Finance and Economics Discussion Series

Federal Reserve Board, Washington, D.C.
ISSN 1936-2854 (Print)
ISSN 2767-3898 (Online)

Demand Uncertainty, Selection, and Trade

Erick Sager, Olga A. Timoshenko

2024-042

Please cite this paper as:
Sager, Erick, and Olga A. Timoshenko (2024). "Demand Uncertainty, Selection, and Trade," Finance and Economics Discussion Series 2024-042. Washington: Board of Governors of the Federal Reserve System, https://doi.org/10.17016/FEDS.2024.042.

Demand Uncertainty, Selection, and Trade *

Erick Sager †
Federal Reserve Board

Olga A. Timoshenko ‡
Temple University

May 16, 2024

Abstract

This paper examines the role of uncertainty on elasticities of trade flows with respect to variable trade costs in a canonical model of trade with monopolistic competition and heterogeneous firms. We identify two channels through which uncertainty impacts trade: through export participation thresholds (the selection effect) and the distribution of shocks governing export selection (the dispersion effect). While the selection effect dampens trade elasticities under uncertainty, the dispersion effect is ambiguous. We develop a methodology for using customs firm-level data to quantify trade elasticities under uncertainty, and the magnitude of each of the two channels through which uncertainty impacts trade. We find that uncertainty amplifies trade elasticities, on average, indicating that the dispersion effect of idiosyncratic firm-level shocks dominates – though the effect is heterogeneous across industries. The overall magnitude of the endogenous selection mechanism on trade elasticities is small, indicating that the main drivers of trade in this class of trade models are overwhelmingly incumbent firms.

Keywords: Demand uncertainty, firm size distribution, extensive margin, selection, trade elasticities, welfare.

JEL: F12, F13.

---

*We thank Arevik Gnutzmann-Mkrtchyan, Mina Kim, Logan Lewis, Martin Lopez-Daneri, Andre Kurmann, Peter Morrow, Nuno Limao, Moritz Ritter, Andrés Rodrígues-Clare, Ina Simonovska, Ben Williams, and Yoto Yotov, as well as seminar participants at Drexel, the George Mason University, the University of Maryland, Temple University, Bank of Canada, Asian Meeting of the Econometric Society in East and Southeast Asia 2023, Canadian Economic Associations Meetings 2018, BEROC International Economics Conference 2018, Southern Economic Association 88th Annual Meetings 2018, Washington Area International Trade Symposium 2018, 1st Mid-Atlantic Trade Workshop 2017, and Midwest International Trade Meeting 2017 for valuable comments and discussions. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Board of Governors or the Federal Reserve System. The work was supported in part by the facilities and staff of the George Washington University Colonial One High Performance Computing Initiative. The paper previously circulated under the title "Uncertainty and Trade Elasticities". First version: March 2017.

†2001 C Street NW, Washington DC, 20551, United States. E-mail: erick.r.sager@frb.gov

‡Corresponding author, Temple University, Department of Economics, 1115 Polett Walk, Philadelphia, PA 19122, United States. E-mail: olga.timoshenko@temple.edu

1 Introduction

When variable trade costs vary, not only do existing exporters change the size of their shipments abroad, but also the set of exporters varies through entry and exit. Participation decisions for exporters – that is, entry into export markets and the subsequent decision of how intensively to produce and ship goods to foreign destinations – is a conceptually and quantitatively important dimension of trade, and factors affecting these decisions can play a central role in determining the social value of trade for an economy. However, the benchmark framework for measuring the gains from trade considers a special set of circumstances surrounding potential exporters' decisions: firms have complete information about the demand for their products in all foreign markets. While this benchmark has elucidated central mechanisms that drive the gains from trade, particularly the role of selection, less is known about the role of selection in the empirically more relevant case under which firms face some amount of uncertainty about how profitable their venture into foreign markets will be.

In this paper, we introduce uncertainty about firm-level idiosyncratic demand in foreign markets into a canonical model of trade à la Melitz (2003), and derive predictions of the model for the partial elasticity of trade with respect to variable trade costs. We show that regardless of whether firms face uncertainty or not, the partial trade elasticity admits the same functional form – it equals the firm-level trade elasticity (the intensive margin) scaled by the effect of endogenous selection (the extensive margin). We demonstrate that while uncertainty has no impact on the intensive margin, these different assumptions about firm-level information do have an ambiguous effect on the endogenous selection component of the partial trade elasticity through their effect on export selection thresholds and distributions of the export selection shocks.

We therefore identify two distinct channels through which uncertainty impacts partial trade elasticities: the selection and the dispersion effects. First, demand uncertainty introduces a wedge between entry thresholds in the two information environments that captures firms' expectations about the unexpected component of idiosyncratic demand shocks. We refer to this effect as the *selection effect* of uncertainty. Second, demand uncertainty introduces a wedge between the export selection shocks in the two information environments that captures realizations of the unexpected component of idiosyncratic demand shocks. We refer to this effect as the *dispersion effect* of uncertainty. These two distinct channels stem from the fact that the information environment has a direct impact on the type of idiosyncratic shocks firms take into account when making export decisions. Under complete information, the decisions are based on realizations of productivity and demand shocks, while under uncertainty firms make their decisions based on productivity and only partial information about demand, namely idiosyncratic expectations about demand shocks.

Using the properties of the model, we show that the selection effect of uncertainty dampens the partial trade elasticity relative to a complete information environment, while the dispersion effect is ambiguous. We therefore proceed by using the theoretical model with demand uncertainty to derive an empirical methodology that quantifies trade elasticities, and to disentangle the selection and dispersion effects of uncertainty. Our empirical methodology is based on the model's prediction that export quantity depends on export election shocks, a combination of ex-ante productivity shocks and expectations about demand, while export sales depend on export selection shocks and a realization of an unanticipated component of demand shocks. This property of the model allows us to simultaneously use export quantity and sales data to recover export selection shocks and the dispersion of the unanticipated component of demand shocks.

We apply the methodology to Brazilian firm-level export data for the period between 1997 and 2000. We find that relative to the complete information environment, uncertainty reduces trade elasticities by an average of 8% due to the selection effect, holding all else constant. While the dispersion effect is theoretically ambiguous, we find that in about ninety seven percent of observations the dispersion effect amplifies trade elasticities but does so by a small amount, holding all else constant.

Overall, we find that uncertainty amplifies trade elasticities in about eighty percent of observations, but does so by a small amount. Moreover, the effect is heterogeneous across products, with larger amplification concentrated among more substitutable products. Uncertainty dampens trade elasticities in twenty percent of observations, and the dampening effect is concentrated among products with low elasticities of substitution across varieties. These results indicate that the dispersion of export selection shocks is the dominant mechanism through which uncertainty impacts trade elasticities among substitutable products, while the selection effect of uncertainty plays a larger role in adjustments to trade costs among inelastic products.

Finally, in the model with uncertainty the endogenous selection effect increases trade elasticities by an average of 2% relative to a benchmark with no endogenous selection indicating that incumbent firms are the main drivers of trade adjustments in this class of models.

This paper is related to several strands of the literature on international trade. First, the benchmark model is based on Melitz (2003) and is further developed in many influential papers, such as Chaney (2008), Bernard, Redding, and Schott (2010), Arkolakis, Costinot, and Rodriguez-Clare (2012), Melitz and Redding (2015). A growing branch of the literature has demonstrated that models incorporating uncertainty along the lines of Jovanovic (1982) are well suited to match salient patterns of empirically observed firm behavior such as firm growth as a function of age and size (Arkolakis, Papageorgiou, and Timoshenko (2018)), firm product switching behavior (Timoshenko (2015b)), and firm input and output pricing behavior (Bastos, Dias, and Timoshenko (2018)). Although models that follow the benchmark have focused on decomposing and measuring trade elasticities, the normative implications of models that incorporate uncertainty, particularly for measurements of trade elasticities are not yet well understood.[1]

In terms of decomposing trade elasticities, this paper shows that selection into exporting (and hence the extensive margin of trade elasticity), depends on the information structure faced by firms. Previous work has shown that the partial elasticity of trade with respect to variable trade costs can be decomposed into an intensive and an extensive margin of adjustment components (Chaney (2008)), and that the extensive margin adjustment crucially depends on the distributional assumptions with respect to the sources of firm-level heterogeneity (Melitz and Redding (2015)). Sager and Timoshenko (2019) characterize a flexible distribution that well describes firm-level heterogeneity and find the extensive margin trade elasticity to be small. With respect to trade elasticity measurement, this paper uses a structural model with alternative assumptions about information and specifies firm-level data requirements necessary for identification. Existing work focuses on full information benchmarks that estimate trade elasticities using aggregate trade flows and prices data (see Eaton and Kortum (2002) and Simonovska and Waugh (2014)) or trade flows and tariff data (Caliendo and Parro (2015)).[2]

This paper relates to several related papers on information asymmetries in trade. The most closely related papers to this one are Timoshenko (2015a) and Dickstein and Morales (2018). Both study information asymmetries in trade by using data and theory to infer information available to firms when making export participation decisions. Notably, those papers focus on firm-level outcomes, while this paper's focus is *macroeconomic* in scope and therefore complementary to this previous work. Specifically, this paper uses insights about export participation decisions from these previous papers to understand the aggregate implications of imperfect information on changes in trade flows due to changes in variable trade costs. Accordingly, this paper makes assumptions that are customized to computing trade elasticities (such as estimating heterogeneity in shocks that lead to sales and quantity outcomes) but does not focus on other assumptions that can characterize the extensive margin of trade (such as heterogeneity in fixed costs of exporting). Despite the difference in focus, this paper's model captures the firm-level relationships found in the previous litera-

[1] A notable exception is Arkolakis, Papageorgiou, and Timoshenko (2018), who characterize constrained efficiency of a model in which firms learn about demand but do not engage in international trade.

[2] This literature further finds elasticities estimated from aggregate trade flows are smaller than those estimated from disaggregated industry-level data (Imbs and Mejean (2015)), and that there is substantial heterogeneity in bilateral trade elasticities due to heterogeneity in countries' industrial production (Imbs and Mejean (2017)).

ture (Timoshenko (2015a) finds that past continuous export history predicts current export choice, and Dickstein and Morales (2018) finds that firm-level sales and industry averages predict exporting for large firms but not for small firms) because firms in our model that have positive ex ante information about productivity levels are more likely to be large and export.

Finally, this paper relates to other recent work on trade policy uncertainty. Handley and Limao (2015) find that trade policy uncertainty lowers entry into foreign markets by reducing the value of the export participation threshold, while we find the opposite result. The distinction arises from differences in the timing of when information is revealed to firms and the option value of waiting such timing may produce. In our framework, uncertainty is revealed *after* entry and production decisions have been made. Therefore, waiting has no impact on a firm's decision relevant information. In contrast, in Handley and Limao (2015) firms first observe a realization of tariff policy and then make their decisions. Handley and Limao (2015) framework therefore features the option value of waiting. Firms can condition their entry decisions on a realization of a shock and only enter when the realization of a shock is high enough, a mechanism absent from our framework. Baley, Veldkamp, and Waugh (2020) develop a model in which firms export more when there is greater uncertainty about the terms of trade in bilateral trade relationships, which can also be thought of as trade policy uncertainty, yet the welfare effects are ambiguous and depend on preferences.[3]

The rest of the paper is organized as follows. Section 2 presents the theoretical framework. Section 3 characterizes the effect of uncertainty on trade elasticities. Section 4 details our empirical methodology for quantifying trade elasticites in an environment with uncertainty. Section 5 describes our data and presents elasticity estimation results. Section 6 performs a counterfactual analysis of trade elasticities in an environment with complete information. Section 7 concludes. All proofs, derivations, and robustness checks are relegated to the Appendix.[4]

[3]There are other papers that consider the effects of information on trade. Bergin and Lin (2012) show that the entry of new varieties increases at the time of the announcement of the future implementation of the European Monetary Union, suggesting that changes in the information available to firms have immediate consequences for firms' decisions; Lewis (2014) studies the effect of exchange rate uncertainty on trade; Allen (2014) shows that information frictions help to explain price variation across locations; Fillat and Garetto (2015) show that aggregate demand fluctuations can explain variation in stock market returns between multinational and non-multinational firms.

[4]Appendix A provides a detailed description of the model with uncertainty and complete information. Appendix B derives properties of the endogenous selection component of the partial trade elasticity. Appendix C details the steps of the counterfactual analysis. Appendix D presents robustness results accounting for the measurement error in the quantity data.

2 Theoretical Framework

This section outlines our main theoretical framework which will serve as the structural benchmark for quantifying trade elasticities in a model with uncertainty. We consider an economic environment in which heterogeneous firms export products to monopolistically competitive markets. This environment is similar to that in Melitz (2003) with an added dimension of demand uncertainty according to Jovanovic (1982) as adapted to a heterogeneous firms framework by Arkolakis et al. (2018). We assume exogenous entry as in Chaney (2008).[5]

2.1 Demand

There are N countries and K sectors in each country. Each country is indexed by j and each sector is indexed by k.

Each country is populated by a mass of L_j identical consumers. Each consumer within country j owns an equal share of domestic firms and is endowed with a unit of labor that is inelastically supplied to the labor market. The preferences of a representative consumer in country j are represented by a nested constant elasticity of substitution utility function

$$U_j \;=\; \prod_{k=1}^{K}\left[\left(\sum_{i=1}^{N}\int_{\omega\in\Omega_{ijk}}\left(e^{z^p_{ijk}(\omega)}\right)^{\frac{1}{\epsilon_k}} c_{ijk}(\omega)^{\frac{\epsilon_k-1}{\epsilon_k}}\,d\omega\right)^{\frac{\epsilon_k}{\epsilon_k-1}}\right]^{\mu_k}, \tag{1}$$

where Ω_{ijk} is the set of varieties in sector k consumed in country j originating from country i, $c_{ijk}(\omega)$ is the consumption of variety $\omega\in\Omega_{ijk}$, ϵ_k is the elasticity of substitution across varieties within sector k, $z^p_{ijk}(\omega)$ is the demand shock for variety $\omega\in\Omega_{ijk}$, and μ_k is the Cobb-Douglas utility parameter for goods in sector k such that $\sum_{k=1}^{K}\mu_k = 1$.

Cost minimization yields a standard expression for the optimal demand for variety $\omega\in\Omega_{ijk}$, given by

$$c_{ijk}(\omega) = e^{z^p_{ijk}(\omega)} p_{ijk}(\omega)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k-1}, \tag{2}$$

where $p_{ijk}(\omega)$ is the price of variety $\omega\in\Omega_{ijk}$, Y_{jk} is total expenditures in country j on varieties from sector k, and P_{jk} is the aggregate price index in country j in sector k.[6]

2.2 Supply

Each variety $\omega\in\Omega_{ijk}$ is supplied by a monopolistically competitive firm f that has access to a linear production technology that transforms labor into output, $q=\exp(z^a)\ell$. Upon entry,

[5] All derivations are relegated to Appendix A.

[6] The assumed Cobb-Douglas utility specification over consumption bundles across sectors implies $Y_{jk} = \mu_k Y_j$, where Y_j is aggregate income in country j.

a firm f selling from country i to country j in sector k is endowed with an idiosyncratic labor productivity level z^a_{fijk} and a set of idiosyncratic destination-sector specific demand shocks, $\{z^p_{fijk}\}_{j=1,\ldots,N}$.[7] Each demand and supply shocks pair (z^p_{fijk}, z^a_{fijk}) is drawn from a joint distribution to be characterized later.

Firms from country i selling output in sector k to country j face fixed costs, f_{ijk}, and variable 'iceberg' trade costs, τ_{ijk}. Fixed and variable costs are denominated in units of labor, and w_j denotes the wage rate in country j.

Each firm can potentially supply one variety of a product from each sector. Firms decide which markets to export to (the extensive margin decision) and how much to export to each of the chosen markets (the intensive margin decision). Without loss of generality, we assume that firms choose a quantity to export. Prices are the result of market clearing given the exported quantity of the variety, and then export sales are realized along with prices.

2.3 Information Structure

We consider an environment with complete information and an environment with uncertainty.

In the environment with complete information, firms observe all idiosyncratic shocks before making decisions. Namely, firms observe their supply, z^a_{fijk}, and demand, z^p_{fijk}, shocks before deciding where to export and how much to export. Denote the firm's decision relevant *export selection* shock in the complete information environment by z^{CI}_{fijk}. As we demonstrate below, z^{CI}_{fijk} is given by

$$z^{CI}_{fijk} = (\epsilon_k - 1) z^a_{fijk} + z^p_{fijk}. \tag{3}$$

In the environment with uncertainty, firms do not observe all idiosyncratic shocks before making export decisions. The timing of the information and firm's decisions follows Arkolakis et al. (2018) and is as follows.

1. First, firms observe their supply side shocks, z^a_{fijk}, and form expectations about demand shocks, $E(z^p_{fijk} | z^a_{fijk})$.

2. Next, firms decide whether and where to export, and how much to export to the chosen destinations.

3. Production takes place and all the quantities are shipped; prices clear in destination markets.

[7] The idiosyncratic demand shocks are realized by consumers, but are a payoff relevant state for the firms. Thus, when firms enter, they draw their realization of the idiosyncratic demand of consumers that determines their sales. Following Foster, Haltiwanger, and Syverson (2008), who document that idiosyncratic firm-level demand shocks, rather than productivity, account for a greater variation of sales across firms, we focus on the demand shocks that are firm specific.

4. Lastly, firms observe their sales and infer their demand shocks, z^p_{fijk}, from the realized observations of prices and sales.

Denote the firm's decision relevant *export selection* shock in the environment with uncertainty by z^U_{fijk}. As we demonstrate below, z^U_{fijk} is given by

$$z^U_{fijk} = (\epsilon_k - 1)z^a_{fijk} + E(z^p_{fijk}|z^a_{fijk}). \tag{4}$$

Observe from equations (3) and (4), that export decision in the environment with uncertainty are based on partial information about the realization of demand shocks. This difference leads to different implications regarding the magnitude of the partial trade elasticities with respect to variable trade costs across information environments, and provides novel insights into which data are suited to structurally identify the partial trade elasticities in the environment with uncertainty.

2.4 Model Validation

2.4.1 Timing Assumption

The timing assumption in the environment with uncertainty implies that firms first produce and deliver goods, and receive payments for those goods after delivery. Such post-shipment payment method, commonly referred to as exporter finance in the trade finance literature, is the most widespread method of financing export transactions. The IMF (2009) reports that globally exporter finance accounts for 42 percent of export transactions. In the context of Latin America in particular, Ahn (2015) finds that exporter finance accounts for 80 to 90 percent of the value of import transactions in Colombia and Chile. While we do not attempt to contribute to the trade finance literature nor do we model export payment methods, its it reassuring that the timing of payments implied by the model in this paper is consistent with empirical evidence on export finance.

In the context of the trade literature, the timing assumption in the environment with uncertainty follows Jovanovic (1982) as adapted to a heterogeneous firms framework by Arkolakis et al. (2018). The framework of Arkolakis et al. (2018) (due to the timing assumption in particular) has been shown to be able to predict within firm and exporter behavior such as gradual growth over time and declining with age survival rates (Timoshenko, 2015a; Ruhl and Willis, 2017), age and size dependence of firm growth rates Arkolakis et al. (2018), and within firm price dynamics (Bastos et al., 2018). In the context of this literature, the goal of this paper is to explore trade elasticities properties of a demand process and uncertainty that has also been shown to deliver properties of firm behavior that are consistent with empirical evidence.

2.4.2 Uncertainty in Demand

Our choice to model uncertainty in demand stems from three recent strands of research. First, the literature on firm growth has robustly rejected the notion that firms operate at optimal scale immediately upon entry. For instance, Ruhl and Willis (2017) find that a new exporter's export sales grow slowly following entry in firm-level Colombian manufacturing data, taking an average of four years to catch up to the (unconditional) average exporter. Learning models deliver this feature of the data, both theoretically and quantitatively. For example, Berman et al. (2019) find that the learning process generates the empirically observed decline in firms' sales growth, exit rates, and the variance of sales growth within a cohort conditional on survival in its market. Moreover, Fitzgerald et al. (2023) find that learning about demand explains the declining exits over time and the observed quantity and price dynamics in their Irish export data.

Second, the canonical model in which firms choose to export based on productivity has been shown to be counterfactual. In particular, in contrast to the canonical model's prediction that the smallest exporter should be larger than the largest non-exporter, Eaton et al. (2011) and Armenter and Koren (2015) find that exporters and non-exporters are not strictly sorted in this way: there is a significant number of exporters that are smaller than non-exporters and non-exporters that are larger than exporters. Hence, Armenter and Koren (2015) conclude that size-independent variation is needed to match the observed frequency and size of exporters. In this paper's model, ex post realizations of demand generate such size-independent variation.

Finally, recent empirical evidence has shown that demand shocks explain a large fraction of the variation in firm sales. For example, Hottman et al. (2016) have shown that variation in firms' product appeal explains between a half to two-thirds of the variance in firm sales. Eaton et al. (2011) and Munch and Nguyen (2014) use French and Danish data, respectively, to estimate that firm-destination idiosyncratic shocks account for almost half of variation in sales. Finally, Foster et al. (2016) find that differences in demand, not productivity, explain size differences between new and incumbent plants.

2.5 Environment with Complete Information

In the complete information environment, a firm f's problem selling from country i to country j in sector k consists of maximizing profit

$$\pi_{fijk}(z^a_{fijk}, z^p_{fijk}) = \max_{q_{fijk}} p_{fijk} q_{fijk} - \frac{w_i \tau_{ijk}}{e^{z^a_{fijk}}} q_{fijk} - w_i f_{ijk}, \tag{5}$$

subject to the demand equation (2). A firm exports if its optimal profit from exporting is positive, $\pi_{fijk}(z^a_{fijk}, z^p_{fijk}) \geq 0$, which yields the following export selection equation

$$e^{(\epsilon_k - 1) z^a_{fijk} + z^p_{fijk}} \geq e^{z^{CI^*}_{ijk}}, \tag{6}$$

where a firm exports if inequality (6) is satisfied, and does not export otherwise. Variable $z^{CI^*}_{ijk}$ denotes the export selection threshold under complete information and is given by

$$z^{CI^*}_{ijk} = \log\left(\frac{\epsilon_k w_i f_{ijk}}{B_{ijk} f^\tau(\tau_{ijk})}\right), \tag{7}$$

where B_{ijk} is an origin-destination-sector fixed effect common across firms exclusive of the variable trade costs, and function $f^\tau(.)$ is a strictly monotonically decreasing function.[8]

A firm's export selection equation (6) implies that a firm's export decision is based on a joint realization of the supply and demand shocks, that together comprise a firm's export *selection shock*. Denote by z^{CI}_{fijk} a firm's export selection shock under complete information. From inequality (6), z^{CI}_{fijk} is defined as

$$z^{CI}_{fijk} = (\epsilon_k - 1) z^a_{fijk} + z^p_{fijk}. \tag{8}$$

The export selection equation (6) can therefore be written as

$$z^{CI}_{fijk} \geq z^{CI^*}_{ijk}. \tag{9}$$

2.6 Environment with Uncertainty

In an environment with uncertainty, a firm f from country i chooses the quantity it will export to country j in sector k in order to maximize its *expected* profit

$$E_{z^p_{fijk}|z^a_{fijk}}[\pi_{fijk}(z^a_{fijk}, z^p_{fijk})] = \max_{q_{fijk}} E_{z^p_{fijk}|z^a_{fijk}}\left(p_{fijk} q_{fijk} - \frac{w_i \tau_{ijk}}{e^{z^a_{fijk}}} q_{fijk}\right) - w_i f_{ijk} \tag{10}$$

subject to the demand equation (2). A firm exports if its optimal expected profit from exporting is positive, $E_{z^p_{fijk}|z^a_{fijk}}[\pi_{fijk}(z^a_{fijk}, z^p_{fijk})] \geq 0$, which yields the following export selection equation

$$e^{(\epsilon_k - 1) z^a_{fijk}} \left[E_{z^p_{fijk}|z^a_{fijk}}\left(e^{\frac{z^p_{fijk}}{\epsilon_k}}\right)\right]^{\epsilon_k} \geq e^{z^{CI^*}_{ijk}}, \tag{11}$$

[8] Specifically, $B_{ijk} = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}$, and $f^\tau(\tau_{ijk}) = \tau_{ijk}^{1-\epsilon_k}$.

where a firm exports if inequality (11) is satisfied, and does not export otherwise. Using the orthogonal projection of z^p_{fijk} on z^a_{fijk} written as

$$z^p_{fijk} = E(z^p_{fijk}|z^a_{fijk}) + \upsilon_{fijk}, \tag{12}$$

where υ_{fijk} are i.i.d., export selection equation (11) can be written as

$$e^{(\epsilon_k-1)z^a_{fijk}+E(z^p_{fijk}|z^a_{fijk})}\left[E\left(e^{\frac{\upsilon_{fijk}}{\epsilon_k}}\right)\right]^{\epsilon_k} \geq e^{z^{CI^*}_{ijk}}, \tag{13}$$

Denote by z^U_{fijk} a firm's export selection shock under uncertainty, and by $z^{U^*}_{ijk}$ the export selection threshold under uncertainty. From inequality (13), z^U_{fijk} and $z^{U^*}_{ijk}$ are defined as

$$z^U_{fijk} = (\epsilon_k - 1)z^a_{fijk} + E(z^p_{fijk}|z^a_{fijk}) \tag{14}$$

and

$$z^{U^*}_{ijk} = z^{CI^*}_{ijk} - \log\left[E\left(e^{\frac{\upsilon_{fijk}}{\epsilon_k}}\right)\right]^{\epsilon_k} \tag{15}$$

The export selection equation (13) can therefore be written as

$$z^U_{ijk} \geq z^{U^*}_{ijk}. \tag{16}$$

3 Characterization of Trade Elasticities

In both information environments, the total trade flows from country i to country j in sector k, X_{ijk}, can be expressed as

$$X_{ijk} = J_i\left[1 - G_{ijk}(z^*_{ijk})\right]\int_{z^*_{ijk}}^{+\infty} B_{ijk} f^\tau(\tau_{ijk}) e^z \frac{g_{ijk}(z)}{1 - G_{ijk}(z^*_{ijk})} dz,$$

where J_i is the exogenous mass of potential entrants in country i, z is the decisions relevant export selection shock as defined in equation (8) for the case of complete information and in equation (14) for the case of uncertainty, z^*_{ijk} is the export selection threshold as defined in equation (7) for the case of complete information and in equation (15) for the case of uncertainty, and $g_{ijk}(z)$ and $G_{ijk}(z)$ are the probability density and cumulative distribution functions of the decision-relevant export selection shock respectively.

The partial elasticity of trade with respect to the iceberg trade costs, τ_{ijk}, can then be written as

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ijk}} = \underbrace{\frac{\partial \log f^\tau(\tau_{ijk})}{\partial \log \tau_{ijk}}}_{\substack{\text{firm-level} \\ \text{trade elasticity}}} \underbrace{\left[1 + \gamma_{ijk}(z^*_{ijk}, g_{ijk}(z))\right]}_{\text{endogenous selection}}, \tag{17}$$

where $\gamma_{ijk}(z^*_{ijk}, g_{ijk}(z))$ is a monotonically increasing hazard rate function associated with the random variable distributed according to the probability density function $h_{ijk}(z^*_{ijk}, g_{ijk}(z))$ given by[9]

$$h_{ijk}(z^*_{ijk}, g_{ijk}(z)) \quad = \quad \frac{e^{z^*_{ijk}} g(z^*_{ijk})}{\int_{-\infty}^{+\infty} e^z g_{ijk}(z) dz}.$$

The first component of the partial trade elasticity in equation (17) is referred to as the firm-level trade elasticity (Bas et al., 2017) and captures the response of incumbent exporters to the changes in variable trade costs. It is determined by the elasticity of substitution across varieties, is given by $(1 - \epsilon_k)$, and does not depend on the information environment. The firm-level trade elasticity is subsequently augmented by the endogenous selection component that arises due to the presence of entry and exit mechanism in export markets. It is the endogenous selection component that is impacted by the information environment, as we elaborate below.

3.1 Canonical Cases

It is helpful to start the analysis by considering two canonical expressions of the partial trade elasticity. First, in the context of Krugman (1980) model, all firms are identical and there is no endogenous selection. In this case the endogenous selection component is zero, and the partial trade elasticity is fully determined by the elasticity of substitution across varieties:

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ijk}}^{\text{Krugman (1980)}} = 1 - \epsilon_k.$$

Second, in the context of Chaney (2008), firms are heterogeneous in their idiosyncratic productivity level which is assumed to be drawn from a Pareto distribution. In this case, $g_{ijk}(z)$ follows a Pareto distribution with the shape parameter denoted by ξ_{ijk}, and the partial trade elasticity takes the following form:

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ijk}}^{\text{Chaney (2008)}} = (1 - \epsilon_k) \cdot \left[1 + \underbrace{\frac{\xi_{ijk}}{\epsilon_k - 1} - 1}_{\gamma_{ijk}} \right]. \tag{18}$$

Notice that even though Chaney (2008) framework features endogenous selection, the partial trade elasticity is independent of the export selection threshold. In this case, the endogenous selection effect on the partial trade elasticity is determined by the shape parameter of the

[9]The proof of monotonically increasing property of $\gamma_{ijk}(z^*_{ijk}, g_{ijk}(z))$ is included in Appendix B.

export sales distribution, $\xi_{ijk}/(\epsilon_k - 1)$.[10]

The shape parameter of firm size distribution has been estimated to lie in the range of 1.01 to 1.2, implying the values of γ_{ijk} in the range of 0.01 to 0.2.[11] Substituting the shape parameter estimates into equation (18) subsequently reveals that endogenous selection increases trade elasticities above the firm-level effect of incumbent firms by 1% to 20%. Notably, the fatter is the tail of the export sales distribution, i.e the closer is the shape parameter to unity, the smaller is the role of the endogenous selection in determining the partial elasticity of trade flows with respect to variable trade costs.

This range will serve us as a reference point against which we will compare our estimates of the endogenous component of the trade elasticity in the model with uncertainty and a generalized distribution of export selection shocks.

Generally, the endogenous selection effect on the partial trade elasticity depends on the *selection effect* through the export selection threshold z^*_{ijk} and the *dispersion effect* through the distribution of the decision relevant selection shock $g_{ijk}(z)$. Both of these channels depend on the information environment as we now discuss.

3.2 The Effect of Uncertainty on Endogenous Selection

Equation (17) highlights two distinct ways in which uncertainty impacts the partial elasticity of trade with respect to variable trade costs. First, uncertainty impacts the elasticity through the export selection threshold, z^*_{ijk}. We will refer to this effect as the *selection effect of uncertainty*. Second, uncertainty impacts the elasticity through the distribution of the export selection shock, $g_{ijk}(z)$. We will refer to this effect as the *dispersion effect of uncertainty*.

3.2.1 The selection effect of uncertainty

Result 1: *Holding all else constant, more stringent selection increases the partial trade elasticity.*

Equations (7) and (15) define the export selection thresholds under complete information,

[10] Using the notation developed in this paper, in Chaney (2008) the export sales are given by $r_{fijk}(z) = B_{ijk}\tau_{ijk}^{1-\epsilon_k}e^{(\epsilon_k-1)z}$. When e^z follows a Pareto distribution with a shape parameter ξ_{ijk}, $r_{fijk}(z)$ follows a Pareto distribution with a shape parameter $\xi_{ijk}/(\epsilon_k - 1)$.

[11] The main benchmark for estimates of the shape parameter of firm size distribution is that of Axtell (2001) with the mean value of 1.06 in the context of U.S. employment firm size distribution. Kondo et al. (2023) provide a more recent analysis of estimates of the shape parameter of firm size distribution and find the estimates to lie in the range of 1.01 to 1.23, for a sufficiently large firm size threshold used to fit a Pareto distribution. Sager and Timoshenko (2019) estimate the shape parameter specifically in the context of export sales distribution for Brazilian exporters, and find the values to lie in the range of 1.08 to 1.42, depending on the firm size threshold.

z_{ijk}^{CI*}, and uncertainty, z_{ijk}^{U*}, respectively. Notice that the two thresholds are related as follows

$$z_{ijk}^{CI*} \quad = \quad z_{ijk}^{U*} + \log\left[E\left(e^{\frac{\upsilon_{fijk}}{\epsilon_k}}\right)\right]^{\epsilon_k}. \tag{19}$$

Therefore, uncertainty introduces a wedge between entry thresholds in the two information environments. This result mirrors the one obtained in Handley and Limao (2015), who refer to the wedge as the "uncertainty factor".[12] The wedge captures expectations about realizations of the unknown uncertainty factor. While in our framework the uncertainty is with respect to an unexpected component of the idiosyncratic demand shock, υ_{fijk}, in Handley and Limao (2015) the uncertainty factor captures expectations about future tariff realizations and the frequency of the tariff regime change, τ_{ijk}.

Provided $\log\left[E\left(e^{\frac{\upsilon_{fijk}}{\epsilon_k}}\right)\right]^{\epsilon_k}$ is positive, the selection threshold under complete information is larger than under uncertainty, implying a more stringent selection mechanism under complete information in our framework.[13] This result is opposite to the one in Handley and Limao (2015), who find that tariff uncertainty leads to a higher entry threshold under uncertainty and therefore lower entry in an environment where future tariffs are uncertain.

This distinction arises from differences in the timing of when information is revealed to firms, and the option value of waiting such timing may produce. In our framework, uncertainty is revealed *after* entry and production decisions have been made. Therefore, waiting has no impact on a firm's decision-relevant information. In contrast, in Handley and Limao (2015) firms first observe a realization of tariff policy and then make their decisions. Handley and Limao (2015) framework therefore features the option value of waiting. Firms can condition their entry decisions on a realization of a shock and only enter when the realization of a shock is high enough, a mechanism absent from our framework.

Given that $\gamma_{ijk}(z_{ijk}^*, g_{ijk}(z))$ is a hazard rate function that is monotonically increasing in z_{ijk}^* for a given distribution $g_{ijk}(z)$, a lower selection threshold under uncertainty implies that the selection effect of uncertainty has a effectively dampens the partial trade elasticity, holding all else constant.

Intuitively, the result can be understood as follows. The endogenous selection effect on the partial trade elasticity captures changes in trade flows due to the entry (or exit) of exporters at the selection margin. Therefore, the size of the selection effect depends on the size of the marginal exporter as well as the mass of firms at the selection threshold. This can be seen when expressing the aggregate trade flows using (expected) export revenue as

[12] Equations (9) and (10) in Handley and Limao (2015) and the discussion therein.

[13] The standard distributional assumptions made in the literature, e.g. the Normal, Exponential, and the Double Exponentially Modified Gaussian distributions, all meet this requirement (with appropriate restrictions on parameters).

follows

$$X_{ijk} \propto \underbrace{\int_{r(z^*)}^{+\infty}}_{\text{the selection effect}} r(z) \underbrace{g(z)}_{\text{the dispersion effect}} dr(z), \tag{20}$$

where $r(z)$ is the (expected) export revenue. The (expected) size of the marginal exporter is given by $r(z^*)$. Therefore, a higher value of the selection threshold will result in a larger size of the marginal exporter, and therefore, larger changes in trade flows as a result of changes in the variable trade costs. Given that the export selection threshold is lower under uncertainty, uncertainty has a dampening effect on the partial trade elasticity through the selection effect.

3.2.2 The dispersion effect of uncertainty

Result 2: *Holding all else constant, the dispersion of a selection shock has an ambiguous effect on the partial trade elasticity.*

Equations (8) and (14) define the export selection shocks under complete information, z^{CI}_{fijk}, and uncertainty, z^{U}_{fijk}, respectively. Notice that the two shocks are related as follows

$$z^{CI}_{fijk} = z^{U}_{fijk} + \upsilon_{fijk}, \tag{21}$$

where υ_{fijk} are i.i.d. Therefore, the selection shock under uncertainty has a lower dispersion than under complete information. This dispersion effect of uncertainty on the partial trade elasticity is ambiguous.[14]

The intuition for this result can similarly be understood from equation (20). In addition to the size of the marginal exporter, aggregate trade flows depend on the mass of firms at any given value of the selection shock, $g(z)$, including the mass of firms at the margin given by $g(z^*)$. As shown above, the information environment impacts the distribution of the underlying selection shocks, and therefore the mass of firms at the margin. The overall effect of dispersion is non-linear and depends on how the curvature of the distribution changes and the value of the threshold where the density is evaluated.

Taken together, **Result 1** and **Result 2** imply that uncertainty has an ambiguous effect on the partial elasticity of trade flows with respect to variable trade costs. We therefore proceed by developing an estimation methodology to quantify the partial elasticity of trade flows with respect to variable trade costs in an environment with uncertainty, and compare those elasticities to counterfactual values obtained under the assumption of complete information.

[14] See Appendix B.

4 Empirical Methodology

In this section we develop an empirical methodology to quantify partial elasticites of trade with respect to variable trade costs in an environment with uncertainty. In doing so we adapt the methodology of Berman et al. (2019) to our framework. Equation (17) informs us about what data are needed to structurally identify partial trade elasticities. First, notice that the overall level of the partial trade elasticity is determined by the direct effect of changes in variable trade costs on the sales of incumbent exporters, the firm-level trade elasticity $\partial \log f^{\tau}(\tau_{ijk})/\partial \log \tau_{ijk}$. This component does not depend on the information structure. In the model with CES preferences considered here, $\partial \log f^{\tau}(\tau_{ijk})/\partial \log \tau_{ijk} = (1-\epsilon_k)$, and hence is entirely determined by preferences, namely the elasticity of substitution across varieties, ϵ_k.

Second, the firm-level trade elasticity is then augmented by the endogenous selection component, which depends on the selection threshold, z^{U*}_{ijk}, and the distribution, $g^{U}_{ijk}(.)$, of the underlying idiosyncratic export selection shock, z^{U}_{ijk}. Hence, to structurally estimate the partial elasticity of trade with respect to variable trade costs, more specifically the endogenous selection component, one needs to recover the firm-level export selection shocks together with the distribution governing the export selection shocks, and quantify the respective export selection threshold. We recover all these objects from the data on export quantities and revenues, as we now explain in detail.

To be consistent with the level of observations in the datasets we use, from hereon we omit the origin subscript i and add a time subscript t where appropriate. The dataset is described in Section 5.1 below and includes an export firm-level panel data for an origin country Brazil.

4.1 Firm-Level Shocks

From the firm's maximization problem (10), the optimal export quantity and realized export revenue for firm f exporting to country j product k in year t are given by

$$q_{fjkt}(z^{a}_{fjkt}, z^{p}_{fjkt}) = \left(\frac{\epsilon_k}{\epsilon_k - 1}\frac{w\tau_{jkt}}{E\left(e^{\frac{\upsilon_{fjkt}}{\epsilon_k}}\right)}\right)^{-\epsilon_k} Y_{jkt}P^{\epsilon_k-1}_{jkt}e^{\epsilon_k z^{a}_{fjkt}+E(z^{p}_{fjkt}|z^{a}_{fjkt})} \tag{22}$$

$$r_{fjkt}(z^{a}_{fjkt}, z^{p}_{fjkt}) = \left(\frac{\epsilon_k}{\epsilon_k - 1}\frac{w\tau_{jkt}}{E\left(e^{\frac{\upsilon_{fjkt}}{\epsilon_k}}\right)}\right)^{1-\epsilon_k} Y_{jkt}P^{\epsilon_k-1}_{jkt}e^{(\epsilon_k-1)z^{a}_{fjkt}+E(z^{p}_{fjkt}|z^{a}_{fjkt})+\frac{\upsilon_{fjkt}}{\epsilon_k}} \tag{23}$$

Notice that the export quantity in equation (22) and revenue in equation (23) depend on two main components: the aggregate market conditions common across all firms exporting product k to country j and idiosyncratic firm-level demand and supply side shocks. We will denote the logarithm of the aggregate market component by FE^q_{jkt} and FE^r_{jkt} respectively, and the weighted sums of firm-level idiosyncratic shocks by ζ^q_{fjkt} and ζ^r_{fjkt} respectively. Log-linearized export quantity and revenue can then be written as

$$\log q_{fjkt} = FE^q_{jkt} + \underbrace{\epsilon_k z^a_{fjkt} + E(z^p_{fjkt}|z^a_{fjkt})}_{\zeta^q_{fjkt}} \tag{24}$$

$$\log r_{fjkt} = FE^r_{jkt} + \underbrace{(\epsilon_k - 1) z^a_{fjkt} + E(z^p_{fjkt}|z^a_{fjkt}) + \frac{\upsilon_{fjkt}}{\epsilon_k}}_{\zeta^r_{fjkt}} \tag{25}$$

Estimating equations (24) and (25) allows to recover residuals $\hat{\zeta}^q_{fjkt}$ and $\hat{\zeta}^r_{fjkt}$ that we use to infer export selection shocks under uncertainty. Using equation (4), notice that the log-revenue residual is comprised of the sum of the export selection shock under uncertainty and the i.i.d. orthogonal component as follows

$$\zeta^r_{fjkt} = z^U_{fjkt} + u_{fjkt}, \tag{26}$$

where

$$u_{fjkt} = \upsilon_{fjkt}/\epsilon_k. \tag{27}$$

To separate the export selection shock from the unanticipated component of the demand shock, υ_{fjkt}, we will utilize the log-quantity residuals that do not encompass the i.i.d. shock. To do so we further assume that the conditional expectation of z^p_{fjkt} is linear in z^a_{fjkt},

$$z^p_{fjkt} = \underbrace{\alpha_{jkt} z^a_{fjkt}}_{E(z^p_{fjkt}|z^a_{fjkt})} + \upsilon_{fjkt}$$

where $\alpha_{jkt} = \rho_{jkt}\left(V^{1/2}_{z^p_{fjkt}}/V^{1/2}_{z^a_{fjkt}}\right)$ and $\upsilon_{fjkt} \sim$ i.i.d. $N[0,(1-\rho^2_{jkt})V_{z^p_{jkt}}]$. Substituting the linear conditional expectation and the log-quantity residual into equation (26) yields

$$\zeta^r_{fjkt} = \beta_{jkt}\zeta^q_{fjkt} + u_{fjkt}, \tag{28}$$

where $\beta_{jkt} = ((\epsilon_k - 1) + \alpha_{jkt})/(\epsilon_k + \alpha_{jkt})$. Estimating equation (28) by destination-year-product triplets allows us to recover the firm-level export selection shocks as follows

$$\hat{z}^U_{fjkt} = \hat{\beta}_{jkt}\hat{\zeta}^q_{fjkt}. \tag{29}$$

Finally, there are several caveats in estimating residuals in equations (24) and (25). First, consistently estimating the residuals requires that they be independent from the aggregate market conditions captured in the fixed effect terms such as the origin's wage, w, and destinations' aggregate conditions, the expenditure level Y_{jkt} and the price level P_{jkt}. This independence assumption implies that the underlying firm-level idiosyncratic labor productivity and demand shocks do not vary systematically with the origin's aggregate costs and destinations' aggregate characteristics. Second, the identifying assumption also rules out the possibility that productivity and demand shocks are correlated across markets. Studying the impact of such spillovers on the partial elasticity of trade flows with respect to variable trade costs lies outside the scope of this paper. The final caveat is that a potential presence of classical measurement error in the export quantity data could bias our estimates. We perform robustness checks to address this possibility in Appendix D.

4.2 The Distribution of Export Selection Shocks

4.2.1 Parameterizing Distributions

To proceed with estimating the partial trade elasticities we, first, need to parametrize the distribution, $g^U_{jkt}(.)$, of the export selection shocks, z^U_{fjkt}.

The majority of the trade literature has relied on either a Pareto distribution (Axtell, 2001; Chaney, 2008) or a log-Normal distribution (Bas et al., 2017; Fernandes et al., 2023) in modeling firm level heterogeneity.[15] However, the Brazilian data reject the assumption of a Pareto distribution and favors a more flexible distribution that can capture left-tail fattness as well as right-tail fatness, or the absence of fat tails at all in some markets. To this end, we parameterize the distributions using a Double Exponentially Modified Gaussian (DEMG) distribution that combines features of both the Normal and double Pareto distributions to obtain cases with left-tail fatness, right-tail fatness or at least one thin tail. Sager and Timoshenko (2019) have shown that a DEMG distribution provides a superior fit to the empirical distribution of the logarithm of export sales compared to an Exponential or a Normal alone (note that the logarithm of a Pareto distribution follows an exponential distribution and the logarithm of a log-Normal follows a Normal distribution).

Hence we proceed by parameterizing distributions $g^U_{jkt}(.)$ with a Double EMG distribution, $DEMG(\mu, \sigma^2, \lambda_L, \lambda_R)$, described by the following cumulative distribution function:

$$\begin{aligned} G(z) = \Phi\left(\frac{z-\mu}{\sigma}\right) \quad &- \quad \frac{\lambda_L}{\lambda_L+\lambda_R} e^{-\lambda_R(z-\mu)+\frac{\sigma^2}{2}\lambda_R^2}\Phi\left(\frac{z-\mu}{\sigma}-\lambda_R\sigma\right) \\ &+ \quad \frac{\lambda_R}{\lambda_L+\lambda_R} e^{\lambda_L(z-\mu)+\frac{\sigma^2}{2}\lambda_L^2}\Phi\left(-\frac{z-\mu}{\sigma}-\lambda_L\sigma\right), \end{aligned} \tag{30}$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal distribution.[16]

[15] A notable exception includes Nigai (2017) who assumes a mixture of log-Normal and Pareto distributions.
[16] For notational compactness we drop the jkt subscripts in this section.

The Double EMG distribution provides a very flexible generalization of common distributional assumptions used in the literature. From equation (30), for example, as $\sigma \to 0$ and $\lambda_L \to 0$, the Double EMG distribution converges to an Exponential (Pareto) distribution, as assumed in Chaney (2008). As $\lambda_L \to +\infty$ and $\lambda_R \to +\infty$, the Double EMG distribution converges to a Normal distribution, as assumed in Bas et al. (2017) and Fernandes et al. (2023). As $\sigma \to 0$, the Double EMG converges to a Double Exponential (Pareto) distribution. By assuming the Double EMG distribution we, therefore, allow the data to recover the best fit of distribution between the Exponential, Normal, Double Exponential or the corresponding convolutions. We estimate parameters of the Double EMG distribution separately for each of the observations in our sample, where an observation is defined as a distribution of export selection shocks in country j for product k at time t.

4.2.2 Distribution Estimation Method

We follow Sager and Timoshenko (2019) in estimating the parameters of the Double EMG distribution using a Generalized Method of Moments (GMM) procedure that minimizes the sum of squared residuals,

$$\min_{(\mu,\sigma^2,\lambda_L,\lambda_R)} \sum_{i=1}^{N_P} \left(x_i^{data} - x_i(\mu,\sigma^2,\lambda_L,\lambda_R)\right)^2,$$

where x_i^{data} is the i-th percentile of the empirical export selection socks distribution for a given product-destination-year, $x_i(\mu,\sigma^2,\lambda_L,\lambda_R)$ is the model implied i-th percentile for given parameters $(\mu,\sigma^2,\lambda_L,\lambda_R)$, and N_P is the number of percentiles used in estimation. We use the 1st through 99th percentiles of the empirical distribution to estimate parameters. In practice, this choice eases computational burden compared to using each data point, without significantly changing the parameter estimates we recover. Furthermore, note that choosing parameters to minimize the sum of squared residuals is equivalent to Head et al.'s (2014) method of recovering parameters from quantile regressions.

Hence, for each product-destination-year observation, we choose distribution parameters $(\mu,\sigma^2,\lambda_L,\lambda_R)$ so that the percentiles of the theoretical distribution of export selection shocks match the percentiles of the respective empirical distribution.

4.2.3 Correcting for Endogenous Selection

In fitting a distribution to the recovered export selection shocks, $\hat{z}^U_{fjkt}$, it is important to note that the model implies truncation in the data. Namely, selection shocks are observed only when $z^U_{fjkt} \geq z^{U*}_{jkt}$. To account for the endogenous selection into exporting, we follow the approach by Sager and Timoshenko (2019). Namely, we proceed by fitting a truncated probability distribution function $g^U_{jkt}(.)$ to the data and take the truncation point z^{U*}_{jkt} to be given by the zeroth percentile of the corresponding empirical distribution of the export selection shocks.

4.3 Selection Thresholds

We adapt the methodology of Bas et al. (2017) to recover the (scaled) export selection thresholds by matching the model-implied average-to-minim ratios of export quantity to those in the data. Using equation (22) and the definition of ζ^q_{fjkt} from equation (24) we can write

$$\frac{\tilde{q}_{jkt}}{q^{\min}_{fjkt}} = e^{-\zeta^{q^*}_{jkt}} \int_{\zeta^{q^*}_{jkt}}^{+\infty} \frac{e^{\zeta^q} g^{\zeta^q}_{jkt}(\zeta^q)}{1 - G^{\zeta^q}_{jkt}(\zeta^{q^*}_{jkt})} d\zeta^q. \tag{31}$$

We solve equation (31) for $\zeta^{q^*}_{jkt}$ to recover the export selection threshold $z^{U*}_{jkt} = \hat{\beta}_{jkt}\zeta^{q^*}_{jkt}$. In solving equation (31), we measure the average-to-minimum ratio of quantity using the average-to-minimum ratio of the exponential of estimated quantity residuals, $\hat{\zeta}^q_{fjkt}$. From equation (29), the distribution $g^{\zeta^q}_{jkt}(.)$ follows the distribution $g^U_{jkt}(.)$ scaled by parameter $1/\beta_{jkt}$. We use the estimates of $1/\hat{\beta}_{jkt}$ and $\hat{g}^U_{jkt}(.)$ obtained in Sections 4.1 and 4.2 respectively.

5 Data and Estimation Results

In this section we use data across Brazilian exporters on the distribution of export quantities and sales by product-destination over time to quantify trade elasticities in an environment with uncertainty. A product is defined as a 6-digit HS code.

5.1 Data

The data come from the Brazilian customs declarations collected by SECEX (*Secretaria de Comercio Exterior*).[17] The data record export value and weight (in kilograms) of the shipments at the firm-product-destination-year level. A product is defined at the 6-digit Harmonized Tariff System (HS) level. We use the data for the period between 1997 and 2000, when both the sales and the weight data are available.

We proxy the theoretical notion of export quantity with an empirical measure of export weight.[18] Since the properties of export weight differ substantially across industries, we further conduct our analysis at the product-destination-year level.

[17]For a detailed description of the dataset see Molinaz and Muendler (2013). The data have further been used in Flach (2016) and Flach and Janeba (2017).

[18]Export weight is used as a measure of export quantity in a number of studies including Manova and Zhang (2012); Bastos et al. (2018).

We define an observation to be a distribution of export quantity or sales across firms for a given product-destination-year triplet, and focus on observations where at least 100 firms export in at least one of the four years for a given product-destination pair.[19] The final sample consists of 288 product-destination-year observations, and covers 14 destinations and 35 industries.[20] For each product-destination-year observation, we clean the data by dropping export sales and export quantity values that fall below the 1st or above the 99th percentiles. Table 1 provides summary statistics of log-export quantities and log-export sales distributions in our final sample.

5.2 Parameter and Threshold Estimates

In this section we present estimates of the distribution parameters of the export selection shocks, and the respective entry threshold estimates.

5.2.1 Parameter Estimates

Table 2 summarizes estimates of distribution parameters across 288 observations for the distributions of the export selection shocks by product-destination-year triplets. As can be seen from Table 2, the average sample value of σ 1.20, which means that we can reject the common assumption of Exponentially (or Double Exponentially) distributed shocks that imply $\sigma = 0$, and consequently consider an alternative distribution to model underlying shocks. Furthermore, as can be inferred from the values of the left and right tail parameters, λ_L and λ_R, distributions exhibit substantial heterogeneity in the fatness of both tails. The value of the right tail parameter, λ_R varies between 0.72 and 76.57, with about 26 percent of observations exhibiting a fat right tail, i.e. $\lambda_R < 2$. These estimates are consistent with the previous empirical research documenting fatness in the right tail of sales or employment distributions across firms.[21] Furthermore, we also find that distributions exhibit fatness in the left tail ($\lambda_L < 2$) in approximately 80 percent of observations.[22]

[19] The thresholds of 100 firms makes our results comparable to other papers in the literature (see Fernandes et al. (2023), Sager and Timoshenko (2019)) and ensures that an empirical distribution can be accurately described by percentiles. The qualitative features and basic quantitative results are not heavily dependent on the exact threshold we select within the neighborhood of 100 firms (results are available upon request).

[20] We note that there are 232,266 product-destination-year observations in the entire data-set. We focus on a sub-sample of 288 observations where at least 100 firms export in at least one of the four years for a given product-destination pair. Among the remaining 231,978 observations, the median and the average number of exporters is 1 and 3.3 respectively. Hence, these markets are unlikely to be characterized by a monopolistic competition environment, and the forces of endogenous market selection that we seek to identify in our paper would not apply.

[21] See Axtell (2001), di Giovanni et al. (2011), and Kondo et al. (2023).

[22] The values of parameter estimates are stable across time with only weak evidence of the right tail getting thinner over time. We do not observe strong systematic variation between parameter values and the elasticity of substitutions cross varieties. Additional details are available upon request.

5.2.2 Entry Thresholds

Figure 1 provides a scatter plot of the entry threshold estimates and the corresponding average-to-minimum ratios of log-export quantity residuals. Each dot in the Figure corresponds to a product-destination-year observation. Figure 1 demonstrates a negative relationship between the average-to-minimum ratio and the entry threshold. The larger is the average-to-minimum ratio, the smaller is the marginal exporter relative an average exporter. Hence, the respective entry threshold must be lower.

5.3 Estimates of Trade Elasticities

Given the estimated distribution parameters and entry thresholds presented in Section 5.2, we compute the partial trade elasticity, $\partial \log X_{jkt}/\partial \log \tau_{jkt}$, and the endogenous selection effect on trade elasticity, γ_{jkt}, according to equation (17). Note from equation (17) that the full endogenous selection effect on partial trade elasticity is captured by $(1+\gamma_{jkt})$. For presentation and clarity purposes, when presenting the quantitative results from hereon, we will omit adding unity to γ_{jkt}, and will refer to γ_{jkt} alone as the endogenous selection effect.

Result 3: *On average, the endogenous selection effect, γ_{jkt}, amounts to 0.02*

Table 3 presents the estimates of partial trade elasticities and the endogenous selection effects in a model with uncertainty. As shown in the first row of the table, the mean endogenous selection effect equals to 0.02. As discussed in Section 3.1, this magnitude is comparable to the one obtained from a standard trade model similar to that of Melitz (2003) where the distribution of export sales follows a Pareto distribution with a shape parameter 1.02, the value which is largely consistent with the shape parameter estimates obtained in the literature.[21]

A useful interpretation of the magnitude of the endogenous selection effect, γ_{jkt}, is the percent by which partial trade elasticity increases relative to a benchmark value without endogenous selection. Notice from equation (17) that in the absence of endogenous selection, the total partial trade elasticity is determined by the firm-level elasticity, $\partial \log f^{\tau}(\tau_{jkt})/\partial \log \tau_{jkt}$. Endogenous selection mechanism subsequently increase the firms-level trade elasticity by a factor of $(1+\gamma_{jkt})$. The mean value of γ_{jkt} of 0.02, therefore, indicates that entrants and exitors change trade flows by an additional 2% relative to the change in trade flows generated by incumbent firms.

Result 4: *The endogenous selection effect, γ_{jkt}, is heterogeneous across products and is higher in products with a larger elasticity of substitution across varieties.*

Figure 2 depicts a relationship between the average selection effect for a given product and that product's elasticity of substitution across varieties.[23] The figure exhibits a weakly positive relationship indicating that selection effect is larger in products where varieties are more substitutable.

In the next section we perform a series of counterfactual experiments to understand the effect of information environment on the endogenous selection effect of the partial trade elasticity.

6 Counterfactuals

As discussed in Section 3.2, uncertainty impacts partial trade elasticity through selection and dispersion effects, with the total effect being ambiguous. We conduct the following three counterfactual experiments to disentangle the two effects and quantify the effect of information on partial trade elasticities.

First, to isolate the selection effect of uncertainty, we compute the counterfactual values of the selection effect and partial trade elasticities by varying the selection thresholds from the baseline values of z_{jkt}^{U*} to their respective counterfactual values of z_{jkt}^{CI*}, while keeping the distribution of the export selection shocks at their baseline values estimated under uncertainty.

Second, to isolate the dispersion effect of uncertainty we compute the counterfactual values of the selection effect and partial trade elasticities by varying the distribution of the selection shocks from the baseline values of $g_{jkt}^{U}(.)$ to their respective counterfactual values, $g_{jkt}^{CI}(.)$, while keeping the entry threshold values at their baseline values estimated under uncertainty.

Finally, we compute the complete counterfactual values of the endogenous selection effects and partial trade elasticities under complete information and compare the obtained values to the baseline estimates under uncertainty.[24]

Result 5: *The selection effect of uncertainty reduces partial trade elasticities by an average of 8% relative to their counterfactual values under complete information.*

Panel A in Table 4 presents counterfactual trade elasticities arising from varying the selection thresholds from the baseline values estimated under uncertainty to the respective counterfactual values computed under complete information, holding all else constant. Notice that the average counterfactual endogenous selection effect, γ_{jkt}, is 0.80. In this

[23]For each 6-digit HS code, the elasticity of substitution across varieties is obtained from Soderbery (2015).
[24]The details on quantifying counterfactual values are included in Appendix C.

counterfactual scenario, entering and exiting exporters contribute an additional 80%, relative to incumbent exporters, to generating new trade flows from a decline in variable trade costs relative to a modest 2% in the baseline estimation. Therefore, uncertainty substantially dampens the selection effect on the partial trade elasticities. This is solely due to a more stringent selection under complete information. As shown in Section 3.2, the export selection thresholds are higher under complete information, which results in a marginal firm being larger. Therefore changes in trade costs will general larger changes in trade volumes due to larger size of marginal firms in an environment with complete information relative to uncertainty.

We subsequently define the amplification effect of uncertainty as the ratio of partial trade elasticities computed in the baseline scenario of uncertainty relative to their counterfactual values computed under complete information. The second row in Panel A Table 4 indicates that the amplification effect on the partial trade elasticity due to selection is 0.92 on average. Hence, the total partial trade elasticities are on average 8% lower due to the selection effect of uncertainty in a model with uncertainty relative to a model with complete information.

Result 6: *The dispersion effect of uncertainty increases partial trade elasticities in about ninety seven percent of observations and decreases partial trade elasticities in the remaining three percent of observations. The magnitude of the dispersion effect is small.*

The dispersion effect of uncertainty captures the mass of firms at the market participation threshold, holding all else constant. This mass depends on how the distribution of export selection shocks changes between information environments. We back out the distributions of shocks from the microdata on export sales and quantities.

Panel B in Table 4 presents counterfactual trade elasticities arising from varying the distribution of the selection shocks from the baseline values estimated under uncertainty to the respective counterfactual values computed under complete information, holding all else constant.

The average amplification effect of dispersion is greater than unity. Notice from the second row in Panel B in Table 4 that the partial trade elasticities are on average 2% higher under uncertainty compared to counterfactual values under complete information. This magnitude is rather small as evident from comparing the average endogenous selection effect: 0.02 in the baseline estimation relative to 0.0005 in the discussed counterfactual.

Result 7: *The total effect of uncertainty on the partial trade elasticity.*

(i) Uncertainty increases partial trade elasticities in about eighty percent of observations and decreases partial trade elasticities in the remaining twenty percent of observations.

(ii) The amplification effect of uncertainty increases with the variance of the unexpected component of the demand shocks.

(iii) The amplification effect of uncertainty increases with the elasticity of substitution across varieties with negative effects concentrated among inelastic products.

The total effect of uncertainty on the partial trade elasticities depends on the interaction of the selection and dispersion effects. Figures 3 provides a scatter plot of the estimates of endogenous selection effects, γ_{jkt}, obtained in the baseline estimation under uncertainty (x-axis) versus the respective counterfactual values under complete information (y-axis). We find that in the majority of observations (80%) the endogenous selection effect is larger under uncertainty. Comparing results in Table 3 and Panel C in Table 4, the average endogenous selection effect under uncertainty, 0.02, is higher than under complete information, 0.001, resulting in on average 1% higher partial trade elasticities under uncertainty. In a subset of observations where the amplification effect is below unity, i.e. uncertainty dampens trade elasticities relative to the complete information environment, the endogenous selection effect is about 23% lower under uncertainty resulting in an insignificant impact on total partial trade elasticities.[25]

The small magnitude of the total amplification effect is largely determined by the dispersion effect of uncertainty. Notice, from Panel A in Table 4 that in the absence of dispersion, the counterfactual trade elasticities are significantly larger: the mean of the endogenous selection component being 0.80 versus 0.02 under uncertainty. The large selection effect is dampened by the dispersion effect of uncertainty. As can be seen from Table 3 and Panel B in Table 4, the dispersion effect alone, reduces the average selection effect from 0.02 under uncertainty to 0.0005 under complete information (relative to 0.001 in the full counterfactual, Panel C in Table 4) resulting in total trade elasticities being on average higher by 2%, which is close to the overall amplification effect of uncertainty on partial trade elasticities noted in Panel C in Table 4 and amounting to 1%.

Panel A in Figure 4 further demonstrates the importance of the distribution of export selection shocks in determining the magnitude of trade elasticities. The figure depicts a relationship between the amplification effect of uncertainty and the standard deviation of the unexpected component of the demand shocks, υ_{fjkt}.[26] The figure demonstrates that the larger is the dispersion of the unexpected component of the demand shocks, the larger is the total amplification effect of uncertainty.

[25] In this subset of observations, the partial trade elasticity declines by an average of one hundredth of a percent.

[26] All values in Panel A in Figure 4 have been normalized by their respective industry averages, where an industry is defined as a 6-digit HS code.

We further find that there exists substantial heterogeneity in the amplification effect of uncertainty across industries. Panel B in Figure 4 depicts a relationship between the amplification effect of uncertainty and the elasticity of substitution across products. Notably, in industries with low elasticity of substitution across varieties, the amplification effect is below unity, meaning that in those products trade elasticities are larger under complete information and that the selection effect plays a dominant role in determining the magnitude of trade elasticities. Hence, when products are less substitutable, the size of the marginal exporter matters more than the mass of firms at any given threshold in predicting how trade flows change in response to changes in trade costs.

7 Conclusion

In this paper, we developed a model that introduces firm-level uncertainty about idiosyncratic demand in foreign markets into a canonical model of trade (c.f. Melitz (2003)), and used the model to study the effect of uncertainty on the partial elasticity of trade with respect to variable costs.

The model predicts that while uncertainty does not change the functional form of the partial trade elasticity relative to an economy with complete information, it changes the forces governing selection into exporting. In particular, we identified two channels through which uncertainty impacts trade – through export participation thresholds (the selection effect) and the distribution of shocks governing export selection (the dispersion effect) – and showed that although the model predicts a lower partial trade elasticity in a model with uncertainty due to the selection effect, the dispersion effect is ambiguous. The total effect of uncertainty on trade elasticities is therefore theoretically ambiguous.

Using the structure of the model, we developed a new empirical methodology to quantify partial elasticities of trade with respect to variable trade costs in an environment with uncertainty using firm-level data. We applied the methodology to the Brazilian firm-level customs data and found that, on average, uncertainty amplifies partial trade elasticities relative to an environment with complete information. This indicates that the dispersion effect of idiosyncratic firm-level shocks has the dominant effect on the partial trade elasticities, although there is heterogeneity in the effect across industries. We also find that the overall magnitude of the endogenous selection mechanism on trade elasticities is small, indicating that the main drivers of trade are overwhelmingly incumbent firms in this class of trade models.

References

Ahn, J. (2015): "Understanding Trade Finance: Theory and Evidence from Transaction-Level Data," International Monetary Fund, mimeo.

Allen, T. (2014): "Information Frictions in Trade," *Econometrica*, 82, 2041–2083.

Arkolakis, C., A. Costinot, and A. Rodriguez-Clare (2012): "New Trade Models, Same Old Gains?" *American Economic Review*, 102, 94–130.

Arkolakis, C., T. Papageorgiou, and O. Timoshenko (2018): "Firm Learning and Growth," *Review of Economic Dynamics*, 27, 146–168.

Armenter, R. and M. Koren (2015): "Economies of Scale and the Size of Exporters," *Journal of the European Economic Association*, 13, 482–511.

Axtell, R. L. (2001): "Zipf Distribution of U.S. Firm Sizes," *Science*, 293, 1818–1820.

Baley, I., L. Veldkamp, and M. Waugh (2020): "Can global uncertainty promote international trade?" *Journal of International Economics*, 126.

Bas, M., T. Mayer, and M. Thoenig (2017): "From Micro to Macro: Demand, Supply, and Heterogeneity in the Trade Elasticity," *Journal of International Economics*, 108, 1–19.

Bastos, P., D. Dias, and O. Timoshenko (2018): "Learning, Prices, and Firm Dynamics," *Canadian Journal of Economics*, 51, 1257–1311.

Bergin, P. R. and C.-Y. Lin (2012): "The dynamic effects of a currency union on trade," *Journal of International Economics*, 87, 191–204.

Berman, N., V. Rebeyrol, and V. Vicard (2019): "Demand learning and firm dynamics: evidence from exporters," *Review of Economics and Statistics*, 101, 91–106.

Bernard, A., S. Redding, and P. Schott (2010): "Multi-Product Firms and Product Switching," *American Economic Review*, 100, 70–97.

Broda, C. and D. Weinstein (2006): "Globalization and the Gains from Variety," *Quarterly Journal of Economics*, 121, 541–585.

Caliendo, L. and F. Parro (2015): "Estimates of the Trade and Welfare Effects of NAFTA," *The Review of Economic Studies*, 82, 1–44.

Chaney, T. (2008): "Distorted Gravity: The Intensive and Extensive Margins of International Trade," *American Economic Review*, 98, 1707–21.

di Giovanni, J., A. A. Levchenko, and R. Rancière (2011): "Power Laws in Firm Size and Openness to Trade: Measurement and Implications," *Journal of International Economics*, 58, 42–52.

Dickstein, M. J. and E. Morales (2018): "What do Exporters Know?" *The Quarterly Journal of Economics*, 133, 1753–1801.

Eaton, J. and S. Kortum (2002): "Technology, Geography, and Trade," *Econometrica*, 70, 1741–1779.

Eaton, J., S. Kortum, and F. Kramarz (2011): "An Anatomy of International Trade: Evidence From French Firms," *Econometrica*, 79, 1453–1498.

Feenstra, R. C. (1994): "New Product Varieties and the Measurement of International Prices," *American Economic Review*, 84, 157–77.

Fernandes, A. M., P. J. Klenow, S. Meleshchuk, M. D. Pierola, and A. Rodríguez-Clare (2023): "The Intensive Margin in Trade: How Big and How Important?" *American Economic Journal: Macroeconomics*, 15, 320–54.

Fillat, J. L. and S. Garetto (2015): "Risk, Returns, and Multinational Production," *The Quarterly Journal of Economics*, 130, 2027–2073.

Fitzgerald, D., S. Haller, and Y. Yedid-Levi (2023): "How Exporters Grow," *The Review of Economic Studies*.

Flach, L. (2016): "Quality upgrading and price heterogeneity: Evidence from Brazilian exporters," *Journal of International Economics*, 102, 282–290.

Flach, L. and E. Janeba (2017): "Income inequality and export prices across countries," *Canadian Journal of Economics*, 50, 162–200.

Foster, L., J. Haltiwanger, and C. Syverson (2008): "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" *American Economic Review*, 98, 394–425.

——— (2016): "The slow growth of new plants: Learning about demand?" *Economica*, 83, 91–129.

Handley, K. and N. Limao (2015): "Trade and Investment under Policy Uncertainty: Theory and Firm Evidence," *American Economic Journal: Economic Policy*, 7, 189–222.

Head, K., T. Mayer, and M. Thoenig (2014): "Welfare and Trade without Pareto," *American Economic Review*, 104, 310–16.

Hottman, C. J., S. J. Redding, and D. E. Weinstein (2016): "Quantifying the sources of firm heterogeneity," *The Quarterly Journal of Economics*, 131, 1291–1364.

Imbs, J. and I. Mejean (2015): "Elasticity Optimism," *American Economic Journal: Macroeconomics*, 7, 43–83.

——— (2017): "Trade Elasticties," *Review of International Economics*, 25, 383–402.

IMF (2009): *World Economic Outlook, October 2009, Sustaining the Recovery*, International Monetary Fund.

Jovanovic, B. (1982): "Selection and the Evolution of Industry," *Econometrica*, 50, 649–670.

Kondo, I., L. Lewis, and A. Stella (2023): "Heavy Tailed, but not Zipf: Firm and Establishment Size in the United States," *Journal of Applied Econometrics*, 38, 767–785.

Krugman, P. (1980): "Scale Economies, Product Differentiation, and the Pattern of Trade," *American Economic Review*, 70, 950–959.

Lewis, L. T. (2014): "Exports Versus Multinational Production Under Nominal Uncertainty," *Journal of International Economics*, 94, 371–386.

Manova, K. and Z. Zhang (2012): "Export Prices Across Firms and Destinations," *The Quarterly Journal of Economics*, 127, 379–436.

Melitz, M. and S. Redding (2015): "New Trade Models, New Welfare Implications," *American Economic Review*, 105, 1105–46.

Melitz, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71, 1695–1725.

Molinaz, D. and M.-A. Muendler (2013): "Preparing to Export," NBER Working Paper No. 18962.

Munch, J. R. and D. X. Nguyen (2014): "Decomposing firm-level sales variation," *Journal of Economic Behavior & Organization*, 106, 317–334.

Nigai, S. (2017): "A tale of two tails: Productivity distribution and the gains from trade," *Journal of International Economics*, 104, 44–62.

Rinne, H. (2014): *The Hazard rate : Theory and inference (with supplementary MATLAB-Programs)*, Justus-Liebig-Universitat.

Ruhl, K. and J. Willis (2017): "New Exporter Dynamics," *International Economic Review*, 58, 703–726.

Sager, E. and O. A. Timoshenko (2019): "The Double EMG Distribution and Trade Elasticities," *Canadian Journal of Economics*, 52, 1523–1557.

Simonovska, I. and M. E. Waugh (2014): "The elasticity of trade: Estimates and evidence," *Journal of International Economics*, 92, 34–50.

Soderbery, A. (2015): "Estimating Import Supply and Demand Elasticities: Analysis and Implications," *Journal of International Economics*, 96, 1–17.

Timoshenko, O. A. (2015a): "Learning versus Sunk Costs Explanations of Export Persistence," *European Economic Review*, 79, 113–128.

——— (2015b): "Product Switching in a Model of Learning," *Journal of International Economics*, 95, 233–249.

Figures and Tables

Table 1: Properties of the log-export quantity and log-export sales distributions across product-destination-year observations over 1997-2000.

Statistic	Mean	Std. Dev.	Min	Max
Panel A: Properties of log-quantity				
Standard Deviation	2.21	0.50	1.06	3.26
Skewness	0.03	0.33	-1.08	0.81
Interquartile Range	3.23	0.81	1.36	5.52
Kelly Skew	0.01	0.14	-0.39	0.55
Panel B: Properties of log-sales				
Standard Deviation	1.94	0.37	0.92	2.75
Skewness	-0.10	0.28	-0.85	1.00
Interquartile Range	2.75	0.57	1.13	4.18
Kelly Skew	-0.02	0.12	-0.30	0.45

Note: the summary statistics are reported across 288 product-destination-year observations. A product is defined as a 6-digit HS code. Export quantity is measured as export weight in kilograms.

Table 2: Double EMG distribution parameter estimates of the distributions of export selection shocks.

Parameter	Mean	Std. Dev.
σ	1.20	0.65
λ_L	4.04	8.39
λ_R	12.64	12.86

Notes: the summary statistics are reported across 288 product-destination-year observations. A product is defined as a 6-digit HS code.

Table 3: Trade elasticity estimates under uncertainty.

Measure	Mean	Std. Dev.
Endogenous selection, γ_{jkt}	0.02	0.10
Total partial trade elasticity, $\partial \log X_{jkt} / \partial \log \tau_{jkt}$	3.44	3.67

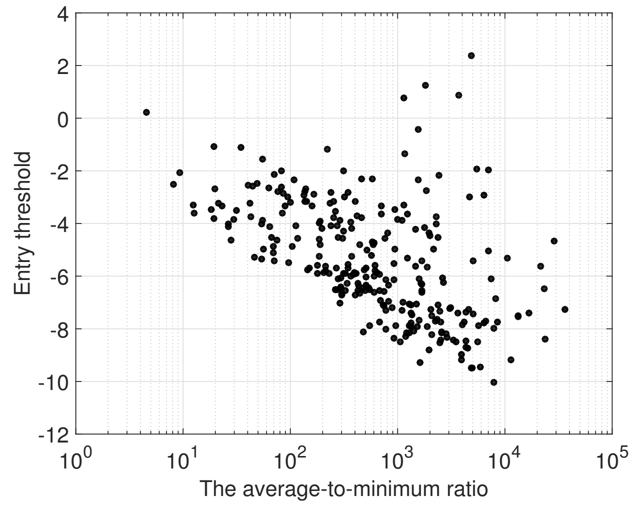
Notes: the summary statistics are reported across 274 product-destination-year observations for which estimates of the Double EMG right tail parameter are greater than unity. The elasticities are not defined otherwise. A product is defined as a 6-digit HS code.

Table 4: Counterfactual trade elasticity estimates under complete information.

	Endogenous Selection γ_{jkt}		Partial Trade Elasticity, $\partial \log X_{jkt} / \partial \log \tau_{jkt}$	
Measure	Mean	Std. Dev.	Mean	Std. Dev.
Panel A: selection effect of uncertainty				
Selection effect	0.80	5.31	7.50	20.09
Amplification due to selection	0.32	0.25	0.92	0.21
Panel B: dispersion effect of uncertainty				
Dispersion effect	0.0005	0.002	3.38	3.64
Amplification due to dispersion	$3.7 \cdot 10^{62}$	$6.2 \cdot 10^{63}$	1.02	0.10
Panel C: total effect of uncertainty				
Total effect	0.001	0.003	3.38	3.64
Total amplification effect	$1.2 \cdot 10^{57}$	$2.0 \cdot 10^{58}$	1.01	0.10

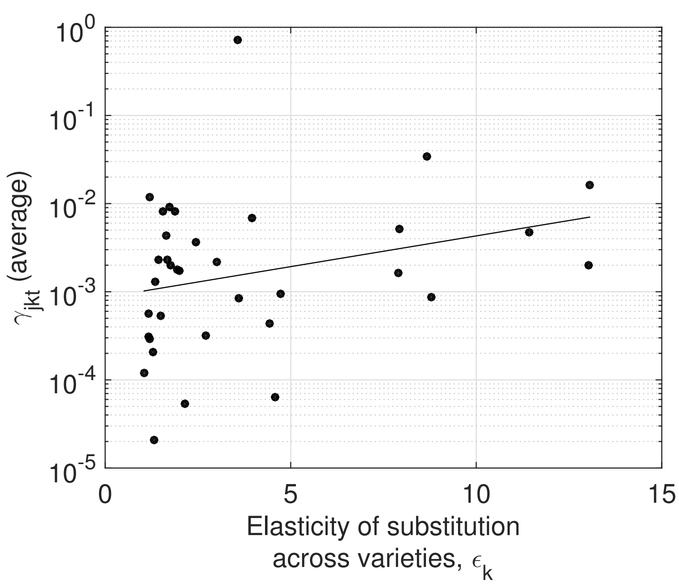
Notes: all summary statistics are reported across 274 destination-year-hs6 observations for which estimates of the Double EMG right tail parameter are greater than unity. The elasticities are not defined otherwise. The amplification effect is computed as the ratio of the baseline estimate of trade elasticity under uncertainty relative to its counterfactual value under complete information for the indicated counterfactual scenario. In Panel A, the counterfactual values are obtained by varying the selection thresholds from the baseline values of $z_{jkt}^{U^*}$ to their respective counterfactual values of $z_{jkt}^{CI^*}$, while keeping the distribution of the export selection shocks at their baseline values estimated under uncertainty. In Panel B, the counterfactual values are obtained by varying the distribution of the selection shocks from the baseline values of g_{jkt}^{U} to their respective counterfactual values, g_{jkt}^{CI}, while keeping the entry threshold values at their baseline values estimated under uncertainty. Panel C computes complete counterfactual values by varying both, the selection thresholds and distributions of selection shocks to their counterfactual values.

Figure 1: The entry thresholds and average-to-minimum ratios.



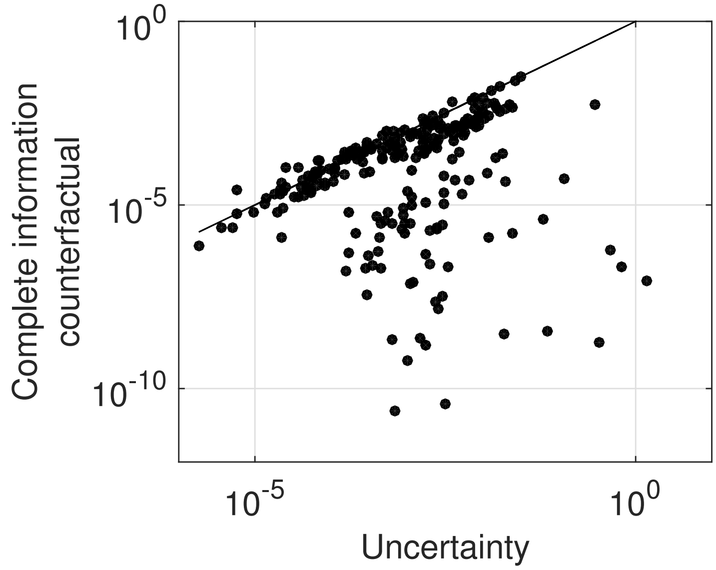
Notes: The figure depicts a scatter plot of the entry threshold estimates and the corresponding average-to-minimum ratios for observation with an estimate of the Double EMG tail parameter $\lambda_R > 1$. The threshold is not defined for $\lambda_R \leq 1$. Each dot corresponds to a product-destination-year observation. Values of the thresholds are demeaned by a corresponding estimate of μ of the Double EMG distribution.

Figure 2: Heterogeneity in endogenous selection effect, γ_{jkt}, across products.



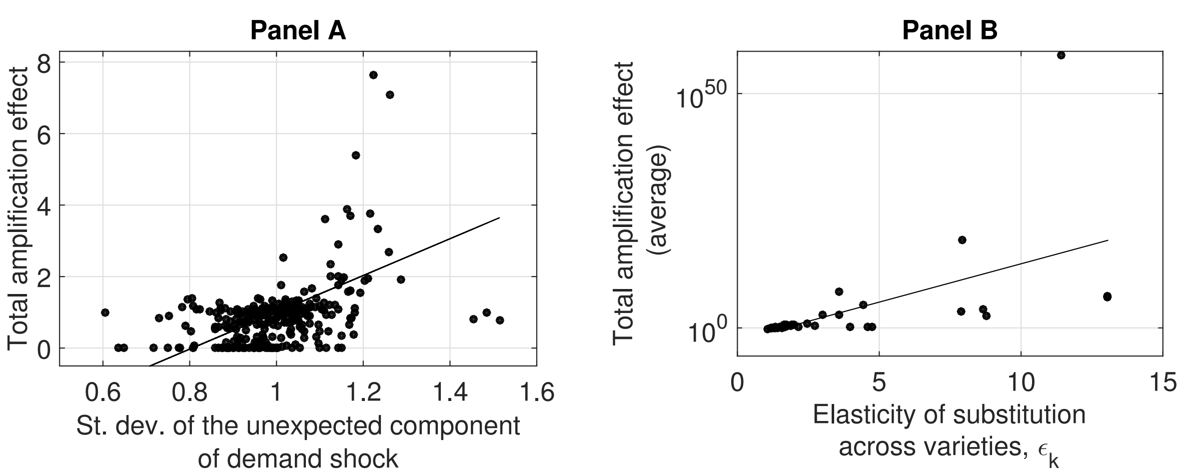
Notes: each dot computes the average across destination-year observations endogenous selection effect, γ_{ijk}, for a given product defined as a 6-digit HS code. The solid line is the OLS best fit line. For each 6-digit HS code, the elasticity of substitution across varieties is obtained from Soderbery (2015).

Figure 3: Estimates of the endogenous selection effect, γ_{jkt}.



Notes: For the ease of visual presentation this graph omits depicting counterfactual values that are below 10^{-12}. There are 30 of such observations. The solid line is the 45-degree line.

Figure 4: Total amplification effect.



Notes: In Panel A, for the ease of visual presentation this graph omits depicting counterfactual values that are above 8. There are two such observations. The solid line is the OLS best fit line. All values are normalized by the respective product averages, a product is a 6-digit HS code. Each dot corresponds to a product-destination-year observation. In Panel B, each dot computes the average across destination-year observations amplification effect for a given product defined as a 6-digit HS code. The solid line is the OLS best fit line. For each 6-digit HS code the elasticity of substitution across varieties are obtained from Soderbery (2015).

A Theoretical Appendix

In this section we provide derivations for the theoretical results in Section 2.

A.1 Environment with Complete Information

The problem of firm f selling from country i to country j in sector k consists of maximizing profit subject to the demand equation (2):

$$\pi_{fijk}(z^a_{fijk}, z^p_{fijk}) = \max_{q_{fijk}} p_{fijk} q_{fijk} - \frac{w_i \tau_{ij}}{e^{z^a_{fijk}}} q_{fijk} - w_i f_{ijk}. \tag{32}$$

The first order conditions with respect to quantity yield the optimal quantity given by

$$q_{fijk}(z^a_{fijk}, z^p_{fijk}) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k} (\tau_{ij} w_i)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} e^{\epsilon_k z^a_{fijk} + z^p_{fijk}}. \tag{33}$$

Using equations (2) and (33), a firm's optimal revenue is further given by

$$r_{fijk}(z^a_{fijk}, z^p_{fijk}) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} Y_{jk} P_{jk}^{\epsilon_k - 1} w_i^{1-\epsilon_k} \tau_{ij}^{1-\epsilon_k} e^{(\epsilon_k - 1) z^a_{fijk} + z^p_{fijk}}. \tag{34}$$

Substituting equations (34) and (33) into equation (32) yields optimal profit given by

$$\pi_{fijk}(z^a_{fijk}, z^p_{fijk}) = \frac{1}{\epsilon_k}\left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} Y_{jk} P_{jk}^{\epsilon_k - 1} w_i^{1-\epsilon_k} \tau_{ij}^{1-\epsilon_k} e^{(\epsilon_k - 1) z^a_{fijk} + z^p_{fijk}} - w_i f_{ijk}. \tag{35}$$

A firm exports if its profit from exporting is positive:

$$\begin{aligned} \pi_{fijk}(z^a_{fijk}, z^p_{fijk}) &\geq 0 \\ e^{(\epsilon_k - 1) z^a_{fijk} + z^p_{fijk}} &\geq \frac{w_i f_{ijk}}{\frac{1}{\epsilon_k}\left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} Y_{jk} P_{jk}^{\epsilon_k - 1} w_i^{1-\epsilon_k} \tau_{ij}^{1-\epsilon_k}} \end{aligned} \tag{36}$$

Denote by $B_{ijk} = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}$ and $f^\tau(\tau_{ij}) = \tau_{ij}^{1-\epsilon_k}$. Then, inequality (36) can be written as

$$e^{(\epsilon_k - 1) z^a_{fijk} + z^p_{fijk}} \geq \frac{\epsilon_k w_i f_{ijk}}{B_{ijk} \tau_{ij}^{1-\epsilon_k}}. \tag{37}$$

Denote by

$$z^{CI^*}_{ijk} = \log\left(\frac{\epsilon_k w_i f_{ijk}}{B_{ijk} f^\tau(\tau_{ij})}\right) \tag{38}$$

and substitute into inequality (37) to obtain export selection equation (6).

Trade Elasticity: Given the endogenous selection into exporting that is based on the realization of profitability shocks, the total trade flows from country i to country j in sector k, X_{ijk}, are defined as

$$X_{ijk} = J_i(1 - G_{ijk}(z_{ijk}^{CI^*})) \int_{z_{ijk}^{CI^*}}^{+\infty} r_{fijk}(z_{fijk}^{CI}) \frac{g_{ijk}(z_{fijk}^{CI})}{1 - G_{ijk}(z_{ijk}^{CI^*})} dz_{fijk}^{CI},$$

where z_{fijk}^{CI} is the decision relevant export selection shocks defined as $(\epsilon_k - 1)z_{fijk}^a + z_{fijk}^p$, and $g_{ijk}(.)$ and $G_{ijk}(.)$ are the respective probability and cumulative density functions of z_{fijk}^{CI}. J_i is the exogenous mass of entrants in country i. Substituting equation (34) for the revenue and omitting subscripts and superscripts on z_{fijk}^{CI} to ease notation yields

$$X_{ijk} = J_i(1 - G_{ijk}(z_{ijk}^{CI^*})) \int_{z_{ijk}^{CI^*}}^{+\infty} B_{ijk} f^\tau(\tau_{ij}) e^z \frac{g_{ijk}(z)}{1 - G_{ijk}(z_{fijk}^{CI^*})} dz,$$

where $B_{ijk} = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} Y_{jk} P_{jk}^{\epsilon_k - 1} w_i^{1-\epsilon_k}$, $f^\tau(\tau_{ij}) = \tau_{ij}^{1-\epsilon_k}$. Differentiating with respect to τ_{ij} yields:

$$\frac{\partial X_{ijk}}{\partial \tau_{ij}} = \frac{\partial f^\tau(\tau_{ij})}{\partial \tau_{ij}} J_i \int_{z_{ijk}^{CI^*}}^{+\infty} B_{ijk} e^z g_{ijk}(z) dz - J_i \frac{\partial z_{ijk}^{CI^*}}{\partial \tau_{ij}} B_{ijk} f^\tau(\tau_{ij}) e^{z_{ijk}^{CI^*}} g_{ijk}(z_{ijk}^{CI^*}). \quad (39)$$

Differentiate equation (38) with respect to τ_{ij} to obtain

$$\frac{\partial z_{ijk}^{CI^*}}{\partial \tau_{ij}} = -\frac{\partial \log f^\tau(\tau_{ij})}{\partial \tau_{ij}}. \quad (40)$$

Substituting equation (40) into equation (39) yields

$$\begin{aligned}
\frac{\partial X_{ijk}}{\partial \tau_{ij}} &= \frac{\partial f^\tau(\tau_{ij})}{\partial \tau_{ij}} J_i \int_{z_{ijk}^{CI^*}}^{+\infty} B_{ijk} e^z g_{ijk}(z) dz + J_i \frac{\partial \log f^\tau(\tau_{ij})}{\partial \tau_{ij}} B_{ijk} f^\tau(\tau_{ij}) e^{z_{ijk}^{CI^*}} g_{ijk}(z_{ijk}^{CI^*}) = \\
&= \frac{\partial \log f^\tau(\tau_{ij})}{\partial \log \tau_{ij}} \left(\tau_{ijk} J_i \int_{z_{ijk}^{CI^*}}^{+\infty} B_{ijk} e^z f^\tau(\tau_{ijk}) g_{ijk}(z) dz + \tau_{ijk} J_i B_{ijk} f^\tau(\tau_{ij}) e^{z_{ijk}^{CI^*}} g_{ijk}(z_{ijk}^{CI^*}) \right) = \\
&= \frac{\partial \log f^\tau(\tau_{ij})}{\partial \log \tau_{ij}} \left(\tau_{ijk}^{-1} X_{ijk} + \tau_{ijk}^{-1} X_{ijk} \frac{e^{z_{ijk}^{CI^*}} g_{ijk}(z_{ijk}^{CI^*})}{\int_{z_{ijk}^{CI^*}}^{+\infty} e^z g_{ijk}(z) dz} \right).
\end{aligned}$$

Hence,

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ij}} = \frac{\partial \log f^\tau(\tau_{ij})}{\partial \log \tau_{ij}} \left(1 + \frac{e^{z_{ijk}^{CI^*}} g_{ijk}(z_{ijk}^{CI^*})}{\int_{z_{ijk}^{CI^*}}^{+\infty} e^z g_{ijk}(z) dz} \right).$$

A.2 Environment with Uncertainty

The problem of firm f selling from country i to country j in sector k consists of maximizing the *expected* profit subject to the demand equation (2):

$$E_{z^p_{fijk}|z^a_{fijk}}[\pi_{fijk}(z^a_{fijk}, z^p_{fijk})] = \max_{q_{fijk}} E_{z^p_{fijk}} \left(p_{fijk}q_{fijk} - \frac{w_i \tau_{ij}}{e^{z^a_{fijk}}} q_{fijk} \right) - w_i f_{ijk}. \quad (41)$$

The first order conditions with respect to quantity yield the optimal quantity given by

$$q_{fijk}(z^a_{fijk}) \quad = \quad \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k} w_i^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \tau_{ij}^{-\epsilon_k} e^{\epsilon_k z^a_{fijk}} \left(E_{z^p_{fijk}|z^a_{fijk}} \left(e^{\frac{z^p_{fijk}}{\epsilon_k}} \right) \right)^{\epsilon_k}. \quad (42)$$

Using equations (2) and (42), a firm's realized revenue is further given by

$$r_{fijk}(z^a_{fijk}, z^p_{fijk}) \quad = \quad \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \tau_{ij}^{1-\epsilon_k} e^{\frac{z^p_{fijk}}{\epsilon_k} + (\epsilon_k - 1) z^a_{fijk}} \left(E_{z^p_{fijk}|z^a_{fijk}} \left(e^{\frac{z^p_{fijk}}{\epsilon_k}} \right) \right)^{\epsilon_k - 1}. \quad (43)$$

Substituting equations (43) and (42) into equation (41) yields optimal expected profit given by

$$\begin{aligned} E_{z^p_{fijk}|z^a_{fijk}}[\pi_{fijk}(z^a_{fijk}, z^p_{fijk})] \quad = \quad & \frac{(\epsilon_k - 1)^{\epsilon_k - 1}}{\epsilon_k^{\epsilon_k}} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \tau_{ij}^{1-\epsilon_k} e^{(\epsilon_k - 1) z^a_{fijk}} \left(E_{z^p_{fijk}|z^a_{fijk}} \left(e^{\frac{z^p_{fijk}}{\epsilon_k}} \right) \right)^{\epsilon_k} - \\ & - w_i f_{ijk}. \end{aligned} \quad (44)$$

A firm exports if its expected profit from exporting is positive:

$$\begin{aligned} E_{z^p_{fijk}|z^a_{fijk}}[\pi_{fijk}(z^a_{fijk}, z^p_{fijk})] \quad & \geq \quad 0 \\ e^{(\epsilon_k - 1) z^a_{fijk}} \left(E_{z^p_{fijk}} \left(e^{\frac{z^p_{fijk}}{\epsilon_k}} \right) \right)^{\epsilon_k} \quad & \geq \quad \frac{w_i f_{ijk}}{\frac{(\epsilon_k - 1)^{\epsilon_k - 1}}{\epsilon_k^{\epsilon_k}} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \tau_{ij}^{1-\epsilon_k}}. \end{aligned} \quad (45)$$

Substituting equation (38) into inequality (45) yields

$$e^{(\epsilon_k - 1) z^a_{fijk}} \left(E_{z^p_{fijk}} \left(e^{\frac{z^p_{fijk}}{\epsilon_k}} \right) \right)^{\epsilon_k} \quad \geq \quad e^{z^{CI^*}_{ijk}}.$$

Trade Elasticity: Using the orthogonal projection of z^p_{fijk} on z^a_{fijk} in equation (12), export revenue (43) can be written as

$$r_{fijk}(z^a_{fijk}, \upsilon_{fijk}) \;=\; \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \tau_{ij}^{1-\epsilon_k} \left(E\left(e^{\frac{\upsilon_{fijk}}{\epsilon_k}}\right)\right)^{\epsilon_k - 1} e^{E(z^p_{fijk}|z^a_{fijk}) + (\epsilon_k - 1) z^a_{fijk} + \frac{\upsilon_{fijk}}{\epsilon_k}}. \tag{46}$$

Using equations (14) and (15), the total trade flows from country i to country j in sector k, X_{ijk}, can be written as

$$\begin{aligned} X_{ijk} &= J_i(1 - G_{ijk}(z^{U*}_{ijk})) \int_{z^{U*}_{ijk}}^{+\infty} \left[\int_{-\infty}^{+\infty} r_{fijk}(z^U_{fijk}, \upsilon_{fijk}) g^{\upsilon}_{ijk}(\upsilon_{fijk}) d\upsilon_{fijk}\right] \frac{g_{ijk}(z^U_{fijk})}{1 - G_{ijk}(z^{U*}_{ijk})} dz^U_{fijk} = \\ &= J_i(1 - G_{ijk}(z^{U*}_{ijk})) \int_{z^{U*}_{ijk}}^{+\infty} B_{ijk} f^{\tau}(\tau_{ij}) e^{z^U_{fijk}} \frac{g_{ijk}(z^U_{fijk})}{1 - G_{ijk}(z^{U*}_{ijk})} dz^U_{fijk}, \end{aligned} \tag{47}$$

where z^U_{fijk} is the decision relevant export selection shocks defined as $E(z^p_{fijk}|z^a_{fijk}) + (\epsilon_k - 1) z^a_{fijk}$, and $g_{ijk}(.)$ and $G_{ijk}(.)$ are the respective probability and cumulative density functions of z^U_{fijk}, z^{U*}_{fijk} is the export selection threshold, $B_{ijk} = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \left(E\left(e^{\frac{\upsilon_{fijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}$, $f^{\tau}(\tau_{ij}) = \tau_{ij}^{1-\epsilon_k}$.

Differentiation equation (47) with respect to τ_{ij} and omitting subscripts and superscripts on z^U_{fijk} to ease notation yields:

$$\frac{\partial X_{ijk}}{\partial \tau_{ij}} \;=\; \frac{\partial f^{\tau}(\tau_{ij})}{\partial \tau_{ij}} J_i \int_{z^{U*}_{ijk}}^{+\infty} B_{ijk} e^z g_{ijk}(z) dz - J_i \frac{\partial z^{U*}_{ijk}}{\partial \tau_{ij}} B_{ijk} f^{\tau}(\tau_{ij}) e^{z^{U*}_{ijk}} g_{ijk}(z^{U*}_{ijk}). \tag{48}$$

Differentiate equation (15) with respect to τ_{ij} to obtain

$$\frac{\partial z^{U*}_{ijk}}{\partial \tau_{ij}} = -\frac{\partial \log f^{\tau}(\tau_{ij})}{\partial \tau_{ij}}. \tag{49}$$

Substituting equation (49) into equation (48) yields

$$\begin{aligned} \frac{\partial X_{ijk}}{\partial \tau_{ij}} &= \frac{\partial f^{\tau}(\tau_{ij})}{\partial \tau_{ij}} J_i \int_{z^{U*}_{ijk}}^{+\infty} B_{ijk} e^z g_{ijk}(z) dz + J_i \frac{\partial \log f^{\tau}(\tau_{ij})}{\partial \tau_{ij}} B_{ijk} f^{\tau}(\tau_{ij}) e^{z^{U*}_{ijk}} g_{ijk}(z^{U*}_{ijk}) = \\ &= \frac{\partial \log f^{\tau}(\tau_{ij})}{\partial \tau_{ij}} \left(\tau_{ij}^{-1} X_{ijk} + \tau_{ij}^{-1} X_{ijk} \frac{e^{z^{U*}_{ijk}} g_{ijk}(z^{U*}_{ijk})}{\int_{z^{U*}_{ijk}}^{+\infty} e^z g_{ijk}(z) dz}\right) \end{aligned}$$

Hence,

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ij}} \;=\; \frac{\partial \log f^{\tau}(\tau_{ij})}{\partial \log \tau_{ij}} \left(1 + \frac{e^{z^{U*}_{ijk}} g_{ijk}(z^{U*}_{ijk})}{\int_{z^{U*}_{ijk}}^{+\infty} e^z g_{ijk}(z) dz}\right).$$

B Properties of γ

Consider function $\gamma(x)$ defined as

$$\gamma(x) = \frac{e^x g(x)}{\int_x^{+\infty} e^u g(u) du},$$

where $g(x)$ is a probability density function defined on $x \in \mathbb{R}$. $\gamma(x)$ can further be expressed as a hazard rate

$$\gamma(x) = \frac{h(x)}{1 - H(x)}, \tag{50}$$

where the probability density function $h(x)$ is defined as

$$h(x) \equiv \frac{e^x g(x)}{\int_{-\infty}^{+\infty} e^u g(u) du}, \tag{51}$$

and the corresponding cumulative distribution function is defined as

$$H(x) = \frac{\int_{-\infty}^{x} e^u g(u) du}{\int_{-\infty}^{+\infty} e^u g(u) du}. \tag{52}$$

Assume $g(x)$ satisfies Assumption 1 below.

Assumption 1 (A1) *The probability density function $g(x)$ has the following properties:*

(i) $E(e^x) \equiv \int_{-\infty}^{+\infty} e^x g(x) dx$ exists and is finite, and

(ii) the function $\log \left(\int_x^{+\infty} e^u g(u) du \right)$ is concave in z.

Assumption (i) ensures that the probability and the cumulative distribution functions $h(.)$ and $H(.)$ are well defined. Assumption (ii) ensures that function $\gamma(x)$ is a monotonically increasing function of x, as we show below. Assumption (ii) states that the log of the conditional expectation of an exponential function is a concave function of the threshold value. Intuitively, this assumption requires that the upper tail of the distribution $g(x)$ does not have too much mass.[27] Without such a restriction, total sales of marginal firms relative to average sales could become very small as the threshold increases, and the extensive margin elasticity, $\gamma(x)$, might not be monotonically increasing in x. The standard distributional assumptions made in the literature all meet this requirement.[28]

Proposition 1 below establishes two properties of function $\gamma(x)$ underlying Result 1 and Result 2.

[27] Heavy-tailed distributions, e.g. distributions that violate assumption (i), are sometimes said to have the property of log-convexity.

[28] For example, the Normal distribution, Exponential distribution (with an appropriate restriction on the scale parameter) and the Double Exponentially Modified Gaussian distribution all satisfy this requirement.

Proposition 1 *Let $g(x)$ be a probability density function satisfying A1. Then the following hold.*

(i) $\gamma(x) \equiv [e^x g(x)]/\int_x^{+\infty} e^u g(u)du$ *is an increasing function of* x.

(ii) Let $\tilde{g}(x)$ *be a mean preserving spread of* $g(x)$*, with an respectively defined* $\tilde{\gamma}(x)$*. Then* $\gamma(x)$ *and* $\tilde{\gamma}(x)$ *satisfy the single crossing property. That is, there exists* x^* *such that* $\tilde{\gamma}(x) \leq \gamma(x)$ *for all* $x \geq x^*$*, and* $\tilde{\gamma}(x) \geq \gamma(x)$ *for all* $x \leq x^*$.

Proof of Proposition 1

Part (i) First, define $h(x) = (e^x g(x))/E$, where $E = \int_{-\infty}^{+\infty} e^u g(u)du$. Notice that $h(x)$ is positive for all x and that $\int_{-\infty}^{+\infty} h(x)dx = 1$. Hence, $h(x)$ is a probability density function. The corresponding cumulative density function is given by $H(x) = \int_{-\infty}^{x} e^u g(u)du/E$. The corresponding survival function is given by $1 - H(x) = \int_x^{+\infty} e^u g(u)du/E$.

Next, function $\gamma(x)$ can then be written as

$$\gamma(x) \quad = \quad \frac{e^x g(x)}{\int_x^{+\infty} e^u g(u)du} = \frac{h(x)}{1 - H(x)}.$$

Hence, $\gamma(x)$ is a hazard rate associated with the distribution $H(x)$. By Theorem 10 in Rinne (2014), the hazard rate $\gamma(x)$ is monotonically increasing in x if and only if its logarithmic survival function, $\log(1 - H(x))$, is concave. Notice that by part (ii) of A1, $\log(1 - H(x))$ is a concave function of x. Hence, $\gamma(x)$ is increasing in x. For completeness, we reproduce the proof of this result below.

Notice that

$$\gamma(x) = -\frac{d\log(1 - H(x))}{dx}.$$

Hence,

$$\frac{d\gamma(x)}{dx} = -\frac{d^2\log(1 - H(x))}{dx^2}.$$

Since $\log(1 - H(x))$ is a concave function of x, $d^2\log(1 - H(x))/dx^2 < 0$. Therefore, $d\gamma(x)/dx > 0$.

Part (ii) Function $\tilde{\gamma}(x)$ is given by

$$\tilde{\gamma}(x) \quad = \quad \frac{e^x \tilde{g}(x)}{\int_x^{+\infty} e^u \tilde{g}(u)du} = \frac{\tilde{h}(x)}{1 - \tilde{H}(x)},$$

where $\tilde{g}(.)$ is a mean preserving spread of $g(.)$, $\tilde{h}(x) = [e^x \tilde{g}(x)]/\int_{-\infty}^{+\infty} e^u \tilde{g}(u)du$, and $\tilde{H}(x)$ is the corresponding cumulative distribution function.

$\gamma(x) > \tilde{\gamma}(x)$ if and only if $H(x) > \tilde{H}(x)$ as follows for the following set of equivalent inequalities:

$$
\begin{aligned}
\gamma(x) = -\frac{d\log(1-H(x))}{dx} &> -\frac{d\log(1-\tilde{H}(x))}{dx} = \tilde{\gamma}(x) \\
d\log(1-H(x)) &< d\log(1-\tilde{H}(x)) \\
\int d\log(1-H(x)) &< \int d\log(1-\tilde{H}(x)) \\
\log(1-H(x)) &< \log(1-\tilde{H}(x)) \\
H(x) &> \tilde{H}(x).
\end{aligned}
$$

We will now show in three steps that $H(x)$ crosses $\tilde{H}(x)$ once from below, and therefore there exists x^* such that $H(x) > \tilde{H}(x)$ holds for $x > x^*$, and therefore (ii) holds.

Step 1: Denote by X and $\tilde{X}$ random variables distributed according to $g(x)$ and $\tilde{g}(x)$ respectively. Since $\tilde{g}(x)$ is a mean preserving spread of $g(x)$, it holds that $\tilde{X} = X + \hat{X}$, where $\hat{X}$ is distributed according to $\hat{g}(x)$ with mean zero, and $\hat{X}$ is independent from X. Hence, $\tilde{g}(.)$ is a convolution of $g(.)$ and $\hat{g}(.)$ and can be written as

$$
\tilde{g}(x) = \int_{-\infty}^{+\infty} g(x-u)\hat{g}(u)du.
$$

Step 2: Denote by X^h, $\tilde{X}^h$, $\hat{X}^h$ random variables distributed according to $h(x)$, $\tilde{h}(x)$, and $\hat{h}(x)$ respectively, where $\hat{h}(x) = [e^x\hat{g}(x)]/\int_{-\infty}^{+\infty} e^x\hat{g}(x)dx$. Similarly, it can be show that $\tilde{h}(.)$ is a convolution of $h(.)$ and $\hat{h}(.)$:

$$
\begin{aligned}
\int_{-\infty}^{+\infty} h(x-u)\hat{h}(u)du &= \frac{\int_{-\infty}^{+\infty} e^{x-u}g(x-u)e^u\hat{g}(u)du}{\left[\int_{-\infty}^{+\infty} e^x g(x)dx\right]\cdot\left[\int_{-\infty}^{+\infty} e^x\hat{g}(x)dx\right]} = \\
&= \frac{\int_{-\infty}^{+\infty} e^x g(x-u)\hat{g}(u)du}{\left[\int_{-\infty}^{+\infty} e^x g(x)dx\right]\cdot\left[\int_{-\infty}^{+\infty} e^x\hat{g}(x)dx\right]} = \\
&= \frac{e^x\tilde{g}(x)}{\left[\int_{-\infty}^{+\infty} e^x g(x)dx\right]\cdot\left[\int_{-\infty}^{+\infty} e^x\hat{g}(x)dx\right]} = \tilde{h}(x).
\end{aligned}
$$

Thus, it hold that $\tilde{X}^h = X^h + \hat{X}^h$, where $\tilde{X}^h$ and $\hat{X}^h$ are independent.

Step 3: Consider a random variable $\bar{X} = X^h + \hat{X}^h - E(\hat{X}^h)$ with the cumulative distribution function denoted by $\bar{H}(x)$. $\bar{X}$ is a mean preserving spread of X^h and therefore the two corresponding cumulative distribution functions satisfy the single-crossing property whereby $H(x) = \bar{H}(x)$ if $x = E(X^h)$; $H(x) < \bar{H}(x)$ for $x < E(X^h)$, and $H(x) > \bar{H}(x)$ for $x > E(X^h)$.

Next, notice that $\tilde{X}^h = \bar{X} + E(\hat{X}^h)$. Therefore the cumulative distribution function of

$\tilde{X}^h$ is a shift of the cumulative distribution function of $\bar{X}$ along the x-axis, namely $\tilde{H}(x) = \bar{H}(x - E(\hat{X}^h))$. Hence $\tilde{H}(x)$ preserves the same single-crossing property with respect to $H(x)$. Namely $\exists x^*$ that that $H(x) = \tilde{H}(x)$ if $x = x^*$; $H(x) < \tilde{H}(x)$ for $x < x^*$, and $H(x) > \tilde{H}(x)$ for $x > x^*$. ■

C Counterfactual Analysis

In Section 6 we use the structure of the model to simulate counterfactual trade elasticities under complete information and compare counterfactual estimates to the baseline estimates to learn about how uncertainty impacts trade elasticities. Here we describe how we obtain counterfactual values of export selection thresholds and counterfactual values of the distribution of export selection shocks.

C.1 Counterfactual Export Selection Thresholds

Equation (15) establishes a relationship between export selection thresholds in the two information environments. Applying the assumption that υ_{fjkt} are i.i.d. $N[0, V(\upsilon_{fjkt})]$ yields

$$z_{jkt}^{CI^*} = z_{jkt}^{U^*} + \frac{1}{2}\frac{V(\upsilon_{fjkt})}{\epsilon_k}. \tag{53}$$

Notice from equation (27) that $V(\upsilon_{fjkt}) = \epsilon_k^2 V(u_{fjkt})$. Therefore, we recover the variance of the unexpected component of the demand shocks, $V(\upsilon_{fjkt})$, from the variance of the residual, u_{fjkt}, in equation (28).

Notice that our quantification method requires assuming values for the elasticities of substitution across varieties, ϵ_k. We proceed by using the values of the elasticities of substitution across varieties from Soderbery (2015), which refines estimates in Feenstra (1994) and Broda and Weinstein (2006).[29] In principle, these estimates for the elasticity of substitution across varieties are estimated under the assumption of complete information and could require an alternative identification assumption to accommodate incomplete information. However, in order to facilitate the comparison between environments with complete and incomplete information, we choose to hold the elasticities of substitution constant at their complete information values. This allows us to cleanly quantify differences in the trade elasticities across information environments that arise directly from the differences in the economic mechanism of selection.

[29] Soderbery (2015) estimates the elasticity of substitution values at the HS-10 digit level using the U.S. import data. To use Soderbery (2015) estimates aggregate the elasticities to the HS-6 digit level equally weighing corresponding HS-10 sub-categories for each HS-6 category.

C.2 Counterfactual Distribution of Export Selection Shocks

Equation (21) establishes a relationship between export selection thresholds in the two information environments. The export selection shock under complete information, z^{CI}_{fjkt} is a mean preserving spread of the selection shock under uncertainty, z^{U}_{fjkt}, where the unexpected component of the demand shock, υ_{fjkt}, is i.i.d. $N[0, V(\upsilon_{fjkt})]$ Therefore, z^{CI}_{fjkt} follows a Double EMG distribution of z^{U}_{fjkt}, with the mean of the Normal component increased by $V(\upsilon_{fjkt})$. As discussed in Section C.1, $V(\upsilon_{fjkt})$ is recovered from the variance of the residual, u_{fjkt}, in equation (28).

D Robustness

A potential concern in our analysis is a measurement error in the quantity data. A classical measurement error would have an ambiguous effect on our baseline and counterfactual results due to an ambiguous effect of the dispersion of export selection shocks on partial trade elasticities.

First, recall that in the baseline calculations, we recover the distribution of the export selection shocks from the distribution of $\hat{z}^{U}_{fjkt} = \hat{\beta}_{jkt}\hat{\zeta}^{q}_{fjkt}$, where $\hat{\zeta}^{q}_{fjkt}$ is the residual from the log-quantity regression

$$\log q_{fjkt} \quad = \quad FE^{q}_{jkt} + \underbrace{\epsilon_k z^{a}_{fjkt} + E(z^{p}_{fjkt}|z^{a}_{fjkt})}_{\zeta^{q}_{fjkt}} \tag{54}$$

A measurement error in the quantity data will increase the dispersion of the error, and therefore the dispersion of the recovered export selection shocks $\hat{z}^{U}_{fjkt}$. As stated in Result 2, the dispersion of a selection shock has an ambiguous effect on the partial trade elasticity.

Further, to calculate counterfactual trade elasticities, we recover the variance of the unexpected component of the demand shock as the variance of the residual in

$$\zeta^{r}_{fjkt} = \beta_{jkt}\zeta^{q}_{fjkt} + u_{fjkt}. \tag{55}$$

A measurement error in quantity data which amplifies the variance of ζ^{q}_{fjkt} will therefore simultaneously attenuate the variance of the unexpected component of the demand shock, which similarly has an ambiguous effect on the counterfactual trade elasticities.

To address concerns with the measurement error, we perform a number of robustness checks. First, all our analysis is conducted after removing severe outliers from the data. Namely in each product-destination-year observation, we drop firm-product-destination-year export sales or export quantity values when they fall below the first or above the 99th percentile of their respective distributions.[30] Removing those helps to reduces the dispersion in the data arising from a severe measurement error among extreme observations.

Second, including an extensive set of fixed effects in a regression of the type presented in equation (24) helps to purge variation in the data most likely to be impacted by a measurement error. In our baseline estimation we include product-destination-year fixed effects that help to account for differences among goods shipped to different destinations in a given year.

Additionally, we perform a robustness check by including an extra set of *firm-product level fixed effects* in the log-quantity regression (24). This helps to alleviate concerns arising from firms potentially shipping different varieties of goods belonging to a given product category. Tables D1 and D2, and Figures D1, D2 and D3 below replicate Tables 3 and 4, and Figures 2, 3 and 4 respectively from the revised manuscript for this robustness check. The qualitative and quantitative results remain largely unchanged. The average values of the endogenous component of the partial trade elasticity changes from the baseline value of 0.02 to 0.01, and the average partial trade elasticity changes from 3.44 to 3.69. (Comparisons are based on Table 3 in the submitted manuscript and Table D1 below.) The average amplification effect remains unchanged at the value of 1.01. (Comparisons are based on Panel C in Table 4 in the submitted manuscript and Panel C in Table D2 below.) Uncertainty increases trade elasticities in about eighty percent of observation in our baseline calculations and in about seventy four percent of observation in this robustness check, with amplification effects below unity similarly concentrated among industries with low elasticities of substitution. (Comparisons are based on panel B in Figure 4 in the submitted manuscript and panel B in Figure D3 below.)

Finally, we focus analysis on products which are less likely to be subjected to a measurement error. Recall that we conduct our analysis at the product-destination-year level, where a product corresponds to a 6-digit HS code. The original data are available at a finer level of disaggregation, 8-digits, where the last two digits are a country specific addition to a standard 6-digit HS code introduced to allow for greater product differentiation where such is needed. For each product-destination-year observation, we therefore look at the number of 8-digit sub-codes within the given hs-6 digit code. The fewer sub-codes there are, the more likely it is that the given exported products are more comparable to each other, and therefore such data will be less likely subjected to a measurement error in the quantity data. Out of 288 product-destination-year observations in our baseline sample, 174 observations have a single 8-digit code corresponding to the given 6-digit HS code. We reproduce our results using the sample of these 174 observations. Tables D3 and D4, and Figures D4, D5

[30] See Manova and Zhang (2012).

and D6 below replicate Tables 3 and 4, and Figures 2, 3 and 4 respectively from the revised manuscript for this robustness check. Qualitative and quantitative results remain robust. The average values of the endogenous component of the partial trade elasticity remains unchanged at the value of 0.02, and the average partial trade elasticity changes from 3.44 to 2.67. (Comparisons are based on Table 3 in the submitted manuscript and Table D3 below.) The average amplification effect changes from the baseline value of 1.01 to 1.02. (Comparisons are based on Panel C in Table 4 in the submitted manuscript and Panel C in Table D4 below.) Uncertainty increases trade elasticities in about eighty percent of observation in our baseline calculations and in about seventy three percent of observation in this robustness check, with amplification effects below unity similarly concentrated among industries with low elasticities of substitution. (Comparisons are based on panel B in Figure 4 in the submitted manuscript and panel B in Figure D3 below.)

Table D1: Trade elasticity estimates under uncertainty (firm-product fixed effects included).

Measure	Mean	Std. Dev.
Endogenous selection, γ_{ijk}	0.01	0.04
Total partial trade elasticity, $\partial \log X_{ijk}/\partial \log \tau_{ij}$	3.39	3.69

Notes: the summary statistics are reported across 281 product-destination-year observations for which estimates of the Double EMG right tail parameter are greater than unity. The elasticities are not defined otherwise. A product is defined as a 6-digit HS code.

Table D2: Counterfactual trade elasticity estimates under complete information (firm-product fixed effects included).

	Endogenous Selection γ_{ijk}		Partial Trade Elasticity, $\partial \log X_{ijk}/\partial \log \tau_{ij}$	
Measure	Mean	Std. Dev.	Mean	Std. Dev.
Panel A: selection effect of uncertainty				
Selection effect	3.07	22.84	22.08	174.57
Amplification due to selection	0.02	0.06	0.63	0.29
Panel B: dispersion effect of uncertainty				
Dispersion effect	0.0006	0.002	3.35	3.64
Amplification due to dispersion	$1.9 \cdot 10^{117}$	$3.1 \cdot 10^{118}$	1.01	0.04
Panel C: total effect of uncertainty				
Total effect	0.003	0.008	3.35	3.64
Total amplification effect	$1.0 \cdot 10^{107}$	$1.7 \cdot 10^{108}$	1.01	0.04

Notes: all summary statistics are reported across 281 product-destination-year observations for which estimates of the Double EMG right tail parameter are greater than unity. The elasticities are not defined otherwise. The amplification effect is computed as the ratio of the baseline estimate of trade elasticity under uncerinaty realtive to its counterfactual value under complete information for the indicated counterfactual scenario.

Table D3: Trade elasticity estimates under uncertainty (single 8-digit subcode).

Measure	Mean	Std. Dev.
Endogenous selection, γ_{ijk}	0.02	0.13
Total partial trade elasticity, $\partial \log X_{ijk}/\partial \log \tau_{ij}$	2.67	3.08

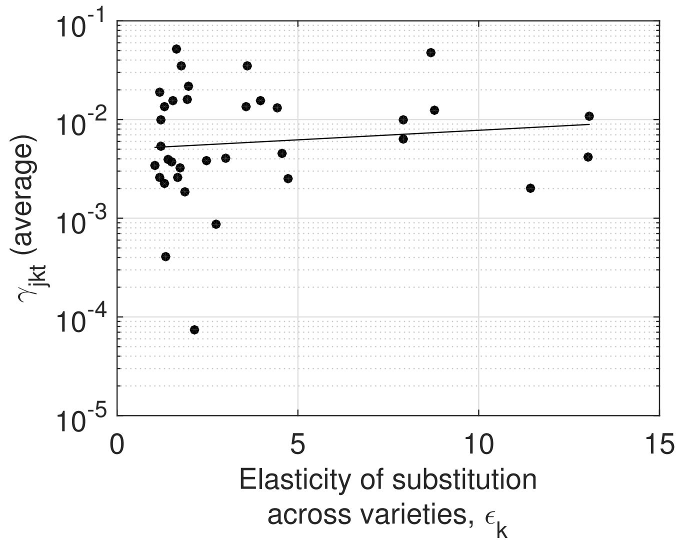
Notes: the summary statistics are reported across 174 product-destination-year observations for which estimates of the Double EMG right tail parameter are greater than unity. The elasticities are not defined otherwise. A product is defined as a 6-digit HS code.

Table D4: Counterfactual trade elasticity estimates under complete information (single 8-digit subcode).

	Endogenous Selection γ_{ijk}		Partial Trade Elasticity, $\partial \log X_{ijk}/\partial \log \tau_{ij}$	
Measure	Mean	Std. Dev.	Mean	Std. Dev.
Panel A: selection effect of uncertainty				
Selection effect	0.85	6.49	4.81	17.00
Amplification due to selection	0.38	0.25	0.96	0.14
Panel B: dispersion effect of uncertainty				
Dispersion effect	0.0006	0.002	2.60	3.00
Amplification due to dispersion	$3.5 \cdot 10^8$	$4.4 \cdot 10^9$	1.02	0.13
Panel C: total effect of uncertainty				
Total	0.001	0.004	2.60	3.00
Total amplification effect	$1.4 \cdot 10^6$	$1.4 \cdot 10^7$	1.02	0.13

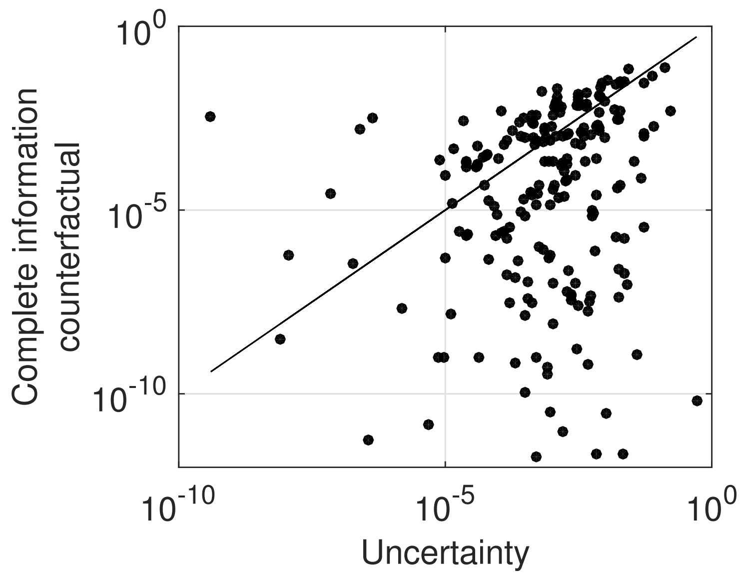
Notes: all summary statistics are reported across 174 product-destination-year observations for which estimates of the Double EMG right tail parameter are greater than unity. The elasticities are not defined otherwise. The amplification effect is computed as the ratio of the baseline estimate of trade elasticity under uncerinaty realtive to its counterfactual value under complete information for the indicated counterfactual scenario.

Figure D1: Heterogeneity in endogenous selection effect, γ_{jkt}, across products (firm-product fixed effects included).



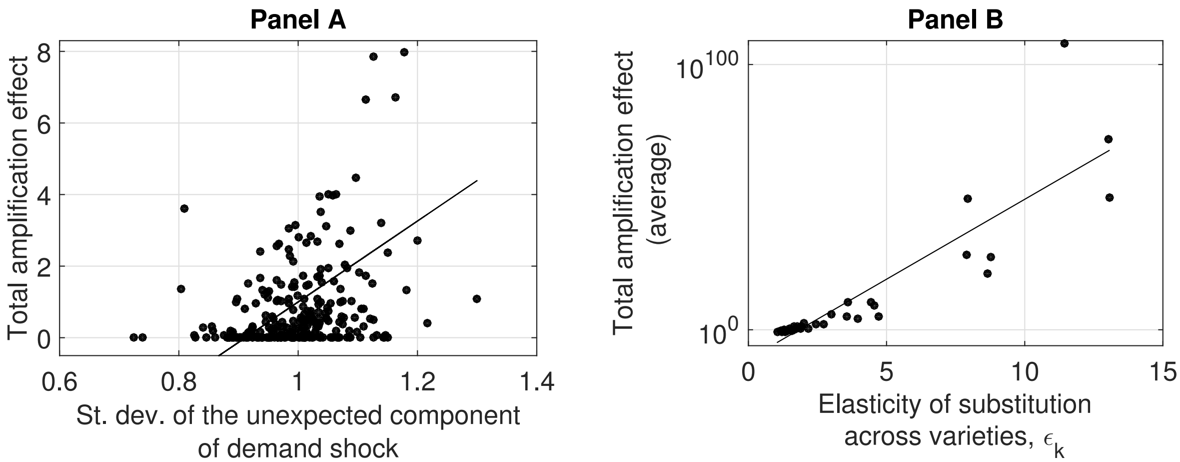
Notes: each dot computes the average across destination-year observations endogenous selection effect, γ_{ijk}, for a given product defined as a 6-digit HS code. The solid line is the OLS best fit line. For each 6-digit HS code, the elasticity of substitution across varieties is obtained from Soderbery (2015).

Figure D2: Estimates of the endogenous selection effect, γ_{jkt} (firm-product fixed effects included).



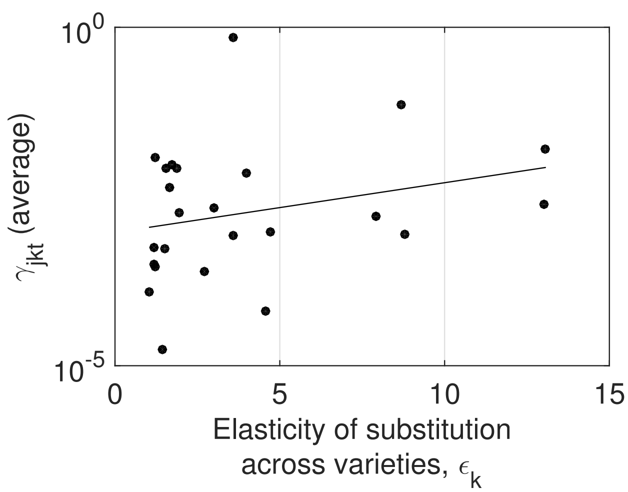
Notes: For the ease of visual presentation this graph omits depicting counterfactual values that are below 10^{-12}. There are 77 of such observations. The solid line is the 45-degree line.

Figure D3: Total amplification effect (firm-product fixed effects included).



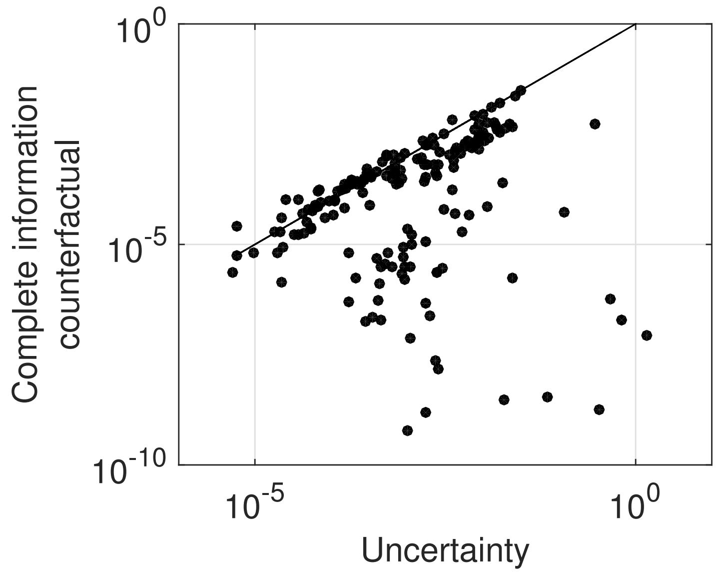
Notes: In Panel A, for the ease of visual presentation this graph omits depicting counterfactual values that are above 8. There are five such observations. The solid line is the OLS best fit line. All values are normalized by the respective product averages, a product is a 6-digit HS code. Each dot corresponds to a product-destination-year observation. In Panel B, each dot computes the average across destination-year observations amplification effect for a given product defined as a 6-digit HS code. The solid line is the OLS best fit line. For each 6-digit HS code the elasticity of substitution across varieties are obtained from Soderbery (2015).

Figure D4: Heterogeneity in endogenous selection effect, γ_{jkt}, across products (single 8-digit subcode).



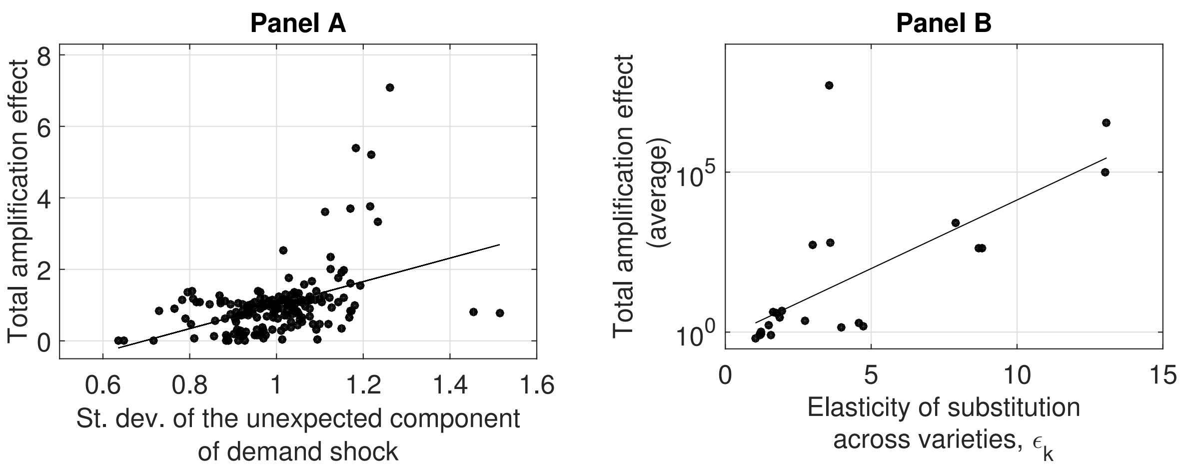
Notes: each dot computes the average across destination-year observations endogenous selection effect, γ_{ijk}, for a given product defined as a 6-digit HS code. The solid line is the OLS best fit line. For each 6-digit HS code, the elasticity of substitution across varieties is obtained from Soderbery (2015).

Figure D5: Estimates of the endogenous selection effect, γ_{jkt} (single 8-digit subcode).



Notes: The solid line is the 45-degree line.

Figure D6: Total amplification effect (single 8-digit subcode).



Notes: In Panel A, the solid line is the OLS best fit line. All values are normalized by the respective product averages, a product is a 6-digit HS code. Each dot corresponds to a product-destination-year observation. In Panel B, each dot computes the average across destination-year observations amplification effect for a given product defined as a 6-digit HS code. The solid line is the OLS best fit line. For each 6-digit HS code the elasticity of substitution across varieties are obtained from Soderbery (2015).