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# Substitution Bias and Fixed-Weight Price Indices in Time-Dependent Pricing Models

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**Abstract**

This paper compares inflation in true price indices to inflation in fixed-weight price indices. We construct model-based inflation measures in time-dependent pricing models that are analogous to measures of inflation in the data, e.g., the Consumer Price Index. In the standard new Keynesian model, when inflation rises rapidly, the differences between inflation in those indices and true price indices are increasing in the degree of price stickiness and the elasticity of substitution across goods. For commonly used parameter values, those differences are large and persistent for increases in inflation of the size seen after 2020 in the U.S.

*The analysis and conclusions set forth are those of the authors and do not indicate concurrence by the Board of Governors or anyone else associated with the Federal Reserve System. Christiano: Northwestern University and NBER, l-christiano@northwestern.edu, Eichenbaum: Northwestern University and NBER, eich@northwestern.edu, Johannsen: Federal Reserve Board: benjamin.k.johannsen@frb.gov. A previous version of this paper was circulated under the title "Measured Inflation and the New-Keynesian Model."

# 1 Introduction

This paper compares true price indices to fixed-weight price indices.[1] True price indices fully reflect consumers' substitution across goods in response to relative price changes. Fixed-weight indices, e.g., Laspeyres indices, do not. Understanding the difference between true price indices and fixed-weight price indices is important for empirically understanding the welfare costs of shocks to the economy and for assessing the empirical plausibility of macro models.

Researchers often compare model-based inflation in the true price index with data-based inflation measures constructed from fixed-weight indices, like the Consumer Price Index (CPI) (see, for example, Christiano et al. (2011), Nakamura et al. (2018), and Blanco et al. (2024)).[2] The first part of this paper studies this mismatch using a relatively general specification of consumer preferences over a fixed continuum of differentiated goods. We assume those preferences are monotone, strictly quasi-concave, and linearly homogeneous in the continuum of underlying consumption goods. We show that under two conditions, the difference between inflation in the true price index and inflation in fixed-weight indices can be unbounded. The first condition is that any strict subset of goods is inessential in the sense that a consumer can reach positive consumption levels without purchasing those goods. The second condition is that a subset of firms cannot change prices. This condition is satisfied in time-dependent pricing models.[3] When these conditions are satisfied, consumers can substitute entirely to goods whose relative price is low. When the growth rate of inflation is high, this substitution can drive a significant wedge between inflation in the true price index and fixed-weight inflation measures.

In the second part of the paper, we focus on the standard new Keynesian (NK) model with Calvo (1983) style price rigidities and constant-elasticity-of-substitution (CES) preferences. That model is of interest because it is widely used in the macroeconomics literature. The standard NK model satisfies the two conditions mentioned above. We construct model analogs to fixed-weight inflation indices and show that when the growth rate of inflation is high, the differences between

[1]See Konüs (1924) for a seminal analysis of true price indices.

[2]Christiano et al. (2011) match inflation in the true price index in the NK model to core CPI inflation. Nakamura et al. (2018) match inflation in the true price index in the Calvo model to CPI inflation excluding shelter in their figures VIII and XIII. Blanco et al. (2024) follow Nakamura et al. (2018) in matching inflation in the true price index to CPI inflation excluding shelter.

[3]Our usage of "time-dependent pricing" follows the Klenow and Kryvtsov (2008), who use that term to mean that the ability of firms to update their prices is based on an exogenous process.

inflation in those indices and inflation in the true price index are increasing in price stickiness, the elasticity of substitution across goods, and the level of inflation.

We then ascertain how large these differences are for inflation increases of the size seen since 2020 in the U.S. Setting the model analogs of inflation in fixed-weight indices equal to observed core CPI inflation, we recover values for inflation in the true price index implied by the model. For parameter values widely used in the literature, the differences between the inflation in fixed-weight indices and the true price index are large and persistent during the past five years when U.S. inflation rose rapidly and then declined. In the decade before this period when inflation was low and stable the differences are small.

Taken as a whole, our results suggest that researchers should use model-based inflation measures that are consistent with how inflation is measured in the data. Doing so is particularly important when using time-dependent pricing models to analyze data in a period of rapid growth in inflation.

## 1.1 Related literature

Our results are related to the large literature on biases in measured price indices. See, for example, Boskin et al. (1996). More recent work includes Redding and Weinstein (2020), Braun and Lein (2021), and Redding and Weinstein (2024). These papers focus on developing measured price indices that take substitution effects and other shocks into account. Our analysis focuses on developing model-based inflation measures in time-dependent pricing models that are consistent with measures of inflation in the data, e.g., the CPI. In addition, we recover model-implied levels of inflation in the true price index.

Our results also relate to Kocherlakota (2024), who analyzes short-run differences between Laspeyres and true price indices in the standard NK model. As in our analysis, these differences arise because of consumers' desire to substitute towards goods whose relative prices are low. By short-run, Kocherlakota (2024) means that people's expectations about future inflation and real marginal costs are fixed at their steady-state values. In contrast, our analysis does not depend on assumptions about people's expectations or many of the details of the standard NK model. Kocherlakota (2024) focuses on the shape of the short-run Phillips curve. We focus on differences between measured price indices and the model-implied true price index, particularly during periods when inflation rises rapidly.

Moulton (1996) argues that substitution bias is not necessarily larger at higher inflation levels. Hausman (2003) argues that substitution bias from an individual price change leads to a second-order difference between the true price index and Laspeyres fixed-weight indices. We provide conditions under which these differences can be large. In addition, we derive expressions for the differences between inflation in fixed-weight price indices and inflation in the true price index implied by the standard NK model, and show how these differences can appear in log-linear approximations of solutions to that model.

Our analysis highlights the importance of price dispersion across goods during a period when inflation rises rapidly, as well as the importance of consumers' willingness to substitute across goods. During periods when inflation rises rapidly, the extent of price dispersion increases in time-dependent pricing models where some firms cannot change their prices. This feature of time-dependent models has been called into question (for related discussion, see Nakamura et al. (2018) and Montag and Villar (2022)). Still, these types of models are widely used to analyze U.S. data. Price dispersion is lower in menu-cost models like the one studied by Golosov and Lucas Jr. (2007). So, in those models, there are fewer opportunities for consumers to substitute across goods.[4]

Our empirical example focuses on inflation during and after the COVID-19 pandemic. Our analysis allows for supply shocks and demand shocks that do not affect the relative demand for different goods. However, we abstract from demand shocks that cause shifts in consumption patterns across sectors (see Redding and Weinstein (2020)). There were likely such shocks during the COVID-19 pandemic (see, for example, Eichenbaum et al. (2022), Ferrante et al. (2023), and Cavallo (2024)), which could have important implications for measured inflation. For example, Cavallo (2024) finds that CPI inflation during the COVID-19 pandemic understated inflation in 2020 relative to measures that take into account shifts in consumption across sectors. These results are consistent with the view that it is important to use model analogs of measured price indices when assessing the empirical plausibility of models.

The remainder of this paper is organized as follows. Section 2 discusses the relationship between inflation rates based on true price indices and fixed-weight price indices. Section 3 applies the analysis of Section 2 to the standard NK model. Section 4 compares the quantitative relationships

[4] Blanco et al. (2024) consider a Calvo-style model in which the number of prices that are reoptimized evolves endogenously. The amount of price dispersion in their model is lower than in a standard Calvo-style model.

between true price indices and fixed-weight price indices using post-2010 U.S. data. Section 5 contains concluding remarks.

# 2 Substitution bias and fixed-weight price indices

In this section, we consider true price indices that emerge from consumers' expenditure choices. We compare inflation computed with these price indices to inflation computed from fixed-weight indices.

## 2.1 Consumer demand

We allow for relatively general demand systems. A representative consumer derives utility from an aggregate consumption good, $C_t$, that is produced using a continuous, strictly increasing, and strictly quasi-concave function that is homogeneous of degree one, which we denote by $\mathcal{C}$, that aggregates a continuum of underlying consumption goods, $x_{i,t}$ for $i \in [0, 1]$.[5] We denote such a continuum by $\{x_{i,t}\}$. Because the aggregate consumption good is produced using a function that is homogeneous of degree one, the true price index, $P_t$, is independent of the level of $C_t$. Therefore, the value of $P_t$ is the expenditure required to produce one unit of $C_t$:

$$P_t = \min_{\{x_{i,t}\}} \int_0^1 P_{i,t} x_{i,t} di \, \text{subject to} \, \mathcal{C}\left(\{x_{i,t}\}\right) \geq 1. \tag{1}$$

Here, $P_{i,t}$ is the price of good $i$. Let $\left\{x^*_{i,t}\right\}$ denote the solution to this problem. We suppose, for simplicity, that for all $t$ and $i$, $x^*_{i,t}$ and $P_{i,t}$ are positive and finite. Note that $P_t$ is constructed using the values $\left\{x^*_{i,t}\right\}$ that change each period as $\{P_{i,t}\}$ changes.

## 2.2 Arithmetic fixed-weight price indices

Define an arithmetic fixed-weight price index as

$$Z_t = \int_0^1 P_{i,t} \omega_i di. \tag{2}$$

[5] We do not consider product entry and exit.

We assume that $\{\omega_i\} \propto \left\{x^*_{i,\tau}\right\}$ for some $\tau < t$ or that the values of $\omega_i$ are set to be proportional to the unconditional mean of expenditure weights over time. We normalize this index so that $Z_{t-1} = P_{t-1}$. The relationship between inflation rates in period $t$ constructed using $P_t$ and $Z_t$ is given by

$$\pi_t - \pi^f_t = \int_0^1 \frac{P_{i,t}}{P_{t-1}} \left(x^*_{i,t} - \omega_i\right) di \tag{3}$$

where, $\pi_t = P_t/P_{t-1}$ and $\pi^f_t = Z_t/Z_{t-1}$.[6] For large increases in the prices of a subset of goods relative to all other goods, $x^*_{i,t}$ could be much lower than $\omega_i$ for those goods. In that case, $\pi_t - \pi^f_t$ could be large and negative. So, the features of preferences that govern the degree of substitutability of different goods play a large role in determining the difference between $\pi_t$ and $\pi^f_t$.

### 2.3 Substitutability, time-dependent pricing, and the difference between inflation rates

In this subsection, we make assumptions about $\mathcal{C}$, which relate to consumers' willingness to substitute between goods, and the way that firms update prices to analyze the difference between $\pi_t$ and $\pi^f_t$.

We assume that one unit of the composite consumption good can be produced using any strict subset with positive measure of the underlying continuum of consumption goods. That is, for any strict subset of the unit interval with positive measure, $\Omega$, if $x_{i,t} > 0$ for $i \in \Omega$ and $x_{i,t} = 0$ for all other $i$,then

$$\mathcal{C}\left(\{x_{i,t}\}\right) > 0. \tag{4}$$

We refer to this assumption as the "inessentiality" of any subset of consumption goods.[7]

Because $\mathcal{C}$ is homogeneous of degree one, there exists some finite $M_\Omega \geq 0$, so that

$$\mathcal{C}\left(\{M_\Omega x_{i,t}\}\right) = 1. \tag{5}$$

In general, $M_\Omega$ depends on the values $\{x_{i,t}\}$.

Macroeconomists often assume that some nominal prices are sticky. Time-dependent models of

[6] The right-hand side of equation (3) would be a covariance between $P_{i,t}/P_{t-1}$ and $x^*_{i,t} - \omega_i$ if $\int_0^1 \left(x^*_{i,t} - \omega_i\right) di = 0$.

[7] Our use of "inessentiality" is similar to the use of the term "inessential" in Matsuyama (2023). But, we use the term with reference to a measure of goods.

nominal rigidities, like those of Calvo (1983) and Taylor (1979, 1980), assume that a subset of prices cannot change in any given period. We now analyze the relationship between $\pi_t$ and $\pi_t^f$ when that assumption holds.

Let $\Theta_t$ be the set of firms that can update their prices at time $t$, and let $\Theta_t^c$ be the complement of that set in the unit interval. We assume $\Theta_t^c$ has positive measure. Given our inessentiality assumption

$$P_t = \int_0^1 P_{i,t} x_{i,t}^* di \leq \int_{\Theta_t^c} P_{i,t-1} x_{i,t-1}^* M_{\Theta_t^c} di \tag{6}$$

where $M_{\Theta_t^c}$ is the finite scalar such that if $x_{i,t} = 0$ for $i \in \Theta_t$ and $x_{i,t} = M_{\Theta_t^c} x_{i,t-1}^*$ for $i \in \Theta_t^c$, $\mathcal{C}\left(\left\{M_{\Theta_t^c} x_{i,t}\right\}\right) = 1$. Recall our assumption that $P_{i,t-1}$ and $x_{i,t-1}^*$ are positive and finite. So, equation (6) implies $P_t$ and $\pi_t$ are bounded in period $t$. Moreover,

$$\pi_t - \pi_t^f \leq -\int_{\Theta_t} \frac{P_{i,t}}{P_{t-1}} \omega_i di + \int_{\Theta_t^c} \frac{P_{i,t-1}}{P_{t-1}} \left(M_{\Theta_t^c} x_{i,t-1}^* - \omega_i\right) di. \tag{7}$$

If the prices chosen by firms that update their prices are high relative to $P_{t-1}$, the first term on the right-hand side of the inequality is negative, large, and increasing in magnitude in the prices set in period $t$. The second term is a constant. So, under our assumptions, $\pi_t^f$ can be arbitrarily larger than $\pi_t$. The exogenous selection of which firms adjust prices is the key to understanding this result. It leads to higher price dispersion as inflation increases and more opportunities for consumers to substitute to goods whose relative price is low. So, the extent to which $\pi_t$ and $\pi_t^f$ differ depends critically on the extent of price dispersion across goods caused by recent price changes and consumers' willingness to substitute between goods.

## 2.4 Geometric mean fixed-weight price indices

Define a geometric mean fixed-weight index as

$$G_t = \exp\left(\int_0^1 \omega_i \log\left(P_{i,t}\right) di\right), \tag{8}$$

where $\omega_i > 0$ and $\int_0^1 \omega_i di = 1$. If preferences are Cobb-Douglas across goods, then $G_t$ is the true price index. More generally, the more substitutable goods are, the more $G_t$ should differ from $P_t$. The Cobb-Douglas case aside, because the weights, $\omega_i$, are fixed, $G_t$ does not fully reflect substitutability induced by relative price changes.

As in the previous sub-section, let $\Theta_t$ be the set of firms that can update their prices at time $t$, let $\Theta_t^c$ be the complement of that set in the unit interval, and let $\Theta_t^c$ have positive measure. To see that $G_t$ can meaningfully differ from $P_t$ in time-dependent pricing models, we write $G_t$ as

$$\log\left(G_t\right) = \int_{\Theta_t} \omega_i \log\left(P_{i,t}\right) di + \int_{\Theta_t^c} \omega_i \log\left(P_{i,t-1}\right) di. \tag{9}$$

If the prices chosen by firms that update their prices are high, the first term on the right-hand side of equation (9) is positive and large. Moreover, that term is increasing in the prices set in period $t$. The second term is a constant. So, under our assumptions, $\pi_t^g = G_t/G_{t-1}$ can be arbitrarily large, even when $\pi_t$ is bounded.

In sum, in this section, we showed that the difference between inflation rates computed using a true price index versus a fixed-weight arithmetic or geometric mean price index can be arbitrarily large. In both cases, the degree to which consumers are willing to substitute across goods is a critical determinant of the difference between inflation rates.

# 3 Application to the NK model with a CES consumption aggregator

It is common in the NK literature to use the CES consumption aggregator of Dixit and Stiglitz (1977) given by

$$\mathcal{C}\left(\{x_{i,t}\}\right) = \left(\int_0^1 x_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{10}$$

Here $\varepsilon > 1$ and the $x_{i,t}$ are differentiated goods produced by monopolists. The CES consumption aggregator has the property that any strict subset of the unit interval is inessential.[8] Equation (10) and cost minimization imply that the ideal price index is given by

$$P_t = \left(\int_0^1 P_{i,t}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}. \tag{11}$$

[8] Some NK models use a consumption aggregator as in Kimball (1995). There are parametric versions of the Kimball (1995) consumption aggregator that have the property that for given parameters a small enough subset of the unit interval is inessential (see, for example, Klenow and Willis (2016)).

The standard NK model assumes Calvo-style frictions in price setting.[9] Specifically, in each period, a measure $0 < \theta < 1$ of randomly selected monopolists cannot re-optimize their price and set $P_{i,t} = P_{i,t-1}$.[10] The complementary measure, $1 - \theta$, of monopolists can re-optimize their price. A standard feature of NK models is that all of the reset prices, $\tilde{P}_t$, chosen by re-optimizing firms are the same. Equation (11) implies

$$P_t^{1-\varepsilon} = (1-\theta)\,\tilde{P}_t^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon}, \tag{12}$$

and equation (12) implies $\tilde{P}_t/P_t = \left(\frac{1-\theta}{1-\theta\pi_t^{\varepsilon-1}}\right)^{\frac{1}{\varepsilon-1}}$.

## 3.1 The upper bound on $\pi_t$ in the NK model

Consistent with equation (6), there is an upper bound on $\pi_t$ in the standard NK model. To derive the bound in the NK model, notice that equations (5) and (10), along with the random selection from the Calvo-style price friction, imply that there exists a finite, positive value $M$ such that if $x_{i,t} = 0$ for goods whose prices change and $x_{i,t} = M x_{i,t-1}^*$ for all other goods then

$$1 = \left(\int_0^1 (x_{i,t}M)^{\frac{\varepsilon-1}{\varepsilon}}\, di\right)^{\frac{\varepsilon}{\varepsilon-1}} = M\left(\theta\int_0^1 \left(x_{i,t-1}^*\right)^{\frac{\varepsilon-1}{\varepsilon}}\, di\right)^{\frac{\varepsilon}{\varepsilon-1}} = M\theta^{\frac{\varepsilon}{\varepsilon-1}}. \tag{13}$$

So, $M = \theta^{-\frac{\varepsilon}{\varepsilon-1}}$. It follows from the inequality in equation (6) that

$$P_t \leq M\theta\int_0^1 P_{i,t-1}x_{i,t-1}^* di = \theta^{-\frac{1}{\varepsilon-1}}P_{t-1} \tag{14}$$

and

$$\pi_t \leq \theta^{-\frac{1}{\varepsilon-1}}. \tag{15}$$

This upper bound on $\pi_t$ is the same as the one in Andreasen and Kronborg (2022), who use equation (12) directly to derive the bound. Consumers' ability to substitute entirely to the goods produced

[9] An alternative that is sometimes used in the NK literature is to assume Rotemberg-style costs of adjusting prices (see Rotemberg (1982)). It is well known that the Calvo- and Rotemberg-style frictions are equivalent up to a first-order approximation. However, Rotemberg-style frictions imply that all prices change every period. This implication is at odds with existing evidence about price changes (see Nakamura et al. (2018)). Because all prices change every period with Rotemberg-style costs of adjustment, the substitution effects that we are stressing are absent.

[10] It would be straightforward to extend our results to account for models in which monopolists that cannot re-optimize change their price using steady-state inflation or past inflation.

by firms that do not re-optimize their prices delivers the bound on $\pi_t$. The bound depends on the parameter $\varepsilon$, which governs the degree to which consumers are willing to substitute across goods. Larger values of $\varepsilon$ imply higher degrees of substitutability and a smaller upper bound on $\pi_t$. The bound also depends on $\theta$, which governs how many monopolists cannot re-optimize their prices. The higher is $\theta$, the smaller is that bound. The reason is that the higher is $\theta$, the larger is the subset of firms that cannot re-optimize their price. So, consumers can substitute to the goods of a larger subset of firms whose price is not re-optimized.[11]

### 3.2 The difference between $\pi_t$ and $\pi_t^f$ in the NK model

Define the arithmetic fixed-weight price index $Z_t = \int_0^1 P_{i,t}\omega_i di$ where $\int_0^1 \omega_i di = 1$. In the standard NK model $Z_t$ evolves according to

$$Z_t = (1-\theta)\,\tilde{P}_t + \theta Z_{t-1}. \tag{16}$$

Define $z_t = Z_t/P_t$ and $\pi_t^f = Z_t/Z_{t-1}$. Equations (12) and (16) imply

$$z_t = (1-\theta)\left(\frac{1-\theta}{1-\theta\pi_t^{\varepsilon-1}}\right)^{\frac{1}{\varepsilon-1}} + \theta\frac{z_{t-1}}{\pi_t}, \tag{17}$$

$$\pi_t - \pi_t^f = -(1-\theta)\left(\frac{1-\theta}{1-\theta\pi_t^{\varepsilon-1}}\right)^{\frac{1}{\varepsilon-1}}\frac{\pi_t}{z_{t-1}} + (\pi_t - \theta). \tag{18}$$

Equation (18) shows how $\pi_t - \pi_t^f$ varies with the level of $\pi_t$ in the NK model. As $\tilde{P}_t$ becomes large, $\pi_t$ approaches its upper bound, $\theta^{-\frac{1}{\varepsilon-1}}$, $\tilde{P}_t/P_t = \left(\frac{1-\theta}{1-\theta\pi_t^{\varepsilon-1}}\right)^{\frac{1}{\varepsilon-1}}$ becomes unboundedly large, and $\pi_t^f$ becomes much larger than $\pi_t$. The reason is that consumers substitute toward goods produced by firms that have not re-optimized their prices, and this substitution effect is not reflected in $\pi_t^f$.

Assuming zero or positive steady-state inflation in the true price index, $Z_t/P_t$ has a finite steady-state value and the steady-state levels of inflation implied by these indices are the same. Intuitively, the distribution of prices does not change over time in a steady state, so substitution bias is not systematically better or worse. In a log-linear approximation of equations (17)-(18) around a zero

[11] The bound on $\pi_t$ in equation (15) also applies to steady-state inflation. King and Wolman (1996), Ascari (2004), and Ascari and Sbordone (2014) also argue that the standard NK model has an upper bound on steady-state price inflation. Ascari and Sbordone (2014) show that steady-state inflation, $\pi$, satisfies $\beta\theta\pi^{\varepsilon} < 1$ and $\beta\theta\pi^{\varepsilon-1} < 1$, where $0 < \beta < 1$ is the rate of time discounting. The bound in equation (15) may be a tighter bound on steady state inflation, depending on parameter values.

inflation steady state, the difference between $\pi_t^f$ and $\pi_t$ is zero.[12] This observation suggests that using linear approximations to analyze inflation in the NK model may lead to misleading inferences in periods when inflation rises rapidly. It also suggests that equation (18) may be useful for assessing the accuracy of linear approximations.

### 3.3 The difference between $\pi_t$ and $\pi_t^g$ in the NK model

Define the geometric mean fixed-weight price index $G_t = \exp\left(\int_0^1 \log\left(P_{i,t}\right)\omega_i di\right)$ where $\int_0^1 \omega_i di = 1$. In the standard NK model $G_t$ evolves according to

$$\log\left(G_t\right) = \left(1-\theta\right)\log\left(\tilde{P}_t\right) + \theta\log\left(G_{t-1}\right). \tag{19}$$

Define $g_t = G_t/P_t$ and $\pi_t^g = G_t/G_{t-1}$. Equations (12) and (19) imply

$$\log\left(g_t\right) = \left(1-\theta\right)\log\left(\left(\frac{1-\theta}{1-\theta\pi_t^{\varepsilon-1}}\right)^{\frac{1}{\varepsilon-1}}\right) + \theta\log\left(g_{t-1}/\pi_t\right), \tag{20}$$

$$\pi_t - \pi_t^g = \pi_t - \left(\frac{\left(\frac{1-\theta}{1-\theta\pi_t^{\varepsilon-1}}\right)^{\frac{1}{\varepsilon-1}}\pi_t}{g_{t-1}}\right)^{1-\theta}. \tag{21}$$

Equation (21) shows how $\pi_t - \pi_t^g$ varies with the level of $\pi_t$ in the NK model. As $\tilde{P}_t$ becomes large, $\pi_t$ approaches its upper bound, $\theta^{-\frac{1}{\varepsilon-1}}$, $\tilde{P}_t/P_t = \left(\frac{1-\theta}{1-\theta\pi_t^{\varepsilon-1}}\right)^{\frac{1}{\varepsilon-1}}$ becomes unboundedly large, and $\pi_t^g$ becomes much larger than $\pi_t$ as consumers substitute away from the goods with the high reset price. This substitution effect is not fully reflected in $\pi_t^g$ as long as $\varepsilon > 1$.[13]

[12] Let $\hat{x}_t = \log\left(x_t/x\right)$, where $x$ is the non-zero steady-state value of $x_t$. Log-linearized versions of equations (17)-(18) around a zero inflation steady state are $\hat{z}_t = \theta\hat{z}_{t-1}$ and $\hat{\pi}_t^f = \hat{\pi}_t - \left(1-\theta\right)\hat{z}_{t-1}$. Setting $\hat{z}_0 = 0$, or assuming that many periods have passed prior to the first period of a simulation gives the result that $\hat{\pi}_t = \hat{\pi}_t^f$. When log-linearized around other steady-state values for inflation, there are first order differences between $\hat{\pi}_t^f$ and $\hat{\pi}_t$.

[13] As was the case for $\pi_t^f$, in a log-linear approximation of equations (20)-(21) around a zero inflation steady state the difference between $\pi_t^g$ and $\pi_t$ is zero (see footnote 12). Additionally, the steady-state levels of $\pi_t^g$ and $\pi_t$ are the same.

# 4 Measured inflation

## 4.1 Model-based measures of inflation that correspond to the CPI

Measured price indices do not correspond to the model-implied true price index. Measured price indices like the CPI are fixed-weight indices. Those weights are updated periodically based on expenditures in prior years (see U.S. Bureau of Labor Statistics (January 30, 2025)). So, over short periods, the weights on different goods in the CPI are effectively fixed. During a period when inflation rises rapidly, our analysis in section 2 indicates that inflation rates implied by fixed-weight price indices could be meaningfully different from inflation in the true price index. To the extent that is true, it is important for quantitative analysis to use a model-based measure of inflation that corresponds to how inflation is measured in the data.

Here, we focus on the CPI to construct a model-based measure of inflation. Roughly, the CPI is constructed in two stages (see U.S. Bureau of Labor Statistics (January 30, 2025)). First, prices within most categories are combined using a geometric average (see Dalton et al. (1998)). The weights in this geometric average come from expenditures in previous periods. Second, categories are combined across sectors using a modified Laspeyres index. The latter price index is an arithmetic fixed-weight index whose weights come from previous-period expenditures across sectors. We expect more substitutability within a sector than across sectors (see, for example, Atkeson and Burstein (2008)).

The standard NK model does not have a rich enough industry structure to mimic the two-stage construction of the CPI. We can think of the different $P_{i,t}$'s as denoting either prices from monopolists within a sector or from monopolists across sectors. So, the analog of core CPI inflation in the model can be thought of as either $\pi_t^f$ or $\pi_t^g$. One would adopt different values of $\varepsilon$ depending on which interpretation one adopts. Below, we investigate different values of $\varepsilon$.

We do not periodically update the weights of the indices in our calculations for two reasons.[14] First, we are particularly interested in the rapid rise in inflation after 2020. Over such a short period, the weights in the CPI would not change to reflect changes in expenditure patterns across goods. Second, our results are not sensitive to choosing different starting dates for $z_{t-1} = 1$ or $g_{t-1} = 1$.

[14] We consider a Laspeyres index in which the weights are updated every period, as in Kocherlakota (2024), in our Appendix.

Recall that those variables are the initial ratio of the fixed weight index to $P_{t-1}$. Updating the weights in the price indices corresponds to setting a new value for $z_{t-1}$ or $g_{t-1}$. The robustness of our results to different starting dates reflects that $\pi_t$, $\pi_t^f$, and $\pi_t^g$ are very similar in the decade before the COVID-19 pandemic.

### 4.2 Model-implied substitution bias since 2011

We assume that core CPI inflation corresponds to $\pi_t^f$ or $\pi_t^g$. We use equations (17)-(18) or (20)-(21) to calculate the quarterly values of $\pi_t$ implied by the model.[15]

Panel (a) of Figure 1 displays the core CPI inflation rate for 2011:Q1 through 2024:Q4 (U.S. Bureau of Labor Statistics 1957-2024 via FRED).[16] Panel (b) displays $\pi_t - \pi_t^f$ for different values of $\varepsilon$ under the assumption that $\pi_t^f$ is the model analog to core CPI inflation. In that panel, $\theta$ is fixed at 0.75. The values of $\varepsilon = 4$, 7, and 10, correspond to the values used in Nakamura et al. (2018), Coibion et al. (2012) and Atkeson and Burstein (2008), respectively. As discussed in Atkeson and Burstein (2008), higher values of $\varepsilon$ are more relevant for thinking about substitution within sectors than across sectors. During the low and stable inflation period from 2011 through 2019, $\pi_t$ and $\pi_t^f$ are similar for the different values of $\varepsilon$. During the post-2020 period, there are substantial and persistent differences between $\pi_t$ and $\pi_t^f$. For example, when $\varepsilon = 7$, the maximal difference between $\pi_t$ and $\pi_t^f$ is roughly one percentage point. Consistent with the discussion above, the differences between $\pi_t$ and $\pi_t^f$ increase in $\varepsilon$, which governs the degree of substitutability among goods.

Panel (c) displays $\pi_t - \pi_t^f$ for different values of $\theta$ assuming that $\varepsilon = 7$. The values of $\theta = 0.65$, 0.75, and 0.85 correspond roughly to the point estimates in Smets and Wouters (2007), Christiano et al. (2014), and Justiniano et al. (2013), respectively. Consistent with the discussion above, the differences between $\pi_t$ and $\pi_t^f$ can be substantial and persistent. The difference is increasing in $\theta$, which governs the degree of price stickiness and is the measure of firms that do not re-optimize their prices to which consumers can substitute.

Panels (a) and (b) in Figure 2 display the analogs of panels (b) and (c) in Figure 1 when we replace $\pi_t^f$ by $\pi_t^g$. Notably, the results for $\pi_t^f$ and $\pi_t^g$ are very similar. At least for increases in

[15] We set $z_t = 1$ in 2010:Q4 and $g_t = 1$ in 2010:Q4. Our results are robust to different starting dates for $z_t$ and $g_t$. If a researcher had a series for $\pi_t$, equations (17)-(18) and (20)-(21) could be used to calculate values for $\pi_t^f$ and $\pi_t^g$.

[16] We average the core CPI index to obtain a quarterly index, and then compute annualized quarterly inflation rates.

inflation of the size experienced after 2020, none of our conclusions regarding the standard NK model depend upon whether we think of the CPI as an arithmetic or geometric mean fixed-weight price index.

## 5 Conclusion

We compare inflation in true price indices to inflation in fixed-weight price indices. As is well known, the differences between these indices depend on the degree to which consumers are willing to substitute across goods. We show that those differences can be unbounded. We apply our analysis to the NK model and show that in the post-2020 period, inflation in model-based fixed-weight price indices differ markedly from the model-based measure of inflation in the true price index. Those differences are increasing in price stickiness and the degree to which consumers are willing to substitute across goods. There may be a smaller mismatch between true price indices and superlative price indices constructed in the model. We leave this issue to future research.

We conclude that researchers should use model-based measures of inflation that are consistent with the way inflation is measured in the data, especially in periods when inflation rises rapidly.

## Appendix

In the context of the standard NK model considered in section 3, define a Laspeyres measure of inflation as in Kocherlakota (2024) as

$$\pi_t^L = \frac{\int_0^1 P_{i,t} x_{i,t-1}^* di}{\int_0^1 P_{i,t-1} x_{i,t-1}^* di} = \frac{(1-\theta)\,\tilde{P}_t \int_0^1 x_{i,t-1}^* di + \theta P_{t-1}}{P_{t-1}} = (1-\theta)\, a_{t-1} \tilde{p}_t \pi_t + \theta \tag{22}$$

where $\tilde{p}_t = \tilde{P}_t / P_t$ and

$$a_t = \int_0^1 x_{i,t}^* di = \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} di = (1-\theta)\,\tilde{p}_t^{-\varepsilon} + \theta \pi_t^{\varepsilon} a_{t-1}. \tag{23}$$

In steady state

$$\pi^L = 1 + (1-\theta)\,\frac{\pi - 1}{1 - \theta \pi^{\varepsilon}}, \tag{24}$$

where $\pi$ is the steady-state value of $\pi_t$ and $\pi^L$ is the steady-state value of $\pi_t^L$. When $\pi > 1$ there is a steady-state difference between $\pi_t^L$ and $\pi_t$, and that difference is increasing in $\pi$. The reason that there is a steady-state difference between $\pi_t^L$ and $\pi_t$, but not between $\pi_t^f$ and $\pi_t$, is that $\pi_t^L$ has updated weights in every period. The weights in $\pi_t^f$ are fixed.

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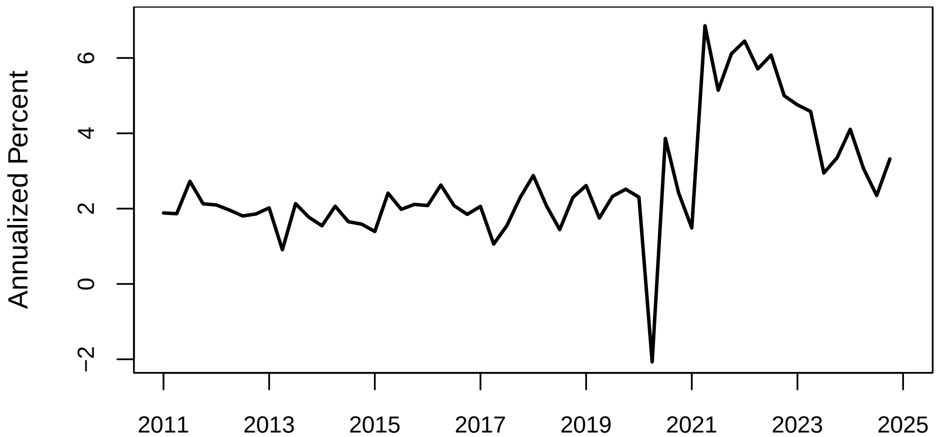
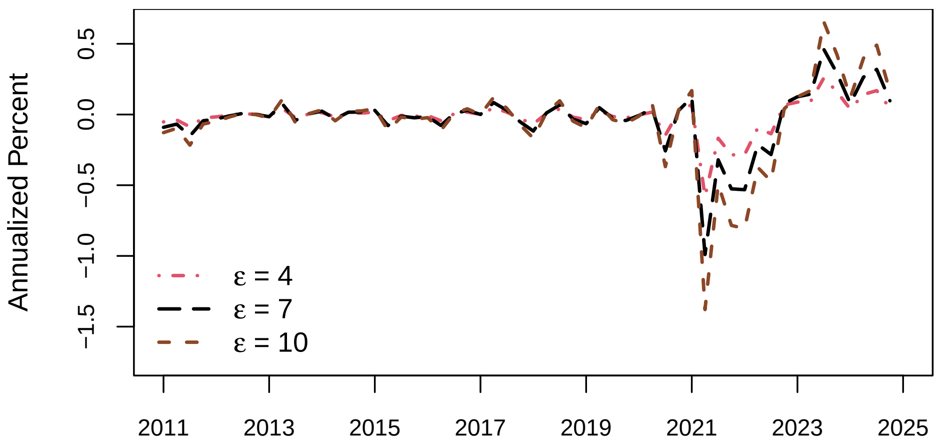
Figure 1: $\pi_t^f$ can be meaningfully different from $\pi_t$

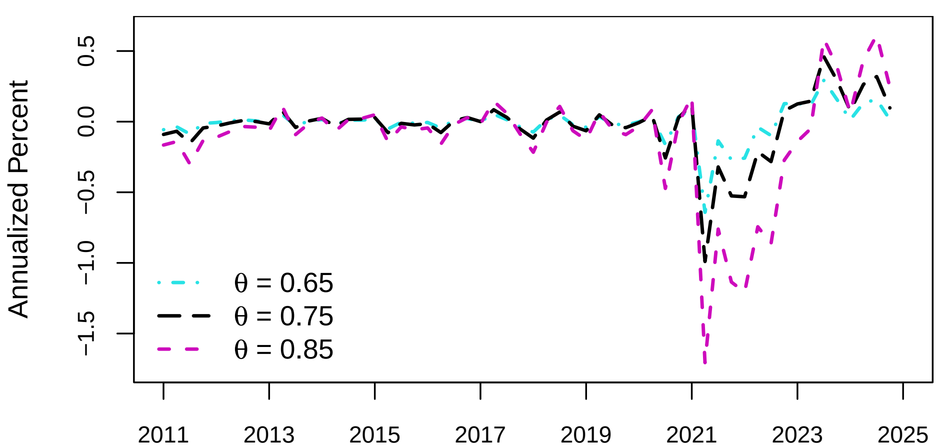
(a) Core CPI inflation




(b) $\pi_t - \pi_t^f$ when $\theta = 0.75$



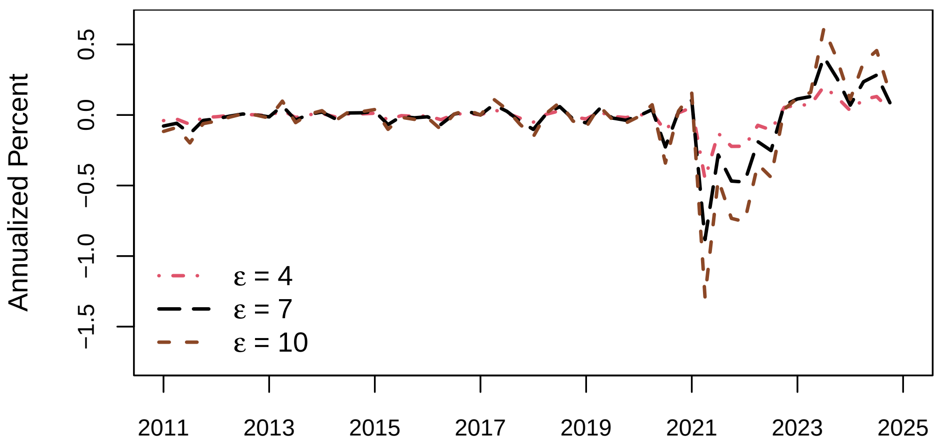

(c) $\pi_t - \pi_t^f$ when $\varepsilon = 7$



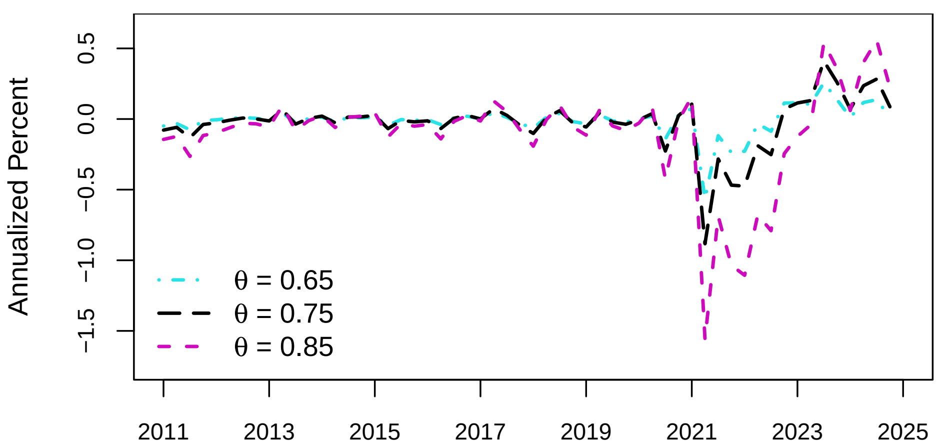

Source: U.S. Bureau of Labor Statistics via FRED and authors' calculations.

Figure 2: $\pi_t^g$ can be meaningfully different from $\pi_t$

(a) $\pi_t - \pi_t^g$ when $\theta = 0.75$




(b) $\pi_t - \pi_t^g$ when $\varepsilon = 7$




Source: U.S. Bureau of Labor Statistics via FRED and authors' calculations.