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Taxation, Social Welfare, and Labor Market Frictions *

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Abstract

Taking inefficiencies from taxation as given, a well-known public finance literature shows that the elasticity of taxable income (ETI) is a sufficient statistic for assessing the deadweight loss (DWL) from taxing labor income in a static neoclassical framework. Using a theoretical approach, we revisit this result from the vantage point of a general equilibrium macroeconomic model with labor search frictions. We show that, in this context, and against the backdrop of inefficient taxation, DWL can be up to 38 percent higher than the ETI under a range of reasonable parametric assumptions. Externalities arising from market participants not taking into account the impact of changes in their search- and vacancy-posting activities on other market participants can amplify this divergence substantially. However, with theoretical precision, we show how the wedge between the ETI and DWL can be controlled for, using readily observable variables.

Keywords: Elasticity of taxable income, deadweight loss from taxation, endogenous amenities, search frictions, social welfare

JEL Classifications: H20, J32

*The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

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1 Introduction

The personal income tax is one of the most important instruments for raising government revenue. As a consequence, this tax is the focus of a large body of public finance research that seeks a theoretical and empirical understanding of the associated deadweight loss (DWL). From a theoretical perspective, taking the inefficiencies that arise from taxation as given, Feldstein (1999) provided a key advancement to the literature by proposing a conceptual underpinning for this research. In particular, Feldstein (1999) demonstrated that, under very general conditions, the elasticity of taxable income (ETI) is a sufficient statistic for evaluating DWL.

This fundamental result was developed within a *microeconomic* framework focusing on partial equilibrium. However, Feldstein's fundamental result is less understood from the perspective of a canonical, contemporary *macroeconomic* model (that is, a representative agent, dynamic, general equilibrium model) with labor search frictions.[1] In such a macroeconomic framework, the following questions arise. Can Feldstein's mapping between the ETI and DWL be replicated? If not, which differences between Feldstein's framework and the search-inclusive canonical macro framework account for this? Finally, how significant can these differences, if any, be quantitatively?

We address these questions using a theoretical approach and standard macroeconomic quantitative analysis. Given the importance of labor markets for aggregate economic activity and the well-known empirical relevance of search frictions, the answers to these questions are critical. In particular, the answer to our third question can point to macroeconomic data that can help inform the welfare implications of aggregate fiscal policy quickly and succinctly. This can help expedite decisions on the implementation of aggregate fiscal policy, which is critical in times of sudden and severe economic turmoils. In terms of broader relevance, it is important to note that the U.S. government raised $3.46 trillion in federal tax receipts in fiscal year 2019 in order to fund the provision of public goods, including programs like Social

[1] Of course, a seminal reference for the canonical macroeconomic model (with fully flexible prices) is Kydland and Prescott (1982), as are Mortensen and Pissarides (1994) and Diamond (1982).

Security ($1.04 trillion), national defense ($688 billion), and Medicare ($651 billion).[2] Most other advanced economies raise even larger sums as a fraction of their economic output. It is well understood that, apart from rarely employed lump-sum taxes and more-common (but quantitatively limited) Pigouvian taxes, revenue-raising tax systems impose efficiency costs by distorting economic outcomes relative to those that would be obtained in the absence of taxation (Harberger (1964); Hines (1999)).

The fundamental result from Feldstein (1999) that the ETI can potentially serve as a perfect proxy for DWL is obtained with in a partial equilibrium microeconomic framework and recapped in Chetty (2009a). Importantly, this result is consistent with the ETI reflecting all taxpayer responses to changes in marginal tax rates, including behavioral changes (e.g., reductions in hours worked) and tax avoidance (e.g., shifting consumption toward tax-preferred goods). Furthermore, this means that it is unnecessary to distinguish between the underlying adjustment mechanisms in order to measure DWL. This is fortuitous because taxable income is easily observed in tax records, whereas tax avoidance or evasion, hours worked, and other relevant variables generally are not. Accordingly, a large empirical literature has provided estimates of the individual ETI, identified based on variation in tax rates and bunching at kinks in the marginal tax schedule. These estimates range from 0.1 to 0.8.[3]

However, researchers have fairly recently come to recognize an important limitation of the finding that the ETI is a sufficient statistic for deadweight loss: taxpayer responses to the income tax must not generate externalities. For example, Chetty (2009a) finds that when part of the expected cost of sheltering activity consists of transfers to other parties (e.g., fines paid to the tax authority), the ETI overstates the welfare consequences of changes in income tax rates. In addition, Doerrenberg et al. (2017) derive a model that shows that the ETI is not a sufficient statistic in the presence of tax deductions that generate externalities and are sensitive to tax rate changes. Several earlier papers identified similar limitations of the sufficient statistic result (Slemrod (1998); Slemrod and Yitzhaki (2002); Saez (2004)).

Turning to our research questions, we address them as follows. First, to build our results

[2]Final Monthly Treasury Statement, September 2019.

[3]An interested reader should review Saez et al. (2012) for a detailed overview. More recent papers include Weber (2014) and Kleven and Schultz (2014).

incrementally and in a disciplined fashion, we recap the result from Feldstein (1999) following Chetty (2009a). Second, we bringing the Chetty (2009a) framework to a *frictionless* canonical macroeconomic environment. We show, in this environment, the Feldstein (1999) result——that is, that the ETI can potentially proxy perfectly for DWL——holds under certain reasonable conditions, *with or without non-pecuniary benefits (for brevity, we refer to these benefits as amenities)*. Finally, we embed labor search frictions into the canonical macroeconomic model. This is our benchmark model, and we show that within this framework, a host of additional information beyond the ETI is needed to infer DWL in the presence of search frictions. In particular, given a change in taxes, this information includes, among other factors, levels and changes in job-finding and job-filling probabilities, the extent to which workers and firms adjust their match-forming behavior, and changes in profits, which, as is well known, are nonzero in typical labor-search environments.

Importantly, we show that once these empirically observable factors are controlled for, DWL can be calculated easily and in a straightforward fashion as the sum of the ETI and additional terms involving these factors. We also show that, given these controls, the labor-search framework calculation of DWL does not require knowledge of non-pecuniary benefits (amenities), just as in Feldstein (1999). This result is important given the well-known relevance of compensating wage differentials for labor-market activity (Rosen (1986); Epstein and Kimball (2019)), the fact that a host of studies find that non-wage compensation (in level and variation) is empirically relevant (Pierce (2001); Becker (2011); Hall and Mueller (2018)), and that many aspects of worker compensation are difficult to measure.[4]

To get a quantitative sense of the deviations that frictional labor markets can generate for the relationship between DWL and the ETI, we operationalize a calibrated version of our (benchmark, labor search) model using standard macroeconomic techniques. We find that the ETI is never a good proxy for DWL once search frictions are introduced, and DWL can be between 7 and 38 percent higher than the ETI under a reasonable calibration. Finally,

[4]Of note, Sullivan and To (2014) assess the relative importance of wage and non-wage job utility and, like Becker (2011), they find non-wage utility to be substantial. Indeed, the analysis of Sullivan and To (2014) suggests that across job matches in non-wage utility is estimated to be roughly as large as variation in wages across matches. Moreover, Hall and Mueller (2018) find that variation in the non-wage component of job *offers* is roughly 50% larger than that of the wage component.

we show that both "congestion externalities" and "thick market externalities," which result from market participants not taking into account the effect of changes in their search and vacancy-posting activities on other market participants (in particular, how their actions affect job-finding and job-filling probabilities in the aggregate), can substantially amplify the difference between DWL and the ETI.

Our model contributes to several important threads of literature across macroeconomics and public finance. Of note, our search model builds on Arseneau and Chugh (2012) and nests all other models that we focus on. Moreover, search models have been used in a recent literature in public finance. In this space, Kroft et al. (2020) and Landais et al. (2018a) are the most related papers. However, as discussed further below, the overlap between these analyses and our own is in methodological spirit. Indeed, our labor-search model and focus are distinctly different from these two papers. This is so, in particular, because these papers focus on optimal policy design—mirroring much of the public finance literature that incorporates search frictions into its analysis. In contrast, our paper revisits the fundamental Feldstein (1999) result from the perspective of a canonical macro model with labor search frictions and, importantly, takes inefficiencies from taxation as given. For this reason, optimal policy design is beyond the scope of our paper.

Importantly, also in general contrast to the public finance literature more broadly related to our paper, recall that *our model is within the representative agent paradigm*. This assumption is critical for getting at a clear, concrete, and disciplined understanding of the implications of the most fundamental aspects of search frictions for the *macroeconomic* relationship between DWL and the ETI. To the best of our knowledge, this understanding has not been arrived at by earlier literature as specifically and concretely as we do in this paper. Moreover, for the present purposes, doing so requires us to steer away from optimal tax formulas or sufficient statistics that are comparatively prominent in the public finance literature and would muddle the highlights of our results as relevant for the macroeconomics literature.

Returning to the papers mentioned earlier, Kroft et al. (2020) focus on optimal income tax policy and adopt a sufficient-statistics approach akin to Chetty (2009b) to do so. The authors conclude that, within this framework, the optimal tax is more in the spirit of a negative income tax compared to income tax credit.[5] Their analytical framework builds on Saez (2002), who focuses on optimal income transfer programs for low incomes. In contrast to our model, the environment in Kroft et al. (2020) involves heterogeneous workers and firms. Therefore, their framework cannot speak directly to the fundamental result from Feldstein (1999), nor be easily adapted to do so, as Feldstein's result is obtained within a representative agent environment.

Landais et al. (2018a) study optimal unemployment insurance embedding the Baily-Chetty replacement rate (Baily (1978) and Chetty (2006)) into a static general equilibrium search and matching model. The model builds on Michaillat and Saez (2015), who in turn build on Barro and Grossman (1971) by embedding a matching component to both the product market and the labor market (Michaillat and Saez (2015) broadly focus on identifying channels of labor market fluctuations, but they do not focus on taxes). Amid this theoretical background, Landais et al. (2018a) find that the Baily-Chetty replacement rate formula is optimal when the level of market tightness is efficient.

It is important to note that, given the objective of their research, the framework developed by Landais et al. (2018a) departs considerably from a canonical search-inclusive macroeconomic model.[6] In addition, as is well known, in the standard matching model, unemployment benefits enter in a way that the model with unemployment benefits is never efficient (in contrast to the results from Landais et al. (2018a)).[7] In sum, like Kroft et al. (2020), the framework of Landais et al. (2018a) cannot get at the issue we study in this

[5] Also within a heterogeneous agent framework, Lavecchia (2018) examines the welfare impact of the minimum wage on low-skill workers against the backdrop of optimal taxes and unemployment.

[6] Among other reasons, this is because of the following. Their benchmark model is, in essence, a "one-shot static game" where all workers begin unemployed and thereafter there is no job destruction (in contrast, the canonical model is dynamic). This implies that search effort is the only worker-side input in the matching function (in contrast, given the ins and outs of unemployment, the worker-side input into the matching function always includes the mass of searchers in the canonical model, which has different matching implications than search effort). Moreover, some workers are devoted to producing while others are actively engaged in posting vacancies (in contrast, all workers are engaged only in production in the canonical model).

[7] In a companion paper, Landais et al. (2018b) study, within a dynamic framework, the applications of the theoretical framework developed by Landais et al. (2018a). Of note, also building on the Michaillat and Saez (2015) framework, Michaillat and Saez (2019) study optimal public expenditure when unemployment is inefficient. Other examples of papers studying optimal policy in contexts with search frictions include, among others, Hungerbühler et al. (2006), Golosov et al. (2013), Lehmann et al. (2011), Lehmann et al. (2016) and Hummel (2019).

paper.

All told, we contribute to the literature by adding to the growing list of departures from the fundamental result, developed by Feldstein (1999), that the ETI is a sufficient statistic for DWL. We highlight the importance of accounting for search frictions when focusing on analysis from the macroeconomic perspective, and we show that the ETI can substantially underestimate the DWL in the presence of these frictions. Moreover, we characterize the information needed to correctly assess DWL in the presence of frictional labor markets. As such, our results can help provide insight to policy makers and, in particular, policy decision weighing social benefits and costs of fiscal policy.

This paper proceeds as follows. In the section 2, we build our theoretical framework—starting from background partial equilibrium model to general equilibrium search model—and assess the implications of a tax change for various model variables, as well as for the relationship between the ETI and DWL. Section 3 introduces calibrated versions of the models developed to assess the quantitative importance of channels of distortion identified in theoretical section. Finally, section 4 concludes.

2 Theory

2.1 Background Partial Equilibrium Model

In this section, for reference, we develop a model in the spirit of the benchmark static framework from Chetty (2009a), in which Chetty uses to recap the critical result from Feldstein (1999) that the elasticity of taxable income can be a sufficient statistic for the deadweight loss from taxing labor income. This benchmark framework is our main modeling reference. In the next section, we extend this framework to a neoclassical dynamic general equilibrium environment, and in the section after that, we extend the framework to a dynamic general equilibrium framework with labor search frictions. Relative to Chetty (2009a), we add the generalization that disutility related to labor market activities potentially depends on variables beyond employment that may respond to changes in taxes. Incorporating this generalization is important, since it helps us to understand results from the models we develop in the following two sections.

For the purposes of the present model, the price of consumption is 1 and the household's problem is to choose consumption c and labor n to maximize $U = u(c) + h(1-n, s)$, where u is increasing in c and h is increasing in leisure $(1-n)$. We make no assumptions on how search activity s affects h and assume that s is not a choice variable for the household. These conditions are without loss of generality for the point we will be making and for relating the present model to the general equilibrium models developed in the following two sections.

The household's constraint is $c \leq (1-\tau)\, wn + \Omega$, where τ is the labor income tax rate, w is the real wage; taxable income is, therefore, wn. Finally, Ω is non-labor income, which can potentially depend on taxes. Each of these is taken as given by the household. The first order conditions imply that $u' = \lambda$ and $h' = (1-\tau)\, n$, where λ denotes the marginal value of real wealth.[8]

Chetty (2009a) defines social welfare SW as the sum of U and T, where T denotes government transfers that are equal to tax revenue τwn. Then, following Chetty:

$$\frac{dSW}{d\tau} = \underbrace{\frac{dU}{d\tau}}_{=-u'wn} + wn + \tau \frac{d\,(wn)}{d\tau},$$

where by the envelope theorem, $\frac{\partial \mathcal{L}}{\partial \tau}$, which is equal to $-u'wn$, is also equal to $dU/d\tau$. Therefore, the deadweight loss from taxation (DWL), which is defined as $DWL \equiv -\frac{dSW}{wn \cdot d\tau}$, satisfies:

$$DWL = -\left(1-u'\right) - \frac{d\ln(wn)}{d\ln\tau},$$

where $\frac{d\ln(wn)}{d\ln\tau}$ is the elasticity of taxable income (ETI). The benchmark model from Chetty (2009a) implies the equivalence of DWL and the ETI under the assumption that utility is quasilinear in consumption. In other words, $u' = 1$. We note that this is an entirely standard assumption in the public finance literature.

[8] The Lagrangian is

$$\mathcal{L} = u(c) + h(1-n) + \lambda\left[(1-\tau)\, wn + \Omega - c\right].$$

It is important to highlight that, in addition to the assumption that $u' = 1$, the sufficiency result hinges on the very strong *partial equilibrium* assumptions that (1) both the wage and non-labor income remain constant after a change in taxes, and (2) there will be no impact on taxes on the variable s, which, as modeled previously, can potentially have an impact on h. Absent these two additional assumptions, as shown in Appendix A.1, it is straightforward to derive that, in fact:

$$DWL = -\left(\frac{1-\tau}{\tau}\right)\frac{d\ln w}{d\ln\tau} - \frac{d\Omega}{wn\cdot d\tau} - h'\frac{ds}{wn\cdot d\tau} - \frac{d\ln(wn)}{d\ln\tau}. \tag{2.1}$$

Importantly, we note that the ETI cannot internalize the impact on household utility of changes in variables that impact utility through channels other than employment, such non-labor income and s.[9] Of note, in the static public finance framework, the presence of a variable such as s is entirely nonstandard, so when we henceforth refer to this static framework, we assume away the presence of this variable. The only reason that we include this variable in the present analysis is to make results from our labor search model more easily understood.

2.2 Neoclassical General Equilibrium Model

In this section, we bring the Chetty (2009a) framework into a dynamic general equilibrium setting, where we also incorporate amenities, which is a catch-all variable standing in for non-pecuniary benefits. To briefly summarize, the economy is inhabited by a continuum of individuals who are grouped into an aggregate household and whose mass is normalized to 1. There is a single consumption good, which is produced by a representative firm that is owned by the household. Labor compensation is comprised of taxable labor income and

[9] In addition, we note that the government does not pass along T to the household in a way by which the household can effectively consume this transfer, else the envelope theorem could not be applied as it is. This is also in line with a partial equilibrium analysis. If the household were able to effectively consume the transfer, then the Lagrangian would be:

$$\mathcal{L} = u(c) + h(1-n) + \lambda\left[(1-\tau)wn + \Omega + T - c\right].$$

The result from the envelope theorem that $dU/d\tau = -\lambda wn$ remains, but in this case, it is clear that $dSW = -\lambda wn$, not $-\lambda wn + dT$.

(non-pecuniary) amenities, and the household obtains utility from consumption, leisure, and amenities. Amenities are produced by the firm at a cost. Finally, all markets are perfectly competitive, and in this first pass at the general equilibrium neoclassical model and for the purposes of generality, we assume that production takes as inputs both capital and labor.

The major result from this section is equation 2.2, which reveals four important points. First, *this equation highlights that even in a neoclassical perfectly competitive environment, general equilibrium itself could potentially result in an endogenous distortion in the relationship between DWL and the ETI as long as wage and non-labor income do not respond to a tax change.* In other words, general equilibrium can yield a result akin to equation 2.1. Second, distortions related to wages and non-labor income will never exist under the assumption of a Cobb-Douglas production function, as we will highlight below—*said differently, the inclusion of capital as a modeling choice is irrelevant for our results.* For this reason, we omit capital from the remainder of the analysis that follows. Third, *this also means that the partial equilibrium assumption that wages and non-labor income do not change given a change in taxes,* which is implicitly in the background of the static model developed in the previous section, is innocuous as far as DWL goes within the model developed in the present section. Both the static model developed in the previous section and the neoclassical model developed in this section establish a direct relationship between *DLW* and *the ETI*. That said, fourth, as shown in Appendix A.1, *the derivation of equation 2.1 implies that the general equilibrium framework does not require the assumption of quasilinear utility in order to establish a direct relationship between DWL and the ETI, which was the case in the partial equilibrium model developed above.*

On returning to the dynamic model, the household's problem is to choose consumption, labor, bonds b, and the desired level of amenities per worker a to maximize:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t [u\left(c_t\right) + h(1 - n_t, s_t) + A(a_t n_t)],$$

where $\mathbb{E}_t$ is the expectation operator conditional on the information available up and through period t, $\beta \in (0, 1)$ is the household's parametric subjective discount factor, and A is an increasing and concave function of total amenities.[10] As in the model developed in section 2.1, we incorporate search activity, s, which potentially has an impact on utility and can also respond to changes in taxes. The household's budget constraint is as follows:

$$c_t + b_t \leq (1 - \tau_t)\, w_t n_t + (1 + r_{t-1})\, b_{t-1} + T_t + \Pi_t,$$

where r is the real interest rate, T is government transfers, and Π denotes profits (an endogenous counterpart to the Chetty (2009a), non-labor income Ω) from the economy's representative firm, which is owned by the household. All markets are perfectly competitive. The firm's problem is:

$$\max_{n_t, a_t, d_t} \mathbb{E}_t \sum_{s=t}^{\infty} \Xi_{s+1|t} \overbrace{[y(n_t, k_t) - w_t n_t - \phi(a_t n_t) - i_t + d_t - (1 + r_{t-1})\, d_{t-1}]}^{\equiv \Pi_t},$$

such that:

$$k_{t+1} = (1 - \delta)\, k_t + i_t$$

where $\Xi_{s+1|t} \beta \frac{u'_{s+1}}{u'_t}$ is the stochastic discount factor, k is the economy's capital stock, which, without loss of generality we assume is owned by the firm, y denotes the production function, which has constant returns to scale[11], i denotes investment, δ is the parametric capital depreciation rate, and d denotes debt. ϕ is the cost of producing total amenities, increasing in its arguments. Of note, assuming that the capital stock is owned by the firm puts the household's problem directly in line with that of Chetty (2009a), where the household effectively does not own any capital.

We assume that government consumption is zero so that $T = \tau w n$ as in Chetty (2009a). That said, unlike Chetty, following standard dynamic general equilibrium frameworks, we assume that, while the household takes T as given, the government gives T to the household in a way that the household can indeed incorporate T into consumption. *This assumption*

[10]In Appendix B.2, we show that, in this neoclassical framework, our results regarding DWL, which is the focus of our analysis, remain unchanged without assuming additive separability and instead using a fully abstract utility function.

[11]Constant returns to scale is a critical and standard assumption in the macroeconomics literature that is consistent with zero profits in equilibrium.

does not have any first-order impact on our results—indeed, as shown in Appendix A.2, if in the Chetty (2009a) framework we assume that T is indeed consumed by the household, then assuming that $u' = 1$ DWL is exactly the same as in equation 2.1.

For ease of comparison with related literature, *throughout the remainder of the paper, we focus on steady states for the purposes of evaluating deadweight loss.* Therefore, we define social welfare SW as simply being equal to U. As shown in Appendix B.1, in the present framework:

$$DWL = -\left(\frac{1-\tau}{\tau}\right)\left(\frac{w_n n}{w}\frac{d\ln n}{d\ln\tau} + \frac{w_k k}{w}\frac{d\ln k}{d\ln\tau}\right) - h'\frac{ds}{u'wn \cdot d\tau} - \frac{u' d\Pi}{u'wn \cdot d\tau} - \frac{d\ln(wn)}{d\ln\tau}. \quad (2.2)$$

The first term above is akin to the first term in 2.1—as shown in Appendix B, the first term in the equation above stems from the derivative of the wage—and it follows that DWL will be equal to the negative of the ETI if and only if $d\ln n$ and/or w_n and/or n are equal to zero *and* w_k and/or k and/or $d\ln k$ are equal to zero and also $d\Pi = 0$ and $ds = 0$. *This means that the relationship between DWL and the ETI can potentially be distorted by the first three terms in equation 2.2 under a general equilibrium framework.*

That said, as shown in Appendix B, with a Cobb-Douglas production function it is always the case that:

$$\frac{w_n n}{w}\frac{d\ln n}{d\ln\tau} + \frac{w_k k}{w}\frac{d\ln k}{d\ln\tau} = 0. \quad (2.3)$$

(Of note and as shown in Appendix B, in the present neoclassical environment $w = y_n - \phi' a$, meaning that $w_a < 0$, which is intuitive.) Moreover, in a neoclassical framework with a constant return to scale production function profits are always zero, meaning that endogenously $d\Pi = 0$ (in line with Chetty (2009a), where the ETI being a perfect proxy for DWL is conditioned on the assumption that non-labor income will not respond to tax changes ($d\Omega = 0$)). Therefore, in this dynamic neoclassical general equilibrium framework, as long as $ds = 0$, then equation 2.2 becomes:

$$DWL = -\frac{d\ln(wn)}{d\ln\tau} \quad (2.4)$$

just like in standard public finance static frameworks, meaning that, in a dynamic general equilibrium framework, the ETI can indeed proxy perfectly for DWL. Moreover, this means *correct appraisal of DWL does not require knowledge of variables that cannot be directly observed, such as amenities. In this way, we recover the sufficiency condition, as in Feldstein (1999)* Also, as shown in Appendix B.2, equation 2.4 also emerges under any abstract utility specification.

Finally, we highlight two things. First, the model developed in this section is efficient from the point of view that its allocations are exactly the same as those that result from a planning version of the economy. We stress, though, that this efficiency is conditional on the environment faced by the planner, meaning that, if the planner must deal with taxes, then even though the outcome is efficient, there is indeed a DWL stemming from taxation. With this caveat in mind and with some abuse of terminology, *we henceforth refer to the cases for which Hosios conditions hold (meaning that the bargaining power of workers should be equal to the elasticity of matches with respect to searchers) and there are no unemployment benefits as being efficient. If either of these two fail, we call the model inefficient.* Second, the presence of a variable like s is entirely nonstandard in a neoclassical framework, so when we henceforth refer to this neoclassical framework, we assume away the presence of this variable. As noted in the previous section, the only reason for which we include this variable in the present analysis is to make results from our labor search model more easily understandable.

2.2.1 Impact of Tax Changes

In this subsection, we analytically assess the impact of tax changes in the neoclassical general equilibrium model we developed above. To this end, we assume, as in standard related public finance literature, that utility is quasilinear in consumption, so that $u' = 1$. Moreover, we assume that production takes labor as an input only and has constant returns to scale in labor so that $y_n > 0$ and $y_{nn} = 0$. As above, our analysis focuses on steady states.

All of the following intuition is confirmed mathematically in Appendixes D.5 and D.6. Suppose that tax rate rises. Then, the economy has an incentive to reallocate worker compensation from taxable income to nontaxable amenities. Therefore, wages decrease and amenities rise. That said, all else equal, higher taxes put downward pressure on labor income and therefore on the household's willingness to work, which drives a decrease in employment. Finally, with lower employment production decreases and, therefore, so does consumption. With both wages and employment lower, taxable income decreases, which per equation 2.2 puts upward pressure on DWL via the ETI. Moreover, in this case, DWL is exactly equal to the ETI since, as noted earlier, with a constant returns to scale production function, changes in labor and capital jointly null the first term in equation 2.2; the second term in this equation is absent by construction in standard neoclassical macroeconomic theory; and the third term is zero as well since profits are always zero with constant returns to scale assumption in this environment.

2.3 General Equilibrium Labor Search Model

In this section, we develop *our benchmark model*. This model adds non-taxable labor income—that is, amenities–to a general equilibrium macroeconomic model with labor search frictions that follows Arseneau and Chugh (2012) regarding assumptions on labor force participation. In the model, firms post vacancies to recruit workers, and the household devotes search activity to find jobs. The overall characterization of the economy is the same as that of the neoclassical general equilibrium labor search model except for the fact that, in the labor search framework, the labor market is not competitive. The reason for this is the following: Given search frictions, workers cannot find jobs instantaneously and firms cannot fill open positions instantaneously. This means that failing to form a match when a worker and firm meet would be costly for either side, which would result in bilateral monopoly power and, therefore, a noncompetitive wage. Given this non-competitiveness, wages are negotiated via Nash bargaining, which is a standard assumption in the search literature.

All told, our model is a dynamic search version of standard static models used to study the ability of the *ETI* to proxy for DWL. Importantly, our labor search model nests the neoclassical model developed above, which results from eliminating all search frictions. Also, recall that for the purposes of straightforward comparison with related literature, our labor search model omits capital. This assumption is justified by the fact that, as noted above, in the neoclassical model the presence of capital does not matter for DWL results. Moreover, having labor as the only input in production puts our model exactly in line with the benchmark theory of equilibrium unemployment (see, for instance, Pissarides (2000)). Finally, the meaning of all notation already introduced remains in the rest of the paper.

2.3.1 The Household

The household's lifetime utility is given by:

$$U_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t [u(c_t) + h(1 - lfp_t) + A(a_t n_t)],$$

where lfp_t is labor force participation, which is equal to the sum of n_t, the mass of employed individuals, and s_t, the mass of (*involuntarily* unemployed) job seekers. The household faces two constraints. First,

$$n_t = (1-\rho)\, n_{t-1} + f_{t-1} s_{t-1}, \tag{2.5}$$

which is completely standard in labor search contexts and amounts to the household's perceived law of motion for employment. In this equation, $f \in (0,1)$ is the per-period job-finding probability, which is endogenous in the model but taken as given by the household; and $\rho \in (0,1)$ is the per-period exogenous job destruction probability. This equation of motion means that, from the household's perspective, contemporaneous employment is equal to the sum of all of the previous period's employed workers whose jobs were not destroyed, $(1-\rho)\, n_{t-1}$ and all of the previous period's successful search activity, $f_{t-1} s_{t-1}$, which is the fraction f_{t-1} of searchers s_{t-1} that transition into employment. Second, the entirely standard budget constraint,

$$c_t + b_t = (1-\tau_t)\, w_t n_t + \chi s_t + (1 + r_{t-1})\, b_{t-1} + T_t + \Pi_t,$$

where χ denotes unemployment benefits. As highlighted below, the wage is also a decreasing function of amenities in this labor search model, just like in the neoclassical general equilibrium model.

The household's choice variables are: c_t, n_t, s_t, b_t, and a_t. The first order conditions for consumption implies that $u'_t = \lambda_t$, meaning the marginal utility of consumption is equal to the marginal value of real wealth, λ. The first order condition for employment yields:

$$\delta_t = u'_t \left(1-\tau_t\right) w_t + A'_t a_t - h'_t + \beta \left(1-\rho\right) \mathbb{E}_t \delta_{t+1}, \tag{2.6}$$

meaning the shadow value of an employed individual in period t, δ_t, is given by: the per-worker utility value of after-tax labor earnings, $u'_t \left(1-\tau_t\right) w_t$; plus the per worker utility value of amenities that accrues to the household as a whole, $A'_t a_t$; net of the foregone per worker value of leisure, $-h'_t$, which also accrues to the entire household; plus its expected continuation shadow value (with probability $1-\rho$ the employment relationship is not destroyed and a currently employed worker remains employed in the following period).

The first order condition for search activity implies that:

$$h'_t = u'_t \chi + \beta f_t \mathbb{E}_t \delta_{t+1}, \tag{2.7}$$

which is akin to a no-arbitrage condition. In particular, this means that the household sets the per worker value of leisure, h'_t, equal to the per worker value of search, which is equal to the sum of the per worker utility value of unemployment benefits, $u'_t \chi$, plus the discounted per worker value of employment, $\beta \mathbb{E}_t \delta_{t+1}$, conditional on finding employment, $\beta f_t \mathbb{E}_t \delta_{t+1}$, which occurs with probability f.

The first order condition for amenities demand implies that

$$A'_t n_t = -u'_t (1-\tau_t) w_{a,t} n_t, \tag{2.8}$$

meaning that the household sets the marginal benefit of amenities in terms of utils, $A'_t n_t$, equal to the marginal cost of amenities in terms of utils, $-u'_t (1-\tau_t) w_{a,t} n_t$.[12]

[12] Recall that, as shown below, the wage is decreasing in amenities so $w_a < 0$).

Finally, the optimality condition for bond holdings implies that

$$1 = \mathbb{E}_t \beta \frac{u'_{t+1}}{u'_t} \left(1 + r_t\right), \tag{2.9}$$

which is a standard Euler equation that defines the stochastic discount factor: $\Xi_{t+1|t} \equiv \beta \frac{u'_{t+1}}{u'_t}$.

2.3.2 The Firm

The firm's problem is:

$$\max_{n_t,\, v_t,\, a_t,\, d_t} \mathbb{E}_t \sum_{t=0}^{\infty} \Xi_{t+1|t} \underbrace{\left\{ y(n_t) - w_t n_t - \phi(a_t n_t) - \gamma v_t + d_t - \left(1 + r_{t-1}\right) d_{t-1} \right\}}_{\equiv \Pi_t}, \tag{2.10}$$

where the production function $y_t = y(n_t)$ delivers final output and has the properties $y_{n,t} > 0$ and $y_{nn,t} = 0$, where this last property is needed to preserve constant returns to scale as in standard search theory and macroeconomic neoclassical models—resulting in zero profits;[13] v_t denotes vacancies, which the firm posts in order to hire workers; γ is the exogenous and constant flow cost of vacancies (in line with standard search theory); and d_t denotes debt. Finally, note that, because the household owns the firm, the firm's discount factor is the stochastic discount factor $\Xi_{t+1|t}$.

The firm's problem is subject to the following constraint, which is its perceived law of motion for employment:

$$n_t = (1 - \rho) n_{t-1} + q_{t-1} v_{t-1}, \tag{2.11}$$

which is completely standard in labor search contexts and amounts to the household's perceived law of motion for employment. In this equation $q \in (0, 1)$ is the per-period probability of filling an open position, which is endogenous in the model but taken as given by the household. This equation of motion means that, from the firm's perspective, contemporaneous employment is equal to the sum of all of the previous period's employed workers whose jobs were not destroyed, $(1 - \rho)\, n_{t-1}$ and all of the previous period's open positions that were

[13] Recall that our assumption on the production function only having labor as an input is consistent with standard labor search theory and also motivated by the fact that, as shown in the preceding section, with constant returns to scale and capital, the presence of capital is irrelevant for results regarding DWL.

successfully filled, $q_{t-1}v_{t-1}$, which is the fraction q_{t-1} of searchers v_{t-1} that result in new hires.

The first order condition for employment yields:

$$\mathbf{J}_t = y_{n,t} - w_t - \phi_t' a_t + (1-\rho)\,\mathbb{E}_t \Xi_{t+1|t} \mathbf{J}_{t+1}, \tag{2.12}$$

meaning that the firm's value shadow of a job, $\mathbf{J}_t$, is equal to the marginal product of labor, $y_{n,t}$, net of marginal labor compensation, $w_t + \phi_t' a_t$, plus the expected continuation value of the job (with probability $1-\rho$, the employment relationship is not destroyed).

The first order condition for vacancies implies that:

$$\frac{\gamma}{q_t} = \mathbb{E}_t \Xi_{t+1|t} \mathbf{J}_{t+1}, \tag{2.13}$$

which means that the expected marginal flow cost of a vacancy, $\frac{\gamma}{q_t}$, is equal to its expected discounted marginal benefit, $\mathbb{E}_t \Xi_{t+1|t} \mathbf{J}_{t+1}$. The optimality condition for amenities supply is:

$$- w_{a,t} n_t = \phi_t' n_t, \tag{2.14}$$

meaning that, in terms of worker compensation, the extra cost of amenities in terms of wages, $w_{a,t} n_t$, is equal to the extra benefit of amenities for workers, $\phi_t' n_t$ (recall that, as shown further below, $w_a < 0$).

Finally, the optimality condition for debt is:

$$1 = \Xi_{t+1|t}\left(1 + r_t\right). \tag{2.15}$$

Note that this last equation and the household's first order condition for bonds are the same, which, as is well known, means that debt and bond holdings are indeterminate in equilibrium.

2.3.3 Closing the Model

We assume that government consumption is zero so that the sum of lump sum transfers to households and unemployment benefits equal total tax revenue:

$$T_t + \chi s_t = \tau_t w_t n_t. \tag{2.16}$$

Therefore, the aggregate resource constraint is given by

$$y_t = c_t + \gamma v_t + \phi. \tag{2.17}$$

Total matches in any given period, m_t are increasing and concave in vacancies and searchers in line with standard search theory. This implies that the job-finding probability satisfies $f_t = m_t/s_t$ and is increasing in the ratio of vacancies to searchers, v_t/s_t (intuitively, the more vacancies per searcher there are, the easier it is for searchers to find jobs), and that the job-filling probability satisfies $q_t = m_t/v_t$ and is decreasing in the ratio of v_t/s_t (intuitively, the more searchers there are, the easier it is for firms to fill open position). Moreover, as in standard search theory, $\theta_t \equiv v_t/s_t$ is *market tightness.* The higher this ratio is, the easier it is for workers to find jobs.

We pause here a moment to highlight the following. The fact that $f'(v/s) > 0$ and $q'(v/s) < 0$ highlights the labor search theory *congestion externality.* This externality reflects the fact that an additional searcher decreases the probability of all searchers finding a job, and an additional vacancy decreases the probability of all vacancies being filled. Since the firm and household take, q and f as given, respectively, then the firm and household do not internalize the labor market impact of additional vacancies and search activity. In addition, we note that there are *"thick market externalities."* This refers to the impact of firm actions on searchers and the impact of searcher actions on firms, which are not internalized by either party. In particular, if search activity rises, then two countervailing outcomes are at play: (1) congestion externalities increase for workers, and (2) thick market externalities decrease for firms because the probability of firms filling positions rises with higher search activity.

If, however, vacancies rise then we see a similar tension: (1) congestion externalities increase for firms, and (2) thick market externalities for workers decrease for workers because, all else being equal, the probability of finding a job rises.

All told, the effective aggregate matching process is:

$$n_t = (1 - \rho)\, n_{t-1} + m_{t-1}. \tag{2.18}$$

This equation of motion means that, from the effective aggregate perspective, contemporaneous employment is equal to the sum of all of the previous period's employed workers whose jobs were not destroyed, $(1 - \rho)\, n_{t-1}$ and all of the previous period's employment matches, $m_{t-1} \in (0, 1)$.

Because the labor market is subject to search frictions, then wage determination is non-competitive. In line with the standard related literature on labor search frictions, we assume that wages are determined via Nash bargaining. In particular, Nash bargaining yields a wage that maximizes the *Nash product*,

$$\left(\mathbf{W_t} - \mathbf{U_t}\right)^{\psi} \left(\mathbf{J_t} - \mathbf{V_t}\right)^{1-\psi},$$

where $\mathbf{W}_t$ is the household's value of a job; $\mathbf{U}_t$ is the household's value of unemployment; $\psi \in (0, 1)$ is the parametric and exogenous bargaining power of workers (therefore, $1 - \psi$ is the parametric and exogenous bargaining power of firms); and $\mathbf{J}_t$ is the firm's value of a job, as defined earlier. Note that assuming free entry into vacancy posting, which we do as in standard search theory, the firm's value of a vacancy $\mathbf{V_t}$ is zero. Moreover, note that, given the definition of these value functions, $\mathbf{W_t} - \mathbf{U_t}$, *which in labor search theory is positive by assumption—else, there would be no search activity*— is the household's capital gain from an additional worker being employed, and $\mathbf{J}_t$, *which in labor search theory is positive by assumption—else, there would be no vacancy postings*— is the firm's capital gain of an additional vacancy being filled.

To arrive at an expression for $\mathbf{W_t} - \mathbf{U_t}$, we plug in h'_t from the household's first order condition for s_t into its first order condition for n_t, divide the entire equation by u'_t, multiply and divide δ_{t+1} by u'_{t+1}, and define $\delta_t/u'_t \equiv \mathbf{W_t} - \mathbf{U_t}$ for all periods t. It follows that:

$$\mathbf{W_t} - \mathbf{U_t} = (1-\tau_t)\, w_t + \frac{A'_t a_t}{u'_t} - \chi + (1-\rho-f_t)\, \mathbb{E}_t \Xi_{t+1|t} \left(\mathbf{W}_{t+1} - \mathbf{U}_{t+1}\right). \tag{2.19}$$

Then, as shown in Appendix C.1, using this equation along with the firm's value of job implies that the wage that maximizes the Nash product is:

$$\begin{aligned} w_t = &(1-\psi)\left[\frac{\chi}{1-\tau_t} - \frac{A'_t a_t}{u'_t\,(1-\tau_t)}\right] \\ &+\psi\left\{y_{n,t} - \phi'_t a_t + \mathbb{E}_t \Xi_{t+1|t}\left[(1-\rho) - (1-\rho-f_t)\,\frac{1-\tau_{t+1}}{1-\tau_t}\right]\mathbf{J}_{t+1}\right\}. \end{aligned} \tag{2.20}$$

From this, we highlight several important intuitive results. To begin, the wage is a weighted average of firm and worker-side employment values and opportunity costs, where the weights are the exogenous bargaining powers of workers and firms. Next, the wage is increasing in unemployment benefits and the marginal product of labor. In addition, the last term on the right-hand side above shows that the wage is increasing in the ratio of vacancies to searchers, since f is increasing in this ratio.[14] Moreover, wage is increasing in the expected value of a job, $\mathbf{J}_{t+1}$, and decreasing in contemporary taxes. Finally, we note two things. First, the ratio of future to contemporary taxes in the last term of the wage equation captures the wage rate's optimal inter-temporal smoothing given changes in taxes.[15] Second, $w_a < 0$, since, at a *given level* of total amenities, $a_t n_t$, a marginal increase in amenities implies that

$$w_{a,t} = -(1-\psi)\,\frac{A'_t}{u'_t\,(1-\tau_t)}.$$

2.3.4 Equilibrium

The model's equilibrium is given by a vector of 17 endogenous variables:

$$[f_t, s_t, a_t, b_t, \Pi_t, q_t, \mathbf{J_t}, v_t, d_t, T_t, y_t, c_t, n_t, \mathbf{W_t} - \mathbf{U_t}, w_t, r_t, \delta_t]$$

[14] Intuitively, the higher v_t/s_t is, the easier it is for workers to find jobs, so their outside options are higher.
[15] This is consistent with the model's dynamic nature and is consistent with Arseneau and Chugh (2012).

that, given the vector of parameters:

$$[\tau_t, \psi, \chi, \beta, \gamma, \rho],$$

satisfies equations 2.5, 2.6, 2.7, 2.8, 2.9, the instantaneous portion of equation 2.10, 2.11, 2.12, 2.13, 2.14, 2.15, 2.16, 2.17, 2.18, 2.19, 2.20, and the bond-market clearing condition:

$$b_t = d_t.$$

Because equations 2.9 and 2.15 are identical and always hold, then equilibrium bond (or debt) is indeterminate. As such, we assume that in every period $b_t = d_t = 0$ for simplicity.

Importantly, note that combining the firm and household first order conditions for amenities implies the following equilibrium condition:

$$A'_t = u'_t(1 - \tau_t)\phi'_t.$$

This condition clearly means that, all else being equal, there is a positive relationship between amenities (per worker) and taxes.[16] Moreover, given this equilibrium condition, it is straightforward to show that the wage equation can be stated as:

$$\begin{gathered} w_t = (1-\psi)\frac{\chi}{1-\tau_t} - \phi'_t a_t \\ +\psi\left\{y_{n,t} + \mathbb{E}_t \Xi_{t+1|t}\left[(1-\rho) - (1-\rho-f_t)\frac{1-\tau_{t+1}}{1-\tau_t}\right]\mathbf{J}_{t+1}\right\}. \end{gathered}$$

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$$A'_t = u'_t(1 - \tau_t)\phi'_t$$

implies that

$$\begin{gathered} A''_t n_t da_t = u'\left(1-\tau_t\right)\phi''_t n_t da_t - u'_t \phi'_t d\tau_t \\ \rightarrow \left[A''_t - u'\left(1-\tau_t\right)\phi''_t\right] n_t da_t = -u'_t \phi'_t d\tau_t. \end{gathered}$$

Therefore,

$$\frac{da_t}{d\tau_t} = \frac{-u'_t \phi'_t}{A''_t - u'\left(1-\tau_t\right)\phi''_t} > 0,$$

since ϕ'_t, ϕ''_t, and u'_t are positive, while A''_t is negative.

2.3.5 Key Equations

While the model has a host of endogenous variables, knowledge of the following six key variables is sufficient to pin down all of the model's endogenous variables: a, c, n, s, v, and w (see, for instance, Pissarides (2000), for a similarly compact set of key equilibrium variables, although without amenities). To develop intuition, consider the model in steady state. In this case, the model's key equilibrium variables are determined, respectively, by the following equations:

$$A' = u'(1-\tau)\phi', \tag{2.21}$$

the equilibrium condition for amenities, which pins down a and follows from combining equations 2.8 and 2.14;

$$y = c + \gamma v + \phi, \tag{2.22}$$

which is the aggregate resource constraint and pins down c;

$$\rho n = m; \tag{2.23}$$

which follows from the aggregate matching process in equation 2.18 and pins down n (in equilibrium inflows into employment equal outflows);

$$h' = \frac{u'\chi\left[1-\beta\left(1-\rho\right)\right] + \beta f\left[u'\left(1-\tau\right)w + A'a\right]}{1-\beta\left(1-\rho\right)+\beta f}, \tag{2.24}$$

which follows from combining equations 2.6 and 2.7 and pins down s;

$$y_n = w + \phi' a + \frac{\left[1-\left(1-\rho\right)\beta\right]}{\beta}\frac{\gamma}{q}, \tag{2.25}$$

which follows from combining equations 2.12 and 2.13, pins down v, and we henceforth refer to as the *job creation condition*. This equation highlights that, from the point of view of firms, the effective wage, that is, the effective compensation of workers is, of course, $w +$

$\phi' a$; and the *wage equation* is:[17]

$$w = (1-\psi)\frac{\chi}{1-\tau} - \phi' a + \psi\left(y_n + \gamma\frac{v}{s}\right). \tag{2.26}$$

To see that our general equilibrium labor search model nests our neoclassical general equilibrium model, consider the impact of removing search frictions. Absent these frictions, workers find jobs instantaneously, and firms fill jobs instantaneously, so $f \to \infty$ and $q \to \infty$. As such, the flow cost of posting vacancies is effectively zero, and there is no concrete notion of search or vacancies. Moreover, there are no unemployment benefits, so $\chi = 0$. As such equation 2.25 becomes:

$$y_n = w + \phi' a,$$

which of course means that the marginal product of labor is equal to its marginal cost, as in the general equilibrium neoclassical model—the wage is competitive. Moreover, with the effective expected cost of posting a vacancy being equal to zero, equation 2.22 becomes:

$$y = c + \phi.$$

Finally, with $\chi = 0$, it is straightforward to show that equation 2.24 becomes:[18]

$$h' = u'(1-\tau)\,w + A' a$$

[17]In steady state:

$$w = (1-\psi)\left[\frac{\chi}{1-\tau} - \frac{A' a}{u'(1-\tau)}\right] + \psi\left(y' - \phi' a + f\beta\mathbf{J}\right).$$

But, from the firm's first order condition for vacancies, in steady state it is also the case that $\gamma/q = \beta\mathbf{J}$. Substituting above gives:

$$w = (1-\psi)\left[\frac{\chi}{1-\tau} - \frac{A' a}{u'(1-\tau)}\right] + \psi\left(y' - \phi' a + \gamma\frac{f}{q}\right).$$

Finally, given constant returns to scale of the matching function, note that $f/q = v/s = \theta$.

[18]With $\chi = 0$, combining the steady state versions of the household's first order conditions for employment and search yields:

$$\frac{h'}{\beta f} = u'(1-\tau_t)\,w + A' a - h' + \beta(1-\rho)\frac{h'}{\beta f},$$

which converges to:

$$0 = u'(1-\tau_t)\,w + A' a - h'$$

with $f \to \infty$.

and pins down employment. This, of course, is simply the general equilibrium neoclassical model's household optimality condition for labor. All told, these last three equations, along with equation 2.21 above, pin down the net-of-capital general equilibrium neoclassical model's four key variables: c, n, a, and w.

2.3.6 Deadweight Loss

To make our model's DWL immediately comparable to the standard static public finance literature that addresses the relationship between DWL and the ETI, we continue to assume steady state when focusing on DWL. Nonetheless, in our quantitative analysis, we show transition dynamics between steady states as implied by our search model and its neoclassical counterpart, which effectively showcase the dynamic aspects of our general equilibrium models.

We define flow social welfare as:

$$SW \equiv u\,(c) + h(1-n-s) + A(an).$$

As shown in Appendix C.2, in our benchmark search model DWL is:

$$\begin{aligned} DWL \;=\; & -\overbrace{\frac{d\ln(wn)}{d\ln\tau}}^{ETI} \\ & \overbrace{-u'\frac{d\Pi}{u'wn\cdot d\tau}}^{D_\Pi} \\ & -\overbrace{\left\{\frac{h'-u'\chi}{\beta\rho}\left[1-\beta\left(1-\rho\right)\right]-h'\right\}\frac{ds}{u'wn\cdot d\tau}}^{D_s} \\ & \overbrace{-\frac{h'-u'\chi}{f\beta\rho}\left[1-\beta\left(1-\rho\right)\right]\left\{sf'+\frac{1-\psi}{\psi}vq'\right\}\frac{d\theta}{u'wn\cdot d\tau}}^{D_\theta}, \end{aligned} \tag{2.27}$$

where, as presented earlier, ETI is the elasticity of taxable income, and θ is the measure of market tightness, which is defined as the ratio of vacancies to searchers. The last three terms D_Π, D_s, and D_θ are distortion terms resulting from endogenous responses of profits, search activity, and market tightness, respectively, to a tax rate change. Note that *if* the distortion terms were zero, then equation 2.27 would be exactly the same as equation 2.4, meaning that just like in the partial equilibrium public finance literature and general equilibrium neoclassical models developed earlier, *the ETI would be a perfect proxy for DWL* in the search framework.

That said, to the extent that the three distortion terms above are not all equal to zero, then labor search introduces important distortions to the relationship between *DLW* and the *ETI*. Clearly, all of these distortions stem from search frictions, as they owe to: changes in search activity, which, as noted earlier, is irrelevant in a neoclassical model; changes in profits, which are trivially zero in a neoclassical model, since profits themselves are zero in a neoclassical model; and changes in market tightness, which as noted earlier, is irrelevant as well in a neoclassical model. In light of the empirical importance of search frictions for the behavior of labor markets, equation 2.27 suggests that much caution must be used when trying to infer DWL from ETI in the presence of search frictions.

Moreover, note that the ds and $d\Pi$ terms that appear endogenously in equation 2.27 are the counterparts of the ds and $d\Omega$ or $d\Pi$ terms that had appeared in the previous two models we developed. And, as highlighted when assessing these two models, the ETI cannot internalize changes in social welfare that stem from non-labor income and factors other than employment that affect the utility of households as related to labor market activity. This result adds to the partial equilibrium evidence documenting deviations from the sufficiency result of Feldstein (1999) driven by externalities (Chetty, 2009a; Doerrenberg et al., 2017).

In equation 2.27, the coefficient on $d\Pi$ is clearly positive, and, as shown in Appendix C.2, the condition for the coefficient on ds being positive is $\frac{\delta f(1-\beta)}{u'\rho} > \chi$, which of course holds trivially when $\chi = 0$. Therefore, for a given increase in taxes: if profits decrease, then this change puts upward pressure on DWL as it means that, all else being equal, the household's consumption decreases. We also show in the Appendices C.2 and D.1 that the signs of ds and its coefficient are ambiguous when $\chi > 0$, making the impact of second distortion term on DWL (D_s) ambiguous as well. On the other hand, when $\chi = 0$, both ds and its coefficient are negative, unambiguously putting upward pressure on DWL.

The coefficient on $d\theta$ requires further discussion as it relates directly to inefficiencies and, in particular, to congestion externalities. As is well known, labor search models are efficient[19] if two conditions hold. First, unemployment benefits χ should be equal to zero. And second, the bargaining power of workers ψ should be equal to the elasticity of matches with respect to searchers (this is the *Hosios condition*—see Hosios (1990)). *If the Hosios condition holds, then in the decentralized economy, all congestion externalities are internalized, just as they are in a centrally planned economy.*

In this case, the term related to $d\theta$ in equation 2.27 drops out, reducing the DWL formulation to:

$$DWL = -\frac{d\ln(wn)}{d\ln\tau} - u'\frac{d\Pi}{u'wn\cdot d\tau} - \left\{\frac{h' - u'\chi}{\beta\rho}\left[1-\beta(1-\rho)\right] - h'\right\}\frac{ds}{u'wn\cdot d\tau}. \tag{2.28}$$

In particular, the coefficient on the $d\theta$ is zero if and only if the Hosios condition holds.[20] In other words, as long as there are no congestion externalities, then no matter how θ may be affected by a tax change, this change will have no impact on the discrepancy between the ETI and DWL. It follows that in the labor search model's DWL equation, any distortions between DWL and the ETI arising from changes in θ will only apply when the Hosios condition *does not* hold. Therefore, the $d\theta$ term captures discrepancies between ETI and DWL that stem exclusively from inefficiencies related to externalities.

All told, given the empirical relevance of labor search frictions, our analysis highlights the importance of accounting for these frictions when trying to infer DWL from the ETI.

2.3.7 Impact of Tax Changes

In this subsection, we assess analytically the impact of tax changes in the labor search general equilibrium model we developed above. To this end, we assume that utility is quasilinear in consumption so that $u' = 1$.[21] Moreover, we also continue to assume that production takes

[19]Efficiency here means that the competitive outcome is the same as the outcome from a social planning framework, so this efficiency is unrelated to inefficiencies arising from exogenous circumstances that the social planner faces such as, in the present case, taxes.

[20]See Appendix C.2 for further details.

[21]This is consistent with our analytical assessment of the neoclassical model and is standard in the related public finance literature.

labor as an input and has constant returns to scale in labor so that $y_n > 0$ and $y_{nn} = 0$. As above, our analysis focuses on steady states.

All of the following intuition is confirmed mathematically in Appendixes D.1 through D.4. Suppose that taxes rise. In the efficient search framework the *effective wage* of the marginal worker remains unchanged as taxable wage compensation is substituted by non-taxable amenities.[22][23] As such, amenities ultimately rise and the wage decreases. In other words, net-of-tax labor income drops because of higher taxes, and this drop results in a decrease in search activity. With lower search activity, it becomes harder for firms to fill jobs, so vacancies decrease, which puts downward pressure on the job finding probability. Ultimately, these dynamics result in lower employment, which leads to lower production and profits, both of which put downward pressure on consumption. All of these results continue to apply in the absence of amenities too.

In the search framework with unemployment benefits, an increase in taxes puts upward pressure on the *effective wage*, $w + \phi' a$, since net-of-tax wages decline relative to non-taxed unemployment benefits. This means that workers' relative outside options rise for a given increase in taxes.[24] All else equal, this higher effective wage makes it more costly for firms to produce, so vacancies ultimately drop, which is consistent with a lower job finding probability. In terms of search incentives, there is now ambiguity. On the one hand, a higher effective wage makes search more appealing, but on the other hand, a lower job finding probability makes search less appealing.[25] Given this ambiguity, quantitative analysis—which we turn to in the next section—is necessary to assess which effect dominates and what ultimately happens in this version of the model. More generally, note that, in both the cases with and without amenities, quantitative analysis is needed to determine the ultimate effect of the tax change on the relationship between DWL and the ETI. This is because the coefficients in equation 2.27 have first-order impact on the extent to which changes in search activity,

[22]Recall, this results with no unemployment benefits χ and the *Hosios condition* that the bargaining power of workers ψ being equal to the elasticity of matches with respect to searchers holds

[23]The effective wage is the sum of the wage and the marginal utility from amenities, $w + \phi' a$. In the case without amenities, this is trivially equally to w.

[24]The same is true absent amenities and, again, note that absent amenities the effective wage is simply equal to the real wage w.

[25]The same is true absent amenities.

profits, and market tightness can distort the relationship between DWL and the ETI.

3 Quantitative Analyses

In this section, we use calibrated versions of the models we developed to explore their quantitative implications. Importantly, recall that, in line with the literature most related to our work, our theoretical framework is quite stylized as, for instance, it omits capital. As such, our quantitative results should be interpreted as similarly stylized. That said, as appropriate, we discuss results from six models: our benchmark labor search model with and without amenities and, alternatively, with and without unemployment benefits, and our neoclassical model with and without amenities.

3.1 Functional Forms

For the household, we assume a constant relative risk aversion utility function for consumption, $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$, where $\sigma > 0$ is the coefficient of relative risk aversion. This functional form is entirely standard in the macroeconomics literature. For simplicity, as a benchmark, we assume $h(1 - lfp_t) = \zeta \ln(1 - lfp_t)$ and $A(a_t n_t) = \zeta_a \ln(a_t n_t)$, where $\zeta > 0$ and $\zeta_a > 0$. Turning to production, we assume linear production technology with $y_t = z n_t$, where z is exogenous productivity.[26] We chose this functional form to preserve the constant returns to scale property of the neoclassical model in the absence of capital. In addition, we assume a general functional form for amenities production technology: in particular, let total amenities $a_t n_t$ be equal to (produced by) the technology $g(y_t^{a_t}) = \Phi y_t^{a_t \Psi}$, where $\Phi > 0$ and $\Psi > 0$. Here, $y_t^{a_t}$ is part of the final output allocated for amenities production. Note that this functional form implies that the cost of total amenities is: $\phi(a_t n_t) = \left[\frac{a_t n_t}{\Phi}\right]^{\frac{1}{\Psi}} \frac{1}{\Psi}$. Finally, regarding the labor market, we assume a Cobb-Douglas matching function $m = \varphi v_t^{\xi} s_t^{1-\xi}$, where $\varphi > 0$ is matching efficiency and $\xi \in (0,1)$ is the elasticity of matches with respect to vacancies. This functional form for the matching function is entirely standard in the search

[26]Recall that, given the irrelevance of capital for our results for the purposes of the neoclassical model, we simply assume that output is a function of labor, so this assumption on the functional form of production applies to all models under consideration.

literature (see, for instance, Pissarides (2000), and Shimer (2005)).

3.2 Calibration

The calibration strategy described below is for the search model with amenities, though the other variants of the model are calibrated in a similar fashion. A period is one month. This choice is based on the fact that, given Cobb-Douglas matching functions, the probabilities f and q are unbounded, and simulations and/or comparative static exercises frequently lead to instances in which these probabilities exceed 1 in models calibrated at quarterly frequency. In line with our one-month time period assumption, we set $\beta = 0.996$, which is consistent with an average yearly interest rate of 5 percent, as is the case in the United States, on average, in the post-war period. Continuing with the household, we choose the leisure parameter ζ so that in equilibrium, the mass of individuals outside the labor force, $1 - lfp$, is equal to 0.38. This target is obtained using data from the Bureau of Labor Statistics (BLS) on the average number of U.S. individuals outside of the labor force in per-population terms. We calibrate the amenities parameter ζ_a by setting the amenities-to-wage ratio $\frac{a}{w}$ to 0.11, following the estimate of Hall and Mueller (2018). As a benchmark, we operationalize the model in line with standard public finance literature by assuming utility to be quasilinear in consumption. This assumption implies that $u' = 1$, which requires setting $\sigma = 0$.

Turning to the firm, we choose the equilibrium value of exogenous productivity, z, so that monthly output, y, equals 1. As a benchmark, we set both Ψ and Φ equal to 1, which implies that $\phi\left(a_t n_t\right) = a_t n_t$.

As related explicitly to the labor market, the wage tax τ is set to 0.189, which is the average U.S. labor tax calculated in McDaniel (2011). Moreover, implementing the methodology from Solon et al. (2009) and Shimer (2012) and using, as they do, BLS data on unemployment and short-term unemployment, we find that the average U.S. monthly probability of finding a job (f) in the post-war period is 0.43. The matching efficiency parameter φ is chosen to hit this target. Then, ρ is chosen to set $s/\left(s+n\right) = 0.058$, which, given BLS data, is the average U.S. unemployment rate in the post-war period.

Of course, the neoclassical version of the model we develop is efficient by construction. In our benchmark calibration, our search models are also efficient in the sense that we assume that there are no unemployment benefits, and the Hosios (1990) condition holds, meaning that the bargaining power of workers, ψ, is set equal to the elasticity of matches with respect to search activity, $1-\xi$. As a benchmark, we set $\psi = 1-\xi = 0.5$, where the value of ξ is in line with the empirical evidence in Pissarides and Petrongolo (2001). Moreover, we explore the implications of efficiency as related to unemployment benefits. As such, we study the cases with $\chi = 0$ and $\chi = 0.9$. The latter value is consistent with the replacement rate being equal to 60 percent and is in line with the fact that replacement rates should target about 60 percent for the average worker under the 2015 Social Security law.

For the purposes of analysis, we recalibrate the other four models we consider where appropriate: the search model without amenities with and without unemployment benefits and the neoclassical model with and without amenities. However, to keep all six models comparable, we set the levels of taxable income in each model to the level of taxable income obtained for the calibrated benchmark search model with amenities and no unemployment benefits instead of setting $y = 1$ to calibrate the productivity parameter z. As such, all the economies we study have the same fundamentals. Table 1 summarizes our benchmark parameter values.

Insert Table 1 about here.

As a robustness check, we calibrate all four models in the same way that we calibrated the search model with amenities and no unemployment benefits, choosing z by setting $y = 1$. In this case, the calibrated parameters are largely unchanged, leaving no visible imprint on our quantitative findings. Hence, we do not report the results from this alternative calibration. That said, we do present results from sensitivity analysis as related to the most critical parameters in our calibration, and *we show that our results are robust to a range of values in these parameters.* In particular, in all robustness checks, the ETI is never a perfect proxy for the DWL and always underestimates it, though at significantly varying degrees.

3.3 DWL versus ETI

As suggested by the theory developed earlier, DWL is always equal to ETI in our neoclassical models, regardless of the provision of amenities. Therefore, we do not report quantitative results for these two models. However, it should be noted that ETI—and hence the DWL—is larger when there are amenities, since, in this case, the agents shift to non-wage income in response to a tax hike.

Tables 2 through 5 present results regarding our labor search model's DWL equation, equation 2.27, when we operationalize the efficient search model with amenities (Table 2, model III), the efficient search model without amenities (Table 3, model V), the search model with amenities and unemployment benefits (Table 4, model IV), and the search model with unemployment benefits but without amenities (Table 5, model VI). In particular, each of these tables shows results for a 1 percent increase in the tax rate compared with our benchmark equilibrium. In each of these tables, the first column shows the negative (since that is the way in which they enter equation 2.27) of the value of the coefficients on the ETI, $d\Pi/d\tau$, $ds/d\tau$, and $d\theta/d\tau$ (rows 1 through 4, respectively). Column 3 shows the values of the ETI, $d\Pi/d\tau$, $ds/d\tau$, and $d\theta/d\tau$ (rows 1 through 4, respectively). Column 4 shows the product of columns 1 and 3 (i.e., each row in column 4 corresponds to the ETI or one of three distortion terms in equation 2.27). Finally, column 5 shows the percent of the ETI of each value in column 4, and the last row of column 5 shows the sum of these percentages, which corresponds to the DWL arising from the tax change. As such, these tables break down DWL by each term on the right-hand side of equation 2.27. Note that Appendix E complements this section by discussing the particular implications of our calibration, which will help the reader understand the magnitudes of ETIs and distortion terms across models, as presented in following tables.

Consider the efficient search model (Tables 2 and 3, with and without amenities, respectively). In the case with amenities, the ETI is about 0.18, and in the case without amenities it is about 0.14. It is, of course, intuitive that taxable income would be more sensitive to a tax change with the existence of amenities, since labor compensation can switch from wages to amenities. In both cases, profits Π and search activity s decrease; there is no change in market tightness, θ,—*in the next section, we elaborate on the reasons behind this and other relevant dynamics*—and the coefficients on $d\Pi$ and ds enter the DWL equation negatively (recall that the coefficient on $d\theta$ is zero, since our benchmark calibrations impose the Hosios condition). Then, as shown in column 5, these "*search distortion*" terms represent, respectively, 3.9, 3.2, and 0 percent of the ETI. The sum across column 5, which is the DWL from taxation, yields 107 percent. This means that, in the efficient search model with amenities, DWL is 7 percent higher than the ETI, so the ETI underestimates DWL.

Insert Table 2 about here.

In turn, inspection of Table 3 shows that, in the efficient search model without amenities, DWL is 8.3 percent higher than the ETI, so, again, the ETI underestimates DWL. This example illuminates the fact that the ETI underestimates DWL by less in the presence of amenities compared with the case in which there are no amenities. To understand this result, consider the following. Given the tax hike, some taxable income can (and does) shifts into compensation from non-taxable amenities when available. Clearly, all else equal, the possibility of this shift in compensation means that taxable income is more sensitive to the tax change when there are amenities compared with the case without amenities. Hence, the ETI with amenities is greater than the ETI without amenities—by roughly 30 percent in our calibration. On the other hand, the distortion terms are fairly similar in level terms with or without amenities (compare columns 4 of Table 2 and Table 3). Because the ETI is smaller without amenities, this means that the search distortions are greater as a fraction of the ETI in the case without amenities than in the case with amenities.

Insert Table 3 about here.

Now, consider the search model with unemployment benefits (Tables 4 and 5, with and without amenities, respectively). Note that, in both cases, the magnitude of the search distortions is notably larger compared with the case without unemployment benefits, and in both cases $d\theta \neq 0$. Importantly, the coefficient on ds flips signs and is now positive unlike the cases without unemployment benefits. This means that our benchmark level of unemployment benefits implies $\frac{\delta f(1-\beta)}{u'\rho} < \chi$.[27] Furthermore, search activity *rises* in response to a tax hike at this level of unemployment benefits, as opposed to the case with no unemployment benefits.[28] Taken together, the search distortions are such that DWL overestimates the ETI by 13–35 percent with and without amenities, respectively. Said differently, the ETI underestimates the DWL by more when there are no amenities, just as in the efficient search model. The intuition behind this result is entirely analogous to its counterpart in the efficient search scenario.

Insert Table 4 and 5 about here.

3.4 Transition Dynamics

In this section, we discuss the transition dynamics implied by a one-time 1 percent permanent increase in taxes in our benchmark model (both with and without unemployment benefits) and the four models nested within it: search without amenities (both with and without unemployment benefits); neoclassical with amenities; and neoclassical without amenities. As such, this exercise complements and speaks to the results in Tables 2 through 5 discussed earlier.

We begin by focusing on the search models without unemployment benefits, with and without amenities. On impact of the shock, search activity drops in both models by roughly the same amount (panel (a) of Figure 1). This similarity owes to the fact that in both models effective compensation remains unchanged (w in the case of the model without amenities and $w + \phi' a$ in the case of model with amenities—see panels (b) and (c) of Figure 1), but the

[27]See section 2.3.6 for a discussion on the implications of this inequality.

[28]Earlier, we highlighted that, with unemployment benefits higher, taxes raised the effective wage, which made the impact of the increase in taxes on search activity in principle ambiguous. As noted earlier, this is the case because, on the one hand, a higher effective wage makes search more appealing, but, on the other hand, lower vacancies, which, all else equal, result in a lower job finding probability, make search less appealing. Therefore, our quantitative analysis suggests that higher effective wages dominate search incentives for an empirically plausible level of unemployment benefits.

higher tax rate puts downward pressure on search incentives since net-of-tax labor income drops. Of note, the reason behind no change in effective compensation is that the change in the tax rate does not have an impact on workers' outside options. With lower search activity, the job filling probability drops, which makes it more costly for firms to hire, so vacancies drop (panel (d) of Figure 1). Because neither the effective wage nor productivity change, then the drop in vacancies is driven entirely by the drop in search activity, which results in vacancies dropping one-for-one with search activity and, therefore, market tightness θ not changing.

Insert Figure 1 about here.

The drop in vacancies drives a jump in profits (panel (a) of Figure 2), which drives a jump in consumption (panel (b) of Figure 2). Because employment is a state variable, then it does not change on impact of the shock (panel (c) of Figure 2). With no on-impact change in employment and an unchanged wage in the model without amenities, then, as shown in panel (d) of Figure 2, taxable income in this model does not change on impact either. As also shown in this figure, because in the model with amenities the wage indeed drops on impact, in spite of unchanged on-impact employment, taxable income drops upon the shock.

Insert Figure 2 about here.

Thereafter, lower search activity and vacancies put downward pressure on employment, which decreases slowly and permanently. Expected lower employment implies an expected increase in the marginal product of labor, which drives a slow-moving recovery in vacancies. With this recovery putting upward pressure on the job finding probability, search activity follows suit and slowly recovers as well. That said, with a permanently higher tax rate, search activity is permanently lower as are vacancies and, therefore, employment as well.

Regarding amenities, note that, because utility from amenities and the cost of producing amenities are functions of total amenities—the product of amenities (per worker) and employment—then amenities rise slowly, mirroring the slow-moving downward dynamic path of employment. Of course, in terms of effective compensation, the slow-moving rise in amenities is offset by the slow-moving decline in wages as a result of the inverse relationship of these two variables. All told, taxable income decreases slowly and permanently in both models, owing to the slow-moving decline in employment in the model without amenities and owing to both the slow-moving decline in employment and the slow-moving decline in wages in the model with amenities.

With permanently lower employment, output is permanently lower as well. Given unchanged effective wages, lower output drives profits down. Of course, in the case of the model with amenities, in terms of profits, the decline in output outweighs the decline in costs owing to vacancy postings. Against this backdrop, consumption declines and is ultimately permanently lower given permanently lower labor income and profits. Finally, Figure 3 shows DWL and the negative of ETI for the models with and without amenities (panel (a) and panel (b), respectively). Note that DWL is actually negative on impact of the shock, which means that on impact of the shock there are actually welfare gains. These gains are in line with the on-impact increase in consumption noted above[29]. Moreover, these gains arrive slowly as the economy begins to settle into its new steady state, eventually turning into welfare losses, or positive DWL.

Insert Figure 3 about here.

As applicable, these figures also show results from our neoclassical model with and without amenities as well. Of course, in the neoclassical model all adjustments are instantaneous. Moreover, the intuition behind the behavior of all variables in the neoclassical model is akin to the behavior of their counterparts in the search models—recall that our search model nests the neoclassical model. These neoclassical model results are reproduced for convenience of reference in Figures 4 through 6, which show the same results as the preceding figures, but now stemming from operationalizing the search model with unemployment benefits. In particular, the intuition developed for the direction and rate-of-chat for all parameters remains as previously discussed. We stress one important deviation, however. In particular, in the

[29] This is tied to the on-impact increase in profits driven, in turn, by the on-impact decline in vacancies.

case with unemployment benefits, search activity ultimately rises while vacancy postings ultimately drops, causing market tightness θ, and hence job finding probability, to eventually fall.

Insert Figure 4, 5, and 6 about here.

3.5 Externalities

Recall that, if the Hosios condition does not hold (i.e., when the bargaining power of workers is not equal to the elasticity of matches with respect to search activity— in our model's notation: $\psi \neq 1 - \xi$), then the competitive version of search models suffers from *congestion externalities*. This leads to the following two facts. First, additional searchers do not internalize that they create a greater competition for jobs by increasing the pool of searchers, and therefore, decrease the probability of finding a job for all workers.[30] Second, firms posting additional vacancies do not internalize that they raise competition for workers by increasing the pool of vacancies, causing the probability of filling a job to decrease for all firms. [31]

While the Hosios condition is a convenient modeling assumption, there is no concrete agreement in the search literature regarding whether or not it is empirically plausible. Therefore, in this section, we consider the impact of relaxing the assumption that the Hosios condition holds. Recall that, in this case, the coefficient on $d\theta$ in the search model's DWL equation 2.27 is no longer zero. Moreover, this coefficient is only zero if the Hosios condition holds. As a result, changes in market tightness, θ, can be interpreted as speaking explicitly to congestion externalities without the Hosios assumption.

Figure 7 shows results from assessing the impact of deviations from the Hosios condition by varying the bargaining power of workers ψ while keeping the value of $1 - \xi$ constant.[32] The left panel of this figure shows results from the search model without unemployment

[30] That said, an additional worker searching increases the probability of all firms of hiring, which is called a *thick market externality*.

[31] That said, an additional vacancy posted increases the probability of all workers finding a job, which also falls into the category of a *thick market externality*.

[32] The model is recalibrated for each value of ψ.

benefits, and the right panel shows results with unemployment benefits. In all graphs, the horizontal axis is ψ, which we vary between 0.4 and 0.6.[33] Within these left and right panels, at each value of ψ, the top panel shows the percentage change in search activity given a 1 percent increase in taxes; at each value of ψ, the middle panel shows the percentage change in market tightness θ given a 1 percent increase in taxes; and at each value of ψ, the bottom panel shows the percent difference between DWL and the ETI given a 1 percent increase in taxes.

Insert Figure 7 about here.

Before addressing the results, it is important to note that over the range $\psi < 1-\xi = 0.5$, the coefficient on $d\theta$ is negative, and over the range $\psi > 1-\xi = 0.5$, it is positive.[34] Moreover, the coefficient on ds is always negative with $\chi = 0$ and always positive with $\chi = 0.9$, which means that in our calibration with unemployment benefits $\frac{\delta f(1-\beta)}{u'\rho} < \chi$ holds. Considering that $ds > 0$ when $\chi = 0.9$, this implies that higher effective wages dominate search incentives when there are unemployment benefits.

In Figure 7, as shown in the left panel, changes in taxes do not have an impact on market tightness θ across ranges of ψ without unemployment benefits. This is in line with our earlier results regarding this variable given our benchmark calibration without unemployment benefits. On the other hand, throughout the range of ψ, values of search activity always decreases. That said, in this case without unemployment benefits, as in Table 2, the effective coefficient on ds is negative, meaning that the consistent drop in s given higher taxes and across ranges of ψ puts upward pressure on DWL.[35] Of course, lower s means that congestion externalities decrease for workers, but thick market externalities rise for firms. Therefore, lower s and therefore upward pressure on DWL can be interpreted as suggesting that the worsening of thick market externalities for firms outweighs the lessening of congestion externalities for workers.

[33] At $\psi = 0.5$, which is our benchmark calibration, $\psi = 1-\xi$ and the Hosios condition holds.

[34] Of course, at $\psi = 1-\xi = 0.5$ the Hosios condition holds, so in Figure 7, our benchmark model results are at $\psi = 0.5$.

[35] See equation 2.27.

That said, the extent to which the ETI underestimates DWL decreases as ψ tends toward 1. The intuition behind this result is straightforward. As ψ tends toward 1, and with no unemployment benefits, the search model's outcome gets closer and closer to the neoclassical outcome. This is because $\psi = 1$ implies that workers obtain all the surplus from a match, just as they do in the neoclassical model. Therefore, with no unemployment benefits, as ψ rises, the search model tends toward the neoclassical model, and, as shown earlier, in the neoclassical framework, the ETI can potentially proxy perfectly for DWL. As such, with no unemployment benefits, the ETI becomes a better proxy for DWL as ψ rises—*this is also the reason behind changes in DWL with positive unemployment benefits (panel f of the figure).*

In the model with unemployment benefits, as shown in the right panels of Figure 7, search activity always rises throughout the range of ψ values given an increase in taxes. That said, in the case with unemployment benefits, as in Table 4, the coefficient on ds is positive. This reflects the fact that the consistent rise in s puts upward pressure on DWL in face of higher taxes and across ranges of ψ.[36] Of course, higher s worsens congestion externalities for workers but alleviates thick market externalities for firms. As such, the fact that changes in s put upward pressure on DWL in this case can be interpreted as worsening in congestion externalities for workers outweighing a lessening of congestion externalities for firms.

Also, recall from the earlier discussion that higher taxes trigger an increase in effective wages in the case with unemployment benefits, which is what drives the increase in search activity. However, higher effective wages ultimately result in firms posting fewer vacancies. This, combined with the increase in search activity, means that the change in market tightness θ is always negative for a given increase in taxes; as such, the job finding probability decreases. It follows that decreases in θ, which put downward pressure on the job finding probability, also reflect a worsening of congestion externalities for workers that can be thought of as outweighing a reduction in thick market externalities for firms.[37]

[36] See equation 2.27.

[37] Lower θ puts upward pressure on the job filling probability.

3.6 Sensitivity Analysis

In this section, we relax some of the assumptions behind the benchmark calibration to check the robustness of implications of the quantitative analyses. We do so by plotting the percentage deviation of DWL from the ETI against different values of certain parameters. To isolate the impact of any one parameter, we keep the rest of the parametric assumptions unchanged. Furthermore, we re-calibrate the entire model each time we change values of a parameter to ensure that the models have the same fundamentals for each case compared. As an example, see the left panel of Figure 8, where we plot the percentage deviation of DWL from the ETI for different values of χ, the unemployment insurance, which we set to 0 in the benchmark for preserving the efficiency. When we set $\chi = 0.5$, for instance, we first re-calibrate the models with this new level of χ, while keeping all other model assumptions unchanged. We then undertake a steady-state comparison after a 1 percent tax rate hike, taking this new steady state as the "before-hike steady state."

In all cases, the benchmark result that the ETI underestimates DWL holds for all parameter values, though at widely varying degrees. Consider first the left panel of Figure 8. Here we replicate our benchmark calibration when $\chi = 0$, where the ETI underestimates DWL by 7–8 percent with and without amenities, respectively. Our sensitivity analysis shows significant variation in this as χ increases. In the case of search with amenities, the difference ranges between 0.5 and 17 percent. In the case of search without amenities, the difference can be as large as 39 percent, which corresponds to about 60 percent of wage income. The dominant driver of this variation comes from D_s in equation 2.27, which is the distortion associated with changes in search activity. As discussed earlier, the level of χ affects the signs of both $\frac{ds}{d\tau}$ and its coefficient.

Insert Figure 8 about here.

Next we consider the impact of ψ, the bargaining power of workers. As the right panel of Figure 8 shows, the wedge between the ETI and DWL shrinks monotonically as the bargaining power of workers increases.[38] The intuition behind this variation is very straightforward.

[38]Note that, different from the Figure 7, we keep Hosios condition here, meaning that we set $1 - \xi = \psi$ as

As the workers' bargaining power increases (ψ rises), the wage rate, per equation 17, converges to the neoclassical wage up to a "constant" determined by a and market tightness v/s, both of which remain unchanged in response to changes in ψ. This causes profits to converge to their neoclassical level of 0, leading to smaller declines in profits in response to a tax rate hike. Thus, as ψ rises, the first distortion term (D_{Π}) gets smaller. Given that the ETI, as well as the second (D_s) and third (D_θ) distortion terms remain unchanged when we change ψ, DWL and ETI gets closer (farther away) as workers' bargaining power rises (falls).

Finally, our benchmark calibration assumes a linear consumption utility function ($\sigma = 0$), consistent with our theoretical analysis and the related literature. Figure 9 depicts the relationship between DWL and the ETI for various levels of risk aversion (σ). As seen in the figure, the magnitude of underestimation remains roughly the same for both models. As the risk aversion increases, consumption becomes more smooth, making $\frac{dc}{d\tau}$ less negative relative to the benchmark case with $\sigma = 0$. This, as can be shown easily using the derivations in Appendix D, would also cause other variables, including taxable income ($w \times n$), profits (Π), and search activity (s), to decline less in response to a tax rate hike. Overall, quantitative results suggest that all variables of interest, including DWL, the ETI, and the distortion terms other than $d\theta$, which is always 0, gets smaller by similar ratios, leaving the degree of underestimation roughly the same.

Insert Figure 9 about here.

4 Conclusion

Governments almost exclusively rely on distortionary taxation for their revenue needs. These taxes induce market participants to alter their behavior, often in substantial ways that reduce social welfare. But just how costly is taxation? The answer is directly relevant to how much taxation (and government spending) is optimally implemented.

we change ψ.

Under some strong conditions in a stylized economic setting, social costs of taxation can be inferred with knowledge of the ETI. A growing literature has characterized deviations from this finding, emphasizing the role of externalities. Using a theoretical approach, we revisit this result from the vantage point of a general equilibrium macroeconomic model with labor search frictions. Employing a calibrated version of our model, we show that, in this context, and against the backdrop of inefficient taxation, DWL can be up to 38 percent higher than the ETI under a range of reasonable parametric assumptions. In addition, we show that the divergence between DWL and ETI is widened in the presence of the typical externalities that can arise in the context of frictional labor markets: congestion externalities and thick market externalities.

That said, we characterize the information needed, in addition to the ETI, in order to characterize DWL in the context of frictional labor markets. In particular, given a change in taxes, this information includes, among other factors, levels and changes in job-finding and job-filling probabilities, the extent to which workers and firms adjust their match-forming behavior, and changes in profits, which, as is well known, are nonzero in typical labor-search environments. Importantly, all of these factors have easily observable empirical counterparts.

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Tables and Figures

Table 1: Benchmark Calibration

		Model	I	II	III	IV	V	VI
Household	β	Discount factor	0.996	0.996	0.996	0.996	0.996	0.996
	ζ	Leisure utility scaling parameter	0.447	0.402	0.443	0.465	0.399	0.421
	ζ_a	Amenities utility scaling parameter	0.075	-	0.075	0.075	-	-
	σ	Coefficient of relative risk aversion	0	0	0	0	0	0
Firm	z	Productivity	1.489	1.341	1.685	1.612	1.518	1.445
	Φ	Efficiency of amenities production	1	-	1	1	-	-
	Ψ	Curvature of amenities production	1	-	1	1	-	-
Labor Market	τ	Tax rate	0.189	0.189	0.189	0.189	0.189	0.189
	φ	Matching efficiency	-	-	0.507	0.507	0.507	0.507
	ξ	Matching function exponent	-	-	0.500	0.500	0.500	0.500
	ψ	Workers' bargaining power	-	-	0.500	0.500	0.500	0.500
	ρ	Job destruction probability	-	-	0.027	0.027	0.027	0.027
	χ	Unemployment benefit	-	-	0	0.900	0	0.900
	γ	Vacancy posting cost	-	-	2.050	0.610	1.847	0.407

Models I and II are neoclassical models with and without amenities, respectively. Models III and IV are search models with amenities with and without unemployment benefit, respectively. Finally, models V and VI are search models without amenities with and without unemployment benefit, respectively.

Table 2: Decomposition of DWL: Search Model with Amenities and $\chi = 0$

For 1% increase in tax rate from baseline

	Coefficient (1)	Variable (2)	Value (3)	Product (4)	% of -ETI (5)
ETI	-1	$\frac{d[\ln(wn)]}{d\ln\tau}$	-0.18	0.18	100.0%
$Distortions$	-1.19	$\frac{d\Pi}{d\tau}$	-0.01	0.01	3.9%
	-0.21	$\frac{ds}{d\tau}$	-0.03	0.01	3.2%
	0	$\frac{d\theta}{d\tau}$	0	0	0.0%
DWL (sum of product column)				0.19	107.1%

The table evaluates equation 2.27 in response to a 1 percent rise in income tax, using the parameter values presented in column III of Table 1. Distortion terms in column 4 correspond to D_Π, D_s and D_θ terms in equation 2.27, respectively, while column 5 reports their magnitudes relative to the ETI.

Table 3: Decomposition of DWL: Search Model without Amenities and $\chi = 0$

For 1% increase in tax rate from baseline

	Coefficient (1)	Variable (2)	Value (3)	Product (4)	% of -ETI (5)
ETI	-1	$\frac{d[\ln(wn)]}{d\ln\tau}$	-0.14	0.14	100.0%
$Distortions$	-1.18	$\frac{d\Pi}{d\tau}$	-0.01	0.01	4.6%
	-0.19	$\frac{ds}{d\tau}$	-0.03	0.01	3.7%
	0	$\frac{d\theta}{d\tau}$	0	0	0.0%
DWL (sum of product column)				0.15	108.3%

The table evaluates equation 2.27 in response to a 1 percent rise in income tax, using the parameter values presented in column V of Table 1. Distortion terms in column 4 correspond to D_Π, D_s and D_θ terms in equation 2.27, respectively, while column 5 reports their magnitudes relative to the ETI.

Table 4: Decomposition of DWL: Search Model with Amenities and $\chi = 0.9$

For 1% increase in tax rate from baseline

	Coefficient (1)	Variable (2)	Value (3)	Product (4)	% of -ETI (5)
ETI	-1	$\frac{d[\ln(wn)]}{d\ln\tau}$	-0.18	0.18	100.0%
$Distortions$	-1.19	$\frac{d\Pi}{d\tau}$	-0.01	0.01	3.5%
	1.00	$\frac{ds}{d\tau}$	0.03	0.03	13.7%
	0	$\frac{d\theta}{d\tau}$	-2.10	0	0.0%
DWL (sum of product column)				0.22	117.2%

The table evaluates equation 2.27 in response to a 1 percent rise in income tax, using the parameter values presented in column IV of Table 1. Distortion terms in column 4 correspond to D_Π, D_s and D_θ terms in equation 2.27, respectively, while column 5 reports their magnitudes relative to the ETI.

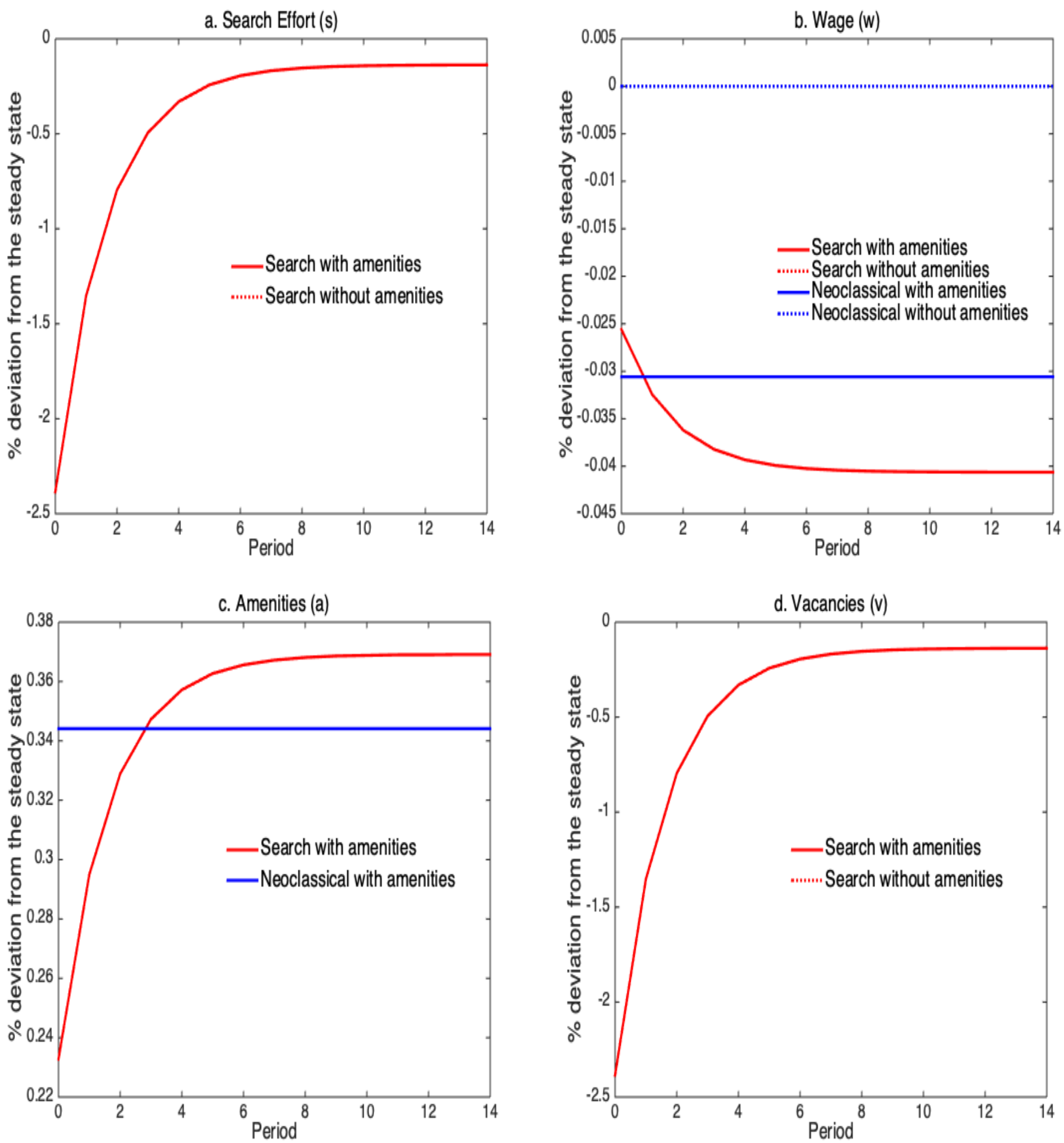
Table 5: Decomposition of DWL: Search Model without Amenities and $\chi = 0.9$

For 1% increase in tax rate from baseline

	Coefficient (1)	Variable (2)	Value (3)	Product (4)	% of -ETI (5)
ETI	-1	$\frac{d[\ln(wn)]}{d\ln\tau}$	-0.15	0.15	100.0%
$Distortions$	-1.18	$\frac{d\Pi}{d\tau}$	-0.01	0.01	3.9%
	1.02	$\frac{ds}{d\tau}$	0.05	0.05	34.7%
	0	$\frac{d\theta}{d\tau}$	-3.15	0	0.0%
DWL (sum of product column)				0.21	138.5%

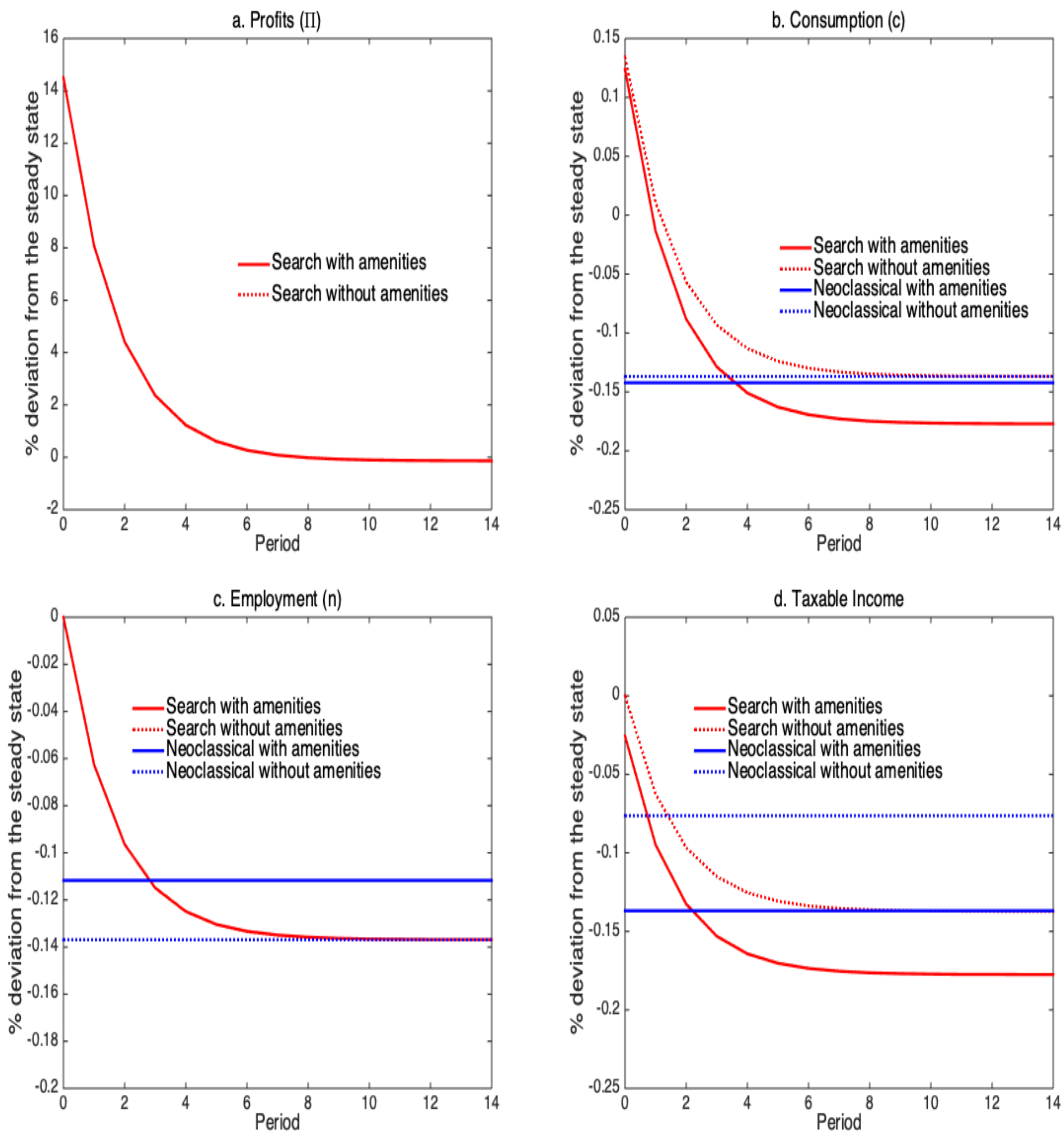
The table evaluates equation 2.27 in response to a 1 percent rise in income tax, using the parameter values presented in column VI of Table 1. Distortion terms in column 4 correspond to D_Π, D_s and D_θ terms in equation 2.27, respectively, while column 5 reports their magnitudes relative to the ETI.

Figure 1: Labor market variables in response to 1% increase in tax rate (no unemployment benefits)



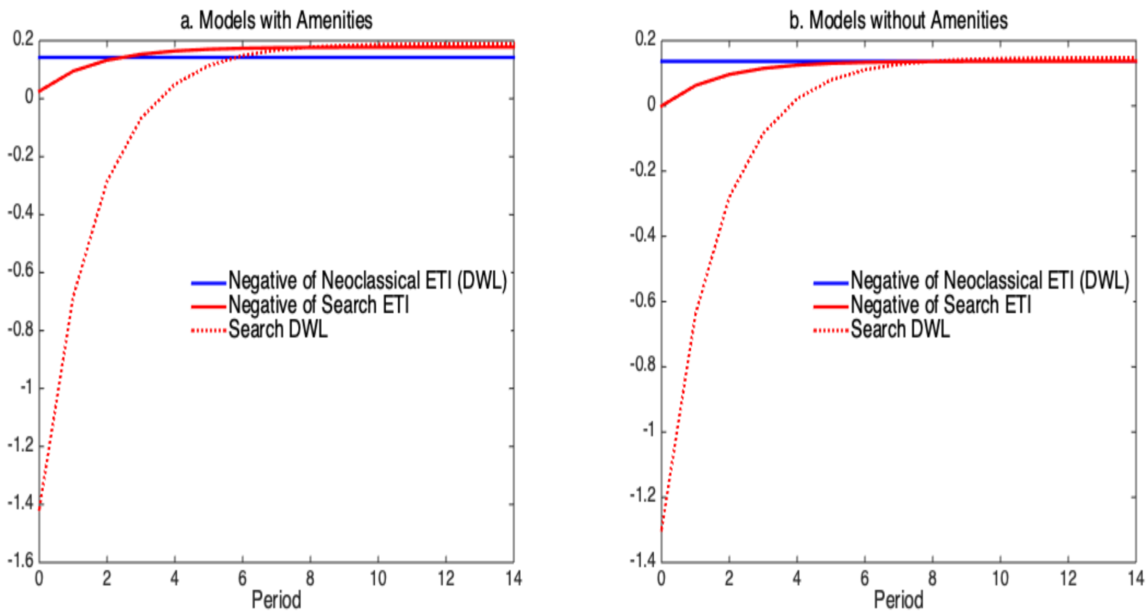
The figure shows the response of selected model variables to a permanent 1-percent rise in income tax when there are no unemployment benefits. The magnitudes are in terms of percentage deviations from the pre-tax hike steady-state levels.

Figure 2: Other variables in response to 1% increase in tax rate
(no unemployment benefits)



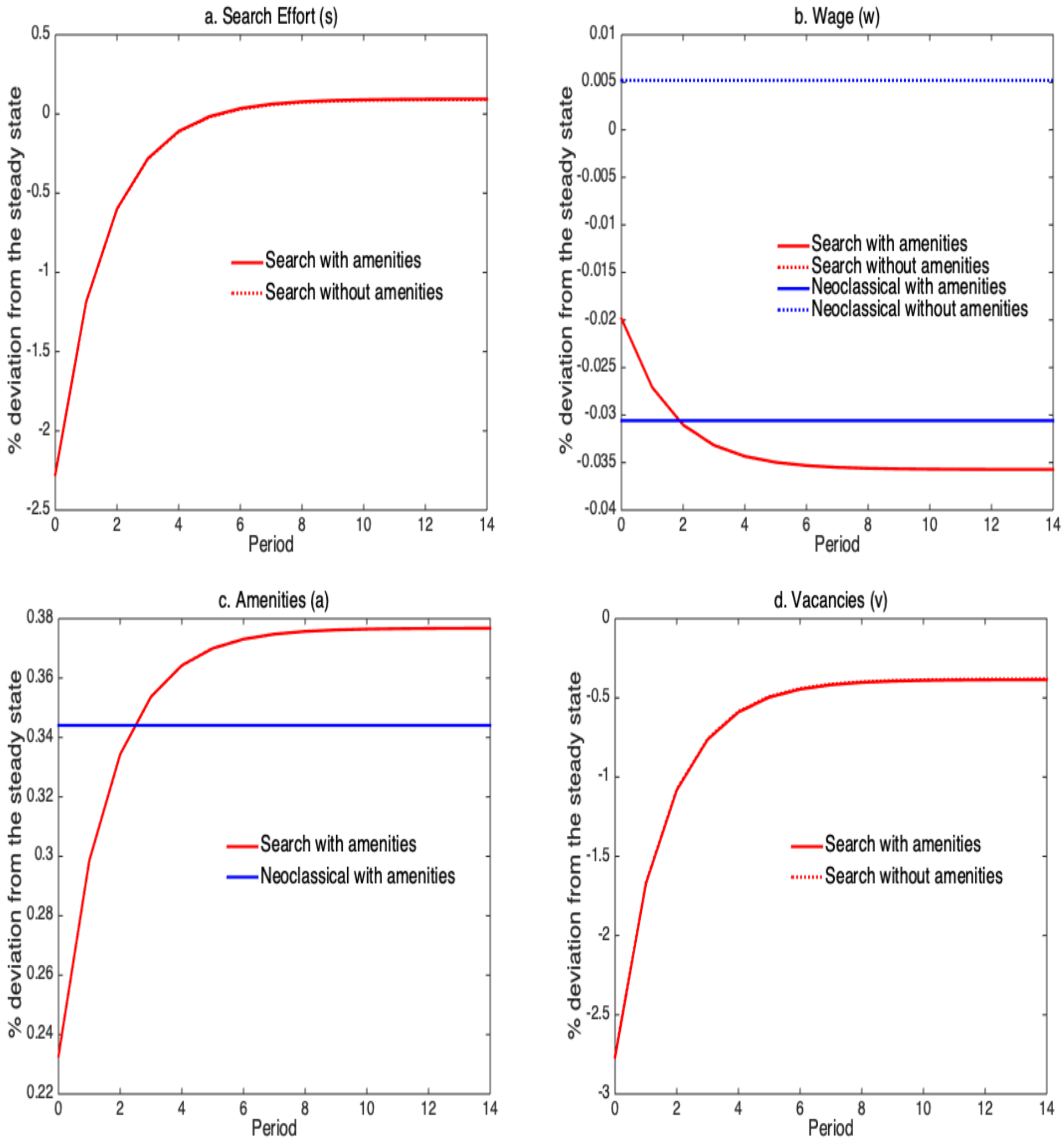
The figure shows the response of selected model variables to a permanent 1-percent rise in income tax when there are no unemployment benefits. The magnitudes are in terms of percentage deviations from the pre-tax hike steady-state levels.

Figure 3: DWL and ETI in response to 1% increase in tax rate
(no unemployment benefits)



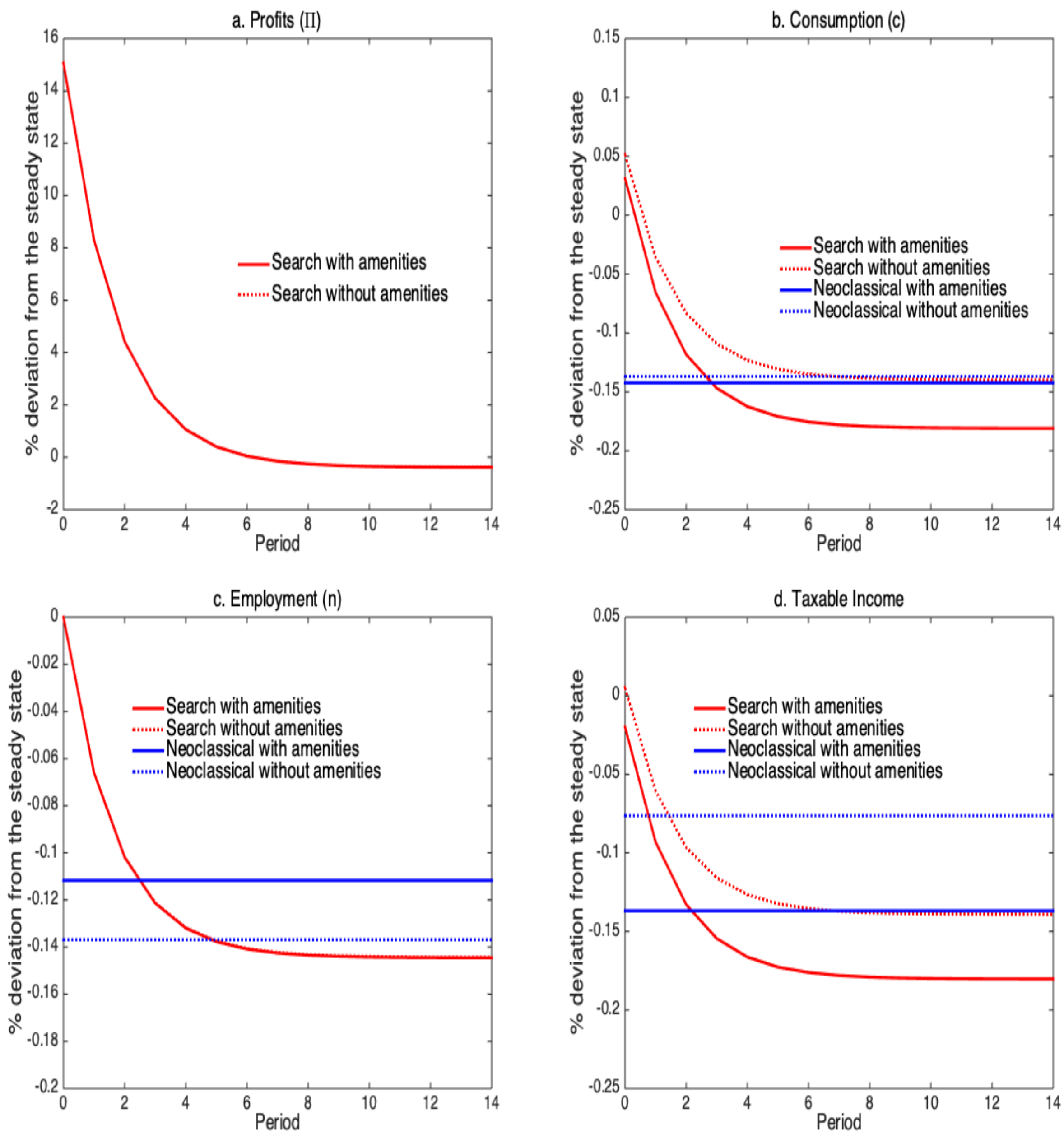
The figure shows the response of the ETI and DWL to a permanent 1-percent rise in income tax when there are no unemployment benefits. The left panel corresponds to the cases with amenities, while the right panel corresponds to the cases without amenities. The magnitudes are in terms of percentage deviations from the pre-tax hike steady-state levels.

Figure 4: Labor market variables in response to 1% increase in tax rate
(with unemployment benefits)



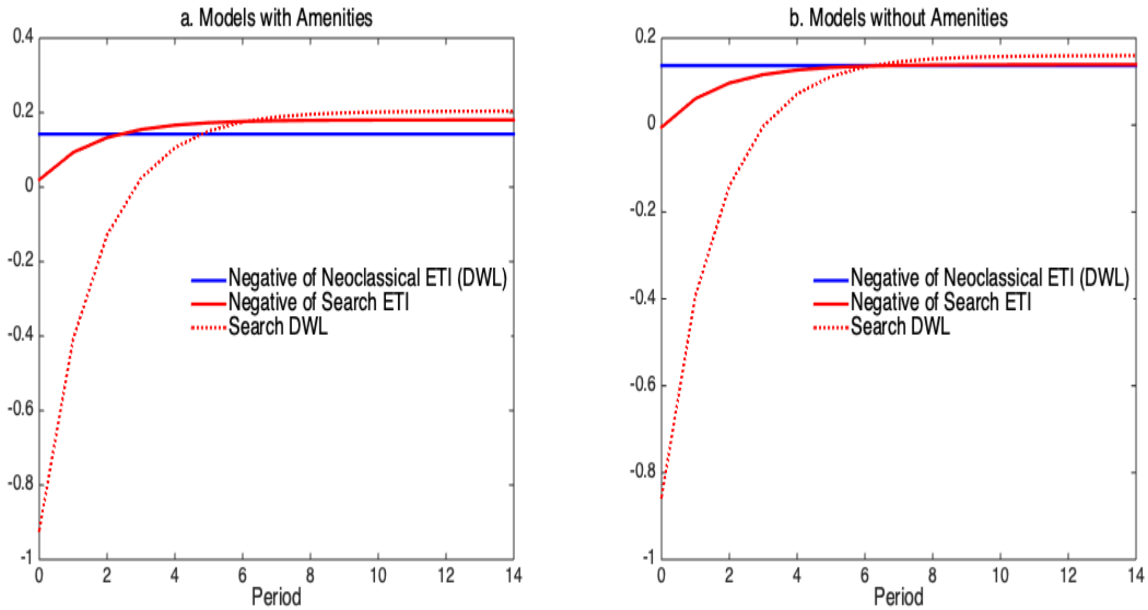
The figure shows the response of selected model variables to a permanent 1-percent rise in income tax when there are unemployment benefits. The magnitudes are in terms of percentage deviations from the pre-tax hike steady-state levels.

Figure 5: Other variables in response to 1% increase in tax rate
(with unemployment benefits)



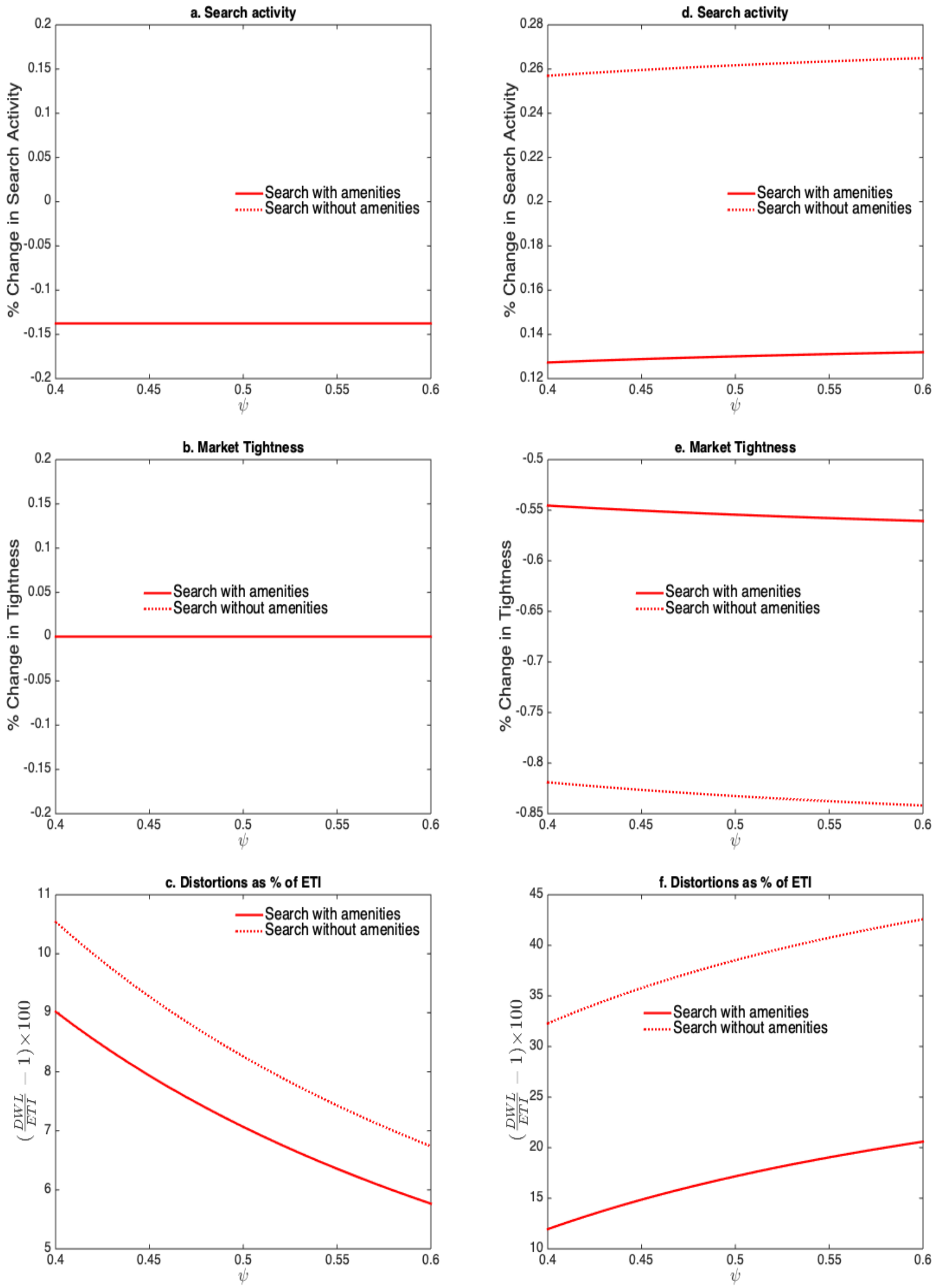
The figure shows the response of selected model variables to a permanent 1-percent rise in income tax when there are unemployment benefits. The magnitudes are in terms of percentage deviations from the pre-tax hike steady-state levels.

Figure 6: DWL and ETI in response to 1% increase in tax rate
(with unemployment benefits)



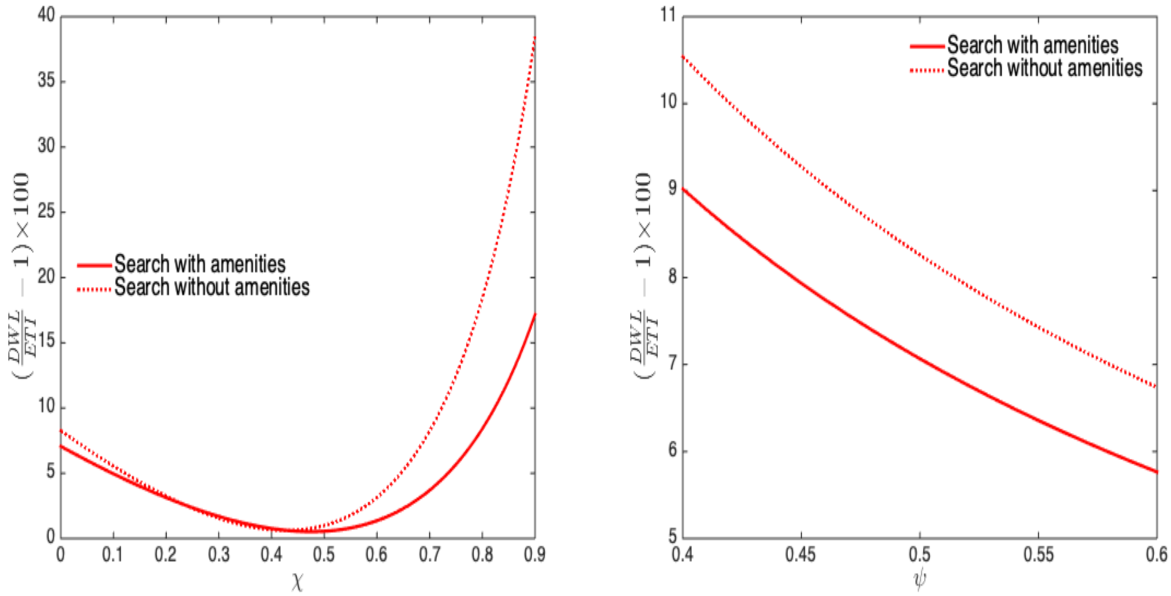
The figure shows the response of the ETI and DWL to a permanent 1-percent rise in income tax when there are unemployment benefits. The left panel corresponds to the cases with amenities, while the right panel corresponds to the cases without amenities. The magnitudes are in terms of percentage deviations from the pre-tax hike steady-state levels.

Figure 7: Impact of deviation from Hosios condition
(Left: No unemployment benefits, Right: With unemployment benefits)



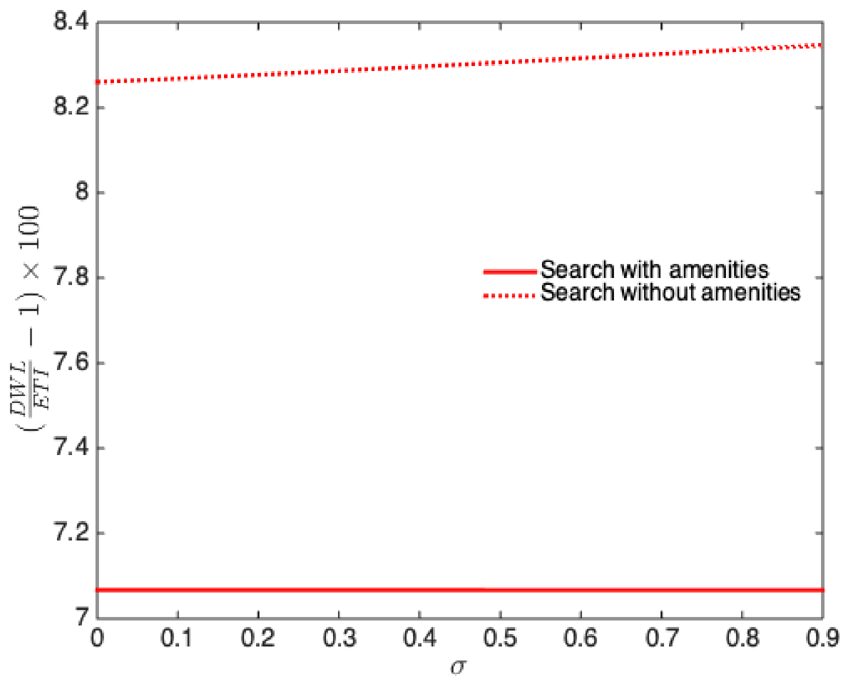
The figure shows selected model variables in response to a 1-percent income tax hike for a range of values of ψ—the bargaining power of workers. When producing the results, $1 - \xi$ is kept constant at 0.5, while the model is recalibrated for each value of ψ. The left panels correspond to the cases without unemployment benefits, while the right panels show the results with unemployment benefits.

Figure 8: Sensitivity to χ and ψ



The figure shows deviation of DWL from the ETI in response to a 1-percent increase in income tax for a range of selected model parameters (χ, the unemployment benefit, on the left; and ψ, the bargaining power of workers, on the right). When producing the results, the models are recalibrated for each value of χ and ψ, respectively. Unlike in Figure 7, $1 - \xi$ is set equal to ψ at each calibration, preserving the Hosios condition.

Figure 9: Sensitivity to risk aversion (σ)



The figure shows deviation of DWL from the ETI in response to a 1-percent increase in income tax for a range of values of relative risk aversion, governed by the parameter σ. When producing the results, the models are recalibrated for each value of σ.

A Background Partial Equilibrium Model

A.1 Derivation of Equation (2.1)

The derivation of equation 2.1 is as follows:

$$\begin{aligned}
dU + dT &= (1-\tau)\, d\,(wn) + \frac{ds}{d\tau} + \frac{d\Omega}{d\tau} - wn \cdot d\tau - h' dn + \tau d\,(wn) + wn \cdot d\tau \\
&= d\,(wn) - (1-\tau)\, wdn + \frac{d\Omega}{d\tau} + h' \frac{ds}{d\tau} \\
&= w \cdot dn + n \cdot dw - (1-\tau)\, wdn + \frac{d\Omega}{d\tau} + h' \frac{ds}{d\tau} \\
&= n \cdot dw + \tau w \cdot dn + (\tau n \cdot dw - \tau n \cdot dw) + \frac{d\Omega}{d\tau} + h' \frac{ds}{d\tau} \\
&= (1-\tau)\, n \cdot dw + \tau d\,(wn) + \frac{d\Omega}{d\tau} + h' \frac{ds}{d\tau},
\end{aligned}$$

where the second line follows from the household's first order condition for n. Therefore:

$$\begin{aligned}
-\frac{dU + dT}{wn \cdot d\tau} &= -(1-\tau)\, \frac{n}{wn} \frac{dw}{d\tau} - \frac{\tau}{wn} \frac{d\,(wn)}{d\tau} - \frac{d\Omega}{wn \cdot d\tau} - h' \frac{ds}{wn \cdot d\tau} \\
&= -(1-\tau)\, \frac{\tau}{\tau} \frac{dw/w}{d\tau} - \frac{d\,(wn)\,/wn}{d\tau/\tau} - \frac{d\Omega}{wn \cdot d\tau} - h' \frac{ds}{wn \cdot d\tau} \\
&= -\left(\frac{1-\tau}{\tau}\right) \frac{dw/w}{d\tau/\tau} - \frac{d\,(wn)\,/wn}{d\tau/\tau} - \frac{d\Omega}{wn \cdot d\tau} - h' \frac{ds}{wn \cdot d\tau} \\
&= -\left(\frac{1-\tau}{\tau}\right) \frac{d\ln w}{d\ln \tau} - \frac{d\Omega}{wn \cdot d\tau} - h' \frac{ds}{wn \cdot d\tau} - \frac{d\ln (wn)}{d\ln \tau}.
\end{aligned}$$

A.2 T Consumed by Household

In the Chetty (2009) framework, assume that:

$$c = (1-\tau)\, wn + \Omega + T,$$

where $T = \tau wn$. Also, assume that $u' = 1$. Then:

$$\begin{aligned}
\frac{dSW}{d\tau} &= \frac{d\left(wn+\Omega\right)}{d\tau} - h'\frac{dn}{d\tau} + h'\frac{ds}{d\tau} \\
&= \left(w\frac{dn}{d\tau} + n\frac{dw}{d\tau}\right) + \frac{d\Omega}{d\tau} - \left(1-\tau\right)w\frac{dn}{d\tau} + h'\frac{ds}{d\tau} \\
&= n\frac{dw}{d\tau} + \tau w\frac{dn}{d\tau} + \frac{d\Omega}{d\tau} + \left(\tau n\frac{dw}{d\tau} - \tau n\frac{dw}{d\tau}\right) + h'\frac{ds}{d\tau} \\
&= \tau\frac{d\left(wn\right)}{d\tau} + \left(1-\tau\right)n\frac{dw}{d\tau} + \frac{d\Omega}{d\tau} + h'\frac{ds}{d\tau},
\end{aligned}$$

where the third line follows by adding and subtracting $\tau n\frac{dw}{d\tau}$. Therefore:

$$\begin{aligned}
-\frac{dSW}{wn\cdot d\tau} &= -\left(1-\tau\right)\frac{n}{wn}\frac{dw}{d\tau} - \frac{\tau}{wn}\frac{d\left(wn\right)}{d\tau} - \frac{d\Omega}{wn\cdot d\tau} - h'\frac{ds}{d\tau} \\
&= -\left(1-\tau\right)\frac{\tau}{\tau}\frac{dw/w}{d\tau} - \frac{d\left(wn\right)/wn}{d\tau/\tau} - \frac{d\Omega}{wn\cdot d\tau} - h'\frac{ds}{d\tau} \\
&= -\left(\frac{1-\tau}{\tau}\right)\frac{dw/w}{d\tau/\tau} - \frac{d\left(wn\right)/wn}{d\tau/\tau} - \frac{d\Omega}{wn\cdot d\tau} - h'\frac{ds}{d\tau} \\
&= -\left(\frac{1-\tau}{\tau}\right)\frac{d\ln w}{d\ln\tau} - \frac{d\Omega}{wn\cdot d\tau} - h'\frac{ds}{d\tau} - \frac{d\ln\left(wn\right)}{d\ln\tau}.
\end{aligned}$$

B Neoclassical Model

The household's problem is:

$$\max_{c_t,\, n_t,\, b_t,\, a_t} \mathbb{E}_t\sum_{t}^{\infty}\beta^t[u\left(c_t\right) + h(1-n_t) + A(a_t n_t)],$$

such that:

$$c_t + b_t = \left(1-\tau_t\right)w_t n_t + \left(1+r_{t-1}\right)b_{t-1} + T_t + \Pi_t.$$

The household's current value Lagrangian is:

$$\mathcal{L} = \mathbb{E}_t\sum_{t=0}^{\infty}\beta^t\{u(c_t) + h\left(1-n_t\right) + A(a_t n_t)$$

$$+\lambda_t[\left(1-\tau_t\right)w_t n_t + \left(1+r_{t-1}\right)b_{t-1} + T_t + \Pi_t - c_t - b_t]\},$$

where λ_t is the marginal value of real wealth. The first order conditions with respect to c_t, n_t, a_t, and b_t are, respectively:

$$c_t\colon\ u'_t = \lambda_t;$$

$$n_t\colon\ h'_t = A'_t a_t + \lambda_t \left(1-\tau_t\right) w_t; \tag{B.1}$$

$$a_t\colon\ -A'_t n_t = \lambda_t (1-\tau_t) w_{a,t} n_t; \tag{B.2}$$

$$b_t\colon\ 1 = \mathbb{E}_t \underbrace{\beta \frac{\lambda_{t+1}}{\lambda_t}}_{\equiv \Xi_{t+1|t}} \left(1+r_t\right). \tag{B.3}$$

Note that combining the first and second first-order conditions yields:

$$h'_t = A'_t a_t + u'_t \left(1-\tau_t\right) w_t,$$

which is equation 2.6 in the main text. Also, combining the first and third first-order conditions yields:

$$A'_t = -u'_t (1-\tau_t) w_{a,t}.$$

The firm's problem is:

$$\max_{n_t,\, a_t,\, d_t} \mathbb{E}_t \sum_t^{\infty} \Xi_{t+1|t} \overbrace{\left[y(n_t,k_t) - w_t n_t - \phi(a_t n_t) - k_{t+1} + (1-\delta) k_t + d_t - (1+r_{t-1}) d_{t-1}\right]}^{\equiv \Pi_t}. \tag{B.4}$$

The first order conditions with respect to n_t, a_t, d_t, and k_{t+1} are, respectively:

$$n_t\colon\ y_{n,t} = w_t + \phi'_t a_t; \tag{B.5}$$

$$a_t\colon\ -w_{a,t} n_t = \phi'_t n_t; \tag{B.6}$$

$$1 = \Xi_{t+1|t} \left(1+r_t\right); \tag{B.7}$$

and

$$1 = \Xi_{t+1|t} \left[y_{k,t+1} + (1-\delta)\right]. \tag{B.8}$$

We assume that government consumption is zero so that the sum of lump sum transfers to households and unemployment benefits equal total tax revenue:

$$T_t = \tau_t w_t n_t. \tag{B.9}$$

Therefore, the aggregate resource constraint is given by:

$$y_t = c_t + i_t + \phi_t \tag{B.10}$$

given bond market clearing.

The model's equilibrium is given by a vector of endogenous variables:

$$[n_t,\ a_t,\ b_t,\ \Pi_t,\ w_t,\ y_t,\ d_t,\ T_t,\ c_t,\ r_t,\ k_{t+1}]$$

that, given the vector of parameters:

$$[\tau_t,\ \beta,]\,,$$

satisfies equations B.1, B.2, B.3, the instantaneous portion of B.4, B.5, B.6, B.7, B.9, B.10, B.8, and the bond-market clearing condition:

$$b_t = d_t.$$

Because equations B.3 and B.7 are identical and always hold, the equilibrium bond/debt is indeterminate. As such, for simplicity we assume that, in every period, $b_t = d_t = 0$.

Lemma. *In a general equilibrium framework, assuming a Cobb-Douglas production function implies that:*

$$\frac{w_n n}{w}\frac{d\ln n}{d\ln\tau} + \frac{w_k k}{w} = 0.$$

Proof. Clearly, showing that this equality is true is the same as showing that:

$$w_n\frac{dn}{d\tau} + w_k\frac{dk}{d\tau} = 0. \tag{B.11}$$

In steady state, equation B.8 implies that:

$$\frac{1}{\beta} = y_k + (1 - \delta) .$$

Therefore:

$$0 = y_{kn}\frac{dn}{d\tau} + y_{kk}\frac{dk}{d\tau} \Rightarrow \frac{dk}{d\tau} = -\frac{y_{kn}}{y_{kk}}\frac{dn}{d\tau}.$$

Using this expression to substitute out dk in equation B.11 implies that:

$$\left(w_n - w_k\frac{y_{kn}}{y_{kk}}\right)\frac{dn}{d\tau} = 0 \Rightarrow \left(w_n - w_k\frac{y_{kn}}{y_{kk}}\right)\frac{dn}{d\tau} = 0,$$

Of note, with a standard Cobb-Douglas production function:

$$y = zk^{\alpha}n^{(1-\alpha)},$$

where z is exogenous productivity and $\alpha \in (0, 1)$, it follows that:

$$w = (1 - \alpha)\, zk^{\alpha}n^{-\alpha} - \phi' a,$$

which implies that:

$$\begin{aligned} w_n &\equiv \frac{\partial w}{\partial n} \\ &= -\alpha\,(1-\alpha)\, zk^{\alpha}n^{-\alpha-1} \end{aligned}$$

and:

$$\begin{aligned} w_k &\equiv \frac{\partial w}{\partial k} \\ &= \alpha\,(1-\alpha)\, zk^{\alpha-1}n^{-\alpha}. \end{aligned}$$

Moreover:

$$y_n = (1-\alpha)\, zk^{\alpha} n^{-\alpha},\ y_{nk} = \alpha(1-\alpha)\, zk^{\alpha-1}n^{-\alpha},\ \text{and}\ y_{nn} = -\alpha(1-\alpha)\, zk^{\alpha}n^{-\alpha-1},$$

and also that:

$$y_k = \alpha zk^{\alpha-1}n^{1-\alpha},\ y_{kn} = \alpha(1-\alpha)\, zk^{\alpha-1}n^{-\alpha},\ \text{and}\ y_{kk} = -\alpha(1-\alpha)\, zk^{\alpha-2}n^{1-\alpha}.$$

Therefore:

$$\begin{aligned}
w_n - w_k\frac{y_{kn}}{y_{kk}} &= -\alpha(1-\alpha)\, zk^{\alpha}n^{-\alpha-1} - \alpha(1-\alpha)\, zk^{\alpha-1}n^{-\alpha}\frac{\alpha(1-\alpha)\, zk^{\alpha-1}n^{-\alpha}}{-\alpha(1-\alpha)\, zk^{\alpha-2}n^{1-\alpha}} \\
&= -\alpha(1-\alpha)\, zk^{\alpha}n^{-\alpha-1} + \alpha(1-\alpha)\, zk^{\alpha-1}n^{-\alpha}\frac{zk^{\alpha-1}n^{-\alpha}}{zk^{\alpha-2}n^{1-\alpha}} \\
&= -\alpha(1-\alpha)\left[zk^{\alpha}n^{-\alpha-1} - zk^{\alpha-1}n^{-\alpha}\frac{zk^{\alpha-1}n^{-\alpha}}{zk^{\alpha-2}n^{1-\alpha}}\right] \\
&= -\alpha(1-\alpha)\, z\left[k^{\alpha}n^{-\alpha-1} - k^{\alpha-1}n^{-\alpha}\frac{k^{\alpha-1}n^{-\alpha}}{k^{\alpha-2}n^{1-\alpha}}\right] \\
&= -\alpha(1-\alpha)\, z\left[k^{\alpha}n^{-\alpha-1} - k^{\alpha-1}n^{-\alpha}\frac{k}{n}\right] \\
&= -\alpha(1-\alpha)\, z\left[k^{\alpha}n^{-\alpha-1} - k^{\alpha}n^{-\alpha-1}\right].
\end{aligned}$$

■

B.1 DWL with Additive Separability

Social welfare is given by:

$$\begin{aligned}
SW &= u\left(\overbrace{(1-\tau)\, wn + \Pi + T}^{=c}\right) + A + h \\
&= u\left(\overbrace{(1-\tau)\, wn + \Pi + \tau wn}^{=c}\right) + A + h,
\end{aligned}$$

where the second line follows by the assumption that government consumption is zero. Therefore:

$$SW = u\left(\overbrace{wn + \Pi}^{=c}\right) + A + h,$$

which implies that:

$$\begin{aligned} dSW &= u'\left(w \cdot dn + n \cdot dw\right) + u'd\Pi + A'\left(n \cdot da + a \cdot dn\right) - h'dn + h'ds \\ &= u'\left(w \cdot dn + n \cdot dw\right) + A'\left(n \cdot da + a \cdot dn\right) - h'dn + h'ds. \end{aligned}$$

Then:

$$\begin{aligned} dSW = & \; u'\left(w \cdot dn + n \cdot dw\right) + u'd\Pi \\ & -u'\left(1-\tau\right) w_a \left(n \cdot da + a \cdot dn\right) \\ & -\left[-u'(1-\tau_t)w_a a + u'\left(1-\tau\right) w\right] \cdot dn + h'ds, \end{aligned}$$

where the second line follows from first order condition for a:

$$-A' = u'(1-\tau)w_a,$$

and the last line follows from the household's first order condition for n:

$$h' = A'a + u'\left(1-\tau\right) w.$$

Simplifying, and adding and subtracting $u'\tau n \cdot dw$ implies that:

$$dSW = u'n \cdot dw - u'\left(1-\tau\right) w_a n \cdot da + u'\tau w \cdot dn + u'\left(\tau n \cdot dw - \tau n \cdot dw\right) + u'd\Pi + h'ds.$$

$$\Rightarrow dSW = u'n \cdot dw - u'\left(1-\tau\right) w_a n \cdot da + u'\tau d\left(wn\right) - u'\tau n \cdot dw + u'd\Pi + h'ds$$

$$\Rightarrow dSW = u'\tau d\left(wn\right) - u'\left(1-\tau\right) w_a n \cdot da + \left(1-\tau\right) u'n \cdot dw + u'd\Pi + h'ds$$

$$\Rightarrow dSW = u'\tau d\left(wn\right) - u'\left(1-\tau\right) w_a n \cdot da + \left(1-\tau\right) u'n \cdot \left(w_n dn + w_k dk + w_a da\right) + u'd\Pi + h'ds,$$

which follows from the fact that in the neoclassical model:

$$w = y_n - \phi' a, \text{ so } w = w\left(n, k, a\right).$$

Thus:

$$dSW = u'\tau d\left(wn\right) + \left(1-\tau\right)u'n\cdot\left(w_n dn + w_k dk\right) + u'd\Pi + h'ds.$$

Implementing the definition of DWL:

$$\begin{aligned}
DWL &\equiv -\frac{dSW}{u'wn\cdot d\tau} = -u'\frac{\tau}{d\tau}d\left(wn\right)/wn - \left(1-\tau\right)\frac{n}{wn}\cdot\left(w_n\frac{dn}{d\tau} + w_k\frac{dk}{d\tau}\right) - \frac{u'd\Pi}{u'wn\cdot d\tau} - \frac{h'ds}{u'wn\cdot d\tau} \\
&= -\frac{d\ln\left(wn\right)}{d\ln\tau} - \left(1-\tau\right)\left(\frac{w_n}{w}\frac{dn}{d\tau} + \frac{w_k}{w}\frac{dk}{d\tau}\right) - \frac{u'd\Pi}{u'wn\cdot d\tau} - \frac{h'ds}{u'wn\cdot d\tau} \\
&= -\frac{d\ln\left(wn\right)}{d\ln\tau} - \left(1-\tau\right)\left(\frac{w_n}{w}\frac{n}{n}\frac{\tau}{\tau}\frac{dn}{d\tau} + \frac{w_k}{w}\frac{k}{k}\frac{\tau}{\tau}\frac{dk}{d\tau}\right) - \frac{u'd\Pi}{u'wn\cdot d\tau} - \frac{h'ds}{u'wn\cdot d\tau} \\
&= -\frac{d\ln\left(wn\right)}{d\ln\tau} - \left(1-\tau\right)\left(\frac{w_n}{w}\frac{n}{\tau}\frac{dn/n}{d\tau/\tau} + \frac{w_k}{w}\frac{k}{\tau}\frac{dk/k}{d\tau/\tau}\right) - \frac{u'd\Pi}{u'wn\cdot d\tau} - \frac{h'ds}{u'wn\cdot d\tau} \\
&= -\frac{d\ln\left(wn\right)}{d\ln\tau} - \left(\frac{1-\tau}{\tau}\right)\left(\frac{w_n n}{w}\frac{d\ln n}{d\ln\tau} + \frac{w_k k}{w}\frac{d\ln k}{d\ln\tau}\right) - \frac{u'd\Pi}{u'wn\cdot d\tau} - \frac{h'ds}{u'wn\cdot d\tau}
\end{aligned}$$

B.2 General Statement of DWL

Here, rather than assuming additively separable utility, we instead consider an entirely abstract utility function. We only discuss the household's problem since the rest of the framework remains as before. This problem is:

$$\max_{c_t,\, n_t,\, b_t,\, a_t} \mathbb{E}_t \sum_{t=0}^{\infty}\beta^t U(c_t, 1-n_t, a_t n_t)],$$

where U is increasing and concave in c, $1-n$, and an, such that:

$$c_t + b_t \leq \left(1-\tau_t\right)w_t n_t + \left(1+r_{t-1}\right)b_{t-1} + T_t + \Pi_t.$$

The household's current value Lagrangian is:

$$\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \{U(c_t, 1-n_t, a_t n_t)$$
$$+\lambda_t[(1-\tau_t)\, w_t n_t + (1+r_{t-1})\, b_{t-1} + T_t + \Pi_t - c_t - b_t]\},$$

where: λ_t is the marginal value of real wealth. The first order conditions with respect to c_t, n_t, a_t, and b_t are, respectively:

$$c_t\colon\ U_{c,t} = \lambda_t;$$

$$n_t\colon\ U_{n,t} = U_{an,t} a_t + u_{c,t}\,(1-\tau_t)\, w_t; \tag{B.12}$$

$$a_t\colon\ -U_{an,t} n_t = U_{c,t}(1-\tau_t) w_{a,t} n_t; \tag{B.13}$$

$$b_t\colon\ 1 = \mathbb{E}_t \underbrace{\beta \frac{U_{c,t+1}}{U_{c,t}}}_{\equiv \Xi_{t+1|t}} (1+r_t)\,. \tag{B.14}$$

Note that combining the first and third first-order conditions yields:

$$U_{n,t} = -U_{c,t}(1-\tau_t) w_{a,t}.$$

Steady-state social welfare is:

$$\begin{aligned} SW &= U\left(\overbrace{(1-\tau)\, wn + \Pi + T + \chi s}^{=c}, 1-n, an\right) \\ &= U\left(\overbrace{(1-\tau)\, wn + \Pi + \tau wn}^{=c}, 1-n, an\right), \end{aligned}$$

where the second line follows by the assumption that government consumption is zero. Therefore:

$$SW = U\left(\overbrace{wn + \Pi}^{=c}, 1-n, an\right).$$

The change in social welfare is thus:

$$\begin{aligned} dSW &= U_c\left(w \cdot dn + n \cdot dw\right) + U_c d\Pi + U_{an}\left(n \cdot da + a \cdot dn\right) - U_n dn \\ &= U_c\left(w \cdot dn + n \cdot dw\right) + U_{an}\left(n \cdot da + a \cdot dn\right) - U_n dn, \end{aligned}$$

where the second line follows from the fact that with a constant returns to scale production function and perfectly competitive markets in a neoclassical framework $d\Pi = 0$.

Then:

$$\begin{aligned} dSW &= U_c\left(w \cdot dn + n \cdot dw\right) \\ &\quad - U_c(1-\tau)w_a\left(n \cdot da + a \cdot dn\right) \\ &\quad - \left[-U_c(1-\tau)w_a a + U_c\left(1-\tau\right)w\right] \cdot dn, \end{aligned}$$

where the second line follows from first order condition for a:

$$-U_{an} = U_c(1-\tau)w_a,$$

and the last line follows from the household's first order condition for n:

$$U_n = U_{an}a + U_c\left(1-\tau\right)w.$$

Simplifying, and adding and subtracting $U_c\tau n \cdot dw$ implies that:

$$dSW = U_c n \cdot dw - U_c\left(1-\tau\right)w_a n \cdot da + U_c \tau w \cdot dn + U_c\left(\tau n \cdot dw - \tau n \cdot dw\right).$$

The remaining steps are as before and, therefore, clearly imply that in this general case:

$$DWL = -\frac{d\ln\left(wn\right)}{d\ln\tau} - \left(\frac{1-\tau}{\tau}\right)\left(\frac{w_n n}{w}\frac{d\ln n}{d\ln\tau} + \frac{w_k k}{w}\frac{d\ln k}{d\ln\tau}\right),$$

as in the additively separable case.

C Search Model

C.1 Derivation of Nash Wage

Since labor markets are frictional, wages are determined via Nash bargaining. The firm and household choose a wage to maximize the Nash product: $(\mathbf{W_t} - \mathbf{U_t})^{\psi} \mathbf{J_t}^{1-\psi}$, where: $\psi \in (0,1)$ is the parametric bargaining power of workers. Taking the first order condition with respect to the wage gives the following generalized expression:

$$\psi (\mathbf{W_t} - \mathbf{U_t})^{\psi-1} \mathbf{J_t}^{1-\psi} \frac{\partial (\mathbf{W_t} - \mathbf{U_t})}{\partial w_t} + (1-\psi) (\mathbf{W_t} - \mathbf{U_t})^{\psi} \mathbf{J_t}^{-\psi} \frac{\partial \mathbf{J_t}}{\partial w_t} = 0,$$

which simplifies to the following:

$$\psi \mathbf{J_t} \frac{\partial (\mathbf{W_t} - \mathbf{U_t})}{\partial w_t} + (1-\psi) (\mathbf{W_t} - \mathbf{U_t}) \frac{\partial \mathbf{J_t}}{\partial w_t} = 0.$$

Given that, as shown in the main text,

$$\mathbf{W_t} - \mathbf{U_t} = (1-\tau_t) w_t - \chi + \frac{A_t' a_t}{u_t'} + (1-\rho-f_t) \mathbb{E}_t \Xi_{t+1|t} (\mathbf{W}_{t+1} - \mathbf{U}_{t+1}) \tag{C.1}$$

and

$$\mathbf{J}_t = y_t' - w_t - \phi_t' a_t + (1-\rho) \mathbb{E}_t \Xi_{t+1|t} \mathbf{J}_{t+1}, \tag{C.2}$$

the relevant first order conditions are: $\frac{\partial (\mathbf{W_t} - \mathbf{U_t})}{\partial w_t} = (1-\tau_t)$ and $\frac{\partial \mathbf{J_t}}{\partial w_t} = -1$. So, we can simplify the generalized Nash sharing rule to: $\psi \mathbf{J_t} (1-\tau_t) - (1-\psi) (\mathbf{W_t} - \mathbf{U_t}) = 0$, which implies that:

$$\mathbf{W_t} - \mathbf{U_t} = \frac{\psi}{1-\psi} (1-\tau_t) \mathbf{J_t}. \tag{C.3}$$

Given this last expression, the Nash wage is obtained as follows. Substitute in the equation above the expression for $\mathbf{W_t} - \mathbf{U_t}$ from equation C.1 into the left-hand side of the Nash sharing rule to obtain:

$$(1-\tau_t) w_t - \chi + \frac{A_t' a_t}{u_t'} + (1-\rho-f_t) \mathbb{E}_t \Xi_{t+1|t} (\mathbf{W}_{t+1} - \mathbf{U}_{t+1}) = \frac{\psi}{1-\psi} (1-\tau_t) \mathbf{J_t}$$

Now substitute in the Nash sharing rule on the left-hand side for $\mathbf{W_{t+1}} - \mathbf{U_{t+1}}$, which yields:

$$(1-\tau_t)\,w_t - \chi + \frac{A_t' a_t}{u_t'} + (1-\rho-f_t)\,\mathbb{E}_t \Xi_{t+1|t} \frac{\psi}{1-\psi}\,(1-\tau_{t+1})\,\mathbf{J_{t+1}} = \frac{\psi}{1-\psi}\,(1-\tau_t)\,\mathbf{J_t}$$

Now, in the equation above, use equation C.2 to substitute out $\mathbf{J_t}$ on its left-hand side. This yields:

$$\begin{gathered}(1-\tau_t)\,w_t - \chi + \frac{A_t' a_t}{u_t'} + (1-\rho-f_t)\,\mathbb{E}_t \Xi_{t+1|t} \frac{\psi}{1-\psi}\,(1-\tau_{t+1})\,\mathbf{J_{t+1}} = \\ \frac{\psi}{1-\psi}\,(1-\tau_t)\left[y_t' - w_t - \phi_t' a_t + (1-\rho)\,\mathbb{E}_t \Xi_{t+1|t} \mathbf{J}_{t+1}\right].\end{gathered}$$

Next, combine all wage-inclusive terms on the left-hand side of this equation and all other terms on the right-hand side to obtain:

$$\begin{gathered}(1-\tau_t)\,w_t + \frac{\psi}{1-\psi}\,(1-\tau_t)\,w_t = \begin{bmatrix} \chi - \frac{A_t' a_t}{u_t'} \\ -(1-\rho-f_t)\,\mathbb{E}_t \Xi_{t+1|t} \frac{\psi}{1-\psi}\,(1-\tau_{t+1})\,\mathbf{J_{t+1}} \end{bmatrix} \\ \frac{\psi}{1-\psi}\,(1-\tau_t)\left[y_t' - \phi_t' a_t + (1-\rho)\,\mathbb{E}_t \Xi_{t+1|t} \mathbf{J}_{t+1}\right],\end{gathered}$$

which implies that:

$$\begin{gathered}\frac{1-\tau_t}{1-\psi} w_t = \begin{bmatrix} \chi - \frac{A_t' a_t}{u_t'} \\ -(1-\rho-f_t)\,\mathbb{E}_t \Xi_{t+1|t} \frac{\psi}{1-\psi}\,(1-\tau_{t+1})\,\mathbf{J_{t+1}} \end{bmatrix} = \\ \frac{\psi}{1-\psi}\,(1-\tau_t)\left[y_t' - \phi_t' a_t + (1-\rho)\,\mathbb{E}_t \Xi_{t+1|t} \mathbf{J}_{t+1}\right].\end{gathered}$$

Now, multiply through by $1-\psi$ and divide through by $1-\tau_t$. This yields:

$$\begin{gathered}w_t = \left\{ \begin{matrix} (1-\psi)\left[\frac{\chi}{1-\tau_t} - \frac{A_t' a_t}{u_t'(1-\tau_t)}\right] \\ -\psi\,(1-\rho-f_t)\,\mathbb{E}_t \Xi_{t+1|t} \frac{1-\tau_{t+1}}{1-\tau_t} \mathbf{J_{t+1}} \end{matrix} \right\} = \\ \psi\left[y_t' - \phi_t' a_t + (1-\rho)\,\mathbb{E}_t \Xi_{t+1|t} \mathbf{J}_{t+1}\right].\end{gathered}$$

Distribute terms:

$$w_t = \left\{ \begin{array}{c} (1-\psi)\left[\frac{\chi}{1-\tau_t} - \frac{A_t' a_t}{u_t'(1-\tau_t)}\right] \\ \psi\left(y_t' - \phi_t' a_t\right) \end{array} \right\} =$$
$$\psi \mathbb{E}_t \Xi_{t+1|t}\left[(1-\rho) - (1-\rho-f_t)\frac{1-\tau_{t+1}}{1-\tau_t}\right]\mathbf{J}_{t+1}.$$

C.2 Deadweight Loss with Additive Separability

Steady-state social welfare is:

$$\begin{aligned} SW &= u\left(\overbrace{(1-\tau)\,wn + \Pi + T + \chi s}^{=c}\right) + A + h \\ &= u\left(\overbrace{(1-\tau)\,wn + \Pi + \tau wn}^{=c}\right) + A + h, \end{aligned}$$

where the second line follows by the assumption that government consumption is zero. Therefore:

$$SW = u\left(\overbrace{wn + \Pi}^{=c}\right) + A + h.$$

This implies that:

$$\begin{aligned} dSW &= u' d\,(wn) + u' d\Pi + dh + d(A) \\ &= u'\,(w \cdot dn + n \cdot dw) + A'n \cdot da + A'a \cdot dn + dh + u'd\Pi, \end{aligned}$$

where, given search frictions and monopoly bargaining power, $d\Pi \neq 0$. Then,

$$dSW = u'\,(w \cdot dn + n \cdot dw) - \underbrace{u'\,(1-\tau)\,w_a}_{=A' \text{ by HH's FOC for } a}\; n \cdot da + A'a \cdot dn + dh + u'd\Pi$$

Now, recall that:

$$w = (1-\psi)\frac{\chi}{1-\tau} - \phi' a + \psi(y_n + \gamma\theta),$$

which implies that:

$$
\begin{aligned}
dw &= w_\tau d\tau + w_a da + y_{nn} dn + \gamma d\theta \\
&= w_\tau d\tau + w_a da + \gamma d\theta
\end{aligned}
$$

given the innocuous assumption that $y_{nn} = 0$. Of course, more concretely and given the present set of assumptions:

$$
\begin{aligned}
dw &= (1-\psi)\frac{\chi}{(1-\tau)^2} \cdot d\tau - \phi' da - \phi'' \left(n \cdot da + a^2 \cdot dn\right) + \psi\gamma \cdot d\theta \\
&= (1-\psi)\frac{\chi}{(1-\tau)^2} \cdot d\tau - \phi' \cdot da + \psi\gamma \cdot d\theta
\end{aligned}
$$

given the assumption that $\phi'' = 0$. Thus, we have:

$$
u' n dw = (1-\psi) u' n \frac{\chi}{(1-\tau)^2} d\tau - u' n \phi' \cdot da + u' n \psi\gamma \cdot d\theta.
$$

Returning to:

$$
dSW = u' \left(w \cdot dn + n \cdot dw\right) - u' \left(1-\tau\right) w_a n \cdot da + A' a \cdot dn + dh + u' d\Pi
$$

and substituting out $u' n dw$ yields:

$$
\begin{aligned}
dSW = \; & u' w \cdot dn + (1-\psi) u' n \frac{\chi}{(1-\tau)^2} d\tau - u' n \phi' \cdot da + u' n \psi\gamma \cdot d\theta \\
& - u' \left(1-\tau\right) w_a n \cdot da + A' a \cdot dn + dh + u' d\Pi.
\end{aligned}
$$

Replacing $\phi' = -w_a$ from firm's FOC:

$$
\begin{aligned}
dSW &= u' w \cdot dn + (1-\psi) u' n \frac{\chi}{(1-\tau)^2} d\tau \\
&\quad + u' n w_a \cdot da + u' n \psi\gamma \cdot d\theta - u' \left(1-\tau\right) w_a n \cdot da + A' a \cdot dn + dh + u' d\Pi \\
&= u' w \cdot dn + (1-\psi) u' n \frac{\chi}{(1-\tau)^2} d\tau \\
&\quad + u' \tau w_a n \cdot da + u' n \psi\gamma \cdot d\theta + A' a \cdot dn + dh + u' d\Pi.
\end{aligned}
$$

Adding and subtracting $u'(1-\tau)w \cdot dn$ implies that:

$$\begin{aligned} dSW &= [u'w \cdot dn - u'(1-\tau)w \cdot dn] + u'(1-\tau)w \cdot dn + (1-\psi)u'n\frac{\chi}{(1-\tau)^2}d\tau \\ &\quad + u'\tau w_a n \cdot da + u'n\psi\gamma \cdot d\theta + A'a \cdot dn + dh + u'd\Pi \\ &= u'\tau w \cdot dn + u'(1-\tau)w \cdot dn + (1-\psi)u'n\frac{\chi}{(1-\tau)^2}d\tau \\ &\quad + u'\tau w_a n \cdot da + u'n\psi\gamma \cdot d\theta + A'a \cdot dn + dh + u'd\Pi. \end{aligned}$$

Adding and subtracting $u'\tau n \cdot dw$ implies that:

$$\begin{aligned} dSW &= (u'\tau w \cdot dn + u'\tau n \cdot dw) - u'\tau n \cdot dw + u'(1-\tau)w \cdot dn + (1-\psi)u'n\frac{\chi}{(1-\tau)^2}d\tau \\ &\quad + u'\tau w_a n \cdot da + u'n\psi\gamma \cdot d\theta + A'a \cdot dn + dh + u'd\Pi. \\ &= u'\tau d(wn) - u'\tau n \cdot dw + u'(1-\tau)w \cdot dn + (1-\psi)u'n\frac{\chi}{(1-\tau)^2}d\tau \\ &\quad + u'\tau w_a n \cdot da + u'n\psi\gamma \cdot d\theta + A'a \cdot dn + dh + u'd\Pi. \end{aligned}$$

Replacing $dh = -h'ds - h'dn$ yields:

$$\begin{aligned} dSW &= u'\tau d(wn) - u'\tau n \cdot dw + u'(1-\tau)w \cdot dn + (1-\psi)u'n\frac{\chi}{(1-\tau)^2}d\tau \\ &\quad + u'\tau w_a n \cdot da + u'n\psi\gamma \cdot d\theta + A'a \cdot dn - h'dn - h'ds + u'd\Pi. \end{aligned}$$

Combining the dn terms, we have:

$$\begin{aligned} dSW &= u'\tau d(wn) - u'\tau n \cdot dw + [u'(1-\tau)w + A'a - h'] \cdot dn + (1-\psi)u'n\frac{\chi}{(1-\tau)^2}d\tau \\ &\quad + u'\tau w_a n \cdot da + u'n\psi\gamma \cdot d\theta - h'ds + u'd\Pi. \end{aligned}$$

With the definition of DWL, it follows that:

$$\begin{aligned} DWL = -\frac{dSW}{u'wn \cdot d\tau} &= -\frac{\tau d(wn)}{wnd\tau} + u'\tau n\frac{dw}{u'wn \cdot d\tau} - [u'(1-\tau)w + A'a - h']\frac{dn}{u'wn \cdot d\tau} \\ &\quad -(1-\psi)u'n\frac{\chi}{(1-\tau)^2}\frac{d\tau}{u'wn \cdot d\tau} \\ &\quad -u'\tau w_a n\frac{da}{u'wn \cdot d\tau} - u'n\psi\gamma\frac{d\theta}{u'wn \cdot d\tau} + h'\frac{ds}{u'wn \cdot d\tau} - u'\frac{d\Pi}{u'wn \cdot d\tau}, \end{aligned}$$

and therefore that:

$$\begin{aligned} DWL &= -\frac{d\ln(wn)}{d\ln\tau} - \left[u'(1-\tau)w + A'a - h'\right]\frac{dn}{u'wn\cdot d\tau} \\ &-u'\frac{d\Pi}{u'wn\cdot d\tau} + h'\frac{ds}{u'wn\cdot d\tau} \\ &-u'\tau w_a n\frac{da}{u'wn\cdot d\tau} - u'n\psi\gamma\frac{d\theta}{u'wn\cdot d\tau} \\ &+u'\tau n\frac{dw}{u'wn\cdot d\tau} - (1-\psi)\frac{\chi}{w(1-\tau)^2}. \end{aligned}$$

Note that, given the employment equation of motion, in steady state $n = m/\rho$. Moreover, $f = m/s$. Therefore:

$$n = \frac{sf}{\rho} \Rightarrow dn = \frac{f}{\rho}ds + \frac{sf'}{\rho}d\left(\frac{v}{s}\right) = \frac{f}{\rho}ds + \frac{sf'}{\rho}d\theta.$$

Using this expression to substitute out dn in the definition of DWL, we have:

$$\begin{aligned} DWL &= -\frac{d\ln(wn)}{d\ln\tau} - \left[u'(1-\tau)w + A'a - h'\right]\frac{\frac{f}{\rho}ds + \frac{sf'}{\rho}d\theta}{u'wn\cdot d\tau} \\ &-u'\frac{d\Pi}{u'wn\cdot d\tau} + h'\frac{ds}{u'wn\cdot d\tau} \\ &-u'\tau w_a n\frac{da}{u'wn\cdot d\tau} - u'n\psi\gamma\frac{d\theta}{u'wn\cdot d\tau} \\ &+u'\tau n\frac{dw}{u'wn\cdot d\tau} - (1-\psi)\frac{\chi}{w(1-\tau)^2}, \end{aligned}$$

which grouping terms yields:

$$\begin{aligned} DWL &= -\frac{d\ln(wn)}{d\ln\tau} - \left\{\left[u'(1-\tau)w + A'a - h'\right]\frac{f}{\rho} - h'\right\}\frac{ds}{u'wn\cdot d\tau} \\ &-u'\frac{d\Pi}{u'wn\cdot d\tau} \\ &-\left\{\left[u'(1-\tau)w + A'a - h'\right]\frac{sf'}{\rho} + u'n\psi\gamma\right\}\frac{d\theta}{u'wn\cdot d\tau} \\ &-u'\tau w_a n\cdot\frac{da}{u'wn\cdot d\tau} + u'\tau n\cdot\frac{dw}{u'wn\cdot d\tau} - (1-\psi)\frac{\chi}{w(1-\tau)^2}, \end{aligned}$$

With the total derivative of the wage:

$$\begin{aligned} dw &= (1-\psi)\frac{\chi}{(1-\tau)^2}\cdot d\tau - \phi' \cdot da + \psi\gamma \cdot d\theta \\ &= (1-\psi)\frac{\chi}{(1-\tau)^2}\cdot d\tau + w_a \cdot da + \psi\gamma \cdot d\theta, \end{aligned}$$

where the second line follows from the firm's first order condition for amenities ($w_a = -\phi'$) implies that:

$$\tau n \frac{dw}{wn \cdot d\tau} = \tau(1-\psi)n\frac{\chi}{(1-\tau)^2}\frac{d\tau}{wn\cdot d\tau} + \tau w_a n \frac{da}{wn \cdot d\tau} + \tau\psi\gamma n \frac{d\theta}{wn\cdot d\tau}.$$

Using this equation to substitute out $\tau n \frac{dw}{wn \cdot d\tau}$ in DWL, we obtain:

$$\begin{aligned} DWL &= -\frac{d\ln(wn)}{d\ln\tau} - \left\{\left[u'(1-\tau)w + A'a - h'\right]\frac{f}{\rho} - h'\right\}\frac{ds}{u'wn\cdot d\tau} \\ &\quad -u'\frac{d\Pi}{u'wn\cdot d\tau} \\ &\quad -\left\{\left[u'(1-\tau)w + A'a - h'\right]\frac{sf'}{\rho} + u'n\psi\gamma\right\}\frac{d\theta}{u'wn\cdot d\tau} \\ &\quad -u'\tau w_a n \cdot \frac{da}{u'wn\cdot d\tau} \\ &\quad +\tau(1-\psi)n\frac{\chi}{(1-\tau)^2}\frac{d\tau}{wn\cdot d\tau} + \tau w_a n\frac{da}{wn\cdot d\tau} + \tau\psi\gamma n\frac{d\theta}{wn\cdot d\tau} \\ &\quad -(1-\psi)\frac{\chi}{w(1-\tau)^2}. \end{aligned}$$

By grouping terms and simplifying, we get:

$$\begin{aligned} DWL &= -\frac{d\ln(wn)}{d\ln\tau} - \left\{\left[u'(1-\tau)w + A'a - h'\right]\frac{f}{\rho} - h'\right\}\frac{ds}{u'wn\cdot d\tau} \\ &\quad -u'\frac{d\Pi}{u'wn\cdot d\tau} \\ &\quad -\left\{\left[u'(1-\tau)w + A'a - h'\right]\frac{sf'}{\rho} + u'n\psi\gamma - u'\tau\gamma n\psi\right\}\frac{d\theta}{u'wn\cdot d\tau} \\ &\quad -(1-\tau)(1-\psi)n\frac{\chi}{(1-\tau)^2}\frac{d\tau}{wn\cdot d\tau}. \end{aligned}$$

Therefore:

$$
\begin{aligned}
DWL &= -\frac{d\ln(wn)}{d\ln\tau} - \left\{ \left[u'(1-\tau)w + A'a - h' \right] \frac{f}{\rho} - h' \right\} \frac{ds}{u'wn \cdot d\tau} \\
& -u' \frac{d\Pi}{u'wn \cdot d\tau} \\
& - \left\{ \left[u'(1-\tau)w + A'a - h' \right] \frac{sf'}{\rho} + u'(1-\tau)n\psi\gamma \right\} \frac{d\theta}{u'wn \cdot d\tau} \\
& -(1-\psi)\frac{\chi}{w(1-\tau)}.
\end{aligned}
$$

Recall the job creation condition:

$$
w + \phi' a = y_n - \frac{1-(1-\rho)\beta}{\beta}\frac{\gamma}{q},
$$

which combined with the wage equation:

$$
w + \phi' a = (1-\psi)\frac{\chi}{1-\tau} + \psi(y_n + \gamma\theta)
$$

implies that:

$$
\begin{aligned}
(1-\psi)\frac{\chi}{1-\tau} &= y_n - \frac{1-(1-\rho)\beta}{\beta}\frac{\gamma}{q} - \psi(y_n + \gamma\theta) \\
\Rightarrow \frac{(1-\psi)\chi}{(1-\tau)^2} d\tau &= (1-\psi)\, y_{nn} dn + \left[\frac{1-(1-\rho)\beta}{\beta}\frac{\gamma}{q^2} q' - \gamma\psi \right] d\theta \\
\Rightarrow \frac{(1-\psi)\chi}{w(1-\tau)}\frac{d\tau}{d\tau} &= (1-\tau)\left[\frac{1-(1-\rho)\beta}{\beta}\frac{\gamma}{q^2} q' - \gamma\psi \right] u'n \frac{d\theta}{u'wn \cdot d\tau}, \\
\Rightarrow \frac{(1-\psi)\chi}{w(1-\tau)} &= (1-\tau)\left[\frac{1-(1-\rho)\beta}{\beta}\frac{\gamma}{q^2} q' - \gamma\psi \right] u'n \frac{d\theta}{u'wn \cdot d\tau},
\end{aligned}
$$

where we used $y_{nn} = 0$. We use this expression to substitute out $\frac{(1-\psi)\chi}{w(1-\tau)}$ in the equation for

DWL:

$$\begin{aligned} DWL &= -\frac{d\ln(wn)}{d\ln\tau} - \left\{ \left[u'(1-\tau)w + A'a - h' \right] \frac{f}{\rho} - h' \right\} \frac{ds}{u'wn\cdot d\tau} \\ &\quad -u'\frac{d\Pi}{u'wn\cdot d\tau} \\ &\quad - \left\{ \left[u'(1-\tau)w + A'a - h' \right] \frac{sf'}{\rho} + u'(1-\tau)n\psi\gamma \right\} \frac{d\theta}{u'wn\cdot d\tau} \\ &\quad -(1-\tau)\left[\frac{1-(1-\rho)\beta}{\beta}\frac{\gamma}{q^2}q' - \gamma\psi \right] u'n\frac{d\theta}{u'wn\cdot d\tau}. \end{aligned}$$

Therefore:

$$\begin{aligned} DWL &= -\frac{d\ln(wn)}{d\ln\tau} - \left\{ \left[u'(1-\tau)w + A'a - h' \right] \frac{f}{\rho} - h' \right\} \frac{ds}{u'wn\cdot d\tau} \\ &\quad -u'\frac{d\Pi}{u'wn\cdot d\tau} \\ &\quad - \left\{ \begin{array}{c} \left[u'(1-\tau)w + A'a - h' \right] \frac{sf'}{\rho} + u'(1-\tau)n\psi\gamma \\ +(1-\tau)\left[\frac{1-(1-\rho)\beta}{\beta}\frac{\gamma}{q^2}q' - \gamma\psi \right] u'n \end{array} \right\} \frac{d\theta}{u'wn\cdot d\tau}. \end{aligned}$$

By canceling terms, we get:

$$\begin{aligned} DWL &= -\frac{d\ln(wn)}{d\ln\tau} - \left\{ \left[u'(1-\tau)w + A'a - h' \right] \frac{f}{\rho} - h' \right\} \frac{ds}{u'wn\cdot d\tau} \\ &\quad -u'\frac{d\Pi}{u'wn\cdot d\tau} \\ &\quad - \left\{ \left[u'(1-\tau)w + A'a - h' \right] \frac{sf'}{\rho} + u'(1-\tau)\frac{1-(1-\rho)\beta}{\beta}\frac{\gamma}{q^2}q'n \right\} \frac{d\theta}{u'wn\cdot d\tau}. \end{aligned}$$

Given the fact that in steady state $n = m/\rho$ and $q = m/v$, $n = vq/\rho$ and therefore $n/q = v/\rho$. Moreover, from the firm's first order condition for vacancies, in steady state $\mathbf{J} = \gamma/(\beta q)$. Substitute out these expressions in the equation for DWL to obtain:

$$\begin{aligned} DWL &= -\frac{d\ln(wn)}{d\ln\tau} - \left\{ \left[u'(1-\tau)w + A'a - h' \right] \frac{f}{\rho} - h' \right\} \frac{ds}{u'wn\cdot d\tau} \\ &\quad -u'\frac{d\Pi}{u'wn\cdot d\tau} \\ &\quad - \left\{ \left[u'(1-\tau)w + A'a - h' \right] \frac{sf'}{\rho} + u'(1-\tau)\left[1-(1-\rho)\beta\right]\mathbf{J}q'\frac{v}{\rho} \right\} \frac{d\theta}{u'wn\cdot d\tau}. \end{aligned}$$

Now, use the Nash sharing rule by which:

$$\begin{aligned}\frac{\psi}{1-\psi}(1-\tau)\mathbf{J} &= \mathbf{W}-\mathbf{U} \\ &\equiv \frac{\delta}{u'},\end{aligned}$$

therefore implying that

$$\mathbf{J}=\frac{1-\psi}{(1-\tau)\psi}\frac{\delta}{u'},$$

to substitute out this expression in the equation for DWL:

$$\begin{aligned}DWL &= -\frac{d\ln(wn)}{d\ln\tau}-\left\{[u'(1-\tau)w+A'a-h']\frac{f}{\rho}-h'\right\}\frac{ds}{u'wn\cdot d\tau} \\ &\quad -u'\frac{d\Pi}{u'wn\cdot d\tau} \\ &\quad -\left\{[u'(1-\tau)w+A'a-h']\frac{sf'}{\rho}+[1-(1-\rho)\beta]\frac{1-\psi}{\psi}\delta q'\frac{v}{\rho}\right\}\frac{d\theta}{u'wn\cdot d\tau}.\end{aligned}$$

Recall from the household's first order condition for employment and search activity, respectively, in steady state:

$$u'(1-\tau)w+A'a-h'=\delta[1-\beta(1-\rho)].$$

Substitute out this expression in the equation for DWL to obtain:

$$\begin{aligned}DWL &= -\frac{d\ln(wn)}{d\ln\tau}-\left\{\delta[1-\beta(1-\rho)]\frac{f}{\rho}-h'\right\}\frac{ds}{u'wn\cdot d\tau} \\ &\quad -u'\frac{d\Pi}{u'wn\cdot d\tau} \\ &\quad -\left\{\delta[1-\beta(1-\rho)]\frac{sf'}{\rho}+[1-(1-\rho)\beta]\frac{1-\psi}{\psi}\delta q'\frac{v}{\rho}\right\}\frac{d\theta}{u'wn\cdot d\tau},\end{aligned}$$

which simplifying implies that:

$$\begin{aligned} DWL &= -\frac{d\ln(wn)}{d\ln\tau} - \left\{\delta\left[1-\beta(1-\rho)\right]\frac{f}{\rho} - h'\right\}\frac{ds}{u'wn\cdot d\tau} \\ &\quad -u'\frac{d\Pi}{u'wn\cdot d\tau} \\ &\quad -\delta\left[1-\beta(1-\rho)\right]\left\{\frac{sf'}{\rho} + \frac{1-\psi}{\psi}\frac{vq'}{\rho}\right\}\frac{d\theta}{u'wn\cdot d\tau}, \end{aligned}$$

Also, from the household's first order condition for search activity in steady state:

$$h' = u'\chi + \beta f\delta \Rightarrow \delta = \frac{h'-u'\chi}{\beta f}.$$

Use this expression to substitute out δ in the equation for DWL:

$$\begin{aligned} DWL &= -\frac{d\ln(wn)}{d\ln\tau} - \left\{\frac{h'-u'\chi}{\beta\rho}\left[1-\beta(1-\rho)\right] - h'\right\}\frac{ds}{u'wn\cdot d\tau} \\ &\quad -u'\frac{d\Pi}{u'wn\cdot d\tau} \\ &\quad -\frac{h'-u'\chi}{\beta\rho}\left[1-\beta(1-\rho)\right]\left\{s\frac{f'}{f} + \frac{1-\psi}{\psi}v\frac{q'}{f}\right\}\frac{d\theta}{u'wn\cdot d\tau}. \end{aligned}$$

Importantly, this means that in the search model, only observables are needed to determine DWL, and, in particular, no knowledge of amenities is required.

Now, note the following. With a constant returns to scale matching function, say, $m = \varphi v^{\xi}s^{1-\xi}$, where φ is matching efficiency and $\xi \in (0,1)$, it follows that:

$$f = \frac{m}{s} = \varphi\frac{v^{\xi}s^{1-\xi}}{s} = \varphi\left(\frac{v}{s}\right)^{\xi},$$

which implies that:

$$f' = \varphi\xi\left(\frac{v}{s}\right)^{\xi-1}$$

and:

$$sf' = s\varphi\xi\left(\frac{v}{s}\right)^{\xi-1} = \varphi\xi v^{\xi-1}s^{-(\xi-2)}.$$

Moreover:

$$q = \frac{m}{v} = \varphi \frac{v^{\xi} s^{1-\xi}}{v} = \varphi \left(\frac{v}{s}\right)^{\xi - 1},$$

which implies that:

$$q' = \varphi \left(\xi - 1\right) \left(\frac{v}{s}\right)^{\xi - 2}$$

and:

$$vq' = v\varphi \left(\xi - 1\right) \left(\frac{v}{s}\right)^{\xi - 2} = \varphi \left(\xi - 1\right) v^{\xi - 1} s^{-(\xi - 2)}.$$

Therefore:

$$\begin{aligned}
sf' + \frac{1-\psi}{\psi} vq' &= \varphi \xi v^{\xi-1} s^{-(\xi-2)} + \frac{1-\psi}{\psi} \varphi \left(\xi - 1\right) v^{\xi-1} s^{-(\xi-2)} \\
&= \varphi v^{\xi-1} s^{-(\xi-2)} \left(\xi + \frac{1-\psi}{\psi} \left(\xi - 1\right)\right) \\
&= \varphi v^{\xi-1} s^{-(\xi-2)} \left(\xi - \left(1 - \xi\right) \frac{1-\psi}{\psi}\right).
\end{aligned}$$

If the Hosios condition holds, then $1 - \xi$ equals ψ and therefore:

$$\begin{aligned}
\xi - \left(1 - \xi\right) \frac{1-\psi}{\psi} &= \xi - \left(1 - \xi\right) \frac{\xi}{1 - \xi} \\
&= 0.
\end{aligned}$$

Of course, the coefficient on $d\theta$ is positive if and only if:

$$\begin{aligned}
\xi &> \left(1 - \xi\right) \frac{1-\psi}{\psi} \\
&\Rightarrow \psi\xi > 1 - \xi - \psi + \psi\xi \\
&\Rightarrow \psi > 1 - \xi.
\end{aligned}$$

Note, moreover, that:

$$h' - u'\chi > 0,$$

since:

$$\delta = \frac{h' - u'\chi}{\beta f} > 0$$

as δ is the households' value of a job. Then:

$$h' = \delta\beta f + u'\chi.$$

Recall that, the DWL coefficient on ds is:

$$\frac{h' - u'\chi}{\beta\rho}\left[1 - \beta\left(1 - \rho\right)\right] - h'$$

can be written as:

$$\begin{aligned}
\frac{\delta\beta f + u'\chi - u'\chi}{\beta\rho}\left[1 - \beta\left(1 - \rho\right)\right] - \delta\beta f - u'\chi &= \frac{\delta\beta f}{\beta\rho}\left[1 - \beta\left(1 - \rho\right)\right] - \delta\beta f - u'\chi \\
&= \frac{\delta f}{\rho}\left[1 - \beta\left(1 - \rho\right)\right] - \delta\beta f - u'\chi \\
&= \delta f\left[1 - \beta\left(1 - \rho\right)\right] - \delta\rho\beta f - u'\rho\chi \\
&= \delta f - \beta\delta f - u'\rho\chi.
\end{aligned}$$

Then:

$$\delta f - \beta\delta f - u'\rho\chi > 0$$

if and only if:

$$\delta f\left(1 - \beta\right) > u'\rho\chi$$

if and only if:

$$\frac{\delta f\left(1 - \beta\right)}{u'\rho} > \chi$$

Of course, this is trivially true when $\chi = 0$.

C.3 General Statement of DWL

Steady-state social welfare is:

$$\begin{aligned} SW &= U\left(\overbrace{(1-\tau)\,wn + \Pi + T + \chi s}^{=c}, an, 1-n-s\right) \\ &= U\left(\overbrace{(1-\tau)\,wn + \Pi + \tau wn}^{=c}, an, 1-n-s\right), \end{aligned}$$

where the second line follows by the assumption that government consumption is zero. Therefore:

$$SW = U\left(\overbrace{wn + \Pi}^{=c}, an, 1-n-s\right).$$

This implies that:

$$\begin{aligned} dSW &= U_c d\,(wn) + U_c d\Pi + U_{an}(n \cdot da + a \cdot dn) - U_{lfp} dn - U_{lfp} ds \\ &= U_c\,(w \cdot dn + n \cdot dw) + U_c d\Pi + U_{an}(n \cdot da + a \cdot dn) - U_{lfp} dn - U_{lfp} ds\ , \end{aligned}$$

where, given search frictions and monopoly bargaining power, $d\Pi \neq\ 0$. Then,

$$dSW = U_c\,(w \cdot dn + n \cdot dw) - \underbrace{U_c\,(1-\tau)\,w_a}_{=U_{an} n \cdot da \text{ by HH's FOC for } a} n \cdot da + U_{an} a \cdot dn - U_{lfp} dn - U_{lfp} ds + U_c d\Pi$$

Following the same steps in Appendix C.2, using the total derivative of wage equation, we obtain:

$$U_c n dw = (1-\psi) U_c n \frac{\chi}{(1-\tau)^2} d\tau - U_c n \phi' \cdot da + U_c n \psi \gamma \cdot d\theta.$$

Substituting out $U_c n dw$ into dSW equation yields:

$$\begin{aligned} dSW = &\ U_c w \cdot dn + (1-\psi) U_c n \frac{\chi}{(1-\tau)^2} d\tau - U_c n \phi' \cdot da + U_c n \psi \gamma \cdot d\theta \\ &- U_c\,(1-\tau)\,w_a n \cdot da + U_{an} a \cdot dn - U_{lfp} dn - U_{lfp} ds + U_c d\Pi. \end{aligned}$$

Replacing $\phi' = -w_a$ from firm's FOC:

$$\begin{aligned}
dSW &= U_c w \cdot dn + (1-\psi) U_c n \frac{\chi}{(1-\tau)^2} d\tau + U_c n w_a \cdot da + U_c n \psi \gamma \cdot d\theta \\
&\quad - U_c (1-\tau) w_a n \cdot da + U_{an} a \cdot dn - U_{lfp} dn - U_{lfp} ds + U_c d\Pi \\
&= U_c w \cdot dn + (1-\psi) U_c n \frac{\chi}{(1-\tau)^2} d\tau + U_c \tau w_a n \cdot da + U_c n \psi \gamma \cdot d\theta \\
&\quad + U_{an} a \cdot dn - U_{lfp} dn - U_{lfp} ds + U_c d\Pi.
\end{aligned}$$

Adding and subtracting $U_c (1-\tau) w \cdot dn$ implies that:

$$\begin{aligned}
dSW &= [U_c w \cdot dn - U_c (1-\tau) w \cdot dn] + U_c (1-\tau) w \cdot dn + (1-\psi) U_c n \frac{\chi}{(1-\tau)^2} d\tau \\
&\quad + U_c \tau w_a n \cdot da + U_c n \psi \gamma \cdot d\theta + U_{an} a \cdot dn - U_{lfp} dn - U_{lfp} ds + U_c d\Pi \\
&= U_c \tau w \cdot dn + U_c (1-\tau) w \cdot dn + (1-\psi) U_c n \frac{\chi}{(1-\tau)^2} d\tau \\
&\quad + U_c \tau w_a n \cdot da + U_c n \psi \gamma \cdot d\theta + U_{an} a \cdot dn - U_{lfp} dn - U_{lfp} ds + U_c d\Pi.
\end{aligned}$$

Adding and subtracting $U_c \tau n \cdot dw$ implies that:

$$\begin{aligned}
dSW &= (U_c \tau w \cdot dn + U_c \tau n \cdot dw) - U_c \tau n \cdot dw + U_c (1-\tau) w \cdot dn + (1-\psi) U_c n \frac{\chi}{(1-\tau)^2} d\tau \\
&\quad + U_c \tau w_a n \cdot da + U_c n \psi \gamma \cdot d\theta + U_{an} a \cdot dn - U_{lfp} dn - U_{lfp} ds + U_c d\Pi \\
&= U_c \tau d(w \cdot n) - U_c \tau n \cdot dw + U_c (1-\tau) w \cdot dn + (1-\psi) U_c n \frac{\chi}{(1-\tau)^2} d\tau \\
&\quad + U_c \tau w_a n \cdot da + U_c n \psi \gamma \cdot d\theta + U_{an} a \cdot dn - U_{lfp} dn - U_{lfp} ds + U_c d\Pi.
\end{aligned}$$

Combining the dn terms, we have:

$$\begin{aligned}
dSW &= U_c \tau d(w \cdot n) - U_c \tau n \cdot dw + [U_c (1-\tau) w + U_{an} a - U_{lfp}] dn + (1-\psi) U_c n \frac{\chi}{(1-\tau)^2} d\tau \\
&\quad + U_c \tau w_a n \cdot da + U_c n \psi \gamma \cdot d\theta - U_{lfp} ds + U_c d\Pi.
\end{aligned}$$

Using the definition of DWL, it follows that:

$$
\begin{aligned}
DWL = -\frac{dSW}{U_c wn \cdot d\tau} \quad &= \quad -\frac{\tau d(w \cdot n)}{wnd\tau} + U_c \tau n \frac{dw}{U_c wn \cdot d\tau} - \left[U_c \left(1-\tau\right) w + U_{an} a - U_{lfp}\right] \frac{dn}{U_c wn \cdot d\tau} \\
& -(1-\psi) U_c n \frac{\chi}{(1-\tau)^2} \frac{d\tau}{U_c wn \cdot d\tau} \\
& -U_c \tau w_a n \frac{da}{U_c wn \cdot d\tau} - U_c n \psi \gamma \frac{d\theta}{U_c wn \cdot d\tau} + U_{lfp} \frac{ds}{U_c wn \cdot d\tau} - U_c \frac{d\Pi}{U_c wn \cdot d\tau},
\end{aligned}
$$

and therefore that:

$$
\begin{aligned}
DWL \quad &= \quad -\frac{\tau d(w \cdot n)}{wnd\tau} - \left[U_c \left(1-\tau\right) w + U_{an} a - U_{lfp}\right] \frac{dn}{U_c wn \cdot d\tau} \\
& -U_c \frac{d\Pi}{U_c wn \cdot d\tau} + U_{lfp} \frac{ds}{U_c wn \cdot d\tau} \\
& -U_c \tau w_a n \frac{da}{U_c wn \cdot d\tau} - U_c n \psi \gamma \frac{d\theta}{U_c wn \cdot d\tau} \\
& +U_c \tau n \frac{dw}{U_c wn \cdot d\tau} - (1-\psi) \frac{\chi}{w(1-\tau)^2}.
\end{aligned}
$$

Following the same steps in Appendix C.2, we substitute

$$
dn = \frac{f}{\rho} ds + \frac{sf'}{\rho} d\theta
$$

into this expression, which yields:

$$
\begin{aligned}
DWL \quad &= \quad -\frac{\tau d(w \cdot n)}{wnd\tau} - \left\{\left[U_c \left(1-\tau\right) w + U_{an} a - U_{lfp}\right] \frac{f}{\rho} - U_{lfp}\right\} \frac{ds}{U_c wn \cdot d\tau} \\
& -U_c \frac{d\Pi}{U_c wn \cdot d\tau} \\
& -\left\{\left[U_c \left(1-\tau\right) w + U_{an} a - U_{lfp}\right] \frac{sf'}{\rho} + U_c n \psi \gamma\right\} \frac{d\theta}{U_c wn \cdot d\tau} \\
& -U_c \tau w_a n \frac{da}{U_c wn \cdot d\tau} + U_c \tau n \frac{dw}{U_c wn \cdot d\tau} - (1-\psi) \frac{\chi}{w(1-\tau)^2}.
\end{aligned}
$$

Using again the total derivative of the wage to substitute out $\tau n \frac{dw}{wn \cdot d\tau}$ in DWL, and grouping and simplifying, as we did in Appendix C.2, we get:

$$\begin{aligned} DWL &= -\frac{d\ln(wn)}{d\ln\tau} - \left\{\left[U_c(1-\tau)w + U_{an}a - U_{lfp}\right]\frac{f}{\rho} - U_{lfp}\right\}\frac{ds}{U_c wn\cdot d\tau} \\ &-U_c\frac{d\Pi}{U_c wn\cdot d\tau} \\ &-\left\{\left[U_c(1-\tau)w + U_{an}a - U_{lfp}\right]\frac{sf'}{\rho} + U_c n(1-\tau)\psi\gamma\right\}\frac{d\theta}{U_c wn\cdot d\tau} \\ &-(1-\psi)\frac{\chi}{w(1-\tau)}. \end{aligned}$$

The rest of the steps are the same as in Appendix C.2. First, recall that using wage and job creation equations, we obtained:

$$\frac{(1-\psi)\chi}{w(1-\tau)} = (1-\tau)\left[\frac{1-(1-\rho)\beta}{\beta}\frac{\gamma}{q^2}q' - \gamma\psi\right]U_c n\frac{d\theta}{U_c wn\cdot d\tau}.$$

Using this expression to substitute out $\frac{(1-\psi)\chi}{w(1-\tau)}$ in the equation for DWL and following the same steps in Appendix C.2, we obtain:

$$\begin{aligned} DWL &= -\frac{d\ln(wn)}{d\ln\tau} - \left\{\left[U_c(1-\tau)w + U_{an}a - U_{lfp}\right]\frac{f}{\rho} - U_{lfp}\right\}\frac{ds}{U_c wn\cdot d\tau} \\ &-U_c\frac{d\Pi}{U_c wn\cdot d\tau} \\ &-\left\{\left[U_c(1-\tau)w + U_{an}a - U_{lfp}\right]\frac{sf'}{\rho} + U_c n(1-\tau)\frac{1-(1-\rho)\beta\gamma}{\beta}\frac{\gamma}{q^2}q'\right\}\frac{d\theta}{U_c wn\cdot d\tau}. \end{aligned}$$

Using the fact that in steady state $n/q = v/\rho$ and $\frac{\gamma}{\beta q} = \mathbf{J} = \frac{1-\psi}{\psi(1-\tau)}\frac{\delta}{U_c}$, following the steps in Appendix C.2, we obtain:

$$\begin{aligned} DWL &= -\frac{d\ln(wn)}{d\ln\tau} - \left\{\left[U_c(1-\tau)w + U_{an}a - U_{lfp}\right]\frac{f}{\rho} - U_{lfp}\right\}\frac{ds}{U_c wn\cdot d\tau} \\ &-U_c\frac{d\Pi}{U_c wn\cdot d\tau} \\ &-\left\{\left[U_c(1-\tau)w + U_{an}a - U_{lfp}\right]\frac{sf'}{\rho} + \left[1-(1-\rho)\beta\right]\frac{1-\psi}{\psi}\delta q'\frac{v}{\rho}\right\}\frac{d\theta}{U_c wn\cdot d\tau}. \end{aligned}$$

Recall from the household's first order condition for employment and search activity, respectively, in steady state:

$$U_c\left(1-\tau\right)w+U_{an}a-U_{lfp}=\delta\left[1-\beta\left(1-\rho\right)\right],$$

and from the household's first order condition for search activity in steady state:

$$U_{lfp}=U_c\chi+\beta f\delta\Rightarrow\delta=\frac{U_lfp-U_c\chi}{\beta f},$$

we have the DWL equation:

$$\begin{aligned}DWL\quad=\quad&-\frac{d\ln\left(wn\right)}{d\ln\tau}-\left\{\frac{U_{lfp}-U_c\chi}{\beta\rho}\left[1-\beta\left(1-\rho\right)\right]-U_{lfp}\right\}\frac{ds}{U_cwn\cdot d\tau}\\&-U_c\frac{d\Pi}{U_cwn\cdot d\tau}\\&-\frac{U_{lfp}-U_c\chi}{\beta\rho}\left[1-\beta\left(1-\rho\right)\right]\left\{s\frac{f'}{f}+\frac{1-\psi}{\psi}v\frac{q'}{f}\right\}\frac{d\theta}{U_cwn\cdot d\tau}.\end{aligned}$$

D Impact of Tax Changes

D.1 Efficient Search Model with Amenities

The Wage Curve Recall that, as highlighted in the main text, the analysis that follows assumes $y_{nn,t}=0$, $u_t=c_t$, which implies that $u'_t=1$, $u''_t=0$ (*for conceptual ease, throughout the paper we use u' rather than 1*), and $\chi=0$. As such, the wage curve, equation 17, becomes:

$$w=-\left(1-\psi\right)\left[\frac{A'a}{u'\left(1-\tau\right)}\right]+\psi\left(y_n-\phi'a+\gamma\theta\right).$$

Recall from the amenities equilibrium condition that:

$$A'a=u'\left(1-\tau\right)\phi'a\rightarrow\frac{A'a}{u'\left(1-\tau\right)}=\phi'a.$$

Making this substitution above implies that:

$$w = -\left(1-\psi\right)\phi' a + \psi\left(y_n - \phi' a + \gamma\theta\right),$$

and therefore

$$w = -\phi' a + \psi\left(y_n + \gamma\theta\right).$$

We rearrange this condition as follows:

$$\underbrace{w + \phi' a}_{\equiv W} = \psi\left(y_n + \gamma\theta\right),$$

where we refer to W as the "effective wage," since it is what matters for the firm on the margin. Moreover, we refer to this last equation as the *effective wage curve*, which we henceforth refer to as WC for short.

Note that:

$$\begin{aligned} dW &= \psi y_{nn} dn + \psi\gamma d\theta \\ &= \psi d\theta, \end{aligned}$$

since by assumption $y_{nn} = 0$. As such, the wage curve is increasing and linear in (θ, W) space.

The Job Creation Condition The job creation condition is equation 2.25 from the main text. This equation can be rearranged as follows:

$$\underbrace{w + \phi' a}_{\equiv W} = y_n - \underbrace{\frac{\left[1-\left(1-\rho\right)\beta\right]}{\beta}\gamma}_{\equiv X_{JC}}\frac{1}{q}.$$

Then,

$$\begin{aligned} dW &= y_{nn} dn - X_{JC} \left(-\frac{1}{q^2} \right) q' d\theta \\ &= X_{JC} \frac{q'}{q^2} d\theta. \end{aligned}$$

Because $q' < 0$ (q' is the derviative of the job filling probability with respect to v/s, of which it is the sole function of), then the job creation condition, which we henceforth refer to as JC for short, is decreasing and convex in in (θ, W) space.

Impact of Tax Changes: $w + \phi' a$ and θ As shown in Figure A1, JC and WC together pin down the model's equilibrium values of $w + \phi' a$ and θ. Note from above that neither JC nor WC are a function of τ and thus, remain unchanged following a change in tax rate. Thus, given a change in τ,

$$\frac{d\theta}{d\tau} = 0 \text{ and } \frac{d\left(w + \phi' a\right)}{d\tau} = 0.$$

Moreover, the last equation above therefore implies that:

$$\frac{dw}{d\tau} = -\frac{d\left(\phi' a\right)}{d\tau}.$$

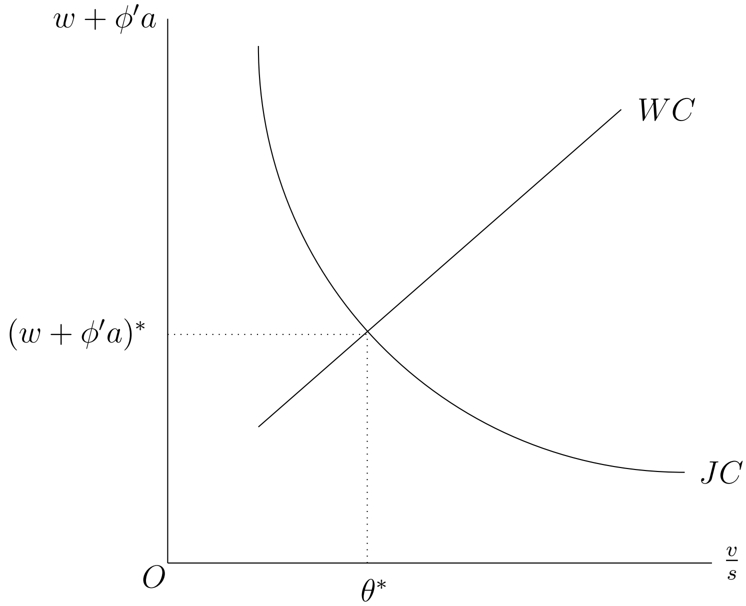
Impact of Tax Changes: lfp With $\chi = 0$, equation 2.24 becomes:

$$h' = \underbrace{\frac{\beta f}{1 - \beta\left(1 - \rho\right) + \beta f}}_{\equiv X_{LFP}} \left[u'\left(1 - \tau\right) w + A' a\right].$$

Recall from the amenities equilibrium condition that:

$$A' a = u'\left(1 - \tau\right) \phi' a.$$

Figure A1: Equilibrium Efficient Wage and Market Tightness without UI



Using this condition above yields:

$$h' = X_{LFP} u' (1-\tau)(w+\phi' a).$$

Then,

$$-h'' dlfp = u'(1-\tau)(w+\phi' a) dX_{LFP} + X_{LFP} \cdot u'(1-\tau) d(w+\phi' a)$$
$$+X_{LFP}(1-\tau)(w+\phi' a) u'' dc - X_{LFP} u' (w+\phi' a) d\tau.$$

Therefore,

$$-h'' \frac{dlfp}{d\tau} = u'(1-\tau)(w+\phi' a) \frac{dX_{LFP}}{d\tau} + X_{LFP}(1-\tau)(w+\phi' a) u'' \frac{dc}{d\tau} - X_{LFP} u' (w+\phi' a),$$

since from above it is an endogenous result that $\frac{d(w+\phi' a)}{d\tau} = 0$.

Now, consider dX_{LFP}. This differential is:

$$\underbrace{\left\{ -\frac{\beta f}{[1-\beta(1-\rho)+\beta f]^2} + \frac{1}{[1-\beta(1-\rho)+\beta f]} \right\}}_{\equiv \bar{X}_{LFP}} \beta f' d\theta,$$

(f' is the derivative of the job filling probability with respect to θ, of which it is the sole function of, and recall that $f' > 0$) and is greater than zero if and only if

$$\frac{1}{\left[1-\beta\left(1-\rho\right)+\beta f\right]} > \frac{\beta f}{\left[1-\beta\left(1-\rho\right)+\beta f\right]^{2}}$$
$$\leftrightarrow 1 > \frac{\beta f}{\left[1-\beta\left(1-\rho\right)+\beta f\right]},$$

which holds, since the numerator βf is smaller than the denominator, which is equal to βf plus the sum of the positive number:

$$1-\beta\left(1-\rho\right).$$

That said, because we already showed that $\frac{d\theta}{d\tau} = 0$, we have:

$$\begin{aligned}\frac{dlfp}{d\tau} &= \frac{X_{LFP}u'\left(w+\phi' a\right)}{h''} - \frac{X_{LFP}(1-\tau)(w+\phi' a)}{h''}u''\frac{dc}{d\tau}\\ &= \frac{X_{LFP}}{h''}(w+\phi' a)[u' - (1-\tau)u''\frac{dc}{d\tau}].\end{aligned}$$

Because we assume that $u' = 1$, and thus, $u'' = 0$, we have:

$$\frac{dlfp}{d\tau} = \frac{X_{LFP}}{h''}(w+\phi' a) < 0.$$

Impact of Tax Changes: n, s, and v Steady-state employment is given by:

$$n = \frac{m}{\rho} = \frac{s}{s}\frac{m}{\rho} = s\frac{f}{\rho},$$

since m/s is equal to f. Then,

$$dn = \frac{f}{\rho}ds + s\frac{f'}{\rho}d\theta.$$

Adding ds to both sides of this equation, we obtain:

$$dn + ds = \left(1+\frac{f}{\rho}\right)ds + s\frac{f'}{\rho}d\theta.$$

Of course, since

$$lfp = s + n,$$

then:

$$dlfp = ds + dn.$$

Therefore,

$$\begin{aligned}\frac{dlfp}{d\tau} &= \left(1 + \frac{f}{\rho}\right)\frac{ds}{d\tau} + s\frac{f'}{\rho}\frac{d\theta}{d\tau} \\ &= \left(1 + \frac{f}{\rho}\right)\frac{ds}{d\tau},\end{aligned}$$

since it is a result that $d\theta/d\tau$ is equal to zero. Moreover, given the result that $dlfp/d\tau$ is negative, this last equation immediately implies the result:

$$\frac{ds}{d\tau} < 0.$$

Finally, since we have from above:

$$\frac{dn}{d\tau} = \frac{f}{\rho}\frac{ds}{d\tau} + s\frac{f'}{\rho}\frac{d\theta}{d\tau},$$

given the results $d\theta/d\tau$ equal to zero and $ds/d\tau$ negative, the following result is immediately implied:

$$\frac{dn}{d\tau} < 0.$$

Of course,

$$\begin{aligned}\frac{d\theta}{d\tau} &= \frac{1}{s}\frac{dv}{d\tau} - \frac{v}{s^2}\frac{ds}{d\tau} \\ &= 0\end{aligned}$$

implies that:

$$\frac{dv}{d\tau} = \theta\frac{ds}{d\tau},$$

which implies the result:

$$\frac{dv}{d\tau} < 0$$

given the result that $ds/d\tau$ is negative.

Impact of Tax Changes: a Consider the equilibrium condition for amenities:

$$A' = u'\left(1-\tau\right)\phi'.$$

This implies that:

$$A''a \cdot dn + A''n \cdot da = \left(1-\tau\right)\phi' \cdot u'''\phi' \cdot d\tau$$
$$+u'\left(1-\tau\right)\left(\phi''a \cdot dn + \phi''n \cdot da\right).$$

Collecting terms implies that:

$$\left[u'\left(1-\tau\right)\phi'' - A''\right]n \cdot da = \left[A'' - u'\left(1-\tau\right)\phi''\right]a \cdot dn + u'\phi' d\tau - (1-\tau)\phi' u'' dc,$$

and therefore:

$$\begin{aligned}
\frac{da}{d\tau} &= \frac{\left[A'' - u'\left(1-\tau\right)\phi''\right]a}{\left[u'\left(1-\tau\right)\phi'' - A''\right]n} \cdot \frac{dn}{d\tau} + \frac{u'\phi'}{\left[u'\left(1-\tau\right)\phi'' - A''\right]n} - \frac{(1-\tau)\phi' u''}{\left[u'\left(1-\tau\right)\phi'' - A''\right]n} \cdot \frac{dc}{d\tau} \\
&= -\frac{a}{n} \cdot \frac{dn}{d\tau} + \frac{u'\phi'}{\left[u'\left(1-\tau\right)\phi'' - A''\right]n} - \frac{(1-\tau)\phi' u''}{\left[u'\left(1-\tau\right)\phi'' - A''\right]n} \cdot \frac{dc}{d\tau}.
\end{aligned}$$

The first term on the right hand side is positive, as we already showed above that $\frac{dn}{d\tau} < 0$. The second term is positive too: since $A'' < 0$, then the denominator terms in square brackets are positive, and because $u'\phi' > 0$, the numerator and the whole term is positive. Finally, the last term is zero, because we assume that $u' = 1$, and thus, $u'' = 0$. Overall, we have:

$$\frac{da}{d\tau} > 0.$$

Impact of Tax Changes: w **and** $\phi' a$ Consider $\phi' a$. The total derivative of this expression implies that:

$$\frac{d(\phi' a)}{d\tau} = \phi' \frac{da}{d\tau} + a\phi'' \frac{d(an)}{d\tau}.$$

Since ϕ' is greater than zero as well as $da/d\tau$, as shown above, the first term on the right hand side above is positive. Therefore, to sign the left hand side of this equation it remains to sign $d(an)/d\tau$. To do so, consider the equilibrium condition for amenities:

$$A' = u'(1-\tau)\phi'.$$

This condition implies that:

$$A'' d(an) = -u'\phi' d\tau + u'(1-\tau)\phi'' d(an) + (1-\tau)\phi' u'' dc.$$

Collecting terms and rearranging, we then have:

$$\frac{d(an)}{d\tau} = -\frac{u'\phi'}{A'' - u'(1-\tau)\phi''} + \frac{(1-\tau)\phi' u''}{A'' - u'(1-\tau)\phi''} \cdot \frac{dc}{d\tau}.$$

The numerator of the first expression in the right hand side is positive, and the denominator is negative since $A'' < 0$ and $\phi'' > 0$. The second term, on the other hand, is zero since we assume $u'' = 0$. Thus, the following result immediately follows:

$$\frac{d(an)}{d\tau} > 0.$$

Returning to the expression:

$$\frac{d(\phi' a)}{d\tau} = \phi' \frac{da}{d\tau} + a\phi'' \frac{d(an)}{d\tau},$$

it immediately follows that:

$$\frac{d(\phi' a)}{d\tau} > 0$$

given $\phi'' > 0$ and the fact that $d(an)/d\tau$ is positive.

Since, as shown earlier,

$$\frac{dw}{d\tau} = -\frac{d(\phi' a)}{d\tau},$$

it immediately follows that:

$$\frac{dw}{d\tau} < 0.$$

Impact of Tax Changes: c From the aggregate budget constraint, we have:

$$\begin{aligned} c &= y - \phi - \gamma v \\ &= y - \phi - s\gamma\theta. \end{aligned}$$

Therefore,

$$\begin{aligned} dc &= y_n dn - (\phi' a \cdot dn + \phi' n \cdot da) - \gamma\theta ds - s\gamma d\theta \\ &= (y_n - \phi' a) \cdot dn - \phi' n \cdot da - \gamma\theta ds - s\gamma d\theta. \end{aligned}$$

From JC, we know that:

$$y_n = w + \phi' a + \gamma \frac{[1-(1-\rho)\beta]}{\beta}\frac{1}{q}.$$

Plugging in above implies that:

$$\begin{aligned} dc &= \left\{ w + \phi' a + \gamma \frac{[1-(1-\rho)\beta]}{\beta}\frac{1}{q} - \phi' a \right\} dn - \phi' n \cdot da - \gamma\theta ds - s\gamma d\theta \\ &= \left\{ w + \gamma \frac{[1-(1-\rho)\beta]}{\beta}\frac{1}{q} \right\} dn - \phi' n \cdot da - \gamma\theta ds - s\gamma d\theta. \end{aligned}$$

Now, recall that:

$$n = \frac{m}{\rho} = \frac{s}{s}\frac{m}{\rho} = s\frac{f}{\rho},$$

since m/s is equal to f, which implies that

$$dn = \frac{f}{\rho} ds + s\frac{f'}{\rho} d\left(v/s\right),$$

and therefore,

$$ds = \frac{\rho}{f} dn - s\frac{f'}{f} d\left(v/s\right).$$

Substituting this into the expression for dc to obtain:

$$dc = \left\{ w + \gamma \frac{\left[1-\left(1-\rho\right)\beta\right]}{\beta}\frac{1}{q} \right\} dn - \phi' n \cdot da - \gamma\theta \left[\frac{\rho}{f} dn - s\frac{f'}{f} d\theta \right] - s\gamma d\theta.$$

Collecting terms, we have:

$$dc = \left\{ w + \gamma \frac{\left[1-\left(1-\rho\right)\beta\right]}{\beta}\frac{1}{q} - \gamma\theta\frac{\rho}{f} \right\} dn - \phi' n \cdot da + \left(s\frac{f'}{f} - s\gamma \right) d\theta.$$

Since, by the assumptions on the matching function, we have:

$$\frac{f}{q} = \theta.$$

It follows that $1/q = \theta/f$. Using this in the expression for dc implies that:

$$dc = \left\{ w + \gamma \frac{\left[1-\left(1-\rho\right)\beta\right]}{\beta}\frac{1}{q} - \gamma\rho\frac{1}{q} \right\} dn - \phi' n \cdot da + \left(s\frac{f'}{f} - s\gamma \right) d\theta.$$

Therefore,

$$\frac{dc}{d\tau} = \left\{ w + \left[\frac{\left[1-\left(1-\rho\right)\beta\right]}{\beta} - \rho \right] \gamma\frac{1}{q} \right\} \frac{dn}{d\tau} - \phi' n \cdot \frac{da}{d\tau} + \left(s\frac{f'}{f} - s\gamma \right) \frac{d\theta}{d\tau}.$$

Now, consider the expression $\frac{[1-(1-\rho)\beta]}{\beta} - \rho$ in the first term of this equation. This expression is greater than zero if and only if

$$\frac{[1-(1-\rho)\beta]}{\beta}-\rho>0$$

$$\leftrightarrow 1-\beta+\beta\rho-\beta\rho>0,$$

which is of course true and immediately implies the result:

$$\frac{dc}{d\tau}<0.$$

Impact of Tax Changes: Π

$$\Pi=y-wn-\phi-\gamma v$$

$$y=c+\phi+\gamma v$$

Combining:

$$\Pi=c-wn$$

Taking the total derivative:

$$d\Pi=dc-w_a n da-w dn$$

Dividing by $d\tau$, and substituting $\frac{dc}{d\tau}$ from above:

$$\frac{d\Pi}{d\tau}=\left\{w+\left[\frac{[1-(1-\rho)\beta]}{\beta}-\rho\right]\gamma\frac{1}{q}\right\}\frac{dn}{d\tau}-\phi' n\frac{da}{d\tau}+\left(s\frac{f'}{f}-s\gamma\right)\frac{d\theta}{d\tau}-w_a n\frac{da}{d\tau}-w\frac{dn}{d\tau}$$

Combining:

$$\frac{d\Pi}{d\tau}=\left\{w+\left[\frac{[1-(1-\rho)\beta]}{\beta}-\rho\right]\gamma\frac{1}{q}-w\right\}\frac{dn}{d\tau}-(w_a)n\frac{da}{d\tau}+\left(s\frac{f'}{f}-s\gamma\right)\frac{d\theta}{d\tau}$$

Cancelling w terms out and substituting $\frac{d\theta}{d\tau}=0$ from above:

$$\frac{d\Pi}{d\tau}=\left\{\left[\frac{[1-(1-\rho)\beta]}{\beta}-\rho\right]\gamma\frac{1}{q}\right\}\frac{dn}{d\tau}-(w_a)n\frac{da}{d\tau}.$$

We know from the equation 2.14 that $w_a + \phi' = 0$. Thus,

$$\frac{d\Pi}{d\tau} = \left\{ \left[\frac{[1-(1-\rho)\beta]}{\beta} - \rho \right] \gamma \frac{1}{q} \right\} \frac{dn}{d\tau}$$

Combining:

$$\frac{d\Pi}{d\tau} = \frac{1-\beta+\rho\beta-\rho\beta}{\beta}\gamma\frac{1}{q}\frac{dn}{d\tau} = \frac{1-\beta}{\beta}\gamma\frac{1}{q}\frac{dn}{d\tau}.$$

Since we already showed that $\frac{dn}{d\tau} < 0$, and $0 < \beta < 1$, $\gamma > 0$, and $q > 0$, we have:

$$\frac{d\Pi}{d\tau} < 0.$$

Impact of Tax Changes: DWL Recall that DWL is given by:

$$\begin{aligned} DWL &= -\left\{ \frac{d\ln(wn)}{d\ln\tau} \right\} - \left\{ [u'(1-\tau)w + A'a - h'] \frac{f}{\rho} - h' \right\} \frac{ds}{u'wn \cdot d\tau} \\ & \quad - [u'(1-\tau)w + A'a - h'] \frac{sf'}{\rho} \frac{d\theta}{u'wn \cdot d\tau} - u' \frac{d\Pi}{u'wn \cdot d\tau}. \end{aligned}$$

Before implementing any of the results obtained thus far, recall that the household's optimality condition for search activity implies that (continuing to assume $\chi = 0$),

$$h' = \underbrace{\frac{\beta f}{1-\beta(1-\rho)+\beta f}}_{\equiv X_{LFP}} [u'(1-\tau)w + A'a].$$

This implies that:

$$u'(1-\tau)w + A'a = \frac{h'}{X_{LFP}}.$$

Substituting in the expression for DWL, this implies that

$$\begin{aligned} DWL &= -\left\{ \frac{d\ln(wn)}{d\ln\tau} \right\} - \left[\left(\frac{h'}{X_{LFP}} - h' \right) \frac{f}{\rho} - h' \right] \frac{ds}{u'wn \cdot d\tau} \\ & \quad - \left(\frac{h'}{X_{LFP}} - h' \right) \frac{sf'}{\rho} \frac{d\theta}{u'wn \cdot d\tau} - u' \frac{d\Pi}{u'wn \cdot d\tau}, \end{aligned}$$

and to simplify further:

$$\begin{aligned} DWL &= -\left\{\frac{d\ln(wn)}{d\ln\tau}\right\} - \left[\left(\frac{1}{X_{LFP}}-1\right)\frac{f}{\rho}-1\right]h'\frac{ds}{u'wn\cdot d\tau} \\ &\quad -\left(\frac{1-X_{LFP}}{X_{LFP}}\right)h'\frac{sf'}{\rho}\frac{d\theta}{u'wn\cdot d\tau} - u'\frac{d\Pi}{u'wn\cdot d\tau}, \end{aligned}$$

which then implies that:

$$\begin{aligned} DWL &= -\left\{\frac{d\ln(wn)}{d\ln\tau}\right\} - \left[\frac{1-X_{LFP}}{X_{LFP}}\frac{f}{\rho}-1\right]h'\frac{ds}{u'wn\cdot d\tau} \\ &\quad -\left(\frac{1-X_{LFP}}{X_{LFP}}\right)h'\frac{sf'}{\rho}\frac{d\theta}{u'wn\cdot d\tau} - u'\frac{d\Pi}{u'wn\cdot d\tau}. \end{aligned}$$

Of course, since X_{LFP} is a number between 0 and 1, then in the third term of DWL,

$$\left(\frac{1-X_{LFP}}{X_{LFP}}\right)h'\frac{sf'}{\rho} > 0.$$

Now, consider the second term of DWL:

$$\left[\left(\frac{1}{X_{LFP}}-1\right)\frac{f}{\rho}-1\right]h'.$$

This can be stated as:

$$\frac{(1-X_{LFP})\frac{f}{\rho}-X_{LFP}}{X_{LFP}}h',$$

which is greater than zero if and only if

$$(1-X_{LFP})\frac{f}{\rho} > X_{LFP}.$$

Using the definition of X_{LFP}, this inequality can be stated as

$$1-\frac{\beta f}{1-\beta(1-\rho)+\beta f}\frac{f}{\rho} > \frac{\beta f}{1-\beta(1-\rho)+\beta f}.$$

This condition can be restated as:

$$1-\beta\left(1-\rho\right)+\beta f-\beta f>\rho\beta,$$

and furthermore as:

$$1-\beta+\beta\rho-\rho\beta>0$$

which of course holds. As such, in the second term of DWL,

$$\left[\frac{1-X_{LFP}}{X_{LFP}}\frac{f}{\rho}-1\right]h'>0.$$

To summarize, we have:

$$\begin{aligned} DWL &= -\left\{\frac{d\ln\left(wn\right)}{d\ln\tau}\right\}-\overbrace{\left[\frac{1-X_{LFP}}{X_{LFP}}\frac{f}{\rho}-1\right]h'}^{(+)}\frac{ds}{u'wn\cdot d\tau} \\ & \quad -\underbrace{\left(\frac{1-X_{LFP}}{X_{LFP}}\right)h'\frac{sf'}{\rho}}_{(+)}\frac{d\theta}{u'wn\cdot d\tau}-\underbrace{u'}_{(+)}\frac{d\Pi}{u'wn\cdot d\tau}. \end{aligned}$$

From the results so far, we know:

$$\frac{ds}{d\tau}<0,\ \frac{d\theta}{d\tau}=0,\text{ and }\frac{d\Pi}{d\tau}<0.$$

It follows that in the efficient version of the model:

$$DWL=\overbrace{-\left\{\frac{d\ln\left(wn\right)}{d\ln\tau}\right\}}^{\equiv ETI>0}\overbrace{-\overbrace{\left[\frac{1-X_{LFP}}{X_{LFP}}\frac{f}{\rho}-1\right]h'}^{(+)}\overbrace{\frac{ds}{u'wn\cdot d\tau}}^{(-)}}^{(+)}\overbrace{-\overbrace{u'}^{(+)}\overbrace{\frac{d\Pi}{u'wn\cdot d\tau}}^{(-)}}^{(+)}.$$

Therefore,

$$DWL>ETI,$$

meaning that *the* ETI always *"underestimates"* the DWL when there are search frictions with amenities, unlike the neoclassical model, for which $ETI=DWL$ in any circumstance.

D.2 Efficient Search Model without Amenities

We proceed with the same assumptions as in the previous section.

The Wage Curve The following analysis is straightforward and follows by netting out all amenities terms from the derivations in the previous section. The wage curve is:

$$w = \psi \left(y_n + \gamma\theta \right),$$

which implies that:

$$\begin{aligned} dw &= \psi y_{nn} dn + \psi\gamma d\theta \\ &= \psi d\theta, \end{aligned}$$

since, by assumption $y_{nn} = 0$. As such, the wage curve is increasing and linear in $(v/s, w)$ space.

The Job Creation Condition The job creation condition is now:

$$w = y_n - \underbrace{\frac{\left[1 - (1-\rho)\beta\right]}{\beta}\gamma}_{\equiv X_{JC}}\frac{1}{q}.$$

Then,

$$\begin{aligned} dw &= y_{nn} dn - X_{JF}\left(-\frac{1}{q^2}\right) q' d\theta \\ &= X_{JC}\frac{q'}{q^2} d\theta. \end{aligned}$$

Because $q' < 0$ (q' is the derivative of the job filling probability with respect to θ, of which it is the sole function of), then the job creation condition is decreasing and convex in in (θ, w) space.

Impact of Tax Changes: w **and** θ Plotting JC and WC in $(v/s, W)$ space (as in Figure A1) pins down the model's equilibrium values of w and θ. Note from above that neither JC nor WC are a function of τ. Thus, given a change in τ,

$$\frac{d\theta}{d\tau} = 0 \text{ and } \frac{d\,(w)}{d\tau} = 0.$$

Moreover, the last equation above therefore implies that:

$$\frac{dw}{d\tau} = 0.$$

Impact of Tax Changes: lfp With $\chi = 0$, following the exact same derivations as earlier, it follows that now:

$$\frac{dlfp}{d\tau} = \frac{X_{LFP}u'w}{h''} < 0.$$

Impact of Tax Changes: n**,** s**, and** v Exactly as in the model with amenities, it is straightforward to show that:

$$\frac{ds}{d\tau} < 0,$$

and therefore, $\frac{dn}{d\tau} < 0$ holds by the same steps above. Moreover, since $d\theta$ equals zero, then,

$$\frac{dv}{d\tau} = \theta\frac{ds}{d\tau} < 0.$$

Impact of Tax Changes: c Using the same methodology as in the previous section,

$$\frac{dc}{d\tau} = \left\{w + \left[\frac{[1-(1-\rho)\,\beta]}{\beta} - \rho\right]\gamma\frac{1}{q}\right\}\frac{dn}{d\tau}X_{LFP}u'w + \left(s\frac{f'}{f} - s\gamma\right)\frac{d\theta}{d\tau}.$$

Now, consider the expression $\frac{[1-(1-\rho)\beta]}{\beta} - \rho$ in the first term of this equation. This expression is greater than zero if and only if

$$\frac{[1-(1-\rho)\beta]}{\beta} - \rho > 0$$

$$\leftrightarrow 1 - \beta + \beta\rho - \beta\rho > 0,$$

which is of course true and immediately implies the result:

$$\frac{dc}{d\tau} < 0.$$

Impact of Tax Changes: Π Recall that:

$$\Pi = c - wn.$$

Taking the total derivative:

$$d\Pi = dc - wdn.$$

Dividing by $d\tau$, and substituting $\frac{dc}{d\tau}$ from above:

$$\frac{d\Pi}{d\tau} = \left\{ w + \left[\frac{[1-(1-\rho)\beta]}{\beta} - \rho \right] \gamma \frac{1}{q} \right\} \frac{dn}{d\tau} + \left(s\frac{f'}{f} - s\gamma \right) \frac{d\theta}{d\tau} - w\frac{dn}{d\tau}.$$

Combining:

$$\frac{d\Pi}{d\tau} = \left\{ w + \left[\frac{[1-(1-\rho)\beta]}{\beta} - \rho \right] \gamma \frac{1}{q} - w \right\} \frac{dn}{d\tau} + \left(s\frac{f'}{f} - s\gamma \right) \frac{d\theta}{d\tau}.$$

Cancelling w terms out and substituting $\frac{d\theta}{d\tau} = 0$ from above:

$$\frac{d\Pi}{d\tau} = \left\{ \left[\frac{[1-(1-\rho)\beta]}{\beta} - \rho \right] \gamma \frac{1}{q} \right\} \frac{dn}{d\tau},$$

which, as shown earlier, equals to:

$$\frac{d\Pi}{d\tau} = \frac{1-\beta}{\beta}\gamma\frac{1}{q}\frac{dn}{d\tau} < 0.$$

Impact of Tax Changes: DWL Recall that in the search model with amenities, DWL is given by:

$$\begin{aligned} DWL &= -\left\{\frac{d\ln(wn)}{d\ln\tau}\right\} - \left\{\left[u'(1-\tau)w + A'a - h'\right]\frac{f}{\rho} - h'\right\}\frac{ds}{u'wn\cdot d\tau} \\ &\quad -\left[u'(1-\tau)w + A'a - h'\right]\frac{sf'}{\rho}\frac{d\theta}{u'wn\cdot d\tau} - u'\frac{d\Pi}{u'wn\cdot d\tau}. \end{aligned}$$

Of course, in the present case:

$$\begin{aligned} DWL &= -\left\{\frac{d\ln(wn)}{d\ln\tau}\right\} - \left\{\left[u'(1-\tau)w - h'\right]\frac{f}{\rho} - h'\right\}\frac{ds}{u'wn\cdot d\tau} \\ &\quad -\left[u'(1-\tau)w - h'\right]\frac{sf'}{\rho}\frac{d\theta}{u'wn\cdot d\tau} - u'\frac{d\Pi}{u'wn\cdot d\tau}. \end{aligned}$$

With $d\theta$ equal to zero, then in this case:

$$DWL = -\left\{\frac{d\ln(wn)}{d\ln\tau}\right\} - \left\{\left[u'(1-\tau)w - h'\right]\frac{f}{\rho} - h'\right\}\frac{ds}{u'wn\cdot d\tau} - u'\frac{d\Pi}{u'wn\cdot d\tau}.$$

As before, substituting

$$h' = \underbrace{\frac{\beta f}{1-\beta(1-\rho)+\beta f}}_{\equiv X_{LFP}}\left[u'(1-\tau)w\right]$$

implies that:

$$DWL = -\left\{\frac{d\ln(wn)}{d\ln\tau}\right\} - \left[\left(\frac{1-X_{LFP}}{X_{LFP}}\right)\frac{f}{\rho} - 1\right]h'\frac{ds}{u'wn\cdot d\tau} - u'\frac{d\Pi}{u'wn\cdot d\tau}.$$

As shown in the previous section, it is straightforward to prove that:

$$\left[\frac{1-X_{LFP}}{X_{LFP}}\frac{f}{\rho}-1\right]h'>0.$$

Therefore, we once again have:

$$DWL=\overbrace{-\left\{\frac{d\ln\left(wn\right)}{d\ln\tau}\right\}}^{\equiv ETI>0}\overbrace{-\overbrace{\left[\frac{1-X_{LFP}}{X_{LFP}}\frac{f}{\rho}-1\right]h'}^{(+)}\overbrace{\frac{ds}{u'wn\cdot d\tau}}^{(-)}}^{(+)}\overbrace{-\overbrace{u'}^{(+)}\overbrace{\frac{d\Pi}{u'wn\cdot d\tau}}^{(-)}}^{(+)},$$

implying that $DWL>ETI$ holds when there are no amenities in our efficient search model as well.

D.3 Unemployment Benefits with Amenities

We now consider the implication of tax changes in the model with positive unemployment benefits.

The Wage Curve Recall that, as highlighted in the main text, the analysis that follows assumes $y_{nn,t}=0$, $u_t=c_t$, which implies that $u'_t=1$, $u''_t=0$ (*for conceptual ease, throughout the paper we use* u' *rather than 1*). With $\chi>0$, equation 17 is:

$$w=\left(1-\psi\right)\left[\frac{\chi}{1-\tau}-\frac{A'a}{u'\left(1-\tau\right)}\right]+\psi\left(y_n-\phi'a+\gamma\theta\right).$$

Recall from the amenities equilibrium condition that:

$$A'a=u'\left(1-\tau\right)\phi'a\rightarrow\frac{A'a}{u'\left(1-\tau\right)}=\phi'a.$$

Making this substitution above implies that:

$$w=\left(1-\psi\right)\frac{\chi}{1-\tau}-\left(1-\psi\right)\phi'a+\psi\left(y_n-\phi'a+\gamma\theta\right),$$

and therefore,

$$w=(1-\psi)\frac{\chi}{1-\tau}-\phi' a+\psi\left(y_n+\gamma\theta\right).$$

We rearrange this condition as follows:

$$\underbrace{w+\phi' a}_{\equiv W}=(1-\psi)\frac{\chi}{1-\tau}+\psi\left(y_n+\gamma\theta\right),$$

where, as before, W is the "effective wage," since it is what matters for the firm on the margin. Moreover, we refer to the *effective wage curve* as WC_χ to distinguish it from the WC with no unemployment insurance.

Note that:

$$\begin{aligned} dW &= \frac{(1-\psi)\,\chi}{(1-\tau)^2}d\tau+\psi y_{nn}dn+\psi\gamma d\theta \\ &= \frac{(1-\psi)\,\chi}{(1-\tau)^2}d\tau+\psi d\theta, \end{aligned}$$

since by assumption $y_{nn}=0$. As such, the wage curve is increasing and linear in (θ, W) space, though it now has an intercept term that depends on tax level and change in taxes.

The Job Creation Condition The job creation condition remains the same as above:

$$\underbrace{w+\phi' a}_{\equiv W}=y_n-\underbrace{\frac{\left[1-(1-\rho)\,\beta\right]}{\beta}}_{\equiv X_{JC}}\gamma\frac{1}{q}.$$

Then,

$$dW=y_{nn}dn-X_{JF}\left(-\frac{1}{q^2}\right)q' d\left(v/s\right)=X_{JC}\frac{q'}{q^2}d\theta.$$

By the same reasoning as above, JC_χ, which is equivalent to JC above, is decreasing and convex in in (θ, W) space.

Impact of Tax Changes: $w+\phi' a$ and θ Plotting JC_χ and WC_χ in (θ, W) space (Figure A2) pins down the model's equilibrium values of $w+\phi' a$ and θ. Note from above that

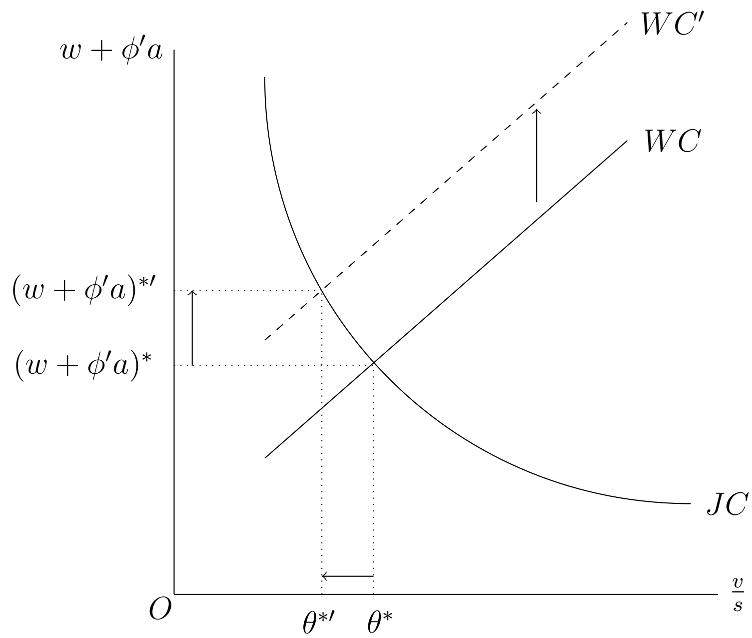
JC_χ is not a function of τ, which implies no change in JC_χ given a tax change. However, from the derivations above WC_χ shifts up given a tax change. Therefore, given a change in τ,

$$\frac{d\theta}{d\tau} < 0 \text{ and } \frac{d\left(w + \phi' a\right)}{d\tau} > 0.$$

Moreover, the last equation above therefore implies that:

$$\frac{dw}{d\tau} > -\frac{d\left(\phi' a\right)}{d\tau}.$$

Figure A2: Equilibrium Efficient Wage and Market Tightness with UI



Impact of Tax Changes: lfp With $\chi > 0$, equation 2.24 is:

$$h' = \underbrace{\frac{u'\left[1 - \beta\left(1 - \rho\right)\right]}{1 - \beta\left(1 - \rho\right) + \beta f}}_{\equiv X_\chi} \chi + \underbrace{\frac{\beta f}{1 - \beta\left(1 - \rho\right) + \beta f}}_{\equiv X_{LFP}} \left[u'\left(1 - \tau\right) w + A' a\right].$$

Recall from the amenities equilibrium condition that:

$$A' a = u'\left(1 - \tau\right) \phi' a.$$

Using this condition above yields:

$$h' = X_\chi + X_{LFP} u' (1-\tau)(w + \phi' a).$$

Then,

$$-h'' dlfp = \chi dX_\chi + u'(1-\tau)(w+\phi' a) dX_{LFP} + X_{LFP} \cdot u'(1-\tau) d(w+\phi' a)$$
$$+X_{LFP}(1-\tau)(w+\phi' a) u' dc - X_{LFP} u'(w+\phi' a) d\tau.$$

Therefore,

$$-h'' dlfp = \chi dX_\chi + u'(1-\tau)(w+\phi' a) dX_{LFP}$$
$$+X_{LFP} \cdot u'(1-\tau) d(w+\phi' a) - X_{LFP} u'(w+\phi' a) d\tau.$$

since by assumption $u'' = 0$, which also implies that:

$$dX_\chi = -\frac{u'[1-\beta(1-\rho)]}{(1-\beta(1-\rho)+\beta f)^2} \beta f' d\theta.$$

It follows that:

$$\begin{aligned} \frac{dlfp}{d\tau} &= -\chi \frac{u'[1-\beta(1-\rho)]}{(1-\beta(1-\rho)+\beta f)^2} \beta f' \frac{d\theta}{d\tau} \\ &\quad + u'(1-\tau)(w+\phi' a) \frac{dX_{LFP}}{d\tau} \\ &\quad + X_{LFP} \cdot u'(1-\tau) \frac{d(w+\phi' a)}{d\tau} \\ &\quad - X_{LFP} u'(w+\phi' a). \end{aligned}$$

From the previous section, $\frac{d\theta}{d\tau} < 0$, so the first term above is positive. As was established in the "Efficient Search Model" section from earlier, $\frac{dX_{LFP}}{d\tau} > 0$, so the second term above is positive. From the previous section, $\frac{d(w+\phi' a)}{d\tau} > 0$, so the third term above is positive. However, the last term above is negative. As such, there is ambiguity in the direction of change of labor force participation given an increase in taxes.

All told, as should be expected given that the presence of unemployment benefits implies that the model is no longer efficient, it follows that the introduction of unemployment benefits introduce a degree of distortions that require parametric assumptions to eradicate ambiguity. Therefore, the case with positive unemployment benefits is best studied quantitatively. Of course, the intuition behind this ambiguity is straightforward. From the household's perspective, the decrease in θ induced by the increase in taxes is a disincentive for search effort because jobs are harder to find. On the other hand, the effective wage rises, which makes it more appealing to search for jobs.

D.4 Unemployment Benefits without Amenities

We proceed with the same assumptions as in the previous section.

The Wage Curve Of course, in this case the wage curve is:

$$w = (1-\psi)\frac{\chi}{1-\tau} + \psi\left(y_n + \gamma\frac{v}{s}\right).$$

Therefore,

$$\begin{aligned} dw &= \frac{(1-\psi)\,\chi}{(1-\tau)^2}d\tau + \psi y_{nn} dn + \psi\gamma d\theta \\ &= \frac{(1-\psi)\,\chi}{(1-\tau)^2}d\tau + \psi d\theta, \end{aligned}$$

since by assumption $y_{nn} = 0$. As such, the wage curve is increasing and linear in (θ, w) space.

The Job Creation Condition The job creation condition is now:

$$w = y_n - \underbrace{\frac{[1-(1-\rho)\,\beta]}{\beta}}_{\equiv X_{JC}}\gamma\frac{1}{q}.$$

Then,

$$
\begin{aligned}
dw &= y_{nn}dn - X_{JF}\left(-\frac{1}{q^2}\right)q'd\theta \\
&= X_{JC}\frac{q'}{q^2}d\theta.
\end{aligned}
$$

Because $q' < 0$ the job creation condition decreasing and convex in in (θ, w) space.

Impact of Tax Changes: $w + \phi' a$ **and** θ Plotting JC_χ and WC_χ in (θ, w) space pins down the model's equilibrium values of w and θ. Note from above that JC_χ is not a function of τ, which implies no change in JC_χ given a tax change. However, from the derivations above WC_χ shifts up given a tax change. Therefore, given a change in τ,

$$
\frac{d\theta}{d\tau} < 0 \text{ and } \frac{dw}{d\tau} > 0.
$$

Impact of Tax Changes: lfp By the same steps above, it is straightforward to show that:

$$
\begin{aligned}
\frac{dlfp}{d\tau} &= -\chi\frac{u'\left[1-\beta\left(1-\rho\right)\right]}{\left(1-\beta\left(1-\rho\right)+\beta f\right)^2}\beta f'\frac{d\theta}{d\tau} \\
&\quad + u'\left(1-\tau\right)w\frac{dX_{LFP}}{d\tau} \\
&\quad + X_{LFP}\cdot u'\left(1-\tau\right)\frac{dw}{d\tau} \\
&\quad - X_{LFP}u'w,
\end{aligned}
$$

where,

$$
X_{LFP} = \frac{\beta f}{1-\beta\left(1-\rho\right)+\beta f}u'\left(1-\tau\right)w.
$$

As such, just like in the case with amenities, there is ambiguity in the direction of change of labor force participation given an increase in taxes.

D.5 Neoclassical Model with Amenities

We continue to assume that $y_{nn,t} = 0$, $u_t = c_t$, which implies that $u'_t = 1$, $u''_t = 0$.

Impact of Tax Changes: n In the neoclassical model, the firm's optimality condition for employment,

$$y_n = w + \phi'_t a,$$

implies that:

$$y_{nn} dn = dw + d(\phi' a)$$

and, therefore, that, as in the efficient search model,

$$dw = -d(\phi' a)$$

given that by assumption $y_{nn} = 0$.

Using the equilibrium condition for amenities, we have:

$$A' = u'(1-\tau)\phi',$$

it follows that:

$$A'a = u'(1-\tau)\phi' a,$$

and therefore,

$$\begin{aligned} d(A'a) &= -u'\phi' a d\tau + u'(1-\tau) d(\phi' a) + (1-\tau)\phi' a u'' dc \\ &= -u'\phi' a d\tau + u'(1-\tau) d(\phi' a), \end{aligned}$$

since by assumption $u'' = 0$. Of note, since from above, $dw = -d(\phi' a)$, it follows that:

$$d(A'a) = -u'\phi' a d\tau - u'(1-\tau) dw.$$

Now consider the household's optimality condition for employment:

$$h' = A'a + u'(1-\tau)w.$$

Given this expression,

$$\begin{aligned}
-h''dn &= d(A'a) - u'wd\tau + u'(1-\tau)dw + (1-\tau)wu''dc \\
&= d(A'a) - u'wd\tau + u'(1-\tau)dw,
\end{aligned}$$

since by assumption $u'' = 0$. Substituting in the above-derived expression:

$$d(A'a) = -u'\phi' a d\tau - u'(1-\tau)dw.$$

This yields:

$$\begin{aligned}
-h''dn &= -u'\phi' a d\tau - u'(1-\tau)dw - u'wd\tau + u'(1-\tau)dw \\
&= -(u'\phi' a + u'w)d\tau.
\end{aligned}$$

Then,

$$\frac{dn}{d\tau} = \frac{-(u'\phi' a + u'w)}{-h''} < 0$$

since $h'' < 0$.

Impact of Tax Changes: a Given the result that $dn/d\tau < 0$, this section is exactly like that in the efficient search model. Consider the equilibrium condition for amenities,

$$A' = u'(1-\tau)\phi'.$$

This implies that:

$$A''a \cdot dn + A''n \cdot da = (1-\tau)\,\phi' \cdot u'''\phi' \cdot d\tau$$
$$+u'\,(1-\tau)\,(\phi''a \cdot dn + \phi''n \cdot da)\,.$$

Implementing $u'' = 0$, which holds by assumption, and collecting terms implies that:

$$[u'\,(1-\tau)\,\phi'' - A'']\,n \cdot da = [A'' - u'\,(1-\tau)\,\phi'']\,a \cdot dn + u'\phi' d\tau,$$

and therefore,

$$\frac{da}{d\tau} = \frac{[A'' - u'\,(1-\tau)\,\phi'']\,a}{[u'\,(1-\tau)\,\phi'' - A'']\,n} \cdot \frac{dn}{d\tau} + \frac{u'\phi'}{[u'\,(1-\tau)\,\phi'' - A'']\,n}.$$

Since $A'' < 0$, the denominator terms in square brackets on the right-hand side of this equation are positive. Of course, $u'\phi' > 0$, so the second term of this equation is positive. The numerator in square brackets in the first term of this equation is negative since $A'' < 0$ and $\phi'' > 0$. As such, the ratio of the bracketed expressions in the first term of this equation is negative. Given the result $dn/d\tau < 0$, the following result immediately follows:

$$\frac{da}{d\tau} > 0.$$

Impact of Tax Changes: w Given the results $dn/d\tau < 0$ and $da/d\tau > 0$, this section is exactly like that in the efficient search model. Consider $\phi' a$. The total derivative of this expression implies that:

$$\frac{d\,(\phi' a)}{d\tau} = \phi' \frac{da}{d\tau} + a\phi'' \frac{d\,(an)}{d\tau}.$$

Since ϕ' is greater than zero as well as $da/d\tau$, as shown above, the first term on the right hand side above is positive. Therefore, to sign the left-hand side of this equation it remains to sign $d\,(an)\,/d\tau$. To do so, consider the equilibrium condition for amenities,

$$A' = u'\,(1-\tau)\,\phi'.$$

This condition implies that:

$$\begin{aligned} A''d(an) &= -u'\phi'd\tau + u'(1-\tau)\phi''d(an) + (1-\tau)\phi'u''du \\ &= -u'\phi'd\tau + u'(1-\tau)\phi''d(an), \end{aligned}$$

since by assumption $u'' = 0$. Collecting terms and rearranging, we then have:

$$\frac{d(an)}{d\tau} = -\frac{u'\phi'}{A'' - u'(1-\tau)\phi''}.$$

The numerator of this expression is positive, and the denominator is negative since $A'' < 0$ and $\phi'' > 0$. Thus, the following result immediately follows:

$$\frac{d(an)}{d\tau} > 0.$$

With the expression

$$\frac{d(\phi'a)}{d\tau} = \phi'\frac{da}{d\tau} + a\phi''\frac{d(an)}{d\tau},$$

it immediately follows that:

$$\frac{d(\phi'a)}{d\tau} > 0.$$

Because, as derived above,

$$dw = -d(\phi'a),$$

it immediately follows that:

$$\frac{dw}{d\tau} < 0.$$

Impact of Tax Changes: c From the aggregate budget constraint, we have:

$$c = y - \phi.$$

Therefore,

$$dc = y_n dn - \phi' d(an).$$

From above, we know that $dn/d\tau < 0$ and $d\left(an\right)/d\tau > 0$. It immediately follows that:

$$\frac{dc}{d\tau} < 0.$$

Of course, another way to see this is to note that from the household's budget constraint,

$$c = wn + \Pi.$$

Therefore,

$$dc = d\left(wn\right) + d\Pi.$$

Of course, though, in the neoclassical environment $d\Pi = 0$. Combined with the fact that with w and n both falling given the rise in taxes, the preceding equation immediately implies that $dc/d\tau < 0$.

D.6 Neoclassical Model without Amenities

We proceed with the same assumptions as in the previous section.

Impact of Tax Changes: n and w In this case,

$$y_{n,t} = w_t,$$

implies that

$$y_{nn}dn = dw$$

and, therefore,

$$dw = 0$$

given that by assumption $y_{nn} = 0$.

Now consider the household's optimality condition for employment:

$$h' = u'\left(1 - \tau\right)w.$$

Given this expression,

$$-h''dn = -u'wd\tau + u'(1-\tau)\,dw,$$

since by assumption $u'' = 0$. Therefore,

$$\frac{dn}{d\tau} = \frac{u'w}{h''} < 0,$$

since $h'' < 0$.

Impact of Tax Changes: c From the aggregate budget constraint, we have:

$$c = y.$$

Therefore,

$$dc = y_n dn.$$

Since $dn/d\tau < 0$, it immediately follows that:

$$\frac{dc}{d\tau} < 0.$$

As above, another way to see this is to note that from the household's budget constraint,

$$c = wn + \Pi.$$

Therefore,

$$dc = d\,(wn) + d\Pi.$$

Of course, though, in the neoclassical environment, $d\Pi = 0$. Combined with the fact that with w and n both falling given the rise in taxes, the preceding equation immediately implies that $dc/d\tau < 0$.

E Implications of Calibration

In this section, we discuss some of the implications of our calibration for results stemming from our quantitative benchmark quantitative analysis.

E.1 Search Models

Of course, in our calibration, taxable income is the same in both search models (amenities versus no amenities), as is v/s ratio and lfp (all of these variables are calibration targets). Let "A" denote the search model with amenities and "NA" the search model without amenities. Because lfp is the same across models, then by definition of lfp and the steady-state value of employment,

$$lfp^A = \left(\frac{m}{\rho}\right)^A + s^A = \left(\frac{m}{\rho}\right)^{NA} + s^{NA} = lfp^{NA},$$

which implies that:

$$\left(s\frac{f}{\rho}\right)^A + s^A = \left(s\frac{f}{\rho}\right)^{NA} + s^{NA},$$

and in turn implies that:

$$s^A\left[1+\left(\frac{f}{\rho}\right)^A\right] = s^{NA}\left[1+\left(\frac{f}{\rho}\right)^{NA}\right].$$

But, since θ is the same across models, then both f and ρ are the same across models as well, so the preceding equation implies that s is the same across models (and therefore n as well, and also v given an equal θ and w given equal taxable income). All told, this last equation immediately implies that $ds/d\tau$ is the same across models, and given $d\theta/d\tau$ being zero across models it follows that $dn/d\tau$ is the same across models and, therefore, $dlfp/d\tau$ as well. [39]

[39] Note as well that the fact that $dlfp/d\tau$ is the same across models implies that:

$$\left(\frac{dlfp}{d\tau}\right)^A = \frac{X_{LFP}\left[u'\left(w+\phi' a\right)\right]^A}{\left(h''\right)^A} = \frac{X_{LFP}\left[u'w\right]^{NA}}{\left(h''\right)^{NA}} = \left(\frac{dlfp}{d\tau}\right)^{NA}$$

Turning to consumption and profits, we first note from the envelope theorem that:

$$\frac{dc}{d\tau} = \frac{-u'wn + h'\frac{dlfp}{d\tau} - A'\frac{d(an)}{d\tau}}{u'}.$$

Therefore, with taxable income being the same across models as well as $dlfp/d\tau$, and by the proof of $d\left(an\right)/d\tau > 0$ in Appendix D, and moreover:

$$\left(h'\right)^{A} = u'\left(1-\tau\right)w + A'a > u'\left(1-\tau\right)w = \left(h'\right)^{NA},$$

it immediately follows that:

$$\left(\frac{dc}{d\tau}\right)^{A} < \left(\frac{dc}{d\tau}\right)^{NA},$$

since consumption drops in both models as shown in Appendix D. As such, consumption drops by more in the model with amenities compared with the model without amenities. Finally, regarding profits, note from Appendix D that:

$$\frac{d\Pi}{d\tau} = \frac{1-\beta}{\beta}\gamma\frac{1}{q}\frac{dn}{d\tau}.$$

With θ equalized across models, q is the same across models as well, and as discussed above, it is also the case that $dn/d\tau$ is the same across models. That said, per our calibration $\gamma^{A} > \gamma^{NA}$, which, given the equation above, immediately implies that:

$$\frac{d\Pi^{A}}{d\tau} < \frac{d\Pi^{NA}}{d\tau}.$$

As such, profits drop by more in the model with amenities compared twith the model without

(of course X_{LFP} is the same across models given θ and therefore f being the same across models). This means that:

$$\frac{\left(h''\right)^{NA}}{\left(h''\right)^{A}} = \frac{\left[u'w\right]^{NA}}{\left[u'\left(w+\phi'a\right)\right]^{A}},$$

ad with $u'=1$ across models, as well as equal wages, then:

$$\frac{\left(h''\right)^{NA}}{\left(h''\right)^{A}} < 1,$$

which means that $\left(h''\right)^{NA} < \left(h''\right)^{A}$, as is the case in our calibration.

amenities.

E.2 Neoclassical Models

In the neoclassical models, with lfp being the same across models (with and without amenities), then n is the same across models. It immediately follows that since taxable income is the same across models, then w is the same across models as well. Moreover, given zero profits consumption is the same across models as well. Of course, with wages and employment equalized across models:

$$\Pi^A = y^A - wn - \phi = 0 = y^{NA} - wn = \Pi^{NA},$$

which implies that:

$$y^A - \phi = y^{NA},$$

and therefore:

$$y^A > y^{NA}.$$

With employment equal across models, then $(y')^A > (y')^{NA}$ as well. Then, given the equation above this last inequality,

$$(y')^A \frac{dn^A}{d\tau} - \phi' a \frac{dn^A}{d\tau} - \phi' n \frac{da}{d\tau} = (y')^{NA} \frac{dn^{NA}}{d\tau}.$$

Using the firm's optimality condition for labor, we can restate this condition as:

$$w\frac{dn^A}{d\tau} - \phi' n \frac{da}{d\tau} = w\frac{dn^{NA}}{d\tau}.$$

Therefore,

$$w\left(\frac{dn^{NA} - dn^A}{d\tau}\right) = -\phi' n \frac{da}{d\tau} < 0,$$

where the inequality follows from Appendix D. It immediately follows that:

$$\frac{dn^{NA}}{d\tau} < \frac{dn^{A}}{d\tau} \rightarrow \frac{dn^{NA}}{dn^{A}} > 1,$$

where the implication follows from Appendix D. Therefore, given an increase in taxes employment falls by more in the model without amenities compared to the model with amenities.

Finally, by the envelope theorem,

$$\frac{dU^{A}}{d\tau} = u'\frac{dc^{A}}{d\tau} + A'\frac{d\left(an\right)}{d\tau} = -\lambda wn = u'\frac{dc^{NA}}{d\tau} = \frac{dU^{NA}}{d\tau}.$$

By $u' = 1$ across models, it follows that:

$$A'\frac{d\left(an\right)}{d\tau} = u'\left(\frac{dc^{NA} - dc^{A}}{d\tau}\right) > 0,$$

where the inequality follows from the proof in Appendix D. As such,

$$\frac{dc^{NA}}{d\tau} > \frac{dc^{A}}{d\tau},$$

which implies that:

$$\frac{dc^{NA}}{dc^{A}} < 1,$$

since, as shown in Appendix D, in both models consumption drops given an increase in taxes. As such, consumption drops by more in the model with amenities compared with the model without amenities.

E.3 The ETIs across Models

Our benchmark calibration implies that, for a given change in tax rate ($d\tau$), the ETI is larger with amenities than in the absence of amenities.

Proof. Consider first the definition of the ETIs for the search models with and without amenities:

$$ETI^A = -\frac{d\ln w^A}{d\ln \tau} - \frac{d\ln n^A}{d\ln \tau},$$

and

$$ETI^{NA} = -\frac{d\ln n^{NA}}{d\ln \tau},$$

which follows from the fact that $dw^{NA} = 0$ as shown in Appendix D.

Note that we showed earlier in Appendix D that both dn^A and dn^{NA} are negative, while $dw^A < 0$.

With our results from Appendix D, it is straightforward to show that:

$$\frac{dn^A}{d\tau} = \frac{f}{f+\rho}\frac{dlfp^A}{d\tau} = \frac{f}{f+\rho}\frac{X^A_{LFP}}{h''^A}(w + \phi'^A) \text{ and,}$$

$$\frac{dn^{NA}}{d\tau} = \frac{f}{f+\rho}\frac{dlfp^{NA}}{d\tau} = \frac{f}{f+\rho}\frac{X^{NA}_{LFP}}{h''^{NA}}w^{NA}.$$

Above, f and ρ (job finding and vacancy filling rates, respectively) would be the same in the initial equilibrium for both models by construction. This would also imply that $X^A_{LFP} = X^{NA}_{LFP}$. Using the equilibrium condition between efficient wage and market tightness (setting $WC = JC$), we can show that the initial equilibrium market tightness would be parametric, and thus, $\theta^A = \theta^{NA}$. Assuming $y' = 1$, this would also imply that $(w + \phi'^A) = w^{NA}$ from equation 2.25. Finally, at the initial steady-state equilibrium, labor force participation would be equal for both models by construction, making $h''^A = h''^{NA}$. All told, we would have $\frac{dn^A}{d\tau} = \frac{dn^{NA}}{d\tau}$. Given that $dw^{NA} = 0$ and $dw^A < 0$, this would imply that $ETI^{NA} < ETI^A$. This is intuitive, as when there are amenities in the model, firms and households can partially substitute wage income with non-wage income (amenities), causing a larger decline in taxable income in response to a tax hike. ■