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Understanding Persistent Stagnation

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Understanding Persistent Stagnation*

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Abstract

We theoretically explore long-run stagnation at the zero lower bound in a representative agent framework. We analytically compare *expectations-driven* stagnation to a *secular stagnation* episode and find contrasting policy implications for changes in government spending, supply shocks and neo-Fisherian policies. On the other hand, a minimum wage policy is expansionary and robust to the source of stagnation. Using Bayesian methods, we estimate a DSGE model that can accommodate two competing hypotheses of long-run stagnation in Japan. We document that equilibrium selection under indeterminacy matters in accounting for model fit.

Keywords: Expectations-driven trap, secular stagnation, inflation expectations, zero lower bound.

JEL Classification: E31, E32, E52.

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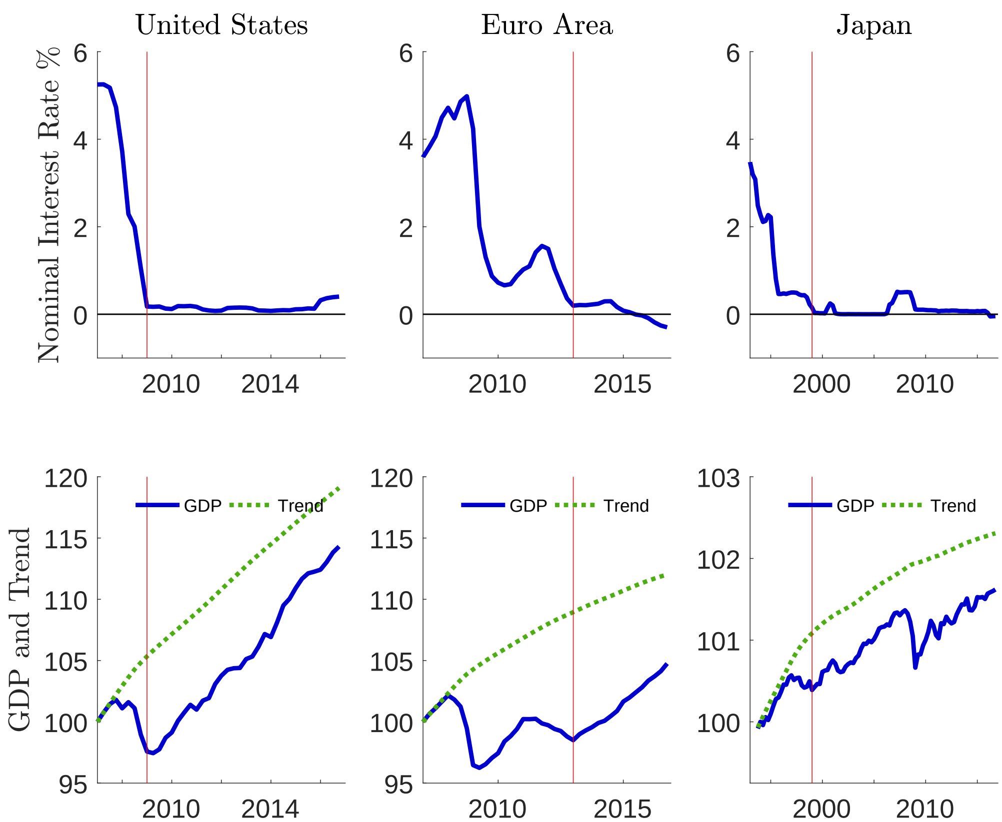
1 Introduction

Concerns of persistent stagnation began with the onset of Japan's prolonged period of zero nominal interest rates in the mid-90s. Since the financial crisis of 2008-09, these concerns have spread to the rest of the advanced world. Figure 1 shows the trajectory of short-term nominal interest rates and output in the United States, the euro area and Japan. In all cases, central banks lowered interest rates to near-zero levels. Yet, output remains stagnant.

Figure 1: Interest Rate and Output Dynamics



Notes: The vertical red lines correspond to the quarter in which the nominal interest rate was below 25 basis points for the first time in the sample shown in each graph. GDP (solid blue line) is indexed equal to 100 in 2007:Q1 for the U.S. and euro area and in 1994:Q1 for Japan. GDP trend (green dotted line) is constructed using a backward looking 10-year moving average of GDP growth.

The macroeconomics literature has primarily sought to reconcile the case of persistent stagnation in two ways: through an *expectations-driven liquidity trap* as in Benhabib, Schmitt-Grohé and Uribe (2001, 2002) or Alvin Hansen's *secular stagnation* hypothesis, which entails a permanent decline in the natural interest rate (Summers, 2013). These distinct concepts about the source of permanent stagnation have only been analyzed in separate theoretical frameworks. In particular, *secular stagnation* has been studied in models with heterogeneous agents (Eggertsson, Mehrotra and Robbins, 2019). In this paper, we build a model that allows us to consider the ideas of *secular stagnation* and *expectations-driven* liquidity traps in a unified setting. These two hypotheses differ in the local determinacy of the equilibrium. Secular stagnation exhibits locally determinate dynamics while the BSGU equilibrium features locally indeterminate dynamics.

We build a tractable model with downward nominal wage rigidity to demonstrate our key results analytically. The representative agent's intertemporal Euler equation is modified such that the steady state features a negative relation between output and real interest rate, similar to a static IS curve. This modification breaks the tight connection between the natural interest rate and the discount factor, thus allowing for the possibility of a permanently negative natural interest rate.[1] In contrast, the textbook representative agent model fixes the steady-state natural rate to equal the inverse of the household's discount factor and cannot accommodate the *secular stagnation* hypothesis.

The *expectations-driven* liquidity trap, commonly known as the BSGU steady state, arises in the presence of a nonlinearity in the monetary policy rule introduced by the zero lower bound constraint on short-term nominal interest rates. Pessimistic inflationary expectations become self-fulfilling in this setting. Combined with long-run money non-neutrality, the BSGU steady state can account for permanent output stagnation, below-target inflation and zero nominal interest rates (Aruoba, Cuba-Borda and Schorfheide, 2017*b*; Schmitt-Grohé and Uribe, 2017). Our framework, thus, accommodates two different explanations for persistent stagnation.

We show that the existence of the BSGU steady state depends on the assumptions on long-run nominal rigidities. The BSGU steady state does not exist when nominal wages are sufficiently rigid downwards. Policies that increase the degree of downward nominal wage rigidity can preclude expectations-driven traps. Conversely, flexible wages expose the economy to the possibility of expectations driven traps. We label this result as the *curse of flexibility*. This result extends the set of tools identified by Benhabib et al. (2002) and Woodford (2003, Ch 2).[2]

[1] We present alternate micro-foundations to modify the Euler equation in an observationally equivalent way and are described shortly.

[2] Benhabib et al. (2002) propose monetary and fiscal policies that violate the households' transversality conditions along the deflationary equilibrium paths.

On the other hand, the *secular stagnation* steady state emerges because of constraints on monetary policy's ability to replicate the permanently low natural interest rate. Due to the zero lower bound constraint on nominal interest rates, output remains permanently stagnant and the central bank cannot attain its inflation target. An increase in liquidity premia associated with government bonds, a reduction in the marginal impatience of the household or an increase in marginal (labor-dependent) tax on interest incomes can generate the decline of the natural rate in our framework. These explanations serve as analytical counterparts for fundamental forces such as aging, inequality, debt deleveraging (Eggertsson et al. 2019), international reserve accumulation, or a scarcity of safe assets (Caballero, Farhi and Gourinchas, 2015; Eggertsson, Mehrotra, Singh and Summers, 2016) in our representative agent framework.

We find that policy interventions to deal with unemployment can lead to contrasting outcomes depending on the source of stagnation. Policy interventions such as increased government spending are effective at reducing unemployment under secular stagnation. Yet, the same intervention is contractionary when implemented around the BSGU steady state. Labor market reforms that increase wage flexibility are expansionary under the BSGU trap but contractionary under secular stagnation (*paradox of flexibility*). Similarly, positive supply shocks that increase productive potential of the economy are contractionary under secular stagnation (*paradox of toil*) but increase employment under the BSGU trap. The local determinacy property is the source of different policy implications in the two theories. Because of the opposite outcomes implied by policy interventions, our exercise suggests caution in implementing policies without identifying the nature of stagnation.

Given the contradictory implications of these interventions, there is a need for robust policies to deal with liquidity traps (Bilbiie, 2018). This is particularly important since it might be hard to identify the source of the trap in real-time. In our model, a minimum wage policy precludes the expectations trap and reduces the severity of a fundamentals-trap. A minimum wage policy installs a lower bound on deflation, hence an expectations-trap can be avoided. Further, it is well known from the literature on fundamentals traps that policies which increase wage rigidity reduce deflationary expectations and stimulate output (Eggertsson, 2011).

This is the first paper to quantitatively compare the two stagnation hypothesis for Japan which has witnessed over two decades of near-zero nominal interest rates. In our estimation of a small-scale quantitative model, using data on output, consumption and inflation over 1995:Q4-2012:Q4, we compare the narrative of secular stagnation to that of the expectations-driven trap. The quantitative models used for policy inference require agents to expect recovery in the medium run (see Bianchi and Melosi 2017 and Gust, Herbst, López-Salido and Smith 2017), while our framework allows agents to expect persistent stagnation.

In the quantitative model, we find that either of the stagnation steady states can exist for reasonable parameters. Like in the analytical model, the BSGU steady state exists when the degree of nominal rigidity is weak enough to allow low levels of inflation in equilibrium. There is a large literature that has modeled liquidity trap as a result of a temporary decline in the natural rate. In those log-linearized models, *deflationary black holes* emerge as the duration of temporary liquidity trap is increased, with inflation and output tending to negative infinity (Eggertsson, 2011).[3] Instead, the modified Euler equation in our setting can give rise to a bounded secular stagnation steady state, thus imposing a lower bound to the deflation in a liquidity trap.

A likelihood-based evaluation of the estimated model suggests that the BSGU trap narrative best fits the Japanese malaise. The indeterminate DSGE model requires the researcher to specify the correlation of sunspot shocks with the fundamental (Lubik and Schorfheide, 2003). We find that the equilibrium selection is crucial in identifying the hypothesis that best explains Japanese stagnation. When these correlations are fixed to zero ('orthogonality' solution) or when there are no sunspot shocks under indeterminacy (Lubik and Schorfheide, 2004), the secular stagnation hypothesis emerges as the dominant hypothesis. When we let the model estimate the correlations to best fit the data, the expectations-driven narrative is the preferred hypothesis for the observed stagnation in Japan.

Our work complements the recent analyses of Michaillat and Saez (2018), Michau (2018), and Ono and Yamada (2018) who use the bonds-in-utility assumption to analyze a unique persistent stagnation scenario. We distinctly focus on understanding the differences between the two stagnation concepts analytically and quantitatively. Following Feenstra (1986) and Fisher (2015), a functional equivalence can be shown between using bonds in the utility and entering transactions costs of bonds in the budget constraint. Additionally, we show that a similar modification can also be brought about by Uzawa-Epstein preferences, or through increase in marginal (labor-dependent) tax

[3] Eggertsson and Singh (2018) show that the massive deflation is an artifact of conducting inference using the log-linearized solution. In the non-linear model, the equilibrium may cease to exist as the expected duration of the liquidity trap is increased beyond few quarters.

rate on interest incomes.[4] These alternate micro-foundations essentially make government bonds matter. In an earlier literature, it has been shown that interest rate pegs can achieve price-level determinacy if government bonds provide transactions services (see Canzoneri and Diba 2005 for closed economy, and Calvo and Végh 1995; Lahiri and Végh 2003 for open economy settings). We emphasize that the modified Euler equation does not, by itself, give rise to price-level determinacy in our setting. This is crucial for the model to exhibit the two different stagnation steady states. Moreover, we show that a breakdown of Ricardian equivalence is also not sufficient for the existence of this secular stagnation steady state. The key element for this existence result is the modified Euler equation which endogenizes the long-run real interest rate.

This paper is also related to the work by Mertens and Ravn (2014) and Aruoba et al. (2017*b*), who contrast the temporary expectations-driven trap to a temporary fundamentals-driven trap. Their model cannot accommodate the permanent fundamentals-driven trap. Our paper is also complementary to Schmitt-Grohé and Uribe (2017), who analyze the case of permanent expectations-driven traps. The policy implication that a flattening of the aggregate supply curve precludes the existence of the BSGU steady state is related to the R&D policies advocated by Benigno and Fornaro (2018) in a New Keynesian model with endogenous growth. Our analysis shows that such policy implications apply even in the case of liquidity traps with exogenous productivity growth. We also contribute to the literature on reference criterion under indeterminacy (Canova and Gambetti, 2010; Justiniano and Primiceri, 2008) and demonstrate that the equilibrium selection criterion matters at the ZLB.

The layout for the remainder of the paper is as follows. Section 2 illustrates key insights from introducing the modified Euler equation in an analytically tractable setting with downward nominal wage rigidity. We show that the bounds on deflation are sufficient to preclude the possibility of expectations trap even with a standard Euler equation. The modified Euler equation, by allowing a negative relation between output and real interest rate, opens up the possibility of a secular stagnation steady state. Section 3 describes a quantitative small-scale DSGE model commonly used in the literature. Section 4 shows the quantitative robustness of the steady-state insights derived from the simple model and compares the dynamics around each steady state. In Section 5,

[4] In this paper, we restrict our quantitative analysis to the bonds-in-utility modification because of its recent popularity. We leave the exploration of differences in the dynamics associated with different micro-foundations away from ZLB to future work.

we estimate the model and discuss the possibility of either scenario for the case of Japan. Section 6 concludes.

2 Key insights in a two equations setup

We begin with a simple setup that analytically captures the key insights. Our model is a variant of the downward-nominal wage rigidity apparatus introduced by Schmitt-Grohé and Uribe (2017). Using this setup, we characterize and formally define the different steady states: *targeted inflation* state, *secular stagnation* steady state, and the *expectations-driven trap* (also referred to as the BSGU steady state).

Time is discrete and there is no uncertainty. Suppose the representative agent supplies labor $\bar{h} = 1$ inelastically and maximizes the following utility function involving a consumption good C_t and (real) stock of one-period risk-free government bonds b_t:

$$\max_{\{C_t, b_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ln(C_t) + \delta_t V(b_t)\right]$$

where $\delta_t \geq 0$ regulates the marginal utility from holding the risk-free bonds supplied by the government. We assume that the felicity from holding these bonds is such that $V'(0) = 1$. Since our objective is to allow the possibility of permanently negative natural rate, we introduce a wedge δ_t in the Euler equation in this particular manner, following Michaillat and Saez (2018). In the following subsection, we discuss alternate micro-foundations for this wedge. The household earns wage income $W_t h_t$, interest income on past bond holdings, dividends Φ_t from firms' ownership and makes transfers T_t to the government.

$$C_t + b_t = \frac{W_t}{P_t} h_t + \frac{R_{t-1}}{\Pi_t} b_{t-1} + \Phi_t + T_t$$

An interior solution to household optimization yields the Euler equation:

$$U'(C_t) = \beta R_t \mathbb{E}_t \left[\frac{U'(C_{t+1})}{\Pi_{t+1}}\right] + \delta_t$$

Consumption goods are produced by competitive firms with labor as the only input using the technology

$$Y_t = F(h_t) = h_t^{\alpha}, \quad \text{where } 0 < \alpha < 1$$

These firms set price of the final good P_t to equate marginal product of labor to the marginal cost.

$$F'(h_t) = \frac{W_t}{P_t}$$

We introduce a very stylized form of downward nominal wage rigidity (following Schmitt-Grohé and Uribe 2017):[5]

$$W_t \geq (\gamma + (1-\gamma)(1-u_t)^{\alpha})W_{t-1} \equiv \tilde{\gamma}(u_t)W_{t-1}$$

where $\gamma \in (0,1)$ and $u_t \equiv 1 - \frac{h_t}{\bar{h}}$ is involuntary unemployment. This downward rigidity implies that employment cannot exceed the total labor supply in the economy i.e. $h_t \leq 1$. We further assume that the following slackness condition holds:

$$(\bar{h} - h_t)(W_t - \tilde{\gamma}(u_t)W_{t-1}) = 0$$

We close the model by assuming a government that balances budget,

$$b_t + T_t = \frac{R_{t-1}}{\Pi_t} b_{t-1}$$

and a monetary authority that sets nominal interest rate on the net zero supply of nominal risk-free one-period bonds using the following Taylor rule

$$R_t = \max\{1, (1+r^*)\Pi^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\alpha_\pi}\}$$

where $(1+r^*) \equiv \frac{1-\delta_t}{\beta}$ is the natural interest rate, Π^* is the inflation target set by the central bank, and $\alpha_\pi > 1$.[6] The zero lower bound (ZLB) constraint on the short-term nominal interest rate introduces an additional nonlinearity in the policy rule. Finally, we assume that the resource

[5] In appendix C, we provide analytical results for the particular downward nominal wage rigidity assumed by Eggertsson et al. (2019).

[6] The natural interest rate is defined as the real interest rate on one-period government bonds that would prevail in the absence of nominal rigidities.

constraints hold in the aggregate:

$$C_t = Y_t, \quad \text{and} \quad b_t = 0.$$

2.1 Equilibrium

Let $w_t \equiv \frac{W_t}{P_t}$ denote the real wage. The competitive equilibrium is given by the sequence of seven endogenous processes $\{C_t, Y_t, R_t, \Pi_t, h_t, w_t, u_t\}$ that satisfy the conditions (1) - (7) for a given exogenous sequence of process $\{\delta_t\}_{t=0}^{\infty}$ and the initial condition w_{-1}:

$$1 = \beta E_t\left[\frac{C_t}{C_{t+1}}\frac{R_t}{\Pi_{t+1}}\right] + \delta_t C_t \tag{1}$$

$$Y_t = h_t^{\alpha}. \tag{2}$$

$$\alpha h_t^{\alpha-1} = w_t \tag{3}$$

$$h_t \leq \bar{h}, \quad w_t \geq (\gamma + (1-\gamma)h_t^{\alpha})\frac{w_{t-1}}{\Pi_t}, \quad (\bar{h} - h_t)\left(w_t - (\gamma + (1-\gamma)h_t^{\alpha})\frac{w_{t-1}}{\Pi_t}\right) = 0 \tag{4}$$

$$u_t = 1 - \frac{h_t}{\bar{h}} \tag{5}$$

$$R_t = \max\{1, (1+r_t^*)\Pi^*\left(\frac{\Pi_t}{\Pi^*}\right)^{\alpha_\pi}\} \tag{6}$$

$$Y_t = C_t \tag{7}$$

where the exogenous sequence of natural interest rate is given by $1 + r_t^* \equiv \frac{1-\delta_t}{\beta}$.

2.2 Non-stochastic steady state

In the steady state, we can simplify the system of equations to an aggregate demand block and an aggregate supply block.

Aggregate Demand (AD) is a relation between output and inflation and is derived by combining the Euler equation and the Taylor rule. Mathematically, the AD curve is given by

$$Y_{AD} = \frac{1}{\delta}\begin{cases} 1 - \beta\Gamma^*\Pi^{\alpha_\pi - 1}, & \text{if R } > 1, \\ 1 - \frac{\beta}{\Pi}, & \text{if R } = 1 \end{cases} \tag{8}$$

where $\Gamma^* = (1 + r^*)(\Pi^*)^{1-\alpha_\pi}$. Away from the ZLB, the AD curve is a downward sloping graph, and it becomes upward sloping when the nominal interest rate is constrained by the ZLB. The kink in the aggregate demand curve occurs at the inflation rate at which monetary policy is constrained by the ZLB.

$$\Pi_{kink} = \left(\frac{1}{(1+r^*)\Pi^*} \right)^{\frac{1}{\alpha_\pi}} \Pi^*$$

The dashed red line in panel a) of Figure 2 plots the aggregate demand curve with a positive natural rate. When $R^* \equiv (1+r^*)\Pi^* > 1$, the kink in the AD curve occurs at an inflation rate below Π^*. For the natural interest rate to be positive, the premium on government bonds must be low enough i.e. $\delta < 1 - \beta$.[7]

Aggregate Supply (AS): Because of the assumptions of downward nominal wage rigidity and capacity constraints on production, the AS curve features a kink at full employment level of output and gross inflation rate equal to one. When inflation rate is less than one, the downward wage rigidity constraint becomes binding. As a result, inflation cannot fall to completely adjust any demand deficiency and firms layoff workers. The aggregate supply curve can be summarized by:

$$Y_{AS} \leq 1, \quad \Pi \geq (\gamma + (1-\gamma)Y_{AS}), \quad (Y_{AS} - 1)\left(1 - (\gamma + (1-\gamma)Y_{AS})\frac{1}{\Pi}\right) = 0 \tag{9}$$

When $h = \bar{h} \equiv 1$, $\Pi \geq 1$. The AS curve is a vertical line at full employment. For $h < \bar{h}$, $\Pi = \gamma + (1-\gamma)h^\alpha$. The AS curve is linear and upward sloping with slope$=1-\gamma$ for $y < 1$. The kink in the AS curve occurs at the coordinate $Y = 1, \Pi = 1$. Because of this assumed linear aggregate supply curve under deflation, the degree of nominal rigidity γ also determines the lower bound on inflation. The solid blue line in both panels of Figure 2 plots the aggregate supply curve.

In the two-equation model, we can characterize and prove the existence of different steady states. The assumptions made to derive the two-equation setup are similar to that made in representative agent models that feature expectations driven traps (Schmitt-Grohé and Uribe, 2017). Nevertheless, we show in proposition 1 that it is possible for the model to feature a unique steady state without a liquidity trap. It formalizes that the unintended steady state can be eliminated by assuming a flat enough Phillips curve i.e. $\gamma > \beta$. This is the first main result of our exercise.

[7]For illustration in this section, we use the following parameters: $\beta = 0.77$, $\alpha = 0.5$, $\alpha_\pi = 2$, $\gamma = \{0.5, 0.9\}$,and $\delta = \{0.1, 0.3\}$.

Proposition 1 *(Disarming the Perils): Let* $\Pi^* = 1$*,* $0 < \delta < 1 - \beta$*, and* $\gamma > \beta$*.There exists a globally unique steady state. It features full employment, inflation at the policy target and positive short term nominal interest rate.*

Proof. The downward sloping portion of aggregate demand always goes through $Y = 1$, $\Pi = 1$ when $\Pi^* = 1$. When $\delta < 1 - \beta$, $R^* > 1$. The kink in the AD curve occurs at inflation rate below 1. There always exists an intersection between the AS and the AD at $Y = 1$ and $\Pi = 1$. To show that there does not exist another equilibrium, note the AS curve is linear for values of inflation less than one (by functional form assumption). At Π_{kink}, $Y_{AD} > 1 > Y_{AS}$. When $\Pi = \gamma$, $Y_{AS} = 0 < Y_{AD}$ (the last inequality uses the assumption of $\gamma > \beta$). Since the downward sloping portion of the AD is strictly convex, it cannot intersect the downward sloping portion of the AS curve. Hence there does not exist an equilibrium with zero nominal interest rates. ■

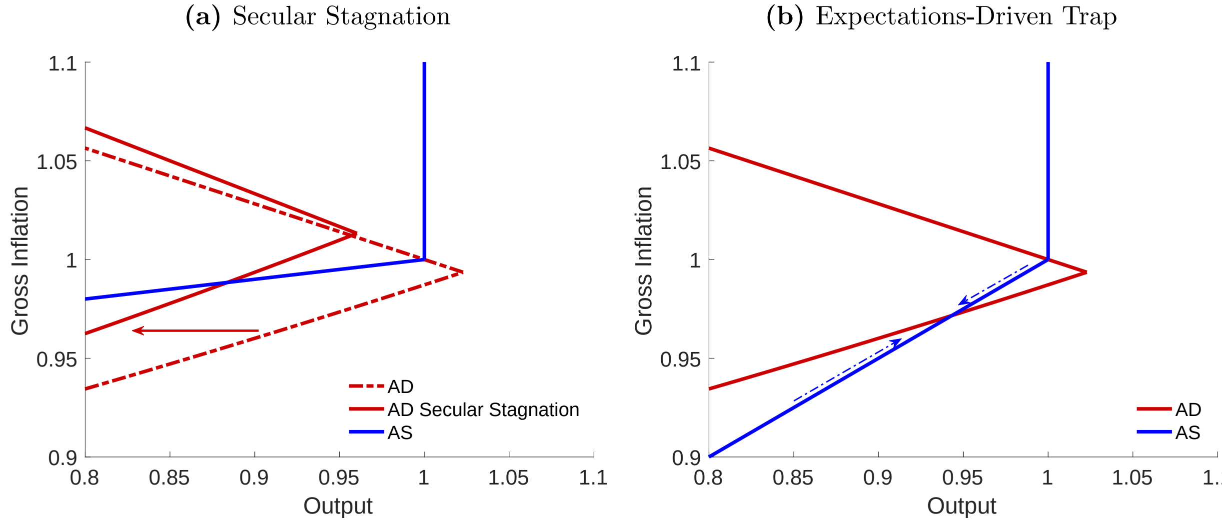
Any steady state at which the central bank can meet its inflation target is *defined* as the *targeted inflation steady state.* The presence of full-employment steady state is contingent on the natural interest rate and the inflation target of the monetary authority. With a unitary inflation target, it must be the case that the natural interest rate be non-negative, which is implied by the assumption of $\delta < 1-\beta$. A high enough nominal rigidity puts a sufficiently high lower bound on deflation, which prohibits the self-fulfilling deflationary expectations to manifest in the steady state. Technically, the set of equilibrium inflation rates is upper hemi-continuous in the lower bound on inflation rate γ.[8]

We now consider the case where adverse fundamentals can push the economy to a permanent liquidity trap. When the natural interest rate is negative, monetary policy can be constrained by the zero lower bound on short-term nominal interest rate and a modest inflation rate target. In that case, the nominal interest rate is permanently zero while there is unemployment and deflation in the economy. We characterize this possibility in Proposition 2.

Proposition 2 *(Secular Stagnation): Let* $\Pi^* = 1$*,* $\delta > 1-\beta > 0$*, and* $\gamma > \beta$*. There exists a unique steady state. It features unemployment and deflation, caused by a permanently negative natural interest rate. The equilibrium dynamics in this steady state's neighborhood are locally determinate.*

[8] The target inflation rate is the unique outcome when $\gamma > \beta$ and continues to be an equilibrium when $\delta = 0$. There is a kink at $\delta = 1 - \beta$, but the equilibrium inflation rate is continuous in $\gamma \leq \beta$.

Figure 2: Steady-State Representation



Proof. When $\delta > 1 - \beta$, $R^* < 1$, thus, the kink in the AD occurs at inflation rate above 1. AS is linear and AD is strictly convex. At Π_{kink}, $Y_{AD} < 1 = Y_{AS}$. When $\Pi = \gamma$, $Y_{AS} = 0 < Y_{AD}$. The last inequality requires the assumption that $\gamma > \beta$. Thus, the AS and AD must intersect at positive unemployment. We leave the proof of local determinacy to the online appendix. It involves log-linearizing the dynamic equilibrium around this secular stagnation steady state and proving that the Blanchard and Kahn (1980) determinacy conditions hold. In this case, the condition is equivalent to the AD curve being steeper than the AS curve at the stagnation steady state. ■

See panel a) in Figure 2 for illustration of this unique steady state. The intersection of the dashed red line(AD) with the solid blue line (AS) depicts the result of proposition 1. The modified Euler equation allows for the possibility of a permanently negative natural interest rate, which is no longer fixed by the discount factor. Instead, the marginal utility of holding government bonds (δ) also determines the steady-state natural interest rate. The stagnation steady state emerges when the premium on government bonds is sufficiently high driving the natural interest rate to negative levels. As a result, the AD curve moves inwards and intersects the AS at zero nominal interest rate, low inflation and positive unemployment.

We formally *define* the *secular stagnation steady state* as the steady state with positive unemployment, zero nominal interest rate on short-term government bonds and exhibiting *locally determinate* equilibrium dynamics in its neighborhood. This local determinacy property, as we show later, is the main difference between the secular stagnation narrative and the expectations-driven narrative.

Propositions 1 and 2 may serve as a counter-example to the folk wisdom that the imposition of a zero bound constraint on the monetary policy rule is sufficient to allow for the possibility of expectations-driven traps. The primary ingredient for the absence of the expectations-driven trap is a sufficiently high lower bound on deflation. Thus, the assumption of long-run money non-neutrality is not necessarily incompatible with the existence of such a locally indeterminate steady state. We show next, in Proposition 3, the conditions conducive for the existence of the expectations-driven trap. Specifically, when the nominal wages are sufficiently downwardly flexible, the economy is open to the possibility of expectations-driven traps (Schmitt-Grohé and Uribe, 2017).

Proposition 3 *(The Peril-ou-s nominal rigidity): Let $\Pi^* = 1$, and $0 < \delta < 1 - \beta$. If $\gamma < \beta$ there exists two steady states:*

1. *A unique full employment steady state, with inflation at the target rate.*
2. *(Expectations-driven trap) A unique unemployment steady state with zero nominal interest rate and deflation. The local dynamics in a small neighborhood around the unemployment steady state are locally indeterminate.*

Without downward rigidity constraint on nominal wages (i.e. $\gamma = 0$), there always exist two steady states: a unique deflationary steady state with zero nominal interest rates and a unique targeted inflation steady state.

Proof. When $\delta < 1 - \beta$, $R^* > 1$. The kink in the AD occurs at inflation rate below 1. Further, note that the downward sloping portion of aggregate demand always goes through $Y = 1$, $\Pi = 1$. This is the unique full employment steady state. At Π_{kink}, $Y_{AD} > 1 > Y_{AS}$. When $\Pi = \gamma$, $Y_{AS} = 0 > Y_{AD}$. Given that AS is linear and AD is strictly convex, there is a unique intersection at positive unemployment. At this second steady state, AS intersects AD from above. In the model, the relative steepness of AS curve is equivalent to locally indeterminate dynamics. We leave the detailed proof following Blanchard and Kahn (1980) to the online appendix. ■

Panel b) in Figure 2 illustrates the unique full employment and a unique stagnation steady state with the modified Euler equation. The co-existence of this second unintended steady state requires that inflation be allowed to fall sufficiently low. We define the *expectations-driven* trap as the steady state with positive natural interest rate, unemployment, and deflation and in whose neighborhood the equilibrium dynamics are *locally indeterminate*. As a shorthand, henceforth, we refer to it as the *BSGU trap*. In Section 2.4.5, we show that similar bounds on deflation are also enough to preclude expectations trap in a model with a textbook Euler equation. In Section 4, we show that putting a high enough lower bound on inflation excludes the BSGU steady state also in the case of a quantitatively calibrated model.

2.3 Transition to Stagnation

We now discuss how an economy may transition from a full employment steady state to a stagnation steady state. Importantly, we document that the transition to a secular stagnation or to expectations-driven trap can look identical.The important difference is that the natural rate of interest falls as the economy falls into a secular stagnation episode, while the economy may transition to an expectations-driven trap without any change in the fundamentals.

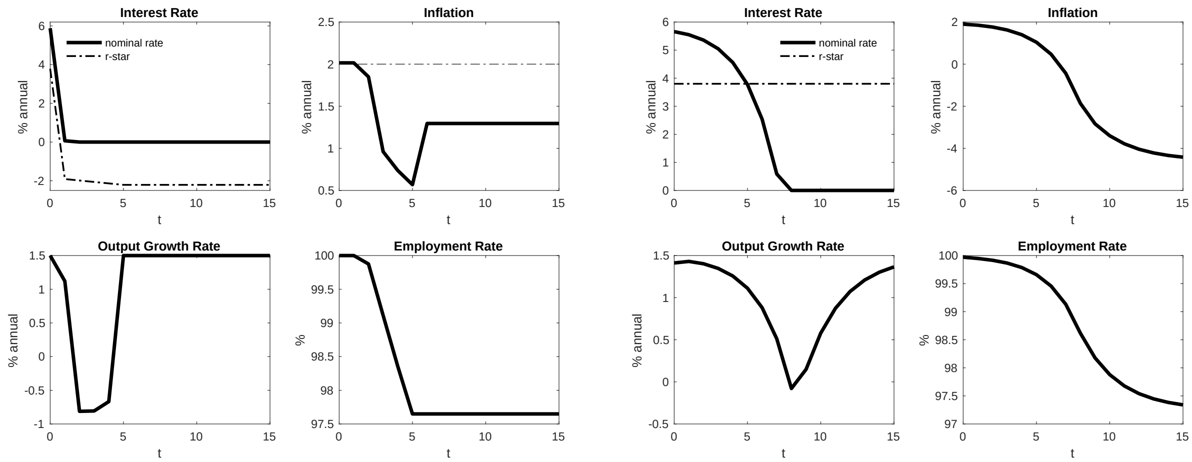
Panel a) of Figure 3 plots the transition from the unique full employment steady state to the unique secular stagnation steady state. A secular increase in the premium associated with government bonds, captured by parameter δ_t, drives the natural rate of interest to negative levels. Since the central bank is unable to lower interest rates below zero, the economy features permanently depressed employment rate and a below-target inflation rate in the presence of downward wage rigidities. Fundamental liquidity traps can replicate the pattern of *jobless recovery*, where output growth recovers to the pre-crisis levels, with permanently depressed employment.

As documented by Schmitt-Grohé and Uribe (2017), an economy at full employment can converge to the second steady state if agents form pessimistic inflationary expectations in the first period. This transition is documented in panel b) of Figure 3. Without any change in fundamentals of the economy, i.e. with the natural rate being constant, the economy can finds its way to stagnation through self-fulfilling deflationary expectations.

We reiterate that the important difference in the two steady states is the local determinacy property of the equilibrium. While the transition to secular stagnation steady state is unique, the transition to the expectations-driven trap may arise through infinite paths. We fix initial inflation expectations to show one transition path associated with expectations-driven jobless recovery.

Figure 3: Transition from Full Employment Steady State to

(a) Secular Stagnation **(b)** Expectations-Driven Trap



2.4 Discussion

2.4.1 Interpreting the modified Euler equation

The key ingredient in deriving the secular stagnation steady state in the representative agent setting is the modified Euler equation. The modification allows the natural interest rate to depend on the marginal utility of holding bonds δ, as well as the discount factor. A high demand for liquidity in the steady state can push the natural interest rate to a permanently negative level. We discuss various alternate modeling devices that give rise to existence of a well-defined secular stagnation steady state. We emphasize that these explanations serve as analytical counterparts for fundamental forces such as aging, debt deleveraging, flight-to-liquidity, savings glut, and inequality micro-founded in heterogeneous agent models more recently. In fact, our result on bounds on deflation apply to heterogenous agent models of secular stagnation with nominal rigidities. In order to keep our discussion focussed, we relegate this result to the following section 2.4.4, where we illustrate the existence of BSGU steady state in the overlapping generation setting of Eggertsson et al. (2019).

We briefly sketch another alternative way to derive the wedge based on inter-temporal discounting à la Uzawa-Epstein preferences that have been prominently used in the small open economy literature (Schmitt-Grohé and Uribe, 2003). The representative agent maximizes the following lifetime-utility function:

$$\max_{\{C_t,B_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \theta_t^t U(C_t)$$
$$\theta_0 = 1;$$
$$\theta_{t+1} = \beta(\tilde{C}_t)\theta_t \quad t \geq 0$$

where θ is the endogenous discount factor, and $\tilde{C}_t$ denotes the average per capita consumption, which the individual household takes as given.[9]

The household budget constraint is the same as before and we assume that government bonds are in net zero supply. Let λ_t be the Lagrange multiplier, associated with the budget constraint. We can derive the following first order conditions:

$$\lambda_t = \beta(\tilde{C}_t)(1 + r_t)E_t\lambda_{t+1}; \quad \lambda_t = U'(C_t)$$

where $1 + r_t = R_t/\Pi_{t+1}$ is the real interest rate. These conditions imply the following Euler equation:

$$1 = \beta(\tilde{C}_t)\frac{R_t}{\Pi_{t+1}}\frac{U'(C_{t+1})}{U'(C_t)}$$

In equilibrium, individual and average per capita variables are identical. That is, $C_t = \tilde{C}_t$. The Euler equation, thus, simplifies to:

$$1 = \beta(C_t)\frac{R_t}{\Pi_{t+1}}\frac{U'(C_{t+1})}{U'(C_t)}$$

In the steady state:

$$1 = \beta(C)\frac{R}{\Pi}$$

If $\beta'(C) > 0$, the Euler equation exhibits a negative relationship between output and real interest rate, as in the static IS-curve. The converse assumption of increasing marginal impatience $\beta'(C) < 0$ is often made in the infinite horizon models to ensure a stable solution.[10] Because we do not allow

[9] This assumption, introduced by Schmitt-Grohé and Uribe (2003), allows us to analytically relate the intertemporal discounting to the bonds-in-utility function. In ongoing work, we compare the dynamics implied by different assumptions in detail.

[10] With capital accumulation, Das (2003) proves that assuming $\beta'(C) > 0$ does not necessarily preclude the existence of a stable solution. We leave the extension with capital to future work.

for accumulation in our model, a stable steady state solution exists. In the case of $\beta(C) = \frac{\beta\frac{R}{\Pi}+\delta C}{\frac{R}{\Pi}}$ (with $\Pi > 0$), the modified Euler equation is exactly similar to the one derived with the bonds-in-utility function specification:

$$1 = \beta\frac{R}{\Pi} + \delta C$$

The δ parameter can thus be interpreted as regulating the marginal impatience of the representative household.

In the previous section, we derived this modification in the Euler equation by adding bonds in the utility function. Fisher (2015) proves an observational equivalence of this assumption with risk-premia shocks assumed by Smets and Wouters (2007) in the budget constraint of the household. Another interpretation of the shocks to δ is that these capture the *flight to liquidity* episode of the recent financial crisis (Krishnamurthy and Vissing-Jorgensen, 2012). A similar wedge in the Euler equation can be associated with the deterioration in liquidity properties of AAA-rated corporate bonds in contrast to Treasury securities during the 2008-09 financial crisis (Del Negro, Eggertsson, Ferrero and Kiyotaki, 2017).[11]

Yet another equivalent way to introduce the modification is to model exogenous changes in transaction costs in trading of financial assets where the liquid government bonds earn the transaction premia (Calvo and Végh 1995; Lahiri and Végh 2003). Finally, modeling labor income dependent taxes on earned interest income on government bonds can provide another meaningful micro-foundation. For example, let the household pay a fraction $\tau(\tilde{C}_t)$ of the interest rate income in taxes. If $\tau'(C) < 0$, we can derive the same Euler equation as with bonds-in-utility/ Uzawa preferences:

$$1 = \beta\frac{(1-\tau(C))R}{\Pi}$$

While these different interpretations follow naturally from extensive literatures, we also view this wedge in the Euler equation as a reduced form for fundamental factors such as aging, savings glut, reserve accumulation, inequality, or debt deleveraging microfounded in the secular stagnation literature (Eggertsson et al. 2016, 2019; Auclert and Rognlie 2018). Such a reduced form modification allows the researchers to employ existing tools to include secular stagnation as a complementary

[11] In the simplest terms, the wedge in the Euler equation can be derived from introducing a binding constraint on the consumption choice of households (see Parker 2016).

explanation in the estimated DSGE models, much like "β shock" is used in the temporary liquidity trap literature.[12]

2.4.2 Ricardian Equivalence and Price level determinacy

We presented alternate ways to micro-found the proposed modification to the Euler equation. Essentially these assumptions make government bonds matter, directly or indirectly. Thus, the Ricardian equivalence proposition (Barro, 1974) does not hold in our setting. However, we emphasize that a mere breakdown of Ricardian equivalence is not enough to generate a secular stagnation steady state. Take for instance the textbook representative agent model. Instead of lump-sum non-distortionary taxation, assume linear distortionary labor taxes. The Euler equation remains unmodified and yet the Ricardian equivalence breaks down in such a setting. The modification to the Euler equation is crucial to allow for the existence of a locally determinate secular stagnation steady state.

Furthermore, it is a well-known result in the literature that interest rate pegs can achieve price level determinacy if government bonds provide transactions services (see Canzoneri and Diba 2005 in closed economy and Calvo and Végh 1995; Lahiri and Végh 2003 in open economy settings). While government bonds matter and provide services to the representative consumer in our setting, we do not obtain price level determinacy just from the breakdown of Ricardian equivalence. This is in fact crucial to allow for the existence of BSGU steady state or the secular stagnation steady state, depending on the bounds on wage flexibility. For example, assume that the felicity from bonds in the utility takes a quadratic form: $\frac{\delta}{2}\left(\frac{B}{P}\right)^2$. Then the Euler equation is:

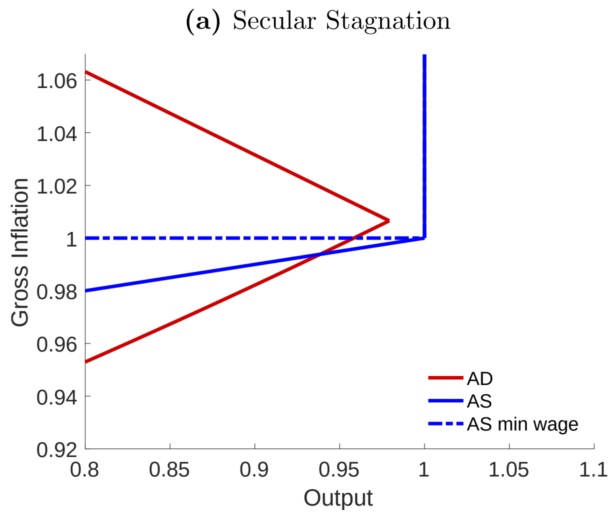
$$1 = \beta \frac{C_t}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} + \delta C_t \frac{B_t}{P_t} \tag{10}$$

Given a path for nominal bonds $\{B_t\}$, with positive nominal bonds in steady state, the price level is uniquely determined even with a fixed nominal interest rate if there exists a steady state.[13]

[12] In the recent literature that augments DSGE models with endogenous growth, (mean zero) shocks to preference for bonds are added to get co-movement of investment and consumption as well to derive the *divine coincidence* benchmark (Garga and Singh, 2016).

[13] Hagedorn (2018) shows that this kind of price-level determinacy is also obtained in the heterogenous agents framework. Furthermore, with positive supply of real bonds and appropriate fiscal policy rules, our model can produce additional amplification that we abstracted from. In response to a positive shock to liquidity demand δ_t, an inflation targeting central bank lowers the real interest rate. This may reduce the interest payments government needs to finance on existing debt. Depending on the assumptions on tax smoothing (Valchev, 2018), the government

Figure 4: A minimum wage policy



Thus, this setting precludes the coexistence of a full employment and a BSGU steady state as shown in our analytical model. This example also clarifies that zero net supply of nominal government bonds is not essential to the existence of secular stagnation steady state. For a given government policy, a linear felicity from real bonds would also modify the Euler equation so as to not make the aggregate price level depend on the supply of nominal bonds as in equation 10.

2.4.3 Minimum wage policy

In models with temporary liquidity traps, a paradoxical result emerges with increasing flexibility of labor markets. A set of structural reforms that increase wage flexibility increase deflationary pressures, worsening the liquidity trap. This is known as the *paradox of flexibility.* We use this insight to show that a minimum wage policy can increase output under secular stagnation and eliminate the expectations-driven trap.

Recall that we introduced downward nominal wage rigidity using the following assumption on the evolution of nominal wages:

$$\frac{W_t}{W_{t-1}} \geq \gamma + (1-\gamma)(1-u_t)^{\alpha}$$

Here, γ indexes the degree of downward wage-rigidity. When $\gamma = 1$, the nominal wages are fixed

may lower the net supply of debt further pushing down the natural interest rate.

to previous period's wages. We interpret $\gamma = 1$ as a minimum wage policy.[14] The left panel of Figure 4 plots the effect of a minimum wage policy in secular stagnation. Because of paradox of flexibility associated with fundamental trap, the minimum wage policy increases the output under secular stagnation.

Since this policy installs a lower bound on deflation, it eliminate the expectations driven trap altogether (see the right panel in Figure 4). Other policies that flatten the Phillips curve by strengthening labor unions during recessions can also preclude the possibility of BSGU trap. A converse implication of this finding is that structural reforms that increase downward flexibility in wages make the economy vulnerable to swings in pessimistic expectations. We label this result as the *curse of flexibility*.

In a recent work, Benigno and Fornaro (2018) construct a model with an expectations-driven trap similar to the BSGU trap in terms of TFP growth and nominal interest rate. Their fiscal policy of precluding the stagnation trap imposes a lower bound on TFP growth rate through R&D subsidies. Thus, their suggested fiscal policy is analogous to a minimum wage policy in our environment. Our analysis shows that policy reforms, which can preclude the BSGU steady state, are valid more broadly even in the textbook representative agent models with nominal rigidities.[15]

2.4.4 Expectations trap in an OLG model

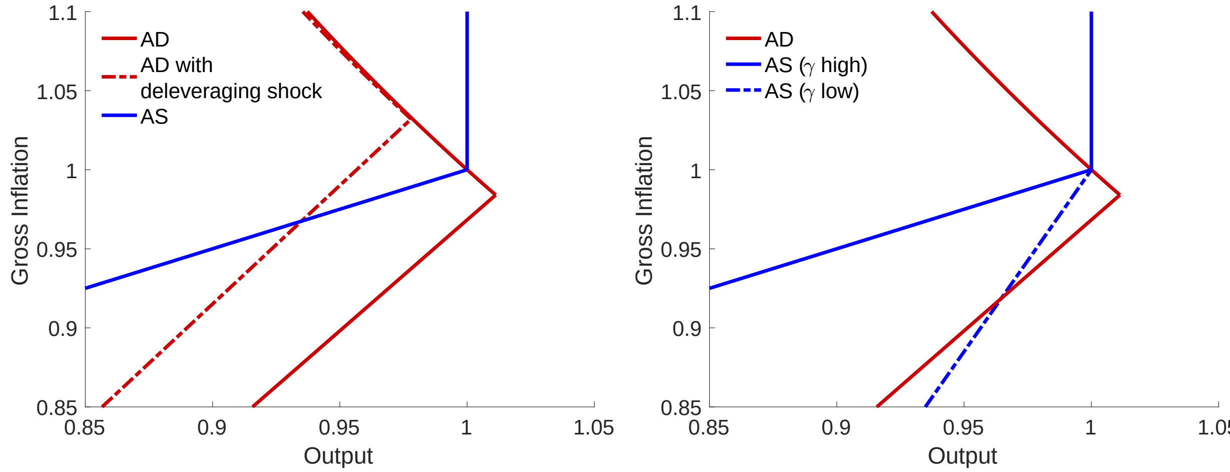
The degree of nominal rigidities also plays a key role in eliminating the locally indeterminate stagnation steady state in the overlapping generations model of Eggertsson et al. (2019) (EMR). We outline the key message here while referring the reader to EMR for detailed model. Agents live for three periods: young, middle and old. Young are borrowing constrained and derive no income. Middle supply labor inelastically to perfectly competitive firms for wages and save for retirement. Old consume the savings made when middle. Supply and demand for savings results

[14] Recently, Glover (2018) proposed a similar mechanism to rationalize the absence of deflation in the US during the Great Recession.

[15] Our analysis does not imply that imposing a lower bound on deflation is enough to preclude the BSGU steady state in more general settings. For example, in Benigno and Fornaro (2018) there is perfect downward nominal rigidity, but endogenous growth opens up the possibility of a stagnation trap along the lines of BSGU steady state. Similarly, Heathcote and Perri (2018) model an economy with perfect downward nominal rigidity, precautionary savings and a liquid asset in net positive supply. Despite perfect downwardly rigid wages, a BSGU steady state exists in their model because of precautionary savings motive.

Figure 5: Stagnation in the OLG model (EMR)

(a) Secular Stagnation/ locally determinate trap **(b)** Expectations-Driven/ locally indeterminate trap



in the following bond market clearing condition.

$$1 + r_t = \frac{1+\beta}{\beta} \frac{D_t}{Y_t - D_{t-1}}$$

where D is the exogenous debt limit faced by the young borrowers. Rest of the equilibrium conditions are same as derived in our benchmark model.

In the steady state, aggregate demand block is given by:

$$Y_{AD} = D + \begin{cases} \frac{1+\beta}{\beta} \frac{D}{\Gamma^*} \Pi^{1-\alpha_\pi}, & \text{if R } > 1, \\ \frac{1+\beta}{\beta} D\Pi, & \text{if R } = 1 \end{cases} \tag{11}$$

where $\Gamma^* = (1 + r^*) (\Pi^*)^{1-\alpha_\pi}$. The aggregate supply block in the steady state simplifies to:

$$Y_{AS} \leq 1, \quad \Pi \geq (\gamma + (1-\gamma)Y_{AS}), \quad (Y_{AS} - 1) \left(1 - (\gamma + (1-\gamma)Y_{AS}) \frac{1}{\Pi}\right) = 0 \tag{12}$$

Figure 5 plots the steady state aggregate demand and aggregate supply curves in the OLG model.[16] Panel a) reproduces the secular stagnation steady state. A debt deleveraging shock

[16] We use the following parameters for illustration: $\beta = 0.77$, $D = 0.31$, $\Pi^* = 1$, $\alpha = 0.5$, $\gamma = 0.5$. With the exception of the debt limit D parameter, the remaining parameters are same as in our benchmark model.

makes the natural rate negative. When combined with the zero lower bound on nominal interest rate and downward nominal wage rigidity, economy enters a locally determinate secular stagnation steady state. Panel b) plots the aggregate supply curve for two values of γ when the natural interest rate is positive. When the lower bound on deflation is sufficiently low, the BSGU steady state coexists along with the full employment steady state (see the dotted blue line). Pessimistic inflationary expectations can lead the economy to this locally indeterminate steady state.

2.4.5 Comparison with the textbook Euler equation

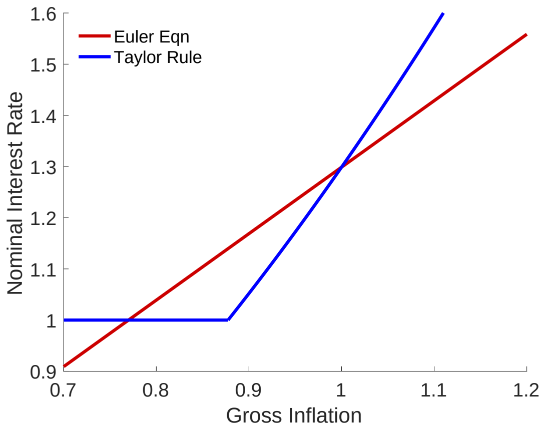
We provide a brief comparison of results for the reader with the textbook Euler equation (Woodford, 2003). This serves to illustrate the use of two concepts in our framework - a) the modified Euler equation and b) bounds on deflation.

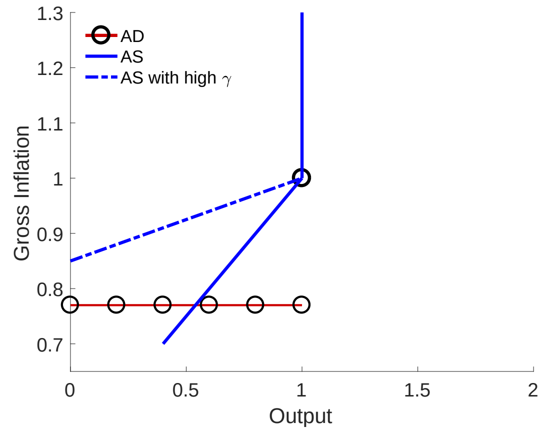
The textbook analysis usually assumes long-run money neutrality. Thus, there always exists a second steady state featuring deflation and zero nominal interest rate on the short-term government bonds. This point can be gleaned from the classic graphical representation of BSGU steady state in the (Π, i) space. Panel a) in figure 6 plots the standard Euler equation along with the nonlinear Taylor rule. There exist two intersections of the Taylor rule with the textbook Euler equation because of the zero lower bound constraint on the short term nominal interest rate. With the assumption of long-run neutrality, it suffices to look at the Euler equation and the Taylor rule to determine the equilibrium.

In an analogous AD-AS representation, we plot the textbook model in panel b) of figure 6. In the textbook model, the natural interest rate is always fixed at $\frac{1}{\beta} > 1$. As a result, the aggregate demand relationship is a horizontal line at $\Pi = \beta < 1$ when the nominal interest rate is constrained by the ZLB. And the aggregate demand is represented by the coordinate $(1, \Pi^*)$ in the (Y, Π) space when monetary policy is unconstrained. The aggregate supply curve in the textbook model is used to pin down the equilibrium output (see for example Schmitt-Grohé and Uribe 2017).

However, the existence of this *unintended* deflationary steady state is contingent on the assumptions regarding the supply side of the economy. If the y-intercept (γ) of the aggregate supply curve is greater than β, then there does not exist a deflationary steady state, as shown in panel b) of figure 6. This discussion clarifies the key distinction of our paper with Schmitt-Grohé and Uribe (2017), who assume a zero intercept for inflation in their downward nominal wage rigidity func-

Figure 6: Steady-state multiplicity in the textbook Euler equation





tional form. With a positive intercept for inflation rate (greater than β), the expectations-driven deflationary steady state can be eliminated. Hence, even the textbook analysis is not complete without sketching out the assumptions on long-run nominal rigidities.

We wish to emphasize that while the modification to the Euler equation is not essential to eliminating the BSGU trap, it is essential for the model to generate a secular stagnation steady state. The modified Euler equation, by endogenizing the long-run natural rate of interest, opens up the possibility of a secular stagnation steady state. This steady state cannot arise in the standard model because the AD curve is a flat line when nominal interest rates are zero, as shown in Figure 6. Furthermore, in the textbook models, β is usually chosen to lie in the range of 0.99 - 0.999 to match the relevant empirical real interest rate. In order to eliminate the BSGU steady state, one needs to assume somewhat extreme form of downward wage rigidity. The model with $\delta > 0$ can allow researchers to match an empirically plausible steady state real interest rate, while imposing realistic restrictions on the extent of downward wage rigidity. Hence, the model with $\delta > 0$ has desirable quantitative features.[17]

2.5 Comparative Statics: BSGU and Secular Stagnation

The BSGU steady state and the secular stagnation steady state have different implications for policy. While government spending is contractionary under BSGU trap, it is expansionary under

[17] In appendix A, we present the (Π, i) space representation of various steady states in our framework with the modified Euler equation, for completeness.

secular stagnation. Similar policy reversals are shown with permanent increases in nominal interest rates, and positive supply shocks. Here, we discuss comparative statics on government spending and increase in nominal interest rates. Other comparative statics are shown in appendix B.

Table 1: Comparative Statics

	Secular Stagnation		BSGU	
	Inflation	Output	Inflation	Output
Fiscal policy: $G\uparrow$	+	+	-	-
Neofisherian exit: $R\uparrow$	-	-	+	+
Higher productivity: $Y^*\uparrow$	-	-	+	+

Because of local determinacy of the secular stagnation steady state, the comparative static experiment is well-defined without the need for additional assumptions. With the BSGU steady state, we assume that inflation expectations do not change drastically to push the economy to the full-employment steady state in response to the experiment. Furthermore, in subsequent comparative statics, there are no labor supply effects because of inelastic labor supply. This is done for analytical tractability. In Section 4, we show the comparative statics in a quantitative model with elastic labor supply.

2.5.1 Government spending multiplier

We introduce government spending financed by lump sum taxes in the model. The resource constraint in the economy is modified to

$$Y = C + G$$

For any given level of output, fiscal policy crowds out private consumption. Lower consumption implies higher marginal utility of consumption, which offsets the increased demand for liquidity. Thus, this permanent change in fiscal policy unambiguously increases the natural rate, as shown below:

$$1 + r^* = \frac{1 - \delta(Y - G)}{\beta} = \frac{1 - \delta(1 - G)}{\beta} \implies \frac{dr^*}{dG} > 0$$

The subsequent rise in natural rate lowers the kink in the AD curve, since the position of the kink depends on the natural interest rate:

$$\Pi_{kink} = \left(\frac{1}{R^*}\right)^{\frac{1}{\alpha_\pi}} \Pi^*$$

At the secular stagnation steady state, the upward sloping portion of the AD curve intersects the AS at a higher level of output since the AD curve is steeper than the AS curve locally. Thus, the policy experiment is expansionary when the economy suffers from fundamentals driven deficiency in aggregate demand. Similarly, when the policy is implemented locally around the BSGU steady state, the policy brings about a permanent decline in steady state output. This is because of a higher steady state natural rate, which implies higher deflation with pessimistic expectations (Mertens and Williams, 2018).

2.5.2 Neo-Fisherian Exit

We now discuss the effects of a permanent increase in nominal interest rate. We model the policy as a permanent change in the intercept of the Taylor rule, a:

$$R^{new} = \max\{1 + a, a + R^* \left(\frac{\Pi}{\Pi^*}\right)^{\alpha_\pi}\} = a + R$$

where a is increased to a positive number from zero. This policy simultaneously increases the lower bound on nominal interest rate and thus does not have any effect on the placement of the kink in the aggregate demand curve. In the aggregate demand curve:

$$Y = \frac{1}{\delta}\left[1 - \frac{\beta(a + R)}{\Pi}\right]$$

Given the inflation rate, an increase in a lowers output demanded. At the secular stagnation steady state, this induces deflationary pressures that increases the real interest rate gap and causes a further drop in output. In contrast, during a BSGU trap, an increase in nominal interest rate anchors agents' expectations to higher levels of inflation, thus obtaining the neo-Fisherian results (Schmitt-Grohé and Uribe, 2017).

3 Quantitative DSGE Model

We present a quantitative analysis based on a stylized small-scale New Keynesian model that has been widely studied in the literature (An and Schorfheide, 2007). Our model economy is composed of households, intermediate good producers, final good producers and a government. We briefly describe each of the decision problems next. A detailed derivation of equilibrium conditions is relegated to the appendix.

3.1 Households

$$\max_{C_t(k),H_t,B_t(k)} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t(k)/A_t)^{1-\tau} - 1}{1-\tau} - \chi_H \frac{H_t^{1+1/\eta}}{1+1/\eta} - \delta \frac{B_t}{A_t P_t} \right) \right],$$

subject to:

$$P_t C_t(k) + B_t(k) + T_t = W_t H_t + R_{t-1} B_{t-1}(k) + P_t D_t(k) + P_t SC_t,$$

The household derives utility from consumption C_t and from holding (real) stock of risk-free nominal bonds B_t, and disutility from hours worked H_t. The parameter χ_H scales the steady-state level of hours worked. The parameter δ regulates the marginal utility from holding bonds. The risk-free nominal bond pays a gross nominal interest rate R_t Each household supplies homogeneous labor services H_t in a competitive labor market taking the aggregate wage W_t as given. It collects interest payments on bond holdings, real profits D_t from intermediate good producers, pays lump sum taxes T_t, and receives payouts SC_t from trading a full set of state(k)-contingent securities.

3.2 Final Good Firms

The perfectly competitive, representative, final-good producing firm combines a continuum of intermediate goods indexed by $j \in [0, 1]$ using the technology:

$$Y_t = \left(\int_0^1 Y_t(j)^{1-\nu_t} dj \right)^{\frac{1}{1-\nu_t}}.$$

Here $1/\nu_t > 1$ represents the elasticity of demand for each intermediate good. Profit maximization implies that the demand for intermediate goods is given by:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-1/\nu_t} Y_t.$$

where the price of the final good P_t is given by :

$$P_t = \left(\int_0^1 P_t(j)^{\frac{\nu_t - 1}{\nu_t}} dj\right)^{\frac{\nu_t}{\nu_t - 1}}.$$

3.3 Intermediate Good Producers

Intermediate good j is produced by a monopolist who has access to the following production technology:

$$Y_t(j) = A_t H_t(j), \quad \text{with} \quad A_t = A_{t-1} z_t,$$

where A_t denotes the aggregate level of technology that is common to all firms, and z_t represents the stochastic (stationary) movements in TFP.

Intermediate good producers buy labor services $H_t(j)$ at a nominal price of W_t. Moreover, they face nominal rigidities in terms of price adjustment costs. These adjustment costs, expressed as a fraction of total output, are defined by the function $\Phi_p(.)$:

$$\Phi_p\left(\frac{P_t(j)}{P_{t-1}(j)}\right) = \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \Pi^*\right)^2 Y_t$$

where Π^* is the inflation rate in the targeted steady state. Taking as given nominal wages, final good prices, the demand schedule for intermediate products and technological constraints, firm j chooses its the price $P_t(j)$ to maximize the present value of future profits:

$$\max_{\{P_{t+s}(j)\}} \quad \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s Q_{t+s|t} \left(\frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - \Phi_p\left(\frac{P_{t+s}(j)}{P_{t+s-1}(j)}\right) Y_{t+s} - \frac{W_{t+s} Y_{t+s}(j)}{z_{t+s} P_{t+s}} \right) \quad ,$$

subject to

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-1/\nu_t} Y_t.$$

Government Policies

The desired policy rate is set according to the following rule:

$$R_t^* = \left[r\Pi^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\psi_1} \left(\frac{Y_t}{Y_{t-1}}\right)^{\psi_2} \right]^{1-\rho_R} R_{t-1}^{\rho_R},$$

Here r is the steady-state real interest rate, Π_t is the gross inflation rate defined as Π_t, and Π^* is the target inflation rate, which in equilibrium coincides with the steady state inflation rate. The actual policy rate relevant for agents decisions is subject to the zero lower bound constraint:

$$R_t = \max\{1, R_t^*\}$$

The government levies a lump-sum tax (subsidy) to finance any shortfalls in government revenues (or to rebate any surplus). The government's budget constraint is given by:

$$P_t G_t + R_{t-1} B_{t-1} = T_t + B_t,$$

where $G_t = \left(1 - \frac{1}{g_t}\right) Y_t$ is the government expenditure.

3.4 Resource constraint

We assume that the price adjustment costs are rebated back to the household in lump-sum fashion as part of the government transfers.[18] Hence, the market-clearing resource constraint is given by:

$$C_t + G_t = Y_t$$

[18] An analogous interpretation would be to consider these costs as mental accounting costs for the firms or model these in the utility function of the representative agent.This assumption allows us to avoid unnatural results commonly associated with resource costs modeled in terms of output.

Finally, we assume nominal bonds are in net zero supply

$$B_t = 0$$

3.5 Shocks

There are three exogenous disturbances in the model: (i) exogenous changes to government expenditure g_t, (ii) exogenous changes to the growth rate of productivity z_t, and (iii) exogenous changes to the inverse demand elasticity for intermediate goods ν_t. We assume that these exogenous components obey the following auto-regressive processes:

$$\ln g_t = (1-\rho_g)\ln(g^*) + \rho_g \log g_{t-1} + \sigma_g \epsilon_{g,t}, \qquad \epsilon_{g,t} \sim N(0,1)$$

$$\ln \nu_t = (1-\rho_\nu)\ln(\nu^*) + \rho_\nu \log \nu_{t-1} + \sigma_\nu \epsilon_{\nu,t}, \qquad \epsilon_{\nu,t} \sim N(0,1)$$

$$\ln z_t = \rho_z \ln z_{t-1} + \sigma_z \epsilon_{z,t}, \qquad \epsilon_{z,t} \sim N(0,1)$$

We allow the g_t and ν_t processes to have a non-zero mean, given by g^* and ν^*, that pin down the steady-state level of government consumption and the steady-state level of price markups, respectively.

3.6 Equilibrium Conditions and Competitive Equilibrium

Our model economy evolves around a stochastic balanced growth path given by the level of technology A_t. In order to solve the model we introduce the following stationary transformation: $\tilde{X}_t = X_t/A_t$.

Definition 1 *A competitive equilibrium in terms the stationary variables is given by the sequence of quantities and prices* $\{\tilde{Y}_t, \tilde{C}_t, R_t, \Pi_{t+1}\}$ *which satisfy equations (13) - (16), given an exogenous* *sequence for processes* $\{g_t, \nu_t, z_t\}$*:*

$$1 = \beta \mathbb{E}_t \left[\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\tau} \frac{R_t}{z_{t+1} \Pi_{t+1}} \right] + \delta \tilde{C}_t^{\tau} \tag{13}$$

$$\zeta_t = \phi \nu_t \beta \mathbb{E}_t \left[\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\tau} \frac{\tilde{Y}_{t+1}}{\tilde{Y}_t} \Pi_{t+1} \left(\Pi_{t+1} - \Pi^* \right) \right] \tag{14}$$

$$R_t = \max \left\{ 1, \left[r \Pi^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\psi_1} \left(\frac{\tilde{Y}_t}{\tilde{Y}_{t-1}} \right)^{\psi_2} \right]^{1-\rho_R} R_{t-1}^{\rho_R} \right\} \tag{15}$$

$$\tilde{C}_t = \frac{\tilde{Y}_t}{g_t}, \tag{16}$$

where $\zeta_t = (1 - \nu_t) - \chi_H \tilde{C}_t^{\tau} \tilde{Y}_t^{1/\eta} + \nu_t \phi \left(\Pi_t - \Pi^* \right) \Pi_t$.

As in our benchmark model in Section 2, equation (13) is the modified Euler equation in which the marginal utility of holding government bonds enters as an additional term.

4 Steady State of the Quantitative Model

4.1 Calibration

Because of the multiplicity of steady states, we calibrate parameters to match observed empirical moments in the data for Japan. Most of the calibrated parameter values are borrowed from Aruoba et al. (2017*b*), and presented in Table 3. The remaining parameters are set such that the economy is either in the secular stagnation steady state or the BSGU steady state.

The Frisch labor supply elasticity is fixed at 0.85, based on micro-level data based estimates of Kuroda and Yamamoto (2008). The (inverse) elasticity of demand for intermediate goods, ν, is set to a value of 0.1 to generate a steady state markup of 11% for the monopolistically competitive firms. Japan did not officially adopt an inflation target until 2013Q2. Consequently, we choose a zero inflation target $\Pi^* = 1$ for convenience. The labor disutility parameter χ_H is chosen so as to normalize the output in the targeted inflation steady-state at one. Steady state government spending ratio is chosen from the consumption output ratio of 85.32% in the Japanese data.

The remaining parameters β, δ, ϕ are chosen to jointly match targets for natural interest rate and average inflation in Japan. For the natural rate, we adopt two different targets depending

Table 2: Fixed Parameters

η	ν	τ	Π^*	g^*
Inverse Frisch elasticity	Price s.s. markup	Intertemporal elasticity of substitution	Inflation target	Government Spending share
0.85	0.1	0.95	1.0025	0.1468

on the steady state. Under secular stagnation we chose an annual rate of -1%. This is based on two studies by Fujiwara et al. (2016) and Iiboshi et al. (2018) that separately estimate a series for the natural rate of interest in Japan based on Laubach and Williams (2003). They find that the quarterly estimate was often -0.5% since late 90s, and -2% at the lowest level. In contrast, the BSGU steady state is calibrated to imply a (annualized) long-run real interest rate of 0%. For the sake of transparency, we fix $\beta = 0.89$ and find the remaining two parameters to target the respective steady-state moments. The calibration implies an inflation rate of -1% for both steady states. Furthermore, the calibration implied slope of the log-linearized Phillips curve κ is 0.003 at the secular stagnation steady state, and 0.011 when calibrate the BSGU steady state. These values lie in the range of the conventional estimates found in the literature (see Aruoba, Bocola and Schorfheide 2017*a*).

Table 3: Calibration

	β	δ	ϕ
	Discount factor	Euler Equation Wedge	Adjustment cost parameter
Secular Stagnation	0.89	0.1319	1.2×10^3
BSGU Trap	0.89	0.1290	631
	Natural Rate	Inflation	Output Gap
Secular Stagnation	-1.1	-1.1	-4.7
BSGU Trap	0	-1.1	-2.4

Notes: The table shows the parameter values of the model for the baseline calibration.

4.2 Steady-state representation

In the last section, we show that the existence of a secular stagnation equilibrium does not depend on our choice of the downward nominal wage rigidity. In fact, a quantitative model with pricing frictions often used in quantitative work can accommodate the permanent stagnation steady state. The important ingredient is the modified Euler equation, which features a negative relationship between output and the short-term real interest rate in steady state. We now show an analogous AD-AS representation of the quantitative steady state.

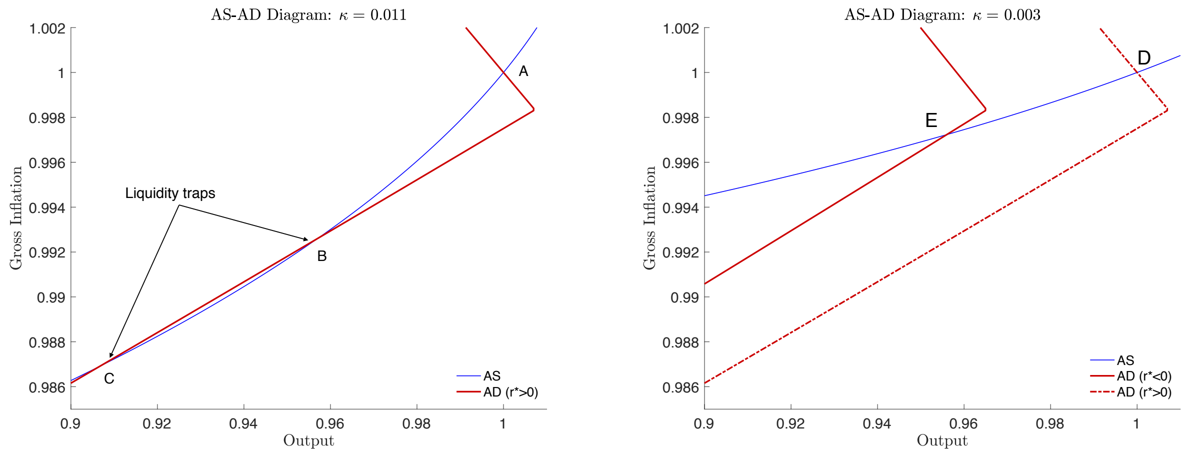
The left panel of figure 7 plots the aggregate demand and the aggregate supply curves for the case in which prices are relatively flexible. As in Section 2, we set $\delta < 1 - \beta$ which implies that the natural rate of interest is positive. The targeted-inflation steady state emerges when the aggregate supply curve intersect the downward sloping portion of the aggregate demand curve, and features target inflation equal Π^*, positive nominal interest rates and output normalized to one (depicted as *point A*).

Along with the targeted inflation steady state, there exist two additional steady states with zero interest rates. The steady state depicted at *point B* is locally indeterminate and satisfies the BSGU steady-state criterion. At this intersection, the aggregate supply curve is steeper than the aggregate demand curve. Furthermore, because intermediate good producers face convex costs of price adjustments, the aggregate supply curve is nonlinear. The curvature of the aggregate supply flattens out as deflation increases the cost of adjusting prices. This opens up the possibility of an additional steady state, as depicted by *point C* where the aggregate supply schedule is flatter relative to the aggregate demand schedule. Although this steady state is locally determinate and features zero nominal interest rates and output permanently below full employment, it does not satisfy our definition of secular stagnation. This is because the natural rate remains positive at this steady state. For this reason, we do not pursue this steady state further in our analysis. We leave the investigation of this interaction of the AS curve and multiple stagnation steady states to future work.[19]

In the right panel of Figure 7, we show the case of the aggregate supply curve with a relatively tight lower bound on inflation. When the natural rate is positive, $\delta < 1 - \beta$, the aggregate demand schedule intersects the aggregate supply curve at the targeted-inflation steady state (denoted by

[19] Important takeaway to note is that multiple steady states with zero nominal interest rates may emerge due to the nonlinearities induced by pricing frictions. Similar multiplicity can be obtained through changes in the curvature of the Euler equation, see for example Heathcote and Perri (2018) for a model with precautionary savings, perfectly downwardly rigid nominal wages and multiple zero interest rate steady states.

Figure 7: Steady-State Equilibria of the Quantitative Model



point D). In contrast, when the natural interest rate is negative, $\delta > 1 - \beta$, aggregate demand intersects aggregate supply at the secular stagnation steady state (denoted by *point E*). Consistent with the results in Section 2, Figure 7 shows that when the long-run aggregate supply curve does not accommodate high enough deflation, in this case because of the higher cost of price adjustment, the BSGU steady state does not exist. This highlights the importance of assumptions imposed on the supply side of the economy when analyzing alternative explanations for prolonged liquidity trap episodes.

5 Reassessing Japan: Expectations Traps vs Secular Stagnation

We illustrate an important quantitative use of our framework. We estimate the model for Japan with zero nominal interest rates using standard rational expectations methods (Herbst and Schorfheide, 2016). The estimations are conducted by log-linearizing the model around the secular stagnation and the BSGU steady state separately. The marginal likelihood evaluation of the estimated models suggests that either of the hypotheses is plausible in explaining the persistent stagnation in Japan. However, the estimated dynamics around either of the steady state feature different transmission mechanisms that we highlight.

Because we consider a period in which the ZLB was binding in Japan, we assume that agents expect to be in the liquidity trap permanently. A long-lasting fundamentals-driven liquidity trap cannot be obtained in the standard models. In contrast, the modified Euler equation allows us to consider such a possibility. While permanent shocks to expectations or fundamentals could move the economy back to the targeted inflation steady state, we do not allow for transitions between steady states. We leave a richer quantitative exercise for future work. Henceforth, ZLB permanently binds and we estimate the local dynamics around each steady state.

5.1 Data

We use data on output, consumption and inflation from Japan while imposing the zero nominal interest rates during the period 1995:Q4 to 2012:Q4. In Section 4.2, we calibrated the structural parameters $\tau, \beta, \delta, \phi, \nu$ to allow the model to be in secular stagnation or the BSGU steady state. Because steady state features ZLB, we do not need to specify the parameters governing the monetary policy rule in equation (15). Instead, we peg the nominal interest rate to its theoretical lower bound of $R_t = 1$ throughout.

5.2 Solution and estimation

Following Lubik and Schorfheide (2004) the first order dynamics to the equilibrium conditions can be summarized by the following system of linear rational expectation equations (LRE):

$$\begin{aligned} \Gamma_0(\theta) s_t &= \Gamma_1(\theta) s_{t-1} + \Psi(\theta)\boldsymbol{\varepsilon}_t + \Pi(\theta)\boldsymbol{\eta}_t, \\ s_t &= \left[\hat{Y}_t, \hat{\Pi}_t, \mathbb{E}_t\hat{Y}_{t+1}, \mathbb{E}_t\hat{\Pi}_{t+1}, \hat{g}_t, \hat{\nu}_t, \hat{z}_t\right]', \\ \boldsymbol{\varepsilon}_t &= \left[\epsilon_{g,t}, \epsilon_{\nu,t}, \epsilon_{z,t}\right]', \\ \boldsymbol{\eta}_t &= \left[\left(\hat{Y}_t - \mathbb{E}_{t-1}\hat{Y}_t\right), \left(\hat{\Pi}_t - \mathbb{E}_{t-1}\hat{\Pi}_t\right)\right]', \end{aligned} \tag{17}$$

where the vector s_t contain endogenous variables including expectations, the vector $\boldsymbol{\varepsilon}_t$ contains the three structural shocks of the model, and the vector $\boldsymbol{\eta}_t$ corresponds to the one-period ahead forecast errors associated with the two expectational variables of the system. The matrices $\Gamma_0, \Gamma_1, \Psi, \Pi$ are functions of the structural parameters denoted by θ. We use standard Bayesian methods to estimate the following set of parameters:

$$\theta^{SecStag} = \{\rho_g, \rho_\nu, \rho_z, \sigma_g, \sigma_\nu, \sigma_z\} \quad \text{and}$$
$$\theta^{BSGU} = \{\rho_g, \rho_\nu, \rho_z, \sigma_g, \sigma_\nu, \sigma_z, \sigma_\zeta, \sigma_{\zeta,\epsilon_g}, \sigma_{\zeta,\epsilon_\nu}, \sigma_{\zeta,\epsilon_z}\}$$

As discussed in Section 2, when the economy is at the secular stagnation steady state, equation 17 satisfies the Blanchard and Kahn (1980) determinacy conditions and a unique solution can be constructed using standard methods. For the secular stagnation model we estimate the persistence and standard deviation of fundamental shocks g_t, ν_t, z_t. For the case in which the economy is near the BSGU steady state the LRE system exhibits self-fulfilling dynamics in which the expectational variables become linear functions of the structural shocks $\boldsymbol{\varepsilon}_t$ and the sunspot shock, which we denote by ζ_t. Following the method of Bianchi and Nicolo (2017), we modify equation 17 to construct a solution to the system of LRE in the presence of self-fulfilling expectations. To account for indeterminacy in the BSGU model we need to estimate the standard deviation of the sunspot shock σ_ζ and the covariances of the sunspot shock with the fundamental shocks: $\sigma_{\zeta,\epsilon_g}, \sigma_{\zeta,\epsilon_\nu}, \sigma_{\zeta,\epsilon_z}$

Table 4 presents the assumed priors and the estimated posteriors for θ approximated around the BSGU and the secular stagnation steady state. The priors for the common parameters across models are identical and we use fairly agnostic priors. We set priors over the correlations, instead of the covariances, of the fundamental shocks and the sunspot shock and use an uninformative prior in order to let the data pin down the preferred correlation structure to describe the dynamics under indeterminacy.

The estimated parameters governing the persistence and volatility of the fundamental shocks are similar across the BSGU and the secular stagnation models, with the exception of the parameters of the technology shock which have a higher persistence and volatility in the secular stagnation model. The standard deviation of the sunspot shock in the BSGU specification is statistically different from zero and with a magnitude similar to that of the technology shock. The estimated correlation between the fundamental and sunspot shocks varies substantially. The data favors a high correlation between the markup shock and the sunspot shock while picking up a small correlation of the sunspot shock with the other two fundamental shocks. These results illustrate that the BSGU model relies on a different transmission mechanism than the secular stagnation model. In the next

Table 4: DSGE Estimated Parameters

Parameter	Distribution	Para (1)	Para (2)	BSGU	Secular Stagnation
			Fundamental shocks parameters		
ρ_g	$\mathcal{B}$	0.6	0.2	0.9605	0.8432
				[0.9273, 0.9949]	[0.7832, 0.9076]
ρ_ν	$\mathcal{B}$	0.6	0.2	0.1824	0.1655
				[0.039, 0.3201]	[0.0469, 0.271]
ρ_z	$\mathcal{B}$	0.4	0.1	0.604	0.6121
				[0.3503, 0.8636]	[0.3459, 0.9062]
σ_g	$\mathcal{IG}$	0.03	2	0.0045	0.0045
				[0.0038, 0.0051]	[0.0038, 0.0051]
σ_ν	$\mathcal{IG}$	0.03	2	0.0172	0.0134
				[0.0142, 0.0202]	[0.011, 0.0156]
σ_z	$\mathcal{IG}$	0.03	2	0.005	0.0061
				[0.0023, 0.0078]	[0.0024, 0.0098]
			Sunspot shock parameters		
σ_ζ	$\mathcal{IG}$	0.03	2	0.0031	-
				[0.0026, 0.0036]	-
$corr(\epsilon_z, \epsilon_\zeta)$	$\mathcal{U}$	-1	1	0.0025	-
				[-0.167, 0.1759]	-
$corr(\epsilon_\nu, \epsilon_\zeta)$	$\mathcal{U}$	-1	1	0.9273	-
				[0.8812, 0.9774]	-
$corr(\epsilon_g, \epsilon_\zeta)$	$\mathcal{U}$	-1	1	-0.2681	-
				[-0.3755, -0.174]	-
$\log(p(\mathcal{Y}_t))$				-280.88	-286.16

Notes: Para (1) and Para (2) list the means and the standard deviations for the Beta ($\mathcal{B}$) distributions; the upper and lower bound of the support for the Uniform distribution ($\mathcal{U}$); and s and v location parameters for the Inverse Gamma ($\mathcal{IG}$) distribution, where $p_{IG}(\sigma|v,s) \propto \sigma^{-v-1}\exp(-vs^2/2\sigma^2)$. Numbers in square brackets denote 90% credible intervals.

section we investigate in more detail the difference in the transmission mechanism of shocks across the two models.

The last row of Table 4 shows the overall fit of the two specifications using the marginal data density estimates, denoted with $\log(p(\mathcal{Y}_t))$. It represents the likelihood function integrated over the prior distribution of the parameters. The results reveal that Japanese data over 1995-2012 seems to be more likely to be explained by the BSGU stagnation hypothesis. Because the dynamics under BSGU are indeterminate, in our estimation we chose a particular solution by allowing the correlations between fundamental and sunspot shocks to be estimated. In Table 5 we explore the role of alternate correlations in accounting for the importance of sunspot shocks in explaining the better fit of the BSGU model. The first row shows the marginal likelihood of the BSGU model when we re-estimate the parameters after fixing one or all of the correlations with the sunspot shock to zero. We find that these correlation are key in improving the fit of the model. In particular it is the correlation of sunspot shocks with markups shocks that largely accounts for the better fit of BSGU over Secular Stagnation. The last row of the table transforms the marginal log-likelihood in terms of posterior probabilities assuming both models are given equal prior odds. In the absence of correlations or when the the correlation of markups with the sunspot are set to zero, we obtain that the data is better explained by the secular stagnation hypothesis. This exercise highlights that the transmission mechanism of shocks under indeterminacy plays a key role in explaining the Japanese experience.

Table 5: Model Comparison

	Baseline	**corr**$(\nu,\zeta)=0$	**corr**$(g,\zeta)=0$	**corr**$(z,\zeta)=0$	**No corr**
$\log(p(\mathcal{Y}_t\|BSGU))$	-280.9	-296.4	-286.2	-278.9	-332.0
$prob(BSGU\|\mathcal{Y}_t)$	0.99	0.00	0.48	1.00	0.00

5.3 Transmission of shocks

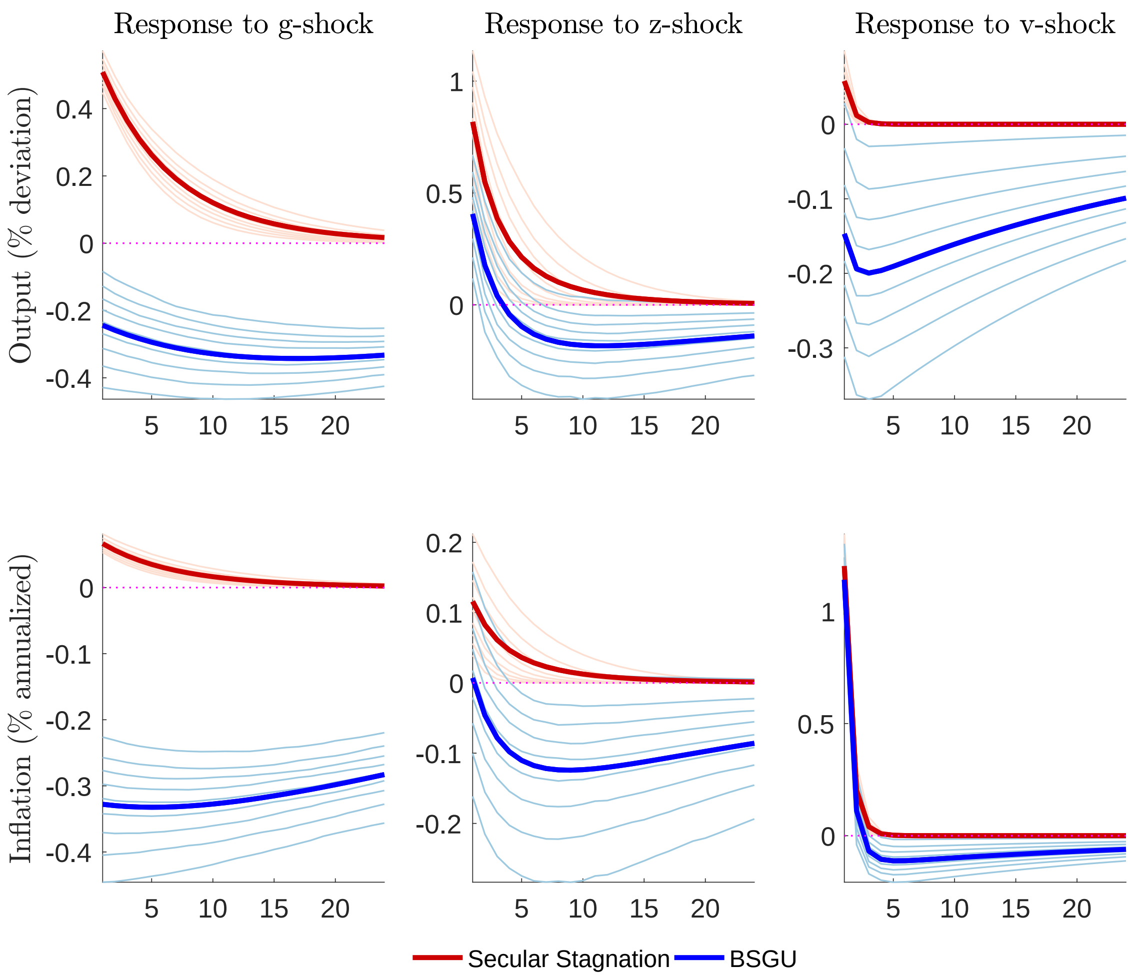
We now illustrate the difference in dynamics of BSGU and secular stagnation through impulse responses. Figure 8 contrasts impulse response function near the Secular Stagnation steady state (red line) to that near the BSGU steady state (blue line) for output and inflation after a one-time unanticipated shock to government expenditure, aggregate productivity and markups. Around the secular stagnation steady state, there is no crowding out of consumption in response to an increase in government expenditure. This is because there is no endogenous monetary policy reaction that counteracts the inflationary pressures of increased government spending. As a result, the government expenditure shock is expansionary. Similarly, technology shocks in the model have permanent effects on the level of TFP, and thus act as positive news shocks about future productivity and lead to an expected increase in future and current demand, which translate into higher inflation. Finally, markup shocks that create exogenous shifts in the Philips curve have an expansionary effect on output. Higher markups put upward pressure on nominal wages and because the economy remains at the zero bound this reduces the real interest rate and increases consumption. This result echoes the implications of transitory increases in productivity that are contractionary (*paradox of toil*) in temporary liquidity traps driven by shocks to the discount rate (Eggertsson, 2010).

The dynamics in the neighborhood of BSGU steady state are depicted by the solid green line in Figure 8. The responses of the economy to an increase in government expenditure are dramatically different. In the presence of deflationary expectations, higher government expenditure signals weaker demand which further reduces inflation expectations. Because the interest rate cannot fall below the zero lower bound, lower expected inflation raises the real rate. This exerts a contractionary pressure on household consumption. These results are similar to dynamic responses documented in Mertens and Ravn (2014) and Aruoba et al. (2017*b*). Technology shocks raise output temporarily but because inflation expectations remain deflationary, actual inflation drops and the economy sinks further into stagnation as the shock unwinds. Similarly, higher markups boost inflation on impact. But the effect quickly reverses turning into deflation as agents expect to remain near the BSGU steady state in the long run. The deflationary expectations dominate leading to an increase in the real interest rate and an output contraction.

5.4 Natural rate of interest and equilibrium selection

For the estimated models of the previous section we made two assumptions that we now discuss. First, we calibrated the model to target a particular value for the steady-state natural rate consistent with each stagnation hypothesis. We now show that the choice of natural rate in our calibration is reasonable. Second, we centered the priors for the correlation between sunspot shocks and

Figure 8: Secular Stagnation vs BSGU

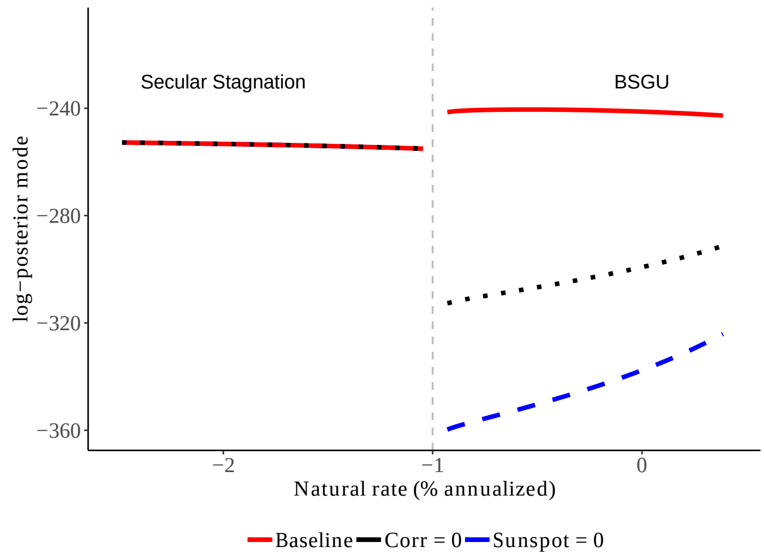


Notes: Thicker lines correspond to the IRF computed at the posterior mean. The lightly colored lines correspond to posterior IRF deciles drawn from the posterior distribution of estimated parameters.

the fundamental shocks around zero (referred to as the 'orthogonality solution' by Lubik and Schorfheide 2003). Under indeterminacy researchers have to specify the correlation of sunspot shocks with the fundamental shocks in order to pin down the equilibrium (Lubik and Schorfheide, 2003). We find that the equilibrium selection is crucial in identifying the hypothesis that best explains the Japanese experience.

In our model, the parameter δ (liquidity premium) regulates the natural interest rate. Using a grid for δ we evaluate the model over a reasonable range of values for the natural rate. We endogenize the price adjustment cost parameter (ϕ) in order to match the steady state inflation rate at the data moment of -1.1%. Since the inflation target is fixed at 1%, values of the natural

Figure 9: Posterior Mode Density: Secular Stagnation vs BSGU



Notes: The red line plots the log-posterior mode of the model when the volatility of the sunspot shock and correlations with the fundamental shocks are estimated. The dotted black line plots the computed likelihood in the BSGU regime imposing zero correlation between sunspot shocks and fundamental shocks. The dashed blue line represents the likelihood in the BSGU regime when sunspot shocks are shutdown $\sigma_u = 0$. Under secular stagnation, there is a unique equilibrium and these lines overlap.

rate (r^*) below -1% lie in the secular stagnation territory while values above -1% lie in the BSGU territory. For each value of the natural rate, we maximize the posterior density of the model described in equation (17).

The red line plots the posterior mode under BSGU with estimated correlations that best fit the data. We label the results as "baseline" and confirm that the choices of empirical moment of natural interest rate in our calibrations are close to the values that maximize the posterior density under each hypothesis. We plot two additional solutions based on alternative equilibrium selection criteria under BSGU. At each natural rate, the dotted black and the dashed blue lines plot the posterior mode density while imposing the restriction of zero correlations between fundamental and sunspot shocks, and no sunspot shocks respectively.

When these correlations are fixed to zero ('orthogonality' solution) or when there are no sunspot shocks under indeterminacy (Lubik and Schorfheide, 2004), the secular stagnation hypothesis emerges as the preferred hypothesis. Consistent with our model comparison results in Section 5.2, when we let the model estimate the correlations to best fit the data, the expectations-driven narrative is the preferred hypothesis for the observed stagnation in Japan. Our exercise further illustrates that sunspot shocks in isolation are not enough to account for the Japanese experience. From the perspective of the model, what makes the BSGU a better fit is the different propagation of fundamental shocks under indeterminacy. This is captured by the estimated correlations between fundamental and sunspot shocks.

6 Conclusion

In this paper, we developed a framework to formally distinguish two hypotheses of stagnation: expectations-driven and fundamentals-driven stagnation. The two hypotheses differ in the local determinacy of the equilibrium. Secular Stagnation is defined as the locally determinate equilibrium with zero nominal interest rates, positive unemployment and below target inflation rate. Expectations-driven trap is defined as the locally indeterminate equilibrium with zero nominal interest rates, positive unemployment and below target inflation rate. The local determinacy property is the source of different policy implications in the two episodes.

This is the first paper to quantitatively compare the two-stagnation hypothesis for Japan. The indeterminate model requires the researcher to specify the correlation of sunspot shocks with the fundamental shocks in order to pin down the equilibrium. We find that the equilibrium selection under indeterminacy is crucial in identifying the hypothesis that explains Japanese stagnation.

When these correlations are fixed to zero ('orthogonality' solution of Lubik and Schorfheide 2003) or when there are no sunspot shocks under indeterminacy (Lubik and Schorfheide, 2004), the secular stagnation hypothesis emerges as the dominant hypothesis. When we let the model pick the correlations to best fit the data, the expectations-driven narrative is the driver of stagnation in Japan.

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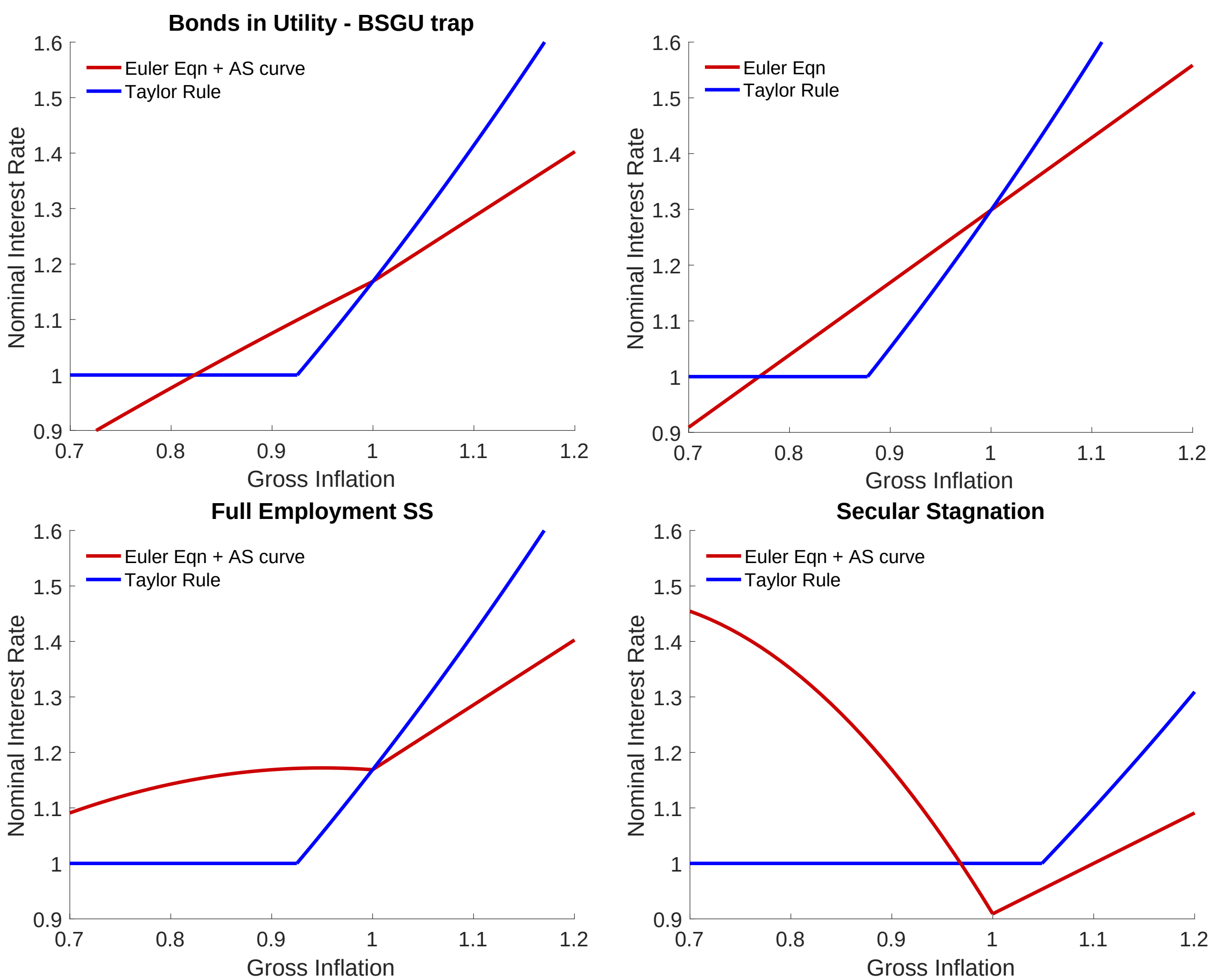
A i-Π graphs representation as in Benhabib et al. (2001)

in our model with $\delta > 0$, the Euler equation cannot be graphed without specifying the aggregate supply block. The top left panel in Figure 10 plots the (Π, i) space representation for the BSGU steady state result highlighted in Proposition 3. There is a kink in the combined Euler equation and AS curve when inflation rate is one. It is a straight line above inflation rate of one, and becomes concave for values of inflation below one. This curvature is inherited form the downward nominal wage rigidity. If this rigidity is strong enough, there does not exist a BSGU steady state (as in Proposition 1), illustrated in bottom left panel. Furthermore, when the constraints on monetary policy are binding such that it cannot replicate the natural rate, there exists a unique secular stagnation steady state as shown in the bottom right panel.

Figure 10

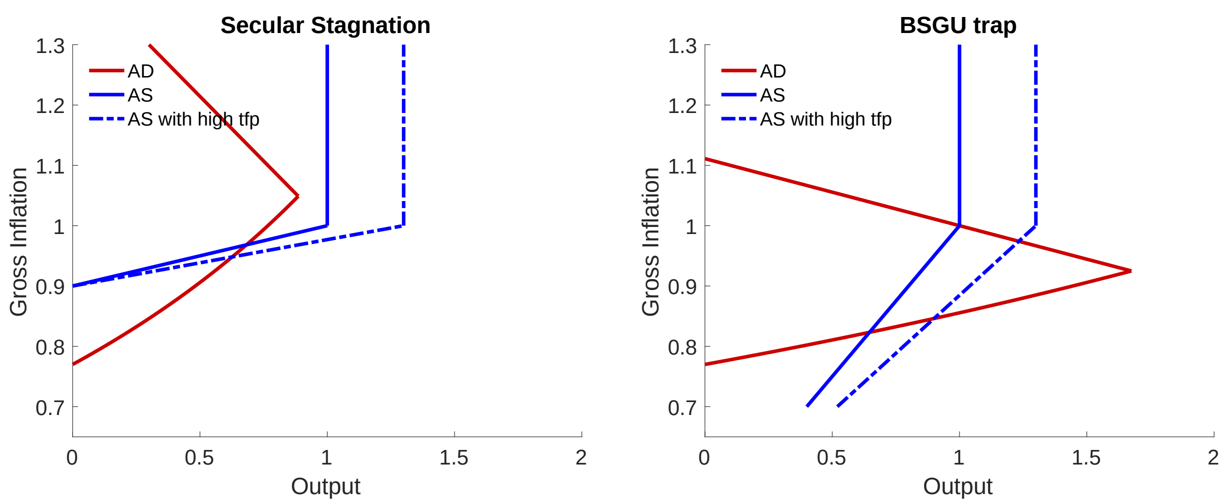


B Other comparative Statics

B.1 Increasing Potential/Paradox of Toil

Eggertsson (2010) found that a temporary positive supply shock (cut in labor taxes, increase in TFP) can be contractionary under a temporary fundamentals driven liquidity trap, provided the supply shock is less persistent than the demand deficiency. This result has been labeled as the *Paradox of Toil*. Since the fundamentals driven trap is of permanent nature, the paradox of toil emerges in our setting regardless of the duration of the supply shock (See the left panel in figure 11). In contrast, a positive TFP shock is expansionary both at the targeted inflation steady state and the BSGU steady state (Mertens and Ravn, 2014).

Figure 11: Permanent increase in TFP



B.2 Changing Lower Bound

Increasing the lower bound from zero makes the BSGU steady state less contractionary and the reverse holds true for the Sec Stag steady state. The normal steady state is not impacted by the changing the lower bound (Figure 12).

B.3 Increasing Inflation Target / Timidity Trap

Increasing inflation target may not change the equilibrium allocation unless it is large enough and credible. The left panel of Figure 13 shows this for secular stagnation, where increasing the inflation target gives rise to the possibility of an equilibrium with full employment only if the

Figure 12: Permanent increase in lower bound

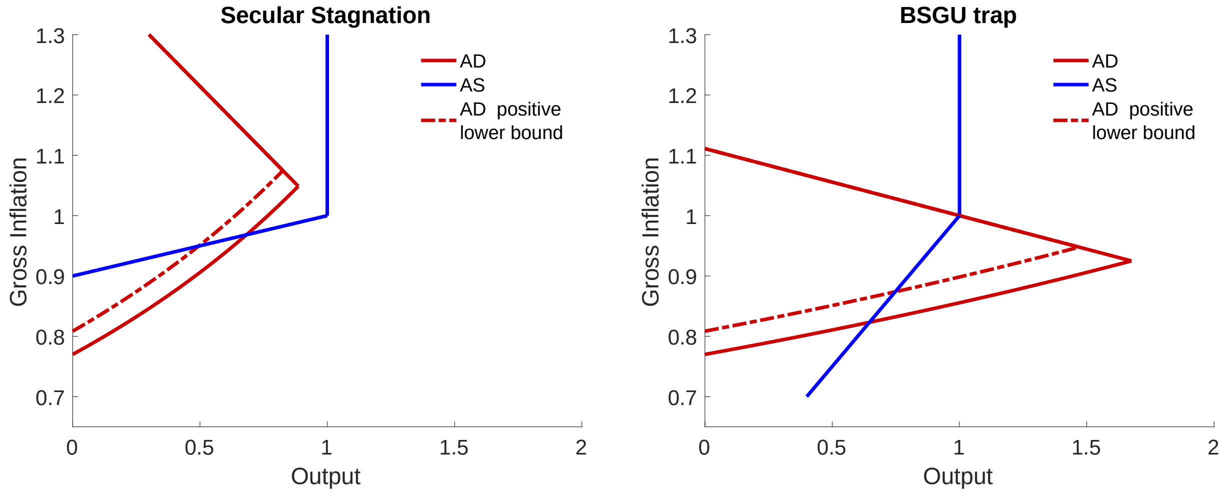
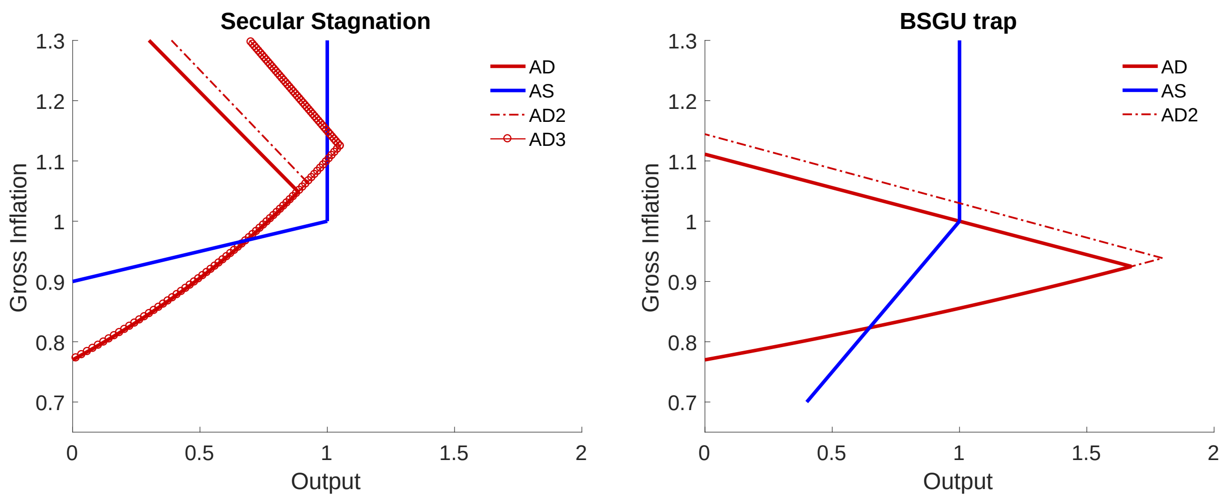


Figure 13: Higher inflation target



inflation target is increased to a sufficiently high level. This formalizes the *timidity trap* argument made by Krugman.[20] The right panel shows that increasing the inflation target might move the economy away from the BSGU steady-state. A robust prediction of our analysis is that in both cases, a rise in inflation target must be credible in order to be expansionary.

[20]See: Krugman, Paul R. "The Timidity Trap." New York Times March 20, 2014. And: Krugman, Paul R. "Timid Analysis (Wonkish)." New York Times March 21, 2014.

C Analytical results with Eggertsson et al. (2019) wage setting

We replace the wage rigidity assumption in the simple model with the specification from Eggertsson et al. (2019) which in the steady-state yields the following:

$$\frac{\gamma}{\Pi} = 1 - (1-\gamma)y^{\frac{1-\alpha}{\alpha}}$$

Steady-state equilibrium is then given by:

$$1 = \frac{\beta R}{\Pi} + \delta C^{\sigma}$$

$$C = F(h)$$

$$F'(h) = w$$

$$\frac{\gamma}{\Pi} = 1 - (1-\gamma)C^{\frac{1-\alpha}{\alpha}} \quad \text{if } \Pi \leq 1$$

$$u = 1 - h$$

$$R = \max\{1, R^* \left(\frac{\Pi}{\Pi^*}\right)^{\alpha_\pi}\}$$

where $\alpha_\pi > 1$.

Aggregate Supply: At $Y = 1$, it becomes a vertical line. For $\Pi < 1$, it is strictly monotonic and upward sloping. At $\Pi = \gamma$, $Y = 0$.

The slope of the upward sloping portion of the aggregate demand is $\frac{\delta\Pi^2}{\beta} > 0$. The second derivative is strictly positive for $y \geq 0$ ($\frac{2\delta^2\Pi^3}{\beta^2}$). As $y \to 0$, $\Pi_{AD} \to \beta$. When $R^* > 1$, $\Pi_{kink} < \Pi^* = 1$.

When $y = 0$, $\Pi_{AD} = \beta$ and $\Pi_{AS} = a$. When $y = 1$, $\Pi_{AD} = \frac{\beta}{1-\delta}$ and $\Pi_{AS} = 1$.

Proposition 4 *(Unique Steady State): Let $\Pi^* = 1$, $1 \geq \gamma > \frac{\beta}{1-\delta}$ and $\sigma = 1$. There exists a unique steady state. It is the full employment steady state.*

Proposition 5 *(Unique Steady State): Let $\Pi^* = 1$, $\delta > 1 - \beta$, $\gamma > \beta$ and $\sigma = 1$. There exists a unique steady state. It is the secular stagnation steady state.*

Proposition 6 *(Two Steady States): Let* $\Pi^* = 1$, $\delta < 1 - \beta$, *and* $\sigma = 1$. *If* $a = \gamma < \beta$ *there exists two steady states. There is a unique full employment steady state, and a BSGU steady state.*

Proofs available upon request.