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Collateral Reuse and Financial Stability *

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Abstract

The isolated effects of collateral reuse on financial stability are ambiguous and understudied. While greater collateral reuse can guarantee more payments with fewer assets, it can also increase the exposure to potential drops in collateral price. To analyze these tradeoffs, we develop a financial network model with endogenous asset pricing, multiple equilibria, and equilibrium selection. We find that more collateral reuse decreases the likelihood of the worst equilibrium (crisis), with varying effects depending on the network structure. Therefore, collateral reuse can unambiguously improve financial stability for a fixed degree of risk-taking behavior. However, with endogenous risk-taking, we show that a higher degree of collateral reuse can worsen financial stability through greater risk-taking. As a result, while crises may occur less frequently, their severity would increase, leading to a lower social surplus during crises.

Keywords: collateral, collateral reuse, financial network, fire sale, multiple equilibria, equilibrium selection, systemic risk

JEL Classification Numbers: D49, D53, G01, G21, G33

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1. Introduction

Securities financing transactions (SFTs)—which include repurchase agreement (repo), reverse repo, securities lending/borrowing, and margin lending transactions— use securities to borrow cash or vice versa, and are crucial for the functioning of the global financial market. Distress in the SFT markets played an important role in the culmination of the Global Financial Crisis (GFC) in 2008 (Allen et al., 2009; Jorion and Zhang, 2009; Gorton and Metrick, 2012). The outstanding amounts of SFTs are substantial: over €15 trillion in the EU,[1] and over $6 trillion in the U.S. (just for repos and reverse repos) as of March 2025.[2]

Most SFTs are collateralized, implying that one side of the trade posts assets to guarantee the payments made to the other side (counterparty) in case of a default. Collateral posted in this way can often be reused by receiving counterparties when they themselves borrow from other counterparties. Such reuse or *rehypothecation* of collateral is prevalent in the markets for SFTs (Fuhrer, Guggenheim, and Schumacher, 2016; Singh, 2017; Infante, Press, and Saravay, 2020).[3] Policy makers have discussed potential systemic risks stemming from collateral reuse.[4] Many academic papers have explored the effects of collateral reuse on leverage, liquidity, safe asset demand, and risks of lender default and collateral runs (Infante, 2019; Gottardi, Maurin, and Monnet, 2019; Park and Kahn, 2019; Infante and Vardoulakis, 2021; Chang, 2021; Maurin, 2022; Brumm, Grill, Kubler, and Schmedders, 2023; Infante and Saravay, 2024).

However, the isolated effects of collateral reuse on financial stability are unclear. When holding counterparty liabilities fixed, greater collateral reuse can guarantee and protect more debt obligations with fewer assets, potentially mitigating defaults. On the contrary, collateral

[1] https://www.icmagroup.org/market-practice-and-regulatory-policy/repo-and-collateral-markets/market-data/sftr-public-data/

[2] https://www.sifma.org/resources/research/statistics/us-repo-statistics/

[3] Infante et al. (2020) find that the collateral multiplier (degree of collateral reuse), measuring SFTs as a multiple of the total amount of collateral owned, for U.S. Treasury securities is around 10 for all contracts and around 5 for repos.

[4] See, for example, Aitken and Singh (2010) and Financial Stability Board (2017).

reuse may harm financial stability as more debt obligations depend on the same collateral, whose value can potentially drop. Furthermore, more collateral reuse may incentivize agents to take on greater risk. Hence, an in-depth analysis is required to understand the financial stability implications of collateral reuse.

The main contribution of this paper is to develop a new model to isolate the effect of collateral reuse on financial stability. We find that an increase in collateral reuse increases financial stability by reducing the likelihood of a crisis (bad equilibrium). However, we also show that agents take on more risk with an increase in collateral reuse since a crisis becomes less likely. Therefore, an increase in collateral reuse leads to a decrease in the frequency of crises but an increase in the severity of crises, leading to a lower social surplus. Thus, this paper primarily contributes to the literature on financial networks and systemic risk (Eisenberg and Noe, 2001; Elliott et al., 2014; Acemoglu et al., 2015; Bernard et al., 2022; Altinoglu and Stiglitz, 2023; Donaldson et al., 2022) and the literature on collateral reuse (Park and Kahn, 2019; Gottardi, Maurin, and Monnet, 2019; Infante and Vardoulakis, 2021; Infante, Press, and Strauss, 2018; Maurin, 2022) by directly linking them together.

Our model is a network model in which agents are connected through collateralized counterparty exposures while allowing for collateral reuse.[5] It is based on the financial networks literature in which links represent payment relationships, stemming from Eisenberg and Noe (2001). In particular, it directly extends Chang and Chuan (2024), which incorporates (re)use of collateral and endogenous collateral price. Therefore, our model allows for contagion through both counterparty exposures and asset prices.

The model of Chang and Chuan (2024) is based on an economy of n agents and three

[5]Typical SFTs take the form of a one-to-one relationship between a borrower and a lender. If the value of the collateral is greater than the face value of the debt (liability), then the payment is always made in full. However, if the value of the collateral is less than the face value of the debt, then the payment depends on both the price of the collateral and the cash balance of the borrowing counterparty. Therefore, a collateralized debt network has two transmission channels of shocks: the collateral price channel and the counterparty channel. The interaction of network structure and collateral prices can dramatically alter a network's systemic risk and, thus, welfare (Chang and Chuan, 2024). For example, the collapse in prices of subprime mortgages during the GFC was exacerbated by the bankruptcy of Lehman Brothers, which spread the initial losses to Lehman's counterparties and further decreased asset prices (Singh, 2017).

periods $t = 0, 1, 2$. Agents are endowed with an asset that can be traded and used as collateral. The price of the asset p is endogenously determined in a competitive Walrasian market in each period. The payoff of the collateralizable asset s is common knowledge and realized in the final period $t = 2$. In the first period $t = 0$, agents borrow from each other using bilateral one-period secured debt contracts.[6] The collection of these bilateral debt contracts is the collateralized debt network. We take the network as exogenously given, as short-term collateralized contracts tend to be relationship-based (Han, Nikolaou, and Tase, 2022; Chang, Klee, and Yankov, 2025). At $t = 0$, agents also invest in a long-term project that generates a non-pledgeable return at the final period $t = 2$. Liquidating the long-term project is costly and thus socially inefficient. However, agents can receive a negative liquidity shock at $t = 1$ and may have to liquidate their long-term projects to pay their debt. If an agent's net wealth is still negative after liquidations, the agent defaults on her inter-agent payments, which can trigger additional liquidations and defaults through the network. Liquidity shocks and defaults can also decrease the collateral asset price, as agents may fire-sell their assets due to liquidity shortages, further exacerbating default losses, as the collateral value declines. In such cases, the equilibrium asset price is determined by cash-in-the-market pricing, as the return on purchasing the collateral asset is greater than the cash return.

As Chang and Chuan (2024) showed, if collateral is sufficient, any network structure is insulated from contagion.[7] Moreover, for a fixed debt network and endowments, a higher degree of collateral reuse (or higher value of the collateral multiplier defined by Aitken and Singh (2010) and Singh (2017)) implies the same liability amount can be covered by more collateral. Therefore, as long as the collateral price remains at its fundamental value s, increased collateral reuse guarantees financial stability.

However, there are multiple equilibria in the model. In line with the literature (Rogers

[6]The debt contract encompasses any type of SFTs and even derivatives.

[7]This result is in line with real-world markets. For example, repo collateral is exempt from the automatic stay of bankruptcy provisions and prevents further spillovers.

and Veraart, 2013; Elliott, Golub, and Jackson, 2014; Bernard, Capponi, and Stiglitz, 2022; Capponi, Corell, and Stiglitz, 2022), the analysis of Chang and Chuan (2024) focuses on the maximum (Pareto-dominant) equilibrium. Nonetheless, multiplicity of equilibria itself can generate interesting venue of research (Dybvig, 2023). As Jackson and Pernoud (2024) show, multiplicity and self-fulfilling defaults can be important in understanding fragility of a financial network.[8] Hence, we analyze the multiplicity of equilibria in the model of Chang and Chuan (2024).

Our first main result is that there can be at most three equilibria in our model, depending on the network structure, when collateral (re)use is large enough. In the best (Pareto-dominant) equilibrium, the collateral asset is maximally priced at its fundamental value $p = s$, and can prevent contagion, resulting in ample liquidity for the agents to buy the asset at its fundamental value. In the worst equilibrium, the collateral asset is priced at zero $p = 0$, maximizing counterparty exposures between agents, when the network is sufficiently connected. This results in full contagion, i.e., all agents default. Because all agents are defaulting, there is no one to buy the asset at a positive price. Finally, in the intermediate equilibrium, the collateral asset is priced at a market-clearing price p^* that equalizes the available cash in the network and the total supply of fire sales.

The existence of three different equilibria implies that the equilibrium can suffer significant swings in social surplus based on changes in coordination of equilibrium. A huge amount of social inefficiency can be realized if agents in the network coordinate into the worst equilibrium in which all agents default and inefficiently liquidate their long-term projects. This result is directly related to the ongoing policy concerns on the resilience of the U.S. Treasury market, the market for the safest and most commonly used collateral assets (Liang, 2025).

Our second main result is that the intermediate equilibrium price p^* is decreasing in the degree of collateral reuse. The intuition is simple: more collateral reuse (c) leads to smaller

[8]For example, Fleming and Keane (2021) and Božić and Zrnc (2023) show that centrally netting out liabilities can greatly reduce the number and severity of defaults, implying that self-fulfilling defaults are real and important (Jackson and Pernoud, 2024).

counterparty exposure for a given price, resulting in an increase in the net wealth of agents. Then, the collateral price p must fall to clear the market, which equalizes the aggregate net wealth with the total value of assets on sale. In other words, the equilibrium price p decreases to make the total value of the collateral, (cp), constant. Although the intermediate equilibrium price p^* may differ across different networks, the price is monotonically decreasing in the degree of collateral reuse c.

We then extend our model to incorporate equilibrium selection. For equilibrium selection, we consider global games, as in Carlsson and Van Damme (1993) and Morris and Shin (1998), and best response dynamics, as in Gilboa and Matsui (1991) and Matsui (1992). Our main results hold using both approaches.

Our third main result is that there is an inverse relationship between the intermediate equilibrium price (p^*) and the likelihood of the best equilibrium. This is because the intermediate equilibrium is unstable compared to the best equilibrium (with $p = s$) or the worst equilibrium (with $p = 0$). If the asset price is perturbed slightly above the intermediate equilibrium price $p > p^*$, then the equilibrium will converge to the best equilibrium. This is because an increase in p increases agents' aggregate net wealth, which further increases p until it reaches its upper bound $p = s$. Conversely, if the asset price is perturbed slightly below the intermediate equilibrium price $p < p^*$, then the equilibrium will converge to the worst equilibrium. Therefore, having a lower intermediate equilibrium price p^* implies that there is a greater range of $(p^*, s]$ in which any realization/perturbation of noisy asset sales would lead to the best equilibrium with $p = s$.

Combining our first three main results, we find that increased collateral reuse improves financial stability for a fixed debt network and endowments. This counterintuitive result alleviates the first-order concern of policymakers that increased collateral reuse increases the financial system's exposure to self-fulfilling price declines and defaults. Even if there exists the possibility of a self-fulfilling crisis (the worst equilibrium), an increase in the degree of collateral reuse actually makes the financial system more stable, not less.

Finally, we further extend our model to incorporate agents' risk-taking decisions. In particular, we endogenize the agents' portfolio choice between cash and long-term investment projects at $t = 0$. Agents can form beliefs about the expected outcome at $t = 1$ and maximize their expected payoff at the final period $t = 2$. Holding more cash can reduce costly liquidation following an agent's default, which occurs during a crisis (the worst equilibrium) or a realization of a large liquidity shock. Alternatively, investing in the long-term project may yield higher returns at the cost of taking on more risk and potentially needing to liquidate more at $t = 1$.

For our fourth and last main results, we find that an increase in collateral reuse leads to more risk-taking behavior, as agents invest more cash in long-term projects. This is because the likelihood of the worst equilibrium decreases as collateral reuse increases. Thus, agents have less incentives to hold cash and instead prefer to invest it in the long-term investment projects, which have a higher payoff if held to maturity. As a result, there will be more socially inefficient liquidations of the long-term projects when a crisis (the worst equilibrium) occurs.[9] Therefore, we find that greater degree of collateral reuse can harm financial stability when agents' portfolio choices are endogenously determined. In particular, it will decrease the resiliency of the financial system as social welfare in the worst equilibrium falls due to more costly liquidations.

Our results have three important policy implications. First, the degree of collateral reuse alone is not a concern for financial stability. Indeed, collateral reuse can alter other relevant factors that may ultimately influence financial stability, such as market liquidity and rate spreads due to safe asset scarcity.[10] However, we find that the direct relationship between collateral reuse and financial stability is positive. Second, the degree of collateral reuse can still negatively impact financial stability through its indirect effects on agents' risk-taking choices, which become concerning in tail events. Therefore, monitoring the degree of

[9] Moreover, the likelihood of a crisis also increases as agents have less cash buffers to absorb shocks.

[10] An increase in the use of collateral, in particular to prevent debt dilution, can exacerbate "collateral overhang" problem, which constrains future borrowing and investment, as Donaldson et al. (2020) show.

collateral reuse remains important, as higher degrees of collateral reuse can be followed by greater leverage and other risk-taking behaviors of market participants. Third, supporting the price and liquidity of the collateral asset can be an effective intervention by a central bank by eliminating a crisis and significantly improving social surplus. Therefore, policy interventions such as a 'dealer of last resort' or a standing facility for repos and other SFTs can be important, especially when collateral reuse is prevalent.[11]

1.1. Relation to the Literature

Many papers have studied the effect of collateral reuse while allowing other factors to change simultaneously. Infante (2019), Gottardi et al. (2019), Park and Kahn (2019), and Brumm et al. (2023) studied how collateral reuse can change liquidity, leverage, and asset prices by allowing more profit-making opportunities to an intermediary. Maurin (2022) and Infante and Saravay (2024) show how collateral reuse can affect safe asset demand and safe asset scarcity, which in turn affect liquidity and prices in the market. Infante and Vardoulakis (2021) find a novel feature of collateral reuse, namely collateral runs, where an ultimate borrower (or lender of collateral) withdraws from its repo with an intermediary due to concerns that the intermediary may default and thus, the borrower is unable to retrieve their collateral. Chang (2021) further studies this aspect of lender default and how it is related to leverage, asset price, and network formation. However, collateral reuse is often a byproduct in such papers, and the effect of collateral reuse is confounded by other factors. Therefore, we make a novel contribution to the literature in our first two main results by formally analyzing the effect of changes in the degree of collateral reuse while holding all else equal, such as available liquidity, network (length of a lending chain), liability, and other risks.

Furthermore, to the best of our knowledge, we are the first to introduce collateral reuse

[11]Central bank interventions to support asset prices are not unprecedented and are quite successful. For example, commercial paper was directly purchased by the Federal Reserve through the Commercial Paper Funding Facility during the GFC and COVID-19 episode in response to investor outflows from money market funds.

to the financial networks literature. Examining collateral reuse through the lens of financial networks is necessary to properly analyze its effect. This is because models with anonymous agents or a simple lending chain are unable to capture the effect of changes in collateral reuse. In a model with anonymous Walrasian markets, the flow of collateral as well as the flow of payments cannot be tracked. In models with a simple lending chain (for example, Glode and Opp (2023) and He and Li (2022)), greater collateral reuse implies that more agents are joining the lending chain, resulting in a higher likelihood of defaults and total endowments. Since our model is a financial network, we allow for changes in the degree of collateral reuse while holding the length of the intermediation chain fixed.

Therefore, we contribute to the financial network literature by directly extending the model of Chang and Chuan (2024) by analyzing the effect of collateral reuse. In particular, our paper is related to the literature on contagion through payment networks, pioneered by Allen and Gale (2000) and Eisenberg and Noe (2001), and extended by Elliott et al. (2014), Acemoglu et al. (2015), Bernard et al. (2022), and Donaldson et al. (2022). Most recently, the literature has incorporated the dimension of collateral as a channel of contagion by Chang and Chuan (2024).

Moreover, our paper contributes to the literature on multiple equilibria in financial networks by Rogers and Veraart (2013), Roukny et al. (2018), and Jackson and Pernoud (2024). In these papers, self-fulfilling defaults can often cause multiplicity of equilibria in financial network models. For example, in one equilibrium, agent 1 is solvent when agent 2 pays agent 1 and is also solvent. In another, both default as they can no longer pay each other back. The source of multiplicity of equilibria in our model is different, as the vector of payments itself is unique for a given asset price. Thus, in our model, the multiplicity of equilibria is indexed by asset price. If the collateral price is high, then no agent suffers significant loss from a default, and the aggregate wealth is enough to buy any collateral asset on sale at its highest price. However, if the collateral price is low, then a default causes significant losses to counterparties, resulting in a lower aggregate wealth, which further decreases the asset price and eventually leads to full contagion, i.e., all agents default and the collateral asset price plummets to zero. Therefore, our model provides a novel source of equilibria multiplicity and analyzes its effect on contagion.

We also incorporate equilibrium selection by applying both global games and best response dynamics (BRD). The global games method in the literature (Carlsson and Van Damme, 1993; Morris and Shin, 1998; Goldstein and Pauzner, 2005; Kuong, 2021; Kashyap et al., 2024; Carapella et al., 2025) provides a natural, interpretable process of equilibrium selection in our financial networks model with endogenous collateral price. We also find an interesting characteristic of fire sales, which makes the threshold type (or marginal buyer) under global games to be the lowest type, contrary to many other models such as bank run models (see Section 4 for more details). Moreover, we examine an alternative model for equilibrium selection, BRD, based on Gilboa and Matsui (1991) and Matsui (1992), inspired by the application of such methods in Mäder (2024).

Our final result finds that greater collateral reuse can increase the riskiness of agents which relates to the literature on systemic risk and excessive risk-taking behavior by individuals. In particular, Elliott et al. (2021), Galeotti and Ghiglino (2021), Altinoglu and Stiglitz (2023), Jackson and Pernoud (2024), and Shu (2024) show that correlation of risks, expectation of bailouts, and correlated payoff structures could lead to a collective increase in risk-taking, resulting in systemic risk-shifting by agents in a financial network. Our result features similar mechanisms, demonstrating how endogenous risk-taking behavior can be exacerbated with an increase in collateral reuse.

2. Model of Contagion

We first present the contagion model based on Chang and Chuan (2024). The model has both the counterparty liability dimension as in Eisenberg and Noe (2001) and the assets that can be used as collateral, whose prices are endogenously determined. The main heterogeneity of interest comes from how agents are connected to each other through collateralized debt relationships.

2.1. Agents and Goods

There are three periods $t = 0, 1, 2$ and two goods, cash and an asset, denoted as e and h, respectively. Cash is the only consumption good, the numeraire good, and storable. The asset can be used as collateral at $t = 0$ and yields s amount of cash at $t = 2$. Agents gain no utility from just holding the asset. All agents know the value of the asset payoff s at $t = 1$; however, the asset payoff is realized at $t = 2$.

There are n different agents in the set $N = \{1, 2, \dots, n\}$. Agents are risk neutral, and their utility is determined by how much cash they consume at $t = 2$. Each agent is investing in a long-term *illiquid* investment project that will give ξ amount of cash at $t = 2$ if it is held until maturity. The payoff from this long-term project is not pledgeable. At $t = 1$, agent j can (partially) liquidate the project by $l_j \in [0, \xi]$ amount to receive the scrap value of ζl_j in cash, where $0 \leq \zeta < 1$ represents the liquidation efficiency. For simplicity, we focus on the case in which $\zeta \to 0$, so liquidation yields infinitesimally small amount of cash.[12] Therefore, agents liquidate the entire ξ when they default, and the total amount of liquidation of long-term projects due to defaults corresponds to the total loss of social surplus, as we show later in Lemma 1.

All information is common knowledge, and the markets for both goods are competitive Walrasian markets. Thus, agents are price-takers with symmetric information. The price of the asset is p_t for $t = 0, 1, 2$. From now on, we use p instead of p_1 for the price of the asset at $t = 1$, as our main focus is analyzing the contagion in $t = 1$.

[12] All the main results remain to hold even when this assumption is relaxed as shown in Chang and Chuan (2024).

2.2. Collateralized Debt Network

At $t = 0$, each agent $j \in N$ holds e_j amount of cash and h_j amount of assets, which are exogenously given, until $t = 1$. We endogenize the amount of cash and the amount of long-term investment later in Section 5. At $t = 1$, agents can buy or sell the asset in a competitive market. Also at $t = 0$, agents borrow or lend cash using assets as collateral. All borrowing contracts are a one-period contract between $t = 0$ and $t = 1$ and are exogenously determined. A borrowing contract consists of the promised cash payment, the ratio of collateral posted per unit of promised cash, and the identities of the borrower and the lender. Denote d_{ij} as the promised cash amount to pay at $t = 1$ to lender i by borrower j. Denote c_{ij} as the collateral ratio per one unit of promised cash. If borrower j pays back the full amount of promised d_{ij}, then the lender returns the collateral in the amount of $c_{ij}d_{ij}$. Otherwise, the lender either keeps or liquidates the collateral, and the cash value of the collateral is $c_{ij}d_{ij}p$. The lender has to return any excess value of the collateral to the borrower, $c_{ij}d_{ij}p - d_{ij}$, if there is any. Normalize $c_{ii} = d_{ii} = 0$ for all $i \in N$ without loss of generality.

Define $C = [c_{ij}]$ and $D = [d_{ij}]$ as the matrices of collateral ratios and promised debt payment amounts, respectively. A collateralized debt network is a weighted, directed multiplex graph that is formed by the set of vertices N and links with two layers C and D. A (collateralized debt) network can be summarized by a double (C, D) given at $t = 0$. Denote the total inter-agent liabilities of agent j as $d_j \equiv \sum_{i \in N} d_{ij}$.

We assume that *collateral constraints* and *resource constraints* hold, implying

$$\sum_{k \in N} c_{jk}d_{jk} + h_j \geq \sum_{i \in N} c_{ij}d_{ij} \qquad \forall j \in N, \tag{1}$$

$$\sum_{i \in N} h_i \geq \sum_{i \in N} c_{ij}d_{ij} \qquad \forall j \in N. \tag{2}$$

The collateral constraint, (1), means that the total amount of collateral a borrowing agent j posts cannot exceed the amount of assets agent j has—either from other agents' collateral that agent j has received as a lender or from the amount of assets agent j purchased outright. This collateral constraint allows for reuse (rehypothecation) of collateral.[13] The resource constraint, (2), means that the total amount of assets an agent is posting cannot exceed the total amount of assets in the economy.[14]

Each agent can be hit by a negative liquidity shock in cash with an absolute value of $\epsilon > 0$ at $t = 1$. Agents should pay the liquidity shock first before paying other agents. We interpret ϵ as a senior debt payment to external creditors, who also have linear utility.[15] A realized state of the liquidity shocks is $\omega \equiv (\omega_1, \omega_2, \ldots, \omega_n)$, and the set of all possible states is Ω. For example, $\omega_j = 1$ if agent j is under a liquidity shock, and $\omega_j = 0$ otherwise. Liabilities other than the liquidity shock are all equal in seniority. Hence, any net wealth left after paying the liquidity shocks will be distributed across all agents on a pro rata basis.

2.3. Timeline and Social Surplus

The timeline of the model, depicted in Figure 1, is the following. Agents' cash and asset holdings as well as debt network are exogenously given at $t = 0$. At the beginning of $t = 1$, asset payoff s is publicly revealed, and liquidity shock arrivals ω are realized. Agents pay their debt, and collateral is returned to the borrower, if not defaulted. If an agent defaults, the agent liquidates its long-term projects, and any remaining assets in the agent's balance sheet will be distributed to all other creditors on a pro rata basis. At the end of $t = 1$, all agents' final asset holdings are determined. At $t = 2$, the payoff of the asset is realized, and agents consume all the cash they have and gain utility.

We define the formal definition of our equilibrium concept, *full equilibrium*, in the next section. In any full equilibrium, define the utilitarian social surplus as the sum of the payoffs

[13] The same collateral can be reused an arbitrary number of times, generalizing typical models of reuse of collateral (Gottardi et al., 2019; Infante, 2019; Park and Kahn, 2019; Infante and Vardoulakis, 2021).

[14] If a resource constraint is not present, then there can be a spurious cycle of collateral justifying any arbitrary amount of collateral circulating in the economy.

[15] Alternative interpretations of negative liquidity shocks include lower-than-expected short-term returns, a sudden increase in deposit withdrawals, wage expenses, taxes, and fines.

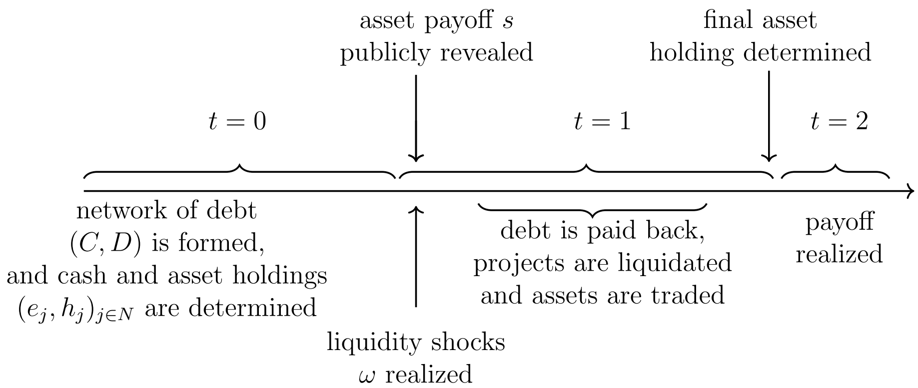

Figure 1: Timeline of the model

of all agents at $t = 2$,

$$U = \sum_{i \in N} (\pi_i + T_i),$$

where $T_i \leq \epsilon$ is the transfer from agent i to its senior creditors (liquidity shock), which simply transfers to $t = 2$, and π_i is the agent's long-term profit evaluated at $t = 2$.[16]

Lemma 1. *For any full equilibrium, the social surplus in the economy is equal to*

$$U = \sum_{j \in N} (e_j + h_j s + \xi) - \sum_{i \in N} l_i.$$

All proofs are relegated to Appendix C. Lemma 1 clarifies that the source of social inefficiency comes from the early liquidation of the long-term project, caused by insufficient liquidity.[17]

[16] This definition of social surplus is consistent with that of Acemoglu et al. (2015).

[17] Technically, agents may liquidate their long-term projects early if the asset price is low enough to make an asset purchase more profitable than the long-term project. By focusing on the case in which ζ is infinitesimally small, we eliminate such cases. See Chang and Chuan (2024) for detailed analysis with a more general setup.

2.4. Full Equilibrium

In this section, we define the equilibrium concept and its relevant elements following Chang and Chuan (2024).

2.4.1. Payment Rules

Let $x_{ij}(p)$ denote the actual payment net of collateral to agent i from agent j when the asset price is p at $t = 1$. This payment will be defined later in (5). The argument p is often omitted from now on. The total *cash inflow* of agent j is

$$a_j(p) \equiv e_j + h_j p + \sum_{k \in N} c_{jk} d_{jk} p - \sum_{i \in N} c_{ij} d_{ij} p + \sum_{k \in N} x_{jk}(p), \tag{3}$$

where the second term is the market value of j's direct asset holdings; the third and fourth terms are the market values of collateral assets posted by j's borrowers and posted to j's lenders, respectively; and the fifth term is the actual payment net of collateral from j's borrowers. The total amount of liabilities net of collateral posted for agent j is

$$b_j(p, \omega) \equiv \sum_{i \in N} (d_{ij} - c_{ij} d_{ij} p) + \omega_j \epsilon, \tag{4}$$

which can be considered the required total *cash outflow*. Note that the first term of the right-hand side can be negative if the contract is overcollateralized—that is, $d_{ij} < c_{ij} d_{ij} p$. The function argument ω is often omitted for simplicity from now on.

If $a_j(p) \geq b_j(p)$, then $x_{ij} = d_{ij} - c_{ij} d_{ij} p$ for any $i \neq j$, i.e., agent j is solvent and pays its obligations in full. If $a_j(p) < b_j(p)$, then agent j defaults and liquidates the long-term project.[18] See Appendix B for the full description of the agent's optimization problem at $t = 1$ and derivation of its solution.

[18] A debt contract references the market price of the collateral, not the fair value s, regardless of whether the lender sells off the asset. While we do this to reflect the default and settlement procedures in the real world, lenders would not accept the collateral at par to begin with, as they would rather demand the full debt amount in cash to purchase potentially cheaper assets priced at market value at $t = 1$.

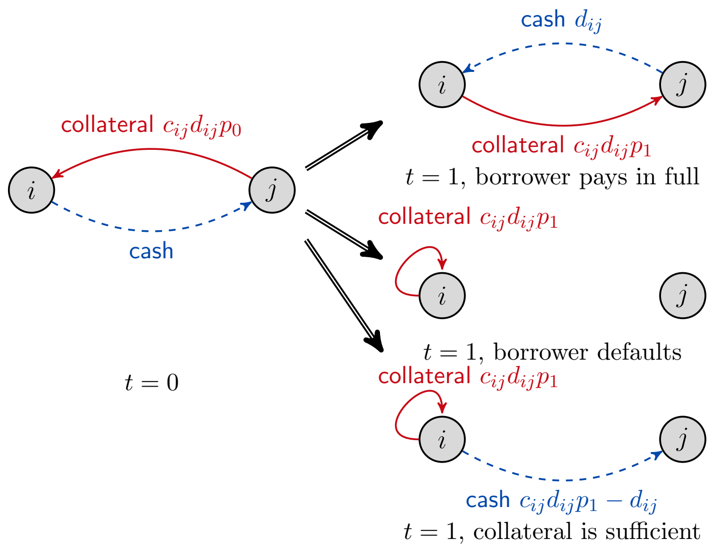

Figure 2: Flows of cash and collateral for three cases

Note: The two nodes, i and j, represent the lender and borrower of a contract, respectively. The blue dashed arrows represent flows of cash, and the red arrows represent flows of collateral. The left figure shows the flows in $t = 0$. The top-right figure shows the flows in the case in which the borrower pays in full in $t = 1$, the middle-right figure shows the flows in the case with borrower default in $t = 1$, and the bottom-right figure shows the flows in the case in which the collateral value exceeds the payment in $t = 1$.

Given the payment rules, the actual payment to lender i from borrower j can be determined. If agent j can pay all of the obligations, then j can pay the promised amount such that $x_{ij}(p) = d_{ij} - c_{ij}d_{ij}p$ as lender i returns the collateral to borrower j, as depicted in the top right of Figure 2. If the total value of collateral $c_{ij}d_{ij}p$ is greater than the debt amount d_{ij}, then the actual net payment $x_{ij}(p)$ can be negative because the more valuable collateral is returned to the borrower from the lender's balance, as depicted in the bottom right of Figure 2.[19] Agent j *defaults* if the payment net of collateral is less than the promised payment, $x_{ij}(p) < d_{ij} - c_{ij}d_{ij}p$ for some $i \in N$. The extreme case is agent j being unable to

[19]The property of collateral directly covering the debt payment is important. For example, if netting the debt with collateral was not possible, then after a negative liquidity shock to the system, all assets posted as collateral would be put on fire sale as all agents liquidate their collateral simultaneously to raise cash for payment obligations. This version of the model is equivalent to having no collateral at all, as in Acemoglu et al. (2015), because collateral plays no direct role in shaping debt. Indeed, this is not the case in the real world, as market participants typically designate particular collateral and can effectively pay their obligations by giving up their collateral as previously described. In other words, collateral plays the role of money across the debt network when agents pay their liabilities.

pay the liquidity shock in which the actual payment will be $x_{ij}(p) = 0$, and the lender keeps the collateral, as depicted in the middle-right section of Figure 2. In an intermediate case, agent j can pay the liquidity shock but cannot pay the inter-agent debt in full. Under such a case, j's remaining wealth is paid out on a pro rata basis. This interaction is formulated as the following *payment rule*:

$$x_{ij}(p) = \min\left\{d_{ij} - c_{ij}d_{ij}p\ ,\quad q_{ij}(p)\left[a_j(p) + \sum_{i\in N}[c_{ij}d_{ij}p - d_{ij}]^+ - \omega_j\epsilon\right]^+\right\}, \tag{5}$$

where $[\cdot]^+ \equiv \max\{\cdot, 0\}$ and $q_{ij}(p)$ is a weight under the *weighting rule*

$$q_{ij}(p) = \frac{[d_{ij} - c_{ij}d_{ij}p]^+}{\sum\limits_{k\in N}[d_{kj} - c_{kj}d_{kj}p]^+} \tag{6}$$

for the pro rata basis. Note that if weights are not defined ($\sum\limits_{k\in N}[d_{kj} - c_{kj}d_{kj}p]^+ = 0$), the weighting rule is never used, because any lender will be paid in full.[20]

[20]The model incorporates the role of explicit collateral, as agents can settle the payments by giving up their collateral to their lenders. This is in line with the standard repo contracts such as the Securities Industry and Financial Markets Association's (SIFMA) Master Repurchase Agreement (MRA), used by most U.S. dealers, and the SIFMA/International Capital Market Association (ICMA) Global Master Repurchase Agreement (GMRA), used for non-U.S. repos (Baklanova et al., 2015). According to both the SIFMA MRA and SIFMA/ICMA GMRA, after determining the market value of the collateral, all repo exposures between the two counterparties are netted off, and whoever owns the residual sum must pay it by the next business day, including the interest on late payment. Hence, the lender has recourse to the borrower's balance sheet and can claim any payment due net of the market value of the collateral (Gottardi et al., 2019). The non-defaulting party may either immediately sell in a recognized market at prices the non-defaulting party reasonably deems satisfactory or give the defaulting party credit for collateral in an amount equal to the price obtained from a generally recognized source. The non-defaulting party may choose the latter option when the market is under stress, as additional sales of collateral would only decrease the price and the value of the collateral is greater than the current market price.

2.4.2. Fire Sales and Market Clearing

For a given network and state realization, or economy, $(N, C, D, e, h, s, \omega)$, where $e \equiv (e_1, e_2, \ldots, e_n)$ and $h \equiv (h_1, h_2, \ldots, h_n)$, the *net wealth* of agent j is

$$m_j(p) \equiv a_j(p) - b_j(p) = e_j + h_j p + \sum_{k \in N} c_{jk} d_{jk} p - \omega_j \epsilon - \sum_{i \in N} d_{ij} + \sum_{k \in N} x_{jk}(p) \tag{7}$$

under the payment rules. Equation (7) consists of the following: cash holdings, the market value of the asset holdings, the market value of collateral received, a negative liquidity shock, the total payment to be paid, and the actual net payment received. If $m_j(p) < 0$, then agent j defaults.

Denote the *fire-sale* amount of agent j as

$$\phi_j(p) = \min \left\{ \left[h_j p - m_j(p) \right]^+, h_j p \right\}. \tag{8}$$

If agent j's net wealth subtracted by j's asset holdings, $m_j(p) - h_j p$, is enough to cover all of the payments (positive), then $\phi_j(p) = 0$ (no fire sales). If agent j's net cash flow is not enough without the sale of asset holdings, then $\phi_j(p) > 0$. If the cash shortage exceeds the total asset holdings ($h_j p - m_j(p) > h_j p$), then the fire-sale amount reaches its upper bound $\phi_j(p) = h_j p$. Note that a defaulting agent would always have $\phi_j(p) = h_j p$.

Recall that the asset market is a perfectly competitive Walrasian market. Unless there is not enough cash to purchase all of the asset sales in the market at the asset's fundamental value s, the market price will always be the fair value s. However, if there is not enough cash in the market, then the asset price can go below its fundamental value as $p < s$, as the market clearing condition becomes a *cash-in-the-market pricing* condition à la Allen and

Gale (1994). The *market clearing condition* can be summarized as

$$\begin{aligned} \sum_{j \notin \mathcal{D}(p)} [m_j(p) - h_j p]^+ &= \sum_{i \in N} \phi_i(p) \qquad \text{if} \;\; 0 \leq p < s \\ \sum_{j \notin \mathcal{D}(p)} [m_j(s) - h_j s]^+ &\geq \sum_{i \in N} \phi_i(s) \qquad \text{iff} \quad\; p = s, \end{aligned} \tag{9}$$

where $\mathcal{D}(p)$ is the set of agents who default under price p. We assume $s < (n-1)e_0/h_0$, i.e., the asset price is at its fundamental value when only one agent defaults.

2.4.3. Equilibrium Properties

For the given rules, the definition of the equilibrium is as follows.

Definition 1. *For given $(N, C, D, e, h, s, \omega)$, if payments $\{x_{ij}(p)\}$ satisfy the payment rule (5), net wealth $\{m_j(p)\}$ is determined by the net wealth equation (7), the fire-sale amount $\{\phi_j(p)\}$ is determined by equation (8), and price p clears the market as in (9), then $(\{x_{ij}\}, \{m_j\}, \{\phi_j\}, p)$ is a full equilibrium.*

In line with the literature, we refer to the interim equilibrium of our full equilibrium without the fire sales and market clearing conditions—that is, the payment decisions, $\{x_{ij}(p)\}$, which satisfy the payment rules, for a given asset price p—as the payment equilibrium of $(N, C, D, e, h, s, \omega)$ and p.

The following proposition from Chang and Chuan (2024) shows that the full equilibrium always exists.

Proposition 1 (Chang and Chuan (2024))**.** *For any given economy $(N, C, D, e, h, s, \omega)$, a full equilibrium always exists and is generically unique for a given equilibrium price. Furthermore, there exists a full equilibrium with the highest price among the set of full equilibria.*

Even though there could be multiple full equilibria, each equilibrium price has a (generically) unique payment equilibrium. Chang and Chuan (2024) focused on the unique maximum full equilibrium that has the highest market clearing price among the set of equilibria. In this paper, we explore all possible equilibria in the following Section 3.

3. Multiple Equilibria and Contagion

In this section, we study the properties of all equilibria in the baseline model. We show the existence of three equilibria for any network in the class of networks we focus on and analyze how contagion patterns and resulting social surpluses differ across these equilibria. Finally, we show the effect of collateral reuse on these equilibria.

3.1. Preliminaries

We focus on *regular* networks in which the total inter-agent claims and liabilities of all agents are equal, i.e., $\sum_{i \in N} d_{ij} = \sum_{i \in N} d_{ji} = d$ for all $j \in N$ for some $d \in \mathbb{R}^+$. Also, we assume that all agents hold the same amounts of cash and assets as $e_i = e_0 > 0$ and $h_i = h_0 > 0$ for all $i \in N$. Similarly, we assume that all agents have the same uniform collateral ratio, $c_{ij} = c$ for all $i, j \in N$. These homogeneity assumptions guarantee that any variation in systemic risk is due to the level of collateral and the interconnectedness of agents while abstracting away from effects from size, balance sheet, or hierarchical heterogeneity (Acemoglu et al., 2015; Chang and Chuan, 2024). For simplicity, we assume that the liquidity shocks are randomly received by only one agent, so $\omega_i = 1$ if agent i receives the shock, and $\omega_j = 0$ for all $j \in N, j \neq i$.

In this setup, the pattern of contagion depends on the level of collateral ratio c. In particular, there are two collateral thresholds, one that prevents any contagion at all, and the other prevents contagion through collateral price in the maximum equilibrium.

Proposition 2 (Collateral Ratio Thresholds). *If $c \geq \bar{c}(s, n) \equiv \frac{1}{s}$, then no agent defaults in the maximum equilibrium for any given network D. If $c \geq \underline{c}(s, n) \equiv \frac{d - (n-1)e_0 + h_0 s}{ds}$, then the asset price is $p = s$ in the maximum equilibrium for any given network D.*

Acemoglu et al. (2015) propose a useful concept for analyzing contagion pattern:

Definition 2. *The harmonic distance from agent i to agent j is*

$$\mu_{ij} = 1 + \sum_{k \neq j} \left(\frac{d_{ik}}{d} \right) \mu_{kj}, \tag{10}$$

with the convention that $\mu_{ii} = 0$ for all i.

As noted by Acemoglu et al. (2015) and Chang and Chuan (2024), the harmonic distance from agent i to agent j depends not only on how far each of its immediate borrowers is from j, but also on the intensity of their liabilities to i, by d_{ik}/d. Chang and Chuan (2024) utilizes this measure to derive implications on contagion in collateralized debt networks.

Proposition 3 (Chang and Chuan (2024))**.** *Suppose that agent j is under a negative liquidity shock of $\epsilon > ne_0$. Then, there exists $\mu^*(p) = (d - cdp)/(e_0 + h_0 p)$, and the following holds:*

1. *If there is a nonempty set $\mathcal{S}$ such that agent $i \in \mathcal{S}$ does not default, then the equilibrium price is either $p = s$ or determined by*

$$\mathbb{1}' G \mu_{sj} = \frac{d - cdp}{e_0 + h_0 p} \mathbb{1}' G \mathbb{1} + \frac{n h_0 p}{e_0 + h_0 p}, \tag{11}$$

   where μ_{sj} is the vector of harmonic distances from agents in $\mathcal{S}$ to j, G is a $|\mathcal{S}| \times |\mathcal{S}|$ non-singular M-matrix,[21] and $\mathbb{1}$ is a vector of ones. Furthermore, if $\mu_{ij} < \mu^*(p)$, then agent i defaults.

2. *If all agents default, then the equilibrium price is $p = 0$ and $\mu_{ij} < \mu^*(0)$ for all i.*

3. *If $\mu^*(p) < 1$ for the equilibrium price p, then no other agents default.*

We restrict our attention to cases in which contagion is meaningful, while collateral can play its role in mitigating contagion but does not fully prevent any contagion. Thus, we

[21] If matrix A can be expressed as $A = sI - B$, $s \geq \rho(B)$, $B \geq 0$, where $\rho(B)$ is the spectral radius of B, then A is an M-matrix (Berman and Plemmons, 1979, p. 133). An M-matrix is non-singular if $s > \rho(B)$.

assume $\epsilon > ne_0$, liquidity shock is significant enough, $\underline{c}(s,n) \leq c \leq \bar{c}(s,n)$, collateral is sufficiently large but not exceedingly large,[22] and $d > (n-1)e_0$, the inter-agent liabilities are large enough (agents are significantly connected). Proposition 3 shows that if $c < \underline{c}(s,n)$, then the equilibrium asset price p can be lower than its fundamental value s even in the maximum equilibrium. In other words, we focus on the case in which the size of the liquidity shock is large enough to result in full contagion in the network, while the amount of collateral is large enough to prevent the asset price from falling below its fundamental value s in the best equilibrium. Finally, we focus on the set of networks such that any network has harmonic distances smaller than $\mu^*(0)$ when $p = 0$. In other words, all agents default when $p = 0$, but because of the region of c we are focusing on, $p = s$ in the maximum equilibrium. We do not focus on disjointed networks, as they have limited relevance to reality and financial stability concerns.[23]

Under such a setup, the collateral ratio c is equivalent to the *degree of collateral reuse* in the network. This is because collateral circulation depends only on c while everything else is fixed. Hence, from now on we will refer to c as the degree of collateral reuse.

3.2. Characterization of Multiple Equilibria

First, we show the existence and characterization of multiple equilibria.

By Lemma 6 of Chang and Chuan (2024), the market clearing asset price can be represented as

$$p = \min \left\{ \frac{\sum_{j \in N} [m_j(p)]^+}{\sum_{j \in N} h_j}, s \right\}. \tag{12}$$

Therefore, (12) implies there are generally three different cases of market clearing price: $p = s$ when the upper bound of the asset price is binding, $p = 0$ when the lower bound of

[22] If $c > \bar{c}(s,n)$, then there will be no default at all, as even the shocked agent can pay its debt by giving up its collateral.

[23] Alternatively, our results can be applied to each of the components of a network that has multiple disjoint components.

the asset price is binding, and $p = \dfrac{\sum_{j \in N} [m_j(p)]^+}{nh_0}$ when the asset price is determined by cash-in-the-market pricing. We first show that these three different equilibria exist in the class of networks we focus on.

Proposition 4 (Three Unique Equilibria)**.** *If $c \geq \underline{c}(s, n)$ and $\mu_{ij} < \mu^*(0)$ for any $i, j \in N$, then three different equilibria exist generically:*

1. ***the maximum equilibrium*** *is when $p = s$ with the least number of defaults;*

2. ***the minimum equilibrium*** *is when $p = 0$ with all agents default; and*

3. ***the intermediate equilibrium*** *is when $0 < p < s$ such that the market clearing condition implies cash-in-the-market pricing, i.e., $p = \dfrac{\sum_{j \in N} [m_j(p)]^+}{nh_0}$.*

The first important property of these multiple equilibria is the uniqueness of the intermediate equilibrium. This is because the aggregate net wealth $\sum_{j \in N} [m_j(p)]^+$ crosses the value of the total assets nh_0p only once before reaching $p = 0$ (see Lemma 4 in Appendix A.2 and the proof of the proposition in Appendix C.4). Therefore, we only need to focus on how this unique intermediate equilibrium varies across different networks D and different degrees of collateral reuse c. Figure 3 provides a visual illustration of this result.

One of the most interesting properties of these multiple equilibria is their self-fulfilling aspect. This property aligns well with the recent literature on safe assets, for example, Brunnermeier, Merkel, and Sannikov (2024), as even the price of a safe asset (such as the asset in our model that has no uncertainty in its fundamental value) depends on the self-fulfilling expectations, and thus, could be fragile. If the market price remains high as $p = s$, then collateral is sufficient to cover counterparty default losses. Therefore, there is limited amount of fire sales and the remaining agents have enough cash to purchase the small amount of asset on sale at its fundamental value s. However, if the market price plummets to $p = 0$, then collateral does not cover any counterparty default losses (even though its fundamental value remains to be s). Therefore, the liquidity shock spreads to all agents in the network

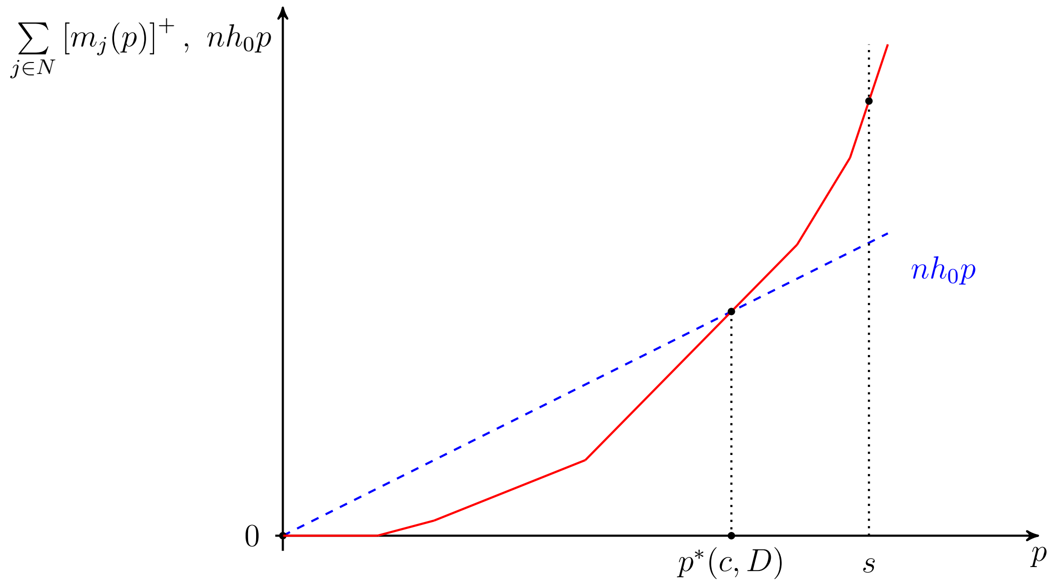

Figure 3: Illustration of multiple equilibria

Note: The x-axis represents asset price p, and the y-axis is the resulting values of the aggregate net wealth and the aggregate supply. The blue dashed line represents the linear function of the value of the aggregate supply nh_0p. The red piecewise linear function represents the aggregate net wealth. The slope of the aggregate net wealth function increases as the number of defaults decreases by Lemma 4 in Appendix A. For the corresponding price at each inflection point of the aggregate net wealth function an agent defaults just below that price. At $p = s$, the aggregate net wealth is above the value of the aggregate supply, so the maximum equilibrium with $p = s$ exists. At $p^*(c, D)$, the two functions intersect, implying that is the intermediate equilibrium. At $p = 0$, all agents default, resulting in the aggregate net wealth of zero, implying that the minimum equilibrium with $p = 0$ exists.

through counterparty default losses, causing all of them to default. Thus, no agent has extra cash to purchase the assets on sale at a positive price, i.e., $p = 0$.

The case of the intermediate equilibrium is more subtle. Even though there can be additional defaults under the intermediate equilibrium compared with the maximum equilibrium, the intermediate equilibrium can still arise even if no additional agents default. In such a case, the market clearing condition could hold in equality $p < s$, which equates to the value of the aggregate net wealth and the value of the total assets. Thus, the asset price can arbitrarily become either $p = s$ or $p < s$ even without additional defaults.

Finally, we emphasize that the source of self-fulfilling defaults and multiplicity of equilibria in our model is fundamentally different from the forces of self-fulfilling insolvencies

Asset Price	Net Debt	Aggregate Net Wealth	Fire-Sales Amount	Liquidations
1	0	16	2	10
0.25	7.5	2.5	8	30
0	10	0	10	50

Table 1: Three equilibria with 5 agents in a ring network

and multiplicity of equilibria in typical models in the literature, such as the one in Jackson and Pernoud (2024). Jackson and Pernoud (2024) propose a model in which self-fulfilling insolvencies and multiplicity arise if a certain type of dependency cycles exists in the network. Such a self-fulfilling insolvency can arise in their model due to the assumption of failure cost, which further erodes the net wealth of a defaulting agent once the agent hits the default threshold. In contrast, our model does not incorporate any additional decline in net wealth other than the deadweight loss of liquidating long-term projects, which cannot be used to pay others anyway. Therefore, our model does not allow multiplicity of equilibria due to self-fulfilling insolvencies due to exogenous failure costs. The source of equilibrium multiplicity in our model is the interaction between collateral asset price and defaults and payments in the network. Hence, we are showing a novel source of multiplicity of equilibria.

3.2.1. Example

Suppose there are 5 agents that form a ring network in which agent 1 owes d to agent 2, who owes d to agent 3, and so on, with agent 5 owing d to agent 1. Each agent is endowed with 2 assets ($h_0 = 2$), 2 units of cash ($e_0 = 2$), and an investment project valued at $\xi = 10$ at $t = 0$. The asset has a fair value of $s = 1$, and its price p is determined as in (9). Each agent owes a total debt amount of $d = 10$ to the next agent along the ring and posts a collateral amount of $cd = 10$, so the debt is fully collateralized. We refer to $d - cdp$ as net debt. Suppose that only agent 1 is under a large liquidity shock of $\epsilon = 15$.

Table 1 summarizes three different equilibria in this example. We describe these equilibria in more detail in the following paragraphs.

For the maximum equilibrium, suppose $p = s = 1$ and therefore, the net debt amount is

$d - cdp = 0$. Agent 1 liquidates, in face of the liquidity shock, and has 0 net wealth. Because the remaining agents have no additional payments to make, they each have a net wealth of $e_0 + h_0 p = 4$. Hence, the aggregate net wealth is 16, which, when divided by 10 (the total assets in the market) yields $\frac{16}{10} > p = 1$, meeting the market clearing condition. Therefore, the maximum price $p = s = 1$ is an equilibrium.

For the intermediate equilibrium, as depicted in the left panel of Figure 4, suppose $p = 0.25$ and the net debt amount is 7.5. Agent 1 still defaults with 0 remaining wealth. But now, agent 2's remaining wealth is $e_0 + h_0 p = 2.5$. Since $2.5 < 7.5$, agent 2 cannot fulfill her debt obligation, defaults, and pays agent 3 her remaining wealth of 2.5 who now has a remaining wealth of 5. Similarly, because $5 < 7.5$, agent 3 defaults and passes her remaining wealth to agent 4. Agent 4 has a remaining wealth of 7.5 and is able to pay agent 5 in full. Agent 5's net wealth after payment to agent 1 is 2.5, which is also the aggregate net wealth since agents 1-4 have 0 net wealth. Dividing 2.5 by 10, the market price is $p = 0.25$, which is the initial starting price. Thus, $p = 0.25$ is also an equilibrium. Because 3 agents are defaulting, they also liquidate their long-term projects, i.e., the total amount of liquidations is $3 \times \xi = 30$.

For the minimum equilibrium, as depicted in the right panel of Figure 4, suppose $p = 0$ and the net debt amount is 10. Agent 1 defaults and pays agent 2 nothing. Agent 2's wealth comprises of only her cash $e_0 = 2$, which is not enough to pay her debt since $2 < 10$. She defaults and pays her remaining wealth to agent 3 whose remaining wealth is now 4. Similarly, agents 3, 4, and 5 default because none are able to fulfill their debt liability of 10. Since everyone defaults, the aggregate net wealth is 0 and also total liquidations are 50. Therefore, the lowest (minimum) price is also an equilibrium.

3.3. Collateral Reuse and Changes in Equilibria

Now, with the full characterization of the multiple equilibria in collateralized debt networks, we examine how changes in collateral reuse affect all equilibria. By Proposition 4,

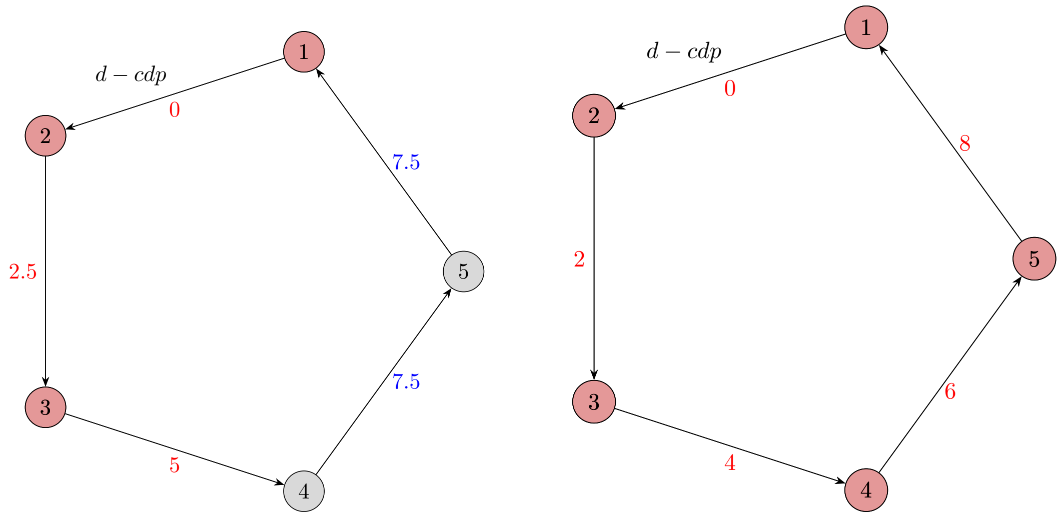

Figure 4: The intermediate equilibrium and the minimum equilibrium

Note: Each node in the figure represents an agent. Red nodes represent defaulting agents and gray nodes represent solvent agents. Arrows represent net debt obligations $d - cdp$, and the numbers next to arrows represent the actual payment amounts. Red numbers represent amounts below $d - cdp$, and blue numbers represent the full payment amount $d - cdp$. The left panel depicts the intermediate equilibrium, and the right panel depicts the minimum equilibrium.

we only need to focus on changes in the intermediate equilibrium. Denote the unique intermediate equilibrium for the given collateral ratio and network as $p^*(c, D)$. The following proposition states that the intermediate equilibrium price is decreasing in the degree of collateral reuse.

Proposition 5 (Collateral Reuse and Price)**.** *Suppose that all the assumptions in Proposition 4 hold. Then, the intermediate equilibrium price $p^*(c, D)$ is decreasing in the degree of collateral reuse c regardless of the network structure D.*

This somewhat counterintuitive result of Proposition 5 is a result of an intricate interaction between the intermediate equilibrium market clearing condition and the solvency condition. Figure 5 provides a visual illustration of the result. We describe the intuition behind this result in the following paragraphs.

First, note that an increase in c implies that agents in the network are receiving greater value of collateral, cp, if price p is fixed, resulting in increases in the net wealth of all agents. Then, the market clearing condition in the intermediate equilibrium does not hold

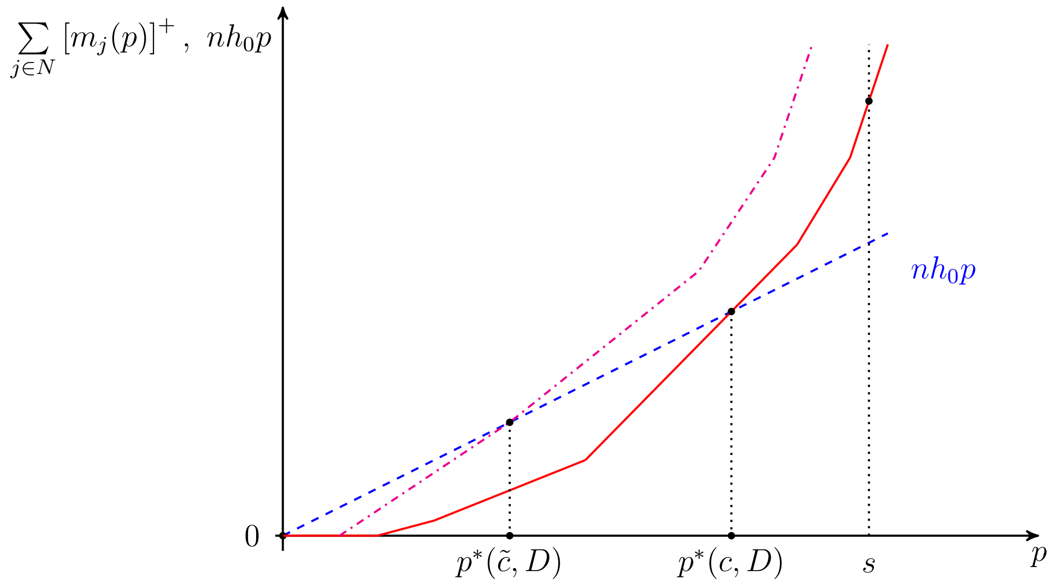

Figure 5: Effect of an increase in collateral reuse

Note: The x-axis represents asset price p, and the y-axis is the resulting values of the aggregate net wealth and the aggregate supply. The blue dashed line represents the linear function of the value of the aggregate supply nh_0p. The red piecewise linear function represents the aggregate net wealth under (c, D). The magenta dash-dotted piecewise linear function represents the aggregate net wealth under $(\tilde{c}, D)$, where $\tilde{c} > c$. The slope of the aggregate net wealth functions increases as the number of defaults decreases by Lemma 4 in Appendix A. For the corresponding price at each inflection point of the aggregate net wealth function an agent defaults just below that price. At $p = s$, the aggregate net wealth in both cases is above the value of the aggregate supply, so the maximum equilibrium with $p = s$ exists. At $p^*(c, D)$ and $p^*(\tilde{c}, D)$, the two aggregate wealth functions intersect with the value of the aggregate supply, respectively, implying they are the intermediate equilibrium prices. At $p = 0$, all agents default, resulting in the aggregate net wealth of zero in both functions, implying that the minimum equilibrium with $p = 0$ exists in both cases.

with equality if p remains the same. In order to satisfy the market clearing condition with equality, the market price p has to fall to offset the effect of higher c, i.e., larger amount of collateral received.

Second, recall that the solvency of agent i depends on its harmonic distance from the shocked agent j, μ_{ij}, which is determined by the network structure D, and the harmonic distance threshold $\mu^*(p) = (d - cdp)/(e_0 + h_0p)$, which is a function of p, by Proposition 3. Even though c increases, if p also decreases, the numerator of $\mu^*(p)$ may not decrease as much (if at all), while the denominator of $\mu^*(p)$ decreases, resulting in an overall increase of $\mu^*(p)$. Therefore, the solvency condition becomes harder to meet with the fixed values of harmonic distances.

Therefore, an increase in c results in a counterintuitive decline in p due to the cash-in-the-market clearing condition being satisfied with equality in the intermediate equilibrium, and a decline in price can only trigger more defaults (despite having higher c), which cause further declines in p. Although higher c can only increase p in the maximum equilibrium, the opposite is true for the intermediate equilibrium because of its market clearing condition. Moreover, this property holds regardless of the network structure D, although the exact degree of price decline may vary across networks. We highlight the inverse relationship between price and collateral ratio using numerical simulations depicted in Figure 6 for three network structures.[24] These results represent a special case in which the prices in the intermediate equilibria at each collateral ratio are the same across all three networks despite having different degrees of liquidations of the long-term investment.

3.4. Policy Implications of Multiple Equilibria

Our results on the multiplicity of equilibria provide a few important policy implications.

First, the result on the existence of three different equilibria in Proposition 4 implies that the equilibrium can suffer significant swings in social surplus based on changes in coordination of equilibrium. A huge amount of social inefficiency can be realized if agents in the network coordinate into the minimum equilibrium in which all agents default and inefficiently liquidate their long-term projects. Such a classic bank-run-like coordination failure has happened countless times throughout history, including most recently the banking crisis in 2023 (Rose, 2023). Therefore, our results show that such a run can materialize in collateralized debt networks, which do not resemble banks at all.

A natural policy implication for runs is the possibility of policy intervention that prevents the realization of a bad equilibrium, for example, the Bank Term Funding Program (BTFP)

[24] In the complete network, each agent owes $\frac{d-cdp}{n-1}$ to every other agent in the network. In the ring network, each agent $i \neq n$ owes $d - cdp$ to $i+1$, and agent n owes $d - cdp$ to agent 1. The γ-convex network $D^\dagger$ is a convex combination of the complete $\tilde{D}$ and ring networks $\hat{D}$, i.e., $D^\dagger = \gamma\tilde{D} + (1-\gamma)\hat{D}$.

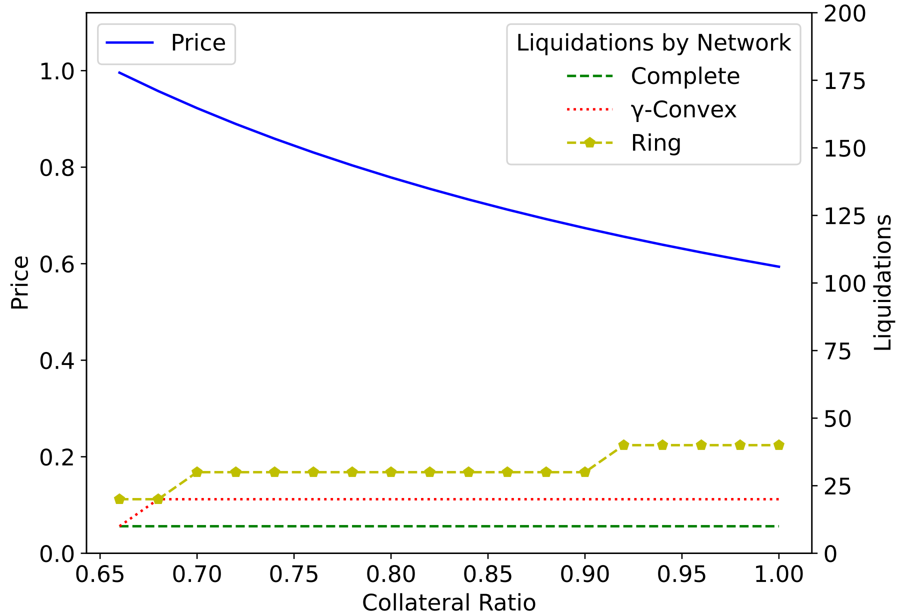

Figure 6: Numerically simulated intermediate price equilibria

Note: We simulate the asset price p and liquidations in the intermediate price equilibrium for each network for each collateral ratio from $\underline{c} = 0.66$ to $\bar{c} = 1$. The x-axis represents collateral ratio (c), the left-hand y-axis represents price (p) up to $s = 1$, and the right-hand y-axis represents aggregate liquidations of long-term projects, where $\xi = 10$ for each agent. As c increases, the intermediate equilibrium price p decreases, and liquidations monotonically increase across all networks. The ring network has the highest aggregate liquidations, while the complete network has the least amount of liquidations throughout the entire collateral range. The γ-convex network, with $\gamma = 0.5$, is in between the two networks in terms of liquidations.

created by the Federal Reserve during the crisis in 2023 (Glancy et al., 2024). A policy intervention like the BTFP can be effective in our model's context as well. A central bank can offer loans at par value s with an interest rate to agents in the network, preventing the asset price from falling below s. Alternatively, the central bank can act as a 'dealer of last resort', highlighted by Altinoglu and Chang (2022), and provide only a small amount of support to boost confidence in the market and bring the asset price to its fundamental value s. However, there is a difference in how a 'dealer of last resort' intervention works in our model, as the intervention works as a coordination device that prevents the minimum equilibrium from emerging instead of a signaling device to reduce information asymmetry.[25]

However, designing a policy intervention and implementing it in time might be challenging in the modern financial system, where realization of a bad equilibrium happens at a very fast pace (Kelly and Rose, 2025). The decisions on whether to intervene and when to intervene can be even more difficult when monitoring and surveillance are not only for examining business models of banks and depositor bases, as highlighted by Kelly and Rose (2025), but also for assessing the interconnectedness of the entire system and its interaction with asset markets. Therefore, establishing a standing facility such as the Federal Reserve's Standing Repo Facility (SRF) can be a more practical and effective solution to prevent the minimum (bad) equilibrium in our model. For example, we can introduce a new agent in the model, the central bank, which purchases the assets at $p = s$ up to a certain amount, preventing the bad equilibrium from emerging. That said, the optimality and optimal design of interventions such as the SRF require a careful cost-benefit analysis (Chang, Klee, and Yankov, 2025), which is beyond the scope of this paper.

4. Equilibrium Selection

With the characterization of the multiple equilibria in the baseline model, we extend our model to incorporate equilibrium selection to fully understand how the changes in the pattern of multiple equilibria alter the realization of equilibrium under equilibrium selection. In particular, we analyze the likelihood of the best (maximum) and worst (minimum) equilibria by incorporating both a global games model and a best response dynamics (BRD) model, separately in each subsection. We find that the main findings are robust to this modeling choice of equilibrium selection.

[25] Jackson and Pernoud (2024) suggest an alternative intervention, a bailout, to prevent self-fulfilling insolvencies along dependency cycles.

4.1. Global Games

In this section, we consider a model of equilibrium selection with global games techniques à la Carlsson and Van Damme (1993). However, there is a crucial difference between typical global games models, which require agents' payoff function to have strategic complementarities such as sequential servicing constraint (Morris and Shin, 1998; Goldstein and Pauzner, 2005; Kashyap et al., 2024), and fire sale models, which naturally have strategic substitutability, as one agent's sale of an asset could be a great opportunity for others to buy the asset. Kuong (2021) also points out this crucial difference and presents a model in which self-fulfilling fire sales occur due to an interaction between moral hazard of borrowers and risk aversion of lenders. In his model, strategic complementarity arises endogenously due to the moral hazard friction. In our model, strategic complementarity arises endogenously due to a different reason, which is the endogenous fire sales due to payment obligations that vary by contagion through networks. Therefore, we make a contribution to the global games literature by providing an alternative source of strategic complementarity through network contagion and endogenous market-clearing price.

We modify the baseline model to incorporate equilibrium selection through global games. In particular, we now assume that the asset payoff is stochastic. The value of asset payoff θ is uniformly distributed in $[0, \bar{\theta}]$, where $\bar{\theta} \geq s$.[26] Each agent receives a noisy signal of θ_i, $\theta_i = \theta + \psi_i$, where ψ_i is independently and identically uniformly distributed in $[-\psi, \psi]$. For ease of exposition and following the tradition in the global games literature, we consider the case in which agents are distributed on a continuum. In our context, we can consider each agent $i \in N$ as consisting of a continuum of identical agents with a mass of one, while each agent in the continuum receives a random signal of the asset payoff.[27]

Under this setup, agents with $\theta_i < -\psi$ sell the asset regardless of other agents' behavior, corresponding to the lower dominance region in typical global games models. However, the

[26] We can simply assume the upper bound is s, and all the results would remain to hold.

[27] Chang (2021) and D'Erasmo et al. (2024) use similar assumptions to maintain competitiveness of markets in their models.

behavior of the rest of the agents is hard to pin down simply by looking into the static best response of each agent based on their beliefs about others', because endogenous asset (fire) sale decisions and endogenous market-clearing asset price are determined simultaneously. Therefore, we focus on a threshold strategy that all agents follow. In equilibrium, the asset price will be endogenously determined by the marginal buyer, who is indifferent between selling the asset and buying the asset, as in Geanakoplos (1997).

First, we modify the endogenous fire-sale amount of agent j in (8) to incorporate the entire continuum of agents as j with additional signals. Denote the belief of the marginal buyer as θ^*, which is $\theta^* \in [\theta - \psi, \theta + \psi]$. Along the continuum, agents with signal less than θ^* sell the asset, while agents with signal greater than θ^* would only sell if they have to. Hence, for a fixed realized asset payoff θ, the proportion of $\dfrac{\theta^* - (\theta - \psi)}{2\psi}$ is the 'full sellers,' who would sell the assets they think the asset is overpriced, while the proportion of $\dfrac{\theta + \psi - \theta^*}{2\psi}$ is the 'reluctant sellers,' who would sell the asset only if necessary. See Figure 7 for a visualization of the proportions. Then, the endogenous fire-sale amount of agent j in the entire continuum is

$$\phi_j(p) = \min\left\{\frac{\theta + \psi - \theta^*}{2\psi}\left[h_j p - m_j(p)\right]^+ + \frac{\theta^* - (\theta - \psi)}{2\psi} h_j p, h_j p\right\}, \tag{13}$$

where the full sellers sell all the assets and the reluctant sellers sell only the necessary portion of their asset holdings. Similarly, only the reluctant sellers would participate in the market as buyers, so the demand side of the market clearing condition (9) becomes

$$\sum_{j \in N} \frac{\theta + \psi - \theta^*}{2\psi}\left[m_j(p) - h_j p\right]^+. \tag{14}$$

Under such a setup, an equilibrium is determined by three factors. First, the asset price p determines the degree of contagion in the network following the results of Proposition 3. Second, for the given degree of contagion, the market-clearing price p is determined. Third, agents who received signals lower than the threshold θ^* become the full sellers in the asset

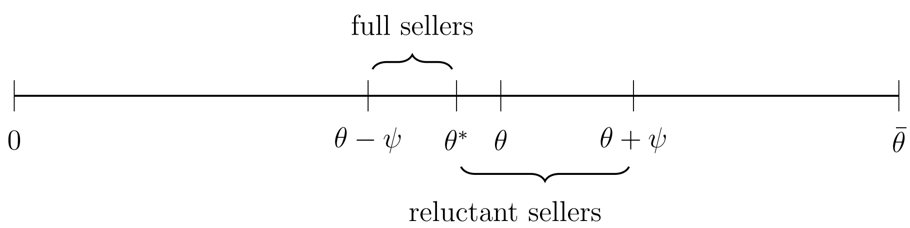

Figure 7: Heterogenous signals and distribution of sellers

market. Hence, the equilibrium under the global games setup should satisfy these three conditions simultaneously.

Interestingly, $\theta^* = \theta - \psi$ is always an equilibrium as long as $\theta - \psi \geq p^*(c, D)$, i.e., the minimum signal is greater than or equal to the intermediate equilibrium price in Proposition 4. In contrast, if $\theta + \psi < p^*(c, D)$, the unique equilibrium is the minimum equilibrium by contagion through defaults and fire sales. The following lemma formalizes this intuition.

Lemma 2. *In the global games setup, the following hold:*

1. *If $\theta - \psi \geq p^*(c, D)$, there is an equilibrium with the threshold for the marginal buyer is $\theta^* = \theta - \psi$, and the equilibrium price is $p > 0$.*

2. *If $\theta + \psi < p^*(c, D)$, then there is a unique equilibrium in which the equilibrium price is $p = 0$ and every agent defaults.*

Finally, we focus on the limit case of $\psi \to 0$ as in typical models in the global games literature. This negligible-noise limit case is not only useful for obtaining a tractable unique equilibrium (Carlsson and Van Damme, 1993), but also helpful in generating results that are robust to the distribution of noise (Morris and Shin, 2003). Then, the equilibrium is simplified as $\theta^* = \theta$ because all agents can correctly infer the true asset payoff from their signal. Hence, the unique threshold strategy equilibrium is simply determined by the realization of the asset payoff θ.

As previously mentioned, a crucial property of the Walrasian market clearing in our model is that there is strategic substitutability from competition—an additional amount of sales will decrease the market price, incentivizing all agents to buy more asset. This market force prevents the full sellers from selling a non-negligible amount in the equilibrium. However, if the realization of the asset payoff is sufficiently low, contagion through debt and collateral market comes into play, making the asset price to go zero and everyone defaults, i.e., $p = 0$.

The following proposition utilizes the unique intermediate equilibrium price defined by $p^*(c, D)$ in Section 3.3 and states its role in equilibrium selection.

Proposition 6 (Global Games Equilibrium)**.** *In the global games setup with $\psi \to 0$, the following hold:*

1. *For any $\theta \geq p^*(c, D)$, the equilibrium price is $p = \theta$.*
2. *For any $\theta < p^*(c, D)$, the equilibrium price is $p = 0$, and all agents default.*
3. *The likelihood of the minimum equilibrium with $p = 0$ decreases as the degree of collateral reuse c increases.*

Proposition 6 implies that the unique intermediate equilibrium price $p^*(c, D)$ plays the role of a threshold for equilibrium selection under global games. The main intuition comes from the proof of Proposition 4. The intermediate equilibrium price is the minimum price at which the aggregate net wealth is at or above the value of the aggregate assets. Hence, any realization of the asset payoff $\theta > p^*(c, D)$ can satisfy the second case of the market clearing condition (9) with strict inequality, i.e., the value of the aggregate net wealth exceeds the value of the aggregate assets. In contrast, any price below $p^*(c, D)$ cannot satisfy the market clearing condition with equality until p hits its lower bound, zero. Hence, the likelihood of the equilibrium price being $p = 0$ is the probability of θ being less than $p^*(c, D)$, i.e., $\dfrac{p^*(c, D)}{\bar{\theta}}$.

Moreover, since the intermediate equilibrium price $p^*(c, D)$ is decreasing in the degree of collateral reuse c by Proposition 5, the probability of the minimum equilibrium decreases, i.e., $\dfrac{\partial p^*(c, D)/\bar{\theta}}{\partial c} < 0$. Hence, the third statement of Proposition 6 holds. In other words, an increase in collateral reuse makes the bad equilibrium less likely to arise, improving financial stability.

4.2. Best Response Dynamics

In this section, we modify the baseline model to incorporate an alternative equilibrium selection model based on best response dynamics (BRD) following Gilboa and Matsui (1991), Matsui (1992), and Mäder (2024) in particular. There are a few advantages of BRD over global games approach. Under global games, agents decide their actions simultaneously based on beliefs. In an environment where agents can respond to each other's mutually observed actions, BRD can capture such interactions, while global games cannot. For example, after observing a sudden price decline, agents who initially decided not to participate in the market, can and optimally choose to participate in the market. Moreover, declines in prices trigger margin calls, leading to further decline of prices by fire sales. Therefore, the actions and responses in the context of our baseline model are well represented by the iterative interactions in BRD. As Mäder (2024) puts it, Walrasian tatônnement itself constitutes a type of BRD.

Following the BRD literature, we assume that all agents first simultaneously select tentative actions (buy or sell) that are publicly observable. Second, agents iteratively revise their tentative actions. The game ends when agents collectively no longer wish to revise—that is, converged to a Nash equilibrium— or when a predetermined time limit has been reached. Further, we assume that initial actions are chosen by randomly predetermined first-order beliefs (RB) of agents following Matsui (1992) and Mäder (2024).

Now we formally define the indeterminate game of BRD as $\Gamma = (N, \mathcal{A}, \pi)$, where each player i in the set of players N maximizes its profits $\pi_i = [\pi]_i$ by choosing a pure strategy (action) $\alpha_i \in A$ such that $A = [0, h_0 p]$ and $\mathcal{A} \equiv \Pi_{i \in N} A$. In other words, agents can choose how much assets to buy or sell after fire-selling the necessary amount for the given asset price. This strategy decision changes the amount of fire sales in (8) to

$$\phi_i(p) = \min\left\{[h_0p - m_i(p)]^+ + \alpha_i, h_0p\right\}. \tag{15}$$

The profit function π_i depends on the asset price, p, which will be determined by the underlying parameters of the baseline model and then the players' strategy profile $\alpha \in \mathcal{A}$. In particular, we follow Mäder (2024) and assume that the Walrasian auctioneer raises or lowers the price in response to excess demand or excess supply. Denote the game's best response correspondence as β. Following Matsui (1992) and Mäder (2024), assume that all players respond to any non-Nash-equilibrium profile by choosing a best response to the prevailing strategic state. Agents continuously revise their actions for each $\tau \in [0, 1]$, which is an internal timing for the BRD path to be realized. Then, a BRD path of the game Γ is a continuous and right-differentiable function $\alpha : [0, 1] \mapsto \mathcal{A}$ such that $\alpha'(\tau) \in \delta\left[\beta(\alpha(\tau)) - \alpha(\tau)\right]$ for some $\delta > 0$, for each $\tau \in [0, 1)$, and for any $\alpha(0) \in \mathcal{A}$. The revision speed δ controls the rate of convergence as well as the actual convergence, as it can prevent a case of oscillating best responses.

The basin of attraction $\mathcal{B}(\alpha^\dagger)$ of an equilibrium $\alpha^\dagger \in \mathcal{E}$, where $\mathcal{E}$ is the set of all equilibria, is the set of all initial conditions giving rise to at least one BRD path ending in $\alpha^\dagger$—that is, $\mathcal{B}(\alpha^\dagger) \equiv \left\{\alpha(0) \in \mathcal{A} | \alpha(1) = \alpha^\dagger\right\}$ with $\alpha'(\tau) \in \delta\left[\beta(\alpha(\tau)) - \alpha(\tau)\right]$ for some $\delta > 0$, for each $\tau \in [0, 1)$. Note that in our fire sales game, each player's best response is a singleton, i.e., β is a function, almost everywhere. Furthermore, we assume that the revision speed δ is chosen to make a unique equilibrium $\alpha^\dagger \in \mathcal{E}$ at the end of its BRD path for every strategic initial conditions, in other words, the game is a 'regular game' (Matsui, 1992).

Finally, we define the random beliefs (RB) as $\kappa : \Theta \mapsto \mathbb{A}$, where Θ is the initial realization of the probability space, and $\mathbb{A} \equiv \prod_{i \in N} \mathcal{A}_{-i}$ is the set of all possible first-order beliefs.

We apply the BRD-RB framework to our baseline model for equilibrium selection, which closely follows that of Mäder (2024). First, note that both the maximum equilibrium and the minimum equilibrium are stable Nash equilibria due to the Walrasian tatônnement and forced fire sales, respectively. In addition, the intermediate equilibrium with $p^*(c, D)$ is unstable in the sense that any deviation from $p^*(c, D)$ will lead the BRD path to converge to either equilibrium with s or 0. For example, if agents buy a little more than the intermediate equilibrium amount, then the price increases to $p > p^*(c, D)$, leading all the way to $p = s$. In contrast, a little downward perturbation to $p < p^*(c, D)$ would lead to a spiral of fire sales and defaults all the way to $p = 0$. Therefore, the following proposition holds.

Proposition 7 (Best Response Dynamics)**.** *In the BRD-RB setup, the likelihood of realization of the minimum (maximum) equilibrium is proportional to $p^*(c, D)$ $(s - p^*(c, D))$. Moreover, the likelihood of the minimum equilibrium decreases as the degree of collateral reuse c increases.*

Proposition 7 implies that the intermediate equilibrium price plays a crucial role in determining the basins of attraction. Any realization above $p^*(c, D)$ leads to a BRD path to the maximum equilibrium, implying that the basin of attraction is $\mathcal{B}(s) = (p^*(c, d), s]$. Therefore, the lower the intermediate equilibrium price, the larger the basin of attraction for the best (maximum) equilibrium. Even though the intermediate equilibrium itself is not a stable equilibrium, it serves an important role in determining *resilience* of the maximum equilibrium as Mäder (2024) highlighted. Figure 8 illustrates the result.

Finally, Proposition 7 implies that collateral reuse decreases the likelihood of the worst (minimum) equilibrium and improves financial stability. As the degree of collateral reuse increases, the financial system is less likely to be perturbed by random noise and end up in the socially inefficient fire sales equilibrium with $p = 0$.

4.3. Discussion on Equilibrium Selection

Both the global games model and BRD model lead to the same conclusion that an increase in the degree of collateral reuse improves financial stability. The main intuition

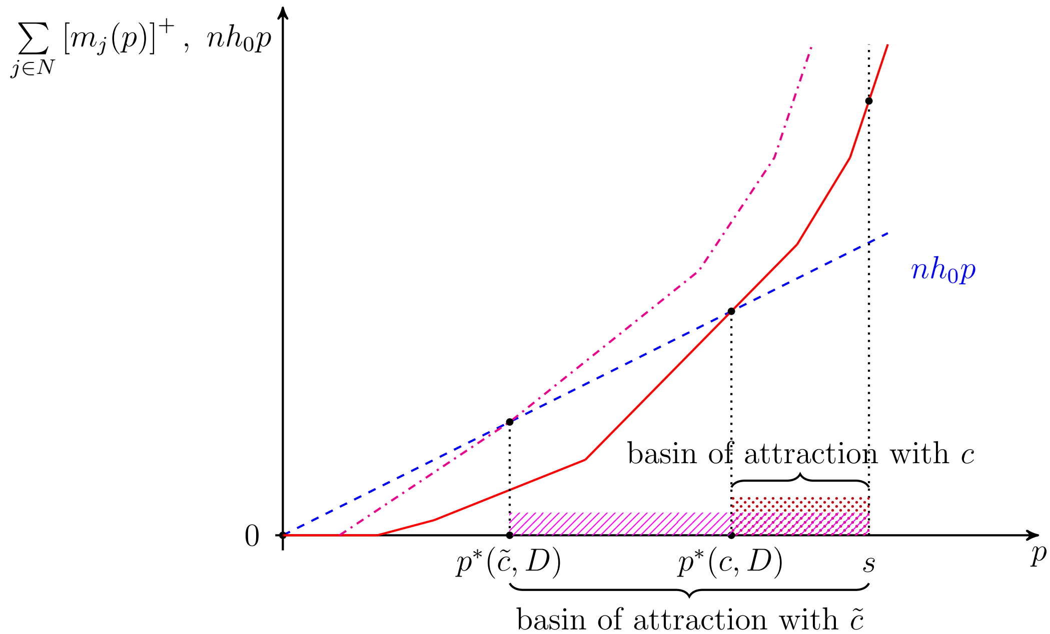

Figure 8: Effect of an increase in collateral reuse

Note: This figure builds upon Figure 5. The red dotted region on the x-axis between $p^*(c, D)$ and s represents the basin of attraction of the maximum equilibrium with the degree of collateral reuse c. The magenta lined region on the x-axis between $p^*(\tilde{c}, D)$ and s represents the basin of attraction of the maximum equilibrium with the degree of collateral reuse $\tilde{c}$, where $\tilde{c} > c$. For other details, see the note below Figure 5.

is the following. If more collateral guarantees inter-agent payments, then a slight increase in collateral asset price p leads to even greater net wealth for all agents. Thus, even at a lower starting price, the equilibrium dynamics in either global games or BRD would lead to realization of the best (maximum) equilibrium once the price enters a certain threshold. And that threshold is exactly the intermediate equilibrium price $p^*(c, D)$ we derived from the baseline model. Hence, an increase in c, which leads to lower $p^*(c, D)$, increases the likelihood of realization of the best equilibrium.

Proposition 6 and Proposition 7 generate a simple empirical prediction: all else being equal, there will be fewer crisis episodes in a market with a higher degree of collateral reuse.

Corollary 1 (Empirical Prediction 1)**.** *Holding all else equal, a higher degree of collateral reuse leads to a lower likelihood of crisis.*

5. Endogenous Risk Taking

From the equilibrium selection models, we found that collateral reuse only increases financial stability by reducing the likelihood of the bad equilibrium, holding all else equal. However, the degree of collateral reuse is often strongly correlated with other important factors such as leverage, collateral circulation and runs, lender default, and length of a lending chain (Chang, 2021; Infante and Vardoulakis, 2021; Maurin, 2022; Brumm et al., 2023). In this section, we extend our model further to incorporate agents' endogenous risk-taking at $t = 0$ to formally analyze the effect of an increase in collateral reuse. This extension differs from approaches in the aforementioned literature, as we focus on the direct effect of an increase in collateral reuse on agents' risk-taking decisions, while other papers have leverage and other factors change simultaneously with the degree of collateral reuse. Hence, we are isolating the effect of collateral reuse itself while holding other factors equal, implying that all other factors in the aforementioned papers would be added on top of the results we find here.

Now we assume that each agent has to optimally choose the size of the investment in the long-term project ξ at $t = 0$. Each agent has a cash endowment in the amount of e_{-1} at the beginning of $t = 0$. During $t = 0$, agents decide how much cash to hold e_0 and how much to invest ξ with the given cash endowment e_{-1}. For each unit of cash invested in the long-term project, an agent receives a payoff of R where $R > 1$. However, if an agent liquidates the long-term project due to default (which is the only case due to our simplifying assumption $\zeta \to 0$), the agent suffers a *liquidation cost* of $K(\xi)$, which is convexly increasing, i.e., $dK/d\xi > 0$ and $d^2K/d\xi^2 > 0$. The convexity assumption ensures that the optimal choice is not a corner solution. We can interpret this cost in various ways, such as a pecuniary cost from resulting litigation and punishment, an additional stake exceeding limited liabilities to have skin-in-the-game due to moral hazard concerns, and cost of effort for managing a project, which is compensated by monetary gains if the project succeeds. For ease of exposition, we use

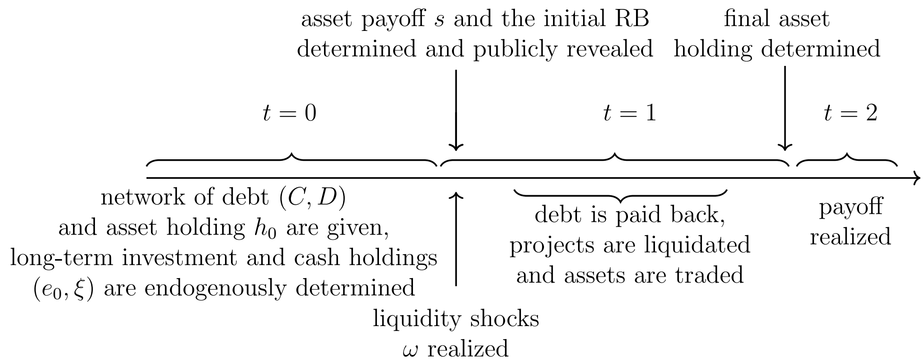

Figure 9: Timeline of the model with endogenous risk-taking

the BRD-RB setup with uniformly distributed RB for equilibrium selection, however, all the main results and mechanism remain the same under the global games setup. Figure 9 summarizes the timeline of the extended model.

For a given amount of long-term project investment ξ, each agent's expected cost of liquidation is

$$p^*(c, D)K(\xi) + (s - p^*(c, D))\frac{1}{n}K(\xi).$$

In other words, with probability $p^*(c, D)$, the economy arrives at the minimum equilibrium, where the agent liquidates and pays the liquidation cost $K(\xi)$. With probability $(s - p^*(c, D))\frac{1}{n}$, the economy arrives at the maximum equilibrium and the agent receives the high liquidity shock, thus the agent still pays $K(\xi)$. In terms of payoffs, the agent increases its return on cash by investing in the long-term project that yields R, so the net return is $R - 1$ per unit of cash invested in the long-term project.

We focus on symmetric equilibrium, as agents are homogeneous ex ante. Then, the resulting cash holdings and long-term investment amount are the solution to the following optimization problem holding others' decisions and resulting macrovariables as given:

$$\max_{e_0,\xi} - p^*(c,D)K(\xi) + (s - p^*(c,D))\left[\frac{n-1}{n}\left(\xi + \tilde{\pi}_j(e_0,D)\right) - \frac{1}{n}K(\xi)\right] \tag{16}$$

$$s.t. \qquad e_0 + \xi/R \leq e_{-1},$$

where $\tilde{\pi}_j(e_0, D) \equiv e_0 + h_0 s - (d - cds) + E_j\left[\sum_{j\neq i} x_{ji}(s)|\omega_j = 0\right]$ is the expected profit of agent j excluding the long-term investment payoff, when j is not the shocked agent.

At the optimum, the budget constraint is binding, hence, $e_0 = e_{-1} - \xi/R$. Then, the first-order condition of (16) with respect to ξ is

$$\left[-p^*(c,D) - (s - p^*(c,D))\frac{1}{n}\right] K'(\xi) + (s - p^*(c,D))\frac{n-1}{n}\left(1 - \frac{1}{R}\right) = 0, \tag{17}$$

which can be rearranged as

$$K'(\xi) = \frac{(s - p^*(c,D))\dfrac{n-1}{n}\left(1 - \dfrac{1}{R}\right)}{\left[p^*(c,D) + (s - p^*(c,D))\dfrac{1}{n}\right]}. \tag{18}$$

By convexity of the liquidation cost function $K(\xi)$, (18) pins down the unique solution ξ and corresponding e_0 to the optimization problem (16). Moreover, the numerator of the right-hand side of (18) is decreasing in $p^*(c, D)$, while the denominator of it is increasing in $p^*(c, D)$. Therefore, the optimal investment amount in the long-term project is increasing when the likelihood of the bad equilibrium decreases, which occurs when c increases. The following lemma summarizes this result.

Lemma 3. *Under the endogenous risk-taking setup, the long-term project investment ξ is increasing in c, if the resulting equilibrium values satisfy the parametric assumptions in the baseline model.*

Lemma 3 implies that agents take more risk by investing more cash into the long-term project, when collateral reuse increases. The intuition is straightforward. As agents know that the bad (minimum) equilibrium is less likely with an increase in the degree of collateral reuse c, they worry less about the liquidation cost of long-term projects and put more weight on the higher return from the long-term project investment. Thus, agents increase their long-term project investment amount. Note that the condition on the resulting equilibrium values in Lemma 3 can be easily satisfied in reverse order—that is, we can set the relevant parametric assumptions on c, d, and ϵ based on the equilibrium e_0 under the endogenous risk-taking model here.

Finally, the endogenous decrease in e_0 by Lemma 3, due to an increase in c, also has a side effect of increasing $p^*(c, D)$. However, this increase in $p^*(c, D)$ cannot exceed the initial decrease in $p^*(c, D)$ caused by c, as it would change the agents' optimal choice of e_0 again—that is, e_0 will increase after an increase in $p^*(c, D)$. Therefore, the resulting probability of the minimum (bad) equilibrium is still below that before the changes in c and e_0.

Proposition 6 and Proposition 7 imply that an increase in collateral reuse c improves financial stability by decreasing the likelihood of the bad (minimum) equilibrium. However, Lemma 3 implies that there is a countervailing effect, which is an increase in risk-taking activities and resulting liquidation amount and deadweight losses. Thus, an increase in the degree of collateral reuse leads to lower social surplus under the minimum equilibrium (during a crisis), although the minimum equilibrium (crisis) is less likely to occur. The following corollary summarizes this result.

Proposition 8. *An increase in the degree of collateral reuse (c) results in lower social surplus during a crisis (realization of the minimum equilibrium), although such crises are less likely to occur.*

This result aligns well with the recent literature on excessive risk-taking behavior by individuals and systemic risk. Elliott et al. (2021), Galeotti and Ghiglino (2021), Altinoglu and Stiglitz (2023), Jackson and Pernoud (2024), and Shu (2024) show that correlation of risks, expectation of bailouts, and correlated payoff structures can lead to a collective increase in risk-taking, resulting in systemic risk-shifting by agents in a financial network. Our results are closely related to this literature, demonstrating how endogenous risk-taking behavior can be exacerbated by an increase in collateral reuse.

In conclusion, we find that a greater degree of collateral reuse can harm financial stability when agents' portfolio choices are endogenously determined. In particular, it decreases the resiliency of the financial system as social welfare in the worst equilibrium falls due to greater liquidations of agents' investment projects. A natural empirical prediction from this result is the following: Holding all else equal, a higher degree of collateral reuse leads to fewer but more severe crisis episodes.

Corollary 2 (Empirical Prediction 2)**.** *Holding all else equal, a higher degree of collateral reuse leads to a lower likelihood of crisis but a greater severity of crisis (lower social surplus) when it occurs.*

6. Extensions

In this section, we discuss the possible extensions of our baseline model and robustness of our main results.

First, we can relax the assumption on the liquidation inefficiency of $\zeta \to 0$, i.e., a trivial liquidation of long-term projects. Then, agents can receive non-trivial liquidation proceeds, i.e., $\zeta > 0$, from partial liquidations of their long-term projects to pay their debt obligations. As Proposition 6 in Chang and Chuan (2024) show, all the main results on the pattern of contagion hold if the size of liquidity shocks is sufficiently large. Therefore, the results in this paper hold when we assume $\epsilon > n\,(e_0 + \zeta\xi)$.

Second, we can allow multiple agents to receive liquidity shocks simultaneously. As shown in the appendix of Chang and Chuan (2024), the main insights of the contagion model hold when liquidity shocks affect multiple agents simultaneously. Therefore, the results in this paper hold if the number of agents receiving liquidity shocks (ν) is greater than or equal to

1 where $\epsilon > ne_0/\nu$.

Third, we can relax the assumption on homogeneity of agent size. In particular, we can introduce a *scale factor* $\sigma_i > 0$ for all $i \in N$. Agent i's liabilities become $d_i = \sigma_i d$ and $\epsilon_i = \sigma_i \epsilon$ if $\omega_i = 1$, while agent i's assets become $e_i = \sigma_i e_0$, $h_i = \sigma_i h_0$, and $\xi_i = \sigma_i \xi$.

We define a size-adjusted harmonic distance similar to that of Acemoglu et al. (2015).

Definition 3. *The weighted harmonic distance from agent i to agent j is*

$$\hat{\mu}_{ij} = \sigma_i + \sum_{k \in N} \left(\frac{d_{ik}}{d} \right) \hat{\mu}_{kj}, \tag{19}$$

with the convention that $\hat{\mu}_{ii} = 0$ for all i.

Under this setup, the results similar to those of Proposition 3 hold.

Proposition 9. *Suppose that agent j is under a liquidity shock such that $\epsilon > ((e_0 + h_0 s)/\sigma_j) \sum_{i \in N} \sigma_i$. Then, the following hold:*

1. *Agent j defaults on its liquidity shock.*

2. *Agent $i \neq j$ defaults if and only if $\hat{\mu}_{ij} < \sigma_i \mu^*(p)$ for a given equilibrium price p.*

Proposition 9 implies that the pattern of contagion remains the same even with the existence of size heterogeneity. All the derivations and conditions in this paper continue to hold as long as agent i's balance sheet is scaled by σ_i for each $i \in N$ (and properly multiplying the matrix representations in the proofs with the appropriate $\sigma_{\mathcal{D}}$ vector for the set of defaulting agents $\mathcal{D}$).

We briefly discuss extensions that may not retain the main findings in our baseline model.

Relaxing the assumption on sufficiently high interconnectedness (i.e., $\mu_{ij} < \mu^*(0)$ for any $i, j \in N$), changes the possible set of equilibria. In particular, if a debt network has two (or more) components that have weak interconnections between them, then the minimum equilibrium with $p = 0$ may not exist. For concreteness, suppose that network D has a subset

$\mathcal{S} \subset N$ such that $\max\{d_{ij}, d_{ji}\} \leq \delta d$ for any $i \in \mathcal{S}, j \notin \mathcal{S}$, and for a small $\delta > 0$. Suppose that agent $j \in N \backslash \mathcal{S} = \mathcal{S}^C$ is hit by a large liquidity shock. All agents in $\mathcal{S}^C$ are subject to default, since they are the most exposed to agent j, but agents in $\mathcal{S}$ remain solvent, if δ is sufficiently small, i.e., $\mu_{ij} > \mu^*(0) = d/e_0$ for $i \in \mathcal{S}$. In such cases, the three unique equilibria result in Proposition 4 does not hold, as there is no minimum equilibrium with $p = 0$. Therefore, we interpret the results in this paper as being most relevant to sufficiently interconnected financial systems as opposed to fragmented ones.

Allowing for heterogeneous collateral ratios is challenging because the difference in payments depends on each individual collateral ratios at different price levels. Thus, there would be many price regions of contagion depending on the shock size and network structure. However, the model with heterogeneous collateral ratios can be solved numerically and could be very useful when combined with empirical data. We leave this direction of extension for future work.

Finally, we take the network structure (C, D) as exogenously given, as do many papers in the financial networks literature. This is mainly for tractability, as endogenizing financial networks, which have non-linear interactions due to defaults, can quickly become intractable as we add more features. Therefore, typical endogenous financial network models require many simplifying assumptions that may hamper comprehensive analysis of contagion in the network itself (e.g., Chang (2021)). That said, if network formation was endogenous, our results on increased systemic risk due to endogenous risk-taking behavior would be further amplified through the network formation incentives, as highlighted by Erol and Vohra (2022).

7. Conclusion

We developed a model of contagion through both debt and collateral market with multiple equilibria. We find that there are three very distinct equilibria for a given network, and the intermediate equilibrium price is decreasing in the degree of collateral reuse. We further extend the model to incorporate equilibrium selection through either global games or best response dynamics to analyze the implications of multiple equilibria and their changing values with respect to collateral reuse. We find that the likelihood of the worst (minimum) equilibrium is decreasing in the degree of collateral reuse. However, if we endogenize the agents' risk-taking choices, then an increase in collateral reuse increases the riskiness of agents' investments, leading to more severe crises with lower social surplus. Therefore, a higher degree of collateral reuse leads to fewer but more severe crises.

Overall, our results have three important policy implications. First, the degree of collateral reuse alone is not a concern for financial stability. Indeed, collateral reuse can alter other relevant factors that may ultimately influence financial stability, such as market liquidity and rate spreads due to safe asset scarcity. However, we find that the direct relationship between collateral reuse and financial stability as a result of self-fulfilling price drops and defaults is positive. Second, the degree of collateral reuse can still negatively impact financial stability through its indirect effects on agents' risk-taking choices, which become concerning in tail events. Therefore, monitoring the degree of collateral reuse is still important, as higher degrees of collateral reuse can be followed by greater leverage and other risk-taking behaviors of market participants. Third, supporting the price and liquidity of the collateral asset can be a critical intervention by a central bank, as such an intervention can significantly improve social surplus by eliminating a crisis of contagion through both debt and collateral price. Therefore, a policy intervention as a 'dealer of last resort' or a standing facility for repos and other SFTs can be an important policy tool, especially when collateral reuse is prevalent.

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Appendix (for online publication only)

A. Preliminaries

A.1. Matrix Representations

For a given network D, without loss of generality, suppose that agent 1 received the liquidity shock. Denote the set of defaulting agents (defaulting set) as $\mathcal{D}$ and the set of surviving agents (solvency set) as $\mathcal{S}$. Denote μ_{d1} as the $|\mathcal{D}| \times 1$ vector of harmonic distances from agents in $\mathcal{D}$ to agent 1, and μ_{s1} as the $|\mathcal{S}| \times 1$ vector of harmonic distances from agents in $\mathcal{S}$ to agent 1. First, note that the weights following the weighting rule do not change with the price p under the uniform collateral ratio because

$$q_{ij}(p) = \frac{d_{ij} - d_{ij}cp}{\sum_{k \in N} d_{kj} - d_{kj}cp} = \frac{d_{ij}(1-cp)}{\sum_{k \in N} d_{kj}(1-cp)} = \frac{d_{ij}}{d}.$$

Hence, we can represent the harmonic distance in (10) from agent i to agent j as

$$\mu_{ij} = 1 + \sum_{k \neq j} q_{ik}\mu_{kj}. \tag{20}$$

Using expression (20), the vectors of harmonic distances can be represented as

$$\mu_{d1} = \mathbb{1} + Q_{dd}\mu_{d1} + Q_{ds}\mu_{s1}, \tag{21}$$

$$\mu_{s1} = \mathbb{1} + Q_{sd}\mu_{d1} + Q_{ss}\mu_{s1}, \tag{22}$$

where $Q_{dd}, Q_{ds}, Q_{sd}, Q_{ss}$ are matrices of weights of liabilities for agents within $\mathcal{D}$, from $\mathcal{S}$ to $\mathcal{D}$, from $\mathcal{D}$ to $\mathcal{S}$, and within $\mathcal{S}$, respectively. The vector of total payments for defaulting agents $\mathbf{x}_d$ is determined as

$$\mathbf{x}_d = Q_{dd}\mathbf{x}_d + (d - cdp)Q_{ds}\mathbb{1}_d + (e_0 + h_0 p)\mathbb{1}_d$$

for a given price p, which can be solved as

$$\mathbf{x}_d = (I - Q_{dd})^{-1} \left[(d - cdp)Q_{ds}\mathbb{1}_d + (e_0 + h_0 p)\mathbb{1}_d\right], \tag{23}$$

because $I - Q_{dd}$ is an M-matrix, which is invertible (Chang and Chuan, 2024). The vector of net wealth of non-defaulting agents is

$$m_s \equiv Q_{sd}\mathbf{x}_d + Q_{ss}(d - cdp)\mathbb{1}_s + (e_0 + h_0 p)\mathbb{1}_s - (d - cdp)\mathbb{1}_s,$$

where we use the fact that non-defaulting agents are paying their debt in full as $d - cdp$. Plugging the payments in (23) into the net wealth vector implies

$$m_s = (e_0 + h_0 p)[\mathbb{1}_s + Q_{sd}(I - Q_{dd})^{-1}\mathbb{1}_d] - (d - cdp)\left[I - Q_{ss} - Q_{sd}(I - Q_{dd})^{-1} Q_{ds}\right]\mathbb{1}_s. \tag{24}$$

Summing up the net wealth over the surviving agents implies

$$\begin{aligned}
&\mathbb{1}_s' \left[I - Q_{ss} - Q_{sd}(I - Q_{dd})^{-1} Q_{ds}\right]\mathbb{1}_s = |\mathcal{S}| - \sum_{\substack{\forall i,j \in S \\ i \neq j}} q_{ij} - \mathbb{1}_s' Q_{sd}(I - Q_{dd})^{-1} Q_{ds}\mathbb{1}_s \\
\implies\ &\mathbb{1}_s' m_s = |\mathcal{S}|(e_0 + h_0 p) - |\mathcal{S}|(d - cdp) + (d - cdp)\sum_{\substack{\forall i,j \in S \\ i \neq j}} q_{ij} \\
&\quad + (e_0 + h_0 p)\mathbb{1}_s' Q_{sd}(I - Q_{dd})^{-1}\mathbb{1}_d + (d - cdp)\mathbb{1}_s' Q_{sd}(I - Q_{dd})^{-1} Q_{ds}\mathbb{1}_s \\
&\quad = |\mathcal{S}|(e_0 + h_0 p) - |\mathcal{S}|(d - cdp) + (d - cdp)\sum_{\substack{\forall i,j \in S \\ i \neq j}} q_{ij} \\
&\quad + \mathbb{1}_s' Q_{sd}(I - Q_{dd})^{-1}[(e_0 + h_0 p)\mathbb{1}_d + (d - cdp)Q_{ds}\mathbb{1}_s].
\end{aligned} \tag{25}$$

Because Q is doubly stochastic and its submatrices are doubly substochastic,

$$\begin{aligned} &\mathbb{1}'_d Q_{dd} + \mathbb{1}'_s Q_{sd} = \mathbb{1}'_d - Q_{1d} \\ &\implies \mathbb{1}'_s Q_{sd} = \mathbb{1}'_d (I - Q_{dd}) - Q_{1d}. \end{aligned} \tag{26}$$

Plugging (26) into (25) implies

$$\begin{aligned} \mathbb{1}'_s m_s &= |\mathcal{S}|(e_0 + h_0 p) - |\mathcal{S}|(d - cdp) + (d - cdp) \sum_{\substack{\forall i,j \in S \\ i \neq j}} q_{ij} + \mathbb{1}'_d[(e_0 + h_0 p)\mathbb{1}_d + (d - cdp) Q_{ds} \mathbb{1}_s] \\ &- Q_{1d}(I - Q_{dd})^{-1}[(e_0 + h_0 p)\mathbb{1}_d + (d - cdp) Q_{ds} \mathbb{1}_s] \\ &= |\mathcal{S}|(e_0 + h_0 p) - |\mathcal{S}|(d - cdp) + (d - cdp) \sum_{\substack{\forall i,j \in S \\ i \neq j}} q_{ij} + |\mathcal{D}|(e_0 + h_0 p) + (d - cdp) \sum_{i \in D, j \in S} q_{ij} \\ &- Q_{1d}(I - Q_{dd})^{-1}[(e_0 + h_0 p)\mathbb{1}_d + (d - cdp) Q_{ds} \mathbb{1}_s] \\ &= (n-1)(e_0 + h_0 p) + (d - cdp)(-|\mathcal{S}| + \sum_{i \in D \cup S, j \in S} q_{ij}) - Q_{1d}(I - Q_{dd})^{-1}[(e_0 + h_0 p)\mathbb{1}_d + (d - cdp) Q_{ds} \mathbb{1}_s] \\ &= (n-1)(e_0 + h_0 p) + (d - cdp)(-|\mathcal{S}| + \sum_{j \in S}(1 - q_{1j})) - Q_{1d}(I - Q_{dd})^{-1}[(e_0 + h_0 p)\mathbb{1}_d + (d - cdp) Q_{ds} \mathbb{1}_s] \\ &= (n-1)(e_0 + h_0 p) + (d - cdp)(-|\mathcal{S}| + |\mathcal{S}| - \sum_{j \in S} q_{1j}) - Q_{1d}(I - Q_{dd})^{-1}[(e_0 + h_0 p)\mathbb{1}_d + (d - cdp) Q_{ds} \mathbb{1}_s]. \end{aligned}$$

Thus, the aggregate net wealth can be represented as

$$\mathbb{1}'_s m_s = (n-1)(e_0 + h_0 p) - (d - cdp) \sum_{j \in S} q_{1j} - Q_{1d}(I - Q_{dd})^{-1}[(e_0 + h_0 p)\mathbb{1}_d + (d - cdp) Q_{ds} \mathbb{1}_s], \tag{27}$$

which essentially determines the market clearing condition

$$\mathbb{1}' m_s = n h_0 p, \tag{28}$$

which holds for the case with $p < s$. Denote the aggregate net wealth less of the aggregate asset value as a function of p, so the market clearing condition is

$$\mathcal{F}(c,p;\mathcal{D}) \equiv {\scriptstyle (n-1)(e_0+h_0p)-(d-cdp)\sum_{j\in S} q_{1j} \;-\; Q_{1d}(I-Q_{dd})^{-1}[(e_0+h_0p)\mathbb{1}_d+(d-cdp)Q_{ds}\mathbb{1}_s]} - nh_0p \;\; = 0. \tag{29}$$

Now consider the solvency conditions for the surviving agents. From Proposition 3, we know that for a given equilibrium price p, any agent i such that

$$\mu_{i1} < \mu^*(p) = \frac{d-cdp}{e_0+h_0p} \tag{30}$$

default.

Therefore, the equilibrium price will be determined by the simplified market clearing condition (29) for a given set of defaulting agents $\mathcal{D}$, and the market clearing price p will determine the set of defaulting agents $\mathcal{D}$ by (30). The equilibrium will be the fixed point of $(\mathcal{D}, p)$ that satisfies the market clearing condition and the solvency condition simultaneously.

A.2. Useful Lemma

The following lemma is useful in proofs for several results in the main text.

Lemma 4 (More Default More Price Decline)**.** *For a fixed set of defaulting agents $\mathcal{D}_1$, define p_1 that satisfies $\mathcal{F}(c,p_1;\mathcal{D}_1)=0$. Suppose that under p_1, the set of defaulting agents increases to $\mathcal{D}_2$ such that $\mathcal{D}_1 \subset \mathcal{D}_2$ and $\mathcal{D}_2 \not\subset \mathcal{D}_1$. Let p_2 be the price that satisfies $\mathcal{F}(c,p_2;\mathcal{D}_2)=0$ for the new defaulting set $\mathcal{D}_2$. Then, $p_2 < p_1$.*

Proof of Lemma 4. By rearranging the terms of $\mathcal{F}(c,p;\mathcal{D})$, we can decompose the terms into coefficients of p and the rest of the constants as

$$\begin{aligned}\mathcal{F}(c,p;\mathcal{D}_1) =& (n-1)e_0 - \sum_{j\in S_1} q_{1j}d - Q_{1d_1}(I-Q_{d_1d_1})^{-1}[e_0\mathbb{1}_{d_1} + dQ_{d_1s_1}\mathbb{1}_{s_1}] \\ &+ (n-1)h_0p + cd\sum_{j\in S_1} q_{1j}p + Q_{1d_1}(I-Q_{d_1d_1})^{-1}[cdQ_{d_1s_1}\mathbb{1}_{s_1} - h_0\mathbb{1}_{d_1}]p - nh_0p.\end{aligned}$$

First, note that each entry of $(I - Q_{d_1 d_1})^{-1} Q_{d_1 s_1} \mathbb{1}_{s_1}$ is less than 1, because otherwise the entry with 1 or higher would have had enough net wealth through

$$\left[(I - Q_{d_1 d_1})^{-1} (e_0 + h_0 p) \mathbb{1}_{d_1} + (d - cdp) (I - Q_{d_1 d_1})^{-1} Q_{d_1 s_1} \mathbb{1}_{s_1}\right]_j > d - cdp,$$

for agent $j \in \mathcal{D}_1$ and agent j would not have defaulted. In other words,

$$(I - Q_{d_1 d_1})^{-1} [(e_0 + h_0 p) \mathbb{1}_{d_1} + (d - cdp) Q_{d_1 s_1} \mathbb{1}_{s_1}] < (d - cdp) \mathbb{1}_{d_1},$$

because all agents in $\mathcal{D}_1$ are defaulting.

Next we show that the coefficients of p is decreasing in the number of defaulting agents. Let $\mathcal{C} \subset \mathcal{D}_2$, $\mathcal{C} \cap \mathcal{D}_1 = \varnothing$, and $\mathcal{D}_2 = \mathcal{D}_1 \cup \mathcal{C}$.

The difference between the coefficients on p in $\mathcal{F}(c, p; \mathcal{D}_1)$ and $\mathcal{F}(c, p; \mathcal{D}_2)$ is

$$\begin{aligned} cd(\sum_{j \in S_1} q_{1j} - \sum_{j \in S_2} q_{1j}) &+ Q_{1d_1}(I - Q_{d_1 d_1})^{-1}[cdQ_{d_1 s_1}\mathbb{1}_{s_1} - h_0 \mathbb{1}_{d_1}] \\ &- Q_{1d_2}(I - Q_{d_2 d_2})^{-1}[cdQ_{d_2 s_2}\mathbb{1}_{d_2} - h_0 \mathbb{1}_{d_2}] \end{aligned}$$

$$= cd \sum_{j \in C} q_{1j} + Q_{1d_1}(I - Q_{d_1 d_1})^{-1}[cdQ_{d_1 s_1}\mathbb{1}_{s_1} - h_0 \mathbb{1}_{d_1}] - Q_{1d_2}(I - Q_{d_2 d_2})^{-1}[cdQ_{d_2 s_2}\mathbb{1}_{s_2} - h_0 \mathbb{1}_{d_2}]. \tag{31}$$

Note that $(I - Q_{d_1 d_1})^{-1}[cdQ_{d_1 s_1}\mathbb{1}_{s_1} - h_0 \mathbb{1}_{d_1}]$ in the second term in equation (31) can be represented as a vector $y(\mathcal{D}_1)$ with entries

$$y_i(\mathcal{D}_1) = \left[\sum_{j \in \mathcal{S}_1} q_{ij} cd - h_0 + \sum_{j \in \mathcal{D}_1} q_{ij} y_j(\mathcal{D}_1)\right].$$

First, note that $y_i(\mathcal{D}_1)$ is increasing in $\sum_{j \in \mathcal{S}_1} q_{ij} cd - h_0$. When all agents in $\mathcal{C}$ default then, $\sum_{j \in \mathcal{S}_1 \setminus C} q_{ij} cd - h_0 \leq \sum_{j \in \mathcal{S}_1} q_{ij} cd - h_0$. Second, since $(I - Q_{dd})^{-1} Q_{ds} \mathbb{1}_s < \mathbb{1}_s$, for any general default set and solvency set, so $y_k(\mathcal{D}_2) < cd$ for any $k \in \mathcal{D}_2$. Hence for any $i \in \mathcal{D}_1$,

$$y_i(\mathcal{D}_2) < y_i(\mathcal{D}_1).$$

Therefore,

$$\begin{aligned}
cd\sum_{j\in C} q_{1j} + Q_{1d_1}(I - Q_{d_1d_1})^{-1}[cdQ_{d_1s_1}\mathbb{1}_{s_1} - h_0\mathbb{1}_{d_1}] - Q_{1d_2}(I - Q_{d_2d_2})^{-1}[cdQ_{d_2s_2}\mathbb{1}_{s_2} - h_0\mathbb{1}_{d_2}] \\
= cd\sum_{j\in C} q_{1j} + Q_{1d_1}y(\mathcal{D}_1) - Q_{1d_2}y(\mathcal{D}_2) \\
= cd\sum_{j\in C} q_{1j}(1 - y_j(\mathcal{D}_2)) + \sum_{i\in\mathcal{D}_1} q_{1i}(y_i(\mathcal{D}_1) - y_i(\mathcal{D}_2)) > 0,
\end{aligned}$$

so the slope of the market clearing condition with $\mathcal{D}_2$ is smaller than that with $\mathcal{D}_1$.

Now suppose that agent $j \in \mathcal{C}$ defaults when the asset price is p_1, i.e., $m_j(p_1) < 0$. Then, by continuity, there exists $\tilde{p}_j$ such that $m_j(\tilde{p}_j) = 0$ and $\tilde{p}_j > p_1$. Now consider $\mathcal{F}(c, p; \mathcal{D}_1 \cup \{j\})$. Because $\mathcal{F}(c, p; \mathcal{D}_1 \cup \{j\})$ has lower coefficients of p than $\mathcal{F}(c, p; \mathcal{D}_1)$ by the previous argument (i.e., considering $\mathcal{C} = \{j\}$), and the value at $\tilde{p}_j$ is the same for the two functions, i.e., $\mathcal{F}(c, \tilde{p}_j; \mathcal{D}_1 \cup \{j\}) = \mathcal{F}(c, \tilde{p}_j; \mathcal{D}_1)$, the price p has to decline further to reach zero, i.e., for p^* such that $\mathcal{F}(c, p^*; \mathcal{D}_1 \cup \{j\}) = 0$, $p^* < p_1$. The same argument applies to any other agent $i \in \mathcal{C}$. Hence, $p_2 < p_1$. ■

B. Agent's Optimization Problem at Date 1 and Its Solution

At $t = 1$, agent j would like to maximize long-term profit, π_j at $t = 2$, which is composed of cash holdings, asset holdings multiplied by the asset payoff, and the payoff from the long-term project net of the liquidation amount. The decision variables are how much cash (e) and assets (h) to hold based on j's belief on the asset payoff θ_j, which is $\theta_j = s$ for all $j \in N$ in the baseline case. All of these decisions are subject to wealth as well as paying off the inter-agent liabilities and liquidity shock, while taking the payments from other agents as given. Hence, agent j solves for the following optimization problem:

$$\begin{aligned} &\max_{e,h} \pi_j = e + h\theta_j + \xi \mathbb{1}\left\{a_j(p) > b_j(p)\right\} \\ \text{s.t.} \quad &e + hp \leq [a_j(p) - b_j(p)]^+ \\ &e \geq 0, h \geq 0, \end{aligned} \tag{32}$$

where the first constraint is the budget constraint, and the second and the third are non-negativity constraints for cash and assets, respectively. Note that the long-term investment payoff is realized only if agent j is solvent, i.e., $a_j(p) > b_j(p)$, because it will be completely liquidated if agent j defaults due to the assumption of $\zeta \to 0$.

There are three possible cases. First, suppose that agent j's available budget is zero as $a_j(p) - b_j \leq 0$. Then, all the constraints are binding, and agent j's portfolio is forced to be $(e, h) = (0, 0)$. Suppose that agent j has some budget available for the rest of the cases. Second, suppose that the asset price is $p = \theta_j$. Then, agent j is indifferent between holding more cash and holding more assets. Hence, j will divide the budget into any arbitrary combination of e and h. Third, suppose that the asset price is $p < \theta_j$. Then, agent j would prefer to buy more assets than cash because the return of buying an asset is θ_j/p, which is greater than the cash return, 1. We present the full solution in the remainder of this section.

The first-order conditions (FOCs) of the optimization problem are

$$\begin{aligned} \partial e: \quad & 1 - \lambda_w + \lambda_e = 0, \\ \partial h: \quad & \theta_j - \lambda_w p + \lambda_h = 0, \end{aligned}$$

where λ_w, λ_e, and λ_h are the Lagrangian multipliers for the budget constraint, non-negativity constraint for e, and non-negativity constraint for h, respectively.

Complementary slackness conditions are

$$\begin{aligned} \lambda_e e = 0, \\ \lambda_h h = 0, \end{aligned}$$

whereas the budget constraint is always binding at the optimum.

Case 1. If $a_j(p) - b_j(p) \leq 0$. The budget constraint becomes

$$e + hp = 0,$$

and by $e, h \geq 0$ and $p \geq 0$, $(e, h) = (0, 0)$.

Case 2. Suppose $a_j(p) - b_j(p) > 0$.

Case 2.1. Suppose that $e > 0$, $h > 0$. FOCs for e and h imply

$$\begin{aligned} \lambda_w = 1, \\ \lambda_w p = \theta_j, \end{aligned}$$

and combining the two implies that $p = \theta_j$. Therefore, the case is optimal only in the case of $p = \theta_j$.

Case 2.2. Suppose that $e > 0$, $h = 0$. FOCs imply

$$\lambda_w = 1,$$
$$\lambda_h = \lambda_w p - \theta_j = p - \theta_j,$$

implying $\lambda_h > 0$ is possible only if $p > \theta_j$. Thus, agent j will sell all the asset and hold only cash if $p > \theta_j$.

Case 2.3. Suppose that $e = 0$, $h > 0$. FOCs imply

$$\lambda_w = \frac{\theta_j}{p},$$
$$\lambda_e = \frac{\theta_j}{p} - 1 > 0,$$

which hold only if $p < \theta_j$. Therefore, this case is possible only if $p < \theta_j$.

To summarize, agent j will be indifferent between cash and asset holdings only if $p = \theta_j$; buy assets using all the available budget if $p < \theta_j$; and sell all the assets and hold only cash if $p > \theta_j$. However, if agent j does not have any available cash, i.e., has non-positive net wealth, then the agent trivially holds zero cash and zero assets.

C. Omitted Proofs

C.1. Social Surplus

Proof of Lemma 1. Lemma 1 in Chang and Chuan (2024) shows that the social surplus of the economy is

$$U = \sum_{j \in N} (e_j + h_j s + \xi) - (1 - \zeta) \sum_{i \in N} l_i. \tag{33}$$

The result automatically follows from plugging $\zeta = 0$ into (33). ■

C.2. Existence

Proof of Proposition 1. See the appendix of Chang and Chuan (2024). ■

C.3. Collateral Thresholds

Proof of Proposition 2. The first statement follows from statement 1 in Proposition 2 of Chang and Chuan (2024). The second statement follows from statement 2 in Proposition 2 of Chang and Chuan (2024). ■

C.4. Multiple Equilibria

Proof of Proposition 3. See the appendix of Chang and Chuan (2024). ■

Proof of Proposition 4. The maximum equilibrium of $p = s$ with a single default exists by Proposition 2, because $c \geq \underline{c}(s, n)$. The minimum equilibrium of $p = 0$, with all agents defaulting, exists by the assumption $\mu_{ij} < \mu^*(0)$ for all $i, j \in N$, and by Proposition 3.

Therefore, the aggregate net wealth, $W(p) \equiv \sum_{j \in N} [m_j(p)]^+$, is $W(s) \geq nh_0 s$ for the maximum equilibrium and $W(0) = 0$ for the minimum equilibrium. By Lemma 4 of Chang and Chuan (2024) the aggregate net wealth is strictly increasing in p. Also, note that $W(p)$ is continuous in p. Therefore, there exists at least one point p^* such that the market clearing condition holds in equality, i.e., $W(p^*) = nh_0p^*$. This p^* is different from s unless $W(s) = nh_0s$, which is non-generic.

Finally, we show that such p^* is unique. By Lemma 4, a default induced by a price decline further decreases the market clearing price. Thus, $\frac{\partial W}{\partial p}$ is increasing in p, i.e., the aggregate net wealth is piece-wise linear, continuous, and convex in p. Now suppose the contrary that there exist two different prices p^* and $\tilde{p}$ with $p^* > \tilde{p}$ and both p^* and $\tilde{p}$ satisfy the market clearing condition with equality, i.e.,

$$W(p^*) = nh_0p^* \tag{34}$$

$$W(\tilde{p}) = nh_0\tilde{p}. \tag{35}$$

If the two equilibria with p^* and $\tilde{p}$ have the same defaulting set $\mathcal{D}$, then the market clearing functions will be the same for both $\mathcal{F}(c, p^*; \mathcal{D}) = \mathcal{F}(c, \tilde{p}; \mathcal{D})$, implying that there should be unique price that clears the market, which contradicts the initial assumption $p^* > \tilde{p}$. Therefore, the two equilibria should have different defaulting sets, $\mathcal{D}^* \subset \tilde{\mathcal{D}}$.

However, from the proof of Lemma 4, the slope of the aggregate net wealth $W(p)$ decreases as the defaulting set increases. The fact that both (34) and (35) hold implies that the average slope of the aggregate net wealth $W(p)$ from p^* to $\tilde{p}$ is nh_0. The slope will decline even further as p goes to zero, as more agents will default (or all agents default at $p = 0$), resulting in

$$W(\tilde{p}) - W(0) < nh_0\tilde{p}$$

$$\Longrightarrow W(0) > 0.$$

Then, there is no equilibrium with $p = 0$, which is a contradiction. Hence, there exists only one intermediate equilibrium with price $0 < p^* < s$ such that the cash-in-the-market price condition is satisfied with equality. ■

C.5. Collateral Reuse and Changes in Contagion

Proof of Proposition 5. We prove this result in three steps. First, we show that the intermediate equilibrium price p is decreasing in c if there is no additional default. Second, we show that number of defaulting agents weakly increases in c. Finally, we claim that the additional defaults will decrease the equilibrium price further by Lemma 4.

Step 1. First, we show that the equilibrium price p in the intermediate equilibrium is decreasing in c if the set of defaulting agents remains the same.

Fix the set of defaulting agents as $\mathcal{D}$. Recall that the market clearing condition can be represented as the market clearing condition function derived in (29),

$$\mathcal{F}(c, p; \mathcal{D}) \equiv (n-1)(e_0+h_0p)-(d-cdp)\sum_{j\in S} q_{1j} - Q_{1d}(I-Q_{dd})^{-1}[(e_0+h_0p)\mathbb{1}_d+(d-cdp)Q_{ds}\mathbb{1}_s] - nh_0p \quad = 0.$$

By the implicit function theorem,

$$\begin{aligned}
\frac{\Delta p}{\Delta c} &= -\frac{\dfrac{\partial \mathcal{F}(c,p;\mathcal{D})}{\partial c}}{\dfrac{\partial \mathcal{F}(c,p;\mathcal{D})}{\partial p}} \\
&= -\frac{dp\sum_{j\in S} q_{1j} + Q_{1d}\left(I-Q_{dd}\right)^{-1} Q_{ds}\mathbb{1}_s dp}{cd\sum_{j\in S} q_{1j} - Q_{1d}\left(I-Q_{dd}\right)^{-1} h_0\mathbb{1}_d + Q_{1d}\left(I-Q_{dd}\right)^{-1} Q_{ds}\mathbb{1}_s cd - h_0} < 0, \qquad (36)
\end{aligned}$$

where the last inequality comes from the fact that the denominator is positive by $cd > h_0$,

$$\begin{aligned}
cd \geq \underline{c}(s,n)d &= \frac{d-(n-1)e_0+h_0 s}{s} \\
&= \frac{d-(n-1)e_0}{s} + h_0 > h_0,
\end{aligned}$$

where the last inequality holds due to $d > (n-1)e_0$. Therefore, p is decreasing in c for a fixed set of defaulting agents.

Step 2. Second, we show that the number of defaults is (weakly) increasing in c in the intermediate equilibrium.

Recall that agent i is defaulting when

$$\mu_{i1} < \mu^*(p) = \frac{d - cdp}{e_0 + h_0 p}.$$

Then, the threshold harmonic distance changes with respect to increase in c as

$$\begin{aligned}\frac{\partial \mu^*}{\partial c} &= \frac{\left(-dp - cd\frac{\Delta p}{\Delta c}\right)(e_0 + h_0 p) - h_0 \frac{\Delta p}{\Delta c}(d - cdp)}{\left((e_0 + h_0)p\right)^2} \\ &= \frac{\left(-dp - cd\frac{\Delta p}{\Delta c}\right) e_0 - dph_0 p - h_0 \frac{\Delta p}{\Delta c} d}{\left((e_0 + h_0)p\right)^2}. \end{aligned} \tag{37}$$

To show that (37) is positive, we calculate a lower bound for $-\frac{\Delta p}{\Delta c}$, which is from (36),

$$-\frac{\Delta p}{\Delta c} \geq \frac{dp\left(\sum_{j \in S} q_{1j} + Q_{1d}\left(I - Q_{dd}\right)^{-1} Q_{ds} \mathbb{1}_s\right)}{cd\left(\sum_{j \in S} q_{1j} + Q_{1d}\left(I - Q_{dd}\right)^{-1} Q_{ds} \mathbb{1}_s\right)} = \frac{p}{c}.$$

Plugging this lower bound into the numerator of (37) implies

$$(-dp - cd\frac{\Delta p}{\Delta c})e_0 - dp^2 h_0 - h_0 \frac{\Delta p}{\Delta c} d \geq -dp^2 h_0 + h_0 d \frac{p}{c} = dph_0(\frac{1}{c} - p)$$

Because $c \leq \bar{c} = \frac{1}{s}$ and $p \leq s$, $\frac{1}{c} - p \geq 0$, implying $dph_0(\frac{1}{c} - p) \geq 0$. Therefore, $\frac{\partial \mu^*}{\partial c} > 0$, i.e., the required threshold harmonic distance is increasing in c, implying more agents will default as c increases by Proposition 3.

Step 3. By Lemma 4, an increase in the number of defaults in Step 2 will decrease the equilibrium asset price p even further. Therefore, an increase in c will decrease the intermediate equilibrium price even without triggering more defaults by Step 1, and can decrease the intermediate equilibrium price even further due to further (possible) increase in defaults by

Step 2. Hence, the intermediate equilibrium price p is decreasing in the degree of collateral reuse c. ■

C.6. Equilibrium Selection

Proof of Lemma 2. Recall that from (13)

$$\phi_j(p) = \min\left\{\frac{\theta+\psi-\theta^*}{2\psi}\left[h_j p - m_j(p)\right]^+ + \frac{\theta^*-(\theta-\psi)}{2\psi} h_j p, h_j p\right\}$$

Similarly, the demand side from (9) becomes (14), which is

$$\sum_{j\in N}\frac{\theta+\psi-\theta^*}{2\psi}\left[m_j(p) - h_j p\right]^+,$$

as only the reluctant sellers would participate the market as buyers. Thus, plugging (13) and (14) into the market clearing condition implies

$$\begin{aligned}
&\sum_{j\in N}\frac{\theta+\psi-\theta^*}{2\psi}\left[m_j(p)-h_j p\right]^+ \\
&\geq \sum_{j\in\mathcal{D}(p)} h_j p + \frac{\theta^*-(\theta-\psi)}{2\psi}\sum_{j\notin\mathcal{D}(p)} h_j p + \frac{\theta+\psi-\theta^*}{2\psi}\sum_{j\notin\mathcal{D}(p)}\left[h_j p - m_j(p)\right]^+ \\
\Longrightarrow &\sum_{j\notin\mathcal{D}(p)}\frac{\theta+\psi-\theta^*}{2\psi}\left(m_j(p)-h_j p\right) \geq \sum_{j\in\mathcal{D}(p)} h_j p + \frac{\theta^*-(\theta-\psi)}{2\psi}\sum_{j\notin\mathcal{D}(p)} h_j p \\
\Longrightarrow &\sum_{j\in N}\frac{\theta+\psi-\theta^*}{2\psi}\left[m_j(p)\right]^+ \geq \sum_{j\in\mathcal{D}(p)} h_j p + \left(\frac{\theta^*-(\theta-\psi)}{2\psi} + \frac{\theta+\psi-\theta^*}{2\psi}\right)\sum_{j\notin\mathcal{D}(p)} h_j p \\
\Longrightarrow &\frac{\theta+\psi-\theta^*}{2\psi}\sum_{j\in N}\left[m_j(p)\right]^+ \geq n h_0 p.
\end{aligned} \tag{38}$$

By the definition of the threshold value of θ_i for the marginal buyer, θ^* determines the asset price, so (38) becomes

$$\frac{\theta+\psi-\theta^*}{2\psi}\sum_{j\in N}\left[m_j(\theta^*)\right]^+ \geq n h_0 \theta^*. \tag{39}$$

If $\theta^* \geq \theta - \psi \geq p^*(c, D)$, then $\sum_{j \in N} [m_j(\theta^*)]^+ > nh_0\theta^*$ for any θ^* by the proof of Proposition 4 and Lemma 4. Moreover, $\frac{\theta + \psi - \theta^*}{2\psi}$ is decreasing in θ^* and its upper bound is $\frac{\theta + \psi - \theta^*}{2\psi} \geq \frac{\theta + \psi - (\theta - \psi)}{2\psi} = 1$. Therefore, the market clearing condition (39) can be satisfied at $p = \theta^* = \theta - \psi$, and thus, an equilibrium.

Finally, for the second statement, recall that from the proof of Proposition 4, the aggregate net wealth is less than the value of the aggregate supply for any $p < p^*(c, D)$. If $\theta^* < p^*(c, D)$ for any $\theta^* \in [\theta - \psi, \theta + \psi]$, then the only equilibrium is $p = 0$ and every agent defaults as in the minimum equilibrium in Proposition 4. ∎

Proof of Proposition 6.

Since $\theta^* \in [\theta - \psi, \theta + \psi]$, $\psi \to 0$ implies $\theta^* = \theta$. From Lemma 2, $\theta^* = \theta - \psi \to \theta$ and $p = \theta$ is an equilibrium with a positive aggregate net wealth. Therefore, $\theta^* = \theta$ is the unique equilibrium.

From the proof of Proposition 4 and Lemma 4,

$$W(p) \geq nh_0 p,$$

where $W(p)$ is the aggregate net wealth, for any $p \geq p^*(c, D)$. Hence, substituting any p with $\theta \geq p^*(c, D)$, which is the upper bound of the asset price, implies

$$W(\theta) \geq nh_0\theta,$$

which satisfies the second case of the market clearing condition (9). In contrast, by the same arguments,

$$W(p) < nh_0 p,$$

for any $0 < p < p^*(c, D)$. Hence, the only price that satisfies the market clearing condition is $p = 0$ for any $\theta < p^*(c, D)$.

For the last statement, it is suffice to calculate the changes in the likelihood of θ being less than $p^*(c, D)$ by the first statement. Recall that the likelihood of $\theta \in [0, p^*(c, D))$ is $\frac{p^*(c, D)}{\bar{\theta}}$. By Proposition 5, $\partial p^*(c, D)/\partial c < 0$, and trivially $\frac{\partial p^*(c, D)/\bar{\theta}}{\partial c} < 0$. ■

Proof of Proposition 7. From the proof of Proposition 4 and Lemma 4, as well as the argument in the main body, any realization of the initial RB that leads to the Walrasian auctioneer's price of $p < p^*(c, D)$ will lead to a BRD path to the minimum equilibrium with $p = 0$ where all agents default. Similarly, any initial RB that leads to $p > p^*(c, D)$ will lead to a BRD path to the maximum equilibrium with $p = s$. Since the size of the range of κ is increasing with greater $p^*(c, D)$, more states in Θ will lead to the realization of RB that leads to the minimum equilibrium. Therefore, the likelihood of realization of the minimum equilibrium is proportional to $p^*(c, D)$. The same argument holds for the maximum equilibrium. Finally, by Proposition 5, the second statement holds. ■

C.7. Endogenous Risk-Taking

Proof of Lemma 3. Because $K'(\xi)$ is increasing in ξ, it is enough to show that the right-hand side of (18) is decreasing in $p^*(c, D)$, which is decreasing in c by Propositions 5 and 7. Differentiating the right-hand side of (18) with respect to $p^*(c, D)$ implies

$$\frac{-\frac{n-1}{n}\left(1-\frac{1}{R}\right)\left[p^*(c,D)+(s-p^*(c,D))\frac{1}{n}\right]-(s-p^*(c,D))\frac{n-1}{n}\left(1-\frac{1}{R}\right)\frac{n-1}{n}}{\left[p^*(c,D)+(s-p^*(c,D))\frac{1}{n}\right]^2} < 0,$$

as long as the resulting ξ and e_0 values satisfy the parametric assumptions in the baseline model, so the contagion pattern remains the same. ■

Proof of Proposition 8. The result follows from Lemma 3 and Propositions 6 and 7. ■

C.8. Extensions

Proof of Proposition 9.

1. Suppose the contrary that agent j can pay its liquidity shock in full. Consider the best-case scenario in which all agents pay their debt in full. Then, for a given equilibrium price p

$$\sigma_i \left(e_0 + h_0 p\right) + \sum_{k \in N} x_{ik}(p) \geq \sigma_i \omega_i \epsilon + \sum_{k \in N} x_{ki}(p)$$

holds for all $i \in N$. However, summing over all $i \in N$ yields

$$(e_0 + h_0 p) \sum_{i \in N} \sigma_i \geq \sigma_j \epsilon,$$

which contradicts $\epsilon > ((e_0 + h_0 s)/\sigma_j) \sum_{i \in N} \sigma_i \geq ((e_0 + h_0 p)/\sigma_j) \sum_{i \in N} \sigma_i$. Therefore, j defaults on its payment to its senior liability, i.e., the liquidity shock.

2. By definition, agent i defaults on its scaled liabilities if $x_i(p) < \sigma_i(d - cdp)$ for a given equilibrium price p, for each $i \in N$, where $x_i(p) \equiv \sum_{k \in N} x_{ki}(p)$. The payment rule implies that

$$x_i(p) = \sigma_i(e_0 + h_0 p) + \sum_{k \in N} q_{ik} x_k(p).$$

Thus, (19) implies that $x_i(p) = (e_0 + h_0 p)\hat{\mu}_{ij}$, because

$$\hat{\mu}_{ij} = \sigma_i + \sum_{k \neq j} q_{ik} \hat{\mu}_{kj}$$
$$\iff \hat{\mu}_{ij}(e_0 + h_0 p) = \sigma_i(e_0 + h_0 p) + \sum_{k \in N} q_{ik} \hat{\mu}_{kj}(e_0 + h_0 p).$$

Then, agent i defaults if and only if $x_i(p) < \sigma_i(d - cdp)$, which is equivalent to $\hat{\mu}_{ij} < \sigma_i \dfrac{d - cdp}{e_0 + h_0 p} = \sigma_i \mu^*(p)$. ■