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What is Measured in National Accounts? [*]

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Abstract

Most statistical agencies construct sectoral real GDP using double deflation and base period prices. When the base period price used for intermediate inputs is not equal to their marginal revenue product, such as when firms apply a markup, real GDP fluctuations become mechanically linked to variations in intermediate inputs. This is because these inputs generate profits that are incorporated into real value added. Taking this channel into account, we demonstrate that real GDP reported in national accounts substantially diverges from a theory-consistent "physical" value added. This, in turn, has implications for the measurement of productivity. Between 1999 and 2021, "physical" productivity cumulative growth in the Finance sector was 15pp lower compared to the Solow Residual, while it was 15pp higher in the Manufacturing sector.

Keywords: Economic Measurement, National Accounts, Markups, Productivity.

JEL Classification: E1, E3, O4, O51

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[*] The views in this paper are solely the responsibility of the authors and should not necessarily be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. All errors are our own.

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1 Introduction

Since the publication of their 2021 “Blue Book”, the UK’s Office for National Statistics started to measure real GDP in the national accounts using double deflation. This methodological update follows the premise that *“double deflation is internationally accepted as the best approach to producing volume estimates of industry Gross Value Added”*. Indeed, double-deflation, taking the difference between gross output and intermediate inputs, has been the international standard since the System of National Accounts (SNA 1993) was adopted by the international community to facilitate international comparisons of national economic statistics. In the United States, this accounting practice has been used since 1992, while it has been the standard practice in the Euro Area since the creation of Eurostat. Although double deflation is the state-of-the-art method for real value-added measurement, its accuracy relies on several assumptions that are not met in practice. As a result, our understanding of relative growth rates across sectors is biased. These limitations are important for researchers that match their quantitative models with the data produced by statistical agencies.

To understand accounting procedures in national accounts, it is useful to come back to the notion of Real value added (RVA). In its original concept described by Fabricant (1940), Sims (1969) and Arrow (1974), RVA is an ideal index of an industry’s net physical output that can be implicitly derived from the production function if it separates primary factors and intermediate inputs. Using Arrow (1974)’s words: we can "*imagine capital and labor cooperating to produce an intermediate good called real value added, which in turn cooperates with materials to produce the final product.*”[1] Concretely, consider an economy in which gross output is produced through the combination Y of labor and capital inputs, as well as intermediate inputs X. According to Sims (1969), the definition of real value added corresponds only to the quantity Y, which can be referred to as Physical Value-Added (PVA). Changes in the quantity of intermediate inputs X only impact PVA when it affects the quantity Y produced in the economy.

In practice, statistical agencies do not observe Physical Value-Added. Instead, national accountants construct a measure of "statistical value added" using double deflation, a method that consists of taking the difference between gross output and intermediate inputs, both valued using base period prices. Using base period prices to measure real GDP is a standard procedure, as merely counting the quantity of goods produced in a country would be meaningless: what would it mean, for example, to compute real value-added as in the automotive sector by taking the difference between the number of cars produced and the number of its

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[1] This quote is taken from Arrow (1974), pp 4-5.

intermediate inputs? One needs a common unit of account, which is why base period prices are necessary for the construction of sectoral real value added, as well as aggregate real GDP.

The main contribution of this paper is to demonstrate a potential bias that may arise from the use of double deflation in comparing theory-based measures of real GDP and productivity with their statistical counterparts. We show that double deflation implicitly assumes that the base period price used to deflate inputs reflects both the marginal cost of inputs and the marginal revenue that their usage can generate. If this assumption holds, then real GDP is indeed an accurate measure of PVA.

The crux of the issue is that while RVA cannot be constructed without prices, the presence of distorted prices creates measurement issues. In the presence of distortions (markups, taxes, or other factors), using prices to construct RVA creates a wedge between the statistical measure and the theory-consistent PVA measure. It is important to note that price distortions need not vary over time to create a bias in RVA measurement. Even with constant markups, intermediate inputs generate more revenues than their cost, meaning using more inputs results in more statistical value-added, even if domestic factors (labor and capital) and technology remain unchanged. This happens because a sector's net operating surplus is included in the statistical agencies' measure of RVA. Consequently, a sector can create statistical value-added by increasing its profits, *ceteris paribus*.

Our analysis shows that real GDP in the national accounts and PVA differ mainly for sectors with three characteristics: (i) markups are large, (ii) the intermediate input share is large, and (iii) variations of PVA and intermediate inputs usage are not collinear. This last condition is interesting because it illustrates why a measurement bias can arise even when markups are constant. To gain clarity, consider first a situation with constant markups and where intermediate inputs are proportional to PVA. In this case, gross output (which is a combination of intermediate inputs and PVA) is also proportional to PVA, and so are profits. This, in turn, implies that a one percent change in PVA is associated with the same proportional change in profits, and hence profits do not bias PVA's measured *growth rate*. Things are different when intermediate inputs and PVA are not collinear though, because in such a case PVA and profits do not have the same growth rate. This wedge then creates a disconnect between the value-added measured by statistical agencies and its theoretical counterpart.

Using data from the Bureau of Economic Analysis (BEA), we then evaluate how profits and distorted prices in general have biased our measure of sectoral output and sectoral productivity in the US. We use the BEA's data on net operating surplus to build a measure of markups at the sector level, as well as standard input-output analysis, to construct a new version of national account statistics.[2] We then compute a measure of PVA for each sector and compare it to the statistical measure of real GDP.

Our results highlight the potential biases in widely used data and how they can affect our understanding of economic activity. We compared the evolution of statistical real value-added (or sectoral real GDP) to our measure of PVA, taking fluctuations in net operating surplus into account. Our analysis of the US economy shows that between 1999 and 2021, there is a 1.24pp difference in cumulative growth between our PVA measurement and real GDP as measured by the BEA. While differences between statistical and physical objects are not significant for the US economy as a whole, individual sectors reveal interesting discrepancies.

We focused on two sectors, Finance and Manufacturing, which illustrate different biases in national statistics. Both sectors heavily rely on intermediate input, with input costs accounting for an average of 55% of total sales in Finance and 70% in Manufacturing between 1999 and 2021. While year-to-year growth rates are highly correlated, the cumulative bias over time leads to significant differences by the end of our sample. Normalizing both measures in 1999, we find that PVA's cumulative growth in 2021 is more than 13pp higher than real GDP growth in the Manufacturing Sector. In Finance, PVA's cumulative growth is 20pp lower than real GDP growth. In other words, national accounts severely underestimated the growth of PVA in Manufacturing, while it overestimated it in Finance.

Equipped with our measure of PVA, we constructed a measure of "Physical Productivity" as the fluctuations in PVA that are not due to observed movements in labor and capital, which we can then compare to the more standard Solow Residual. Results showed that by the end of our sample in 2021, cumulative growth of physical productivity is about 15pp smaller than that of the Solow residual in Finance, while it is 15pp larger than Solow Residual in Manufacturing. Once again, national accounts' data overestimated productivity growth in Finance, while it underestimated productivity growth in Manufacturing.

Relation to the literature The importance of using base period prices in real GDP measurement has been analyzed in several contexts, most notably Kehoe and Ruhl (2008) and Burstein and Cravino (2015). The role of markups in generating a link between intermediate input and measured productivity has been discussed in several papers such as Hall (1988) and Basu and Fernald (2002), and more recently in Gopinath and Neiman (2014). The mismatch between the

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[2] The BEA defines its measure of net operating surplus as: *"a profits-like measure that shows business income after subtracting the costs of compensation of employees (received), taxes on production and imports less subsidies, and consumption of fixed capital from value added, but before subtracting financing costs and business transfer payments."*

theory-consistent measure of real value added (PVA, defined from the production function) and statistical real GDP measured in the national accounts is significant in many contexts. For instance, using double deflation to measure real GDP in a macro model has a substantial impact on the statistical properties of real GDP in the model's simulations. In de Soyres and Gaillard (2021), we argue that double deflation is critical to understanding cross-country real GDP co-movement. It is a way to reconcile data and model-based simulations and can help solve the "Trade Comovement Puzzle" in an International Business Cycle model with markups.

Overall, our findings underscore the importance of paying close attention to the measurement of economic variables used in policy making decisions and academic research, especially regarding the rise of market power and markup in the US (De Loecker et al., 2020).

Finally, it is important to note that the analysis aims to quantify how the presence of markups creates a measurement issue in national accounts data, which is conceptually distinct from the topic of resource misallocation. Recent papers, such as Baqaee and Farhi (2020), highlighted how markup heterogeneity across firms implies that the allocative efficiency of the US economy has changed over time. As production factors are reallocated to high-markup firms, this reallocation process accounts for approximately half of aggregate TFP growth from 1997 to 2015. While both misallocation and measurement are significant issues, they highlight different aspects of how markups impact the economy.

2 An Accounting Framework with Input-Output Linkages

Consider an economy with J sectors indexed by j. We present a simple accounting framework and show how the presence of a wedge between total sales and total cost at the sectoral level gives rise to a disconnect between Real Value Added (RVA or real GDP) (as measured by statistical agencies) and what we call "Physical Value Added". For any variable A, we define the proportional change as $\widehat{A}_t = \frac{\Delta A_t}{A_{t-1}}$.

2.1 Production

Gross output (GO_{jt}) in sector j is produced using labor L_{jt}, physical capital K_{jt}, productivity Z_{jt}, and intermediate inputs X_{jt}, such that:

$$GO_{jt} = \left(Z_{jt} L_{jt}^{\alpha_j} K_{jt}^{1-\alpha_j}\right)^{\gamma_j} \cdot X_{jt}^{1-\gamma_j}, \tag{1}$$

where $\alpha_j \in (0,1)$ and $\gamma_j \in (0,1)$. We consider the presence of a *markup wedge* (μ_{jt}) which measures the difference between total sales and total cost (TC_{jt}):

$$\mu_{jt} = \frac{P_{jt} GO_{jt}}{TC_{jt}} \,. \tag{2}$$

The situation where $\mu_{jt} > 1$ can arise because firms are making profits or for other reasons, for example the presence of sales taxes. The definition of μ_{jt} and the production structure provides a relationship between the intermediate input share of total cost, $(1 - \gamma_j)$, and the share of total sales, $\frac{P^X_{jt-1} X_{jt-1}}{P_{jt-1} GO_{jt-1}}$, such that:

$$\frac{P^X_{jt-1} X_{jt-1}}{P_{jt-1} GO_{jt-1}} = \frac{1 - \gamma_j}{\mu_{jt-1}} \,. \tag{3}$$

2.2 From sectoral GDP to aggregate GDP

At the sectoral level, we can define the change in sectoral real GDP (denoted $sRGDP_{jt}$ for sector j) between $t-1$ and t by keeping prices at their base period value and using the change in gross outputs and inputs so that:

$$\widehat{sRGDP}_{jt} = \frac{P_{jt-1} \Delta GO_{jt} - P^X_{jt-1} \Delta X_{jt}}{sRGDP_{jt-1}} = \underbrace{\frac{P_{jt-1} GO_{jt-1}}{sRGDP_{jt-1}}}_{= \omega_{jt-1}} \left[\frac{\Delta GO_{jt}}{GO_{jt-1}} - \frac{P^X_{t-1} \Delta X_{jt}}{P_{jt-1} GO_{jt-1}} \right] , \tag{4}$$

where ω_{jt-1} is the ratio of total sales to value added and is therefore larger or equal to one. When total sales equal total cost, ω_{jt-1} is simply equal to the inverse of the value added share in gross output, $\omega_{jt-1} = \frac{P_{jt-1} GO_{jt-1}}{w_{jt-1} L_{jt-1} + r_{jt-1} K_{jt-1}} = 1/\gamma_j$. However, in the presence of a wedge between sales and cost, we can use the definition of μ_{jt-1} and equation (3) to obtain:

$$\omega_{jt-1} = \frac{P_{jt-1} GO_{jt-1}}{P_{jt-1} GO_{jt-1} - P^X_{jt-1} X_{jt-1}} = \frac{1}{1 - \frac{P^X_{jt-1} X_{jt-1}}{P_{jt-1} GO_{jt-1}}} = \frac{\mu_{jt-1}}{\gamma_j + \mu_{jt-1} - 1} \,. \tag{5}$$

Note that when the markup wedge $\mu_{jt-1} > 1$, the ratio of total sales to value added ω_{jt-1} is smaller than $1/\gamma_j$.

At the national level, the change in aggregate real GDP of the country can be defined as the sum of the value added in each sector, given by:

$$\widehat{RGDP}_t = \frac{\sum_{j=1}^{J} P_{jt-1}\Delta GO_{jt} - P^X_{jt-1}\Delta X_{jt}}{RGDP_{t-1}} = \sum_{j=1}^{J} \underbrace{\frac{P_{jt-1}GO_{jt-1}}{RGDP_{t-1}}}_{=\, d_{jt-1}} \left[\frac{\Delta GO_{jt}}{GO_{jt-1}} - \frac{P^X_{t-1}\Delta X_{jt}}{P_{jt-1}GO_{jt-1}}\right], \tag{6}$$

where d_{jt-1} defines the Domar weights which is the ratio of sector j's sales to aggregate value added.

The Domar weights d_{jt-1} combine industry-level growth into aggregate growth by weighting each sector's output share by its value added share. By construction, since the numerator is gross output (sales) and the denominator is value added, the weights sum up to more than one by construction. To better understand the Domar weights, we decompose them into a sales-to-value-added ratio at the sectoral level and a sector weight, resulting in:

$$d_{jt-1} = \frac{P_{jt-1}GO_{jt-1}}{sRGDP_{jt-1}} \cdot \frac{sRGDP_{jt-1}}{RGDP_{t-1}} = \omega_{jt-1} \cdot \frac{sRGDP_{jt-1}}{RGDP_{t-1}}. \tag{7}$$

Using (7) together with (4) and (6), we can reconcile industry and country-level RGDP:

$$\widehat{RGDP}_t = \sum_{j=1}^{J} \frac{sRGDP_{jt-1}}{RGDP_{t-1}} \cdot \widehat{sRGDP}_{jt}. \tag{8}$$

Equation (8) indicates that the growth rate of country-level RGDP is the weighted sum of industry-level RGDP growth rates, with weights proportional to each industry's share of aggregate value added.

2.3 A New Real GDP Decomposition

Physical Value Added (PVA) In order to provide a more complete decomposition of real GDP changes, we follow the approach of Fabricant (1940) and Arrow (1974) and define Physical Value Added (PVA) as the "quantity" of the value-added bundle used in gross output production. Specifically, we define PVA as the sum of labor and capital inputs weighted by their respective shares in total factor income, plus a term for total factor productivity:

$$\widehat{PVA}_{jt} = \widehat{Z}_{jt} + \alpha_j \widehat{L}_{jt} + (1-\alpha_j)\widehat{K}_{jt}. \tag{9}$$

Note that, unlike the traditional definition of value added, which subtracts intermediate inputs from gross output, PVA measures the physical quantity of value added used in production.

Real Value Added (RVA) In Equation (8), real GDP change in sector j can be expressed as a weighted sum of changes in labor, productivity, physical capital, and intermediate inputs usage from equation (1) using standard log linearization. Using the PVA definition (9) and (3), real GDP fluctuations can finally be decomposed into (i) fluctuations in Physical Value Added (PVA) and (ii) fluctuations in profits derived from intermediate input usage:

$$\begin{aligned}
\widehat{RGDP}_t &= \sum_{j=1}^{J} d_{jt-1}\left[\gamma_j\left(\widehat{Z}_{jt}+\alpha_j\widehat{L}_{jt}+(1-\alpha_j)\widehat{K}_{jt}\right)+(1-\gamma_j)\widehat{X}_{jt}-\frac{P^X_{jt-1}X_{jt-1}}{P_{jt-1}GO_{jt-1}}\widehat{X}_{jt}\right] \\
&= \sum_{j=1}^{J} d_{jt-1}\left[\gamma_j\left(\widehat{PVA}_{jt}\right)+(1-\gamma_j)\underbrace{\left(\frac{\mu_{jt-1}-1}{\mu_{jt-1}}\cdot\widehat{X}_{jt}\right)}_{\text{Markup Effect}}\right]. \qquad (10)
\end{aligned}$$

The presence of markups ($\mu_{jt-1} > 1$) creates a mismatch between RGDP movements and fluctuations in the "quantity of value added" as measured by PVA. This disconnect appears in (10) in two ways: (i) the "markup effect" term shows that any change in intermediate input usage affects statistical value added, and (ii) markups affect the Domar weight d_{jt-1} in (5) such that the ratio of nominal value added to sales (ω_{jt-1}) is not equal to $1/\gamma_j$.

2.4 Solow Residual (SR) and Physical Productivity (Z)

The real GDP decomposition presented in (10) also bears important implications for the measure of productivity. We define the Solow Residual (SR) at the sectoral level as:

$$\widehat{SR}_{jt} = s\widehat{RGDP}_{jt} - \alpha_j\widehat{L}_{jt} - (1-\alpha_j)\widehat{K}_{jt} \qquad (11)$$

Which can be rewritten as:

$$\begin{aligned}
\widehat{SR}_{jt} \;&=\; \gamma_j\omega_{jt-1}\widehat{Z}_{jt} + \left(\gamma_j\omega_{jt-1}-1\right)\left(\alpha_j\widehat{L}_{jt}+(1-\alpha_j)\widehat{K}_{jt}\right) \qquad (12) \\
&\quad +\; \omega_{jt-1}(1-\gamma_j)\left(\frac{\mu_{jt-1}-1}{\mu_{jt-1}}\widehat{X}_{jt}\right)
\end{aligned}$$

Expression (12) highlights how the presence of markups affects the estimation of physical productivity (Z) and emphasizes the distinction between SR and Z. $\widehat{SR}_{jt}$ captures the fluctuations in RGDP that go beyond the physical changes of domestic factor supply (i.e., beyond changes in $L_{jt}^{\alpha}K_{jt}^{1-\alpha}$). We can use the definition (11) and expression (12) to derive physical productivity $\widehat{Z}_{jt}$ as a function of *observable variables*:

$$\widehat{Z}_{jt} = \frac{\widehat{sRGDP}_{jt}}{\gamma_j \omega_{jt-1}} - \alpha_j \widehat{L}_{jt} - (1-\alpha_j)\widehat{K}_{jt} - \frac{(1-\gamma_j)}{\gamma_j}\frac{(\mu_{jt-1}-1)}{\mu_{jt-1}}\widehat{X}_{jt}\,. \tag{13}$$

2.5 Taking the Accounting Measures to the Data

To fully grasp the extent of the measurement bias discussed earlier, we proceed to quantify it. This will provide a thorough and precise understanding of the discrepancies between the statistically measured real value added and productivity measures, and their theoretical counterparts used in macroeconomic models.

2.5.1 Data

We rely on the Bureau of Economic Analysis (BEA) National Accounts data to conduct our analysis. Table 1 presents the specific data from the BEA that we use, and Table 4 in Appendix provides a mapping between our model variables and the corresponding BEA data. It is important to note that our computations are not affected by the level of "real" variables, which allows us to use BEA variables expressed in indexed quantities.

Table. 1. Description of BEA data used throughout the analysis.

Object	**Indicator**[a]	**Unit**
$GO_{jt}^{nominal}$	Nominal Gross Output	Millions of Dollars, Annual.
$VA_{jt}^{nominal}$	Nominal Value Added	Millions of Dollars, Annual.
$II_{jt}^{nominal}$	Nominal Intermediate Inputs	Millions of Dollars, Annual.
NOS_{jt}	Nominal Net Operating Surplus	Millions of Dollars, Annual.
$COMP_{jt}$	Employee Compensation by Industry	Millions of Dollars, Annual.
GO_Q_{jt}	Gross Output	Index 2012= 100, Quantities, Annual.
VA_Q_{jt}	Value Added	Index 2012= 100, Quantities, Annual.
CAP_QI_{jt}	Capital Stock	Index 2012= 100, Quantities, Annual.
II_QI_{jt}	Intermediate Inputs	Index 2012= 100, Quantities, Annual.
EMP_{jt}	Full-Time and Part-Time Employees	Thousands of People, Annual.

[a] *Notes:* `https://www.bea.gov/resources/guide-interactive-gdp-industry-accounts-tables`

2.5.2 Construction of Variables

The procedure involves several steps to obtain the necessary variables for the mapping of the model. First, the markup ratio μ_{jt-1} is computed using data on Net Operating Surplus (NOS).

The markup ratio measures the wedge between total cost and total sales at the industry level, and it can be recovered using data on nominal gross output (=total sales) and NOS using:

$$\mu_{jt} = \frac{GO_{jt}^{nominal}}{GO_{jt}^{nominal} - NOS_{jt}} \tag{14}$$

Next, the data on total sales and intermediate input payment is used to back out γ_j as follows:

$$1 - \gamma_j = \mu_{jt-1} \cdot \frac{P_{jt-1}^{X} X_{jt-1}}{P_{jt-1} GO_{jt-1}} = \mu_{jt-1} \cdot \frac{II_{jt-1}^{\text{nominal}}}{GO_{jt-1}^{\text{nominal}}} \tag{15}$$

where $II_{jt-1}^{\text{nominal}}$ is the BEA variable recording nominal spending on intermediate inputs. In practice, we take the average over time for each sector. In practice, the average over time is taken for each sector.

In the data, payments to capital are computed as the difference between measured sectoral value added and payments to labor, which means payment to capital cannot be used in the calibration. Using the definition of μ_{jt-1} and the fact that the labor share of total cost is equal to $\gamma_j \alpha_j$, we can obtain the expression the capital share α_j using:

$$\frac{w_{jt-1} L_{jt-1}}{P_{jt-1} GO_{jt-1}} = \frac{\gamma_j \alpha_j}{\mu_{jt-1}} \qquad \Rightarrow \qquad \alpha_j = \frac{\mu_{jt-1}}{\gamma_j} \frac{w_{jt-1} L_{jt-1}}{P_{jt-1} GO_{jt-1}} = \frac{\mu_{jt-1}}{\gamma_j} \frac{COMP_{jt-1}}{GO_{jt-1}^{nominal}} \tag{16}$$

The average over time is again taken for each sector.

Finally, the Domar weights ω_{jt-1} and d_{jt-1} are computed directly from the data using current-year prices of gross output and value added, such that:

$$\omega_{jt-1} = \frac{P_{jt-1} GO_{jt-1}}{\text{Sectoral RGDP}_{jt-1}} = \frac{GO_{jt-1}^{\text{nominal}}}{VA_{jt-1}^{\text{nominal}}} \quad , \qquad d_{jt-1} = \omega_{jt-1} \cdot \frac{VA\_Q_{jt}}{VA\_Q_t} \tag{17}$$

Table 4 in Appendix summarizes the construction of all variables.

3 Results: Quantifying the Measurement Bias

In this section, we focus on the United States and examine the consequences of measuring real value-added inaccurately at both the aggregate and sectoral levels. To achieve this, we follow a three-step process. Firstly, we construct all variables as outlined in Table 4 in Appendix and estimate $\widehat{Z}$ using (13). Secondly, we use our computed $\widehat{Z}$ to calculate $\widehat{PVA}$ using (9), and also obtain the time series for the level of these variables from their growth rates (the "hat variables"). Finally, we compare the properties of statistical variables, such as $\widehat{RGDP}$ and $\widehat{SR}$, with their "physical" counterparts, $\widehat{PVA}$ and $\widehat{Z}$.

3.1 A First Glance into the Main Statistics

In general, statistical variables such as $\widehat{RGDP}$ and $\widehat{SR}$ may be either more or less volatile than their theoretical counterparts, $\widehat{PVA}$ and $\widehat{Z}$. The differences arise from the presence of profits, as measured by Net Operating Surplus, and intermediate inputs. Depending on the sector, profits can be positively or negatively correlated with PVA and/or Z, leading to divergent behavior between the statistical and physical variables.

We present a comprehensive overview of our findings in Table 2, which shows selected moments of standard variables like real GDP and Solow Residual, as well as our own physical variables (PVA and Z). Table 5 in Appendix also displays all other sectors of the US economy.

Table. 2. Selected properties of "statistical" and "Physical" variables.[a]

Sector	Average		Relative volatility (σ)		Correlation (ρ)	
	$\widehat{RGDP}$	$\widehat{PVA}$	$\frac{\sigma(\widehat{PVA})}{\sigma(\widehat{RGDP})}$	$\frac{\sigma(\widehat{Z})}{\sigma(\widehat{SR})}$	$\rho(\widehat{PVA},\widehat{RGDP})$	$\rho(\widehat{Z},\widehat{SR})$
US Aggregate	2.20%	2.21%	0.92	0.95	1.00	0.97
Finance	2.77%	2.41%	1.19	1.18	0.99	0.99
Manufacturing	2.00%	2.54%	1.07	1.12	0.99	0.99
Retail Trade	1.68%	1.52%	1.02	1.06	0.99	0.98
Computer Systems	8.90%	8.91%	1.01	1.00	1.00	1.00
Agriculture	2.12%	3.32%	1.76	1.71	0.98	0.98

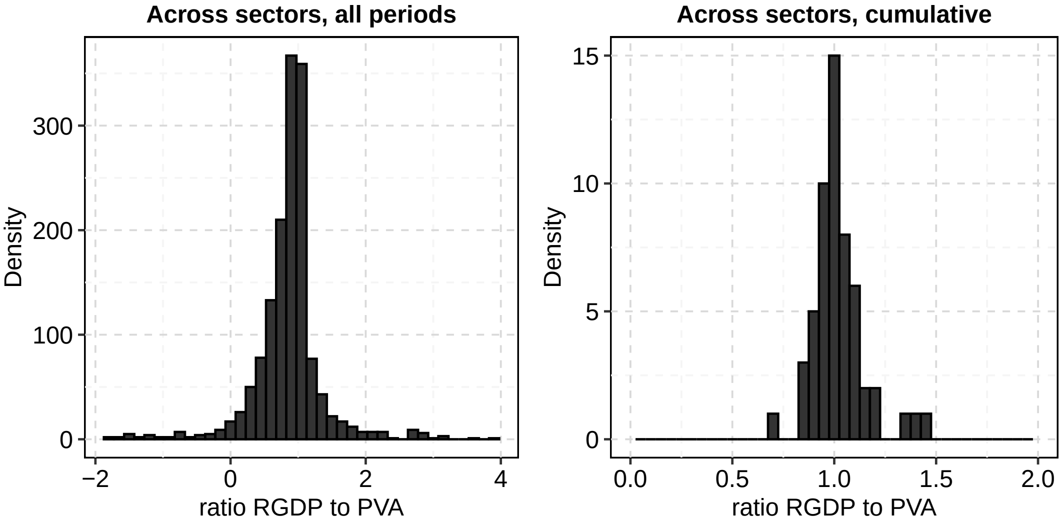
[a] Variables constructed using data from the BEA.

The study's findings indicate that there are significant differences between the measured real gross domestic product (RGDP) and the theoretical production value added (PVA) in the finance, manufacturing, and agricultural sectors. Furthermore, the theoretical measures are generally more volatile than the actual measures at the sector level, especially in the agricultural sector. Finally, there is a significant correlation between the theoretical and actual measures. Surprisingly, we find that the measurement bias can be positive or negative. As a result, when we aggregate the positive and negative biases, they tend to cancel each other out.

Figure 1 illustrates that while the average ratio of measured aggregate GDP to true GDP is close to unity, there is a significant and noteworthy dispersion within sectors, even after aggregating growth rates across all periods. Specifically, the mean difference between $\prod_t (1 +$

$\widehat{RGDP}_t)$ and $\prod_t(1 + \widehat{PVA}_t)$ is 6pp across sectors, and the standard deviation is 0.33, indicating considerable measurement bias dispersion.[3] Therefore, it is important to carefully consider the impact of this dispersion on the interpretation of the data.

Figure 1. Dispersion of statistical RGDP vs PVA.



3.2 Tracing the Evolution of Value Added and Productivity

We now go beyond presenting tables and delve deeper into the results of our study. Using the estimated growth rates of Physical Value Added and Physical Productivity, we construct the time series for these variables and compare them with their standard statistical counterparts. This comparison is presented in Figures 2 for the US aggregate and for key sectors.

Our findings, which are consistent with those presented in Table 2, reveal significant biases in three key sectors of the economy: Finance and Insurance, Manufacturing, and Agriculture, Fishing, Forestry. At almost all periods of time, when these biases are aggregated across sectors, their impact on the overall economy is relatively minor, a fact that is again somewhat surprising.

Specifically, we find that the measured real gross domestic product (RGDP) in 2020 exhibited a bias of 17 percentage points higher in the Finance and Insurance sector, while it was 21 percentage points lower in the Manufacturing sector. In addition, we observed more pronounced biases in technology of 15 percentage points in the Finance sector and 15 percentage points in the Manufacturing sector.

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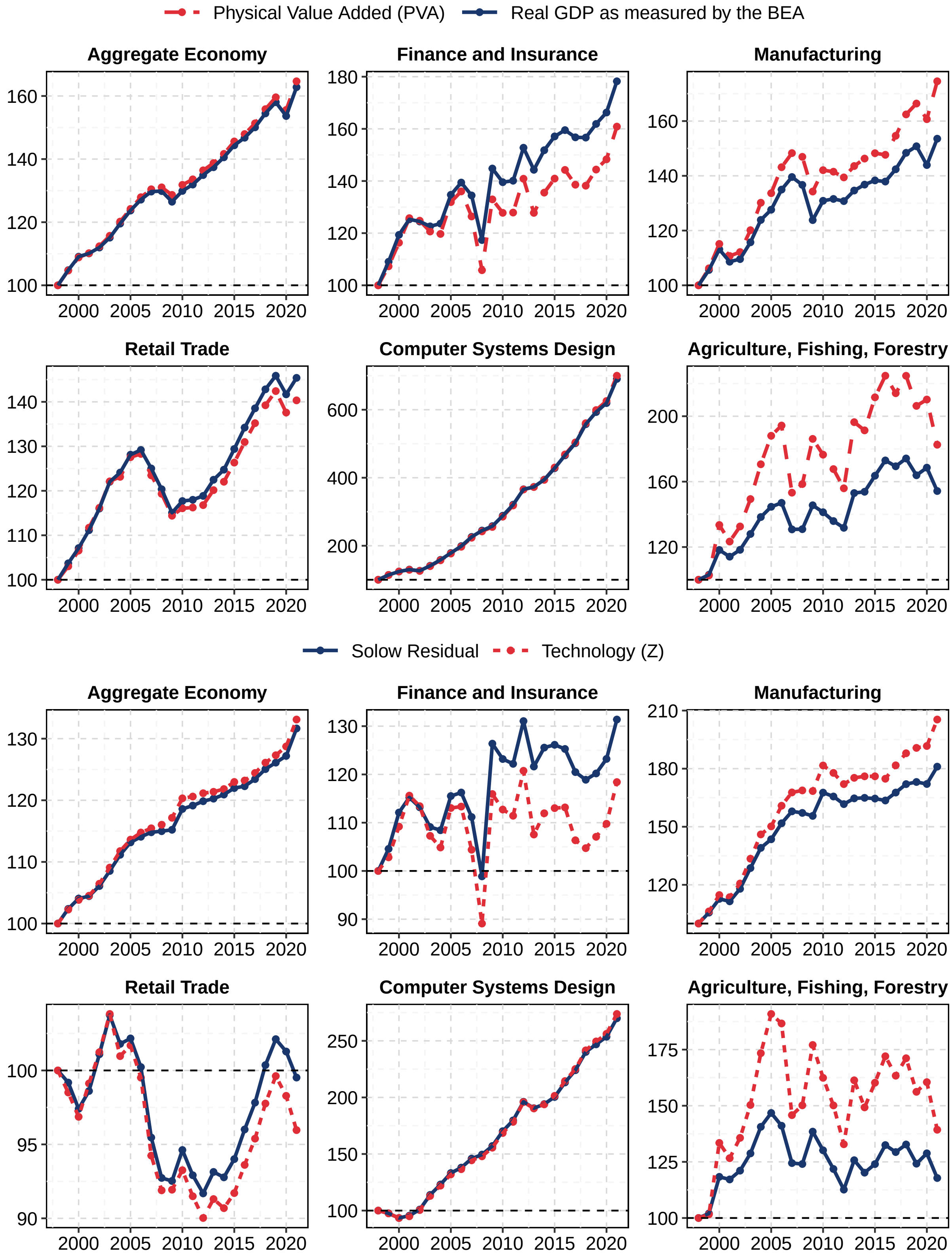
[3] It is worth noting that the aggregate measurement bias is weighted by sector importance. If we take this into account, the weighted mean would be around about 0.

Figure 2. Statistical RGDP vs PVA (top panel) and SR vs Technology (bottom panel) (index 100 = 1997).



These results suggest that the current methods used to measure economic activity in these sectors may not accurately capture the true value added. As a consequence, the interpretation of the drivers of economic growth and performance in these areas may require a better mapping with the metric used in economic models.

3.3 The (absence of) Role of Markup Fluctuations

The above analysis aims to quantify how the presence of markups creates a measurement issue in national accounts data, which is conceptually distinct from the topic of resource misallocation. Recent papers such as Baqaee and Farhi (2020) highlight how markup heterogeneity across firms implies that the allocative efficiency of the US economy changed over time, as production factors are reallocated to high-markup firms. According to their estimate, this reallocation process accounts for about half of aggregate TFP growth over the period 1997-2015. While both misallocation and measurement are important issues, they highlight different aspects of how markups impact the economy.

In our case, the source of the bias is not the heterogeneity of markups, nor their fluctuations over time. To emphasize this point, we recompute the growth rate of physical productivity (Z) and Physical Value Added (PVA) using a constant markup value of 1.15 – hence assuming that all sectors share the exact same time-invariant markup of 15% (which is contrary to what our data on Net Operating Surplus suggests, but this is an illustration). The main messages hold with constant and homogeneous markups.

3.4 Decomposing the Measurement Bias

Using the sectoral version of equation (10), we can decompose sectoral real GDP ($sRGDP_{jt}$) into two terms that capture sectoral PVA change and sectoral markup effect.

$$\widehat{sRGDP}_{jt} = \omega_{jt-1}\gamma_j\left(\widehat{PVA}_{jt}\right) + \frac{\omega_{jt-1}(1-\gamma_j)}{\mu_{jt-1}}\left(\underbrace{(\mu_{jt-1}-1)\cdot\widehat{X}_{jt}}_{\text{Markup Effect}}\right). \tag{18}$$

Using $\omega_{jt-1} = \mu_{jt-1}/(\gamma_j + \mu_{jt-1} - 1)$ and reorganizing terms, we can compute the ratio of real GDP growth ($\widehat{sRGDP}_{jt}$) to Physical Value Added growth ($\widehat{PVA}_{jt}$), such that:

$$\frac{\widehat{sRGDP}_{jt}}{\widehat{PVA}_{jt}} = \frac{\widehat{X}_{jt}}{\widehat{PVA}_{jt}} - \frac{\mu_{jt-1}\gamma_j}{\mu_{jt-1}-1+\gamma_j}\left(\frac{\widehat{X}_{jt}}{\widehat{PVA}_{jt}} - 1\right) \tag{19}$$

Equation (19) reveals that the direction of the wedge between real GDP growth and PVA growth is a priori ambiguous, and real GDP growth can be larger or smaller than PVA growth depending on the level of markup (μ_{jt-1}), the share of intermediate inputs (γ_j), and the ratio $\frac{\widehat{X}_{jt}}{\widehat{PVA}_{jt}}$. In table 3, we report the average value of each of term of (19) for selected sectors.

Table. 3. Investigation of Direction of Bias between $\widehat{PVA}$ and $\widehat{RGDP}$, sectoral time average.

	Term in equation (19)			Correlation
Sector	$(1-\gamma_j)$	μ_{jt-1}	$\widehat{X}_{jt}/\widehat{PVA}_{jt}$	$\rho\left(\widehat{X}_{jt}, \widehat{PVA}_{jt}\right)$
US Aggregate	0.51	0.13	0.93	0.88
Finance	0.54	0.16	0.47	-0.57
Manufacturing	0.70	0.09	0.09	0.56
Retail Trade	0.39	0.10	3.67	0.22
Computer Sys.	0.34	0.03	0.08	0.16
Agriculture	0.74	0.20	-0.10	-0.63

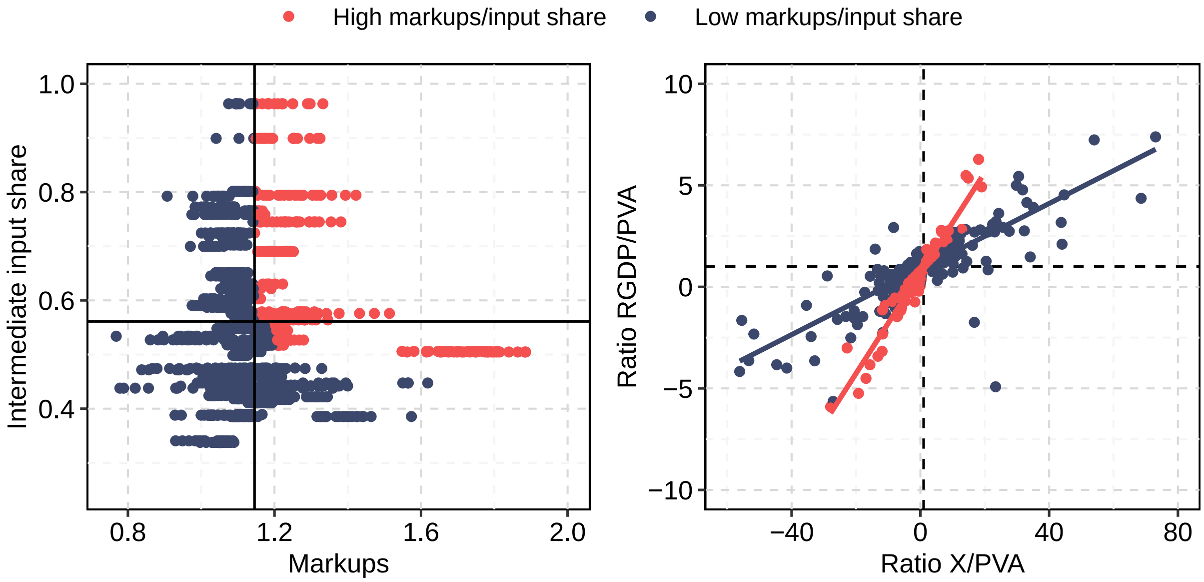
Clearly, the size of markups and the share of intermediate goods is the highest in the Finance, Manufacturing and Agriculture sectors, three sectors in which the bias was reported to be the highest in Table 2.

How can we understand this? From equation (19), it is clear that when markups are zero ($\mu = 1$) or if there is no intermediate good usage ($\gamma = 1$), the two measures ($sRGDP_{jt}$ and PVA_{jt}) would be equivalent, i.e. there is no bias.

Suppose now that $\mu > 1$ and $\gamma < 1$. In this case, the two measures would be systematically biased downward or upward if PVA do not proportionally move with intermediate inputs usage. As such, the ratio $\frac{\widehat{X}_{jt}}{\widehat{PVA}_{jt}}$ is a key element. Even with $\mu > 1$ and $\gamma < 1$, there would be no bias if gross output and physical value added were moving proportionally. Generally, the key property of this equation is that a bias between the theory-consistent measure of GDP and its empirical counterpart arises if: (i) there is no one-to-one mapping between intermediate input fluctuations and PVA, (ii) intermediate inputs are used in the production, (iii) there are markups. Moreover, the sign of the bias is determined by $\frac{X_{jt}}{PVA_{jt}} - 1$.

It follows that sectors with a high share of intermediate inputs and high markups, such as Finance, Manufacturing, and Agriculture, are likely to exhibit the largest absolute size of the measurement bias by amplifying movements in $\frac{\widehat{X}jt}{\widehat{PVA}jt} - 1$. To illustrate this point, we distinguish between two types of sectors-year couples. Those with a large intermediate input share and others. The first category is defined as sectors–year in which either γ_j and μ are both larger than their average value, or ($1 - \gamma_j > 0.5$) and ($\mu_{jt-1} > 1.2$). Figure 3 displays the relationship between the measurement bias (the LHS) and the ratio $\frac{\widehat{X}jt}{\widehat{PVA}jt}$ on the RHS.

Figure 3. The role of markups and intermediate input share and relation between $\widehat{X}/\widehat{PVA}$ and the bias



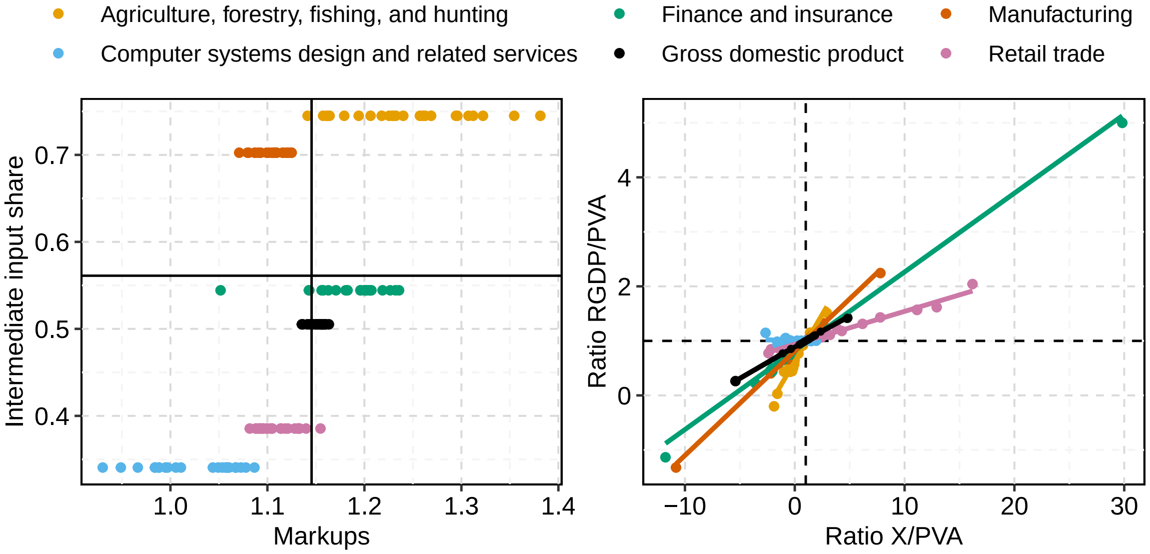
Each dot corresponds to a couple (sector,year). Left plot: the solid black lines represent the average μ and $1-\gamma$. The horizontal dashed black line in the right plot refers to the case without any measurement bias, i.e. $s\widehat{RGDP} = \widehat{PVA}$, while the vertical dashed black refers to the case where $\widehat{X} = \widehat{PVA}$.

It can be seen that the sensitivity of the measurement bias largely depends on the sectoral markups and intermediate shares. When both components are high, even small variations in the ratio $\frac{\widehat{X}jt}{\widehat{PVA}jt}$ can result in substantial changes in the magnitude of the bias, as illustrated by the large red slope in the right chart in Figure 3. Sectors in which fluctuations in intermediate inputs are large but fluctuations in technology, capital, and labor inputs are low, exhibit a strong bias if they also exhibit large markups and if intermediate inputs account for a significant proportion of production.

To further illustrate the relationship between intermediate inputs and the measurement bias, it is helpful to examine how this sensitivity appears in our selected sectors. Figure 4 displays the relationship between $\widehat{X}/\widehat{PVA}$ and the measurement bias. As expected, there is a clear relationship between the sectoral share of intermediate inputs, the presence of markups, and the slope of the bias as intermediate inputs fluctuate relative to the PVA.

At the aggregate level (in black), we observe that markups are relatively high, with a value close to 1.15, while the share of intermediate inputs is also high, exceeding 0.5. However, the variations in terms of $\widehat{X}/\widehat{PVA}$ (summed over all sectors) are relatively small and centered around the value that generates exactly a measurement bias of zero (a ratio $\widehat{RGDP}/\widehat{PVA}$ of 1), such that there is not much happening in terms of the relationship between intermediate inputs and the measurement bias at the aggregate level.

Figure 4. The role of markups and intermediate input share and relation between $\widehat{X}/\widehat{PVA}$ and the bias



Each dot corresponds to a couple (sector,year). Left plot: the solid black lines represent the average μ and $1-\gamma$. The horizontal dashed black line in the right plot refers to the case without any measurement bias, i.e. $s\widehat{RGDP} = \widehat{PVA}$, while the vertical dashed black refers to the case where $\widehat{X} = \widehat{PVA}$.

In the Appendix, we also present the time series of the measurement bias in the selected sector. As already shown, the Finance and Manufacturing sectors exhibit the strongest volatility in the measurement bias.

These findings underscore the importance of considering how the data are constructed when comparing them to the properties of a model. This is particularly relevant for frameworks that aim to assess the impact of markups. Therefore, a careful analysis that accounts for these issues is necessary to draw accurate conclusions about the effects of markups on the economy.

4 Conclusion

This paper provides new insights into the discrepancies between statistically measured real value added and productivity measures and their theoretical counterparts used in macroeconomic models. By quantifying the measurement bias, we obtain a comprehensive and accurate understanding of the impact of mis-measurement of real value-added at the aggregate as well as sectoral level. Our results highlight the substantial and persistent measurement biases in the current methods of measuring real value added and productivity, especially in the finance and manufacturing sector. Overall, this paper emphasizes the importance of accurate measurement in understanding economic growth and fluctuations.

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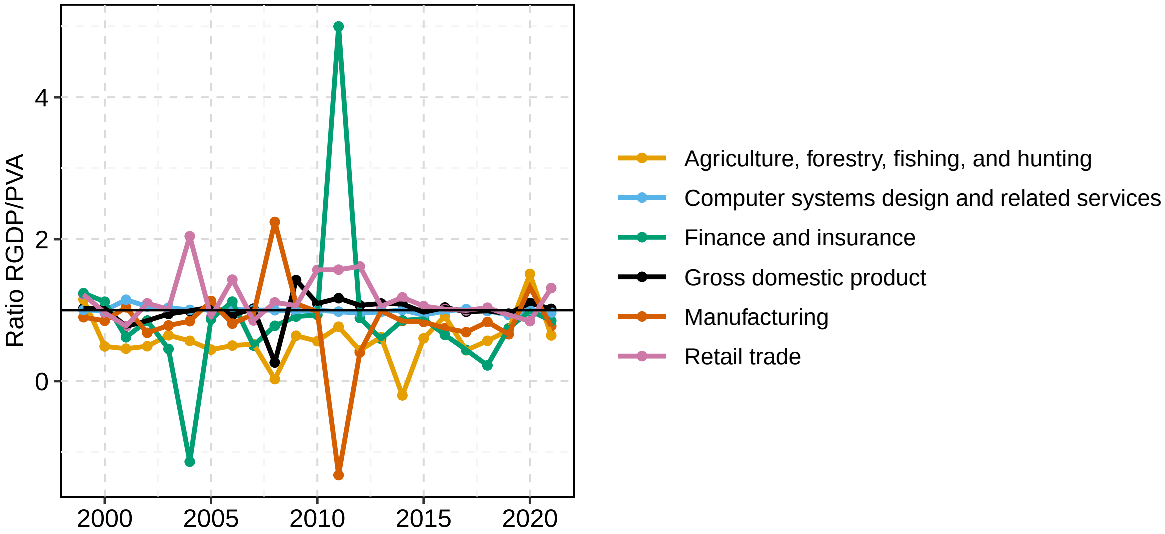
A Data

Table. 4. Variable construction using BEA data.

	Construction with BEA	**Description**
μ_{jt}	$\frac{GO^{nominal}_{jt}}{GO^{nominal}_{jt} - NOS_{jt}}$	Markup
γ_j	$1 - \left(\frac{II^{nominal}_{jt}}{GO^{nominal}_{jt}}\right) \cdot \mu_{jt}.$	Intermediate input share of Gross Output. In practice, we take the time-period average for each sector.
α_j	$\left(\frac{COMP_{jt}}{GO^{nominal}_{jt}}\right) \cdot \frac{\mu_{jt}}{\gamma_j}.$	Labor Share of Value Added. In practice, we take the average over time for each sector.
$\widehat{X}_{jt}$	$\frac{II\_QI_{jt} - II\_QI_{jt-1}}{II\_QI_{jt-1}}$	Proportional change in intermediate input usage for sector j at time t.
$\widehat{K}_{jt}$	$\frac{CAP\_QI_{jt} - CAP\_QI_{jt-1}}{CAP\_QI_{jt-1}}$	Proportional change in Capital input usage for sector j at time t.
$\widehat{L}_{jt}$	$\widehat{L}_{jt} = \frac{EMP_{jt} - EMP_{jt-1}}{EMP_{jt-1}}$	Proportional change in Labor usage (measured as hours worked) for sector j at t.
ω_{jt-1}	Two possible methods: 1. $\frac{\mu_{jt-1}}{\gamma_j + \mu_{jt-1} - 1}$, as in (5) 2. $\frac{GO_{jt-1}}{VA_{jt-1}}$, as in (17)	Sectoral part of the domar weight – ratio of sales to value added – for each sector j at t.
$\widehat{sRGDP}_{jt}$	$\frac{VA\_QI_{jt} - VA\_QI_{jt-1}}{VA\_QI_{jt-1}}.$	Proportional change in sectoral real GDP for each sector j at time t.
d_{jt-1}	$\omega_{jt-1} \cdot \frac{VA\_QI_{jt-1}}{VA\_QI_{t-1}}$	Domar weight for each sector j at time t.

Figure 5. Time serie of the measurement bias



Each deviation from the horizontal solid black line at 1 is a bias.

Table. 5. Sectoral deviation between *RGDP* and *PVA*.

Industry	$\prod_t(1+g_t)$			$\overline{\mu}$	$\overline{\gamma}$
	$\widehat{RGDP_t}$	$\widehat{PVA_t}$	diff. (pp)		
Gross domestic product	1.63	1.65	-1.86	1.15	0.51
Accommodation	1.22	1.21	1.23	1.14	0.44
Accommodation and food services	1.34	1.30	3.90	1.10	0.50
Administrative and support services	2.16	2.12	3.60	1.12	0.42
Administrative and waste management services	2.07	2.04	3.03	1.12	0.43
Agriculture, forestry, fishing, and hunting	1.54	1.83	-28.30	1.25	0.74
Air transportation	0.98	0.97	0.93	1.03	0.53
Ambulatory health care services	2.28	2.33	-5.81	1.12	0.39
Amusements, gambling, and recreation industries	1.11	1.08	3.03	1.07	0.44
Apparel and leather and allied products	0.37	0.38	-1.19	1.01	0.59
Arts, entertainment, and recreation	1.38	1.37	1.32	1.15	0.43
Broadcasting and telecommunications	3.08	3.34	-26.27	1.22	0.58
Chemical products	1.27	1.51	-23.42	1.20	0.69
Computer and electronic products	10.13	10.35	-21.95	1.03	0.39
Computer systems design and related services	6.91	7.00	-9.27	1.02	0.34
Construction	0.92	0.86	6.23	1.17	0.58
Durable goods	2.06	2.28	-21.74	1.07	0.65
Educational services	1.51	1.50	1.61	1.05	0.34
Electrical equipment, appliances, and components	1.34	1.38	-4.16	1.12	0.62
Fabricated metal products	0.97	0.99	-2.21	1.10	0.64
Farms	1.51	1.64	-12.69	1.26	0.79
Finance and insurance	1.78	1.61	17.38	1.19	0.54
Food and beverage and tobacco products	1.23	1.30	-6.99	1.11	0.80
Food services and drinking places	1.40	1.35	4.52	1.09	0.52
Forestry, fishing, and related activities	1.55	1.65	-9.90	1.14	0.44
Funds, trusts, and other financial vehicles	1.55	0.12	143.04	1.18	0.96
Furniture and related products	0.76	0.76	0.25	1.08	0.65
Information	3.84	4.16	-32.70	1.20	0.53
Insurance carriers and related activities	1.98	1.39	58.62	1.24	0.56
Legal services	0.95	0.79	15.54	1.39	0.39
Machinery	1.11	1.14	-2.89	1.08	0.65
Manufacturing	1.54	1.75	-20.98	1.10	0.70
Mining	1.50	1.65	-15.20	1.10	0.47
Mining, except oil and gas	0.70	0.61	9.61	1.24	0.58
Miscellaneous manufacturing	1.69	1.86	-16.79	1.14	0.57

Table. 6. Sectoral deviation between $RGDP$ and PVA.

Industry	$\prod_t(1+g_t)$			$\overline{\mu}$	$\overline{\gamma}$
	$\widehat{RGDP_t}$	$\widehat{PVA_t}$	diff. (pp)		
Motion picture and sound recording industries	2.26	2.04	21.76	1.12	0.48
Motor vehicles, bodies and trailers, and parts	1.88	1.84	3.49	1.04	0.79
Nondurable goods	1.06	1.13	-7.31	1.14	0.77
Nonmetallic mineral products	1.03	1.06	-2.89	1.13	0.63
Oil and gas extraction	1.62	2.28	-66.10	1.07	0.44
Other services, except government	0.83	0.78	5.70	1.12	0.43
Other transportation equipment	1.46	1.52	-6.31	1.10	0.60
Paper products	0.79	0.74	5.23	1.09	0.72
Performing arts, spectator sports, museums, etc.	1.62	1.62	0.61	1.23	0.42
Petroleum and coal products	0.74	0.04	70.65	1.20	0.90
Pipeline transportation	1.93	0.79	114.49	1.23	0.45
Plastics and rubber products	1.16	1.23	-6.45	1.08	0.72
Primary metals	1.35	1.56	-21.56	1.05	0.77
Printing and related support activities	0.94	1.00	-5.93	1.08	0.59
Private industries	1.67	1.71	-3.33	1.17	0.53
Professional, scientific, and technical services	2.23	2.26	-2.06	1.17	0.41
Real estate	1.67	1.47	20.60	1.75	0.50
Real estate and rental and leasing	1.66	1.49	16.60	1.71	0.51
Retail trade	1.45	1.40	5.06	1.11	0.39
Securities, commodity contracts, and investments	1.68	1.14	53.50	1.00	0.53
Social assistance	1.68	1.62	6.35	1.07	0.42
Support activities for mining	2.89	2.53	36.21	1.05	0.47
Textile mills and textile product mills	0.57	0.57	0.49	1.02	0.70
Transit and ground passenger transportation	1.45	1.21	23.94	1.28	0.44
Transportation and warehousing	1.26	1.23	3.54	1.09	0.55
Truck transportation	1.10	1.01	8.68	1.11	0.61
Utilities	1.29	1.19	9.48	1.17	0.52
Warehousing and storage	3.58	2.65	93.43	1.11	0.46
Waste management and remediation services	1.40	1.30	9.75	1.14	0.55
Water transportation	1.44	1.45	-0.70	1.06	0.76
Wholesale trade	1.57	1.41	15.48	1.19	0.42
Wood products	1.17	1.15	1.76	1.06	0.72