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Federal Reserve Board of Governors†‡

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Abstract

This paper provides evidence that shifts in the occupational composition of the U.S. workforce are the most important factor explaining the trend decline in the labor share over the past four decades. Estimates suggest that while there is unitary elasticity between equipment capital and non-routine tasks, equipment capital and routine tasks are highly substitutable. Through the lenses of a general equilibrium model with occupational choice and the estimated production technology, I document that the fall in relative price of equipment capital alone can explain 72 percent of the observed decline in the U.S. labor share. In addition, I find that differences in labor share trends across sectors can be accounted for by varying sensitivities of cost of production to the price of equipment capital.

Keywords: Labor share, technological change, capital-task complementarity, elasticity of substitution, job polarization, Bayesian estimation.

JEL Codes: C11, E22, E23, E25, J24, J31.

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1 Introduction

> ". . . the stability of the proportion of the national income dividend accruing to the labor is one of the most surprising yet best established facts in the whole range of economic statistics."
>
> *J. M. Keynes, 1939*

Since Keynes and Kaldor (1957), macroeconomists have been accustomed to assume that the labor share of income is constant. Over the past four decades, however, the labor share in the United States has fallen nearly 10 percent, and similar trends have been documented for other economies.[1] A wide range of explanations have been proposed to shed light on the causes of this recent phenomenon, of which the most popular has been technological progress and an accompanying rise in the aggregate capital-to-labor ratio. This story relies on the precise substitutability between aggregate capital and labor, over which existing studies have reached conflicting conclusions.[2] One potential source of disagreement is the focus of the existing literature on the relationship between factors of production at the aggregate level, which masks substantial changes in the occupational composition of the labor force. Behind this aggregate picture, these occupational shifts have played a key role in linking capital accumulation with the decline of the labor share. Two observations support this conjecture: First, it is only the labor associated with jobs involving routine tasks that experienced a loss in its share of income, while income share of non-routine task labor relative to that of capital has been relatively stable.[3] Second, sectors with the largest decline in their labor shares are also the ones experiencing the most drastic changes in occupational composition of their total wage bills.

Motivated by these facts, in this paper I take a more disaggregated view of the economy than the existing literature and examine the long-run effect of technology-induced shifts in employment shares and the relative wages of various types of labor on the la-

[1] Some studies documenting the decline in the labor share are Elsby et al. (2013), Karabarbounis and Neiman (2014b), and Armenter (2015).

[2] A vast majority of existing studies document the aggregate capital and labor to be complements, implying a counterfactual rise in the labor share. While a few recent studies, such as Karabarbounis and Neiman (2014b) and Piketty (2014), present evidence on the contrary, Rognlie (2015) argues that the elasticities required in these studies are theoretically implausible—especially for Piketty (2014).

[3] Routine task occupations represent medium-skill jobs that can be codified, and are thus vulnerable to automation. Non-routine task occupations are divided into two types: cognitive and manual. Examples of the former are high-skill managerial and professional jobs, while low-skill personal service jobs, such as child care and home health care, are common examples of the latter.

bor share.[4] To do so, I first estimate a production function for the U.S. economy that is consistent with job and wage polarizations observed in past few decades, which together summarize the aforementioned employment and wage shifts across occupations.[5][6] The estimation step is essential in the sense that the elasticities of substitution between equipment capital and each type of labor play a key role in shaping the response of aggregate labor share to technological progress. When estimating these elasticities, which are not available in the related literature, I do not employ the labor share as one of the targeted series. Abstracting from the labor share in estimation allows me to assess the consistency of the observed decline in the labor share with technological progress and the accompanying shifts in the U.S. labor market during the same period.

There are two key findings standing out from the estimation analysis. First, I find a larger substitution elasticity between routine labor and equipment capital than what has generally been used in the job polarization literature. Second, and even more importantly, the elasticity between equipment capital and labor devoted to non-routine tasks is close to unity. Together, these two findings confirm that it is only the disappearance of routine jobs that has been depressing the labor share, whereas equipment capital and labor working in non-routine task occupations have been capturing constant shares of income lost by this group. This means that income losses of labor devoted to routine task intensive jobs would always dominate the gains of labor in non-routine task occupations, making the decline in the aggregate labor share a natural outcome of equipment-specific technological progress and capital-task complementarity—which I define as equipment capital being more substitutable with routine task jobs than non-routine task jobs. On the other hand, however, it also implies that, in the absence of other trend changes, labor share would asymptotically go back to a constant, as these routine jobs gradually disappear.

This partially unitary-elastic (Cobb-Douglas) production function tells us that changes in the labor share can solely be summarized by the changes in the wage-bill ratio, which

[4]I particularly focus on equipment-specific technological change. Gort et al. (1999) define this type of technological change as technological progress embodied in the form of new equipment capital goods, which makes this particular type of capital relatively cheaper and, hence, changes the distribution of capital used in production in favor it.

[5]I used a two-stage Bayesian estimation technique based on the works of Krusell et al. (2000) and Polgreen and Silos (2009).

[6]Here, job polarization refers to the growth of the employment shares of high-skill (cognitive) and low-skill (manual) occupations, while that of medium-skill (routine) occupations erodes. Similarly, the term "wage polarization" is used to describe the decrease in the relative wages of medium-skill jobs.

is defined as the ratio of wage income of labor doing non-routine tasks to that of labor devoted to routine tasks. This demonstrates that the trend decline of the U.S. labor share has been consistent with, if not a result of, changes in the occupational composition of the total wage bill over the past four decades.[7] Two factors determine, quantitatively, the magnitude of the decline in the labor share. First, higher substitutability between routine tasks and the composite output of equipment capital and non-routine tasks makes it easier to switch from routine tasks to other factors, resulting in a larger decline in the labor share. Second, higher shares of equipment capital in production make the production costs more sensitive to equipment-specific technological progress, resulting in a larger fall of the labor share.

Focusing on the interactions between equipment capital and task groups of labor—rather than educational groups—considerably improves our understanding of the link between technological progress and the labor share. When the I repeat the estimation, this time using an education-based skill classification, the estimated production function fails to generate a decline in the labor share since early 2000s, which marks the beginning of a significant acceleration in the decline of the labor share. Overall, the production function consistent with the traditional capital-skill complementarity theory can explain around 40 percent less of the total decline observed in the labor share compared with the baseline production function.[8]

I next present a general equilibrium model with occupational choice that features a production function consistent with my estimates to quantify the importance of equipment-specific technological change and capital-task complementarity in accounting for the decline of the labor share. Calibrated with the U.S. data, this framework shows that equipment-specific technological change alone can account for nearly 75 percent of the trend decline in the labor share over the past four decades. This number is roughly equivalent to the within-sector component of the decline in the labor share during the same period. A counterfactual experiment reveals that the technology boom experienced during the 1996–2003 period alone is responsible for around 22 percent of the observed decline. An interesting implication of this model is that the response of the labor share to permanent technology shocks diminishes as the employment share of labor working in routine task occupations declines. As a consequence, the labor share is projected to

[7]The estimated production function successfully generates the entire decline of the labor share since 1967, when it is fed with observed series of labor and capital inputs.

[8]Capital-skill complementarity is a concept similar to capital-task complementarity when workers are grouped on the basis of years of education rather than tasks associated with their occupations.

stabilize at about 55 percent in the very long run, even if technological progress continues at its current pace.

This paper is most closely related to the works of Karabarbounis and Neiman (2014b), Piketty and Zucman (2014), and Lawrence (2015) as it also focuses on technological progress as the source of the decline of the labor share. However, it differs from the existing studies in several aspects: First, the labor share is not used in deriving the elasticities of substitution between capital and labor. Abstracting from the labor share in the estimation ensures that the estimation results and the inference about the labor share are not driven by the labor share itself. Second, while the other studies relate technological progress to the labor share directly, this paper focuses on the effect of changes in skill and occupational structure of labor force.[9] Third, unlike the existing studies, the elasticity of substitution between aggregate capital and labor—over which there is conflicting findings—plays no direct role in this paper in linking technological progress and decline in the labor share. As a consequence, this paper provides new insights into understanding the labor share trends, and thus, would potentially have different policy implications.[10]

This study also contributes to the labor share literature by providing an alternative explanation for the substantial differences in labor share trends across sectors. This heterogeneity has previously been studied by Alvarez-Cuadrado et al. (2015), who account for these differences by allowing the substitutabilities of capital and labor to vary across sectors. In contrast, this paper shows that it is the differences in the sensitivities of cost of production to equipment capital prices across sectors, which in turn are affected by a sector's load of routine employment, that causes this heterogeneity in sectoral labor share trends. When the final output in the general equilibrium model is decomposed into two sectors, the wage-bill ratio channel generates a decline of around 22 percent in the labor share for the goods sector and a decline of only 3 percent for the services sector, which are consistent with what we observe in the data.

[9] Eden and Gaggl (2015) also disaggregate the labor share into routine and non-routine components. Unlike this paper, however, they focus on information and communication technology (ICT) capital and show that ICT technology had a significant effect on the income distribution within labor, while having only a moderate effect on the aggregate labor share. Overall, they document that the rise in the income share of ICT capital accounts for half of the decline in the U.S. labor share.

[10] For instance, the findings of study imply that policies helping labor acquire skills compatible with the non-routine cognitive tasks might be the most appropriate options when addressing economic inequality, rather than measures such as taxing capital.

This paper is organized as follows. In the next section, I briefly discuss where this paper stands in each of three strands of literature—namely, labor share, capital-skill complementarity and job polarization. The literature survey is followed by Section 3, which presents data sources and motivating facts. Section 4 is the empirical part of the paper in which I present the production function to be estimated, the estimation methodology, and the results. Then I describe the general equilibrium model in Section 5 before presenting the quantitative findings of the model in Section 6, in which I also discuss the model's implications for sectoral differences in terms of labor share trends. Section 7 concludes.

2 Related Literature

2.1 Labor share

Even though the timing and magnitude of the decline in the labor share might change based on how one defines it, there is almost a consensus over the existence of a declining trend in most countries and sectors.[11] Armenter (2015) constructs a wide range of alternative definitions of the labor share to address measurement issues and confirms the overall trend decline, regardless of the measure used. Thus, rather than questioning or documenting the decline of the labor share, I take one of the definitions and analyze the effect of technological progress on trend changes in the labor share as quantified by this particular measure.

There is a wide range of possible explanations proposed for the decline of the labor share.[12] This paper stands among those that relate the decline of the labor share to technological progress. A general conclusion among these studies is that factor prices,

[11] See, for example, Guerriero (2012), Karabarbounis and Neiman (2014b), and Elsby et al. (2013).

[12] These include: trade and offshoring (Elsby et al. (2013)), foreign direct investment inflows and mechanization (Guerriero and Sen (2012)), structural change and heterogeneity (Alvarez-Cuadrado et al. (2015)), increased international trade (globalization) and the resulting micro-level restructuring (Böckerman and Maliranta (2011)), and measurement issues related to depreciation and production taxes (Bridgman (2014)), housing sector (Rognlie (2015)), and capitalization of intellectual property products (Koh et al. (2015)). In their recent study, Autor et al. (2017) attribute the decline in the labor share to increasing concentration within industries. They document that the market share of a small number of "superstar firms" with high profitability has been growing since 1982. As these firms have the lowest labor shares, industries with larger increases in concentration have experienced a larger decline in the labor share, thereby driving the aggregate labor share down as well.

which are a proxy for technological progress, alone cannot account for the labor share's decline, as most of the studies document aggregate capital and labor to be substitutes. Both Chirinko (2008) and Leon-Ledesma et al. (2010) provide excellent reviews of studies estimating the elasticity of substitution between capital and labor. While Chirinko (2008) report that most estimates of the elasticity of substitution lie within the range of 0.4 and 0.6, Leon-Ledesma et al. (2010) document that most of the estimates range between 0.5 and 0.8.[13] In a recent paper, Oberfield and Raval (2014), document a similar aggregate elasticity and thus, conclude that capital prices must have played a positive effect on the labor share, if any. Hence, attempts to explain the decline in the labor share using the factor prices require extending the dimension of the story to include other factors, such as automation.[14]

Nonetheless, there are a few recent exceptions, such as Karabarbounis and Neiman (2014a), Karabarbounis and Neiman (2014b), and Piketty (2014), that attribute the decline in the labor share to changes in factor prices—namely, falling relative price of capital. In their pioneering paper, Karabarbounis and Neiman (2014b) use cross-sectional variation across countries in terms of the labor share trends and capital prices and estimate the elasticity of substitution between capital and labor as 1.25. They find that the relative price of investment goods explains almost half of the decline in the labor share.[15]

One source of these conflicting results is differences in methodologies and data employed. My goal is neither to address those differences nor to come up with a new measure of elasticity of substitution between aggregate capital and labor. Rather, by taking changes observed in occupational composition of labor force into account, my study implies that the elasticity of substitution between aggregate capital and labor is of little importance in explaining the decline of the labor share. What matters the most when it comes to linking technological progress and the labor share are the elasticity of substitution between equipment capital and labor devoted to routine tasks and sensitivity

[13] See Table 1 in Leon-Ledesma et al. (2010) and Table 1 in Chirinko (2008) for empirical estimates of the aggregate elasticity of substitution between capital and labor from various studies. Moreover, Leon-Ledesma et al. (2015) find this elasticity to be 0.7.

[14] Taking the general view on the complementarity of capital and labor as given, Lawrence (2015) argues that this complementarity does not contradict the decline in the labor share, as he claims that effective capital-labor ratios have actually fallen in the sectors and industries that account for the largest portion of the declining labor share in income since 1980.

[15] Rognlie (2015) argues that Piketty (2014) argument is both theoretically and empirically implausible; while, despite remaining theoretically viable, Karabarbounis and Neiman (2014b) explanation is inconsistent with time-series evidence.

of the cost function to equipment-capital prices.

2.2 Capital-skill complementarity

The second line of literature this paper fits in is the broad capital-skill complementarity literature.[16] Among the existing studies, this paper is mostly related to Krusell et al. (2000), who also estimate a similar production function for the U.S. economy and show that equipment-specific technological change can explain both skill premium and the labor share trends up to the year 1992. In the years following their study, the skill premium kept rising in a period when the labor share was falling. Ohanian and Orak (2016) analyze the Krusell et al. (2000) model and extend their data set through 2013; they find that their model implies a counterfactual rise in the labor share since the 1990s, despite successfully capturing the major changes in the skill premium since then. When they reestimate the production function for the entire period up to 2013, the counterfactual rise in the labor share is curbed, but the production function still fails to account for the recent acceleration of the decline in the labor share since early 2000s.[17] This paper accounts for the inconsistency between the wage premium and the labor share trends that is implied by the traditional capital-skill complementarity story.

2.3 Job polarization

Job polarization has been documented for the U.S. economy in many studies exploring its existence, causes, and consequences as well as its implications for income inequality.[18] Similar trends have been found for other countries as well.[19]

Even though job polarization can be considered a nuanced version of capital-skill complementarity, the two differ in the sense that in the job polarization literature labor is divided into skill groups in terms of task content of their occupations. Thus, elasticity

[16] Pioneering works in this literature are Katz and Murphy (1992), Greenwood et al. (1997), and Krusell et al. (2000).

[17] See Hansen and Ohanian (2016) for a more detailed description of Ohanian and Orak (2016).

[18] A few examples are Autor et al. (2003), Acemoglu and Autor (2011), Acemoglu and Autor (2012), Autor and Dorn (2013), Beaudry et al. (2013), and Autor (2015). Papers studying cyclical properties of job polarization are Jaimovich and Siu (2012), Albanesi et al. (2013), Smith (2013), and Foote and Ryan (2014).

[19] See Goos and Manning (2007) and Bisello (2013) for the United Kingdom; Spitz-Oener (2006), Dustmann et al. (2009), and Kampelmann and Rycx (2011) for Germany; Ikenaga and Kambayashi (2010) for Japan, and Goos et al. (2011), Nellas and Olivieri (2012), and Fernandez-Macias (2012) for other European countries.

measures consistent with this classification need to be used, and this paper provides those to the job polarization literature. Furthermore, to my knowledge, the link between job polarization and the labor share has not been explored yet, and this paper is the one of the first to establish this link.

3 Data and Facts

In this section, I first briefly introduce classification used for disaggregating labor input. Later, data sources and an outline of the methodology used in calculation of labor input and wages, details of which can be found in Appendix D, are described. Then, key motivating facts from the data are presented.

3.1 Classification of labor

In this paper, workers are classified based on the tasks associated with their occupations, rather than whether or not they possess a college degree. Occupations are grouped into three categories following the works of Autor et al. (2003), Acemoglu and Autor (2011), Jaimovich and Siu (2012), and Autor and Dorn (2013) among many others: (non-routine) cognitive, routine, and (non-routine) manual.[20] The main reason for this choice is the inability of education-based skill definition to fully account for the trend decline in the labor share, as discussed earlier.

In addition, there are several other advantages of task-based definition of skill over the traditional skill classification based on college attainment when studying the link between technological progress and the labor share. First, it is more responsive to technological progress than education-based definition. Share of college degree attainment was on the rise long before the onset of the decline of the labor share, and this process continues with no noticeable trend shift. However, polarization away from routine-task

[20]Details of the occupational classification can be found in Appendix D. In short, routine tasks are the ones that can be summarized as a set of specific activities accomplished by following well-defined instructions and procedures. These tasks can be replaced by machines or computers or even be offshored. In contrast, if a task requires instant decision making, flexibility, taking initiatives, problem solving, creativity, and interpersonal interaction, then it is classified as non-routine. Cognitive non-routine occupations mainly consist of cognitive tasks that require extensive mental activity. A few examples in this category are managerial, professional, and technical occupations. Furthermore, manual non-routine service jobs that mostly require physical activity but also require taking initiative, flexibility, interpersonal interaction, and decision-making fall into this category as well.

occupations is a more recent phenomenon, and it coincides with the trend decline in the labor share. Furthermore, switching between occupations is a more dynamic choice, as it responds to business cycles and labor market conditions. On the other hand, moving up the educational ladder takes time, while moving down is practically impossible within a generation.

Second, when the Current Population Survey (CPS) data are analyzed, one can see that around 22 percent of college graduates already work in routine-task occupations with lower pay.[21] If we assume that wage paid in an occupation reflects the productivity of labor, then years of education does not seem to be a perfect measure of the skills possessed by labor. Finally, data reveal that the labor share is falling solely due to the decline in the income share of workers in routine occupations, while the income share of workers in non-routine occupations is on the rise in all sectors. Hence, it is important to separate these two groups from each other to identify the causes of the overall decline in the labor share.

3.2 Data sources

3.2.1 Labor input and wages

The derivation of labor input and wages closely follows Krusell et al. (2000) and Domeij and Ljungqvist (2006). The main source of wage and occupational composition data is the CPS March Supplement for the years between 1967 and 2013.[22] I use all of the person-level data, excluding the agents who are younger than 16 or older than 70, the self-employed, unpaid family workers, and those working in either agriculture or the military. Observations with reported annual working hours fewer than 260 (quarter of part-time work) in the past year and hourly wages below half of the minimum federal wage rate are dropped to remove outliers and misreporting in the data. After these adjustments, we are left with annual observations ranging from $55,721$ (1996) to $96,184$ (2001).

The main variables of interest are the nominal hourly wage rates and inputs of three

[21] The number is larger for high school or lower degree graduates in non-routine occupations, as these jobs also include low-skill service jobs. Yet, even in managerial positions, there are a significant portion of workers without a college degree.

[22] CPS data are compiled and published by IPUMS (Flood et al. (2015)). For the data between 1967 and 2013, I use CPS files from 1968 to 2014. Even though the capital stock and price series are available for earlier years, I restrict the period of study to start from 1967, as consistent occupational data are available only starting from this year.

types of labor. The details of obtaining hourly wages and the resulting task premiums are described in Appendix D. In short, I first calculate total hours worked last year by multiplying the usual hours worked by the weeks worked last year.[23] Later, annual wage and salary income is divided by total hours worked last year to obtain the nominal hourly wage rate of each agent. Finally, each agent is assigned to one of 198 groups broken down by by age, sex, race, and task.[24] I calculate average hours and wage rates of each group and then aggregate them into three task groups using the group wages of a particular year for the aggregation.[25] The task premium between any two task groups is simply obtained by dividing the wage rate of one group by the wage rate of another.

CPS income data are top-coded, meaning that the observations above a certain threshold are censored to ensure confidentiality. The top-coding practice has changed from time to time, thereby causing jumps in the task premium series. Fortunately, in 2012, the Census Bureau released a series of revised income top-codes files that reconciled the top-coded income values between the CPS years 1976 and 2010 with the top-coded values based on the Income Component Rank Proximity Swap methodology that was introduced in year 2011.[26] Having a consistent top-coding method throughout the 1975–2013 period enables us to eliminate the jumps resulting from the switches in top-coding methodology. For earlier years, no adjustments are made to top-coded values.[27]

3.2.2 Capital stock and price series

The construction of structural capital series is straightforward. First, the nonresidential structural capital investment series from the national income and product accounts (NIPA) Table 5.2.5 are deflated and, using these series and a depreciation rate of 0.0275, I recursively construct structural capital stock series. The initial structural and equipment capital stock series are chosen to match the work of Krusell et al. (2000), who start from

[23] These series are not available for CPS years before 1976. For those years, I used hours worked last week instead of usual hours worked and I had to use the intervalled weeks worked last year. See the Appendix D for details of how these series are used.

[24] Alternatively, I doubled the size of groups by controlling for education as well. This did not affect key findings of the paper.

[25] Here, there is an assumption that the groups within a class are perfect substitutes (Krusell et al. (2000)).

[26] IPUMS (Flood et al. (2015)) published these swap values along with IPUMS identifiers and income variable names.

[27] I also multiplied the top-coded values by 1.45, as suggested by Krusell et al. (2000), and the key findings of the paper remained intact. As a matter of fact, top-coding is a trivial concern for this study, as its sole focus is on long-term trend, rather than short-term fluctuations.

a value of capital that matches the investment-to-capital ratio as in Gordon (1990).

To obtain investment in equipment capital in efficiency units, we need to use quality-adjusted measures of equipment capital prices. Fortunately, capital price series of Krusell et al. (2000) are updated and published by DiCecio (2009) until 2013. These price series are used in deflating the investment in nonresidential equipment capital and intellectual property products in NIPA Table 5.2.5. The efficient equipment capital stock is then constructed in the same way as structural capital series, using a calibrated depreciation rate of 0.1460.

3.2.3 Aggregate and disaggregated labor shares

I follow Krusell et al. (2000) and calculate the aggregate labor share as the ratio of labor income to the sum of labor income and capital income (depreciation, corporate profits, net interest, and rental income of people) from NIPA Table 1.10. Unfortunately, it is not possible to measure the labor shares of different task groups using the NIPA tables. To approximate those, I calculate the total wage bill of three groups of labor using the CPS data and obtain the share of each group in total wage bill.[28] Assuming that non-wage labor income across groups is proportional to the wage income of each group, these shares are then used as the weights of each group in the aggregate labor share.

Even though the central focus of this paper is the aggregate labor share, sectoral data are used in calibration as well as testing some of the predictions of the model. The sectoral labor shares are calculated from the labor file of World-KLEMS data set of Jorgenson (2012), while sectoral capital stock and compensation series are taken from the capital files of EU-KLEMS data set constructed by O'Mahony and Timmer (2009).[29] I combine these data with my calculations of hours and wage—and hence, task-specific labor shares—data from CPS to obtain sector-specific wage-bill ratios and share of equipment capital in sectoral production functions.[30]

[28] I used a wider CPS sample than the one used for obtaining efficiency hours and wages. In particular, I did not exclude even the agents who reported below 260 hours in calculation of hours. Furthermore, I used raw hours here rather than the efficiency hours, as the aim here is to calculate the wage bill of each group.

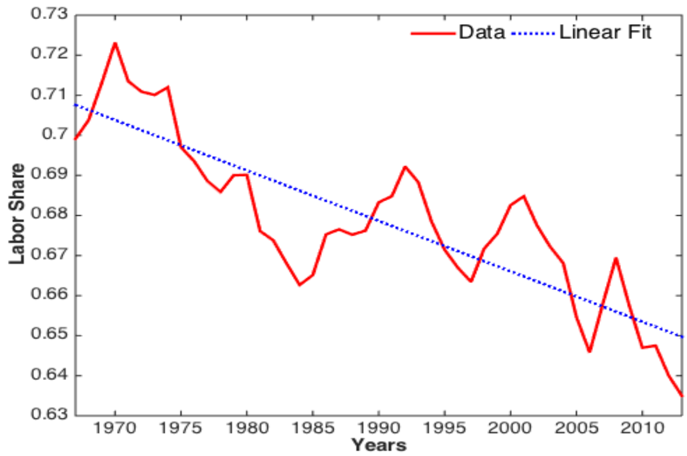
[29] EU-KLEMS and World-KLEMS data sets provide capital stock and the labor share series for up to 72 industries. I aggregate those sectors into 27 groups, excluding agriculture, public administration, and military and real estate activities.

[30] Sectoral classification of CPS data and EU-KLEMS data do not match one-to-one as the former is based on NAICS classification while the latter uses ISIC Rev. 4 to classify the industries. To match these two, I took the EU-KLEMS aggregations and aggregated the CPS sub-industries at the three-digit

3.3 Motivating facts

Fact 1: The aggregate labor share has declined substantially in last few decades. The total decline in the labor share between the years 1967 and 2013 accumulates up to 9.6 percent. The slope of the labor share trend depicted in Figure 3.1 corresponds to 0.13 percent decline per year.

Figure 3.1: The U.S. Labor Share of Income



Source: Author's calculations from NIPA tables.

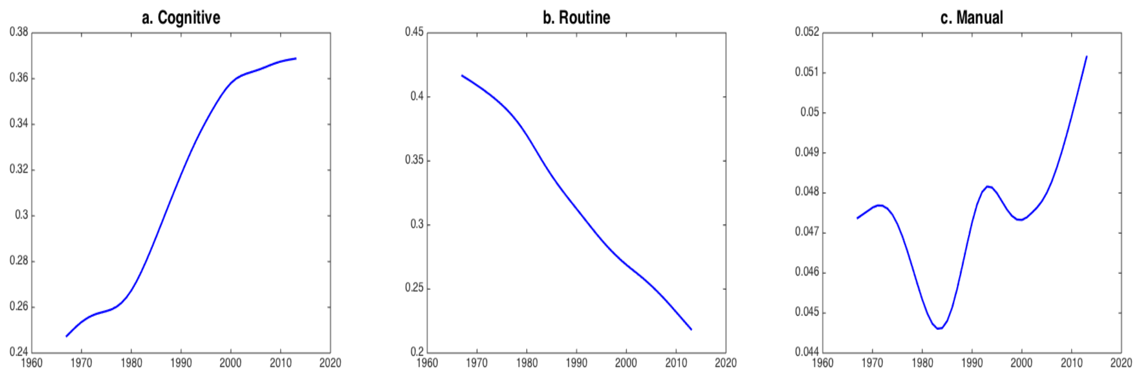
Most of the trend decline in the labor share has taken place in two periods: from 1967 to 1984 and from 2003 to 2013. As it will be discussed later, the former corresponds to a period in which between-sector changes were the main driver of the aggregate labor share. The latter period, on the other hand, follows the ICT boom that occurred between the years 1996 and 2003. As Figure A.1 demonstrates, during this period, the average annual percentage decline in equipment capital prices doubled from around 3.5 percent to above 7 percent, while the rise in equipment capital stock reached historically high levels. The acceleration in the decline in labor share following the ICT boom strengthens the conjecture that decline in the labor share might be closely related to equipment-specific technological progress and its interaction with various task groups.

Fact 2: Decline in the aggregate labor share is driven solely by the decline in the income share of labor working in routine-task occupations, while labor devoted to non-routine tasks enjoyed a substantial rise in their share of

level into the two-digit level to be consistent with ISIC classification.

income. Figure 3.2 splits the aggregate labor share into three groups: cognitive, routine, and manual. These plots are obtained by calculating the wage bills of each group from CPS and using their share in total wage bill as their weight in the aggregate labor share. As shown in panel (b) Figure 3.2, routine-task labor experienced a substantial and continuous fall in labor share, which accumulates to up to 50 percent in the past 47 years. Panel (a) shows that the cognitive labor share rapidly rose between 1980 and 2000 and has slowed down since then.[31] Finally, as depicted in panel (c), manual service labor experienced a limited rise in its share of income.[32] Figure A.3 shows that the fall in routine and gain in non-routine labor shares are common patterns across all sectors.

Figure 3.2: The Labor Share, by Tasks



The series are HP-trends.

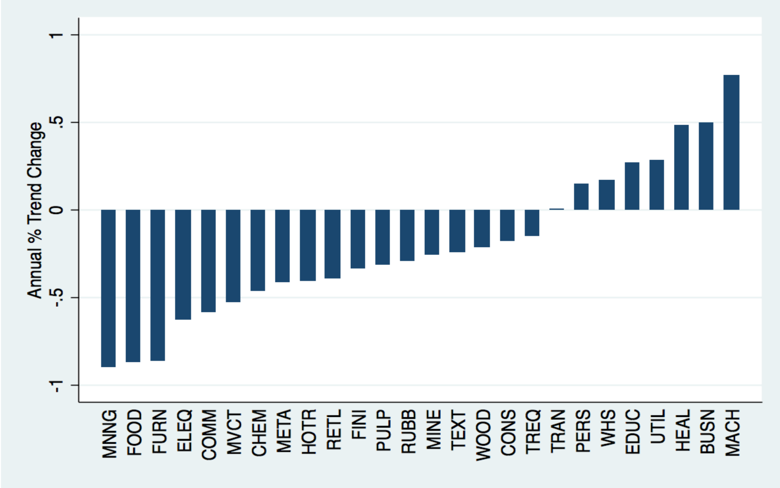
Source: Author's calculations from CPS and NIPA.

Fact 3: The decline in the labor share mainly stems from within-sector changes. Even though the share of traditionally high labor share sectors, such as manufacturing, in value-added had been persistently falling, the labor share had been declining in most of the sectors. As a matter of fact, in general, manufacturing industries experienced the largest decline in the labor share. This can be seen clearly in Figure 3.3, which plots the labor share trend coefficients for each sector.[33][34]

[31]This recent slowdown is consistent with the findings of Beaudry et al. (2013), who document that the demand for skill has recently declined.

[32]Income share of non-routine labor has been roughly constant relative to that of capital except for a brief period of increase between the late '80s and early '90s (Figure A.2). When the compensation share of structural capital is assumed to be constant at around 10 percent as in the literature, the ratio of income shares of non-routine labor and equipment capital does not exhibit a significant trend over the

Figure 3.3: The Labor Share Trend Coefficients, by Sectors



The y-axis shows the coefficient of the regression of log of the labor share on a constant and a time variable. In this sense, this coefficient shows the annual growth rate of the labor share. Source: Author's calculations from labor and output files of the World-KLEMS data set.

To prove more formally that the decline in the labor share cannot be accounted for by the structural transformation in the economy, I conduct a standard decomposition analysis of the labor share. The within/between decomposition formula used follows Karabarbounis and Neiman (2014b):

$$\Delta LS_{agg} = \sum_i \left(\bar{\omega}_i \Delta LS_i\right) + \sum_i \left(\bar{LS}_i \Delta\omega_i\right),$$

where, i represents 25 non-agricultural and non-public sectors, data for which are compiled from the World-KLEMS data set.[35] The sectors included can be found in Table B.1. LS denotes the labor share while ω_i is the share of the value-added of the sector i in total value-added. Variables with a bar stand for the average value for the entire period between 1967 and 2010. The first component on the right hand side measures the within component of the decline in the labor share, while the second term represents the between component.

period of this study. This particular data is what drives the Cobb-Douglas finding mentioned earlier.

[33] See Table B.1 in Appendix B for the abbreviations.

[34] Figure 3.3 excludes Coke and Refined Petroleum Products industry as it has three times larger trend coefficient than the next sector with largest trend coefficient.

[35] The aggregate labor share I calculated from the NIPA tables and the series I obtained from the World-KLEMS data set are different by definition and hence, not directly comparable. However, they both foresee a similar decline at the aggregate level. Therefore, since World-KLEMS is the only detailed source of the labor share at the sectoral level and what matters for the purpose of this study is the general trends at the sectoral level, I use them in sectoral analysis as a proxy to NIPA tables.

Table 3.1 shows that 73.7 percent of the decline in the aggregate labor share in U.S. stems from the within-sector changes during the entire period of study. This number rises to 95 percent for the second half of the study. However, for the earlier periods, the between-sector component drives the entire fall in the labor share. Since this paper does not encompass sectoral shifts in its analysis, the story provided here is expected to describe the post-1990 experience more accurately.

Table 3.1: Decomposition of the Labor Shares for Various Periods

Period	Within	Between
1967–1989	−36.8%	136.8%
1990–2010	95.0%	5.0%
1967–2010	73.7%	22.3%

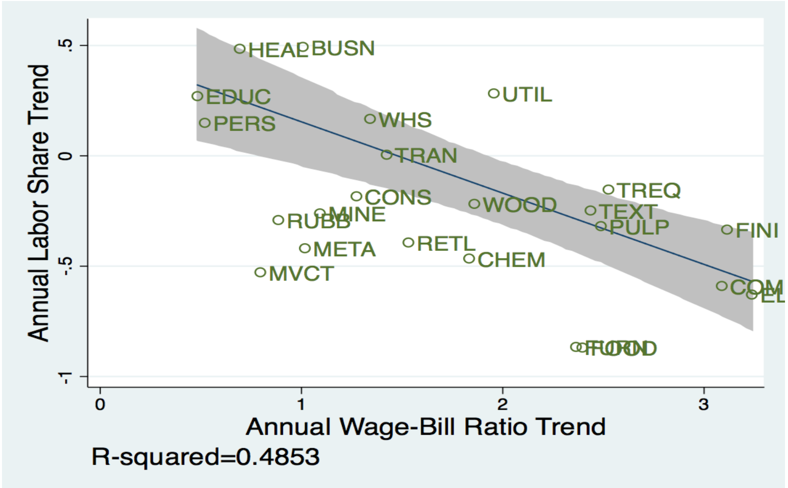
Fact 4: There is a strong correlation between trend changes in the labor shares and wage-bill ratios across sectors. Figure 3.4 summarizes this relationship.[36] Sectors with a larger rise in their wage-bill ratios in favor of non-routine occupations are the ones that started with a larger share of routine jobs and thus, have experienced a stronger polarization in response to rapid technological progress. This strengthens the idea that changes in the occupational composition of the labor force might have played a crucial role in shaping the major trends of the labor share.

4 Empirical Analysis: Estimation and Implications

In this section, the theoretical model to be estimated is introduced first. Later, the estimation strategy is presented, which is followed by an overview of the algorithm. Finally, the estimation results and their implications are discussed.

[36]The weighted correlation coefficient between two trend coefficients is -0.70 and significant with a p-value of around 0. When producing the figure and correlation coefficient, I excluded four outlier sectors: Mining, Coke and Refined Petroleum, Machinery Equipment, and Hotels and Restaurants. See Table B.1 for the abbreviations. Finally, average value-added shares of the sectors are used as weights in the regression and the scatter plot.

Figure 3.4: The Labor Share and Wage-Bill Ratio Trend Coefficients across Sectors



Trend coefficients are obtained from regressions of log series on a constant and a time variable.

4.1 Theoretical Model

4.1.1 Model environment

Two of the equations employed in the estimation come from the profit maximization decision rules of a representative firm that produces using a general production function, which allows, but does not enforce, job polarization characteristics. The firm chooses among five factors of production: structural capital; equipment capital; and the three types of labor input in efficiency units–namely, cognitive, routine, and manual. The elasticities of substitution between equipment capital and each type of labor are allowed to differ.

Even though not modeled explicitly, two other sectors give input to our model equations. First, a household sector is assumed to own the two types of capital used in production. The investment decision rules of the household between two types of capital provide us with a no-arbitrage condition between the returns on them. Combined with the firm's first order conditions, this no-arbitrage condition constitutes our third equation used in the estimation. Second, I implicitly assume that there are perfectly competitive firms producing equipment capital, and their profit maximization decision allows us to model technological advancement as a decline in the relative price of equipment capital. All these sectors are introduced in more detail in the upcoming subsections.

4.1.2 Final good production technology

The final good, Y_t, is produced using five factors of production:

$$Y_t = A_t G\left(K_{str,t}, K_{eq,t}, h_{c,t}, h_{r,t}, h_{m,t}; \varphi_{c,t}, \varphi_{r,t}, \varphi_{m,t}; \Upsilon\right),$$

where, K_{str} is structural capital, K_{eq} is equipment capital, and h_c, h_r, and h_m are raw labor input in cognitive, routine, and manual task occupations, respectively. Similarly, φ_c, φ_r, and φ_m are efficiencies of these labor groups. Furthermore, A denotes the neutral technological change. Finally, Υ is the set of model parameters, most of which will be estimated.

Below is the specific functional form used in baseline estimation:

$$Y_t = A_t K_{str,t}^{\alpha}\left(\mu L_{r,t}^{\sigma} + (1-\mu)\left(\theta\left[\lambda K_{eq,t}^{\rho} + (1-\lambda)L_{c,t}^{\rho}\right]^{\frac{\beta}{\rho}} + (1-\theta)L_{m,t}^{\beta}\right)^{\frac{\sigma}{\beta}}\right)^{\frac{1-\alpha}{\sigma}}. \tag{4.1}$$

Here, L_c, L_r, and L_m denote the efficiency hours for the respective task groups, which are defined as follows:

$$L_{i,t} = e^{\varphi_{i,t}} h_{i,t} \qquad \forall i \in \{c, r, m\}. \tag{4.2}$$

This production function differs from that in Krusell et al. (2000) in the sense that it has three skill groups rather than two. I tried several alternative functional forms differing especially in the position of manual service hours.[37] As it generated the best fit of the data among all other alternatives, and is also consistent with the job polarization literature in the sense that manual tasks are weakly complementary to equipment capital, equation 4.1 is chosen as the baseline functional form.[38]

The complete set of parameters are $\Upsilon \in \{\sigma, \rho, \beta, \alpha, \mu, \lambda, \theta\}$. The latter three are weight parameters and thus not at the center of our focus. The first three, on the other hand, are the key parameters governing the elasticities of substitution between equipment capital and different task groups. Using the definition of Krusell et al. (2000), the elasticity of substitution between routine (unskilled) hours and the composite product of equipment capital and non-routine hours (cognitive and manual) is $\frac{1}{1-\sigma}$, while the elasticity of substitution between equipment capital and cognitive (skilled) hours is defined

[37] Among several others, two main alternative functional forms estimated are as follows:
(1) $G(.) = A_t K_{str,t}^{\alpha}\left(\mu\left[\theta L_r^{\beta} + (1-\theta)L_m^{\beta}\right]^{\frac{\sigma}{\beta}} + (1-\mu)\left[\lambda K_{eq}^{\rho} + (1-\lambda)L_c^{\rho}\right]^{\frac{\sigma}{\rho}}\right)^{\frac{1-\alpha}{\sigma}}$, and
(2) $G(.) = A_t K_{str,t}^{\alpha}\left(\mu L_r^{\sigma} + (1-\mu)\left[\lambda K_{eq}^{\rho} + (1-\lambda)L_c^{\rho}\right]^{\frac{\sigma}{\rho}}\right)^{\frac{1-\alpha-\beta}{\sigma}} L_m^{\beta}$.

[38] Basically, the elasticities between equipment capital and labor associated with routine and cognitive tasks did not change significantly with other specifications considered. Thus, my choice of functional form is based on the consistency of the model generated and data series of the manual labor share.

as $\frac{1}{1-\rho}$.[39] Similarly, $\frac{1}{1-\beta}$ is the elasticity of substitution between manual tasks and the combined output of cognitive tasks and equipment capital. Finally, the parameter α determines the share of structural capital compensation in value-added.

I define capital-task complementarity as $\sigma > \rho$ (equivalently, $\frac{1}{1-\sigma} > \frac{1}{1-\rho}$). Therefore, capital-task complementarity implies that when there is equipment-specific technological improvement, which is proxied by a fall in the relative price of equipment capital, and equipment capital intensity in production increases, we would expect the share of routine hours in total hours to fall, while that of non-routine cognitive hours is expected to rise. Meanwhile, if the increase in the relative supply of non-routine cognitive labor does not keep up with the rise in the relative demand for it, we would expect the task premium and the wage-bill ratio (defined as the ratio of total wage bill of non-routine labor to the total wage bill of routine labor) to rise as well.[40] Considering the large decline in equipment capital prices (from substantial improvement in technology) over the previous decades, the demand effect is expected to dominate, thereby increasing the task premium and wage-bill ratio between non-routine and routine task occupations.

The representative firm maximizes its profits by choosing five factors of production $K_{str,t}$, $K_{eq,t}$, $h_{c,t}$, $h_{r,t}$ and $h_{m,t}$, while taking the efficiencies of three task groups—$\varphi_{c,t}$, $\varphi_{r,t}$ and $\varphi_{m,t}$—and wage rates and rental rates as given. Using the final good and structural capital as the numeraire, and setting the price of final good as 1, the profit of the representative firm at time t is as follows:

$$\Pi_t = Y_t - r_{str,t}K_{str,t} - r_{eq,t}K_{eq,t} - w_{c,t}h_{c,t} - w_{r,t}h_{r,t} - w_{m,t}h_{m,t}, \tag{4.3}$$

where, $r_{str,t}$ and $r_{eq,t}$ are the rental rates of structural and equipment capital, respectively. Similarly, $w_{c,t}$, $w_{r,t}$, and $w_{m,t}$ denote the hourly wage rates of cognitive, routine, and manual task occupations at time t.[41]

[39] This definition of elasticities are based on the assumption that all other factors are constant. When other factors change, elasticities of substitution might change as well. However, Polgreen and Silos (2009) report that none of the findings are significantly altered if Allen and Morishima elasticities of substitution are used instead.

[40] Note that here I assume, as in the data, the share of manual non-routine labor in total income to be very small.

[41] We are assuming here that one unit of final good can be transformed into one unit of structural capital, while equipment capital is produced using a different technology that will be described in next subsection. Validating this assumption, data reveal that price of final good and structural capital have a correlation of almost 1.

Two of the equations I use in the estimation process are two task premium equations:

$$tp_{cr} = \frac{w_c}{w_r} = \frac{(1-\mu)\theta(1-\lambda)}{\mu} \left(\theta \left[\lambda K_{eq}^{\rho} + (1-\lambda) L_c^{\rho} \right]^{\frac{\beta}{\rho}} + (1-\theta) L_m^{\beta} \right)^{\frac{\sigma}{\beta}-1} \times \left[\lambda K_{eq}^{\rho} + (1-\lambda) L_c^{\rho} \right]^{\frac{\beta}{\rho}-1} \frac{L_c^{\rho} h_r}{L_r^{\sigma} h_c} \quad (4.4)$$

$$tp_{cm} = \frac{w_c}{w_m} = \frac{\theta(1-\lambda)}{1-\theta} \left[\lambda K_{eq}^{\rho} + (1-\lambda) L_c^{\rho} \right]^{\frac{\beta}{\rho}-1} \frac{L_c^{\rho} h_m}{L_m^{\beta} h_c}, \quad (4.5)$$

which are derived from the first order conditions of the firm's problem that can be found in equations C.1-C.5 in Appendix C.[42] The task premium between cognitive and routine occupations (equation 4.4) is obtained by dividing equation C.1 by equation C.2. Similarly, the second task premium I target is the relative wages of cognitive and manual tasks, and it is calculated by dividing equation C.1 by equation C.3 (equation 4.5). Targeting these two automatically implies targeting the premium between routine and manual tasks as well; and thus, the third task premium is not among my targets. Finally, the third equation of the estimation is derived using equations C.4 and C.5 from the firm's problem and a no-arbitrage condition coming from the household problem as described in subsection 4.1.4.[43]

4.1.3 Equipment capital producing technology

The final output is used for three purposes: consumption C_t, investment in equipment capital $I_{eq,t}$, and investment in structural capital $I_{str,t}$:

$$Y_t = C_t + I_{eq,t} + I_{str,t}. \quad (4.6)$$

Production technology of structural capital is quite standard: one unit of investment good is converted into one unit of structural capital. Hence, prices of both final good and structural capital are normalized as 1. On the other hand, equipment capital is produced by perfectly competitive firms, converting part of the final good that is invested in equipment capital into q_t units of equipment capital, where q_t is the relative productivity of equipment capital producing sector.

Perfect competition in this sector guarantees that

$$p_{eq,t} = \frac{1}{q_t}. \quad (4.7)$$

[42]For the sake of notational simplicity, I do not use time subscript most of the time. The reader, however, should keep in mind that each equation is time specific.

[43]As one of the motivations of this paper is to investigate whether the decline of the labor share is consistent with job and wage polarizations, the labor share is kept as a non-targeted series.

In our model, equipment-capital-biased technological advancements are formalized by increases in q_t. However, as q_t is hard to measure in data, following equation 4.7, and consistent with the literature, I use the decline in the relative price of equipment capital as the proxy for technological progress.

4.1.4 The representative household's problem

Even though, as is common in the literature, we abstract from the households' labor supply decisions and take them as given for the representative firm in the estimation, households play a crucial role with their decisions in determining how investment is allocated between two types of capital. The representative household faces the following budget constraint:

$$C_t + I_{str,t} + I_{eq,t} \leq W_t L_t + R_{str,t} K_{str,t} + R_{eq,t} K_{eq,t} \tag{4.8}$$

along with the laws of motion for two types of capital

$$\begin{aligned} K_{str,t+1} &= I_{str,t+1} - \delta_{str} K_{str,t} && (4.9)\\ K_{eq,t+1} &= q_t I_{eq,t+1} - \delta_{eq} K_{eq,t}. && (4.10) \end{aligned}$$

where, δ_{str} and δ_{eq} are the depreciation rates of structural and equipment capital respectively, W_t is the wage per hour worked, and $R_{str,t}$ and $R_{eq,t}$ are the gross returns of each type of capital.[44]

Household's utility maximization yields two Euler equations: one for each type of capital. Equilibrium requires that the expected rates of return on these two types of capital should be equalized. Otherwise, households would invest only in one type of capital. Thus, manipulating the two Euler equations, we obtain the third—and last—equation used in estimation:

$$E\left(\frac{p_{eq,t+1}}{p_{eq,t}}\right) = \frac{1}{(1-\delta_{eq})}\left[MPK_{str,t+1} + (1-\delta_{str}) - \frac{MPK_{eq,t+1}}{p_{eq,t}}\right], \tag{4.11}$$

where E denotes the expected value and $MPK_{str,t+1}$ and $MPL_{eq,t+1}$ are marginal products of capital for structural and equipment capital. These are equivalent to $r_{str,t+1}$ and $r_{eq,t+1}$ coming from the first order conditions of the firm depicted in equations C.4 and C.5.

[44] Note that we are assuming that households supply a certain amount of hours, which are then assigned into tasks by the representative firm. The wage rate W_t, then, can be considered a weighted average of wage rates of each type of labor.

4.2 Estimation Strategy

The estimation strategy is in the spirit of Krusell et al. (2000), while the methodology is mostly borrowed from Polgreen and Silos (2008). The theoretical model introduced earlier is reduced into a nonlinear state space model with three observation and three state equations. The observation equations are equations 4.4, or the task premium between cognitive and routine tasks; 4.5, or the task premium between cognitive and manual tasks; and 4.11, or the no-arbitrage condition in capital markets, all summarized in the following form:

$$Z_t = f(X_t, \varphi_t, \varepsilon_t; \phi), \tag{4.12}$$

where, the function $f(.)$ contains the three nonlinear observational equations, while $X_t = \{K_{str,t}, K_{eq,t}, h_{c,t}, h_{r,t}, h_{m,t}, p_{eq,t}\}$ is the set of observed covariates as described earlier. $\phi = \{\alpha, \sigma, \rho, \beta, \mu, \lambda, \theta, \varphi_{c,0}, \varphi_{r,0}, \varphi_{m,0}, \delta_s, \delta_e, \sigma_\varepsilon^2, \sigma_\eta^2\}$ is the set of parameters, most of which will be estimated, although some are calibrated from the data. ε_t is the 3×1 vector of measurement error with a multivariate normal distribution with a zero mean and variance $\Omega = \sigma_\varepsilon^2 I_3$, while I_3 is 3×3 identity matrix. σ_ε^2 will be estimated along with the rest of the parameters. Finally, Z_t is a 3×1 vector of observations: two task premiums and the growth rate of the relative equipment capital price.

One source of estimation error stems from the relative price of equipment capital, which is related to the returns on both types of capital, as summarized in equation 4.11. This error is accounted for in the measurement error vector ε_t. The second source of uncertainty is the three efficiency factors for each types of labor. Even though these factors are known to the firm, they are not observable to us from the data. Hence, we have to define a stochastic process for efficiency factors. Following Krusell et al. (2000), I rule out the trend variation in labor quality of each type and focus only on the effects of observable variables, such as capital deepening and labor supply on the income shares and the task premiums. The stochastic processes for the three task groups comprise our state equations:

$$\log(\varphi_{j,t}) = \log(\varphi_{j,0}) + \eta_t, \tag{4.13}$$

where, $\varphi_{j,0}$ is the vector of parameters specifying the average efficiencies of cognitive, routine, and manual labor and η_t is a 3×1 vector of shocks, which is assumed to have a multivariate normal distribution with a zero mean and covariance matrix $\Sigma = \sigma_\eta^2 I_3$. σ_η^2 will also be an outcome of our estimation process.

Table 4.1 presents the list of parameters that need to be estimated. Since μ, λ θ,

Table 4.1: Parameters to be Estimated

$\frac{1}{1-\rho}$	Elasticity of subst. b/w L_c and K_{eq}
$\frac{1}{1-\sigma}$	Elasticity of subst. b/w L_r and composite of L_c, L_m and K_{eq}
$\frac{1}{1-\beta}$	Elasticity of subst. b/w L_m and composite of L_c and K_{eq}
μ	Share of L_r in CES b/w L_r and composite of L_c, L_m and K_{eq}
λ	Share of K_{eq} in CES b/w L_c and K_{eq}
$1-\theta$	Share of L_m in CES b/w L_m and composite of L_c and K_{eq}
$\varphi_{r,0}$	Average efficiency of L_r
$\varphi_{m,0}$	Average efficiency of L_m
σ_ε^2	Variance of the measurement errors
σ_η^2	Variance of the state equations errors

$\varphi_{c,0}$, $\varphi_{r,0}$, and $\varphi_{m,0}$ are all scaling parameters, one of them is fixed as a normalization as Krusell et al. (2000) do: $\varphi_{c,0}$. Depreciation rates are both capital specific and I calibrate them using NIPA tables for capital stock and consumption. The resulting depreciation rates are 0.0275 for structural and 0.1460 for equipment capital. Since incorporating the manual labor into the model raises the number of parameters to be estimated, I prefer calibrating α using the EU-KLEMS capital and output files.[45] This suggests that $\alpha = 0.1058$, which is consistent with the finding of Krusell et al. (2000) as well.

4.3 Algorithm

The estimation algorithm is not described in detail here, as it is heavily based on the work of Polgreen and Silos (2009).[46] Different from Polgreen and Silos (2009), I follow an approach closer to Krusell et al. (2000) and use instrumental variables to allow for the possible dependence of hours worked on shocks. In this framework, cognitive, routine, and manual task hours are considered to be endogenous and hence regressed on current

[45] Alternatively, I also estimated α along with rest of the parameters. However, this required targeting the labor share as well, as the original targets does not have enough information to recover α. Estimating α with rest of the parameters did not alter the estimation results for other parameters.

[46] See Polgreen and Silos (2005) and Polgreen and Silos (2008) for a detailed description of the algorithm. I also estimate the parameters of the production function using two-stage simulated pseudo-maximum likelihood estimation (SMPLE) algorithm of Krusell et al. (2000).The estimation results are consistent between two methods. Therefore, the differences in the findings of this paper and the others mentioned above are data-driven, rather than methodological differences.

and lagged stocks of both types of capital, lagged relative equipment capital price, a time trend, and the lagged value of OECD's leading business cycle indicator. The fitted values are later used in the methodology of Polgreen and Silos (2009), which is why I name the methodology I use here a two-stage Bayesian Estimation.[47]

As listed in Polgreen and Silos (2009), there are several advantages to using Bayesian inference techniques, especially when there is high-dimensionality problem and it is possible to partition the set of parameters to be estimated. The method used is based on the explicit assumption of measurement error and it uses a Metropolis Hastings algorithm, which is a Markov Chain Monte Carlo (MCMC) method of obtaining a sequence of random samples from a probability distribution. Here, a special sampling method is used—namely, Gibbs sampling, which involves choosing different samples for different dimensions rather sampling for the whole dimension of the study. Gibbs sampling is especially possible when some of the random variables can be conditioned on some of the others. Considering that we have a large set of parameters to estimate but a relatively low number of observations, Gibbs sampling makes the process more efficient by breaking the complex high-dimensional problem into simpler low-dimensional ones.

Let $\phi = \{\sigma, \rho, \beta, \mu, \lambda, \theta, \varphi_{r,0}, \varphi_{m,0}, \sigma_\varepsilon, \sigma_\eta\}$ be the set of parameters to be estimated. The goal is to characterize the distribution of the parameter set conditional on the covariates and the endogenous variables on the left hand side of the observational equations. I start with an initial guess of probability distribution of parameters, which is called prior distribution: $p(\phi)$. Using Bayes' theorem, we know that:

$$p(\phi|Z, X) \propto p(\phi)L(Z|\phi, X),$$

where, X is the set of covariates as described earlier, Z is the vector of endogenous variables, $p(\phi|Z, X)$ is the posterior distribution and $L(Z|\phi, X)$ is the likelihood function. Our goal is to characterize $p(\phi|Z, X)$ by using the proportionality property above and random sampling. The parameter set ϕ is partitioned into three blocks: $\{\sigma, \rho, \beta, \mu, \lambda, \theta\}$, $\{\Omega\}$, $\{\varphi_{r,0}, \varphi_{m,0}, \Sigma\}$ (conditioning on ϕ_{-i}).

The estimation is done in three steps and at each step we apply to a Metropolis Hasting MCMC algorithm. In step 1, the unobserved state is sampled. We take the

[47] Apart from this, I deviate from Polgreen and Silos (2009) in two dimensions. First, the production function used is different as it has three types of labor, rather than two. Second, I incorporate total factor productivity into the estimation process as well. They, however, ignore total factor productivity and thus have to restrict the parameter governing the compensation share of structural capital around the level found by Krusell et al. (2000).

Table 4.2: Priors

Parameter	Distr.	Mean	Std.	Range
σ	Normal	0.5	0.25	$[-2, 1]$
ρ	Normal	-0.5	0.25	$[-2, 1]$
β	Normal	-0.5	0.25	$[-2, 1]$
μ	Normal	0.5	0.2	$[0, 1]$
λ	Normal	0.5	0.2	$[0, 1]$
θ	Normal	0.5	0.2	$[0, 1]$
$\varphi_{r,0}$	Normal	0	0.25	$[-5,5]$
$\varphi_{m,0}$	Normal	-2	0.25	$[-5,5]$
σ_e^2	Gamma	0.3	0.01	$(0,\infty)$
σ_η^2	Gamma	0.4	0.01	$(0,\infty)$

drawn parameters and an initial state space $\{\{\varphi_{i,t}^{k-1}\}_{t=1}^{T}\} \quad \forall i \in \{c, r, m\}$ and update the state space applying to a Metropolis Hasting MCMC step. Step 2 involves sampling the parameters of the measurement equations: first the parameters of the production function and then updating the matrix of measurement errors (Ω), taking the parameters as given. Finally, in step 3, the parameters of transition equations are sampled, first the efficiency parameters and then variances of shock processes.

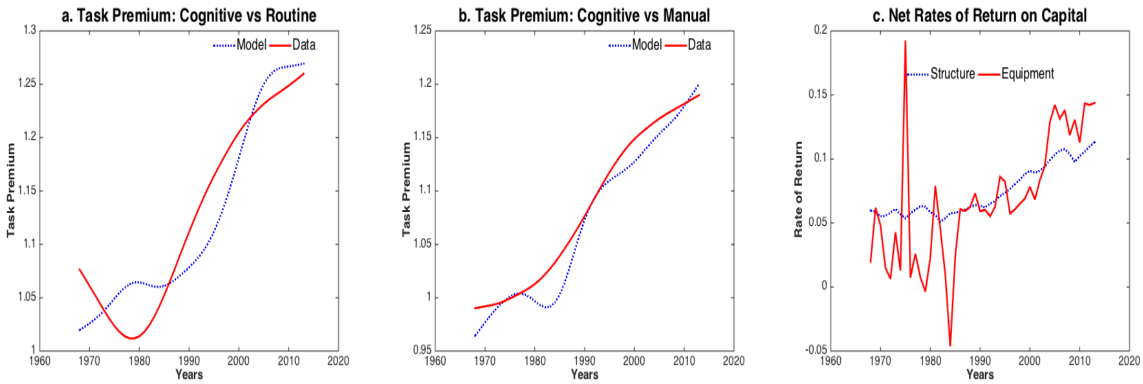
The priors chosen are close to the ones used by Polgreen and Silos (2009) and presented in Table 4.2. All the findings are strongly robust to the choice of priors as the methodology does not heavily depend on the priors or the initial starting points. Furthermore, as I iterate the process for 1 million times and burn the first half of the draw before calculating the moments, the effects of the initial choice of parameter means vanish. Finally, an acceptance rate between 20 percent and 40 percent is aimed for each block of the estimation.

4.4 Estimation Results

4.4.1 Targeted and non-targeted series

Panels (a) and (b) of Figure 4.1 show the data and model fit for the task premium between cognitive and routine and cognitive and manual tasks, respectively. To remove noise in the data, I present the series in trends, as the focus of this paper is long-term study of the labor share. The dotted blue lines showing model fit in panels (a) and (b) are plotted with the assumption of zero shocks to the efficiencies, and hence, they show the ability of the model and observable covariates such as capital and hours series to account for the changes in the data. As panel (a) implies, the model can account for most of the rise in the task premium between cognitive and routine task occupations since the 1980s but fails to fully capture the fall prior to that year. Panel (c), on the other hand, shows the net rates of returns on two types of capital, which is obtained by rearranging the no-arbitrage condition.

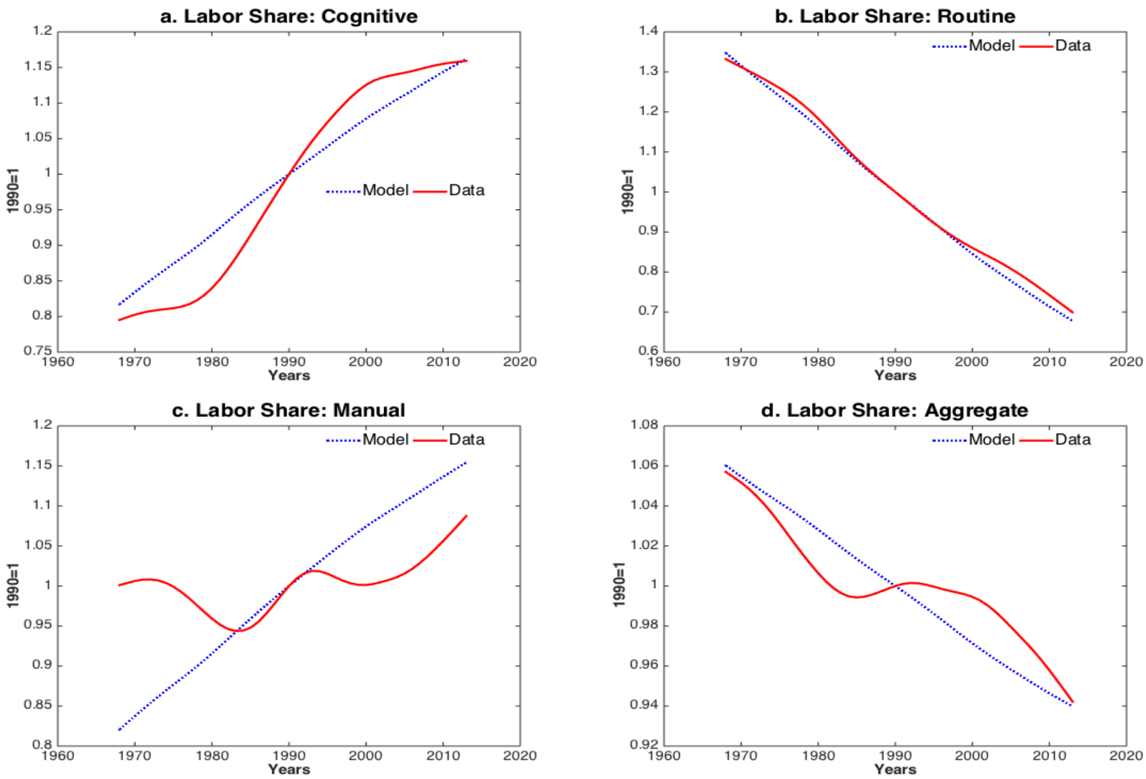
Figure 4.1: Model Targets: Data versus Model



Panels (a) and (b) are HP-trends. Panel (c) shows the expected rates of return on two types of capital in model.

Figure 4.2 plots the model's predictions in terms of disaggregated and aggregate labor shares. As panels (a) and (b) demonstrate, the model can fully predict the trend rise in cognitive labor share and the decline in routine labor share. However, it overshoots the rise in manual labor share, as it predicts an approximately 15 percent rise since 1990, while the actual increase is below 10 percent in the same period. Prior to 1990 on the other hand, the share of this group in total income remained roughly constant while the model predicts 25 percent rise. The fact that the model is inconsistent with the data in terms of the manual labor share for the pre-1990 period is not surprising, as the complementarity between cognitive and manual tasks is a relatively new phenomenon, and the model treats the entire period of the study in the same way. Furthermore, in the job polarization literature, manual task occupations are usually not included as one of the factors of the production of the final good. However, the share of this group in total wage bill and hours is very small. Besides, the success of our model in capturing the trend decline in the labor share stems from task-based definition, rather than inclusion of manual tasks separately from cognitive non-routine tasks.[48] Therefore, I prefer keeping the manual-task as a part of the final good production for computational purposes; however, I define a second sector employing only manual task labor in the general equilibrium version of the model.

Figure 4.2: Labor Shares: Data versus Model



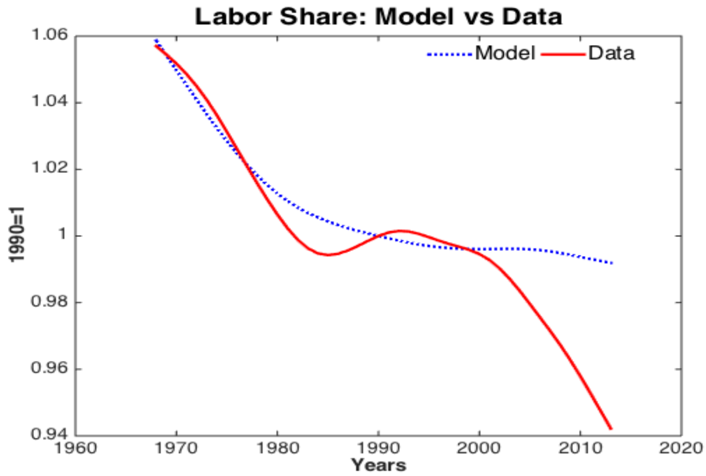
The series are HP-trends.

Panel (d) of Figure 4.2 shows that a model describing the job and wage polarizations can fully account for the trend decline in the aggregate labor share. At this point, the reader might wonder whether the estimation strategy mechanically implies targeting the labor share or not. In the end, we are targeting relative prices of types of labor and feeding their quantities. Can this guarantee that the labor share trend will be captured in any case? As Figure 4.3 depicts, the answer to this question turns out to be no. This figure shows the model-generated labor share when I estimate the model with the same

[48] The fit of the aggregate labor share significantly improves when we switch from the education-based skill classification to the task-based classification. On the other hand, when I estimate the model by aggregating cognitive and manual tasks as "non-routine," the fit of the aggregate labor share is not noticeably altered.

strategy but define the skill on the basis of having a college degree or not. As in our baseline estimation, I targeted the skill premium and the regular no-arbitrage condition, thereby making the labor share an outcome of the model. Figure 4.3 shows that the model is unable to foresee the decline in the aggregate labor share in the last decade. During the entire period, the model predicted only around 58 percent decline in the labor share, which proves that switching to the task-based skill definition significantly improves our understanding of the decline of the labor share.

Figure 4.3: Model's Prediction with Education-Based Skill Classification



The series are HP-trends.

The dotted blue line shows the fit of the model when the labor is split as skilled and unskilled on the basis of years of education.

4.4.2 Elasticities of substitution and shape of the production function

One of the goals of this paper is providing the elasticities of substitution that need to be used in job polarization literature. The estimation results for the parameters governing those elasticities are presented in Table 4.3. The elasticity of substitution between equipment capital and labor devoted to routine tasks is measured as 2.40, while equipment capital and labor employed in non-routine occupations are found to be neither substitutes nor complements.[49] Finally, as can be seen in the last column of Table 4.3,

[49]Even though not directly comparable due to different classifications of labor, these elasticities are stronger than the commonly used measures in the capital-skill complementarity literature. Krusell et al. (2000) report the elasticity of substitution between equipment capital and unskilled labor, which is most closely corresponds to our routine labor, as 1.67. Also, they estimate a strong complementarity between

Table 4.3: Estimation Results

	σ	ρ	β
Mean	0.5833	0.0240	0.0420
Mode	0.5930	-0.0044	0.0462
Standard Deviation	0.0160	0.0198	0.0252
Elasticity	2.40	1.02	1.04

there is neither strong complementarity nor substitutability between manual service tasks and the composite of equipment capital and cognitive tasks, which is consistent with the common view in the job polarization literature.

Significance tests conducted at 95 percent confidence level give p-values higher than 0.05 for the parameters ρ and β.[50] In other words, the elasticities of substitution between equipment capital and cognitive task and between manual tasks and the composite of equipment capital and cognitive tasks are not significantly different from 1. This finding implies that we can model job/wage polarization and the labor share with a functional form of production that has a Cobb-Douglas relationship between equipment capital, the cognitive task and the manual task. In more aggregate terms, I document that the production function is Cobb-Douglas between equipment capital and labor associated with non-routine task occupations.

4.4.3 Implications

The partially Cobb-Douglas production function has important implications. First of all, it shows that the neo-classical view of constancy of the labor share is still valid, albeit at a smaller scale. Any part of the income that is lost by labor employed in routine task occupations is proportionately captured by equipment capital and labor devoted to non-routine tasks. This implies that the economy will converge to another steady state with a constant but lower labor share once this transformation is completed. Second, as will be formally proven in next subsection, with this Cobb-Douglas structure, the changes in the labor share can be pinned down only to the changes in the composition of the wage

equipment capital and skilled labor, which can be considered as our cognitive labor. They estimate the elasticity of substitution between these two factors as 0.67.

[50] p-value reported for ρ is 0.2265 and for β is 0.0952. Here, the null hypotheses are ρ and β are separately not different from zero.

bill. To be more specific, the decline in the labor share can be explained and modeled by capital-task complementarity that will distort the wage-bill ratio between non-routine and routine tasks in favor of the non-routine labor following technological advances. In the upcoming sections, I will introduce a simple general equilibrium model following from this finding and show that equipment-specific technological progress and capital-task complementarity alone can account for most of the decline of the labor share. Finally, using the Cobb-Douglas specification significantly reduces the burden of the estimation, as we now have two less parameters to estimate.

Since the estimation of the general functional form indicates a Cobb-Douglas specification instead of inner CES aggregators, we can use the following production function:

$$Y = AK_{str}^{\alpha}\left(\mu L_r^{\sigma} + (1-\mu)\left[K_{eq}^{\gamma}L_c^{\delta}L_m^{1-\gamma-\delta}\right]^{\sigma}\right)^{\frac{1-\alpha}{\sigma}}. \tag{4.14}$$

When I estimate the model using the production function specification in equation 4.14, neither the model fit, nor the prediction of the model in terms of the labor shares, is visibly altered. Figure A.4 replicates Figures 4.1 and 4.2 for the Cobb-Douglas version of the model, and the reader can verify that the model fit does not worsen, if not improve.

Table 4.4 reports the estimation outcome for the key parameters. The elasticity of substitution between routine task and the composite of equipment capital and non-routine tasks is 2.33, which is close to the value obtained for the general functional form. As reported in the last column, 35 percent of the total income that is left after the compensations of structural capital and the routine tasks are paid goes to equipment capital, while the remaining 65 percent goes to labor employed in non-routine task occupations. Hence, the 65 percent to 35 percent balance between labor and capital in neoclassical theory turns out to be still valid at a smaller scale than the aggregate economy.

Table 4.4: Estimation Results for the Cobb-Douglas Specification

	σ	δ	γ
Mean	0.5708	0.5566	0.3528
Mode	0.5610	0.5530	0.3567
Standard Deviation	0.0057	0.0198	0.0066
Elasticity	2.33	-	-

4.4.4 Labor share in the model

The production function in equation 4.14 yields the following labor share equation:

$$LS_t = (1-\alpha)\frac{\mu L_{r,t}^{\sigma} + (1-\mu)(1-\gamma)K_{eq,t}^{\sigma\gamma}L_{c,t}^{\sigma\delta}L_{m,t}^{\sigma(1-\delta-\gamma)}}{\left(\mu L_{r,t}^{\sigma} + (1-\mu)\left[K_{eq,t}^{\gamma}L_{c,t}^{\delta}L_{m,t}^{1-\delta-\gamma}\right]^{\sigma}\right)}. \tag{4.15}$$

When this equation is rearranged, details of which can be found in equations C.11-C.18 in Appendix C, I obtain the following relationship between the labor share and the wage-bill ratio between non-routine and routine tasks:

$$LS_t = (1-\alpha)\frac{1+wbr_t}{1+\frac{wbr_t}{1-\gamma}}. \tag{4.16}$$

In growth terms,[51]

$$\widehat{LS}_t = -\widehat{wbr}_t\left[\frac{\gamma}{wbr_t + (2-\gamma) + \frac{(1-\gamma)}{wbr_t}}\right], \tag{4.17}$$

where, the growth rate of the wage-bill ratio between non-routine and routine tasks is given as:

$$\widehat{wbr}_t = \sigma\left[\delta\widehat{L_{ct}} + \gamma\widehat{K_{eq_t}} + (1-\delta-\gamma)\widehat{L_{mt}} - \widehat{L_{rt}}\right]. \tag{4.18}$$

Equation 4.17 reduces the growth rate of the labor share down only to the negative of the growth rate of the wage-bill ratio and a key parameter γ, which is the weight of equipment capital in Cobb-Douglas segment of the production function. The wage-bill ratio, on the other hand, rises faster if (i) the possibilities to switch from routine task to non-routine tasks and equipment capital are higher and (ii) the cost function is more sensitive to the changes in equipment capital prices. The former condition corresponds to a larger elasticity of substitution between routine task and composite output of equipment capital and non-routine tasks, while the latter corresponds to a larger weight of equipment capital (γ) in the Cobb-Douglas specification.[52]

Equation 4.17 shows that anything that will raise the wage-bill ratio in favor of non-routine labor will cause a decline in the labor share as well. Thus, in an environment where price of equipment capital declines rapidly, capital-task complementarity with unit elasticity between equipment capital and non-routine labor is sufficient to distort the wage-bill ratio in favor the labor working in non-routine task occupations, thereby causing

[51] Steps of converting equation 4.16 into growth terms are presented in equations C.19-C.25.

[52] Note that this former condition is also equivalent to having a larger capital-task complementarity, which is measured by σ in our case, as the parameter governing the elasticity of substitution between non-routine tasks and equipment capital is documented to be zero. Hence, $\sigma - 0 = \sigma$ is at the same time our capital-task complementarity measure.

a decline in the labor share. Nonetheless, the effect of the rise in the wage-bill ratio on the labor share decreases over time, even if the wage-bill ratio continues to grow. This can be seen from the second term in equation 4.17: $D_t = \frac{\gamma}{wbr_t + (2-\gamma) + \frac{(1-\gamma)}{wbr_t}}$. The term D_t first rises in wbr_t and then starts falling as wbr_t passes a certain threshold, as shown in Figure A.5. The reverse U-shape of the coefficient D_t strengthens our conjecture that the U.S. economy is currently at early stages of a long-term transition period that is likely to cool down in the medium-term, even if the composition of the wage bill continues to change at its current pace.

4.4.5 Testing the estimation outcome using the aggregate and sectoral data

If the functional form estimated is correct, we should be able to replicate the decline of the labor share by feeding equation 4.17 with the wage-bill ratio series calculated from the CPS data. Figure 4.4 plots the model fit obtained from this exercise. It is clear from the figure that the wage-bill ratio series can account for almost all of the trend decline of the labor share in the long run.[53]

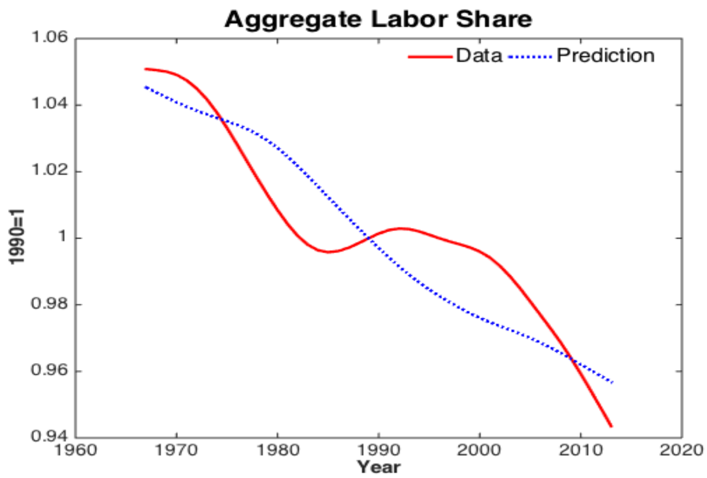
A similar exercise is repeated for some selected sectors. To do so, I first calibrate γs for each sector.[54] Then, I feed equation 4.17 with the sectoral wage-bill ratios. As panels (a) and (b) of Figure A.6 show, the wage-bill ratio does remarkably well in replicating the decline in labor shares in some sectors such as Food, Tobacco and Beverages, and Finance and Insurance. However, as panels (c) and (d) depict, equation 4.17 fails to fully explain why the labor share rises in some of the sectors; yet, it does not predict a fall in these sectors either. This failure most probably stems from ruling out other sector-specific factors, such as an increase in relative demand for those sectors or heterogeneity in sector-specific technological progresses or substitution possibilities. The success of the model for other sectors lies somewhere between these two extremes. Considering that I assumed same substitutability at each sector, the model's predictions of the labor share

[53]It should be reminded here that this paper views the changes in the labor share from a long-term perspective. Thus, the model does not capture short term cyclical or trend changes in the labor share.

[54]To calibrate sector specific γs, I first calculate the wage-bill of labor employed in non-routine task occupations from the CPS data. Then I use the ratio of this to total wage bill as the weight of this group in the aggregate labor share. Multiplying this weight by the labor share of this sector in EU-KLEMS data set gives me income share of labor devoted to non-routine tasks in that sector. The share of equipment capital is also calculated from the EU-KLEMS data set. γ is then obtained as the ratio of the equipment capital compensation share to the sum of the compensation shares of equipment capital and labor associated with non-routine tasks.

should be considered as a lower bound of our ability to explain the labor share trends by equipment-specific technological change and capital-task complementarity.

Figure 4.4: Labor Share Fit, by Wage-Bill Ratio Series Alone



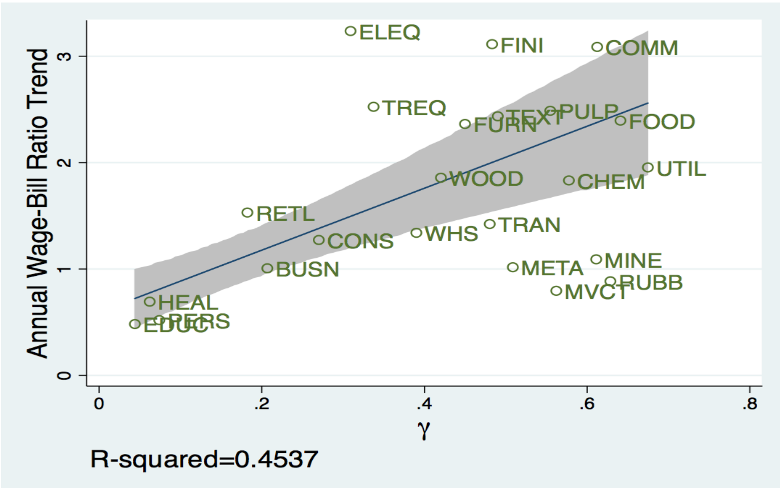
The series are HP-trends.

The dotted blue line is generated by feeding equation 4.17 with the wage-bill ratio series calculated from the CPS data and using the parameter $\gamma = 0.3528$.

Being able to replicate the labor share trend based only on the wage-bill ratio series and a single parameter significantly reduces the burden of the analysis. To be more specific, we do not need to make assumptions about a whole set of capital stocks and prices and labor input series if we want to predict the future path of the labor share. For instance, if we assume that the wage-bill ratio continues to grow at its current average rate of 2 percent per year, equation 4.17 tells us that the labor share should stabilize around 55 percent. Sole dependence of the labor share on the wage-bill ratio also simplifies the general equilibrium model that I will use to quantify the decline of the labor share that can be attributed to equipment-specific technological progress and capital-task complementarity. As long as we have the correct elasticity of substitution between routine jobs and other factors of production, and a correct weight of equipment capital in the Cobb-Douglas specification, the decline in relative equipment capital price will be sufficient to raise the wage-bill ratio in favor of labor working in non-routine task occupations. The rise in the wage-bill ratio, in turn, is expected to generate a trend decline in the labor share at a magnitude similar to the data if the model parameters are well identified.

Before proceeding to the general equilibrium model, I test the key mechanism implied by the estimation using the sectoral data. Figure 4.5 shows that for a given level of elasticity of substitution between routine task and composite output of equipment capital and non-routine tasks, in sectors for which the weight of equipment capital compensation in the Cobb-Douglas specification is larger, the rise in the wage-bill ratio is larger as well. The weighted coefficient of correlation between the annual trend change of the wage-bill ratio and the parameter γ is 0.67 and significant with a p-value of almost 0. Figure 3.4, which was presented earlier, on the other hand, confirms the remaining part of the mechanism: in sectors where the wage-bill ratio rose more, the trend decline of the labor share was larger as well. To sum up, considering that we ruled out possible differences in elasticities of substitution across sectors, the estimated aggregate production function and the implied key channel driving the labor share trend does remarkably well in explaining differences across sectors as well.

Figure 4.5: Weights of Equipment Capital and Wage-Bill Ratios across Sectors



Trend coefficients are obtained from regressions of log series on a constant and a time variable.

5 General Equilibrium Model

The estimation analyses have provided us with two important ingredients for modeling the decline of the labor share. First, it gave us the shape of production function that is consistent with major employment and wage shifts observed over the past few decades as well as the decline in the labor share. Second, and more importantly, it revealed that we can model the labor share decline by a rise in the wage-bill ratio, which is triggered by technological advances and capital-task complementarity. Up until now, all of the variables of the model were exogenous in our analysis. From now on, I will only keep technological change as exogenous and all the remaining variables will be determined endogenously within a dynamic general equilibrium environment. This way, we can quantify part of the labor share trend that can be accounted for by the decline in relative equipment capital prices only. This model can also be used for conducting counterfactual exercises, labor share projections, and policy analysis.

5.1 Model Environment

The household sector consists of two types of agents: skilled and unskilled. The representative household of each type has a continuum measure of unit 1 individuals differing in their routine task efficiencies. The representative households supply their labor inelastically and, taking the wages as given, assign their members to occupations. Members of the skilled household can work in either cognitive or routine task occupations, while the unskilled household makes a choice between routine and manual task occupations. Cognitive and routine hours in efficiency are combined with both structural and equipment capital to produce the final good that is consumed and invested. In addition, there also exists a sector producing manual services by employing only labor for manual tasks.[55] Finally, technological progress enters the model exogenously in the form of a rise in the productivity of equipment capital-specific production.

5.2 The Households' Labor Supply and Investment Decisions

There is a measure of S skilled and a measure of U unskilled households in the economy. Both types of agents face similar problems in the sense that they assign their members into occupations and save an asset that is used in purchasing both types of capital. However, each type of agent differs in a couple of dimensions. First, members of the skilled household do not have access to the manual labor market, while the unskilled agents cannot be employed in cognitive task occupations.[56] On the other hand, both types can work in routine task occupations. Secondly, manual services are assumed to be consumed only by the skilled agents. This assumption is consistent with the proposed

[55] The manual services in the model represent the personal services such as child care, cleaning, and home health care.

[56] Even if the skilled agents were allowed to work in manual task occupations, they wouldn't prefer doing so unless manual sector wages are too high. Thus, the set-up is simplified by ruling out the manual employment of the members of the skilled household.

relationship between cognitive task labor and manual services in the job polarization literature. Yet, this is a trivial assumption and does not affect the quantitative findings of the paper.

5.2.1 Problem of the skilled agent

The skilled household has members with the vector of efficiencies $(\zeta, \tau_i, 0)$ for cognitive, routine, and manual task occupations respectively. In other words, skilled individuals have access to two labor markets. If they work in cognitive task occupations, their efficiency is ζ, and thus they receive an hourly wage rate of $\zeta w_{c,t}$, where $w_{c,t}$ is the wage rate for cognitive task occupations. Alternatively, they can prefer working in routine task occupations. Their efficiencies in routine task occupations are ranked by τ_i, which is assumed to be distributed uniformly between 0 and 1: $\tau_i \sim U[0,1]$. These identifiers are translated into routine task efficiencies through the function $h(\tau_i)$. Thus, the skilled individuals working in routine task occupations receive efficiency wages equivalent to $h(\tau_i)w_{r,t}$, where $w_{r,t}$ is the wage rate for these occupations.[57] Following Beaudry et al. (2013), I choose $h(\tau_i) = \tau_i^{-\frac{1}{2}}$ as the functional form of routine task efficiency.

The skilled household consumes a composite of final good and manual services; save an asset and chooses a cutoff $\bar{\tau}_t$ that determines the allocation of its members between occupations. The household's inter-temporal problem is:

$$\max_{c_{s,g,t}, c_{s,m,t}, a_{s,t+1}, \bar{\tau}_t} E\left[\sum_{t=0}^{\infty} \Gamma^t log(c_{s,t})\right],$$

subject to

$$c_{s,g,t} + p_{m,t}c_{s,m,t} + \gamma_z a_{s,t+1} \leq w_{c,t}\int_{\bar{\tau}_t}^{1} \zeta d\tau + w_{r,t}\int_{0}^{\bar{\tau}_t} h(\tau_i)d\tau + R_t a_{s,t}. \tag{5.1}$$

Here, $c_{s,g,t}$ and $c_{s,m,t}$ are the skilled household's consumption of final good and manual services, respectively. $a_{s,t+1}$ is the household's assets at time $t+1$, while R_t is the gross return on the asset between periods t and $t+1$. γ_z, on the other hand, is a common growth factor, used in de-trending the variables, and Γ is the discount factor. Finally, $c_{s,t}$ is the composite consumption of the skilled household, such that

$$c_{s,t} = \left(\chi c_{s,g,t}^{\kappa} + (1-\chi)c_{s,m,t}^{\kappa}\right)^{\frac{1}{\kappa}}, \tag{5.2}$$

[57]Instead of allowing the members of the skilled household differ in their cognitive task efficiencies, I preferred the current set-up for modeling purposes. Thus, having a high routine task efficiency can be considered as having comparative advantage in routine tasks. Therefore, from a reverse perspective, these members of the skilled group can be thought of as having a comparative disadvantage in occupations requiring cognitive skills.

where, the demand elasticity of substitution between final good and manual services is $\frac{1}{1-\kappa}$.

Utility maximization requires that the marginal utility per dollar should be equalized between the final good and manual services consumptions:

$$p_{m,t} = \frac{1-\chi}{\chi}\left(\frac{c_{s,g,t}}{c_{s,m,t}}\right)^{1-\kappa}. \tag{5.3}$$

The first order condition with respect to $\bar{\tau}_t$ gives us the rule for cutoff efficiency rank as shown below:

$$\frac{w_{c,t}}{w_{r,t}} = \frac{\bar{\tau}_t^{-\frac{1}{2}}}{\zeta}. \tag{5.4}$$

.

Equation 5.4 shows that the member of the household with efficiency rank $\bar{\tau}_t$ will be indifferent between working in cognitive and routine task occupations. The members with efficiency rank above $\bar{\tau}_t$, on the other hand, will supply their labor endowment to occupations associated with cognitive tasks. When the advances in technology raises the relative wage of cognitive task occupations, cutoff efficiency rank will fall, and thus more skilled labor will be employed in these occupations.

Finally, we have the usual Euler equation that makes the skilled household indifferent between consuming one more unit today and saving that one unit and consuming it in the next period:

$$\frac{c_{s,g,t}^{\kappa-1}}{c_{s,t}^{\kappa}}\gamma_z = \Gamma E_t\left[\frac{c_{s,g,t+1}^{\kappa-1}}{c_{s,t+1}^{\kappa}}R_{t+1}\right]. \tag{5.5}$$

5.2.2 Problem of the unskilled agent

The problem of the unskilled household is similar. The unskilled household has members with the vector of efficiencies $(0, \psi_i, 1)$ for cognitive, routine, and manual task occupations, respectively. Thus, unlike the skilled agent, the unskilled does not have access to the labor market for cognitive occupations. However, it competes with the skilled agents for the routine task occupations. The members of the household are uniformly ranked between 0 and 1. Their rank, ψ_i, determines their efficiency in routine task occupations. Similar to the skilled agents, these identifiers are translated into routine task efficiencies through the function $g(\psi_i)$. As before, I assume that the efficiency is decreasing in ψ_i, meaning that $g(\psi_i') < 0$. However, it is assumed, on average, that the routine task efficiency of the unskilled labor is lower than that of skilled labor. In particular, I specify that for each rank level $\psi_i = \tau_i$, $g(\psi_i) = bh(\tau_i)$, where $0 \leq b \leq 1$. Thus, the unskilled individuals doing routine tasks receive $g(\psi_i)w_{r,t}$ worth of efficiency wage. The members of the unskilled household also have the option of supplying their labor endowments to manual service production, where the wage rate is $w_{m,t}$. Comparing these two wage rates in efficiency units, the household chooses an efficiency rank cutoff that determines the fraction of its members assigned to routine and manual task occupations.

The unskilled household consumes only the final good, and, similar to the skilled agent, it saves an asset and chooses a cutoff $\bar{\psi}_t$ that determines the allocation of its individuals between tasks. The household's inter-temporal problem is:

$$\max_{c_{u,g,t}, a_{u,t+1}, \bar{\psi}_t} E\left[\sum_{t=0}^{\infty} \Gamma^t log(c_{u,g,t})\right],$$

subject to

$$c_{u,g,t} + \gamma_z a_{u,t+1} \leq w_{r,t} \int_0^{\bar{\psi}_t} g(\psi_i) d\psi + w_{m,t} \int_{\bar{\psi}_t}^1 1 d\psi + R_t a_{u,t}, \tag{5.6}$$

where, $c_{u,g,t}$ is the unskilled household's consumption of final good, $a_{u,t+1}$ is its asset holdings at the end of period t, and $\bar{\psi}_t$ is the efficiency rank cutoff that determines the allocation of members of the household across occupations. This cutoff is determined by the first order condition with respect to $\bar{\psi}_t$. When there is technological advancement, the demand for cognitive tasks is expected to rise, which, in turn, will raise the total income of the skilled group. As they are the sole demanders of manual services, the demand and relative price of these services will rise. Hence, wage paid to manual task occupation will rise relative to the occupations associated with routine tasks, and, as equation 5.7 shows, this will raise the cutoff, leaving less unskilled workers in routine task occupations.

$$\frac{w_{m,t}}{w_{r,t}} = b\bar{\psi}_t^{-\frac{1}{2}} \tag{5.7}$$

Finally, we have our usual Euler equation coming from the agent's inter-temporal saving decisions:

$$\frac{1}{c_{u,g,t}}\gamma_z = \Gamma E_t\left[\frac{1}{c_{u,g,t+1}} R_{t+1}\right]. \tag{5.8}$$

5.3 Final Good Production

The representative firm produces the final good that is either consumed or invested in either structural or equipment capital. The final good Y_g is produced using four factors of production: structural capital (K_{str}), equipment capital (K_{eq}), cognitive (L_c), and routine task (L_r) hours in efficiency units. Consistent with the estimation outcome, there is a Cobb-Douglas relationship between equipment capital and cognitive task, while there is non-unitary elasticity of substitution between the composite of these two factors and routine task:

$$Y_{g,t} = A_t K_{str,t}^{\alpha} \left(\mu L_{r,t}^{\sigma} + (1-\mu) L_{c,t}^{(1-\gamma)\sigma} K_{eq,t}^{\gamma\sigma} \right)^{\frac{1-\alpha}{\sigma}}, \tag{5.9}$$

where, A_t is the total factor productivity (tfp) term, for which the following stochastic process is defined:[58]

$$\ln A_t = \rho_A \ln A_{t-1} + \sigma_{A,t}.$$

The final good is chosen as the numeraire, and thus its price is set as 1. The representative firm takes the wages and rental rates of capitals as given and maximizes its profits by choosing quantities of four factors of production. The problem of the firm at time t is described as:

$$\max_{K_{str,t}, K_{eq,t}, L_{c,t}, L_{r,t}} Y_{g,t} - w_{c,t} L_{c,t} - w_{r,t} L_{r,t} - r_{str,t} K_{str,t} - r_{eq,t} K_{eq,t}.$$

The firm's optimal decision rules are given by the following first order conditions:

$$w_{c,t} = (1-\alpha)(1-\mu)(1-\gamma) \frac{Y_{g,t}}{\mu L_{r,t}^{\sigma} + (1-\mu) L_{c,t}^{(1-\gamma)\sigma} K_{eq,t}^{\gamma\sigma}} K_{e,t}^{\gamma\sigma} L_{c,t}^{(1-\gamma)\sigma-1} \tag{5.10}$$

$$w_{r,t} = (1-\alpha)\mu \frac{Y_{g,t}}{\mu L_{r,t}^{\sigma} + (1-\mu) L_{c,t}^{(1-\gamma)\sigma} K_{eq,t}^{\gamma\sigma}} L_{r,t}^{\sigma-1} \tag{5.11}$$

$$r_{eq,t} = (1-\alpha)(1-\mu)\gamma \frac{Y_{g,t}}{\mu L_{r,t}^{\sigma} + (1-\mu) L_{c,t}^{(1-\gamma)\sigma} K_{eq,t}^{\gamma\sigma}} K_{eq,t}^{\gamma\sigma-1} L_{c,t}^{(1-\gamma)\sigma} \tag{5.12}$$

$$r_{str,t} = \alpha \frac{Y_{g,t}}{K_{str,t}}. \tag{5.13}$$

5.4 Personal (Manual) Services Production

The manual services sector represents the home production. This sector is essential, as it helps the model generate a larger decline in labor devoted to routine tasks in the response to declining demand for this type of labor following the fall in equipment capital prices. In the job polarization literature, manual task occupations are neither complements nor substitutes to equipment capital. However, equipment capital and manual task occupations are indirectly linked through the consumption behavior of labor employed in cognitive task occupations. As demand for these high-skill types of task rises, their income rises as well, which, in turn, increases the demand for manual services by labor employed in cognitive task occupations. Therefore, the share of labor devoted to manual tasks in total income and hours shows a modest rise.

[58] This tfp process will be fed with the shocks, which I obtain from the estimation process, when solving for the general equilibrium model under perfect foresight.

Consistent with the job polarization literature, including Autor and Dorn (2013), the production function for manual services is assumed to be linear. To be more specific, one unit of manual labor input produces one unit of manual services:

$$Y_{m,t} = L_m. \tag{5.14}$$

Letting p_m denote the price of one unit of manual service and w_m the hourly wage rate in this market, profit maximization under perfect competition yields:

$$p_m = w_m. \tag{5.15}$$

5.5 Technological Progress

The final good is either consumed or invested in structural or equipment capital. One unit of investment in structural capital is converted into one unit of this type of capital. On the other hand, investment in equipment capital is scaled by a factor q_t before being added to equipment capital stock. q_t here represents the equipment-specific technological progress and will be assumed to be exogenous in the model. As it had already been discussed in the estimation part, perfect competition drives the relative price of equipment capital to:

$$p_{eq,t} = \frac{1}{q_t}. \tag{5.16}$$

5.6 Equilibrium

For a given level of equipment-specific technology q_t, an equilibrium for our model economy is a collection of prices $\{w_{c,t}, w_{r,t}, w_{m,t}, r_{s,t},\ r_{e,t}, R_t, p_{m,t}, p_{eq,t}\}$, capital stocks and assets $\{K_{str,t}, K_{eq,t}, a_{s,t}, a_{u,t}\}$, quantities of consumption and output $\{Y_{g,t}, Y_{m,t}, c_{s,t}, c_{s,g,t}, c_{s,m,t}, c_{u,g,t}\}$, labor hours in efficiency units $\{L_{c,t}, L_{r,t}, L_{m,t}\}$, and efficiency rank cutoffs $\{\bar{\tau}_t, \bar{\psi}_t\}$ such that: (1) households maximize their discounted utilities subject to their budget constraints, (2) both the final good producers and manual service producers maximize their profits, (3) labor markets clear:

$$L_{c,t} = \int_{\bar{\tau}_t}^{1} \zeta d\tau S = (1-\bar{\tau}_t)\zeta S \tag{5.17}$$

$$L_{r,t} = \int_{0}^{\bar{\tau}_t} \tau_i^{-1/2} d\tau S + b\int_{0}^{\bar{\psi}_t} \psi_i^{-1/2} d\psi U = 2\bar{\tau}_t^{\frac{1}{2}} S + 2b\bar{\psi}_t^{\frac{1}{2}} U \tag{5.18}$$

$$L_{m,t} = \int_{\bar{\psi}_t}^{1} 1 d\psi U = (1-\bar{\psi}_t)U, \tag{5.19}$$

(4) capital and asset markets clear:

$$a_{s,t}S + a_{u,t}U = K_{str,t} + p_t K_{eq,t} \tag{5.20}$$

$$\frac{p_{eq,t+1}}{p_{eq,t}} = \frac{1}{1-\delta_{eq}}\left[r_{str,t+1} + (1-\delta_{str}) - \frac{r_{eq,t+1}}{p_{eq,t}}\right] \tag{5.21}$$

$$R_t = 1 - \delta_{str} + r_{str,t}, \tag{5.22}$$

and (5) manual service market clears:

$$c_{s,m,t}S = Y_{m,t}. \tag{5.23}$$

Households invest in one type of asset, which is then purchased by a zero-profit intermediary and converted into two types of capital. Thus, market clearing requires that the total amount of savings should be equal to the total value of the capital (equation 5.20). The intermediary's investment decision will be such that the expected return on two types of capital should be equal (equation 5.21). Otherwise, there would be investment in only one type of capital in the model economy. Finally, gross returns on the asset and each type of capital should be equalized as well (equation 5.22).

In summary, there are 23 unknowns in our model $\{w_{c,t}, w_{r,t}, w_{m,t}, r_{s,t}, r_{e,t}, R_t, p_{m,t}, p_{eq,t}, K_{str,t}, K_{eq,t}, a_{s,t}, a_{u,t}, Y_{g,t}, Y_{m,t}, c_{s,t}, c_{s,g,t}, c_{s,m,t}, c_{u,g,t}, L_{c,t}, L_{r,t}, L_{m,t}, \bar{\tau}_t, \bar{\psi}_t\}$, and all 23 equations between 5.1 and 5.23 solve for these unknowns.

6 Quantitative Analysis

In this section, the general equilibrium model introduced above is used to quantify the contribution of the equipment-specific technological progress to the trend decline in the labor share. First, I discuss the parameterization of the model. Later, I briefly present the solution method and report the model's findings. Finally, I discuss the implications of the model for heterogeneity in labor share trends across sectors.

6.1 Parameterization

The model parameters are set in three main blocks: estimation, regression, and calibration. Below, I describe each of these in detail. The reader can refer to Table B.2 in Appendix B, which lists all the parameters of the model and the sources of parameterization.

6.1.1 Estimation

The first set of parameters are already available from the estimation part in Section 4. In particular, $\sigma = 0.5708$, which is the parameter that governs the elasticity of substitution between labor employed in routine task occupations and the composite output of equipment capital and non-routine tasks. This value of σ implies an elasticity of substitution of 2.33. Furthermore, the choice of production function in equation 5.9 is based on the estimation result that equipment capital and cognitive tasks are neither complements, nor substitutes. Finally, I borrow the parameter γ, which is the share of equipment capital in the partial Cobb-Douglas specification between equipment capital and cognitive task, from Table 4.4.[59]

6.1.2 Regression

There are two parameters in the CES consumption aggregator in equation 5.2: κ is the parameter that shapes the elasticity of substitution between consumption of the final good and manual services. The other parameter, χ, is the share of the final good in aggregate consumption of the skilled agent. To recover these two parameters, I combine the first order conditions of the skilled household with respect to each types of consumption, which gives:

$$p_{m,t} = \frac{1-\chi}{\chi}\left(\frac{c_{s,g,t}}{c_{s,s,t}}\right)^{1-\kappa}.$$

Then, I take the logarithm of this equation and run the following simple regression:

$$\ln p_{m,t} = \ln \frac{1-\chi}{\chi} + (1-\kappa)\ln\left(\frac{c_{s,g,t}}{c_{s,s,t}}\right) + u_t.$$

In this regression, I use personal consumption of goods and services (apart from the "other services" category) from NIPA Table 2.3.6 as consumption of the final good in our model, $c_{s,g,t}$, and personal consumption expenditures on other services as manual services, $c_{s,m,t}$. The price series, on the other hand, are taken from NIPA Table 2.3.4 for the 1967–2013 period. This regression gives us $\chi = 0.9410$ and $\kappa = -0.1547$. This level of κ corresponds to an elasticity of substitution between final good and manual services equal to 0.87, which implies complementarity between these two consumption items.

[59] However, here an adjustment is needed since the production function for the final good in general equilibrium model does not encompass manual tasks. Thus, I distribute the share of labor employed in manual task occupations reported in Table 4.4 proportionally to equipment capital and labor devoted to cognitive task occupations, which gives us $\gamma = 0.3879$.

6.1.3 Calibration

Data: Some parameters are taken directly from the data. To begin with, depreciation rates for each type of capital are easily recoverable from NIPA fixed assets tables. For the entire period 1967–2013 period, I divide the consumption of capital series from Table 2.4 by the capital stock series from Table 2.1. The average depreciation rates found are $\delta_s = 0.0275$ for structural capital and $\delta_e = 0.1460$ for equipment capital. Furthermore, discount rate Γ is taken as 0.96 to match approximately 4 percent annual interest rate in equilibrium. Measure of the unskilled agents is normalized as 1. Then I calculate the average share of hours worked by college and above educated labor in total hours from the CPS data set for the 1967–2013 period, which gives $S = 0.361$. Finally, γ_z, the common growth factor, is set equal to the growth rate of the total factor productivity obtained as a residual from the estimation in Section 4.

Targeting initial values: There remain four parameters to calibrate: μ is the share of routine task occupations in CES between routine task occupations and the composite of equipment capital and cognitive task occupations as in production function in equation 5.9; b is the scale parameter between the routine task efficiencies of the skilled and the unskilled agents; ζ is the cognitive task efficiency of the skilled household; and $p_{eq,0}$ is the initial relative price of equipment capital. To set these parameters, I set year 1966 as the initial steady state and target levels of some of the key variables in that year. To be more specific, I target the wage-bill ratio between cognitive and routine tasks in 1966 to recover μ. To obtain two efficiency parameters, I target the task premiums. Finally, the ratio of real equipment capital stock to real structural capital stock gives us $p_{eq,0}$. These targets are shown in the left two columns of Table 6.1. The right two columns, on the other hand, show the data and model levels of other selected variables for the initial year.

These targets give us $\mu = 0.4160$, $\zeta = 1.8681$, $b = 0.6493$, and $p_{eq,0} = 0.8302$.

6.2 Perfect Foresight Solution

It is well known in the literature that models with capital-augmenting technological growth do not have balanced growth path properties. Therefore, I rule out the growth in equipment-specific technology and define a random-walk process for it, thereby making every technology shock persistent. Hence, following each shock to the equipment-specific technology, the economy moves towards a potentially new steady state over time.

Table 6.1: Targets and Other Selected Moments for 1966

	Data	Model		Data	Model
Targeted			Not Targeted		
wbr_{cr}	0.58	0.58	LS_{agg}	0.70	0.73
tp_{cr}	1.46	1.46	LS_{cog}	0.24	0.26
tp_{mr}	0.70	0.70	LS_{rtn}	0.41	0.45
$\frac{K_{eq}}{K_{str}}$	0.78	0.78	LS_{man}	0.05	0.03
			wbr_{mr}	0.11	0.10

I solve the model under perfect foresight assumption. In other words, I assume that at the beginning of the first period, agents are fully aware of future paths of q_t and shocks to the total factor productivity. This solution method requires imposing initial and terminal conditions and solving the path from one to the other. To do so, I assume that the relative equipment capital price was constant up to year 1966 at its price in this year. Then, the year 1966 is imposed as the initial steady state. Later, agents are assumed to learn the future path of both technology and total factor productivity shocks beginning from 1967. In the baseline scenario, I assume that equipment capital-specific technology stays constant at its 2013 level and the economy reaches its new steady state at this level of technology T periods after this. Then, I solve $23 \times (T + 47)$ equations simultaneously.[60]

6.3 Findings

The paths of the key variables under the perfect foresight solution are presented in Figure 6.1. The three figures in the top row show the labor share trends for three task groups. Both panels (a) and (b) show that the task-based explanation for the trend decline in the labor share is quite successful for labor employed in cognitive and routine task occupations for the period after 1990. However, in the first half of the period of study, the model can account for only around 40 percent of the trend changes in these labor shares. Furthermore, even though the model is unable to capture the large cycles observed in the data, it successfully captures the total rise in the income share of labor

[60] I tried various Ts ranging from 50 to 500 periods to check if the choice of the terminal year affects the results. Fortunately, there was no noticeable differences in results for different choices of Ts in this range.

associated with manual tasks as depicted in panel (c) of the Figure 6.1.

Consistent with the task-level labor shares, the model's success in predicting the aggregate labor share is different in the first and second halves of our period of study. As panel (d) of Figure 6.1 shows, 87 percent of the labor share trend after 1990 can be explained by the advances in equipment-specific technology. Up to year 1990, on the other hand, we can attribute only 45 percent of the decline to this channel. Overall, for the entire period 1967–2013 period, we can account for 72.2 percent of the trend decline in the labor share by equipment-specific technological progress and capital-task complementarity.

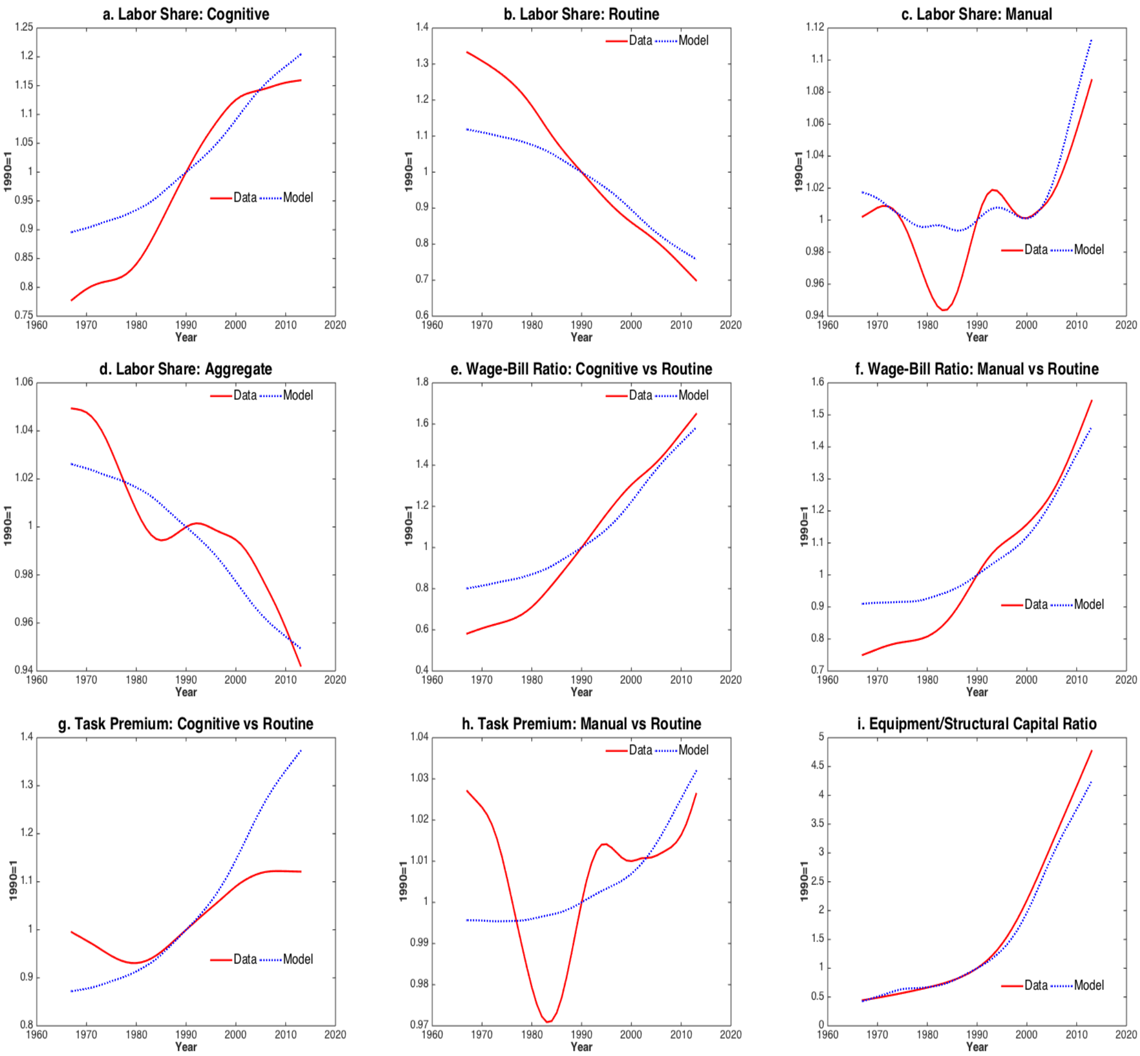
The model's performance in predicting the paths of the wage-bill ratios between various tasks groups is similar to the labor shares (panels (e) and (f) of Figure 6.1). This is not a surprising outcome, as the advances in technology affect the labor share through changes in the wage-bill ratio in my model. However, the model foresees a substantially larger growth in task premiums, especially between cognitive and routine tasks, than what we observe in the data. This overshooting of the task premiums results from the assumption of exogenous (and fixed measures) of skill groups. My model does not allow for skill accumulation, and hence, the measures of the skilled and the unskilled agents are fixed at some pre-given levels. Normally, when there is technological progress and capital-task complementarity, and thus, the demand for cognitive tasks rises, we would expect to see both a rise in relative wage and also, over time, rise in total supply of skilled agents, who are the sole suppliers of the cognitive hours. However, because the skill supply channel is shut off in our model, the rise in the wage-bill ratio mostly stems from the rise in the task premium, rather than a combination of relative hours and wages.[61]

6.3.1 Counterfactual: the effect of the technological boom on labor share trend

The model's inability to fully account for the decline in the labor share for the pre-1990 period is consistent with the data. As discussed earlier, the between-sector component was the dominant factor in this period, and my model does not allow structural shifts. On the other hand, most of the decline in the labor share is due to the within-sectoral changes for the post-1990 period. As a matter of fact, during the eight years between 1996 and 2003, the growth rate of technological progress has doubled (Table 6.2). This

[61] Incorporating a simple educational choice, like in Orak (2013), into the baseline model would potentially eliminate the overshooting of the task premium.

Figure 6.1: Prefect Foresight Solution: Data versus Model



The series are HP-trends.

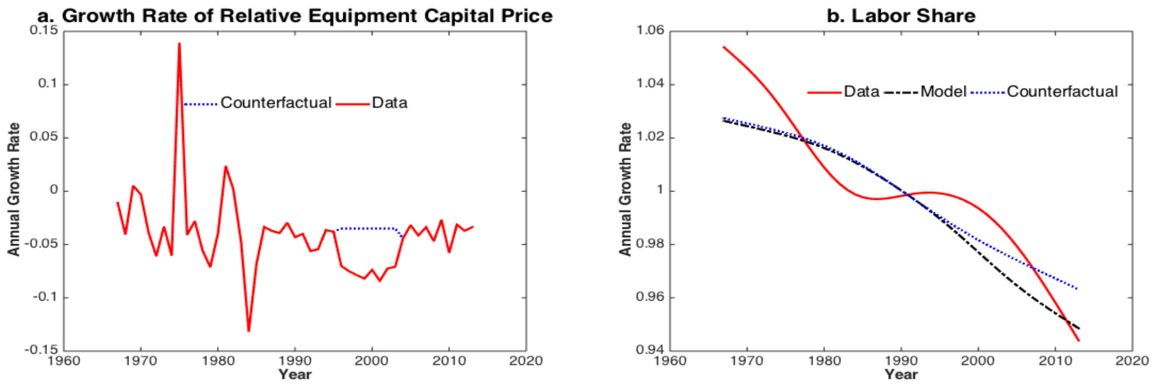
technological boom has been followed by an acceleration of the decline in the labor share. As this is consistent with the proposed channel in my model, its performance in explaining the labor share trend is substantially stronger for this period.

The model enables us to quantify the contribution of this technological boom alone on the labor share trend. To do so, as depicted in panel (a) of Figure 6.2, for the 1996–2003 period, I fixed the growth rate of the relative equipment capital price at its average level. Panel (b) of Figure 6.2 shows that the labor share would have been around 2 points higher today if we had not had the technological boom of 1996–2003. These 2 points correspond to about 22 percent of the decline in the labor share.

Table 6.2: Annual Growth Rate of Relative Equipment Capital Prices

Period	Growth Rate
1967–1995	$-.0342$
1996–2003	$-.0759$
2004–2013	$-.0378$
1967–2013	$-.0424$

Figure 6.2: Counterfactual: the Effect of the Technological Boom on the Labor Share



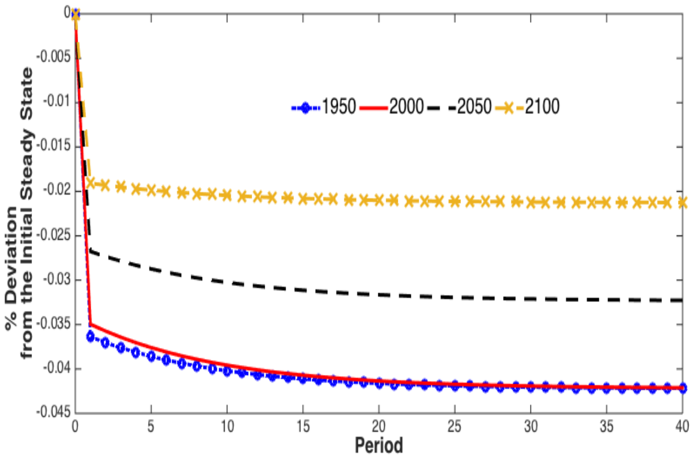
The labor share series in panel (b) are HP-trends.

6.3.2 Projection of the labor share

My model has a unique prediction for the future path of the labor share. The studies relating the decline in the labor share to the substitutability between aggregate capital and labor, such as Karabarbounis and Neiman (2014b), imply that the labor share should converge to 0 over time if technological progress continues at its ongoing pace. However, this paper documents that there is a specific group of labor that is losing due to technological progress, and a constant share of their losses is captured by the other group of labor. Thus, the explanation proposed in this paper foresees a stabilization of the labor share in the long run as the economy completes its transition to a new steady state with a very low share of the labor employed in occupations associated with routine tasks.

To visualize this convergence to a new economy with lower but stable labor share, I analyze how the labor share would respond to a 1 percent positive shock to equipment-specific technology at various stages of technological progress. To do so, I impose a level of technology for each year after 2013 based on the assumption that the relative equipment capital price would continue to decline at its current average annual rate of 4.24 percent for another 200 years. Then, I independently give a 1 percent technological shock at each level and plot the impulse responses of the labor share to these shocks at various years.[62] Each line in Figure 6.3 corresponds to the response of the labor share to a 1 percent technological shock at the level of equipment-specific technology in the specified year. As the figure depicts, the effect of technological progress on the labor share gradually fades ,even if technological progress maintains its strength.

Figure 6.3: Impulse Response of the Labor Share to a 1 percent Technological Shock



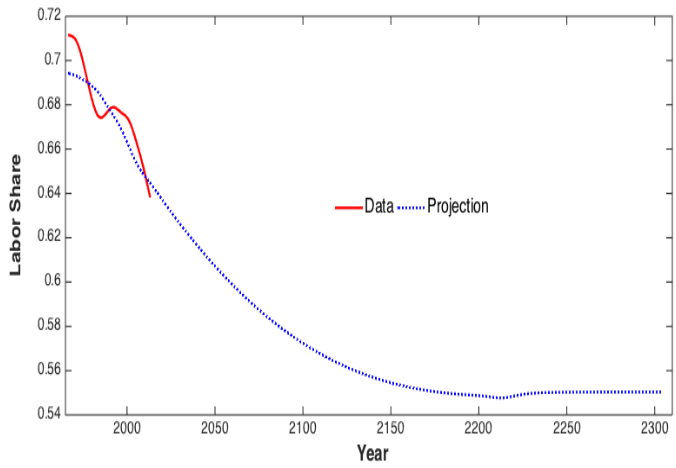
To project a lower bound for the U.S. labor share, I use the imposed future path for the relative equipment capital price and solve the model under the perfect foresight assumption. Even under this extreme case, the model predicts that the labor share will stabilize around 55 percent in the long run, as Figure 6.4 shows. However, this number should be interpreted with caution, as the nature of technological progress might evolve over time, and what we define as routine might drastically change.

6.4 Sectoral Decomposition

As discussed earlier, there has been substantial heterogeneity in the labor share trends across sectors (Figure 3.3). Most of the goods sectors, including the manufacturing industries, have experienced large trend decline in the labor share, while service sectors

[62] To solve for the stochastic version of the model, I defined a random-walk process for the equipment-specific technology. Thus, each shock to the technology is permanent and the economy slowly moves to a new steady state after the shock. It should be clarified at this point that each shock is given separately for each year under the assumption that the economy is at the steady state before the shock.

Figure 6.4: Projection of the Labor Share



The series are HP-trends.

have shown a limited decline, if not a rise. How consistent is the sectoral experience with our technological process and capital-task complementarity story? In the end, all the sectors have faced similar equipment-specific technological progress. Likewise, task premiums between various tasks have been rising at similar magnitudes in most of the sectors. If sectors were to operate under same technology, we would expect the labor shares to exhibit similar trends across sectors as well.

Two papers try to provide an explanation for this heterogeneity in labor share trends across sectors. Orak (2015) shows that if the elasticities of substitution, and hence the degrees of capital-task complementarity, are allowed to differ across sectors, we can account for trend differences in the labor shares across sectors. Similarly, in their recent paper, Alvarez-Cuadrado et al. (2015) claim that differences in the degree of aggregate capital-labor substitutability across sectors can explain the decline in the sectoral and aggregate labor shares. However, a three-sector extension of our model shows that we can mimic the sectoral differences in the labor share trends even when the elasticities between equipment capital and various types of labor are the same across sectors. In other words, our key channel working through the rise in the wage-bill ratio is consistent with the sectoral experiences as well, and this consistency does not rely on heterogeneity in the capital-task complementarities. The only factor that causes sectors to react differently to technological change is the share of equipment capital in the partial Cobb-Douglas specification. When the extended model is calibrated to the U.S. data, I show that in sectors where the share of equipment capital is larger (such as manufacturing), advances in technology cause a larger rise in equipment capital per efficiency hour and in the wage-bill ratio between labor devoted to cognitive and routine tasks, thereby leading to a larger fall in the labor share.

6.4.1 Extended Model

The problem of the households is the same as in the baseline model and thus is not presented again. Furthermore, the manual service sector is kept as before as well. The key difference between baseline model and the extended model is the introduction of two intermediate outputs that are aggregated into our final good used in consumption and investment. The production functions for these intermediate sectors are similar except for three parameters that will be calibrated.

Intermediate sectors: Each intermediate sector $i \in \{1,2\}$ has the following production function:

$$Y_{g,i,t} = A_t K_{str,i,t}^{\alpha_i} \left(\mu_i L_{r,i,t}^{\sigma} + (1-\mu_i) L_{c,i,t}^{(1-\gamma_i)\sigma} K_{eq,i,t}^{\gamma_i \sigma} \right)^{\frac{1-\alpha_i}{\sigma}}. \tag{6.1}$$

Equation 6.1 implies that two intermediate sectors face the same total factor productivity shocks (A_t). Furthermore, the elasticity of substitution between routine task occupations and the composite output of equipment capital and non-routine task occupations ($\frac{1}{1-\sigma}$) is the same in both sectors as well. Moreover, I assume that capital and labor are not sector-specific. Thus, wages and rental rates are equalized across sectors.

Nevertheless, sectors differ in three parameters: μ_i, which governs the compensation shares of factors; α_i, which determines the compensation share of structural capital; and γ_i, which is the share of equipment capital in the partial Cobb-Douglas specification. However, as summarized in equations 4.17 and 4.18, the first two of these parameters do not play any role in the changes of the labor share. On the other hand, γ_i is the key parameter in shaping the reaction of the wage-bill ratio and the labor share, as it shows how sensitive the cost function is to changes in the price of equipment capital. In addition, it also determines how a given change in the wage-bill ratio is translated into changes in the labor share.

Letting $p_{i,t}$ be the price of intermediate output i, the representative firm in sector i takes the wage rates and rental rates as given and solves the following profit maximization problem:

$$\max_{K_{str,i,t}, K_{eq,i,t}, L_{c,i,t}, L_{r,t}} p_{i,t} Y_{g,i,t} - w_{c,t} L_{c,i,t} - w_{r,t} L_{r,i,t} - r_{str,t} K_{str,i,t} - r_{eq,t} K_{eq,i,t}.$$

The first order conditions of the firm's problem are:

$$\frac{w_{c,t}}{p_{i,t}} = (1-\alpha_i)(1-\mu_i)(1-\gamma_i)\frac{Y_{g,i,t}}{\mu_i L_{r,i,t}^{\sigma}+(1-\mu_i)L_{c,i,t}^{(1-\gamma_i)\sigma}K_{eq,i,t}^{\gamma_i\sigma}}L_{c,i,t}^{(1-\gamma_i)\sigma-1} \tag{6.2}$$

$$\frac{w_{r,t}}{p_{i,t}} = (1-\alpha_i)\mu_2\frac{Y_{g,i,t}}{\mu_i L_{r,i,t}^{\sigma}+(1-\mu_i)L_{c,i,t}^{(1-\gamma_i)\sigma}K_{eq,i,t}^{\gamma_i\sigma}}L_{r,i,t}^{\sigma-1} \tag{6.3}$$

$$\frac{r_{eq,t}}{p_{i,t}} = (1-\alpha_i)(1-\mu_i)\gamma_i\frac{Y_{g,i,t}}{\mu_i L_{r,i,t}^{\sigma}+(1-\mu_i)L_{c,i,t}^{(1-\gamma_i)\sigma}K_{eq,i,t}^{\gamma_i\sigma}}K_{eq,i,t}^{\gamma_i\sigma-1} \tag{6.4}$$

$$\frac{r_{str,t}}{p_{i,t}} = \alpha_i\frac{Y_{g,i,t}}{K_{str,i,t}}. \tag{6.5}$$

Final good: Two intermediate outputs are aggregated into the final good that is used for consumption and investment by perfectly competitive intermediaries using the following CES aggregator:

$$Y_{g,t} = \left(\xi Y_{g,1,t}^{\nu}+(1-\xi)Y_{g,2,t}^{\nu}\right)^{\frac{1}{\nu}}. \tag{6.6}$$

The final good aggregating firms take the prices of the intermediates as given. As before, the final good is the numeraire in the model, and hence its price is set as 1. Thus, the problem of the representative final good aggregating firm is as follows:

$$\max_{Y_{g,1,t},Y_{g,2,t}} Y_{g,t}-p_{1,t}Y_{g,1,t}-p_{2,t}Y_{g,2,t}.$$

The firm's problem gives the following relationship between prices and quantities of two intermediates:

$$\frac{p_{1,t}}{p_{2,t}} = \frac{\xi}{1-\xi}\left(\frac{Y_{g,1,t}}{Y_{g,2,t}}\right)^{\nu-1}. \tag{6.7}$$

Equilibrium: Our equilibrium definition is the same as before. Here, I present only the market clearing conditions that are different from the ones in our baseline model. First of all, we assumed that capital is perfectly mobile across sectors. Hence, the asset market clearing condition now becomes:

$$a_{s,t}+a_{u,t} = K_{str,1,t}+K_{str,2,t}+p_t\left(K_{eq,1,t}+K_{eq,2,t}\right). \tag{6.8}$$

Two of the labor market clearing conditions now also have to reflect the mobility of labor across two intermediate sectors:

$$L_{c,1,t}+L_{c,2,t} = (1-\bar{\tau}_t)\zeta S \tag{6.9}$$

$$L_{r,1,t}+L_{r,2,t} = 2\bar{\tau}_t^{\frac{1}{2}}S+2b\bar{\psi}_t^{\frac{1}{2}}U. \tag{6.10}$$

6.4.2 Parameterization of the three-sectors model

I keep the parameters of the baseline model the same for the extended version. Those parameter values can be found in the first block of Table B.2. Keeping the baseline parameters intact leaves eight parameters specific to the three-sectors version to calibrate. Six of those are sector specific parameters governing the compensation shares of factors: $\alpha_1, \alpha_2, \mu_1, \mu_2, \gamma_1$ and γ_2. The other two parameters come from the CES aggregator in equation 6.6: $\frac{1}{1-\nu}$ is the elasticity of substitution between two intermediates, and ξ shapes the shares of the intermediate sectors in aggregation. The calibration of each parameter is described below in detail, and the second block of Table B.2 reports the calibrated values.

First of all, two sectors chosen for parameterization are goods sectors including manufacturing and service sectors, excluding personal services. The goods sector (sector 1) experienced, on average, 0.6 percent annual decline in the labor share, while in the service sector (sector 2), the labor share has risen 0.12 percent each year for the years between 1977 and 2007.[63][64] To obtain α_1 and α_2, I use the capital and output files of EU-KLEMS for the years between 1977 and 2007 and divide the compensation of the non-residential structural capital by the value-added. Then I take the average as the compensation share of the structural capital in that sector., which gives $\alpha_1 = 0.1355$ and $\alpha_2 = 0.0921$. Similarly, to derive γ_1 and γ_2, I take the equipment capital compensation shares from EU-KLEMS capital file and calculate the weight of the income of the labor associated with cognitive tasks in aggregate labor share from the CPS data. Multiplying this weight by the labor share in the EU-KLEMS file gives me the cognitive labor share. Then, γ_is are found by dividing the equipment capital compensation share by the total of the compensation shares of equipment capital and labor employed in cognitive task occupations. The obtained parameters are $\gamma_1 = 0.5043$ for the goods sector and $\gamma_2 = 0.2653$ for the service sectors. In other words, equipment capital has a substantially larger share in production relative to cognitive tasks in the goods sector.

Similar to the parameters of the consumption aggregator, I use a simple regression for parameterizing the final good aggregator in equation 6.6 as well. To begin, I take the

[63]If we exclude public services such as education and health, then service sector exhibits a small trend decline as well.

[64]The choice of period for parameterizing the sectors is constrained by the availability of detailed capital compensation data by sectors in EU-KLEMS files.

Table 6.3: Targets and Other Selected Moments for 1966

Targeted			Not Targeted		
	Data	**Model**		**Data**	**Model**
$wbr_{cr,sec1}$	0.29	0.29	LS_{agg}	0.70	0.75
$wbr_{cr,sec1}$	1.02	1.02	LS_{sec1}		0.70
			LS_{sec2}		0.77
			tp_{cr}	1.46	1.62
			tp_{mr}	0.70	0.70
			wbr_{mr}	0.11	0.06
			$\frac{K_{eq}}{K_{str}}$	0.78	0.66

logarithm of both sides of equation 6.7, which yields:

$$\ln \frac{p_{1,t}}{p_{2,t}} = \ln \frac{\xi}{1-\xi} + (\nu - 1) \ln \left(\frac{Y_{g,1,t}}{Y_{g,2,t}} \right).$$

Using NIPA tables 1.5.5 for quantity and 1.6.4 for prices for the 1967–2013 period, the estimated regression is:

$$\ln \frac{p_{1,t}}{p_{2,t}} = -.5204601 - 1.161871 \ln \left(\frac{Y_{g,1,t}}{Y_{g,2,t}} \right).$$

This regression gives us: $\xi = 0.3728$ and $\nu = -0.1619$. This level of ν implies complementarity between goods and services.

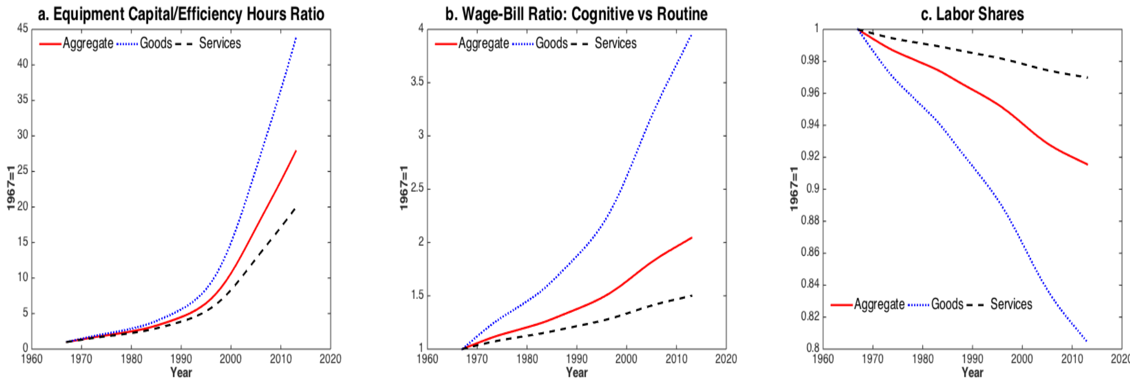
Finally, there are two weight parameters to be calibrated: μ_1 and μ_2, which are the share of routine-task-associated occupations in the outer CES production function. These parameters only affect the level of the wage-bill ratio and have no role in changes of the labor share. As in the baseline model, I assume that the year 1966 was the initial steady state and choose μ_1 and μ_2 to match the wage-bill ratios of each sector in 1966. The targeted and non-targeted starting points are reported in Table 6.3. The calibrated parameters are $\mu_1 = 0.4039$ and $\mu_2 = 0.3276$.

6.4.3 Solution of the extended model

The extended model is solved under perfect foresight in the same way the baseline model is solved. As before, agents know the future paths of the relative equipment capital price and the total factor productivity. The year 1966 is taken as the initial steady state, and capital prices are assumed to stay at their 2013 level after this year. Later, the economy is assumed to reach to a terminal equilibrium point in T years. Using the value of assets at the initial year and consumptions $T+47$ years later, I simultaneously solve the paths of every variable between two steady states.

Two sectors face the same price and total factor productivity shocks. Furthermore, since both labor and capital are perfectly mobile across sectors, the relative task and capital prices are the same for each sector as well. Sectors only differ in their initial levels of the wage-bill ratios and the parameters governing the compensation shares of factors. As discussed before, the parameter γ_i, which is the share of equipment capital in the partial Cobb-Douglas between equipment capital and cognitive tasks, is the key parameter affecting the changes in the wage-bill ratio and the labor share.

Figure 6.5: Extended Model's Solution under Perfect Foresight



Series are HP-trends.

Panel (a) of Figure 6.5 shows equipment capital stock per efficient hour at the aggregate and sectoral levels. The large increases in this variable are comparable to the rise observed in the data.[65] Since equipment capital per efficient hour rises faster in the goods sector and there is capital-task complementarity, the wage-bill ratio between cognitive and routine tasks also increases significantly more in this sector, as shown in panel (b). This rise in the wage-bill ratio is translated into around 20 percent decline in the labor share of this sector, as depicted in panel (c) of Figure 6.5. On the other hand, both the rise in the wage-bill ratio and the fall in the labor share remains limited in the service sector.

Even though equipment capital per efficient hour and the labor share series generated by the model are in line with the data, the model significantly overshoots the rise in the

[65]For instance, equipment capital per efficient hour at the aggregate level has risen 26 times in the data and 28 times in the model.

wage-bill ratio of the goods sector. To be more specific, the rise in the wage-bill ratio in goods sector is 300 percent in the model, whereas it is only around 150 percent in the data. However, model's overshooting of the wage-bill ratio does not invalidate our explanation for the decline in the labor share through the changes in the task composition of the wage bills. Despite quantitatively overshooting the wage-bill ratios, model qualitatively captures two facts derived from data: (i) the larger the share of equipment capital in the Cobb-Douglas specification, the larger the rise in the wage-bill ratio and, (ii) the larger the rise in the wage-bill ratio, the larger the fall in the labor share. These facts were presented earlier in Figures 4.5 and 3.4, respectively.

To summarize, model is quite successful in capturing the cross-sectional differences in the labor share in response to advances in technology. At this point, one can wonder whether this is the result of differences in initial employment of labor devoted to routine task occupations or weights of equipment capital in the Cobb-Douglas specification. To answer this question, I set the initial wage-bill ratios close to each other by setting $\mu_1 = \mu_2$. The resulting transition paths of the wage-bill ratios and labor shares were similar to those in Figure 6.5. A reverse exercise setting $\gamma_1 = \gamma_2$, on the other hand, eliminates sectoral differences generated by the model. Thus, sectoral differences in terms of the labor share trends are solely driven by the sensitivities of cost function to equipment capital prices.

7 Conclusion

This paper assesses the relationship between technological progress and the decline of the aggregate and sectoral labor shares in the U.S. over the last few decades through the lenses of capital-task complementarity and job polarization. With this goal in mind, an aggregate production function for the U.S. economy—which treats three types of labor, classified by the particular tasks associated with their occupations, as different inputs—is estimated. The estimation analysis reveals that it is not labor per se that is being substituted away. Rather, the substitution is solely away from routine tasks to both equipment capital and non-routine tasks, and the income losses of this group account for the entire decline of the labor share during the 1967–2013 period. When the estimated production function is nested into a general equilibrium model, in which the where rest of the model parameters are calibrated to the U.S. data, it shows that the decline in equipment capital prices alone can reproduce around three-fourths of the decline in the labor share between 1967 and 2013. The model also successfully captures heterogeneous labor share trends observed across sectors.

The key insight of this paper is that changes in the occupational composition of the labor force have played a key role in shaping the response of the labor share to technological progress. This implies that the decline of the labor share can be considered a natural outcome of a transition process of the U.S. economy to a new steady state with a very low level employment share of routine task labor. Both the estimation analysis and the dynamic general equilibrium model predict that the decline of the labor share should come to a halt in the long run and reach a lower bound, even if equipment-specific technological progress continues at its current pace. Furthermore, it also implies that policies to train routine task labor to acquire skills compatible with non-routine cognitive tasks would be more effective than tax policies in alleviating the social and economic damages of inequalities.

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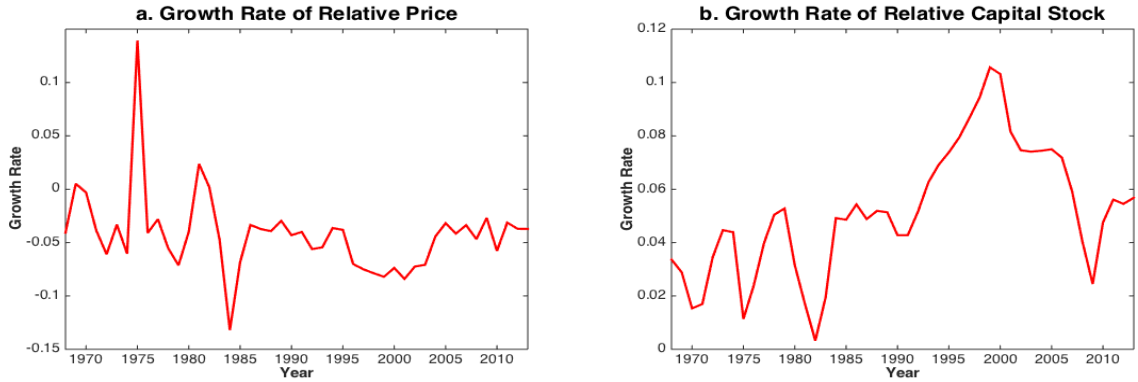
APPENDIX

A Additional Figures

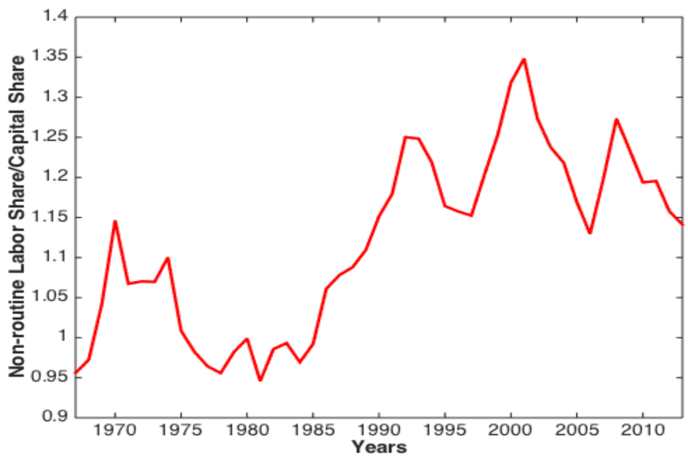
Figure A.1: Growth of Relative Price and Stock of Equipment Capital



Both the price and stock data are relative to structural capital.

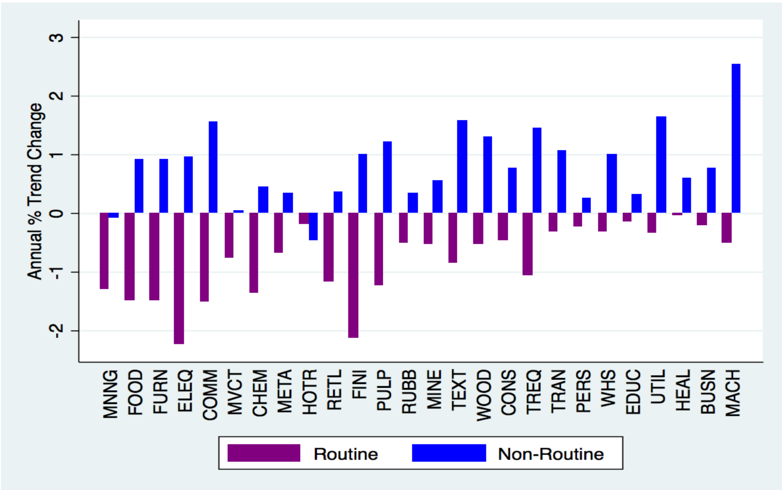
Source: Author's calculations from the data set of DiCecio (2009) and NIPA tables.

Figure A.2: Income Share of Non-Routine Labor relative to Capital Share



Source: Author's calculations from NIPA and CPS.

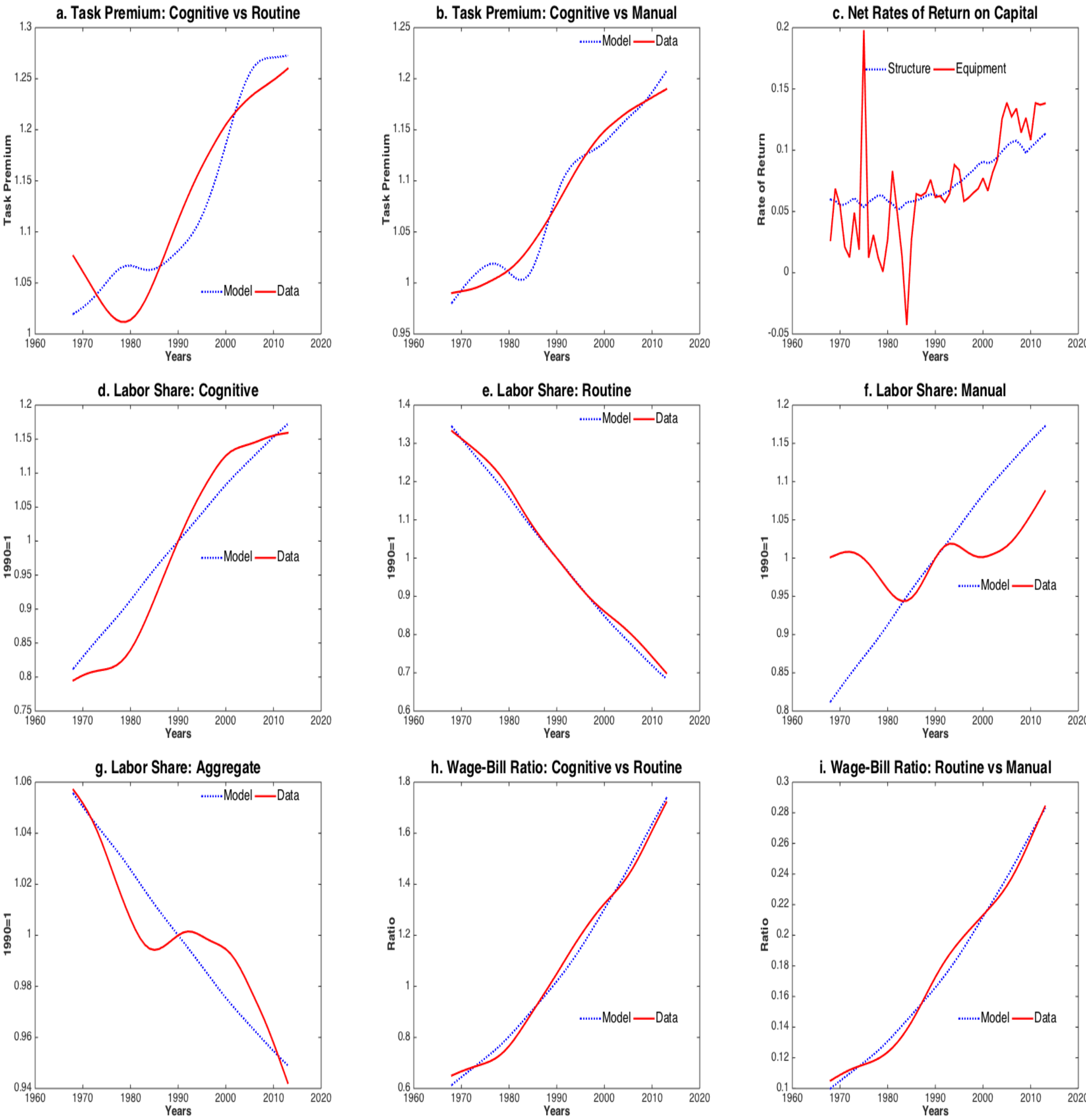
Figure A.3: Labor Share Trend Coefficients, by Sectors and Tasks



The y-axis shows the coefficient of the regression of log of the labor share on a constant and a time variable. In this sense, this coefficient shows the annual growth rate of the labor share. Due to the small number of data for manual labor for many sectors, cognitive and manual labor are aggregated as non-routine labor.

Source: Author's calculations from labor and output files of the WORLD-KLEMS and CPS data sets.

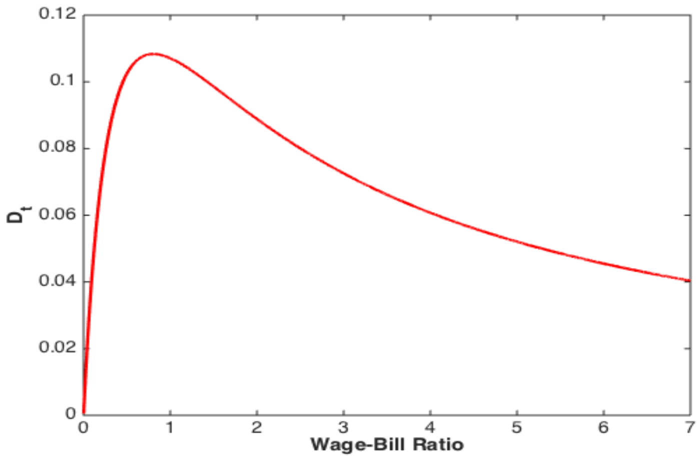
Figure A.4: Model Fit and Data for the Cobb-Douglas Specification



The series are HP-trends.

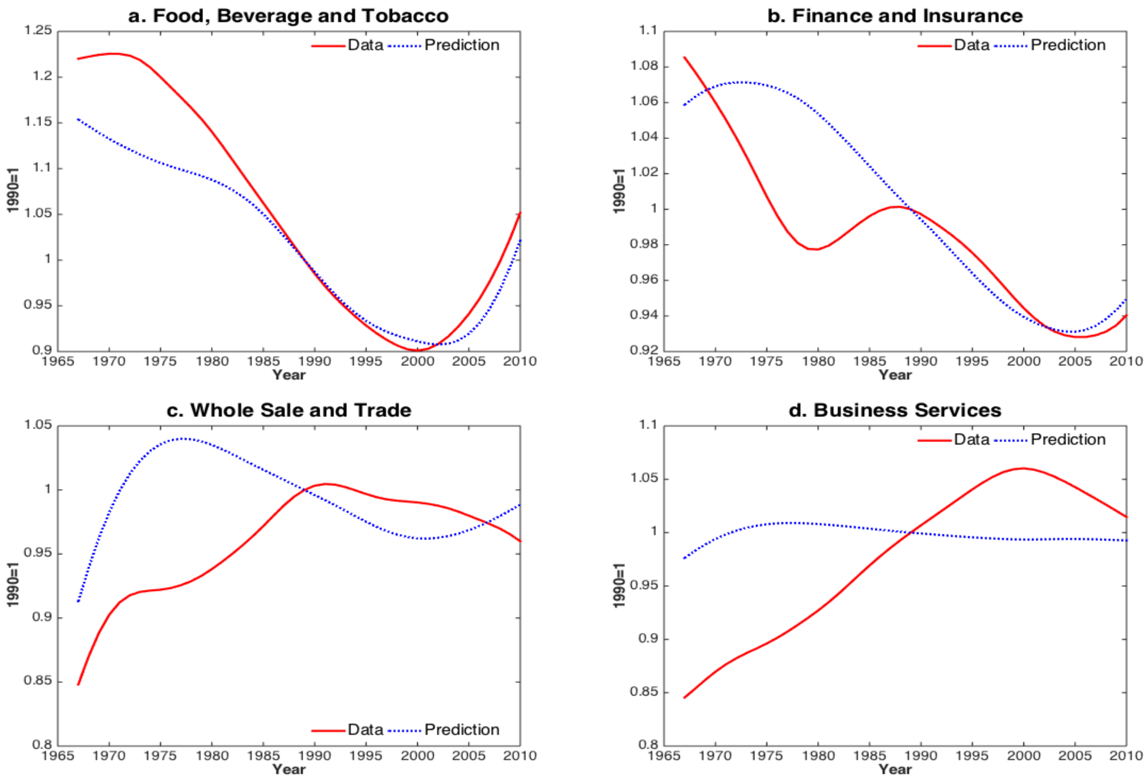
The model plots are derived from the estimation of the model with the Cobb-Douglas specification as in equation 4.14

Figure A.5: Effect of the Changes in Wage-Bill Ratio on Labor Share Growth



The figure shows the coefficient of the changes in wage-bill ratio in equation 4.17.

Figure A.6: Labor Share Generated by the Wage-Bill Ratio Series for Selected Sectors



The series are HP-trends.

The figure shows the model fit when equation 4.17 is fed with the wage-bill ratios and γs calibrated for each sector.

B Additional Tables

Table B.1: Abbreviations of the Sectors

Abbreviation	**Sector**
AGRI	Agriculture, Hunting and Fishery
BUSN	Real Estate and Business
CHEM	Chemicals and Chemical Products
COMM	Post and Telecommunication
CONS	Construction
EDUC	Educational Services
ELEQ	Electrical and Optical Equipment
FINI	Financial Intermediation
FOOD	Food, Tobacco and Beverage
FURN	Furniture and Recycling
HEAL	Health Services
HOTR	Hotels and Restaurants
MACH	Machinery Equipment
META	Basic and Fabricated Metal
MINE	Non-Metallic Products
MING	Mining and Quarrying
MVCT	Motor Vehicles Trade
PERS	Other Personal Services
PETR	Coke and Refined Petroleum
PULP	Pulp, Paper and Printing
RETT	Retail Trade and Repair
RUBB	Rubber and Plastic Products
TEXT	Textile, Leather and Footwear
TRAN	Transportation and Storage
TREQ	Transport Equipment
UTIL	Utilities
WHST	Wholesale Trade and Commission
WOOD	Wood and Cork Products

Table B.2: List of Model Parameters

Parameter	Description	Value	Source
α	Compensation share of structural capital	0.1058	Bayesian Inference
$\frac{1}{1-\sigma}$	Elasticity of subst. b/w routine task and equipment capital	2.40	Bayesian Inference
γ	Share of equipment capital in partial Cobb-Douglas	0.3879	Bayesian Inference
$\frac{1}{1-\kappa}$	Elasticity of subst. b/w final good and manual service consumption	0.87	Regression
χ	Share of final good in consumption composite	0.9410	Regression
δ_s	Depreciation rate of structural capital	0.0275	BEA Tables
δ_e	Depreciation rate of equipment capital	0.1460	BEA Tables
Γ	Discount factor	0.96	Literature
S	Measure of skilled households	0.361	CPS Data
μ	Share of routine task in outer CES	0.4160	Calibration
b	Relative scale of routine task efficiency of unskilled agents	0.6493	Calibration
ζ	Cognitive task efficiency of skilled agents	1.8681	Calibration
p_0	Initial relative equipment capital price	0.8302	Calibration
	Parameters Specific to Extended Model		
α_1	Compensation share of structural capital in goods sector	0.1355	EU-KLEMS Data
α_2	Compensation share of structural capital in service sector	0.0921	EU-KLEMS Data
γ_1	Share of equipment capital in partial Cobb-Douglas in goods sector	0.5043	EU-KLEMS Data
γ_2	Share of equipment capital in partial Cobb-Douglas in service sector	0.2653	EU-KLEMS Data
μ_1	Share of routine task in outer CES in good sector	0.4039	Calibration
μ_2	Share of routine task in outer CES in service sector	0.3276	Calibration
$\frac{1}{1-\nu}$	Elasticity of subst. b/w intermediate sectors	0.87	Regression
ξ	Share of good sector in final good aggregator	0.3727	Regression

C Derivations and Equations

C.1 Representative Firm's First Order Conditions

The first order conditions below are with respect to cognitive, routine, and manual tasks and structural and equipment capitals in this specific order.

C.1.1 General functional form

$$w_c = (1-\alpha)(1-\mu)\theta(1-\lambda)\frac{Y}{\left(\mu L_r^{\sigma} + (1-\mu)\left(\theta\left[\lambda K_{eq}^{\rho} + (1-\lambda)L_c^{\rho}\right]^{\frac{\beta}{\rho}} + (1-\theta)L_m^{\beta}\right)^{\frac{\sigma}{\beta}}\right)} \times \left(\theta\left[\lambda K_{eq}^{\rho} + (1-\lambda)L_c^{\rho}\right]^{\frac{\beta}{\rho}} + (1-\theta)L_m^{\beta}\right)^{\frac{\sigma}{\beta}-1}\left[\lambda K_{eq}^{\rho} + (1-\lambda)L_c^{\rho}\right]^{\frac{\beta}{\rho}-1}\frac{L_c^{\rho}}{h_c} \quad (C.1)$$

$$w_r = (1-\alpha)\mu\frac{Y}{\left(\mu L_r^{\sigma} + (1-\mu)\left(\theta\left[\lambda K_{eq}^{\rho} + (1-\lambda)L_c^{\rho}\right]^{\frac{\beta}{\rho}} + (1-\theta)L_m^{\beta}\right)^{\frac{\sigma}{\beta}}\right)}\frac{L_r^{\sigma}}{h_r} \quad (C.2)$$

$$w_m = (1-\alpha)(1-\mu)(1-\theta)\frac{Y}{\left(\mu L_r^{\sigma} + (1-\mu)\left(\theta\left[\lambda K_{eq}^{\rho} + (1-\lambda)L_c^{\rho}\right]^{\frac{\beta}{\rho}} + (1-\theta)L_m^{\beta}\right)^{\frac{\sigma}{\beta}}\right)} \times \left(\theta\left[\lambda K_{eq}^{\rho} + (1-\lambda)L_c^{\rho}\right]^{\frac{\beta}{\rho}} + (1-\theta)L_m^{\beta}\right)^{\frac{\sigma}{\beta}-1}\frac{L_m^{\beta}}{h_m} \quad (C.3)$$

$$r_{str} = \alpha\frac{Y}{K_{str}} \quad (C.4)$$

$$r_{eq} = (1-\alpha)(1-\mu)\theta\lambda\frac{Y}{\left(\mu L_r^{\sigma} + (1-\mu)\left(\theta\left[\lambda K_{eq}^{\rho} + (1-\lambda)L_c^{\rho}\right]^{\frac{\beta}{\rho}} + (1-\theta)L_m^{\beta}\right)^{\frac{\sigma}{\beta}}\right)} \times \left(\theta\left[\lambda K_{eq}^{\rho} + (1-\lambda)L_c^{\rho}\right]^{\frac{\beta}{\rho}} + (1-\theta)L_m^{\beta}\right)^{\frac{\sigma}{\beta}-1}\left[\lambda K_{eq}^{\rho} + (1-\lambda)L_c^{\rho}\right]^{\frac{\beta}{\rho}-1}K_{eq}^{\rho-1} \quad (C.5)$$

C.1.2 Cobb-Douglas specification

$$w_c = (1-\alpha)(1-\mu)\delta\frac{Y}{\left(\mu L_r^{\sigma} + (1-\mu)\left[L_c^{\delta}K_{eq}^{\gamma}L_m^{1-\gamma-\delta}\right]^{\sigma}\right)}K_{eq}^{\sigma\gamma}L_m^{\sigma(1-\delta-\gamma)}\frac{L_c^{\sigma\delta}}{h_c} \quad (C.6)$$

$$w_r = (1-\alpha)\mu\frac{Y}{\left(\mu L_r^{\sigma} + (1-\mu)\left[L_c^{\delta}K_{eq}^{\gamma}L_m^{1-\gamma-\delta}\right]^{\sigma}\right)}\frac{L_r^{\sigma}}{h_r} \quad (C.7)$$

$$w_m = (1-\alpha)(1-\mu)(1-\delta-\gamma)\frac{Y}{\left(\mu L_r^{\sigma} + (1-\mu)\left[L_c^{\delta}K_{eq}^{\gamma}L_m^{1-\gamma-\delta}\right]^{\sigma}\right)}K_{eq}^{\sigma\gamma}L_c^{\sigma\delta}\frac{L_m^{\sigma(1-\delta-\gamma)}}{h_m} \quad (C.8)$$

$$r_{str} = \alpha\frac{Y}{K_{str}} \quad (C.9)$$

$$r_{eq} = (1-\alpha)(1-\mu)\gamma \frac{Y}{\left(\mu L_r^\sigma + (1-\mu)\left[L_c^\delta K_{eq}^\gamma L_m^{1-\gamma-\delta}\right]^\sigma\right)} K_{eq}^{\sigma\gamma-1} L_m^{\sigma(1-\delta-\gamma)} L_c^{\sigma\delta} \quad \text{(C.10)}$$

C.2 Relationship between Labor Share and Wage-Bill Ratio

$$LS_t = \frac{w_{r,t}h_{r,t} + w_{c,t}h_{c,t} + w_{m,t}h_{m,t}}{Y_t} \quad \text{(C.11)}$$

$$LS_t = (1-\alpha)\frac{\mu L_{r,t}^\sigma + (1-\mu)\delta K_{eq,t}^{\sigma\gamma} L_{c,t}^{\sigma\delta} L_{m,t}^{\sigma(1-\delta-\gamma)} + (1-\mu)(1-\gamma-\delta) K_{eq,t}^{\sigma\gamma} L_{c,t}^{\sigma\delta} L_{m,t}^{\sigma(1-\delta-\gamma)}}{\left(\mu L_{r,t}^\sigma + (1-\mu)\left[K_{eq,t}^\gamma L_{c,t}^\rho L_{m,t}^{1-\delta-\gamma}\right]^\sigma\right)} \quad \text{(C.12)}$$

$$LS_t = (1-\alpha)\frac{\mu L_{r,t}^\sigma + (1-\mu)(1-\gamma) K_{eq,t}^{\sigma\gamma} L_{c,t}^{\sigma\delta} L_{m,t}^{\sigma(1-\delta-\gamma)}}{\left(\mu L_{r,t}^\sigma + (1-\mu)\left[K_{eq,t}^\gamma L_{c,t}^\rho L_{m,t}^{1-\delta-\gamma}\right]^\sigma\right)} \quad \text{(C.13)}$$

Multiplying both denominator and numerator of the right hand side by $\frac{1}{\mu L_{r,t}^\sigma}$

$$LS_t = (1-\alpha)\frac{1 + \frac{(1-\mu)(1-\gamma) K_{eq,t}^{\sigma\gamma} L_{c,t}^{\sigma\delta} L_{m,t}^{\sigma(1-\delta-\gamma)}}{\mu L_{r,t}^\sigma}}{1 + \frac{(1-\mu) K_{eq,t}^{\sigma\gamma} L_{c,t}^{\sigma\delta} L_{m,t}^{\sigma(1-\delta-\gamma)}}{\mu L_{r,t}^\sigma}} \quad \text{(C.14)}$$

Multiplying the second term in the denominator by $\frac{1-\gamma}{1-\gamma}$:

$$LS_t = (1-\alpha)\frac{1 + \frac{(1-\mu)(1-\gamma) K_{eq,t}^{\sigma\gamma} L_{c,t}^{\sigma\delta} L_{m,t}^{\sigma(1-\delta-\gamma)}}{\mu L_{r,t}^\sigma}}{1 + \frac{(1-\mu) K_{eq,t}^{\sigma\gamma} L_{c,t}^{\sigma\delta} L_{m,t}^{\sigma(1-\delta-\gamma)}}{\mu L_{r,t}^\sigma}\frac{1-\gamma}{1-\gamma}} \quad \text{(C.15)}$$

$$LS_t = (1-\alpha)\frac{1 + \frac{(1-\mu)(1-\gamma) K_{eq,t}^{\sigma\gamma} L_{c,t}^{\sigma\delta} L_{m,t}^{\sigma(1-\delta-\gamma)}}{\mu L_{r,t}^\sigma}}{1 + \frac{\frac{(1-\mu)(1-\gamma) K_{eq,t}^{\sigma\gamma} L_{c,t}^{\sigma\delta} L_{m,t}^{\sigma(1-\delta-\gamma)}}{\mu L_{r,t}^\sigma}}{1-\gamma}} \quad \text{(C.16)}$$

Substituting the wage-bill ratio between non-routine and routine tasks:

$$wbr_t = \frac{(1-\mu)(1-\gamma) K_{eq,t}^{\sigma\gamma} L_{c,t}^{\sigma\delta} L_{m,t}^{\sigma(1-\gamma-\delta)}}{\mu L_{r,t}^\sigma} \quad \text{(C.17)}$$

$$LS_t = (1-\alpha)\frac{1 + wbr_t}{1 + \frac{wbr_t}{1-\gamma}} \quad \text{(C.18)}$$

C.3 Labor Share in Difference Terms

Below, the variables with a hat denote growth rate of the variable from period $t-1$ to t. Equation C.18 can be written in growth terms as follows:

$$\widehat{LS}_t = (\widehat{1 + wbr_t}) - \left(\widehat{1 + \frac{wbr_t}{1-\gamma}}\right) \quad \text{(C.19)}$$

$$\widehat{LS}_t = \frac{wbr_t}{1+wbr_t}\widehat{wbr}_t - \frac{\frac{wbr_t}{1-\gamma}}{1+\frac{wbr_t}{1-\gamma}}\widehat{wbr}_t \tag{C.20}$$

$$\widehat{LS}_t = \frac{wbr_t}{1+wbr_t}\widehat{wbr}_t - \frac{wbr_t}{1-\gamma+wbr_t}\widehat{wbr}_t \tag{C.21}$$

$$\widehat{LS}_t = \frac{(1-\gamma)wbr_t + wbr_t^2}{(1-\gamma)+wbr_t+(1-\gamma)wbr_t+wbr_t^2}\widehat{wbr}_t - \frac{wbr_t + wbr_t^2}{(1-\gamma)+wbr_t+(1-\gamma)wbr_t+wbr_t^2}\widehat{wbr}_t \tag{C.22}$$

$$\widehat{LS}_t = \widehat{wbr}_t\left[\frac{wbr_t - \gamma wbr_t + wbr_t^2 - wbr_t - wbr_t^2}{(1-\gamma)+wbr_t+(1-\gamma)wbr_t+wbr_t^2}\right] \tag{C.23}$$

$$\widehat{LS}_t = -\widehat{wbr}_t\left[\frac{\gamma wbr_t}{(1-\gamma)+(2-\gamma)wbr_t+wbr_t^2}\right] \tag{C.24}$$

$$\widehat{LS}_t = -\widehat{wbr}_t\left[\frac{\gamma}{wbr_t+(2-\gamma)+\frac{(1-\gamma)}{wbr_t}}\right] \tag{C.25}$$

D Labor Input and Wage Data

D.1 Measuring Labor Input and Wage Rates

This methodology closely follow from Krusell et al. (2000) and Domeij and Ljungqvist (2006). Labor data is obtained from March Current Population Survey (CPS) integrated by IPUMS (Flood et al. (2015)). The availability of a consistent occupational definition limits the period of study to 1967 to 2013. Number of annual observations in raw data ranges from 130,476 (1996) to 218,269 (2001). However, after dropping out some observations, we are left with numbers ranging from 55,721 (1996) to 96,184 (2001). I record every person whose age is between 16 and 70. Consistent with the literature, self-employed people, unpaid family workers and agricultural and military workers are dropped from the sample. Furthermore, people who reported zero usual hours worked per week last year are dropped as well.

For each person, I record their personal characteristics: age, sex, race, occupation (*occ50ly*); employment statistics: class of worker (*classwly*), weeks worked last year (*wkswork1* and *wkswork2*), usual hours worked per week last year (*uhrswork* and *hrswork1*); income: total wage and salary income last year (*incwage*) and CPS personal supplement weights: *wtsupp*.[66] Three task groups are obtained from the occupation series based on the classifications in the literature. The details of this can be found in the following subsection. Then, each person is assigned to one of 198 groups created by age, race, sex and task. Age is divided into 11 five-year groups: 16-20, 21-25, 26-30, 31-35,

[66] IPUMS recodes the CPS series on occupation into the 1950 Census Bureau occupational classification system. This provides consistent occupational codes for the jobs respondents reported working during the previous calendar year, for 1968 forward.

36-40, 41-45, 46-50, 51-55, 56-60, 61-65, 66-70. Race is divided into three: white, black, others. Finally, occupation is divided into three task groups: cognitive, routine, and manual.

Before calculating the annual hours worked and hourly wage rates, top-coded income values are adjusted using the "revised income top-codes files" published by Census Bureau (and compiled by IPUMS) to swap top-coded values in 1976–2010 CPS files with these revised values. This procedure replaces the top-coded values with new values based on the Income Component Rank Proximity Swap method, which was introduced in 2011. Hence, throughout the whole 1975–2013 period, the top-coding methodology is consistent and comparable. This also eliminates the jumps in task premium that researchers observe due to the change in topcoding methodology in some years. Unfortunately, swap values are not available for earlier years. Therefore, following Krusell et al. (2000), I tried both leaving the top-coded values as they are and also multiplying them by 1.45 for the years before 1975. Switching from one practice to the other does not change any of the results of the paper.

For the CPS years after 1975, CPS has usual hours worked per week and weeks worked last year. Thus, calculating the annual hours for a person is straightforward: I simply multiply weeks worked last year by usual hours worked. Hence, for CPS years 1976 and afterwards, total hours are:

$$hours_{i,t-1} = wkswork1_{i,t-1} \times uhrswork_{i,t-1},$$

where, i is individual observation and t is the CPS year.

For earlier years, I need to do two adjustments. First, weeks worked are available only as intervals and I need to approximate a scalar value for each interval. Fortunately, both the intervals and actual weeks are available for years after 1975. Therefore, we can calculate the average weeks worked after year 1975 for each interval and replace the earlier years with those values. In this context, the following approximations are used:

Interval	Approximation
1-13	8.1
14-26	21.5
27-39	33.6
40-47	42.6
48-49	48.3
50-52	51.8

Second, I have to use hours worked last week variable as a proxy to usual hours worked per week last year. However, there are many agents who were not employed last week or who were employed but not at work for some reason, despite reporting a positive income for last year. Rather than dropping those observations, I replaced the hours they worked per week with the average of the hours worked by the people in their group in that particular year. I also paid attention to whether the person was employed part time or full time when doing this replacement.

Hourly wage is calculated as:

$$wage_{i,t-1} = \frac{incwage\_swapped_{i,t-1}}{hours_{i,t-1}}.$$

Later, observations with $hours_{i,t-1} < 260$ (quarter of part time work) and $wage_{i,t-1} < 0.5 \times minimum\_wage_{t-1}$ are dropped to smooth out the effect of externalities and misreporting. Following this, for each groups and year, I calculate group weights as: $\mu_{g,t} = \sum_{i \in g} \mu_{i,t}$ where, $g \in G$ is set of groups. Then average hours and wage measures for each group and year are calculated:

$$\begin{aligned} hours_{g,t-1} &= \frac{\sum_{i \in g} \mu_{i,t} \times hours_{i,t-1}}{\mu_{g,t}} \\ wage_{g,t-1} &= \frac{\sum_{i \in g} \mu_{i,t} \times wage_{i,t-1}}{\mu_{g,t}}. \end{aligned}$$

To aggregate across groups into aggregate task groups, the group wages of 1980 are used as the weights. Letting $j \in \{C, R, M\}$, where C, R and M represent the cognitive, routine and manual occupations respectively, then for each task group j, we have total hours:

$$N_{j,t-1} = \sum_{g \in G_j} hours_{g,t-1} \times \mu_{g,t} \times wage_{g,80},$$

and average hourly wage is:

$$W_{j,t-1} = \frac{\sum_{g \in G_j} hours_{g,t-1} \times \mu_{g,t} \times wage_{g,t-1}}{N_{j,t-1}}.$$

Finally task premium between cognitive and routine tasks is calculated as $\frac{W_{C,t-1}}{W_{R,t-1}}$.. The other task premiums are found in the same way.

D.2 Occupational Classification

I follow Autor et al. (2003), Acemoglu and Autor (2011), Jaimovich and Siu (2012) Autor and Dorn (2013)—among many other studies in the literature—to classify the occupations into different tasks. Occupations are divided along two dimensions: cognitive versus manual and routine versus non-routine. Table D.1 provides detailed explanations and examples for each of these categories.[67] In short, cognitive and manual jobs are characterized by the differences in the extent of mental versus physical activity. On the other hand, routine and non-routine jobs are differentiated based on how easily the work can be codified. Routine tasks are the ones that can be summarized as a set of specific activities accomplished by following well-defined instructions and procedures. In contrast, when a task requires instant decision making, flexibility, taking initiatives, problem solving, creativity, and personal interaction, then it is classified as non-routine.

Table D.1: Occupational Classification

	Non-Routine	Routine
Cognitive	Managerial, professional, technical workers (namely, physicians, public relations managers, financial analysts, computer programmers, economists)	Sales and office and administrative support (namely, secretaries, bank tellers, retail salespeople, travel agents, mail clerks, data entry keyers)
Manual	Service jobs (namely, janitors, gardeners, manicurists, bartenders, home health asides)	Blue collar jobs (namely, machine operators and tenders, mechanics, dress makers, fabricators and assemblers, cement masons, and meat processing workers)

For the purpose of this study—and as common in the literature—I classify all the routine occupations in one single category: "Routine", whether they are cognitive or manual. This group represents the tasks implemented by the "middle-skill" labor. These

[67] This section and table are mostly borrowed from the work of Jaimovich and Siu (2012).

occupations are not necessarily mechanical nor computerized and they might even require personal interaction. The common characteristic of the tasks labeled as routine is that they can be replaced by technological improvement or can be offshored. For instance, as online sales became more widespread, many sale jobs have been lost. As another example, most of the call centers started employing labor that is physically in India.

Non-routine occupations are divided into two categories. I label cognitive non-routine occupations as “Abstract”, which tends to be “high-skill” labor. These are mainly managerial, professional, executive, and technical jobs. Finally, manual non-routine occupations are classified as “Manual” occupations. These occupations represent the lowest tail of the skill distribution and they are termed as “service occupations” by Autor and Dorn (2013) and Firpo et al. (2011).