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FINANCING REPEAT BORROWERS: DESIGNING CREDIBLE INCENTIVES FOR TODAY AND TOMORROW

ANIL JAIN* AND PIRUZ SABOURY†

Abstract. We analyze relational contracts between a lender and borrower when borrower cash flows are not contractible and the costs of intermediation vary over time. Because lenders provide repayment incentives to borrowers through the continuation value of the lending relationship, borrowers will condition loan repayment on the likelihood of receiving loans in the future. Therefore, the borrower's beliefs about the lender's future liquidity and profitability become an important component of the borrower's repayment decision. Consequently, the possibility of high lending costs in the future weakens repayment incentives and can cause the borrower to strategically default in some states and an inefficient under-provision of credit. We characterize the optimal relational contract and discuss the application of our model to the case of microfinance and trade credit.

JEL Classification: G21, G33, O16

Keywords: Relationship Lending, Dynamic Incentives, Microfinance, Repeated Games, Trade Credit

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1. Introduction

In many forms of financial intermediation, such as microfinance and trade credit, a lender has little formal recourse or enforcement power to procure repayment. This friction is especially prevalent in developing countries with weaker institutions and fewer creditor protections. Therefore, to stop ex-post moral hazard and ensure repayment, a lender must design the contract to be incentive-compatible. This paper investigates the use of dynamic incentives with a lender whose funding cost varies over time. We propose a simple model of relational contracting—informal agreements sustained by the value of the future relationships (Baker et al. [2002])—where borrowers derive utility from the future value of lending relationships, which in turn, induces repayment incentives today. In other words, the long-run value from the bilateral relationship provides short-run repayment incentives. Specifically, we focus on how changes in a lender's capacity to provide future loans affects the borrower's incentive to repay today, and in turn, the equilibrium loan contract.

We consider a world with two frictions: First, borrowers cannot commit to repay their loans and lenders cannot commit to providing loans in future periods. Second, lenders face shocks to the cost of intermediation that are uncorrelated with the borrower's repayment ability. We believe the latter to be consistent with the case of microfinance, where global credit markets are virtually uncorrelated with the profits of micro-enterprises, and with the case of trade credit, where costs faced by suppliers may not be directly related to the demand for final goods faced by purchasers.

We characterize how changes in the future value of a relationship affect today's actions and subsequently the utility of both participants. We describe how the lender structures incentives to maximize profits within the constraint of being unable to write binding long-term contracts.

We show that the inability to commit to long-term actions, combined with changing intermediation costs, can lead to reduced lending and may even prohibit lending. Furthermore, we detail how the lack of enforceable contracts can change the distribution of economic rents shared between the two parties. For instance, environments with more variable lender costs (e.g. an economy characterized by deep financial crises and strong booms versus an economy with relative stable economy with shallow recessions and modest economic growth) cause the lending environment to be more likely to be characterized by only partial lending and some strategic borrower default.

To gain intuition for our model, we model repeated lending where the lender's cost of lending does not change over time. Due to moral hazard, no lending is possible without the prospect of future trading. We solve for the lender's optimal stationary relational contract and characterize the equilibrium. We then consider the lending environment when the lender's costs vary over time (according to a given Markov process). This added friction tempts the lender to not offer loans in some periods because of the higher realized costs in those periods. We investigate equilibria where lending is possible in all periods as well as equilibria where the lender may choose not to give loans in some periods. We show that in the latter case, partial default occurs along the equilibrium path solely due to the lender being unwilling to provide loans in some periods.

After solving the equilibrium of the game, we detail how the equilibrium changes as we alter the game's parameters. For instance, more persistent cost environments lead to a greater likelihood of the relationship breaking down (even when holding the ex-ante probability of each state constant). Therefore, this result would support the idea that infrequent long recessions may be worse for relational contracting than frequent short recessions.

This paper is related to several strands of the economics and finance literature. First, it calls upon models of banking and intermediation under ex post moral hazard. Many related papers such as Townsend [1979] and Bernanke and Gertler [1989] show that under informational frictions, debt is the optimal type of financing by outside, uninformed investors. Bolton and Scharfstein [1996] consider the financing decision of the firm and the optimal debt structure when control can be transferred in different states of the world. In our setting, we also consider the characteristics of optimal financing, but we assume that borrowers have no net worth, that lenders cannot improve the contracting environment through monitoring, and that the borrowers cannot transfer control of their projects to creditors.

Furthermore, a large financial literature (starting with Rajan [1992], Petersen and Rajan [1994], Berger and Udell [1995], Boot and Thakor [2000]) studies how strong borrower-lender relationships can help solve informational asymmetries. We contribute to this literature by modeling a lending relationship that is only possible through relationship lending—specifically, without the potential for a future lending relationship, the borrower has no incentive to repay the lender.

In this paper, we consider a lender that faces liquidity risk in the capital markets. In some periods, financing costs for the intermediary are high, while in other states, these financing costs are low. The liquidity channel and the potential benefits of securitization have been studied by Carlstrom and Samolyk [1995] and Parlour and Plantin [2008]. Drucker and Puri [2009] consider the potential diversification benefits of loan sales by financial intermediaries and show that borrowers whose loans are sold appear to receive more future credit from the same originator, potentially increasing the value of the lending relationship. Consistent with our model predictions, Berlin and Mester [1999] empirically show relationships between the structure of bank liabilities and relationship lending. Specifically, banks with greater deposit funding (relative to wholesale funding) provide greater loan smoothing than other banks.

Our paper builds on the insights and tools developed in the relational contracting literature (see Levin [2003], Halac [2012], McAdams [2011], Macchiavello and Morjaria [2012], Li and Matouschek [2013], and Barron and Powell [2019] for example), which studies repeated contracting and reputation-building between parties. Our paper uses the relational lending literature tools to explain the relationship dynamics that we see in microfinance and trade finance.

1.1. **Motivating examples.** While our framework applies to any setting where contract enforcement between two parties today depends on the future value of the relationship between them, we highlight two applications—microfinance and trade credit.

Microfinance. Microfinance has been a rapidly growing and well-publicized financial tool in developing countries.[1] Microloans are intended for poor clients who generally do not have any pledgeable collateral and who live in countries where the formal legal institutions have little to no ability to enforce small credit contracts. To overcome these constraints, microlenders have developed two important contractual innovations to make lending feasible: social collateral and dynamic incentives. Microfinance is able to harness the power of social networks in a variety of ways

[1]According to mixmarket.org, microfinance institutions disbursed more than $150 billion in loans to over 30 million customers worldwide in September 2018.

to provide repayment incentives.[2] However, in this paper we focus on the use of repeated contracting to provide repayment incentives. The standard microloan is around $100 and primarily aimed to finance the purchase of assets or working capital for microenterprises. Borrowers make frequent installment payments for one year and upon completion, are disbursed a new loan.[3] If borrowers default on their loans, then they are not offered future access to credit. This dynamic incentive is often credited as one of the main innovations of microfinance.[4]

In connection with our model, we highlight the potential fragility of microfinance institutions (MFIs) as a result of the reliance on future lending activities to provide repayment incentives today.[5] Namely, if microlenders themselves become liquidity-constrained in the future, does this constraint have feedback effects on the repayment of existing, outstanding loans? In this type of scenario, common financing structures such as short-term debt or loan securitization may be suboptimal. This analysis is relevant to recent developments in microfinance in India, one of the largest microfinance markets, which has faced a similar crisis in the past.[6] Throughout the crisis, microlenders faced difficulties accessing credit markets, despite the fact that the borrowers' economic conditions remained unchanged. Some smaller MFIs failed to obtain financing and walked away from their loan portfolios. Larger MFIs were forced to delay new disbursements while still paying staff costs and assuring clients that their new loans were indeed coming.

[2]Many microfinance contracts have joint liability, where borrowers must pay for their peers if those peers decide to default (see Gine and Karlan [2006], Gine and Karlan [2009], and Giné et al. [2011] for empirical evidence on the effects of joint liability.) Research has also shown (see Feigenberg et al. [2010] and Breza [2012]) that even without contractual linkages between borrowers, there still may be social effects at play.

[3]In many settings the standard maturity for a microloan is 50 weeks, but some lenders offer variations on this product.

[4]See Morduch [1999]. Using an innovative empirical design, Karlan and Zinman [2009] find that borrowers do respond to dynamic incentives and are more likely to repay loans if they know a new loan is forthcoming.

[5]In recent empirical work, Gertler et al. [2021] show how digital collateral can also be used to improve the long-term borrower incentives within microfinance contracts.

[6]In October 2010, the Indian state of Andhra Pradesh suffered from a debilitating wave of microfinance defaults sending repayment rates from close to 100 percent to 10 percent and below. The default crisis began when the state government passed an emergency ordinance to rein in microfinance institutions. The ordinance set out to curb usurious interest rates, cases of borrower harassment by credit officers, and over borrowing by the state's poor. Stories of borrower suicides permeated the local and international press (see, for example, Indian Express [2010] and International Business Times [2010]). When banks refused to lend to MFIs, the localized crisis spread to all of India through a national liquidity crisis.

To apply our model to microfinance, we make two financial friction assumptions. First, we assume that MFIs are monopolist lenders. We argue that this assumption is innocuous because some sort of monopoly power is required to maintain repeated lending relationships (Petersen and Rajan [1995]). Furthermore in the case of India, regulation gives MFIs some degree of market power, specifically, borrowers are allowed to take loans from a maximum of two lenders.[7] Second, in our model, we assume that an MFI must have at least some debt financing. This assumption is consistent with the microfinance industry where the average reported capital to asset ratio for Indian MFIs is just over 27 percent (World Bank [2019]).

Trade credit and supplier relationships. Our model and explanation is focused on a traditional lender-borrower relationship; however, relabelling the agents in our model show that it can also be used to describe trade credit and supplier relationships. Specifically, we can analyze a buyer's (borrower's) incentive to renege on repaying the supplier (lender) as a function of the supplier's capacity to continue to deliver future products (loans).

Seminal early work on trade credit and supplier relationships, such as McMillan and Woodruff [1999] and Banerjee and Duflo [2000] analyze the value of reputation, repeated interactions, and relational contracts. We contribute to this literature by considering how a borrower's incentive to repay trade credit depends on a supplier's capacity to either finance or supply more goods—a channel that is missing from recent contributions to the literature such as Boissay and Gropp [2013], Jacobson and Von Schedvin [2015], McGuinness et al. [2018] as well as major prior contributions such as Burkart and Ellingsen [2004] and Cunat [2007]. This channel of potential default risk is particularly relevant to the developing world where contractual protections are weaker and expected loss due to default is significantly higher. In our model, we show how changes in the supplier's financing—through changes in the value of the future relationship—directly affects the set of possible relationship's today.

In the next section, we describe our environment. Section (3) presents the baseline case where the lender's cost does not change over time. Section (4) extends the analysis to our full model by allowing the cost of lending to vary over time. Section

[7]To protect borrowers against price gouging by monopolists, interest rates are also regulated at a maximum rate of 10 percent to 12 percent above the lender's cost of capital.

(5) analyzes how the optimal contract changes with different parameters and section (6) concludes.

2. Model

This section describes the model, assumptions, timing, and solution concept. Section (3) characterizes the solution for a basic version of the model that has no change in the lender's funding cost and section (4) extends the analysis to the full model by allowing the lender's funding cost to follow a Markov process.

2.1. **Set-up.** There is an infinitely repeated game between a risk-neutral, profit-maximizing, monopolist lender and a risk-neutral capital-constrained agent.

Each period, the agent has an investment project that requires a unit of capital and produces a deterministic output of v at the end of the period. The lender can provide a loan to the agent for the project. The lender's cost of a unit of capital is c_t, which takes two possible values, c_l and c_h, where c_l is strictly less than c_h.[8] Upon lending, the lender requests a repayment of r_t to be made at the end of the period.[9]

When the agent invests in their project, the output is observable and verifiable but, crucially, the lender does not have any enforcement power. Rather, the bank's sole tool to extract repayment for the loan is through a relational contract and ensuring the value of a long-term banking relationship outweighs the short-term incentive to renege on payment.

To highlight the importance of the future relationship between the lender and the agent, we assume that the lender's cost of capital for the next period, c_{t+1}, is publicly observable before the agent repays the lender. Therefore, before the agent decides whether to repay, the agent can infer whether the lender will offer a future loan and, if so, at what interest rate. This assumption is motivated by two different phenomena: first, microfinance lenders have borrowers on different loan cycles, therefore, if a borrower's friend is unable to renew their loan, this information will influence the

[8] c_t can be thought of as a reduced form formalization of the cost of lending in each period. In particular, $c_t \equiv \kappa c_{e,t} + (1-\kappa)c_{d,t}$ such that κ is between 0 and 1. $c_{e,t}$ is the cost of equity financing, $c_{d,t}$ is the cost of debt financing, and κ is the bank's capital-to-assets ratio.

[9] r_t includes both the loan's principal and interest payment.

borrower's expectation of whether they will get a loan tomorrow. Second, the information concerning microfinance institutions is well documented both in newspapers and by politicians.[10]

The evolution of lending cost (c_t) follows a Markov process whose transition matrix is common knowledge:

$$\begin{bmatrix} p_l & 1-p_l \\ 1-p_h & p_h \end{bmatrix}$$

where $p_l = \text{Prob}(c_{t+1} = c_l | c_t = c_l)$ is the probability of moving from a low-cost state to a low-cost state and $p_h = \text{Prob}(c_{t+1} = c_h | c_t = c_h)$ is the probability of moving from a high-cost state to a high-cost state.[11] Thus, the long-term stationary (invariant) distribution of states is:

$$\Theta = \begin{bmatrix} \theta_l \\ \theta_h \end{bmatrix} = \begin{bmatrix} \frac{1-p_h}{2-p_l-p_h} \\ \frac{1-p_l}{2-p_l-p_h} \end{bmatrix} \tag{1}$$

where θ_l is the long-run probability of low cost and θ_h is the long-run probability of high cost.

For ease of exposition, we make a number of simplifying assumptions. The agent's and the lender's outside options are both normalized to zero and the borrower is credit-constrained such that they cannot pay any amount ahead of time. The agent also has no storage technology to transfer wealth across time and we assume that the agent and the lender have the same common discount rate, $\delta \in (0, 1)$.

While the bank and the agent cannot enter into any form of binding contract, before the game begins, the lender offers a "relational contract" that lays out the conditions under which the lender will lend the required capital to the agent and the required repayment amount in any given time period.[12] The relational contract also specifies the action undertaken by the lender or the borrower if the other party fails to undertake their specified action in the relational contract.

Once the lender offers the relational contract, if the borrower accepts the contract, the repeated game begins. The lender's goal is to choose a relational contract

[10]For example, during the recent Indian microfinance crisis, journalists and politicians wrote damning articles about MFIs and bankrolled television commercials to further undermine the portfolios of MFIs after the onset of the crisis. See Banerjee and Duflo [2011] and Breza [2012] for discussions of the recent Indian microfinance crises.

[11]The probability of moving from a low state to a high state is therefore $(1 - p_l)$ and the probability of transitioning from a high state to a low state is $(1 - p_h)$.

[12]This lending structure can entail lending in every period or just in some periods.

that maximizes their expected discounted stationary profits in the repeated game, conditional on being accepted by the borrower and being incentive-compatible at all times.

We solve for the perfect public equilibrium of this game which requires that play following each history be a Nash equilibrium.[13] Moreover, for simplicity and ease of exposition, we focus solely on stationary relational contracts where the loan schedule and repayment amounts are independent of time.

2.2. **Timing.** The timing of the game is as follows:

(1) The lender offers a relational contract to the potential borrower.
(2) The borrower decides whether to accept the relational contract. If the borrower rejects the contract, the game ends and the lender and the agent both receive zero payoff. Otherwise, the game proceeds.
(3) Both parties observe the lending cost in the first period (c_1) and the repeated game begins.
(4) At the beginning of each period t (including $t = 1$):
    (a) The lender chooses whether to offer a loan. Their decision is denoted by $l_t \in \{0, 1\}$, with $l_t = 1$ for lending. If the lender offers a loan, the lender also chooses a required repayment amount r_t.
    (b) The agent either accepts or rejects the loan offer. Their decision is denoted by $d_t \in \{0, 1\}$ with $d_t = 1$ for accepting.
(5) At the end of each period t (including $t = 1$):
    (a) The lending cost for the next period (c_{t+1}) is observed by all parties.
    (b) The borrower then decides how much to repay, $\hat{r}_t \in [0, r_t]$.
    (c) Payoffs are realized.
(6) The game enters period $t + 1$ and repeats from step (4).

[13]In this model, perfect public equilibrium imposes the same sequential rationality requirement that subgame perfection would impose in a complete information model [Fudenberg et al., 1994]. Hence, once the repeated game starts, neither the lender nor the borrower can commit to the relational contract. Each agent will unilaterally renege on the agreement in any period if the benefits from reneging are larger than the expected benefits from continuing with the relational contract. In other words, the bank's choices are limited to relational contracts that are incentive-compatible—that is, perfect public equilibria. Simply put, the pre-game step works as an equilibrium refinement criterion for cases where the repeated game has more than one equilibrium. Such setting is consistent with the reality of microfinance where formal contracting is not possible but the bank can implement dynamic repayment incentives by putting preconditions on subsequent loans.

We first analyze a benchmark case where lending cost is constant in section (3). Then we turn our attention to finding the equilibrium of the full model in section (4).

3. Equilibrium: Relational contracts with no change in cost over time (baseline)

To gain intuition into the lender and borrower's dynamic incentives, we begin with a simple environment that excludes any uncertainty. Specifically, we start with a constant cost of lending in all time periods, that is, $c_t = c$.

By abstracting from the time-varying cost structure in the full model, we consider a simple stationary version of our problem to demonstrate what inefficiencies in lending may arise because of the absence of binding contracts.

In the simple baseline model, the lender chooses a feasible relational contract that maximizes their total discounted profits:

$$\Pi = \sum_{t=0}^{\infty} \delta^t l_t (r_t - c) \tag{2}$$

Recall that the relational contract is not a binding contract and thus has to be sustainable in equilibrium—that is, it must be incentive-compatible for the lender and also satisfy the borrower's repayment incentive compatibility constraint (ICC) at every point in time. In other words, the borrower must be willing to make the repayment the bank requires in every period, that is, $\hat{r}_t = r_t$. Using Abreu [1988] one-shot deviation principle, we know that the optimal lender punishment for non-compliance is breaking off trade for all future rounds. Therefore, the borrower's long-term payoff for complying at the end of each period must be greater than or equal to the borrower's outside option:

$$\sum_{t=1}^{\infty} \delta^t l_t (v - r_t) - r_0 \geq 0 \tag{3}$$

where $\sum_{t=1}^{\infty} \delta^t l_t (v - r_t)$ is the discounted value of future borrower surplus from borrowing and r_0 is the required repayment at time $t = 0$. Thus, the lender's constrained maximization problem is as follows:

$$\max_{\{l_t, r_t\}_{t=0}^{\infty}} \Pi \text{ (as defined by equation [2])}$$

$$\text{s.t. equation [3] holds.} \tag{4}$$

Furthermore, recall that we concentrate solely on stationary relational contracts where contract parameters are time-invariant—that is, $l_t = l$ and $r_t = r$ for all t. Therefore, there are only two possible types of relational contracts: no lending in any period ($l = 0$) or the following:

i. The lender extends a loan and requires repayment r in the first period.
ii. After the first period, at all histories where the lender has always received a repayment of at least r in the past, the lender will continue to extend a loan and require repayment r. At any other history, the lender will not offer any loans.
iv. The borrower accepts a loan and repays r in the first period.
v. After the first period, at all histories where a loan has been extended in all previous periods, the borrower accepts a loan and repays r. At any other history, the borrower accepts a loan and repays zero.

The relational contract described above will be a perfect public equilibrium so long as both parties' long-run discounted payoff is non-negative. However, because we assume that the lender makes the initial offer, the lender opts for the equilibrium that generates the largest profits, which is the equilibrium with the highest repayment (r^*) that is acceptable to the borrower. The resulting interest rate would be the solution to the lender's maximization problem laid out in equation [4].

To pin down r^*, we begin by noting that in a stationary relational contract, the borrower's ICC (equation [3]) simplifies to:

$$\frac{\delta}{1-\delta}(v - r) - r \geq 0 \tag{5}$$

The left-hand side of equation [5] is the value of sustaining the relationship and the right-hand side is the borrower's outside option. The latter is the result of the fact that, as discussed earlier, the optimal punishment for reneging is breaking the relational contract so that the game reverts to static Nash equilibrium actions, in which no loans are offered.

Solving equation [5] for r reveals that the maximum repayment, r^*, that can be sustained in a stationary equilibrium is δv.

Regarding the lender's problem of choosing the most profitable stationary relational contract, equation [4] simplifies to maximizing $r - c$ conditional on the expected profits in the sustained relationship being higher than the outside option of zero. Therefore, if δv is greater than or equal to c, the profit-maximizing monopolist lender will lend (that is, $l^* = 1$) and require a repayment of δv in every period. Otherwise, no stationary relational contract with positive lending is possible because even with maximal punishment (no future loans), the borrower is unwilling to make a repayment that is high enough for the bank to cover its costs.

Proposition 1. *When intermediation costs are fixed over time at c, lending is possible if and only if δv is greater than or equal to c. Under this condition, the bank will lend ($l^* = 1$) and require a repayment of $r^* = \delta v$ in every period. Otherwise, no lending occurs in equilibrium.*

Proposition 1 states that even though the lender is a monopolist, it cannot expropriate the full rent from the relationship by charging $r = v$ because of the lack of formal contractual protection. In fact, the only obstacle that prevents the borrower from strategic defaulting on their loan is the "shadow value of the future," equal to δv. The latter is only sufficient to constrain the borrower from reneging if the repayment does not exceed the shadow value of the future. Thus, the borrower retains a positive per-period payoff equal to $(1 - \delta)v$. Moreover, as a result of this limitation, lending does not occur at costs above δv even though it is socially efficient to lend if the cost of lending, c, is less than borrower's value from a loan, v.

4. Equilibrium: Relational contracts with changing costs

In this section we turn our attention to the full model where the cost parameter changes over time. In particular, we concentrate on the interesting case where the lender's cost of lending c_t, take two possible values, c_l and c_h, such that c_l is less than δv and c_h is greater than δv (that is, $c_l < \delta v < c_h$).[14] Note that lending in the

[14]In the other two possible cases, the equilibrium characterization is trivial. If cost is always below the borrower's shadow value of the future ($c_l < c_h < \delta v$), then lending in every period is profitable for the bank and the lending equilibrium characterized in section (3) continues to hold. If cost is always more than the borrower's shadow value of the future ($\delta v < c_l < c_h$), then lending is never profitable and no lending occurs in equilibrium.

high cost state is efficient if the borrower's benefit, v, is greater than the lender's cost of lending, c_h, but we do not place any parameter restrictions to ensure this occurs.

Recall the assumption that at the end of each time period t, the borrower observes tomorrow's cost realization c_{t+1} *before* deciding whether to repay today's loan. This assumption, combined with the variability of lending costs, captures the possibility that the borrower's moral hazard—the borrower's incentive to renege—changes over time.

As before, the lender and the borrower cannot enter into a binding contract. Nonetheless, they can agree on a relational contract (l_t, r_t) if it is incentive-compatible for both the lender and the borrower. The borrower's ICC is that for all periods where the borrower received a loan, the borrower's expected value of the future lending relationship is at least as large as their outside option at time t (not repaying the loan).[15] Formally:

$$\forall t \text{ s.t. } l_t = 1, \quad \sum_{k=1}^{\infty} \delta^k E[l_{t+k}(v - r_{t+k})] - r_t \geq 0 \tag{6}$$

Moreover, the lender should also find it profitable to abide by the relational contract at any point in the game, that is, for all periods the expected profits from lending are greater than or equal to zero. Formally:

$$\forall t \quad \Pi_t = \sum_{k=0}^{\infty} \delta^k E[l_{t+k}(r_{t+k} - c_{t+k})] \geq 0 \tag{7}$$

At the beginning of the game, the lender's goal is to choose a relational contract that maximizes the lender's long-term expected stationary profits, conditional on both ICCs (equations [6] and [7]) being satisfied.

$$\begin{aligned} &\max_{\{l_t, r_t\}_{t=0}^{\infty}} \Pi = E_\Theta(\Pi_0) \\ &\text{s.t. equations [6] and [7] hold.} \end{aligned} \tag{8}$$

As before, we limit our attention to stationary relational contracts, where lending and repayment are independent of time but can be state-dependent.

[15]In Abreu [1988], the optimal punishment for non-compliance is breaking off trade for all future periods.

Definition 1. A relational contract (l_t, r_t) is stationary if, for all t,

$$l_t = \begin{cases} l_h \text{ if } c_t = c_h \\ l_l \text{ if } c_t = c_l \end{cases} \quad \& \ r_t = \begin{cases} r_h \text{ if } c_{t+1} = c_h \\ r_l \text{ if } c_{t+1} = c_l \end{cases} .$$

Stated differently, the lender can offer a loan and promise to continue lending in both or one of the states as long as the borrower repays the agreed amount. Such an offer would be accepted by the borrower and respected by both parties only if it satisfies both the borrower's and the lender's ICCs (equations [6] and [7])—the relational contract's feasibility conditions.[16] Then, conditional on the relational contract being feasible, the lender offers the relational contract that generates the largest expected profit for the lender. There are three possible types of stationary relational contracts that the lender can choose from:

(1) **always-lending**—offer to lend in all states.
(2) **partial-lending**—offer to lend in some states, which is state-dependant.
(3) **no-lending**—not offer to lend in any state.

To determine the equilibrium of the game, we need to complete two steps. First, we need to solve for the parameter space where each relational contract is feasible. Second, from the set of feasible relational contracts, we need to solve for the contract that confers the largest expected profit for the lender. Since we assume the lender is able to choose their preferred relational contract (conditional on it being incentive compatible), the equilibrium of the game can be characterised using this contract.

In proposition (2), we state the lender's optimal relational contract and subsequently explain the intuition for the lender's preferred strategy and the game's characterization. Then in sections (4.1) and (4.2), we explain in detail the lender's and borrower's strategies for each relational contract. The no-lending relational contract's feasibility and equilibrium characterization are trivial.

Proposition 2. *At the beginning of the game, the lender's optimal relational contract is one of three possible relational contracts: "always-lending," "partial-lending," or "no-lending," according to the following conditions.*

Always-lending relational contract

The lender offers to lend in all states if, and only if, conditions (1) and (2) hold:

[16]Note by requiring the ICC to hold for all periods, including t equal to 0, we are implicitly subsuming the lender and borrower's participation constraint into the ICC.

(1) $c_h - \delta v \leq \delta \left(\frac{1-p_h}{1-\delta p_l}\right)(\delta v - c_l)$*, which guarantees always-lending to be feasible.*

(2) $\theta_h c_h \leq \delta v - \delta v \theta_l \left(p_l + (1-p_l)\frac{\delta(1-p_h)}{1-\delta p_h}\right)$*, which guarantees always-lending to be more profitable than partial-lending.*

Partial-lending relational contract

The lender offers to lend only in the low-cost state if, and only if, at least one of conditions (1) and (2) does not hold, and the following condition holds:

(3) $c_l \leq \delta v \left[p_l + (1-p_l)\frac{\delta(1-p_h)}{1-\delta p_h}\right]$*, which guarantees partial-lending to be feasible.*

No-lending relational contract

Otherwise, if both conditions (1) and (3) do not hold, then lending is not feasible and the lender does not lend.

The proof of proposition (2) is in the appendix.

Conditions (1) and (3) in proposition (2) are the feasibility conditions for the always-lending relational contract and the partial-lending relational contract, respectively.[17] Condition (2) is the lender's expected profit maximising condition, and if it holds, the always-lending relational contract is more profitable than partial-lending relational contract. Conversely, if condition (2) does not hold, partial-lending is more profitable than always-lending. Finally, if the always-lending and partial-lending relational contracts are both infeasible (conditions (1) and (3) do not hold), then the lender makes no loans (no-lending).

For convenience and clarity, table (4) summarizes the equilibrium strategies, payoffs, and feasibility conditions.

For completeness, sections (4.1) and (4.2) describe and characterize the feasibility of the always-lending and partial-lending relational contracts respectively. Section (5) analyses how the lender's optimal relational contract varies in different economic environments.

[17]Recall that the feasibility conditions require that the relational contract is incentive compatible for both the lender and the borrower for any point in the game.

TABLE 1. Summary of Relational Contract Characteristics

	Partial-lending	Always-lending
Lending Strategy	$l_t = 1$ if $c_t = c_l$ $l_t = 0$ if $c_t = c_h$	$l_t = 1$ for all t
Gross Repayment	δv if $c_{t+1} = c_l$ $\delta v \frac{\delta(1-p_h)}{(1-\delta p_h)}$ if $c_{t+1} = c_h$	$r^* = \delta v$ for all t
Expected Stationary Profits	$\frac{\theta_l \left[p_l r_l + (1-p_l) r_h - c_l\right]}{1-\delta}$	$\frac{r^* - \bar{c}}{1-\delta}$
Additional Feasibility Condition	None	$(1-\delta p_l)(r^* - c_h) + \delta(1-p_h)(r^* - c_l) \geq 0$
Stationary Borrower Payoff	$\frac{\theta_l [v - p_l r_l - (1-p_l) r_h]}{1-\delta}$	v

4.1. **Always-lending: The lender offers a loan in all states.** In this subsection we start by walking through the conditions that determine the feasibility of the always-lending relational contract and the underlying intuition. Finally, we characterize the always-lending relational contract.

Starting with the borrower's incentive compatibility condition: If the borrower believes that the lender will offer a loan in every period, then the borrower's participation decision problem will be identical to the case where the lender's cost is deterministic, as discussed in section (3). Thus, the borrower's maximum incentive-compatible repayment is δv for all time periods.

Turning to the lender's incentive compatibility conditions: Using the borrower's maximum willingness to repay and the lender's expected profit (as shown in equation [7]) reveals that by extracting a maximum repayment of $r_t = \delta v$ in every period, the lender's expected discounted profit at time $t = 0$ simplifies to

$$\Pi_0 = E\left[\sum_{t=0}^{\infty} \delta^t(\delta v - c_t)|c_0\right] \tag{9}$$

To ensure that offering loans in every period is a credible strategy, equation [9] must be greater than or equal to zero for both possible initial cost realizations, $c_0 = c_l$ and $c_0 = c_h$. Otherwise, the lender could not credibly offer loans in all states. This equation can be written in recursive form by denoting the lender's value function for each state as V_s, where s is the current state, and s' is the state in the next period. For the lender to avoid reneging on the relational contract, the following lender ICC must hold:

$$V_s = (\delta v - c_s) + \delta E(V_{s'}|s) \geq 0 \quad \forall s = \{l, h\}$$

This ICC requires the current value of the relationship to be sufficiently large such that the lender would prefer not to renege on the contract by stopping lending. Using this ICC we can state the following corollary of proposition (2) and the proof is in the appendix.

Corollary 1. *The lender will only be able to credibly offer loans in all states if the following condition is satisfied:*

$$(1 - \delta p_l)(\delta v - c_h) + \delta(1 - p_h)(\delta v - c_l) \geq 0$$

Corollary 1 states the intuitive condition for the always-lending relational contract to be feasible: If the lender realizes a high cost state, the lender's expected discounted losses from continuing to lend in this high state and future possible high states must be smaller than the lender's expected discounted profits from future low states. This condition is the same as condition 1 in proposition (2), but rearranged in a more intuitive manner.

To sum, the always-lending relational contract equilibrium is characterized by the following:

- The lender lends in all states ($l_s = 1$ for all $s \in \{l, h\}$) and the borrower always repays δv, i.e., $r_s = \delta v$ for all $s \in \{l, h\}$.

- This relational contract is feasible only if the loss from lending in high cost states does not outweigh the expected profit from future low cost states:

$$(1-\delta p_l)(\delta v - c_h) + \delta(1-p_h)(\delta v - c_l) \geq 0$$

- The lender's ex-ante expected profit is:

$$\Pi_{AL} = \frac{\delta v - \theta_l c_l - \theta_h c_h}{1-\delta}$$

- The borrower's welfare (ex ante expected utility) is:

$$U_{AL} = v$$

- This relational contract is socially efficient if the borrower project's payoff is greater than the cost of capital in the high state ($v > c_h$).

4.2. **Partial-lending: Lender offers a loan only in the low-cost state.** In this subsection we analyze the partial-lending relational contract. We start by walking through the conditions that determine the feasibility of the partial-lending relational contract and the underlying intuition. Finally, we characterize the partial-lending relational contract.

Starting with the lender's incentives: The cost of lending may be prohibitively high in the high cost state causing either the lender's feasibility condition for lending in all periods to not be met (corollary (1)) or the profits from lending in only low cost states to be higher than always-lending.

Regarding the borrower's incentives, we assume that the borrower observes tomorrow's state *before* deciding to repay; therefore, if the lender lends in only low cost states, the borrower's willingness to repay a loan will always be relatively lower if tomorrow's state realization is a high cost state rather than a low cost state.

Taking the borrower and lender's incentives together, the stationary partial-lending relational contract consists of lending only in the low-cost state and the required loan repayment being conditional on tomorrow's state of the world.[18] Specifically, the lender sets an interest rate (r_l) for all loans, which the borrower has to pay in full if tomorrow's state is a low cost state ($s_{t+1} = c_l$), or lose access to all future

[18]To be precise, there is also a feasible stationary partial-lending relational contract that involves lending only in high cost states. However, that contract would always be weakly dominated by lending in all states and partial-lending in low cost states; therefore, for convenience that relational contract is ignored.

loans. However, if tomorrow is a high cost state ($s_{t+1} = c_h$), the borrower is allowed to partially default and only repay $r_h < r_l$. If the borrower repays less than r_h then the lender stops future lending.[19] This partial-lending relational contract can be formally represented with (r_l, r_h) such that:

(1) the lender's obligation within the relational contract is described as
   i. offer a loan in the first period if, and only if, the cost is c_l,
   ii. at all histories where (a) the lender has offered a loan when the cost was c_l, and (b), received at least r_s when tomorrow's state was s, continue to offer a loan whenever the cost is c_l,
   iii. at any other history, do not offer a loan,
(2) the borrower's obligation within the relational contract is described as:
   i. accept a loan and repay $r_{s'}$ in the first period if a loan is offered, where s' is the next period's state,
   ii. at all histories where the lender has offered a loan whenever the cost was c_l, accept a loan and repay $r_{s'}$, where s' is the next period's state,
   iii. at any other history, accept a loan and repay zero.

For this relational contract to be feasible, we have to verify the lender's and borrower's ICCs are satisfied.

The borrower's incentive compatibility requires:

$$E\left[\sum_{t=1}^{\infty} \delta^t l_t(v - r_t)|c_1\right] \geq r_0(c_1)\ \forall c_1 \in \{c_l, c_h\} \tag{10}$$

where $r_0(c_s) = r_s$.

Equation [10] can be rewritten in the following recursive forms where the subscripts denote today's state and tomorrow's state respectively:

$$\begin{aligned} U_{LL} = &\quad v - r_l + \delta\left[p_l U_{LL} + (1 - p_l)U_{LH}\right] &&\geq v \\ U_{LH} = &\quad v - r_h + \delta\left[p_h U_{HH} + (1 - p_h)U_{HL}\right] &&\geq v \\ U_{HH} = &\quad 0 + \delta\left[p_h U_{HH} + (1 - p_h)U_{HL}\right] &&\geq 0 \\ U_{HL} = &\quad 0 + \delta\left[p_l U_{LL} + (1 - p_l)U_{LH}\right] &&\geq 0 \end{aligned} \tag{11}$$

Because U_{HL} is a weighted average of U_{LL} and U_{LH}, and U_{HH} is a fraction of U_{HL}, the last two inequalities are non-binding. Intuitively, the borrower has more incentive

[19] We do not rule out situations where $r_h = 0$.

to renege when the current state is low and the borrower has already received a loan than when the borrower has not received a loan. Thus, we restrict our attention to the first two constraints in equation [11] that are potentially binding. They can be rewritten as:

$$\begin{aligned} &(12)\quad v\delta(1-\delta p_h) \;\geq\; [(1-\delta p_h)-\delta^2(1-p_l)(1-p_h)]r_l+\delta(1-\delta p_h)(1-p_l)r_h \\ &(13)\quad v\delta^2(1-p_h) \;\geq\; \delta^2 p_l(1-p_h)r_l+(1-\delta p_h)(1-\delta p_l)r_h \end{aligned}$$

Intuitively, inequalities (12) and (13) require that, conditional on getting a loan, the borrower is sufficiently incentivised to repay r_l when tomorrow's state is low and to repay r_h when tomorrow's state is high, respectively.

The lender's expected discounted profit at time $t = 0$ simplifies to:

$$\Pi_0 = E\left[\sum_{t=0}^{\infty}\delta^t l_t(r_t - c_t)|c_0\right]\ \forall c_0 \in \{c_l, c_h\} \tag{14}$$

Because, in a partial-lending relational contract, loans are only disbursed in the low-cost state, the lender's problem (that is, maximize Π_0 as described in equation [14]) can be simplified to maximizing the expected per-period profit in the low cost state:

$$\begin{aligned} \max_{r_l, r_h} \quad & p_l r_l + (1-p_l)r_h - c_l \\ & \text{s.t. inequalities [12] and [13] hold} \\ & \text{and } r_l \geq r_h \geq 0 \end{aligned} \tag{15}$$

The solution to this optimization problem can be solved with a linear program. Using this solution, we can state the contract's feasibility condition in lemma (1).

Lemma 1. *The lender will be able to credibly offer loans only in the low-cost state, if, and only if, the following condition is satisfied:*

$$\delta v\left[p_l + \frac{\delta(1-p_l)(1-p_h)}{1-\delta p_h}\right] \geq c_l$$

The proof of lemma (1) is in the appendix.

To sum, the partial-lending relational contract equilibrium is characterized by the following:

- The lender only lends in the low cost state ($l_l = 1$ and $l_h = 0$).
- The size of borrower's repayment is conditional on the next period's state:

$$\begin{aligned} r_l^* &= \delta v \\ r_h^* &= \delta v \frac{\delta(1-p_h)}{(1-\delta p_h)} \leq r_l^* \end{aligned}$$

- This relational contract is feasible (and profitable) only if the expected repayment covers the cost of lending in the low cost state:

$$\delta v \left[p_l + \frac{\delta(1-p_l)(1-p_h)}{1-\delta p_h} \right] \geq c_l$$

- The lender's ex-ante expected profit is:

$$\Pi_{PL} = \frac{\theta_l}{1-\delta} \left[\left(p_l + \frac{\delta(1-p_l)(1-p_h)}{1-\delta p_h} \right) \delta v - c_l \right]$$

- The borrower's welfare (ex ante expected utility) is:

$$U_{PL} = \frac{\theta_l}{1-\delta} \left[v - \left(p_l + \frac{\delta(1-p_l)(1-p_h)}{1-\delta p_h} \right) \delta v \right]$$

- This relational contract is socially efficient if the borrower project's payoff is less than the cost of capital in the high state ($v < c_h$).

5. Comparative statics

Relational contracts are often less robust to greater economic volatility than other contracts. To gain both more intuition for the relational contract characterization and to understand how the optimal relational contracts change, this section analyzes how the feasibility and profitability of the always-lending and partial-lending relational contracts are affected by different economic parameters, such as persistence of economic recessions or depth of depressions. We introduce a number of comparative statics.

5.1. **Deep financial crisis versus a shallow recessions.** Economic depressions often make maintaining relational contracts less feasible. In this subsection, we analyze how volatility in lender costs affect always-lending and partial-lending. To do so, we examine the experiment where the lender's expected cost does not change — only the dispersion in the cost varies. We show that with greater dispersion in costs, the lender's optimal lending contract is more likely to be a partial-lending contract than an always-lending contract because, with greater cost dispersion, the lender's expected profits from partial-lending rise and the feasibility of always-lending falls.

To start, we introduce the parameter, Δ, that measures the dispersion in the lender's cost between the low and high cost states while holding the lender's expected cost, $\bar{c}$, and the stationary distribution of states (θ_l and θ_h) fixed. Formally, Δ is defined by the following two equations:

$$\Delta \equiv c_h - c_l \text{ such that } \bar{c},\ \theta_l, \text{ and } \theta_h, \text{ are constants} \quad (16)$$
$$\text{where } \bar{c} \equiv \theta_l c_l + \theta_h c_h$$

Therefore, as we increase Δ we increase the dispersion in the lender's possible realization of costs. In effect, we consider large values of Δ simulate deep financial crises and large economic booms whereas small values of Δ simulate a calm economic period with only shallow recessions.

Lemma 2. *As the measure of cost dispersion, Δ, rises:*

(1) the expected profitability of the always-lending contract is unchanged but the expected profitability of the partial-lending contract increases,
(2) the feasibility of the always-lending contract decreases whereas the feasibility of the partial-lending contract increases.

To gain intuition for lemma (2), figure (1) plots the expected discounted profits for always-lending (blue line) and partial-lending (orange line), as well as the expected discounted profit from always-lending starting from a high state (dashed blue line) as we vary the cost dispersion parameter, Δ (x-axis).

Starting with the profitability and feasibility of the always-lending relational contract: The expected profitability of always-lending does not change because the stationary distribution of low and high cost states do not change (horizontal blue line). However, the feasibility of this contract decreases as dispersion rises because the expected future profits of lending when starting in a high cost state, linearly fall (the dashed blue line). The intuition for this result follows from corollary (1). Specifically, for the always-lending contract to be feasible, the lender must make non-negative expected profits in all periods; but as dispersion rises, if the lender is in a high state, the initial expected losses from lending rise.

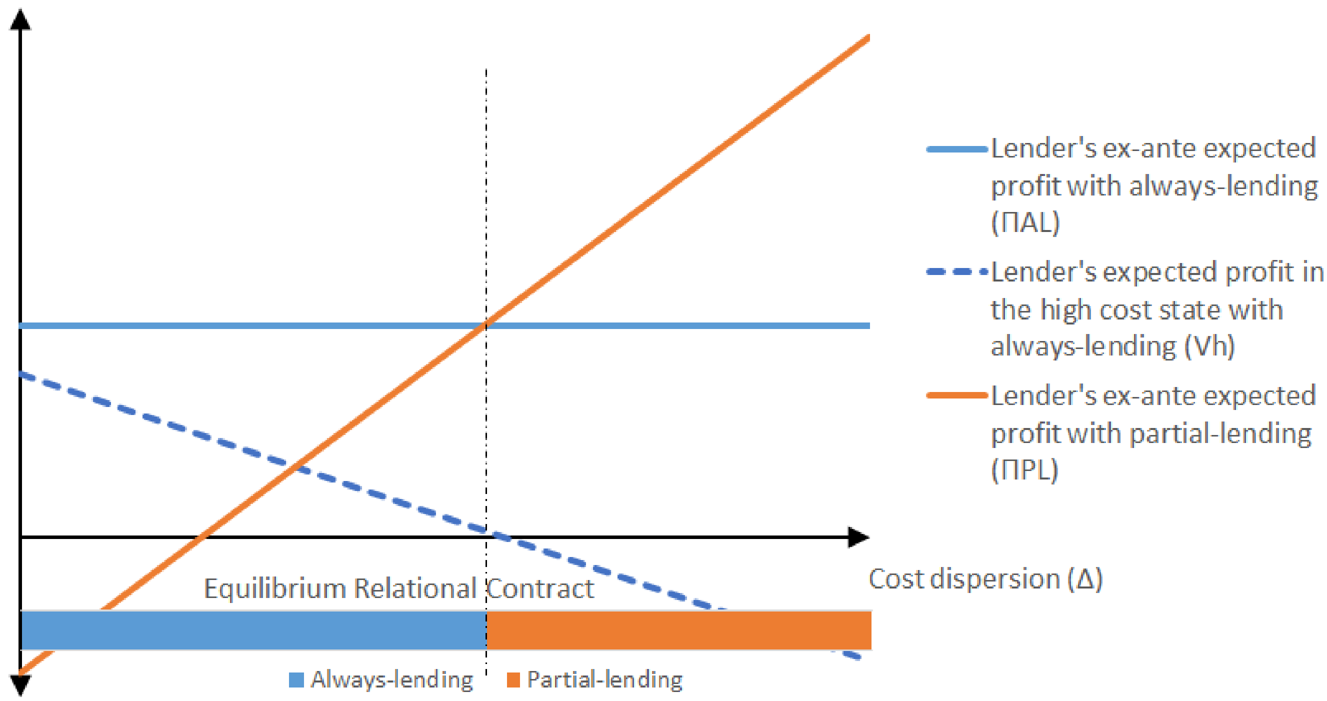


FIGURE 1. Lender's expected profit as a function of cost dispersion

Regarding the partial-lending relational contract, as dispersion rises, the partial-lending contract is both more profitable and likely to be feasible (the orange line). The intuition for this result follows from that the lender only lends in the low cost state, therefore, as dispersion rises, the lender's realized cost of lending (c_l) linearly falls and consequently the lender's profitability rises. Moreover, since the feasibility of partial-lending solely requires that the lender makes positive profits in a low cost state (equation 15), the feasibility of the partial-lending relational contract also rises.

Finally, we see that as the cost dispersion rises sufficiently, the equilibrium relational contract switches from always-lending to partial-lending. In figure (1), the switch occurs because partial lending is more profitable (orange line is higher than blue line), however, it is also possible that the switch occurs because always lending becomes non-feasible (dashed blue line goes below zero).

5.2. **Persistence of economic recessions.** In addition to the depth of the economic crisis, the persistence or stickiness of economic crisis can effect the feasibility and profitability of relational contracts. This subsection analyzes how greater persistence (that is, tomorrow's state is more likely to be the same as today's state) affects the equilibrium. Moreover, to isolate persistence, we vary the probability of moving between states without changing the invariant distribution of states, that is, the ex-ante probability of either state. We show that greater state persistence is more likely to lead to a partial-lending relational contract than an always-lending relational contract.

To start, we introduce the parameter, ρ, that measures the stationary probability of observing two consecutive periods with the same state.

$$\rho \equiv \theta_l p_l + \theta_h p_h \tag{17}$$

Therefore, by varying ρ but ensuring that the stationary distribution of low and high cost states (θ_l and θ_h) does not change, we can examine how the feasibility and profitability of the different relational contracts responds to changes in stability.

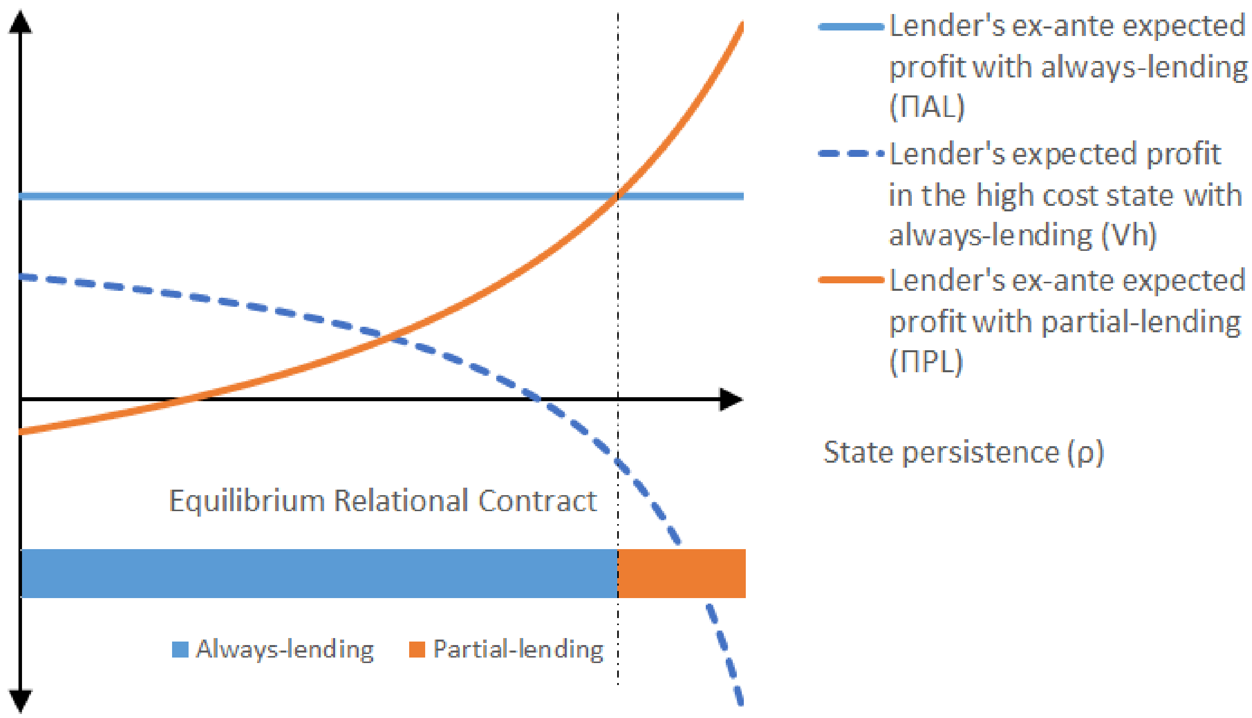
Lemma 3. *As the measure of state persistence, ρ, rises but the stationary distribution of each state (Θ) does not change:*

1. *the expected profitability of the always-lending contract is unchanged but the expected profitability of the partial-lending contract increases,*
2. *the feasibility of the always-lending contract decreases whereas the feasibility of the partial-lending contract increases.*

To gain intuition for lemma (3), figure 2 plots the expected discounted profits for always-lending (blue line) and partial-lending (orange line), as well as the expected discounted profit from always-lending starting from a high state—the always lending feasibility condition (dashed blue line) as we vary the persistence parameter (x-axis).

Starting with the profitability and feasibility of the always-lending relational contract. Similar to varying the cost dispersion parameter, the expected profitability of always-lending does not change because the stationary distribution of low and high cost states do not change. However, the feasibility of this contract decreases as persistence rises because the expected future profits of continuing to lend when starting in a high cost state falls (the dashed blue line). The intuition for this result follows from corollary (1). Specifically, for the always-lending contract to be feasible, the lender must make non-negative expected profits in all periods; but as

Figure 2. Lender's expected profit as a function of state persistence



persistence rises, if the lender is in a high state, the expected losses from continuing to lend rise.

Regarding the partial-lending relational contract, as persistence rises, the partial-lending contract is both more profitable and likely to be feasible (orange line is increasing in persistence). Recall, the lender only lends in the low cost state, therefore, as persistence rises, the lender is more likely to receive the higher repayment associated with a low cost state (r_l) than the partial defaulted payment (r_h). In effect, this persistence causes the lender's profitability to rise. In turn, since the feasibility of partial-lending solely requires that the lender makes positive profits in a low cost state (equation 15), the feasibility of the partial-lending relational contract also rises.

To sum, as state persistence rises, the feasibility of an always-lending contract falls, whereas, both the feasibility and profitability of a partial-lending contract rises. Therefore, as persistence rises, the equilibrium lending strategy is more likely to be a partial-lending relational contract.

6. Discussion

The model describes how the future affects the borrower's decision today, and in turn affects the strategy undertaken by the lender. Furthermore, the potential to renege occurs for both parties: The borrower may renege on repaying the lender; the lender could renege on the implicit promise to offer loans. Thus, even if, ex-ante the lender's profits were maximized by offering loans in all states, this outcome may not be credible due to the potential to renege when the cost of lending is high. The equilibrium of the game may stipulate states of the world where no loans are given and where full repayment is not expected.

One implication is that the lender may have difficulties financing its operations and its loan origination activities. Because of the nature of the equilibria described above, equity financing on the part of the lender would lessen these concerns because the long-run equity investor would be able to reap the benefits of offering loans at a loss in some periods. In many real world settings, however, relationship lenders frequently access the short term debt markets for financing. In this case, our model suggests that short term debt, and therefore the securitization of individual loans, may be a problematic financing structure for loans that rely on dynamic repayment incentives.

References

Dilip Abreu. On the theory of infinitely repeated games with discounting. *Econometrica*, 56:383–396, 1988.

George Baker, Robert Gibbons, and Kevin J Murphy. Relational contracts and the theory of the firm. *The Quarterly Journal of Economics*, 117(1):39–84, 2002.

A. Banerjee and E. Duflo. *Poor economics: a radical rethinking of the way to fight global poverty*. PublicAffairs, 2011.

A.V. Banerjee and E. Duflo. Reputation effects and the limits of contracting: A study of the indian software industry. *The Quarterly Journal of Economics*, 115 (3):989–1017, 2000.

Daniel Barron and Michael Powell. Policies in relational contracts. *American Economic Journal: Microeconomics*, 11(2):228–49, 2019.

Allen N Berger and Gregory F Udell. Relationship lending and lines of credit in small firm finance. *Journal of Business*, pages 351–381, 1995.

Mitchell Berlin and Loretta J Mester. Deposits and relationship lending. *The Review of Financial Studies*, 12(3):579–607, 1999.

Ben Bernanke and Mark Gertler. Agency costs, net worth, and business fluctuations. *American Economic Review*, 79:14–31, 1989.

Frederic Boissay and Reint Gropp. Payment defaults and interfirm liquidity provision. *Review of Finance*, 17(6):1853–1894, 2013.

P. Bolton and D.S. Scharfstein. Optimal debt structure and the number of creditors. *Journal of Political Economy*, pages 1–25, 1996.

Arnoud WA Boot and Anjan V Thakor. Can relationship banking survive competition? *The journal of Finance*, 55(2):679–713, 2000.

E. Breza. Peer pressure and loan repayment: Evidence from the krishna default crisis. *Working Paper*, 2012.

M. Burkart and T. Ellingsen. In-kind finance: A theory of trade credit. *American Economic Review*, pages 569–590, 2004.

C.T. Carlstrom and K.A. Samolyk. Loan sales as a response to market-based capital constraints. *Journal of Banking & Finance*, 19(3):627–646, 1995.

V. Cunat. Trade credit: suppliers as debt collectors and insurance providers. *Review of Financial Studies*, 20(2):491–527, 2007.

S. Drucker and M. Puri. On loan sales, loan contracting, and lending relationships. *Review of Financial Studies*, 22(7):2835–2872, 2009.

B. Feigenberg, E.M. Field, and R. Pande. Building Social Capital Through Micro-Finance. *NBER Working Papers*, 2010.

D. Fudenberg, D. Levine, and E. Maskin. The folk theorem with imperfect public information. *Econometrica: Journal of the Econometric Society*, pages 997–1039, 1994.

Paul Gertler, Brett Green, and Catherine Wolfram. Digital collateral. Technical report, National Bureau of Economic Research, 2021.

X. Giné, K. Krishnaswamy, and A. Ponce. Strategic default in joint liability groups: Evidence from a natural experiment in india. 2011.

Xavier Gine and Dean Karlan. Group versus individual liability: Evidence from a field experiment in the Philippines. Yale University Economic Growth Center working paper 940, 2006.

Xavier Gine and Dean Karlan. Group versus individual liability: Long term evidence from philippine microcredit lending groups. Yale University working paper, 2009.

M. Halac. Relational contracts and the value of relationships. *The American Economic Review*, 102(2):750–779, 2012.

Indian Express. Don't repay microfinance loans: Tdp. November 2, 2010.

International Business Times. 54 committed suicide in ap due to microfinance debts, says serp: report. October 29, 2010.

Tor Jacobson and Erik Von Schedvin. Trade credit and the propagation of corporate failure: An empirical analysis. *Econometrica*, 83(4):1315–1371, 2015.

D. Karlan and J. Zinman. Observing unobservables: Identifying information asymmetries with a consumer credit field experiment. *Econometrica*, 77(6):1993–2008, 2009. ISSN 1468-0262.

J. Levin. Relational incentive contracts. *The American Economic Review*, 93(3): 835–857, 2003.

Jin Li and Niko Matouschek. Managing conflicts in relational contracts. *American Economic Review*, 103(6):2328–51, 2013.

R. Macchiavello and A. Morjaria. The value of relational contracts: Evidence from a supply shock to kenyan flower exports. 2012.

D. McAdams. Performance and turnover in a stochastic partnership. *American Economic Journal: Microeconomics*, 3(4):107–142, 2011.

Gerard McGuinness, Teresa Hogan, and Ronan Powell. European trade credit use and sme survival. *Journal of Corporate Finance*, 49:81–103, 2018.

J. McMillan and C. Woodruff. Interfirm relationships and informal credit in vietnam. *The Quarterly Journal of Economics*, 114(4):1285–1320, 1999.

J. Morduch. The microfinance promise. *Journal of economic literature*, 37(4):1569–1614, 1999.

C.A. Parlour and G. Plantin. Loan sales and relationship banking. *The Journal of Finance*, 63(3):1291–1314, 2008.

Mitchell A Petersen and Raghuram G Rajan. The benefits of lending relationships: Evidence from small business data. *The Journal of Finance*, 49(1):3–37, 1994.

Mitchell A Petersen and Raghuram G Rajan. The effect of credit market competition on lending relationships. *The Quarterly Journal of Economics*, 110(2):407–443, 1995.

Raghuram G Rajan. Insiders and outsiders: The choice between informed and arm's-length debt. *The Journal of finance*, 47(4):1367–1400, 1992.

R. Townsend. Optimal contracts and competitive markets with costly state verification. *Journal of Economic theory*, 21(2):265–293, 1979.

World Bank. Mixmarket. 2019. URL `https://datacatalog.worldbank.org/search/dataset/0038647`.

Appendix

Proof. of proposition (2)

The stationary expected per-period profits when lending in all states is $\delta v - \theta_l c_l - \theta_h c_h$ and when lending only in the low-cost state is $\theta_l(p_l r_l^* + (1-p_l)r_h^* - c_l)$. Thus, after plugging in for r_l^* and r_h^*, the lender's expected profit is higher when lending in every period if, and only if,:

$$\delta v - \theta_l c_l - \theta_h c_h \geq \theta_l \left[\left(p_l + \frac{\delta(1-p_l)(1-p_h)}{1-\delta p_h} \right) \delta v - c_l \right]$$

which with some algebraic manipulation gives:

$$\delta v \left[1 - \theta_l \left(p_l + \frac{\delta(1-p_l)(1-p_h)}{1-\delta p_h} \right) \right] \geq (1-\theta_l) c_h$$

The remainder is based on lender credibility conditions (ICCs) given by corollary (1) and lemma (1). □

Proof. of corollary (1)

Assuming the lender always offers loans we can write V as:

$$\begin{aligned} V_s &= (\delta v - c_s) + \delta[p_s V_s + (1-p_s)V_{-s}] \qquad \forall s \\ &= \frac{(\delta v - c_s) + \delta(1-p_s)V_{-s}}{1-\delta p_s} \\ &= \frac{(\delta v - c_s)}{1-\delta p_s} + \frac{\delta(1-p_s)}{1-\delta p_s} \times \left[\frac{(\delta v - c_{-s}) + \delta(1-p_{-s})V_s}{1-\delta p_{-s}} \right] \Rightarrow \end{aligned}$$

$$V_s \left[\frac{(1-\delta p_s)(1-\delta p_{-s}) - \delta^2(1-p_s)(1-p_{-s})}{1-\delta p_{-s}} \right] = (\delta v - c_s) + \delta(1-p_s)\left(\frac{\delta v - c_{-s}}{1-\delta p_{-s}} \right) \Rightarrow$$

$$\begin{aligned} &V_s \left[(1-\delta p_s)(1-\delta p_{-s}) - \delta^2(1-p_s)(1-p_{-s}) \right] \\ &\quad = (1-\delta p_{-s})(\delta v - c_s) + \delta(1-p_s)(\delta v - c_{-s}) \end{aligned} \tag{18}$$

Since $\delta \in (0,1)$ and $p_s \in (0,1)$, the term that multiples V_s on left hand side of equation [18] is greater than zero. Thus, the right hand side of equation [18] tells us that the sign of V_s depends on the weighted values from the profits in each state.

Since we have assumed that $c_l < c_h$, then it can be shown that equation [18] implies that $V_l > V_h$.[20] Therefore to ensure ICC for the lender to give loans in all states we

[20]We can write equation [18] as $\kappa V_s = (1-\delta p_{-s})u_s + \delta(1-p_s)u_{-s}$, solving this equation and noting that $u_l > u_h$ we can conclude that $V_l > V_h$.

only need to verify that:

$$
\begin{aligned}
V_h &\geq 0 \Rightarrow \\
(1-\delta p_l)(\delta v - c_h) + \delta(1-p_h)(\delta v - c_l) &\geq 0 \qquad (19)
\end{aligned}
$$

If equation [19] is not satisfied, then the lender would renege on offering loans if a high cost state ($c_s = c_h$) is observed. □

Proof. of lemma (1)

The constraints in the lender's problem in equation [15] form a non-empty compact set of feasible (r_l, r_h) pairs, and the linear objective function is continuous. Thus, by the Weierstrass extreme value theorem, an optimal (r_l^*, r_h^*) pair exists. Moreover, since the objective function is (weakly) convex, a local maximum is also the global maximum. Therefor, the problem in equation [15] has a unique solution that takes one of the three following forms:

(1) $r_l = 0$, $r_h > 0$, only one of the constraints in equation [13] binds.
(2) $r_l > 0$, $r_h = 0$, only one of the constraints in equation [13] binds.
(3) $r_l > 0$, $r_h > 0$, both constraints in equation [13] bind.

Direct payoff comparison for the lender between the above three possibilities reveals the optimal repayment rates. The first case imposes that $r_l^{(1)} = 0$, hence the objective function in equation [15] simplifies to $(1-p_l)r_h - c_l$ and equation [13] simplifies to $r_h \leq \frac{v}{1-p_l}$ and $r_h \leq \frac{\delta^2 v(\frac{1-p_h}{1-\delta p_h})}{1-\delta p_l}$. Since the latter inequality implies the former, the maximizer will be $r_h^{(1)} = \frac{\delta^2 v(\frac{1-p_h}{1-\delta p_h})}{1-\delta p_l}$. Thus, the maximum expected repayment will be: $(1-p_l)r_h^{(1)} = \frac{\delta^2 v(1-p_l)(1-p_h)}{(1-\delta p_l)(1-\delta p_h)}$

The second case imposes that $r_h^{(2)} = 0$, hence the objective function in equation [15] simplifies to $p_l r_l - c_l$ and equation [13] simplifies to $r_l \leq \frac{v}{p_l}$ and $r_l \leq \frac{\delta v(1-\delta p_h)}{(1-\delta p_h)-\delta^2(1-p_l)(1-p_h)}$. Since the latter inequality implies the former, the maximizer will be $r_l^{(2)} = \frac{\delta v(1-\delta p_h)}{(1-\delta p_h)-\delta^2(1-p_l)(1-p_h)}$. Thus, the maximum expected repayment will be: $p_l r_l^{(2)} = \frac{\delta v p_l(1-\delta p_h)}{(1-\delta p_h)-\delta^2(1-p_l)(1-p_h)}$

In the third case, both constraints in equation [13] bind. Thus, the optimal repayment rates can be found by solving the system of equation when both inequalities in equation [13] bind. The solution is $r_l^{(3)} = \delta v$ and $r_h^{(3)} = \delta^2 v(\frac{1-p_h}{1-\delta p_h})$. The maximum expected repayment will be: $p_l r_l^{(3)} + (1-p_l)r_h^{(3)} = \frac{\delta v p_l(1-\delta p_h)+\delta^2 v(1-p_l)(1-p_h)}{1-\delta p_h}$, which is higher than the maximum expected repayment in the first and second cases. As a result, $r_l^* = r_l^{(3)}$ and $r_h^* = r_h^{(3)}$.

Finally, in order for the lender's commitment to lend when cost is low to be credible, the low cost should not exceed the expected repayment:

$$\delta v[p_l + \frac{\delta(1-p_l)(1-p_h)}{1-\delta p_h}] \geq c_l \tag{20}$$

□