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Diego Comin, Robert Johnson, Callum Jones

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Supply Chain Constraints and Inflation*

Diego Comin† Robert C. Johnson‡ Callum Jones§

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Abstract

We develop a multisector, open economy, New Keynesian framework to evaluate how potentially binding capacity constraints, and shocks to them, shape inflation. We show that binding constraints for domestic and foreign producers shift domestic and import price Phillips Curves up, similar to reduced-form markup shocks. Further, data on prices and quantities together identify whether constraints bind due to increased demand or reductions in capacity. Applying the model to interpret recent US data, we find that binding constraints explain half of the increase in inflation during 2021-2022. In particular, tight capacity served to amplify the impact of loose monetary policy in 2021, fueling the inflation takeoff.

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†Dartmouth College, NBER, and CEPR. Email: diego.comin@dartmouth.edu.

‡University of Notre Dame & NBER. Email: rjohns24@nd.edu.

§Federal Reserve Board. Email: callum.j.jones@frb.gov.

In the later half of 2021 and into 2022, the United States experienced a burst of inflation as it emerged from the COVID-19 pandemic, led by a large increase in goods price inflation. Popular narratives suggest that strong consumer demand bumped up against constraints on the supply of goods, fueling inflation.[1] Further, in their public statements, policymakers frequently blamed disruptions in both domestic and foreign segments of supply chains for restraining the supply of goods.[2] Despite the plausibility of this narrative, it has been difficult to evaluate the quantitative importance of supply chain constraints for inflation, not least because we lack models that capture their impact.

In this paper, we investigate how potentially binding capacity constraints for domestic and foreign producers shape inflation in a multisector, open economy, New Keynesian (NK) model with imported inputs and input-output linkages across sectors. Solving for the model's non-linear equilibrium dynamics via piecewise linear approximations, we develop a Bayesian maximum likelihood procedure to estimate key parameters and infer when constraints bind. We then apply the model to quantify how constraints in the supply chain, and potential shocks to them, have influenced recent data outcomes. We find that binding constraints account for about half (two percentage points) of the increase in inflation during 2021-2022. Interestingly, no single set of shocks can explain the inflation takeoff. Rather, shocks that tightened capacity set the stage for demand shocks – most importantly, monetary policy shocks – to trigger binding constraints and accelerate inflation in 2021. Relaxation of the constraints, in part due to monetary tightening, then also explains the rapid decline in goods price inflation in the latter half of 2022.

The framework we develop features occasionally binding constraints in two different places. The first is a constraint that applies at the level of individual foreign firms, whereby foreign producers are able to supply output at constant marginal costs up to a predetermined level, at which point production is quantity-constrained. Motivated by evidence on disruptions in markets for imported inputs, we devote particular attention to binding constraints on foreign input supply. The second constraint is a similar limit on production capacity for domestic firms, which impacts both downstream firms and consumers. These dual constraints allow us to separately capture the role of domestic versus foreign supply chain disruptions on inflation.

Further, this framework features a distinction between supply-side versus demand-side explanations for binding constraints, with potentially important implications for policy. On the supply side, we assume the levels of the capacity constraints are exogenous and subject to stochastic

[1] See The Economist (2021), Rees and Rungcharoenkitkul (2021), and de Soyres, Santacreu and Young (2023).

[2] In International Monetary Fund (2021), Gita Gopinath writes: "Pandemic outbreaks in critical links of global supply chains have resulted in longer-than-expected supply disruptions, further feeding inflation in many countries." Smialek and Nelson (2021) characterize the views of the US Federal Reserve chair as follows: "[Jerome Powell] noted that while demand was strong in the United States, factory shutdowns and shipping problems were holding back supply, weighing on the economy and pushing inflation above the Fed's goal." See Lane (2022) for a discussion of views at the European Central Bank, and Goodman (2021) for a narrative of supply chain breakdowns.

shocks.[3] This formulation captures the type of time-varying input shortages that occurred during the COVID period, both in the United States and abroad.[4] On the demand side, an increase in demand may also exhaust excess capacity and induce capacity constraints to bind in the model. This alternative mechanism is salient, because the abrupt recovery of demand in 2021 seemed to stress existing supply chain capacity.

Separating these two mechanisms – that binding constraints may be the result of strong demand, or disruptions to capacity – represents a key quantitative challenge. Breaking the challenge into two pieces, we must ascertain whether constraints bind, while also identifying why they bind.

To shed light on how binding constraints may be detected, we note that binding constraints impact pricing decisions. In the model, constraints are internalized by each firm as it sets its price, such that the firm's optimal markup differs depending on whether the constraint is binding. Assuming that both exports and imports are invoiced in US Dollars, and prices are subject to adjustment frictions, then domestic and import price inflation satisfy Phillips Curve type relationships. When the domestic constraint binds, we show that there is an additional term in sector-level, domestic price Phillips Curves that resembles a markup (equivalently, cost-push) shock. Similarly, there is a quasi-markup shock in the import price Phillips Curve when the import constraint binds. Thus, our framework provides a structural interpretation for reduced-form markup/cost-push shocks, based on binding constraints.

This "markup shock" interpretation of the role of binding constrains dovetails well with related work by Bernanke and Blanchard (2023), which uses an empirical model to argue that product market shocks (which raise prices given wages) explain a large share of recent US inflation. Importantly, our work investigates the structural origins of these empirically plausible shocks.[5] The markup shock interpretation also highlights the contrast between binding constraints and other competing mechanisms that work through marginal costs, such as factor reallocation frictions or labor shortages. Finally, the markup shock interpretation is also prima facie consistent with the fact that US profit margins increased as inflation took off in 2021, so binding constraints can also

[3]Pre-determined production capacity is shaped by past decisions about organization, installed capital, investments in worker capabilities, and the firm's stock of buyer/supplier relationships. Though we treat capacity as an exogenous stochastic variable, one could extend the model to allow for endogenous capacity investment decisions. We eschew this extension for now, because it distracts from our main focus on accounting for recent inflation dynamics.

[4]One source of these shocks would be pandemic-related factory shutdowns, as occurred in the US, China, Vietnam and elsewhere. They also capture shortages of inputs due other disruptions to global supply relationships (e.g., cancellation of supply contracts early in the pandemic led to shortages of foreign-supplied semiconductors that curtailed US auto production). Other historical shocks are also plausibly thought of as shocks to capacity – for example, the 2011 Tōhoku earthquake/tsunami took production capacity offline in Japan. We note here that disruptions in the global shipping industry (e.g., port congestion) and bottlenecks in distribution networks (e.g., trucking shortages) also made it difficult to deliver goods to buyers during the pandemic recovery. We focus on constraints on the supply of goods, rather than distribution of them, in our model.

[5]In a blog post, Del Negro et al. (2022) also argue that markup shocks are important, based on analyzing US data through the lens of a closed economy model without capacity constraints (the NYFed model).

help rationalize concerns about "greedflation" in the U.S.

Turning to the second challenge, we demonstrate that data on quantities and prices together serve to identify the reasons why constraints bind – i.e., to disentangle whether demand shocks or supply-side constraint shocks lead constraints to bind. While either a positive demand shock or negative constraint shock may trigger binding constraints and thus lead inflation to rise, these shocks have distinct implications for quantities. A demand shock pushes both inflation and output quantity up, while a negative constraint shock raises inflation whilst lowering output. Put differently, adverse constraint shocks lead to negative comovement between inflation and quantities (of output or imports) in the model. In contrast, there is positive comovement in these variables following a goods-biased demand shock. Implicitly, we use these quantitative patterns to identify shocks when applying the model to filter data. In particular, constraints help the model explain why both US goods output and imports of intermediate inputs did not rise in response to strong US demand.

To lay out the structure of the paper, we start by collecting stylized facts in Section 1, which both motivate elements of the framework and serve as inputs into quantification. Some are well known: headline consumer price inflation rose a lot, more for goods than services. And consumer expenditure shifted from services to goods, driving real goods expenditures above trend. On the import side, prices for imported industrial materials (inputs) rose rapidly in 2021, while prices for imported consumer goods were essentially flat. As for quantities, production of goods has recovered from its temporary pandemic downturn, but it has not increased in response to the surge in consumer demand for goods.[6] Stagnant domestic production in the face of surging demand (and the corresponding lack of imported inputs) hints at potentially binding constraints, whether domestic or foreign in nature.

In Section 2, we develop a model to organize our interpretation of these facts, in which we study the impact of constraints for domestic goods producers and foreign goods input suppliers. Using impulse responses from the model in Section 3, we describe how prices and quantities respond to demand shocks and shocks to constraints. In Section 4, we then apply the model to filter shocks from US national accounts data. To capture the rich data dynamics, we allow for a number of different shocks, including shocks to aggregate demand (time preference), demand for goods (preferences for goods versus services), monetary policy, capacity levels at home and abroad, sector-specific productivity, and foreign production costs. In an extended version of the model, we also allow for labor supply shocks (disutility of labor) and stochastic constraints on labor supply.

As a key intermediate step, we develop a Bayesian Maximum Likelihood estimation procedure

[6]Correspondingly, imports of final goods have risen by 40%, while imports of industrial materials and other intermediates have barely recovered to pre-pandemic levels.

to infer when constraints are binding and estimate structural parameters.[7] Our model presents several challenges for estimation. One challenge is that it features capacity shocks, and capacity is a latent variable that has no first order impact on other potentially observable equilibrium variables when constraints are slack. As a result, prior estimation routines (e.g., Guerrieri and Iacoviello (2017)) that use inversion filters to construct the likelihood function are not applicable in our context. Instead, our estimation procedure builds on prior work by Kulish, Morley and Robinson (2017), Kulish and Pagan (2017), and Jones, Kulish and Rees (2022), which treats the duration of binding constraints as a parameter to be estimated. In this, a second challenge is that the duration of binding constraints is an equilibrium outcome in our model, unlike prior applications of the duration-based estimation approach. Therefore, we adapt the maximum likelihood procedure to impose constraints on admissible duration parameter draws.[8] Overall, our estimated model fits the data well; most importantly, it captures the evolution of inflation for goods, services, and imports during the post-2020 period, making it a useful laboratory for analysis. Further, smoothed values for multipliers on the constraints imply that constraints bind during most of 2021-2022, and how tight they are fluctuates over time.

With the model and estimates in hand, we evaluate the role of binding constraints in explaining the evolution of inflation through a sequence of counterfactual exercises. The first counterfactual allows all shocks to be active, but exogenously relaxes the capacity constraints in all periods. Comparing this counterfactual to the data, we find that binding constraints explain about half of the increase in inflation in 2021-2022, about two percentage points of the four percentage point increase in overall inflation. Further, easing of constraints in the latter half of 2022 helps explain recent declines in goods and import price inflation.

To evaluate the role of individual shocks, we run a series of counterfactuals in which we introduce shocks one at a time and in combination. We find that tight capacity, in part due to negative capacity shocks, set the stage for expansionary monetary policy – looser policy than suggested by an extended Taylor rule – to generate excess inflation in 2021. By implication, neither aggregate nor goods-biased consumer demand shocks play an important role in 2021, though they do account for inflation dynamics in 2020.[9] As monetary policy was tightened in 2022, demand shocks then

[7] The structural parameters we estimate are substitution elasticities between home and foreign inputs, coefficients in the monetary policy rule, the mean level of capacity, and the stochastic processes for shocks.

[8] In Kulish, Morley and Robinson (2017) and Jones, Kulish and Rees (2022), the binding constraint is the zero lower bound on interest rates, so the duration to be estimated reflects beliefs about how long the central bank will hold the interest rate at zero. Because this is a free policy variable, these papers treat durations as unconstrained in the estimation. In our application, the anticipated duration of binding capacity constraints is determined by the realized shock today and the state of the economy. Thus, we adapt the estimation procedure to this new environment; see Sections 4.1 and A.3.2 for full details.

[9] Though we do not directly account for fiscal policy, we note that tax and transfer policy changes enacted during the pandemic largely worked by supporting consumption. Thus, the consumption demand shocks that we infer from data partly capture the impact of these fiscal policies.

play a larger role in accounting for sustained inflation.

Probing the robustness of these results, we show that these results are not spuriously driven by fluctuations in energy prices, by re-estimating and simulating the model using inflation data that excludes energy. We also investigate how our mechanism compares to a leading alternative – labor market shocks – in accounting for inflation. Specifically, we enrich the labor market to allow for wage rigidity, labor supply shocks, and (novel) potentially binding constraints on labor supply. While these additional features help us account for labor market dynamics (labor quantities and real wages) and the absence of disinflation in 2020, binding capacity constraints continue to play an important role in explaining inflation dynamics in 2021-2022.

In addition to work cited above, our paper is related to two distinct strands of work. First, our approach to modeling capacity constraints is related to models developed in Álvarez-Lois (2006) and Boehm and Pandalai-Nayar (2022), which feature heterogeneous firms that differ in terms of their exogenous capacity constraints on output.[10] As Boehm and Pandalai-Nayar emphasize, aggregating across these heterogeneous firms yields convex industry supply curves, in which industry price indexes increase with industry output, since it is related to the share of firms whose constraints are binding. In contrast to these papers, we employ a homogeneous firms framework, which has pedagogical advantages for comparability to textbook models. Further, we allow for binding aggregate constraints, which give rise to kinked convex supply (Phillips) curves with vertical segments where capacity is exhausted.

Second, our themes are related to recent work on how global value chains may have played a role in transmitting shocks during the pandemic crisis, including Bonadio et al. (2021), Lafrogne-Joussier, Martin and Mejean (2023*b*), Gourinchas et al. (2021), Alessandria et al. (2023), and Lafrogne-Joussier, Martin and Mejean (2023*a*).[11] Celasun et al. (2022) provide a comprehensive analysis of the global scope of disruptions and bottlenecks in supply chains during the pandemic, and attribute large output losses to them.

Several contributions specifically study the role of supply chain distributions in explaining price changes during the pandemic period. Amiti, Heise and Wang (2021), Young et al. (2021), and Santacreu and LaBelle (2022) demonstrate industry-level exposure to input price changes and/or supply chain disruptions are related to differences in output price changes across industries in the United States. Relatedly, Benigno et al. (2022) develop an index of global supply chain pressures from survey data and transportation indicators, and they find it has predictive power for inflation during the pandemic using a local projections empirical framework. di Giovanni et al. (2022) examine the role of disruptions to input markets and trade linkages on inflation during

[10] Fagnart, Licandro and Portier (1999) studies the endogenous determination of capacity, in a model that provides a microfoundation for capacity constraints on output. As we discuss below, output-based constraints are related, but somewhat different than, prior work on capital utilization.

[11] See also a discussion of the impact of Chinese shutdowns on US sourcing from China by Heise (2020).

the pandemic, using a sufficient statistics approach in a two period, multi-country, multi-sector input-output framework.[12] Amiti et al. (2023) study how the combination of domestic labor market shocks and import supply chain disruptions contribute to inflation. Additional contributions focus on the impacts of fiscal policy on inflation, including di Giovanni et al. (2023), de Soyres, Santacreu and Young (2023), and Bianchi, Faccini and Melosi (forthcoming).

Relative to this literature, our paper is the first (to our knowledge) to analyze occasionally binding capacity constraints in the supply chain, within a complete DSGE model. In this, our paper extends the new literature on monetary policy in economies with production networks [Ozdagli and Weber (2021); La'o and Tahbaz-Salehi (2022)] to accommodate supply chain constraints. Thus, we believe it opens the door to further study of the implications of supply chain bottlenecks for the conduct of policy.

1 Collecting Facts

We begin by collecting several key facts about recent inflation, consumer expenditure, production, and imports that motivate various elements of the framework we construct.

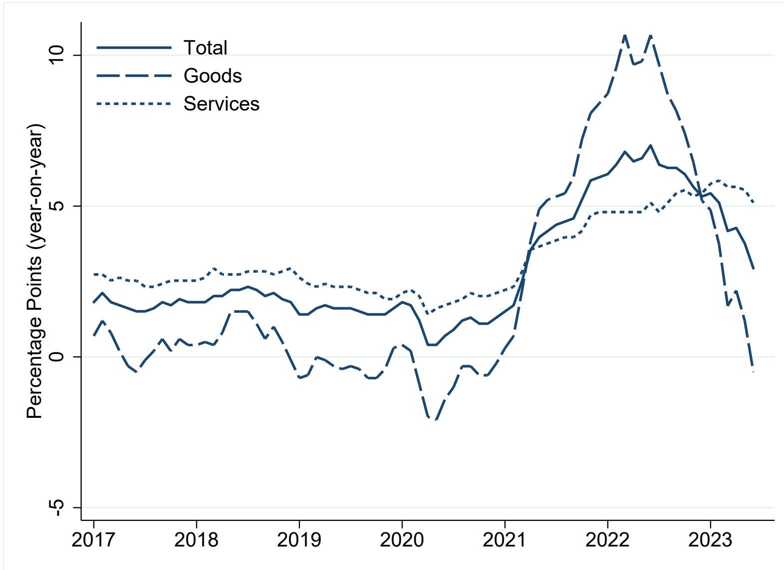
The first facts about consumer price inflation are well known: consumer price inflation rose substantially in 2021, led by inflation for goods. In Figure 1, we plot year-on-year growth in the price deflator for US personal consumption expenditure (PCE), as well as separate series for goods and services. The rise in headline inflation – from roughly 2 percent in 2021 to 6 percent as of early 2022 – is obviously startling. Importantly, this rise in inflation was led by goods price inflation, which rose from near zero to 10 percent in 2021 and then plummeted in the second half of 2022.

A second set of facts concerns import price inflation: prices for imported inputs rose dramatically in 2021, while price changes for imported consumer goods were modest. Plotting import price inflation by end use in Figure 2, we see that inflation for imported industrial materials rose substantially in 2021, peaking at 50% year on year.[13] While the price of oil and derivative fuels doubled during this period, the price of industrial materials excluding fuels also rose over 30%

[12] The sufficient statistics approach has previously been used to study macroeconomic shock propogation in Baqaee and Farhi (2019) in general, and the impacts of changes in trade on inflation by Comin and Johnson (2020). While well-suited to analyze shocks of foreign origin, the sufficient statistics approach is less useful for studying how trade mitigates/exacerbates the impacts of shocks that are domestic in origin, because domestic shocks have both direct effects and indirect effects through the import share. Further, as Comin and Johnson (2020) discuss, conclusions drawn about inflation from the sufficient statistics approach are sensitive to assumptions about the timing and persistence of changes in domestic sourcing shares.

[13] This data is from the International Price Program of the Bureau of Labor Statistics. The source data consist primarily of free on board (FOB) prices (i.e., prices received by foreign producers at foreign dock). During 2021-2022, transport costs also increased dramatically, which then would be added to these FOB prices to arrive at CIF prices (inclusive of cost, insurance, and freight) paid by the importer. We abstract from these additional transport margins, in order to focus on changes in supply prices.

Figure 1: Consumer Price Inflation



Note: Consumer prices are measured using the Personal Consumption Expenditure (PCE) price index, from the US Bureau of Economic Analysis (series identifiers DPCERGM, DGDSRGM, and DSERRGM.

in 2021. In contrast, inflation for imported consumer goods was subdued. This large difference between import price inflation for inputs versus consumer goods motivates our ensuing focus on disruptions impacting markets for imported inputs, rather than consumer goods.[14] In 2022, imported input price inflation dissipates rapidly, even excluding volatile fuels prices.

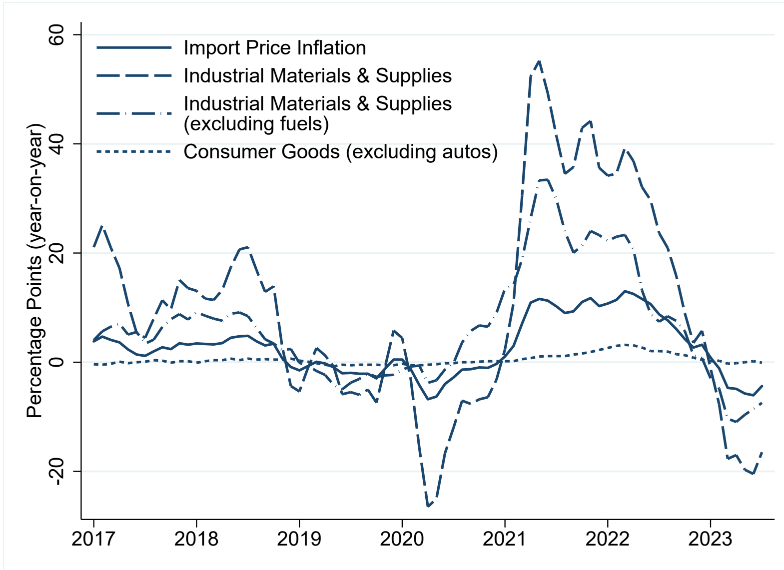
Tying the first and second set of facts together, goods production relies heavily on imported materials, relative production of services. Thus, the large increase in imported materials prices may play a role in explaining the surge of inflation in the goods sector discussed above. This observation is consistent with Amiti, Heise and Wang (2021), which documents that sectors that were more exposed to recent imported input price changes experienced higher producer price inflation in the U.S.[15] Our model framework will include this potential mechanism, alongside other competing drivers of inflation.

The third set of facts relate to consumer expenditures. While consumer expenditure collapsed during the lockdown phase of the pandemic, it returned to trend by the end of 2021. At the same time, the sector composition of consumer expenditures changed dramatically, as consumers reallocated away from services toward goods. This is illustrated in terms of nominal expenditure

[14]We have omitted several categories of imports from the figure for clarity, including capital goods imports (IR2), imports of automotive vehicles, parts, and engines (IR3), and foods, feeds, and beverages (IR0). To verbally summarize, inflation for capital goods imports was generally low, similar to imported consumer goods. Inflation for the automotive sector was also very low, and inflation for foods tracked total import price inflation closely. Thus, the behavior of imported materials prices stands out.

[15]In a related vein, Santacreu and LaBelle (2022) find that sectors more exposed to global supply chain disruptions (as measured an index of backlogs and delivery times) also experienced higher producer price inflation.

Figure 2: Import Price Inflation by End Use



Note: Import price indexes are obtained from the US Bureau of Labor Statistics (series identifiers: IR for total imports, EIUIR1 for industrial materials, EUIIR1EXFUEL for industrial materials excluding fuels, and EIUIR4 for consumer goods).

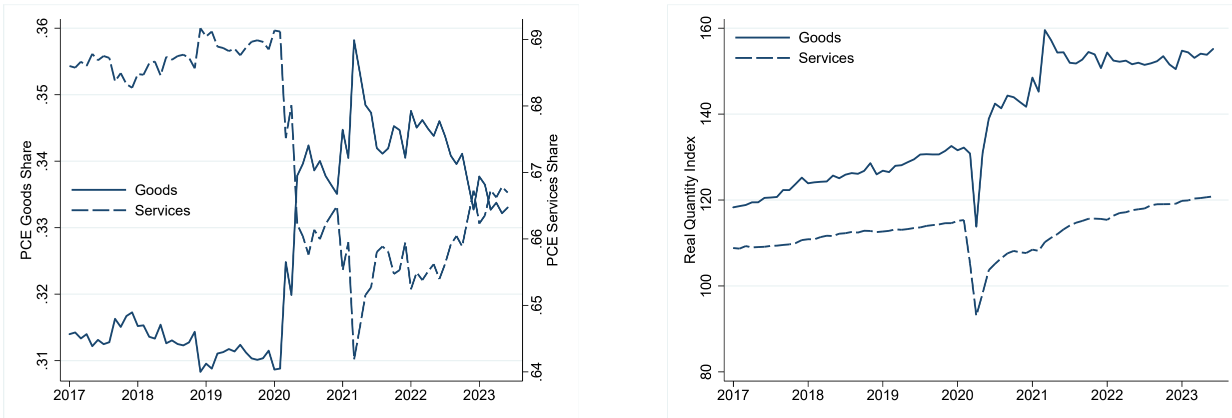
shares in Figure 3a, and in terms of real quantities consumed for goods and services in Figure 3b. Further, note that the change in composition has proven remarkably persistent: real consumption of goods (correspondingly, the goods share in expenditure) remains high relative to pre-pandemic levels through 2023.

The final set of facts point to potential supply-side constraints. In Figure 4a, we plot real US gross output by broad sector. The key fact is that real production of goods (already stagnant before the pandemic) only just recovered and then trended slightly down in 2021-2022, which contrasts sharply with services output. Stagnant goods production in the face of high domestic demand for goods immediately suggests that US producers may have faced binding constraints. Correspondingly, consumer demand for goods was filled by imports: in Figure 4b, imported quantities for consumer goods (excluding autos) surge. In contrast, imports of industrial materials are flat, recovering only to its 2017 levels by the end of 2021 and plateauing there.

Deficient US goods production and stagnant imports of industrial materials are naturally connected, though the direction of causality is not immediately clear. Limited supplies of imported materials may have constrained domestic production, or distinct binding constraints of domestic origin may have curtailed production and indirectly depressed demand for imported inputs. Moreover, both these mechanisms might be active simultaneously. Below, we discuss how quantity and price data together help distinguish between binding domestic versus foreign supply constraints in our model. With this background in mind, we turn to details of the model.

Figure 3: Consumption by Sector

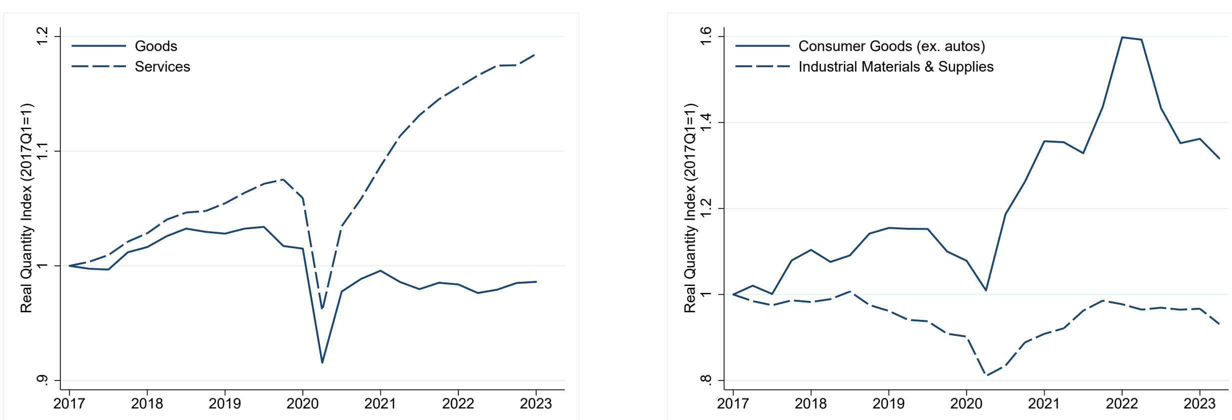
(a) Sector Shares in Expenditure (b) Real Quantities Consumed by Sector



Note: Personal Consumption Expenditure shares and real quantity indexes by sector are obtained from the US Bureau of Economic Analysis (series identifiers: DPCERC, PCES, DGDSRA3, and DSERRA3).

Figure 4: Production and Import Quantities

(a) Real Gross Output by Sector (b) Import Quantities by End Use



Note: Real gross output is constructed using data from the US Bureau of Economic Analysis (GDP by Industry, Table 17). Real quantity indexes for imports are obtained from the US Bureau of Economic Analysis (series identifiers: IB0000043 and B652RA3).

2 Model

This section presents a small open economy model with many sectors, $s \in \{1, \ldots, S\}$, which are are connected through input-output linkages. Within each sector, there is a continuum of monopolistically competitive firms, who set prices subject to Rotemberg-type adjustment costs. As in Gopinath et al. (2020), we assume that both exports and imports for the Home country are denominated in Home currency (i.e., US Dollars). Motivated by the data, we also allow import prices to differ for final goods and inputs.

The principal new features of the model are the output capacity constraints, for foreign and domestic firms. In writing down the model here, we allow these constraints to be potentially binding in any domestic sector, and we distinguish constraints that apply to foreign final versus input producing firms. Looking forward, we then restrict attention to particular constraints in quantitative analysis of the model for reasons of both tractability and empirical relevance. We also assume that the constraints are exogenously determined and (potentially) time varying, subject to stochastic shocks. This sets up a framework in which constraints may bind either due to negative shocks to capacity, or because other shocks lead firms to exhaust their excess capacity.

2.1 Consumers

There is a representative Home consumer, with preferences over labor supply L_t and consumption of sector composite goods $\{C_t(s)\}_{s \in S}$ represented by:

$$U\left(\{C_t, L_t\}_{t=0}^{\infty}\right) = \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \Theta_t \left[\frac{C_t^{1-\rho}}{1-\rho} - \frac{L_t^{1+\psi}}{1+\psi}\right] \tag{1}$$

$$\text{with} \quad C_t = \left(\sum_s \zeta_t(s)^{1/\vartheta} C_t(s)^{(\vartheta-1)/\vartheta}\right)^{\vartheta/(\vartheta-1)}$$

$$\text{and} \quad C_t(s) = \left(\sum_s \gamma(s)^{1/\varepsilon(s)} C_{Ht}(s)^{(\varepsilon(s)-1)/\varepsilon(s)} + (1-\gamma(s))^{1/\varepsilon(s)} C_{Ft}(s)^{(\varepsilon(s)-1)/\varepsilon(s)}\right)^{\varepsilon(s)/(\varepsilon(s)-1)},$$

where $C_t(s)$ is consumption of a sector-composite good, which is comprised of domestic ($C_{Ht}(s)$) and foreign ($C_{Ft}(s)$) sub-composite goods. The parameter $\beta < 1$ is the usual time discount rate, $\rho \geq 0$ controls intertemporal substitution, $\psi > 0$ governs the elasticity of labor supply, $\vartheta \geq 0$ is the elasticity of substitution across sectors, and $\varepsilon(s) \geq 0$ is the elasticity of substitution between home and foreign consumption composites.

The parameter $\zeta_t(s)$ is a time-varying parameter that controls tastes for goods from sector s, and we require that $\sum_s \zeta_t(s) = 1$ throughout, so $\zeta_t(s)$ should be interpreted as a relative sectoral demand shock. The parameter Θ_t is an aggregate preference (discount rate) shock at date t. Though our setting does not directly consider fiscal policy shocks, fiscal shocks would be subsumed in these discount rate shocks when Ricardian equivalence fails, as in recent models by Gabaix (2020) and Angeletos, Lian and Wolf (2023). Therefore, our framework parsimoniously captures the combined effect of fiscal policy and other drivers of discount rates in this single exogenous variable.

Financial markets are complete, and the agent's budget constraint is given by:

$$P_t C_t + \mathbf{E}_t \left[S_{t,t+1} B_{t+1} \right] \leq B_t + W_t L_t, \tag{2}$$

where $P_t C_t = \sum_s P_t(s) C_t(s)$, with P_t being the price for one unit of the composite consumption good and and $P_t(s)$ being the price of the sector composite good. B_t denotes the portfolio of Arrow-Debreu securities that pay off in domestic currency, and $S_{t,t+1}$ is the Home consumer's stochastic discount factor (defined below). Further, sectoral consumption expenditure is $P_t(s)C_t(s) = P_{Ht}(s)C_{Ht}(s) + P_{Ft}(s)C_{Ft}(s)$, where $P_{Ht}(s)$ and $P_{Ft}(s)$ are the prices of the home and foreign consumption composites.

Given prices $\{P_t, P_t(s), P_{Ht}(s), P_{Ft}(s), S_{t,t+1}, W_t\}$ and initial asset holdings B_0, the consumer chooses consumption, labor supply, and asset holdings to maximize Equation 1 subject to Equation 2 and the standard transversality condition. Optimal consumption and labor choices satisfy:

$$C_t^{-\rho} \left(\frac{W_t}{P_t} \right) = L_t^{\psi} \tag{3}$$

$$C_t(s) = \zeta_t(s) \left(\frac{P_t(s)}{P_t} \right)^{-\vartheta} C_t \tag{4}$$

$$C_{Ht}(s) = \gamma(s) \left(\frac{P_{Ht}(s)}{P_t(s)} \right)^{-\varepsilon(s)} C_t(s) \tag{5}$$

$$C_{Ft}(s) = (1-\gamma(s)) \left(\frac{P_{Ft}(s)}{P_t(s)} \right)^{-\varepsilon(s)} C_t(s) \tag{6}$$

$$1 = \mathbf{E}_t \left[S_{t,t+1} \frac{P_t}{P_{t+1}} (1+i_t) \right] \tag{7}$$

where $S_{t,t+1} = \beta \frac{\Theta_{t+1}}{\Theta_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho}$ is the stochastic discount factor, $P_t = \left(\sum_s \zeta_t(s) P_t(s)^{1-\vartheta} \right)^{1/(1-\vartheta)}$ is the aggregate price index, $P_t(s) = \left(\gamma(s) \left(P_{Ht}(s)\right)^{1-\varepsilon(s)} + (1-\gamma(s)) \left(P_{Ft}(s)\right)^{1-\varepsilon(s)} \right)^{1/(1-\varepsilon(s))}$ is the sector-composite price index, and i_t is a one period, risk free nominal interest rate.

2.2 Domestic Producers

There is a continuum of firms within each sector in Home, each of which produces a differentiated good (indexed by ω). In addition, there exist competitive intermediary firms that aggregate these varieties into composite goods, which are then consumed, used as inputs, and exported. We start by describing these intermediaries, and then turn to individual firms.

2.2.1 Composite Domestic Good

Each competitive intermediary firm purchases output from domestic producers to form a domestic composite. The production function for the intermediary is: $Y_t(s) = \left(\int_0^1 Y_t(s,\omega)^{(\varepsilon-1)/\varepsilon} d\omega\right)^{\varepsilon/(\varepsilon-1)}$, where $Y_t(s,\omega)$ is the amount of output purchased from firm ω in sector s, and $\varepsilon > 1$ is the elasticity of substitution. Given prices $P_t(s,\omega)$ for individual domestic varieties, cost minimization implies demands $Y_t(s,\omega) = \left(\frac{P_t(s,\omega)}{P_{Ht}(s)}\right)^{-\varepsilon} Y_t(s)$, where $P_{Ht}(s) = \left[\int_0^1 P_t(s,\omega)^{1-\varepsilon} d\omega\right]^{1/(1-\varepsilon)}$ is the price of the sector composite good.

2.2.2 Domestic Firms

Each domestic producer in sector s is able to supply output up to a pre-determined capacity of $\bar{Y}_t(s)$, which we refer to as a firm-level capacity constraint. We assume this capacity level is exogenously determined and equal across firms within each sector.

The production function for domestic variety ω in sector s is:

$$Y_t(s,\omega) = Z_t(s,\omega) A(s) \left(L_t(s,\omega)\right)^{1-\alpha(s)} \left(M_t(s,\omega)\right)^{\alpha(s)} \tag{8}$$

$$M_t(s,\omega) = \left(\sum_{s'} \left(\alpha(s',s)/\alpha(s)\right)^{1/\kappa} M_t(s',s,\omega)^{(\kappa-1)/\kappa}\right)^{\kappa/(\kappa-1)} \tag{9}$$

$$M_t(s',s,\omega) = \left[\xi(s',s)^{\frac{1}{\eta(s')}} M_{Ht}(s',s,\omega)^{\frac{\eta(s')-1}{\eta(s')}} + (1-\xi(s',s))^{\frac{1}{\eta(s')}} M_{Ft}(s',s,\omega)^{\frac{\eta(s')-1}{\eta(s')}}\right]^{\frac{\eta(s')}{\eta(s')-1}}, \tag{10}$$

where $L_t(s,\omega)$ is the quantity of labor used by the firm, $M_t(s,\omega)$ is the firm's use of a composite input, $Z_t(\omega)$ is productivity, and $A(s) = \alpha(s)^{-\alpha(s)}(1-\alpha(s))^{-(1-\alpha(s))}$ is a normalization constant. The composite input combines inputs purchased from upstream sectors $M_t(s',s,\omega)$, with elasticity of substitution $\kappa \geq 0$. And those upstream inputs are themselves a CES composite of Home ($M_{Ht}(s',s,\omega)$) and Foreign ($M_{Ft}(s',s,\omega)$) composite inputs. The parameters $\eta(s) \geq 0$ are elasticities of substitution across country sources for inputs (conventionally termed the Armington elasticity), while $\xi(s',s) \in (0,1)$ controls relative demand for home inputs conditional on prices.

Producers set prices in domestic currency under monopolistic competition, and they select the input mix to satisfy the implied demand. These two problems can be analyzed separately. The firm chooses $\left\{L_t(s,\omega), M_t(s,\omega), M_t(s',s,\omega), M_{Ht}(s,\omega), M_{Ft}(s',s,\omega)\right\}$ to minimize the cost of producing $Y_t(s,\omega)$, which is $W_t L_t(s,\omega) + P_{Mt}(s) M_t(s,\omega)$, with $P_{Mt}(s) M_t(s,\omega) = \sum_{s'} P_t(s',s) M_t(s',s,\omega)$

and $P_t(s',s)M_t(s',s,\omega) = P_t(s')M_{Ht}(s',s,\omega) + P_{Ft}(s')M_{Ft}(s',s,\omega)$, where $P_{Ft}(s')$ is the (domestic currency) price of the foreign composite input from sector s' . The first order conditions to this problem can be written as follows:

$$W_tL_t(s,\omega) = \alpha(s)MC_t(s,\omega)Y_t(\omega) \tag{11}$$

$$P_{Mt}(s)M_t(s,\omega) = (1-\alpha(s))MC_t(s,\omega)Y_t(s,\omega) \tag{12}$$

$$M_t(s',s,\omega) = \frac{\alpha(s',s)}{\alpha(s)}\left(\frac{P_t(s',s)/P_t}{P_{Mt}(s)/P_t}\right)^{-\kappa} M_t(s,\omega) \tag{13}$$

$$M_{Ht}(s',s,\omega) = \xi(s',s)\left(\frac{P_{Ht}(s')/P_t}{P_t(s',s)/P_t}\right)^{-\eta(s')} M_t(s',s,\omega) \tag{14}$$

$$M_{Ft}(s',s,\omega) = (1-\xi(s',s))\left(\frac{P_{Ft}(s')/P_t}{P_t(s',s)/P_t}\right)^{-\eta(s')} M_t(s',s,\omega), \tag{15}$$

where $P_{Mt}(s) = \left(\sum_{s'}\left(\frac{\alpha(s',s)}{\alpha(s)}\right)P_t(s',s)^{1-\kappa}\right)^{1/(1-\kappa)}$ is the price of the composite input, $P_t(s',s)$ is the price of $M_t(s,\omega)$, and the firm's marginal cost is $MC_t(s,\omega) = (Z_t(s,\omega))^{-1} W_t^{1-\alpha(s)} (P_{Mt}(s))^{\alpha(s)}$.

Given this solution for marginal costs, the domestic firm chooses a sequence of prices to maximize profits, with knowledge of the demand curve for its output, and subject to quadratic adjustment cost for prices [Rotemberg (1982*a*,*b*)]. This pricing problem can be written as:

$$\max_{\{P_t(s,\omega)\}} \quad \mathbf{E}_0 \sum_{t=0}^{\infty} \frac{S_{0,t}}{P_t}\left[P_t(s,\omega)Y_t(s,\omega) - MC_t(s,\omega)Y_t(s,\omega) - \frac{\phi(s)}{2}\left(\frac{P_t(s,\omega)}{P_{t-1}(s,\omega)}-1\right)^2 P_{Ht}(s)Y_t(s)\right]$$
$$\text{s.t.} \quad Y_t(s,\omega) \le \bar{Y}_t(s),$$

where the discount rate for profits reflects the domestic agent's stochastic discounting.[16] The final term in the first line captures the adjustment costs, where $\phi(s)$ governs the degree of price rigidity. Note also that the firm accounts for the potentially binding constraint in its pricing decisions. Denoting the Lagrange multiplier attached to the capacity constraint $\mu_t(s,\omega)$, optimal prices satisfy:

$$\begin{aligned} 0 = 1 - \varepsilon\left(1 - \frac{MC_t(s,\omega)+\mu_t(s,\omega)}{P_t(s,\omega)}\right) - \phi(s)\left(\frac{P_t(s,\omega)}{P_{t-1}(s,\omega)}-1\right)\frac{P_{Ht}(s)Y_t(s)}{P_{t-1}(s,\omega)Y_t(s,\omega)} \\ + E_t\left[\beta\frac{\Theta_{t+1}}{\Theta_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\frac{P_t}{P_{t+1}}\phi(s)\left(\frac{P_{t+1}(s,\omega)}{P_t(s,\omega)}-1\right)\frac{P_{Ht+1}(s)Y_{t+1}(s)}{P_t(s,\omega)Y_t(s,\omega)}\frac{P_{t+1}(s,\omega)}{P_t(s,\omega)}\right]. \end{aligned} \tag{16}$$

[16] Of course, with complete markets, it is immaterial whether domestic or foreign agents own the firm.

The corresponding complementary slackness condition is:

$$\mu_t(s,\omega)\left[Y_t(s,\omega) - \bar{Y}_t(s)\right] = 0. \tag{17}$$

And we require $\mu_t(s,\omega) \geq 0$ and the constraint to hold in equilibrium ($Y_t(s,\omega) \leq \bar{Y}_t(s)$) as usual. When the constraint binds, then $\mu_t(s,\omega) > 0$. In Equation 16, we see this is equivalent to an increase in the marginal cost of the firm, which drives up the optimal price. When the capacity constraint is slack, such that $\mu_t(s,\omega) = 0$, and is expected to remain slack, then Equation 16 collapses to a standard intertemporal pricing equation.

2.3 Foreign Producers

Turning to foreign producers, we again start with aggregation of varieties by competitive intermediaries, and then we present the pricing problem for foreign firms. Here we distinguish between producers of foreign consumption goods versus inputs, which allows us to to analyze data on import prices by end use.

2.3.1 Composite Foreign Goods

For each end use $u \in \{C, M\}$, where C and M denote consumption and intermediate use respectively, there is a unit continuum of foreign firms that produce foreign inputs, indexed by ϖ. A competitive intermediary firm aggregates output produced by each foreign firm, and bundles it into the foreign composite according to the production function: $Y^*_{ut}(s) = \left(\int_0^1 Y^*_{ut}(s,\varpi)^{(\varepsilon-1)/\varepsilon} d\varpi\right)^{\varepsilon/(\varepsilon-1)}$. Demand for each variety then takes takes the standard CES form: $Y^*_{ut}(s,\varpi) = \left(\frac{P_{uFt}(s,\varpi)}{P_{uFt}(s)}\right)^{-\varepsilon} Y^*_{ut}(s)$, where $P_{uFt}(s,\varpi)$ is the price of variety ϖ and $P_{uFt}(s) = \left(\int_0^1 P_{uFt}(s,\varpi)^{1-\varepsilon} d\varpi\right)^{1/(1-\varepsilon)}$ is the price of the foreign composite, both denominated in Home currency.

2.3.2 Foreign Firms

Each foreign firm (in sector s, producing for end use u) is able to supply output up to a predetermined capacity of $\bar{Y}^*_{ut}(s)$, and this capacity is exogenous and equal across firms. Foreign marginal costs are given by $MC^*(s,\varpi)$, and we assume this cost is exogenous (as in a small open economy), denominated in foreign currency, and equal across end uses.

Each firm chooses a sequence for the price of its variety in Home currency $\{P_{uFt}(s,\varpi)\}$, subject to price adjustment frictions, to solve:

$$\max_{\{P_{Ft}(s,\varpi)\}} \quad \mathbf{E}_0 \sum_{t=0}^{\infty} \frac{S^*_{0,t}}{P^*_t E_t} \left[P_{uFt}(s,\varpi) Y^*_{ut}(s,\varpi) - E_t MC^*_t(s) Y^*_{ut}(s,\varpi) - \frac{\phi(s)}{2} \left(\frac{P_{uFt}(s,\varpi)}{P_{uFt-1}(s,\varpi)} - 1 \right)^2 P_{uFt}(s) Y^*_{ut}(s) \right]$$

$$\text{s.t.} \quad Y^*_{ut}(s,\varpi) \leq \bar{Y}^*_{ut}(s),$$

with knowledge of the demand curve for its output specified above. Here $S^*_{0,t} = \beta^t \left(\frac{C^*_t}{C^*_0} \right)^{-\rho}$ is the foreign stochastic discount factor (with C^*_tdenoting foreign consumption), P^*_t is the foreign price level (in foreign currency), and E_t is a the nominal exchange rate (units of home currency to buy one unit of foreign currency).

Denoting the Lagrange multiplier attached to the capacity constraint $\mu^*_{ut}(s,\varpi)$, then the first order condition is:

$$1 - \varepsilon \left(1 - \frac{E_t \left(MC^*_t(s,\varpi) + \mu^*_{ut}(s,\varpi) \right)}{P_{uFt}(s,\varpi)} \right) - \phi(s) \left(\frac{P_{uFt}(s,\varpi)}{P_{uFt-1}(s,\varpi)} - 1 \right) \frac{P_{uFt}(s) Y^*_{ut}(s)}{P_{uFt-1}(s,\varpi) Y^*_{ut}(s,\varpi)}$$
$$+ \mathbf{E}_t \left[\beta \left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\rho} \left(\frac{E_t P^*_t}{E_{t+1} P^*_{t+1}} \right) \phi(s) \left(\frac{P_{uFt+1}(s,\varpi)}{P_{uFt}(s,\varpi)} - 1 \right) \frac{P_{uFt+1}(s) Y^*_{ut+1}(s)}{P_{uFt}(s,\varpi) Y^*_{ut}(s,\varpi)} \frac{P_{uFt+1}(s,\varpi)}{P_{uFt}(s,\varpi)} \right] = 0. \quad (18)$$

The complementary slackness condition is:

$$\mu^*_{ut}(s,\varpi) \left[Y^*_{ut}(\varpi) - \bar{Y}^*_{ut} \right] = 0. \quad (19)$$

In equilibrium, $\mu^*_{ut}(\varpi) \geq 0$ and $Y^*_{ut}(\varpi) \leq \bar{Y}^*_{ut}$.

2.4 Closing the Model

We assume that demand for exports of the home composite good takes the CES form:

$$X_t(s) = \left(\frac{P_{Ht}(s)}{P_t Q_t} \right)^{-\sigma(s)} X^*_t(s), \quad (20)$$

where $Q_t \equiv \frac{E_t P^*_t}{P_t}$ is the real exchange rate and $X^*_t(s)$ is an exogenous foreign sector-demand factor.

The market clearing condition for the home composite good is:

$$Y_t(s) = C_{Ht}(s) + \sum_{s'=1}^{S} \int_0^1 M_{Ht}(s,s',\omega) d\omega + X_t(s) + \int_0^1 \left[\frac{\phi(s)}{2} \left(\frac{P_t(s,\omega)}{P_{t-1}(s,\omega)} - 1 \right)^2 Y_t(s) \right] d\omega, \quad (21)$$

where the composite good is sold to consumers and domestic producers, exported, and used to cover price adjustment costs. For the foreign composite goods, we impose similar market clearing conditions:

$$Y^*_{Ct}(s) = C_{Ft}(s) + \int_0^1 \left[\frac{\phi(s)}{2} \left(\frac{P_{CFt}(s,\varpi)}{P_{CFt-1}(s,\varpi)} - 1 \right)^2 Y^*_{Ct}(s) \right] d\varpi \tag{22}$$

$$Y^*_{Mt}(s) = \sum_{s'} M_{Ft}(s,s') + \int_0^1 \left[\frac{\phi(s)}{2} \left(\frac{P_{MFt}(s,\varpi)}{P_{MFt-1}(s,\varpi)} - 1 \right)^2 Y^*_{Mt}(s) \right] d\varpi. \tag{23}$$

Labor market clearing is given by:

$$L_t = \sum_{s=1}^{S} L_t(s) \quad \text{with} \quad L_t(s) = \int_0^1 L_t(s,\omega) d\omega. \tag{24}$$

Trade in Arrow-Debreu securities implies that Home and Foreign consumers share risk, such that:

$$\Theta_t \left(\frac{C_t}{C^*_t} \right)^{-\rho} Q_t = \Xi, \tag{25}$$

where Ξ is a constant.

Turning to monetary policy, we specify an extended inflation-targeting rule for interest rates. Since we allow for sector-specific preference shocks, we now distinguish measured price inflation from changes in the welfare-theoretic price index. We define an auxiliary price index under the assumption that preferences are constant over time: $\bar{P}_t = \left(\sum_s \zeta_0(s) \left(P_t(s) \right)^{1-\vartheta} \right)^{1/(1-\vartheta)}$, where $\zeta_0(s)$ are steady-state CES weights. Then $\bar{\Pi}_t = \bar{P}_t / \bar{P}_{t-1}$ is the ratio of measured prices across periods, and the approximate inflation rate is given by $\bar{\pi}_t = \sum_s \left(\frac{P_0(s)C_0(s)}{P_0 C_0} \right) \left[\ln P_t(s) - \ln P_{t-1}(s) \right]$.[17] We write the monetary policy rule in terms of measured inflation:

$$1 + i_t = (1 + i_{t-1})^{\rho_i} \bar{\Pi}_t^{\omega(1-\rho_i)} \left(Y_t / Y_0 \right)^{(1-\rho_i)\rho_y} \Psi_t \tag{26}$$

where $Y_t = \sum_s P_0(s) Y_t(s)$ is aggregate real gross output and Ψ_t is a monetary policy shock. The parameters ω and ρ_y determine how aggressively the central bank responds to inflation and the output gap (defined as the deviation of output from steady state), while the parameter ρ_i controls the degree of interest rate inertia.

[17] The following relationship holds between the ratios of measured and welfare-based price indexes across periods: $\bar{\Pi}_t = \frac{\bar{P}_t / P_t}{\bar{P}_{t-1} / P_{t-1}} \Pi_t$, where $\frac{\bar{P}_t}{P_t} = \left(\sum_s \zeta_0(s) \left(\frac{P_t(s)}{P_t} \right)^{1-\vartheta} \right)^{1/(1-\vartheta)}$ and the ratio of aggregate prices across periods is $\Pi_t \equiv \frac{P_t}{P_{t-1}}$. We include these among auxiliary price definitions in the model equilibrium.

2.5 Equilibrium with Symmetric Firms

We focus on an equilibrium with symmetric producers within each sector and country. The model parameters are $\{\rho,\psi,\chi,\vartheta,\beta,\kappa,\varepsilon,\Xi,i_0,\omega,\rho_i,\rho_y\}$, $\{\varepsilon(s),\alpha(s),\eta(s),\phi(s),\sigma(s),\phi(s),\gamma(s)\}_s$, and $\{\alpha(s',s),\xi(s',s)\}_{s,s'}$. Further, values for domestic variables $\{\Theta_t,\{\zeta_t(s),Z_t(s)\}_s,\Psi_t\}$, foreign variables $\{C_t^*,\{X_t^*(s),MC_t^*(s)/P_t^*\}_s\}$ and constraints $\{\bar{Y}_t(s),\bar{Y}_{Ct}^*(s),\bar{Y}_{Mt}^*(s)\}_s$ are exogenously given. We write all prices relative to the domestic price level, and we define $\Pi_t \equiv \frac{P_t}{P_{t-1}}$.

Given parameters and exogenous variables, an equilibrium is a sequence of aggregate quantities $\{C_t,L_t\}$, sector-level quantities $\{C_t(s),C_{Ht}(s),C_{Ft}(s),L_t(s),Y_t(s),M_t(s),X_t(s),Y_{Ct}^*(s),Y_{Mt}^*(s)\}_s$, input use $\{\{M_t(s',s),M_{Ht}(s',s),M_{Ft}(s',s)\}_{s'}\}_s$, aggregate prices $\{W_t/P_t,i_t,Q_t,\Pi_t,\bar{\Pi}_t,\bar{P}_t/P_t\}$, sector-level prices $\{\Pi_t(s),\Pi_{CFt}(s),\Pi_{MFt}(s),P_t(s)/P_t,MC_t(s)/P_t,P_{Mt}(s)/P_t,P_{Ht}(s)/P_t,P_{CFt}(s)/P_t,P_{MFt}(s)/P_t\}_s$, input prices $\{\{P_t(s',s)/P_t\}_{s'}\}_s$, and (normalized) multipliers $\{\mu_t(s)/P_t,\mu_{Ct}^*(s)/P_t^*,\mu_{Mt}^*(s)/P_t^*\}_s$ that satisfy the equilibrium conditions collected in Table 1. This system is $8+21S+4S^2$ equations in the same number of unknowns.

Table 1: Equilibrium Conditions

Labor Supply	$C_t^{-\rho}\frac{W_t}{P_t}=\chi L_t^{\psi}$
Consumption Allocation	$C_t(s)=\zeta_t(s)\left(\frac{P_t(s)}{P_t}\right)^{-\vartheta}C_t$
	$C_{Ht}(s)=\gamma(s)\left(\frac{P_{Ht}(s)/P_t}{P_t(s)/P_t}\right)^{-\varepsilon(s)}C_t(s)$
	$C_{Ft}(s)=(1-\gamma(s))\left(\frac{P_{CFt}(s)/P_t}{P_t(s)/P_t}\right)^{-\varepsilon(s)}C_t(s)$
Euler Equation	$1=E_t\left[\beta\frac{\Theta_{t+1}}{\Theta_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left(\frac{1+i_t}{\Pi_{t+1}}\right)\right]$
Consumer Prices	$1=\left(\sum_s\zeta_t(s)\left(\frac{P_t(s)}{P_t}\right)^{1-\vartheta}\right)^{1/(1-\vartheta)}$
	$\frac{P_t(s)}{P_t}=\left(\gamma(s)\left(\frac{P_{Ht}(s)}{P_t}\right)^{1-\varepsilon(s)}+(1-\gamma(s))\left(\frac{P_{CFt}(s)}{P_t}\right)^{1-\varepsilon(s)}\right)^{1/(1-\varepsilon(s))}$
Labor Demand	$\frac{W_t}{P_t}L_t(s)=(1-\alpha(s))\frac{MC_t(s)}{P_t}Y_t(s)$
	$\frac{P_{Mt}(s)}{P_t}M_t(s)=\alpha(s)\frac{MC_t(s)}{P_t}Y_t(s)$
Input Demand	$M_t(s',s)=\frac{\alpha(s',s)}{\alpha(s)}\left(\frac{P_t(s',s)/P_t}{P_{Mt}(s)/P_t}\right)^{-\kappa}M_t(s)$
	$M_{Ht}(s',s)=\xi(s',s)\left(\frac{P_{Ht}(s')/P_t}{P_t(s',s)/P_t}\right)^{-\eta(s')}M_t(s',s)$
	$M_{Ft}(s',s)=(1-\xi(s',s))\left(\frac{P_{MFt}(s')/P_t}{P_t(s',s)/P_t}\right)^{-\eta(s')}M_t(s',s)$
Marginal Cost	$\frac{MC_t(s)}{P_t}=\frac{1}{Z_t(s)}\left(\frac{W_t}{P_t}\right)^{1-\alpha(s)}\left(\frac{P_{Mt}(s)}{P_t}\right)^{\alpha(s)}$
Input Prices	$\frac{P_{Mt}(s)}{P_t}=\left(\sum_{s'}\left(\frac{\alpha(s',s)}{\alpha(s)}\right)\left(\frac{P_t(s',s)}{P_t}\right)^{1-\kappa}\right)^{1/(1-\kappa)}$
	$\frac{P_t(s',s)}{P_t}=\left[\xi(s',s)\left(\frac{P_{Ht}(s')}{P_t}\right)^{1-\eta(s')}+(1-\xi(s',s))\left(\frac{P_{MFt}(s')}{P_t}\right)^{1-\eta(s')}\right]^{1/(1-\eta(s'))}$

Table 1: Equilibrium Conditions

Domestic Pricing	$0 = 1 - \varepsilon\left(1 - \frac{MC_t(s)/P_t + \mu_t(s)/P_t}{P_{Ht}(s)/P_t}\right) - \phi(s)\left(\Pi_{Ht}(s)-1\right)\Pi_{Ht}(s)$ $+E_t\left[\beta \frac{\Theta_{t+1}C_{t+1}^{-\rho}}{\Theta_t C_t^{-\rho}}\frac{1}{\Pi_{t+1}}\phi(s)\left(\Pi_{Ht+1}(s)-1\right)\Pi_{Ht+1}(s)^2\frac{Y_{t+1}(s)}{Y_t(s)}\right]$
Consumption Import Pricing	$0 = 1-\varepsilon\left(1-\frac{Q_t}{P_{CFt}(s)/P_t}\frac{MC_t^*(s)+\mu_{Ct}^*(s)}{P_t^*}\right) - \phi(s)\left(\Pi_{CFt}(s)-1\right)\Pi_{CFt}(s)$ $+E_t\left[\beta\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\rho}\frac{Q_t}{Q_{t+1}}\frac{1}{\Pi_{t+1}}\phi(s)\left(\Pi_{CFt+1}(s)-1\right)\Pi_{CFt+1}(s)^2\frac{Y_{Ct+1}^*(s)}{Y_{Ct}^*(s)}\right]$
Input Import Pricing	$0 = 1-\varepsilon\left(1-\frac{Q_t}{P_{MFt}(s)/P_t}\frac{MC_t^*(s)+\mu_{Mt}^*(s)}{P_t^*}\right) - \phi(s)\left(\Pi_{MFt}(s)-1\right)\Pi_{MFt}(s)$ $+E_t\left[\beta\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\rho}\frac{Q_t}{Q_{t+1}}\frac{1}{\Pi_{t+1}}\phi(s)\left(\Pi_{MFt+1}(s)-1\right)\Pi_{MFt+1}(s)^2\frac{Y_{Mt+1}^*(s)}{Y_{Mt}^*(s)}\right]$
Comp. Slackness and Firm Constraints	$\min\left\{\mu_t(s), \bar{Y}_t(s) - Y_t(s)\right\} = 0$ $\min\left\{\mu_{Ct}^*(s), \bar{Y}_{Ct}^*(s) - Y_{Ct}^*(s)\right\} = 0$ $\min\left\{\mu_{Mt}^*(s), \bar{Y}_{Mt}^*(s) - Y_{Mt}^*(s)\right\} = 0$
Market Clearing	$Y_t(s) = C_{Ht}(s) + \sum_{s'} M_{Ht}(s,s') + X_t(s) + \frac{\phi(s)}{2}\left(\frac{P_t(s)}{P_{t-1}(s)}-1\right)^2 Y_t(s)$ $X_t(s) = \left(\frac{P_{Ht}(s)}{P_t Q_t}\right)^{-\sigma(s)} X_t^*(s)$ $Y_{Ct}^*(s) = C_{Ft}(s) + \frac{\phi(s)}{2}\left(\Pi_{CFt}(s)-1\right)^2 Y_{Ct}^*(s)$ $Y_{Mt}^*(s) = \sum_{s'} M_{Ft}(s,s') + \frac{\phi(s)}{2}\left(\Pi_{MFt}(s)-1\right)^2 Y_{Mt}^*(s)$ $\Theta_t\left(\frac{C_t}{C_t^*}\right)^{-\rho} Q_t = \Xi$ $\sum_s L_t(s) = L_t$
Monetary Policy Rule	$1+i_t = (1+i_{t-1})^{\rho_i}\bar{\Pi}_t^{\omega(1-\rho_i)}\left(Y_t/Y_0\right)^{(1-\rho_i)\rho_y}\Psi_t$ with $Y_t = \sum_s P_0(s) Y_t(s)$
Auxiliary Inflation and Price Definitions	$\Pi_{Ht}(s) = \left(\frac{P_{Ht}(s)/P_t}{P_{Ht-1}(s)/P_{t-1}}\right)\Pi_t$ $\Pi_{CFt}(s) = \left(\frac{P_{CFt}(s)/P_t}{P_{CFt-1}(s)/P_{t-1}}\right)\Pi_t$ $\Pi_{MFt}(s) = \left(\frac{P_{MFt}(s)/P_t}{P_{MFt-1}(s)/P_{t-1}}\right)\Pi_t$ $\bar{\Pi}_t = \frac{\bar{P}_t/P_t}{\bar{P}_{t-1}/P_{t-1}}\Pi_t$ $\frac{\bar{P}_t}{P_t} = \left(\sum_s \zeta_0(s)\left(\frac{P_t(s)}{P_t}\right)^{1-\vartheta}\right)^{1/(1-\vartheta)}$

2.6 Discussion

We briefly discuss some technicalities associated with solving the model. We then describe Phillips Curves in the model, which contain an important insight for interpreting simulation results.

2.6.1 Solving the Model

Because the model features occasionally binding constraints, we need to adopt an appropriate solution technique that captures the non-linearities induced by them. Among alternatives, we adopt the piecewise linear solution technique developed by Guerrieri and Iacoviello (2015). The perturbation-based solution algorithm combines first order approximations to the model equilibrium for both the unconstrained and constrained equilibria, where the point of approximation is the unconstrained equilibrium in all cases.[18] The log-linear approximation for the model used in our quantitative analysis, and details regarding the solution procedure, are presented in Appendix A.

Collecting log deviations from steady state for endogenous (both control and state) variables in the vector X_t, the general solution for the model can be written as:

$$X_t = \mathbf{J}\left(X_{t-1}, \varepsilon_t; \theta\right) + \mathbf{Q}\left(X_{t-1}, \varepsilon_t; \theta\right) X_{t-1} + \mathbf{G}\left(X_{t-1}, \varepsilon_t; \theta\right) \varepsilon_t, \tag{27}$$

where ε_t is the vector of exogenous shocks in period t, θ is a collection of structural parameters, and $\mathbf{J}(\cdot)$, $\mathbf{Q}(\cdot)$, and $\mathbf{G}(\cdot)$ are time-varying matrices (dependent on the state and current shocks) that describe the optimal policy function. Given parameters and the initial steady-state shares needed to parameterize the approximate model, as well as lagged values X_{t-1} and a realization for ε_t, we solve for the policy functions using the OccBin toolbox in Dynare.

2.6.2 Domestic and Import Price Phillips Curves

It is instructive to examine log-linear approximations for the dynamic pricing equations for domestic and imported goods. Noting that $\mu_t(s)/P_t$ and $\mu^*_{ut}(s)/P^*_t$ for $u \in \{C, M\}$ take on zero values in the unconstrained equilibrium, we define auxiliary variables $\tilde{\mu}_t(s) \equiv \mu_t(s)/P_t + 1$ and $\tilde{\mu}^*_{ut}(s) \equiv \mu^*_{ut}(s)/P^*_t + 1$, and then we log-linearize the equilibrium with respect to these auxiliary variables. The resulting approximate pricing equations are:

$$\pi_{Ht}(s) = \left(\frac{\varepsilon - 1}{\phi(s)}\right)\left(\widehat{rmc}_t(s) - \widehat{rp}_{Ht}(s)\right) + \left(\frac{\varepsilon}{\phi(s)} \frac{P_0}{P_{H0}(s)}\right)\hat{\tilde{\mu}}_t(s) + \beta E_t\left[\pi_{Ht+1}(s)\right] \tag{28}$$

$$\pi_{uFt}(s) = \left(\frac{\varepsilon - 1}{\phi(s)}\right)\left(\widehat{rmc}^*_t(s) + \hat{q}_t - \widehat{rp}_{uFt}(s)\right) + \left(\frac{\varepsilon}{\phi(s)} \frac{P_0}{P_{uF0}(s)}\right)\hat{\tilde{\mu}}^*_{ut}(s) + \beta E_t\left[\pi_{uFt+1}(s)\right], \tag{29}$$

where hat-notation denotes deviations from steady state, $\pi_t(s) \equiv \ln P_t(s) - \ln P_{t-1}(s)$, $\pi_{Ft}(s) \equiv \ln P_{Ft}(s) - \ln P_{Ft-1}(s)$, $rmc_t(s) = \ln\left(MC_t(s)/P_t\right)$, $rmc^*_t(s) = \ln\left(MC^*_t(s)/P^*_t\right)$, $rp_{Ht}(s) = \ln\left(P_{Ht}(s)/P_t\right)$,

[18] The solution procedures requires that the model satisfies two important conditions. First, it is assumed that the model returns to the unconstrained equilibrium in finite time after a once-off shock, if agents expect future shocks to be zero. Second, the unconstrained equilibrium must be stable, in the usual Blanchard-Kahn sense. Both requirements are satisfied for our baseline model and parameter values.

$rp_{uFt}(s) = \ln\left(P_{uFt}(s)/P_t\right)$, and $q_t = \ln Q_t$. Equations 28-29 are sector-level domestic and import price Phillips curves.

Binding Constraints as Markup Shocks An important conceptual point is that binding constraints – when $\mu_t(s)$ or $\mu^*_{ut}(s)$ are strictly positive – appear as "markup shocks" in reduced form. That is, binding constraints lead inflation to be higher than can be accounted for given parameters, real marginal costs, and expected inflation. Thus, one can identify whether constraints bind in our model using the same approaches that would typically be used to identify exogenous, reduced-form markup shocks in standard New Keynesian models.

Whereas exogenous markups shocks in New Keynesian models typically are micro-founded by assuming that there are shocks to the elasticity of demand, the endogenous "markups shocks" in our model have a different structural interpretation. Markup shocks arise in our model not because the competitive environment per se has changed – i.e., market structure and demand elasticities are time invariant – rather firm conduct changes when constraints bind. Firms cease to make price changes to target their ideal (flexible price, CES) markups; they instead "price to demand," based on willingness to pay for their constrained output.[19] Further, markups may rise and fall sharply (reflecting non-linearities) as constraints are triggered and relaxed, such that the evolution of markups in times of binding constraints will be different than in normal times when constraints are always slack.

This "markup shock" interpretation of binding constraints also serves to highlight how the mechanism we emphasize is distinct from alternative explanations for the inflation surge. First, much attention has focused on the role of labor shortages. At the aggregate level, these may reflect changes in worker preferences for supplying labor, or other constraints on labor supply. At the sector level, worker shortages may be explained by impediments to reallocating workers in response to differential changes in demand across sectors.[20] In either case, demand for workers outstripping supply ought to manifest as higher wages, which would then drive marginal costs higher. Thus, one would expect to see that changes in real marginal costs explain inflation outcomes, not markups (one might even expect markups to be compressed where labor shortages are tightest). To the extent that constraints masquerading as markup shocks explain inflation, this then limits the scope for these alternative labor market mechanisms. All that said, we will discuss exactly how

[19] The conclusion that capacity constraints influence firm conduct, holding market structure fixed, is not unique to our particular monopolistic competition model. For example, in oligopoly models with symmetric firms, it is well-known that Bertrand competition leads to competitive (marginal cost) pricing when firms are unconstrained. In contrast, when firms are capacity constrained such that they cannot collectively meet total market demand when prices equal marginal cost, then the Bertrand equilibrium features prices set above marginal cost.

[20] We have assumed that factors are perfectly mobile across sectors in our model, with a common economy-wide wage. This contrasts with Ferrante, Graves and Iacoviello (forthcoming), who analyze how asymmetric demand shocks lead to inflation when there are worker reallocation frictions.

incorporating labor market shocks and constraints affects inflation in Section 5.2.

In a related vein, the approach we adopt for modeling capacity differs from prior literature, which has emphasized variable capital utilization rather than output-based capacity constraints [Greenwood, Hercowitz and Huffman (1988); Gilchrist and Williams (2000)]. In this literature, it is typically assumed that higher rates of capital utilization lead capital to depreciate faster. As a result, higher utilization raises the effective marginal cost for the firm (including wages, user costs of capital, and increased capital depreciation), so utilization affects inflation through marginal costs. Further, with the functional form assumptions in Greenwood, Hercowitz and Huffman, the standard log-linear Phillips Curve relationship between marginal costs and inflation (equivalently, utilization and inflation) holds. Thus, this alternative approach to capacity utilization will struggle to explain the highly non-linear response of inflation observed in recent data, as well as the role of reduced-form markup shocks in explaining it.

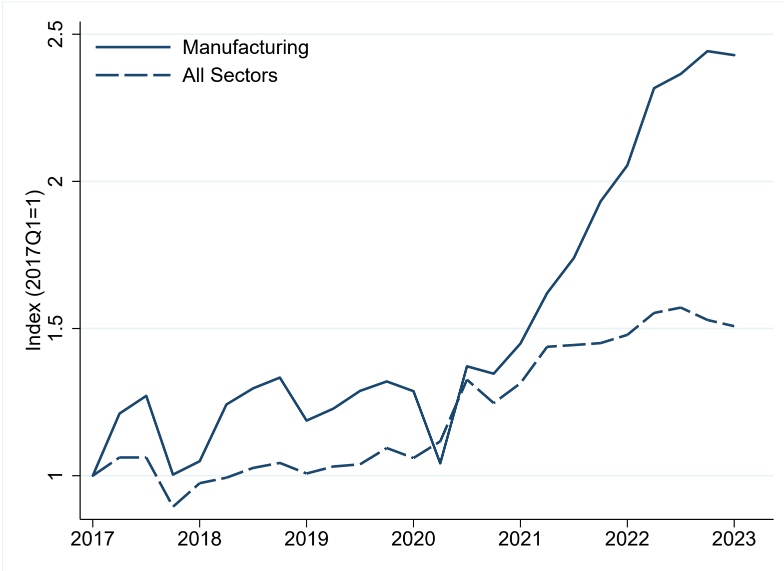
Profits Our model implies that price-cost margins (realized markups) are high when firms face binding constraints. To examine the plausibility of this channel, we turn to data on profits per unit of output, which serves as an observable proxy for price-cost margins. To formalize this link, note that the absolute markup is equal to profits per unit of output in the steady state: $P_t(s) - MC_t(s) = \frac{\Xi_t(s)}{Y_t(s)}$, where $\Xi_t(s) \equiv P_t(s)Y_t(s) - MC_t(s)Y_t(s)$ is the profit of the representative producer in sector s. Thus, tracking profits per unit over time sheds light on how markups are changing.

In Figure 5, we plot indexes of US corporate profits per unit of gross output for both the manufacturing sector and the aggregate private sector.[21] The takeaway is that profits per unit escalated sharply for manufacturing firms during the pandemic recovery, coinciding with the takeoff in goods price inflation and widespread complaints about binding (supply chain) constraints that limited production. Further, total profits (profits per unit times quantity sold) were at historically high levels in 2021. This pattern of high profitability alongside high inflation is a natural outcome of binding (domestic) constraints in our model. It is also remarkably consistent with concerns about "greedflation" in the U.S., wherein corporations have been criticized for fueling inflation by gouging consumers [DePillis (2022)].[22] More recently, profit margins appear to be falling as

[21]This corporate profit measure omits profits attributable to non-corporate entities; We focus on corporate profits because data is available for manufacturing on a quarterly frequency in the national accounts.

[22]Greedflation is often attributed to the secular rise of market power, which has potentially increased the ability of corporations to pass cost shocks through to consumers. Our mechanism is not about pass-through of cost shocks; it is about how firms set prices conditional on costs when constraints bind. To explain the difference in emphasis, it is useful to refer to the automobile industry. High auto prices during the pandemic did not reflect high rates of pass-through from dealer costs (production costs) to retail prices. Rather, the constrained supply of new automobiles led to higher dealer markups and robust profitability [Smialek (2022)]. To interpret this, think about dealers as having a Leontief production function, combining cars with dealer services, such that constrained access to inputs (new cars) effectively constrained dealer output. Unlike this auto example, our model allows primary factors to substitute for inputs, and allows domestic inputs to substitute for foreign inputs, so output constraints are distinct from input constraints.

Figure 5: Corporate Profits per unit of Gross Output



Note: Corporate profits (with inventory valuation adjustments) and gross output are obtained from the US Bureau of Economic Analysis (series identifiers N400RC and A390RC). The figure contains the ratio of corporate profits to gross output in each quarter, expressed as an index number (values are measured relative to the ratio in 2017Q1).

inflation has declined in 2023 [Kerr (2023)].

3 Impulse Response Analysis

To illustrate how the model works, we turn to impulse response functions. We first discuss the quantitative setup, which puts restrictions on the general model presented above. We then analyze responses to particular demand and capacity constraints that play important roles in accounting for variation in the data below. Further, looking forward to the full quantitative analysis, we pay particular attention to illustrating how data may be useful in identifying shocks.

3.1 Quantitative Setup

While the model (as written above) allows for many sectors and many potentially binding constraints, we now hone in on a two sector structure with two potentially binding constraints, motivated by the stylized facts presented above. As for numbering sectors, let $s = 1$ be the goods sector and $s = 2$ be the services sector. We then focus on equilibria with potentially binding constraints for the goods sector, since the anomalous behavior of goods price inflation in recent data requires explanation.[23] Further, motivated by data that shows a surge of consumption goods imports during 2020-2021, we assume that the constraints for foreign consumer goods firms are slack as well. That is, we allow for potentially binding constraints for domestic firms and foreign input producing firms – i.e., constraints in the supply chain for goods.

We set parameters in the model based on external calibration and an estimation procedure that we describe in Section 4.2, with the full set of parameters provided in Appendix A. Further, exogenous variables follow independent first-order autoregressive processes, with parameters estimated below.

3.2 Analyzing a Demand Shock

To start, we consider a consumer discount rate (Θ_t) shock, which raises the desire to consume in the current period. For concreteness, let $\hat{\Theta}_t = \lambda_\Theta \hat{\Theta}_{t-1} + \varepsilon_{\Theta t}$, with $\text{var}(\varepsilon_{\Theta t}) = \sigma_\theta^2$. To scale the shock, we assume that the initial innovation to Θ_t is $0.11\sigma_\theta$ for the simulation when the domestic constraint binds and $0.065\sigma_\theta$ for the simulation when the foreign constraint binds, where σ_θ and λ_Θ are estimated below. These are relatively small shocks, so our focus is on qualitative results in this section, rather than magnitudes. We contrast the impacts of this shock in an unconstrained equilibrium to those in two alternative equilibria, in which only the domestic constraint binds, or only the foreign constraint binds on impact. In each case, we set the level of the constraint so that the constraint binds for only one period (i.e., on impact) in response to the shock, and is slack thereafter. While a special case, this serves to highlight the key impacts of hitting constraints.

3.2.1 Unconstrained Benchmark

Figure 6 collects impulse responses for key variables from the unconstrained model (i.e., assuming constraints do not bind after the shock). In Figure 6a, we see that the demand shock raises overall consumer price inflation, and inflation for services is about double that for goods after the shock. To break this down, sector-level inflation is a weighted average of domestic price inflation ($\pi_{Ht}(s)$) and import price inflation for consumption goods ($\pi_{CFt}(s)$), so we plot $\pi_{Ht}(s)$ and $\pi_{CFt}(s)$ in Figure 6b.[24]

First, we note that import price inflation for consumer goods is negative on impact in Figure 6b, so this also lowers overall consumption price inflation for goods relative to services, since the

[23] Taking the constraints for sector 2 to infinity – i.e., $\bar{Y}_t(2) \to \infty$, $\bar{Y}^*_{Ct}(2) \to \infty$, and $\bar{Y}^*_{Mt}(2) \to \infty$ – is sufficient to ensure constraints are only potentially binding for goods. One ought not over-interpret this assumption. This is a sufficient, but not necessary condition. Given any sequence of shocks, one can back out the level of finite constraints needed to prevents these constraints from binding.

[24] Sector-level consumer price inflation is given by: $\pi_t(s) = \left(\frac{P_{H0}(s)C_{H0}(s)}{P_0(s)C_0(s)}\right)\pi_{Ht}(s) + \left(\frac{P_{F0}(s)C_{F0}(s)}{P_0(s)C_0(s)}\right)\pi_{CFt}(s)$, where $\pi_t(s) = \hat{p}_t(s) - \hat{p}_{t-1}(s)$ is inflation in sector s.

goods sector has a higher import share. Glancing forward, note that import price inflation for goods inputs is also negative on impact in Figure 6d, matching the dynamics for consumer import prices. The reason is that on impact the exchange rate appreciates in response to the demand shock, and this appreciation lowers import price inflation symmetrically across sectors and end uses ($\pi_{uFt}(s)$ coincide across u and s).

Second, domestic price inflation is also lower for goods than services. The reason is that real wages rise, and services use labor more intensively than do goods. To illustrate this, we iterate the Phillips Curves in Equation 28 forward to yield the decomposition:

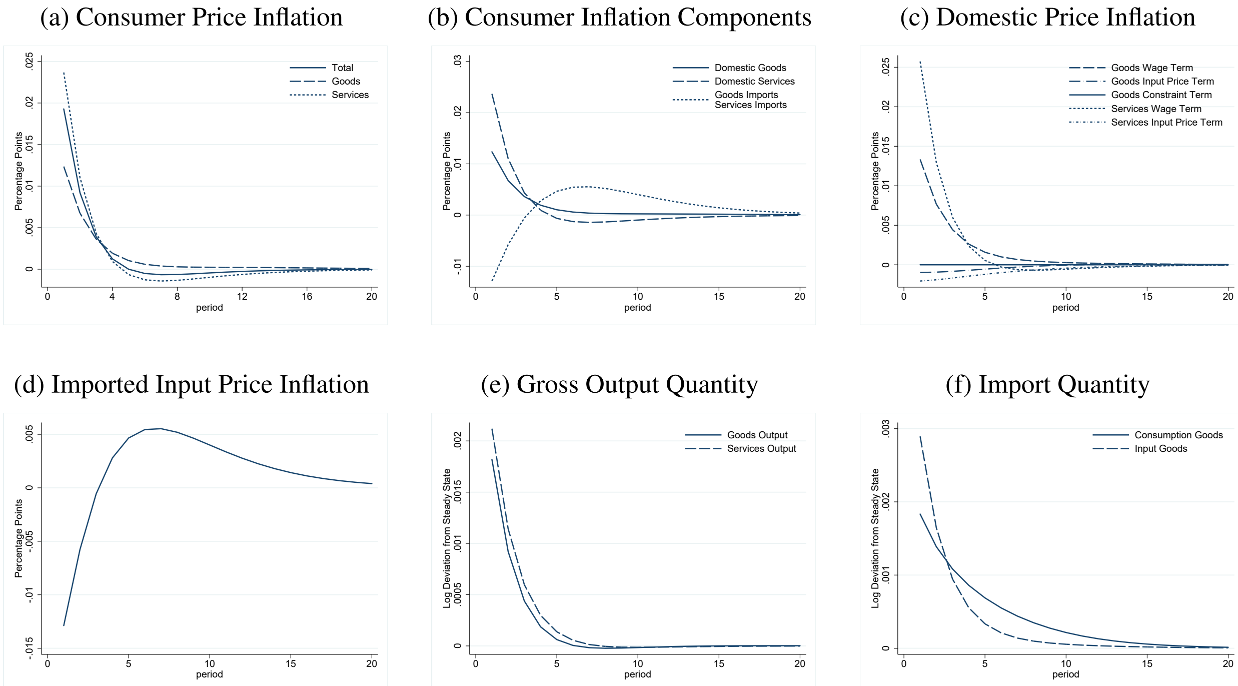
$$\pi_{Ht}(s) = \underbrace{(1-\alpha(s))\left(\frac{\varepsilon-1}{\phi(s)}\right)\sum_{r=0}^{\infty}\beta^r E_t\left[\hat{w}_{t+r}-\hat{p}_{Ht+r}(s)\right]}_{\text{real wage term}} + \underbrace{\alpha(s)\left(\frac{\varepsilon-1}{\phi(s)}\right)\sum_{r=0}^{\infty}\beta^r E_t\left[\hat{p}_{Mt+r}(s)-\hat{p}_{Ht+r}(s)\right]}_{\text{real input price term}}$$
$$+\underbrace{\left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{H0}(s)}\right)\sum_{r=0}^{\infty}\beta^r E_t\left[\hat{\tilde{\mu}}_{t+r}(s)\right]}_{\text{binding constraint term}}. \quad (30)$$

where we have substituted for $\widehat{rmc}_{t+r}(s)-\widehat{rp}_{Ht+r}(s)$ and defined "real" values here in term of domestic output prices. The first term captures the role of real wages, where the labor share of gross output $(1-\alpha(s))$ is higher for services than goods. The second term then accounts for real input prices in costs. The third term captures the impact of binding constraints on markups, which is identically zero in this simulation with slack constraints. We plot this decomposition by sector in Figure 6c. The real wage term for services clearly drives inflation for services beyond that for goods, reflecting the higher labor content of services. The second point to note in the figure is that input costs actually restrain inflation in both sectors, though these effects are small in magnitude.

Looking at quantities, we plot real gross output by sector in Figure 6e and real goods imports in Figure 6f. The quantity of both domestic goods and services produced rises in response to increased demand in the unconstrained equilibrium. Further, both the quantity of imported consumer goods and inputs rise, with the increase in imported inputs outstripping consumption goods.

In all, while the demand shock raises overall inflation, it yields a mix of results that are inconsistent with recent data. Whereas goods price inflation exceeds services inflation in data, the opposite is true in the unconstrained model. Further, import price inflation is negative on impact, in contrast to data. Finally, both real goods output and imported inputs rise in the simulation, while they are largely flat in the data [see Figure 4]. With these puzzles in hand, we turn to versions of the model with binding constraints.

Figure 6: Demand Shock: Unconstrained Equilibrium



3.2.2 Binding Constraints

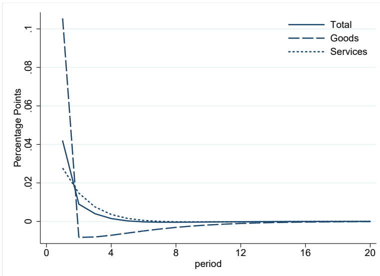
We now turn to discuss the impacts of the demand shock when constraints bind. We illustrate the impact of binding domestic constraints in Figure 7, and the impact of binding foreign constraints in Figure 8. For comparison to Figure 6, recall that we set the values of the constraints so that they bind only in the initial post-shock period, and are slack thereafter.

In 7a, we see that binding domestic constraints lead to about twice as much inflation on impact. Importantly, goods price inflation now rises more than inflation in services, increasing about ten times as much as in the unconstrained equilibrium. This reflects high inflation in domestic goods prices, in Figure 7b.[25] Unpacking domestic prices using Equation 30 in Figure 7c, domestic price inflation surges due to the markup shocks induced by binding constraint, where $\hat{\tilde{\mu}}_1(1) > 0$ on impact. In Figures 7d and 7b, we again see that import price inflation falls on impact in response to the demand shock, reflecting an exchange rate appreciation and slack imported input constraints. Turning to quantities, goods output rises on impact (as there is surplus capacity in steady state), but its rise is capped at about half the unconstrained response, so goods output rises by significantly less than services output. Further, reflecting lower domestic goods output, the quantity of imported goods inputs is dampened as well, while imports of consumption goods imports increase by more

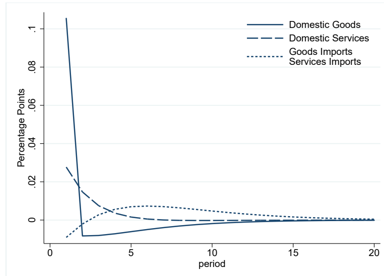
[25] Like in the unconstrained equilibrium, low (positive) import price inflation for consumer goods actually attenuates overall goods price inflation; equivalently, domestic goods prices rise more than the consumer price index for goods.

Figure 7: Demand Shock: Domestic Constraint Binds

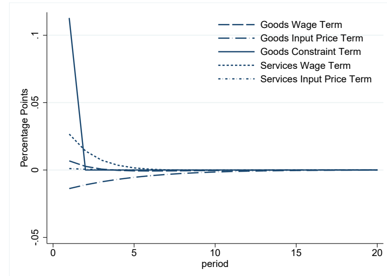
(a) Consumer Price Inflation



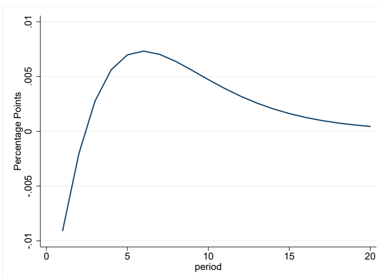
(b) Consumer Inflation Components



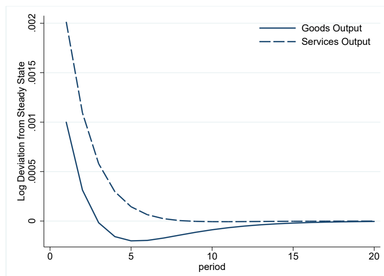
(c) Domestic Price Inflation



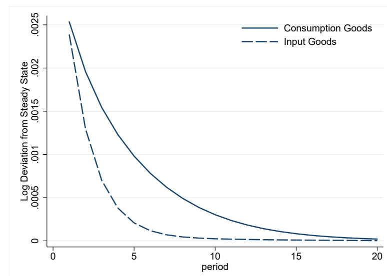
(d) Imported Input Price Inflation



(e) Gross Output Quantity



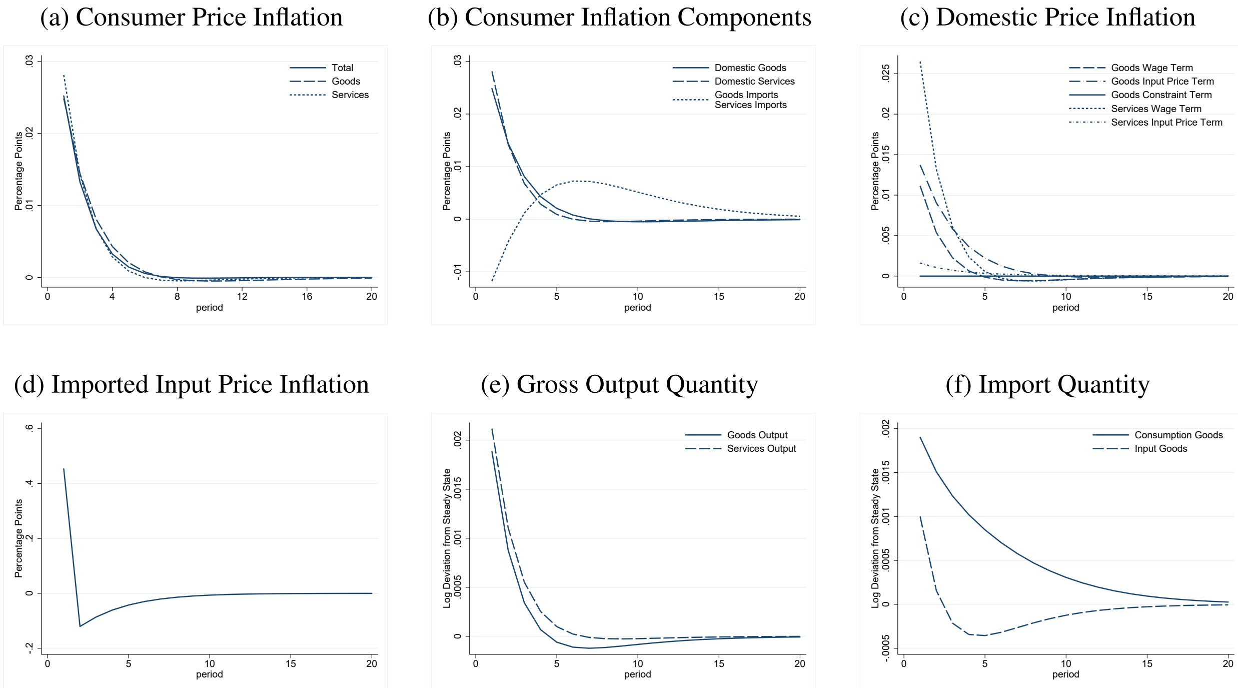
(f) Import Quantity



than in the unconstrained equilibrium.

Turning to the case where only foreign constraints are binding in Figure 8, we note first that binding foreign constraints raise both consumer goods and domestic goods price inflation in Figures 8a and 8b. Looking at 8c, higher goods price inflation reflects the fact that prices for inputs in the goods sector increase on impact, and these are passed into domestic goods prices. In turn, the price of inputs for goods producers rises because imported goods inputs price inflation now spikes on impact in Figure 8d, due to the binding foreign constraints. The behavior of imported input price inflation differs from both the unconstrained equilibrium and the equilibrium with binding domestic constraints. It also differs from import price inflation for consumer goods in Figure 8b, where the effects of binding constraints on imported input prices dominates the impacts of the exchange rate appreciation, which leads imported consumer goods inflation to fall on impact. Turning to quantities, note that domestic goods output rises even though the foreign constraint binds, nearly the same amount as in the unconstrained equilibrium. Although foreign input constraints limit input availability for domestic producers, they also trigger substitution toward domestic goods producers, with offsetting effects for total production. Finally, imports of goods inputs are obviously constrained in Figure 8f, relative to the prior two cases.

Figure 8: Demand Shock: Foreign Constraint Binds



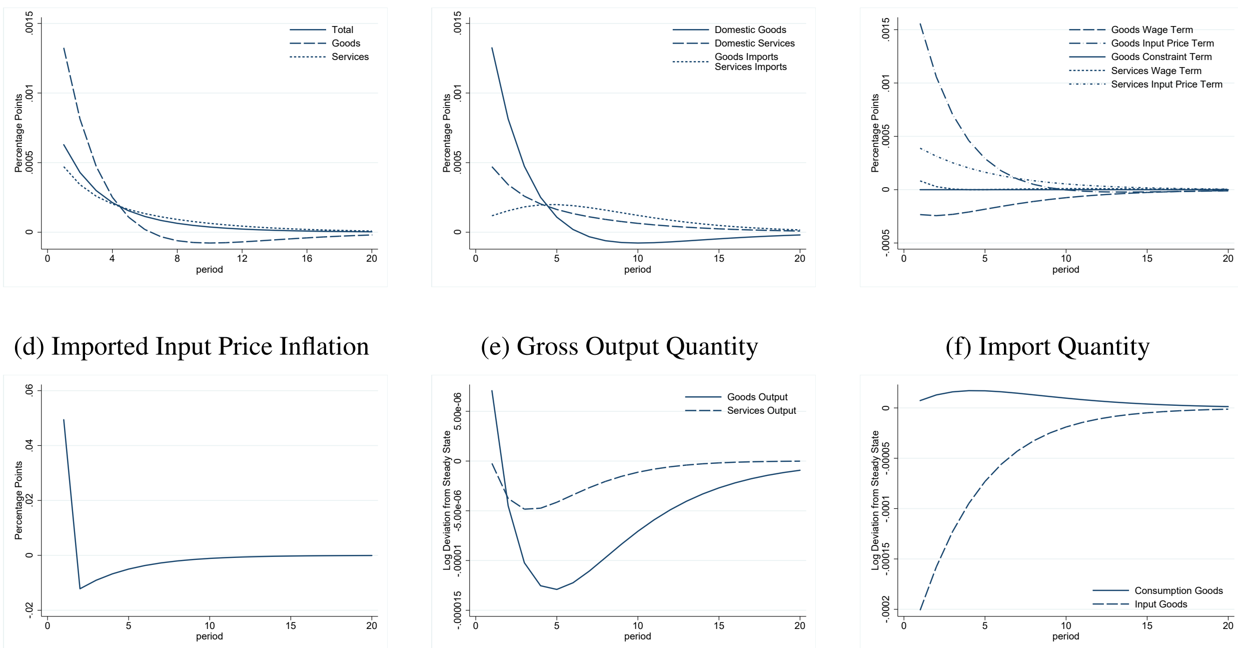
3.3 Shocks to Constraints

We now briefly summarize the impacts of shocks to the foreign and domestic firm-level constraints. To anchor the magnitudes, deviations in domestic and foreign capacity from steady state are given by: $\hat{\bar{y}}_t(1) = \lambda_{\bar{y}}\hat{\bar{y}}_{t-1}(1) + \varepsilon_{\bar{y}t}$, $\hat{\bar{y}}^*_{Mt}(1) = \lambda^*_{\bar{y}}\hat{\bar{y}}^*_{Mt-1}(1) + \varepsilon_{\bar{y}^*t}$, with $\text{var}(\varepsilon_{\bar{y}t}) = \sigma^2_{\bar{y}}$ and $\text{var}(\varepsilon_{\bar{y}^*t}) = \sigma^2_{\bar{y}^*}$, and autocorrelation and shock variance parameters set based on estimates below.

In Figure 9, we plot responses to a shock to the foreign capacity constraint, equal to $-0.15\sigma_{\bar{y}^*}$ in magnitude. As above, we set the initial steady state capacity level so that the constraint binds for only one period after the shock, and we assume that the domestic constraint is slack in all periods. Following the negative foreign capacity shocks, there is a sharp rise in imported input price inflation in Figure 9d. This feeds through to domestic goods prices, and in turn to overall goods price inflation, which rises more than services price inflation in this scenario (Figures 9a and 9b). Nonetheless, the overall change in inflation is modest, reflecting the relatively small share of imported inputs in domestic input use, as well possibilities to substitute domestic for foreign inputs. Reflecting this substitution, domestic goods output actually rises slightly. Despite the fact that the real quantity of imported inputs falls on impact, due to reduced foreign capacity. Thus, this constraint shock leads to rising import price inflation together with falling quantities of imports.

In Figure 9d, we plot responses to a $-0.15\sigma_{\bar{y}}$ shock to the domestic capacity constraint, under the assumption that the foreign constraint is slack. There is again a rise in goods price inflation in

Figure 9: Foreign Firm Capacity Shock



(a) Consumer Price Inflation (b) Consumer Inflation Components (c) Domestic Price Inflation

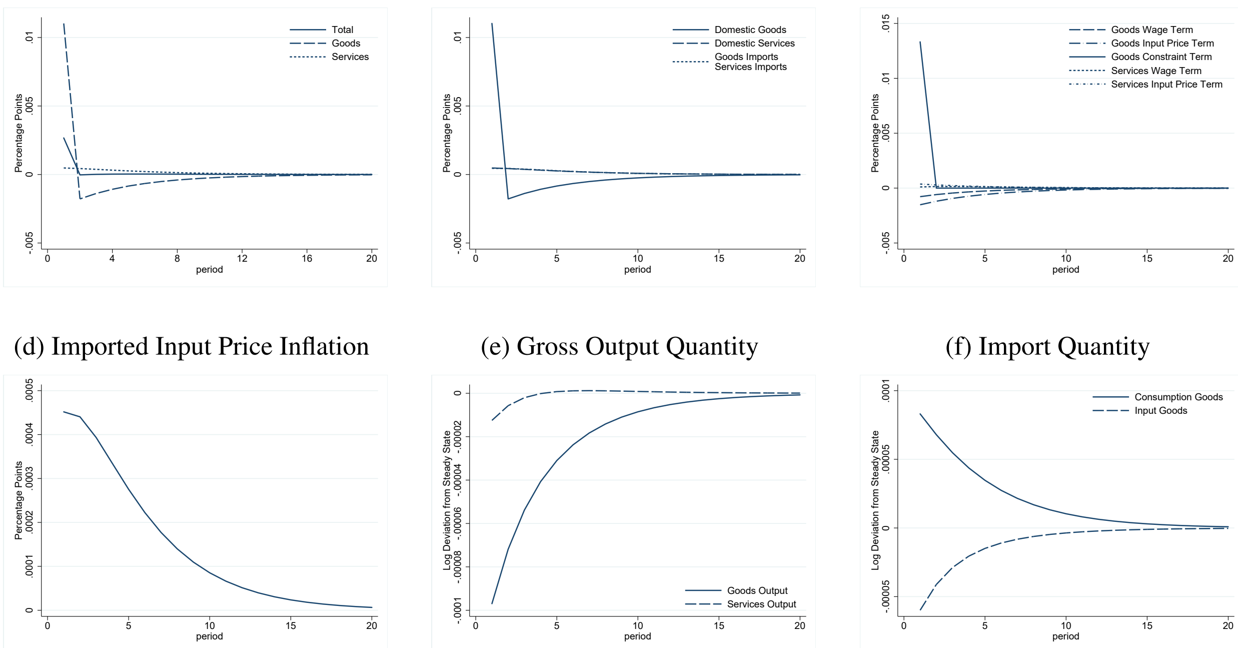
(d) Imported Input Price Inflation (e) Gross Output Quantity (f) Import Quantity

Figure 10a, driven by an increase in domestic goods price inflation (Figure 10b). The constraint shock leads the multiplier on the constraint to be greater than zero, whose effects on domestic price inflation are captured in the goods constraint term in Figure 10c. Due to the fall in domestic goods capacity, actual realized goods output falls in this case (Figure 10e), and imports of intermediate goods fall on impact as well (Figure 10f). In contrast, imports of consumption goods increase, reflecting substitution from domestic to import sources. The negative comovement between inflation and real output/imports is a distinctive feature of this shock.

3.4 Summing Up

Stepping back, we collect a few summary results that shed light on how the model will identify which constraints are binding and the structure of underlying shocks. First, the sector-composition of inflation depends on the configuration of shocks and constraints. With slack constraints, inflation for services outstrips that for goods. Binding foreign constraints raise goods price inflation to equal services price inflation, and binding domestic constraints lead goods price inflation to outstrip services inflation, which is a prominent feature of recent data. Second, imported goods input price inflation is high only when the foreign firm constraint binds, either following a demand shock or a shock to the constraint itself. Put differently, while a binding domestic constraint may explain excess inflation for goods, it cannot also generate a sharp increase in import price inflation on its

Figure 10: Domestic Firm Capacity Shock



(a) Consumer Price Inflation (b) Consumer Inflation Components (c) Domestic Price Inflation

(d) Imported Input Price Inflation (e) Gross Output Quantity (f) Import Quantity

own. Third, co-movement in prices and quantities differs depending on whether constraints are binding or slack, as well as the nature of the shocks. In the case of a demand shock, gross output rises along with inflation, as does the quantity of imported inputs. For constraint shocks, however, inflation and output comove negatively: a shock to the foreign firm constraint raises import price inflation while lowering imported input quantities, and a shock to the domestic constraint raises domestic goods price inflation while lowering quantities produced.

4 Accounting for Recent Inflation Experience

We now apply the model to parse recent data. We describe the procedure we use to estimate the model in Section 4.1, with additional details in Appendix A. Then, we discuss data, calibration, and estimated parameters in Section 4.2. In Section 4.3, we review model fit. We then discuss what the model tells us about recent inflation in Section 4.4.

4.1 Estimation Framework

Referring back to Section 2.6.1, the impact of a given structural shock in the model depends on whether constraints bind today following the shock, as well as the expected duration that constraints are expected to continue to bind into the future following that shock. To make this dependence explicit, let us define a set of regimes ($\mathbb{R}_t$), which record which constraints are binding at a given point in time: $\mathbb{R}_t = \{\mathbf{1}(Y_t(1) = \bar{Y}_t(1)), \mathbf{1}(Y^*_{Mt}(1) = \bar{Y}^*_{Mt}(1))\}$, where the indicator functions switch on when individual constraints bind. Given a sequence $\{\mathbb{R}_{t+j}\}$ for $0 \leq j \leq J$, together with the assumption that $\{\mathbb{R}_{t+j}\} = \{0,0\}$ for $j > J$, we can solve for an equilibrium path for $\{X_t\}$, using the method described in Cagliarini and Kulish (2013) and Kulish and Pagan (2017) (see Appendix A for details).

Building on this idea, we re-parameterize the model solution in a convenient way. Specifically, let us define the duration that constraints are expected to bind from date t forward as $\mathbf{d}_t = [d_t, d^*_t]$, where each entry is a non-negative integer that records the number of periods that the domestic (d_t) or foreign constraint (d^*_t) binds. By convention, d_t and d^*_t take on zero values when constraints are slack today and expected to remain so in the absence of future shocks, and they are positive when they are binding today. As in Guerrieri and Iacoviello (2015), we construct policy matrices under the assumptions that agents know the state (X_{t-1}) and the current realization of the shocks (ε_t), but that they do not anticipate that future shocks will occur. Under these assumptions, $\mathbf{d}_t$ summarizes all the information about the anticipated sequence of regimes that is needed to solve for equilibrium responses to a one-time shock in our model. Specifically, constraints may switch on immediately in response to shock at date t, then bind for some (non-negative) number of consecutive periods, and switch off thereafter. In the absence of future shocks, constraints do not then switch on again in periods after they switch off (e.g., following a shock ε_t, constraints cannot be slack at date t and then binding at date $t+1$).[26] With these observations, we re-write the model solution directly in terms of durations:

$$X_t = \mathbf{J}(\mathbf{d}_t, \theta) + \mathbf{Q}(\mathbf{d}_t, \theta) X_{t-1} + \mathbf{G}(\mathbf{d}_t, \theta)\, \varepsilon_t, \tag{31}$$

where duration $\mathbf{d}_t$ implies a specific anticipated sequence of regimes over time. We refer the reader to Appendix A.3.2 for details on this result.

Following Kulish, Morley and Robinson (2017), Kulish and Pagan (2017), and Jones, Kulish and Rees (2022), our estimation framework exploits the fact that durations enter the policy function like parameters. As is standard, let us assume that observables (S_t) are linearly related to the unobserved state, as in $S_t = H_t X_t + \nu_t$, where ν_t is an i.i.d. vector of normally distributed measurement errors. Given $\mathbf{d} \equiv \{\mathbf{d}_t\}_{t=1}^T$ and θ, we can construct the piecewise linear solution with time-varying coefficients, and then apply the Kalman filter to construct the Likelihood function $\mathscr{L}(\theta, \mathbf{d} | \{S_t\}_{t=1}^T)$. We put priors over structural parameters and independent priors over durations

[26] To be careful, this is not a general property of models with potentially binding constraints, but rather one that holds given the structural assumptions in our model about behavior and shock processes. While we lack a general proof of this property, we verify it holds numerically in the model in practice, and we can demonstrate that imposing this criterion in the estimation procedure is reasonable via simulation analysis. One could capture a more complex structure of potential regime changes via introduction of additional parameters (e.g., durations for binding constraints that start one period forward), at the cost of added computational complexity.

to construct the posterior, and then estimate the model via Bayesian Maximum Likelihood.

In implementing this approach to estimation, we are careful to account for the fact that the duration of binding constraints is an equilibrium object in the model – i.e., d_t depends on both the state X_{t-1} and current shock ε_t in our model. Thus, we impose a rational expectations equilibrium restriction on admissible durations, which requires that agents' forecasts about how long constraints bind following a given shock are consistent with equilibrium model responses. To impose this restriction, we proceed as follows. For each proposed duration and parameter draw, we filter the data for smoothed shocks. We then evaluate whether the equilibrium model response to those smoothed shocks is consistent with the proposed duration draw. We retain the proposed draw if this requirement is satisfied; otherwise, we reject it and draw again.

In Appendix A.3.3, we study the performance of this procedure using simulated data, for which we know the true data generating process and the exact incidence of endogenously binding constraints. First, we confirm that our estimation procedure is able to recover unobserved durations from the observables that we use, by directly examining likelihood functions. Then, we also show that the reduced-form multipliers implied by the duration and parameter estimates align with true latent multipliers, which summarize the impacts of binding constraint on inflation, our key outcome.

Lastly, as a practical matter to restrict the size of the parameter space, we impose priors that allow capacity constraints to bind only periods from 2020:Q2 forward. Put differently, we impose dogmatic priors that assign zero probably to binding constraints prior to 2020:Q2, thus focusing on the role of capacity in explaining the unusual post-pandemic inflation dynamics.[27]

4.2 Data and Parameters

To populate Y_t, we collect standard macro variables together with particular series that serve to identify whether constraints are binding and shocks to them. Among standard macro variables, we include consumption price inflation and the growth rates of consumption expenditure for goods and services. We also use data on aggregate nominal GDP growth, the growth rate of (real) industrial production (which we treat as a proxy for output of the goods sector), and labor productivity growth by sector (measured as real value added per worker).[28] On the international side, we use data on

[27] As a robustness check, we have estimated the model allowing constraints to potentially bind starting in 2018:Q1, prior to the pandemic. We find that the mode of estimated durations before 2020:Q2 is zero, and that the mode of estimated durations after 2020:Q2 is not affected by the initial date when capacity constraints are allowed to bind.

[28] We use data on labor productivity growth in manufacturing and total (private sector) labor productivity growth from the Bureau of Labor Statistics. We assume that labor productivity growth in manufacturing coincides with goods labor productivity (growth in real value added per worker) in the model, while also matching aggregate (economy-wide) labor productivity growth in the model. While the definition of industrial production and goods output do not align exactly(industrial production includes manufacturing, mining, and electrical/gas utilities, while the BEA-defined goods sector excludes utilities and includes agriculture and construction), the dynamics of gross output for

import price inflation for consumption goods, and we proxy input price inflation in the model using data on inflation for imported industrial materials (excluding fuels). We then also use data on the growth of total expenditure on imported consumption goods and imported materials inputs (again excluding fuels), which we associate with imported inputs of goods.[29]

These data are all obtained from quarterly US national accounts produced by the Bureau of Economic Analysis, with the exception of labor productivity data from the Bureau of Labor Statistics and industrial production from the Federal Reserve Board (G.17 program). Having constructed growth rates for individual variables from the first quarter of 1990 through the first quarter of 2022, we detrend the data by removing the mean growth rate from each series. Finally, because our estimation sample includes a significant period during which interest rates are at the zero lower bound, we use data on the "shadow Fed Funds rate" to estimate parameters in the monetary policy rule.[30]

We present the full set of parameters for the model in Appendix A, which we obtain through a mix of estimation and calibration. We calibrate key value shares in the model – e.g., consumer expenditure, input use, export and import shares, etc. – to match US national accounts and input-output data (see Table A.4). We set a subset of the structural parameters to standard values from the literature, including preference parameters and some elasticities of substitution (see Table A.3).

We also calibrate the level of excess capacity for domestic and foreign firms, setting $\bar{Y}_0(1) = 1.05Y_0(1)$ and $\bar{Y}^*_{M0}(1) = 1.10Y^*_{M0}(1)$. These levels are chosen to be sufficiently high that constraints are slack prior to 2020:Q2.[31] Further, note that the model and data allows us to estimate the level of capacity that actually prevailed during the pandemic. Alternative values for steady state capacity then re-scale the size of the capacity shocks needed to achieve this realized capacity level. Consistent with this observation, the level of calibrated steady state capacity is not an important parameter in understanding the key quantitative results. To demonstrate this robustness, we estimate steady-state capacity levels directly in Appendix A.6, using data from the pandemic period,

the goods sector and industrial production are similar. We opt for industrial production data here, because we require a sufficiently long time series to estimate model parameters; quarterly gross output data is only available after 2005, while industrial production data is available from 1947.

[29] We use data for consumer goods (except food and automotive) to proxy for consumption imports, and we construct proxies for imported inputs (excluding fuels) by removing the subcategory of petroleum and products from industrial materials and supplies using standard chain index formulas and auxiliary NIPA data on the sub-categories of imports.

[30] During periods where the nominal Fed Funds rate is at zero, we replace it with the shadow rate from Wu and Xia (2016): `https://www.atlantafed.org/cqer/research/wu-xia-shadow-federal-funds-rate`. Changes in the shadow rate capture the consequences of unconventional policy actions taken by the Federal Reserve, such as forward guidance or quantitative easing policies. We have checked the results using an alternative shadow rate series from Jones, Kulish and Morley (2022) as well, which yields similar results.

[31] This amount of domestic excess capacity is consistent with historical fluctuations in capacity utilization for the US, as measured by the Federal Reserve's G.17 data series, for which the maximal value for capital utilization about five percent higher than the minimum. Further, cyclical fluctuations in this capacity utilization measure are almost entirely driven by changes in industrial production itself, rather than the Fed's estimate of capacity (based on firm survey data). Thus, our calibration accommodates historically normal fluctuations in industrial production, absent shocks to capacity.

and show that our main counterfactual results go through with this alternative parameterization.

Turning to the final set of parameters, we estimate (a) the elasticities of substitution between home and foreign goods, in consumption and production separately; (b) the parameters in the extended Taylor rule governing the response of interest rates to inflation and output, as well as interest rate inertia, (c) parameters governing the stochastic processes for exogenous variables, and (d) the variance of measurement errors. Regarding (c), we assume that exogenous variables evolve according to AR1 stochastic processes.

We obtain an estimated mean value for the elasticity of substitution between home and foreign goods of about 1.5 in consumption and 0.5 for inputs, so consumer goods are substitutes while inputs are complements. These values are not far from standard values estimated using aggregate time series variation in the macroeconomic literature, though there is limited prior work that distinguishes consumption and input elasticities. We find that the policy rule displays inertia, and it responds to both inflation and output gaps with reasonable magnitudes.[32] There is significant autoregressive persistence in most exogenous variables, and measurement error variances are plausible. See Table A.5 for the full set of estimated parameters.

4.3 Model Fit

Applying the quantitative model framework to the data, we construct Kalman-smoothed values for endogenous variables and observables. In Figure 11, we plot data and smoothed values for several key observables – goods, services, and aggregate price inflation for consumers, and imported input price inflation – over the 2017-2022 period, where each data point is the annualized value of quarterly inflation. To compute the smoothed inflation series, we take 1000 draws from the posterior distribution for model parameters, compute Kalman-smoothed inflation for each draw, and then plot statistics (the median, 5th, and 95% percentiles) for the distribution of smoothed values.

The model fits the dynamics of aggregate consumer price inflation well, accounting for essentially all of the four percentage point increase in headline inflation after 2020 (Figure 11a).[33] It also accounts well for the two percentage point rise in inflation for the services sector (Figure 11b). Because goods price inflation is substantially more volatile than that for services, the model attributes more of its variation to measurement error. Nonetheless, smoothed values for goods

[32] In unreported analysis, we have verified that our results are robust to alternative formulations and parameters for the monetary policy rule.

[33] Recall that aggregate consumer price inflation is treated as an unobserved variable. In the model, it is constructed by aggregating sector-level consumer price growth using fixed (steady-state) expenditure weights. In the data, however, the PCE deflator is a chain-weighted index, which features time-varying weights. Thus, part of the discrepancy between aggregate inflation in the model and data is likely due to differing index number concepts. Specifically, the dramatic increase in the goods expenditure share, combined with high goods price inflation, likely pushed measured inflation up relative to our fixed-weight index. Going forward, we focus entirely on decomposing model-based measures of inflation, so we do not belabor this point.

price inflation also track the data well (Figure 11c). The model replicates the initial (roughly six percentage point) surge in goods price inflation in 2021, and goods price inflation then remains elevated into 2022. While the model captures its transitory (up/down) dynamics, it moderately undershoots the level of goods price inflation in 2022, meaning that the model attributes the gap to measurement error. The model also matches inflation for imported goods inputs well (Figure 11d), matching both levels and dynamics closely.

For brevity here, we present similar figures illustrating model fit for the remaining observables in Appendix A.5. Together with the inflation figures here, we assess that the model captures the behavior of economic variables well during the pandemic, so it is a useful laboratory for exploring the driving forces underlying the inflation surge.

4.4 Explaining the Inflation Surge

We provide three sets of results. The first two illustrate the role of constraints in explaining inflation. First, we examine the dynamics of the multipliers on the constraints. Second, we present counterfactuals in which we switch off the constraints, comparing model responses to the same set of shocks with and without constraints. The third set of results focuses on how individual shocks and constraints shape inflation outcomes, both individually and via interactions between them.

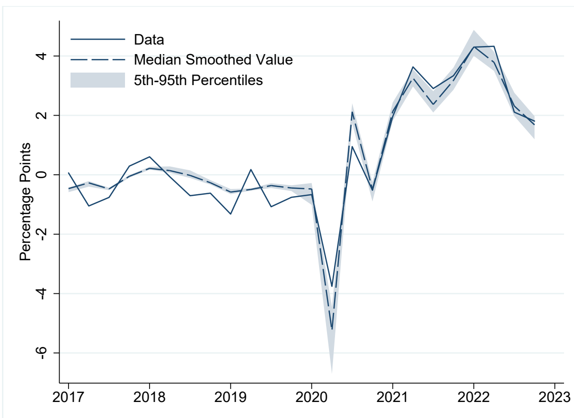
4.4.1 Multipliers on Constraints

To start, we can directly illustrate the impact of constraints by examining the smoothed value of multipliers on the domestic and foreign constraints. Because the multipliers themselves do not have intuitive economic units, we plot the reduced-form markup shocks implied by the value of the multipliers – given by $\left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{H0}(s)}\right)\hat{\mu}_t(s)$ in Equation 28 and $\left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{uF0}(s)}\right)\hat{\mu}^*_{ut}(s)$ in Equation 29 – which summarize the impulse of binding constraints for domestic and import price inflation. As is evident, the values of the multipliers rise in 2021, coincident with the rise in headline inflation.[34] On the import side, constraints appear to be slack in 2020, then bind sharply at the start of 2021, relax somewhat, then bind sharply again into 2022, and ease in the latter half of 2022. Domestic multipliers fluctuate in 2020 with gyrations in the US economy, but are near zero heading into

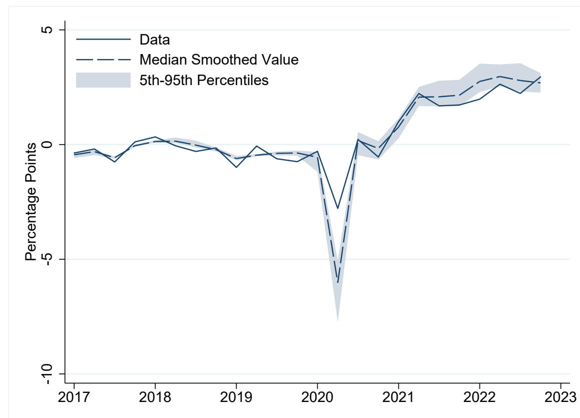
[34] As a technical note, recall we place positive mass on values $d_t > 0$ in our priors only starting in 2020:Q2, so the reduced-form markup shock (tied to the multiplier) in the figure is identically zero before that date. A second note is that while multipliers and hence reduced-form markup shocks are typically positive, they dip into negative territory at times in the simulations. This primarily reflects the fact that there is approximation error in the piecewise linear solution technique that we employ. When constraints bind, the value of the multipliers are computed as residuals in the log-linearized Phillips Curves. As such, while the quantity constraint binds exactly, the computed multipliers are approximations to the exact equilibrium multipliers; further, we do not impose a zero lower bound on them, as would be required in the full non-linear solution to the model. Despite this, we find that the estimated multipliers are typically positive, consistent with the underlying theory.

Figure 11: Consumer Price Inflation in Model and Data

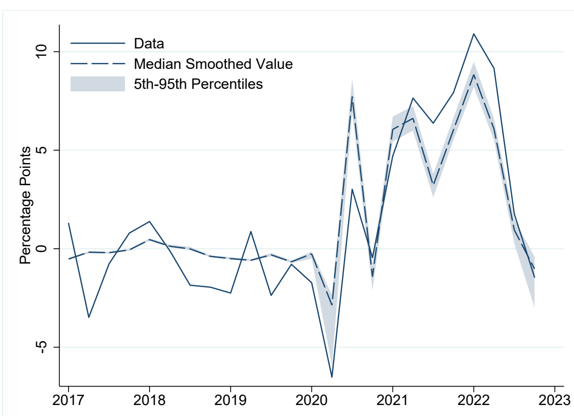
(a) Aggregate Consumer Inflation



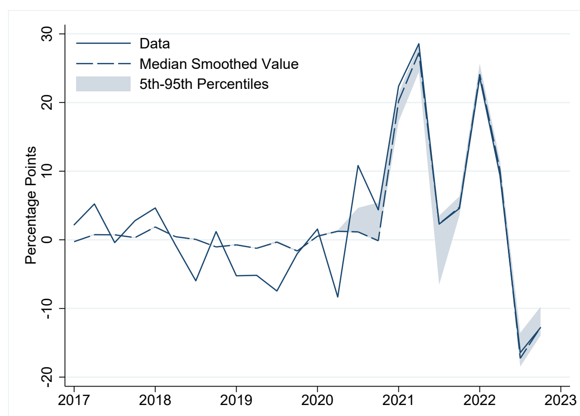
(b) Consumer Services Inflation



(c) Consumer Goods Inflation

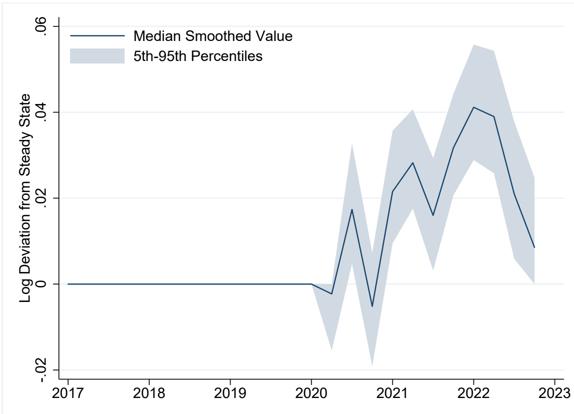


(d) Inflation for Imported Goods Inputs

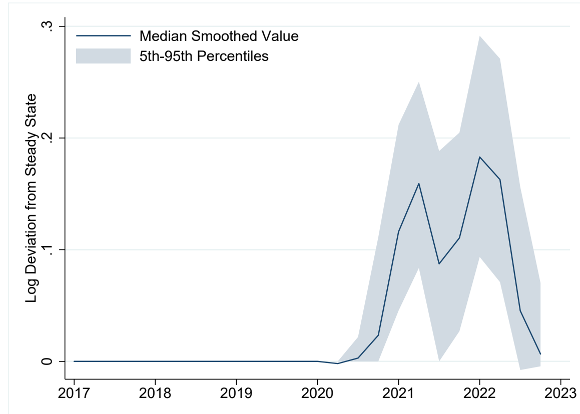


Note: Inflation at each date is the annualized value for demeaned quarterly inflation, in percentage points. If demeaned quarterly inflation is $\pi_t(s) = \ln P_t(s) - \ln P_{t-1}(s)$ where t indexes quarters, then the annualized inflation rate is $4\pi_t(s)$. Data is raw data. We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the dashed line. We shade the area covering the 5% to 95% percentile for smoothed values (the interval is imperceptibly small prior to 2020).

Figure 12: Smoothed Values for the Reduced-Form Markup Shock Implied by the Multipliers on Constraints



(a) Domestic Constraint



(b) Foreign Constraint

Note: Figure 12a plots composite variable $\left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{H0}(s)}\right)\hat{\mu}_t(s)$ and Figure 12b plots composite variable $\left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{uF0}(s)}\right)\hat{\mu}^*_{ut}(s)$, which are the reduced-form markup shocks in domestic and import price Phillips Curves induced by binding constraints. We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the solid line. We shade the area covering the 5% to 95% percentile for smoothed values.

2021. They rise steadily through 2021 into 2022, and then slacken (though still bind) through 2022:Q3.

While there is limited external data to which we can benchmark the estimated multipliers, we note that their joint dynamics align well with fluctuations in the New York Federal Reserve's Global Supply Chain Pressure Index over the post-2020 period, as we illustrate in Appendix A.5. For both multipliers, the high frequency dynamics also correspond to fluctuations in goods price inflation and imported input price inflation in Figure 11, which foreshadows the quantitative role of the constraints in explaining inflation. Further, the large absolute size of increases in multipliers, and their volatility translate into large, abrupt shifts in the Phillips Curves. In Appendix A.5, we show that these quasi-markup shocks are substantially larger than would be consistent with a stochastic process for (exogenous) markup shocks estimated from pre-pandemic data. Thus, our model appears to capture a source of markup variation that is distinct from run-of-the-mill markup (elasticity of demand) shocks. We turn to model counterfactuals to parse the role of constraints further.

4.4.2 Relaxing Constraints

We now provide counterfactual analysis as to how inflation would have evolved in the absence of capacity constraints, given the path of realized shocks that we infer hit the US economy after 2020.

To describe this exercise more precisely, the mechanics of each iteration are as follows. We first draw model parameters from the estimated posterior distributions, including the durations for binding constraints. Given these parameters, we apply the Kalman-filter to the data and construct smoothed model outcomes and shocks. Note that we construct smoothed shocks here assuming that constraints are potentially binding, in line with posterior duration estimates. Using these smoothed shocks, we then simulate the path of the economy under the counterfactual assumption that constraints are slack throughout, such that the solution conforms to the unconstrained equilibrium dynamics of the model. We repeat this procedure for one thousand posterior draws, and we plot statistics (means and percentiles) across these simulations in Figures 13 and 14.

Figure 13 presents results for consumer price inflation. The figures present raw data on annualized values of (de-meaned) quarterly inflation, along with data from counterfactual simulations in which we allow for measurement error in these observables.[35] In Figure 13a, we see that realized inflation for consumer goods is substantially higher than counterfactual inflation with slack capacity constraints during 2021 and into 2022, with the absolute gap peaking near six percentage points in early 2021. Put differently, given the shocks we infer from data, binding constraints account for about half of the acceleration in goods price inflation from 2020:Q2 through 2021:Q2. Likewise, they appear to explain about half of the decline in goods price inflation in the latter half of 2022.

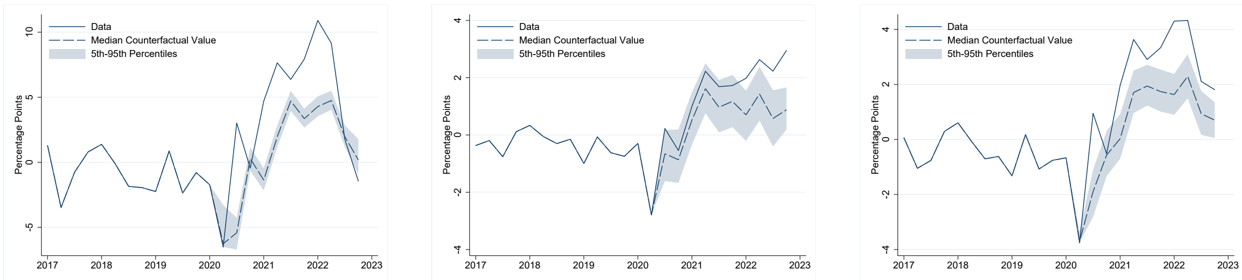
Under the hood, these inflation outcomes are tied to the impact of binding constraints in holding back production of domestic goods and foreign goods inputs. In Figure 14a, we plot the path for smoothed domestic goods output along with counterfactual output. As is evident, in the absence of constraints, goods output would have risen significantly in 2021 relative to its pre-pandemic level, as a result of the other shocks (principally, demand shocks) that hit the economy. The fact that output did not rise in reality speaks directly to the role of constraints. Output of foreign goods inputs is similarly constrained in Figure 14b. Correspondingly, smoothed inflation for both domestically-produced goods and foreign-produced inputs is substantially higher than counterfactual inflation in Figures 14c and 14d.

Interestingly, binding constraints also play an important role in driving price inflation for services in Figure 13b. While services price inflation initially accelerates due to the underlying shocks, it is between one and two percentage points higher in 2021 as a result of binding con-

[35] In the procedure described above, we draw the variance of the measurement error from the posterior, and then filter the data given this draw. We then add a draw from the measurement error to the smoothed counterfactual endogenous variables to get counterfactual values for the observables that are comparable to data. An alternative approach to presenting the results would be to compare smoothed observables to model counterfactuals without measurement error; naturally, this alternative leads to similar conclusions.

Figure 13: Counterfactual Consumer Price Inflation without Capacity Constraints

(a) Goods Inflation (b) Services Inflation (c) Aggregate Inflation



Note: We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, add measurement error to the observables, and then plot the median smoothed value as the solid line. We shade the area covering the 5% to 95% percentile for smoothed values.

straints. In the background, this reflects both the fact services use goods as inputs, so there is a direct inflation spillover from binding constraints in the goods sector via input-output linkages. Further, binding constraints serve to tighten the labor market as well, as the price increases they generate trigger substitution from goods inputs toward labor in production.

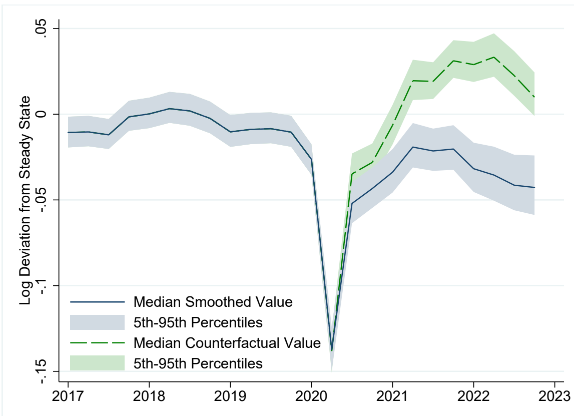
Adding up these results in Figure 13c, headline consumer price inflation is between one and two percentage points higher than counterfactual inflation during 2021-2022. And binding constraints account for about one third of the acceleration in headline goods price inflation from 2020:Q2 through 2021:Q2. Note further that the effect of constraints is substantially diminished late in 2022, as actual and counterfactual inflation converge again.

Finally, we revisit the discussion surrounding profits per unit in this counterfactual exercise. In Figure 5, we presented an index of nominal profits per unit of gross output for manufacturing and the aggregate economy. In Figure 15 we present analogous results from the model for goods and services.[36] Similar to the data, the smoothed data from our model yields a sharp increase in profits for the goods sector during the 2021-2022 period, even though this is not a targeted data moment. In contrast, the counterfactual economy with slack constraints yields no such goods profit surge. Moreover, profits per unit are essentially flat through the pandemic period (outside the 2020 spike), for both the economies with and without capacity constraints. We conclude that the model provides a plausible explanation for the run-up in profits for goods producers that occurred alongside the

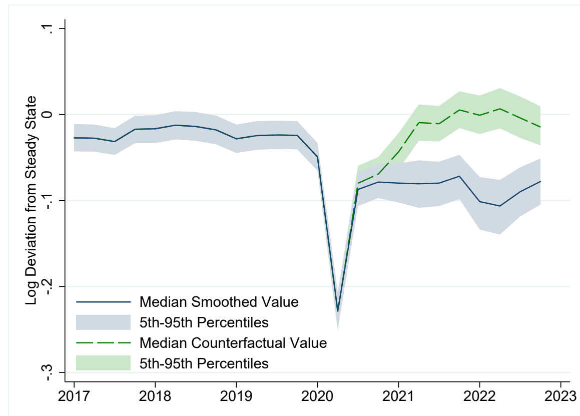
[36] In the model, the log change in nominal profits per unit of output from a given base period ($t = 0$) is given by: $\left[\hat{\Xi}_t(s) - \hat{y}_t(s)\right] - \left[\hat{\Xi}_0(s) - \hat{y}_0(s)\right] = [\hat{p}_{Ct} - \hat{p}_{C0}] + \varepsilon\,[\widehat{rp}_t(s) - \widehat{rp}_0(s)] - (\varepsilon - 1)\,[\widehat{rmc}_t(s) - \widehat{rmc}_0(s)]$, where $\hat{p}_{Ct} - \hat{p}_{C0} = \sum_{s=0}^{t} \pi_{Cs}$. We add trend inflation to these log changes, to make it comparable to the date in Figure 5, and then we convert the log change to levels to plot the index. While we discuss manufacturing and aggregate profits in Figure 5 due data availability in the national accounts, the goods sector in our calibration includes manufacturing and other non-manufacturing goods sectors. Further, again for data reasons, we focused on corporate profits in Figure 5, while we have no distinction between corporate and non-corporate entities in the model. This implies that one ought to focus on qualitative comparisons between the figures, rather than a more precise quantitative comparison.

Figure 14: Counterfactual Quantities and Inflation without Capacity Constraints

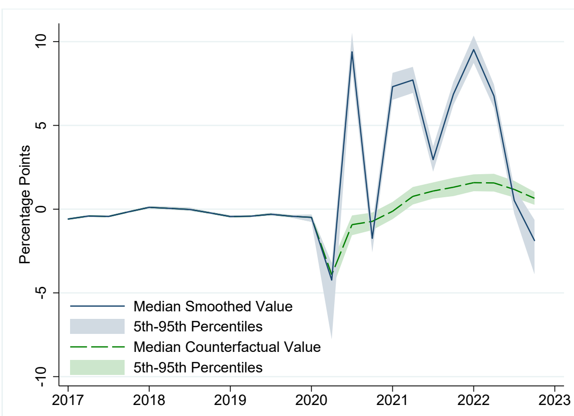
(a) Domestic Goods Output ($Y_t(1)$)



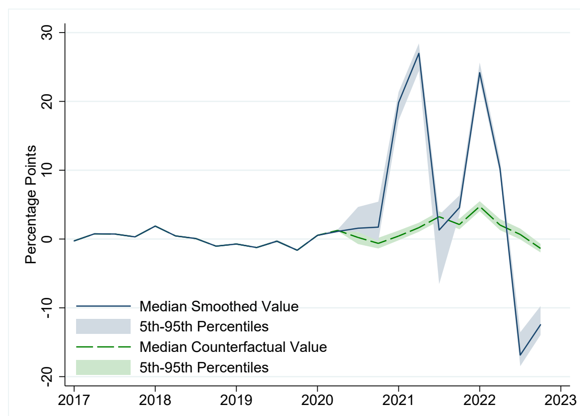
(b) Imported Goods Inputs $Y^*_{Mt}(1)$



(c) Domestic Goods Price Inflation ($\pi_t(1)$)



(d) Imported Goods Input Inflation ($\pi_{Mt}(1)$)

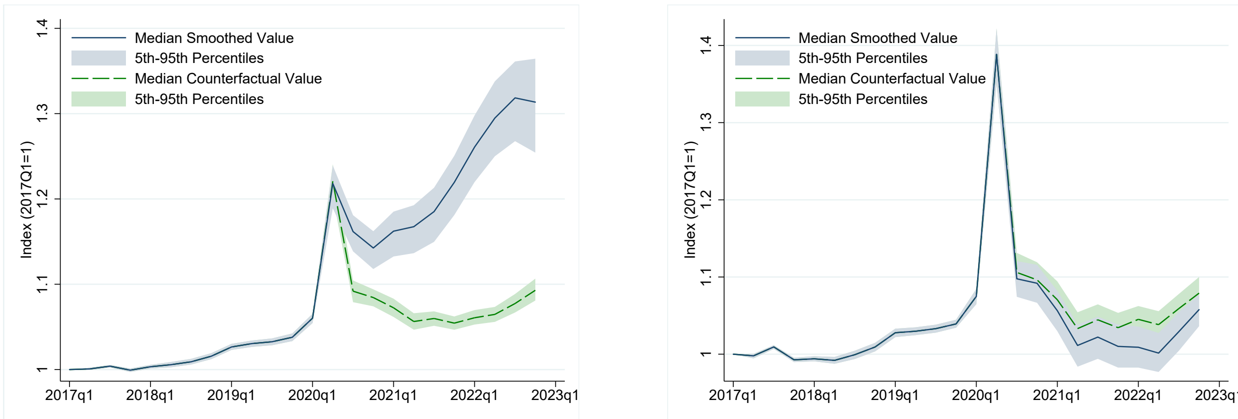


Note: We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the solid line. We shade the area covering the 5% to 95% percentile for smoothed values. Counterfactual assumes that constraints are slack in all periods.

Figure 15: Counterfactual Profits per Unit without Capacity Constraints

(a) Goods

(b) Services



Note: We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the solid line. We shade the area covering the 5% to 95% percentile for smoothed values.

inflation takeoff, where both are explained in large measure by binding capacity constraints.

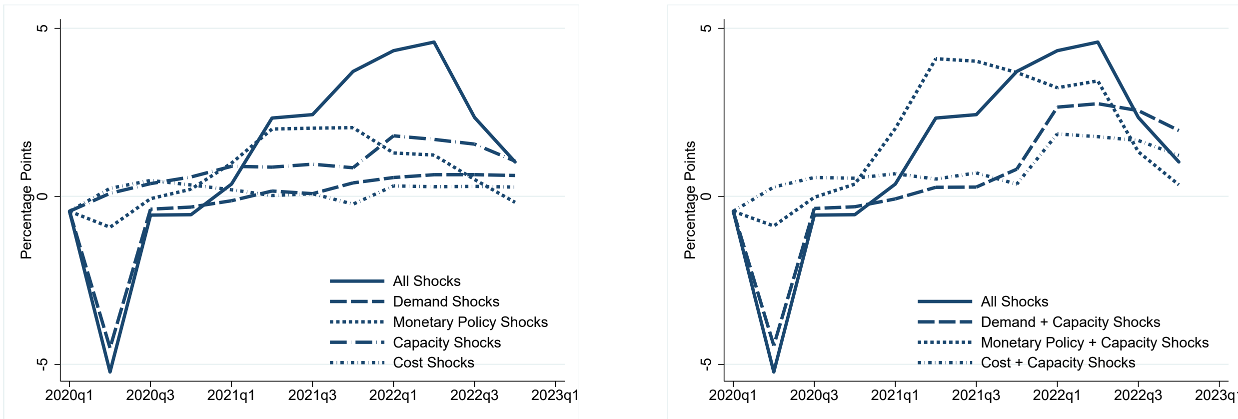
4.4.3 Decomposing the Role of Individual Shocks and Constraints

We now examine the role of individual shocks in explaining inflation outcomes. To construct the counterfactual series, we take a draw from the posterior distributions for structural parameters and durations. Using this draw to parameterize the state equation (Equation 31), we Kalman filter the data to obtain smoothed shocks. We then feed a subset of these shocks into the structural model (summarized by Equation 27) to compute counterfactual model outcomes. In each case, we solve for the simulated equilibrium path using Dynare's OccBin procedure. By doing so, we ensure that whether constraints bind at particular points in time in response to shocks is endogenous. We repeat this procedure 1000 times and compute the median across the simulated counterfactual series, and these medians are plotted in Figure 16.

In Figure 16a, we plot the path of aggregate consumer price inflation following four types of shocks, each fed individually into the model: demand shocks (including both the discount rate and goods-biased preference shocks), monetary policy shocks, capacity shocks, and cost shocks (including domestic productivity and foreign cost shocks). The final line is the value for inflation when all shocks are fed simultaneously into the model. At the outset, temporary negative demand shocks yield a decline then rebound of inflation in 2020. Into 2021, however, no single shock appears to play a particularly important role in explaining the path of inflation on its own. The underlying reason is that no single shock is capable of causing capacity constraints to bind, so model outcomes conform closely to those observed in the prior counterfactuals in which we exogenously

Figure 16: Counterfactual Consumer Price Inflation for Individual Shocks

(a) Individual Shocks (b) Capacity Shocks plus Individual Shocks



Note: Each series represents the simulated path of consumer price inflation (quarterly value, annualized) for the indicated subset of smoothed shocks during 2020-2022. See text for definition of the counterfactuals.

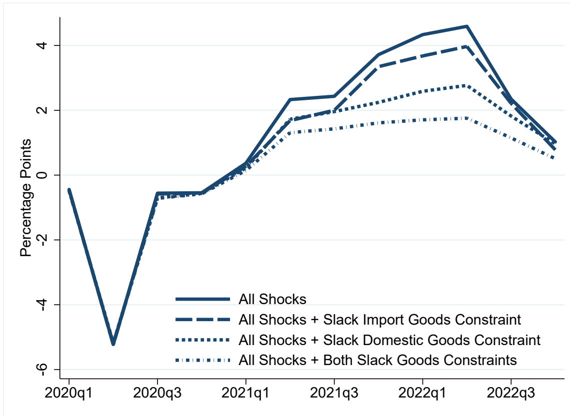
relaxed the constraints.

In Figure 16b, we plot a second set of counterfactuals, in which individual shocks are fed into the model in combination with shocks to capacity. In contrast to the prior figure, monetary policy stands out here. While monetary policy shocks play essentially no role in 2020, expansionary monetary policy shocks in 2021 in combination with prevailing negative capacity shocks led to a surge in inflation of about 4 percentage points in 2021. Put differently, while negative capacity shocks are insufficient on their own to trigger binding constraints, negative capacity shocks set the stage for demand-side shocks – especially expansionary monetary policy – to trigger the constraints. In turn, as monetary shocks dissipate in 2021 (i.e., as the Federal Reserve raises interest rates to bring them back in line with the extended Taylor rule), then inflation falls rapidly in 2022 as constraints are relaxed again. In contrast, the role of the demand shock is much more muted in 2021 possibly reflecting the smaller expansionary effects of fiscal vs. monetary policy in that year, but continued to stoke inflation in 2022.[37] Thus, we conclude that the dynamics of monetary policy during this period interacted with shocks to capacity, are the driving force behind the rapid rise and subsequent fall in inflation during the post-pandemic period.

As a final set of counterfactuals, recall that there are both domestic and import constraints. In counterfactual results above, we have relaxed these in tandem, but it is natural to wonder what the relative contribution of each constraint is to the results. Thus, we now plot counterfactuals in which we relax one constraint at a time, and then in tandem, following the same approach to generate the

[37] Both in variance and historical decompositions we observe a predominant role of shocks to the discount rate in driving output consistent with a realistic fiscal multiplier. However, in 2021 the historical decompositions for goods and overall output reveal a stronger role for monetary than discount rate shocks.

Figure 17: Counterfactual Consumer Price Inflation With Relaxed Domestic versus Import Constraints



Note: Each series represents the simulated path of consumer price inflation (quarterly value, annualized) for all shocks and the indicated set of constraints during 2020-2022. See text for definition of the counterfactuals.

counterfactuals as in the prior figure. As in Figure 13c, relaxing both constraints evidently lowers the amount of realized inflation resulting from the shocks. The domestic constraint plays a more important role in explaining the joint gap, accounting for roughly three-quarters of the overall impact of constraints. This is to be expected, in that imports account for a minority of overall spending on inputs, which limits the quantitative role of import constraints relative to domestic input constraints. Nonetheless, both constraints play independent roles.

5 Extensions

In this section, we present two sets of additional results, which probe the robustness of our findings. First, we examine whether our results change when we account more carefully for energy shocks. Second, we enrich the labor market structure of the model, to account for labor market stress during the pandemic.

5.1 Accounting for Energy Shocks

During 2021-2022, global energy prices escalated, as strong demand for energy combined with supply disruptions (e.g., following from the Ukraine war) to drive energy prices up. Further, since the middle of 2022, energy prices have receded rapidly as inflation has cooled. A natural question arises then about whether the dynamics of inflation that we attribute to occasionally binding constraints might instead be driven by these energy price fluctuations.

To frame this discussion, note that our model abstracts from the peculiar features of energy markets – i.e., we do not attempt to model energy prices, production, and demand explicitly. Therefore, we think it reasonable to estimate our model using data that also excludes energy prices. In part, we have already done this in prior sections, in that we have stripped out petroleum and fuels when we constructed the price index for imported materials. Here we also remove energy prices from the domestic price indexes used in estimation – constructing PCE inflation for goods and services, excluding energy. Specifically, we remove prices for "gasoline and other energy goods" (which includes motor vehicle fuels and lubricants, fuel oil, and other fuels) from the goods PCE price index, and then we remove prices for electricity and gas utilities from the services PCE price index. We then re-estimate the model using the modified domestic price indexes.

In Figure 18a, we plot the adjusted PCE inflation series for goods prices and overall consumption.[38] Goods price inflation is virtually indistinguishable with/without energy through 2021:Q3, during the initial inflation takeoff. Thereafter, energy prices push inflation up during early 2022, and then rapidly bring goods price inflation down thereafter. Nonetheless, the basic inverted U-shape for goods price inflation appears in both series, with non-energy goods price inflation falling from 8 percent to near zero during the course of 2022. Overall PCE price inflation then reflects these deviations in goods price inflation.

In Figure 18b, we investigate the role of these differences for our conclusions about the role of constraints in explaining inflation dynamics. The simulations here follow the same scheme as in Section 4.4.3: we compare simulated inflation when all shocks are fed through the model to counterfactual inflation when one or both constraints are relaxed. As in the prior counterfactuals, binding constraints continue to play a large quantitative role in driving inflation. One difference is that the foreign constraint plays a somewhat larger role in explain inflation here than in the prior simulation. To understand this, recall that we removed energy prices from import prices in all the analysis above, but we left them in domestic prices. Now, we also remove energy from domestic prices, so this increases measured import price inflation relative to domestic price inflation, and thus the import constraint appears more important. Overall, however, we read these results as confirming that constraints play an important role in explaining inflation, above and beyond any separate impacts of energy price shocks.

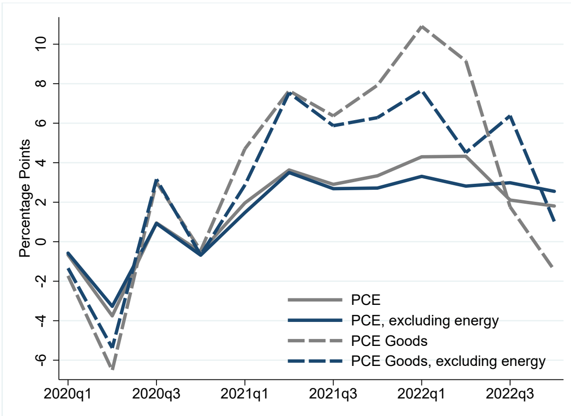
5.2 Enriching the Labor Market

Motivated by pervasive discussion of labor markets during the pandemic period and recovery, we enrich the labor market of the model in three ways. First, we allow for adjustment frictions for nominal wages, in addition to price adjustment frictions. Second, we introduce shocks to the disu-

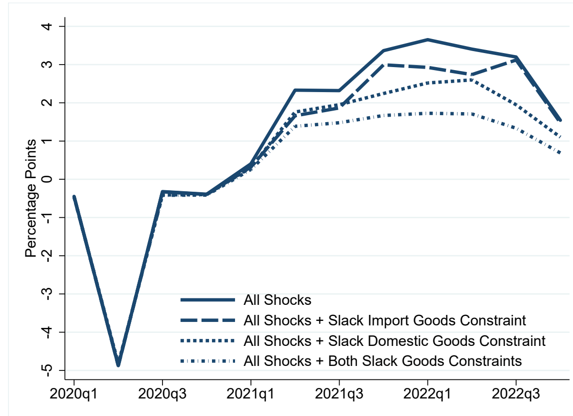
[38] Services inflation looks very similar with and without energy prices, so we omit it for clarity in the figure.

Figure 18: Accounting for Energy Shocks

(a) PCE Inflation with and without Energy



(b) Capacity Shocks plus Individual Shocks



Note: In Figure 18b, each series represents the simulated path of consumer price inflation (quarterly value, annualized) for all shocks and the indicated set of constraints during 2020-2022. See text for definition of the counterfactuals.

tility of labor supply, which stand in for various pandemic-related supply shocks (e.g., responses to disease risk, the great resignation, etc.). Third, we incorporate an occasionally-binding constraint on labor supply, in addition to the goods market capacity constraints considered previously. Unlike normal times, labor supply constraints plausibly loomed large during the COVID period, where stay-at-home orders, school closures, and other abnormal policies constrained households' ability to supply labor to the market.

For brevity, we consign the details about this extended model to Appendix B, and we instead focus on one key result here. The model yields a wage Phillips Curve that takes the form:

$$\pi_{Wt} = \left(\frac{\varepsilon_L - 1}{\phi_W}\right)\left[\widehat{mrs}_t - \widehat{rw}_t\right] + \left(\frac{\varepsilon_L}{\phi_W}\frac{P_0}{W_0}\right)\hat{\tilde{\mu}}_{Lt} + \beta E_t\left(\pi_{Wt+1}\right), \tag{32}$$

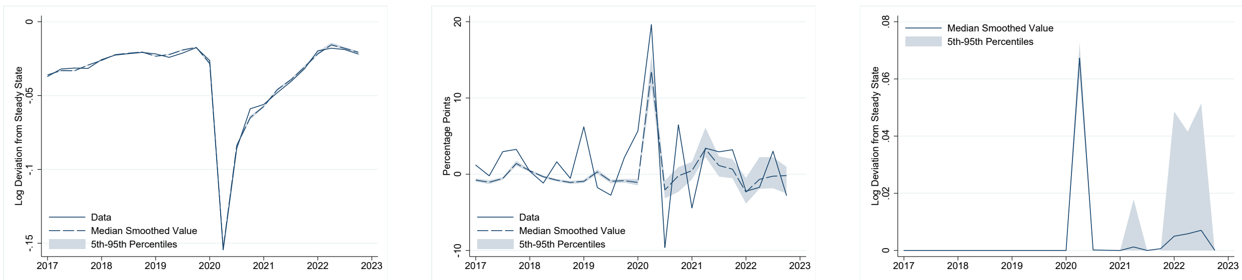
where π_{Wt} is nominal wage inflation, mrs_t is the log of the marginal rate of substitution between labor supply and consumption in preferences, rw_t is the log real wage, and $\tilde{\mu}_{Lt} \equiv 1 + (\mu_{Lt}/C_t{}^{-\rho})$ is a function of the multiplier on the labor constraint (μ_{Lt}).[39]

Two important results follow from inspection of Equation 32. The first (standard) result is that labor (disutility) supply shocks enter the wage Phillips curve via the marginal rate of substitution ($\widehat{mrs}_t$), where increased disutility of supplying labor raises $\widehat{mrs}_t$ and thus wage inflation. Elsewhere in the model, increases in the disutility of labor supply also naturally lower the equilibrium quantity of labor employed as well. The second (non-standard) result is that binding labor constraints appear

[39]For completeness, the parameter ε_L controls steady-state wage markups (the degree of market power exercised by workers) and the parameter ϕ_W controls the flexibility of wages. See Appendix B for the details underlying derivation of Equation 32, and how it fits into the remainder of the model.

Figure 19: Model Fit with Labor Market Extensions

(a) Hours Worked (b) Real Wage Growth (c) Reduced-Form Wage Markup Shock



Note: We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the solid line. We shade the area covering the 5% to 95% percentile for smoothed values. In Figure 19c, we plot the reduced form labor markup shock term $\left(\frac{\varepsilon_L}{\phi_W}\frac{P_0}{W_0}\right)\hat{\mu}_{Lt}$.

as reduced-form "markup shocks" in the wage Phillips Curve. As a result, binding labor constraints drive up wage inflation, conditional on the other labor market fundamentals.

With these results in hand, we turn to quantitative analysis. We calibrate several new parameters (e.g., ε_L and ϕ_W) based on external references. We then re-estimate the extended model along with stochastic processes for labor disutility and labor constraint shocks using two new observable data series: aggregate hours worked and real wage growth, which are constructed using data from the US Bureau of Labor Statistics. Details on these steps are provided in Appendix B.

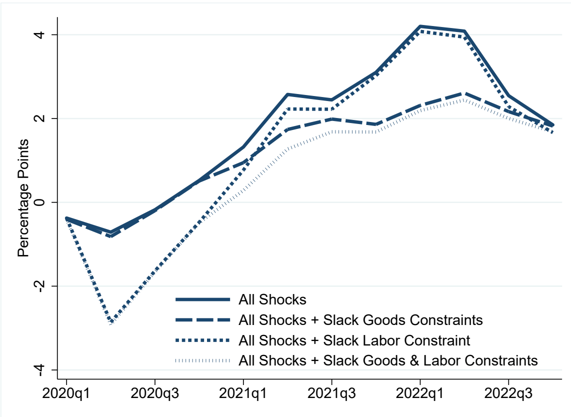
Turning to results, we illustrate model fit and smoothed multipliers on the labor constraint in Figure 19. In Figure 19a, there is an obvious dramatic collapse in hours in early 2020:Q2, a rapid partial rebound in Q3, and then a slow recovery thereafter through 2021. The model matches these dynamics well, in large part through shocks to labor supply. In addition, Figure 19b illustrates that there were sharp gyrations in real wage growth during the early pandemic. However, real wage growth from 2020:Q4 forward was similar to the pre-pandemic period. Turning to Figure 19c, the model clearly favors a binding labor constraint in 2020:Q2, in order to explain the spike and subsequent collapse in real wage growth. Labor constraints then play a less important role in 2021-2022. The median simulation has a slack or nearly slack labor constraint in most periods, though labor constraints do appear to bind in 2022 for a non-trivial share of the simulations.

To evaluate how incorporating labor supply shocks and constraints affect our prior results, we present two sets of counterfactuals.[40] First, in Figure 20a, we illustrate how relaxing the goods

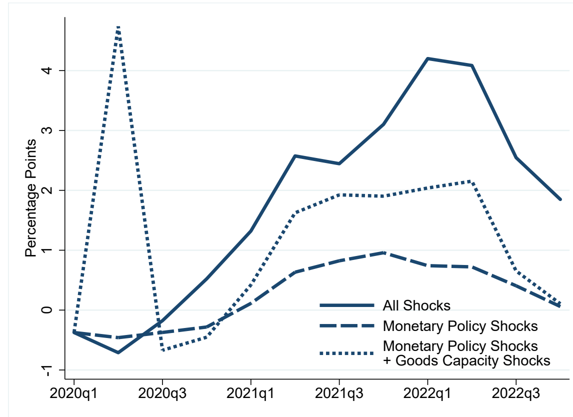
[40] Like prior counterfactuals, we draw form the posterior to parameterize the model and filter smoothed shocks from data, and we then simulate responses to subsets of the smoothed shocks under various assumptions about whether constraints bind. Repeating this procedure 1000 times, we report median outcomes in the figures. As a technical matter, we allow goods constraints to bind endogenously in all these simulations. The labor constraint is a third con-

Figure 20: Counterfactuals with Labor Market Extensions

(a) Relaxing Goods Capacity or Labor Constraints



(b) Monetary Policy Shocks and Capacity Shocks



Note: Each series represents the simulated path of consumer price inflation (quarterly value, annualized) for the indicated subset of smoothed shocks and constraints during 2020-2022. See text for definition of the counterfactuals.

and labor constraints separately and in combination affects inflation. When the labor constraint is assumed to be slack, inflation falls substantially at the outset of the pandemic, which is counterfactual; thus, binding labor constraints help explain the absence of disinflation in 2020. However, their impact dissipates rapidly, such that inflation is essentially similar across versions of the model with and without labor constraints in 2021-2022. In contrast, assuming goods constraints are slack has little impact on inflation in 2020, but then inflation would have been significantly lower in 2021-2022 with slack goods constraints (this echoes Figure 13c). Further, we point out that the quantitative impact of removing the goods market constraints is essentially the same in this model with labor supply (disutility) shocks as in the baseline without them.

Second, in Figure 20b, we investigate again how monetary policy interacts with constraint shocks in this version of the model. In these simulations, the first simulation shuts off all shocks except for the monetary policy shocks, and the second considers the joint impact of monetary policy shocks and capacity shocks for both domestic and imported goods. As in the prior simulations, monetary policy alone has a moderate effect on inflation, while monetary policy combined with capacity shocks lead to a rapid increase in inflation in 2021, sustained high inflation through 2021 into 2022, and then a collapse in inflation from 2022:Q3 forward.

straint, which complicates simulation, as the Dynare implementation of OccBin only admits two potentially binding constraints. Therefore, we impose the labor constraint by assuming that there are reduced-form wage markup shocks, which are tied to the smoothed values of the multiplier on the labor constraint. We then solve for whether the two goods constraints are binding endogenously.

6 Concluding Remarks

We have developed a quantitative framework to study inflation that places potentially binding capacity constraints at center stage. We show that binding constraints alter the Phillips Curve relationship between inflation and real marginal costs, because firms take these constraints into account when setting prices. Specifically, when constraints bind, firms set prices to equate demand to their constrained capacity, rather than targeting their optimal unconstrained markup over marginal costs. This implies that binding constraints introduce a term that looks like a markup shock in both domestic and import price Phillips Curves. Applying the quantitative framework to interpret recent US data, we find that binding constraints are quantitatively important drivers of inflation, explaining half of the rise in US inflation during 2021-2022. We also find that negative capacity shocks tightened constraints during the pandemic period, which set the stage for modestly-sized demand shocks to have outsized impacts on inflation. In particular, monetary policy shocks loom large in driving inflation in 2021.

Going forward, there are various extensions of this framework that would be useful to consider. While the model includes demand-side (discount rate) shocks that capture important aspects of fiscal policy, it would be useful to extend the model to provide a more careful treatment of fiscal shocks, especially to study the tax policy instruments that supported consumption in the early stages of the pandemic. Further, we have included capacity as an exogenous, stochastic variable in our framework. We also see high returns to extending the model to include endogenous capacity investment. Lastly, while we have focused on applying the framework to analyze US data in this paper, it would clearly be interesting to parse data for other countries (e.g., the UK and euro area) that experienced similar high inflation episodes. Because energy prices likely played a larger role in these related contexts, we also see value in extending the model to treat energy supply and use more carefully.

More generally, the framework we have developed can be deployed to study optimal policy, and by extension potential policy mistakes during the pandemic recovery. In our framework, binding constraints imply that demand shocks work through both the IS and Phillips Curves, appearing like a markup shock. This would appear to complicate policy design, relative to canonical frameworks in which shocks to the IS and Phillips Curves are unrelated. Further, when reduced-form markups may reflect either the influence of exogenous markup shocks, or the impact of binding constraints, optimal policy will depend on the central bank's ability to discriminate between them. Given the importance of monetary policy shocks in our quantitative analysis, a critical analysis of policy is warranted.

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A Quantitative Model

In this appendix, we discuss the quantitative version of the general model described in Section 2. We start by presenting the log-linear approximation of the model equilibrium conditions and the stochastic processes for exogenous variables. We then proceed to present the full details of the calibration and model estimation procedure. We provide supplemental results on parameter estimates, model fit, and robustness checks in later sections.

A.1 Log-Linearization of the Model Equilibrium Conditions

In Table 1, we wrote out the full nonlinear model equilibrium. In practice, we solve a piecewise linear approximation to the model, as in Guerrieri and Iacoviello (2015). This entails log-linearizing the model equilibrium conditions for both the unconstrained and constrained equilibria around the steady state.

We normalize Home prices relative to the domestic price level, and we denote "real" prices with the letter r attached to the price. Further, lower case variables with hats denote log deviations from steady state. For example, the log deviation in the real wage from steady state is given by $\widehat{rw}_t = \hat{w}_t - \hat{p}_t$, while the real price of home output in sector s is $\widehat{rp}_{Ht}(s) = \hat{p}_{Ht}(s) - \hat{p}_t$, and so on.[41] Foreign currency prices (denoted by stars) are normalized relative to the foreign price level; for example, foreign real marginal costs are $\widehat{rmc}^*_t(s) = \widehat{mc}^*_t - \hat{p}^*_t$. We also define deviations in the value of constraints from steady state: $\hat{y}_t(1) = \ln \bar{Y}_t(1) - \ln \bar{Y}_0(1)$ and $\hat{y}^*_t(1) = \ln \bar{Y}^*_t(1) - \ln \bar{Y}^*_0(1)$. Finally, to reduce the number of potential foreign shocks, we assume that foreign export demand is given by $X^*_t(s) = \varpi(s) \left(\frac{P^*_t}{P^*_t(s)} \right)^{-\sigma(s)} C^*_t$, where we treat $\frac{P^*_t}{P^*_t(s)}$ and $\varpi(s)$ as constants, so $\hat{x}^*_t(s) = \hat{c}^*_t$.

We present the log-linear equilibrium conditions in Tables A.1 and A.2. Table A.1 contains equilibrium conditions that hold in both unconstrained and constrained equilibria. Table A.2 collects equilibrium conditions that differ across equilibria, depending on which constraints are slack or binding.

A.2 Stochastic Processes

We collect log deviations in exogenous domestic and foreign variables – including $\hat{\Theta}_t$, $\hat{\zeta}_t(1)$, $\hat{c}^*_t$, and $\left\{\hat{z}_t(s), \widehat{rmc}^*_t\right\}_s$ – into vector $\hat{F}_t$, and we assume that $\hat{F}_t$ is a first-order vector autoregressive process, as in $\hat{F}_t = \Lambda \hat{F}_{t-1} + \varepsilon_t$, where Λ is a diagonal matrix that contains autoregressive coefficients for each series (denoted λ_x for variable x) and ε_t is a vector of shocks.[42] We assume the vector of

[41] For completeness, $\widehat{rp}_t(s) = \hat{p}_t(s) - \hat{p}_t$, $\widehat{rp}_{Ft}(s) = \hat{p}_{Ft}(s) - \hat{p}_t$, $\widehat{rmc}_t(s) = \widehat{mc}_t(s) - \hat{p}_t$, $\widehat{rp}_{Mt}(s) = \hat{p}_{Mt}(s) - \hat{p}_t$, $\widehat{rp}_{Mt}(s', s) = \hat{p}_{Mt}(s', s) - \hat{p}_t$.

[42] Note that we have imposed the restriction that foreign real marginal costs are the same for goods and services: $\widehat{rmc}^*_t(s) = \widehat{rmc}^*_t$. We estimate the stochastic process for this variable using data for goods imports, roughly speaking.

Table A.1: Common Equilibrium Conditions across Unconstrained and Constrained Equilibria

Labor Supply	$-\rho\hat{c}_t + \widehat{rw}_t = \psi\hat{l}_t$
Consumption Allocation	$\hat{c}_t(s) = \hat{\zeta}_t(s) - \vartheta\widehat{rp}_t(s) + \hat{c}_t$ with $\sum_s \zeta_0(s)\hat{\zeta}_t(s) = 0$ $\hat{c}_{Ht}(s) = -\varepsilon(s)\left(\widehat{rp}_{Ht}(s) - \widehat{rp}_t(s)\right) + \hat{c}_t(s)$ $\hat{c}_{Ft}(s) = -\varepsilon(s)\left(\widehat{rp}_{Ft}(s) - \widehat{rp}_t(s)\right) + \hat{c}_t(s)$
Euler Equation	$0 = \mathbf{E}_t\hat{\Theta}_{t+1} - \hat{\Theta}_t - \rho\left(\mathbf{E}_t\hat{c}_{t+1} - \hat{c}_t\right) + i_t - \mathbf{E}_t\pi_{t+1}$
Consumer Prices	$0 = \sum_s \left[\zeta_0(s)\left(\frac{P_0(s)}{P_0}\right)^{1-\vartheta}\right]\left[\widehat{rp}_t(s) + \frac{1}{1-\vartheta}\hat{\zeta}_t(s)\right]$ $\widehat{rp}_t(s) = \gamma(s)\left(\frac{P_{H0}(s)}{P_0(s)}\right)^{1-\varepsilon(s)}\widehat{rp}_{Ht}(s) + (1-\gamma(s))\left(\frac{P_{CF0}(s)}{P_0(s)}\right)^{1-\varepsilon(s)}\widehat{rp}_{Ft}(s)$
Labor Demand	$\widehat{rw}_t + \hat{l}_t(s) = \widehat{rmc}_t(s) + \hat{y}_t(s)$
Input Demand	$\widehat{rp}_{Mt}(s) + \hat{m}_t(s) = \widehat{rmc}_t(s) + \hat{y}_t(s)$ $\hat{m}_t(s',s) = -\kappa\left(\widehat{rp}_{Mt}(s',s) - \widehat{rp}_{Mt}(s)\right) + \hat{m}_t(s)$ $\hat{m}_{Ht}(s',s) = -\eta(s')\left(\widehat{rp}_{Ht}(s') - \widehat{rp}_{Mt}(s',s)\right) + \hat{m}_t(s',s)$ $\hat{m}_{Ft}(s',s) = -\eta(s')\left(\widehat{rp}_{FMt}(s') - \widehat{rp}_{Mt}(s',s)\right) + \hat{m}_t(s',s)$
Marginal Cost	$\widehat{rmc}_t(s) = -\hat{z}_t(s) + (1-\alpha(s))\widehat{rw}_t(s) + \alpha(s)\widehat{rp}_{Mt}(s)$
Input Prices	$\widehat{rp}_{Mt}(s) = \sum_{s'}\left(\frac{\alpha(s',s)}{\alpha(s)}\right)\left(\frac{P_0(s',s)}{P_{M0}(s)}\right)^{1-\kappa}\widehat{rp}_{Mt}(s',s)$ $\widehat{rp}_{Mt}(s',s) =$ $\xi(s',s)\left(\frac{P_{H0}(s')}{P_0(s',s)}\right)^{1-\eta(s')}\widehat{rp}_{Ht}(s') + (1-\xi(s',s))\left(\frac{P_{MFt}(s')}{P_0(s',s)}\right)^{1-\eta(s')}\widehat{rp}_{FMt}(s')$
Consumption Import Pricing	$\pi_{Ft}(s) = \frac{\varepsilon-1}{\phi(s)}\left(\widehat{rmc}^*_t(s) + \hat{q}_t - \widehat{rp}_{Ft}(s)\right) + \beta\mathbf{E}_t\pi_{Ft+1}(s)$
Domestic Pricing for Services	$\pi_{Ht}(2) = \frac{\varepsilon-1}{\phi(2)}\left(\widehat{rmc}_t(2) - \widehat{rp}_{Ht}(2)\right) + \beta\mathbf{E}_t\pi_{Ht+1}(2)$
Input Import Pricing for Services	$\pi_{MFt}(2) = \frac{\varepsilon-1}{\phi(2)}\left(\widehat{rmc}^*_t(2) + \hat{q}_t - \widehat{rp}_{FMt}(2)\right) + \beta\mathbf{E}_t\pi_{FMt+1}(2)$
Market Clearing	$\hat{y}_t(s) = \left(\frac{C_{H0}(s)}{Y_0(s)}\right)\hat{c}_{Ht}(s) + \sum_{s'}\left(\frac{M_{H0}(s,s')}{Y_0(s)}\right)\hat{m}_{Ht}(s,s') + \left(\frac{X_0(s)}{Y_0(s)}\right)\hat{x}_t(s)$ $\hat{x}_t(s) = -\sigma(s)\left(\widehat{rp}_{Ht}(s) - \hat{q}_t\right) + \hat{c}^*_t$ $\hat{y}^*_{Ct}(s) = \hat{c}_{Ft}(s)$ $\hat{y}^*_{Mt}(s) = \sum_{s'}\left(\frac{M_{F0}(s,s')}{Y^*_{M0}(s)}\right)\hat{m}_{Ft}(s,s')$ $\hat{\Theta}_t - \rho\left(\hat{c}_t - \hat{c}^*_t\right) + \hat{q}_t = 0$ $\sum_s\left(\frac{L_0(s)}{L_0}\right)\hat{l}_t(s) = \hat{l}_t$
Monetary Policy Rule	$i_t = \rho_i i_{t-1} + \omega(1-\rho_i)\hat{\bar{\pi}}_t + (1-\rho_i)\rho_y\hat{y}_t + \hat{\Psi}_t$ with $\hat{y}_t = \sum_s\left(\frac{P_0(s)Y_0(s)}{Y_0}\right)\hat{y}_t(s)$
Auxiliary Inflation Definitions	$\pi_{Ht}(s) = \widehat{rp}_{Ht}(s) - \widehat{rp}_{Ht-1}(s) + \pi_t$ $\pi_{Ft}(s) = \widehat{rp}_{Ft}(s) - \widehat{rp}_{Ft-1}(s) + \pi_t$ $\pi_{FMt}(s) = \widehat{rp}_{FMt}(s) - \widehat{rp}_{FMt-1}(s) + \pi_t$ $\bar{\pi}_t = \pi_t + \sum_s\zeta_0(s)\left(\frac{P_0(s)}{P_0}\right)^{1-\vartheta}\left(\widehat{rp}_t(s) - \widehat{rp}_{t-1}(s)\right)$

Table A.2: Equilibrium Conditions with Binding Constraints for Goods

Panel A: Only Domestic Constraint Binds	
Domestic Pricing	$\pi_{Ht}(1)=\left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}_t(1)-\widehat{rp}_{Ht}(1)\right)+\left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{H0}(1)}\right)\hat{\mu}_t(1)+\beta\mathbf{E}_t\pi_{Ht+1}(1)$
Input Import Pricing	$\pi_{MFt}(1)=\left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}^*_t(1)+\hat{q}_t-\widehat{rp}_{MFt}(1)\right)+\beta\mathbf{E}_t\pi_{FMt+1}(1)$
Domestic Constraint	$\hat{y}_t(1)=\hat{\bar{y}}_t(1)+\ln(\bar{Y}_0(1)/Y_0(1))$
Panel B: Only Foreign Constraint Binds	
Domestic Pricing	$\pi_{Ht}(1)=\left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}_t(1)-\widehat{rp}_{Ht}(1)\right)+\beta\mathbf{E}_t\pi_{Ht+1}(1)$
Input Import Pricing	$\pi_{MFt}(1)=\left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}^*_t(1)+\hat{q}_t-\widehat{rp}_{MFt}(1)\right)+\left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{MF0}(1)}\right)\hat{\mu}^*_t(1)+\beta\mathbf{E}_t\pi_{MFt+1}(1)$
Import Constraint	$\hat{y}^*_t(1)=\hat{\bar{y}}^*_t(1)+\ln(\bar{Y}^*_0(1)/Y^*_0(1))$
Panel C: Both Constraints Bind	
Domestic Pricing	$\pi_{Ht}(1)=\left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}_t(1)-\widehat{rp}_{Ht}(1)\right)+\left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{H0}(1)}\right)\hat{\mu}_t(1)+\beta\mathbf{E}_t\pi_{Ht+1}(1)$
Input Import Pricing	$\pi_{MFt}(1)=\left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}^*_t(1)+\hat{q}_t-\widehat{rp}_{MFt}(1)\right)+\left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{MF0}(1)}\right)\hat{\mu}^*_t(1)+\beta\mathbf{E}_t\pi_{MFt+1}(1)$
Domestic Constraint	$\hat{y}_t(1)=\hat{\bar{y}}_t(1)+\ln(\bar{Y}_0(1)/Y_0(1))$
Import Constraint	$\hat{y}^*_t(1)=\hat{\bar{y}}^*_t(1)+\ln(\bar{Y}^*_0(1)/Y^*_0(1))$

shocks has a multivariate normal distribution, with $var(\varepsilon_t)=\Sigma$ having diagonal elements σ_x^2 for each variable x and zeros off diagonal, and $cov(\varepsilon_t,\varepsilon_{t+s})=0$ at all leads and lags ($s\neq 0$).

Turning to constraints, we assume that the constraint for imports of consumption goods is not binding in all periods. In the first order approximate model, a sufficient condition to guarantee the constraint is never binding is to take $\bar{Y}^*_{Ct}(s)$ to infinity.[43] Similarly, we assume that constraints are not binding for services, for which taking $\bar{Y}_t(2)$ and $\bar{Y}^*_{Mt}(2)$ to infinity would be sufficient. This leaves $\bar{Y}_t(1)$ and $\bar{Y}^*_{Mt}(1)$ as the remaining constraints. We specify a stochastic process for them here, consistent with how we treat them as exogenous in the model.[44] We assume they follow an

Because the services sector is relatively closed, this cross-sector restriction has little substantive import.

[43]In the first order approximation, no decisions depend on the distance between the an endogenous variable and its constrained value. Thus, taking $\bar{Y}^*_{Ct}(s)$ to infinity ensures the constraint is always slack, without further indirect consequences for the approximate equilibrium.

[44]Reliable data on capacity at high frequencies is generally not available, so we cannot include capacity among the observable variables. Existing data on capacity, such as the series compiled by the Federal Reserve Board to produce its G.17 series, are not well suited to our exercise. One problem concerns data frequency. The Federal Reserve relies on underlying survey data collected at an annual frequency, so this data sheds little direct light on the dynamics of capacity at higher frequencies (monthly or quarterly). A second problem concerns how capacity survey questions are posed to firms. Specifically, the survey instrument asks firms to report how much they could produce if they had access to all the labor and materials they need to produce. This survey question fails to capture key aspects of production that effectively limit true capacity. For example, firms make predetermined choices about essential labor, material inputs, and other aspects of the production process that limit their ability to produce today, but this would be not be picked up by the survey. Related to these concerns, we note two features of the actual G.17 capacity data. First, capacity utilization is well below 1 in historical data (typically near 0.75 in recent data) – taken literally, capacity constraints are never even close to binding, which seems implausible. Further, measures of capacity utilization have trended down over time, as if firms are carrying more excess capacity now than in the past. This is prima facie inconsistent with auxiliary evidence of decreased slack in other dimensions of the production process (e.g., the rising prevalence of “lean production” methods, such as just-in-time inventory management).

Table A.3: Calibration

Parameter	Value	Reference/Target
ψ	2	Labor supply elasticity of 0.5
ρ	2	Intertemporal elasticity of substitution of 0.5
β	.995	Annual risk-free real rate of 2%
ϑ	0.5	Elasticity of substitution across sectors in consumption
ε	4	Elasticity of substitution between varieties
κ	0.3	Elasticity of substitution for inputs across sectors
$\sigma(s)$	1.5	Export demand elasticity
ϕ	35.468	To yield first order equivalence to Calvo pricing, with average price duration of 4 quarters [Sims and Wolff (2017)].

autoregressive process:

$$\hat{\bar{y}}_t(1) = \rho_{\bar{y}}\hat{\bar{y}}_{t-1}(1) + \varepsilon_{\bar{y}t}(1) \tag{A.1}$$

$$\hat{\bar{y}}^*_t(1) = \rho_{\bar{y}^*}\hat{\bar{y}}^*_{t-1}(1) + \varepsilon_{\bar{y}^*t}(1), \tag{A.2}$$

where $\gamma \in (0,1)$ and $\varepsilon_{\bar{y}t}(1)$ and $\varepsilon_{\bar{y}^*t}(1)$ denote capacity shocks. We assume the capacity shocks are independent, mean zero normal random variables, with variances $var(\varepsilon_{\bar{y}t}(1)) = \sigma^2_{\bar{y}}$ and $var(\varepsilon_{\bar{y}^*t}(1)) = \sigma^2_{\bar{y}^*}$, and $cov(\varepsilon_{\bar{y}t}(1), \varepsilon_{\bar{y},t+s}(1)) = cov(\varepsilon_{\bar{y}^*t}(1), \varepsilon_{\bar{y}^*,t+s}(1)) = 0$ at all leads and lags ($s \neq 0$).

A.3 Quantitative Implementation

We set parameters for quantitative analysis through a mix of calibration and estimation. We describe calibrated parameters first, and then we provide details regarding the estimation procedure.

A.3.1 Calibration

We set values for a subset of the structural parameters based on standard values in the literature, which we collect in Table A.3. We use input-output data compiled by the US Bureau of Economic Analysis to pin down values for steady-state expenditure shares. We report these shares, which reflect mean values over the 1997-2018 period, in Table A.4, along with their corresponding definitions in the model.

Table A.4: Steady State Shares

Model and Data	Description
$\begin{bmatrix} \zeta_0(1)\left(\frac{P_0(1)}{P_0}\right)^{1-\vartheta} \\ \zeta_0(2)\left(\frac{P_0(2)}{P_0}\right)^{1-\vartheta} \end{bmatrix} = \begin{bmatrix} 0.26 \\ 0.74 \end{bmatrix}$	Sector shares in consumption expenditure
$\begin{bmatrix} \gamma(1)\left(\frac{P_{H0}(1)}{P_0(1)}\right)^{1-\varepsilon} \\ \gamma(2)\left(\frac{P_{H0}(2)}{P_0(2)}\right)^{1-\varepsilon} \end{bmatrix} = \begin{bmatrix} 0.80 \\ 0.995 \end{bmatrix}$	Home shares in consumption expenditure by sector
$\begin{bmatrix} \alpha(1) \\ \alpha(2) \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$	Input expenditure share of gross output
$\begin{bmatrix} \left(\frac{\alpha(1,1)}{\alpha(1)}\right)\left(\frac{P_0(1,1)}{P_{M0}(1)}\right)^{1-\kappa} & \left(\frac{\alpha(1,2)}{\alpha(2)}\right)\left(\frac{P_0(1,2)}{P_{M0}(2)}\right)^{1-\kappa} \\ \left(\frac{\alpha(2,1)}{\alpha(1)}\right)\left(\frac{P_0(2,1)}{P_{M0}(1)}\right)^{1-\kappa} & \left(\frac{\alpha(2,2)}{\alpha(2)}\right)\left(\frac{P_0(2,2)}{P_{M0}(2)}\right)^{1-\kappa} \end{bmatrix} = \begin{bmatrix} 0.70 & 0.20 \\ 0.30 & 0.80 \end{bmatrix}$	Sector shares in input expenditure
$\begin{bmatrix} \xi(1,1)\left(\frac{P_{H0}(1)}{P_0(1,1)}\right)^{1-\eta} & \xi(1,2)\left(\frac{P_{H0}(1)}{P_0(1,2)}\right)^{1-\eta} \\ \xi(2,1)\left(\frac{P_{H0}(2)}{P_0(2,1)}\right)^{1-\eta} & \xi(2,2)\left(\frac{P_{H0}(2)}{P_0(2,2)}\right)^{1-\eta} \end{bmatrix} = \begin{bmatrix} 0.77 & 0.84 \\ 0.99 & 0.98 \end{bmatrix}$	Home shares in input expenditure
$\begin{bmatrix} \frac{C_{H0}(1)}{Y_0(1)} & \frac{M_{H0}(1,1)}{Y_0(1)} & \frac{M_{H0}(1,2)}{Y_0(1)} & \frac{X_0(1)}{Y_0(1)} \\ \frac{C_{H0}(2)}{Y_0(2)} & \frac{M_{H0}(2,1)}{Y_0(2)} & \frac{M_{H0}(2,2)}{Y_0(2)} & \frac{X_0(2)}{Y_0(2)} \end{bmatrix} = \begin{bmatrix} 0.41 & 0.32 & 0.16 & 0.11 \\ 0.61 & 0.07 & 0.29 & 0.03 \end{bmatrix}$	Domestic output allocation
$\begin{bmatrix} \frac{M_{F0}(1,1)}{Y^*_{M0}(1)} & \frac{M_{F0}(1,2)}{Y^*_{M0}(1)} \\ \frac{M_{F0}(2,1)}{Y^*_{M0}(2)} & \frac{M_{F0}(2,2)}{Y^*_{M0}(2)} \end{bmatrix} = \begin{bmatrix} 0.76 & 0.24 \\ 0.08 & 0.92 \end{bmatrix}$	Foreign output allocation for inputs
$\begin{bmatrix} \frac{P_{H0}(1)Y_0(1)}{P_{H0}(1)Y_0(1)+P_{H0}(2)Y_0(2)} \\ \frac{P_{H0}(2)Y_0(2)}{P_{H0}(1)Y_0(1)+P_{H0}(2)Y_0(2)} \end{bmatrix} = \begin{bmatrix} 0.29 \\ 0.71 \end{bmatrix}$	Sector shares in gross output

A.3.2 Estimation Procedure

As described in the main text, we build on papers by Kulish, Morley and Robinson (2017), Kulish and Pagan (2017), and Jones, Kulish and Rees (2022) that estimate models with occasionally binding constraints by treating the duration of those binding constraints as an estimable parameter. To explain the method in more detail, we first discuss how to solve the model for given durations for the binding constraints. We then describe the estimation procedure in greater detail, including how we constrain admissible values for durations to be consistent with model equilibrium constraints.

Solving the Model for Given Durations As in Guerrieri and Iacoviello (2015), we construct a piecewise linear approximation to the model. This consists of taking linear approximations of the model equilibrium for four regimes: the unconstrained regime, a second regime in which only domestic constraints bind, a third regime in which foreign constraints bind, and a fourth regime in which both constraints bind. Further, the linear approximations for all these regimes are taken around the non-stochastic (unconstrained) steady state of the model. The solution procedure then combines these local approximations to solve for the policy function.

The linear approximation to the unconstrained system can be written as:

$$\mathbf{A}X_t = \mathbf{C} + \mathbf{B}X_{t-1} + \mathbf{D}\mathbb{E}_t X_{t+1} + \mathbf{F}\varepsilon_t,$$

where x_t is an $n \times 1$ vector of model variables, ε_t is an $l \times 1$ vector of structural shocks, and $\mathbf{A}$, $\mathbf{C}$, $\mathbf{B}$, $\mathbf{D}$, and $\mathbf{F}$ are conformable matrices determined by the structural equations. If agents expect the economy to remain unconstrained from date t forward, then standard rational expectations solution procedures imply that the reduced-form solution is given by: $X_t = \mathbf{J} + \mathbf{Q}X_{t-1} + \mathbf{G}\varepsilon_t$, where $\mathbf{J}$, $\mathbf{Q}$, and $\mathbf{G}$ describe the policy function and model dynamics.

There are three regimes in which one or both constraints bind, and let us index these by $r \in \{1,2,3\}$. Then we can express the linear approximation to the model equilibrium in each case as:

$$\bar{\mathbf{A}}_r X_t = \bar{\mathbf{C}}_r + \bar{\mathbf{B}}_r X_{t-1} + \bar{\mathbf{D}}_r \mathbb{E}_t X_{t+1} + \bar{\mathbf{F}}_r \varepsilon_t,$$

where $\bar{\mathbf{A}}_r$, $\bar{\mathbf{C}}_r$, $\bar{\mathbf{B}}_r$, $\bar{\mathbf{D}}_r$, and $\bar{\mathbf{F}}_r$ are conformable matrices that correspond to the structural equations for each.

We summarize the expected evolution of regimes from a given date t forward by the durations that the individual constraints are expected to bind, as in $\mathbf{d}_t = [d_t, d_t^*]$. To fix ideas, suppose that $d_t = 1$, which means that the domestic constraint binds today, and then is expected to be slack in the future. Further, suppose that $d_t^* = 0$, so the foreign constraint is slack today and in the future. This implies that the constrained system governs model responses in period t and then the unconstrained system applies thereafter. Working backwards from the unconstrained solution, then $\mathbb{E}_t X_{t+1} = \mathbf{J} + \mathbf{Q} X_t$, so then $\bar{\mathbf{A}}_1 X_t = \bar{\mathbf{C}}_1 + \bar{\mathbf{B}}_1 X_{t-1} + \bar{\mathbf{D}}_1 \left(\mathbf{J} + \mathbf{Q} X_t\right) + \bar{\mathbf{F}}_1 \varepsilon_t$, where $r = 1$ is the system that applies when the domestic constraint binds and the foreign constraint is slack. Solving this linear equation yields the reduced form solution for X_t.

Generalizing this idea, the system will evolve according to:

$$\mathbf{A}_t X_t = \mathbf{C}_t + \mathbf{B}_t X_{t-1} + \mathbf{D}_t \mathbb{E}_t X_{t+1} + \mathbf{F}_t \varepsilon_t, \tag{A.3}$$

where $\mathbf{A}_t$, $\mathbf{C}_t$, $\mathbf{B}_t$, $\mathbf{D}_t$, and $\mathbf{F}_t$ are the structural matrices that apply at date t. Then the piecewise linear solution is given by:

$$X_t = \mathbf{J}_t + \mathbf{Q}_t X_{t-1} + \mathbf{G}_t \varepsilon_t, \tag{A.4}$$

where $\mathbf{J}_t$, $\mathbf{Q}_t$, and $\mathbf{G}_t$ are determined via the following backward recursion, which is initialized as starting from the unconstrained solution:

$$\begin{aligned}
\mathbf{Q}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \mathbf{B}_t \\
\mathbf{J}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \left(\mathbf{C}_t + \mathbf{D}_t \mathbf{J}_{t+1}\right) \\
\mathbf{G}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \mathbf{F}_t.
\end{aligned} \tag{A.5}$$

At this point, it is useful to note that this recursive solution coincides with the recursion employed by the OccBin toolkit [Guerrieri and Iacoviello (2015)] to obtain policy functions for a given guess about the sequence of regimes. The Occbin toolkit then proceeds to verify whether the guess about the sequence of regimes is consistent with model equilibrium, given the current value of the shocks. Put differently, it solves for endogenous constraint durations given ε_t. While we do not discuss this second step here, we do solve for endogenous durations (using Occbin) when we analyze counterfactual responses to shocks in the model. We also take the dependence of $\mathbf{d}_t$ on ε_t into account in the estimation procedure, with details below.

While Equations A.4 and A.5 present the model solution for a given anticipated sequence of regimes, it is important to note that the anticipated sequence changes as durations evolve over time. The duration $\mathbf{d}_t$ implies a particular sequence of regimes anticipated at dates $t+1$, $t+2$, etc. Given this sequence and the maintained assumption that agents do not anticipate future shocks, one then uses the recursion above to solve for the associated policy matrices: $\mathbf{J}\left(\mathbf{d}_t, \theta\right), \mathbf{Q}\left(\mathbf{d}_t, \theta\right)$, and $\mathbf{G}\left(\mathbf{d}_t, \theta\right)$, where the notation captures the dependence of these matrices on $\mathbf{d}_t$. At date $t+1$, a new value for durations ($\mathbf{d}_{t+1}$) will be realized, and one then solves the recursion anew to obtain $\mathbf{J}\left(\mathbf{d}_{t+1}, \theta\right), \mathbf{Q}\left(\mathbf{d}_{t+1}, \theta\right)$, and $\mathbf{G}\left(\mathbf{d}_{t+1}, \theta\right)$. And so on. The state (transition) equation of the model then features time-varying coefficients:

$$X_t = \mathbf{J}(\mathbf{d}_t, \theta) + \mathbf{Q}(\mathbf{d}_t, \theta) X_{t-1} + \mathbf{G}(\mathbf{d}_t, \theta) \varepsilon_t. \tag{A.6}$$

When $\mathbf{d}_t = 0$, the unconstrained solution applies, so $\mathbf{J}(\mathbf{d}_t, \theta) = \mathbf{J}(\theta)$, $\mathbf{Q}(\mathbf{d}_t, \theta) = \mathbf{Q}(\theta)$, and $\mathbf{G}(\mathbf{d}_t, \theta) = \mathbf{G}(\theta)$ are time invariant.

Joint Estimation of Durations and Structural Parameters We assume that a vector of observables (S_t) are linked to underlying model states via the measurement equation: $S_t = \mathbf{H}_t X_t + \nu_t$, where ν_t is an i.i.d. vector of normally distributed measurement errors and $\mathbf{H}_t$ is a conformable (potentially time-varying) matrix linking states to observables. Using this state space representation of the model, we can apply the Kalman filter to construct the Likelihood function $\mathscr{L}(\theta, \mathbf{d} | \{S_t\}_{t=1}^T)$, where $\mathbf{d} = \{\mathbf{d}\}_{t=1}^T$ is the sequence of durations.

We put priors over structural parameters and independent priors over durations to construct the posterior, and then estimate the model via Bayesian Maximum Likelihood. We construct draws from the joint posterior distribution $p\left(\theta, \mathbf{d} | \{S_t\}_{t=1}^T\right)$ using a Metropolis-Hastings algorithm with two blocks – one for the structural parameters, which are continuous, and a second for the discrete duration parameters – as in Kulish, Morley and Robinson (2017). We use a uniform proposal density for the durations, between 0 (unconstrained) and a sufficiently large maximum duration. We discuss the priors in Section A.3.4 below.

In evaluating proposed parameter and durations draws, we recognize that it is desirable for posterior estimates of constraint durations to be consistent with agents' forecasts about how long constraints will endogenously bind given shocks. To this end, we constrain admissible draws to enforce this constraint, in an approximate sense. For a given proposed joint parameter (θ^i) and duration draw ($\mathbf{d}^i$), we construct the piecewise linear solution for the model and use the Kalman filter to obtain smoothed structural shocks $\{\tilde{\varepsilon}_t^i\}_{t=1}^T$ and equilibrium variables $\{\tilde{X}_t^i\}_{t=1}^T$ given the data. At each sample period $\tau \in [1, \ldots, T]$, we then use the piecewise linear solution to project model outcomes forward given the state and current shock – $\left(\tilde{X}_{\tau-1}^i, \tilde{\varepsilon}_\tau^i\right)$, assuming that there are no anticipated future shocks.[45] We then check for violations of the output capacity constraints. If projected home or foreign output violates the constraints, then we reject the proposed parameter draw as inconsistent with model equilibrium. Otherwise, we accept the parameter draw, evaluate the likelihood, and proceed through the estimation algorithm. Under this procedure, we accept about 25% of the proposed parameter/duration draws, so the estimation proceeds at reasonable computational pace.

[45] Recall that in the absence of future shocks, agents anticipate that the model will return to the unconstrained state over time, where the duration of binding constraints ticks down toward zero in each passing period. We project model outcomes forward using this expected path for durations.

In this procedure, note that we reject the proposed draw when it implies that constraints will be violated in expectation. In turn, we accept draws for which constraints are satisfied. Strictly speaking, we do not explicitly check whether the duration $\mathbf{d}_\tau$ is equal to the endogenous equilibrium duration consistent with $\left(\tilde{X}^i_{\tau-1}, \tilde{\varepsilon}^i_\tau\right)$ in the model. Nonetheless, our approach to estimation provides a good approximation to model outcomes with endogenously binding constraints. To demonstrate this in greater detail, we turn to simulation evidence.

A.3.3 Validating the Estimation Procedure

We provide results for two exercises to evaluate the accuracy of our estimation procedure. First, using simulated data, we demonstrate that our estimation procedure is capable of identifying latent durations. Moreover, we show that it accurately recovers the corresponding multipliers on the constraints. Second, using results from the full estimation of the model with real world data, we compare smoothed inflation to simulated model results.

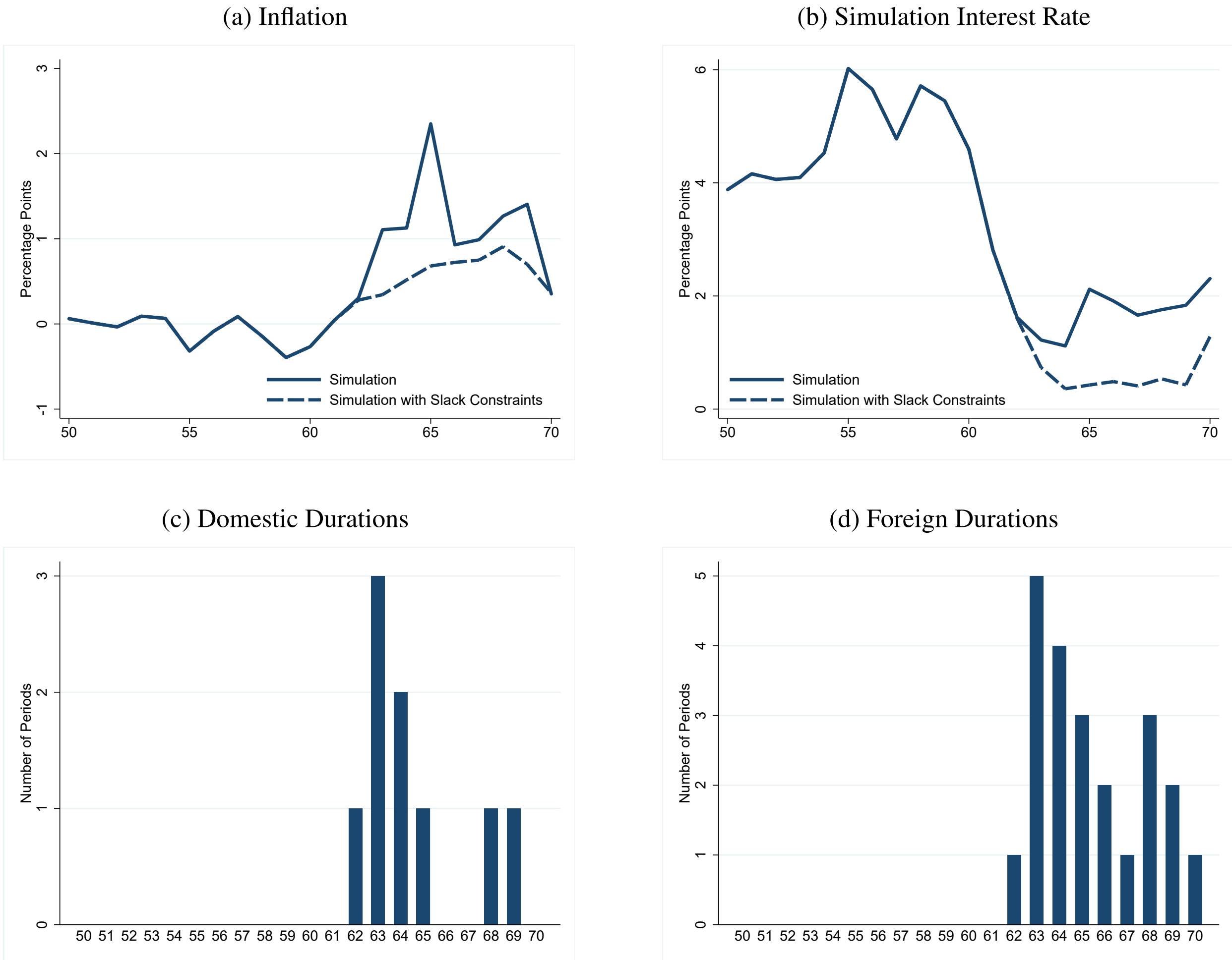
Estimation using Simulated Data The first step is to generate simulated data from the model, for given parameters.[46] Specifically, we draw a set of i.i.d. shocks for all variables over 70 quarters, and then impose a sequence of large, expansionary monetary policy shocks for quarters 61 to 69 of the simulation (the size of the monetary policy shocks is chosen to be three standard deviations). These large monetary shocks are set to trigger the capacity constraints. Since we can identify when the constraints bind in the simulated data, we thus know the true sequence of durations.

We plot several simulated data series in Figure A.1 to illustrate this set up, under both the maintained assumption that constraints are potentially binding and the counterfactual assumption that constraints are slack in all periods. The top two panels contain simulated inflation and the policy interest rate, while the implied durations for domestic and foreign constraints are recorded in the bottom two panels. The expansionary policy shocks evidently cause the policy rate to be low in periods 62 through 70. Further, when capacity constraints bind, inflation more than doubles at its peak relative to a simulation without capacity constraints.

Treating the simulated series as observable data, we illustrate that our empirical model is capable of identifying the true durations by directly examining model likelihood functions. Setting all parameters in the state and observation equations (other than durations) to their true values used to generate the simulated data, we compute the likelihood of the model for different values of the

[46] In contrast to the main quantitative model, we assume there is zero measurement error, so observable variables are equal to corresponding objects in the simulated data. Further, we set steady-state capacity levels so that there is 4% excess capacity for both home and foreign goods firms, which is lower than the main model. This tighter capacity implies that we can trigger binding constraints with demand shocks alone (i.e., without negative capacity shocks). Remaining parameters are set to the mode of our baseline estimates, and (as elsewhere) we use Dynare's OccBin toolbox to simulate the model.

Figure A.1: Simulation



Note: Inflation is reported at a quarterly rate in percentage points. The interest rate is also reported in percentage points.

domestic and foreign durations, at given points in time. For example, setting the duration of the foreign constraint to its true value in a given period, we then trace out the likelihood over alternative values of the duration of the domestic constraint. And vice versa. We present the results from period 60, before the constraints become binding, through period 70, when the domestic capacity constraint stops binding and the foreign capacity constraint binds for one more quarter.

Figure A.2 plots the inverse of the likelihood value across durations of the domestic constraint, where each panel corresponds to a period and the vertical line identifies the true duration. Figure A.3 plots the corresponding results for the foreign constraint. As both figures illustrate, the inverse likelihood is minimized at the true values in every quarter, which confirms that the likelihood procedure we implement is able to discriminate between durations of different length. Importantly, for periods when the constraint does not bind, the likelihood is maximized at a duration value of zero.

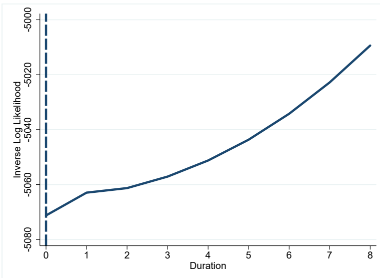
Turning to estimation of the multipliers, we conduct a full estimation of the model using the simulated data, in which we estimate both the structural parameters and durations, as in the main analysis.[47] The posterior distributions for the structural parameters are generally well behaved, and their peaks lie close to the true values. Here we focus on the estimated (smoothed) multipliers on the capacity constraints, as these play a key role in the framework. In Figure A.4, we plot the true paths for the multipliers in the simulation, along with smoothed multipliers recovered via estimation. As is evident, the smoothed values of the multipliers match the exact simulation values closely, meaning the procedure does a good job at pinning down the reduced-form impact of constraints on inflation.

Smoothed vs. Simulated Inflation Drawing on results presented below and in the main text, we briefly compare smoothed inflation outcomes obtained via our estimation procedure with outcomes from the full structural model with endogenously binding constraints. This comparison serves to check that the empirical model with estimated durations replicates the outcomes of the structural model with endogenously binding constraints. Specifically, suppose we feed the structural shocks $\{\tilde{\varepsilon}_t^i\}_{t=1}^T$ obtained from our estimation procedure through the model, where structural parameters are set to their modal values and we use the OccBin procedure to solve for the endogenous duration of binding constraints in each period following the realization of shocks. We then plot this simulated inflation series to the smoothed inflation series from our estimation in Figure A.5. As is evident, the two series track each other closely, so we conclude that our approach to capturing endogenously binding constraints in the estimation routine performs well.

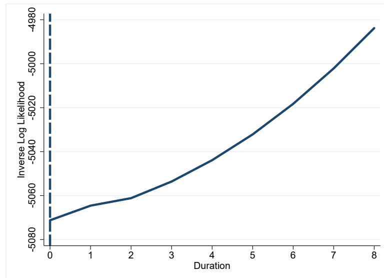
[47] In this estimation, we allow constraints to potentially bind for two quarters before the first period in which they actually bind in the simulated data. Further, we use the same priors here as in the baseline estimation.

Figure A.2: Likelihood Over Domestic Durations

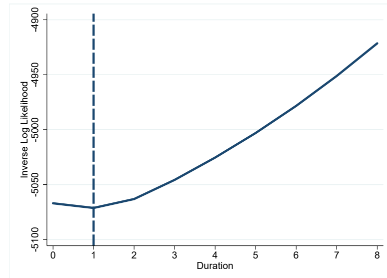
(a) Period 60



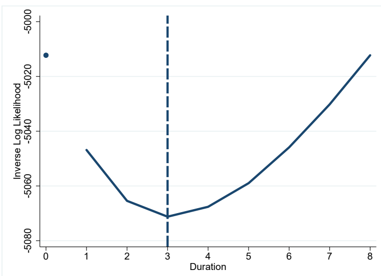
(b) Period 61



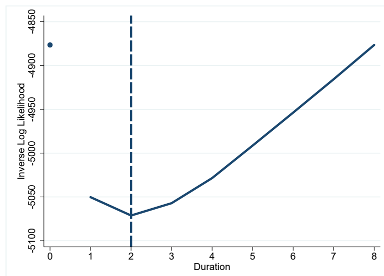
(c) Period 62



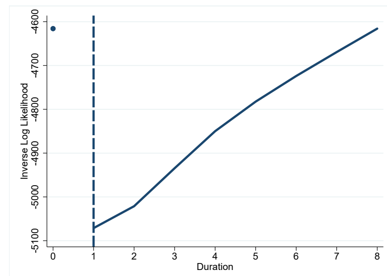
(d) Period 63



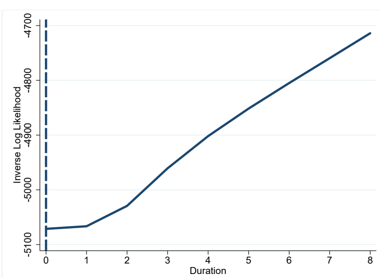
(e) Period 64



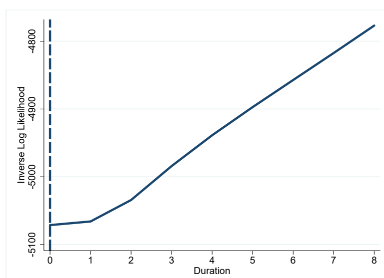
(f) Period 65



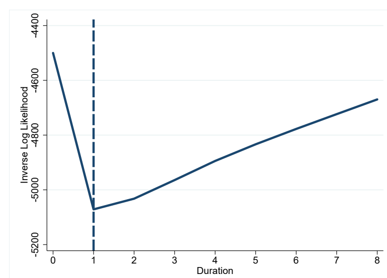
(g) Period 66



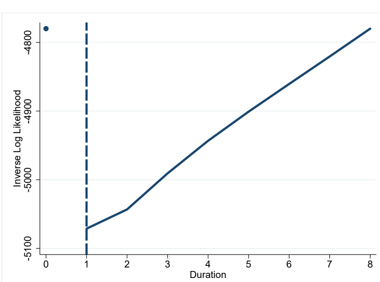
(h) Period 67



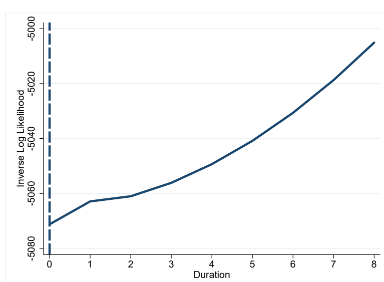
(i) Period 68



(j) Period 69



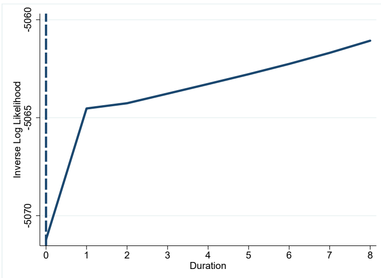
(k) Period 70



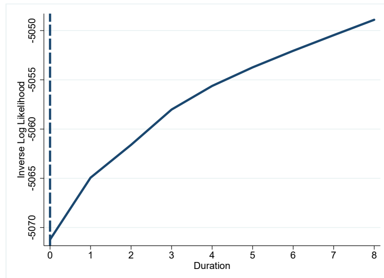
Note: The vertical dashed line marks the true duration of the constraint in the simulation for each period. In some figures, the dot denotes a value of the inverse likelihood that is substantially higher than the other values plotted in the figure; the dot is located at the maximal value depicted in the figure for visual reference.

Figure A.3: Likelihood Over Foreign Durations

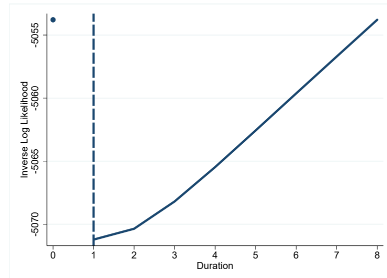
(a) Period 60



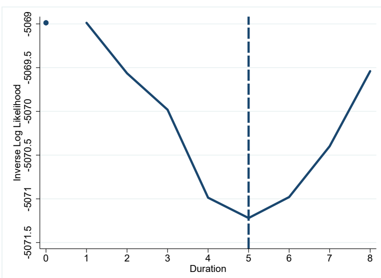
(b) Period 61



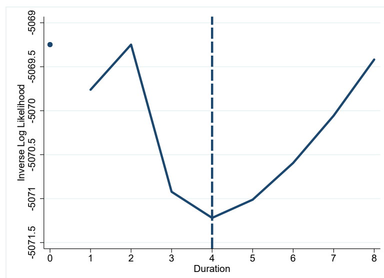
(c) Period 62



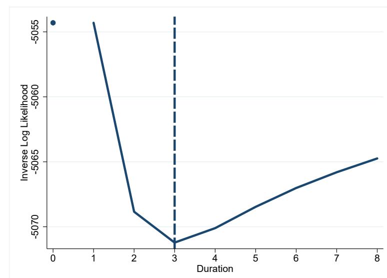
(d) Period 63



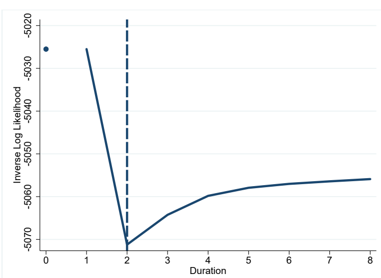
(e) Period 64



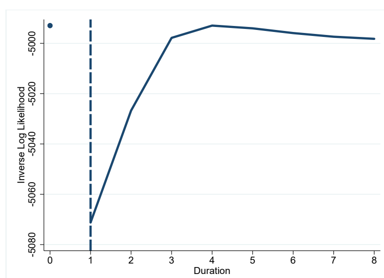
(f) Period 65



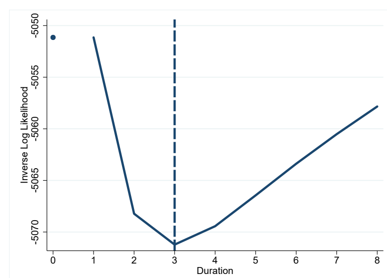
(g) Period 66



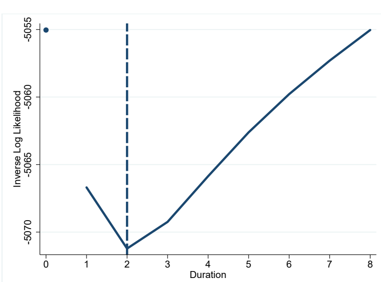
(h) Period 67



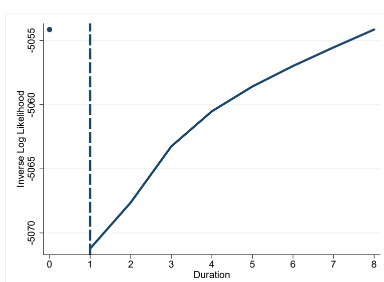
(i) Period 68



(j) Period 69



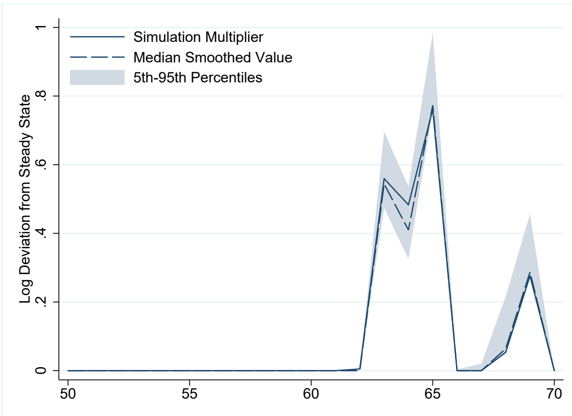
(k) Period 70



Note: The vertical dashed line marks the true duration of the constraint in the simulation for each period. In some figures, the dot denotes a value of the inverse likelihood that is substantially higher than the other values plotted in the figure; the dot is located at the maximal value depicted in the figure for visual reference.

Figure A.4: Multipliers Capacity Constraints: Simulation vs. Estimation

(a) Multiplier on the Domestic Constraint



(b) Multiplier on the Foreign Constraint

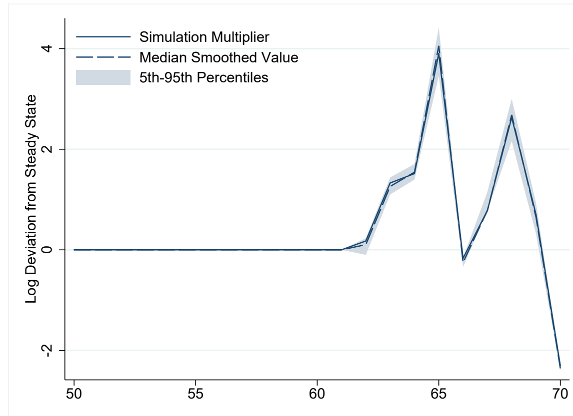
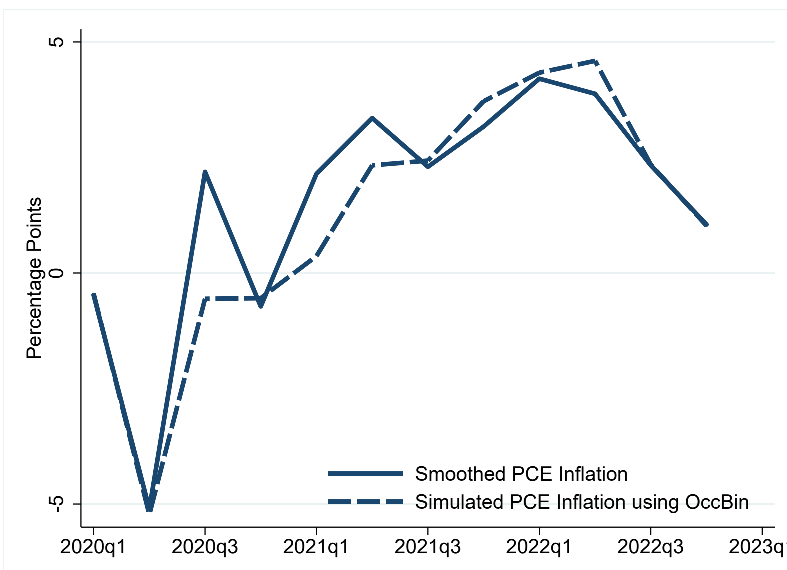


Figure A.5: Comparison Between Smoothed Inflation and OccBin Simulated Inflation



Note: Smoothed PCE Inflation is Kalman-smoothed consumer price inflation, where the filter is parameterized using the modal values of structural parameters and durations from the empirical estimation. Simulated PCE Inflation using OccBin is obtained by simulating model responses to smoothed shocks (see the text for further description).

A.3.4 Priors

The full set of priors for structural parameters is included in Table A.5. We use standard priors on autoregressive persistence of exogenous variables, parameters in the monetary policy rule, elasticities, and the standard deviations of most structural shocks. We set priors on the persistences of the exogenous capacity shocks that are wider than the priors on the other exogenous variables, as well as wide (uniform) priors on the standard deviations of the capacity shocks, since these are nonstandard parameters.

We set uniform priors on measurement errors associated with three inflation series – consumer price inflation for goods, imported input price inflation, and imported consumption goods price inflation. Further, looking forward, we will report below that the posterior estimates are pushed toward the boundary of the allowed parameter space for these parameters. The logic for constraining the measurement error parameters in this way is twofold. First, because our focus is on inflation outcomes and the role of constraints in driving them, we want to lean heavily on the realized data here. Second, we estimate the model using both pre-2020 and post-2020 data. As is evident in raw data series, the post-2020 COVID period features extreme variability in outcomes relative to the pre-2020 data. One way for the model to make sense of this is to assign very high measurement errors to the data. This is unpalatable from our perspective, as we wish to parse the actual data for this period. Thus, we effectively constrain the model to treat the post-2020 inflation data as an accurate representation of latent unobserved model variables. We envision experimenting with alternatives to this approach (e.g., allowing for different measurement error or shock processes before and after 2020), as thinking about how to model the COVID period evolves.

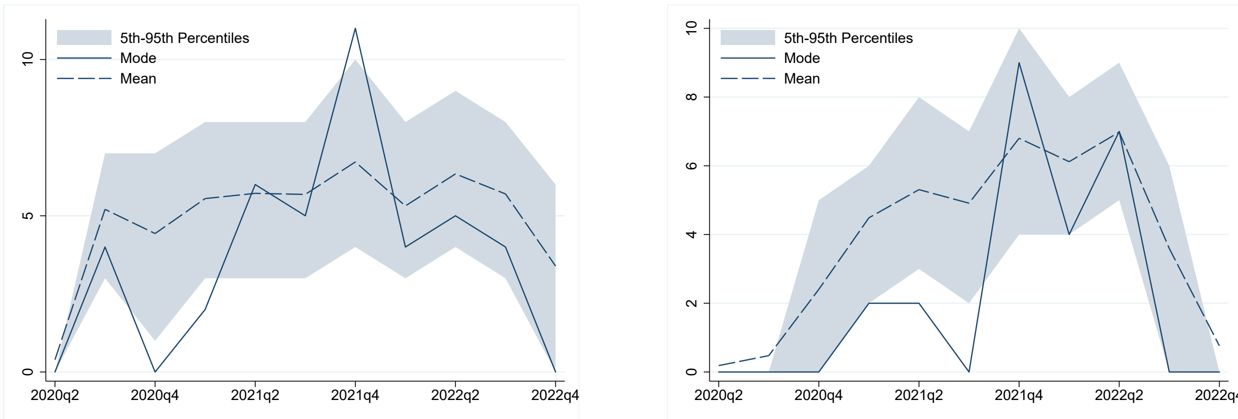
As noted in the main text, we allow constraints to potentially bind only starting in the second quarter of 2020. That is, we put zero mass on positive durations at all dates at/before 2020:Q1, which can be thought of as a dogmatic prior that constraints were not substantively important prior to the pandemic. Thereafter in each period, we place equal mass on durations of 0 to 4 quarters, summing to 60% total (12% on each discrete duration). We place 30% mass on durations of 5, 6, 7, and 8 quarters, again equally spread (7.5% each). The remaining 10% mass is spread equally over durations 9 through 12, and we place zero mass on durations longer than 12 quarters.

A.4 Estimation Results

In Table A.5, we provide the mode, mean, and 5th-95th percentiles for the posterior distributions of the structural parameters. As noted in the text, we find that domestic and foreign goods inputs are complements on the production side, while domestic and foreign goods are substitutes in consumption. The Taylor rule coefficient on inflation is near 1.5, which is standard. Interest rates also depend positively on deviations of output from steady state, and the policy rule features

Figure A.6: Posterior Distributions for Constraint Durations

(a) Domestic Constraint Durations (b) Foreign Constraint Durations



Note: At each date, there is a posterior distribution for constraint durations. Each figure presents the mean, mode, and interquartile range for this posterior distribution.

a significant degree of inertia. The stochastic processes for shocks generally feature persistence, with auto-regressive coefficients generally between 0.7 and 0.9. Building on the discussion of measurement error above, we note that posterior estimates for measurement errors on consumer goods price inflation and import price inflation are pushed toward the boundary of their prior distributions, reflecting tension in the model between fitting data before and during the COVID period. For all the other parameters, posterior distributions are generally well behaved, with single peaks well inside the allowable parameter space and reasonably tight distributions.

Turning to duration estimates, we plot statistics for the posterior distributions of domestic and foreign constraint durations in Figure A.6. Due to skewness in the distributions, modal values for the duration (our preferred approach to summarizing the posterior distribution) are below the mean value in most periods. The time path for the duration estimates mimics the path of estimated multipliers on the constraints, as reported in Figure 12.

A.5 Model Fit

In the main text, we presented results on model fit for core inflation series in Figure 11. To evaluate model fit more broadly, we present data and smoothed values for the remaining observable variables in Figure A.7.[48] For legibility in the figures, we focus on the 2017-2022 period – the key period leading up to and through our analysis. The model fits most series well, even capturing the whiplash dynamics of the data in 2020. The model struggles to replicate data on US labor produc-

[48] While we treat the interest rate as an observable variable, we assume it is measured without error, so it is omitted here.

Table A.5: Prior and Posterior Distributions for Structural Parameters

Panel A: Elasticity and Taylor Rule Parameters

	Prior			Posterior			
Parameter	Dist	Mean	SD	Mode	Mean	5%	95%
Consumption Armington Elasticity: ι	G	1.5	0.25	1.500	1.469	1.124	1.825
Input Armington Elasticity: η	G	0.5	0.15	0.549	0.563	0.362	0.796
Taylor Rule Inflation: ω	N	1.5	0.12	1.553	1.545	1.354	1.745
Taylor Rule Inertia:α_i	B	0.75	0.1	0.877	0.872	0.850	0.892
Taylor Rule Output: α_y	G	0.12	0.05	0.249	0.246	0.173	0.331

Panel B: Stochastic Processes

	Prior			Posterior			
Parameter	Dist	Mean	SD	Mode	Mean	5%	95%
Preference for Goods: σ_ζ	IG	1	2	0.314	0.403	0.192	0.642
Discount Rate: σ_Θ	IG	1	2	3.373	3.487	3.120	3.913
Foreign Costs: σ_{rmc^*}	IG	1	2	2.194	2.297	1.955	2.717
Goods Productivity: $\sigma_{z(1)}$	IG	1	2	0.192	0.196	0.128	0.273
Services Productivity: $\sigma_{z(2)}$	IG	1	2	0.205	0.206	0.139	0.282
Foreign Constraint: $\sigma_{\bar{y}^*}$	U	1	0.58	0.079	0.086	0.041	0.143
Domestic Constraint: $\sigma_{\bar{y}}$	U	1	0.58	0.019	0.025	0.012	0.050
Monetary Policy Shock: σ_i	IG	1	2	0.153	0.154	0.134	0.176
Preference for Goods: ρ_ζ	B	0.5	0.15	0.825	0.715	0.424	0.906
Discount Rate: ρ_Θ	B	0.5	0.15	0.727	0.727	0.670	0.780
Foreign Costs: ρ_{rmc^*}	B	0.5	0.15	0.922	0.917	0.875	0.952
Goods Productivity: $\rho_{z(1)}$	B	0.5	0.10	0.542	0.538	0.371	0.697
Services Productivity: $\rho_{z(2)}$	B	0.5	0.15	0.911	0.813	0.434	0.946
Foreign Constraint: $\rho_{\bar{y}^*}$	B	0.5	0.20	0.715	0.683	0.454	0.882
Domestic Constraint: $\rho_{\bar{y}}$	B	0.5	0.20	0.914	0.863	0.698	0.957

Panel C: Measurement Error

	Prior			Posterior			
Parameter	Dist	Mean	SD	Mode	Mean	5%	95%
Goods PCE: $\sigma^{\text{me}}_{\text{pceg}}$	IG	1	2	1.014	1.032	0.879	1.194
Services PCE: $\sigma^{\text{me}}_{\text{pces}}$	IG	1	2	0.664	0.663	0.558	0.773
Goods PCE Inflation: $\sigma^{\text{me}}_{\pi(1)}$	U	0.25	.14	0.499	0.497	0.492	0.500
Services PCE Inflation: $\sigma^{\text{me}}_{\pi(2)}$	IG	1	2	0.155	0.163	0.124	0.213
Imp. Input Goods Expenditure: $\sigma^{\text{me}}_{\text{inp}}$	IG	1	2	3.205	3.253	2.929	3.629
Imp. Consumption Goods Expenditure: $\sigma^{\text{me}}_{\text{finp}}$	IG	1	2	2.868	2.948	2.659	3.287
Imp. Input Goods Inflation: $\sigma^{\text{me}}_{\text{inpp}}$	U	0.75	.43	1.498	1.487	1.463	1.499
Imp. Consumption Goods Inflation: $\sigma^{\text{me}}_{\text{fimp}}$	U	0.075	0.043	0.145	0.112	0.033	0.148
Goods Productivity: $\sigma^{\text{me}}_{\text{prod1}}$	IG	1	2	1.151	1.163	1.012	1.324
Services Productivity: $\sigma^{\text{me}}_{\text{prod2}}$	IG	1	2	1.053	1.059	0.930	1.203
Industrial Production: $\sigma^{\text{me}}_{\text{ip}}$	IG	1	2	0.930	0.961	0.836	1.100
Aggregate Nominal GDP: $\sigma^{\text{me}}_{\text{nva}}$	IG	1	2	0.476	0.474	0.397	0.556

Note: G denotes the gamma distribution, IG denotes the inverse gamma distribution, U denotes the uniform distribution, B denotes the beta distribution, and N denotes the normal distribution.

tivity, particularly in 2020 for services. Through the lens of the model, this implies that the data contains substantial measurement error during the pandemic period, which seems plausible to us. More broadly, a more sensitive treatment of the impact of lockdowns on the services sector would likely be needed to match data in the middle quarters of 2020. Nonetheless, referring back to the main text, the model is able to capture the dynamics of services inflation well overall, particularly in 2021-2022 when inflation escalates.

Turning to "non-targeted data," we now compare smoothed values for multipliers attached to the constraints to an external measure of supply chain disruptions. Specifically, we use the Global Supply Chain Pressure Index (GSCPI), developed by the New York Federal Reserve [Benigno et al. (2022)], which combines data on transportation costs (sea and air freight rates) with elements of Purchasing Managers' Index surveys pertaining to supply chain management from major industrial countries (China, the Eurozone, Japan, United States, etc.). To be clear, this data is not tightly related to the theoretical construct that we recover from the data; it also is not scaled in way that is directly comparable to our estimates.[49] Further, it is a proxy for global conditions, which doesn't distinguish between US-based and foreign supply chain constraints, so we compare it to a weighted mean of the median multipliers on the domestic and foreign constraints. With all these caveats, we plot the GSCPI and the weighted mean multiplier in Figure A.8. As is evident, both the composite multiplier and the GSCPI index rise and fall in tandem.[50]

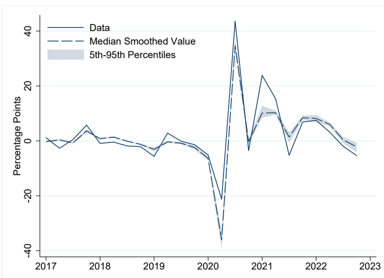
Lastly, in the text, we noted that fluctuations in the reduced-form markup shocks in the Phillips Curves implied by binding constraints do not behave like standard markup shocks estimated from historical data. To illustrate this, we introduce an exogenous markup shock into the domestic and foreign price Phillips Curves of the baseline model, and we assume the markup shocks follow an AR1 stochastic process. We then re-estimate the model including exogenous markup shocks using only data from 1990:Q1-2019:Q4, under the assumption that constraints are slack throughout this period. We then filter the data to recover smoothed values for the markup shocks. In Figure A.9, we plot the median smoothed values for the exogenous markup shocks over the period 1990-2020. We then also plot the median smoothed values for the reduced-form markup shocks implied by binding constraints during the 2020:Q2-2022:Q4 period, obtained from the estimation above. As is evident, constraints induce markups shocks that are substantially larger than those that are consistent with

[49] The raw GSCPI index is reported as deviations from its mean value, in units of the standard deviation of the series. The NY Fed does not report either the mean or standard deviation, only the summary index, so we cannot compute log changes in the underlying index over time. Further, economically speaking, there is no obvious relationship between units attached to the multipliers – which summarize impacts of constraints on inflation – and units on the GSCPI. Because the GSCPI is reported at the monthly frequency, we take simple means across three month intervals to form quarterly values.

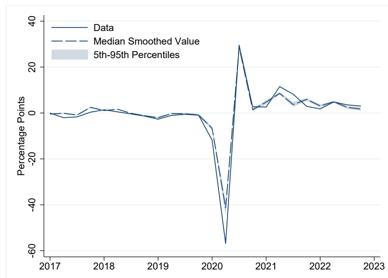
[50] The largest deviation occurs in early 2020, when the GSCPI index escalates rapidly then falls back. While our estimated multipliers here to not reflect this, we note that the multipliers we recover from the extended model that incorporates labor supply constraints and shocks does a better job of fitting these early-pandemic dynamics.

Figure A.7: Data and Smoothed Model Observables

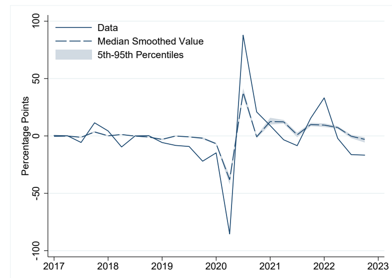
(a) Goods Cons. Expenditure



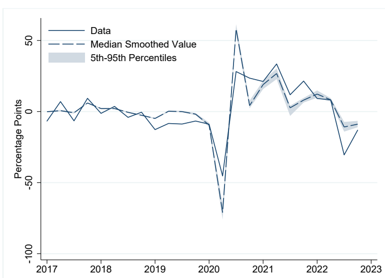
(b) Services Cons. Expenditure



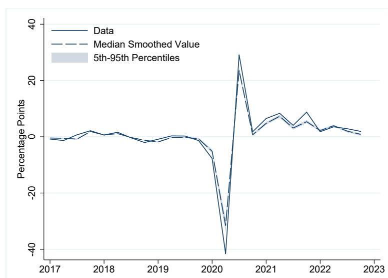
(c) Import Cons. Goods Expenditure



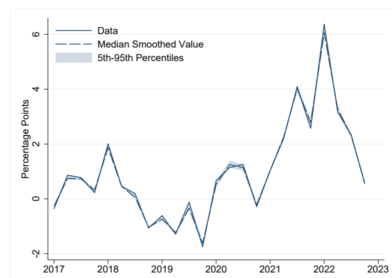
(d) Import Goods Input Expenditure



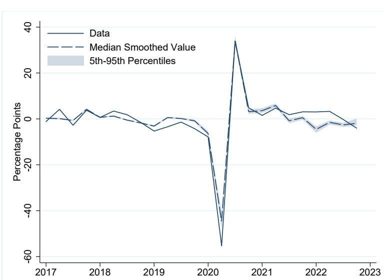
(e) Nominal GDP



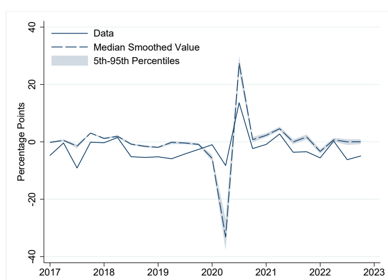
(f) Import Consumer Goods Inflation



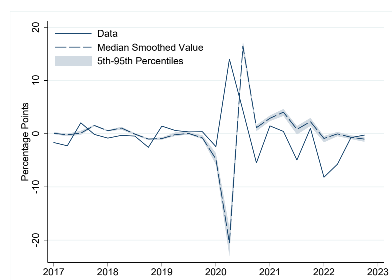
(g) Industrial Production



(h) Goods Productivity

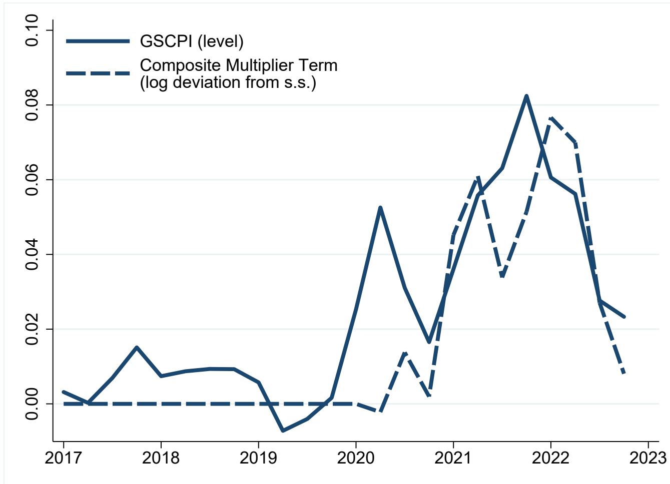


(i) Aggregate Productivity



Note: All data and simulated series are annualized values for de-meaned quarterly growth rates in percentage points. Data is raw data. We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the dashed line. We shade the area covering the the 5% to 95% percentile for smoothed values.

Figure A.8: Comparing the NY Fed GSCPI to the Weighted Mean of Multipliers on Domestic and Foreign Constraints



Note: To make the scale of the GSCPI index comparable to the multiplier, we plot the raw level of the GSCPI index divided by 50. The Composite Multiplier is computed as $0.75\left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{H0}(s)}\right)\hat{\bar{\mu}}_t(s) + 0.25\left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{uF0}(s)}\right)\hat{\bar{\mu}}^*_{ut}(s)$. The weight on the domestic term is 0.75 and the weight on the foreign term is 0.25, which roughly correspond to shares of total spending allocated to domestic and foreign goods.

historical data; further, the reduced-form markup shocks are also less persistent than the historical exogenous markup process.

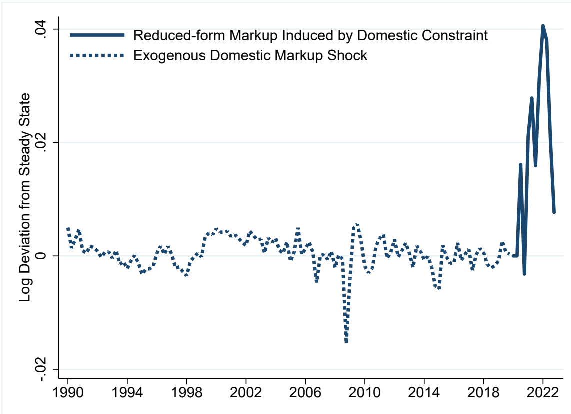
A.6 Estimated Capacity Levels

In the preceding (main) model, we calibrated the levels of domestic and foreign goods capacity in steady state. However, we could instead estimate those levels, with an important caveat. The caveat is that we allow constraints to bind only after 2020 in the estimation. The "steady-state" capacity level is the level to which capacity reverts in the long run, in the absence of shocks. We are able to estimate this level conditional on the data in periods in which constraints are potentially binding. Thus, if we estimate capacity levels, we are attempting to infer the capacity level only using post-2020 data. Naturally, since constraints were binding for much of this period, plausibly due to negative shocks that pushed realized capacity down, using only this data will tend to lead us to estimate a relatively low level for steady-state capacity. And in fact, this is that we find when we treat capacity levels as parameters to be estimated: steady state goods capacity is roughly 1% above the steady state level of goods output, which is lower than the calibrated value we have used previously. Nonetheless, this difference in the level of steady-state capacity has little import for our quantitative assessment, as we noted in the main text.

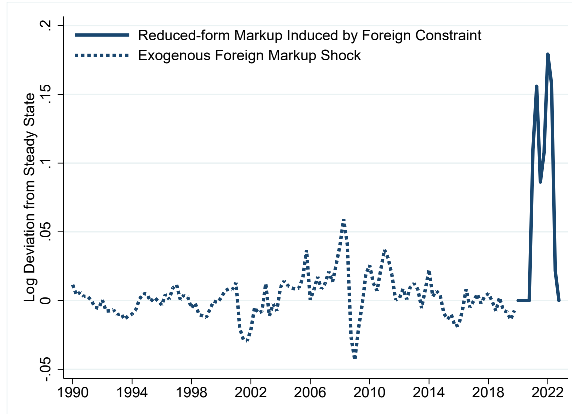
To demonstrate this, we provide supplemental figures illustrating results from a version of

Figure A.9: Exogenous and Reduced-Form (Binding Constraint) Markup Shocks

(a) Domestic Markup Shocks



(b) Foreign Markup Shocks

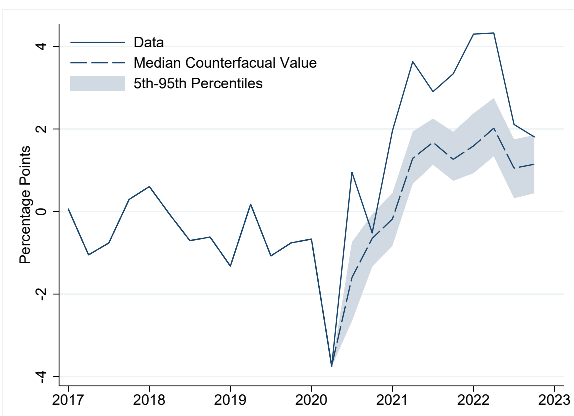


Note:

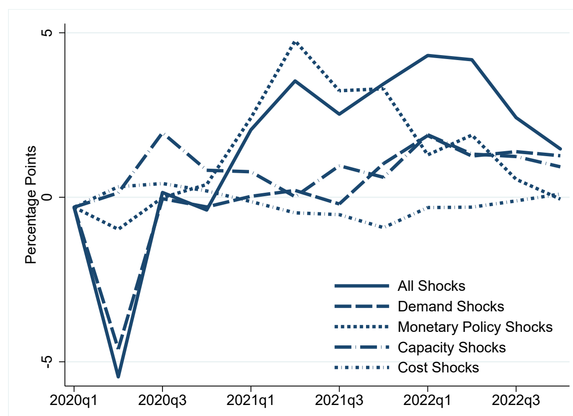
The solid lines depict reduced-form markup shocks induced by binding constraints; these are the same data as in Figures 12a and 12b. The dashed lines are exogenous markup shocks obtained by estimating the model with exogenous markup shocks and slack constraints using only data for 1990:Q1-2019:Q4. We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed values.

Figure A.10: Counterfactual Consumer Price Inflation in Model with Estimated Steady-State Capacity Levels

(a) Aggregate Consumer Price Inflation in Model and Data



(b) Counterfactual Consumer Price Inflation for Individual Shocks



Note: In Figure A.10a, we take 1000 draws from the posterior distribution of model parameters (re-estimated for this application including estimated capacity levels), compute the Kalman-smoothed values for model variables for each draw, add measurement error to the observables, and then plot the median smoothed value as the solid line. We shade the area covering the 5% to 95% percentile for smoothed values. In Figure A.10b, each series represents the simulated path of consumer price inflation (quarterly value, annualized) for the indicated subset of smoothed shocks during 2020-2022. See text for definition of the counterfactuals.

the model in which capacity is estimated in Figure A.10.[51] In Figure A.10a, we replicate the counterfactual in which we relax both the domestic and import goods constraints, as in Figure 13c. In Figure A.10b, we replicate the simulated impact of individual shocks on inflation, as presented in Figure 16. To interpret this figure, we note that these counterfactuals are comparable to those in which we feed individual shocks into the model together with capacity shocks. The reason is that capacity shocks essentially lower the average capacity level to near the estimated steady-state capacity level recovered using only post-2020 data. The results are both qualitatively and quantitatively similar to prior results, which further demonstrates that the core counterfactual results are largely robust to the level of steady-state capacity.

[51] We present only two key results here for brevity sake; a full set of results for this model is available on request.

B Labor Market Extension

This appendix provides details regarding how we extend the model to address the labor market, and then additional details about quantitative implementation of this extended model.

B.1 Model Extension

Referring back the text, we now add sticky wages, potentially binding labor market constraints, and shocks to the disutility of labor to the baseline model. Conveniently, all three extensions can be formalized by re-writing the consumer-side of the model as follows.

We now assume there is a unit continuum of consumers, indexed by $j \in (0,1)$. Consumers are identical, with one exception: each is the monopolistic supplier of its own differentiated labor services to the market. Further, the amount of labor that each consumer is able to supply in a given period is bound above by $\overline{L}_t$, which is exogenous and time varying. Differentiated labor services supplied by consumers are costlessly aggregated into a composite bundle by competitive intermediaries and sold to firms. The labor aggregation technology is given by $L_t = \left(\int_0^1 L_t(j)^{(\varepsilon_L-1)/\varepsilon_L} dj\right)^{\varepsilon_L/(\varepsilon_L-1)}$, where $\varepsilon_L > 1$ is the elasticity of substitution between differentiated labor services and the price index for the labor composite is $W_t = \left(\int_0^1 W_t(j)^{1-\varepsilon_L} dj\right)^{1/(1-\varepsilon_L)}$. Finally, each consumer faces pays Rotemberg-type adjustment costs to modify the nominal wage at which it supplies labor, as in Born and Pfeifer (2020).

Consumer j chooses its consumption, wage, and asset holdings to maximize utility, subject to its budget constraint, the demand curve for its labor, and the labor supply constraint:

$$\max_{\{C_t(j), W_t(j), B_{t+1}(j)\}_{t=0}^{\infty}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \Theta_t \left[\frac{(C_t(j))^{1-\rho}}{1-\rho} - \Lambda_t \frac{L_t(j)^{1+\psi}}{1+\psi} \right] \tag{B.1}$$

$$\text{s.t.} \quad P_t C_t(j) + E_t\left[S_{t,t+1} B_{t+1}(j)\right] \leq B_t(j) + W_t(j) L_t(j) - \frac{\phi_W}{2}\left(\frac{W_t(j)}{W_{t-1}(j)} - 1\right)^2 W_t L_t, \tag{B.2}$$

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_L} L_t, \quad \text{and} \quad L_t(j) \leq \overline{L}_t, \tag{B.3}$$

where ϕ_W is a parameter governing wage adjustment costs and Λ_t governs the time-varying disutility of labor supply. In a symmetric equilibrium, the first order condition for the optimal wage is:

$$1-\varepsilon_L\left(1-\frac{MRS_t+\left(\mu_{Lt}/C_t{}^{-\rho}\right)}{\left(\frac{W_t}{P_t}\right)}\right)-\phi_W\left(\Pi_{Wt}-1\right)\Pi_{Wt}$$
$$+E_t\left[\beta\frac{\Theta_{t+1}}{\Theta_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\frac{1}{\Pi_{t+1}}\phi_W\left(\Pi_{Wt+1}-1\right)\Pi_{Wt+1}^2\frac{L_{t+1}}{L_t}\right]=0, \quad \text{(B.4)}$$

where μ_{Lt} is the multiplier on the labor constraint, $\Pi_{Wt}\equiv\frac{W_t}{W_{t-1}}$, and $MRS_t=\frac{\Lambda_t L_t^{\psi}}{C_t^{-\rho}}$ is the marginal rate of substitution between consumption and labor supply in preferences. Further, the complementary slackness condition applies: $\left(L_t-\overline{L}_t\right)\mu_{Lt}=0$, with $\mu_{Lt}\geq 0$.

Taking a log linear approximation for this equation, we arrive at the wage Phillips Curve presented in the main text:

$$\pi_{Wt}=\left(\frac{\varepsilon_L-1}{\phi_W}\right)\left[\widehat{mrs}_t-\widehat{rw}_t\right]+\left(\frac{\varepsilon_L}{\phi_W}\frac{P_0}{W_0}\right)\hat{\tilde{\mu}}_{Lt}+\beta E_t\left(\pi_{Wt+1}\right), \quad \text{(B.5)}$$

where $\pi_{Wt}\equiv\hat{w}_t-\hat{w}_{t-1}=\widehat{rw}_t-\widehat{rw}_{t-1}+\pi_t$ is nominal wage inflation, $\widehat{rw}_t\equiv\hat{w}-\hat{p}_t$, $\widehat{mrs}_t=\hat{\lambda}_t+\psi\hat{l}_t-\rho\hat{c}_t$ with $\hat{\lambda}_t\equiv\ln\Lambda_t-\ln\Lambda_0$, and $\hat{\tilde{\mu}}_{Lt}\equiv\ln\tilde{\mu}_{Lt}-\ln\tilde{\mu}_{L0}$ where $\tilde{\mu}_{Lt}\equiv 1+\left(\mu_{Lt}/C_t{}^{-\rho}\right)$ is a function of the multiplier on the labor constraint.

To define equilibrium in this model, we modify the equilibrium conditions from Tables A.1 and A.2 as follows. First, we drop the "labor supply" condition from the baseline model, as labor supply is no longer determined by equating the marginal rate of substitution to the real wage. Second, we add the equilibrium conditions specified in Table A.6, where Panel A corresponds to an equilibrium when labor constraints are slack at date t, and Panel B corresponds to the case when they are binding. The new endogenous variables in the equilibrium system are: $\{\pi_{Wt},\widehat{mrs}_t\}$ when the labor constraint are slack (when $\hat{\tilde{\mu}}_{Lt}=0$), and $\left\{\pi_{Wt},\widehat{mrs}_t,\hat{\tilde{\mu}}_{Lt}\right\}$ when the labor constraint binds. Combined with the goods constraints, this defines eight model regimes with different combinations of binding and slack constraints.

B.2 Quantitative Details

Starting with calibrated parameters, we set $\varepsilon_L=21$, following Christiano, Eichenbaum and Evans (2005). We then choose ϕ_W so that the slope of the wage Phillips Curve is equivalent to a Calvo model with wage adjustment parameter 0.4, when $\varepsilon_L=21$. This Calvo wage adjustment target is taken from Fitzgerald et al. (forthcoming), who estimate it based on state-level data. The implied slope of the wage Phillips Curve is then about 0.02, which is relatively flat. We calibrate the level of the labor constraint ($\bar{L}_0$) to be 1% higher than steady state labor supply. Because the actual level

Table A.6: Equilibrium Conditions with Binding Constraints for Labor

Panel A: Labor Constraint is Slack	
Wage Setting	$\pi_{Wt} = \left(\frac{\varepsilon_L - 1}{\phi_W}\right)\left[\widehat{mrs}_t - \widehat{rw}_t\right] + \beta E_t\left(\pi_{Wt+1}\right)$
Marginal Rate of Substitution	$\widehat{mrs}_t = \hat{\lambda}_t + \psi \hat{l}_t - \rho \hat{c}_t$
Auxiliary Inflation Definition	$\pi_{Wt} = \widehat{rw}_t - \widehat{rw}_{t-1} + \pi_t$
Panel B: Labor Constraint Binds	
Wage Setting	$\pi_{Wt} = \left(\frac{\varepsilon_L - 1}{\phi_W}\right)\left[\widehat{mrs}_t - \widehat{rw}_t\right] + \left(\frac{\varepsilon_L}{\phi_W}\frac{P_0}{W_0}\right)\hat{\tilde{\mu}}_{Lt} + \beta E_t\left(\pi_{Wt+1}\right)$
Marginal Rate of Substitution	$\widehat{mrs}_t = \hat{\lambda}_t + \psi \hat{l}_t - \rho \hat{c}_t$
Auxiliary Inflation Definition	$\pi_{Wt} = \widehat{rw}_t - \widehat{rw}_{t-1} + \pi_t$
Labor Market Constraint	$\hat{l}_t = \hat{\bar{l}}_t + \ln\left(\bar{L}_0 / L_0\right)$

of the constraint at a given point in time is a realization of a stochastic process, results are not sensitive to this value.

We assume the disutility of labor evolves according to $\hat{\lambda}_t = \rho_\lambda \hat{\lambda}_{t-1} + \varepsilon_{\lambda t}$, where $var\left(\varepsilon_{\lambda t}\right) = \sigma_\lambda^2$ and $cov\left(\varepsilon_{\lambda t}, \varepsilon_{\lambda t+s}\right) = 0$ for $s \neq 0$, and we estimate ρ_λ and σ_λ. Further, we assume that the labor constraint is subject to shocks, such that $\ln \bar{L}_t - \ln \bar{L}_0 \equiv \hat{\bar{l}}_t = \varepsilon_{\bar{l}t}$ with $var\left(\varepsilon_{\bar{l}t}\right) = \sigma_{\bar{l}}^2$ and $cov\left(\varepsilon_{\bar{l}t}, \varepsilon_{\bar{l},t+s}\right) = 0$ for $s \neq 0$, and we estimate $\sigma_{\bar{l}}$.[52] We assume observables (aggregate hours worked and real wage growth) are measured with error and estimate the variance of the measurement errors. We also re-estimate all the same structural parameters and stochastic processes using this version of the model.

We construct data on aggregate hours worked and real wage growth from raw data provided by the US Bureau of Labor Statistics.[53] To construct real wage growth, we use hourly compensation data for the non-farm business sector to proxy for nominal wage growth (FRED series id: COMPNFB), taking log growth rates of that quarterly index. We then deflate this nominal wage growth using the aggregate PCE price index, used in prior sections. To build an aggregate hours series, we combine several series. We use average weekly hours of production and nonsupervisory works in the private sector (FRED series id: AWHNONAG) to proxy hours per worker. We then compute the ratio of employment (FRED series id: CE16OV) to population (FRED series id: CNP16OV), were we smooth population estimates by taking means within two-year moving windows in order to eliminate jumps due to data revisions. We then multiply average weekly hours by the employment to population ratio, take logs of that index, and compute deviations from the sample mean of the index over the 1992:Q2 to 2019:Q4 (the pre-COVID sample). This provides an index of the

[52] Generalizing this description, we could alternatively think about the constraint as evolving according to $\hat{\bar{l}}_t = \rho_{\bar{l}} \hat{\bar{l}}_{t-1} + \varepsilon_{\bar{l}t}$, where $\rho_{\bar{l}} \in (0,1)$, $var\left(\varepsilon_{\bar{l}t}\right) = \sigma_{\bar{l}}^2$, and $cov\left(\varepsilon_{\bar{l}t}, \varepsilon_{\bar{l},t+s}\right) = 0$ for $s \neq 0$. Our model implementation calibrates $\rho_{\bar{l}} = 0$, because low persistence in the constraint helps the model explain the dramatic swings in the data in 2020.

[53] We retrieve these data from the FRED database, maintained by the Federal Reserve Bank of St. Louis: https://fred.stlouisfed.org/. So, we provide FRED series identifies here.

level of labor over time.

Results for the new model parameters for this extended model, and re-estimation of the remaining parameters, are included in Table A.7. Parameters estimated previously are little changed, in general. We find a reasonable degree of persistence in labor shocks, with the modal autocorrelation parameter near 0.7.

B.3 Additional Results

In Figure A.11, we present supplemental counterfactual results for the labor market extension. Figure A.11a contains the model response to constraint shocks in isolation. The remaining figures collect model responses to labor supply, demand, and cost (productivity and foreign cost) shocks in isolation, as well as combined with simultaneous constraint shocks. These are analogous to Figure 20b in the main text. We note that demand and productivity shocks both depress inflation below zero in isolation and combined with constraint shocks during most of 2020-2021. As a result, labor and monetary policy shocks together account for more than 100% of the overall increase in inflation. However, because shocks interact with one another, we note that it is not correct to simply add their effects together to assess their relative importance.

Table A.7: Prior and Posterior Distributions for Structural Parameters, Labor Market Extension

Panel A: Elasticity and Taylor Rule Parameters

	Prior			Posterior			
Parameter	Dist	Mean	SD	Mode	Mean	5%	95%
Consumption Armington Elasticity: ι	G	1.5	0.25	1.244	1.237	0.940	1.578
Input Armington Elasticity: η	G	0.5	0.15	0.597	0.632	0.405	0.888
Taylor Rule Inflation: ω	N	1.5	0.12	1.379	1.400	1.214	1.587
Taylor Rule Inertia: α_i	B	0.75	0.1	0.865	0.861	0.836	0.883
Taylor Rule Output: α_y	G	0.12	0.05	0.158	0.153	0.106	0.202

Panel B: Stochastic Processes

	Prior			Posterior			
Parameter	Dist	Mean	SD	Mode	Mean	5%	95%
Preference for Goods: σ_ζ	IG	1	2	0.554	0.552	0.332	0.741
Discount Rate: σ_Θ	IG	1	2	4.166	4.233	3.782	4.724
Foreign Costs: σ_{rmc^*}	IG	1	2	2.096	2.129	1.745	2.528
Goods Productivity: $\sigma_{z(1)}$	IG	1	2	0.143	0.153	0.105	0.209
Services Productivity: $\sigma_{z(2)}$	IG	1	2	0.330	0.327	0.266	0.392
Foreign Constraint: $\sigma_{\bar{y}^*}$	U	1	0.58	0.042	0.056	0.025	0.111
Domestic Constraint: $\sigma_{\bar{y}}$	U	1	0.58	0.068	0.088	0.049	0.153
Monetary Policy Shock: σ_i	IG	1	2	0.149	0.151	0.134	0.170
Labor Preference: σ_λ	IG	1	2	0.123	0.133	0.093	0.184
Labor Constraint: $\sigma_{\bar{l}}$	U	1	0.58	0.109	0.546	0.079	1.659
Preference for Goods: ρ_ζ	B	0.5	0.15	0.833	0.789	0.578	0.918
Discount Rate: ρ_Θ	B	0.5	0.15	0.827	0.822	0.791	0.849
Foreign Costs: ρ_{rmc^*}	B	0.5	0.15	0.937	0.930	0.891	0.964
Goods Productivity: $\rho_{z(1)}$	B	0.5	0.10	0.556	0.551	0.381	0.707
Services Productivity: $\rho_{z(2)}$	B	0.5	0.15	0.880	0.868	0.791	0.924
Foreign Constraint: $\rho_{\bar{y}^*}$	B	0.5	0.20	0.903	0.854	0.669	0.962
Domestic Constraint: $\rho_{\bar{y}}$	B	0.5	0.20	0.435	0.441	0.233	0.641
Labor Preference: ρ_λ	B	0.5	0.20	0.712	0.709	0.621	0.788

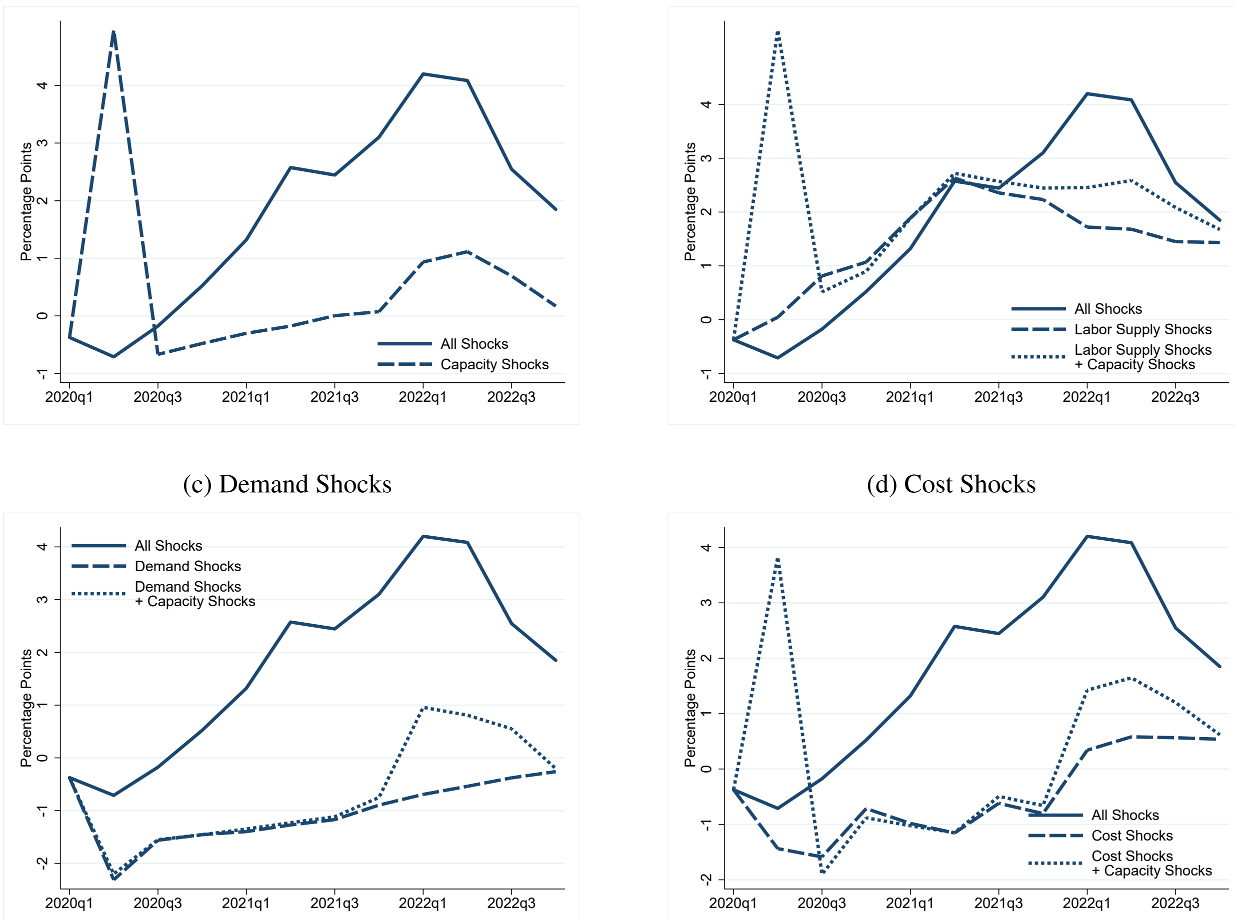
Panel C: Measurement Error

	Prior			Posterior			
Parameter	Dist	Mean	SD	Mode	Mean	5%	95%
Goods PCE: $\sigma^{\text{me}}_{\text{pceg}}$	IG	1	2	1.034	1.042	0.887	1.202
Services PCE: $\sigma^{\text{me}}_{\text{pces}}$	IG	1	2	0.612	0.605	0.530	0.684
Goods PCE Inflation: $\sigma^{\text{me}}_{\pi(1)}$	IG	1	2	0.779	0.800	0.713	0.898
Services PCE Inflation: $\sigma^{\text{me}}_{\pi(2)}$	IG	1	2	0.100	0.102	0.079	0.129
Imp. Input Goods Expenditure: $\sigma^{\text{me}}_{\text{inp}}$	IG	1	2	3.300	3.298	2.971	3.658
Imp. Consumption Goods Expenditure: $\sigma^{\text{me}}_{\text{finp}}$	IG	1	2	2.864	2.846	2.542	3.167
Imp. Input Goods Inflation: $\sigma^{\text{me}}_{\text{inpp}}$	IG	1	2	2.022	2.043	1.844	2.270
Imp. Consumption Goods Inflation: $\sigma^{\text{me}}_{\text{fimp}}$	IG	1	2	0.210	0.207	0.138	0.281
Goods Productivity: $\sigma^{\text{me}}_{\text{prod1}}$	IG	1	2	0.957	0.976	0.865	1.101
Services Productivity: $\sigma^{\text{me}}_{\text{prod2}}$	IG	1	2	0.468	0.466	0.393	0.546
Industrial Production: $\sigma^{\text{me}}_{\text{ip}}$	IG	1	2	0.990	1.010	0.886	1.147
Aggregate Nominal GDP: $\sigma^{\text{me}}_{\text{nva}}$	IG	1	2	0.212	0.210	0.150	0.273
Real Wages: $\sigma^{\text{me}}_{\text{drw}}$	IG	1	2	0.982	0.974	0.872	1.088
Hours: $\sigma^{\text{me}}_{\text{l}}$	IG	1	2	0.033	0.033	0.027	0.039

Note: G denotes the gamma distribution, IG denotes the inverse gamma distribution, U denotes the uniform distribution, B denotes the beta distribution, and N denotes the normal distribution.

Figure A.11: Supplemental Counterfactuals with Labor Market Extensions

(a) Constraint Shocks (b) Labor Supply Shocks (c) Demand Shocks (d) Cost Shocks



Note: Each series represents the simulated path of consumer price inflation (quarterly value, annualized) for the indicated subset of smoothed shocks and constraints during 2020-2022. See text for definition of the counterfactuals.