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Explaining World Savings

Colin Caines[*] and Amartya Lahiri[†] [‡]

Abstract

Saving rates are significantly different across countries and remain different for long periods of time. This paper provides an explanation for this phenomenon. We formalize a model of a world economy comprised of open economies inhabited by heterogeneous agents endowed with recursive preferences. Our assumed preferences imply increasing marginal impatience of agents as their consumption rises relative to average consumption of a reference group. Using measured productivity as the sole exogenous driver, we show that the model can not only reproduce the sustained long run differences in average saving rates across countries, but also provides a good fit to the time series behavior of saving observed in the data.

Keywords: World savings, recursive preferences

JEL Classification: E2, F3, F4

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1 Introduction

Data on the world saving distribution reveals that cross-country differences in saving rates are significant and persistent. This is problematic for the standard model with time-additive preferences. Without equal rates of time preference, the asymptotic distribution of world wealth is typically degenerate under additively separable preferences. While models with equal rates of time preference but cross-country differences in demographics and productivity have had some success in accounting for part of the cross-country dispersion in saving rates, a substantial amount of variation still remains unexplained in these models.

This paper provides an alternative explanation for the observed saving patterns. We formalize a model of the world economy that is comprised of open economies inhabited by infinitely-lived agents. Our main point of departure from the standard exogenous growth neoclassical model is that we endow agents with recursive preferences.[1] Specifically, we follow Farmer and Lahiri (2005) and use a modified version of recursive preferences. The key implication of the Farmer-Lahiri specification is that it generates a determinate steady state wealth distribution within a growing world economy, a feature that typical models with recursive preferences cannot generate.[2]

One might of course consider the issue of balanced growth to be irrelevant to understanding savings behavior. We however believe that the inconsistency of standard recursive preferences with balanced growth is problematic if one's goal is to explain saving rates. In particular, given the constancy of both long run average growth rates and saving rates for

[1] The dynamic properties of models with recursive preferences and multiple agents were analyzed in a celebrated paper by Lucas and Stokey (1984). Assuming bounded utility, Lucas-Stokey studied an endowment economy. Consequently, the results of Lucas-Stokey cannot be directly applied to growing economies. There is a small literature on recursive preferences with unbounded aggregators. Boyd (1990) has developed a version of the contraction mapping theorem that can be used to generalize Lucas-Stokey's proof of existence of a utility function to the unbounded case. If preferences are time-separable, King et al. (1988) showed that the period utility function must be homogenous in consumption and Dolmas (1996) has generalized their result to the case of recursive utility. Farmer and Lahiri (2005) have applied the Dolmas result to a multi-agent economy and have established that homogeneity rules out the assumption of increasing marginal impatience. Hence, the existence of an endogenous stationary wealth distribution is inconsistent with balanced growth.

[2] The problem with the standard recursive preference specification is that it is inconsistent with balanced growth. The existence of balanced growth requires homothetic preferences. However, under homothetic preferences, the asymptotic wealth distribution in multi-agent environments is either degenerate or reflects the initial wealth distribution. Farmer and Lahiri (2005) provided a reformulation of the standard recursive preference specification that avoids this problem.

most groups of countries (regions or continents), understanding long run patterns of savings would appear to be intrinsically linked to long run steady state dynamics. As is well known, the Kaldor growth facts are quite stark in suggesting balanced growth to be a robust feature of long run growth. Hence, we feel that any model attempting an explanation of dispersions in long run saving rates across countries should be consistent with balanced growth.

We follow Farmer and Lahiri (2005) and construct a model of recursive utility in which agents care about *relative* consumption. We assume that preferences are described by an aggregator that contains current consumption, future utility, and a time-varying factor that is external to the agent but grows at the common growth rate in a balanced growth equilibrium. This time dependence allows for preferences to exhibit increasing marginal impatience, which is a necessary condition for a non-degenerate asymptotic wealth distribution. A positive productivity shock in our model induces a rise in saving which ultimately reverts back to its prior level due to the increasing marginal impatience of agents as their wealth rises relative to world wealth, thereby preserving a determinate asymptotic wealth distribution. Equally importantly, this specification implies that different preferences induce different steady state consumption-to-wealth ratios of different agents. This implies that countries operating in the same world bond market and facing a common world interest rate have different steady state saving rates.

Can the modified recursive preferences of Farmer and Lahiri (2005) account for the observed differences in average long run saving rates between regions for the period 1970-2014? Can they also explain the time series behavior of region-specific saving rates in an open economy environment? In the context of a four-region, heterogenous agent world economy where the external factor in preferences is indexed to the common world per capita consumption level, we demonstrate that our model with recursive preferences can achieve both these goals.

The paper first calibrates the baseline open economy, four-region model to match the 1970 average regional saving rates and the capital shares in the G7, the Newly Industrialized economies (NIE), the Latin American & Caribbean economies (LAC), and sub-Saharan African countries (SSA). This list comprises 86 countries of the world. We are able to match these average saving rates by allowing one preference parameter to vary across regions. We test the model by examining its time series behavior and compare that with the data.

We examine the time series properties of the model by first estimating a region specific productivity process for each of our four regions using data between 1970 and 2014. The model is simulated by taking 100,000 correlated draws from the estimated process for each region. The model's time series fit is examined by comparing the simulated moments on regional savings from the model with their data counterparts. In order to establish a frame of reference for our results, we also compute the model-induced saving rates under the more traditional CRRA preferences. Our focus is on three moments of the time series behavior of regional saving rates: long run mean saving rates, volatility of savings and the correlation of saving rates in the model and the data.

The recursive preference specification outperforms CRRA preferences in its predictions for long run average saving rates. We also find that the saving rates generated by the model in response to the actual in-sample productivity shocks between 1970 and 2014 correlate more strongly with the actual regional saving data for almost all regions. In terms of savings volatility, the CRRA specification generates lower standard deviations than the recursive model. However, since the CRRA model also generates lower average saving rates, the predicted coefficient of variation of the two models are similar.

We view these results as suggesting that the recursive preference specification provides a better description of saving behavior relative to the traditional CRRA specification since it better matches the time series behavior of savings while also being able to match the long-run differences in saving rates between the regions. Recall that the additively-separable CRRA preferences cannot generate steady state differences in saving rates between countries operating in an integrated world economy with free cross-country flow of goods and capital.

Another striking feature of the cross-country saving data is the sudden increase in saving rates (or saving miracles) that are often observed in specific regions. Can our model generate saving miracles? In order to generate sudden switches in saving rates, we propose a new mechanism. Specifically, we allow the external factor in preferences to be different in *levels* for the regional groups. The basic idea behind this is that all societies have role models/peer groups that they want to keep up with or imitate. This approach to explaining miracles amounts to a hypothesis that these sudden transformations of economies occur due to changes in their aspirations. Under our formalization, steady state saving rates are functions of the external factor in preferences which describes the benchmark relative to whom the country evaluates itself. Changes in this reference level can cause an immediate and sharp change in desired long run consumption which can only be brought about through sustained changes in saving rates.

As an example, we show that our 4-region model can, both qualitatively and quantitatively, generate the observed saving miracle of the NIEs if the benchmark external factor of NIEs is switched in 1985 from their own regional per capita consumption level to the average per capita consumption level of the world instead. The model predicts that saving rates rise as the economy starts building its consumption towards its new aspiration level. As consumption rises however, increasing marginal impatience starts to become stronger over time which eventually induces the saving rates to come back down. We show that these predictions match the facts for the NIEs. We believe this aspirations based explanation for sudden increases in saving rates is novel and is worth investigating further in future work.

Our work is related to two different strands of literature. The first is the relatively large body of research focused on explaining the dispersion of saving rates across countries. Explanations for the observed variation in cross-country savings have typically focused on variations in per capita incomes, productivity growth, financial market development, and demographic factors such as fertility rates or the age distribution. For example, in empirical work Horioka and Terada-Hagiwara (2012) and Loayza et al. (2000) find significant explanatory power for age-dependency ratios, and some explanatory power for both per capita income and financial market development (though the direction of causality there is somewhat unclear in these cases). Quantitative work by Tobing (2012) finds that a sizeable portion of the cross-country variation in savings can be explained by the impact of fertility rates and lifespan on age-dependency ratios. In addition, a variety of models have been used to explore the role of financial frictions in explaining aggregate saving, both through the impact of capital misallocation on the marginal product of capital, as in Bai et al. (2018), as well as through the precautionary savings channel, as in Fernandez-Villaverde et al. (2023). However, a significant part of cross-country saving variability in the data remains unaccounted for, while persistent structural differences across countries cannot account for higher-frequency time series behavior of saving.

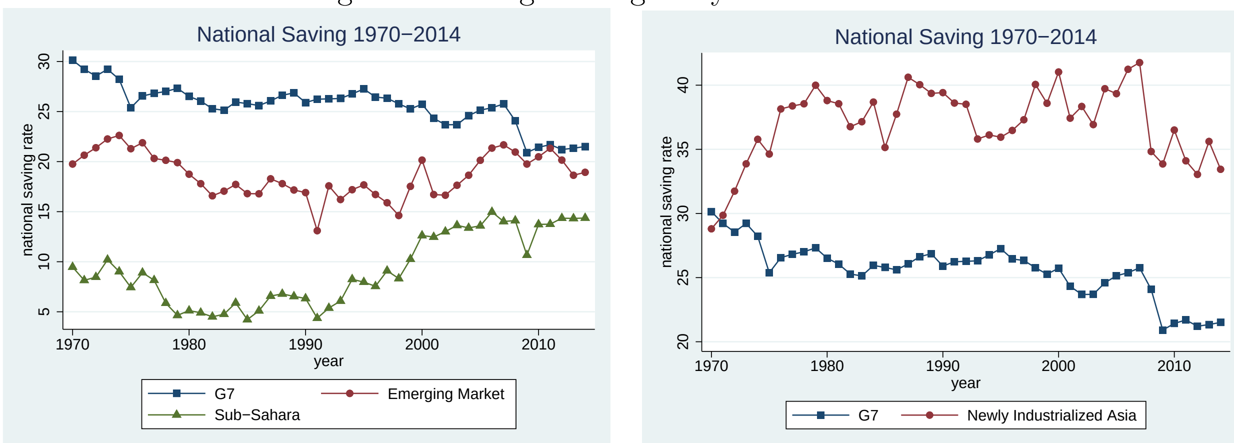
This paper also connects to a long-running effort to explain non-degenerate stationary wealth distributions in models of capital accumulation. One strand of this literature relys on idiosyncratic risks, such as in Aiyagari (1994), as the explanatory mechanism. An alternative approach has turned to financial frictions, examples of whch can be found in Aghion and Bolton (1997), Chatterjee (1994), Piketty (1997), Galor and Zeira (1993), and Sorger (2000). Our paper more directly connects to an alternate stand that explains non-denegerate wealth distirbutions via agent heterogeneity, which goes back in its modern form to Lucas and Stokey (1984) and Epstein and Hynes (1983). Of particular relevance to our work are the contributions of Boyd (1990), Dolmas (1996) and Ben-Gad (1998) who focused on characterizing the stationary wealth distribution in growing economies using recursive preferences. Finally, also relevant to our work are the papers by Uzawa (1969), Mendoza (1991) and others who examined the effects of endogenously varying discount rates on the equilibrium dynamics of the neoclassical growth model.

In the next section we describe some of the key data features that motivate our study. Section 3 quickly reviews the key issues associated with recursive preferences under balanced growth as well as the "fix" to the problem suggested by Farmer and Lahiri (2005). Section 4 presents and develops the model. In section 5 we calibrate the model and examine its quantitative fit to average saving rates in a two-region economy. Section 6 discusses miracles while the last section concludes.

2 Two Facts on Cross-Country Saving

There are two features of the data that we want to draw attention to. First, we highlight the sustained differences in saving rates across groups of countries. To do this we collect countries into three groups: the G7, the Emerging Market, and Sub-Saharan Africa.[3] Panel (a) of Figure 1 plots the savings rates of these three groups of countries between 1970 and 2014. The figure illustrates that savings rates are different for different countries for long periods of time. Further, they show little or no evidence of convergence.

Figure 1: Saving heterogeneity in the world



(a) Differences in saving rates across regions (b) Saving miracles

Note: The graph depicts mean saving rates in different regions of the world between 1970 and 2014. Panel (a) plots the saving rates in the G7, Emerging economies and sub-Saharan Africa. Panel (b) plots the saving rates in the G7 and Newly Industrialized Asian Economies).

While the overall pattern suggests that saving rates are persistent, the data has another important feature: in some countries saving rates show sudden and sharp swings over relatively short periods of time. Panel (b) of Figure 1 highlights this by plotting the saving rates in the Newly Industrialized Economies (NIEs) between 1970 and 2014. Clearly, saving rates in the NIEs increased very sharply from 1970 onward. In a short time period of time, their saving rates rose by over 10 percentage points. Since the mid-1980s, the average saving rate in the NIEs has routinely exceeded the average saving rate in the G7 countries by over 10 percentage points.

We believe this data can be explained by allowing the rate of time-preference to vary across countries using a modified version of recursive preferences. In the standard model of recursive preferences studied by Lucas and Stokey (1984) and Epstein and Hynes (1983), agents become less patient as they become richer in an absolute sense. We adapt this idea

[3]The list of countries in each group is in the Appendix.

to the case of a growing economy with the assumption that agents become less patient as they become *relatively* richer.

3 Recursive preferences and balanced growth

As discussed in Section 1, a key goal of the paper is to examine the ability of a modified version of recursive preferences to rationalize the cross-country saving facts. Before presenting the model it is worthwhile to review why a modification is needed at all. In a nutshell, the need to modify the standard recursive specification arises because we are interested in analyzing environments with steady state balanced growth. Balanced growth is not only one of the celebrated Kaldor facts but also happens to characterize the modern data.

The baseline recursive preference structure however is not consistent with balanced growth.[4] To see this, consider the following recursive aggregator of preferences

$$u_t = W\left(c_t, u_{t+1}\right)$$

With heterogenous agents, there exists a stationary asymptotic wealth distribution if along a steady state balanced growth path all agents equate

$$W_u^i\left(c^i, u^i\right) = W_u^j\left(c^j, u^j\right) = 1/R \text{ for all } i, j \tag{3.1}$$

where c is consumption, u is utility and R is the steady state interest factor common to all agents. Dolmas (1996) showed that this can only occur if the aggregator W is homogenous of the form

$$W\left(\lambda x, \lambda^{\gamma} y\right) = \lambda^{\gamma} W\left(x, y\right) \tag{3.2}$$

When this homogeneity condition is satisfied W_u becomes a constant along a balanced growth path thereby making it possible for the endogenous rate of time preference to remain constant

[4] We should note that the celebrated Lucas and Stokey (1984) paper that studied recursive preferences in a heterogenous agent economy did not examine an environment with long run growth. Hence, this inconsistency was not germane to their work.

and equal to the constant interest rate along a balanced growth path. More fundamentally, the asymptotic value of W_u is independent of the values of c and u along the balanced growth path.

While the condition above is intuitively obvious, Farmer and Lahiri (2005) showed that this homogeneity condition also implies that the stationary wealth distribution in a heterogenous agent economy is generically degenerate. It admits a non-degenerate steady state wealth distribution only in the knife-edged case of $W_u^i = W_u^j$. In this case, however, the independence of W_u^i from c and u in the long run implies that any distribution of wealth/savings across agents will satisfy the asymptotic equilibrium conditions and so the wealth distribution alng the balanced growth path will be indeterminate. These are exactly the implications inthe case of additively separable preferences. In other words, the key Farmer-Lahiri result is that recursive preferences do not add anything to our understanding of stationary wealth distributions beyond what we already know from additively separable preferences.

In the context of recursive preferences in environments without steady state growth, Lucas and Stokey (1984) proved the existence of a stationary wealth distribution as long as preferences exhibited increasing marginal impatience, i.e., agents became more impatient as they grew wealthier. Intuitively, rising impatience bounds the desire to accumulate assets by raising the desire to consume. The problem of the specification in equation (3.2) is that there is no force akin to the increasing marginal impatience of Lucas-Stokey that can endogenously equate it across agents. Consequently, the equilibrium has a knife-edge property to it.

Farmer and Lahiri (2005) showed however that the introduction of an externality into preferences could fix this problem. In particular, they considered preferences of the form

$$u^i = W^i \left(c_t^i, u_{t+1}^i, \bar{\zeta}_t \right)$$

where $\bar{\zeta}_t$ is a factor that is external to the individual. This could stand in for habits, the average consumption level of the economy, or any other factor provided it grows at the rate of steady state growth and, as formalized here, is external to the individual household. Farmer-Lahiri showed that as long as the aggregator W was homogenous in all three arguments so that $W(\lambda x, \lambda^\gamma y, \lambda z) = \lambda^\gamma W$, an economy with heterogenous agents would give rise to an endogenously determined stationary wealth distribution with different agents choosing different saving levels in order to equate $W_u^i(\tilde{c}^i, g^{\gamma_i}\tilde{u}^i, 1) = W_u^j(\tilde{c}^j, g^{\gamma_j}\tilde{u}^j, 1)$ where $\tilde{x} = \frac{x}{\zeta}$, and g denotes the steady state growth. Moreover, the homogeneity property also ensures that W_u would be constant in steady state.

Hence, this specification can generate steady state differences in saving rates across agents facing a common vector of prices. In the rest of the paper we shall examine the potential of these preferences to account for the disparity in saving rates across the world.

4 The Model

We consider a one good world economy consisting of N small economies. Each country i is populated by L_i identical individuals and a continuum of firms of measure 1. The measures of individuals and firms remains constant over time. Introducing population growth into the model is a straightforward extension that does not change any fundamental result. With no loss of generality, we normalize the world population to unity so that $\sum_i L_i = 1$ and set the world price of the good to one.

The world economy is open to frictionless trade in the final good and risk free real bonds denominated in terms of the final good. The assumption of a world economy with an integrated bond market implies that there are no impediments to savings flows throughout the world. This assumption is in stark contrast to closed economy literature on savings which forces saving to equal investment. Consequently, those models are equally well described as explaining investment rather than saving behavior.

4.1 Individuals

Individual agents in every country are endowed with one unit of labor time which they supply inelastically to the market. We assume that all individual agents within a country have identical preferences but preferences of agents across countries maybe different. The preference of the representative agent in country i is described by the recursive representation

$$u_{it} = \frac{c_{it}^{\theta_i} \bar{\zeta}_{it}^{1-\theta_i}}{\theta_i} + \mathbb{E}_t \left[\frac{\beta_i u_{it+1}^{\delta_i} \bar{\zeta}_{it}^{1-\delta_i}}{\delta_i} \right] \tag{4.3}$$

where c denotes consumption and u denotes utility.

This recursive preference specification is standard except for the argument $\bar{\zeta}_{it}$ which stands for an externality in preferences. It is external to the individual but is indexed by i since we allow this externality parameter to vary across countries.

This externality could represent a number of different things including external habits, relative consumption ("keeping up with the Joneses"), etc.. Allowing it to vary across agents implies, for example, that the relative consumption targets could vary across countries. Note that these preferences reduce to the standard additively separable across time specification in the special case where when $\delta = 1$. *Ceteris paribus*, a higher δ makes agents more patient by raising the discount factor. It is easy to check that this aggregator is linearly homogenous, thereby satisfying the homogeneity and regularity conditions needed for the existence of a Balanced Growth Path (BGP) as shown in Farmer and Lahiri (2005).

Individuals have five sources of income: wage income from working, capital income earned by renting out their capital to firms, government transfers, dividend earnings from firms that they own, and interest earned on risk-free one-period bonds. Individual agents save by accumulating capital or by purchasing bonds. Income can be used for either consumption or saving. The budget constraint for individual i is thus given by

$$c_{it} + \iota_{it} + b_{it} + \tau_{it} = r_{it} k_{it} + w_{it} + R_{t-1} b_{it-1} + \pi_{it} \tag{4.4}$$

where k is the capital stock of individual i at the beginning of period t, ι denotes investment in capital, b denotes bond holdings, τ denotes lump sum taxes paid by individuals, and π denotes dividends. The rental rate on capital for country i is denoted by r_i, while w_i is the wage rate of labor for country i, and R is the world risk-free rate on bonds. The capital stock of the agent evolves according to the accumulation equation

$$k_{it+1} = (1 - d_i)\, k_{it} + \iota_{it} - k_{it-1}\phi\left(\frac{i_{it}}{k_{it}}\right) \;, k_{i0} \text{ given for } i = 1, ..., N \tag{4.5}$$

where d is the depreciation rate and the function ϕ represents capital adjustment costs which is increasing and convex. In the following we shall assume that ϕ is given by

$$\phi\left(\frac{i}{k}\right) = \frac{b}{2}\left(\frac{i}{k} + 1 - d - g\right)^2 \tag{4.6}$$

where g is the gross rate of trend growth in aggregate productivity. This specification implies that in a non-stochastic steady state of the model, the adjustment costs would be zero.

Individuals maximize utility (4.3) by choosing sequences for c, k and b subject to the budget constraint (4.4) and the capital accumulation equation (4.5). This problem gives two first-order conditions:

$$\left(\frac{c_{it}}{\bar{\zeta}_{it}}\right)^{\theta_i - 1} = \beta_i \mathbb{E}_t\left[\left(\frac{c_{it+1}}{\bar{\zeta}_{it+1}}\right)^{\theta_i - 1}\left(\frac{u_{it+1}}{\bar{\zeta}_{it+1}}\right)^{\delta_i - 1} R^I_{it+1}\right] \tag{4.7}$$

$$0 = \mathbb{E}_t\left[\left(\frac{c_{it+1}}{\bar{\zeta}_{it+1}}\right)^{\theta_i - 1}\left(\frac{u_{it+1}}{\bar{\zeta}_{it+1}}\right)^{\delta_i - 1}\left(R^I_{it+1} - R_t\right)\right] \tag{4.8}$$

Equation 4.7 is the Euler equation governing the optimal consumption-saving decision while equation 4.8 is the first order condition for the individual's optimal portfolio allocation between capital and risk-free international bonds. Note that in equation 4.8 above, R^I is the effective gross return on capital investment. It is given by

$$R^I_{it+1} \equiv (1 - \phi'_t)\left(r_{it+1} + \frac{1 - d_i - \phi_t + \phi'_t \cdot \left(\frac{i_{it}}{k_{it}}\right)}{1 - \phi'_t}\right) \quad ; \qquad \phi_t \equiv \phi\left(\frac{i_{it}}{k_{it}}\right)$$

4.2 Firms

Firms in each country produce an identical final good which is freely traded across countries. Firms operate in perfectly competitive local factor markets. Since all firms are identical, the production technology for the economy can be represented with the aggregate production function

$$Y_{it} = A_{it} K_{it}^{\alpha_i} L_i^{1-\alpha_i}$$

where Y_i is aggregate output, K_i is the aggregate capital stock and L_i is aggregate labor, and A_i is the total factor productivity in country i.

The representative firm rents capital and labor in competitive factor markets to maximize profits given by

$$\Pi_t = Y_{it} - r_{it} K_{it} - w_{it} L_{it}.$$

Optimality in factor markets dictates that factor prices are given by:

$$r_{it} = \alpha_i \frac{y_{it}}{k_{it}} \tag{4.9}$$

$$w_{it} = (1 - \alpha_i)\, y_{it} \tag{4.10}$$

where y and k denote per capita output and per capita capital, respectively.

The evolution of the country-specific productivity of the technology, A_{it}, is given by

$$A_{it} = e^{z_{it}} a_{it}^{1-\alpha}$$

where a and z are productivity processes described by

$$a_{it} = g a_{it-1} \tag{4.11}$$

$$z_{it} = \bar{z}_i + \rho_i^z z_{it-1} + \sigma_i^z \varepsilon_{it}^z \tag{4.12}$$

Thus, a_{it} is the long run trend in TFP with g being the trend growth of productivity (which is common across regions) while z_{it} represents TFP fluctuations around the trend.

4.3 Equilibrium

Any world equilibrium must clear aggregate world goods and bond markets

$$\sum_{i=1}^{N} L_i \left(c_{it} + \iota_{it}\right) = \sum_{i=1}^{N} Y_{it}, \tag{4.13}$$

$$\sum_{i=1}^{N} L_i = 1 \tag{4.14}$$

$$\sum_{i=1}^{N} L_i b_{it} = 0, \tag{4.15}$$

country-specific factor markets:

$$K_{it} = k_{it} L_{it}, \tag{4.16}$$

$$\Pi_i = \pi_{it} L_i, \tag{4.17}$$

and the government meets its budget constraint:

$$G_{it} = T_{it} = \tau_{it} L_i \tag{4.18}$$

Equation (4.13) is the goods market clearing condition which dictates that the total demand for consumption and investment by the world must equal the world GDP. Equation (4.14) says that total world labor supply must equal the total labor endowment. Equation (4.15) is implied by clearing in the international bond market. Equation (4.16) is the domestic capital market clearing condition in each country i while equation 4.17 gives the clearing condition for dividend distribution in each country.

Definition 4.1 *A world equilibrium is a set of allocations* $\{c_{it}, k_{it}, \iota_{it}, b_{it}, y_{it}\}$ *and prices* $\{w_{it}, r_{it}, R_t\}$ *such that at each date* t *(a) all individuals in all* i *solve their optimization problem given prices; (b) firms maximize profits given prices; and (c) the allocations clear all markets.*

For a growing economy characterized by agents with such heterogenous preferences, Farmer and Lahiri (2005) used the results of Lucas and Stokey (1984) to prove that there exists a unique convergent path to a unique steady state with a stationary distribution of saving rates provided c and u are both "non-inferior"[5], and preferences display increasing marginal impatience, i.e., W_u is decreasing in c. Our specification satisfies all the conditions of Farmer and Lahiri (2005). Hence, their results apply to our model as well.

4.4 Saving heterogeneity

Before proceeding further it is worth sketching out a brief description of how the recursive preference specification generates heterogenous saving rates across countries in steady state. Let $\tilde{x} = \frac{x}{\zeta_{it}}$. In steady state, the rate of time preference for agent i is given by

$$W_u^i = \beta_i \left(g\tilde{u}^i\right)^{\delta_i - 1}$$

while steady state normalized utility is

$$\tilde{u}^i = \frac{\left(R\beta_i\right)^{\frac{1}{1-\delta_i}}}{g}$$

where $R = 1 + r - d$. Using the definition of the aggregator, we also get a steady state expression for normalized consumption:

$$\tilde{c}^i = \left[\theta_i \tilde{u}^i - \frac{\theta_i \beta_i}{\delta_i}\left(\tilde{u}^i g\right)^{\delta_i}\right]^{\frac{1}{\theta_i}}$$

Hence, each $\tilde{u}$ maps into a different $\tilde{c}$. In this set-up, different $\delta's$ and $\beta's$ imply different steady state $\tilde{u}'s$. The rate of time preference W_u is equated across agents by different steady state $\tilde{c}'s$ and $\tilde{u}'s$. Hence, different $\delta's$ and $\beta's$ across agents induce a dispersion in steady state saving rates across agents.

In our calibration strategy below, we will set the β to be identical across countries and calibrate δ_i to match the initial saving rate in 1970 of each region i in our sample. We will then test the model by examining the closeness of fit of the model generated time series of

[5] c and u are non-inferior if $c < c'$ and $u > u' \Longrightarrow \frac{\tilde{W}_c^i(c,u)}{\tilde{W}_u^i(c,u)} > \frac{\tilde{W}_c^i\left(c',u'\right)}{\tilde{W}_u^i(c',u')}$, $i = 1, 2$.

regional saving rates in response to measured shocks with the corresponding time series data on regional saving for the period 1970-2014.

4.5 Super-martingale property of the Euler equation

Under the conventional time additive preferences, the first order condition for optimal savings in a riskless international bond for a small open economy facing a constant world interest rate R_t is

$$u'(c_t) \geq \beta R \mathbb{E}'_t(c_{t+1})$$

As is well known, defining $M_t \equiv (\beta R)^t u'(c_t)$, this expression can be rewritten as $M_t \geq \mathbb{E}_t M_{t+1} > 0$ which is a super-martingale that converges to some finite $M > 0$. For $\beta R > 1$ this implies that u' must converge to zero which, via the Inada condition on utility, implies that c and assets b go to infinity. This is true independent of whether income is deterministic or stochastic.

How do recursive preferences affect this result? The short answer is that recursive preferences make the discount factor endogenous. This renders the sign of βR irrelevant for characterizing the dynamics of the economy. Crucially, as long the recursive preference aggregator exhibits *increasing marginal impatience*, equilibrium consumption and asset holding remain bounded for all values of βR. The key condition for increasing marginal impatience is that $\frac{d\tilde{\beta}}{d\tilde{c}} < 0$ where $\tilde{\beta} \equiv W_u = \beta \tilde{u}^{\delta-1}$. This condition is always satisfied in the model as long as $\delta < 1$.[6]

5 Quantifying the model

The model is calibrated using data from the Penn World Tables (PWT 9.0, Feenstra et al. (2015)). Unless otherwise stated our sample period is 1970-2014. We divide the world into four groups: G7, Newly Industrialized economies (NIE), Latin American and Caribbean

[6] Farmer and Lahiri (2005) work out two detailed examples with these preferences to illustrate how *increasing marginal impatience* endogenously generates a non-degenerate stationary wealth distribution. The examples work out the case of a heterogenous agent closed economy and multiple small open economy cases.

economies (LAC) and sub-Saharan Africa (SSA). Details regarding the countries in each region, the data and the series construction are contained in the Data Appendix. When quantifying the model, we approximate for the rational expectations equilibrium along the non-stochastic balanced growth path. In particular, we solve for linear approximations of the policy functions using perturbation methods.[7]

5.1 Mapping model to data

Our model is one of private sector saving behavior with fiscal policy playing a passive background role with a balanced budget specification $G = T$. Private saving as a share of disposable income in each region i is given by

$$s_{it} = \frac{S_{it}}{Y_{it} - T_{it}} = \frac{Y_{it} - C_{it} - T_{it}}{Y_{it} - T_{it}}$$

Substituting in the balanced budget condition of the model implies that

$$s_{it} = \frac{Y_{it} - C_{it} - G_{it}}{Y_{it} - G_{it}}$$

. This expression for private saving can be used to derive the aggregate saving rate for the economy:

$$s_{it}^{A} = \frac{S_{it}}{Y_{it}} = s_{it}\left(1 - \frac{G_{it}}{Y_{it}}\right) \tag{5.19}$$

In what follows we will consider a world composed of regional groupings of countries. For each region we use the Penn World Tables (PWT) to construct data analogues of s_{it}^{A}, s_{it} and $\frac{G_{it}}{Y_{it}}$. We then calibrate the baseline recursive model to match the private saving rate s_i in each region i in 1970. Since we take government consumption G_{it} as an exogenous series in the model, we should note that matching the private saving rate in 1970 implies that the model can also match the aggregate regional saving rate if one uses the exogenous $\frac{G_i}{Y_i}$ in equation 5.19.

[7]We use Eric Swanson's *Petrubation AIM* algorithm to obtain the model's policy functions. The details of this method are provided in the Appendix.

The only stochastic variable in the model is regional total factor productivity (TFP) A_{it}. To construct a productivity series for each region, we conduct a growth decomposition exercise using the constructed data on regional GDP, capital and labor for 1970-2014 and recover a time series for the growth rate of TFP for each region for this period.

We calibrate the level of regional TFP in 1970, A_{i1970}, to match the data for regional per capita GDP in 1970 to that predicted by the model. Combining this initial level of regional TFP with the growth rate of TFP derived from the decomposition exercise, we get a time series for TFP for each region. We detrend this series using a linear trend and set the linear trend equal to $a_{it}^{1-\alpha}$. We set the detrended series equal to z and use the derived series for z to estimate

$$z_{it} = \hat{\bar{z}}_i + \hat{\rho}_i^z z_{it-1} + \hat{\sigma}_i^z \hat{\varepsilon}_{it}^z$$

We use the computed residuals $\hat{\varepsilon}_{it}$ as shocks to the model. There are thus four parameters to be estimated per region: z_i, ρ_i^z and σ_i^z. The estimated parameters are reported in Table 1 below:

Table 1: Estimated productivity parameters

	productivity parameters		
	$\hat{z}_i$	$\hat{\rho}_i^z$	$\hat{\sigma}_i^z$
G7	0	0.948	0.049
LAC	-0.019	0.970	0.078
NIE	-0.042	0.954	0.104
SSA	-0.486	0.664	0.155

This table reports the estimated productivity parameters for the four regions. G7 denotes the G7 countries, NIE the newly industrialized economies, LAC the Latin American and Caribbean economies while SSA denotes sub-Saharan African economies.

A key variable in the model is the externality process. For our baseline case we set

$$\bar{\zeta}_{it} = c_{it} \text{ for all } i \text{ and } t$$

as this is a neutral starting point where households use the per capita consumption in their own region as the reference consumption target.

We choose to set the exogenous discount factor $\beta = 0.97$, the capital share $\alpha = 0.33$, the capital adjustment cost parameter $b = 4.7$ and the trend gross growth rate of productivity $g = 1.03$. These parameters are assumed to be common across regions. The values for β and α are standard.[8] We calibrate b such that the model reproduces the standard deviation of world investment per worker. Lastly, we set the world trend growth of productivity g equal to the estimated world trend growth during the period 1970-2014. By letting these parameters be identical across countries allows us to retain the focus of the analysis on recursive preferences and the key aspect of heterogeneity in preferences emanating from the non-additively separable component of preferences.

We estimate the regional labor shares L_i from the data (after normalizing the total population of the four regions to one). The depreciation rate of capital, d_i is estimated as the average across all countries in each region i from the depreciation series reported in the PWT 9.0. Both these numbers are estimated based on data for 1970.

This then leaves the two key preference parameters θ_i and δ_i. We set the vector $(\delta_{G7}, \delta_{NIE}, \delta_{LAC}, \delta_{SSA})$ so that the steady state values of the regional saving rates are equal to those observed in 1970. Note that our procedure targets the four regional private saving rates, hence we choose four parameters. We then set θ_i such that the steady state relative risk aversion for each region i is 5. These parameter choices are summarized in Table 2 below.[9]

It is important to note that we need a frame of reference in order to assess the value added by our model with recursive preferences. The most obvious reference point for this purpose is the standard model with time additive, constant relative risk aversion (CRRA) preferences

[8] For the capital share numbers we tried a number of alternative approaches ranging from a constant 1/3 share of output for all countries to the numbers computed by Caselli and Feyrer (2007) as well as those from Bernanke and Grkaynak (2002). The results are robust to these alternative approaches, hence in the following we shall set the capital share to a common 1/3 for all countries.

[9] In the Appendix 8.1, we derive the expression for relative risk aversion in steady state under recursive preferences. As we show, relative risk aversion under recursive preferences is a non-linear function of the growth rate, the interest rate, the steady state normalized consumption $\tilde{c}$ and the preference parameters θ and δ. Hence, given a target value for relative risk aversion that is common to all regions, one can back out the implied value for θ_i once we have values for all the other parameters listed above. Preference parameters could alternatively be set to match a desired intertemporal elasticity of substitution, however we choose risk aversion as a calibration target so as to minimize assumptions about model dynamics in our parameterization.

Table 2: Parameterization of baseline model

	Parameters		
	labor share l_i	preference parameters $\{\delta_i, \theta_i\}$	depreciation rate d_i
G7	0.593	$\{0.979, -3.643\}$	0.049
LAC	0.171	$\{0.980, -3.647\}$	0.046
NIE	0.038	$\{0.980, -3.646\}$	0.052
SSA	0.198	$\{0.980, -3.645\}$	0.038
$(\beta, \alpha, b, g) =$		$(0.97, 0.33, 4.3, 1.012)$	

This table summarizes the regional parameter choices to enable the baseline model to match the calibration targets for regional saving rates and regional shares of world capital in 1970. G7 denotes the G7 countries, NIE the newly industrialized economies, LAC the Latin American and Caribbean economies while SSA denotes sub-Saharan African economies.

since it is the typical workhorse macro model that is used widely for both quantitative and qualitative work. For the CRRA model we consider standard preferences of the form

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \qquad \sigma > 0$$

where σ is the coefficient of relative risk aversion.

For our baseline calibration of the CRRA model we assume that σ is the same across all regions and equal to the coefficient of relative risk aversion of five which was assumed for the model with recursive preferences. As before, the adjustment cost parameter b is set to match world per capita investment volatility to its value in the data. The rest of the model parameterization is exactly as before. As in the main exercise reported above, we feed in measured shocks from the TFP processes for each of the regions into the model and generate the model implied time series for the saving rates for the four regions.

Table 3 reports the regional savings rates for 1970 that are generated by the model under the baseline calibration for both the recursive and CRRA versions of the model. The table shows that the recursive model fits the level of regional savings rates while the CRRA model fails to do so. The reason is simple: the recursive model has an extra parameter δ with which to target the savings rate which the CRRA model doesn't have.

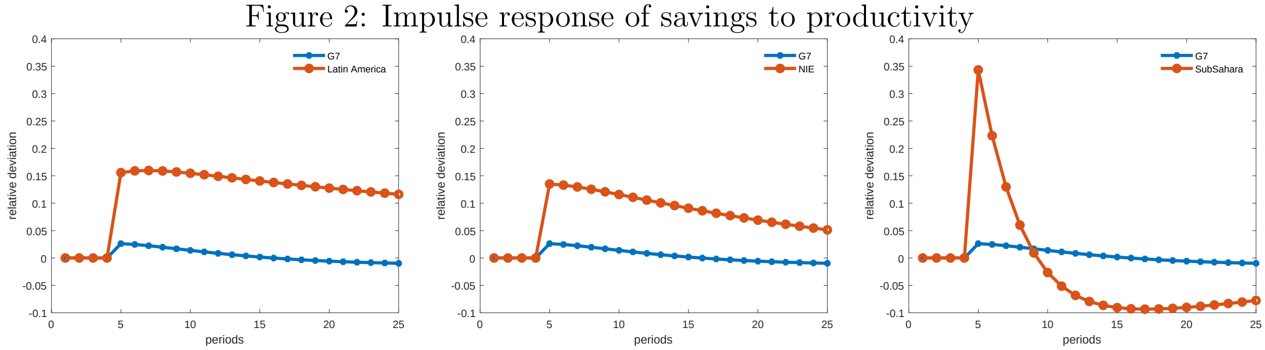
To illustrate the mechanics of the model we start by plotting the response of the saving

Table 3: Saving Rates in 1970

	Data	Model	CRRA
G7	0.326	0.326	0.140
Latin America	0.189	0.189	0.136
NIE	0.294	0.294	0.144
Sub-Sahara	0.223	0.224	0.123

rates in the G7, SSA and NIE regions to a one standard deviation positive shock to the regional productivity processes z_i. Figure 2 shows the regional responses to the shocks. The size of the savings response just reflects the size of the shock. Since the SSA region has the highest measured standard deviation of productivity (see 1), their savings respond the most while the G7 savings respond the least since it has the lowest standard deviation of productivity. Independent of the size of the initial response, savings in all regions then declines back down to their long run stationary levels. The reason for this is the *increasing marginal impatience* that is built into these preferences. The stationary world wealth distribution is non-degenerate precisely due to this feature of preferences.

Figure 2: Impulse response of savings to productivity



(a) G7 and Latin America (b) G7 and NIE (b) G7 and sub-Sahara

Note: The graph depicts the response of saving rates in the four regions to a positive one standard deviation shock to the regional productivity process z_{it}.

5.2 Baseline results

So, how well does the model explain world saving behavior? We examine this by simulating the response of saving in the model to the measured productivity shocks shocks in the data between 1970 and 2014. In particular, we compute moments from the model from data that is generated by feeding in the estimated productivity processes for the four regions for the period 1970-2014. Recall that the model was parameterized to mimic data in 1970, not to explain actual saving movements between 1970 and 2014. We keep all parameters fixed across time for these simulations while feeding the measured regional productivities during 1970-2014 into the model. The model is simulated by taking 100,000 correlated draws from the estimated process for each region.

Table 4 presents a comparative analysis of the two models. The results emphatically indicate the better performance of the recursive preference model in explaining saving behavior. The recursive model generates average saving rates that are much closer to the data for all regions except SSA where it marginally underperforms relative to the CRRA model. In addition, the recursive model on average outperforms the CRRA model in terms of the correlation between the model and data saving rates for all the regions except for the LAC.[10]

Table 4: Regional Saving Rates: 1970-2014

	Mean			Standard Deviation			Correlation(data,model)	
	Data	Recursive	CRRA	Data	Recursive	CRRA	Recursive	CRRA
G7	0.288	0.320	0.148	0.029	0.037	0.022	0.727	0.690
LAC	0.229	0.236	0.147	0.015	0.113	0.022	0.065	0.117
NIE	0.399	0.271	0.132	0.034	0.150	0.054	-0.012	-0.051
SSA	0.203	0.140	0.179	0.053	0.212	0.155	0.533	0.463

Notes: The table reports moments of regional saving rates in the data for the period 1970-2014 and simulated from the model using measured shocks from the TFP process. G7 denotes the G7 countries, NIE the newly industrialized economies, LAC the Latin American and Caribbean economies while SSA denotes sub-Saharan African economies. Results are shown for both the recursive and CRRA versions of the model under the baseline calibration. $Correlation(\text{data}, \text{model})$ denotes the correlation between the savings rates in the data and the model using measured shocks from 1970-2014 data.

To assess the performance of the models in terms of their predicted volatilities, it is important to note that relative to the recursive preference model, the CRRA model generates

[10]To interpret the weaker correlations of saving rates in the CRRA model with the data, note that the unit root structure of the CRRA model implies that all productivity shocks have permanent effects on consumption, and by extension, on savings. The lower correlations of the savings rate from the CRRA model for most of the regions, when interpreted through the lens of these two models, would indicate that shocks do not tend to have permanent effects in the data.

both lower mean saving rates as well as lower standard deviation of savings. As a result, the coefficient of variation of savings generated by the two models are similar even though their means and standard deviations are not.[11]

5.3 The Current Account

One of the key features of the model that we have formalized is its open economy structure. The ability of the model to generate realistic predictions for its external sector is thus of independent interest. In Table 5 we report the key moments of the regional current account balances and compare them to the data. Specifically, we report the regional current account to GDP ratios in 1970, the standard deviation of current account balance during 1970-2014, and the correlation between the data and the model of the current account to GDP ratio during this period.

Table 5: Current Account in the Model and Data

	1970 Level (%)		St. Dev.		*corr*(Data,Model)
	Data	Model	Data	Model	
G7	-0.9	1.7	0.009	0.012	0.354
LAC	-2.3	-11.2	0.024	0.076	0.271
NIE	-0.4	-1.5	0.034	0.115	-0.486
SSA	0.0	-8.1	0.061	0.268	0.301

Notes: The table reports moments of the current account to GDP ratio in the data for the period 1970-2014 and simulated from the model using measured shocks from the TFP process. G7 denotes the G7 countries, NIE the newly industrialized economies, LAC the Latin American and Caribbean economies while SSA denotes sub-Saharan African economies

The results in Table 5 indicate that the model's performance on its open economy dimension is slightly mixed. On the one hand, for all regions except the NIEs the model generates saving rates that are positively correlated with the data. The model also does a reasonable job of predicting the initial current account to GDP ratios of all the regions except for sub-Saharan Africa where the model predicts a much larger deficit than in the data. On the

[11] One should point out though that the volatility in the recursive model would become much lower if we took many draws from the estimated productivity processes rather than the actual draws during a short 45 year period. On the other side, taking many draws from the estimated productivity process would make the predicted standard deviation of saving rates much larger in the CRRA model due to its unit root characteristic. The upshot of this is that the longer the period over which one computes the moments for saving rates the better will be the relative performance of the recursive model.

other hand, the model generates excess volatility of the current account relative to the data for all regions except the G7.

5.4 Summary

We view these results as being generally supportive of the recursive preference specification in explaining the observed world saving heterogeneity, especially in comparison to the more common CRRA specification. The model does a good job of reproducing the long run levels of regional saving rates. It also generates time series variation in the regional saving rates that tracks the data reasonably well. We find this especially encouraging since the model generates these moments with only productivity shocks. Adding more sources of stochastic variations will only improve the fit of the model.

6 Saving miracles

The second motivation of this paper was the observation that some countries have shown sudden and dramatic increases in their average saving rates. We showed an example of this in Figure 1 which depicted the sharp pickup in the average saving rate of the NIEs. We now turn to demonstrating how our model can accommodate these dramatic saving miracles.

Our principal idea is that saving behavior is dictated by aspirations which, in turn, is often determined by one's position relative to a comparison group. If a society begins to aspire to have the consumption and wealth levels of a much richer comparison group then its saving levels have to respond to achieve that new goal. The recursive preference structure that we have used in this paper is well suited to examining the relevance of this mechanism since one of its key features is the presence of relative consumption.

In our model, the relative consumption level in preferences was just the average per capita consumption in the household's own region. In this section we shall examine the consequences of a country undergoing a change in its relative comparison group from the average per capita level in its own region to the average per capita consumption level in the world. Given that world per capita consumption was greater than the average per capita consumption in the NIEs till the 1990s, could such a change in aspirations generate an increase in saving rates similar in magnitude to the rise in NIE savings that we saw in Figure 1?[12]

Consider the same world economy as before comprising four regions: the G7, LAC, NIE and SSA. Emerging economies, and the NIEs. The list of countries in each of these groupings is given in Table 8 in the Appendix. Let average per capita world consumption be

$$C = l_{G7}c_{G7} + l_{LAC}c_{LAC} + l_{NIE}c_{NIE} + l_{SSA}c_{SSA}$$

where c_i is the per capita consumption of region i.

Consider two regimes:

$$\text{Regime 1: } \zeta_i = c_i \ \ \forall\ i$$

$$\text{Regime 2: } \zeta_{NIEs} = C, \ \ \zeta_i = c_i \ \ \forall\ i \neq NIEs$$

where ζ_i denotes the externality in preferences of region i at date t. Regime 1 corresponds to our baseline model. Under Regime 2 however, the NIEs switch to valuing their own consumption relative to the world per capita consumption level. We consider an environment where at some date t^*, the regime switches from Regime 1 to Regime 2. Given that per capita world consumption is higher, such a regime switch represents a switch to a higher aspiration level for the NIEs. In the context of our model, could such a regime switch account for the 10 percentage point increase in the saving rate of the NIEs since 1970?

To answer this question we calibrate the model by choosing parameter values such that the model reproduces the steady state saving rates, world capital shares and world labor shares of the three regions in 1970 under the maintained assumption of the world economy being in Regime 1. Keeping the parameters underlying the initial calibration unchanged, we then feed the estimated regional productivity processes for 1970-2014 into the model and simulate

[12] We should clarify that we are not building a theory of aspirations in this paper. Rather, we view this exercise as a quantitative exploration of the dynamic general equilibrium consequences of a change in aspirations.

the three economies under two scenarios: (a) an unanticipated, one-time, permanent switch from Regime 1 to Regime 2 at some date t^*; and (b) the world economy stays in Regime 1 at all dates $t \geq 1970$.

Table 6 presents the quantitative effects of this aspirational regime switch for NIE saving rates. In order to examine the sensitivity of the results to the date of the regime switch, we report results from three different switch dates: $t^* = 1970, 1980, 1985$. The last three columns give the predicted saving rates under the three different regime switch dates. The Table demonstrates the power of changes in aspirations. Productivity shocks, on their own, have barely any effect on the average saving rate for 1970-2014, for both the recursive and the CRRA models. In contrast, the regime change for aspirations has a dramatically strong effect with the predicted saving rate rising by 10 to 15 percentage points.

Table 6 also shows that the earlier is the switch date for the regime, the greater is increase in the average saving rate predicted by the model. This is intuitively clear: the earlier the switch the longer is the duration of higher savings, hence the greater is the average saving rate for the entire 1970-2014 period.

Table 6: Saving Rate in NIEs After Regime Change

	Savings in 1970	Mean Savings 1970-2014		
		Shift in 1970	Shift in 1980	Shift in 1985
Data	0.294	0.399	0.399	0.399
Recursive Preferences				
Regime shift only	0.294	0.447	0.423	0.409
Productivity shocks only	0.294	0.271	0.271	0.271
Regime shift + productivity shocks	0.294	0.435	0.414	0.401
CRRA Preferences				
Productivity shocks only	0.144	0.132	0.132	0.132

In summary, the switch to a higher aspiration is key for the model to reproduce the sharp increase in the saving rates of the NIEs. Given that the actual increase in the average NIE saving rate was around 10.5 percent, which is similar to the predictions from the regime switch version of the model, we find these results to be indicative of the promise of the aspiration mechanism to explain the rapid growth of savings in the NIEs.

7 Conclusion

The variation in saving behavior across countries has long been a puzzle and a challenge to explain for standard neoclassical models. In this paper we have built on the work of Farmer and Lahiri (2005) to explore the explanatory potential of recursive preferences and preference heterogeneity in jointly accounting for the cross-country saving data. The key aspect of the preferences that we used is that utility depends on consumption relative to the consumption of a reference group. Our specification implies that when countries are poor they display high patience and high saving rates. As their consumption gets closer to the levels of their reference group however they become more impatient, a property that Lucas and Stokey (1984) called "increasing marginal impatience". This feature of the preferences keeps the wealth distribution from becoming degenerate even when preferences are heterogenous across countries.

We have applied these preferences to a four-region world economy model that is calibrated to match regional saving rates in 1970. Using only region-specific productivity shocks as an exogenous driver, we have showed that the calibrated model outperforms the CRRA model in matching the time series behavior in saving rates of the different regions in terms of both the mean level of regional saving rates as well as their correlations with the saving rates in the data. Given that our recursive preference structure can also account for the long run differences in saving rates across regions, a feature that CRRA preferences cannot match under the our environment, we view this as being strongly supportive evidence in favour of the recursive preference specification for understanding world saving behavior.

In addition, we have shown that a change in the aspirations of societies, as captured by a change in the reference consumption basket they use to value their own utility, can account for sudden and sharp changes in saving rates. Thus, our model can account for the rapid increase in saving rates in NIEs and its overall behavior between 1970 and 2014 by allowing for a change in the reference basket being used by the NIEs from the average world consumption level to the G7 consumption level in 1985. Intuitively, a higher reference consumption level induces greater saving as accumulating greater wealth is the only way to achieve a higher steady state consumption.

We believe this class of models has great potential in also helping us understand changes in the wealth distribution within countries over time. Wealth evolves as a function of saving. Accounting for differential saving rates is thus key to explaining wealth distributions and changes therein. We hope to address this issue in future work.

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8 Appendix

8.1 Risk aversion under recursive preferences

We have assumed that the representative agent's preferences are described by the recursive representation

$$u_t = \frac{c_t^\theta \bar{\zeta}_t^{1-\theta}}{\theta} + \beta \mathbb{E}_t \left[\frac{u_{t+1}^\delta \bar{\zeta}_t^{1-\delta}}{\delta} \right] \tag{8.20}$$

The two key preference parameters here are θ and δ. We have calibrated δ to match the initial savings ratio in 1970. We calibrated θ such that the relative risk aversion in steady state is 2. In this appendix we derive the expression for relative risk aversion under the recursive preference specification that we use and show its mapping to the parameters δ and θ. Our approach closely follows Swanson (2012) augmented with growth.

Consider a simplified representation of our model where the representative agent's periodic budget constraint is

$$b_{t+1} = R_t b_t + y_t - c_t.$$

Here, b denotes real bonds, y is income and c is consumption. Assume that the exogenous factor $\bar{\zeta}_{t+1}$ grows at a constant rate g so that $\bar{\zeta}_{t+1} = g\bar{\zeta}_t$.[13]

The agent maximizes lifetime utility subject to the flow budget constraint. Since the only state variable in this environment is b, we can write this problem using the value function representation

$$V(b_t) = \max \left\{ \frac{c_t^\theta \bar{\zeta}_t^{1-\theta}}{\theta} + \beta \mathbb{E}_t \left(\frac{V(b_{t+1})^\delta \bar{\zeta}_t^{1-\delta}}{\delta} \right) \right\}$$

The first order condition for this problem is

$$\left(\frac{c_t}{\bar{\zeta}_t}\right)^{\theta-1} = \beta \mathbb{E}_t \left[\left(\frac{V_{t+1}}{\bar{\zeta}_{t+1}}\right)^{\delta-1} \left(\frac{c_{t+1}}{\bar{\zeta}_{t+1}}\right)^{\theta-1} \left(\frac{\zeta_{t+1}}{\zeta_t}\right)^{\delta-1} R_{t+1} \right]$$

[13] In the model in the paper this steady state growth rate of $\bar{\zeta}$ will just be the rate of growth of trend productivity z.

Using the notational convention $\tilde{x}_t = \frac{x_t}{\zeta_t}$, this first order condition can be written as

$$\tilde{c}_t^{\theta-1} = \beta g^{\delta-1} \mathbb{E}_t \left[\tilde{V}_{t+1}^{\delta-1} \tilde{c}_{t+1}^{\theta-1} R_{t+1} \right] \tag{8.21}$$

Note that under this normalization all variables with tildes on top are rendered stationary in steady state. Using the same convention we can write the budget constraint as

$$g\tilde{b}_{t+1} = R_t \tilde{b}_t + \tilde{y}_t - \tilde{c}_t$$

Lastly, the preferences themselves can be also be written in stationary form as

$$\tilde{u}_{t+1} = \frac{\tilde{c}_t^\theta}{\theta} + \beta g^\delta \mathbb{E}_t \left(\frac{\tilde{u}_{t+1}^\delta}{\delta} \right)$$

Now consider the following gamble:

$$b_{t+1} = R_t b_t + y_t - c_t + \sigma \bar{\zeta}_{t+1} \epsilon_{t+1}$$

where $\epsilon_{t+1} \sim N(0,1)$ is independent of all date t choice variables. Importantly, this is a one-time gamble (*ie.* $\epsilon_{t+j} = 0 \ \ \forall\ j > 1$) that is realized after choices are made in period t. We index the gamble to the exogenous growth factor $\bar{\zeta}$ in order to keep the relative size of the gamble constant over time in a growing economy. Normalizing for growth, this gamble can be written as

$$\tilde{b}_{t+1} = \frac{R_t}{g} \tilde{b}_t + \frac{\tilde{y}_t - \tilde{c}_t}{g} + \sigma \epsilon_{t+1}$$

We denote the normalized value function associated with this gamble by $\tilde{V}^*(\tilde{b}_t, \sigma)$. The Bellman equation for this problem is

$$\tilde{V}^*(\tilde{b}_t, \sigma) = \max \left\{ \frac{\tilde{c}_t^\theta}{\theta} + \beta \mathbb{E}_t \left(\frac{\tilde{V}(\tilde{b}_{t+1})^\delta}{\delta} \right) \right\} \tag{8.22}$$

Note that $\tilde{V}_{t+1}$ is independent of σ since this is a one-period, one-time gamble.[14]

[14]Under our preferences, the optimal plans derived by working under the normalized value function $\tilde{V}$

To determine the agent's aptitude for risk, one can ask how much she is willing to pay in order to avoid the gamble. To formalize this, consider an alternative gamble:

$$b_{t+1} = R_t b_t + y_t - c_t - \bar{\zeta}_{t+1}\mu$$

Normalizing this by the growth factor $\bar{\zeta}_t$, the flow budget constraint can be written as

$$\tilde{b}_{t+1} = \frac{R_t}{g}\left(\tilde{b}_t - \frac{g}{R_t}\mu\right) + \frac{\tilde{y}_t - \tilde{c}_t}{g}$$

Denote the normalized value function associated with this gamble by $\tilde{V}\left(\tilde{b}_t - \frac{g}{R_t}\mu\right)$.

The one-time payment μ that makes the agent indifferent between taking or rejecting the gamble is her coefficient of absolute risk aversion. The condition

$$\tilde{V}\left(\tilde{b}_t - \frac{g}{R_t}\mu\right) = \tilde{V}^*(\tilde{b}_t, \sigma)$$

yields a solution $\mu^a(\tilde{b}_t, \sigma)$. This is the payment that makes the agent indifferent between taking the gamble or rejecting it. Following the Arrow-Pratt definitions of risk, the coefficient of absolute relative aversion is

$$\tilde{R}^a \equiv \lim_{\sigma \to 0} \frac{\mu^a(\tilde{b}_t, \sigma)}{\sigma^2/2}$$

$\tilde{R}^a$ measures the willingness of the agent to pay in order to avoid the risky gamble as the variance of the gamble becomes vanishingly small.

Correspondingly, the coefficient of relative risk aversion can be derived by determining the certainty equivalent cost $\mu(\sigma)$ that makes the agent indifferent between the following two gambles that are proportional to the agent's normalized aggregate wealth $\tilde{A}$:

$$\tilde{b}_{t+1} = \frac{R_t}{g}\tilde{b}_t + \frac{\tilde{y}_t}{g} - \frac{\tilde{c}_t}{g} + \sigma \tilde{A}_t \epsilon_{t+1}$$
$$\tilde{b}_{t+1} = \frac{R_t}{g}\left(\tilde{b}_t - \frac{g}{R_t}\tilde{A}_t\mu\right) + \frac{\tilde{y}_t - \tilde{c}_t}{g}$$

(which is $\frac{V}{\zeta}$) along with the normalized budget constraint are identical to those derived by working with V and the un-normalized budget constraint.

Let μ^r be the one-time payment that makes the agent indifferent between these two wealth-weighted gambles. Hence, μ^r solves

$$\tilde{V}\left(\tilde{b}_t - \frac{g}{R_t}\tilde{A}_t\mu^r\right) = \tilde{V}^*(\tilde{b}_t, \tilde{A}_t\sigma)$$

The coefficient of relative risk aversion is given by

$$\tilde{R}^r \equiv \lim_{\sigma\to 0}\frac{\mu^r(\tilde{b}_t, \sigma)}{\sigma^2/2}$$

To determine the precise expressions for these two measures of risk aversion, we start by examining the effect of the gamble on the value function. Let $\tilde{c}_t^*$ and $\tilde{b}_{t+1}^*$ denote the optimal values of normalized consumption and bonds that solve the dynamic program given in equation (8.22). The value from the gamble can then be written as

$$\tilde{V}^*(\tilde{b}_t, \sigma) = \left\{\frac{(\tilde{c}_t^*)^\theta}{\theta} + \beta\mathbb{E}_t\left(\frac{\tilde{V}(\tilde{b}_{t+1}^*)^\delta}{\delta}\right)\right\}$$

Differentiating $\tilde{V}^*(\tilde{b}_t, \sigma)$ with respect to σ gives[15]

$$\frac{d\tilde{V}^*(\tilde{b}_t, \sigma)}{d\sigma} = \beta g^\delta \mathbb{E}_t\left(\tilde{V}_{t+1}^{\delta-1}\tilde{V}_{t+1}'\epsilon_{t+1}\right) = 0$$

The last equality with zero follows from the fact that this is a one period gamble which implies that $\tilde{V}_{t+1}$ is independent of ϵ_{t+1}. Hence,

$$\mathbb{E}_t\left(\tilde{V}_{t+1}^{\delta-1}\tilde{V}_{t+1}'\epsilon_{t+1}\right) = \mathbb{E}_t\left(\tilde{V}_{t+1}^{\delta-1}\tilde{V}_{t+1}'\right)\mathbb{E}_t\epsilon_{t+1} = 0.$$

This implies that the first-order effect of the gamble on $\tilde{V}^*(\tilde{b}_t^*, \sigma)$ is zero.

The second-order effect of the gamble is

$$\beta g^\delta\left[(\delta-1)\mathbb{E}_t\tilde{V}_{t+1}^{\delta-2}\tilde{V}_{t+1}'^2 + \mathbb{E}_t\tilde{V}_{t+1}^{\delta-1}\tilde{V}_{t+1}''\right].$$

[15]There are additional terms which cancel out by the envelope condition.

The other second-order terms drop out because we are evaluating the second order expansion around the optimum and $\sigma = 0$. Hence, the second-order expansion of the value of the gamble to the agent around $\sigma = 0$ is

$$\tilde{V}^*(\tilde{b}_t^*, \sigma) = \tilde{V}(\tilde{b}_t^*) + \beta g^\delta \left[(\delta - 1)\mathbb{E}_t \tilde{V}(\tilde{b}_{t+1}^*)^{\delta-2} \left(\frac{\partial \tilde{V}_{t+1}}{\partial \tilde{b}_{t+1}^*} \right)^2 + \mathbb{E}_t \tilde{V}(\tilde{b}_{t+1}^*)^{\delta-1} \frac{\partial^2 \tilde{V}_{t+1}}{\partial (\tilde{b}_{t+1}^*)^2} \right] \frac{d\sigma^2}{2} \tag{8.23}$$

The effect on the value of the agent of paying μ to avoid the gamble, evaluated around $\mu = 0$, is

$$\tilde{V}(\tilde{b}_t^* - \frac{\mu}{R_t}) = \tilde{V}(\tilde{b}_t^*) - \beta g^\delta \mathbb{E}_t \tilde{V}(\tilde{b}_{t+1}^*)^{\delta-1} \frac{\partial \tilde{V}_{t+1}}{\partial \tilde{b}_{t+1}^*} d\mu \tag{8.24}$$

Equating equations 8.23 and 8.24 gives

$$\frac{d\mu}{d\sigma^2/2} = \frac{(1-\delta)\mathbb{E}_t \tilde{V}(\tilde{b}_{t+1})^{\delta-2} \left(\frac{\partial \tilde{V}_{t+1}}{\partial \tilde{b}_{t+1}} \right)^2 - \mathbb{E}_t \tilde{V}(\tilde{b}_{t+1})^{\delta-1} \frac{\partial^2 \tilde{V}_{t+1}}{\partial \tilde{b}_{t+1}^2}}{\mathbb{E}_t \tilde{V}(\tilde{b}_{t+1})^{\delta-1} \frac{\partial \tilde{V}_{t+1}}{\partial \tilde{b}_{t+1}}}$$

Around a non-stochastic steady state, this expression reduces to

$$\frac{d\mu}{d\sigma^2/2} = (1-\delta)\frac{\tilde{V}'(\tilde{b})}{\tilde{V}(\tilde{b})} - \frac{\tilde{V}''(\tilde{b})}{\tilde{V}'(\tilde{b})}$$

Recall that this derivative is being evaluated around $\sigma = 0$ and $\lim_{\sigma \to 0} \frac{\mu(\tilde{b}_t, \sigma)}{\sigma^2/2} = \frac{d\mu}{d\sigma^2/2}$. Hence, we have

$$\tilde{R}^a = (1-\delta)\frac{\tilde{V}'(\tilde{b})}{\tilde{V}(\tilde{b})} - \frac{\tilde{V}''(\tilde{b})}{\tilde{V}'(\tilde{b})}$$

From the expressions above, it is straightforward to check that the coefficient of relative risk aversion in a non-stochastic steady state is given by

$$\tilde{R}^r = \tilde{A} \left[(1-\delta)\frac{\tilde{V}'(\tilde{b})}{\tilde{V}(\tilde{b})} - \frac{\tilde{V}''(\tilde{b})}{\tilde{V}'(\tilde{b})} \right]$$

In order to make these expressions operational within our model, we need steady state expressions for $\tilde{V}, \tilde{V}', \tilde{V}''$ and $\tilde{A}$. In the following, we shall use the subscript s to denote steady state values of variables.

Firstly, from equation 8.21 the steady state normalized value function is

$$\tilde{V}_s = \frac{(\beta R_s)^{\frac{1}{1-\delta}}}{g}$$

where we have used the fact that $\tilde{c}_t = \tilde{c}_s \; \forall \; t$ in steady state. Next, around the optimum, the envelope condition gives $\tilde{V}'(\tilde{b}_t) = R_t \tilde{c}_t^{\theta-1}$. In steady state this becomes

$$\tilde{V}'(\tilde{b}_s) = R_s \tilde{c}_s^{\theta-1}$$

Differentiating $\tilde{V}'(\tilde{b}_t)$ with respect to $\tilde{b}_t$ and evaluating it around the steady state gives

$$\tilde{V}''(\tilde{b}_s) = (\theta - 1) R_s \tilde{c}_s^{\theta-2} \frac{\partial \tilde{c}_s}{\partial \tilde{b}_s} = (\theta - 1) R_s \tilde{c}_s^{\theta-2} (R_s - g)$$

where the second equality follows from noting that the steady state normalized budget constraint is

$$\tilde{c}_s = (R_s - g)\tilde{b}_s + \tilde{y}_s$$

Substituting these expressions into the formula for absolute risk aversion derived above gives

$$\tilde{R}^a = \frac{R_s g (1-\delta) \tilde{c}_s^{\theta-1}}{(\beta R_s)^{\frac{1}{1-\delta}}} + \frac{(1-\theta)(R_s - g)}{\tilde{c}_s}$$

Aggregate normalized wealth is given by

$$\tilde{A}_t = \left(\frac{g}{1+r_t}\right) \mathbb{E}_t \sum_{\tau=t}^{\infty} M_\tau \tilde{c}_\tau$$

where M_τ is the stochastic discount factor. From the Euler equation in steady state we have

$$M_\tau = \prod_t^\tau \beta g^\delta \tilde{V}_s^{\delta-1} = \prod_t^\tau \left(\frac{g}{R_s}\right)$$

Hence,

$$\tilde{A}_s = \left(\frac{g}{R_s - g}\right) \tilde{c}_s \tag{8.25}$$

Substituting this into the expression for the coefficient of relative risk aversion gives

$$\tilde{R}^c = g(1-\theta) + \left(\frac{gR_s}{R_s - g}\right) \frac{(1-\delta) g \tilde{c}_s^{\theta}}{(\beta R_s)^{\frac{1}{1-\delta}}}$$

From equation 8.20 we have

$$1 = \frac{\tilde{c}_s^{\theta}}{\theta \tilde{V}_s} + \beta g^{\delta} \tilde{V}_s^{\delta-1}$$

while the Euler equation in steady state gives

$$\beta g^{\delta} \tilde{V}_s^{\delta-1} = \frac{g}{R_s}$$

. Combining these two expressions and substituting the result into the expression for $\tilde{R}^c$ gives

$$\tilde{R}^c = g\left[1 - \theta + \theta\left(\frac{1-\delta}{\delta}\right)\left(\frac{\delta R_s - g}{R_s - g}\right)\right] \tag{8.26}$$

We can now use equation 8.26 to solve out for θ in terms of parameters and the coefficient of relative risk aversion:

$$\theta = \frac{\frac{\tilde{R}^c}{g} - 1}{\left(\frac{1-\delta}{\delta}\right)\left(\frac{\delta R_s - g}{R_s - g}\right) - 1} \tag{8.27}$$

Given the parameters δ, g and the steady state value of the real interest factor R_s, this equation can be used to calibrate θ to match a given target for the coefficient of relative risk aversion $\tilde{R}^c$.

8.2 Model solution method

To compute the equilibrium decision rules in the model we use perturbation methods, which can provide solutions to discrete-time systems of rational expectations equations of arbitrarily high order. Provided a solution exists in a neighborhood of a non-stochastic steady state – and if the system of equations is smooth around the steady state point – perturbation methods are guaranteed to converge to the true solution in the relevant region.

We can summarize the model's equilibrium conditions by

$$\mathbb{E}_t F(x_{t-1}, x_t, x_{t+1}; \epsilon_t, \epsilon_{t+1}, \epsilon_{t+2}, ...) = 0 \tag{8.28}$$

where x_t denotes the vector of model variables, the time-invariant equilibrium solution is given by

$$x_t = g(x_{t-1}, x_{t-1}, ...; \epsilon_t) \tag{8.29}$$

The perturbation algorithm solves a model of the form

$$\mathbb{E}_t F(x_{t-1}, x_t, x_{t+1}; \epsilon_t, \epsilon_{t+1}, \epsilon_{t+2}, ...; \sigma) = 0 \tag{8.30}$$

where $\sigma \in [0, 1]$ is a scaling parameter that ratchets the level of uncertainty up or down, so that $\sigma = 1$ corresponds to the original model (8.28) and $\sigma = 0$ corresponds to a version of the model with no uncertainty. The algorithm thereby yields a family of solutions

$$x_t = g(x_{t-1}, x_{t-1}, ...; \epsilon_t; \sigma) \tag{8.31}$$

Assuming that the non-stochastic steady state, x_{ss} is known with certainty,

$$F(x_{ss}, x_{ss}, x_{ss}; 0, 0, 0, ...; 0) = 0 \tag{8.32}$$

then a first-order approximation of $F(\cdot)$ around $\{x, \sigma\} = \{x_{ss}, 0\}$ can be used to solve for the coefficients in

$$x_t = g^{(1)}(x_{t-1}, x_{t-1}, ...; \epsilon_t; \sigma) \tag{8.33}$$

which is the first-order approximation of $g(\cdot)$ around $\{x, \sigma\} = \{x_{ss}, 0\}$. Iteratively taking higher-order approximations of $F(\cdot)$ around $\{x_{ss}, 0\}$, solutions can then be found for the coefficients of

$$x_t = g^{(N)}(x_{t-1}, x_{t-1}, ...; \epsilon_t; \sigma) \tag{8.34}$$

for arbitrarily high N. However, in practice we work with first order approximations as higher-order approximations provide little benefit for our purposes. To implement the algorithm we use Eric Swanson's *Perturbation AIM* program.

8.3 Data

Table 7: List of Countries in Figure 1

Region	Countries
G7	Canada, France, Germany, Italy, Japan, United Kingdom, United States
Emerging Market	Afghanistan, Albania, Algeria, Antigua & Barbuda, Argentina, Armenia, Azerbaijan, Bahamas, Bahrain, Bangladesh, Barbados, Belarus, Belize, Bhutan, Bolivia, Bosna & Herzegovina, Brazil, Brunei, Bulgaria, Cambodia, Chad, Chile, Colombia, Costa Rica, Croatia, Djibouti, Dominica, Dominican Republic, Ecuador, Egypt, El Salvador, Fiji, Georgia, Grenada, Guatemala, Guyana, Haiti, Honduras, Hungary, Kiribati, Kosovo, Kuwait, Iran, Iraq, Jamaica, Kazakhstan, Kyrgystan, Laos, Latvia, Lebanon, Liberia, Libya, Lithuania, Macedonia, Maldives, Mauritania, Mexico, Moldova, Mongolia, Montenegro, Morocco, Myanmar, Nepal, Nicaragua, Oman, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Poland, Qatar, Romania, Russia, Samoa, Saudi Arabia, Serbia, Solomon Islands, Sri Lanka, St. Kitts & Nevis, St. Lucia, St. Vincent & the Grenadines, Suriname, Syria, Tajikistan, Timor-Leste, Tonga, Trinidad & Tobago, Tunisia, Turkey, Turkmenistan, Tuvalu, Ukraine, United Arab Emirates, Uruguay, Uzbekistan, Vanuatu, Venezuela, Yemen
Sub Sahara	Angola, Benin, Botswana, Burkina Faso, Cameroon, Cape Verde Central Africa Republic, Comoros, Democratic Republic of Congo, Republic of Congo, Cote d'Ivoire, Equatorial Guinea, Eritrea, Ethiopia, Gabon, Gambia, Ghana, Guinea, Guinea-Bissau, Kenya Lesotho, Liberia, Madagascar, Malawi, Mali, Maritius, Mozambique Namibia, Niger, Nigeria, Rwanda, Sao Tome Principe, Senegal, Seychelles, Sierra Leone, South Africa, Sudan, Swaziland, Tanzania, Togo, Uganda, Zambia, Zimbabwe

Table 8: List of Countries in 4-Region Model

Region	Countries
G7	Canada, France, Germany, Italy, Japan, United Kingdom, United States
Newly Industrialized Asia	Hong Kong, Korea, Singapore, Taiwan,
Latin America & Caribbean	Antigua & Barbuda, Argentina, Bahamas, Barbados, Belize, Bolivia Brazil, Chile, Colombia, Costa Rica, Dominica, Dominican Republic Ecuador, El Salvador, Grenada, Guatemala, Guyana, Haitia, Honduras Jamaica, Mexico, Nicaragua, Panama, Paraguay, Peru St. Kitts & Nevis, St. Lucia, St. Vincent and the Grenadines, Suriname Trinidad & Tobago, Uruguay, Venezuala
Sub Sahara	Angola, Benin, Botswana, Burkina Faso, Cameroon, Cape Verde Central Africa Republic, Comoros, Democratic Republic of Congo, Republic of Congo, Cote d'Ivoire, Equatorial Guinea, Eritrea, Ethiopia, Gabon, Gambia, Ghana, Guinea, Guinea-Bissau, Kenya Lesotho, Liberia, Madagascar, Malawi, Mali, Maritius, Mozambique Namibia, Niger, Nigeria, Rwanda, Sao Tome Principe, Senegal, Seychelles, Sierra Leone, South Africa, Sudan, Swaziland, Tanzania, Togo, Uganda, Zambia, Zimbabwe