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Settlement Speed and Financial Stability ^{*}

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June 2, 2026

Abstract

This paper investigates how settlement speed affects financial stability in payment networks, accounting for netting benefits, liquidity costs, and counterparty risks. Faster settlement reduces crisis likelihood but amplifies crisis severity. The net welfare effect depends on network topology and proximity to default threshold points—settlement times at which the number of defaulting agents changes discontinuously. The optimal settlement speed is not universal: it depends on payment network structure and liquidity conditions. Deteriorating liquidity shifts the optimum toward slower settlement, even when faster settlement reduces counterparty default probability.

Keywords: settlement, payment systems, financial network, financial stability, systemic risk

JEL Classification Numbers: D49, D53, G01, G21, G33

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1. Introduction

The Securities and Exchange Commission (SEC) implemented the conversion of the U.S. securities market to a T+1 (one business day after the trade date) settlement cycle starting May 28, 2024. Following this transition, there has not been a noticeable change in settlement fails other than trading halts due to technical malfunctions at the New York Stock Exchange on June 4, 2024. However, the question of “whether shorter settlement lags would increase the likelihood or severity of systemic failures in payment systems or not” remains open.¹ This question is particularly timely, given the recent expansion of real-time payment systems such as FedWire, FedNow, and Real-Time Payments (RTP), as well as the ongoing trend toward faster deferred net settlement systems like the Clearing House Interbank Payments System (CHIPS).² Additionally, emerging payment systems utilizing distributed ledger technology (DLT) for crypto assets and stablecoins are fundamentally reshaping the settlement speed tradeoff (Bech, Shimizu, and Wong, 2017; McLaughlin, 2023).

Despite extensive study, research on the systemic risk implications of altering the settlement cycle remains scarce. In particular, payment systems can amplify initial shocks because of their critical role in the financial system and their complex network structure, which can propagate shocks rapidly through cascades of defaults.³ Therefore, understanding how the interconnectedness of payment systems affects financial stability, and how this relationship changes with settlement speed, is crucial for both policymakers and researchers.

We develop a network model to study the role of settlement in payment systems. Our

¹Throughout this paper, we use the term “*payment system*” to refer broadly to payment, clearing, and settlement systems for a variety of financial transactions, including payments in cash, loans, securities, repurchase agreements, and even derivatives.

²As McLaughlin (2023) points out, the congressional hearing on the “meme” stock episode in 2022 raised quicker settlement as a priority to mitigate market volatility.

³We focus on systemic settlement failures, in which liquidity shocks lead to actual defaults (bankruptcies or resolutions) rather than operational delays. While payment systems routinely experience settlement fails that are typically resolved through unwinding, delay penalties, or fails charges, our analysis concerns tail events where settlement stress escalates to counterparty insolvency and cascading defaults. In other words, we analyze systemic risk transmitted through payment systems. See Section 2.5 for more details. For an example of payment-cascade-induced stress episodes, see Kotidis and Schreft (forthcoming).

model incorporates four core features of payment systems: (i) networked payment flows, (ii) netting of obligations, (iii) liquidity costs of funding payments, and (iv) idiosyncratic liquidity shocks, including counterparty defaults (not simple operational failures).

Our key contribution is to incorporate *settlement time* into a financial network model with varying netting efficiency, liquidity costs, and counterparty risk. Our central finding is that faster settlement and financial stability are not monotonically related. Faster settlement reduces crisis likelihood but amplifies crisis severity. The net welfare effect therefore depends on network topology and on proximity to default threshold points, namely settlement times at which the defaulting set changes discontinuously. In contrast, most existing work analyzes welfare conditional on shock arrival or fixed shock arrival rate, abstracting from changes in the likelihood of the shock itself (Allen and Gale, 2000; Eisenberg and Noe, 2001; Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015). This reveals a fundamental tradeoff between crisis likelihood and crisis severity that we characterize across network structures.

Each agent in the network holds a cash buffer and has senior debt obligations. Agents are interconnected through a payment network. The key parameter of our model is τ , which represents the settlement time for all transactions in the network. As τ decreases, the settlement speed increases, with $\tau = 0$ implying instant settlement. The settlement time τ impacts the liquidity cost function and the netting efficiency function. The liquidity cost function, which is decreasing in τ , captures the costs associated with raising liquidity or posting collateral to finance transactions. The netting function, which is also decreasing in τ , represents the proportion of liabilities that are not netted in the network. We provide institutional background and empirical evidence supporting our monotonicity assumptions in Section 1.1.

We incorporate counterparty risk through a random liquidity shock that can hit some of the agents in the network before settlement occurs. The probability of this shock arriving before settlement is increasing in τ . When the shock arrives, it reduces the available cash of affected agents, potentially leading to defaults, which can propagate and cause further de-

faults through the network. For defaults, we consider systemic failures such as bankruptcies or resolutions rather than operational failures, as we are focusing on systemic events. Such defaults result in deadweight losses to the economy that are proportional to the amount of payment shortfalls.

The model implies a fundamental tradeoff: *faster settlement* (smaller τ) reduces the likelihood of default contagion, but increases liquidity costs and reduces netting efficiency. The ex-post welfare implication when a shock arrives is clear, as we show in Lemma 1, because a smaller τ reduces available cash and amplifies contagion via higher gross (less netted) exposures, thereby lowering ex-post welfare. By contrast, the *ex ante* welfare effect is generally ambiguous: while faster settlement may reduce the probability of stress events, it increases the severity of contagion when they occur.

We establish several key results characterizing this tradeoff and its implications for financial stability.

First, we show that settlement speed can generate discontinuous contagion patterns that depend sharply on network topology. In sparse networks, faster settlement may increase the number of defaults gradually, while in dense networks contagion may remain limited until a threshold is crossed, after which all agents default simultaneously (see Figure 3).

Second, we introduce the concept of *default threshold points*—settlement times at which the number of defaulting agents changes discretely. These threshold points vary across network structures and prove critical for determining the ex-ante welfare implications of settlement speed. The welfare effects of marginal changes in settlement speed differ sharply depending on whether the system operates at or above such threshold points. Furthermore, we show that default threshold points can be identified in polynomial time.

Third, we characterize the differential welfare impact of settlement speed across network structures. We prove that faster settlement can improve ex ante social welfare in the ring network but worsen it in the complete network. This heterogeneity in welfare effects underscores the importance of considering network structure when designing settlement systems

or implementing changes to settlement speeds.

Fourth, we show that the node depth centrality is a key measure of an agent’s systemic importance. This measure captures how the initial impact of a default by one agent is amplified through the network of defaulting agents, providing a useful tool for identifying systemically important institutions in payment networks.

Finally, we show that liquidity conditions play a crucial role in determining the optimal settlement speed. Deteriorating liquidity conditions make slower settlement socially preferable, as netting and liquidity conservation may dominate the benefits of reduced likelihood of counterparty defaults. This interaction between liquidity conditions and settlement speed has direct implications for payment system design or implementing changes to settlement cycles. Furthermore, this interaction also highlights an understudied aspect of liquidity, relating to the recent policy discussions regarding intraday liquidity provision for various payment systems.

Altogether, our results provide new insights into the design of optimal settlement systems and emphasize the importance of nuanced interactions between various factors and interconnectedness for policymakers. Our findings have direct policy implications. We show that a one-size-fits-all approach to settlement speed may be suboptimal, given the heterogeneity in network structures. Moreover, our results provide a theoretical underpinning for targeted liquidity support to central entities of payment systems during times of stress. Furthermore, we show that faster settlement can decrease the likelihood of crisis events but increase their severity, potentially leading to more frequent government interventions due to stronger incentives to intervene. Finally, we show that slower settlements can improve financial stability when liquidity conditions worsen.

1.1. Related Literature

Our paper contributes most directly to the literature on the fundamental economic trade-off of settlement speed in clearinghouses and payment systems. [McLaughlin \(2023\)](#) extends

Duffie and Zhu (2011)’s random network model of clearinghouses, deriving a netting benefit function that increases with settlement delay and identifying the key tradeoff: slower settlement allows more netting but increases counterparty risk through longer exposure periods.⁴

Our main contribution is to examine how this tradeoff depends on *network structure*. Rather than deriving average effects from random network models, we analyze how specific network architectures (complete networks, ring networks, and general structures) fundamentally alter the costs and benefits of faster settlement. Moreover, our model incorporates cascades of settlement fails along the given network structure, allowing us to examine financial stability implications of settlement speed. By explicitly modeling network topology, we find that the optimal settlement speed is not universal but depends critically on the pattern of interconnections, as the network structure significantly alters the costs and benefits of faster settlement. Therefore, we can provide guidance for system design that accounts for network characteristics and also a practical tool for identifying the key network characteristics (see Proposition 4).

Beyond network structure, our analysis extends the literature in two key dimensions. First, we evaluate *ex ante social welfare*, accounting for both crisis likelihood and conditional severity of crisis, a distinction crucial for policy. Most financial network models focus on ex post welfare holding shock probabilities fixed (Allen and Gale, 2000; Eisenberg and Noe, 2001; Rogers and Veraart, 2013; Elliott, Golub, and Jackson, 2014; Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015; Glasserman and Young, 2015; Bernard, Capponi, and Stiglitz, 2022; Capponi, Corell, and Stiglitz, 2022; Donaldson, Piacentino, and Yu, 2022; Jackson and Pernoud, 2024; Chang and Chuan, 2024).⁵

⁴McLaughlin (2023) also finds that DLT settlement systems can circumvent liquidity challenges, as they can achieve up to 90% of the benefit of multilateral netting if they operate with a 10-minute netting window. We can consider this as an expansion of the technological frontier, changing the functional forms or parameters of netting efficiency and liquidity cost. Nevertheless, the tradeoff of faster settlement itself remains the same.

⁵Chang and Chuan (2025) examine ex ante welfare in networks, finding that collateral reuse decreases crisis likelihood but increases severity. While similar to our finding, their uncertainty stems from equilibrium selection, whereas ours arises from exogenous shock timing, a more fundamental source directly tied to netting efficiency and liquidity costs.

Second, our framework incorporates explicit time dimension through settlement time parameter τ , enabling comparative statics that account for interactions between counterparty risk, netting efficiency, and liquidity costs. This complements [Khapko and Zoican \(2020\)](#), who study settlement speed without network effects or systemic risk.

Our paper relates to work on clearing system design and liquidity provision in payment systems ([Williamson, 1998, 2003](#); [Fujiki, Green, and Yamazaki, 2008](#); [Koepl, Monnet, and Temzelides, 2008, 2012](#)). [Martin and McAndrews \(2008\)](#) show how liquidity savings mechanisms may or may not improve welfare by attenuating the issue of payment delays and costs of funds with netting and reduction of liquidity cost. We extend this literature by incorporating financial networks and explicitly analyzing how settlement speed affects systemic risk and financial stability across network structures.

The portfolio compression and netting literature ([Duffie and Zhu, 2011](#); [Duffie, Scheicher, and Vuillemeys, 2015](#); [D’Errico and Roukny, 2021](#); [Veraart, 2022](#); [Chang, 2021](#)) provides theoretical and empirical context for the magnitude of netting benefits. For example, at the NSCC, multilateral netting reduces approximately \$2 trillion in daily transactions to \$35 billion in settlement—a 98% reduction ([Bodson, 2021](#)). Furthermore, [Carapella and Mills \(2011\)](#) show CCPs mitigate informational frictions through delayed settlement and netting. When CCPs mutualize losses, longer horizons can improve liquidity conditions and netting efficiency by alleviating information asymmetries ([Capponi, Wang, and Zhang, 2022](#)). Our time-varying liquidity cost and netting function, interacted with network structure, capture how such compression opportunities decrease with settlement speed.

Extensive empirical research supports our liquidity cost and netting efficiency assumptions. [Rochet and Tirole \(1996\)](#) show that net settlement systems like the Clearing House Interbank Payment Systems (CHIPS) alleviate liquidity pressures through netting, while real-time gross settlement (RTGS) systems like Fedwire prevent failure propagation but face greater liquidity constraints. [Copeland and Garratt \(2019\)](#) document that Fedwire requires large cash for small net transactions, whereas CHIPS clears large gross obligations with much

smaller net cash outlays due to netting services. Furthermore, [Copeland and Garratt \(2019\)](#) find that dozens of large banks conduct substantial volumes of transactions through CHIPS, whereas thousands of banks rely on Fedwire to settle smaller net amounts. [Ding et al. \(2025\)](#) find Brazilian banks increased liquid holdings after adopting instant payments, confirming the tradeoff between settlement speed and liquidity costs.

Studies of payment timing further validate our liquidity cost assumptions. Empirical work documents that daylight overdraft costs and liquidity constraints drive delayed payments in RTGS systems, with trades concentrated late in the day ([VanHoose and Sellon Jr, 1989](#); [Hancock and Wilcox, 1996](#); [McAndrews and Rajan, 2000](#); [Mills Jr and Nesmith, 2008](#)). Theoretical literature shows that RTGS and delivery-versus-payment (DVP) systems require substantial transaction-by-transaction liquidity, potentially creating gridlock that deferred net settlement (DNS) systems avoid through netting and delayed settlement ([Bech and Garratt, 2003](#); [Kahn, McAndrews, and Roberds, 2003](#); [Mills Jr and Nesmith, 2008](#)). RTGS and DVP systems settle payments and securities one at a time, so liquidity is needed to complete each transaction, incentivizing participants to concentrate trades later in the day. [Afonso and Shin \(2011\)](#) also show that the Fedwire system requires large amounts of liquidity for participating banks that can trigger precautionary demand in liquidity, which could result in a systemic event.

During stress periods, liquidity concerns amplify. [Brunnermeier and Pedersen \(2009\)](#) uncover a self-reinforcing feedback loop between a trader's funding liquidity and an asset's market liquidity. [Andolfatto \(2020\)](#) show that during periods of stress, market participants tend to hoard liquidity, leading to lost trading opportunities. [Acharya and Merrouche \(2013\)](#) provide empirical evidence that large settlement banks hoarded liquidity during the 2007-2008 subprime crisis, because of increased uncertainty about counterparty risk, leading to a freeze of the interbank lending market. All of these studies support our focus on systemic risk stemming from liquidity shortfalls and contagion through payment networks.

2. Model

We develop a general model that can be applied to both payment systems and securities settlement systems. The following is an overview of the model, which is designed to capture a slice of the continuous transactions over time in a parsimonious way. For every slice or snapshot of time, there is a set of transactions that are scheduled to be settled sometime later. Some amount of liquidity is needed for each slice of transactions to settle them. Over time, some transactions can be netted. However, over time, some counterparties may receive shocks and default. Although there will be new transactions in the following periods, not being able to settle the initial slice of transactions would lead to default. Thus, our model captures a dynamic tradeoff for given transactions at a particular point in time.

2.1. Agents and Payment Network

There are n different agents, and the set of all agents is $N = \{1, 2, \dots, n\}$ with $n > 2$. Time is continuous and denoted as $t \in [0, \bar{T}]$, where \bar{T} is finite. Agents are risk neutral, do not discount future up to \bar{T} ,⁶ and their utility is determined by how much net payments they receive less liquidity cost (described below) at $t = \bar{T}$. Each agent i holds $e_i > 0$ amount of cash buffer (or operating net cash flow as in Eisenberg and Noe (2001)), and has senior debt (such as deposits) to pay in the amount of $s_i > 0$. Each agent is connected to each other through transaction agreements, which are exogenously given at $t = 0$.⁷ The amount of j 's payment promised (liability) to i is denoted by d_{ij} . The payment network, which is defined as the network of payments promised, is denoted by the matrix $D \equiv [d_{ij}]_{i,j \in N}$. Denote the sum of payment obligations or the column sums of D as $d_j \equiv \sum_{i \in N} d_{ij}$. Define $n \times 1$

⁶Although this is a simplifying assumption, we can consider \bar{T} as a relatively short horizon, which is typically the case, so the effective discount rate is negligible.

⁷For example, banks do not have much control over how payments are made across their own depositors and depositors of other banks. Another example is that large dealers tend to trade with counterparties with whom they have long-term relationships, which are costly to adjust even when there are clear arbitrage opportunities (Chang, Klee, and Yankov, 2025).

vectors $d \equiv (d_i)_{i \in N}$, $e \equiv (e_i)_{i \in N}$, and $s \equiv (s_i)_{i \in N}$. The payment matrix can be represented as $D = Q \circ d^T$, where \circ is the Hadamard (Schur) product operator, and Q is the matrix of weights with its (i, j) -element as

$$q_{ij} = \frac{d_{ij}}{d_j}. \quad (1)$$

2.2. Settlement Speed, Netting, and Liquidity Cost

Settlement speed. The time to settle all transactions in D is denoted by τ , which is the key parameter of the model. By construction, $\tau \in [0, \bar{T}]$. As τ becomes smaller (larger), the settlement speed increases (decreases), and $\tau = 0$ implies instantaneous settlement.

Liquidity cost. Agents need to raise liquidity (or post collateral) to finance their transactions. Such liquidity needs may decrease over time when settlement happens in later time. For example, if a transaction is the purchase of a security, an agent (dealer) may find a trader (outside of the network) who is willing to purchase the security from the agent over time. However, if an agent (dealer) needs to settle the transaction right away, the agent has to use its own funds to purchase or borrow the security. These effects are captured by a reduced-form *liquidity cost function* denoted as $L(\tau)$, which is differentiable and strictly decreasing in τ and $L(\tau) \leq e_i - s_i$, $\forall i \in N$ and $\forall \tau \in [0, \bar{T}]$.⁸ Further, we assume that the liquidity cost is senior to payments in the network. In sum, $L(\tau)$ represents the amount of the opportunity cost of cash and the external netting of payments.

Netting. The matrix of transactions may have cycles or simply a path, which could be compressed through netting of transaction liabilities. Define the *full netting matrix* \underline{D} as the

⁸Khapko and Zoican (2020) provide a microfoundation for this liquidity cost. For example, broker-dealers have to prefund their trades and face inventory risk (or opportunity cost of funds) or borrow securities that they need to settle, and costs associated with both increase as settlement speed increases. In other cases, agents in a payment system may have to rely on external short-term borrowing because mobilizing internal liquidity takes time as settlement speed increases. Even if there are a lot of ways to support liquidity (e.g., near-free intraday credit in the FedWire or zero-cost borrowing with eligible collateral), such liquidity sources have limits and opportunity cost. Moreover, shorter payment delay means that agents cannot use expected incoming funds to cover outgoing payments. In this paper, we are agnostic about the source of the cost and take the decreasing liquidity cost as a reduced-form functional property.

matrix of transactions when all possible netting is conducted for the original network D , and $\underline{D} \leq D$. The exact details of what type of netting is conducted are discussed in Appendix B.

However, the actual netting may not reach the full netting matrix for many reasons, such as limited information and staggered maturities or covenants (Donaldson, Piacentino, and Yu, 2022; Jackson and Pernoud, 2024).⁹ The degree of netting can be greater when the settlement occurs later, as agents and the payment system can identify more netting opportunities or simply because some of the transactions in D are actually formed later (Kahn and Roberds, 1998). In other words, as the settlement speed decreases (τ increases), the total required transactions to be settled decrease. We represent this degree of netting by the *netting function* $\alpha(\tau)$, which is differentiable and decreasing in τ , and $\alpha(\tau) \in [0, 1]$ for any $\tau \in [0, \bar{T}]$.¹⁰ We assume the actual required transactions are a convex combination of the full netting matrix and the original transaction matrix as

$$\hat{D} \equiv (1 - \alpha(\tau))\underline{D} + \alpha(\tau)D, \quad (2)$$

which we call as the *partial netting matrix* of D for given τ .¹¹ We use corresponding notations for the full netting and partial netting matrices: $\underline{d}_{ij} \equiv \underline{D}_{ij}$, $\underline{d}_j \equiv \sum_{i \in N} \underline{d}_{ij}$, $\underline{d} \equiv (\underline{d}_i)_{i \in N}$, $\hat{d}_{ij} \equiv \hat{D}_{ij}$, $\hat{d}_j \equiv \sum_{i \in N} \hat{d}_{ij}$, and $\hat{d} \equiv (\hat{d}_i)_{i \in N}$. Also, \hat{Q} denotes the partial netting matrix weights with its (i, j) -element as $\hat{q}_{ij} = \hat{d}_{ij}/\hat{d}_j$. Therefore, $(1 - \alpha(\tau))$ represents the degree of internal netting of payments within the payment network.

⁹For example, D’Errico and Roukny (2021) find that even the most conservative compression scenario eliminates on average, more than 85% of the CDS positions in the European Union and all their counterparties.

¹⁰Our reduced-form netting function $\alpha(\tau)$ is agnostic about the specific netting mechanism, bilateral, multilateral, or CCP-intermediated. It is intended to capture the aggregate degree of obligation compression achievable at settlement horizon τ . The institutional details of each mechanism would affect the functional form of $\alpha(\tau)$ but not the qualitative structure of our results.

¹¹This general representation of netting encompasses a reduced-form representation of netting in Donaldson, Piacentino, and Yu (2022).

2.3. Counterparty Risk and Default Spillovers

At any point in time, there is a small probability that some agents receive liquidity shocks and default. The arrival time of liquidity shocks t_l is a random variable with full support $[0, \bar{T}]$. Therefore, the probability of the arrival time being less than or equal to τ (i.e. the probability of liquidity shocks arriving before the settlement of payments) is $F(\tau) \equiv \Pr(t_l \leq \tau)$, which is differentiable and increasing in τ . In other words, the likelihood of a liquidity shock disrupting the payment system is increasing in τ . Once liquidity shocks arrive, each agent i experiences an immediate reduction in available cash in the amount of $\epsilon_i \geq 0$. Denote the $n \times 1$ vector of liquidity shocks as $\epsilon = (\epsilon_i)_{i \in N}$. For exposition purposes, we slightly abuse notation and denote $\epsilon_i = 0$ when agent i does not receive a liquidity shock. The vector of liquidity shocks ϵ is a random variable with the probability mass function $G(\epsilon)$ for any $\epsilon \in \mathbb{R}_+^n$. The liquidity shock ϵ_i captures severe idiosyncratic stress sufficient to push agents into insolvency, not routine liquidity management issues.

Due to the liquidity cost and liquidity shocks, some agents may default, as their obligations may exceed their available liquidity. Therefore, agent j can pay the full promised amount (after netting) \hat{d}_j when j has enough liquidity, or j would default when the full promised amount exceeds the total net liquidity—the sum of payments from other agents and cash buffer less of senior debt and liquidity cost paid.¹² Denote the actual total payment made by agent j as $x_j(\tau, D, \epsilon)$. The arguments τ , D and ϵ are often omitted in the following for expositional simplicity. Agent j will pay the total amount of $x_j < \hat{d}_j$, if agent j is defaulting, or $x_j = \hat{d}_j$, if agent j is solvent. Following the literature, we assume the pro rata rule for distributing the remaining wealth of a defaulting agent to other agents; hence the payment to agent i from agent j is $\hat{q}_{ij}x_j$.¹³

¹²Here we are simply treating transactions in \hat{D} as being made in terms of cash. This is without loss of generality, as for any transactions with securities, we can just re-normalize everything with the securities price p .

¹³This is a standard assumption in the literature, such as in [Eisenberg and Noe \(2001\)](#) and [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2015\)](#). [Csóka and Herings \(2021\)](#) provide an axiomatization of the proportional rule in financial networks. Our model encompasses situations where initial settlement fails (inability to

Finally, we assume that a default incurs deadweight loss in terms of real cost as in Rogers and Veraart (2013), Elliott, Golub, and Jackson (2014), and Glasserman and Young (2015) among many others. The deadweight loss can be interpreted as legal costs or costs of delays and inefficient allocation of resources during the bankruptcy procedure. In particular, following Bernard, Capponi, and Stiglitz (2022) and Capponi, Corell, and Stiglitz (2022), the default of agent j inflicts a deadweight loss of $\min\{\beta\xi_j(\tau, D, \epsilon), A_j(\tau, D, \epsilon)\}$, where $A_j(\tau, D, \epsilon) \equiv e_j + \sum_{k \in N} \hat{q}_{jk}x_k(\tau, D, \epsilon)$ is the value of agent j 's assets (liquidity buffer plus payments received from other agents), $\xi_j(\tau, D, \epsilon) \equiv [\hat{d}_j + s_j + \epsilon_j + L(\tau) - A_j(\tau, D, \epsilon)]^+$ is agent j 's shortfall (total payment obligations + senior debt + liquidity shock + liquidity cost - assets), and $\beta > 0$ is a scaling parameter. The arguments τ , D , and ϵ are often omitted in the following. Note that the deadweight loss is bounded above by the total value of the agent's assets. The deadweight loss can cause either a decline in the defaulting agents' remaining asset value (hence less payment to other agents) or purely a societal cost to the un-modeled external agents. Denote the modeled agents' share of the deadweight loss as γ .

Mathematically, the total payments made by agent j is

$$x_j(\tau, D, \epsilon) \equiv \begin{cases} \hat{d}_j, & \text{if } A_j(\tau, D, \epsilon) \geq \hat{d}_j + L(\tau) + s_j + \epsilon_j \\ [A_j(\tau, D, \epsilon) - L(\tau) - s_j - \epsilon_j - \gamma \min\{\beta\xi_j(\tau, D, \epsilon), A_j(\tau, D, \epsilon)\}]^+, & \text{otherwise,} \end{cases} \quad (3)$$

where $[\cdot]^+ \equiv \max\{\cdot, 0\}$ denotes the positive part. Thus, in a matrix-vector notation, the payment that clear the payment network is

$$x = \left[\min \left\{ \hat{d}, \hat{Q}x + e - L(\tau)\mathbf{1} - s - \epsilon - \gamma \min\{\beta\xi, A\} \right\} \right]^+, \quad (4)$$

deliver at scheduled time) escalate to actual defaults when agents cannot meet their obligations even after accounting for all available liquidity. The pro rata rule applies once limited liability is binding, consistent with bankruptcy and resolution procedures. This differs from routine settlement fails where unwinding or delay penalties apply without insolvency (see Section 2.5 for details). Iyer and Macchiavelli (2017) and Gurrola-Perez, He, and Harper (2019) show that settlement fails along a chain of payments (daisy chain) can have systemic implications.

where $x \equiv (x_i(\tau, D, \epsilon))_{i \in N}$, $\xi \equiv (\xi_i(\tau, D, \epsilon))_{i \in N}$, $A \equiv (A_i(\tau, D, \epsilon))_{i \in N}$, and $\mathbf{1}$ is a vector of ones for the appropriate dimension.

The (ex post) equilibrium concept we use is *payment equilibrium*, which satisfies (4) for the given realization of liquidity shocks, liquidity costs, and netting. This equilibrium concept is consistent with that of Eisenberg and Noe (2001) and many other papers in the financial networks literature. The existence of multiple Pareto-ranked payment equilibrium follows directly from Proposition 3 in Glasserman and Young (2015) using the standard fixed-point theorem arguments. There could be multiple equilibria due to the fact that defaults can generate discontinuous drops in the asset values, resulting in self-fulfilling defaults.¹⁴ However, all equilibria can be Pareto-ranked, so we focus on the unique Pareto dominant payment equilibrium, i.e. the equilibrium with the lowest total payment shortfalls and social welfare loss.

2.4. Timeline

Figure 1 summarizes the timeline of the model. The nature decides the arrival time of the shock t_l . If the settlement time τ is less than t_l , then there will be no defaults, payments are netted by $1 - \alpha(\tau)$, reducing the actual payment network as $\hat{D} = (1 - \alpha(\tau))\underline{D} + \alpha(\tau)D$, and liquidity costs $L(\tau)$ are paid by agents. Agents will settle all the payments, and surviving agents will consume their resulting wealth at $t = \bar{T}$. If the settlement time τ is greater than or equal t_l , then some agents may default (possibly from spillovers), depending on the equilibrium payments x . Surviving agents consume but defaults also incur deadweight loss of β multiplied by the amount of payment shortfalls.

¹⁴See Chang and Chuan (2025) for a detailed discussion of this property of financial network models and an analysis of equilibrium selection.

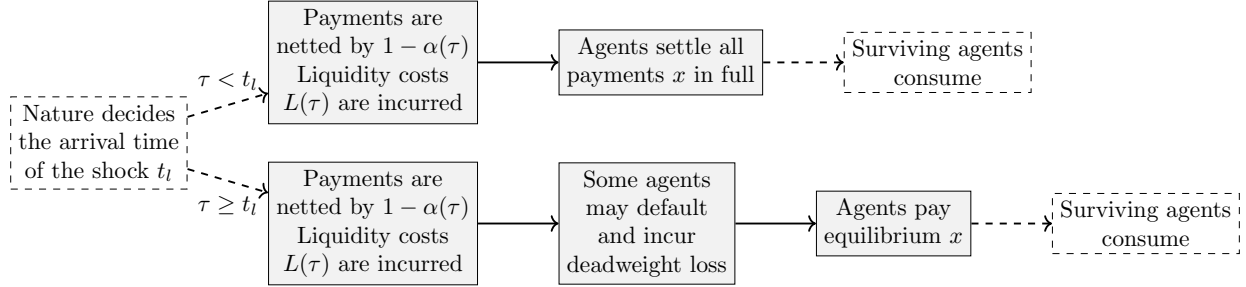


Figure 1: Timeline of the model.

Note: Nature determines the shock arrival time t_l . If $\tau < t_l$, settlement occurs before the shock: payments are netted, liquidity costs are incurred, all payments are made in full, and surviving agents consume. If $\tau \geq t_l$, the shock arrives before settlement: agents may default, deadweight losses may arise, equilibrium payments x are made, and surviving agents consume.

2.5. Model Scope and Interpretation

Our model focuses on systemic events that have significant financial stability implications. We distinguish between routine settlement fails (e.g., operational delays, temporary liquidity mismatches) and the systemic defaults we model. Routine settlement fails are typically resolved through unwinding, delay penalties, or fails charges. In contrast, our model focuses on severe stress scenarios where liquidity shocks exceed agents' capacity to meet obligations, leading to actual defaults under limited liability. Under such circumstances, there are limited or no opportunities to access liquidity from repos (e.g., due to lack of collateral or no lenders available or willing to provide cash), the discount window (due to lack of collateral or no prepositioned collateral at the Fed, as in the case of the banking turmoil in 2023), daylight overdrafts, or other sources of liquidity. This scenario aligns with the concept of a tail event in which liquidity from all agents and other markets and sources is depleted. In such cases, the pro rata distribution rule is consistent with bankruptcy procedures and legal frameworks.

Regarding the timing of events, our model analyzes a specific set of payment obligations D that are promised at $t = 0$ and settled at $t = \tau$. These obligations are exogenous to our model, as they arise from client trade requests, contractual obligations, or other economic activities that occur at various points in time and in exogenously determined amounts. In

this sense, our framework captures a “slice” of continuous-time economic activity. At any given moment, there exists a portfolio of outstanding obligations (or cohort of payments) awaiting settlement, and our model examines how the choice of settlement time τ affects the stability and efficiency of clearing these obligations.

In this context, the liquidity cost function $L(\tau)$ has a dual interpretation. First, it represents the direct costs of funding payments when settlement occurs at time τ (e.g., opportunity costs of capital, collateral costs, or borrowing costs as in [Khapko and Zoican \(2020\)](#)). Second, $L(\tau)$ can capture the benefits of *external netting*—transactions with parties within the modeled network or outside the network that occur between $t = 0$ and $t = \tau$ and that help offset payment obligations. As settlement is delayed, agents have more opportunities to net obligations against incoming payments from external sources (outside of payments in D at $t = 0$), effectively reducing their funding needs.

We acknowledge that substantial changes in settlement speed could induce behavioral responses that alter the structure of D itself. For instance, a shift from T+2 to T+0 settlement might fundamentally change trading patterns, relationship formation, or risk management practices. However, modeling such endogenous network formation is beyond our scope. Instead, our analysis is most naturally interpreted as examining *marginal* changes in settlement speed. For instance, moving from T+2 to T+1, or comparing settlement systems that differ moderately in speed. For such marginal changes, treating D as fixed is a reasonable approximation, and our comparative statics provide meaningful guidance for policy and system design.¹⁵

2.6. Ex Ante Social Welfare Loss

Agents’ utility is determined by the net payment they receive less the liquidity cost. Hence, the expected utility depends on the resulting equilibrium vector of payments that

¹⁵Our results on changes in the likelihood and severity of crises can be exacerbated when agents in the network can reoptimize their base cash buffers e_i . For example, agents may care less about maintaining sufficient cash buffers when a crisis is less likely to occur, but once it happens, it would wipe out the entire network regardless. See, for example, [Chang and Chuan \(2025\)](#).

varies by whether there is a liquidity shock or not. Note that the amounts of cash buffer, senior debt, liquidity cost, and payment obligations remain the same with or without the arrival of a liquidity shock. Without loss of generality, we assume that all obligations in D can be made in full if no agent defaults.¹⁶ Then, by construction, all payments are made in full if no liquidity shock has arrived. With probability $F(\tau)$, a liquidity shock arrives before the settlement, and some agents may default and incur deadweight losses. Therefore, the ex ante expected utility of agent i for a given settlement time τ is

$$\begin{aligned}
U_i(\tau, D) = & e_i - s_i - L(\tau) + (1 - F(\tau)) \left[\sum_{j \in N} \hat{q}_{ij} \hat{d}_j - \hat{d}_i \right] \\
& + F(\tau) E \left[\sum_{j \in N} \hat{q}_{ij} x_j - x_i - \epsilon_i - \gamma \min \{ \beta \xi_i(\tau, D, \epsilon), A_i(\tau, D, \epsilon) \} \right], \tag{5}
\end{aligned}$$

where the payments and values relevant to deadweight losses, $(x_i, \xi_i, A_i)_{i \in N}$, are determined by the payment equilibrium when liquidity shocks arrive, and $E[\cdot]$ is the expectation over different realizations of these liquidity shocks with probability $G(\epsilon)$.

Define the ex ante (expected) social welfare losses for a given τ and network D as

$$W(\tau, D) \equiv nL(\tau) + F(\tau) E \left[\sum_{j \in N} \min \{ \beta \xi_j(\tau, D, \epsilon), A_j(\tau, D, \epsilon) \} \right], \tag{6}$$

where the first-term is the liquidity cost for settling trades at τ , and the second term is the sum of the total deadweight losses for internal network agents (γ proportion) and for external agents ($(1 - \gamma)$ proportion) multiplied by the probability of shock arrival $F(\tau)$. Note that the payment shortfall to the liquidity shock ϵ is not included in the welfare loss, as the payments to ϵ are simple transfers of wealth from the shocked agents to some unmodeled external agents (senior creditors), i.e., it is zero sum.¹⁷ See Appendix C for a detailed discussion.

¹⁶This assumption helps us to focus on additional social welfare losses due to changes in the counterparty default probabilities resulting from changes in settlement time and network structure. Our results can be extended to the case in which some agents default on their obligations even if they are paid in full.

¹⁷This definition of social welfare loss generalizes the typical definition of ex post social surplus in the literature. The ex post social surplus in our model is consistent with that of [Acemoglu, Ozdaglar, and](#)

Equation (6) implies that the ex ante social welfare loss is determined by the liquidity costs, and the expected deadweight losses from payment shortfalls, multiplied by the probability of liquidity shock arrival. As τ increases, the likelihood of liquidity shock state increases, the inter-agent liabilities decrease, and the liquidity cost decreases. Hence, the resulting ex ante social welfare loss depends on the tradeoff and functional forms of $\alpha(\tau)$ and $L(\tau)$ as well as the structure of the network D and resulting spillovers. Note that increase in τ also decreases the liquidity cost when there is no liquidity shock.

3. Settlement Speed and Contagion: Stylized Cases

We begin by analyzing the interaction between settlement speed and network structure in stylized cases, demonstrating how they jointly determine contagion patterns and social welfare. After establishing the core intuition in the following subsections, we generalize our results in Section 4.

3.1. Discontinuous Contagion Patterns

We establish our first main result, which shows the emergence of discontinuous contagion patterns. Throughout, we use *discontinuous contagion* to refer to situations in which a marginal change in τ causes a discrete jump in the number of defaulting agents, and *phase transition* to refer specifically to the all-or-nothing default pattern in the complete network, whereby either exactly one agent or all agents default depending on whether τ is above or below the threshold τ^* . When the settlement time τ changes, the social welfare may drastically increase or decrease depending on the network structure and whether a phase transition is crossed. To illustrate the main mechanism clearly, we impose a few simplifying assumptions in this subsection.

Following the literature, we focus on *regular networks*, which have homogeneous agents,

Tahbaz-Salehi (2015) and Chang and Chuan (2024).

i.e., $e_j = \bar{e}$ and $s_j = \bar{s}$ for any $j \in N$, with the same in-degree and out-degree of payments, i.e., $\sum_{i \in N} d_{ij} = \sum_{i \in N} d_{ji} = \bar{d}$ (Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015; Donaldson, Piacentino, and Yu, 2022).¹⁸ Further, we assume the full netting matrix would eliminate all cycles in the network.¹⁹ Specifically, for any connected regular network, the full netting matrix will be $\underline{D} = \mathbf{0}$, where $\mathbf{0}$ is the zero matrix (therefore, $\hat{Q} = Q$). To simplify the feedback from deadweight losses, we focus on the indirect deadweight loss case in which agents in the network do not suffer the deadweight loss from payment shortfalls directly, i.e. $\gamma = 0$. Finally, we assume that only one of the n agents is hit by the liquidity shock once it arrives, and the size of the liquidity shock is fixed as a single value $\bar{\epsilon}$. Hence, the liquidity shock vector will be $\epsilon_i = \bar{\epsilon}$ and $\epsilon_j = 0$ for any $j \neq i$. Also, we focus on the case with $\bar{\epsilon} > \bar{e} - \bar{s} - L(\bar{T})$ to exclude trivial cases of no default even after receiving a liquidity shock. This simplified setup enables us to compare our model directly with other key models in the literature, most notably that in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015).

First, we focus on two special cases of regular networks: ring and complete networks, depicted in Figure 2. The ring network is a structure in which agent i is the only agent to be paid by agent $i - 1$ for each $i > 1$ (for the entirety of \bar{d}), and agent 1 is the only agent to be paid by agent n . The complete network is a structure in which every agent owes the same amount to each other agent, $\bar{d}/(n - 1)$. Denote the ring network and complete network as D_r and D_c , respectively.

The first proposition establishes the discontinuous patterns of contagion in payment equilibrium for the complete network, which contrasts with the pattern of contagion in the ring network. In particular, we consider two moving parts, $L(\tau)$ and $\alpha(\tau)$, to prepare for comparative statics with the settlement time τ . As τ increases, $L(\tau)$ decreases, and agents

¹⁸In addition to the benefits of bringing simplicity and tractability due to symmetry, this setup allows us to focus on the network connection itself rather than the size or asymmetry of payment flows. Given that payment networks tend to have net zero flows in a steady state (because otherwise an agent would be depleted of cash over time), this assumption is relatively in line with real-world payment networks.

¹⁹The notion of netting or portfolio compression here is “conservative compression” defined by D’Errico and Roukny (2021). A “nonconservative compression” is possible if new edges of liabilities can be formed. For a practical example, if transactions are centrally cleared, they can be netted even without cycles. See Appendix Section B for more discussion.

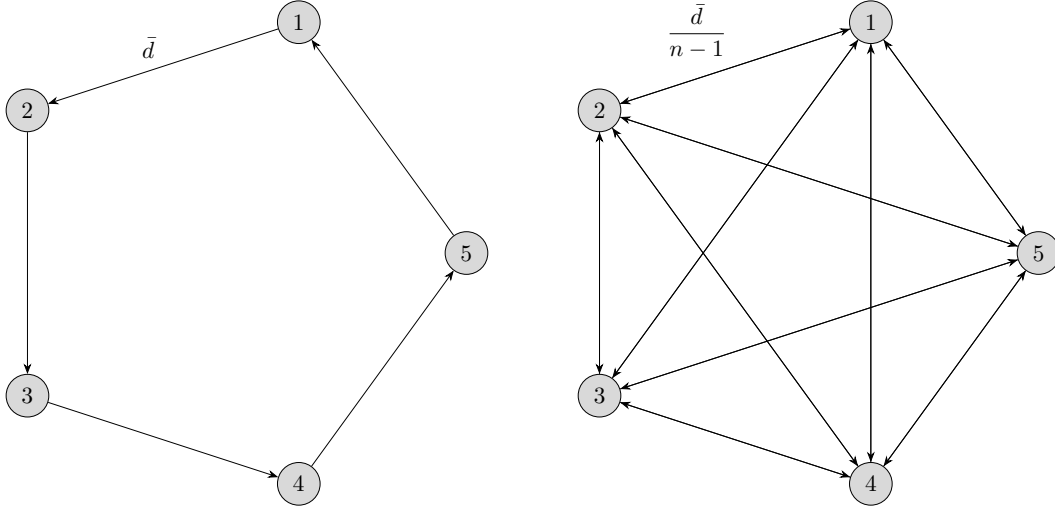


Figure 2: The ring network and the complete network

Note: The left graph depicts the ring network, in which each agent owes the full liability amount of \bar{d} to the next agent. The right graph depicts the complete network, in which each agent owes $\frac{\bar{d}}{n-1}$ to every other agent.

have more liquidity. Therefore, agents can absorb larger shocks with their liquidity buffer. In addition, as τ increases, more netting can be done (lower $\alpha(\tau)$), reducing the total inter-agent liability amount $\alpha(\tau)\bar{d}$. This further reduces the inter-agent exposures, decreasing the contagion throughout the network. Therefore, understanding the roles of the liquidity cost, $L(\tau)$, and the netting function, $\alpha(\tau)$, is crucial for the ultimate comparative statics with the settlement time, τ .

The proposition shows the following: For the complete network, if either the size of the liquidity shock is relatively small (compared to the liquidity buffer) or the size of the total liabilities is relatively small (compared to the liquidity buffer), then only one agent defaults. In comparison, the number of defaulting agents is greater than one in the ring network under the same conditions.

In contrast, when both the relative size of the liquidity shock and the relative size of the total liabilities are large, all agents in the complete network default, and the same happens in the ring network. Thus, the model exhibits a phase transition in contagion patterns across

network structures.

However, there is a subtle difference between the results in our model and those in the literature. In typical models, such as those in [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2015\)](#) and [Chang and Chuan \(2024\)](#), the complete network and the ring network have the same level of social welfare when full contagion (i.e., all agents defaulting) occurs, because both networks have the same number of defaults. In contrast, our model shows that the complete network has a smaller total welfare loss than the ring network when the deadweight losses don't hit their upper bound, i.e., not all of the assets disappear due to deadweight losses. This novel result further identifies a subtle difference in contagion pattern even when the number of defaults is the same.

The main factor is our definition of deadweight loss, which is different from simply counting the number or the amount of defaults in [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2015\)](#) and [Chang and Chuan \(2024\)](#). Even when all agents default, the total deadweight loss depends on the distribution of payment shortfalls in our model. In the ring network each defaulting agent is simply a conduit of the payments from previous agents to the next agent, until the payment flow reaches the agent under shock. However, in the complete network, each agent is using its payment to pay everyone else, who then reuses the payment to pay everyone else, and so on. The excess propagation of payments reduces payment shortfalls in the complete network, reducing the deadweight loss. This novel mechanism is relevant to deadweight loss in the real world, as deadweight losses would be smaller in more complexly interconnected payment systems.

Proposition 1. *For a fixed settlement time τ , suppose that a liquidity shock $\bar{\epsilon}$ hits one agent, and denote $\epsilon^*(\tau) \equiv n(\bar{\epsilon} - \bar{s} - L(\tau))$ and $d^*(\tau) \equiv (n - 1)(\bar{\epsilon} - \bar{s} - L(\tau))$.*

1. *Suppose that $D = D_c$, the complete network.*

(a) *If either the relative size of the liquidity shock is small or the relative size of the total liabilities is small, i.e. $\bar{\epsilon} \leq \epsilon^*(\tau)$ or $\alpha(\tau)\bar{d} \leq d^*(\tau)$, then only one agent*

defaults and the total deadweight loss from default is

$$\min \{ \beta (\bar{s} + \bar{\epsilon} - \bar{e} + L(\tau)), \bar{e} + \alpha(\tau)\bar{d} \}. \quad (7)$$

(b) If both the relative size of the liquidity shock is large and the relative size of the total liabilities is large, i.e. $\bar{\epsilon} > \epsilon^*(\tau)$ and $\alpha(\tau)\bar{d} > d^*(\tau)$, then all agents default and the total deadweight loss from defaults is

$$\begin{aligned} & \min \{ \beta [\alpha(\tau)\bar{d} + \bar{\epsilon} - n(\bar{e} - \bar{s} - L(\tau))], \bar{e} + (n-1)(\bar{e} - \bar{s} - L(\tau)) \} \\ & + (n-1) \min \{ \beta [\alpha(\tau)\bar{d} - (n-1)(\bar{e} - \bar{s} - L(\tau))], \bar{e} + (n-2)(\bar{e} - \bar{s} - L(\tau)) \}. \end{aligned} \quad (8)$$

2. Suppose that $D = D_r$, the ring network.

(a) If either the relative size of the liquidity shock or the relative size of the total liabilities is small, i.e., $\bar{\epsilon} \leq \epsilon^*(\tau)$ or $\alpha(\tau)\bar{d} \leq d^*(\tau)$, then ψ other agents default in addition to the shocked agent, and the total deadweight loss from defaults is

$$\begin{aligned} & \min \{ \beta (\bar{s} + \bar{\epsilon} - \bar{e} + L(\tau)), \bar{e} + \alpha(\tau)\bar{d} \} \\ & + \sum_{m=1}^{\psi} \min \{ \beta (\alpha(\tau)\bar{d} - m(\bar{e} - \bar{s} - L(\tau)) - \phi), \bar{e} + (m-1)(\bar{e} - \bar{s} - L(\tau)) + \phi \}, \end{aligned} \quad (9)$$

where $\phi = [\bar{e} - \bar{s} - L(\tau) + \alpha(\tau)\bar{d} - \bar{\epsilon}]^+$, and $\psi = \min \left\{ n-1, \left\lfloor \frac{\alpha(\tau)\bar{d} - \phi}{\bar{e} - \bar{s} - L(\tau)} \right\rfloor \right\}$.²⁰

(b) If both the relative size of the liquidity shock is large and the relative size of the total liabilities is large, i.e. $\bar{\epsilon} > \epsilon^*(\tau)$ and $\alpha(\tau)\bar{d} > d^*(\tau)$, then all agents default, and the total deadweight loss from default is

$$\begin{aligned} & \min \{ \beta [\alpha(\tau)\bar{d} + \bar{\epsilon} - n(\bar{e} - \bar{s} - L(\tau))], \bar{e} + (n-1)(\bar{e} - \bar{s} - L(\tau)) \} \\ & + \sum_{m=1}^{n-1} \min \{ \beta (\alpha(\tau)\bar{d} - m(\bar{e} - \bar{s} - L(\tau))), \bar{e} + (m-1)(\bar{e} - \bar{s} - L(\tau)) \}, \end{aligned} \quad (10)$$

²⁰ $\lfloor \cdot \rfloor$ denotes the floor function, i.e. the greatest integer that is less than or equal to \cdot .

which is greater than (8) for a small enough β . Otherwise, the deadweight loss is the same as that in (8).

All proofs are relegated to Appendix Section A.

First, consider the case in which either the shock size relative to the amount of liquidity is small or the relative size of the total liabilities is small. For the complete network, the deadweight loss amount (7) comes from only the shocked agent, as all other agents can pay each other in full by dispersing the liquidity shock among themselves equally. In contrast, there can be other defaulting agents in the ring network, resulting in additional deadweight losses as in the second term of (9). This is because the shock size can be still greater than what a single agent can bear with its own cash buffer.

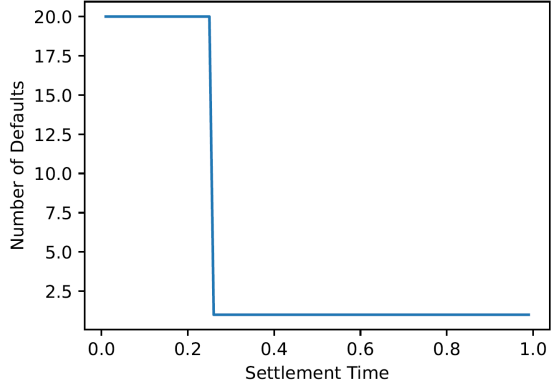
Suppose, without loss of generality, that agent 1 is the agent under liquidity shock. In the ring network, agent 1 pays only ϕ to agent 2 after absorbing the shock. This shortfall cascades: agent 2 can pay $\bar{e} - \bar{s} - L(\tau) + \phi$, agent 3 can pay $2(\bar{e} - \bar{s} - L(\tau)) + \phi$, and so on. Defaults continue until agent $\psi + 1$ accumulates sufficient buffers to pay in full.

Now suppose that both the shock size relative to the amount of liquidity and the relative size of the total liabilities are large. All agents default for both the ring and complete networks. However, the two networks experience different levels of changes in deadweight losses. This is mainly because there are already additional ψ number of agents (other than the shocked agent) who are defaulting in the ring network even in case 2.(a), so only the additional $n - \psi - 1$ agents are defaulting, whereas the additional number of defaulting agents is $n - 1$ in the complete network.²¹ Figure 3 illustrates the results in Proposition 1.

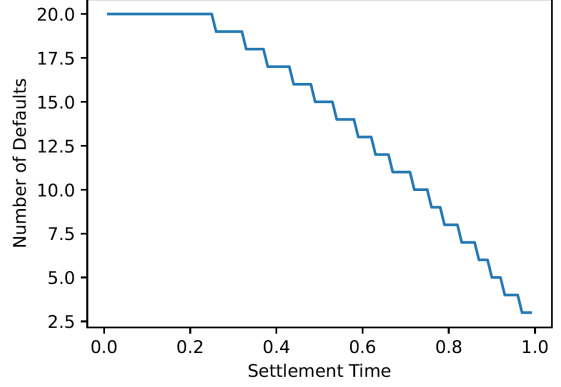
Finally, we discuss the two key functions, the netting function $\alpha(\tau)$ and the liquidity cost function $L(\tau)$, and how they interact with each other.

First, the liquidity cost function $L(\tau)$ plays a role in determining the threshold liquidity

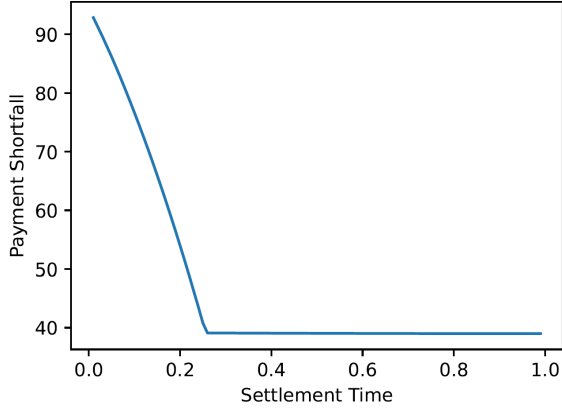
²¹On the deadweight loss comparison between the two networks when all agents default: Intuitively, when deadweight losses are small relative to asset values (small β), the ring network's sequential contagion structure generates larger aggregate shortfalls than the complete network's simultaneous loss-sharing. For large β , deadweight losses deplete asset values faster in both networks, compressing the difference.



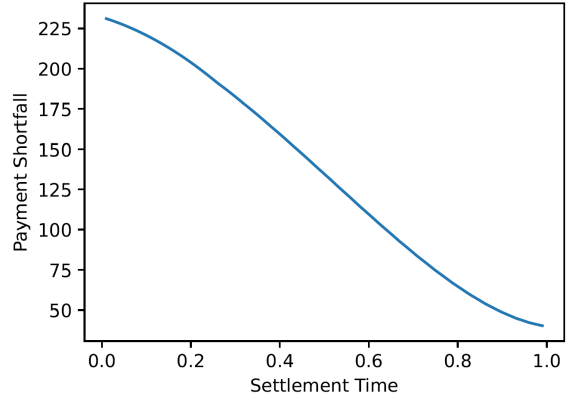
(a) Number of defaults: Complete network



(b) Number of defaults: Ring network



(c) Aggregate payment shortfall: Complete network



(d) Aggregate payment shortfall: Ring network

Figure 3: Number of defaults and aggregate payment shortfall: Complete vs Ring

Note: This figure illustrates the results in Proposition 1. The top panels have the number of defaults on the y-axis and the settlement time τ on the x-axis. The top-left panel illustrates the phase transition property of contagion in the complete network, as the number of defaults jumps at the threshold τ value. In contrast, the top-right panel illustrates the gradual increase in the number of defaults for the ring network as τ decreases. The bottom panels have the aggregate payment shortfall on the y-axis and the settlement time τ on the x-axis. The bottom-left panel illustrates the phase transition property in the complete network. The bottom-right panel illustrates smooth changes in the payment shortfall in the ring network as τ decreases.

shock size $\epsilon^*(\tau) = n(\bar{e} - \bar{s} - L(\tau))$. Hence, as τ increases, $\epsilon^*(\tau)$ increases; i.e., the given liquidity shock \bar{e} is less likely to exceed the threshold, due to the decrease in $L(\tau)$. In other words, agents will have a relatively larger cash buffer if they are paying less liquidity cost when the settlement is slower. Similarly, $L(\tau)$ also plays a role in increasing the threshold total liability amount $d^*(\tau) = (n - 1)(\bar{e} - \bar{s} - L(\tau))$ in a similar fashion. As the liquidity cost decreases with slower settlement, the given total liability amount is less likely to exceed the

threshold.

The netting function $\alpha(\tau)$ plays a role in determining only the total liability threshold. As τ increases, the effective netting increases, i.e., $\alpha(\tau)$ decreases, effectively increasing the required total liability amount. This is because the inequality $\bar{d} \leq \frac{d^*(\tau)}{\alpha(\tau)}$ becomes more likely to hold as $\alpha(\tau)$ decreases for a given \bar{d} .

Both $L(\tau)$ and $\alpha(\tau)$ move in the same direction, making cases 1.(a) and 2.(a) more likely as τ increases. We can rearrange the threshold conditions and combine the two as presented in the following corollary.

Corollary 1. *All agents default in the complete network if and only if $\tau < \tau^*$, where τ^* satisfies*

$$L(\tau^*) \equiv \bar{e} - \bar{s} - \min \left\{ \frac{\bar{e}}{n}, \alpha(\tau^*) \frac{\bar{d}}{n-1} \right\}. \quad (11)$$

We refer to τ^* as the threshold point (for the complete network) from now on. Corollary 1 implies that the threshold point is decreasing in the cash buffer net of senior debt $\bar{e} - \bar{s}$, as the required liquidity cost amount for the threshold condition increases with it. For example, if there is no net cash buffer at all ($\bar{e} - \bar{s} = 0$), then full contagion occurs in the complete network even if there is no liquidity cost ($L(\tau) = 0$), regardless of other values. Moreover, the threshold point, τ^* , decreases, as the shock size divided by the number of agents decreases. In other words, if the shock can be absorbed by the aggregate net cash buffer of all agents, $n(\bar{e} - \bar{s}) > \bar{e}$, then some degree of liquidity cost $L(\tau)$ can still be absorbed by the system, thus there is no full contagion with a faster settlement speed (lower settlement time).

Finally, Corollary 1 implies that there is an interaction between $L(\tau)$ and $\alpha(\tau)$ when determining the threshold condition. As the degree of netting increases (i.e., $\alpha(\tau)$ decreases), the network can withstand a larger value of liquidity cost $L(\tau)$, allowing for a faster settlement speed (lower τ^*) without full contagion. However, if the degree of netting is small ($\alpha(\tau)$ is relatively large) and the total liabilities for each agent are large (\bar{d} is large), then

the interaction between $\alpha(\tau)$ and $L(\tau)$ becomes irrelevant. This is because the size of the liquidity shock becomes the minimum of the last term in the right-hand side of (11). In other words, in this case, the relative size of the total liabilities is already large, so the size of the liquidity shock is the marginal factor that determines the value of the liquidity cost that the network can bear while absorbing the liquidity shocks and preventing full contagion.

3.2. Heterogeneous Ex Ante Social Welfare Implications

Proposition 1 shows the phase transition in ex post contagion patterns. Now we utilize the phase transition property to evaluate ex ante social welfare implications of different networks and settlement times. The next proposition builds on Proposition 1 and shows that decreasing the settlement time τ has different effects across different network structures. In particular, we show that faster settlement can improve ex ante social welfare of the ring network but worsen ex ante social welfare of the complete network.

For expositional simplicity, we focus on the case in which the upper bound on each agent's deadweight loss is not binding, i.e., β and ξ_i are small enough so that the defaulting agent's assets A_i are not completely wiped out due to bankruptcy losses. While this assumption is mainly for tractability, it is also consistent with reality. Legal costs of bankruptcy or costs of delays in allocating remaining assets of a defaulting agent are unlikely to be as large as the total assets themselves. Nevertheless, our model can be easily extended to incorporate cases with $\beta\xi_i > A_i$ in a numerical model.

A marginal change in the settlement time τ affects the ex ante welfare loss in three different ways. It affects:

- (a) The aggregate liquidity cost $nL(\tau)$;
- (b) The likelihood of the liquidity shock and resulting deadweight losses due to counterparty defaults by changing the shock arrival probability $F(\tau)$; and
- (c) The total deadweight losses $\sum_{j \in N} \beta\xi_j(\tau, D, \epsilon)$.

Therefore, for a given τ and network D , the marginal change in the ex ante welfare loss is:

$$\frac{\partial W(\tau, D)}{\partial \tau} = nL'(\tau) + F(\tau) \sum_{j \in N} \beta \frac{\partial \xi_j(\tau, D, \epsilon)}{\partial \tau} + F'(\tau) \sum_{j \in N} \beta \xi_j(\tau, D, \epsilon). \quad (12)$$

The first term of (12) represents the decrease in liquidity cost for all n agents. The second term represents the marginal change of the deadweight losses multiplied by the default probability. The third term represents an increase in the default probability multiplied by the deadweight losses due to default contagion. We later show that the second term is always negative in Lemma 1.

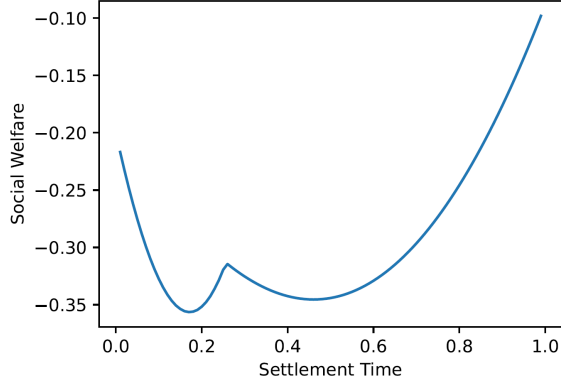
The following proposition demonstrates the heterogeneous changes in ex ante welfare loss when τ changes.

Proposition 2. *For a fixed $\tau \in (0, \bar{T})$, and the complete network D_c and the ring network D_r , the following statements hold:*

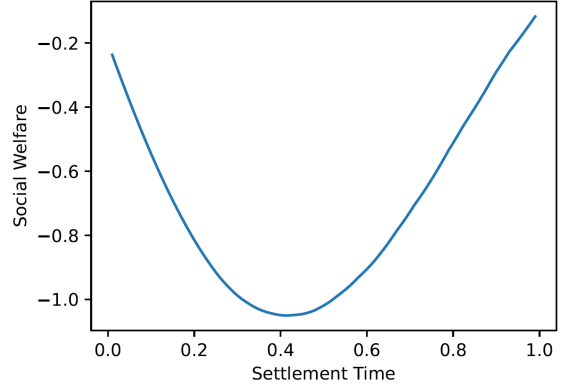
1. *Suppose that $L(\tau) < L^*(\tau)$. The comparison between the marginal change in the ex ante welfare loss for the complete network and that for the ring network is ambiguous, i.e., both $\frac{\partial W(\tau, D_c)}{\partial \tau} > \frac{\partial W(\tau, D_r)}{\partial \tau}$ and $\frac{\partial W(\tau, D_c)}{\partial \tau} \leq \frac{\partial W(\tau, D_r)}{\partial \tau}$ are possible depending on the parameter values and functional forms.*
2. *Suppose that $L(\tau) \geq L^*(\tau)$. Then, the marginal change in the ex ante welfare loss for the complete network is less than that for the ring network, i.e., $\frac{\partial W(\tau, D_c)}{\partial \tau} < \frac{\partial W(\tau, D_r)}{\partial \tau}$. Moreover, an increase in τ can make the complete network better off, while making the ring network worse off, i.e.,*

$$\frac{\partial W(\tau, D_c)}{\partial \tau} < 0 < \frac{\partial W(\tau, D_r)}{\partial \tau}. \quad (13)$$

Proposition 2 implies that there can be cases, where faster settlement speed can make the ring network better off while making the complete network worse off. In particular, such concrete ex ante social welfare comparison is possible once the liquidity cost exceeds the phase



(a) Ex ante Social Welfare: Complete network



(b) Ex ante Social Welfare: Ring network

Figure 4: Ex ante Social Welfare: Complete vs Ring

Note: This figure illustrates the results in Proposition 2. Both panels have the ex ante social welfare on the y-axis and the settlement time τ on the x-axis. The left panel shows the changes in ex ante social welfare for the complete network. The ex ante social welfare shows a kink, a discrete jump in the slope, at the threshold value. The right panel shows the changes in the ex ante social welfare for the ring network that changes smoothly. Thus, if τ is just below the threshold value, the marginal change in the ex ante social welfare is positive for the complete network, while it is negative for the ring network.

transition threshold $L^*(\tau)$. If the liquidity cost is below the threshold, or equivalently τ is large enough, then the comparison depends on all parameter values and functional forms, as we have many moving parts. However, this intricacy surprisingly disappears once the liquidity cost reaches its threshold, or equivalently τ is small enough. Thus, our result highlights the critical role of sudden jumps in the number of defaulting agents when considering the ex ante social welfare of a given network and settlement speed τ .

Figure 4 shows that a decrease in settlement time can either increase or decrease the ex ante social welfare depending on the current settlement time τ . Although the global maximum of the ex ante social welfare is attained at the maximum τ value in both cases under this specific parameter space of simulations, it's important to note that the global maximum depends on the functional form and parameter values of $\alpha(\tau)$, $L(\tau)$, $F(\tau)$, and others, as stated in the first part of Proposition 2.

4. General Networks and Ex Ante Social Welfare

We continue to explore the effect of jumps in the number of defaulting agents on marginal ex ante social welfare in a general setup with general regular networks. In this section, we allow for the case with $\gamma > 0$, i.e., deadweight losses from defaults further reduce the payments from defaulting agents. For simplicity, we focus on the case of $\gamma = 1$; however, our results can be easily extended to any $0 < \gamma < 1$. Similarly, we focus on cases in which $\epsilon_j < e_j$ for any $j \in N$ and any realization of ϵ , for expositional simplicity.²² Throughout this section, unless otherwise stated, we maintain the assumption that β is sufficiently small so that the asset cap in the deadweight-loss function does not bind for the relevant states.

4.1. Aggregate Deadweight Loss and Welfare Decomposition

Consider any general regular network D .²³ Denote the set of defaulting agents and solvent agents as $\mathcal{D}(\tau, \epsilon)$ and $\mathcal{S}(\tau, \epsilon)$, respectively, for a given settlement time τ and liquidity shock vector ϵ . First, we show that the deadweight loss from payment shortfall of any $j \in N$,

$$\beta \xi_j(\tau, D, \epsilon) = \beta \left[\alpha(\tau) \bar{d} + \bar{s} + \bar{\epsilon}_j - \bar{e} + L(\tau) - \sum_{k \in N} q_{jk} x_k(\tau, D, \epsilon) \right]^+,$$

is decreasing in τ . In other words, for a fixed probability of shock arrival, $F(\tau)$, the ex ante welfare loss always decreases as τ increases.

Lemma 1. *The aggregate deadweight loss conditional on the arrival of a liquidity shock decreases as τ increases. Furthermore, the number of defaulting agents weakly decreases as τ increases, i.e., $\mathcal{D}(\tau', \epsilon) \subseteq \mathcal{D}(\tau, \epsilon)$ for any $\tau < \tau'$, D , and ϵ .*

The intuition is that the payment shortfall for each agent decreases as its liability and

²²This assumption is similar to the assumption on liquidity shock in [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2015\)](#).

²³While we don't explicitly analyze some prominent structures like core-periphery networks, our focus encompasses any general structure of the "core" in such networks. This approach suffices for systemic risk analysis, as peripheral agents are typically not systemically important.

liquidity cost decrease through decreases in $\alpha(\tau)$ and $L(\tau)$. Even though the payments from others also decrease in $\sum_{k \in N} q_{jk} x_k$ if $x_k = \alpha(\tau) \bar{d}$, the sum of q_{jk} over $j \in N$ cannot exceed 1, which is the coefficient for the liability of agent j itself. In other words, the decrease in liabilities to be paid by j exceeds the decrease in payments from solvent counterparties.

Lemma 1 clarifies the tradeoff associated with changes in τ . As τ increases, the aggregate deadweight loss always decreases, either through a smaller amount of payment shortfalls or a reduced number of defaulting agents. However, an increase in the likelihood of shock arrival counteracts that effect. The following result summarizes the decomposition of net welfare effect of an increase in τ :

Proposition 3. *For a given settlement time τ , network structure D , and the liquidity shock state ϵ , suppose that $\mathcal{D}(\tau, \epsilon) = \mathcal{D}(\tau + \delta, \epsilon)$ for a small enough $\delta > 0$. Then, the effect of an increase in the settlement lag on the ex ante welfare loss depends on netting benefits, liquidity costs, and counterparty risks, as shown in the below decomposition:*

$$\underbrace{\underbrace{nL'(\tau)}_{\text{decrease in liquidity cost}} + \underbrace{F(\tau)}_{\text{probability of shock arrival}} \underbrace{\sum_{j \in \mathcal{D}(\tau, \epsilon)} \beta \frac{\partial \xi_j(\tau, D, \epsilon)}{\partial \tau}}_{\text{decrease in aggregate deadweight loss}}}_{\text{Benefit}} + \underbrace{F'(\tau)}_{\text{increased likelihood of shock arrival}} \underbrace{\sum_{j \in \mathcal{D}(\tau, \epsilon)} \beta \xi_j(\tau, D, \epsilon)}_{\text{aggregate deadweight loss}}. \quad (14)$$

The proposition provides us several insights. For example, if a default does not incur deadweight losses, i.e., $\beta = 0$, the aggregate welfare loss is decreasing in τ due to decreases in liquidity costs $nL'(\tau)$. Similarly, if the payment shortfall, $\xi_j(\tau, D, \epsilon)$, is very small, the ex ante welfare loss given ϵ decreases as τ increases. The welfare implication becomes ambiguous only if the conditional payment shortfall is large, and the increase in the likelihood of shock arrival $F'(\tau)$ is significant.

4.2. Default Threshold Points, Centrality, and Ex Ante Welfare

Now, we can finally analyze the changes in the ex ante welfare loss by utilizing the monotone effect of the settlement time τ from Lemma 1 and the decomposition of net welfare effect from Proposition 3. In particular, we show that default threshold points, the settlement times at which one or more agents are on the verge of default, play an important role in comparing ex ante welfare losses across two different settlement times. For example, τ^* introduced in Corollary 1 is a default threshold point. Moreover, we show that the node depth centrality, which represents the systemic importance of an agent, is key for measuring the effect of payment shortfalls. The formal definitions and statements are below.

First, we compute the total amount of deadweight loss for a given shock vector ϵ .

Node depth centrality. Recall that the difference between the total liability and the actual payment of a defaulting agent j is

$$\alpha(\tau)\bar{d} - x_j(\tau, D, \epsilon) = (1 + \beta)\xi_j(\tau, D, \epsilon), \quad (15)$$

i.e., the fundamental payment shortfall

$$\xi_j(\tau, D, \epsilon) \equiv \left[\alpha(\tau)\bar{d} + \bar{s} + \epsilon_j + L(\tau) - \bar{e} - \sum_{k \in N} q_{jk}x_k(\tau, D, \epsilon) \right]^+,$$

multiplied by $(1 + \beta)$ due to the deadweight loss. We can use (15) to represent the aggregate deadweight losses, similar to Capponi, Corell, and Stiglitz (2022), as:

$$\begin{aligned} & \beta \sum_{j \in \mathcal{D}(\tau, \epsilon)} \xi_j(\tau, D, \epsilon) \left[1 + (1 + \beta) \sum_{k \in \mathcal{D}(\tau, \epsilon)} q_{kj} + (1 + \beta)^2 \sum_{k \in \mathcal{D}(\tau, \epsilon)} \sum_{l \in \mathcal{D}(\tau, \epsilon)} q_{lk}q_{kj} \right. \\ & \left. + (1 + \beta)^3 \sum_{k \in \mathcal{D}(\tau, \epsilon)} \sum_{l \in \mathcal{D}(\tau, \epsilon)} \sum_{m \in \mathcal{D}(\tau, \epsilon)} q_{ml}q_{lk}q_{kj} + \dots \right]. \end{aligned} \quad (16)$$

The expression in (16) captures the following:

1. The initial deadweight loss for a defaulting agent j : $\beta\xi_j(\tau, D, \epsilon)$.
2. The direct first-order effect of payment shortfalls and the deadweight loss on agent j 's counterparties: $\beta\xi_j(\tau, D, \epsilon)(1 + \beta) \sum_{k \in \mathcal{D}(\tau, \epsilon)} q_{kj}$.
3. The second-order effect of these counterparties affecting their own counterparties: $\beta\xi_j(\tau, D, \epsilon)(1 + \beta)^2 \sum_{k \in \mathcal{D}(\tau, \epsilon)} \sum_{l \in \mathcal{D}(\tau, \epsilon)} q_{lk}q_{kj}$.
4. All the higher-order effects from iterating this process infinitely.

Denote $Q_{\mathcal{D}(\tau, \epsilon)}$ as a submatrix of Q for the subset of $\mathcal{D}(\tau, \epsilon)$. If the spectral radius of $(1 + \beta)Q_{\mathcal{D}(\tau, \epsilon)}$ is less than one, then we can represent the sum as a Neumann series

$$\beta \sum_{j \in \mathcal{D}(\tau, \epsilon)} \xi_j(\tau, D, \epsilon) \underbrace{\left(1 + (1 + \beta) \sum_{k \in \mathcal{D}(\tau, \epsilon)} q_{kj} + (1 + \beta)^2 \sum_{k \in \mathcal{D}(\tau, \epsilon)} \sum_{l \in \mathcal{D}(\tau, \epsilon)} q_{lk}q_{kj} + \dots \right)}_{C_j(\mathcal{D}(\tau, \epsilon))}, \quad (17)$$

where $C_j(\mathcal{D}(\tau, \epsilon))$ is the *node depth centrality* among $\mathcal{D}(\tau, \epsilon)$ as defined by [Glasserman and Young \(2015\)](#). Note that $C_j(\mathcal{D}(\tau, \epsilon))$ only depends on the set of defaulting agents.²⁴

We assume that the spectral radius assumption, the spectral radius of $(1 + \beta)Q_{\mathcal{D}(\tau, \epsilon)}$ is less than one, holds for the following analysis, as it allows us to derive crisp analytical results, which bring useful insights. In particular, the node depth centrality is useful to derive a closed-form solution for the defaulting amount in matrix notation. However, the insights can be generalized to cases in which the spectral radius assumption does not hold. All the main results we show later (Theorems 1 and 2, and Proposition 4) continue to hold without the spectral radius assumption.

²⁴The technical condition for the spectral radius being less than one also has a clear economic interpretation. For the node depth centrality to be well defined, if there are cycles in the subnetwork $\mathcal{D}(\tau, \epsilon)$, the entries of the relative liability matrix $Q_{\mathcal{D}(\tau, \epsilon)}$ have to be sufficiently small. In other words, if a payment within the subnetwork can be amplified through cycles, then it must be sufficiently small, because otherwise there would be too much amplification of welfare losses within the set of defaulting agents and the infinite sum (16) does not converge, resulting in a complete depletion of one or more agents' assets. For example, the ring network has a spectral radius greater than one, and its node depth centrality could not be computed using matrix inversion.

As it can be seen from Lemma A.2 in the Appendix, the aggregate deadweight loss can be represented in terms of the node depth centrality. The (node depth) centrality $C_j(\mathcal{D}(\tau, \epsilon))$ can be interpreted as a “default contagion multiplier.” This multiplier captures the amplification effect of defaults on the aggregate deadweight losses throughout the network, as $C_j(\mathcal{D}(\tau, \epsilon))$ shows how the initial impact of a default by agent j is magnified across the network of defaulting agents. The magnitude of $C_j(\mathcal{D}(\tau, \epsilon))$ represents the importance of j , in terms of amplification of deadweight losses, in the given network structure D for a given settlement time τ . Therefore, the multiplier $C_j(\mathcal{D}(\tau, \epsilon))$ quantifies the systemic importance of agent j in terms of its payment shortfall-induced deadweight losses.

Finally, we consider comparative statics of decreasing τ to τ' on the ex ante welfare loss. There can be two different cases following this effect: with or without the changes in the set of defaulting agents.

Case with no additional defaults. First, suppose that $\mathcal{D}(\tau', \epsilon) = \mathcal{D}(\tau, \epsilon)$, i.e., the set of defaulting agents under the same liquidity shock state ϵ would be the same as that under τ . The centrality of an agent $C_j(\mathcal{D}(\tau, \epsilon))$ would remain the same, but the payment shortfall amount will change due to the following factors:

- (a) The initial decline in $x_j(\tau, D, \epsilon)$ due to $L(\tau)$ increasing to $L(\tau')$, resulting from higher liquidity cost to facilitate quicker trades;
- (b) The decrease in inter-agent payments $x_j(\tau, D, \epsilon)$ to $x_j(\tau', D, \epsilon)$ that amplifies the initial increase in cost of $L(\tau') - L(\tau)$; and
- (c) The increase from $\alpha(\tau)\bar{d}$ to $\alpha(\tau')\bar{d}$ due to a lesser degree of netting of liabilities.

Case with additional defaults. Second, suppose that $\mathcal{D}(\tau, \epsilon) \subset \mathcal{D}(\tau', \epsilon)$, i.e., there are additional agents who default under this new regime under the same ϵ . Then, there will be an additional jump in deadweight losses due to reaching this default threshold for previously solvent agents. This impacts the deadweight losses in two ways:

1. There will be more agents included in the summation of $\sum_{j \in \mathcal{D}(\tau', \epsilon)}$ compared to $\sum_{j \in \mathcal{D}(\tau, \epsilon)}$.
2. The multiplier for the payment shortfalls, $C_j(\mathcal{D}(\tau, \epsilon))$, will jump up to $C_j(\mathcal{D}(\tau', \epsilon))$ as the summation of the weighting matrix $\sum_{k \in \mathcal{D}(\tau', \epsilon)} Q_{kj}$ increases for each $j \in \mathcal{D}(\tau', \epsilon)$.

Consequently, even if the payment shortfall for each agent $\alpha(\tau)\bar{d} - x_j(\tau, D, \epsilon)$ is continuously decreasing in τ , there will be discontinuous effects from the change in the set of defaulting agents $\mathcal{D}(\tau', \epsilon)$. This effect increases as the number of additional defaults increases.

After considering both changes, we also discount the deadweight losses by the lower likelihood of liquidity shocks, as $F(\tau') < F(\tau)$. Hence, the tension between the two factors—an increase in deadweight losses versus a decrease in the likelihood of liquidity shocks—will determine whether faster payments (lower τ') increase or decrease social welfare.

Default threshold points. Combining the three factors, we argue that focusing on the *default threshold points*—the values of τ where a marginal decrease in τ would increase the number of defaulting agents—is critical in evaluating comparative statics with ex ante welfare losses. Formally, we define the set of default threshold points for a given network D and liquidity shock ϵ as $\mathcal{T}(D, \epsilon)$, where $\forall \tilde{\tau} \in \mathcal{T}(D, \epsilon)$, $\mathcal{D}(\tilde{\tau}, \epsilon) \subset \mathcal{D}(\tilde{\tau} - \delta, \epsilon)$ and $\mathcal{D}(\tilde{\tau} - \delta, \epsilon) \not\subset \mathcal{D}(\tilde{\tau}, \epsilon)$, while $\mathcal{D}(\tilde{\tau} + \delta, \epsilon) = \mathcal{D}(\tilde{\tau}, \epsilon)$, for any small $\delta > 0$. Define the union of the set of default threshold points for every possible realization of ϵ (i.e., $\forall \epsilon$ with positive probability) as the default threshold points $\mathcal{T}(D)$. We can define the set of settlement times such that the number of defaulting agents remains constant over the set up to the default threshold points. In other words, for any $\tilde{\tau} \in \mathcal{T}(D)$, we define the set of constant defaults as $\underline{\mathcal{T}}(\tilde{\tau}, D)$ such that $\mathcal{D}(\tau, \epsilon) = \mathcal{D}(\tilde{\tau}, \epsilon)$ if and only if $\tau \in \underline{\mathcal{T}}(\tilde{\tau}, D)$ for every possible ϵ .

Theorem 1. *If the settlement time τ is a default threshold point, i.e., $\tau \in \mathcal{T}(D)$ for a given network D , the left-hand derivative of the ex ante welfare loss is strictly less than the right-hand derivative of the ex ante welfare loss at τ , i.e., $\frac{\partial W(\tau, D)}{\partial \tau^-} < \frac{\partial W(\tau, D)}{\partial \tau^+}$, due to additional defaults. Furthermore, there can be cases in which an increase in τ improves*

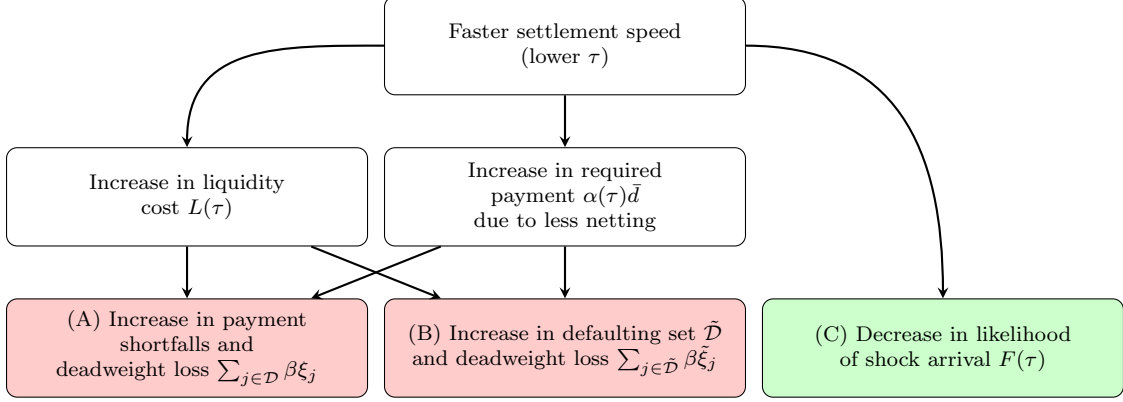


Figure 5: Impact of a faster settlement speed on financial stability

Note: This figure visualizes the effect of a faster settlement speed (lower τ) on financial stability through three different channels—the cost channels, (A) and (B), and the benefit channel (C). Faster settlement increases liquidity cost and required payment due to less netting, while decreasing the likelihood of shock arrival, i.e., the benefit channel (C). However, the first two effects lead to an increase in payment shortfalls and deadweight loss, i.e., the cost channel (A). Moreover, if τ is a default threshold point, the first two effects can lead to an increase in the number of defaults, leading to a further increase in deadweight loss, i.e., the cost channel (B). Therefore, whether τ is a default threshold point or not is important, as it determines the importance of the cost channel (B), highlighted by Theorem 1.

welfare exactly at a default threshold point.

Theorem 1 implies that evaluating the changes in ex ante social welfare should account for both the continuous changes in the likelihood of shocks and the resulting aggregate deadweight loss and the *discrete jump* in the aggregate deadweight loss due to changes in the number of defaulting agents at the default threshold points in $\mathcal{T}(D)$. An extreme case of the effect of a default threshold point is the phase transition of the complete network shown in Proposition 2 and τ^* in Corollary 1. There is only one threshold point for the complete network due to its perfect symmetry. Figure 5 visualizes the tradeoff and the role of default threshold points highlighted by Theorem 1.

Note that if τ is not a default threshold point, the effect of faster settlement speed is ambiguous. A marginal change in ex ante welfare loss $W(\tau, D)$ with respect to τ can be positive or negative, depending on the functional forms ($\alpha(\tau)$, $L(\tau)$, and $F(\tau)$), the parameters ($\beta, \bar{\epsilon}, \bar{s}, G(\epsilon)$), and the centrality of each agent under each realization of ϵ ($C_j(\mathcal{D}(\tau, \epsilon))$). In

other words, the tradeoff depends on many moving parts—netting benefits, liquidity costs, counterparty risks, and network structure—as characterized by Proposition 3.

Figure 4 illustrates a great example of Theorem 1 and the tradeoff of faster settlement speed. First, it shows that the settlement time does not have a monotone effect on the ex ante social welfare. When settlement time τ is close to zero, an increase in τ leads to a decrease in welfare. The ex ante social welfare starts to increase around $\tau \approx 0.18$. This is because of the interaction between the cost and benefit of a marginal change in τ , summarized in Proposition 3. The cost of increased likelihood of shock arrival dominates the benefits of decreases in liquidity cost and aggregate deadweight loss, resulting in a U-shaped graph. However, the ex ante social welfare forms another larger U-shaped graph once τ reaches the default threshold point at which the set of defaulting agents becomes just one shocked agent. Therefore, the ex ante social welfare has a stark kink at the threshold point, where the left-hand derivative and the right-hand derivative have different signs. This is due to the sudden shift in the set of defaulting agents from all agents to only one agent.

4.3. Liquidity Conditions and Optimal Settlement Speed

The amount of available liquidity is a critical factor that determines the likelihood of a liquidity crunch. For example, Copeland, Duffie, and Yang (2025) find that the likelihood of a future liquidity crunch in wholesale U.S. dollar funding markets can be predicted by the levels of reserve balances at the most active financial intermediaries. Even though there are various channels of intraday liquidity provision and general trends towards ample liquidity Kabadjova et al. (2023), there has been a recent reversal in liquidity conditions, which could be concerning. Thus, we analyze how liquidity conditions influence the relationship between settlement speed and financial stability.

In particular, for this exercise, we conduct comparative statics with respect to liquidity cost, $L(\tau)$, and cash buffer, \bar{e} . The payment system liquidity conditions worsen, or become more scarce, if $L(\tau)$ increases and \bar{e} decreases. The following result considers a worsening of

liquidity conditions from an ample (liquidity) regime to a scarce (liquidity) regime denoted as subscripts A and S , respectively.

Theorem 2. *If liquidity conditions worsen from an ample regime A to a scarce regime S by an increase in liquidity cost from $L_A(\tau)$ to $L_S(\tau)$ and/or a decrease in cash buffer from \bar{e}_A to \bar{e}_S , then the following holds:*

1. **Shifting of default threshold points:** *For each default threshold point $\tilde{\tau}_A \in \mathcal{T}_A(D)$ in the ample regime, there exists a corresponding threshold point $\tilde{\tau}_S \in \mathcal{T}_S(D)$ in the scarce regime at which the same agent is on the verge of default. Furthermore, this corresponding threshold point is greater under the scarce regime, $\tilde{\tau}_S > \tilde{\tau}_A$.*
2. **Higher social welfare loss:** *The ex ante social welfare loss is strictly higher under the scarce liquidity regime for any given settlement time τ and network D , i.e., $W_S(\tau, D) > W_A(\tau, D)$.*
3. **Slower optimal settlement speed:** *Suppose that the optimal settlement time under the scarce regime is interior and locally unique. Then for a sufficiently small deterioration in liquidity conditions, the optimal settlement speed is strictly slower than under the ample regime: $\tau_S^{opt} > \tau_A^{opt}$.*

The first two statements of Theorem 2 provide a theoretical underpinning of the empirical findings of Copeland, Duffie, and Yang (2025). First, scarce liquidity makes the marginal agent, who was on the verge of default at default threshold point, in the ample liquidity regime, deeper into default, as the agent has less liquidity, i.e., larger payment shortfall. The default threshold point for the same agent under the scarce liquidity regime should then be larger. Similarly, the ex ante social welfare loss is greater under the scarce liquidity regime, as more agents default with the same τ , and any payment shortfall is larger, as agents have less liquidity in general. These results are straightforward, as liquidity plays an important role in mitigating any potential shocks and contagion through counterparty defaults.

Moreover, Theorem 2 provides a novel insight from our model of settlement speed with payment network: as liquidity conditions deteriorate, adopting slower settlement can improve financial stability. Under worsening liquidity conditions, financial stability concerns favor slower settlement systems that prioritize netting efficiency and liquidity conservation over rapid settlement, despite the lower likelihood of counterparty defaults with faster settlements. This result highlights the important relationship between liquidity conditions and optimal settlement speed for financial stability. Furthermore, it can also explain market participants' choice of payment systems with different settlement times (e.g., RTGS vs DNS), depending on the liquidity conditions and payment amounts (Copeland and Garratt, 2019).

The intuition of this result of *slower optimal settlement speed* is the following. If the optimal settlement time under the scarce regime τ_S^{opt} is at a default threshold point, the discrete jump in deadweight loss due to an additional default, highlighted by Theorem 1, plays a significant role in determining the optimal settlement speed. If the optimal settlement time under the ample regime τ_A^{opt} is also at the corresponding default threshold point, τ_S^{opt} , then we already showed that the optimal settlement time under the scarce regime is greater than that under the ample regime by statement 1 in Theorem 2.

Now consider the other case, i.e., τ_A^{opt} is not a corresponding default threshold point. We claim that any time greater than the optimal settlement time under the scarce regime τ_S^{opt} is not optimal under the ample regime. This is because in the absence of a discrete change in welfare loss, the marginal change in ex ante welfare loss is positive by the fact that the optimal settlement time under the scarce regime is exactly at the default threshold point τ_S^{opt} but no more. In other words, from the decomposition (14), the benefit is outweighed by the cost, in the absence of a change in default set. However, that implies the same tradeoff would also tip the scale to the cost, as the change in the liquidity conditions are small. Therefore, the marginal change in the ex ante welfare loss is positive at any τ greater than or equal to τ_S^{opt} under the ample regime. Thus, the optimal settlement time under the ample regime should be less than that under the scarce regime, i.e., $\tau_A^{opt} < \tau_S^{opt}$.

4.4. Identification of Default Threshold Points

Given the importance of default threshold points on the evaluation of ex ante social welfare with respect to settlement speed, finding the default threshold points of a given network is crucial. However, if identification of default threshold points can be computationally complex, our results, especially Theorem 1 and Theorem 2, would be difficult to utilize in practice. Hence, we characterize the tractability of default threshold point identification. In particular, we show that identification of default threshold points is solvable in polynomial time (i.e., a problem in complexity class P).

First, we introduce a finite discretization of a continuous distribution $G(\epsilon)$. This is, in practice, the only feasible way to simulate and compute continuous distributions. Formally, a finite discrete approximation of $G(\epsilon)$ is $\tilde{G}(\epsilon)$, which is supported on a finite set $\{\epsilon^{(1)}, \dots, \epsilon^{(M)}\}$. This approximation can be done by several methods including quantization (divide the support into bins and take representative values) and Monte Carlo simulation. For any desired level of precision, we can approximate the shock distribution $G(\epsilon)$ arbitrarily close to that level of precision by adjusting the finite number M .

Proposition 4. *Identification of the set of default threshold points is solvable in polynomial time to any desired precision $\delta > 0$. Moreover, each node's default indicator is monotone in τ , so the number of threshold points is at most n per shock realization ϵ .*

Proposition 4 implies that the seemingly complex problem of identifying default threshold points of a network is computationally tractable in polynomial time. In addition to simply checking which points are the default threshold points in $[0, \bar{T}]$, we can also see how many additional defaults are occurring at each default threshold point. Thus, we can identify whether a network would have a sudden jump in deadweight loss, as in the case of the complete network, or not. Therefore, the key insight of considering default threshold points for the ex ante welfare calculations, highlighted by Theorem 1, provides guidance for numerical implementation in stylized network settings. While the computational complexity

grows with the number of shock realizations M and the desired precision δ , Proposition 4 establishes that the problem is tractable, offering guidance for evaluating the effect of faster settlement speed on financial stability.

5. Empirical Implications

The model suggests that faster settlement can reduce the likelihood of stress events but can increase the severity of stress events when they happen. This could be tested by comparing the likelihood and severity of systemic events across markets or time periods with different settlement speeds, controlling for other factors.

Although we do not model government intervention explicitly, the role of the government or central bank is crucial in maintaining stable payment systems, and our results provide insights on this as well. An implicit benefit of a slower settlement speed (larger τ) is that the government has less incentive to intervene in the ex-post event of a shock arrival. Conversely, if settlement speed is fast and a shock arrives, despite its lower likelihood, the government has a much stronger incentive to intervene, as the ex-post cost of the systemic event is much larger. This implies that faster settlement systems might paradoxically lead to more frequent government interventions. Researchers could investigate this by comparing the frequency and scale of government interventions in markets with different settlement speeds.

Lemma A.2 provides a justification for ex-post liquidity injection to central agents when a shock arrives and causes a systemic event. This implies that policymakers and central banks should focus on identifying and supporting the most systemically important institutions during stress times. Historical examples of various emergency facilities for central entities in payment systems, such as the Primary Dealer Credit Facility, are well justified by our results and by various previous papers in the literature on bailouts.

Our model's focus on ex ante welfare, incorporating both the likelihood and severity of shocks, suggests a new framework for empirical assessments of payment system stability.

Further studies could compare traditional ex post measures with ex ante measures that account for the probability of shock arrivals.

On a related note, our results suggest that optimal settlement speed may vary depending on network structure and other parameters. This implies that a one-size-fits-all approach to settlement speed regulation may be suboptimal. Empirical studies could identify the parameter values and analyze whether a faster settlement speed can improve or worsen the financial stability of a payment system.

Finally, our model’s insights on the relationship between liquidity conditions and optimal settlement speed offer several testable hypotheses. Researchers could examine how changes in liquidity conditions, such as variations in reserve balances or funding costs, correlate with changes in the likelihood and severity of liquidity crunches. Moreover, the prediction that deteriorating liquidity conditions could lead to adjustments towards slower settlement speeds could be tested by analyzing historical data on settlement cycles and liquidity measures across different payment systems or financial markets. Conversely, one could test how changes in liquidity conditions can affect market participants’ choice of payment systems with different settlement speeds.

These empirical implications suggest a set of empirically testable hypotheses and new approaches to analyzing payment systems and financial networks. They also highlight the importance of considering network structure, settlement speed, and systemic risk in an integrated framework when designing and evaluating financial market infrastructures.²⁵

6. Conclusion

This paper provides a comprehensive analysis of the implications of settlement speed in payment systems, taking into account the complex interactions between network structure,

²⁵Note that our policy implications apply primarily to systemic stress scenarios where settlement disruptions threaten default cascades. For routine settlement fails without solvency implications, different policy tools such as operational improvements and penalty structures (e.g., fails charge) may be more appropriate. Thus, our model and results are relevant for financial stability implications rather than operational efficiency.

netting efficiency, liquidity costs, and counterparty risks. We highlight the potential tradeoffs and identify key factors that influence the effect of a change in settlement speed. Our results contribute to the ongoing debate about optimal settlement speeds in financial markets and offer several key insights in the design and regulation of payment systems.

We show that faster settlements do not always lead to improved financial stability. While quicker settlement can reduce the likelihood of counterparty defaults, it can also increase liquidity costs and reduce netting opportunities. Hence, faster settlements can decrease the likelihood of crisis events but increase their severity, and might paradoxically lead to more frequent government interventions due to stronger incentives to intervene when a crisis is more severe. Thus, we show that the optimal settlement speed depends critically on the structure of the payment network, the distribution of liquidity shocks, and the rates of change in liquidity costs and netting efficiency.

We identify the existence of default threshold points, and show that the welfare effects of changing settlement speed can differ dramatically depending on whether the system is operating at or above these threshold points.

We demonstrate that the node depth centrality is measure of an agent’s systemic importance in the context of payment systems, consistent with the literature on financial networks. This measure captures how the initial impact of a default by one agent is amplified through the network of defaulting agents, providing a useful tool for identifying systemically important institutions in payment networks.

Our results illustrate that the optimal settlement speed depends on liquidity conditions, with slower settlement being preferable when liquidity is scarce. This finding has important implications for the design of resilient payment systems, intraday liquidity provision, and the implementation of changes in settlement cycles, as a combination of faster settlement speed and scarce liquidity can worsen financial stability.

Our paper also suggests several directions for empirical work. The model’s predictions, specifically that faster settlement reduces crisis frequency but increases conditional severity,

and that this tradeoff depends on network density and liquidity conditions, could be tested by exploiting cross-market or cross-country variation in settlement cycle reforms, such as the staggered adoption of $T + 1$ across equity markets, or the differential adoption of real-time gross settlement systems across payment networks. We leave formal empirical investigation to future work.

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Internet Appendix (for online publication only)

A. Omitted Proofs

A.1. Proof of Proposition 1

Proof. Without loss of generality, assume that agent 1 receives the liquidity shock.

1. First, consider the complete network.

(a) **Case 1.1.** Suppose that the relative size of the liquidity shock is small as $\bar{\epsilon} \leq n(\bar{e} - \bar{s} - L(\tau))$. First consider the case in which the relative size of the total liabilities is small as $\alpha(\tau)\bar{d} \leq \bar{\epsilon} - (\bar{e} - \bar{s} - L(\tau))$. Hence, even with all the payments received, agent 1 does not have any excess cash to cover the liquidity shock and the senior debt, i.e. agent 1 fully defaults on all the liabilities towards other agents in the network.

Then, for any agent $j \geq 2$ to survive, the following should hold:

$$\bar{e} - \bar{s} - L(\tau) + \frac{n-2}{n-1}\alpha(\tau)\bar{d} \geq \alpha(\tau)\bar{d}. \quad (18)$$

Rearranging (18) yields

$$(n-1)(\bar{e} - \bar{s} - L(\tau)) \geq \alpha(\tau)\bar{d}.$$

This holds because

$$\alpha(\tau)\bar{d} \leq \bar{\epsilon} - (\bar{e} - \bar{s} - L(\tau)) \leq (n-1)(\bar{e} - \bar{s} - L(\tau)),$$

where the first inequality comes from the relative size of the total liabilities, and the second inequality comes from the relatively small size of the liquidity shock. Therefore, (18) holds, so only agent 1 defaults.

Since all other agents are paying in full, $A_1 = \bar{e} + \alpha(\tau)\bar{d}$. Then,

$$\begin{aligned} \xi_1 &= \alpha(\tau)\bar{d} + \bar{s} + \epsilon - \bar{e} + L(\tau) - \alpha(\tau)\bar{d} \\ &= \bar{s} + \epsilon - \bar{e} + L(\tau), \end{aligned}$$

and the total deadweight loss from default in this case is

$$\min \{ \beta(\bar{s} + \epsilon - \bar{e} + L(\tau)), \bar{e} + \alpha(\tau)\bar{d} \}.$$

Case 1.2. Now consider the case in which the relative size of the total liabilities is large as $\alpha(\tau)\bar{d} > \bar{\epsilon} - (\bar{e} - \bar{s} - L(\tau))$. Then, all the payments received from agents 2 through n will exceed the cash amount required to pay for the liquidity shock for agent 1. Then, all agents survive if

$$n(\bar{e} - \bar{s} - L(\tau)) \geq \bar{\epsilon},$$

which is immediately satisfied by the small shock assumption. Therefore, only agent 1 defaults in this case as well.

Since agent 1 is receiving all the liabilities from other agents in full, $A_1 = \bar{e} + \alpha(\tau)\bar{d}$. Therefore,

$$\begin{aligned} \xi_1 &= \alpha(\tau)\bar{d} + \bar{s} + \bar{\epsilon} - \bar{e} + L(\tau) - \alpha(\tau)\bar{d} \\ &= \bar{s} + \bar{\epsilon} - \bar{e} + L(\tau). \end{aligned}$$

Hence, the total deadweight loss from default is

$$\min \{ \beta(\bar{s} + \bar{\epsilon} - \bar{e} + L(\tau)), \bar{e} + \alpha(\tau)\bar{d} \}.$$

Case 1.3. Next, suppose that the relative size of the liquidity shock is large as $\bar{\epsilon} > n(\bar{e} - \bar{s} - L(\tau))$ but the relative size of the total liabilities is small as $\alpha(\tau)\bar{d} \leq (n-1)(\bar{e} - \bar{s} - L(\tau))$. Because the liquidity shock exceeds the aggregate cash in the network, agent 1 will be fully defaulting on its payments to other agents. Therefore, (18) should hold for the remaining agents to survive. Rearranging (18) implies

$$(n-1)(\bar{e} - \bar{s} - L(\tau)) \geq \alpha(\tau)\bar{d},$$

which is immediately satisfied by the assumption on the relative size of the total liabilities. Therefore, no other agents default in this case.

Since all other agents are paying in full, $A_1 = \bar{e} + \alpha(\tau)\bar{d}$. Then,

$$\begin{aligned} \xi_1 &= \alpha(\tau)\bar{d} + \bar{s} + \bar{\epsilon} - \bar{e} + L(\tau) - \alpha(\tau)\bar{d} \\ &= \bar{s} + \bar{\epsilon} - \bar{e} + L(\tau), \end{aligned}$$

and the total deadweight loss from default in this case is

$$\min \{ \beta (\bar{s} + \epsilon - \bar{e} + L(\tau)), \bar{e} + \alpha(\tau)\bar{d} \}.$$

- (b) Suppose that the relative size of the liquidity shock is large as $\bar{e} > n(\bar{e} - \bar{s} - L(\tau))$, and the relative size of the total liabilities is large as $\alpha(\tau)\bar{d} > (n-1)(\bar{e} - \bar{s} - L(\tau))$. Again, due to the size of the liquidity shock, agent 1 fully defaults on its payments to other agents, and the no default condition for other agents is (18). Again, (18) is equivalent to

$$(n-1)(\bar{e} - \bar{s} - L(\tau)) \geq \alpha(\tau)\bar{d},$$

which contradicts the condition for the relative size of the total liabilities. Therefore, all agents default in this case.

Finally, we compute the total deadweight loss amount from defaults in this case. First, note that $x_1 = 0$, as the liquidity shock is larger than the total amount of cash in the network, i.e. there will be no remaining asset value even if all other agents pay their full cash amount to agent 1. Second, due to symmetry, all other agents will pay the same amount, i.e. $x_2 = x_3 = \dots = x_n = \bar{x}$. Plugging this into (4) implies

$$\bar{x} = \bar{e} - \bar{s} - L(\tau) + \frac{(n-2)\bar{x}}{n-1}.$$

Thus, we derive

$$\bar{x} = (n-1)(\bar{e} - \bar{s} - L(\tau)).$$

Then, plugging this into the definition A_j yields

$$\begin{aligned} A_j &= \bar{e} + \frac{n-2}{n-1}(n-1)(\bar{e} - \bar{s} - L(\tau)) \\ \Rightarrow A_j &= \bar{e} + (n-2)(\bar{e} - \bar{s} - L(\tau)) \end{aligned} \tag{19}$$

for any $j > 1$. Thus, the total payment shortfall is

$$\begin{aligned}
\xi_j &= \alpha(\tau)\bar{d} + \bar{s} + L(\tau) - A_j \\
\Rightarrow \xi_j &= \alpha(\tau)\bar{d} + \bar{s} - (n-1)(\bar{e} - L(\tau)) + (n-2)\bar{s} \\
\Rightarrow \xi_j &= \alpha(\tau)\bar{d} - (n-1)(\bar{e} - L(\tau) - \bar{s})
\end{aligned} \tag{20}$$

for each $j > 2$ after plugging in (19). By construction, $A_1 = \bar{e} + (n-1)(\bar{e} - \bar{s} - L(\tau))$, and

$$\begin{aligned}
\xi_1 &= \alpha(\tau)\bar{d} + \bar{s} + L(\tau) + \bar{e} - A_1 \\
&= \alpha(\tau)\bar{d} + \bar{e} - n(\bar{e} - \bar{s} - L(\tau)).
\end{aligned}$$

Thus, the total deadweight loss from defaults is

$$\begin{aligned}
&\min \left\{ \beta \left[\alpha(\tau)\bar{d} + \bar{e} - n(\bar{e} - \bar{s} - L(\tau)) \right], \bar{e} + (n-1)(\bar{e} - \bar{s} - L(\tau)) \right\} \\
&+ (n-1) \min \left\{ \beta \left[\alpha(\tau)\bar{d} - (n-1)(\bar{e} - L(\tau) - \bar{s}) \right], \bar{e} + (n-2)(\bar{e} - \bar{s} - L(\tau)) \right\}.
\end{aligned}$$

2. Now consider the ring network.

- (a) **Case 2.1.** Suppose that the relative size of the liquidity shock is small as $\bar{\epsilon} \leq n(\bar{e} - \bar{s} - L(\tau))$. Hence, if all agents combine their available liquidity buffers, they can either pay the promised payment in full even without any payment from agent 1, when the total liabilities for each agent is small, or can reuse the payments received by agent 1 and exceed the liquidity shock amount by combined payments, when the total liabilities for each agent is large. In either case, agent 1 will receive the full payment $\alpha(\tau)\bar{d}$ from agent n . Therefore, agent 1 pays

$$x_1 = \phi \equiv [\bar{e} - \bar{s} - L(\tau) + \alpha(\tau)\bar{d} - \bar{\epsilon}]^+$$

to agent 2. Then, agent 2 can reuse this payment amount on top of its own cash buffer $\bar{e} - \bar{s} - L(\tau)$ to pay agent 3, and agent 3 can reuse the whole payment on top of its own cash buffer $\bar{e} - \bar{s} - L(\tau)$, and so on. In other words, by the same

iterative steps,

$$\begin{aligned}
x_2 &= \bar{e} - \bar{s} - L(\tau) + \phi \\
x_3 &= 2(\bar{e} - \bar{s} - L(\tau)) + \phi \\
x_4 &= 3(\bar{e} - \bar{s} - L(\tau)) + \phi \\
&\vdots \\
x_k &= (k - 1)(\bar{e} - \bar{s} - L(\tau)) + \phi.
\end{aligned}$$

By the initial assumption on the shock size, there exists an integer k such that

$$k(\bar{e} - \bar{s} - L(\tau)) + \phi \geq \alpha(\tau)\bar{d},$$

and rearranging the condition implies the smallest k is equivalent to

$$\psi = \min \left\{ n - 1, \left\lfloor \frac{\alpha(\tau)\bar{d} - \phi}{\bar{e} - \bar{s} - L(\tau)} \right\rfloor \right\},$$

where $\lfloor \cdot \rfloor$ denotes the floor function, i.e., the greatest integer that is less than or equal to the argument \cdot . Now we calculate the total deadweight loss from the defaults of ψ agents and agent 1. First, note that agent 1 receives the full payment amount, so $A_1 = \bar{e} + \alpha(\tau)\bar{d}$, and $\xi_1 = \bar{s} + \bar{e} - \bar{e} + L(\tau)$. Thus, the deadweight loss from agent 1's default is

$$\min \{ \beta(\bar{s} + \bar{e} - \bar{e} + L(\tau)), \bar{e} + \alpha(\tau)\bar{d} \}.$$

Second, note that agent 2 receives ϕ from agent 1, so $A_2 = \bar{e} + \phi$. Thus, $\xi_2 = \alpha(\tau)\bar{d} + \bar{s} - \bar{e} + L(\tau) - \phi$. From this, we can also compute $A_3 = \bar{e} + x_2 = 2\bar{e} - \bar{s} - L(\tau) + \phi$, and $\xi_3 = \alpha(\tau)\bar{d} - 2(\bar{e} - \bar{s} - L(\tau)) - \phi$. Similarly, for any $m \leq \psi$,

$$\begin{aligned}
A_{m+1} &= \bar{e} + (m - 1)(\bar{e} - \bar{s} - L(\tau)) + \phi \\
\xi_{m+1} &= \alpha(\tau)\bar{d} - m(\bar{e} - \bar{s} - L(\tau)) - \phi.
\end{aligned}$$

Therefore, the total deadweight loss from defaults is

$$\begin{aligned}
&\min \{ \beta(\bar{s} + \bar{e} - \bar{e} + L(\tau)), \bar{e} + \alpha(\tau)\bar{d} \} \\
&+ \sum_{m=1}^{\psi} \min \{ \beta(\alpha(\tau)\bar{d} - m(\bar{e} - \bar{s} - L(\tau)) - \phi), \bar{e} + (m - 1)(\bar{e} - \bar{s} - L(\tau)) + \phi \},
\end{aligned}$$

Case 2.2. Finally, suppose that the relative size of the liquidity shock is large as $\bar{\epsilon} > n(\bar{\epsilon} - \bar{s} - L(\tau))$, but the relative size of the total liabilities is small as $\alpha(\tau)\bar{d} \leq (n-1)(\bar{\epsilon} - \bar{s} - L(\tau))$. Then, agent n can still pay its full liabilities to agent 1, and the rest of the payment structure would be the same. Therefore, all the previous arguments go through and the total deadweight loss from defaults is the same.

- (b) Suppose that the relative size of the liquidity shock is large as $\bar{\epsilon} > n(\bar{\epsilon} - \bar{s} - L(\tau))$, and the relative size of the total liabilities is also large as $\alpha(\tau)\bar{d} > (n-1)(\bar{\epsilon} - \bar{s} - L(\tau))$.

Then, all agents default, and agent 1 cannot even pay the liquidity shock and senior debt, as shown in the complete network case (Statement 1). Hence, $\phi = 0$, and even agent n cannot pay its payment obligation in full as

$$(n-1)(\bar{\epsilon} - \bar{s} - L(\tau)) < \alpha(\tau)\bar{d}.$$

Thus, all agents default in this case.

Finally, we compute the total deadweight loss in this case. As agents 2 through n will accumulate all their available liquidity and pay agent 1, $x_n = (n-1)(\bar{\epsilon} - \bar{s} - L(\tau))$, implying

$$\begin{aligned} A_1 &= \bar{\epsilon} + (n-1)(\bar{\epsilon} - \bar{s} - L(\tau)) \\ \xi_1 &= \alpha(\tau)\bar{d} + \bar{\epsilon} - n(\bar{\epsilon} - \bar{s} - L(\tau)). \end{aligned}$$

Thus, the deadweight loss from agent 1's default is

$$\min \left\{ \beta \left[\alpha(\tau)\bar{d} + \bar{\epsilon} - n(\bar{\epsilon} - \bar{s} - L(\tau)) \right], \bar{\epsilon} + (n-1)(\bar{\epsilon} - \bar{s} - L(\tau)) \right\}.$$

Since agent 2 receives nothing from agent 1,

$$\begin{aligned} A_2 &= \bar{\epsilon} \\ \xi_2 &= \alpha(\tau)\bar{d} - (\bar{\epsilon} - \bar{s} - L(\tau)). \end{aligned}$$

Similarly, agent 3 receives $x_2 = \bar{\epsilon} - \bar{s} - L(\tau)$, so

$$\begin{aligned} A_3 &= \bar{\epsilon} + (\bar{\epsilon} - \bar{s} - L(\tau)) \\ \xi_2 &= \alpha(\tau)\bar{d} - 2(\bar{\epsilon} - \bar{s} - L(\tau)). \end{aligned}$$

Iteratively, we obtain

$$\begin{aligned} A_{m+1} &= \bar{e} + (m-1)(\bar{e} - \bar{s} - L(\tau)) \\ \xi_{m+1} &= \alpha(\tau)\bar{d} - m(\bar{e} - \bar{s} - L(\tau)). \end{aligned}$$

Therefore, the total deadweight loss from default is

$$\begin{aligned} &\min \{ \beta [\alpha(\tau)\bar{d} + \bar{e} - n(\bar{e} - \bar{s} - L(\tau))], \bar{e} + (n-1)(\bar{e} - \bar{s} - L(\tau)) \} \\ &+ \sum_{m=1}^{n-1} \min \{ \beta (\alpha(\tau)\bar{d} - m(\bar{e} - \bar{s} - L(\tau))), \bar{e} + (m-1)(\bar{e} - \bar{s} - L(\tau)) \}, \end{aligned}$$

which is greater than (8) for a small enough β , because $m \leq n-1$. If β is large enough, then the deadweight loss from default becomes simply the asset value, i.e., the second term in the minimum function. Thus, the above expression collapses to

$$\begin{aligned} &\bar{e} + (n-1)(\bar{e} - \bar{s} - L(\tau)) + \sum_{m=1}^{n-1} (\bar{e} + (m-1)(\bar{e} - \bar{s} - L(\tau))) \\ &= \bar{e} + (n-1)(\bar{e} - \bar{s} - L(\tau)) + (n-1)\bar{e} + (n-1)(n-2)(\bar{e} - \bar{s} - L(\tau)), \end{aligned}$$

which is equivalent to (8) when β is large.

■

A.2. Proof of Corollary 1

Proof. Recall that $\epsilon^*(\tau) = n(\bar{e} - \bar{s} - L(\tau))$ and $d^*(\tau) = (n - 1)(\bar{e} - \bar{s} - L(\tau))$ and the threshold conditions are

$$\begin{aligned}\bar{\epsilon} &> n(\bar{e} - \bar{s} - L(\tau)) \\ \alpha(\tau)\bar{d} &> (n - 1)(\bar{e} - \bar{s} - L(\tau)).\end{aligned}$$

Thus, by rearranging both threshold conditions we obtain

$$\begin{aligned}L(\tau) &> \bar{e} - \bar{s} - \frac{\bar{\epsilon}}{n} \\ L(\tau) &> \bar{e} - \bar{s} - \alpha(\tau)\frac{\bar{d}}{n - 1},\end{aligned}$$

where the first inequality comes from rearranging the threshold condition for $\bar{\epsilon}$ and the second inequality comes from the rearranged threshold condition for \bar{d} . Since both conditions should be simultaneously satisfied, we can combine the two conditions as the following

$$\begin{aligned}L(\tau) &> \max \left\{ \bar{e} - \bar{s} - \frac{\bar{\epsilon}}{n}, \bar{e} - \bar{s} - \alpha(\tau)\frac{\bar{d}}{n - 1} \right\} \\ \Rightarrow L(\tau) &> \bar{e} - \bar{s} - \min \left\{ \frac{\bar{\epsilon}}{n}, \alpha(\tau)\frac{\bar{d}}{n - 1} \right\}.\end{aligned}\tag{21}$$

Note that τ^* such that $L(\tau^*) \equiv \bar{e} - \bar{s} - \min \left\{ \frac{\bar{\epsilon}}{n}, \alpha(\tau^*)\frac{\bar{d}}{n - 1} \right\}$ holds is unique, because $L(\tau)$ is strictly decreasing in τ , while $\alpha(\tau)$ is decreasing in τ .

Finally, we check the boundary conditions. If $L(0) \leq \bar{e} - \bar{s} - \min \left\{ \frac{\bar{\epsilon}}{n}, \alpha(0)\frac{\bar{d}}{n - 1} \right\}$, then $\tau^* < 0$, implying that the threshold condition is never satisfied and the given statement is trivially true. If $L(\bar{T}) > \bar{e} - \bar{s} - \min \left\{ \frac{\bar{\epsilon}}{n}, \alpha(\bar{T})\frac{\bar{d}}{n - 1} \right\}$, then $\tau^* > \bar{T}$, and the threshold condition is always satisfied under the feasible settlement time. Thus, the statement is trivially true.

Therefore, for any $\tau > \tau^*$, the combined threshold condition, (21), is satisfied, and vice versa. ■

A.3. Proof of Proposition 2

Proof. First, note that the identity of the agent hit by a shock is irrelevant due to the symmetry of the network structure for both the complete and the ring networks. Suppose that τ is large enough so that $L(\tau) < L^*(\tau)$. If D is the complete network, then the marginal change in the ex ante welfare loss is

$$\frac{\partial W(\tau, D_c)}{\partial \tau} = nL'(\tau) + F'(\tau)\beta(\bar{s} + \bar{\epsilon} - \bar{e} + L(\tau)) + F(\tau)\beta L'(\tau). \quad (22)$$

If D is the ring network, then the marginal change in the ex ante welfare loss is

$$\begin{aligned} \frac{\partial W(\tau, D_r)}{\partial \tau} = & nL'(\tau) + F'(\tau) \left[\beta(\bar{s} + \bar{\epsilon} - \bar{e} + L(\tau)) \right. \\ & \left. + \sum_{m=1}^{\psi} \beta(\alpha(\tau)\bar{d} - m(\bar{e} - \bar{s} - L(\tau))) \right] \\ & + F(\tau) \left[\beta L'(\tau) + \sum_{m=1}^{\psi} \beta(\alpha'(\tau)\bar{d} + mL'(\tau)) \right], \end{aligned} \quad (23)$$

if $\phi \equiv [\bar{e} - \bar{s} - L(\tau) + \alpha(\tau)\bar{d} - \bar{\epsilon}]^+ = 0$, and

$$\begin{aligned} \frac{\partial W(\tau, D_r)}{\partial \tau} = & nL'(\tau) + F'(\tau) \left[\beta(\bar{s} + \bar{\epsilon} - \bar{e} + L(\tau)) \right. \\ & \left. + \sum_{m=1}^{\psi} \beta(\bar{\epsilon} - (m+1)(\bar{e} - \bar{s} - L(\tau)) - \phi) \right] \\ & + F(\tau) \left[\beta L'(\tau) + \sum_{m=1}^{\psi} \beta((m+1)L'(\tau) - \alpha'(\tau)\bar{d}) \right], \end{aligned} \quad (24)$$

if $\phi > 0$, where $\psi \equiv \min \left\{ n-1, \left\lfloor \frac{\alpha(\tau)\bar{d} - \phi}{\bar{e} - \bar{s} - L(\tau)} \right\rfloor \right\}$.

The comparison between the marginal change in the ex ante welfare loss for the complete network and that for the ring network is ambiguous. For example, (23) subtracted by (22) implies

$$F'(\tau) \sum_{m=1}^{\psi} \beta [\alpha(\tau)\bar{d} - m(\bar{e} - \bar{s} - L(\tau))] + F(\tau) \sum_{m=1}^{\psi} \beta [\alpha'(\tau)\bar{d} + mL'(\tau)],$$

which can be positive or negative depending on the parameter values $\beta, \bar{e}, \bar{s}, \bar{d}$ as well as the functional forms of $\alpha(\tau), F(\tau)$, and $L(\tau)$, because $F'(\tau) < 0$ and $F(\tau) > 0$.

Now suppose that τ is small enough so that $L(\tau) \geq L^*(\tau)$. We first show that every agent in the ring network defaults even if $L(\tau) = L^*(\tau)$.

Case 1. Suppose that $\frac{\bar{\epsilon}}{n} \leq \alpha(\tau) \frac{\bar{d}}{n-1}$. Then, $n(\bar{e} - \bar{s} - L(\tau)) = \bar{\epsilon}$ and $(n-1)(\bar{e} - \bar{s} - L(\tau)) \leq \alpha(\tau) \bar{d}$. Therefore, $\phi = 0$ and $\psi = n - 1$.

Case 2. Suppose $\frac{\bar{\epsilon}}{n} > \alpha(\tau) \frac{\bar{d}}{n-1}$. Then, $(n-1)(\bar{e} - \bar{s} - L(\tau)) = \alpha(\tau) \bar{d}$ and $n(\bar{e} - \bar{s} - L(\tau)) < \bar{\epsilon}$. Therefore, $\phi = 0$ and $\psi = n - 1$.

Therefore, even at the verge of the threshold condition, the ring network already has full contagion.

Recall that the aggregate deadweight loss in the complete network is

$$\begin{aligned} & \beta [\alpha(\tau) \bar{d} + \bar{\epsilon} - n(\bar{e} - \bar{s} - L(\tau))] + (n-1)\beta [\alpha(\tau) \bar{d} - (n-1)(\bar{e} - L(\tau) - \bar{s})] \\ &= \beta [n\alpha(\tau) \bar{d} + \bar{\epsilon} - (n + (n-1)^2) (\bar{e} - \bar{s} - L(\tau))]. \end{aligned}$$

The marginal change in ex ante welfare loss with respect to τ is

$$\begin{aligned} \frac{\partial W(\tau, D_c)}{\partial \tau} &= nL'(\tau) + F'(\tau)\beta [n\alpha(\tau) \bar{d} + \bar{\epsilon} - (n + (n-1)^2) (\bar{e} - \bar{s} - L(\tau))] \\ & \quad + F(\tau)\beta [n\bar{d}\alpha'(\tau) + (n + (n-1)^2) L'(\tau)]. \end{aligned} \quad (25)$$

Now recall that the aggregate deadweight loss in the ring network is

$$\begin{aligned} & \beta \left[\alpha(\tau) \bar{d} + \bar{\epsilon} - n(\bar{e} - \bar{s} - L(\tau)) + \sum_{m=1}^{n-1} (\alpha(\tau) \bar{d} - m(\bar{e} - \bar{s} - L(\tau))) \right] \\ &= \beta \left[n\alpha(\tau) \bar{d} + \bar{\epsilon} - n(\bar{e} - \bar{s} - L(\tau)) - \sum_{m=1}^{n-1} m(\bar{e} - \bar{s} - L(\tau)) \right] \\ &= \beta \left[n\alpha(\tau) \bar{d} + \bar{\epsilon} - n(\bar{e} - \bar{s} - L(\tau)) - \frac{n(n-1)}{2} (\bar{e} - \bar{s} - L(\tau)) \right] \\ &= \beta \left[n\alpha(\tau) \bar{d} + \bar{\epsilon} - \frac{n(n+1)}{2} (\bar{e} - \bar{s} - L(\tau)) \right], \end{aligned}$$

Therefore, the marginal change in the ex ante welfare loss with respect to τ is

$$\begin{aligned} \frac{\partial W(\tau, D_r)}{\partial \tau} &= nL'(\tau) + F'(\tau)\beta \left[n\alpha(\tau) \bar{d} + \bar{\epsilon} - \frac{n(n+1)}{2} (\bar{e} - \bar{s} - L(\tau)) \right] \\ & \quad + F(\tau)\beta \left[n\bar{d}\alpha'(\tau) + \frac{n(n+1)}{2} L'(\tau) \right], \end{aligned} \quad (26)$$

because $\phi \equiv [\bar{e} - \bar{s} - L(\tau) + \alpha(\tau) \bar{d} - \bar{\epsilon}]^+ = 0$ by the assumption.

Subtracting (25) from (26) implies

$$\begin{aligned} \frac{\partial W(\tau, D_r)}{\partial \tau} - \frac{\partial W(\tau, D_c)}{\partial \tau} = & F'(\tau)\beta \left[(n + (n-1)^2) - \frac{n(n+1)}{2} \right] (\bar{e} - \bar{s} - L(\tau)) \\ & - F(\tau)\beta \left[(n + (n-1)^2) - \frac{n(n+1)}{2} \right] L'(\tau). \end{aligned} \quad (27)$$

Since $(\bar{e} - \bar{s} - L(\tau))$ is positive and $-L'(\tau)$ is positive, (27) is positive, because

$$\left[(n + (n-1)^2) - \frac{n(n+1)}{2} \right] > 0. \quad (28)$$

The previous inequality (28) holds because

$$\begin{aligned} n + (n-1)^2 &> \frac{n(n+1)}{2} \\ 2n^2 - 2n + 2 &> n^2 + n \\ n^2 + 2 &> 3n, \end{aligned}$$

which holds for any $n > 2$. Therefore, $\frac{\partial W(\tau, D_c)}{\partial \tau} < \frac{\partial W(\tau, D_r)}{\partial \tau}$ holds.

Moreover, since there is a gap between $\frac{\partial SW(\tau, D_c)}{\partial \tau}$ and $\frac{\partial SW(\tau, D_r)}{\partial \tau}$, the following inequalities

$$\begin{aligned} & nL'(\tau) + F'(\tau)\beta \left[n\alpha(\tau)\bar{d} + \bar{e} - \frac{n(n+1)}{2}(\bar{e} - \bar{s} - L(\tau)) \right] \\ & + F(\tau)\beta \left[n\bar{d}\alpha'(\tau) + \frac{n(n+1)}{2}L'(\tau) \right] > 0 \\ > nL'(\tau) + F'(\tau)\beta \left[n\alpha(\tau)\bar{d} + \bar{e} - (n + (n-1)^2)(\bar{e} - \bar{s} - L(\tau)) \right] \\ & + F(\tau)\beta \left[n\bar{d}\alpha'(\tau) + (n + (n-1)^2)L'(\tau) \right] \end{aligned} \quad (29)$$

hold, depending on the parameters and functional forms. Thus, for the same τ increase, the complete network can be better off while the ring network can be worse off. ■

A.4. Proof of Lemma 1

Proof. Recall that the payment vector is

$$x = [\min \{ \alpha(\tau)\bar{d}\mathbf{1}, Qx + \bar{e}\mathbf{1} - \bar{s}\mathbf{1} - L(\tau)\mathbf{1} - \epsilon - \beta\xi \}]^+,$$

where the payment shortfall is

$$\xi(\tau) = [\alpha(\tau)\bar{d}\mathbf{1} + \bar{s}\mathbf{1} + \epsilon - \bar{e}\mathbf{1} + L(\tau)\mathbf{1} - Qx]^+.$$

Suppose that $\tau < \tau'$. We compare the payment shortfalls and the sets of defaulting agents for these two different settlement times.

Case 1. If $x_j = \alpha(\tau)\bar{d}$, then $\xi_j = 0$, and x_j remains $\alpha(\tau)\bar{d}$ as τ increases, and thus, ξ_j remains to be zero.

Case 2. Suppose that the agent under liquidity shock defaults on its senior liabilities with τ but not with τ' . If $x_j < \alpha(\tau)\bar{d}$ and $x_j = \alpha(\tau')\bar{d}$ due to the decrease in liabilities through more netting by $\alpha(\tau')$, then ξ_j decreases to zero under τ' .

Case 3. Now consider the last case such that $x_j < \alpha(\tau')\bar{d}$. As noted previously, $\mathcal{S}(\tau) \subseteq \mathcal{S}(\tau')$ and $\mathcal{D}(\tau') \subseteq \mathcal{D}(\tau)$, as agents who are solvent in τ remains to be solvent in τ' .

Case 3.1. Suppose that the agent hit by liquidity shock j does not default on its senior liabilities with τ' . We can permute the network into a matrix such that the payment equilibrium vector is $x = \begin{pmatrix} x_{\mathcal{D}} \\ \alpha(\tau')\bar{d}\mathbf{1} \end{pmatrix}$, where $x_{\mathcal{D}}$ is payments by agents in the defaulting set $\mathcal{D}(\tau')$. The payments by agents in the defaulting set is

$$x_d = \alpha(\tau')\bar{d}Q_{ds}\mathbf{1} + Q_{dd}x_d\mathbf{1} + [(\bar{e} - \bar{s} - L(\tau'))\mathbf{1} - \epsilon_d - \beta\xi_d],$$

where Q_{ds} is the liability weight matrix from $\mathcal{S}(\tau')$ to $\mathcal{D}(\tau')$, Q_{dd} is the liability weight matrix within $\mathcal{D}(\tau')$, and ϵ_d and ξ_d are subvectors of ϵ for $\mathcal{D}(\tau)$ and ξ for $\mathcal{D}(\tau')$, respectively. Because Q is a stochastic matrix, $Q_{ds} + Q_{dd} = 1$, so the payment vector becomes

$$\begin{aligned} x_d &= (I - Q_{dd})^{-1} [\alpha(\tau')\bar{d}Q_{ds}\mathbf{1} + (\bar{e} - \bar{s} - L(\tau'))\mathbf{1} - \epsilon_d - \beta\xi_d] \\ \Rightarrow x_d &= (I - Q_{dd})^{-1} [\alpha(\tau')\bar{d}(I - Q_{dd})\mathbf{1} + (\bar{e} - \bar{s} - L(\tau'))\mathbf{1} - \epsilon_d - \beta\xi_d] \\ \Rightarrow x_d &= \alpha(\tau')\bar{d}\mathbf{1} + (I - Q_{dd})^{-1} [(\bar{e} - \bar{s} - L(\tau'))\mathbf{1} - \epsilon_d - \beta\xi_d] < \alpha(\tau')\bar{d}\mathbf{1}, \end{aligned} \quad (30)$$

where I denotes the identity matrix of the appropriate dimension, and the last inequality of (30) holds, because agents in $\mathcal{D}(\tau')$ default.

Now consider the case with τ . We show that even when there are no additional defaults,

i.e., $\mathcal{D}(\tau') = \mathcal{D}(\tau)$, the aggregate deadweight loss is greater than that under τ' , resulting in even greater payment shortfall due to an increase in $\beta\xi_d$.

If any agent in $\mathcal{D}(\tau)$ can no longer meet their senior liabilities, then we have an increase in deadweight losses. Suppose the contrary and agents can still meet their senior liabilities. By $L(\tau) > L(\tau')$, we have

$$\begin{aligned} & (I - Q_{dd})^{-1} [(\bar{e} - \bar{s} - L(\tau)) \mathbf{1} - \epsilon_d - \beta\xi_d] \\ & < (I - Q_{dd})^{-1} [(\bar{e} - \bar{s} - L(\tau')) \mathbf{1} - \epsilon_d - \beta\xi_d] < 0, \end{aligned}$$

where the last inequality holds by (30). Then, it is immediate that $x_d(\tau) < \alpha(\tau)\bar{d}\mathbf{1}$. In addition, the payment shortfall increases as

$$\begin{aligned} \alpha(\tau)\bar{d}\mathbf{1} - x_d &= (I - Q_{dd})^{-1} [(\bar{e} - \bar{s} - L(\tau)) \mathbf{1} - \epsilon_d - \beta\xi_d(\tau)] \\ &\geq (I - Q_{dd})^{-1} [(\bar{e} - \bar{s} - L(\tau)) \mathbf{1} - \epsilon_d - \beta\xi_d(\tau')] \\ &> (I - Q_{dd})^{-1} [(\bar{e} - \bar{s} - L(\tau')) \mathbf{1} - \epsilon_d - \beta\xi_d(\tau')], \end{aligned}$$

where $\xi_d(\tau)$ and $\xi_d(\tau')$ denote the payment shortfall amounts with τ and τ' , respectively. Therefore, the aggregate deadweight loss is greater with τ .

Case 3.2. Suppose that the agent under liquidity shock defaults on its senior liabilities for both τ and τ' . First, we show the following lemma.

Lemma A.1. *Suppose that the agent under liquidity shock f defaults on its senior liabilities. If $\mathcal{D}(\tau)$ denotes the set of all other defaulting agents, then*

$$(I - Q_{dd})^{-1} [(\bar{e} - \bar{s} - L(\tau)) \mathbf{1} - \beta\xi_d(\tau) - \alpha(\tau)\bar{d}Q_{df}] < 0. \quad (31)$$

Further, if (31) is satisfied for a subset of agents $\mathcal{D}(\tau)$, then all agents in $\mathcal{D}(\tau)$ default.

Proof of Lemma A.1. For the first statement, suppose that the set of defaulting agents except agent f is $\mathcal{D}(\tau)$ and the complement set, i.e., the set of solvent agents is $\mathcal{S}(\tau)$. Then, by the assumption,

$$\begin{aligned} x_d &= Q_{dd}x_d + \alpha(\tau)\bar{d}Q_{ds}\mathbf{1} + (\bar{e} - \bar{s} - L(\tau)) \mathbf{1} - \beta\xi_d(\tau) \\ \Rightarrow x_d &= (I - Q_{dd})^{-1} [\alpha(\tau)\bar{d}Q_{ds}\mathbf{1} + (\bar{e} - \bar{s} - L(\tau)) \mathbf{1} - \beta\xi_d(\tau)]. \end{aligned}$$

Since Q is a stochastic matrix, $Q_{ds}\mathbf{1} + Q_{dd}\mathbf{1} + Q_{df} = \mathbf{1}$, implying that

$$\begin{aligned} Q_{ds}\mathbf{1} + Q_{df} &= (I - Q_{dd})\mathbf{1} \\ \Rightarrow Q_{ds}\mathbf{1} - (I - Q_{dd})\mathbf{1} &= -Q_{df}. \end{aligned}$$

Since agents in $\mathcal{D}(\tau)$ default, the payment vector x_d is less than the promised payment $\alpha(\tau)\bar{d}$. Therefore,

$$\begin{aligned} &(I - Q_{dd})^{-1} [\alpha(\tau)\bar{d}Q_{ds}\mathbf{1} + (\bar{e} - \bar{s} - L(\tau))\mathbf{1} - \beta\xi_d(\tau)] < \alpha(\tau)\bar{d}\mathbf{1} \\ \Rightarrow (I - Q_{dd})^{-1} [\alpha(\tau)\bar{d}(Q_{ds}\mathbf{1} - (I - Q_{dd})\mathbf{1}) + (\bar{e} - \bar{s} - L(\tau))\mathbf{1} - \beta\xi_d(\tau)] &< 0 \\ \Rightarrow (I - Q_{dd})^{-1} [-\alpha(\tau)\bar{d}Q_{df} + (\bar{e} - \bar{s} - L(\tau))\mathbf{1} - \beta\xi_d(\tau)] &< 0, \end{aligned}$$

which shows the first statement.

For the second statement, suppose that (A.1) is satisfied for the agents in $\mathcal{D}(\tau)$. Again using the fact that Q is a stochastic matrix, we obtain

$$\begin{aligned} &(I - Q_{dd})^{-1} [(\bar{e} - \bar{s} - L(\tau))\mathbf{1} - \beta\xi_d(\tau) - \alpha(\tau)\bar{d}Q_{df}] < 0 \\ \Rightarrow (I - Q_{dd})^{-1} [\alpha(\tau)\bar{d}Q_{ds}\mathbf{1} + (\bar{e} - \bar{s} - L(\tau))\mathbf{1} - \beta\xi_d(\tau)] &< \alpha(\tau)\bar{d}\mathbf{1}, \end{aligned}$$

which implies that even if no other agents in $N \setminus \mathcal{D}(\tau)$ default, all agents in $\mathcal{D}(\tau)$ cannot pay their liabilities. Therefore, all agents in $\mathcal{D}(\tau)$ default. ■

As in the assumption of Lemma A.1, denote the agent under liquidity shock, the set of defaulting agents other than the agent under liquidity shock, and the set of solvent agents as f , $\mathcal{D}(\tau')$, and $\mathcal{S}(\tau')$, respectively. By Lemma A.1,

$$(I - Q_{dd})^{-1} [(\bar{e} - \bar{s} - L(\tau'))\mathbf{1} - \beta\xi_d(\tau') - \alpha(\tau')\bar{d}Q_{df}] < 0.$$

Because entries in Q_{dd} is a sub-stochastic matrix with non-negative real numbers, $I - Q_{dd}$ is an M-matrix, which is a matrix whose off-diagonal entries are less than or equal to zero and whose eigenvalues have nonnegative real parts.²⁶ Therefore, $(I - Q_{dd})^{-1}$ is an inverse M-matrix and has nonnegative elements, implying

$$(I - Q_{dd})^{-1} [(\bar{e} - \bar{s} - L(\tau))\mathbf{1} - \beta\xi_d(\tau') - \alpha(\tau)\bar{d}Q_{df}] < 0,$$

²⁶If matrix A can be expressed as $A = sI - B$, $s \geq \rho(B)$, $B \geq 0$, where $\rho(B)$ is the spectral radius of B , then A is an M-matrix (Berman and Plemmons, 1979, p. 133). An M-matrix is non-singular if $s > \rho(B)$. See the appendix of Chang and Chuan (2024) for more details of how the concept of M-matrix can be applied to financial contagion.

as $L(\tau) > L(\tau')$ and $\alpha(\tau) > \alpha(\tau')$. Then, the payment shortfalls are greater under τ , hence,

$$\begin{aligned} & (I - Q_{dd})^{-1} [(\bar{e} - \bar{s} - L(\tau)) \mathbf{1} - \beta \xi_d(\tau) - \alpha(\tau) \bar{d} Q_{df}] \\ & < (I - Q_{dd})^{-1} [(\bar{e} - \bar{s} - L(\tau)) \mathbf{1} - \beta \xi_d(\tau') - \alpha(\tau) \bar{d} Q_{df}] < 0, \end{aligned}$$

and the set of defaulting agents under τ is a superset of the set of defaulting agents with τ' , as in the second statement of Lemma A.1. Hence, the aggregate deadweight loss is greater with τ .

Case 3.3. Finally, suppose that the agent under liquidity shock defaults on its senior liabilities with τ' but not with τ . In this case, the agent under liquidity shock, agent f , receives more payments from others, as $\alpha(\tau) \bar{d} > \alpha(\tau') \bar{d}$ for the payments from solvent agents, and able to pay its senior liabilities under τ . From Cases 1 and 2, $\mathcal{D}(\tau') \subseteq \mathcal{D}(\tau)$ as noted in the beginning of Case 3. Then, by following the same steps in Case 3.1,

$$\alpha(\tau) \bar{d} \mathbf{1} - x_d(\tau) > \alpha(\tau') \bar{d} \mathbf{1} - x_d(\tau') \geq \sum_{j \in N} \xi_j,$$

where $x_d(\tau')$ includes the negative payment from agent f with $x_f(\tau') = 0$ by the initial assumption of Case 3.3. Therefore, the aggregate deadweight loss is greater with τ . ■

A.5. Statement and Proof of Lemma A.2

Lemma A.2. *The vector of defaulting amounts $\alpha(\tau)\bar{d}\mathbf{1} - x_d$, where each entry is $\alpha(\tau)\bar{d} - x_j(\tau, D, \epsilon)$ for each $j \in \mathcal{D}(\tau, \epsilon)$ can be represented as*

$$\alpha(\tau)\bar{d}(\mathbf{1} + \beta C(\mathcal{D}(\tau, \epsilon))\mathbf{1}) + (1 + \beta)(C(\mathcal{D}(\tau, \epsilon))L(\tau)\mathbf{1} - C(\mathcal{D}(\tau, \epsilon))(e - s - \epsilon)), \quad (32)$$

where $C(\mathcal{D}(\tau, \epsilon)) \equiv (I - (1 + \beta)Q_{\mathcal{D}(\tau, \epsilon)})^{-1}$, and $C(\mathcal{D}(\tau, \epsilon))\mathbf{1}$ is the $|\mathcal{D}(\tau, \epsilon)| \times 1$ vector of node depth centrality.

Proof. We can express (15) in a matrix notation as

$$\alpha(\tau)\bar{d}\mathbf{1} - x_d = (1 + \beta)(\alpha(\tau)\bar{d}\mathbf{1} + s + \epsilon + L(\tau)\mathbf{1} - e - Q_{\mathcal{D}(\tau, \epsilon)}x_d).$$

Rearranging the expression to collect x_d term yields

$$\begin{aligned} (I - (1 + \beta)Q_{\mathcal{D}(\tau, \epsilon)})x_d &= -\beta\alpha(\tau)\bar{d}\mathbf{1} + (1 + \beta)(e - s - \epsilon - L(\tau)\mathbf{1}) \\ \Rightarrow x_d &= -\beta\alpha(\tau)\bar{d}(I - (1 + \beta)Q_{\mathcal{D}(\tau, \epsilon)})^{-1}\mathbf{1} + (1 + \beta)(I - (1 + \beta)Q_{\mathcal{D}(\tau, \epsilon)})^{-1}(e - s - \epsilon - L(\tau)\mathbf{1}) \\ \Rightarrow x_d &= -\beta\alpha(\tau)\bar{d}C(\mathcal{D}(\tau, \epsilon))\mathbf{1} + (1 + \beta)C(\mathcal{D}(\tau, \epsilon))(e - s - \epsilon - L(\tau)\mathbf{1}). \end{aligned} \quad (33)$$

Plugging (33) into $\alpha(\tau)\bar{d}\mathbf{1} - x_d$ implies

$$\begin{aligned} \alpha(\tau)\bar{d}\mathbf{1} - x_d &= \alpha(\tau)\bar{d}[\mathbf{1} + \beta C(\mathcal{D}(\tau, \epsilon))\mathbf{1}] + (1 + \beta)L(\tau)C(\mathcal{D}(\tau, \epsilon))\mathbf{1} - (1 + \beta)C(\mathcal{D}(\tau, \epsilon))(e - s - \epsilon), \end{aligned}$$

i.e., the expression in (32). ■

A.6. Proof of Theorem 1

Proof. For the first part, by Proposition 3, we know that the cost and benefits of an increase in τ depends on the functional forms of the liquidity cost, the probability of shock arrival, and the netting efficiency. From (32), we know that the conditional aggregate deadweight loss depends on the centrality of agents in the defaulting set.

Furthermore, from (32), the decrease in the aggregate deadweight loss $\frac{\partial \xi_j(\tau, D, \epsilon)}{\partial \tau}$ equals

$$\frac{1}{1 + \beta} \sum_{j \in \mathcal{D}(\tau, \epsilon)} [(1 + \beta C_j(\mathcal{D}(\tau, \epsilon))) \alpha'(\tau) \bar{d} + (1 + \beta) C_j(\mathcal{D}(\tau, \epsilon)) L'(\tau)].$$

Then, the expression (14) has to be multiplied by the probability of the realization of the liquidity shock ϵ , $G(\epsilon)$.²⁷

For the second part, we first show that an increase in the set of defaulting agents results in a discontinuous jump in the aggregate deadweight loss. The definition of node depth centrality in (17) implies that an increase in the set of defaulting agents leads to a discrete increase in the centrality, because

$$\begin{aligned} & 1 + (1 + \beta) \sum_{k \in \mathcal{D}(\tau, \epsilon)} q_{kj} + \dots \\ & < 1 + (1 + \beta) \sum_{k \in \mathcal{D}(\tau', \epsilon)} q_{kj} + \dots, \end{aligned}$$

if $\mathcal{D}(\tau, \epsilon) \subset \mathcal{D}(\tau', \epsilon)$ and $\mathcal{D}(\tau', \epsilon) \not\subset \mathcal{D}(\tau, \epsilon)$. Furthermore, the initial summation of (17) includes more agents in $\mathcal{D}(\tau', \epsilon)$. Therefore, the aggregate deadweight loss shows a discrete jump when there is an increase in the set of defaulting agents.

Because τ is a default threshold point, by definition, $\mathcal{D}(\tau, \epsilon) \subset \mathcal{D}(\tau - \delta, \epsilon)$ and $\mathcal{D}(\tau - \delta, \epsilon) \not\subset \mathcal{D}(\tau, \epsilon)$ for any small $\delta > 0$. Therefore, a change from $\tau - \delta$ to τ will result in a change in the defaulting set and discrete decrease in the aggregate deadweight loss. Such a change is multiplied by the shock realization probability $G(\epsilon)$ and the probability of shock arrival $F(\tau)$.

Because all functions are continuously changing in τ and are linearly combined for the ex ante welfare loss and payment shortfall function, they are continuous in τ . Thus, a marginal increase in τ from a threshold point does not result in an additional solvency, as there is already a discrete jump in payments due to the reduction of defaulting set of agents. Therefore, the right-hand derivative of the ex ante welfare loss does not involve a discrete

²⁷Note that the effect is the same even when the spectral radius assumption does not hold, making the centrality expression infeasible.

change in welfare loss. Hence, $\frac{\partial W(\tau, D)}{\partial \tau^-} < \frac{\partial W(\tau, D)}{\partial \tau^+}$. ■

A.7. Proof of Theorem 2

Proof. First, recall that an agent j defaults if and only if: $\alpha(\tau)\bar{d} + \bar{s} + \epsilon_j + L(\tau) > \bar{e} + \sum_{k \in N} q_{jk}x_k(\tau, D, \epsilon)$.

1. **Shifting of default threshold points:** Under scarce liquidity regime, $L_A(\tau)$ increases to $L_S(\tau)$ and \bar{e}_A decreases to \bar{e}_S , making the default condition more likely to be satisfied for any agent for the given τ . Consider any default threshold point $\tilde{\tau}_A \in \mathcal{T}_A(D)$ in the ample regime, where the default sets under the ample regime are $\mathcal{D}_A(\tilde{\tau}_A - \delta, \epsilon) \supsetneq \mathcal{D}_A(\tilde{\tau}_A, \epsilon)$ for any small $\delta > 0$. Denote by j the marginal agent who transitions from defaulting to solvent at this threshold. Under scarce liquidity regime, this agent requires a greater liquidity buffer (through more netting and lower liquidity cost) to remain solvent, which is achieved only at a higher value of τ by Lemma 1. Thus, the corresponding default threshold point shifts to a higher value in the scarce regime: $\tilde{\tau}_S > \tilde{\tau}_A$.
2. **Higher social welfare loss:** Recall that the ex ante social welfare loss is: $W(\tau, D) = \sum_{i \in N} (L(\tau) + F(\tau)E[\beta\xi_j(\tau, D, \epsilon)])$. For any fixed τ , the difference in ex ante social welfare loss between the scarce regime and the ample regime is

$$\begin{aligned} & W_S(\tau, D) - W_A(\tau, D) \\ &= \sum_{i \in N} ([L_S(\tau) - L_A(\tau)] + F(\tau)[E[\beta\xi_{i,S}(\tau, D, \epsilon)] - E[\beta\xi_{i,A}(\tau, D, \epsilon)]]. \end{aligned}$$

Since $L_S(\tau) > L_A(\tau)$ by the definition of scarce liquidity, and the expected deadweight loss increases due to more frequent and severe defaults due to less liquidity (either from high liquidity cost or less cash buffer), the overall social welfare loss is strictly higher in the scarce liquidity regime.

3. **Slower optimal settlement speed: Case 1.** Suppose τ_S^{opt} is at a default threshold point $\tau^* \in \mathcal{T}_S(D)$. By Theorem 1, at a default threshold point, the left-hand derivative of the ex ante welfare loss is strictly less than the right-hand derivative: $\frac{\partial W_S(\tau^*, D)}{\partial \tau^*} < \frac{\partial W_S(\tau^*, D)}{\partial \tau^{*+}}$.

Since τ_S^{opt} minimizes the welfare loss, it must be that $\frac{\partial W_S(\tau_S^{opt}, D)}{\partial \tau^-} < 0 \leq \frac{\partial W_S(\tau_S^{opt}, D)}{\partial \tau^+}$. From statement 1, we know that this default threshold point corresponds to a threshold point τ_A^* in the ample liquidity regime such that $\tau_S^* > \tau_A^*$.

Case 1.1. If the corresponding default threshold point under the ample regime is also the optimal settlement time for the ample regime, i.e., $\tau_A^{opt} = \tau_A^*$, then, it is trivial that

$\tau_S^{opt} = \tau_S^* > \tau_A^* = \tau_A^{opt}$. Thus, the optimal settlement speed is slower under the scarce regime compared to the ample regime.

Case 1.2. If the corresponding default threshold point under the ample regime is not the optimal settlement time for the ample regime, we need to show that any settlement time $\tau \geq \tau_S^{opt}$ is not the optimal settlement time. Recall that the marginal change in the ex ante social welfare loss under the R regime, where $R \in \{A, S\}$ is

$$MW_R(\tau) \equiv \underbrace{nL'_R(\tau) + F(\tau) \sum_{j \in \mathcal{D}_R(\tau, \epsilon)} \beta \frac{\partial \xi_{j,R}(\tau, D, \epsilon)}{\partial \tau}}_{= \text{Benefit}_R(\tau) \leq 0} + \underbrace{F'(\tau) \sum_{j \in \mathcal{D}_R(\tau, \epsilon)} \beta \xi_{j,R}(\tau, D, \epsilon)}_{= \text{Cost}_R(\tau) \geq 0}$$

by Proposition 3. Also, by the functional assumptions and Lemma A.2, which shows the aggregate deadweight loss is linear in τ in terms of payment shortfalls and the corresponding node depth centrality, every term in MW_R is continuous in τ , as long as the default set remains the same. In other words, the welfare function is continuous in τ except at default threshold points. Since τ_S^{opt} is a default threshold point, $MW_S(\tau_S^{opt} + \delta) > 0$, i.e., $\text{Cost}_S(\tau_S^{opt} + \delta) > \text{Benefit}_S(\tau_S^{opt} + \delta)$. In other words, in the absence of the discrete jump in $\text{Benefit}_S(\tau_S^{opt})$, the marginal ex ante social welfare loss is increasing, so the optimal settlement speed is exactly at τ_S^{opt} . By continuity and the assumption of small changes, $MW_A(\tau_S^{opt} + \delta) > 0$ for any small $\delta \geq 0$. Thus, $\tau_A^{opt} < \tau_S^{opt}$, and the optimal settlement speed is slower under the scarce regime than the ample regime.

Case 2. Suppose τ_S^{opt} is not at a default threshold point under the scarce regime $\tau_S^{opt} \notin \mathcal{T}_S(D)$. If this same settlement time, τ_S^{opt} , corresponds to a default threshold point under the ample regime, then the optimal settlement time under the ample regime must be strictly less than τ_S^{opt} . Otherwise, the logic in **Case 1.2** applies to this case as well. Therefore, $\tau_A^{opt} < \tau_S^{opt}$, implying that the optimal settlement speed is slower under the scarce regime than under the ample regime.

■

A.8. Proof of Proposition 4

Proof. Step 1. Solving payment equilibrium is polytime. For any given τ , computing the payment equilibrium clearing vector $x(\tau, D, \epsilon)$ reduces to solving a fixed-point iteration in the framework of Eisenberg and Noe (2001) with a default cost extension. Rogers and Veraart (2013) show that this algorithm finds a fixed point at most in n steps of iteration. Therefore, $\mathcal{D}(\tau, \epsilon)$ can be computed in n steps or in polynomial time for any τ .

Step 2. Monotonicity. By Lemma 1, $\mathcal{D}(\tau, \epsilon)$ is monotonically (weakly) decreasing in τ . Thus, each agent's status of being default or solvent switches only once as τ increases.

Step 3. Checking existence of a default threshold point. For the given network D and parameters and functional forms $(\beta, \bar{e}, \bar{s}, \alpha(\cdot), L(\cdot))$, and shock realization ϵ , fix an interval $[a, b] \subset \mathbb{R}^+$. We first ask whether there exists $\tilde{\tau} \in [a, b]$ such that $\tilde{\tau} \in \mathcal{T}(D, \epsilon)$. This is equivalent to computing $\mathcal{D}(a, \epsilon)$ and $\mathcal{D}(b, \epsilon)$, which is polytime as shown in Step 1. If $\mathcal{D}(a, \epsilon) = \mathcal{D}(b, \epsilon)$, then by Step 2, no threshold exists in $[a, b]$. If $\mathcal{D}(a, \epsilon) \neq \mathcal{D}(b, \epsilon)$, then at least one default threshold point exists. This entire procedure is again solvable in polynomial time.

Step 4. Binary search. If an agent j 's status switches within the interval $[a, b]$, i.e., $j \in \mathcal{D}(a, \epsilon)$ and $j \notin \mathcal{D}(b, \epsilon)$, then there exists a default threshold point. By Step 2, there are at most n thresholds. Each can be located to precision δ by binary search: iteratively evaluating $\mathcal{D}(\tau, \epsilon)$ at midpoints until the switch point is bracketed within a subinterval. This requires $O\left(\log\left(\frac{b-a}{\delta}\right)\right)$ of payment equilibrium solves per agent. Thus, for any arbitrary precision $\delta > 0$, the overall procedure is solvable in polynomial time.

Step 5. Approximation by discretization. By step 4, for any arbitrary precision $\delta > 0$, we can compute the default threshold points for a given ϵ , $\mathcal{T}(D, \epsilon)$. If the distribution $G(\epsilon)$ is discrete, then we are done. Now suppose that $G(\epsilon)$ is a continuous distribution. Then, an approximation of $G(\epsilon)$ is $\tilde{G}(\epsilon)$, which is supported on a finite set $\{\epsilon^{(1)}, \dots, \epsilon^{(M)}\}$. Then, the union of default threshold points for the approximation, i.e.,

$$\tilde{\mathcal{T}}(D) \equiv \bigcup_{m=1}^M \mathcal{T}(D, \epsilon^{(m)}) \quad (34)$$

is computable in polynomial time. ■

B. Details of Netting

In this section, we discuss the details of various netting procedures and the resulting full netting matrices.

The simplest netting procedure is bilateral netting—netting the payments between two agents in all existing pairs. The full netting matrix under bilateral netting of the payment network D is

$$\underline{D} = [D - D^T]^+ . \quad (35)$$

There are multiple ways to perform multilateral netting or portfolio compression. One of the more relevant and realistic netting procedure is simply eliminating cycles that [D’Errico and Roukny \(2021\)](#) defined as conservative portfolio compression. For example, [Veraart \(2022\)](#) builds upon [D’Errico and Roukny \(2021\)](#) and considers the effect of eliminating all cycles by a degree of μ . Therefore, our model is generalizing the netting (or compression) in her model by allowing time-varying degree of netting, $\alpha(\tau)$. The exact procedure of obtaining the full netting matrix under this cycle elimination is the following:²⁸

1. Fix the index $\kappa = 1$ and define $D^0 \equiv D$.
2. From the matrix $D^{\kappa-1}$, find a cycle, indexed by κ , which is a sequence of agents i_0, \dots, i_K , for some $K \geq 2$ such that $i_0 = i_K$, $i_\ell \neq i_0$, and $d_{i_{\ell+1}i_\ell}^{\kappa-1} \geq \underline{d}^\kappa > 0$ for each $\ell < K$, where $\underline{d}^\kappa \equiv \max_\delta \left\{ \delta \leq d_{i_{\ell+1}i_\ell}^{\kappa-1} : \forall 0 \leq \ell < K \right\}$.
3. Subtract \underline{d}^κ from each $d_{i_{\ell+1}i_\ell}^{\kappa-1}$ along the cycle κ . Denote the new matrix as D^κ .
4. If there is a cycle in D^κ , return to step 2. If there is no cycle, define the resulting matrix as the full netting matrix $\underline{D} \equiv D^\kappa$.

If a network is regular, then the full netting matrix under this procedure will be $\underline{D} = \mathbf{0}$.

However, the full netting matrix can be a non-zero matrix for non-regular networks. For example, consider a simple ring network as depicted in the left panel of [Figure 6](#). Agent A owes 1 to agent B , agent B owes 2 to agent C , and agent C owes 1 to agent A . The netting procedure can eliminate the cycle up to the maximum of 1. Therefore, as in the right panel of [Figure 6](#), agent B still owes 1 to agent C after the full netting procedure. The full netting matrix is not a zero matrix in this example.

²⁸This procedure is equivalent to obtaining the μ -compressed liabilities matrix with $\mu = \mu^{\max}$ in [Veraart \(2022\)](#). Note that instead of changing the parameter μ , we set the time varying value of $1 - \alpha(\tau)$ for the degree of netting. Our procedure is closer to how to design numerical procedures, which might be computationally intensive as shown by [Jackson and Pernoud \(2024\)](#).

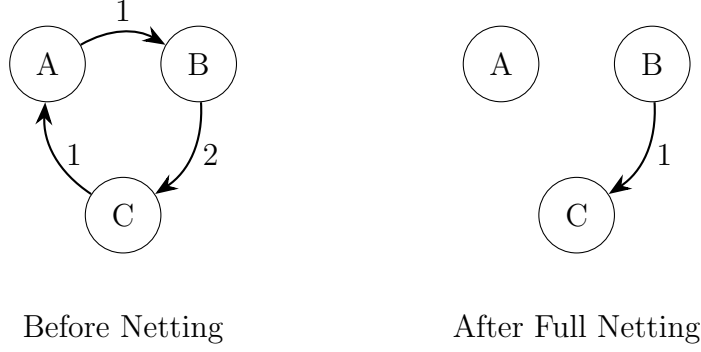


Figure 6: An example of non-zero full netting matrix

Note: Before netting (left), agents form a cycle: A owes B 1, B owes C 2, C owes A 1. After full netting (right), only B owes C 1.

C. Discussion of Social Welfare

In this section, we define the ex ante (expected) social welfare and discuss how it relates to the ex ante welfare loss defined in (6).

Define the expected transfer from agent i to the sources of liquidity shock as $T_i \equiv F(\tau)1/n\epsilon$. As previously mentioned, we consider transfers to liquidity shock as simple transfers of wealth from agents to senior creditors and vice versa. The sum of agents' expected utilities plus the expected transfers to liquidity shock net of the deadweight loss incurred to external agents is

$$\begin{aligned}
& \sum_{i \in N} (U_i(\tau, D) + T_i - F(\tau)(1 - \gamma)E[\min\{\beta\xi_i(\tau, D, \epsilon), A_i(\tau, D, \epsilon)\}]) \\
&= \sum_{i \in N} (e_i - s_i) - nL(\tau) + (1 - F(\tau)) \left[\sum_{i \in N} \sum_{j \in N} \hat{q}_{ij} \hat{d}_j - \sum_{i \in N} \hat{d}_i \right] \\
&\quad + F(\tau) E \left[\sum_{i \in N} \sum_{j \in N} \hat{q}_{ij} x_j - \sum_{i \in N} x_i - \sum_{i \in N} (\epsilon_i - \epsilon_i) - \sum_{i \in N} \min\{\beta\xi_i, A_i\} \right] \\
&= \sum_{i \in N} (e_i - s_i) - nL(\tau) - F(\tau) E \left[\sum_{j \in N} \min\{\beta\xi_j, A_j\} \right], \tag{36}
\end{aligned}$$

because $\sum_{j \in N} \hat{q}_{ij} = 1$. Since the first term of (36) does not depend on τ , D , or ϵ , we can focus on the second and third terms. The ex ante (expected) social welfare losses defined in (6) therefore capture the relevant changes in social welfare from varying the settlement time τ and the network structure D .