Board of Governors of the Federal Reserve System

International Finance Discussion Papers

ISSN 1073-2500 (Print)
ISSN 2767-4509 (Online)

Number 1340

March 2022

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Please cite this paper as:
Queralto, Albert (2022). “Monetary Policy in a Model of Growth,” International Finance Discussion Papers 1340. Washington: Board of Governors of the Federal Reserve System, https://doi.org/10.17016/IFDP.2022.1340.

Monetary Policy in a Model of Growth

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Federal Reserve Board

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Abstract

Empirical evidence suggests that recessions have long-run effects on the economy's productive capacity. Recent literature embeds endogenous growth mechanisms within business cycle models to account for these "scarring" effects. The optimal conduct of monetary policy in these settings, however, remains largely unexplored. This paper augments the standard sticky-price New Keynesian (NK) to allow for endogenous dynamics in aggregate productivity. The model has a representation similar to the two-equation NK model, with an additional condition linking productivity growth to current and expected future output gaps. Absent state contingency in the subsidies that correct the externalities associated with productivity growth, optimal monetary policy sets inflation above target whenever the subsidies fall short of the externalities. In the recovery from a spell at the ZLB, the optimal discretionary policy sets inflation temporarily above target, helping mitigate the long-run damage. Following a cost-push shock that creates inflationary pressure, the central bank tolerates a larger rise in inflation than in a model with exogenous productivity. The gains from commitment include the central bank's ability to make credible promises about future output gaps in a way that allows it to manipulate current productivity growth.

Keywords: Business cycles; Growth; Optimal monetary policy; Hysteresis; Scarring.
JEL classification: E32; E43; E52; E58; O31; O42.

*I thank Matteo Iacoviello and seminar participants at the Federal Reserve Board for useful comments. I am especially grateful to Narayana Kocherlakota for conversations at several stages of this project and for many valuable suggestions. I thank Mike McHenry for excellent research assistance. This paper was partly written while I was visiting the Department of Economics at the University of Rochester, whose hospitality is gratefully acknowledged. Any views expressed are those of the author and do not reflect the views of the Board of Governors of the Federal Reserve or the Federal Reserve System. E-mail: albert.queralto@frb.gov.

1 Introduction

A growing literature combines models used to study business cycles and monetary policy with mechanisms that endogenize aggregate productivity dynamics.[1] One goal of that literature is to capture the so-called *scarring* effect of recessions: the phenomenon that recessions sometimes appear to induce long-term damage on the supply side of the economy. Over the past two decades, an extensive empirical literature has emerged documenting these scarring effects. For example, Cerra and Saxena (2008) first showed that financial and other crises are associated with output losses that are typically not reversed. Ball (2014), Blanchard, Cerutti and Summers (2015), and Martin, Munyan and Wilson (2015) document a similar fact for recessions in advanced economies. Queralto (2020) finds that a large part of these losses are attributable to declines in labor productivity and in total factor productivity (TFP). Reifschneider, Wascher and Wilcox (2015) show that the Great Recession in the United States was associated with adverse effects of aggregate demand on potential output and TFP.

The question of how monetary policy should be conducted in an environment with endogenous productivity growth remains, however, largely unexplored.[2] This paper seeks to contribute to filling this gap by generalizing the *textbook* version of the New Keynesian (NK) model to allow for the presence of productivity-enhancing investments—which imply an endogenous rate of aggregate TFP growth—and by studying the optimal conduct of monetary policy in that setting. The goal is to provide an analytically tractable setup that tracks closely the standard NK economy, and to focus on whether, and how, the presence of endogenous productivity dynamics affects the main lessons regarding the conduct of monetary policy that emerge from the textbook NK model.

In the economy studied here, the creation of new varieties of intermediate goods is the source of endogenous productivity growth, as in Romer (1990). Intermediate goods are used as inputs in the production of final goods. The producers of the latter type of goods, in turn, are subject to price-setting frictions as in Calvo (1983). An entrepreneurial sector develops new types of intermediates. As is common in models of endogenous growth, an entrepreneur's ability to successfully develop a new product involves *spillovers*, in that it depends in part on the activity of other entrepreneurs or other firms. The model allows for both knowledge spillovers (whereby the knowledge created by a given entrepreneur benefits subsequent entrepreneurs) and for learning-by-doing spillovers (whereby experience in the production of existing innovations also creates practical knowledge that is useful for subsequent entrepreneurs). These spillovers, or externalities, generally imply that the market equilibrium is inefficient.

The model has a representation similar to the two-equation NK model (consisting of a "dynamic IS" curve and a Phillips curve), with an additional condition through which productivity growth depends positively on both current and expected future output gaps. This dependence arises for two reasons: first, lower future output gaps imply lower monopoly profits from new innovations,

[1]See Cerra, Fatás and Saxena (forthcoming) for a recent survey.

[2]Some notable exceptions include Benigno and Fornaro (2018), Garga and Singh (2021), and Fornaro and Wolf (2020). I clarify below how the present paper differs from these contributions.

and therefore weaken the incentives of entrepreneurs to develop new intermediate varieties. Second, lower current and future activity leads to smaller learning-by-doing spillovers, also slowing growth. The model's representation is highly tractable, and can be used to derive analytical results for a first-order approximation of the dynamic effects of shocks—as in the textbook NK model. A monetary shock in this economy, by lowering the path of the output gap, triggers temporarily slower productivity growth, and leads to permanently lower output and TFP. These dynamics are consistent with recent evidence (Moran and Queralto 2018, Jordà, Singh and Taylor 2020) that monetary tightenings lead to persistent supply-side damage.

I next turn to optimal policy. If state-contingent labor subsidies are available to correct the externalities, then the optimal monetary policy sets inflation to target at all times, and the resulting allocation is efficient. By contrast, if state-contingent subsidies are not available, optimal monetary policy obeys the following principle: allow inflation to rise above target whenever the growth externalities are high relative to the subsidies. For example, following an exogenous preference shock that temporarily lowers the value of the representative household's utility flow—a shock that makes growth more valuable, and is associated with larger externalities—it is optimal to allow inflation to rise above target. I also show how the presence of the zero lower bound may result in a permanent TFP and output loss, as the central bank's inability to prevent a decline in the output gap leads to a slowdown in productivity growth. Still, even a discretionary central bank may have incentives to allow for temporarily elevated inflation in the aftermath of a spell at the ZLB, which helps mitigate some of the long-run damage.

I also examine the optimal monetary policy response to cost-push shocks that initiate inflationary pressure. I find that the presence of endogenous growth tilts the central bank's incentives toward allowing for a larger rise in inflation (and a smaller decline in the output gap) compared with the standard model with exogenous productivity. The reason is that by preventing the output gap from turning too negative, the central bank is able to reduce the extent of supply-side damage.

Finally, I examine optimal policy under commitment, and identify a benefit from the ability to commit that arises due to the forward-looking nature of productivity growth. Thus, a central bank that can commit itself to future actions is able to boost current productivity growth by promising higher future output gaps—in a way that would not be credible absent the ability to commit, due to the ex-post costs of higher inflation.

Related literature. This paper relates to work that endogenizes productivity dynamics within business cycle and monetary models. Examples include Fatás (2000), Comin and Gertler (2006), Kung and Schmid (2015), Moran and Queralto (2018), Benigno and Fornaro (2018), Anzoategui et al. (2019), Guerron-Quintana and Jinnai (2019), Bianchi et al. (2019), Queralto (2020), Garga and Singh (2021), Ates and Saffie (2021), Gornemann et al. (2020), and Fornaro and Wolf (2020). Of these, the closest to the present paper are Benigno and Fornaro (2018), Garga and Singh (2021), and Fornaro and Wolf (2020), who also focus on the conduct of of monetary policy. There are a number of important differences between these studies and the present paper. First, here I focus on the textbook NK model with staggered price setting, as laid out in Woodford (2003) and Galí (2015)—a framework that remains the main workhorse for monetary policy analysis. By contrast, Benigno and Fornaro (2018), Garga and Singh (2021), and Fornaro and Wolf (2020) focus on frameworks that feature either fully rigid wages or staggered wage setting. Second, the present paper emphasizes how the externalities inherent in models of endogenous productivity dynamics shape the desirable course of monetary policy. The behavior of these externalities is key to the optimal conduct of monetary policy in the present framework, but it has not received much attention in the existing literature. Finally, and less importantly, I endogenize productivity dynamics by allowing for an expanding variety of intermediate products as in Romer (1990), while much of the related literature relies on quality-ladders models as in Aghion and Howitt (1992). One advantage of the present formulation is that it conveniently nests the textbook NK economy, when the intermediate varieties come close to being perfect substitutes.

This paper also relates to the literature that endogenizes firm entry within business cycle models, for example Bilbiie et al. (2007), Bilbiie, Ghironi and Melitz (2012), or Bilbiie et al. (2019). The present study shares the emphasis of these papers on endogenous product creation, but differs in that I highlight the central role played by the externalities associated with the productivity growth process. Finally, a recent set of related papers also focuses on endogenizing the supply-side effects of monetary policy, via heterogeneity among producers. Examples include Baqaee, Farhi and Sangani (2021) and Reinelt and Meier (2020).

2 Model

The two central features of the model are the following: First, firms set prices on a staggered basis as in the textbook New Keynesian model (e.g., Galí 2015), implying that monetary policy has real effects. Second, productivity growth results from the purposeful innovation activity of entrepreneurs, and thus evolves endogenously.

Time is discrete and indexed by $t = 0, 1, \dots$. The economy is inhabited by households, final goods firms, intermediate goods firms, and entrepreneurs. Intermediate goods are used in the production of final goods. Entrepreneurs use "skilled" labor to create new varieties of intermediate goods.

2.1 Households

Assume a representative infinitely-lived household that seeks to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{\zeta_t} \left(\log(C_t) - \frac{N_t^{1+\varphi}}{1+\varphi} - \frac{\chi S_t^{1+\varphi}}{1+\varphi} \right), \tag{1}$$

where $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$ is an index of final good consumption, with $C_t(i)$ denoting the quantity of good i consumed by the household; N_t is the amount of goods labor (or "unskilled" labor) supplied to producers of intermediate goods; S_t is the amount of ("skilled") labor supplied to the entrepreneurial sector; and ζ_t is an exogenous shifter of the representative household's discount rate, which is assumed to follow a first-order autoregressive process. The period budget constraint takes the form

$$\int_0^1 P_t(i)C_t(i)di + Q_tB_t \leq B_{t-1} + W_{n,t}N_t + W_{s,t}S_t + \overline{D}_t - T_t, \tag{2}$$

where Q_t is the price of a one-period discount bond, B_t is holdings of these bonds, $W_{n,t}$ is the unskilled labor wage, $W_{s,t}$ is the skilled labor wage, $\overline{D}_t$ is dividends from the ownership of firms, and T_t is lump-sum taxes.

Letting

$$P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon}di\right)^{\frac{1}{1-\epsilon}} \tag{3}$$

denote the final goods price index, the optimality conditions are as follows:

$$Q_t = \beta\mathbb{E}_t\Big\{\frac{e^{\zeta_{t+1}}}{e^{\zeta_t}}\frac{C_{t+1}^{-1}}{C_t^{-1}}\frac{P_t}{P_{t+1}}\Big\}, \tag{4}$$

$$N_t^{\varphi}C_t = \frac{W_{n,t}}{P_t}, \tag{5}$$

$$\chi S_t^{\varphi}C_t = \frac{W_{s,t}}{P_t}. \tag{6}$$

2.2 Final goods firms

Each final goods firm may reset its price only with probability $(1-\theta)$, and therefore each period a measure θ of firms keep their price unchanged. The production function of any final goods firm $i \in [0,1]$ is

$$Y_t(i) = \left(\int_0^{A_t} Z_t(i,j)^{\frac{\vartheta-1}{\vartheta}}dj\right)^{\frac{\vartheta}{\vartheta-1}}, \tag{7}$$

where $Y_t(i)$ is output of final good i, $Z_t(i,j)$ is the amount of intermediate variety j, for $j \in [0, A_t]$, used by final goods firm i, A_t is the range of intermediate varieties that exist in period t, and $\vartheta > 1$ is the elasticity of substitution between intermediate varieties.

Letting $P_{z,t}(j)$ denote the price of a unit of intermediate type j, cost minimization by firm i can be shown to yield the following conditions. First, demand for intermediate type j from firm i is characterized by

$$Z_t(i,j) = \left(\frac{P_{z,t}(j)}{P_{Z,t}}\right)^{-\vartheta}Y_t(i), \tag{8}$$

where

$$P_{Z,t} = \left(\int_0^{A_t} P_{z,t}(j)^{1-\vartheta} dj \right)^{\frac{1}{1-\vartheta}} \tag{9}$$

is a price index of intermediate goods. Second, the real marginal cost of production for firm i, denoted MC_t, is given by

$$MC_t = \frac{P_{Z,t}}{P_t}, \tag{10}$$

and is therefore the same for all final goods firms.

Given marginal cost, a firm re-optimizing in period t chooses a price P_t^* to maximize the present value of profits generated while that price remains effective. This leads to the conventional optimality condition

$$\mathbb{E}_t \left\{ \sum_{j=0}^{\infty} U_{C,t+j} \beta^j \theta^j \left(\frac{P_t^*}{P_{t+j}} - \frac{\epsilon}{\epsilon - 1} MC_{t+j} \right) Y_{t,t+j} \right\} = 0, \tag{11}$$

where $U_{C,t+j} \equiv \frac{\partial U_t}{\partial C_t} = e^{\zeta_t} C_t^{-1}$ is marginal utility of consumption (with U_t denoting instantaneous utility), and $Y_{t,t+j} = \left(\frac{P_t^*}{P_{t+j}} \right)^{-\epsilon} Y_{t+j}$ is the firm's output in period $t + j$.

2.3 Intermediate goods firms

A set of producers employs unskilled labor to manufacture intermediate goods, using a linear technology:

$$Z_t(j) = N_t(j), \tag{12}$$

where $Z_t(j)$ is the amount produced of intermediate variety $j \in [0, A_t]$.

Let $D_t(j)$ denote the *real* profit (or *dividend*) in period t of the producer of intermediate good j. This producer solves the following problem:

$$D_t(j) = \max_{P_{z,t}(j), Z_t(j)} \left(\frac{P_{z,t}(j)}{P_t} - \frac{(1 - \tau_{n,t}) W_{n,t}}{P_t} \right) Z_t(j) \tag{13}$$

subject to

$$Z_t(j) = \left(\frac{P_{z,t}(j)}{P_{Z,t}} \right)^{-\vartheta} \int_0^1 Y_t(i) di, \tag{14}$$

where $\tau_{n,t}$ is a (possibly time-varying) *unskilled labor subsidy*, and where (14) combines the market-clearing condition for intermediate j, $Z_t(j) = \int_0^1 Z_t(i, j) di$, with (8).

Solving the problem above yields the usual pricing rule $P_{z,t}(j) = \frac{\vartheta}{\vartheta - 1}(1 - \tau_{n,t}) W_{n,t}$, which holds for any $j \in [0,1]$. This condition can be combined with the intermediate goods price index (9) to yield the following expression for final goods firms' real marginal cost:

$$MC_t = \frac{\vartheta}{(\vartheta - 1)} \frac{1}{A_t^{\frac{1}{\vartheta-1}}} \frac{(1-\tau_{n,t})W_{n,t}}{P_t}. \tag{15}$$

Plugging the previous equations into (27) yields the following expression for dividends:

$$D_t = \frac{1}{\vartheta} MC_t \int_0^1 Y_t(i) di. \tag{16}$$

2.4 Entrepreneurs

A large set of entrepreneurs employs skilled labor as input in their innovation activity. The output of this activity is the creation of new *designs* of intermediate goods. An entrepreneur who discovers a new design receives a perpetual patent on the newly discovered good. A design discovered in period t becomes available for production starting in period $t+1$. After its initial discovery, a design becomes (exogenously) obsolete with probability $\phi \in [0,1]$. Thus, the payoff from a new discovery, denoted Γ_t, is

$$\begin{aligned} \Gamma_t &= \mathbb{E}_t \left\{ \sum_{i=1}^{\infty} \frac{\beta^i U_{C,t+i}}{U_{C,t}} \phi^{i-1} D_{t+i} \right\} \\ &= \mathbb{E}_t \left\{ \frac{\beta U_{C,t+1}}{U_{C,t}} \Big(D_{t+1} + \phi \Gamma_{t+1} \Big) \right\}. \end{aligned} \tag{17}$$

The production function of new designs takes the following form. One unit of skilled labor can be used to create Ψ_t new designs, where

$$\Psi_t = e^{\psi_t} A_t N_t^{\eta}, \tag{18}$$

where the variable ψ_t is an exogenous random disturbance that follows a first-order autoregressive process. This production function allows for two types of "spillovers" emphasized in the endogenous growth literature. The first, embodied in the presence of A_t in (18), is a *knowledge* spillover as in Romer (1990), whereby the efforts of previous innovators result in knowledge that is useful for current innovators. The second is a *learning-by-doing* spillover in the spirit of Arrow (1962) and Romer (1986), whereby experience in the production floor, as captured by the amount of labor engaged in the production of existing intermediates $N_t = \int_0^{A_t} N_t(j) dj$, also has the by-product of generating knowledge that is useful in the development of subsequent innovations. Parameter $\eta > 0$ regulates the strength of the learning-by-doing spillover. I will assume that this parameter is bounded above by the curvature of the disutility of labor: $\eta < \varphi$. This is a technical assumption that ensures that the social planner's problem is well-behaved.

Entrepreneurs' optimality condition is then given by

$$\Gamma_t \Psi_t = (1 - \tau_{s,t}) \frac{W_{s,t}}{P_t}, \tag{19}$$

where $\tau_{s,t}$ is a skilled-labor subsidy.

2.5 Stationary Equilibrium

Plugging the pricing rule from intermediate producers into (8) yields $Z_t(i,j) = A_t^{\frac{-\vartheta}{\vartheta-1}} Y_t(i)$. Combining this condition with the market-clearing condition for goods labor, $N_t = \int_0^1 \int_0^{A_t} N_t(i,j) dj di = \int_0^1 \int_0^{A_t} Z_t(i,j) dj di$, we obtain $\int_0^1 Y_t(i) di = A_t^{\frac{1}{\vartheta-1}} N_t$. Finally, using the expression for demand for any final good i, $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t$, yields the following production function for final output $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$:

$$Y_t = A_t^{\frac{1}{\vartheta-1}} N_t / v_t \tag{20}$$

where v_t is and index of price dispersion among final goods producers:

$$v_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di \geq 1. \tag{21}$$

According to (20), aggregate output increases with aggregate goods labor supply, N_t, and with the measure of available intermediates, A_t. The effect of the latter on output is larger when the intermediate types are less substitutable (ϑ is small). When intermediates are perfect substitutes, $\vartheta \to \infty$, an increase in the number of available intermediates does not raise output. In addition, aggregate output falls when price dispersion v_t rises.

In turn, the range of available intermediates evolves according to

$$A_{t+1} = \phi A_t + \Psi_t S_t, \tag{22}$$

which states that the mass of goods available in $t+1$ is the sum of the size of the set of period-t designs that do not become obsolete, plus the new designs discovered in period t.

The economy exhibits positive long-run growth. It is convenient to express the equilibrium conditions in terms of variables that are stationary. To this end, define

$$\left\{\mathcal{G}_t, \mathcal{Y}_t, \mathcal{D}_t, \mathbf{\Gamma}_t\right\} \equiv \left\{\frac{A_{t+1}}{A_t}, \frac{Y_t}{A_t^{\frac{1}{\vartheta-1}}}, \frac{D_t}{A_t^{\frac{\vartheta-2}{\vartheta-1}}}, \frac{\Gamma_t}{A_t^{\frac{\vartheta-2}{\vartheta-1}}}\right\}. \tag{23}$$

Here $\mathcal{G}_t \geq 1$ is the (gross) growth rate of technology, and $\mathcal{Y}_t$, $\mathcal{D}_t$, and $\mathbf{\Gamma}_t$, are the *detrended* versions of aggregate output, dividends, and the value of varieties, respectively. It is then possible to re-write the equilibrium conditions pertaining to the *growth* block of the model as

$$\mathcal{G}_t = \phi + e^{\psi_t} N_t^{\eta} S_t, \tag{24}$$

$$\mathbf{\Gamma}_t e^{\psi_t} N_t^{\eta} = (1-\tau_{s,t})\chi S_t^{\varphi} \mathcal{Y}_t, \tag{25}$$

$$\mathbf{\Gamma}_t = \mathbb{E}_t \left\{ \frac{\beta e^{\zeta_{t+1}-\zeta_t} \mathcal{Y}_t}{\mathcal{G}_t \mathcal{Y}_{t+1}} \left[\mathcal{D}_{t+1} + \phi \mathbf{\Gamma}_{t+1} \right] \right\}, \tag{26}$$

$$\mathcal{D}_t = \frac{1}{\vartheta} MC_t \mathcal{Y}_t v_t, \tag{27}$$

where we have used the definitions in (23), and invoked (6) to eliminate the skilled-labor wage $W_{s,t}$.

The remaining equilibrium conditions are as follows. Combining (28) and (5) and using $C_t = Y_t$ yields the following expression for real marginal cost:

$$MC_t = \frac{\vartheta}{(\vartheta - 1)}(1-\tau_{n,t}) N_t^{\varphi} \mathcal{Y}_t. \tag{28}$$

The detrended version of the aggregate production function (20) is simply

$$\mathcal{Y}_t = N_t / v_t. \tag{29}$$

Finally, the Euler equation (4) can be re-written as

$$Q_t = \beta \mathbb{E}_t \Big\{ \frac{e^{\zeta_{t+1}}}{e^{\zeta_t}} \frac{\mathcal{Y}_t}{\mathcal{Y}_{t+1}} \frac{P_t}{P_{t+1}} \mathcal{G}_t^{\frac{-1}{\vartheta-1}} \Big\}. \tag{30}$$

Equilibrium conditions (24)-(30), along with (11) and (3), must be satisfied by the endogenous vector $\{\mathcal{G}_t, \mathcal{Y}_t, \mathcal{D}_t, \mathbf{\Gamma}_t, N_t, S_t, MC_t, P_t, P_t^*\}$, given policy $\{Q_t, \tau_{n,t}, \tau_{s,t}\}$ and exogenous shocks $\{\zeta_t, \psi_t\}$.

The model nests the standard New Keynesian economy: if $\vartheta \to \infty$ (intermediate varieties are perfect substitutes), or if $\chi \to \infty$ and $\phi = 1$ (so that $S_t = 0$ and A_t is constant), the growth block of the model becomes irrelevant, and the equilibrium conditions mimic those of the standard New Keynesian model.

2.6 Effects of monetary and discount rate shocks

This section shows that a first order approximation yields a tractable and intuitive representation of the model. For the remainder of the section, I assume that the subsidies $\tau_{n,t}$ and $\tau_{s,t}$ are constant and set so that the economy's balanced-growth path (BGP) is efficient. (The efficient allocation is described in Section 3).

A first-order approximation of (24) yields

$$\hat{g}_t = (1 - \overline{\phi})(\psi_t + \eta \hat{y}_t + \hat{s}_t) \tag{31}$$

where $\overline{\phi} \equiv \phi / \mathcal{G}^* \in (0, 1)$, with $\mathcal{G}^* \geq 1$ denoting the efficient growth rate. (Lowercase hatted variables denote log-deviations from BGP of the *detrended* variables defined in (23): $\hat{g}_t \equiv g_t - g \equiv$

$\log(\mathcal{G}_t) - \log(\mathcal{G})$, $\hat{y}_t \equiv y_t - y \equiv \log(\mathcal{Y}_t) - \log(\mathcal{Y})$, and so on, where a variable without time subscript indicates its value in the BGP).[3]

In log-deviations, (25) is

$$\psi_t + \hat{\gamma}_t + \eta \hat{y}_t = \varphi \hat{s}_t + \hat{y}_t \tag{32}$$

Log-linearizing (26) yields

$$\hat{\gamma}_t = -\hat{g}_t + \mathbb{E}_t\left\{\Delta\zeta_{t+1} - \Delta\hat{y}_{t+1}\right\} + (1-\overline{\beta})\mathbb{E}_t\{\hat{d}_{t+1}\} + \overline{\beta}\mathbb{E}_t\left\{\hat{\gamma}_{t+1}\right\}, \tag{33}$$

where $\overline{\beta} \equiv \beta\phi/\mathcal{G}^* \in (0,1)$. Finally, (27) can be written in log-deviations as

$$\hat{d}_t = \delta \hat{y}_t, \tag{34}$$

where $\delta \equiv (2+\varphi)$ captures the elasticity of dividends with respect to (detrended) output. Combining (31) and (32) to eliminate $\hat{s}_t$, and inserting the resulting expression (along with (34)) into (33), yields

$$\hat{g}_t = \mathbf{g}_t + \eta\omega\left[\hat{y}_t - \overline{\beta}\mathbb{E}_t\left\{\hat{y}_{t+1}\right\}\right] + \overline{\delta}\mathbb{E}_t\left\{\hat{y}_{t+1}\right\} + \frac{\overline{\beta}\overline{\varphi}}{1+\overline{\varphi}}\mathbb{E}_t\left\{\hat{g}_{t+1}\right\}, \tag{35}$$

where $\overline{\varphi} \equiv \varphi/(1-\overline{\phi}) > 0$, $\omega \equiv (1+\varphi)/(1+\overline{\varphi}) > 0$, $\overline{\delta} \equiv (\delta-1)(1-\overline{\beta})/(1+\overline{\varphi}) > 0$, and where

$$\mathbf{g}_t \equiv (1+\overline{\varphi})^{-1}\Big[(1+\varphi)(\psi_t - \overline{\beta}\mathbb{E}_t\left\{\psi_{t+1}\right\}) + \mathbb{E}_t(\Delta\zeta_{t+1})\Big] \tag{36}$$

depends only on the exogenous shocks ψ_t, ζ_t.

Equation (35) indicates that fluctuations in technology growth reflect (1) the direct effect of the exogenous disturbances, with higher entrepreneurial productivity (high ψ_t) or higher household patience (higher $\mathbb{E}_t\left\{\Delta\zeta_{t+1}\right\}$) leading to higher growth; (2) the impact of learning-by-doing, given by the second term in the right-hand side of (35), whereby above-trend output yields faster productivity growth; (3) the impact of higher future output on expected dividends, captured by the third term in the RHS of (35) (note that the parameter composite $\overline{\delta}$ is an increasing function of the elasticity of dividends to output, δ); and (4) expectations of future growth.

The remaining equilibrium conditions can be combined into a New Keynesian Phillips Curve (NKPC, for short) and a "dynamic IS" (DIS) equation. The model thus has a representation similar to the textbook NK economy, with an additional condition determining technology growth. Letting $i_t \equiv -\log(Q_t)$ denote the short-term nominal interest rate and $\pi_t \equiv \log(P_t) - \log(P_{t-1})$ the inflation rate, the proposition below collects the result.

Proposition 1 (New Keynesian growth model). To a first order, the model's endogenous

[3] I treat $\mathcal{G}^*$ as a parameter in much of the paper. It is always possible to calibrate the skilled labor disutility parameter χ to hit any desired target for $\mathcal{G}^*$ (see Appendix B). In the example calibration studied later, $\mathcal{G}^*$ is set to match the average rate of TFP growth in the United States.

variables $\hat{g}_t, \hat{y}_t, \pi_t$ satisfy the conditions

$$\text{(DIS)} \quad \hat{y}_t = -\left[\hat{i}_t - \mathbb{E}_t\left\{\pi_{t+1}\right\} - (1-\rho_\zeta)\zeta_t\right] + \frac{1}{(\vartheta-1)}\hat{g}_t + \mathbb{E}_t\left\{\hat{y}_{t+1}\right\}, \tag{37}$$

$$\text{(NKPC)} \quad \pi_t = \lambda(1+\varphi)\hat{y}_t + \beta\mathbb{E}_t\left\{\pi_{t+1}\right\}, \tag{38}$$

$$\text{(Growth)} \quad \hat{g}_t = \mathbf{g}_t + \eta\omega\left[\hat{y}_t - \overline{\beta}\mathbb{E}_t\left\{\hat{y}_{t+1}\right\}\right] + \overline{\delta}\mathbb{E}_t\left\{\hat{y}_{t+1}\right\} + \frac{\overline{\beta}\overline{\varphi}}{1+\overline{\varphi}}\mathbb{E}_t\left\{\hat{g}_{t+1}\right\}, \tag{39}$$

where $\lambda \equiv (1-\theta)(1-\theta\beta)/\theta$.

Proof: In Appendix A.

The above equations can be supplemented with a condition determining how the monetary policy rate i_t evolves over time to determine the economy's dynamics. In the remainder of this section the policy rate is assumed to follow a simple Taylor rule:

$$\hat{i}_t = \phi_\pi \pi_t + \nu_t, \quad \phi_\pi > 1, \tag{40}$$

where $\nu_t \sim \text{AR}(1)$ captures a monetary policy shock.

I next illustrate the model's behavior by showing the effects of monetary and discount rate shocks, given the parameter values shown in Table 1. The parameters $\beta, \varphi, \theta, \epsilon$, and ϕ_π are standard in the literature, and are set to standard values. The remaining parameters govern the growth process. The elasticity of substitution between intermediate good varieties, ϑ, is set to 3, a value much lower than its final-good counterpart—consistent with the evidence in Broda and Weinstein (2006). I set the technology survival rate, ϕ, to a value somewhat lower than found in related papers. Lower values of this parameter are associated with a greater elasticity of $\hat{g}_t$, as made clear by (31). I set a relatively low value for this parameter to partly compensate for the absence of endogenous technology diffusion lags, which (as emphasized by Comin and Gertler (2006)) make aggregate technology more cyclical. The learning-by-doing spillover η, for which there are no readily available estimates, is set so that the model's implications for the path of aggregate TFP following a monetary shock are roughly consistent with the empirical evidence—which suggests a medium-run effect on TFP of between a third and a sixth of the near-term effect on GDP, as shown later. All shock autoregressive parameters are set to 0.5, consistent with a moderate amount of persistence. I emphasize, however, that the results I describe below qualitatively hold in general, and are not dependent on the specific calibration.

The first experiment is a 0.25 percent increase in the monetary shock ν_t (which would result in a 100 basis point increase in the policy rate absent any endogenous reaction of inflation), shown in Figure 1. The resulting increase in real rates drives down inflation and the output gap, as in the standard NK model. The current and lower expected future output gaps, in turn, reduce the technology growth rate $\hat{g}_t$. This leads to a permanent loss in aggregate TFP, and to an output level that is permanently depressed relative to its pre-shock trend.

These effects of monetary shocks are qualitatively consistent with the empirical evidence. Figure

Table 1: Parameter Values

Symbol	Value	Description
β	0.99	Household discount factor
φ	2	Inverse Frisch labor supply elasticity
θ	0.75	Probability of keeping price fixed
ϵ	9	Elasticity of substitution between final goods varieties
ϑ	3	Elasticity of substitution between intermediates varieties
ϕ	0.925	Survival rate of varieties
$\mathcal{G}^*$	$1.02^{(\vartheta-1)/4}$	TFP growth rate along BGP (back out χ)
η	1.5	Learning-by-doing spillover
ϕ_π	1.5	Response to inflation in Taylor rule
ρ_ν	0.5	Persistence of monetary shock
ρ_ζ	0.5	Persistence of discount rate shock
ρ_ψ	0.5	Persistence of innovation shock

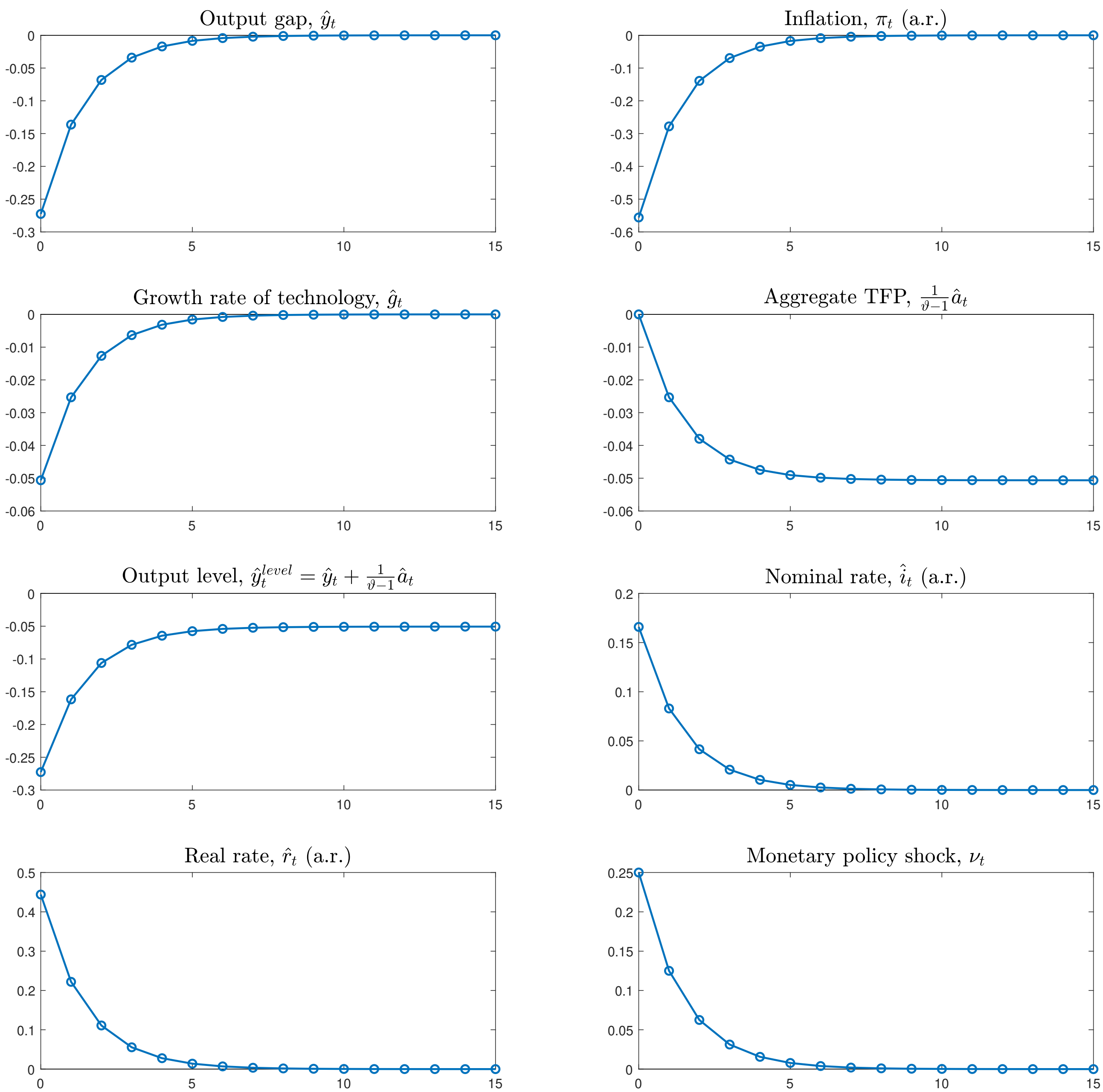
2 shows the effects of a one-standard-deviation increase in the Federal Funds rate, obtained from a VAR that includes, in additional to a standard set of macroeconomic variables, private-sector R&D expenditure (a proxy for innovation), and utilization-adjusted TFP from Fernald (2014). A tightening in U.S. monetary policy slows R&D and leads to a persistent loss in TFP. Forty quarters out, GDP is still significantly below its pre-shock path, by an amount similar to the shortfall in TFP. These effects are consistent with those found by Jordà et al. (2020), who use a different methodology (relying on the policy "trilemma" in open economies) to identify the long-run effects of monetary policy.

The second experiment is a decrease in the discount rate ζ_t of size 0.5, which given $\rho_\zeta = 0.5$, involves a decrease in the $(1-\rho_\zeta)\zeta_t$ term in equation (37) of 25 basis points—similar to the size of the monetary shock studied earlier. The resulting dynamics are shown in Figure 3. The shock decreases the output gap, given consumers' reduced desire for current consumption. The effects on the output gap and inflation are very close to those seen in the monetary shock. As in that case, lower current and expected output gap impacts TFP growth negatively, and the economy experiences a permanent productivity and output loss. The slowdown in productivity growth, however, is somewhat smaller than in the case of a monetary shock. The reason is the presence in (39) of the term $\mathbf{g}_t$, which increases following a lower ζ_t.

2.7 A "gap" representation

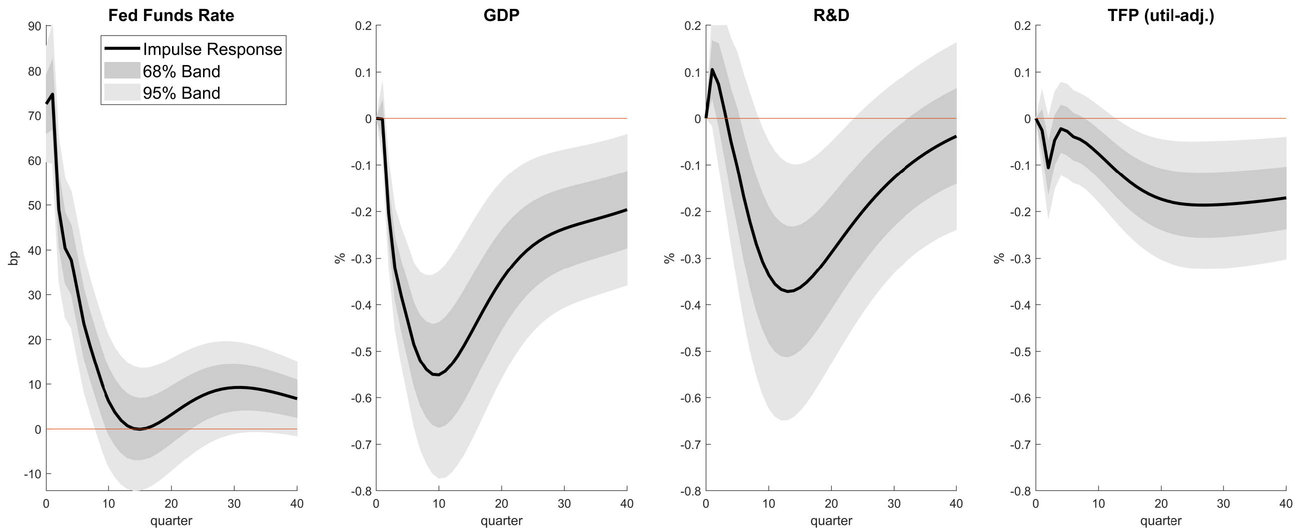
The equilibrium conditions (37)-(39) can be expressed in terms of deviations from the flexible-price (or "natural") allocation, denoted with superscript n. Let $\tilde{x}_t = x_t - x_t^n$ for any variable x_t. The flexible-price allocation features a constant level of (detrended) output, as made clear by combining

Figure 1: Dynamic responses to a monetary policy shock



Note: Impulse responses to a positive monetary shock ν_t. Responses of inflation, the nominal rate, and the real rate have been multiplied by 4 so they are expressed in annual terms.

Figure 2: VAR-based effects of a monetary shock



Note: Dynamic effects of shock to Federal funds rate from an identified VAR. Source: Moran and Queralto (2018).

(28) and (29) given a constant real marginal cost. Thus, $\tilde{y}_t = \hat{y}_t$. We can write (37) as

$$\tilde{y}_t = -\left[i_t - \mathbb{E}_t\left\{\pi_{t+1}\right\} - r_t^n\right] + \frac{1}{(\vartheta - 1)}\tilde{g}_t + \mathbb{E}_t\left\{\tilde{y}_{t+1}\right\}, \tag{41}$$

where $r_t^n \equiv \log(\frac{1}{\beta}) + (1-\rho_\zeta)\zeta_t + \frac{1}{(\vartheta-1)}g_t^n$ is the natural rate of interest (the rate that would prevail in an economy with flexible prices), with $\frac{1}{(\vartheta-1)}g_t^n$ capturing the rate of TFP growth in the flexible-price equilibrium. The latter variable satisfies $\hat{g}_t^n = \mathbf{g}_t + \beta_\varphi \mathbb{E}_t\left\{\hat{g}_{t+1}^n\right\}$, $\beta_\varphi \equiv \frac{\overline{\beta}\overline{\varphi}}{1+\overline{\varphi}}$, which can be used along with (36) to solve for g_t^n solely as a function of the exogenous shocks: $g_t^n = \log(\mathcal{G}^*) + \kappa_\psi \psi_t - \kappa_\zeta \zeta_t$, with $\kappa_\psi, \kappa_\zeta > 0$.[4]

The NKPC can be written in terms of the output gap,

$$\pi_t = \lambda_\varphi \tilde{y}_t + \beta \mathbb{E}_t\left\{\pi_{t+1}\right\}, \tag{42}$$

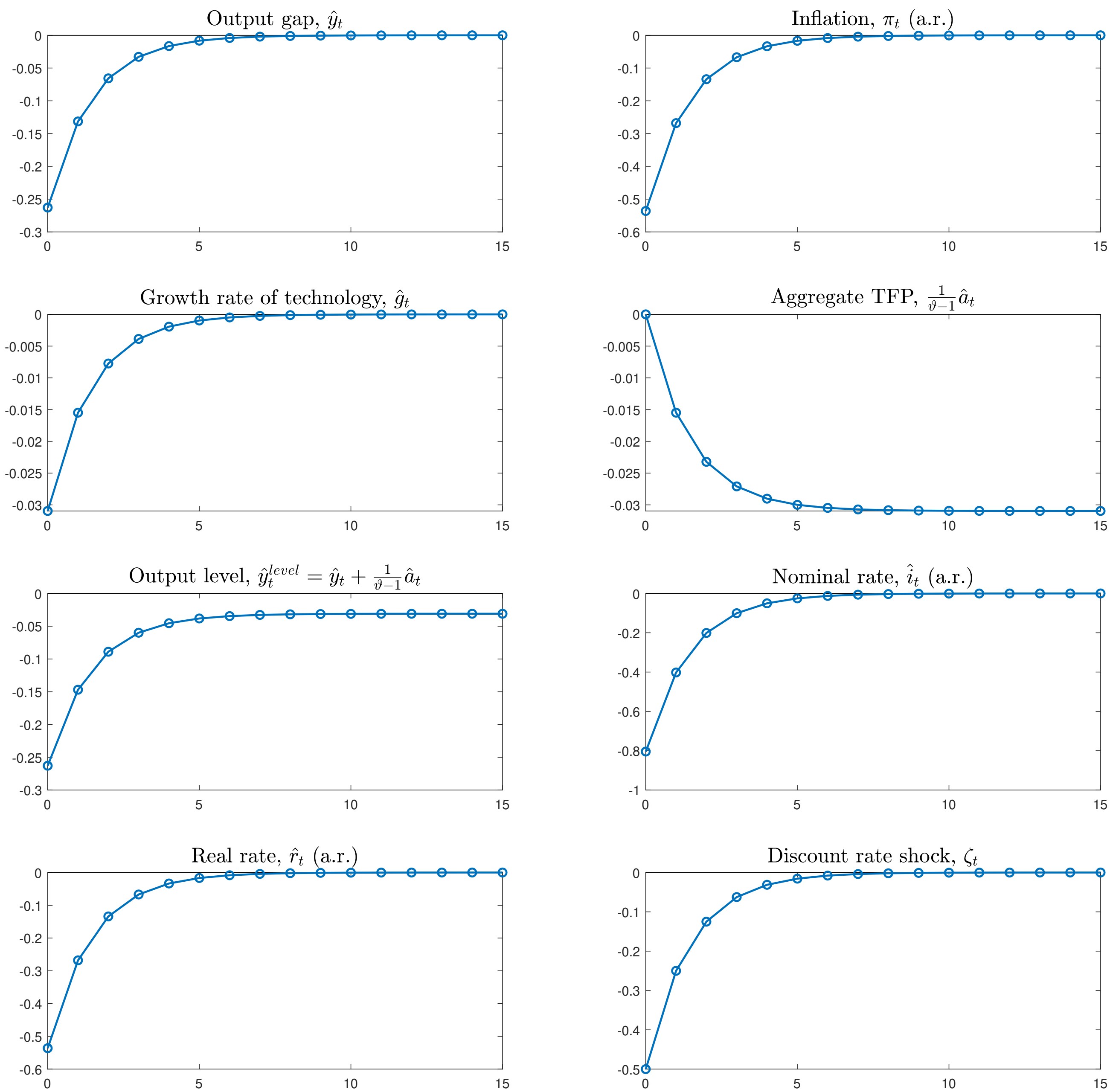
$\lambda_\varphi \equiv \lambda(1+\varphi)$. Finally, the *growth gap* $\tilde{g}_t$ satisfies

$$\tilde{g}_t = \eta\omega\left[\tilde{y}_t - \overline{\beta}\mathbb{E}_t\left\{\tilde{y}_{t+1}\right\}\right] + \overline{\delta}\mathbb{E}_t\left\{\tilde{y}_{t+1}\right\} + \beta_\varphi \mathbb{E}_t\left\{\tilde{g}_{t+1}\right\}. \tag{43}$$

We thus have three equations for the three variables $\tilde{y}_t, \pi_t, \tilde{g}_t$. The natural rate of interest r_t^n captures the influence on the system of the two exogenous shocks ζ_t, ψ_t.

[4]The expressions for coefficients κ_ψ, κ_ζ are $\kappa_\psi \equiv \frac{(1+\varphi)(1-\overline{\beta}\rho_\psi)}{(1+\overline{\psi})(1-\beta_\varphi\rho_\psi)}$, $\kappa_\zeta = \frac{(1-\rho_\zeta)}{(1+\overline{\varphi})(1-\beta_\varphi\rho_\zeta)}$.

Figure 3: Dynamic responses to a discount rate shock under a Taylor rule



Note: Impulse responses to a negative discount rate shock ζ_t. Responses of inflation, the nominal rate, and the real rate have been multiplied by 4 so they are expressed in annual terms.

3 The Efficient Allocation

The efficient allocation associated with this economy (henceforth denoted using *) can be determined by solving the problem facing a social planner that seeks to maximize household welfare. This allocation has the following features. First, the planner sets $C_t^*(i) = C_t^*(j)$ for any pair $i, j \in [0,1]$: the assumed form of the consumption index C_t implies that it is optimal to produce and consume the same quantity of all final goods. Second, the path of the aggregate variables $\left\{Y_t^* = C_t^*, N_t^*, S_t^*, A_{t+1}^*\right\}$ maximizes household utility given technological constraints, and therefore solves the following problem:

$$\max_{\left\{Y_t^*, N_t^*, S_t^*, A_{t+1}^*\right\}} \quad \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t e^{\zeta_t} \left(\log(Y_t^*) - \frac{N_t^{*1+\varphi}}{1+\varphi} - \frac{\chi S_t^{*1+\varphi}}{1+\varphi} \right) \tag{44}$$

subject to

$$Y_t^* = A_t^{*\frac{1}{\vartheta-1}} N_t^*, \tag{45}$$

$$A_{t+1}^* = \phi A_t^* + e^{\psi_t} A_t^* N_t^{*\eta} S_t^*. \tag{46}$$

It is possible to show that the problem (44)-(46) has a unique solution that satisfies the second-order conditions for a maximum. In addition, the first-order dynamics around the BGP can be easily characterized. The following Proposition collects the result.

Proposition 2 (Planner's problem). For low enough χ, there exists a unique solution to the social planner's problem with positive growth along the BGP. To a first order, the solution's local dynamics around the BGP satisfy

$$\hat{s}_t^* = -\frac{(1-\rho_\zeta)}{(1-\beta\rho_\zeta)\Lambda_s}\zeta_t + \frac{\overline{\phi}}{\Lambda_s}\psi_t \tag{47}$$

$$\hat{n}_t^* = \hat{y}_t^* = \Lambda_{ns}\hat{s}_t^* \tag{48}$$

where $\Lambda_{ns} \equiv \frac{\eta\kappa(1-\frac{\phi}{\mathcal{G}^*})}{1+\eta\kappa(1-\frac{\phi}{\mathcal{G}^*})} \in (0,1)$, $\Lambda_s \equiv 1 + \varphi - \frac{\phi}{\mathcal{G}^*}(1+\eta\Lambda_{ns}) > 0$, with $\kappa \equiv \frac{\beta}{(1-\beta)(\vartheta-1)} > 0$, and where $\mathcal{G}^* > 1$ is the efficient (gross) growth rate along the BGP.

Proof: In Appendix B.

Thus optimal innovation effort, as measured by the use of skilled labor $\hat{s}_t$, increases when innovation productivity ψ_t rises, and when the discount rate shock ζ_t falls (leading households to become more patient). Because fluctuations in growth $\hat{g}_t$ are increasing in $\hat{s}_t$, the same observations apply to technology growth. In addition, optimal goods labor $\hat{n}_t$ (and therefore detrended output $\hat{y}_t$) co-moves positively with innovation labor, with the magnitude of the effect stronger the larger the learning-by-doing spillover η.

It is useful to describe the externalities present in the model. These externalities arise due to the knowledge and learning-by-doing spillovers embedded in (18). Consider the value of an innovation for the planner, in contrast to that facing innovators in the decentralized economy (assuming fully flexible prices and zero subsidies):

$$\text{Planner: } \mathbf{\Gamma}_t^* = \mathbb{E}_t \left\{ \frac{\beta e^{\zeta_{t+1}-\zeta_t} \mathcal{Y}_t^*}{\mathcal{G}_t^* \mathcal{Y}_{t+1}^*} \Big[\mathcal{D}_{t+1}^* + \mathbf{\Gamma}_{t+1}^* \left(\phi + e^{\psi_{t+1}} N_{t+1}^{*\eta} S_{t+1}^* \right) \Big] \right\}. \tag{49}$$

$$\text{Market: } \mathbf{\Gamma}_t = \mathbb{E}_t \left\{ \frac{\beta e^{\zeta_{t+1}-\zeta_t} \mathcal{Y}_t}{\mathcal{G}_t \mathcal{Y}_{t+1}} \Big[\mathcal{D}_{t+1} + \phi \mathbf{\Gamma}_{t+1} \Big] \right\}. \tag{50}$$

Above, the term highlighted in blue is present in the planner's problem, but absent in the decentralized allocation. This term captures the knowledge spillover: the planner internalizes the fact that an additional innovation today enhances entrepreneurs' future ability to create new products.

Consider next the optimality condition for goods labor N_t:

$$\text{Planner: } \quad \mathcal{Y}_t^* N_t^{*\varphi} = 1 + \eta \mathbf{\Gamma}_t^* e^{\psi_t} N_t^{*\eta - 1} S_t^*. \tag{51}$$

$$\text{Market: } \frac{\epsilon}{(\epsilon-1)} \frac{\vartheta}{(\vartheta-1)} \mathcal{Y}_t N_t^{\varphi} = 1. \tag{52}$$

Thus, there are two sources of inefficiency in the market equilibrium's condition for N_t. The first is a static distortion due to monopoly power on the part of final goods and intermediate goods firms, whereby the marginal product of labor (equal to unity in detrended terms) is set to a markup $\frac{\epsilon}{(\epsilon-1)} \frac{\vartheta}{(\vartheta-1)}$ over its social marginal cost (given by the household's marginal rate of substitution). The second distortion arises due to learning-by-doing: to the extent that $\eta > 0$, higher goods production enhances innovation productivity—an effect also not internalized in the decentralized equilibrium.

The knowledge and the learning-by-doing spillovers both imply that S_t and N_t in the market economy are inefficiently low. They also imply that the gap between the optimal and the market levels of these variables are generally *time-varying*: the magnitude of the externalities is larger whenever the planner wants to set faster current and future growth, as in those times the value of both knowledge and learning spillovers are larger. This occurs whenever ψ_t rises or ζ_t falls.

These observations imply that there is a rationale for policy interventions to improve on the market allocation. I turn to this issue next.

4 State-Contingent Labor Subsidies

It is useful to first consider the case in which the government can set the labor subsidies $\{\tau_{s,t}, \tau_{n,t}\}$ in a time-varying manner (and finance them via lump-sum taxes). We have the following result.

Proposition 3 (Optimal subsidies). The efficient allocation can be decentralized by means of time-varying labor subsidies $\{\tau_{s,t}, \tau_{n,t}\}$ and strict inflation targeting ($\pi_t = 0$ for all t). The required labor subsidies are positive. If there are no learning-by-doing spillovers ($\eta = 0$), the optimal goods labor subsidy $\tau_{n,t}$ is constant, and the innovation labor subsidy $\tau_{s,t}$ increases when ψ_t rises and when ζ_t falls. If there are learning-by-doing spillovers, both subsidies increase when ψ_t rises and when ζ_t falls.

Proof: In Appendix C.

Thus the optimal labor subsidies are adjusted upward when innovation productivity is high or when households become more patient, as hinted by the discussion in the preceding section. The reason is that in those instances, the values of the knowledge and learning spillovers rise. In this decentralization, the monetary regime consists of strict inflation targeting: inflation is set to its target (zero) at all times. This implies a constant final goods price level, ensuring the optimality of the consumption allocation across final good types.

As a corollary of the above result, suppose that the subsidies are restricted to be constant, and the central bank continues to practice strict inflation targeting ($\pi_t = 0$) at all times. Suppose further that the subsidies are at their optimal level absent any shocks. Then, if ζ_t declines, or if ψ_t rises, the subsidies will fall short of the externality, and thus the labor inputs S_t and N_t will be inefficiently low. This creates an incentive for the monetary authority to deviate from full price stability. We turn to this issue next.

5 Optimal Monetary Policy with Constant Subsidies

Suppose that τ_n, τ_s are constant and set so that the BGP of the market economy is efficient. I also restrict attention to discount rate shocks, so in what follows $\psi_t = 0$ for all t. Under these assumptions, the following result obtains.

Proposition 4 (Approximate welfare loss). A second-order approximation to the consumer's welfare losses relative to the BGP can be expressed as a fraction of BGP consumption (up to additive terms independent of policy) as

$$\mathbb{W} = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\epsilon}{\lambda}\pi_t^2 + (1+\varphi)\hat{y}_t^2 + \kappa \Big\{ (1+\overline{\varphi})\, \hat{g}_t^2 + 2\frac{1-\rho_\zeta}{1-\beta\rho_\zeta}\zeta_t \hat{g}_t + \eta(1+\varphi)\big[(1+\eta)(1-\overline{\phi})\hat{y}_t^2 - 2\hat{g}_t\hat{y}_t\big] \Big\} \right\} \tag{53}$$

where $\kappa \equiv \frac{\beta}{(1-\beta)(\vartheta-1)} > 0$, $\overline{\phi} \equiv \frac{\phi}{\mathcal{G}^*} \in (0,1)$, $\overline{\varphi} \equiv \frac{\varphi}{1-\overline{\phi}} > 0$.

Proof: In Appendix D.

The first two terms in the loss function, $\frac{\epsilon}{\lambda}\pi_t^2 + (1+\varphi)\hat{y}_t^2$, are identical to those in the standard NK model. The term in brackets multiplying κ arises due to the presence of endogenous growth. Thus, the household dislikes volatility in the growth rate $\hat{g}_t$, as it is associated with more-volatile consumption and skilled labor. Negative co-movement between ζ_t and $\hat{g}_t$ improves welfare, as future consumption is more valuable (and therefore growth more desirable) when ζ_t is more negative. With learning-by-doing ($\eta > 0$), there is an additional penalty to output volatility, due to the fact that now $\hat{y}_t = \hat{n}_t$ also impacts growth dynamics directly. Finally, also to the extent that there is learning-by-doing, positive co-movement between $\hat{g}_t$ and $\hat{y}_t$ also improves welfare.

5.1 Optimal monetary policy under discretion

I next analyze the optimal monetary policy assuming the central bank cannot commit itself to any future actions. The policy problem in this case consists in solving

$$\min_{\pi_t,\hat{y}_t,\hat{g}_t} \quad \frac{\epsilon}{\lambda}\pi_t^2 + (1+\varphi)\hat{y}_t^2 + \kappa\Big\{ (1+\overline{\varphi})\,\hat{g}_t^2 + 2\frac{1-\rho_\zeta}{1-\beta\rho_\zeta}\zeta_t\hat{g}_t + \eta(1+\varphi)\big[(1+\eta)(1-\overline{\phi})\hat{y}_t^2 - 2\hat{g}_t\hat{y}_t\big]\Big\} \tag{54}$$

subject to (38) and (39), with the conditional expectations $\mathbb{E}_t\left\{\hat{\pi}_{t+1}\right\}, \mathbb{E}_t\left\{\hat{y}_{t+1}\right\}, \mathbb{E}_t\left\{\hat{g}_{t+1}\right\}$ in (38)-(39) taken as given. By combining the first-order conditions of the corresponding Lagrangian and eliminating multipliers, it can be shown that the condition

$$(1+\eta_y)\,\hat{y}_t + \epsilon\pi_t = -\frac{\kappa\eta}{1+\frac{\varphi}{1-\overline{\phi}}}\frac{1-\rho_\zeta}{1-\beta\rho_\zeta}\zeta_t \tag{55}$$

must hold, where $\eta_y \equiv \eta\kappa(1-\overline{\phi})\left[\frac{1+\varphi-(1+\eta)\overline{\phi}}{1-\overline{\phi}+\varphi}\right] > 0$. Plugging this condition in the NKPC, it follows that

$$\pi_t = -\Lambda_{\pi\zeta}\zeta_t,$$

with $\Lambda_{\pi\zeta} > 0$. Thus, the central bank allows inflation to run above target whenever $\zeta_t < 0$.

The blue circled line in Figure 4 illustrates the resulting dynamics following a decline in ζ_t. By letting the output gap $\hat{y}_t$ turn positive, the central bank is able to boost productivity growth. As a result, the path of (log) TFP, $\frac{1}{(\vartheta-1)}\hat{a}_t$, is closer to the one that obtains in the efficient allocation (the green line with diamonds) than if the central bank followed a strict inflation targeting policy ($\pi_t = 0$ for all t), shown by the yellow crossed line. The latter would be the optimal policy if productivity dynamics were exogenous, as in the standard NK model.

As Figure 4 makes clear, monetary policy alone is not sufficient to attain the efficient allocation. There is a tension between the two features of the efficient allocation described earlier: the desire to eliminate dispersion in the production of final goods on the one hand, and the desire to correct for the growth externalities on the other. In Figure 4, the optimal policy allows for some inflation (resulting in some inefficient price dispersion) to partly correct for the growth externalities, thereby making the path of TFP closer to its efficient counterpart.

Observe that absent learning-by-doing spillovers, the optimal policy under discretion is strict inflation targeting: combining (55) (given $\eta = 0$) with the NKPC, it follows that $\pi_t = \hat{y}_t = 0$ for all t. Thus, a discretionary central bank facing a persistent decrease in ζ_t will not provide monetary stimulus, even if such a decrease makes the knowledge spillover larger, and is therefore associated with inefficiently low productivity growth. The reason is that a central bank operating under discretion has no effective means of boosting productivity growth (even if it is desirable to do so), because $\eta = 0$ implies that productivity growth in period t is unaffected by the period-t output gap $\hat{y}_t$, and instead depends only on future expected output gaps.[5] The central bank cannot credibly commit to increase future output gaps to raise current productivity growth, because the required inflation is ex-post undesirable. We return to this issue in Section 5.4, which studies the optimal policy problem under commitment.

5.2 Optimal policy under the zero lower bound

I next study optimal discretionary policy under the zero lower bound (ZLB). The goal is to show how a shortfall in consumption demand, triggered by the combination of a decline in households' discount rate and a central bank that is constrained by the ZLB, may result in a permanent TFP loss. In addition, I also illustrate how it may be desirable to run above-target inflation after a ZLB spell, even for a central bank that operates under discretion.

Consider the following experiment. At $t = 0$, the natural rate r_t^n in (41) falls to $K < 0$ for t_K periods, due to decline in the discount rate ζ_t. From t_K onward, r_t^n recovers gradually:

$$r_t^n - i = \rho_\zeta (r_{t-1}^n - i).$$

The path of r_t^n is fully anticipated by all agents as of $t = 0$. This experiment follows the one analyzed in Chapter 5 of Galí (2015), with the only difference consisting of the additional "recovery" phase in which the natural rate slowly returns to its long-run value (rather than immediately reverting back after t_K periods, as in Galí (2015)).

The optimal discretionary policy takes the following form. From t_K onward, set

$$\pi_t = -\Lambda_{\pi\zeta}\zeta_t$$

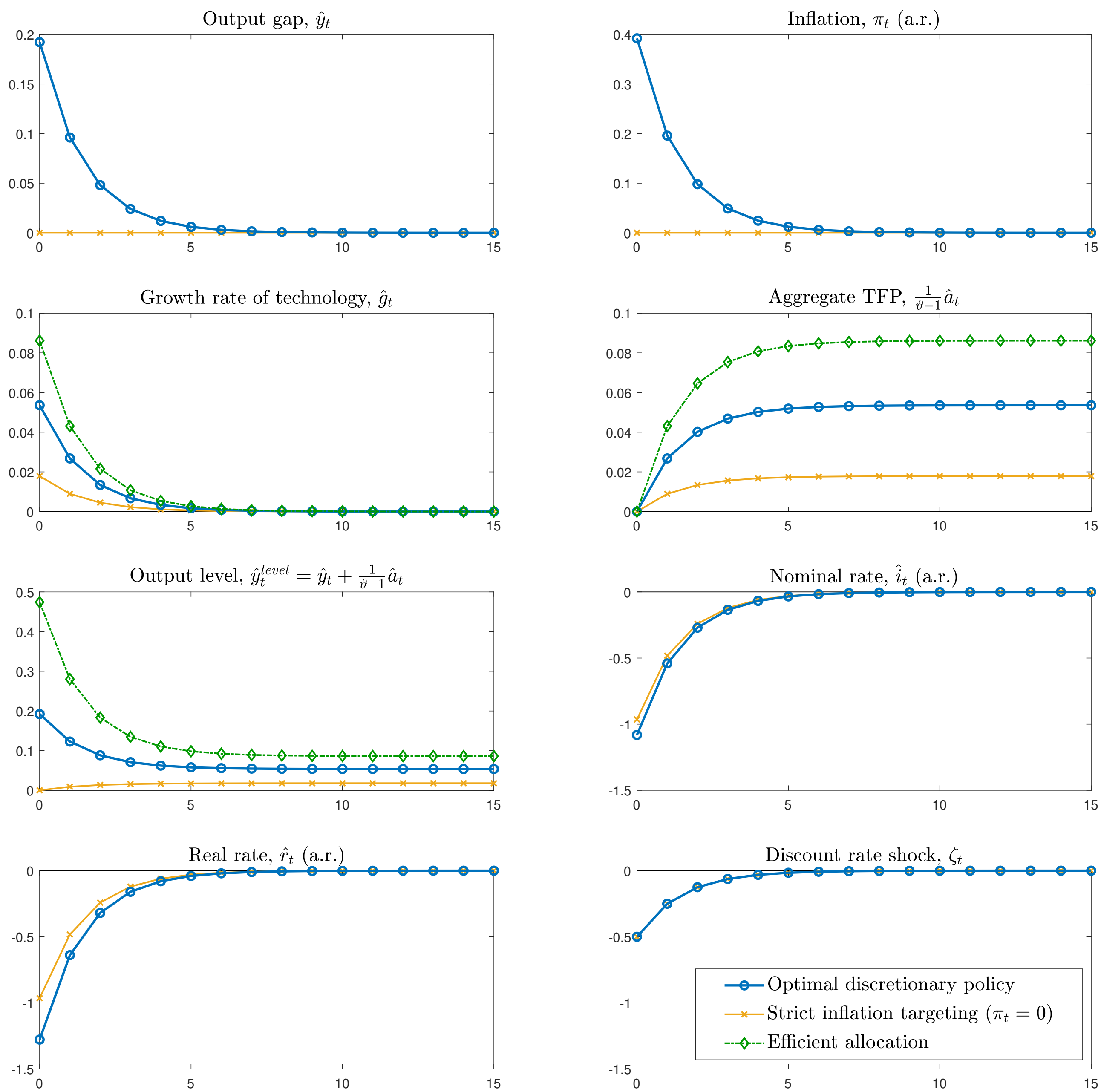
if feasible; otherwise set $i_t = 0$. For $t < t_K$, set $i_t = 0$.

Figure 5 illustrates the resulting dynamics, with $t_K = 5$ and $K = 5$ percent annually. Inflation runs above target for about for several quarters quarters after its initial decline, resulting from a positive output gap starting around the time the policy rate lifts off the ZLB. This allows for some recovery in aggregate TFP. Still, the latter variable exhibits a permanent decline, in stark contrast to what would occur in the case without the ZLB, which features an *increase* in TFP under the optimal policy following a decline in ζ_t (as made clear by Figure 4).

It is useful to contrast the above results with the resulting dynamics if the central bank follows a strict inflation targeting policy when outside the ZLB, shown by the yellow crossed line in Figure 6. The magnitude of the permanent decline in TFP is much larger in that case, as are the paths of

[5]Note, however, that this would not be the case if the output gap featured some form of endogenous inertia, for example due to habits in consumption.

Figure 4: Dynamic responses to discount rate shock: optimal discretionary policy



Note: Impulse responses to a negative discount rate shock ζ_t under the optimal discretionary policy (blue circled line), under strict inflation targeting (yellow line with crosses), and under the efficient allocation (green line with diamonds). Responses of inflation, the nominal rate, and the real rate have been multiplied by 4 so they are expressed in annual terms.

output and inflation, compared with the optimal discretionary policy (the blue circled line): when the shock hits, agents recognize that the optimal discretionary policy will involve some degree of accommodation in the future, while this is not the case under the strict inflation targeting regime.

5.3 Cost-push shocks

How does the presence of endogenous growth affect the central bank's trade-off in the face of inflationary pressures? To address this question, let the NKPC be modified to allow for a *cost-push* shock, denoted u_t:

$$\pi_t = \lambda(1+\varphi)\hat{y}_t + \beta\mathbb{E}_t\left\{\pi_{t+1}\right\} + u_t \tag{56}$$

The shock u_t could result, for example, from an increase in in final goods firms' desired markups, resulting from (exogenous) downward variation in the substitution elasticity ϵ.

The optimal discretionary policy is now characterized by the first-order condition

$$(1+\eta_y)\hat{y}_t = -\epsilon\pi_t, \tag{57}$$

where $\eta_y > 0$.

Compare the previous solution to the (well-known) condition in the textbook NK model, denoted with superscript ^{nk}, and given by

$$\hat{y}_t^{nk} = -\epsilon\pi_t^{nk}. \tag{58}$$

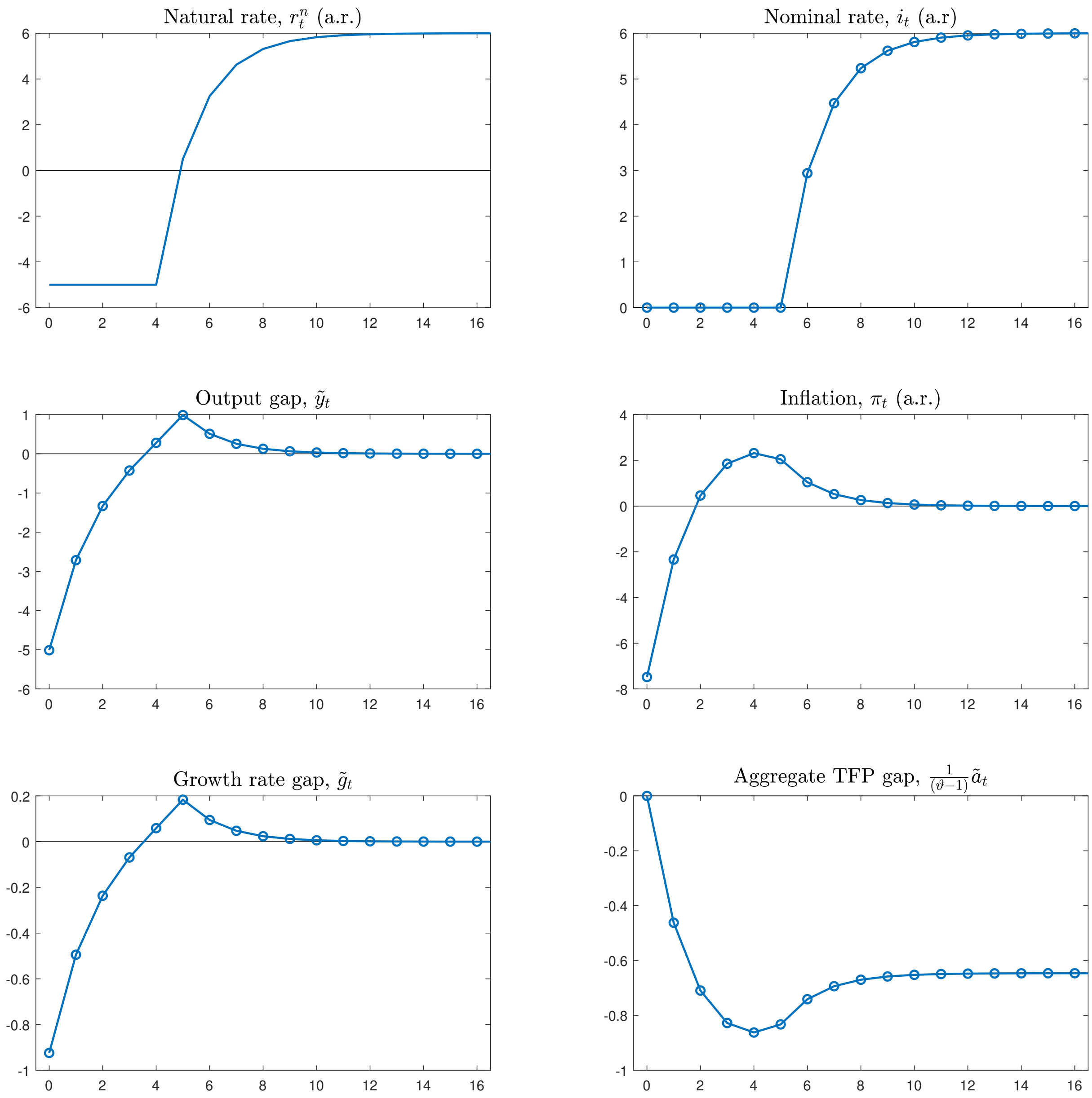
Thus, $\pi_t > \pi^{nk}$: the optimal discretionary policy allows for higher inflation than in the textbook NK case (in which productivity dynamics are exogenous). The reason is that in the model with endogenous growth, the decline in the output gap that is required to contain inflation has additional costs, as it implies a lower rate of productivity growth.

Figure 7 illustrates the result conditional on a transitory increase in u_t. The drop in the output gap is smaller, and the increase in inflation accordingly larger, under the optimal policy (the blue lines), compared with the case in which the central bank treats productivity growth as exogenous (and thus runs the policy (58)). The economy experiences a smaller TFP loss as a result.

5.4 Gains from commitment

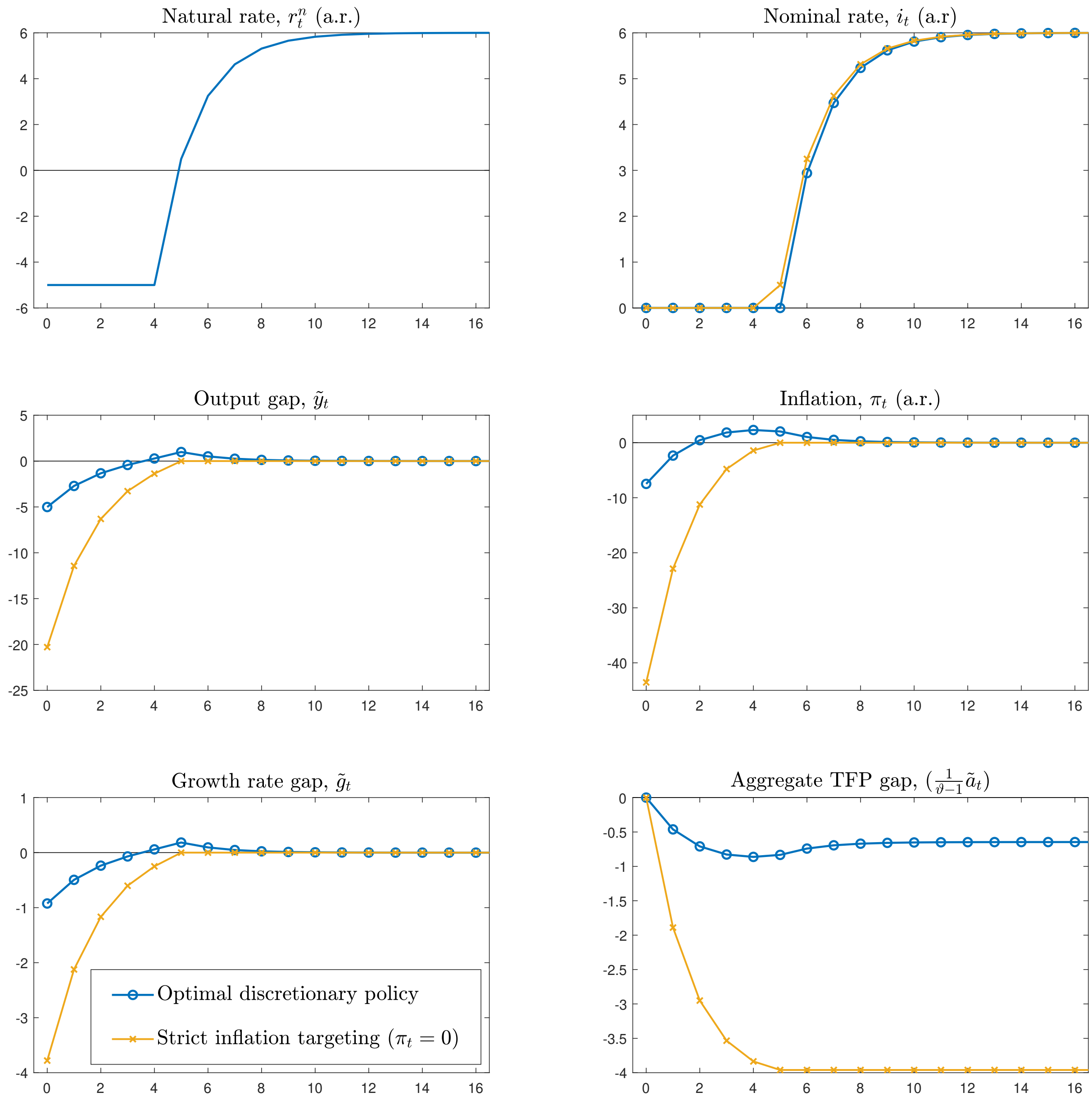
I have focused so far on optimal monetary policy under discretion. This section turns to the case in which the central bank is able to commit to future policy actions. The analysis reveals that the ability to commit affords the central bank an additional means to influence productivity growth, namely, the ability to affect expectations of future economic activity—which, through their impact on the expected profitability of innovations, exert a positive influence on current innovation activity, and therefore on current productivity growth.

Figure 5: Optimal discretionary policy in the presence of a ZLB



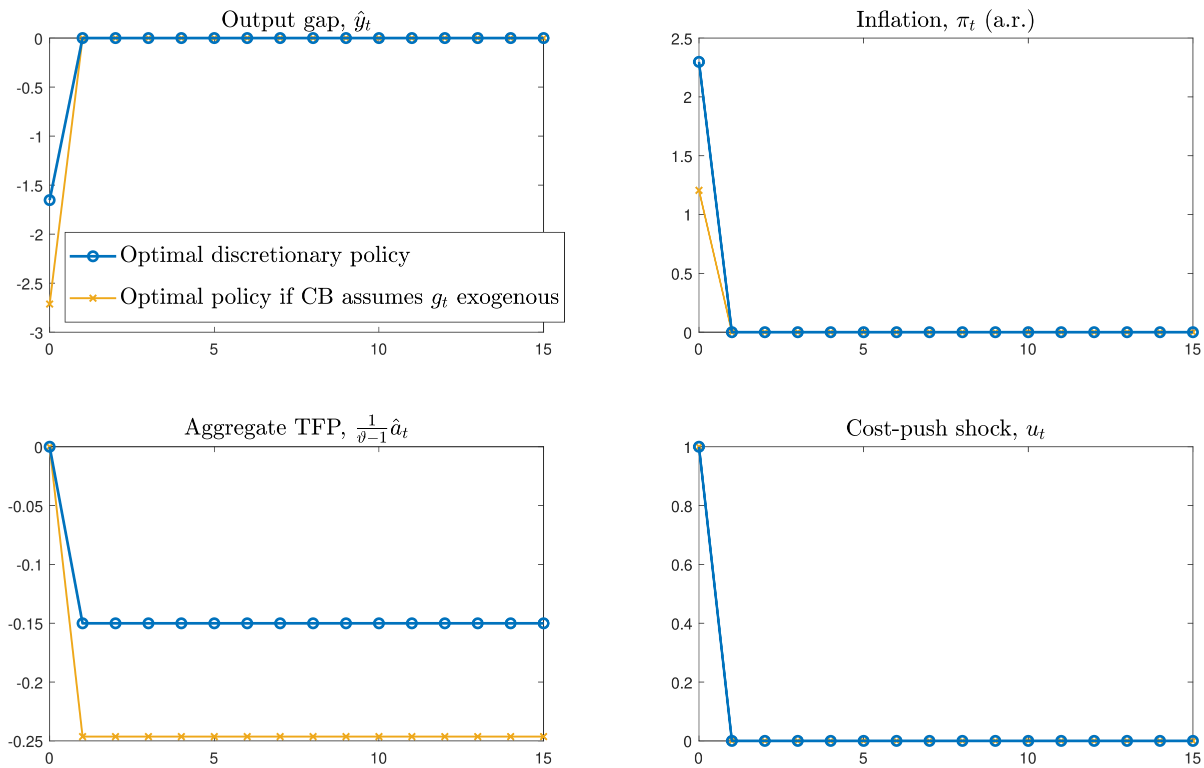
Note: Effects of a decrease in the natural rate (driven by a lower discount rate ζ_t) under the optimal discretionary monetary policy in the presence of a ZLB constraint.

Figure 6: Optimal discretionary policy in the presence of a ZLB v. strict inflation targeting



Note: Effects of a decrease in the natural rate (driven by a lower discount rate ζ_t) under strict inflation targeting (yellow line with crosses) and under the optimal discretionary policy (blue line with circles) in the presence of a ZLB constraint.

Figure 7: Dynamic responses to a cost-push shock



Note: Effects of a cost-push shock u_t under the optimal discretionary policy (blue circled line) and under the optimal policy if the central bank takes productivity growth g_t as exogenous (yellow line with crosses).

To illustrate this point, I now assume that $\eta = 0$, that is, that learning-by-doing spillovers are absent. As discussed in Section 5.1, this assumption implies that the optimal policy under discretion features $\pi_t = \hat{y}_t = 0$ at all times, just as in the standard NK framework—precisely because the absence of a contemporaneous effect of $\hat{y}_t$ on $\hat{g}_t$ implies that a discretionary central bank effectively has no means of influencing productivity growth. I also assume that the obsolescence parameter ϕ is equal to zero, implying that new innovations last for only one period. While not essential, this assumption simplifies the problem and makes its solution more transparent.

Under the simplifying assumptions made above, the household's welfare losses are

$$\mathbb{W} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2}\frac{\epsilon}{\lambda}\pi_t^2 + \frac{(1+\varphi)}{2}\hat{y}_t^2 + \frac{\kappa\,(1+\varphi)}{2}\hat{g}_t^2 + \kappa\frac{(1-\rho_\zeta)}{(1-\beta\rho_\zeta)}\zeta_t\hat{g}_t \right\}, \tag{59}$$

with $\kappa \equiv \frac{\beta}{(1-\beta)(\vartheta-1)}$. A central bank under commitment chooses a state-contingent sequence

$\{\pi_t, \hat{y}_t, \hat{g}_t\}_{t=0}^{\infty}$ that minimizes (59) subject to

$$\pi_t = \lambda(1+\varphi)\hat{y}_t + \beta\mathbb{E}_t\{\pi_{t+1}\}, \tag{60}$$

$$\hat{g}_t = \frac{-(1-\rho_\zeta)}{1+\varphi}\zeta_t + \mathbb{E}_t\{\hat{y}_{t+1}\}. \tag{61}$$

The associated Lagrangian is given by

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \frac{1}{2}\frac{\epsilon}{\lambda}\pi_t^2 + \frac{(1+\varphi)}{2}\hat{y}_t^2 + \frac{\kappa\,(1+\varphi)}{2}\hat{g}_t^2 + \kappa\frac{(1-\rho_\zeta)}{(1-\beta\rho_\zeta)}\zeta_t\hat{g}_t$$
$$+\,\xi_{\pi,t}\big[\pi_t - \lambda(1+\varphi)\hat{y}_t - \beta\pi_{t+1}\big]$$
$$+\xi_{g,t}\big[\hat{g}_t + \frac{(1-\rho_\zeta)}{1+\varphi}\zeta_t - \hat{y}_{t+1}\big]\Big\}, \tag{62}$$

where $\{\xi_{\pi_t}, \xi_{g,t}\}_{t=0}^{\infty}$ is a sequence of Lagrange multipliers associated with the constraints (60)-(61).

Differentiating the Lagrangian with respect to π_t, $\hat{y}_t$, and $\hat{g}_t$ yields

$$\frac{\epsilon}{\lambda}\pi_t + \xi_{\pi,t} - \xi_{\pi,t-1} = 0, \tag{63}$$

$$(1+\varphi)\hat{y}_t - \lambda(1+\varphi)\xi_{\pi,t} - \beta^{-1}\xi_{g,t-1} = 0, \tag{64}$$

$$\kappa(1+\varphi)\hat{g}_t + \kappa\frac{(1-\rho_\zeta)}{(1-\beta\rho_\zeta)}\zeta_t + \xi_{g,t} = 0, \tag{65}$$

for $t = 0, 1, 2, ...$, with $\xi_{\pi,-1} = \xi_{g,-1} = 0$. Combining the above conditions to eliminate multipliers yields

$$\hat{y}_t = -\Big\{\epsilon\hat{p}_t + \frac{1}{(1-\beta)(\vartheta-1)}\left[\hat{g}_{t-1} + \frac{(1-\rho_\zeta)}{(1+\varphi)(1-\beta\rho_\zeta)}\zeta_{t-1}\right]\Big\} \tag{66}$$

for $t = 0, 1, 2, ...$, with $\hat{g}_{-1} = \zeta_{-1} = 0$, and where $\hat{p}_t \equiv \log(P_t/P)$ denotes deviations of the price *level* from the BGP.

Equation (66) shows that the optimal commitment policy can be characterized by a targeting "rule" that relates the output gap inversely with the price level, as in the standard NK model (see Galí 2015, section 5.2.2), but that includes two additional terms, whereby the output gap $\hat{y}_t$ is also linked inversely with previous-period productivity growth, $\hat{g}_{t-1}$, and with the previous-period discount rate shock, ζ_{t-1}. Both terms reflect the influence of productivity growth on household welfare. The $\hat{g}_{t-1}$ term reflects that fluctuations in growth are themselves undesirable: the central bank sets lower $\hat{y}_t$ if previous-period growth $\hat{g}_{t-1}$ was high, with the goal of curbing the volatility of the latter (note from (61) that the expectation of lower $\hat{y}_t$, as of period $t-1$, works to depress $\hat{g}_{t-1}$). At the same time, lower ζ_{t-1} makes growth $\hat{g}_{t-1}$ more desirable, which can be achieved via higher expected output in period t.

Using (66) to eliminate $\hat{y}_t$ from (60) and (61) yields

$$\hat{g}_t = -\nu_\zeta \zeta_t - \nu_p \mathbb{E}_t \{\hat{p}_{t+1}\}, \tag{67}$$

$$\hat{p}_t = \delta \hat{p}_{t-1} - \delta_g \hat{g}_{t-1} - \delta_\zeta \zeta_{t-1} + \beta\delta \mathbb{E}_t \{\hat{p}_{t+1}\}, \tag{68}$$

where $\nu_\zeta \equiv \frac{(1-\rho_\zeta)}{(1+\varphi)} \frac{((1-\beta)(\vartheta-1)+(1-\beta\rho_\zeta)^{-1})}{((1-\beta)(\vartheta-1)+1)} > 0$, $\nu_p \equiv \frac{\epsilon}{(1+\frac{1}{(1-\beta)(\vartheta-1)})} > 0$, $\delta \equiv \frac{1}{(1+\beta+\lambda(1+\varphi)\epsilon)} \in (0,1)$, $\delta_g \equiv \frac{\delta\lambda(1+\varphi)}{(1-\beta)(\vartheta-1)} > 0$, and $\delta_\zeta \equiv \frac{\delta\lambda(1-\rho_\zeta)}{(1-\beta)(\vartheta-1)(1-\beta\rho_\zeta)} > 0$.

The above expressions make clear that full price stability, $\hat{p}_t = 0$ for all t, is not optimal as long as ζ_t fluctuates: if $\zeta_t \neq 0$, $\hat{p}_t = 0$ for all t is only a solution of (67)-(68) if $\nu_\zeta = \delta_\zeta / \delta_g$, which is the case only if $\rho_\zeta = 0$.[6] The system (67)-(68) suggests that a negative shock to ζ_t tends to push $\hat{g}_t$ and $\mathbb{E}_t \{\hat{p}_{t+1}\}$ up; the latter, in turn, exerts positive impact on $\hat{p}_t$, whose dynamics are partly backward-looking.

Figure 8 shows the effects of a decline in ζ_t implied by (67)-(68) (the blue circled line), as well as the effects under the optimal discretionary policy (the yellow line with crosses) and the corresponding time paths in the efficient allocation. The optimal discretionary policy, $\pi_t = \hat{y}_t = 0$, delivers a path for $\hat{g}_t$ that is well below its efficient counterpart. As a result, once the shock dies out, the level of aggregate TFP is much lower than in the first best. The central bank operating under commitment remedies this by promising a positive future output gap, which remains elevated for several periods starting in period 1 (one period after the shock hits). Such a commitment provides incentives for increased innovation already in period 0. As a result, the initial path of $\hat{g}_t$ is higher than under discretion, and closer to its efficient counterpart. The gap in aggregate TFP relative to the first best is, therefore, much lower. In period 0, the central bank can afford to let $\hat{y}_t$ turn negative, which helps curb inflation without any negative influence on innovation incentives (which depend on future but not current activity).

The fact that the output gap remains elevated is associated with inflationary pressure. Such high inflation is subsequently reversed: in later periods, starting around period 6, the output gap goes modestly below zero, implying below-zero inflation for a prolonged period—in a way that ensures that the price level eventually returns to its original value. This feature of the optimal policy under commitment is similar to that in the textbook case following a cost-push shock (see Galí 2015, section 5.2.2).

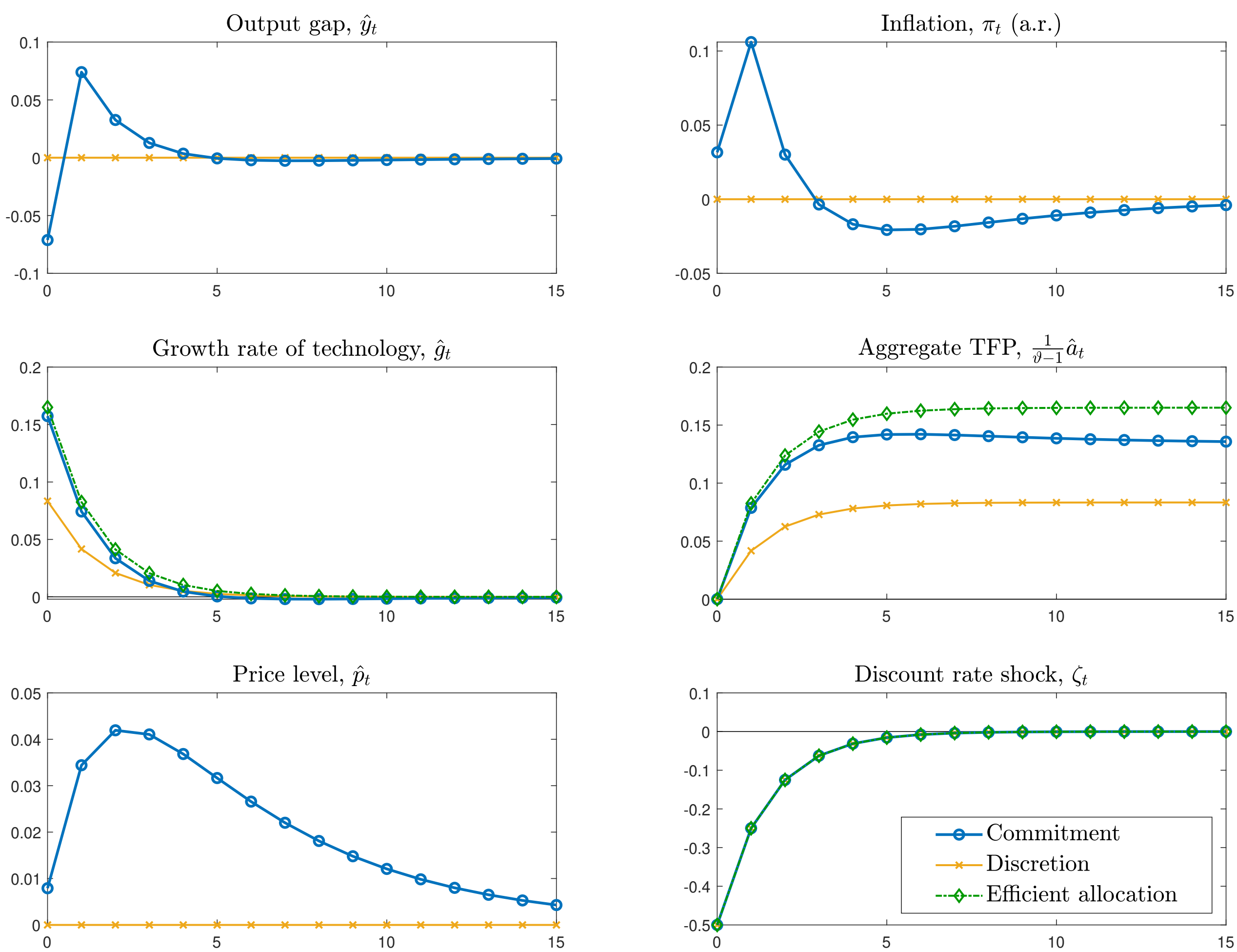
[6] If $\rho_\zeta = 0$, time variation in ζ_t does not induce time variation in the knowledge spillover—the highlighted term in (49), which captures the impact of an innovation developed in period t on entrepreneurs' productivity in period $t+1$—because all variables return to their balanced-growth-path values after one period. In this case—and only in this case—the market allocation with full price stability replicates the first best, given the presence of subsidies that render the balanced growth path efficient.

6 Conclusion

This paper develops a tractable version of the New Keynesian model augmented to allow for endogenous productivity dynamics, and uses it to analyze optimal monetary policy. Unlike textbook versions of the New Keynesian model (e.g. Galí 2015), in which strict inflation targeting emerges as the optimal policy and delivers the efficient allocation (a property known as the *divine coincidence*), such policy is generally not optimal in the economy studied here, and monetary policy alone can never deliver the efficient allocation. The optimal policy in this setting allows inflation to run above target whenever the externalities associated with productivity growth are high—which occurs, for example, when households' preferences discount future utility flows at a lower rate, or when the efficiency of the entrepreneurial sector rises. I also show that it can be optimal to run inflation above target in the recovery from a ZLB episode, even under discretion. When cost-push shocks are allowed for that induce inflationary pressure, the presence of endogenous growth tilts the central bank's incentives toward allowing for a bigger rise in inflation (and a lower drop in the output gap) compared with a model in which productivity evolves exogenously. The welfare gains from commitment include the ability of the central bank to make credible promises about future output gaps in a way that allows it to manipulate current productivity growth.

The present paper restricts attention to optimal monetary policy in the presence of subsidies that replicate the efficient allocation along the balanced growth path. In future work, it would be desirable to extend the analysis to the case of an inefficient balanced growth path.

Figure 8: Optimal monetary policy following a discount rate shock: Discretion v. Commitment



Note: Effects of a negative discount rate shock ζ_t with no learning spillovers, under the optimal policy under commitment (blue line with circles), under discretion (yellow line with crosses), and in the efficient allocation (green line with diamonds).

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Appendix

A Proof of Proposition 1

Proof. From (3), $P_t \equiv \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)P_t^*\right]^{\frac{1}{1-\epsilon}}$. Log-linearizing this condition and combining with the log-linear versions of (11) and (28) yields (38). Log-linearizing (30) yields (37). Equation (39) follows from derivations in the main text. ■

B Proof of Proposition 2

Proof. The first-order conditions associated with the planning problem (44)-(46) are as follows:

$$Y_t^* N_t^{*\varphi} = A_t^{*\frac{1}{\vartheta-1}} + \eta \Gamma_t^* e^{\psi_t} A_t^* N_t^{*\eta-1} S_t^* \tag{69}$$

$$\chi Y_t^* S_t^{*\varphi} = \Gamma_t^* e^{\psi_t} A_t^* N_t^{*\eta} \tag{70}$$

$$\Gamma_t^* = \beta \mathbb{E}_t \frac{e^{\zeta_{t+1}}/Y_{t+1}^*}{e^{\zeta_t}/Y_t^*} \left[\frac{1}{\vartheta-1} A_{t+1}^{*\frac{1}{\vartheta-1}-1} N_{t+1}^* + \Gamma_{t+1}^* \left(\phi + e^{\psi_{t+1}} N_{t+1}^{*\eta} S_{t+1}^* \right) \right] \tag{71}$$

Define

$$\left\{ \mathcal{G}_t^*, \boldsymbol{\Gamma}_t^*, \mathcal{Y}_t^* \right\} \equiv \left\{ \frac{A_{t+1}^*}{A_t^*}, \frac{\Gamma_t^*}{A_t^{*\frac{1}{\vartheta-1}-1}}, \frac{Y_t^*}{A_t^{*\frac{1}{\vartheta-1}}} \right\}. \tag{72}$$

The conditions characterizing the solution to the planning problem can then be expressed in terms of the stationary variables $\{\mathcal{Y}^*, \mathcal{G}_t^*, \boldsymbol{\Gamma}_t^*, N_t^*, S_t^*\}$ as follows:

$$\mathcal{Y}_t^* = N_t^* \tag{73}$$

$$\mathcal{G}_t^* = \phi + e^{\psi_t} N_t^{*\eta} S_t^* \tag{74}$$

$$\mathcal{Y}_t^* N_t^{*\varphi} = 1 + \eta \boldsymbol{\Gamma}_t^* e^{\psi_t} N_t^{*\eta-1} S_t^* \tag{75}$$

$$\chi \mathcal{Y}_t^* S_t^{*\varphi} = \boldsymbol{\Gamma}_t^* e^{\psi_t} N_t^{*\eta} \tag{76}$$

$$\boldsymbol{\Gamma}_t^* = \beta \mathcal{G}_t^{*-1} \mathbb{E}_t \frac{e^{\zeta_{t+1}}}{e^{\zeta_t}} \frac{\mathcal{Y}_t^*}{\mathcal{Y}_{t+1}^*} \left[\frac{1}{\vartheta-1} N_{t+1}^* + \boldsymbol{\Gamma}_{t+1}^* \left(\phi + e^{\psi_{t+1}} N_{t+1}^{*\eta} S_{t+1}^* \right) \right] \tag{77}$$

The steady-state of system (73)-(77) can be collapsed to one equation in $\mathcal{G}^*$:

$$\chi \left(\mathcal{G}^* - \phi \right)^{\varphi} \left[1 + \eta \kappa (1 - \phi \mathcal{G}^{-1}) \right]^{-\eta} = \kappa \mathcal{G}^{*-1} \tag{78}$$

where

$$\kappa \equiv \frac{\beta}{(1-\beta)} \frac{1}{(\vartheta-1)}. \tag{79}$$

The right-hand side (RHS) of (78) is decreasing in $\mathcal{G}^*$. Given the parametric assumption

$$\eta < \varphi, \tag{80}$$

the left-hand side (LHS) of (78) is increasing in $\mathcal{G}^*$. For χ low enough, the LHS evaluated at $\mathcal{G}^* = 1$ is smaller than the RHS evaluated at $\mathcal{G}^* = 1$. In any such instance the LHS and RHS (expressed as functions of $\mathcal{G}^*$) cross only once at a value $\mathcal{G}^* > 1$. Thus, for χ low enough there exists a unique BGP with positive growth.

To verify that (73)-(77) characterize a solution, we must check that the second-order conditions are also satisfied along the BGP. (If they hold along the BGP, they also hold locally around it).

To this end, express the steady-state of the system (73)-(77) in terms of the two optimality conditions for the two labor types:

$$F_N(N^*, S^*) = -N^{*\varphi} + N^{*-1} + \eta\kappa(\phi + S^* N^{*\eta})^{-1} S^* N^{*\eta-1} = 0, \tag{81}$$
$$F_S(N^*, S^*) = -\chi S^{*\varphi} + \kappa(\phi + S^* N^{*\eta})^{-1} N^{*\eta} = 0, \tag{82}$$

where $F(N^*, S^*)$ is used to denote the planner's objective in steady state as a function of N^* and S^*. The conditions (81), (82) equate the marginal disutility of each type of labor to its social benefit.

The second-order conditions for a maximum are $F_{NN} \leq 0$, $F_{SS} \leq 0$, and $F_{NN}F_{SS} \geq F_{NS}F_{SN}$. $F_{NN} \leq 0$ holds *iff*

$$F_{NN}N^{*2} = \eta\kappa(1 - \phi\mathcal{G}^{*-1}) \left[\eta\phi\mathcal{G}^{*-1} - (1+\varphi)\right] - (1+\varphi) \leq 0$$
$$\leftrightarrow$$
$$\eta \frac{\eta}{\kappa^{-1}(1 - \phi\mathcal{G}^{*-1})^{-1} + \eta} \leq \frac{1+\varphi}{\phi\mathcal{G}^{*-1}} \tag{83}$$

which holds as long as (80) holds. $F_{SS} \leq 0$ holds *iff* $F_{SS}S^{*2} \leq 0$, which is always satisfied:

$$F_{SS}S^2 = -\kappa(1 - \phi\mathcal{G}^{*-1})[\varphi + \kappa(1 - \phi\mathcal{G}^{*-1})] < 0 \tag{84}$$

Finally, $F_{NN}F_{SS} \geq F_{NS}F_{SN}$ *iff*

$$(F_{NN}N^{*2})(F_{SS}S^{*2}) \geq (F_{NS}N^*S^*)^2, \tag{85}$$

which can be written as

$$\eta \frac{\eta}{\kappa^{-1}(1 - \phi\mathcal{G}^{*-1})^{-1} + \eta} \leq \frac{(1+\varphi)}{\phi\mathcal{G}^{*-1}} \left(\frac{\varphi + \kappa(1 - \phi\mathcal{G}^{*-1})}{\phi\mathcal{G}^{*-1} + \varphi + \kappa(1 - \phi\mathcal{G}^{*-1})} \right), \tag{86}$$

which also holds given (80).

By log-linearizing the system (73)-(77) around its steady state and using the method of undetermined coefficients, it is then possible to express the variables $\hat{s}_t, \hat{g}_t, \hat{n}_t, \hat{y}_t$ (where $\hat{x}_t \equiv \log(X_t/X)$ for any variable X_t) as a function of the fundamental shocks ζ_t, ψ_t, leading to the expressions (47)-(48) in the main text. ■

C Proof of Proposition 3

Proof. If π_t=0, and therefore $P_t = P_{t-1}$ for all t, there is no price dispersion, and we have $N_t(i) = N_t(j)$ for any pair (i, j) as required by efficiency. The equilibrium conditions become

$$\mathcal{Y}_t = N_t, \tag{87}$$

$$\mathcal{G}_t = \phi + e^{\psi_t} S_t N_t^{\eta}, \tag{88}$$

$$\frac{(\epsilon-1)}{\epsilon}\frac{(\vartheta-1)}{\vartheta} = (1-\tau_{n,t}) N_t^{\varphi} \mathcal{Y}_t, \tag{89}$$

$$\mathbf{\Gamma}_t e^{\psi_t} N_t^{\eta} = (1-\tau_{s,t})\chi S_t^{\varphi} \mathcal{Y}_t, \tag{90}$$

$$\mathbf{\Gamma}_t = \beta \mathcal{G}_t^{-1} \mathbb{E}_t \frac{e^{\zeta_{t+1}}}{e^{\zeta_t}} \frac{\mathcal{Y}_t}{\mathcal{Y}_{t+1}} \Big[\frac{1}{\vartheta}\mathcal{Y}_{t+1} + \phi \mathbf{\Gamma}_{t+1}\Big], \tag{91}$$

in the variables $\{\mathcal{Y}, \mathcal{G}_t, \mathbf{\Gamma}_t, N_t, S_t\}$. (Equation (4) can then be used to back out the required i_t).

Suppose first that $\eta = 0$. Comparing (87)-(91) with (73)-(77) reveals that the decentralized equilibrium conditions match their efficient counterparts if the subsidies are set as follows:

$$1 - \tau_{n,t} = \frac{\epsilon - 1}{\epsilon}, \tag{92}$$

$$\frac{\mathbf{\Gamma}_t}{1-\tau_{s,t}} = \mathbf{\Gamma}_t^* \tag{93}$$

for all t. The subsidy $\tau_{n,t}$ is constant over time and equal to $1-(1-\epsilon^{-1})(1-\vartheta^{-1})$, which undoes the overall monopoly distortion (in both final and intermediate goods). The subsidy $\tau_{s,t}$ fluctuates around $\tau_s \in (0,1)$. To determine how $\tau_{s,t}$ depends on fundamental shocks, let

$$\hat{\tau}_{s,t} \equiv \frac{\tau_s}{1-\tau_s}[\log(\tau_{s,t}) - \log(\tau_s)], \tag{94}$$

so that from (93),

$$\hat{\tau}_{s,t} = \hat{\gamma}_t^* - \hat{\gamma}_t \tag{95}$$

where $\hat{\gamma}_t^* \equiv \log(\mathbf{\Gamma}_t^*/\mathbf{\Gamma^*})$, $\hat{\gamma}_t \equiv \log(\mathbf{\Gamma}_t/\mathbf{\Gamma})$. Then use a fist-order approximation and undetermined coefficients to write $\hat{\gamma}_t$ as function of the fundamental shocks, which yields

$$
\begin{aligned}
\gamma_t &= \Lambda_{\gamma\zeta}\zeta_t + \Lambda_{\gamma\psi}\psi_t, \\
\Lambda_{\gamma\zeta} &\equiv -\frac{(1-\rho_\zeta)}{(1-\beta\phi\mathcal{G}^{*-1}\rho_\zeta)} + \frac{\Lambda_{g\zeta}}{1-\beta\phi\mathcal{G}^{*-1}\rho_\zeta}, \\
\Lambda_{\gamma\psi} &\equiv \frac{-\Lambda_{g\psi}}{1-\beta\phi\mathcal{G}^{*-1}\rho_\psi},
\end{aligned}
\tag{96}
$$

with

$$
\begin{aligned}
\Lambda_{g\zeta} &\equiv (1-\phi\mathcal{G}^{*-1})\frac{(1-\rho_\zeta)}{1-\beta\rho_\zeta}\frac{1}{\Lambda_s} > 0, \\
\Lambda_{g\psi} &\equiv (1-\phi\mathcal{G}^{*-1})\left(1+\frac{\phi\mathcal{G}^{*-1}}{\Lambda_s}\right) > 0,
\end{aligned}
$$

with Λ_s defined as in Proposition 1.

Using the same steps for γ_t^* yields

$$
\begin{aligned}
\gamma_t^* &= \Lambda_{\gamma\zeta}^*\zeta_t + \Lambda_{\gamma\psi}^*\psi_t, \\
\Lambda_{\gamma\zeta}^* &\equiv -\frac{(1-\rho_\zeta)}{(1-\beta\rho_\zeta)} + \Lambda_{g\zeta}, \\
\Lambda_{\gamma\psi}^* &\equiv -\Lambda_{g\psi}.
\end{aligned}
\tag{97}
$$

Thus the dynamics of τ_{st} are governed by

$$
\begin{aligned}
\hat{\tau}_{st} &= \gamma_t^* - \gamma_t \\
&= (\Lambda_{\gamma\zeta}^* - \Lambda_{\gamma\zeta})\zeta_t + (\Lambda_{\gamma\psi}^* - \Lambda_{\gamma\psi})\psi_t,
\end{aligned}
\tag{98}
$$

where is straightforward to show that $(\Lambda_{\gamma\zeta}^* - \Lambda_{\gamma\zeta}) < 0$ and $(\Lambda_{\gamma\psi}^* - \Lambda_{\gamma\psi}) > 0$. The innovation labor subsidy is thus adjusted upward in response to a higher ψ_t, and downward in response to higher ζ_t.

Now suppose $\eta > 0$. In this case, there is a time-varying inefficiency in goods labor as well. To restore efficiency, the subsidy $\tau_{n,t}$ must be set so that $N_t = N_t^*$ for all t. This means setting $\tau_{n,t}$ so that

$$
\frac{1}{(1-\tau_{n,t})}\frac{(\epsilon-1)}{\epsilon}\frac{(\vartheta-1)}{\vartheta} = 1 + \eta\mathbf{\Gamma}_t^* e^{\psi_t} N_t^{*\eta-1} S_t^*. \tag{99}
$$

In the BGP, clearly $\tau_n > 1 - (1-\epsilon^{-1})(1-\vartheta^{-1})$ (the value needed to correct the monopoly distortion) because the learning-by-doing spillover introduces a second source of suboptimality of N.

To determine the first-order dynamics of $\tau_{n,t}$, note that (99) can be written

$$\frac{1}{(1-\tau_{n,t})}\frac{(\epsilon-1)}{\epsilon}\frac{(\vartheta-1)}{\vartheta}=N_t^{*1+\varphi}. \tag{100}$$

Thus, $\tau_{n,t}$ is set as an increasing function of N_t^*, which from Proposition 1 varies proportionately with S_t^* (which increases with ψ_t and falls with ζ_t). The dependency of the goods labor subsidy on shocks is thus of the same sign as the innovation labor subsidy. ■

D Proof of Proposition 4

Proof. The first step is to re-write lifetime expected utility in terms of stationary variables. We have the following:

$$\begin{aligned}
&\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t e^{\zeta_t}\left(\log(C_t)-\frac{N_t^{1+\varphi}}{1+\varphi}-\frac{\chi S_t^{1+\varphi}}{1+\varphi}\right)\\
&=\frac{\beta\log(A_0)}{(1-\beta)(\vartheta-1)}+\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t e^{\zeta_t}\left(\log(\mathcal{C}_t)+\frac{\beta}{\vartheta-1}X_t\log(\mathcal{G}_t)-\frac{N_t^{1+\varphi}}{1+\varphi}-\frac{\chi S_t^{1+\varphi}}{1+\varphi}\right)
\end{aligned} \tag{101}$$

where $\mathcal{C}_t\equiv C_t/A_t^{\frac{1}{\vartheta-1}}$, $\mathcal{G}_t\equiv A_{t+1}/A_t$, $X_t\equiv\mathbb{E}_t\left(\sum_{i=1}^{\infty}\beta^{i-1}e^{\zeta_{t+i}-\zeta_t}\right)$, and where A_0 (the initial state of technology) is given.

We seek to approximate the welfare loss $\mathbb{W}\equiv-\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left(\frac{\bar{U}_t-\bar{U}}{\bar{U}_{\mathcal{C}}\mathcal{C}}\right)$, where the function

$$\bar{U}(\mathcal{C},\mathcal{G},N,S;\zeta,X)\equiv e^{\zeta}\left(\log(\mathcal{C})+\frac{\beta}{\vartheta-1}X\log(\mathcal{G})-\frac{N^{1+\varphi}}{1+\varphi}-\frac{\chi S^{1+\varphi}}{1+\varphi}\right). \tag{102}$$

A second-order approximation to the latter around the BGP yields

$$\begin{aligned}
\bar{U}_t-\bar{U}\simeq\ &\hat{c}_t(1+\zeta_t)-N^{1+\varphi}\hat{n}_t(1+\zeta_t)-N^{1+\varphi}\frac{1+\varphi}{2}\hat{n}_t^2\\
&+\kappa\hat{g}_t(1+\zeta_t)-\chi S^{1+\varphi}\hat{s}_t(1+\zeta_t)-\chi S^{1+\varphi}\frac{1+\varphi}{2}\hat{s}_t^2+\kappa\hat{g}_t\hat{x}_t,
\end{aligned} \tag{103}$$

where we ignore terms independent of policy, and where $\hat{c}_t\equiv\log(\mathcal{C}_t/\mathcal{C})$, $\kappa\equiv\frac{\beta}{(1-\beta)}\frac{1}{(\vartheta-1)}$.

Next, use goods market clearing ($\hat{c}_t=\hat{y}_t$), equation (29), and a second-order approximation of equation (24) to eliminate $\hat{c}_t$, $\hat{n}_t$, and $\hat{s}_t$ from (103), along with the steady-state equations characterizing the steady-state allocation (which are identical to the social planner's first-order conditions), yielding

$$\bar{U}_t-\bar{U}\simeq-\frac{1}{2}\left\{\epsilon var_i\left\{p_t(i)\right\}+(1+\varphi)\left[1+\eta(1+\eta)\kappa(1-\overline{\phi})\right]\hat{y}_t^2+\kappa(1+\overline{\varphi})\hat{g}_t^2+2\kappa\frac{1-\rho_\zeta}{1-\beta\rho_\zeta}\hat{g}_t\zeta_t-2\eta(1+\varphi)\kappa\hat{g}_t\hat{y}_t\right\}. \tag{104}$$

Using the result $\sum_{t=0}^{\infty} \beta^t var_i \{p_t(i)\} = \frac{1}{\lambda} \sum_{t=0}^{\infty} \beta^t \pi_t$ (shown in Woodford (2003), Chapter 6) and rearranging yields the expression for $\mathbb{W}$ shown in Proposition 4. ■