system of Equations: (6), (7), (8) and (9) for banks; (10) and (11) for the central bank; (12), (13), (14), (15) and (16) for the private sector; and (17) for the government. Figure 2 illustrates the interaction between different sectors in the model, and Appendix B shows how these equations can be used to compute equilibrium prices and quantities.

### 3 Discussion

Before discussing quantitative results, we first highlight how the impact of CBDC issuance on the size and composition of the Federal Reserve’s balance sheet varies greatly depending on key aspects of the model. These factors include the extent to which households and firms substitute CBDC for cash and deposits, how banks are affected by and respond to the introduction of CBDC, and how the Fed chooses to manage its balance sheet.

#### 3.1 Substitutability of CBDC with cash and deposits

An important factor determining the balance sheet impact of CBDC is its substitutability with cash and deposits. If CBDC were largely a substitute for cash but not for deposits, due to either the underlying preference of households or CBDC’s design, then the Fed’s liability composition would change to reflect the exchange of cash for CBDC. Other than this change in liabilities, the balance sheet would be largely unaffected; the level of reserves would not change and the commercial banks in the model would not be impacted if holders of bank deposits were largely uninterested in using CBDC to conduct transactions instead of using their deposit accounts.
This figure illustrates the interaction between different agents and sectors in the model. The government issues treasury securities that are held by either the private sector or the central bank. The central bank issues cash and CBDC that can be held by the private sector and reserves, which can be held by banks. Banks can raise funding from the private sector by either deposits or wholesale funding. Lastly, the private sector borrows loans from banks.

To see this using the model, assume that $D_0 = 0$ and $r_C = r_D = 1$. In that case, there is only a reallocation of household money demand between cash and CBDC, except possibly in response a change in the deposit rate, $i_D$. Also, let $\beta_1 = \beta_2 = 0$ so that the central bank keeps the supply of reserves constant, $\Delta(Reserves) = 0$. In this case, there is no change in the equilibrium deposit rate, $\Delta i_D = 0$ (see Equation A-41). Intuitively, with reserve supply and deposit demand (function) unchanged, there is no change in the banks’ portfolio problem, and the equilibrium deposit rate as well as loan and wholesale funding rates remain unchanged. Changes in the private sector’s money holdings are simply $\Delta(Cash) = -C_0, \Delta(CBDC) = C_0$, and $\Delta(Deposits) = 0$. None of the other quantities such as Treasury holdings by the central bank and the private sector, bank loans and wholesale funding change following the introduction of CBDC.

In contrast, if CBDC were largely a substitute for deposits, it could have a significant impact on the balance sheet of the banking sector. The effect on the size and composition of the Federal Reserve’s balance sheet would depend critically on how the Federal Reserve manages its reserves.
3.2 Deposit outflow without reserve management purchases

If the Federal Reserve did not undertake reserve management purchases, then the exchange of deposits for CBDC would result in an increase in CBDC on the Federal Reserve’s balance sheet with a corresponding decline in reserves. Accordingly, there would be a change in the composition of liabilities while the overall size of the balance sheet would remain unchanged.

If commercial banks did not seek to obtain additional funding, their balance sheets would shrink, with decreases in reserve holdings as assets and deposits as liabilities. To the extent that there was a positive synergy between banks’ loan-making and deposit-taking activities, the decrease in deposits might precipitate some decline in bank loans.

In the model, this case can be characterized by setting \( \beta_1 = 0 \) and \( \beta_2 = 1 \) so that the central bank lets reserves drain as deposits are exchanged for CBDC: \( \Delta(\text{Reserves}) = \Delta(\text{Deposits}) \). In this example, households and firms exchange deposits for CBDC with \( r_D = 1 \), and we abstract from substitution for cash \( (C_0 = 0) \). As a result, both deposits and bank loans shrink:

\[
\Delta(\text{Deposits}) = -\eta D_0. \tag{18}
\]
\[
\Delta(\text{Loans}) = -\alpha sL \eta D_0. \tag{19}
\]

In Appendix B, we define \( \eta \equiv h_D / (g_D + h_D) \), where \( h_D \equiv [(c_B - \alpha) \alpha sL - c_B \beta_1 r_D + c_B \beta_2]^{-1} \). The parameter \( \eta \) captures the decrease in the equilibrium amount of deposits due to a downward shift in deposit demand by one unit; \( g_D \) is the slope of deposit demand by the private sector and \( h_D \) is the slope of deposit supply by banks, endogenously determined by banks’ profit maximization behavior described earlier. As usual in this type of setup, \( 0 < \eta < 1 \) because the reduced demand for bank deposits puts upward pressure on the interest rate offered on bank deposits, which partially moderates the effect of the downward shift in deposit demand.

Since the commercial banks hold central bank reserves and loans as assets, their balance sheet size changes by \( \Delta(\text{Reserves}) + \Delta(\text{Loans}) = -(1 + \alpha sL) \eta D_0 \), which is negative, indicating that banks’ balance sheets shrink. The size of the central bank’s balance sheet is unchanged, as total liabilities remain the same with the decrease in reserves offset by an increase in CBDC.

With banks’ deposits falling, banks are willing to pay more for wholesale funds, and the wholesale funding rate, the model’s proxy for the federal funds rate, increases: \( \Delta i_E = (1 + \alpha sL) c_B \eta D_0 > 0 \). In principle, the increase in the wholesale funding rate relative to the IORB rate could be sizeable if the public’s substitution of CBDC for deposits were
large. In that case, the Federal Reserve might want to engage in reserve management purchases to maintain effective interest rate control and keep reserves ample. Although not explicitly recognized in the model, concerns about reserve scarcity would reinforce the case for reserve management purchases.

### 3.3 With reserve management purchases

To alleviate the potential upward pressure on wholesale funding rates associated with a large demand for CBDC and a corresponding large decline in reserves, the Federal Reserve could purchase assets such as Treasury securities to maintain the level of reserves at a level that had prevailed prior to the issuance of CBDC.\(^{20}\) To illustrate this with an example, suppose that depositors initially exchanged $100 of deposits for CBDC, resulting in a $100 decrease in reserves and a $100 increase in CBDC among the Federal Reserve’s liabilities with no change in its assets. Furthermore, imagine that the Federal Reserve countered the decrease in reserves by buying $100 of Treasury securities.\(^{21}\) Then, on net, the Federal Reserve’s Treasury holdings and CBDC liability would both increase by $100 with no change in its reserve liability. As a result, the balance sheets of commercial banks would not shrink as much. Also, relative to the case in which the Federal Reserve did not make reserve management purchases, there would be less upward pressure on the wholesale funding rate.

The model does not explicitly account for additional factors that may affect the level of reserves that a central bank seeks to maintain. In particular, the Federal Reserve may have to address the impact of CBDC on the variability of reserve supply and changes in reserve demand associated with factors not captured by the model. Variability in CBDC balances that result from flows between CBDC and bank deposits or other liquid assets could increase the variability in total non-reserve liabilities and, by extension, the volatility of reserve supply. In that case, the Federal Reserve might respond by expanding the supply of reserves to reduce the likelihood that changes in reserve balances result in significant upward pressure on short-term interest rates. With regard to reserve demand, the model

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\(^{20}\) These reserve management asset purchases could in principle have similar effects as quantitative easing with decreasing term premiums leading to more accommodative financial conditions. If the Fed was concerned about these purchases affecting the stance of policy, it could expand its balance sheet by primarily purchasing short-duration securities, which would decrease the overall duration of the Fed’s portfolio as its size expands. Even if term premiums fell in the short and medium run because these purchases consisted of longer-duration securities, term premiums in the long run might remain unaffected if longer-duration Treasury issuance increased over time.

\(^{21}\) In the model, we assume that when the Federal Reserve purchases the assets from the private sector, the household or firm can receive payment from the proceeds of the sale of the security in terms of deposits, CBDC, or cash. In the model, we only keep track of the net change in the equilibrium quantities in each of these three different forms of money in response to the introduction of CBDC that leads the private sector on net to substitute CBDC for deposits and cash. The model is not designed to keep track of changes in gross quantities or payment flows on individual transactions.
abstracts from precautionary or related regulatory motives that banks might have for holding reserves, and if the introduction of CBDC increased banks’ reliance on less stable sources of funding, their demand for reserves could increase. However, banks might demand less reserves if their balance sheets contracted sufficiently with the decline in deposits, and in that case, it might be appropriate for the Federal Reserve to reduce the supply of reserves.\(^{22}\)

4 Numerical exercise

We calibrate the model in order to estimate the long-run impact of introducing CBDC on the Federal Reserve’s balance sheet and income.\(^{23}\) The parameters are chosen to be in line with existing literature and are shown in Table 1.

For the parameters determining how much the private sector substitutes CBDC for bank deposits, we consider a range of estimates as they are crucial for determining the impact of introducing CBDC on the Federal Reserve’s balance sheet and the banking sector. For the lower end of the range, we assume that the quantity of bank deposits that are exchanged for CBDC is 0.4% of (annual) GDP, which is about $100 billion as of 2022, in the absence of a change in the deposit rate. For the higher end, we assume that the degree of substitution is ten times higher at 4% of GDP, on the order of $1 trillion as of 2022. The lower end of the range is roughly in line with estimates for moderate CBDC demand as a substitute for bank deposits in Adalid et al. (2022). Higher estimates for CBDC demand include about 3% of GDP for Sweden and Norway from Segendorf (2018) and Norges Bank (2021) and upper range of estimates from Adalid et al. (2022) of roughly 20% of Euro-area GDP. Thus, our assumption for the high end of the range is consistent with existing literature while being not as large as some of the more extreme estimates. Further, we assume that the demand for CBDC as a substitute for cash is 0.4% of GDP, about $100 billion as of 2022, which is also roughly in line with estimates of moderate take-up in Adalid et al. (2022).

For the deposit-rate sensitivity parameter \((g_D)\), we use 0.04, which means that one

\(^{22}\)While the model may capture some regulatory aspects related to the size of a bank’s balance sheet, other bank regulations do not depend only on a bank’s size. For example, regulatory requirements such as the Liquidity Coverage Ratio require banks to maintain a liquidity buffer as a cushion against outflows in deposits and other liabilities, and banks may demand less reserves to satisfy such requirements if the stock of their deposits declines with the introduction of CBDC. In practice, unless the immediate effect on the federal funds rate is undesirably large, the Federal Reserve could keep the size of its balance sheet unchanged rather than sell assets to reduce reserve supply, with the expectation that the general growth in non-reserve liabilities will reduce reserve supply, while the growth in the size of the banking sector will increase reserve demand.

\(^{23}\)In focusing on the long-run impact of CBDC, we are abstracting from the effects that may occur as new users begin to adopt CBDC and have in mind a time frame in which CBDC’s usage pattern has become firmly established.
Table 1: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_0$</td>
<td>Substitution for cash</td>
<td>0.4% of (annual) GDP</td>
</tr>
<tr>
<td>$D_0$</td>
<td>Substitution for deposits</td>
<td>0.4% / 4% of GDP</td>
</tr>
<tr>
<td>$g_D$</td>
<td>Deposit rate sensitivity</td>
<td>0.04</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Loan-deposit synergy</td>
<td>1.75 / 3.5</td>
</tr>
<tr>
<td>$c_B$</td>
<td>Marginal balance sheet costs</td>
<td>3 / 6</td>
</tr>
<tr>
<td>$s_L$</td>
<td>Slope of loan demand</td>
<td>0.06</td>
</tr>
<tr>
<td>$r_C$</td>
<td>Cash conversion rate</td>
<td>1</td>
</tr>
<tr>
<td>$r_D$</td>
<td>Deposit conversion rate</td>
<td>1</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>Policy parameter for CBDC</td>
<td>0</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>Policy parameter for deposits</td>
<td>0 / 1</td>
</tr>
<tr>
<td>$i_R$</td>
<td>Interest rate on reserves (IORB)</td>
<td>2.5%</td>
</tr>
<tr>
<td>$i_P$</td>
<td>Term premium</td>
<td>0.5%</td>
</tr>
</tbody>
</table>

The parameter values are chosen with the assumption that all monetary quantities are expressed as a fraction of annual GDP and all interest rates are expressed in percents. For example, $g_D = 0.04$ implies that one percentage point increase in deposit rate increases the demand for deposit by 0.04, or 4 percent, of annual GDP.

A percentage point decrease in the deposit rate reduces deposit demand of the private sector by 4 percent of GDP. Multiplying this by the ratio of deposits to GDP as of 2022, which was roughly 1.4, implies a semi-elasticity of about -5.6 (of deposits with respect to a decrease in deposit rates). This is in line with existing literature on deposit rate sensitivity: Drechsler et al. (2017) estimate the semi-elasticity of deposits with respect to EFFR-deposit rate spread to be -5.3. However, the sensitivity of money-like asset holdings to interest rates can vary to a great extent depending on context: Lam et al. (1989) estimate that one percentage point increase in money market fund yields leads to a 0.21 percent increase in assets under management, much smaller than what is implied for deposits based on our parameter choice.

We jointly calibrate the loan-deposit synergy parameter ($\alpha$) and the slope of the loan demand curve ($s_L$). We set the (negative) slope of the loan demand curve equal to 0.06, which implies that a one percentage point increase in the interest on bank loans reduces demand by 6 percent of GDP. This value is in line with those in the literature and modestly larger than the point estimates of Arseneau and Rappoport (2021) or Bassett et al. (2014). In order to parameterize the loan-deposit synergy, we examine the ratio of the change in bank lending due to the change in bank deposits implied by the model, which is determined by $\alpha s_L$. Here, results vary across studies in the literature. Whited et al. (2022) specifically study the loan-deposit synergy in the presence of a CBDC. In their calibrated model, a one

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24 This is based on the finding that one percentage point increase in the EFFR leads to a 0.61 percentage point increase in the EFFR-deposit rate spread and to a decrease in deposits of 3.25 percent.
A dollar increase in CBDC leads to a decline in bank deposits by about 0.7-0.8 dollar and in bank lending by to about 0.2-0.3 dollar. We set the loan-deposit synergy parameter $\alpha$ equal to 1.75, implying $\alpha s_L = 0.105$, which is somewhat smaller than what is implied by Whited et al. (2022). In the “greater disintermediation” scenario, we use a larger loan-deposit synergy parameter of 3.5, implying $\alpha s_L = 0.21$.

For the marginal balance sheet cost parameter ($c_B$), we choose 3 for the baseline scenario, which implies that a contraction of the banks’ aggregate balance sheet by one percent of GDP decreases the marginal balance sheet cost by 3 percentage points. To put this number into perspective, if banks lost $1$ trillion of reserves while maintaining all other assets in 2022, the marginal balance sheet cost would decrease by about 12 basis points. Such a change is broadly in line with the estimates of Armenter and Lester (2017), who find an estimated mean balance sheet cost equal to 15 basis points.25 We also conduct sensitivity analysis with regard to the balance sheet cost parameter, and in the “greater disintermediation” scenario, we double $c_B$ to 6.

In characterizing the private sector’s demand over different forms of money, we assume $r_C = r_D = 1$ for simplicity. This means that the private sector’s demand for money—CBDC, deposits and cash—is fixed in total quantity and only reallocation over these different forms of money occurs when CBDC is introduced. While many papers also make a similar assumption, $r_C$ and $r_D$ can in theory differ from 1. To our knowledge, there is very little existing literature that estimates these hypothetical “conversion rates” between cash or deposits and CBDC other than Whited et al. (2022), which estimates that a one-dollar increase in CBDC reduces bank deposits only by about 0.7-0.8 dollar, implying $r_D$ of about 1.3.

The policy parameters $\beta_1$ and $\beta_2$ are chosen to represent alternative scenarios about how a central bank may manage its reserve supply. We set the parameter $\beta_1 = 0$. If the presence of CBDC implied greater fluctuation or volatility in reserves, a central bank might choose to set this parameter positive in order to increase the size of its reserve buffer in response to the introduction of CBDC. Given that we do not model reserve volatility, we set the parameter equal to zero. The parameter $\beta_2$ determines the reaction of the central bank with respect to changes in bank deposits. Here, we consider two values. For the scenarios with reserve management purchases, we set $\beta_2 = 0$, which implies that the central bank engages in reserve management purchases to keep the level of reserves fixed. For the scenarios without reserve management purchases, we set $\beta_2 = 1$, which implies that the central bank is willing to let the level of reserves decrease with the outflow of

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25 In our model, $c_B$ refers to the slope of the marginal balance sheet cost with respect to the size of the banks’ aggregate balance sheet, and thus does not map directly into the the estimate of the mean balance sheet cost from Armenter and Lester (2017) or other empirical measures of balance sheet costs in the literature.
deposits. Note that this is equivalent to assuming that the central bank keeps the size of its balance sheet unchanged given $r_C = r_D = 1$.

In order to calculate the effect on net income, we set the IORB rate to 2.5%, which is consistent with the long-run median estimate of the federal funds rate in the Summary of Economic Projections (SEP) of September 2022. The term premium is set to 50 basis points, which is in line with historical estimates of term premiums for longer-dated Treasury securities (Adrian et al., 2013).

We present the quantitative results for three scenarios. In the first, we assume that the Federal Reserve responds to the introduction of CBDC by conducting reserve management purchases to keep reserves fixed at their steady-state level prior to introduction of CBDC. In the second, we modify this assumption and instead assume that the Federal Reserve does not conduct reserve management purchases and allows the level of reserves to decline with the fall in deposits. Finally, in the third scenario, we assume that there are greater synergies between bank deposits and bank loans so that the introduction of CBDC leads to greater disintermediation of the banking sector and significantly more upward pressure on the wholesale funding rate than in the other scenarios. We also assume that the reduction in the banks’ balance sheet cost associated with a balance sheet contraction is greater in this scenario, which augments the upward rate pressure.

4.1 Quantitative Results with Reserve Management Purchases

Tables 2 - 4 display the changes in various quantities and interest rates in equilibrium following the introduction of CBDC, assuming annual GDP of $25 trillion, roughly its value in 2022. The T-accounts—assets and liabilities—for the central bank and the commercial banking sector on the left-hand side show the effects when there is low substitution of CBDC for bank deposits and the T-accounts on the right show the effects when there is high substitution. In both cases, it is assumed that the Federal Reserve engages in reserve management purchases so that the level of reserves is fixed at its steady-state level prior to the introduction of CBDC. Accordingly, any decline in reserves due the exchange of bank deposits for CBDC is offset by increased Federal Reserve asset holdings. Under this assumption, total assets of the Federal Reserve increase by $99 billion, in the case of low substitution, or by $995 billion, in the case of high substitution. In both cases, the increase in CBDC on the Fed’s balance sheet also reflects $100 billion substitution of CBDC for cash. The substitution of CBDC for bank deposits ranges from $99 billion to $995 billion.

In both cases, the changes in deposits reflect the direct reduction in deposit demand in

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27 Assets and liabilities may not balance due to rounding.
favor of CBDC and the offsetting indirect effect of banks’ responding by increasing deposit rates to retain deposits. As shown in Table 4, the deposit rate increases a bit in both the low and high substitution scenarios. In addition, the model’s calibration is such that the private sector’s demand for deposits is relatively insensitive to the deposit rate so the higher deposit rates do not have large offsetting effects on the level of deposits. Because of the synergy between bank deposits and loans, the decline in bank deposits leads to a decline in the supply of bank credit that ranges from $10 billion to $104 billion. With the supply of bank credit contracting, banks’ lending rate increases, ranging from 0.7 to 7 basis points.

Facing reduced balance sheet costs with smaller balance sheets, banks are more willing to raise funds in wholesale markets. As a result, the interest rate on wholesale funding relative to the IORB rate increases by roughly 0.1 and 1 basis points in the low and high substitution scenarios, respectively. Moreover, the volume of wholesale funding rises by approximately $89 for the low substitution scenario, and $890 billion in the high substitution scenario. This increase offsets some of the decline in deposit funding in banks’ liabilities.

We also consider the effect of the introduction of a CBDC on the net income of the central bank. In our analytical framework, the change in (annual) net income, \( \Delta(Net\ income) \) can be written as

\[
\Delta(Net\ income) = (i_R + i_P)\Delta(Treasury_{CB}) - i_R\Delta(Reserves),
\]

where \( i_P \) denotes the term premium. The change in net income has two components. The first component is the change in interest income due to the change in Treasury securities holdings. To represent the long-run average effect on the net income, we assume that Treasury securities pay interest rate tied to the current level of IORB (plus the term premium). The second component is the change in interest expenses due to the change in reserves.

Under the active reserve management scenario, \( \Delta(Reserves) = 0 \). Thus, the change in net income can be written as

\[
\Delta(Net\ income) = (i_R + i_P)\Delta(Treasury_{CB}) = -(i_R + i_P)\Delta(Deposits).
\]

Under this scenario, the central bank purchases Treasury securities to offset the decline in reserve supply due to the conversion of deposits into CBDC by the private sector. The amount of purchases needs to be identical to the decline in the amount of outstanding deposits to keep reserve supply unchanged on net. In our calibration, the Fed’s net income increases by about $3 billion in the low substitution scenario and about $30 billion in the high substitution scenario. With asset holdings expanding so that reserves remain

21
unchanged in these scenarios, the increase in net income reflects the additional interest income associated with the Fed’s larger Treasury securities holdings, while interest expense remains unchanged since CBDC is unremunerated by assumption.

<table>
<thead>
<tr>
<th>Central Bank</th>
<th>Central Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSY (CB) +99</td>
<td>TSY (CB) +995</td>
</tr>
<tr>
<td>Reserves 0</td>
<td>Reserves 0</td>
</tr>
<tr>
<td>Gov Cash 0</td>
<td>Gov Cash 0</td>
</tr>
<tr>
<td>Cash -100</td>
<td>Cash -100</td>
</tr>
<tr>
<td>CBDC +199</td>
<td>CBDC +1,095</td>
</tr>
</tbody>
</table>

(a) Low substitution  (b) High substitution

Table 2: Changes in the central bank’s balance sheet (in billions) with reserve management purchases

<table>
<thead>
<tr>
<th>Banks</th>
<th>Banks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves 0</td>
<td>Reserves 0</td>
</tr>
<tr>
<td>Deposits -99</td>
<td>Deposits -995</td>
</tr>
<tr>
<td>Loans -10</td>
<td>Loans -104</td>
</tr>
<tr>
<td>Wholesale funds +89</td>
<td>Wholesale funds +890</td>
</tr>
</tbody>
</table>

(a) Low substitution (b) High substitution

Table 3: Changes in the balance sheet of the banking sector (in billions) with reserve management purchases

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Low substitution</th>
<th>High substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in wholesale funding rate</td>
<td>+0.1 bp</td>
<td>+1.3 bp</td>
</tr>
<tr>
<td>Change in deposit rate</td>
<td>+0.1 bp</td>
<td>+0.5 bp</td>
</tr>
<tr>
<td>Change in loan rate</td>
<td>+0.70 bp</td>
<td>+7.0 bp</td>
</tr>
<tr>
<td>Change in net income (in billions)</td>
<td>+3</td>
<td>+30</td>
</tr>
</tbody>
</table>

Table 4: Effects on interest rates and central bank income with reserve management purchases

4.2 Quantitative Results without Reserve Management Purchases

Tables 5 - 7 display the results for scenarios with a low and a high substitution of CDBC for deposits under the assumption that the Federal Reserve does not engage in reserve management, letting the level of reserves decline as bank deposit balances are exchanged for CBDC. Under this assumption, Federal Reserve asset holdings are unchanged and the decline in reserves ranges from $89 billion to $889 billion. With reserves declining, the overall size of banks’ balance sheet declines more than in the corresponding scenarios with reserve management purchases. With a larger decline in the associated balance sheet costs, banks respond more aggressively to retain deposits than in the corresponding scenarios with
reserve management purchases and thus the deposit rates rise by more and deposit and loan balances fall by less. However, because of the relative insensitivity of the private sector’s demand for bank deposits to the deposit rate, the effect on bank deposits is small, and with the aggregate level of bank reserves falling, aggregate bank assets decline by more than in the corresponding scenarios with reserve management purchases. This decline in bank assets reduces banks’ need to raise wholesale funds, which decline by $9 billion in the low substitution scenario and by about $93 billion in the high substitution scenario.\textsuperscript{28} With banks’ balance sheets contracting more, banks are willing to pay more for wholesale funds and the wholesale funding rate rises more than in the scenarios with reserve management purchases. As shown in Table 7, in the high substitution scenario, the wholesale funding rate rises about 12 basis points, compared to 1 basis point in the corresponding scenario in which the Fed makes reserve management purchases.\textsuperscript{29}

Without reserve management purchases, the Fed’s net income from Equation (20) simplifies to

$$\Delta(\text{Net income}) = i_R \Delta(\text{Reserves}) = -i_R \Delta(\text{Deposits}),$$

since $\Delta(Treasury_{CB}) = 0$. As in the previous scenario, the change in reserves is equal to the change in bank deposits. In our calibration, the Fed’s net income increases by $2 billion and $22 billion in the low and high substitution scenarios, respectively, reflecting the shift away from interest-bearing reserves into non-interest-bearing CBDC. Net income increases less than in the case of reserve management purchases because the contribution to net income associated with earning a term premium on a larger portfolio of assets does not occur in this case.

<table>
<thead>
<tr>
<th>Central Bank</th>
<th>Central Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSY (CB) 0</td>
<td>TSY (CB) 0</td>
</tr>
<tr>
<td>Reserves</td>
<td>Reserves</td>
</tr>
<tr>
<td>-89</td>
<td>-889</td>
</tr>
<tr>
<td>Gov Cash</td>
<td>Gov Cash</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cash</td>
<td>Cash</td>
</tr>
<tr>
<td>-100</td>
<td>-100</td>
</tr>
<tr>
<td>CBDC</td>
<td>CBDC</td>
</tr>
<tr>
<td>+189</td>
<td>+989</td>
</tr>
</tbody>
</table>

(a) Low substitution (b) High substitution

Table 5: Changes in the central bank’s balance sheet under no reserve management purchases (in billions)

\textsuperscript{28}While the decline in aggregate reserves is important in accounting for the decline in wholesale funding, the synergy between loan and deposits also plays an important role since it implies that the decline in bank deposits leads to a decline in bank lending, reducing banks’ overall demand for funding.

\textsuperscript{29}With the wholesale funding rate rising 12 basis points in the high substitution scenario, in principle a technical adjustment could be made by lowering the IORB rate to put downward pressure on the wholesale funding rate.
Banks
Reserves −89
Deposits −89
Loans −9
Wholesale funds −9
(a) Low substitution

Reserves −889
Deposits −889
Loans −93
Wholesale funds −93
(b) High substitution

Table 6: Changes in the balance sheet of the banking sector under no reserve management purchases (in billions)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Low substitution</th>
<th>High substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in wholesale funding rate</td>
<td>+1.2 bp</td>
<td>+11.8 bp</td>
</tr>
<tr>
<td>Change in deposit rate</td>
<td>+1.1 bp</td>
<td>+11.1 bp</td>
</tr>
<tr>
<td>Change in loan rate</td>
<td>+0.6 bp</td>
<td>+6.2 bp</td>
</tr>
<tr>
<td>Change in net income (in billions)</td>
<td>+2</td>
<td>+22</td>
</tr>
</tbody>
</table>

Table 7: Effects on interest rates and central bank income under no reserve management purchases

4.3 Greater Disintermediation Scenario

The earlier scenarios highlight that without reserve management purchases, there will be more upward pressures on the wholesale funding rate. This upward pressure will be more significant if the introduction of CBDC implies greater disintermediation of the banking sector. To explore this possibility, we present a third scenario, where we assume that there are greater synergies between loans and deposits than assumed earlier. Moreover, we assume that banks’ balance sheet costs decrease more with a contraction in their balance sheets, which makes the wholesale funding rate more sensitive to changes in the size of the banks’ balance sheets. These assumptions correspond to the use of larger values of $\alpha$ and $c_B$ than in the baseline, as listed in Table 1. The T-accounts on the left-hand side of Tables 8 - 10 show the effects with reserve management purchases that keep reserves fixed at their level prior to the introduction of CBDC and the T-accounts on the right-hand side show the effects without reserve management purchases. We assume a high degree substitution of CBDC for deposits in both cases.

In this scenario, the outflow of money from bank deposits to CBDC leads to a larger decline in bank credit than in the previous scenarios due to the greater synergy between loans and deposits. As a result, banks’ balance sheets also shrink more. In the case without reserve management purchases, the wholesale funding rate rises by about 23 basis points. With the wholesale funding rate proxying for the EFFR, such an increase could push the EFFR out of its target range. Accordingly, the scenario suggests that reserve management purchases might be necessary to maintain effective control of the federal funds rate if there was a large outflow of bank deposits to CBDC that led to greater disintermediation. As shown in Table 10, such purchases would alleviate much of the upward pressure on the
wholesale funding rate.

<table>
<thead>
<tr>
<th>Central Bank</th>
<th>Central Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSY (CB)</td>
<td>TSY (CB)</td>
</tr>
<tr>
<td>Reserves</td>
<td>Reserves</td>
</tr>
<tr>
<td>0</td>
<td>-793</td>
</tr>
<tr>
<td>Gov Cash</td>
<td>Gov Cash</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cash</td>
<td>Cash</td>
</tr>
<tr>
<td>-100</td>
<td>-100</td>
</tr>
<tr>
<td>CBDC</td>
<td>CBDC</td>
</tr>
<tr>
<td>+1,079</td>
<td>+893</td>
</tr>
</tbody>
</table>

(a) Active reserve management

(b) No reserve management

Table 8: Changes in the central bank’s balance sheet (in billions) under greater disintermediation

<table>
<thead>
<tr>
<th>Banks</th>
<th>Banks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves</td>
<td>Reserves</td>
</tr>
<tr>
<td>0</td>
<td>-793</td>
</tr>
<tr>
<td>Deposits</td>
<td>Deposits</td>
</tr>
<tr>
<td>-979</td>
<td>-793</td>
</tr>
<tr>
<td>Loans</td>
<td>Loans</td>
</tr>
<tr>
<td>-206</td>
<td>-167</td>
</tr>
<tr>
<td>Wholesale funds</td>
<td>Wholesale funds</td>
</tr>
<tr>
<td>+774</td>
<td>-167</td>
</tr>
</tbody>
</table>

(a) Active reserve management

(b) No reserve management

Table 9: Changes in the balance sheet of the banking sector (in billions) under greater disintermediation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Reserve mgt</th>
<th>No reserve mgt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in wholesale funding rate</td>
<td>+4.9 bp</td>
<td>+23.0 bp</td>
</tr>
<tr>
<td>Change in deposit rate</td>
<td>+2.1 bp</td>
<td>+20.7 bp</td>
</tr>
<tr>
<td>Change in loan rate</td>
<td>+13.7 bp</td>
<td>+11.1 bp</td>
</tr>
<tr>
<td>Change in net income (in billions)</td>
<td>+29</td>
<td>+20</td>
</tr>
</tbody>
</table>

Table 10: Effects on interest rates and central bank income under greater disintermediation

5 Conclusion

We have developed a model to characterize changes to the Federal Reserve’s balance sheet following a hypothetical issuance of CBDC. While stylized, the model is tractable and captures important and practically relevant factors that determine changes to the Federal Reserve’s balance sheet. We calibrate the parameters of the model in light of past findings in the banking literature and show that a wide range of balance sheet outcomes are possible. We show that an important factor in determining balance sheet outcomes is the extent to which CBDC is substituted for cash and bank deposits by the private sector and how banks react to the potential outflow of deposits.

Our analysis highlights that the potential effects of CBDC on the financial sector depend crucially on how the Federal Reserve manages its balance sheet. If the Fed did not
conduct reserve management purchases, there could potentially be a larger reduction in bank assets and more upward pressure on bank funding costs that could push the policy rate (EFFR) outside of its target range. To ensure interest rate control, the Fed may have to conduct reserve management purchases, expanding the size of its balance sheet. Thus, it is important that policy makers take into account balance sheet management considerations when contemplating the introduction of a CBDC.

\[30\] Such purchases would be distinct from asset purchases that would occur in the absence of a CBDC in order to accommodate the general growth of the economy and an associated increase in demand for reserves and non-reserve liabilities.
References


David M Arseneau and David E Rappoport. Optimal design of funding for lending programs. mimeo, 2021.


Online Appendix

A Money Demand in the Private Sector

In this section of the appendix, we outline a stylized model of money choice that gives rise to the characterization of money demand consistent with Equations (12), (13) and (14). In the model, “households” have different “types,” represented by a vector $y$. The household of type $y$ chooses holdings of cash, $C(y)$, deposits, $D(y)$, and CBDC, $X(y)$, to maximize its utility:

$$\max_{C(y), D(y), X(y) \geq 0} u(M(y)) - f_C(y)C(y) - (f_D(y) - \gamma_D(y)i_D)D(y) - (f_X(y) - \gamma_X(y)i_X)X(y).$$  \hfill (A-1)

$M(y) \equiv v_C(y)C(y) + v_D(y)D(y) + v_X(y)X(y)$ is the total money holding by household $y$. We interpret $v_j$, $j \in \{C, D, X\}$ as representing the efficiency or convenience of each type of money. $f_j$ represents the cost of holding each type of money, which may represent either opportunity cost or heterogeneous individual preferences over different types of money. $i_D$ and $i_X$ are interest rates on deposit and CBDC, respectively, and $\gamma_j$’s are positive constants that relate higher rates to effectively lower cost for holding deposits or CBDC.

The utility of money is defined as follows:

$$u(M) = 2M - \frac{1}{2}M^2.$$  \hfill (A-2)

Note that $u(M)$ is decreasing over $M > 2$; to ensure non-decreasing utility, we can assume $u(M) = u(2) = 2$ for $M > 2$. This assumption is not important because in the remainder of the paper, we only consider parameter values that make households choose $0 < M < 2$.

The type vector $y$ has two elements: $y = (y_1, y_2)$, where $y_1 \in \{C, D\}$ and $y_2 \in [0, 1]$. The density of households is given as follows:

$$\mu(C, y_2) = m_C;$$  \hfill (A-3)

$$\mu(D, y_2) = m_D.$$  \hfill (A-4)

In other words, household density is uniform for each $y_1$. 

A-1
Next, we assume the following about the effectiveness of money, $v_j$:

\[ v_C(C, y_2) = v_X(C, y_2) = 1, v_D(C, y_2) = 0; \quad (A-5) \]
\[ v_D(D, y_2) = v_X(D, y_2) = 1, v_C(D, y_2) = 0. \quad (A-6) \]

These assumptions imply a complete separation between households with $y_1 = C$ and those with $y_2 = D$, in that $y_1 = C$ households will always choose $D = 0$ and $y_1 = D$ households will always choose $C = 0$. The only substitution is between $C$ and $X$ or between $D$ and $X$, depending on $y_1$.

Moreover, we assume $f_C = f_D = 1$ for all types $y$. The choice of $f_X$ is interpreted as a design choice of CBDC and will be discussed later.

**Optimal choice with no CBDC and no interest rate sensitivity:** This is the simplest case which helps illustrate how the model works. We suppress any interest rate response by setting $\gamma_D = \gamma_X = 0$. And we effectively assume that there is no CBDC in the economy by setting $f_X = +\infty$.

With these assumptions, the problem for households with $y_1 = C$ is simplified:

\[ \max_{C,D \ge 0} u(C) - C - D. \quad (A-7) \]

The optimal choice is such that $u'(C) = 1$, implying $C = 1$. The optimal choice of $D$ is 0 because there is no benefit from increasing $D$, as $D$ does not enter the utility function $u$. Similarly, for households with $y_1 = D$, the optimal choice is $C = 0$ and $D = 1$.

The aggregate demand for cash and deposit is obtained by integrating individual choices over the type space:

\[ (Cash) = \int C(y)\mu(y)dy = \int C(C, y_2)\mu(C, y_2)dy_2 = m_C. \quad (A-8) \]
\[ (Deposits) = \int D(y)\mu(y)dy = m_D. \quad (A-9) \]

**Introduction of CBDC with no interest rate sensitivity:** We introduce CBDC by assuming that $f_X$ is no longer prohibitively high. We suppose that the central bank introduces a CBDC with a linear $f_X$: For $y_1 = C$,

\[ f_X(C, y_2) = f_X(C, 0) + f'_X(C, 0)y_2 = f_X(C) + f'_X(C)y_2. \quad (A-10) \]

We assume $f_X(C) < 1$ and $f'_X(C) > 0$. Furthermore, we assume that $f_X(C)$ and $f'_X(C)$ are such that $f_X(C, y_2)$ is smaller than 1 for some $y_2$ and greater than 1 for some other $y_2$. This will make sure that some but not all cash-holding households choose to hold CBDC
Similarly, 
\[
    f_X(D, y_2) = f_X(D) + f'_X(D)y_2. \tag{A-11}
\]

Among the households with \( y_1 = C \), those with \( y_2 \in [0, (1 - f_X(C))/f'_X(C)] \) prefer CBDC to cash and demand only CBDC while the remainder demand only cash. A household demanding only CBDC solves the following problem:

\[
    \max_X u(X) - [f_X(C, 0) + f'_X(C)y_2]X. \tag{A-12}
\]

The first-order condition implies \( X = 2 - f_X(C) + f'_X(C)y_2 \). Given the linear relationship between the optimal \( X \) and \( y_2 \), the average demand for CBDC among households with \( y_1 = C \) and \( y_2 \in [0, (1 - f_X(C))/f'_X(C)] \) is identical to the average demand of two types of households, \( y_2 = 0 \) and \( y_2 = (1 - f_X(C))/f'_X(C) \):

\[
    \frac{1}{2}[(2 - f_X(C)) + 1] = 1 + \frac{1}{2}(1 - f_X(C)). \tag{A-13}
\]

Each household holding positive cash still holds 1 unit of cash, as before.

Similarly, among the households with \( y_1 = D \), those with \( y_2 \in [0, (1 - f_X(D))/f'_X(D)] \) prefer CBDC to deposits and hold on average \( 1 + (1/2)(1 - f_X(D)) \) units of CBDC.

Thus, the overall demand for each type of money is as follows:

\[
    (\text{Cash}) = (1 - \frac{1 - f_X(C)}{f'_X(C)})m_C. \tag{A-14}
\]

\[
    (\text{Deposit}) = (1 - \frac{1 - f_X(D)}{f'_X(D)})m_D. \tag{A-15}
\]

\[
    (\text{CBDC}) = r_C(m_C - (\text{Cash})) + r_D(m_C - (\text{Deposit})). \tag{A-16}
\]

We define \( r_C = 1 + (1/2)(1 - f_X(C)) \) and \( r_D = 1 + (1/2)(1 - f_X(D)) \).

We can write the change from the previous case with no CBDC as follows:

\[
    \Delta(\text{Cash}) = -C_0. \tag{A-17}
\]

\[
    \Delta(\text{Deposit}) = -D_0. \tag{A-18}
\]

\[
    \Delta(\text{CBDC}) = -r_C\Delta(\text{Cash}) - r_D\Delta(\text{Deposit}). \tag{A-19}
\]

Note \( C_0 = (\text{Cash}) - m_C \), where the expression for \( (\text{Cash}) \) is given by the expressions for money demand just derived. \( D_0 = (\text{Deposit}) - m_D \) is similarly defined.

**Introduction of CBDC with interest rate sensitivity:** Next, we allow interest rate
sensitivity. We only consider interest rate sensitivity by depositors and make the response to the deposit rate and the CBDC remuneration rate symmetric. That is,

\[ \gamma_D(C, y_2) = \gamma_X(C, y_2) = 0; \]  
\[ \gamma_D(D, y_2) = \gamma_X(D, y_2) = \gamma. \]  
\[ (A-20) \]
\[ (A-21) \]

\( \gamma \) is a positive constant.

This affects both the choice between deposit and CBDC and the choice of the total amount of money to demand. Note that for households with \( y_1 = D \), the cost of holding a unit of deposit is \( 1 - \gamma i_D \) and that of holding a unit of CBDC is \( f_X(D) - \gamma i_X + f_X'(D)y_2 \).

Comparing these two costs, households with \( y_1 = D \) demand only CBDC but not deposits if

\[ y_2 < \frac{1 - f_X(D) - \gamma(i_D - i_X)}{f_X'(D)}. \]  
\[ (A-22) \]

These households demand the following amount of CBDC on average, conditional on demanding a positive amount:

\[ 1 + \frac{1}{2}(1 - f_X + \gamma i_X). \]  
\[ (A-23) \]

Each household demanding a positive amount of deposits demands the following amount:

\[ 1 + \gamma i_D. \]  
\[ (A-24) \]

Before aggregating individual choices, we make the following assumption to make the problem simpler: \( i_X = 0 \). Alternatively, we can think of this as redefining the value of \( f_X(D) \) as \( f_X(D) - \gamma i_X \). Either way, we do not care much about this in the paper because we consider \( i_X \) an exogenous choice by the central bank.

With no CBDC (\( v_X = 0 \) or \( f_X = +\infty \)), households demand the following amount of deposits in aggregate:

\[ (Deposit) = (1 + \gamma i_D)m_D. \]  
\[ (A-25) \]

The expression for cash or CBDC demand is unchanged from the case with no interest rate sensitivity.
With CBDC, households demand the following amounts of money:

\[
(\text{Deposit}) = \left[1 - \frac{1 - f_X(D) - \gamma(i_D + \Delta i_D)}{f_X'(D)}\right][1 + \gamma(i_D + \Delta i_D)]m_D. \tag{A-26}
\]

\[
(CBDC) = r_C(m_C - (\text{Cash})) + \left[1 - \frac{1 - f_X(D) - \gamma(i_D + \Delta i_D)}{f_X'(D)}\right][1 + \frac{1}{2}(1 - f_X(D))]m_D. \tag{A-27}
\]

Again the expression for cash demand is omitted because it is unchanged. Similarly, the term related to the choice of cash holders (\(y_1 = C\)) in the expression for CBDC remains the same as that in the previous case. Assuming a relatively small change in \(i_D\) by \(\Delta i_D\), we ignore the second order term \(\Delta i_D^2\). Then, the level of deposit demand after the introduction of CBDC is

\[
(\text{Deposit}) = (1 + \gamma i_D)m_D - D_0 + D_1 \Delta i_D + D_2 \Delta i_D. \tag{A-29}
\]

Note the following definition:

\[
D_0 = \left[1 - \frac{1 - f_X(D) - \gamma i_D}{f_X'(D)}\right](1 + \gamma i_D)m_D. \tag{A-30}
\]

\[
D_1 = \frac{\gamma}{f_X(D)}(1 + \gamma i_D)m_D. \tag{A-31}
\]

\[
D_2 = \left[1 - \frac{1 - f_X(D) - \gamma i_D}{f_X'(D)}\right]\gamma m_D. \tag{A-32}
\]

These terms have a straightforward interpretation. \(D_0\) is how much deposit demand would decline due to households switching from deposit to CBDC if there were no change in the deposit rate. \(D_1\) characterizes the increase in the mass of households choosing deposit over CBDC associated with an increase in deposit rate. \(D_2\) captures the increase in deposit demand associated with an increase in deposit rate by those households that choose deposits over CBDC.

The change in deposit demand between pre- and post-CBDC is:

\[
\Delta(\text{Deposit}) = -D_0 + D_1 \Delta i_D + D_2 \Delta i_D. \tag{A-33}
\]

The change in CBDC demand is:

\[
\Delta(CBDC) = -r_C\Delta(\text{Cash}) + r_D(D_0 - D_1 \Delta i_D). \tag{A-34}
\]

Note that \(r_D\) is defined as follows, differently from before:

\[
r_D = \frac{1}{1 + \gamma i_D}[1 + \frac{1}{2}(1 - f_X(D))]. \tag{A-35}
\]
Consistent with the earlier interpretation, $D_1$ appears both in deposit demand and CBDC demand as it characterizes the decrease in the mass of population choosing CBDC over deposit in response to an increase in the deposit rate. $D_2$ appears only in deposit demand because it characterizes the behavior of households that do not demand CBDC.

We ignore $D_2$ in the rest of the paper because we are mainly interested in capturing the substitution between deposits and CBDC. We can easily change our assumptions about households to bring out this result if we wish; for example, we may assume that each household demands exactly 1 unit of money, in which case only the substitution term $D_1$ will be nonzero.\(^{31}\)

Thus we have the following set of equations characterizing households’ demand for different forms of money:

\[
\begin{align*}
\Delta(\text{Cash}) &= -C_0. \quad \text{(A-36)} \\
\Delta(\text{Deposit}) &= -D_0 + D_1 \Delta i_D. \quad \text{(A-37)} \\
\Delta(\text{CBDC}) &= -r_C \Delta(\text{Cash}) - r_D \Delta(\text{Deposit}). \quad \text{(A-38)}
\end{align*}
\]

These equations are identical to the assumptions we make about households’ money demand in Equations (12), (13) and (14), except that we use $g_D$ in place of $D_1$. In the paper, we generally assume $r_C = r_D = 1$ while based on the model, $r_C > 1$; however it can be made arbitrarily close to 1. If needed, we can change $r_C$ and $r_D$ by modifying the functional form of $f_X$ or assuming $v_X > 1$; higher $v_X$ tends to result in lower demand for CBDC among households that choose CBDC over cash and deposits given the curvature in the utility function of money. We do not explicitly solve for the case of $v_X > 1$ in this section, as it is not necessary to motivate the equations characterizing money demand in the main text.

### B Solving the Model

In the section, we explicitly characterize the equilibrium. Combining Equations (6) and (7) characterizing the banks’ behavior; Equations (12), (14) and (15) describing the private sector’s cash, CBDC and loan demand; and Equation (10) describing the central bank’s reserve management, we obtain an equation that effectively captures the banking sector’s supply of deposit liability:

\[
\Delta(\text{Deposits}) = -h_D \Delta i_D - h_0. \quad \text{(A-39)}
\]

\(^{31}\)Generally, $D_1$ is based on the characteristic of the marginal household that is roughly indifferent between deposit and CBDC, while $D_2$ is based on the characteristics of non-marginal households.
In the equation, $h_D \equiv [(c_B - \alpha)\alpha s_L - c_B\beta_1 r_D + c_B\beta_2]^{-1}$ and $h_0 \equiv h_D c_B \beta_1 r_C C_0$. Broadly speaking, this equation captures the effect of banks’ assets—reserves and loans—on deposit demand through balance sheet costs and the synergy between loan and deposit. In the rest of the paper, we consider parameter values consistent with $h_D > 0$; justifying this assumption, we see that this condition holds if we assume reasonable values for parameters based on existing literature in Section 4.

The intersection between the banking sector’s supply of deposits and the private sector’s demand for deposits from Equation (13) gives

$$\Delta(\text{Deposits}) = -\eta D_0 - (1 - \eta)h_0.$$  \hspace{1cm} \text{(A-40)}

$$\Delta i_D = \frac{1}{g_D + h_D} (D_0 - h_0).$$  \hspace{1cm} \text{(A-41)}

We define $\eta \equiv h_D / (g_D + h_D)$. In the equation for the change in deposits, the attenuation of $-D_0$ by the less-than-one factor $\eta$ represents the offsetting response by the banking sector to deposit outflow, while the term $(1 - \eta)h_0$ captures how the central bank expanding its reserve supply buffer in response to cash-to-CBDC conversion by the private sector may increase the burden of balance sheet cost on banks and discourage banks from taking in deposits. As expected, the latter effect is present only if $\beta_1 > 0$.

From Equation (14), the take-up of CBDC, identical to the change in CBDC (because the amount of CBDC in the pre-CBDC economy is zero), is

$$\Delta(CBDC) = r_C C_0 + r_D (\eta D_0 + (1 - \eta)h_0).$$  \hspace{1cm} \text{(A-42)}

The central bank’s reserve management in Equation (10) gives us

$$\Delta(\text{Reserves}) = \beta_1 [r_C C_0 + r_D (\eta D_0 + (1 - \eta)h_0)] - \beta_2 (\eta D_0 + (1 - \eta)h_0).$$  \hspace{1cm} \text{(A-43)}

The change in the central bank’s Treasury holdings equals the sum of changes in its liabilities, as in Equation (11):

$$\Delta(\text{Treasury}_C) = (1 + \beta_1)[r_C C_0 + r_D (\eta D_0 + (1 - \eta)h_0)] - \beta_2 [\eta D_0 + (1 - \eta)h_0] - C_0.$$  \hspace{1cm} \text{(A-44)}

The private sector’s bank loan demand in Equation (15), along with Equation (6), gives

$$\Delta(\text{Loans}) = \alpha s_L \Delta(\text{Deposits}) = -\alpha s_L (\eta D_0 + (1 - \eta)h_0).$$  \hspace{1cm} \text{(A-45)}

The change in the amount of loans is related to the change in deposits through loan-deposit synergy and the slope of the private sector’s loan demand. The change in the loan interest
rate is
\[ \Delta i_L = -\frac{1}{s_L} \Delta (Loans) = \alpha(\eta D_0 + (1 - \eta)h_0). \] (A-46)

Changes in banks’ assets and liabilities are the same, as in Equation (9), and this lets us calculate the change in wholesale bank funding:
\[ \Delta (Wholesale \ funding) = (1 + \beta_1 r_D - \beta_2 - \alpha s_L)(\eta D_0 + (1 - \eta)h_0) + \beta_1 r_C C_0. \] (A-47)

The change in the wholesale funding rate is the negative value of the change in marginal balance sheet cost faced by banks (Equation 8):
\[ \Delta i_E = c_B(\alpha s_L - \beta_1 r_D + \beta_2)(\eta D_0 + (1 - \eta)h_0) - c_B \beta_1 r_C C_0. \] (A-48)

The government does not change net debt or its cash holdings at the central bank, thus the sum of Treasury holding by the central bank and the private sector is unchanged (Equation 17):
\[ \Delta (Treasury_{Private}) = -\Delta (Treasury_{CB}) \\
= -(1 + \beta_1)[r_CC_0 + r_D(\eta D_0 + (1 - \eta)h_0)] + \beta_2[\eta D_0 + (1 - \eta)h_0] + C_0. \] (A-49)