Board of Governors of the Federal Reserve System

International Finance Discussion Papers

ISSN 1073-2500 (Print)
ISSN 2767-4509 (Online)

Number 1437

April 2026

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Please cite this paper as:
Krolikowski, Pawel M. and Andrew H. McCallum (2026). "Tariffs and Goods-Market Search Frictions," International Finance Discussion Papers 1437. Washington: Board of Governors of the Federal Reserve System, https://doi.org/10.17016/IFDP.2026.1437.

Tariffs and Goods-Market Search Frictions *

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April 7, 2026

Abstract

We study tariffs in a general equilibrium dynamic model with search frictions between heterogeneous exporting producers and importing retailers. We show the model has a unique equilibrium and analytically characterize home unilateral import tariffs that maximize welfare given a passive foreign country. Search frictions add two terms to the standard optimal tariff expression: One lowers tariffs when contact rates are low; another when private export costs exceed social opportunity costs. Search frictions also introduce new incentives to subsidize imports due to market thickness effects. We calibrate our baseline to U.S. and Chinese 2016 data. We compare this baseline to a counterfactual with international search costs reduced to domestic levels but with all other parameters fixed. We find that higher baseline search costs reduce optimal U.S. unilateral and Nash tariffs and attenuate welfare responses to tariff changes.

JEL codes: C78, D62, D83, F12, F13.

Keywords: Optimal tariffs, trade policy, efficiency, welfare, social planner, bargaining, market tightness.

*We thank Svetlana Demidova (discussant), Dave Donaldson, Lukasz Drozd (discussant), Doireann Fitzgerald (editor), Agustin Gutierrez, Andrei Levchenko, Robert Staiger, and an anonymous referee for helpful advice. We also thank seminar participants at the 2024 Dynamic Quantitative Trade Conference and the 2026 ASSA Annual Meeting. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors, the Federal Reserve Bank of Cleveland, or the Federal Reserve System.

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1 Introduction

We understand little about how tariff policy interacts with search frictions despite the fact that connecting with overseas buyers is a prevalent firm-level search friction faced by exporters (Kneller and Pisu, 2011) and that tariff policy can have large effects on welfare (Costinot and Rodríguez-Clare, 2014).

To improve our understanding, we study uniform tariffs in a model with random search between heterogeneous exporting producers and importing retailers in general equilibrium. Our model introduces an endogenous fraction of unmatched exporters that are actively searching for importing partners, but otherwise nests Melitz (2003). These unmatched exporters alter the levels of aggregate variables and the changes in aggregate variables in response to tariffs because when producers are unmatched their associated varieties cannot be traded.

We derive three analytical results in this framework. First, we show that the model admits a unique equilibrium. Second, we analytically characterize the unilateral uniform import tariff that maximizes domestic welfare in a two-country setting. With search frictions, the optimal tariff depends not only on the offer curve elasticity—as it does in a model without search frictions—but also on two new terms. An import markdown term relates to the difference between import prices and final consumer prices. A second term relates to additional externalities introduced by search frictions. Third, we characterize the offer curve elasticity in our model and show that search frictions introduce a new incentive to subsidize imports because of endogenous market thickness effects. These results extend those in Costinot, Rodríguez-Clare, and Werning (2020), henceforth CRW, to a setting that includes search frictions.

We also obtain two important quantitative results using a realistic calibration that matches U.S. and Chinese data in 2016. First, the optimal U.S. unilateral tariff with baseline search frictions is about 20 percentage points below that in a counterfactual exercise in which international search costs are reduced to domestic search costs, but we otherwise retain the rest of the baseline calibration, "reduced search frictions". Second, search frictions attenuate welfare gains so that optimal tariff effects are not as large as predicted by models with reduced search frictions. Intuitively, this is because search frictions imply fewer matched varieties and mainly tariffs affect aggregates through the intensive margin of only matched varieties. For example, moving from baseline tariffs to optimal unilateral tariffs raises U.S. welfare by about 0.03 with baseline search frictions. But with reduced frictions, moving to optimal tariffs raises U.S. welfare by 1.0 percent.

Our new optimal tariff expression implies that the country social planner subsidizes import markets in which the levels of import markdowns are large. The social planner's solution involves a manipulation of the terms of trade (TOT), as in Dixit (1985) and CRW.

With search frictions, the TOT include import markdowns so the optimal tariff includes these as well. Intuitively, producers are willing to accept low negotiated prices (large import markdowns) when they face poor outside options driven by low contact rates (high unmatched rates). The social planner stimulates retailer entry in these search markets by setting lower tariffs to increase contact rates.

Search frictions also introduce new externalities that affect optimal tariffs. In the decentralized equilibrium of a model without search frictions, there are no allocative inefficiencies. The (negative of the) slope of the production possibility frontier (PPF), the marginal rate of transformation (MRT), is equal to the ratio of the negotiated price indexes received by producers. Search frictions, however, introduce new externalities because producers do not consider their effect on unmatched rates when deciding whether to search or not and how much to produce. In contrast, the social planner explicitly considers the social opportunity cost of production including any effects on unmatched rates, which affect the MRT. Our new optimal tariff expression implies that the social planner sets higher import tariffs when the social opportunity cost of exports exceeds the relative private cost because higher import tariffs reduce exports via balanced trade.

Search frictions also create an incentive for country d to subsidize imports from country o. In a model without search frictions, selection introduces increasing returns to scale in the PPF (aggregate production nonconvexities), which affects the offer curve elasticity, as in CRW. In that model, as exports from country o to d rise, producers enter that market, and domestic goods production in o falls, which lowers producer entry in the domestic market. Both of these effects lower the opportunity cost of o exports in terms of domestic goods, which gives rise to increasing returns to scale. These scale effects create an incentive for country d to subsidize imports because subsidies lower the opportunity cost, and therefore the price, of exports from o to d. Search frictions strengthen this effect on the offer curve elasticity for two reasons. First, increasing country o's exports to country d lowers search frictions in this market because of increased retailer entry in country d, which leads to a higher matched rate. The higher matched rate lowers the opportunity cost of exports. Second, as exports from o to d rise, domestic goods production in country o falls, which lowers retailer entry and the domestic matched rate, and increases the opportunity cost of producing domestic goods.

To provide intuition for our analytical results, we present numerical examples with simple assumptions—two symmetric countries and search frictions in only one of four possible markets. These examples confirm our new optimal tariff decomposition and the effects that search frictions have on the offer curve elasticity. The numerical examples also have the following six features. First, the unilateral optimal tariff in a model with search frictions is below that in a model without search frictions, for any positive search cost. Second, the optimal tariff declines rapidly as search costs increase. Third, the offer curve elasticity is quantitatively the largest contribution to the level of optimal tariffs. Fourth, in a model with exogenous (finite) unmatched rates and no selection, the economy attains allocative efficiency and the offer curve elasticity formula is the same as in a model without search frictions. Nevertheless, optimal tariffs fall with unmatched rates in this restricted model through their effect on import markdowns. As such, even exogenous search frictions affect the level of optimal tariffs in these examples, because they affect the levels of aggregate variables. Fifth, search frictions also reduce optimal tariffs in a Nash equilibrium. Sixth, optimal tariffs yield a smaller welfare gain relative to free trade in the model with search frictions than without them, echoing the result in Krolikowski and McCallum (2021), henceforth KM, that search frictions attenuate welfare responses to shocks. We stress that these six results are useful for understanding the model's mechanisms but are not general ones because they are based on our simple numerical example.

To obtain a quantitatively realistic environment, we match our model to U.S. and Chinese data in 2016 using the approach in KM. In short, to calibrate international and domestic search costs we use the fraction of firms that export in each country and manufacturing capacity utilization rates. Estimates of tariff and distance elasticities from the literature inform the elasticity of matches with respect to the number of searching producers.

In this calibration, search frictions substantially affect optimal tariffs and attenuate the welfare gains from optimal tariff policy. With baseline search frictions, the U.S. and Chinese unilateral optimal tariffs are about 20 percentage points below that in a counterfactual exercise in which international search costs are reduced to domestic search costs, but we otherwise retain the rest of the baseline calibration. This exercise could be motivated by technological innovations that reduce international search costs. For the United States (China), the respective optimal unilateral tariff with baseline search frictions would increase welfare by only 0.03 (0.01) percent relative to the welfare at 2016 tariff levels. In contrast, with reduced search frictions, the optimal unilateral tariffs would raise welfare by about 1 percent in both countries. We also solve for Nash equilibrium tariffs in the model with baseline and reduced search frictions. The optimal U.S. Nash tariff in the model with baseline search frictions is 1.15, below the optimal tariff with reduced search frictions, 1.33. In the Nash equilibrium of the model with baseline frictions, U.S. (Chinese) welfare is 0.2 percent higher (0.7 percent lower) than welfare at 2016 tariff levels.

Related research

Search frictions in international goods markets are motivated by direct evidence as in Eaton, Jinkins, Tybout, and Xu (2026). In addition, Kneller and Pisu (2011) find that "identifying the first contact" and "establishing initial dialogue" are more common obstacles to exporting than "dealing with legal, financial and tax regulations overseas" in a survey of U.K. firms. The broad relevance of search frictions is established for labor markets, but also motivated by a variety of other contexts. For example, Wasmer and Weil (2004) study search frictions in credit markets, Lagos and Wright (2005) study search frictions in monetary economics, and Piazzesi, Schneider, and Stroebel (2020) study search frictions in housing markets. Search frictions extend to the contexts of marriage markets (Smith, 2006), insurance markets (Cebul, Rebitzer, Taylor, and Votruba, 2011), and goods markets (Drozd and Nosal, 2012), among others.

We also compare our results to past studies of tariff policy, efficiency, and welfare in settings without and with search frictions. Particularly relevant work is by Demidova and Rodríguez-Clare (2009). That paper extends the optimal-tariff results in Gros (1987) to a small country Melitz model. Both models are special cases of the model studied in Felbermayr, Jung, and Larch (2013), which characterizes optimal tariffs in cooperative and noncooperative games for two large countries with heterogeneous producers. CRW generalizes these results beyond homogeneous firms and Pareto-distributed productivity, and to tariffs that vary with firm productivity. Our work complements Grossman, Helpman, and Redding (2024), who study how unexpected tariff hikes affect welfare through renegotiation and new trading relationships in a context of global supply chains. Brancaccio, Kalouptsidi, Papageorgiou, and Rosaia (2023) study global efficiency in markets with search but focus on the international transportation sector. Finally, our work contributes to a vast literature about the welfare gains from trade in models without search frictions (Eaton and Kortum, 2002; Broda and Weinstein, 2006; Broda, Limao, and Weinstein, 2008; Arkolakis, Costinot, and Rodríguez-Clare, 2012; Ossa, 2011, 2014).

2 The model, aggregation, and steady-state equilibrium

We present an extension of the continuous-time model of KM with additional details and equations included in Appendix A.

2.1 Model

We add tariffs and endogenous wages to the environment in KM, which features search frictions between producers and retailers in domestic and international goods markets. We focus on steady-state implications. The model features D countries. The first subscript denotes the importing (destination) country d and the second subscript denotes the exporting (origin) country o. For example, imports by d from o are denoted IM_{do}.

2.1.1 Consumers

A representative consumer in country d has utility over a constant elasticity of substitution (CES) aggregate of differentiated varieties, $q_{do}(\omega)$, from all origins, with elasticity of substitution, $\sigma > 1$. The consumer's problem is

$$\max_{q_d(1), q_{dk}(\omega)} \left[\sum_{k=1}^{O} Q_{dk}^{\frac{1}{\mu}}\right]^{\mu} \tag{1a}$$

$$s.t.\ Q_{do} = \left[\int_{\omega\in\Omega_{do}} q_{do}(\omega)^{\frac{1}{\mu}}\, d\omega\right]^{\mu}, \tag{1b}$$

$$C_d = \sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)\, q_{dk}(\omega)\, d\omega, \tag{1c}$$

in which $\mu = \sigma/(\sigma-1)$, Eq. (1b) defines the subutility from consuming differentiated goods in destination d from origin o, Q_{do}, and Eq. (1c) constrains the value of total consumption in destination country d, C_d, evaluated at prices paid by final consumers, $p_{dk}(\omega)$. Solving this problem yields the following demand for each differentiated variety

$$q_{do}(\omega) = C_d \frac{p_{do}(\omega)^{-\sigma}}{P_d^{1-\sigma}}, \tag{2}$$

in which P_d is the price index for the bundle of differentiated varieties in country d, with more details in Section 2.3.2. Recall that the (negative of the) slope of an indifference curve is the marginal rate of substitution (MRS) between exports, Q_{od}, and domestic goods, Q_{dd}. We denote this MRS in country d as MRS_{od}^{d}, in which the first subscript refers to the imported good from country o and the second to the domestic good in country d. Appendix A.1 shows how to solve the model if we extend the utility function in Eq. (1a) to include a homogeneous good.

The representative consumer problem defined by Eq. (1) is static but solving a dynamic problem with discounting yields the same demand functions as in Eq. (2) because there is no storage technology in our model, as shown in Appendix A.1.1. We allow for consumers to transfer resources across time by using the mass of matched relationships in the dd or do market as a storage technology in Appendix A.1.2. We relegate these extensions to appendixes because they reduce tractability and obscure our main results about optimal tariffs. Additionally, the static problem in Eq. (1) is comparable to other static settings, such as CRW.

2.1.2 The matching function

A costly process of search governs how producers and retailers find one another, similar to that in Diamond (1982), Pissarides (1985), and Mortensen (1986). As in many studies of the labor market (e.g., Shimer, 2005), we assume that the matching function takes a Cobb-Douglas form:

$$m\left(u_{do}N_o^x, v_{do}N_d^m\right) = \xi\left(u_{do}N_o^x\right)^{\eta}\left(v_{do}N_d^m\right)^{1-\eta}, \tag{3}$$

in which ξ is the matching efficiency, η is the elasticity of matches with respect to the number of searching producers, and $u_{do}N_o^x$ and $v_{do}N_d^m$ are the stocks of unmatched producers and retailers, respectively. N_o^x and N_d^m represent the total mass of varieties in country o and retailing firms in country d regardless of their match status. The fraction of producing (retailing) firms in country o (d) looking for retailers (producers) in country d (o) is u_{do} (v_{do}).

The matching function in Eq. (3) is homogeneous of degree one. Therefore, market tightness—the ratio of the mass of searching retailers to the mass of searching producers, which we denote $\kappa_{do} = v_{do}N_d^m/u_{do}N_o^x$—is sufficient to determine contact rates on both sides of each do search market. The Poisson rate at which retailers in country d contact producers in country o and the contact rate at which producers in country o contact retailers in country d are respectively given by:

$$\chi\left(\kappa_{do}\right) = \xi\kappa_{do}^{-\eta}, \qquad \kappa_{do}\chi\left(\kappa_{do}\right) = \xi\kappa_{do}^{1-\eta}. \tag{4}$$

Only the number of vacancies matters in our model, not the number of retailers. Vacancies can originate from one retailer posting all vacancies, all retailers posting one vacancy each, or anything in between. Nonetheless, we interpret matches as one retailer to one producer, as in Pissarides (2000), and we refer to vacancies and retailers interchangeably. Details about the matching function are in Appendix A.1.3 with details about continuous time Poisson processes in Appendix A.2 of KM.

2.1.3 Producers

We index differentiated goods producers by their permanent productivity, φ. We assume this productivity is exogenous and has the same distribution in all countries: Pareto with cumulative distribution function $G\left(\varphi\right) = 1 - \varphi^{-\theta}$ so that $\varphi = 1$ is the minimum possible value of productivity. We make the standard assumption that $\theta > \sigma - 1$ to ensure that integrals that define aggregate variables are bounded.

There are two production costs for differentiated goods. First, producers face a variable cost indexed by productivity

$$v\left(q_{do}, w_o, \tau_{do}, \varphi\right) = q_{do} w_o \tau_{do} \varphi^{-1}. \tag{5}$$

This cost function implies a constant-returns-to-scale production function in which labor is the only input. w_o is the wage in the exporting (origin) country; $\tau_{do} \geq 1$ is an iceberg cost such that one unit of the differentiated good arrives in destination d when τ_{do} units are sent from origin o and $\tau_{do} - 1$ units are lost to physical destruction; and q_{do} is the amount traded. Second, producers face a fixed cost of production, $w_o f_{do}$, in which f_{do} is in labor units, so that the total production cost is $v\left(q_{do}, w_o, \tau_{do}, \varphi\right) + w_o f_{do}$.

At any instant in time, each producer is in one of three mutually exclusive states. First, the producer could be matched with a retailer with value $X_{do}\left(\varphi\right)$ defined by,

$$rX_{do}\left(\varphi\right) = n_{do} q_{do} - v\left(q_{do}, w_o, \tau_{do}, \varphi\right) - w_o f_{do} + \lambda\left(U_{do}\left(\varphi\right) - X_{do}\left(\varphi\right)\right). \tag{6}$$

In this state, the flow payoff is the revenue obtained from selling q_{do} units of the good at negotiated price n_{do} to retailers, less the variable cost, $v\left(q_{do}, w_o, \tau_{do}, \varphi\right)$, and fixed cost of production, $w_o f_{do}$. The negotiated price, n_{do}, and the quantity traded, q_{do}, are determined through a bargaining process that we describe in Section 2.2. Matches end exogenously at rate λ, which leads to a capital loss as the producer becomes unmatched. The future is discounted at rate r.

Second, the producer could be unmatched but searching with value $U_{do}\left(\varphi\right)$ defined by,

$$rU_{do}\left(\varphi\right) = -w_o l_{do} + \kappa_{do}\chi\left(\kappa_{do}\right)\left(X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right) - w_o s_{do}\right). \tag{7}$$

The producer pays a flow cost, $w_o l_{do}$, to generate contacts with retailers. At endogenous Poisson rate $\kappa_{do}\chi(\kappa_{do})$ the producer contacts a retailer and becomes matched, after paying the sunk cost, $w_o s_{do}$, of starting up the relationship.

Third, the producer could be idle and not expend resources to look for a retailer with value $I_{do}\left(\varphi\right)$ defined by,

$$rI_{do}\left(\varphi\right) = w_o h_{do}. \tag{8}$$

Idle producers receive a constant flow payoff, $w_o h_{do}$. We include an idle state because without it, all producers would search in all markets, even if they expect to reject all contacts. Allowing producers to optimally choose not to search in each market is both more general and more intuitive.

We assume that creating each producer with heterogeneous productivity, φ, requires a one-time sunk "exploration" cost, e_d^x, similar to di Giovanni and Levchenko (2012). Appendix A.1.4 has more details about the producers' value functions.

2.1.4 Retailers

Each retailer is in one of two states. First, the retailer could be matched with a producer and receive value $M_{do}(\varphi)$ defined by,

$$rM_{do}(\varphi) = p_{do}q_{do} - t_{do}n_{do}q_{do} + \lambda\left(V_{do} - M_{do}(\varphi)\right). \tag{9}$$

The flow payoff is the revenue, $p_{do}q_{do}$, generated by selling q_{do} units of the differentiated good at a final sales price, p_{do}, paid by the consumer less the tariff-inclusive cost of acquiring these goods, $t_{do}n_{do}q_{do}$. The retailer pays the *ad valorem* tariff, t_{do}, on the imported value, $n_{do}q_{do}$, to the government. The tariff creates a potential wedge between producer revenue, $n_{do}q_{do}$, in Eq. (6) and retailer cost, $t_{do}n_{do}q_{do}$, in Eq. (9). Tariff revenues are rebated lump-sum from the government to consumers in the destination country as discussed in Section 2.3. When the relationship is destroyed exogenously, at rate λ, the retailer loses the capital value of being matched. All retailers are identical before matching but have differential matched values because of productivity heterogeneity among producers.

Second, a retailer could be unmatched with value V_{do} defined by,

$$rV_{do} = -w_d c_{do} + \chi(\kappa_{do}) \int \left[\max\left\{V_{do}, M_{do}(\varphi)\right\} - V_{do}\right] dG(\varphi). \tag{10}$$

The retailer pays a flow search cost, $w_d c_{do}$, to generate contacts with producers. At endogenous Poisson rate $\chi(\kappa_{do})$, retailing firms meet a producer and, before consummating a match, learn the productivity of the producer. Retailers then choose between matching with that producer or continuing to search. Because they are uncertain about the productivity of the producer they might meet, retailers take the expectation over all productivities they might encounter when computing their continuation value of searching. There is an unbounded mass of potential retailers that could decide to search. Appendix A.1.5 has more details about the retailers' value functions.

2.2 Solving the partial-equilibrium search problem

Retailing and producing firms use backward induction to maximize their value. The second-stage solution results from jointly Nash bargaining over quantity, q_{do}, and negotiated price, n_{do}, after a retailer and producer meet. In the first stage, retailers and producers—taking the solution to the second-stage bargaining problem as given—choose whether to search for a business partner, or to remain idle. Appendix A.2 solves the search problem in detail.

2.2.1 Match surplus

Define the total private surplus as the value of the relationship to the retailer and the producer less their outside options,

$$S_{do}(\varphi) = X_{do}(\varphi) - U_{do}(\varphi) + M_{do}(\varphi) - V_{do}. \tag{11}$$

Importantly, $S_{do}(\varphi)$ excludes the government's value of collecting tariffs from each match and the government is passive during bargaining. Bargaining over quantity, q_{do}, will maximize total private surplus and bargaining over price, n_{do}, will divide the surplus between the producer and retailer. Appendix A.2.1 derives the surplus in terms of appropriately discounted profits. That appendix also derives the value of a relationship and discusses the expected duration of matches.

2.2.2 Bargaining over quantity

Bargaining over quantity implies that the quantity exchanged within matches equates marginal revenue obtained by retailers from consumers with marginal production cost inclusive of tariffs incurred by producers. Our assumptions about the utility and variable cost functions result in an equivalent definition for negotiated quantity in terms of the final consumer price as a markup over marginal production and tariff costs,

$$p_{do}(\varphi) = t_{do}\mu w_o \tau_{do}\varphi^{-1}. \tag{12}$$

Negotiated quantity is obtained by substituting Eq. (12) into the demand curve, Eq. (2). Appendix A.2.2 discusses bargaining over quantity in detail.

2.2.3 Bargaining over the negotiated price

Bargaining over the negotiated price, n_{do}, will divide the private surplus, $S_{do}(\varphi)$, between producers and retailers according to the "surplus sharing rule", which is:

$$X_{do}(\varphi) - U_{do}(\varphi) = \frac{\beta}{\beta + t_{do}(1-\beta)} S_{do}(\varphi), \quad M_{do}(\varphi) - V_{do} = \frac{t_{do}(1-\beta)}{\beta + t_{do}(1-\beta)} S_{do}(\varphi). \tag{13}$$

in which β is producers' bargaining power. Eq. (13) nests the sharing rule in KM (Eq. 13) when $t_{do} = 1$. In addition, as the tariff rises, retailers receive a larger fraction of the surplus to account for their increased import costs: As $t_{do} \to \infty$, the fraction of the surplus received by retailers approaches 1.

The negotiated price that splits the surplus according to Eq. (13), if we assume free entry into retailer vacancies, $V_{do} = 0$, is

$$n_{do} = (1 - \gamma_{do}) \left(\frac{p_{do}}{t_{do}} \right) + \gamma_{do} \left(\frac{v\left(q_{do}, w_o, \tau_{do}, \varphi\right) + w_o f_{do} - w_o l_{do} - \kappa_{do} \chi\left(\kappa_{do}\right) w_o s_{do}}{q_{do}} \right), \quad (14)$$

in which

$$\gamma_{do} \equiv \frac{(r + \lambda)(1 - \beta)}{r + \lambda + \beta \kappa_{do} \chi\left(\kappa_{do}\right)} \in [0, 1]. \quad (15)$$

The equilibrium negotiated price, n_{do}, is a convex combination of the tariff-adjusted final sales price, p_{do}/t_{do}, and the average total production cost less producers' search costs. It maps to the price of products at the dock or the import price in the data. Eq. (14) is like Eq. 14 in KM, but includes tariffs and endogenous wages. Notice that if the sunk cost, s_{do}, is set to zero, and if producers find retailers immediately (no search frictions) so that the contact rate $\kappa_{do}\chi\left(\kappa_{do}\right) \to \infty$, then the negotiated price converges to the final sales price less tariffs, $n_{do} \to p_{do}/t_{do}$, which is the markup over marginal cost. Appendix A.2.3 shows that these equations hold for any arbitrary utility function and importer production function.

The expression for the negotiated price in Eq. (14) relates to several papers. We make three observations. First, the bilateral price markup in Alviarez, Fioretti, Kikkawa, and Morlacco (2025a,b) is closely related to the negotiated price in Eq. (14). Second, Eq. (14) implies that the relationship between tariffs and import prices is not log linear in general, as assumed in many papers (e.g., Cavallo, Gopinath, Neiman, and Tang, 2021; Javorcik, Pierce, and Wisniewski, 2025). Notice that tariffs cancel from the first term on the RHS, and tariffs affect import prices indirectly through the quantity traded in the denominator of the second term because tariffs affect prices. But the relationship between import prices and tariffs is not log linear because Eq. (14) includes an intercept. Third, and related, the model implies a specification that can be estimated using observed data to recover the tariff pass-through to import prices, as studied in Fajgelbaum, Goldberg, Kennedy, and Khandelwal (2019) and Flaaen, Hortaçsu, Tintelnot, Urdaneta, and Xu (2025), for example.

2.2.4 Producers' search productivity threshold

In the first stage, producers, taking as given the solution to this second-stage bargaining problem from Eqs. (14) and (12), choose whether to search for a business partner or to remain idle. As such, a zero-value condition, $U_{do}\left(\bar{\varphi}_{do}\right) - I_{do}\left(\bar{\varphi}_{do}\right) = 0$, which can be written as

$$\left(\frac{p_{do}\left(\bar{\varphi}_{do}\right)}{t_{do}} \right) q_{do}\left(\bar{\varphi}_{do}\right) - v\left(q_{do}\left(\bar{\varphi}_{do}\right), w_o, \tau_{do}, \bar{\varphi}_{do}\right) = w_o F\left(\kappa_{do}\right), \quad (16)$$

determines producers' minimum productivity threshold, $\bar{\varphi}_{do}$, that makes searching worthwhile. Eq. (16) equates tariff-adjusted variable profits from the match with the "effective entry cost." In labor units, the latter is defined as

$$F(\kappa_{do}) \equiv f_{do} + \left(\frac{r+\lambda}{\beta\kappa_{do}\chi(\kappa_{do})}\right) l_{do} + \left(1 + \frac{r+\lambda}{\beta\kappa_{do}\chi(\kappa_{do})}\right) h_{do} + \left(\frac{r+\lambda}{\beta}\right) s_{do}, \tag{17}$$

which is the sum of the fixed cost of production, f_{do}, and the (appropriately discounted) flow cost of searching for a retailer, l_{do}, the opportunity cost of remaining idle, h_{do}, and the sunk cost of starting up a relationship, s_{do}. Sometimes we refer to $F(\kappa_{do})$ as F_{do}.

Solve Eq. (16) using our functional forms to get the threshold explicitly as

$$\bar{\varphi}_{do} = \max\left\{1, \mu\sigma^{\frac{1}{\sigma-1}}\left(\frac{w_o\tau_{do}}{P_d}\right)\left(\frac{w_oF(\kappa_{do})}{C_d}\right)^{\frac{1}{\sigma-1}} t_{do}^{\mu}\right\}, \tag{18}$$

in which the max operator ensures that the threshold does not fall below the lower bound of the productivity distribution. Detailed discussion of the threshold productivity is in Appendix A.2.4.

2.2.5 Retailer free entry and equilibrium market tightness

We assume free entry into the market of unmatched retailers so that $V_{do} = 0$ in Eq. (10), as in Pissarides (1985) and Shimer (2005). This assumption implies that

$$\frac{w_d c_{do}}{\chi(\kappa_{do})} = \int_{\bar{\varphi}_{do}} M_{do}(\varphi)\, dG(\varphi). \tag{19}$$

This equation defines the equilibrium market tightness, κ_{do}, that equates the expected cost of being an unmatched retailer (left) with the expected benefit from matching (right).

To get intuition from Eq. (19), notice that as the expected benefit from retailing rises, free entry implies that retailers enter the search market. This entry raises market tightness, κ_{do}, and, through congestion effects, reduces the rate at which searching retailers contact searching producers, $\chi(\kappa_{do})$. This increases retailers expected cost of search (the left-hand side) so that, with free entry into retailing, κ_{do} always satisfies Eq. (19) in equilibrium. With free entry into retailer search, market tightness, κ_{do}, is finite if and only if retailers' search cost, $w_d c_{do}$, is positive, as proved in Appendix A.8 of KM.

2.3 Aggregation

2.3.1 Fraction of unmatched producers

Because of search frictions, in steady state there exists a fraction of unmatched producers (mass of unmatched product varieties) that are actively looking for a retail partner. This fraction of unmatched producers is given by

$$\frac{u_{do}}{1-i_{do}} = \frac{\lambda}{\lambda + \kappa_{do}\chi\left(\kappa_{do}\right)}, \tag{20}$$

in which u_{do} is the fraction of producers that are unmatched and searching and $u_{do}/\left(1-i_{do}\right)$ is the fraction of active producers that are unmatched adjusted by the fraction of producers that will ever search, $1-i_{do}$. The fraction of idle producers, i_{do}, that choose not to search is defined by the steady-state productivity threshold, $\bar{\varphi}_{do}$, and the exogenous productivity distribution:

$$i_{do} = \int_{1}^{\bar{\varphi}_{do}} dG\left(\varphi\right) = G\left(\bar{\varphi}_{do}\right) = 1 - \bar{\varphi}_{do}^{-\theta}. \tag{21}$$

The unmatched producers characterized by Eq. (20) imply associated unmatched varieties that cannot be consumed and are therefore absent from imports, the indirect utility (welfare) function, and all other aggregates. Appendix A.3.1 derives the elasticity of the fraction of unmatched producers in Eq. (20) with respect to exogenous parameters.

2.3.2 Price index

Using the optimal final sales price from Eq. (12) and the other assumptions in Sections 2.1 and 2.2, we derive the price index for differentiated goods in country d:

$$P_d = \lambda_2 C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}}\rho_d, \quad \rho_d \equiv \left(\sum_{k=1}^{D}\left(1-\frac{u_{dk}}{1-i_{dk}}\right)N_k^x\left(w_k\tau_{dk}\right)^{-\theta}\left(w_kF_{dk}\right)^{-\left[\frac{\theta}{\sigma-1}-1\right]}t_{dk}^{1-\mu\theta}\right)^{-\frac{1}{\theta}}, \tag{22}$$

in which $\lambda_2 \equiv \left(\theta/\left(\theta-\left(\sigma-1\right)\right)\right)^{-\frac{1}{\theta}}\sigma^{\frac{1}{\sigma-1}-\frac{1}{\theta}}\mu$. More details appear in Appendix A.3.2.

Eq. (22) uses the fact that if an unordered set of varieties, Ω_o, has measure $N_o^x = |\Omega_o|$, then the set of varieties above the threshold has measure $\left(1-G\left(\bar{\varphi}_{do}\right)\right)N_o^x = \left(1-i_{do}\right)N_o^x$ and the set of matched varieties that are above the threshold has measure $\left(1-u_{do}/\left(1-i_{do}\right)\right)N_o^x$. Appendix A.11.1 of KM has details. This measure of goods consumed features prominently in aggregate model quantities.

2.3.3 Imports

The gravity equation gives total imports by destination d from origin o in the differentiated goods sector, which is the total value of all imported varieties evaluated at negotiated prices, $n_{do}q_{do}$. As we show in Appendix A.3.3, imports are:

$$IM_{do} = \left(1-\frac{u_{do}}{1-i_{do}}\right)\left(1-b_{do}\right)N_o^xC_d\left(\frac{w_o\tau_{do}}{\rho_d}\right)^{-\theta}\left(w_oF_{do}\right)^{-\left(\frac{\theta}{\sigma-1}-1\right)}t_{do}^{-\mu\theta}, \tag{23}$$

in which the fraction of matched exporters, $1-u_{do}/\left(1-i_{do}\right)$, and the import markdown, $1-b_{do}$, reduce imports relative to a model without search, as shown in KM. The import markdown is a function of parameters, but also market tightness so that $b_{do}=b\left(\kappa_{do}\right)$ as described Eq. (A58). The markdown term is below one because negotiated import prices, n_{do}, are lower than final sales prices, p_{do}, with positive search costs, and imports are evaluated at negotiated import prices.

Eq. (23) shows that the exponent on iceberg trade costs ($-\theta$) differs from the exponent on tariffs ($-\mu\theta$). One reason these differ is that higher iceberg trade costs have a direct negative effect on productivity by raising production costs (Eq. 5), whereas tariffs do not affect productivity. Appendix A.3.4 contains details about the tariff exponent and elasticity in our model and in one without selection.

The total amount paid by consumers in d for imports from o, C_{do}, equals the value of all imported varieties evaluated at final sales prices. Appendix A.3.5 shows that $C_{do}=t_{do}IM_{do}/\left(1-b_{do}\right)$, so that as retailers' search costs go to zero, $c_{do}\to 0$, import markdowns vanish, $\left(1-b_{do}\right)\to 1$, and $IM_{do}\to C_{do}/t_{do}$. That appendix also shows that $C_{do}=P_{do}Q_{do}$, in which P_{do} is the price index for the bundle of varieties produced in country o and consumed in country d. As such, we can define a negotiated price index

$$\bar{N}_{do}=\frac{\left(1-b_{do}\right)}{t_{do}}P_{do}, \tag{24}$$

so that $IM_{do}=\bar{N}_{do}Q_{do}$ and the dd bundle of varieties satisfies $IM_{dd}=\bar{N}_{dd}Q_{dd}$.

2.3.4 Profits

In our framework, monopolistic producers generate operating profits. To determine these aggregate profits in country d, Π_d, we assume that country-d consumers own retailers and producers in their country so that profits are attributed by location. As such, profits in country d satisfy

$$\Pi_d=\Pi_d^r+\Pi_d^p=\sum_k \Pi_{dk}^r+\sum_k \Pi_{kd}^p=\sum_k C_{dk}-\sum_k t_{dk}IM_{dk}+\sum_k IM_{kd}-\sum_k \frac{C_{kd}}{\mu t_{kd}}, \tag{25}$$

in which $\Pi_d^r=\sum_k \Pi_{dk}^r$ is the total retailer profits from all varieties sold by retailers in country d who source their products from country k and $\Pi_d^p=\sum_k \Pi_{kd}^p$ is the total producer profits from all varieties sold by producers in country d to country k. We discuss five alternative ownership structures of firms in Appendix A.3.6. We assume an exogenous number of producers in country d, N_d^x, as in CRW. Appendix A.9.2 in KM derives additional restrictions imposed by free entry into production.

2.3.5 Expenditure and income approaches to national accounting

The aggregate resource constraint using the expenditure approach can be written as,

$$\begin{aligned} Y_d = & \underbrace{p_d(1)\, q_d(1) + \sum_{k=1}^{D}\left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} p_{dk}(\varphi)\, q_{dk}(\varphi)\, dG(\varphi)}_{\text{Consumption } (C_d)} \\ & \underbrace{+ N_d^x w_d e_d^x + \sum_{k=1}^{D} \kappa_{dk} u_{dk} N_k^x w_d c_{dk} + u_{kd} N_d^x \left(w_d l_{kd} + w_d s_{kd} \kappa_{kd} \chi(\kappa_{kd})\right) + (1 - u_{kd} - i_{kd})\, N_d^x w_d f_{kd},}_{\text{Investment } (I_d)} \end{aligned} \tag{26}$$

with details in Appendix A.3.7. Consumption expenditure, C_d, is defined in Eq. (1c). Investment expenditure, I_d, is the resources devoted to creating producers, to creating retailer-producer relationships, and to paying for the per-period fixed costs of production. We define investment costs as those that must be paid before producing the first unit of output and that do not scale with output. We discuss the equilibrium wage in Section 2.3.6. The government budget is balanced by rebating tariff revenue to consumers as income. This assumption means that the government does not make any purchases with tariff revenue and there is no government term on the right hand side of Eq. (26). Government payments to idle producers are financed by a lump-sum tax on consumers so that they cancel out on the right hand side of Eq. (26). Net exports do not appear in Eq. (26) because trade is balanced in equilibrium, as discussed in Section 2.4.1.

The aggregate resource constraint using the income approach can be written as

$$Y_d = w_d L_d + \Pi_d + G_d, \tag{27}$$

in which L_d is the exogenous labor endowment, w_d is the equilibrium wage, Π_d are profits, and G_d is income raised by tariffs on retailers and rebated to consumers:

$$G_d = \sum_{k=1}^{D}\left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}}^{\infty} (t_{dk} - 1)\, n_{dk}(\varphi)\, q_{dk}(\varphi)\, dG(\varphi) = \sum_{k=1}^{D} (t_{dk} - 1)\, IM_{dk}. \tag{28}$$

Government payments to idle producers cancel out on the right hand side of Eq. (27) because they enter workers' income positively but are subtracted from government income as subsidies. We present details about the income approach to national accounting in Appendix A.3.8. Appendix A.3.9 uses the two approaches to national accounting to define the aggregate resource constraint for consumers.

2.3.6 Labor market clearing

Labor demand in country d, LD_d, in units of labor is defined by

$$LD_d = \frac{I_d}{w_d} + q_d(1) + \sum_o \left(1 - \frac{u_{od}}{1 - i_{od}}\right) N_d^x \int_{\bar{\varphi}_{od}} q_{od}(\varphi)\, \tau_{od} \varphi^{-1} dG(\varphi), \qquad (29)$$

with details in Appendix A.3.10. Labor demand is the sum of three terms. First, the cost to create firms, pay fixed costs, and form matches captured by the investment term, I_d, from Eq. (26). Second, the labor used to produce the homogeneous good. Third, the labor used to produce domestic and exported differentiated goods. Labor supply is immobile and equal to a country's labor endowment, L_d.

Recall that the production possibility frontier (PPF) for country d traces out how much of the domestic good, Q_{dd}, can be produced for each level of exports, Q_{od}, given a level of imports, Q_{do}, and labor endowment, L_d. The (negative of the) slope of the PPF is the marginal rate of transformation (MRT) between exports, Q_{od}, and domestic goods, Q_{dd}, denoted as MRT_{od}^d.

2.4 Steady-state general equilibrium

2.4.1 Defining the competitive equilibrium

A competitive equilibrium is defined as an allocation of goods and a set of prices such that firms maximize profits, consumers maximize utility, and all markets clear. It is convenient to express our model's equilibrium in terms of threshold productivities, $\bar{\varphi}_{do}$, and market tightnesses, κ_{do}, $\forall do$, wages, w_d, and aggregate consumptions, C_d, $\forall d$. These will jointly satisfy producers' zero profit conditions (Eq. 18), retailers' free-entry conditions (Eq. 19), aggregate resource constraints (Eqs. 26 and 27), and labor market clearing conditions (Eq. 29). Walras' Law states that if all but one markets in an economy clear, the last one must also clear. We normalize the wage in one country to 1, $w_D = 1$. The exogenous parameters are β, λ, r, η, ξ, θ, σ, α, e_d^x, L_d, t_{do}, c_{do}, f_{do}, h_{do}, l_{do}, and s_{do}, in which d and o vary by countries. Tariffs, t_{do}, are exogenous parameters to economic agents, except if they are chosen by a social planner, as discussed in Section 3.

Formally, the decentralized equilibrium solves a system of nonlinear equations in the equilibrium variables in which the equilibrium conditions are constraints, model parameters are taken as given, and the objective function is any constant, including zero:

$$\left(\boldsymbol{\kappa}^c, \bar{\boldsymbol{\varphi}}^c, \vec{C}^c, \vec{w}^c\right) = \underset{\boldsymbol{\kappa}, \bar{\boldsymbol{\varphi}}, \vec{C}, \vec{w}}{\arg\max}\, 0 \tag{30a}$$

subject to:

$$\frac{w_d c_{do}}{\chi\left(\kappa_{do}\right)} = \left(\frac{1}{r+\lambda}\right) \frac{\Pi^r_{do}\left(\vec{\kappa}_{d*}, \vec{\bar{\varphi}}_{d*}, \vec{C}, \vec{w}, \vec{t}_{d*}\right)}{\left(1 - u_{do}\left(\kappa_{do}\right) / \left(1 - i_{do}\left(\bar{\varphi}_{do}\right)\right)\right) N^x_o} \;\forall do, \tag{30b}$$

$$\bar{\varphi}_{do} = \max\left\{1, \mu\sigma^{\frac{1}{\sigma-1}} \left(\frac{w_o \tau_{do}}{P_d\left(\vec{\kappa}_{d*}, \vec{\bar{\varphi}}_{d*}, \vec{C}, \pi, \vec{w}, \vec{t}_{d*}\right)}\right) \left(\frac{w_o F\left(\kappa_{do}\right)}{C_d}\right)^{\frac{1}{\sigma-1}} t^{\mu}_{do}\right\} \;\forall do, \tag{30c}$$

$$w_d L_d + \Pi_d + G_d\left(\vec{\kappa}_{d*}, \vec{\bar{\varphi}}_{*d}, \vec{C}, \vec{t}_{d*}\right) = C_d + I_d\left(\vec{\kappa}_{d*}, \vec{\kappa}_{*d}, \vec{\bar{\varphi}}_{*d}, \vec{C}\right) \;\forall d, \tag{30d}$$

$$w_d = \frac{I_d\left(\vec{\kappa}_{d*}, \vec{\kappa}_{*d}, \vec{\bar{\varphi}}_{*d}, \vec{C}\right) + \frac{1}{\mu}\left(\vec{C}_{*d}\left(\vec{\kappa}_{*d}, \vec{\bar{\varphi}}_{*d}, \vec{C}, w_d, \vec{t}_{*d}\right) / \vec{t}_{*d}\right)' \vec{\iota}}{L_d} \;\forall d, \tag{30e}$$

$$w_D = 1, \tag{30f}$$

$$\vec{t}_{d*} = \vec{t}^c_{d*} \;\forall d, \tag{30g}$$

with details in Appendix A.4.1. We denote the solutions to the decentralized competitive equilibrium defined by Eq. (30) with "c" superscripts. We also define vectors as collections of the variables across subindexes and matrices are denoted as bold. For example, search market tightnesses are collected into the following

$$\vec{\kappa}_{*o} = \begin{pmatrix} \kappa_{1o} \\ \kappa_{2o} \\ \vdots \\ \kappa_{Do} \end{pmatrix}, \qquad \vec{\kappa}_{d*} = \begin{pmatrix} \kappa_{d1} & \kappa_{d2} & \dots & \kappa_{dD} \end{pmatrix}, \qquad \boldsymbol{\kappa} = \begin{bmatrix} \kappa_{11} & \dots & \kappa_{1D} \\ \vdots & \ddots & \vdots \\ \kappa_{D1} & \dots & \kappa_{DD} \end{bmatrix}, \tag{31}$$

so that rows index destinations and columns index origins. $\vec{\kappa_{d*}}$ is the dth row of $\boldsymbol{\kappa}$ and $\vec{\kappa_{*o}}$ is the oth column of $\boldsymbol{\kappa}$. The column vector of D aggregate consumption expenditures in each country d is collected in $\vec{C}$, $\vec{\iota}$ is a $D \times 1$ column vector of ones, and $\vec{L}$ is a column vector of D labor endowments. Division of matrices is element by element.

Utility maximization requires balanced trade so that

$$\sum_{k=1}^{D} IM_{kd} = \sum_{k=1}^{D} IM_{dk}, \tag{32}$$

in which IM_{do} is the value of imports by country d from country o. Eq. (32) implies that net exports do not appear in Eqs. (26) and (30d).

The main difference between our model's equilibrium definition and the definitions in trade models without search frictions is that we introduce market tightnesses, κ_{do}. Our model nests trade models without search if market tightnesses are infinite. Specifically, our model exactly reproduces Chaney (2008) if retailers' search costs are zero, $w_d = 1$ and $G_d = 0$, $\forall d$, and we make the same parameter value restrictions that he does ($s_{do} = h_{do} = e^x_d = 0$, $\forall d$, and, $\forall o$), among other assumptions. We provide more details for this result in Appendix A.4.2.

2.4.2 Equilibrium existence and uniqueness: A graphical depiction

We depict the general equilibrium with six graphs: two for bargaining over the negotiated price and the quantity, two for producer and retailer entry decisions, and two for goods and labor market clearing. These heuristic graphs are for an arbitrary good, φ, bilateral market do, and country d, so that our discussion is without loss of generality. The equilibrium in our model exists and is unique as shown in each figure with details in Appendix A.4.3. All endogenous variables are jointly determined.

First, Fig. 1a depicts the equilibrium final sales price, $p^*_{do}(\varphi)$, and the quantity traded within a relationship, $q^*_{do}(\varphi)$, resulting from bargaining over the quantity (Section 2.2.2). This bargaining implies that the quantity exchanged *within* matches equates retailers' marginal revenue from consumers with the marginal production cost times the tariff (Appendix A.2.2). The demand curve (Eq. 2) slopes down because demand for a particular variety falls when its price rises. Monopolistic competition then implies that the marginal revenue curve slopes down as well. The marginal cost curve is not a function of quantity because producers' cost function is linear in quantity (Eq. 5).

Second, Fig. 1b depicts the equilibrium negotiated price for one good in the do market, $n^*_{do}(\varphi)$, resulting from bargaining over the negotiated price (Section 2.2.3). The negotiated price curve (Eq. 14) slopes up because a tighter market means that it is easier for unmatched producers to find retailers. This raises producers' outside option and allows them to negotiate a higher price. The negotiated price of any single atomistic variety does not affect tightness in the do search market, so the market tightness curve is a vertical line at κ^*_{do} from Fig. 1d.

Third, Fig. 1c depicts the equilibrium threshold productivity, $\bar{\varphi}^*_{do}$, resulting from producers' entry decision (Section 2.2.4). This figure takes as given equilibrium aggregate consumption, C^*_d, from Fig. 1e, depicted as a vertical line. The productivity threshold curve slopes down because higher consumption in the destination market raises profitability of each variety, which makes the threshold producer less productive (Eq. 18).

Fourth, Fig. 1d depicts the equilibrium mean imports, $\mathbb{E}_\varphi\left[n_{do}(\varphi)\, q_{do}(\varphi)\right]^*$, and goods-market tightness, κ^*_{do}, in the do search market, resulting from retailers' entry decision (Section 2.2.5). The mean imports curve (derived from Eq. 14 in Appendix

A.4.3.4) slopes up for the same reason that the negotiated price curve slopes up in Fig. 1b: a tighter market means that producers can negotiate a higher price. The retailers' free entry curve (Eq. 19) slopes down because a higher expected negotiated cost lowers the value of being a matched retailer and so leads to less retailer entry, lowering market tightness.

Fifth, Fig. 1e depicts the equilibrium aggregate consumption, C_d^*, resulting from goods market clearing (Section 2.3.5). This figure takes as given the equilibrium wage, w_d^*, from Fig. 1f, depicted as a horizontal line. The consumption curve slopes up because the expenditure and income approaches to national accounting (Eqs. 26 and 27, respectively) imply that a higher wage generates higher income and higher consumption.

Finally, Fig. 1f depicts the equilibrium wage, w_d^*, and labor, L_d^*, resulting from labor market clearing (Section 2.3.6). The labor demand curve slopes down because a higher wage reduces real investment and lowers labor demand (Eq. 29). Labor supply is a vertical line at a country's labor endowment, L_d.

Proposition 1 summarizes this discussion.

Proposition 1. *The equilibrium defined by Eq.* (30) *exists and is unique.*

Proof. See Appendix A.4.3. □

The model delivers a tractable framework with a unique equilibrium. It extends KM by introducing labor market clearing and endogenous wages because there is no freely-traded homogeneous good.

3 Optimal uniform import tariffs

3.1 The unilateral country social planner problem

This section considers a country social planner that unilaterally chooses import tariffs to maximize their own country's welfare without considering the welfare of other countries. While this country can set import tariffs, t_{d*}, we assume it cannot choose domestic taxes and the taxes and tariffs of other countries. Without loss of generality, we set these to the numbers given in Eq. (30g) so that $t_{dd} = t_{dd}^c$ and $\vec{t}_{o*} = \vec{t}_{o*}^{\,c}$, $\forall o \neq d$. Country d's social planner remains constrained by the decentralized retailer entry condition and the other equilibrium constraints in all countries defined in Eq. (30). Formally, this problem is given by

$$\left(\boldsymbol{\kappa}^u, \bar{\boldsymbol{\varphi}}^u, \vec{C}^u, \vec{w}^u, \vec{t}^u_{d*}\right) = \underset{\vec{t}_{d*}}{\arg\max} \left(\frac{C_d}{P_d\left(\vec{\kappa}_{d*}, \vec{\bar{\varphi}}_{d*}, \vec{C}, \vec{w}, \vec{t}_{d*}\right)} \right) \tag{33a}$$

subject to: Eqs. (30b) through (30f),

$$\vec{t}_{o*} = \vec{t}^c_{o*} \ \forall o \neq d, \tag{33b}$$

$$t_{dd} = t^c_{dd} \ \forall d. \tag{33c}$$

We denote the variables that solve the unilateral problem defined by Eq. (33) with "u" superscripts. Welfare (indirect utility) in country d is equal to real consumption, the objective function Eq. (33a), because preferences represented by utility in Eq. (1) are homothetic (Appendix B.1.1).

If instead of Eq. (33a), the social planner maximizes the present discounted value of welfare, the solution is unchanged. This follows because if there is no storage technology, the solution to the consumer's static and dynamic problems are identical (Appendix A.1.2). In that case, the present discounted value of welfare is real consumption divided by the constant interest rate, r, as shown in Appendix B.1.2. As such, the main text uses current value welfare for simplicity.

3.2 Analytical solutions

In this section we derive analytical solutions for the optimal tariff and the offer curve elasticity for the two-country case.

3.2.1 Optimal tariff solution

This section presents the analytical solution to Eq. (33). Our main finding is that in the model with search frictions, the optimal tariff depends not only on the elasticity of the offer curve—as it does in a model without frictions—but also on two new terms. One term relates to the import markdowns introduced in Section 2.3.3 and the second term relates to the allocative inefficiency of the decentralized economy because search frictions introduce additional externalities.

The following assumption helps yield analytical solutions.

Assumption 1. *Assume that 1) $D = 2$ so that there are two countries, d and o; 2) there are no tariffs or subsidies except for tariffs in the do market; 3) and the consumer optimization problem yields an interior solution.*

Proposition 2 characterizes the optimal tariff that maximizes domestic welfare.

Proposition 2. *The optimal import tariff for country d satisfies*

$$t_{do}^{u}=\left(\frac{1-b_{do}^{u}}{1-b_{dd}^{u}}\right)\left(\frac{MRT_{od}^{d,u}}{\bar{N}_{od}^{u}/\bar{N}_{dd}^{u}}\right)\frac{1}{H_{do}^{u}}, \tag{34a}$$

in which

$$H_{do}^{u}=\frac{d\ln Q_{do}^{u}\left(Q_{od}^{u}\right)}{d\ln Q_{od}^{u}}, \tag{34b}$$

is the elasticity of the offer curve for exports from country o to country d and we use Assumption 1.

Proof. See Appendix B.2.1. □

We make four remarks about Eq. (34). First, the optimal tariff is a fixed point because it is a function of the general equilibrium of the model. Specifically, the superscript "u" denotes variables that are in equilibrium at the optimal tariff and so, in general, computing the optimal tariff in Eq. (34) requires a fully-specified general equilibrium model. The optimal tariff expression in CRW (Eq. 15) shares this property, as do the large economy models in Gros (1987) and Felbermayr et al. (2013).

Second, Eq. (34) nests the optimal tariff formula in a model without search frictions. Without search, there are no import markdowns so that $\left(1-b_{do}^{u}\right)/\left(1-b_{dd}^{u}\right)=1$ and hence $\bar{N}_{od}^{u}/\bar{N}_{dd}^{u}=P_{od}^{u}/P_{dd}^{u}$. Also, the decentralized solution features allocative efficiency so that $MRT_{od}^{d,u}=P_{od}^{u}/P_{dd}^{u}$ and Eq. (34) simplifies to

$$t_{do}^{u}=\frac{1}{H_{do}^{u}}, \tag{35}$$

which corresponds to Eq. (15) in CRW. Section 3.3 discusses optimal tariffs and efficiency in greater detail.

Third, the tariff in Eq. (34) depends on the ratio of markdowns in the do market to the dd market, $\left(1-b_{do}^{u}\right)/\left(1-b_{dd}^{u}\right)$. The social planner's solution involves a manipulation of the terms of trade (TOT), $\bar{N}_{od}/\bar{N}_{do}$, so that the wedge between the TOT and the MRT-MRS ratio is equal to the inverse of the elasticity of the offer curve, as in Dixit (1985) and CRW (see Appendix B.2.1.1 and Eq. B146). With search frictions, the TOT include the import markdown terms so the optimal tariff includes these as well. Intuitively, producers are willing to accept low negotiated prices (large import markdowns) when they face poor outside options driven by low contact rates (high unmatched rates). The social planner stimulates retailer entry in these search markets by setting lower tariffs to reduce unmatched rates.

Fourth, the tariff in Eq. (34) depends on the ratio of the MRT in country d to the ratio of negotiated price indices in country d, $MRT_{od}^{d,u}/\left(\bar{N}_{od}^{u}/\bar{N}_{dd}^{u}\right)$. This follows because search frictions introduce additional externalities that interact with tariffs. For example, in the decentralized equilibrium, each producer decides whether to search or not, and how much to produce, without considering their effect on equilibrium market tightnesses, $\kappa_{do}\forall do$. Their choices influence the ratio of negotiated price indexes, $\bar{N}_{od}^{u}/\bar{N}_{dd}^{u}$. In contrast, the social planner explicitly considers the social opportunity cost of producing Q_{do} including any effects on market tightnesses, which affects the PPF and therefore the ratio $MRT_{od}^{d,u}/\left(\bar{N}_{od}^{u}/\bar{N}_{dd}^{u}\right)$. Our new optimal tariff expression implies that the social planner sets higher import tariffs when the social opportunity cost of exports exceeds the relative private cost. Higher import tariffs reduce exports via balanced trade. In an economy without search frictions, the decentralized economy features allocative efficiency so that $MRT_{od}^{d,u}/\left(\bar{N}_{od}^{u}/\bar{N}_{dd}^{u}\right)=1$, as in CRW.

To elucidate these analytical results, we present numerical examples for two symmetric countries labeled as Home (h) and Foreign (f) without search costs, $c_{do}=0\ \forall do$, and with search costs in only the hf market so that $c_{hf}>0$ but $c_{do}=0\ \forall do\neq hf$. Appendix B.2.2 contains parametrization details.

Fig. 2 computes the general equilibrium defined in Eq. (30) for the numerical example for a range of Home import tariffs, t_{hf}, without and with search costs. Using these equilibria, Panel A shows Home welfare as a function of Home import tariffs without and with search costs (Figs. 2a and 2b). The left (right) vertical axes are in units of real consumption (welfare percent change from free trade, $t_{hf}=1$). Welfare is globally concave and peaks at 1.32 (1.23) in the model without (with) search costs. Panel B plots the right hand side (RHS) of Eqs. (35) and (34) (blue lines) for various Home import tariffs and the left hand side (LHS) 45 degree line (solid black) in Figs. 2c and 2d. These figures show that the optimal tariffs are fixed points and satisfy Eqs. (35) and (34): At the optimal tariffs, the RHS of those equations evaluate to the optimal tariffs on the LHS. Figs. A1a and A1b show that Foreign welfare falls monotonically with higher Home import tariffs in the model without and with search costs.

Fig. 3a computes the Home optimal tariff defined in Eq. (33) for different search costs on the horizontal axis, $c_{hf}>0$ but $c_{do}=0\ \forall do\neq hf$. The vertical axis intercept corresponds to the case without search costs (dotted black line) shown in Fig. 2a. The optimal tariff of $t_{hf}^{u}=1.23$ corresponds to a search cost of $c_{hf}\times 10^{4}=5$ used in Fig. 2b. We highlight that the optimal tariff in a model with any positive search cost is below that in the model without them (orange circles below intercept) and that it declines rapidly as search costs increase for this simple parameterization.

We also depict how the decomposition of the optimal unilateral tariff in Eq. (34) varies with search costs in Fig. 3c. We make three observations. First, the offer curve elasticity term (dotted blue line) is quantitatively the most relevant for the level of optimal tariffs in this numerical example, regardless of the search cost level. At the vertical axis, the figure confirms that if search costs are zero, the optimal tariff is fully determined by the equilibrium offer curve elasticity because Eq. (34) simplifies to Eq. (35). Second, the ratio of markdowns in the hf market to the hh market falls with search costs in the hf market (green line below its intercept) because the import markdown in the hf market falls, as discussed in Section 2.3.3. Third, allocative inefficiency increases optimal tariffs with search costs in the hf market (orange dashed line above its intercept) in this numerical example. As search costs in the hf market rise, the PPF steepens in country f because producing one more unit of the exported good requires giving up more of the domestic good. This effect also steepens the PPF in country h through balanced trade. Because aggregate producer prices do not fully reflect externalities, the MRT exceeds the ratio of negotiated prices in country h.

3.2.2 Offer curve elasticity solution

Proposition 3 characterizes the inverse of the elasticity of the offer curve in Eq. (34b).

Proposition 3. *The inverse of the elasticity of the offer curve satisfies*

$$\frac{1}{H^u_{do}} = 1 + \mu\epsilon^u_{\mathcal{P}_{do},Q_{do}} + \frac{1}{(\sigma-1)\,x^u_{oo}} + \mu\epsilon^u_{\mathcal{B}_{do},Q_{do}}, \tag{36a}$$

in which

$$\epsilon^u_{\mathcal{P}_{do},Q_{do}} = \frac{d\ln\left[\left(P^u_{do}/t^u_{do}\right)/P^u_{oo}\right]}{d\ln Q^u_{do}}, \tag{36b}$$

$$x^u_{oo} = \left(1 - \frac{d\ln Q^u_{oo}}{d\ln Q^u_{do}}\right)^{-1}, \tag{36c}$$

$$\epsilon^u_{\mathcal{B}_{do},Q_{do}} = \frac{d\ln\left[\left(1-b^u_{do}\right)/\left(1-b^u_{od}\right)\right]}{d\ln Q^u_{do}}, \tag{36d}$$

and we use Assumption 1.

Proof. See Appendix B.2.3. □

We make two observations about Eq. (36). First, balanced trade implies that the offer curve relates closely to the TOT because $Q_{do} = Q_{od}\bar{N}_{od}/\bar{N}_{do}$. As such, the offer curve elasticity, Eq. (36a), includes price ratio elasticities, Eq. (36b), and markdown ratio elasticities, Eq. (36d).

Second, Proposition 3 nests the model without search frictions so that Eq. (37) recovers Eq. (22) in CRW

$$t^u_{do} = \frac{1}{H^u_{do}} = 1 + \mu\epsilon^u_{\mathcal{P}_{do},Q_{do}} + \frac{1}{(\sigma-1)\,x^u_{oo}} = 1 + \frac{1+\epsilon^u_{\mathcal{P}_{do},Q_{do}}\sigma x^u_{oo}}{(\sigma-1)\,x^u_{oo}}, \tag{37}$$

in which we use Eq. (35) for the first equality, we use Eq. (36) with $\epsilon^{u}_{\mathcal{B}_{do},Q_{do}} = 0$ because of no search frictions for the second equality, and we simplify for the third equality. Additionally, without search frictions, $MRT^{o,u}_{do} = \left(P^{u}_{do}/t^{u}_{do}\right)/P^{u}_{oo}$. As such, the elasticity of transformation (EoT) in country o—defined as the elasticity of the MRT in country o with respect to exports Q_{do}— coincides with Eq. (36b). In other words, without search, $\epsilon^{u}_{\mathcal{P}_{do},Q_{do}} = \partial \ln MRT^{o,u}_{do}/\partial \ln Q^{u}_{do}$, as discussed in Appendix B.2.4. In that appendix, we also show that, without search frictions, x^{u}_{oo} in Eq. (36c) simplifies to the domestic consumption share in country o and so we refer to it as such throughout, as in CRW.

Fig. 3e clarifies how search costs affect the inverse of the elasticity of the offer curve and its decomposition in Eq. (36) for our numerical example. The inverse of the elasticity of the offer curve (dotted blue line), Eq. (36a), falls with search costs. We make four observations about its decomposition. First, the consumption share term (orange dashed line), Eq. (36c), contributes the most to the level of the inverse of the offer curve elasticity for all search cost levels in this numerical example. Second, the decline in the inverse of the offer curve elasticity as search costs rise is chiefly because the price elasticity (dashed-dotted red line), Eq. (36b), falls with search costs. We discuss this result below and we characterize this elasticity in Proposition 4. Third, Eq. (36c) also falls slightly with search costs—although that is difficult to see in the figure—because higher search costs reduce imports by h and therefore reduce imports by f through trade balance, which raises the domestic consumption share in country f. Simultaneously, optimal tariffs fall as search costs increase and this tends to reduce the domestic consumption share, but the search cost effect dominates in this numerical example. Fourth, the elasticity of the ratio of markdowns (solid green line), Eq. (36d), rises slightly with search costs because changes in Q_{hf} affect retailer entry—and unmatched rates and import markdowns—directly in the hf market and only indirectly in the fh market.

Next, we discuss how search frictions affect Eq. (36b), which is the EoT in the model without search. Proposition 4 decomposes Eq. (36b).

Proposition 4. *The price elasticity satisfies*

$$\epsilon^{u}_{\mathcal{P}_{do},Q_{do}} = \underbrace{-\frac{\theta-(\sigma-1)}{[\sigma\theta-(\sigma-1)]\,x^{u}_{oo}}}_{\text{Selection}} \underbrace{-(1-\eta)\left[\frac{(\sigma-1)}{\sigma\theta-(\sigma-1)}\right]\left[\left(\frac{u^{u}_{do}}{1-i^{u}_{do}}\right)\frac{d\ln\kappa^{u}_{do}}{d\ln Q^{u}_{do}}-\left(\frac{u^{u}_{oo}}{1-i^{u}_{oo}}\right)\frac{d\ln\kappa^{u}_{oo}}{d\ln Q^{u}_{do}}\right]}_{\text{Search frictions}}, \quad (38)$$

which uses Assumption 1 and that $l_{ko} = -h_{ko}$ for $k = d, o$ so that $F\left(\kappa_{ko}\right)$ are parameters.

Proof. See Appendix B.2.5. □

Eq. (38) is the standard selection term plus a new search-frictions term that captures the effect of country o exports on endogenous goods-market tightnesses. Specifically, the first term in Eq. (38) is the EoT in a model with a Pareto distribution for firm productivity and constant fixed exporting cost, but without endogenous search frictions, as in Felbermayr et al. (2013). This EoT is negative because the PPF is bowed into the origin as a result of increasing returns. This aggregate nonconvexity arises because of self-selection into exporting, as discussed in CRW. Specifically, as Q_{do} rises, there is more producer entry in the do market, as well as fewer domestic goods, Q_{oo}, which lowers producer entry in the oo market. Both of these effects lower the opportunity cost of exports in terms of domestic goods. As the productivity distribution has more mass at the threshold ($\theta \to \infty$), this selection channel becomes more relevant and the PPF becomes more bowed into the origin. This EoT term is zero in a model without selection, as in Gros (1987).

The search-frictions term in Eq. (38) captures the effects of changes in endogenous goods-market tightnesses. The first derivative term captures the effects of country o's exports on tightness in the do market. This term is positive because an increase in exports increases retailer entry and tightness in the do market, which increases the matched rate and decreases the differentiated-goods price index. Simply put, by increasing exports, country o lowers search frictions, which lowers the opportunity cost of exports. The second derivative term is negative because higher exports imply substitution away from the domestic market, which lowers the domestic matched rate and increases the opportunity cost of producing domestic goods. These two effects imply that the search-frictions term in Eq. (38) is negative. Naturally, these search friction effects are larger when unmatched rates in the do and oo markets are high and when the number of matches is more responsive to the number of searching retailers; that is, when $(1-\eta)$ is closer to one. These findings explain why the price elasticity (dashed-dotted red line), Eq. (38), falls with search costs in Fig. 3e.

A sufficient condition that ensures that Eq. (36) in the model with search (s) frictions is below that in the model with no search (ns) frictions is $x_{oo}^{s,u} \geq x_{oo}^{ns,u}$, as shown in Appendix B.2.6.

3.3 Optimal tariffs and efficiency

Notice that the optimal tariffs in the model with and without search frictions in Eqs. (34) and (35) differ because search frictions introduce the ratio of import markdowns, $(1-b_{do}^u)\,/\,(1-b_{dd}^u)$, and they change the elasticity of the offer curve (Eqs. 36a and 38), but also because of different efficiency properties of the decentralized equilibria in the two models through $MRT_{od}^{d,u}/\left(\bar{N}_{od}^u/\bar{N}_{dd}^u\right)$ in Eq. (34). Recall that without search frictions, the competitive equilibrium attains allocative efficiency, as in CRW, so that $MRT_{od}^{d,u}/\left(\bar{N}_{od}^u/\bar{N}_{dd}^u\right)=1$.

A model with search can still attain allocative efficiency at a competitive equilibrium. Appendix B.2.8 considers optimal tariffs in a restricted search model with exogenous (finite) unmatched rates and exogenous idle rates, which is equivalent to an exogenous threshold, Eq. (18). With these restrictions, each economy attains allocative efficiency. Nevertheless, optimal tariffs are determined by Eq. (34) instead of (35) and therefore differ from the model without search. Appendix B.2.8 also derives sufficient conditions for a search model with these restrictions to give the same optimal tariff as the model without search. Namely, restrictions on parameters that result in the import markdown ratio being one and that country d is a small open economy (so that $w_o/w_d \to 0$) and the domestic consumption share is one.

To illustrate that a search model can attain allocative efficiency, our numerical example shows that these inefficiencies disappear when unmatched and idle rates are exogenous. Figs. 3b, 3d, and 3f depict the optimal tariff, its decomposition, and the decomposition of the inverse of the offer curve elasticity, respectively, in the model with exogenous unmatched and idle rates. For each of these figures, we fix the unmatched and idle rates to the endogenous equilibrium unmatched and idle rates that obtain given search costs in the corresponding figure in the left column of Fig. 3. Fig. 3b shows that the optimal tariff (orange circles) falls with search costs even with exogenous unmatched and idle rates. Fig. 3d shows that this decline is due to the markdown term (green line). Notably, $MRT^{d,u}_{od} / \left(\bar{N}^u_{od}/\bar{N}^u_{dd}\right) = 1$ (dashed orange line) because of allocative efficiency. Fig. 3f shows that the markdown elasticity is zero because markdowns are exogenous, and the price elasticity, Eq. (36b), is also zero because the PPF is linear and the search-frictions term in Eq. (38) is zero. The consumption share term falls slightly as tariffs rise by the same logic as discussed in Section 3.2.

Optimal tariffs may also differ in a model without search frictions and with endogenous search frictions because the latter is not globally efficient. This inefficiency results from the standard matching externalities in search models; namely, retailers and producers do not internalize how searching affects equilibrium matching probabilities. In addition, our model also has composition externalities because the threshold producer does not internalize their effect on average match productivity, as in Albrecht, Navarro, and Vroman (2010) and Julien and Mangin (2017).

There exist conditions like those in Hosios (1990) that internalize congestion and market thickness externalities, as well as conditions that internalize composition externalities. Brancaccio et al. (2023) derive related conditions under which the decentralized equilibrium is globally efficient in an environment similar to ours that has random search between customers and shipping carriers. We discuss those conditions more fully in Appendix B.2.9. This appendix also explains that formalizing the global efficiency properties of our endogenous search model and how they interact with optimal tariffs, as well as deriving the relationship between country-level allocative efficiency and global efficiency, and comparing and contrasting to the models in Moen (1997), Albrecht et al. (2010), Mangin and Julien (2021), and Brancaccio et al. (2023) is beyond the scope of this paper.

3.4 Optimal tariffs with retaliation

This section considers optimal unilateral tariffs in a strategic environment. We define and solve for a D-country pure-strategy Nash equilibrium in which countries choose import tariffs. We assume countries cannot choose domestic taxes, which are set to their competitive levels, without loss of generality. The Nash equilibrium import tariffs are defined by tariffs that maximize each country's welfare, subject to the equilibrium conditions and the Nash tariffs set by other countries. Formally, this problem is given by

$$\text{Find } \left\{\boldsymbol{\kappa}^n, \bar{\boldsymbol{\varphi}}^n, \vec{C}^n, \vec{w}^n, \boldsymbol{t}^n\right\} \text{ subject to} \tag{39a}$$

$$\left\{\boldsymbol{\kappa}^n, \bar{\boldsymbol{\varphi}}^n, \vec{C}^n, \vec{w}^n, \vec{t}^n_{d*}\right\} = \underset{\vec{t}_{d*}}{\arg\max} \left(\frac{C_d}{P_d\left(\vec{\kappa}_{d*}, \vec{\bar{\varphi}}_{d*}, \vec{C}, \vec{w}, \vec{t}_{d*}\right)}\right) \; \forall d, \tag{39b}$$

subject to: Eqs. (30b) through (30f),

$$\vec{t}_{o*} = \vec{t}^n_{o*} \; \forall o \neq d, \tag{39c}$$

$$t_{dd} = t^c_{dd} \; \forall d. \tag{39d}$$

We denote the variables that solve the Nash problem defined by Eq. (39) with "n" superscripts. Appendix B.3 describes how to solve for the Nash equilibrium using the Nikaidô-Isoda function (Nikaidô and Isoda, 1955). The intuition for the mechanisms for optimal tariffs remain largely the same in this context as in Section 3.2. However, instead of one fixed point equation that determines the optimal tariff (Eq. 34), there exist similar equations for each country. In these equations, the equilibrium variables on the RHS are functions of tariffs. The optimal tariffs satisfy all the equations simultaneously and imply no incentive to deviate for any country.

We use the same numerical example as in Section 3.2 to illustrate the Nash equilibrium. Fig. 4a (Fig. 4b) depicts Home's optimal import tariff for each Foreign import tariff (dashed blue line) and Foreign's optimal tariff for each Home import tariff (dashed red line) in a model without (with) search costs in the hf market. The intersection of these two best response curves identifies the Nash equilibrium import tariffs for which neither country has an incentive to deviate. In the model without search frictions, the Nash equilibrium import tariff is 1.32 in both Home and Foreign because the countries are symmetric. In the model with search frictions, Home's best response line shifts down relative to the model without search frictions and Foreign's best response and Nash equilibrium import tariff are little changed. Home's Nash equilibrium import tariff is 1.18. In short, optimal import tariffs are lower in a model with search frictions than without them, even with strategic considerations.

4 Calibration

We use data for China and the United States in 2016 to calibrate our model, as in KM, but we can generalize our approach to include more trading partners or a different time period. The calibration proceeds in two steps and follows KM closely. First, we externally calibrate parameters that can be normalized or that are standard in the literature, with details in Appendix C.1. Second, we internally calibrate the remaining parameters by minimizing the distance between moments in the data and the decentralized model (Eq. 30). Formally, this minimization is accomplished by solving a mathematical program with equilibrium constraints (MPEC) following Dubé, Fox, and Su (2012) and Su and Judd (2012). Appendix C.2 provides a discussion of our calibration and intuition for identification of each internally calibrated parameter.

We present the calibrated parameters in Table 1 and discuss a few of the most important ones here. To calibrate retailers' flow search costs, c_{do}, we use the fraction of firms that export and manufacturing capacity utilization rates, similar to Armenter and Koren (2014), Eaton, Eslava, Jinkins, Krizan, and Tybout (2014), and Eaton, Jinkins, Tybout, and Xu (2016). To calibrate domestic retailers' search costs, we use manufacturing capacity utilization rates in each country, as in Michaillat and Saez (2015), Petrosky-Nadeau and Wasmer (2017) and Petrosky-Nadeau, Wasmer, and Weil (2018). Coefficients from a log-linear regression of imports on tariffs and distance inform the elasticity of matches with respect to the number of searching producers, η, as described in Appendix C.3.

Without search, the tariff elasticity is important for determining the optimal tariff. In fact, it solely determines the optimal tariff in country d if it is a small open economy. For example, in Demidova and Rodríguez-Clare (2009) the optimal tariff for d is given by $t_{do}^{u} = 1 + 1/\left(\mu\theta + 1\right)$ and the tariff elasticity is $-\mu\theta$, which is also the tariff coefficient in a log-linear import regression. In contrast, with a large open economy, the tariff elasticity does not solely determine the optimal tariff but it remains an important ingredient, as discussed in Appendix B.2.7. Likewise, in a model with search, the import elasticity with respect to tariffs remains important. Additionally, as first pointed out in KM, standard log-linear regressions that counterfactually omit search frictions yield a biased tariff coefficient (KM, Appendix C.3). As such, we account for this bias when targeting the distance and tariff coefficients, as discussed in Appendix C.3. Without search, this bias disappears and the tariff coefficient in a log-linear import specification correctly recovers the parameters, $-\mu\theta$. Therefore, to obtain the same tariff coefficient in a model without search as in the model with search, the parameters σ or θ would need to change.

Our baseline calibration fits economic aggregates, such as GDP, consumption, investment, wages, and international trade flows, and exporter failure rates. Table 2 presents the moments from the model and shows that the model matches the data well and provides a realistic environment for quantitative exercises, a topic we pursue in the next section.

5 Quantitative results

5.1 Search frictions reduce optimal tariffs

Higher search costs significantly reduce optimal unilateral import tariffs for both the United States and China in the calibrated model. Table 3 shows results from our calibrated model for baseline search frictions and Table 4 shows results for a counterfactual exercise in which international search costs are reduced to domestic search costs, but we otherwise retain the rest of the baseline calibration ("reduced search frictions"). This counterfactual exercise could be motivated by innovations that reduce international search costs.

In both tables, the Rows A and B show import tariffs, Rows C and D show welfare, and Rows E and F show welfare changes from the baseline calibration. Column 1 shows results for the baseline U.S. and Chinese tariffs. Column 2 (3) shows results for the optimal unilateral tariff for the U.S. (Chinese) country social planner if China (the United States) sets tariffs passively at the baseline values from Column 1. Column 4 shows results for the Nash equilibrium tariffs.

In Column 2 (3), the U.S. (Chinese) optimal unilateral tariff is 1.14 (1.15) with baseline search frictions, Table 3, and 1.38 (1.35) with reduced search frictions, Table 4. Nonetheless, higher search costs reduce U.S. (Chinese) welfare at the optimal tariff because the economy devotes more resources to overcoming them (compare Rows C and Rows D across the tables). In Column 2, both tables show that the U.S. optimal unilateral tariffs raises U.S. welfare relative to the baseline for the United States (Row E) and lowers welfare for China (Row F) because the U.S. optimal tariff (Row B) is higher than the baseline tariff in Column 1. In Column 3, the Chinese optimal tariffs are higher than the baseline and so welfare is higher for China but lower for the United States in Rows E and F of Table 4. In contrast, Chinese optimal unilateral tariffs in Column 3 are lower than the baseline in Column 1 of Table 3 and so both countries gain in Rows E and F. We depict U.S. and Chinese welfare as a function of U.S. unilateral tariffs for baseline and reduced search frictions in Fig. 5. We show the same for Chinese unilateral tariffs in Fig. A2.

International search frictions also reduce significantly equilibrium tariffs in a strategic setting. Specifically, international search costs reduce U.S. (Chinese) Nash equilibrium tariffs from 1.33 to 1.15 (1.3 to 1.13), if we compare the calibration with reduced search frictions and baseline search frictions (compare Rows A and B in Column 1 and 4 within each table). For the baseline and counterfactual, Figs. A3a and A3b show the best response functions, Figs. A3c and A3d show U.S. welfare, and Figs. A3e and A3f show Chinese welfare, respectively. In the welfare figures for a country, we vary the import tariff for that country while holding the tariff in the other country at the Nash tariff.

5.2 Welfare gains with and without international search frictions

U.S. and Chinese welfare would increase only slightly with optimal unilateral import tariffs and baseline search frictions relative to observed 2016 tariff levels. At the optimal U.S. tariff, U.S. welfare would rise by 0.03 percent and Chinese welfare would fall by 0.57 percent (Table 3, Column 2). Similarly, at the optimal Chinese unilateral tariff, Chinese welfare would rise by 0.01 percent and U.S. welfare would rise by 0.09 percent (Table 3, Column 3).

Predicted welfare gains are significantly larger in the counterfactual than in the baseline. For example, optimal unilateral U.S. and Chinese tariffs in this counterfactual calibration imply welfare gains of one percent for each country (Table 4, Columns 2 and 3, Rows E and F).

More generally, search frictions attenuate the welfare response to tariff changes and the gains from trade. Intuitively, this is because, search frictions imply fewer matched varieties and tariffs mainly affect aggregates through the intensive margin of matched varieties. This result is consistent with our findings in Section 3.2 (Fig. 2) and echos one of the main conclusions of KM. Comparing the welfare changes from baseline between the left and right panels in Figs. 5 and A2 highlights this point. For example, in the model with reduced search frictions, varying U.S. unilateral tariffs from 1 to 2 yields U.S. welfare that is at most 1 percent above and below the baseline level (Fig. 5a). And moving from baseline tariffs to optimal tariffs raises welfare by 1.0 percent. In comparison, in the model with baseline search frictions, varying tariffs from 1 to 2 yields welfare that is no more than 0.1 percent above, and at most 1.1 percent below, the baseline level (Fig. 5b). And moving from baseline tariffs to optimal tariffs raises welfare by about 1.0 percent. That is, search frictions render tariffs far less potent for welfare changes. Figs. A2a and A2b show the same results for China, and Fig. A3 shows that it holds true in a strategic environment.

We note that the observed U.S. import tariff in 2016 was below that implied by our model, including the optimal unilateral or Nash tariff and whether we consider search frictions or not. The welfare gains from moving from observed to optimal tariffs are small, except in the case with reduced search frictions and U.S. unilateral tariffs.

6 Conclusion

We study optimal import tariffs in an environment with search frictions between exporting producers and importing retailers. Our model nests standard trade models and it admits a unique equilibrium. We analytically characterize the unilateral uniform import tariff that maximizes domestic welfare in a two-country setting. With search frictions, this optimal tariff depends not only on the offer curve elasticity—as it does in a model without search frictions—but also on two new terms. An import markdown term relates to the difference between import prices and final consumer prices. A second term relates to additional externalities introduced by search frictions. We also characterize the offer curve elasticity in our model and show that search frictions introduce a new incentive to subsidize imports because of endogenous market thickness effects. We elucidate these analytical results with simple numerical examples.

Quantitative results using 2016 U.S. and Chinese data suggest that the optimal U.S. unilateral tariff with search frictions is about 20 percentage points below that in a model with reduced search costs. Changes in welfare in response to changes in tariffs are smaller in the model with baseline search costs than in the model with reduced search costs. For example, moving from baseline to optimal unilateral tariffs raises U.S. welfare by about 0.03 (1) percent in a model with baseline (reduced) search costs. In the Nash equilibrium with baseline search costs, U.S. (Chinese) tariffs are 7 (4) ppt. higher (lower) and welfare is 0.2 (0.7) percent higher (lower) than result from 2016 tariff levels.

Our study points to a few directions for future research. First, empirical work could use trade flows between many countries, the fraction of exporting firms, and capacity utilization rates to estimate the parameters of our model. Second, after estimating, Sections 3.1 and 3.4 could characterize optimal unilateral and Nash tariffs in a D-country version of our model. Third, search frictions could be used to rationalize and model the pervasiveness of trade promotion programs, like the State Trade Expansion Program (STEP, 2024). Fourth, international search frictions could vary over time, as they do over the business cycle in labor markets, for example. If variation in search frictions over space and time is important, the framework in this paper would imply that optimal tariffs should vary in these dimensions as well. Our framework provides a foundation for analyzing the welfare implications of tariffs in the presence of goods-market search frictions.

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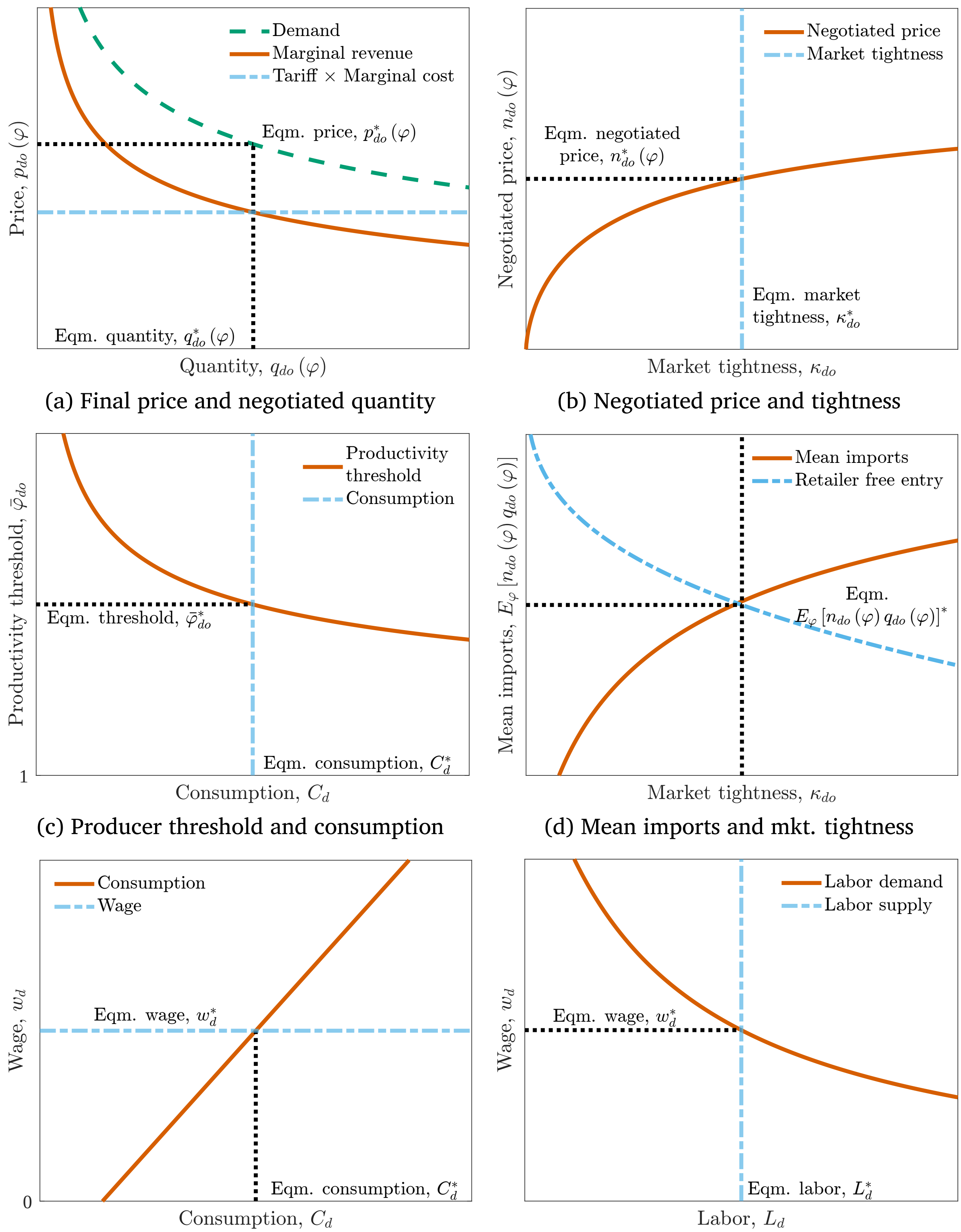
Fig. 1: Unique general equilibrium of the model



(a) Final price and negotiated quantity

(b) Negotiated price and tightness

(c) Producer threshold and consumption

(d) Mean imports and mkt. tightness

(e) Goods market clearing

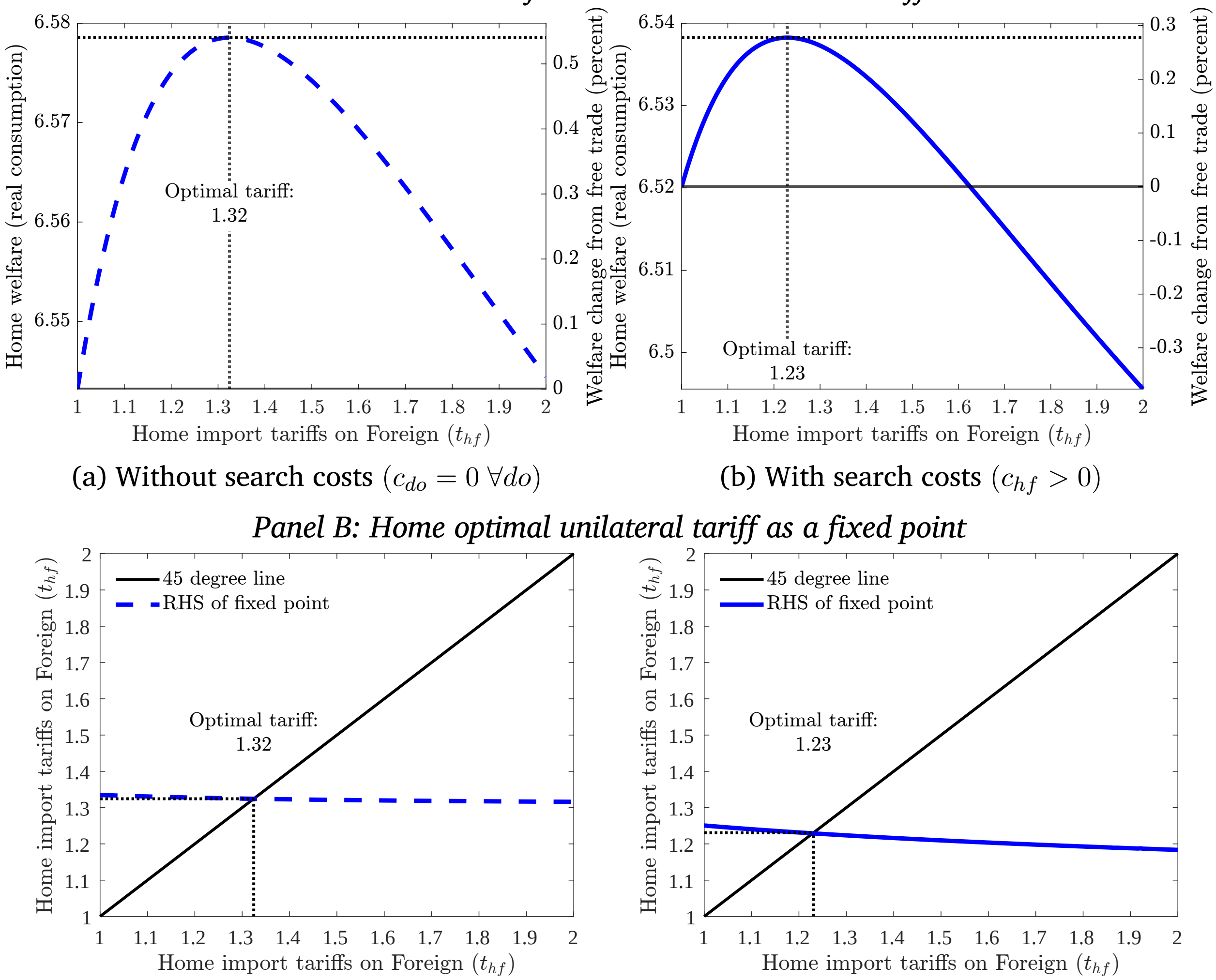
(f) Labor market clearing

Note: Fig. 1b depicts bargaining over negotiated price from Eq. (14) given market tightness from Fig. 1d (Section 2.2.3). Fig. 1a depicts bargaining over quantity with demand and marginal revenue from Eq. (2) and marginal cost from Eq. (5) (Section 2.2.2). Fig. 1c depicts the producer entry decision from Eq. (18) given consumption from Fig. 1e (Section 2.2.4). Fig. 1d depicts the retailer entry decision from Eq. (19) given negotiated prices from Fig. 1b (Section 2.2.5). Fig. 1e depicts goods market clearing using Eqs. 26 and 27 given wage from Fig. 1f (Section 2.3.5). Fig. 1f depicts labor market clearing using Eq. (29) and the country's endowment (Section 2.3.6). Section 2.4.2 and Appendix A.4.3 discuss the equilibrium.

Fig. 2: Home welfare and optimal tariffs without and with search costs: Numerical example

Panel A: Home welfare and Home unilateral tariffs

Panel B: Home optimal unilateral tariff as a fixed point



(a) Without search costs ($c_{do} = 0 \; \forall do$)

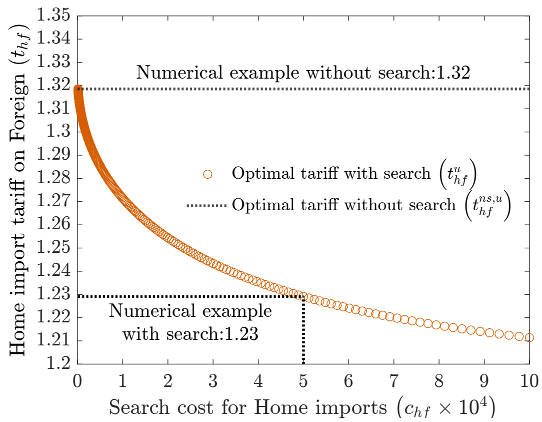
(b) With search costs ($c_{hf} > 0$)

(c) Without search costs ($c_{do} = 0 \; \forall do$)

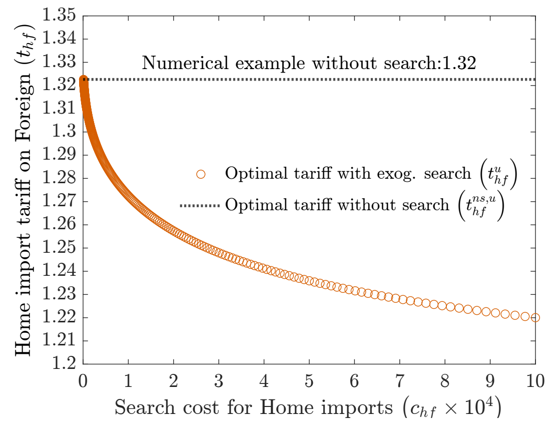
(d) With search costs ($c_{hf} > 0$)

Note: The left panels present results for the numerical example without search costs, $c_{do} = 0 \; \forall do$. The right panels present results for the numerical example with search costs, $c_{hf} > 0$ but $c_{do} = 0 \; \forall do \neq hf$. Fig. 2a plots Home's welfare as function of Home's import tariff, t_{hf}, without search costs. Fig. 2c plots the LHS and the RHS of Eq. (35). The solid black lines depicts the LHS of Eq. (35) (which is a 45 degree line) and the dashed blue line depicts the RHS of Eq. (35) for various levels of Home's import tariff. The intersection of the two lines identifies the optimal tariff. Fig. 2b plots Home's welfare as a function of Home's import tariff, t_{hf}, with search costs. Fig. 2d plots the LHS and the RHS of Eq. (34). The solid black line depicts the LHS of Eq. (34) (which is a 45 degree line) and the solid blue line depicts the RHS of Eq. (34) for various levels of Home's import tariff. The intersection of the two identifies the optimal tariff. See Section 3.2 for further details.

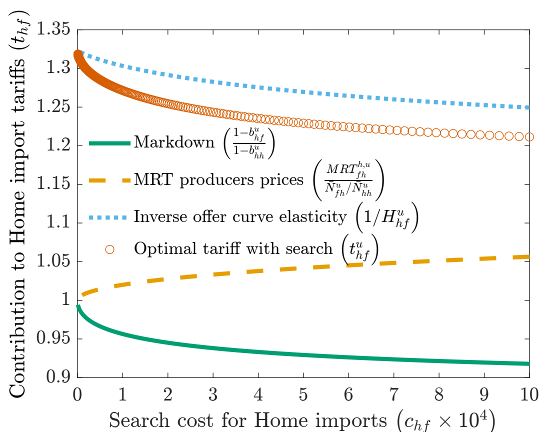
Fig. 3: Optimal unilateral tariffs for endogenous & exogenous frictions: Numerical example



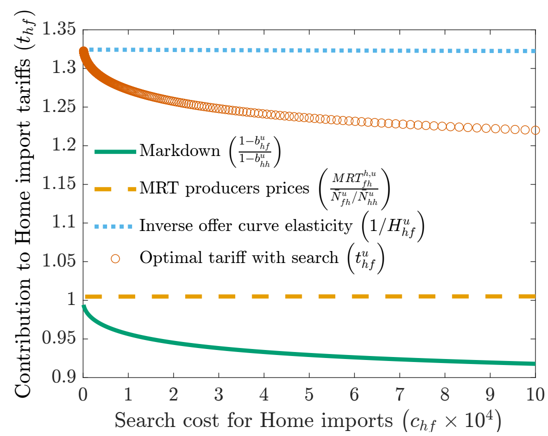
(a) Optimal tariffs (endo. frictions)



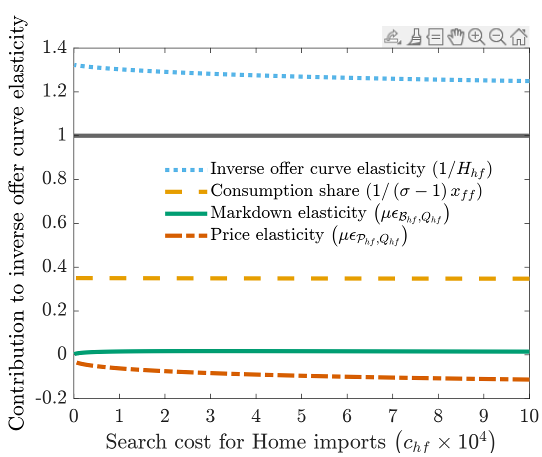
(b) Optimal tariffs (exo. frictions)



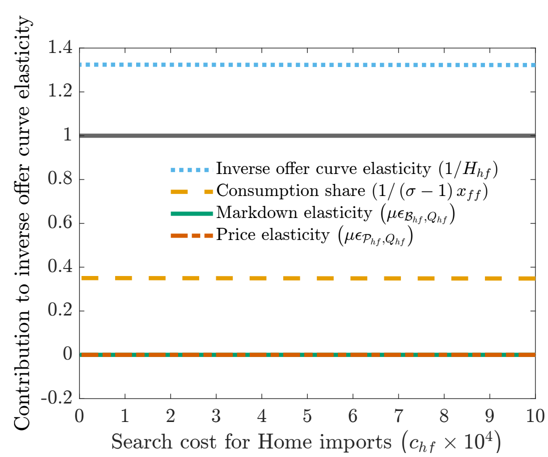
(c) Optimal tariff decomposition (endo.)



(d) Optimal tariff decomposition (exo.)



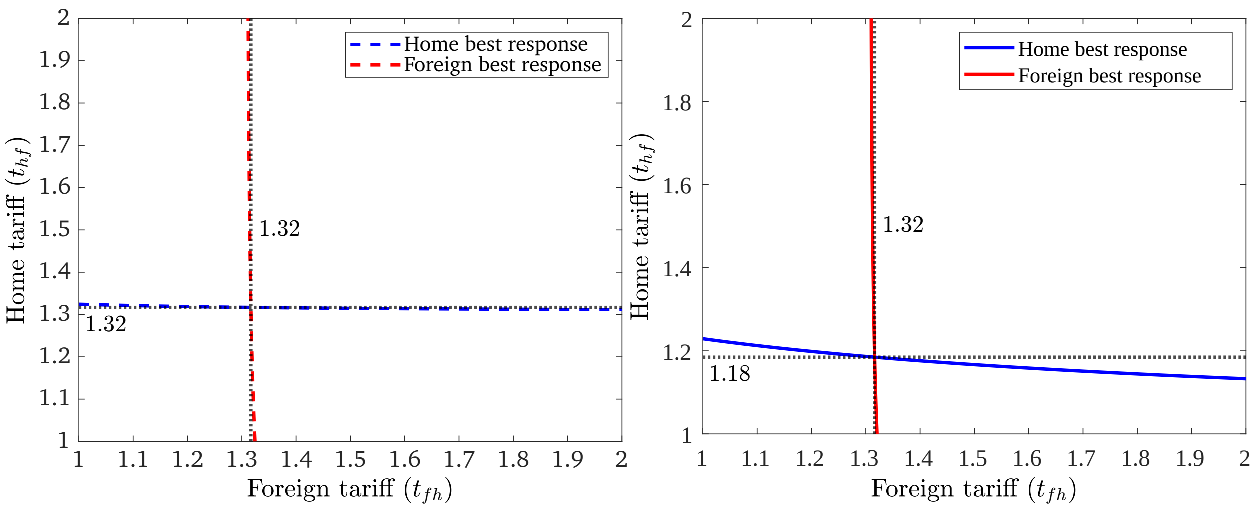
(e) Offer curve elasticity decomposition (endo.)



(f) Offer curve elasticity decomposition (exo.)

Note: All figures show results for different levels of search costs $c_{hf} > 0$ and $c_{do} = 0\ \forall do \neq hf$. The panels of the left show results for a model with endogenous search frictions. The panels on the right show results in which we fix the matching and idle rates for each search cost to the endogenous matching and idle rates in the corresponding figure on the left. The first row shows the optimal unilateral import tariff, t^u_{hf}, from solving the Home country social planner's problem (Eq. 33). The second row shows the decomposition of the optimal tariff (Eq. 34). The third row shows the decomposition of the inverse of the offer curve elasticity (Eq. 36). The vertical axis intercepts of the various lines correspond to the case without search costs, $c_{do} = 0\ \forall do$. See Sections 3.2 and 3.3 for further details.

Fig. 4: Pure strategy Nash equilibrium: Numerical example



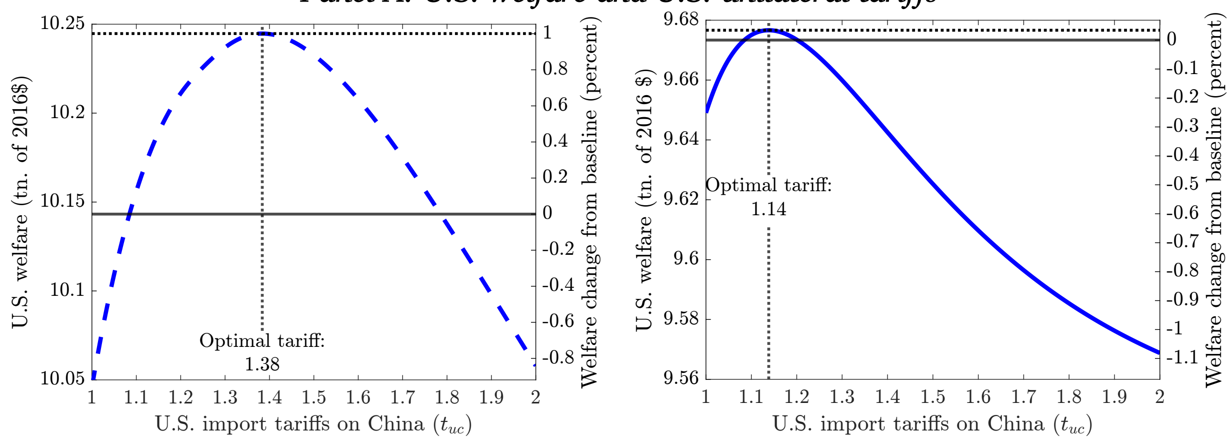
(a) Without search costs $(c_{do} = 0 \; \forall do)$

(b) With search costs $(c_{hf} > 0)$

Note: Fig. 4a plots import tariff best response functions of Home and Foreign in the numerical example without search costs, $c_{do} = 0 \; \forall do$. Fig. 4b plots best response functions of Home and Foreign in the numerical example with search costs, $c_{hf} > 0$ and $c_{do} = 0 \; \forall do \neq hf$. The best response tariff is the tariff that maximizes welfare in country h (f) conditional on a tariff in country f (h). See Section 3.4 for further details.

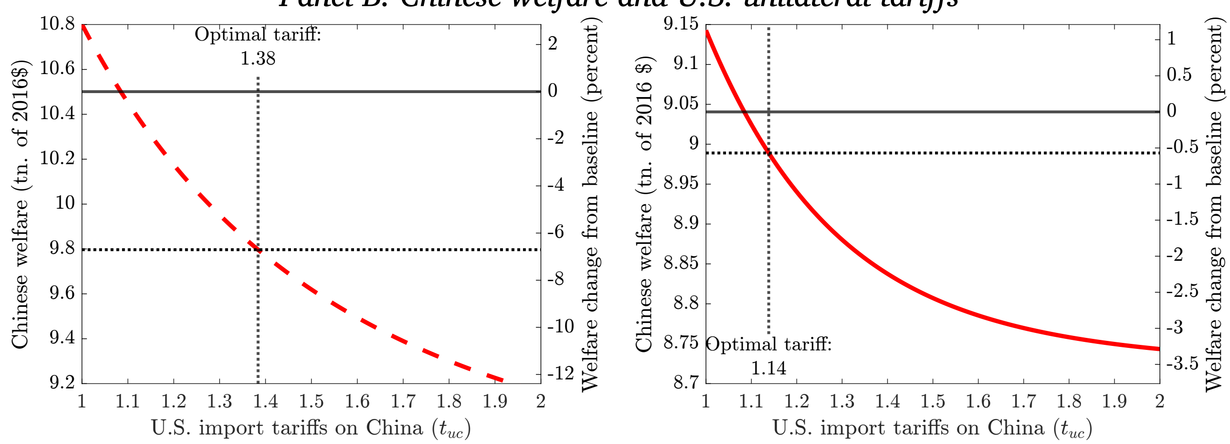
Fig. 5: U.S. unilateral tariffs and U.S. and Chinese welfare for reduced and baseline search frictions

Panel A: U.S. welfare and U.S. unilateral tariffs



(a) Intl. search costs reduced to domestic levels (b) Baseline search costs

Panel B: Chinese welfare and U.S. unilateral tariffs



(c) Intl. search costs reduced to domestic levels (d) Baseline search costs

Note: Fig. 5a (Fig. 5c) plots U.S. (Chinese) welfare as a function of U.S. import tariffs with all parameters as in the baseline but with international search costs reduced to their domestic levels and given by $c_{uu} \times 10^4 = 10.43$, $c_{cu} \times 10^4 = 15.28$, $c_{uc} \times 10^4 = 10.43$, $c_{cc} \times 10^4 = 15.28$. Fig. 5b (Fig. 5d) plots U.S. (Chinese) welfare as a function of U.S. import tariffs in the baseline calibrated model, as in Table 1. The left vertical axis denotes levels and the right vertical axis denotes the welfare change from observed 2016 tariff levels. See Sections 5.1 and 5.2 for further details.

Table 1: Calibrated model parameters

Parameter	Value	Unit	Reason
Panel A. Externally calibrated parameters			
Producers' bargaining power (β)	0.50	fraction	Benchmark
Risk-free rate ($r \times 10^2$)	5.00	percent	Interest rate
Separation rate (λ)	1.00	Poisson rate	Among trading partners
Elasticity of substitution (σ)	4.00	elasticity	Demand estimation
Pareto shape parameter (θ)	3.18	unitless	U.S. firm size distribution
Efficiency of matching function (ξ)	1.00	elasticity	Normalization
Labor endowment in US (L_u)	213.72	mn. people	Working age population
Labor endowment in CH ($L_c \times$)	988.05	mn. people	Working age population
Firm endowment in US ($N_u^x \times 10^{-1}$)	500.00	num. varieties	Consumption level
Firm endowment in CH ($N_x^c \times 10^{-1}$)	500.00	num. varieties	Consumption level
Iceberg origin US ($A_{o=u}$)	1.00	multiple	Gravity equation
Iceberg scale (A)	1.00	multiple	Gravity equation
US domestic tax (t_{uu})	1.06	multiple	Sales tax rate
CH import tariff (t_{cu})	1.17	multiple	Import VAT rate
US import tariff (t_{uc})	1.08	multiple	Tariffs plus sales tax
CH domestic tax (t_{cc})	1.11	multiple	VAT rate
Internal distance US to US (D_{uu})	1.85	kkm	Distance
Distance to CH from US (D_{cu})	11.18	kkm	Distance
Distance to US from CH (D_{uc})	11.18	kkm	Distance
Internal distance CH to CH (D_{cc})	1.02	kkm	Distance
Panel B. Internally calibrated parameters			
US domestic search cost ($c_{uu} \times 10^4/\chi(\kappa_{uu})$)	18.14	labor	US mfg. capacity utilization
CH importers' search cost ($c_{cu} \times 10^4/\chi(\kappa_{cu})$)	171.43	labor	Percent of US firms exp. to CH
US importers' search cost ($c_{uc} \times 10^4/\chi(\kappa_{uc})$)	8.14	labor	Percent of CH firms exp. to US
CH domestic search cost ($(c_{cc}10^4)/\chi(\kappa_{cc})$)	33.82	labor	CH mfg. capacity utilization
US domestic fixed cost ($f_{uu} \times 10^4$)	6.74	labor	US business failure rate
US export fixed cost ($f_{cu} \times 10^4$)	8.17	labor	CH-US exporter failure rate
CH export fixed cost ($f_{uc} \times 10^4$)	9.47	labor	US-CH exporter failure rate
CH domestic fixed cost ($f_{cc} \times 10^4$)	16.36	labor	CH business failure rate
US exploration cost ($e_u^x \times 10^2$)	1.84	labor	Investment expenditure
CH exploration cost ($e_c^x \times 10^2$)	12.90	labor	Investment expenditure
Iceberg destination US ($A_{d=u} \times 10^{-2}$)	464.31	multiple	Gravity equation
Iceberg destination CH ($A_{d=c} \times 10^{-2}$)	201.49	multiple	Gravity equation
Iceberg origin CH ($A_{o=c}$)	7.73	multiple	Gravity equation
Iceberg distance (a_1)	0.07	elasticity	Gravity equation
Elasticity of matching function (η)	0.33	elasticity	Log-linear import elasticity

Note: The "Reason" column provides the reason for externally calibrated parameters and the main source of identification for internally calibrated parameters. The levels of the retailer search costs, $c_{uu} \times 10^4 = 10.43$, $c_{cu} \times 10^4 = 632.03$, $c_{uc} \times 10^4 = 15.34$, $c_{cc} \times 10^4 = 15.28$, do not have meaning because they depend on the normalization of the matching efficiency, ξ, as in Shimer (2005). As such, we report average retailer search costs, $c_{do}/\chi(\kappa_{do})$, which have intrinsic meaning. Parameters not shown are $h_{do} = l_{do} = s_{do} = 0\ \forall do$. Calibrated parameters of the model are at annual frequency. We discuss the calibration methodology in Section 4 and intuition for parameter identification in Appendix C.2. "CH" stands for China and "US" stands for the United States.

Table 2: Model fit

Moment description	Data	Model	Unit
Log-log imports distance	-0.9	-0.9	elasticity
Log-log imports tariffs	-5.0	-5.2	elasticity
US mfg. capacity utilization	75	76	percent
US firms exporting to CH	6	6	percent
CH firms exporting to US	21	21	percent
CH mfg. capacity utilization	74	84	percent
US business failure rate	20	20	percent
CH-US exporter failure rate	60	56	percent
US-CH exporter failure rate	40	40	percent
CH business failure rate	20	20	percent
US dom. consump. share ($IM_{uu}/\left(IM_{uu}+IM_{uc}\right)$)	86	95	percent
CH dom. consump. share ($IM_{cc}/\left(IM_{cc}+IM_{cu}\right)$)	85	91	percent
US wage	52	60	$ thsnd.
CH wage	11	11	$ thsnd.
CH imports from US (IM_{cu})	463	438	$ bn.
US imports from CH (IM_{uc})	463	438	$ bn.
US investment	5,980	6,034	$ thsnd.
CH investment	6,773	7,532	$ thsnd.
US consumption	12,727	9,673	$ bn.
CH consumption	4,418	5,451	$ bn.
US GDP	18,707	15,708	$ bn.
CH GDP	11,191	12,983	$ bn.
US price index (P_u)	1,000	1,000	percent
CH price index (P_c)	603	603	percent

Note: The model matches the empirical targets relatively well. The "Data" and "Model" columns present the value of the corresponding moment in the data and model at the calibrated parameter values given in Table 1. "CH" stands for China, "US" for the United States, and "GDP" for Gross Domestic Product. The country d consumption share is $IM_{dd}/\left(IM_{dd}+IM_{do}\right)$. The parameter A_d in Eq. (C183) by itself defines the level of the price index Ξ_d without changing any other value and gets zero weight in the MPEC objective function. The normalization value is chosen to express real GDP in trillions of dollars and satisfy purchase power parity of 60 percent between the United States and China. We discuss model fit and other details in Section 4.

Table 3: Optimal tariffs with baseline search frictions

	(1) Baseline (t^c_{cu}, t^c_{uc})	(2) US CSP (t^c_{cu}, t^u_{uc})	(3) CH CSP (t^u_{cu}, t^c_{uc})	(4) Nash (t^n_{cu}, t^n_{uc})
(A) Chinese import tariffs (t_{cu})	1.17	1.17	1.15	1.13
(B) U.S. import tariffs (t_{uc})	1.08	1.14	1.08	1.15
(C) U.S. welfare (tn. of 2016 $)	9.67	9.68	9.68	9.69
(D) Chinese welfare (tn. of 2016 $)	9.04	8.99	9.04	8.98
(E) U.S. welfare change from baseline (%)	0.00	0.03	0.09	0.20
(F) Chinese welfare change from baseline (%)	0.00	-0.57	0.01	-0.66

Note: Results with baseline search frictions. The rows of the table show Chinese and U.S. import tariffs, the level of welfare in both countries, and the change in welfare from the baseline calibration. Column 1 shows results when tariffs are set to their baseline level, as in Table 1. Column 2 (3) shows results when the Chinese (U.S.) tariff is set at the baseline level and the U.S. (Chinese) tariff is unilaterally set to maximize U.S. (Chinese) welfare. Column 4 shows results for the Nash equilibrium when both countries choose tariffs optimally. "CSP" stands for country social planner. See Sections 5.1 and 5.2 for further details.

Table 4: Optimal tariffs with international search costs reduced to domestic levels

	(1) Baseline (t^c_{cu}, t^c_{uc})	(2) US CSP (t^c_{cu}, t^u_{uc})	(3) CH CSP (t^u_{cu}, t^c_{uc})	(4) Nash (t^n_{cu}, t^n_{uc})
(A) Chinese import tariffs (t_{cu})	1.17	1.17	1.36	1.30
(B) U.S. import tariffs (t_{uc})	1.08	1.38	1.08	1.33
(C) U.S. welfare (tn. of 2016 $)	10.14	10.24	9.99	10.11
(D) Chinese welfare (tn. of 2016 $)	10.50	9.80	10.61	9.94
(E) U.S. welfare change from baseline (%)	0.00	1.00	-1.49	-0.30
(F) Chinese welfare change from baseline (%)	0.00	-6.71	1.00	-5.37

Note: Results for a calibration in which all parameters are as in the baseline but with international search costs reduced to their domestic levels and given by $c_{uu} \times 10^4 = 10.43$, $c_{cu} \times 10^4 = 15.28$, $c_{uc} \times 10^4 = 10.43$, $c_{cc} \times 10^4 = 15.28$. The rows of the table show Chinese and U.S. import tariffs, the level of welfare in both countries, and the change in welfare from the baseline calibration. Column 1 shows results when tariffs are set to their baseline level, as in Table 1. Column 2 (3) shows results when the Chinese (U.S.) tariff is set at the baseline level and the U.S. (Chinese) tariff is unilaterally set to maximize U.S. (Chinese) welfare. Column 4 shows results for the Nash equilibrium when both countries choose tariffs optimally. "CSP" stands for country social planner. See Sections 5.1 and 5.2 for further details.

Appendix to "Tariffs and Goods-Market Search Frictions"

Pawel M. Krolikowski* Andrew H. McCallum†

March 12, 2026

A Model appendix: Model details, model solution, and aggregation

In this appendix we study the same consumer problem as in the main text but assume that the representative consumer combines differentiated varieties with a homogeneous good using a Cobb-Douglas utility function:

$$\max_{q_d(1), q_{dk}(\omega)} q_d(1)^{1-\alpha} \left[\sum_{k=1}^{O} Q_{dk}^{\frac{1}{\mu}} \right]^{\alpha\mu} \tag{A1a}$$

$$s.t.\ Q_{do} = \left[\int_{\omega \in \Omega_{do}} q_{do}(\omega)^{\frac{1}{\mu}} d\omega \right]^{\mu}, \tag{A1b}$$

$$C_d = p_d(1)\, q_d(1) + \sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega)\, q_{dk}(\omega)\, d\omega, \tag{A1c}$$

The homogeneous good has price $p_d(1)$. The solution results in the following demand for the homogeneous good and each differentiated variety, respectively

$$q_d(1) = \frac{(1-\alpha) C_d}{p_d(1)}, \qquad q_{do}(\omega) = \alpha C_d \frac{p_{do}(\omega)^{-\sigma}}{P_d^{1-\sigma}}. \tag{A2}$$

We assume that the homogeneous good is produced with one unit of labor under constant returns to scale in each country. As such, the price of the homogeneous good must equal the wage in each country, $p_d(1) = w_d, \forall d$. To obtain results in the main text, set $\alpha = 1$, which removes the homogeneous good.

A.1 Model details

A.1.1 Consumer problem with no storage technology

Define P_d as the price index for the bundle of differentiated varieties and P_{do} as the price index for the bundle of varieties produced in country o and consumed in country d:

$$P_d = \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} = \left[\sum_{k=1}^{O} P_{dk}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{A3}$$

The ideal price index including the homogeneous good that minimizes expenditure to obtain utility level $U_d = 1$ is

$$\Xi_d = \left[p_d(1) / (1-\alpha) \right]^{1-\alpha} \left[P_d / \alpha \right]^{\alpha}. \tag{A4}$$

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Notice that if $\alpha = 1$ then $\Xi_d = P_d$. We solve the consumer's utility maximization and expenditure minimization problems explicitly in KM, Appendix A.1.

The approach in the main text yields the same consumer demand functions as the solution to a dynamic problem with discounting if we assume that the consumer cannot use trade relationships to transfer resources across time.

In this case, the consumer problem is:

$$\begin{aligned} \max_{q_d(1,t), q_{dk}(\omega,t)} \quad & \int_0^\infty e^{-rt} q_d(1,t)^{1-\alpha} \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega,t)^{\left(\frac{\sigma-1}{\sigma}\right)} d\omega \right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)} dt \qquad \text{(A5)} \\ s.t.\ C_d(t) \quad &= \quad p_d(1,t)\, q_d(1,t) + \sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega,t)\, q_{dk}(\omega,t)\, d\omega, \end{aligned}$$

in which the consumer chooses the quantity of the homogeneous and differentiated varieties at each point in time to maximize discounted utility, subject to their budget constraint. This problem has no state variable because if we included a state variable that is affected by a choice variable, the consumer would implicitly have access to a storage technology.

The associated current value Hamiltonian is

$$\begin{aligned} \mathcal{H} &= q_d(1,t)^{1-\alpha} \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega,t)^{\left(\frac{\sigma-1}{\sigma}\right)} d\omega \right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)} \\ &- \mu(t) \left[p_d(1,t)\, q_d(1,t) + \sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega,t)\, q_{dk}(\omega,t)\, d\omega - C_d(t) \right]. \end{aligned}$$

Notice that this Hamiltonian is identical to the Lagrangian for the one-period problem in Appendix A.1.1 of KM (2021), except the Hamiltonian includes t indices. The first order necessary conditions are

$$\begin{aligned} \mathcal{H}_{q_d(1,t)} &= (1-\alpha)\, q_d(1,t)^{-\alpha} \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega,t)^{\left(\frac{\sigma-1}{\sigma}\right)} d\omega \right] - \mu(t)\, p_d(1,t) = 0 \\ \mathcal{H}_{q_{dk}(\omega,t)} &= q_d(1,t)^{1-\alpha} \alpha \left(\frac{\sigma}{\sigma-1}\right) \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega,t)^{\left(\frac{\sigma-1}{\sigma}\right)} d\omega \right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)-1} \left(\frac{\sigma}{\sigma-1}\right) q_{dk}(\omega,t)^{\left(\frac{\sigma-1}{\sigma}\right)-1} - \mu(t)\, p_{dk}(\omega,t) = 0. \end{aligned}$$

Using the steps in Appendix A.1.1 of KM (2021) yields the demand functions:

$$q_d(1,t) = \frac{(1-\alpha)\, C_d(t)}{p_d(1,t)}, \qquad q_{do}(\omega,t) = \alpha C_d(t) \frac{p_{do}(\omega,t)^{-\sigma}}{P_d(t)^{1-\sigma}}, \qquad \text{(A6)}$$

which replicates Eq. (2) in the main text if we drop the t index.

Appendix B.1 shows that welfare in the dynamic and static settings are trivially different and do not affect any of the main results.

A.1.2 Consumer problem with a storage technology

If we allow the consumer to transfer resources across time by using domestic (dd) relationships as an asset, the consumer problem is:

$$\max_{q_d(1,t),q_{dk}(\varphi,t)} \int_0^\infty e^{-rt} q_d(1,t)^{1-\alpha} \left[\sum_{k=1}^{O} \left(1 - \frac{u_{dk}}{1-i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} q_{dk}(\varphi,t)^{\left(\frac{\sigma-1}{\sigma}\right)} dG(\varphi)\right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)} dt \tag{A7a}$$

$$s.t.\ C_d(t) = p_d(1,t)\, q_d(1,t) + \sum_{k=1}^{O} \left(1 - \frac{u_{dk}}{1-i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} p_{dk}(\varphi,t)\, q_{dk}(\varphi,t)\, dG(\varphi), \tag{A7b}$$

$$Y_d(t) = C_d(t) + I_d(t), \tag{A7c}$$

$$(u_{do}\dot{N}_o^x) = \lambda(1 - u_{do} - i_{do}) N_o^x - \kappa_{do}\chi(\kappa_{do}) u_{do} N_o^x\ \forall do, \tag{A7d}$$

in which we have used KM Appendix A.11.1 to move from an index ω to a distribution of goods over φ to highlight dependence on u_{do}. The consumer maximizes discounted utility, Eq. (A7a), subject to their budget constraint, Eq. (A7b). The consumer can also use investment in Eq. (A7c) to transfer consumption across time, subject to the equation of motion for the number of unmatched producers in each market, Eq. (A7d). Because the consumer uses only domestic relationships as an asset, we maintain balanced trade and set $NX_d = 0$ in the resource constraint, Eq. (A7c). If the consumer could also use foreign relationships (do) as an asset then trade might not be balanced in each period. We discuss this approach below.

The state variables for the problem in Eq. (A7) are the unmatched fraction in each do market, u_{do}, from Eq. (A7d). This follows because N_o^x is a constant so that Eq. (A7d) yields

$$\dot{u}_{do} = \lambda(1 - u_{do} - i_{do}) - \kappa_{do}\chi(\kappa_{do}) u_{do}. \tag{A8}$$

The current value Hamiltonian associated with Eq. (A7) is therefore

$$\begin{aligned}
\mathcal{H} &= q_d(1,t)^{1-\alpha} \left[\sum_{k=1}^{O} \left(1 - \frac{u_{dk}}{1-i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} q_{dk}(\varphi,t)^{\left(\frac{\sigma-1}{\sigma}\right)} dG(\varphi)\right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)} \\
&+ \lambda_C(t) \left[C_d(t) - p_d(1,t)\, q_d(1,t) - \sum_{k=1}^{O} \left(1 - \frac{u_{dk}}{1-i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} p_{dk}(\varphi,t)\, q_{dk}(\varphi,t)\, dG(\varphi)\right] \\
&+ \lambda_Y(t) \left[Y_d(t) - C_d(t) - I_d(t)\right] \\
&+ \sum_d \sum_o \mu_{do}(t) \left[\lambda(1 - u_{do} - i_{do}) - \kappa_{do}\chi(\kappa_{do}) u_{do}\right].
\end{aligned}$$

The first order necessary conditions with respect to the control variables are:

$$\begin{aligned}
\mathcal{H}_{q_d(1,t)} &= 0 \\
\mathcal{H}_{q_{dk}(\varphi,t)} &= 0
\end{aligned}$$

in which

$$\begin{aligned}\mathcal{H}_{q_d(1,t)} &= (1-\alpha)\, q_d(1,t)^{-\alpha} \left[\sum_{k=1}^{O} \left(1-\frac{u_{dk}}{1-i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} q_{dk}(\varphi,t)^{\left(\frac{\sigma-1}{\sigma}\right)} dG(\varphi)\right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)} \\ &- \lambda_C(t)\, p_d(1,t) + \lambda_Y(t)\left[\frac{\partial Y_d(t)}{\partial q_d(1,t)} - \frac{\partial I_d(t)}{\partial q_d(1,t)}\right] + \sum_d \sum_o \mu_{do}(t)\left[-\lambda \frac{\partial i_{do}}{\partial q_d(1,t)} - \frac{\partial \kappa_{do}\chi(\kappa_{do})}{\partial q_d(1,t)} u_{do}\right],\end{aligned} \tag{A9}$$

and

$$\begin{aligned}\mathcal{H}_{q_{dk}(\varphi,t)} &= q_d(1,t)^{1-\alpha}\,\alpha\left(\frac{\sigma}{\sigma-1}\right)\left[\sum_{k=1}^{O}\left(1-\frac{u_{dk}}{1-i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} q_{dk}(\varphi,t)^{\left(\frac{\sigma-1}{\sigma}\right)} dG(\varphi)\right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)-1} \\ &\times \left(1-\frac{u_{dk}}{1-i_{dk}}\right) N_k^x \left(\frac{\sigma}{\sigma-1}\right) q_{dk}(\varphi,t)^{\left(\frac{\sigma-1}{\sigma}\right)-1} \\ &- \lambda_C(t)\left(1-\frac{u_{dk}}{1-i_{dk}}\right) N_k^x p_{dk}(\varphi,t) + \lambda_Y(t)\left[\frac{\partial Y_d(t)}{\partial q_{dk}(\varphi,t)} - \frac{\partial I_d(t)}{\partial q_{dk}(\varphi,t)}\right] \\ &+ \mu_{dk}(t)\left[-\lambda\frac{\partial i_{dk}}{\partial q_{dk}(\varphi,t)} - \frac{\partial \kappa_{dk}\chi(\kappa_{dk})}{\partial q_{dk}(\varphi,t)} u_{do}\right].\end{aligned} \tag{A10}$$

In Eq. (A9), homogeneous good consumption affects utility directly. It also affects aggregate resources, $Y_d(t)$, as well as investment, $I_d(t)$, through its effects on the equilibrium wage, among other things. In Eq. (A10), differentiated good consumption affects utility directly. It also affects aggregate resources through its effects on profits and government revenue, as well as investment through its effects on the equilibrium wage. The necessary first order conditions with respect to the state variables are

$$\mathcal{H}_{u_{do}} = -\dot{\mu}_{do}(t) + r\mu_{do}(t)\,\forall d, o,$$

which implies that

$$\begin{aligned}-\dot{\mu}_{do}(t) + r\mu_{do}(t) &= -q_d(1,t)^{1-\alpha}\,\alpha\left(\frac{\sigma}{\sigma-1}\right)\left[\sum_{k=1}^{O}\left(1-\frac{u_{dk}}{1-i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} q_{dk}(\varphi,t)^{\left(\frac{\sigma-1}{\sigma}\right)} dG(\varphi)\right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)-1} \\ &\times\left(\frac{1}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} q_{do}(\varphi,t)^{\left(\frac{\sigma-1}{\sigma}\right)} dG(\varphi) \\ &+\lambda_C(t)\left(\frac{1}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} p_{do}(\varphi,t)\, q_{do}(\varphi,t)\, dG(\varphi) \\ &+\lambda_Y(t)\left[\frac{\partial Y_d(t)}{\partial u_{do}} - \frac{\partial I_d(t)}{\partial u_{do}}\right] + \mu_{do}(t)\left[-\lambda - \kappa_{do}\chi(\kappa_{do})\right].\end{aligned}$$

The unmatched rate affects utility directly through the measure of goods that are consumed, but it also affects aggregate resources and investment.

The unknowns for this problem in each time t are the quantities of the homogeneous and differentiated goods, and the costate variables λ_C, λ_Y, and $\mu_{do}\forall d, o$. The equations are the the budget constraint (Eq. A7b), the aggregate resource constraint (Eq. A7c), the equation of motion for each state variable (Eq. A8), the FOC for the homogeneous good (Eq. A9), and the FOCs for the differentiated goods (Eq. A10). These variables and equations define a system that defines the consumer's problem out of steady state. If we focus on outcomes in the steady state we would impose the additional condition that $\dot{\mu}_{do}(t) = 0\forall d, o$.

If the consumer can use domestic (dd) and international (do) relationships as assets, the consumer problem is similar to the maximization problem in Eq. (A7) above, but trade between economies need not be balanced in each period. In this case, the resource constraint, Eq. (A7c), would also include $NX_d \neq 0$.

A.1.3 The matching function

The rate at which retailers in country d contact producers in country o, $\chi(\kappa_{do})$, is the number of matches formed each instant over the number of searching retailers:

$$\chi(\kappa_{do}) = \frac{m(u_{do}N_o^x, v_{do}N_d^m)}{v_{do}N_d^m} = \frac{\xi(u_{do}N_o^x)^\eta (v_{do}N_d^m)^{1-\eta}}{v_{do}N_d^m} = \xi\kappa_{do}^{-\eta}. \tag{A11}$$

Notice that retailers' contact rate falls with market tightness ($d\chi(\kappa_{do})/d\kappa_{do} < 0$) because with more retailers relative to producers, the search market becomes congested with retailers.

The rate at which producers in country o contact retailers in country d is the number of matches formed each instant over the number of searching producers, so that the producer contact rate is,

$$\kappa_{do}\chi(\kappa_{do}) = \xi\kappa_{do}^{1-\eta}. \tag{A12}$$

Producers' contact rate rises with tightness ($d\kappa_{do}\chi(\kappa_{do})/d\kappa_{do} > 0$), also called a market thickness effect. Market tightness is defined from the perspective of producers so that the market is tighter when there are relatively more retailers than producers. Eqs. (A11) and (A12) are restated in Eq. (4) of the main text.

A.1.4 Producers' value functions

The value of a producer with productivity φ being matched to a retailer, $X_{do}(\varphi)$, can be summarized by a value function in continuous time defined in Eq. (6). That asset equation states that the flow return at the risk-free rate, r, from the value of producing must equal the flow payoff plus the expected capital gain from operating as an exporting producer. Each producer is indexed by exogenous productivity, φ. The flow payoff consists of $n_{do}q_{do}$, the revenue obtained from selling q_{do} units of the good at negotiated price n_{do} to retailers, less the variable, Eq. (5), and fixed costs of production, $w_o f_{do}$. The last term in Eq. (6) is the value from the dissolution of the match, which occurs at exogenous rate λ and leads to a capital loss of $U_{do}(\varphi) - X_{do}(\varphi)$ as the producer loses value $X_{do}(\varphi)$ but gains the value of being an unmatched producer, $U_{do}(\varphi)$. In writing Eq. (6), we explicitly write the value $X_{do}(\varphi)$ as a function of the producer's productivity, φ, but we conserve on notation by omitting this argument from the negotiated price, n_{do}, and traded quantity, q_{do}.

The value that an unmatched producer receives from looking for a retail partner without being in a business relationship is defined by Eq. (7). The flow search cost, $w_o l_{do}$, is what the producer pays when looking for a retailer. Examples of which are the costs of maintaining foreign sales offices, sending sales representatives abroad, researching potential foreign buyers, and paying for a web presence. The second term captures the expected capital gain, in which $\kappa_{do}\chi(\kappa_{do})$ is the endogenous rate at which producing firms contact retailers, and $w_o s_{do}$ is the sunk cost of starting up the relationship. The producer considers the difference between being in a business relationship, $X_{do}(\varphi)$, and searching,

$U_{do}(\varphi)$, rather than these quantities separately. As such, any additive term that enters both Eqs. (6) and (7) will not affect producers' decisions.

The producing firm also has the option of remaining idle and not expending resources to look for a retailer. For producers, the value of not searching, $I_{do}(\varphi)$, is given by Eq. (8). The value to a producer of remaining idle can be interpreted, for example, as the value of the stream of payments after liquidation or the flow payoff from home production if these firms are viewed as entrepreneurs.

A.1.5 Retailers' value functions

The value of a retailing firm in a business relationship with a producer of productivity φ, is defined by the asset Eq. (9) The flow payoff from being in a relationship is the revenue generated by selling q_{do} units of the product to a representative consumer at a final sales price, p_{do}—determined in Appendix A.2.2—less the tariff inclusive cost of acquiring these goods from producers at negotiated price n_{do}. As stated in the main text, tariff revenue is collected from the retailers by the government and rebated lump-sum to consumers. When the relationship is destroyed exogenously, at rate λ, the retailing firm loses the capital value of being matched. All retailers are identical before matching but have differential matched values because producers are heterogeneous in their productivity.

Retailers do not use the product as an input in another stage of production but only facilitate the match between producers and consumers and collect tariffs that are paid to the government. In the event that the relationship undergoes an exogenous separation, at rate λ, the retailing firm loses the capital value of being matched, $V_{do} - M_{do}(\varphi)$.

The value of being an unmatched retailer, V_{do}, satisfies Eq. (10). Retailers need to pay a flow cost, $w_d c_{do}$, to search for a producing affiliate. At endogenous Poisson rate $\chi(\kappa_{do})$, retailing firms meet a producer of unknown productivity. Producers' productivities are ex-ante unknown to retailers so retailers take the expectation over all productivities they might encounter when computing the expected continuation value of searching. As a result, the value, V_{do}, is not a function of a producer's productivity, φ, but rather a function of the expected payoff. We assume that upon meeting, but before consummating a match, retailers learn the productivity of the producer. Depending on the producer's productivity, φ, retailers choose between matching with that producer, which generates value $M_{do}(\varphi)$, or continuing the search, which generates V_{do}. Hence, the capital gain to retailers from meeting a producer with productivity φ can be expressed as $\max\{V_{do}, M_{do}(\varphi)\} - V_{do}$.

A.2 Solving the partial-equilibrium search problem

A.2.1 The surplus, value, and expected duration of a relationship

To derive the surplus in terms of model primitives, substitute Eqs. (6), (7), (9), and free entry for retailers, $V_{do} = 0$, into Eq. (11) to write the surplus as,

$$\left(r + \lambda + \frac{\beta \kappa_{do} \chi(\kappa_{do})}{\beta + t_{do}(1-\beta)}\right) S_{do}(\varphi) = p_{do} q_{do} + n_{do} q_{do}(1 - t_{do}) - v(q_{do}, w_o, \tau_{do}, \varphi) - w_o \delta_{do}, \tag{A13}$$

in which we use Eq. (4) to get $\kappa_{do}\chi(\kappa_{do}) = \xi \kappa_{do}^{1-\eta}$ and we define,

$$\delta_{do} \equiv \delta_{do}(\xi, \eta, f_{do}, l_{do}, s_{do}, \kappa_{do}) = f_{do} - l_{do} - \xi \kappa_{do}^{1-\eta} s_{do}. \tag{A14}$$

Now substitute the negotiated price from Eq. (14) into Eq. (A13) and Eq. use (15) written as

$$\gamma_{do} = \frac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\xi\kappa_{do}^{1-\eta}} \tag{A15}$$

to write surplus as,

$$S_{do}(\varphi) = \left(\frac{\beta + t_{do}(1-\beta)}{r+\lambda+\beta\kappa_{do}\chi(\kappa_{do})}\right)\left(\frac{p_{do}q_{do}}{t_{do}} - v(q_{do}, w_o, \tau_{do}, \varphi) - w_o\delta_{do}\right). \tag{A16}$$

The surplus created by a match is the appropriately discounted after-tariff flow profit, with the search cost $w_o l_{do}$ and the sunk cost $w_o s_{do}$ also entering the surplus equation (through δ_{do}) because being matched avoids paying these costs.

There are four things to notice about Eq. (A16). First, if $t_{do} = w_o = 1$, it becomes the surplus in Appendix A.3 Eq. (A33) of KM. Second, the surplus from a match is a function of productivity. We show in Appendix A.2.4.3 that matches that include a more productive exporting firm lead to greater surplus, that is, $S'_{do}(\varphi) > 0$. Third, the value of a relationship depends on aggregate endogenous quantities such as the price index, consumption, and finding rate $\kappa_{do}\chi(\kappa_{do})$, among others. Finally, surplus is greater than or equal to zero if and only if after-tariff total profits are. That is, when

$$\frac{p_{do}q_{do}}{t_{do}} - v(q_{do}, w_o, \tau_{do}, \varphi) - w_o f_{do} + w_o l_{do} + w_o s_{do}\kappa_{do}\chi(\kappa_{do}) \geq 0. \tag{A17}$$

The value of the relationship to the producer is $X_{do}(\varphi)$ and to the retailer $M_{do}(\varphi)$. Therefore, the total value of a matched relationship is,

$$R_{do}(\varphi) = X_{do}(\varphi) + M_{do}(\varphi). \tag{A18}$$

We can express Eq. (A18) in terms of surplus and then Eq. (11) with $V_{do} = 0$, by adding and subtracting Eq. (7), substituting in Eq. (13) for $X_{do}(\varphi) - U_{do}(\varphi)$ and then simplifing to get

$$R_{do}(\varphi) = \left[\frac{r(\beta + t_{do}(1-\beta)) + \beta\kappa_{do}\chi(\kappa_{do})}{r(\beta + t_{do}(1-\beta))}\right] S_{do}(\varphi) - \left[\frac{w_o l_{do} + \kappa_{do}\chi(\kappa_{do}) w_o s_{do}}{r}\right]. \tag{A19}$$

Eq. (A19) can be expressed in terms of model primitives using (A16) and the definitions for those functions provided in Eqs. (12), (5), and (2). Relationships are destroyed at Poisson rate λ in the model, which implies the average duration of each match is $1/\lambda$. Because the destruction rate is exogenous and does not vary in our model, the average duration of each match is constant. The value of a relationship in product markets has been of recent interest in Monarch and Schmidt-Eisenlohr (2023) and Heise (2016). Finally, Eq. (A19) is the same as the $R_{do}(\varphi)$ equation on page 7 of Appendix A.3 of KM when $t_{do} = 1$ and $w_o s_{do} = 0$.

A.2.2 Bargaining over the quantity

Upon meeting, the retailer and producer bargain over the negotiated price, n_{do}, and quantity, q_{do}, simultaneously. We assume that these objects are determined by the generalized Nash bargaining solution, which, as shown by Nash (1950) and Osborne and

Rubinstein (1990), is equivalent to maximizing the following Nash product:

$$\max_{q_{do},n_{do}} \left[X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right)\right]^{\beta} \left[M_{do}\left(\varphi\right) - V_{do}\right]^{1-\beta}, 0 \leq \beta < 1, \tag{A20}$$

in which β is producers' bargaining power. To solve Eq. (A20), first solve for $X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right)$ by combining Eqs. (6) and (7) to get that:

$$X_{do}(\varphi) - U_{do}(\varphi) = \frac{n_{do}q_{do} - v\left(q_{do}, w_o, \tau_{do}, \varphi\right) - w_o f_{do} + w_o l_{do} + \kappa_{do}\chi(\kappa_{do})w_o s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})}. \tag{A21}$$

Next rearrange Eq. (9) to get that:

$$M_{do}\left(\varphi\right) - V_{do} = \frac{p_{do}\left(q_{do}\right)q_{do} - t_{do}n_{do}q_{do} - rV_{do}}{r + \lambda}. \tag{A22}$$

Substitute Eqs. (A21) and (A22) into (A20), then log and differentiate with respect to q_{do} to get the relevant first order condition for quantity:

$$\beta \frac{\left(n_{do} - \dfrac{\partial v\left(q_{do}, w_o, \tau_{do}, \varphi\right)}{\partial q_{do}}\right)}{\left[X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right)\right]\left(r + \lambda\right)} + (1 - \beta)\frac{\left(p_{do}\left(q_{do}\right) + \dfrac{\partial p_{do}\left(q_{do}\right)}{\partial q_{do}}q_{do} - t_{do}n_{do}\right)}{\left[M_{do}\left(\varphi\right) - V_{do}\right]\left(r + \lambda\right)} = 0. \tag{A23}$$

We do not need to calculate the partial derivative with respect to κ_{do}, w_o, or other endogenous variables, because we assume individual varieties are too small to influence aggregate values. Hence, when they meet, the firms bargain taking everything but n_{do} and q_{do} as given. Furthermore, the partial of the value of a vacancy, $\partial V_{do}/\partial n_{do} = 0$, because bargaining takes place over each variety, φ, individually. As long as the distribution of varieties is continuous, $\partial V_{do}/\partial n_{do}$ does not have an effect on the expectation in the continuation value in Eq. (10).

Considering only solutions with positive values for Eq. (A22) and $q_{do} > 0$, we plug Eq. (A28) into (A23) rearrange to get:

$$p_{do}\left(q_{do}\right) + \frac{\partial p_{do}\left(q_{do}\right)}{\partial q_{do}}q_{do} = t_{do}\frac{\partial v\left(q_{do}, w_o, \tau_{do}, \varphi\right)}{\partial q_{do}}. \tag{A24}$$

This expression says that the quantity produced and traded equates marginal revenue earned from consumers to tariff-inclusive marginal production cost paid by producers. Eq. (A24) is the same in a model with or without search frictions implying that search does not change the quantity traded within each match. In a model of search, parties agree upon a quantity that equates marginal revenue and marginal tariff-inclusive cost because that quantity maximizes surplus.

CES utility implies the consumer's price elasticity of demand from Eq. (2) is

$$\frac{\partial q_{do}}{\partial p_{do}}\frac{p_{do}}{q_{do}} = -\sigma. \tag{A25}$$

Indexing an individual variety by ω is equivalent to indexing by φ and we have treated these interchangeably when using Eq. (2) here. The equivalence of indexing variables contrasts with changing from a measure over a set of goods indexed by ω to a distribution of goods indexed by φ, which is subtle and discussed in detail in KM Appendix A.11.1.

Combining Eq. (A25) with the fact that $\partial p_{do}/\partial q_{do} = 1/\left(\partial q_{do}/\partial p_{do}\right)$, we can write Eq. (A24) as

$$p_{do}\left(q_{do}\right) + \frac{\partial p_{do}\left(q_{do}\right)}{\partial q_{do}} q_{do} = p_{do}\left(\frac{\sigma - 1}{\sigma}\right) = t_{do}\frac{\partial v\left(q_{do}, w_o, \tau_{do}, \varphi\right)}{\partial q_{do}}. \tag{A26}$$

Rearranging Eq. (A26) and computing marginal costs from Eq. (5) gives Eq. (12).

Finally, setting price equal to average total cost (ATC) gives zero profit for any variety. As such, Eq. (12) is always at least as high as ATC for all traded varieties above the threshold defined in Eq. (18) and therefore defines the equilibrium price.

A.2.3 Bargaining over the negotiated price

Substitute Eqs. (A21) and (A22) into (A20), then log and differentiate with respect to the n_{do} to get the relevant first order condition:

$$\beta\frac{q_{do}/\left(r+\lambda\right)}{X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right)} + \left(1-\beta\right)\frac{-t_{do}q_{do}/\left(r+\lambda\right)}{M_{do}\left(\varphi\right) - V_{do}} = 0, \tag{A27}$$

in which we use the same reasoning to impose $\partial V_{do}/\partial n_{do} = 0$ as in Appendix A.2.2.

For any variety that is traded, $q_{do} > 0$, Eq. (A27) can be written as,

$$\beta\left(M_{do}\left(\varphi\right) - V_{do}\right) = \left(1-\beta\right)t_{do}\left(X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right)\right). \tag{A28}$$

Using Eqs. (A28) and (11) delivers the surplus sharing rule, Eq. (13). To find the negotiated price show in Eq. (14), use the equilibrium free entry condition $V_{do} = 0$ and substitute (A21) and (A22) into (A28), then solve for n_{do}.

A.2.4 Producers' search productivity threshold

There are two productivity thresholds to consider. First, there is a productivity threshold, $\bar{\varphi}_{do}$, that makes the producer indifferent between searching and remaining idle defined by, $U_{do}\left(\bar{\varphi}_{do}\right) - I_{do}\left(\bar{\varphi}_{do}\right) = 0$. Second, there is a weakly lower productivity threshold, $\underline{\varphi}_{do}$, which makes that producer indifferent between consummating a relationship upon contacting a retailer and continuing to search defined by, $X_{do}\left(\underline{\varphi}_{do}\right) - U_{do}\left(\underline{\varphi}_{do}\right) = 0$. We derive these two thresholds and show in Appendix A.2.4.2 that the binding threshold is defined by $\bar{\varphi}_{do}$, because $\bar{\varphi}_{do} \geq \underline{\varphi}_{do}$ if and only if $w_o l_{do} + w_o h_{do} + \kappa_{do}\chi\left(\kappa_{do}\right)w_o s_{do} \geq 0$.

The productivity threshold nests the threshold from KM (Eq. 18) and we compare it to productivity thresholds in other models in Appendix A.6.4 of the same paper.

A.2.4.1 Solving for the binding productivity threshold

Combine Eqs. (7) and (8) to get

$$U_{do}\left(\varphi_{do}\right) - I_{do}\left(\varphi_{do}\right) = \frac{-w_o l_{do} + \kappa_{do}\chi\left(\kappa_{do}\right)\left(X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right) - w_o s_{do}\right) - w_o h_{do}}{r} \tag{A29}$$

The threshold productivity, $\bar{\varphi}_{do}$, is given by $U_{do}(\bar{\varphi}) - I_{do}(\bar{\varphi}_{do}) = 0$ so evaluate Eq. (A29) at $\bar{\varphi}_{do}$, set the left hand side to zero, and rearrange to get

$$X_{do}(\bar{\varphi}_{do}) - U_{do}(\bar{\varphi}_{do}) = \frac{w_o l_{do} + w_o h_{do}}{\kappa_{do}\chi(\kappa_{do})} + w_o s_{do}. \tag{A30}$$

Substitute Eq. (A21) into Eq. (A30) and suppress $\bar{\varphi}_{do}$ for simplicity to derive

$$n_{do}q_{do} - v(q_{do}, w_o, \tau_{do}, \varphi) - w_o f_{do} = \left(\frac{r+\lambda}{\kappa_{do}\chi(\kappa_{do})}\right) w_o l_{do} + \left(1 + \frac{r+\lambda}{\kappa_{do}\chi(\kappa_{do})}\right) w_o h_{do} + (r+\lambda) w_o s_{do}.$$

Now, use the negotiated price, n_{do}, from Eq. (14), to get

$$\begin{aligned}(1-\gamma_{do})\left(\frac{p_{do}}{t_{do}}\right) q_{do} + \gamma_{do}\left(v(q_{do}, w_o, \tau_{do}, \varphi) + w_o f_{do} - w_o l_{do} - \kappa_{do}\chi(\kappa_{do}) w_o s_{do}\right) - v(q_{do}, w_o, \tau_{do}, \varphi) - w_o f_{do} \\ = \left(\frac{r+\lambda}{\kappa_{do}\chi(\kappa_{do})}\right) w_o l_{do} + \left(1 + \frac{r+\lambda}{\kappa_{do}\chi(\kappa_{do})}\right) w_o h_{do} + (r+\lambda) w_o s_{do}.\end{aligned}$$

which can be rearranged to obtain

$$\begin{aligned}&\left(\frac{p_{do}}{t_{do}}\right) q_{do} - v(q_{do}, w_o, \tau_{do}, \varphi) - w_o f_{do} \\ &= (1-\gamma_{do})^{-1}\left[\left(\frac{r+\lambda}{\kappa_{do}\chi(\kappa_{do})}\right) w_o l_{do} + \left(1 + \frac{r+\lambda}{\kappa_{do}\chi(\kappa_{do})}\right) w_o h_{do} + (r+\lambda) w_o s_{do} + \gamma_{do}\left[w_o l_{do} + \kappa_{do}\chi(\kappa_{do}) w_o s_{do}\right]\right].\end{aligned}$$

Further simplification of the terms on the right hand side with γ_{do} delivers Eq. (16) in the main text.

Using the price charged to consumers by retailers from Eq. (12) we can write retailer revenue as proportional to variable production costs, Eq. (5):

$$p_{do}(\varphi) q_{do}(\varphi) = \left(t_{do}\mu w_o \tau_{do}\varphi^{-1}\right) q_{do}(\varphi) = t_{do}\mu v(q_{do}, w_o, \tau_{do}, \varphi). \tag{A31}$$

Then Eq. (A31) implies that after-tariff variable profits are,

$$\left(\frac{p_{do}(\varphi_{do})}{t_{do}}\right) q_{do}(\varphi_{do}) - v(q_{do}, w_o, \tau_{do}, \varphi_{do}) = \frac{p_{do}(\varphi) q_{do}(\varphi)}{\sigma t_{do}}. \tag{A32}$$

Or alternatively,

$$\left(\frac{p_{do}(\varphi_{do})}{t_{do}}\right) q_{do}(\varphi_{do}) - v(q_{do}, w_o, \tau_{do}, \varphi_{do}) = (\mu - 1) v(q_{do}, w_o, \tau_{do}, \varphi). \tag{A33}$$

Substitute Eqs. (12) and (2) into Eq. (A32) and then substitute the resulting expression into the left hand side of Eq. (16) to get that:

$$\frac{\alpha}{\sigma} C_d P_d^{\sigma-1} (\mu w_o \tau_{do})^{1-\sigma} t_{do}^{-\sigma} \bar{\varphi}_{do}^{\sigma-1} = w_o F(\kappa_{do}). \tag{A34}$$

Solving this expression for $\bar{\varphi}_{do}$ gives Eq. (18).

Finally, all matches must have positive surplus so we can check that $S_{do}(\bar{\varphi}_{do}) \geq 0$ by using Eqs. (13) and (A30) to write,

$$S_{do}\left(\bar{\varphi}_{do}\right)=\left(\frac{\beta+t_{do}\left(1-\beta\right)}{\beta}\right)\left(\frac{w_{o}l_{do}+w_{o}h_{do}}{\kappa_{do}\chi\left(\kappa_{do}\right)}+w_{o}s_{do}\right). \tag{A35}$$

Eq. (A35) puts restrictions on the parameters because they must be such that $S_{do}\left(\bar{\varphi}_{do}\right)\geq 0$. For example, $w_o h_{do}$ cannot be so negative as to make Eq. (A35) negative.

A.2.4.2 Solving for the non-binding productivity threshold

The threshold productivity that is indifferent between matching and not, $\underline{\varphi}_{do}$, is defined by

$$X_{do}\left(\underline{\varphi}_{do}\right)-U_{do}\left(\underline{\varphi}_{do}\right)=0. \tag{A36}$$

We can be sure that $X_{do}\left(\bar{\varphi}_{do}\right)-U_{do}\left(\bar{\varphi}_{do}\right)\geq X_{do}\left(\underline{\varphi}_{do}\right)-U_{do}\left(\underline{\varphi}_{do}\right)$ as long as $\left(w_o l_{do}+w_o h_{do}\right)/\kappa_{do}\chi\left(\kappa_{do}\right)+w_o s_{do}\geq 0$. This result implies that as long as $X_{do}\left(\varphi\right)-U_{do}\left(\varphi\right)$ is increasing in φ, then $\bar{\varphi}_{do}\geq\underline{\varphi}_{do}$. In Appendix A.2.4.3, we show the very general conditions under which $X_{do}\left(\varphi\right)-U_{do}\left(\varphi\right)$ is increasing in φ. The binding productivity threshold defining the mass of producers that have retail partners is the greater of these two thresholds and hence $\bar{\varphi}_{do}$. In other words, the productivity necessary to induce a producer to search for a retail partner is greater than the productivity necessary to consummate a match after meeting a retailer due to the costs that are incurred while searching. Similarly, the productivity necessary to form a match is greater than the productivity to maintain one already in place. Note that $\bar{\varphi}_{do}>\underline{\varphi}_{do}$ if $\left(w_o l_{do}+w_o h_{do}\right)/\kappa_{do}\chi\left(\kappa_{do}\right)+w_o s_{do}>0$, which is true if and only if the cost of forming a relationship is positive, $w_o l_{do}+w_o h_{do}+\kappa_{do}\chi\left(\kappa_{do}\right)w_o s_{do}>0$.

A.2.4.3 The value of importing is strictly increasing in productivity

Here we show that the value of importing, $M_{do}(\varphi)$, is strictly increasing with the producer's productivity level, φ. This result leads to three implications. First, it allows us to replace the integral of the max over V_{do} and $M_{do}\left(\varphi\right)$ in Eq. (10) with the integral of $M_{do}\left(\varphi\right)$ from the threshold from Eq. (18). Second, in equilibrium, because $M'_{do}(\varphi)>0$, Eq. (13) implies that $S'_{do}\left(\varphi\right)>0$ and therefore that $X'_{do}\left(\varphi\right)-U'_{do}\left(\varphi\right)>0$. Third, it allows us to show that $\bar{\varphi}_{do}\geq\underline{\varphi}_{do}$, as we did in Appendix A.2.4.2.

Starting with Eq. (9) and $V_{do}=0$, substituting in negotiated prices from Eq. (14), and using the relationship between retailer revenue and variable costs from Eq. (A31) we can write,

$$M_{do}\left(\varphi\right)=\frac{\sigma^{-1}\gamma_{do}p_{do}\left(\varphi\right)q_{do}\left(\varphi\right)-t_{do}\gamma_{do}w_o\delta_{do}}{r+\lambda} \tag{A37}$$

Remember that δ_{do} from Eq. (A14) and γ_{do} from Eq. (A15) are functions of tightness, κ_{do}, but not productivity, φ. It is clear from the integral in the import relationship creation, Eq. (19), that κ_{do} is not a function of φ. Similarly, w_o is not a function of φ (Eq. 29). Given these facts, we can prove our result by differentiating both sides of Eq. (A37) with respect to φ and showing that $M'_{do}\left(\varphi\right)=\left(\partial M_{do}\left(\varphi\right)/\partial q_{do}\left(\varphi\right)\right)\cdot\left(\partial q_{do}\left(\varphi\right)/\partial\varphi\right)>0$. Using demand from Eq. (2), first write inverse demand $p_{do}\left(q_{do}\left(\varphi\right)\right)$ then

$$M'_{do}\left(\varphi\right)=\frac{\sigma^{-1}\gamma_{do}}{r+\lambda}\left(p_{do}\left(q_{do}\left(\varphi\right)\right)+\frac{\partial p_{do}\left(\varphi\right)}{\partial q_{do}\left(\varphi\right)}q_{do}\right)\frac{\partial q_{do}\left(\varphi\right)}{\partial\varphi}. \tag{A38}$$

The partial derivative in parentheses is marginal revenue, which we know in equilibrium will be equal to marginal cost times the tariff as shown in Eq. (A24). Using this fact and applying the chain rule to $\partial q_{do}(\varphi)/\partial\varphi$ leads to our final expression,

$$M'_{do}(\varphi) = \frac{\sigma^{-1}\gamma_{do}}{r+\lambda}\left(t_{do}\frac{\partial v\left(q_{do}, w_o, \tau_{do}, \varphi\right)}{\partial q_{do}(\varphi)}\right)\frac{\partial q_{do}(\varphi)}{\partial p_{do}(\varphi)}\frac{\partial p_{do}(\varphi)}{\partial\varphi}. \tag{A39}$$

As long as $\gamma_{do} > 0$ (which holds for finite κ_{do} and $\beta < 1$), marginal cost is positive, demand is downward sloping, and higher productivity varieties cost less, then $M'_{do}(\varphi) > 0$. These general conditions are satisfied for the functional forms of our model.

We can use the fact that $M'_{do}(\varphi) > 0$ to demonstrate the way in which many other important quantities depend on the producer's productivity level, φ. The surplus sharing rule, Eq. (13), can be rewritten as

$$\beta M_{do}(\varphi) = (1-\beta)\, t_{do}\left(X_{do}(\varphi) - U_{do}(\varphi)\right), \tag{A40}$$

We know that in equilibrium, because $M'_{do}(\varphi) > 0$, it must be that $X'_{do}(\varphi) - U'_{do}(\varphi) > 0$. Differentiating both sides of Eq. (7) gives $rU'_{do}(\varphi) = \kappa_{do}\chi(\kappa_{do})\left(X'_{do}(\varphi) - U'_{do}(\varphi)\right) > 0$. We can combine these facts to show $X'_{do}(\varphi) > U'_{do}(\varphi) > 0$. Using the definition of the joint surplus of a match, Eq. (11), we get $S'_{do}(\varphi) > 0$. Likewise, the value of a relationship, $R_{do}(\varphi) = X_{do}(\varphi) + M_{do}(\varphi)$, has $R'_{do}(\varphi) > 0$.

A.3 Aggregation

A.3.1 The elasticity of the fraction of unmatched producers

Many aggregate results in our paper rely on the elasticity of Eq. (20) with respect to some variable, call it x. Using Eq. (20) and the functional form in Eq. (A12) gives

$$\frac{\partial}{\partial\ln(x)}\ln\left(1-\frac{u_{do}}{1-i_{do}}\right) = \left(\frac{u_{do}}{1-i_{do}}\right)(1-\eta)\frac{\partial\ln\kappa_{do}}{\partial\ln(x)}, \tag{A41}$$

by the chain rule and the fact that

$$\frac{\partial\ln\kappa_{do}\chi\left(\kappa_{do}\right)}{\partial\ln\kappa_{do}} = 1-\eta.$$

A.3.2 The ideal price index with our productivity distribution

The ideal price index is provided in Eq. (A4) and is a function of the homogeneous good price and the differentiated goods price index price index in Eq. (A3), which indexes over an unordered set of varieties. We can move from an unordered set of varieties to an index over a distribution of productivities using the steps in Appendix A.11.1 of KM so that the differentiated goods price index is given by:

$$P_d = \left[\sum_{k=1}^{O} P_{dk}^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = \left[\sum_{k=1}^{O}\left(1-\frac{u_{dk}}{1-i_{dk}}\right)N_k^x\int_{\bar{\varphi}_{dk}}^{\infty} p_{dk}(\varphi)^{1-\sigma}\,dG(\varphi)\right]^{\frac{1}{1-\sigma}}, \tag{A42}$$

in which $G(\cdot)$ is a Pareto cumulative density function from Section 2.1.3. Using the final consumer price from Eq. (12) gives

$$P_d = \left[\sum_{k=1}^{O} \left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}}^{\infty} \left(\frac{t_{dk}\mu w_k \tau_{dk}}{\varphi}\right)^{1-\sigma} dG(\varphi) \right]^{\frac{1}{1-\sigma}}, \tag{A43}$$

in which

$$P_{dk} = \left[\left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}}^{\infty} \left(\frac{t_{dk}\mu w_k \tau_{dk}}{\varphi}\right)^{1-\sigma} dG(\varphi) \right]^{\frac{1}{1-\sigma}}. \tag{A44}$$

The relevant moment is,

$$\int_{\bar{\varphi}_{dk}}^{\infty} z^{\sigma-1} dG(z) = \frac{\theta \bar{\varphi}_{dk}^{\sigma-\theta-1}}{\theta - \sigma + 1}, \tag{A45}$$

which is bounded because we assume that $\theta > \sigma - 1$. The threshold from Eq. (18) raised to $\sigma - 1$ is

$$\bar{\varphi}_{do}^{\sigma-1} = \mu^{\sigma-1} \left(\frac{\sigma}{\alpha}\right) \left(\frac{w_o \tau_{do}}{P_d}\right)^{\sigma-1} \left(\frac{w_o F(\kappa_{do})}{C_d}\right) t_{do}^{\mu(\sigma-1)}. \tag{A46}$$

The threshold from Eq. (18) raised to $-\theta$ is

$$\bar{\varphi}_{do}^{-\theta} = \mu^{-\theta} \left(\frac{\sigma}{\alpha}\right)^{\frac{-\theta}{\sigma-1}} \left(\frac{w_o \tau_{do}}{P_d}\right)^{-\theta} \left(\frac{w_o F(\kappa_{do})}{C_d}\right)^{\frac{-\theta}{\sigma-1}} t_{do}^{-\mu\theta}. \tag{A47}$$

The product of Eqs. (A46) and (A47) give the relevant moment,

$$\bar{\varphi}_{do}^{\sigma-1-\theta} = P_d^{\theta-(\sigma-1)} \mu^{\sigma-1-\theta} \left(\frac{\sigma}{\alpha}\right)^{1-\frac{\theta}{\sigma-1}} (w_o \tau_{do})^{\sigma-1-\theta} \left(\frac{w_o F_{do}}{C_d}\right)^{1-\frac{\theta}{\sigma-1}} t_{do}^{\sigma-\mu\theta}. \tag{A48}$$

Because the threshold is a function of P_d, Eq. (A43) is itself a function of P_d too. Using Eq. (A48) in Eq. (A43) and simplifying gives,

$$\begin{aligned} P_d &= P_d^{1-\frac{\theta}{\sigma-1}} \\ &\times \left[\left(\frac{\theta}{\theta - (\sigma - 1)}\right) \left(\frac{\sigma}{\alpha}\right)^{1-\frac{\theta}{\sigma-1}} \mu^{-\theta} C_d^{\frac{\theta}{\sigma-1}-1} \sum_{k=1}^{O} \left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x (w_k \tau_{dk})^{-\theta} (w_k F_{dk})^{1-\frac{\theta}{\sigma-1}} t_{dk}^{1-\mu\theta} \right]^{\frac{1}{1-\sigma}}. \end{aligned} \tag{A49}$$

Solving for P_d and rearranging gives

$$\begin{aligned} P_d &= \left(\frac{\theta}{\theta - (\sigma - 1)}\right)^{-\frac{1}{\theta}} \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1} - \frac{1}{\theta}} \mu \\ &\times C_d^{\frac{1}{\theta} - \frac{1}{\sigma-1}} \\ &\times \left(\sum_{k=1}^{O} \left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x (w_k \tau_{dk})^{-\theta} (w_k F_{dk})^{-\left[\frac{\theta}{\sigma-1} - 1\right]} t_{dk}^{-(\mu\theta-1)} \right)^{-\frac{1}{\theta}}. \end{aligned} \tag{A50}$$

We define,

$$\lambda_2 \equiv \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}} \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}} \mu, \tag{A51}$$

and the “multilateral resistance term” as,

$$\rho_d \equiv \left(\sum_{k=1}^{O} \left(1-\frac{u_{dk}}{1-i_{dk}}\right) N_k^x \left(w_k \tau_{dk}\right)^{-\theta} \left(w_k F_{dk}\right)^{-\left[\frac{\theta}{\sigma-1}-1\right]} t_{dk}^{-(\mu\theta-1)}\right)^{-\frac{1}{\theta}}. \tag{A52}$$

These definitions deliver the final expression of the differentiated goods price index presented in Eq. (22).

The price index in our model closely resembles the price index in Chaney (2008, Eq. 8). In that model, the price index is an equilibrium object in wages, GDP, iceberg and fixed entry costs, whereas in our model it is an equilibrium object in wages, the number of producers, market tightness (through u_{dk} and F_{dk}), iceberg and fixed entry costs, and also tariffs.

A.3.3 Deriving the gravity equation

The total amount paid by the consumers in d for imports from o, C_{do}, is the sum of the value of imports, the profits to retailers, and tariffs paid to the government:

$$C_{do} = IM_{do} + \Pi_{do}^r + G_{do},$$

in which C_{do} is defined in Eq. (A76), Π_{do}^r is defined in Eq. (A78), and

$$G_{do} = \left(1-\frac{u_{do}}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} \left(t_{do}-1\right) n_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi). \tag{A53}$$

Rearranging gives

$$IM_{do} = C_{do} - \Pi_{do}^r - G_{do},$$

and then using Eqs. (A76), (A78), and (A53) proves that the value of total imports is:

$$IM_{do} = \left(1-\frac{u_{do}}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} n_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi), \tag{A54}$$

which is the same definition as Proposition 1 in KM. With imports defined in (A54), government revenue from each do market can be written as

$$G_{do} = \left(t_{do}-1\right) IM_{do} \tag{A55}$$

and total government revenue in d is $G_d = \sum_{k=1}^{D} G_{dk}$.

We need to integrate Eq. (A54) over product varieties to get the total value of imports going into the domestic market, before tariffs are applied. Using the negotiated price in Eq. (14), the value of each import variety is:

$$n_{do}(\varphi)\, q_{do}(\varphi) = \left[1-\gamma_{do}\right] \left(\frac{p_{do}(\varphi)\, q_{do}(\varphi)}{t_{do}}\right) + \gamma_{do} \left[v\left(q_{do}, w_o, \tau_{do}, \varphi\right) + w_o \delta_{do}\right],$$

in which δ_{do} is defined in Eq. (A14) and is not a function of φ but is potentially a function of κ_{do}. Using Eq. (A31) and simplifying, we can write the imported value of each variety as

$$n_{do}(\varphi)\, q_{do}(\varphi) = \left(\frac{\sigma - \gamma_{do}}{\sigma t_{do}}\right) p_{do}(\varphi)\, q_{do}(\varphi) + \gamma_{do} w_o \delta_{do}. \tag{A56}$$

Demand for a variety in the differentiated goods sector is given in Eq. (2). The optimal final price from Eq. (12) implies that

$$p_{do}(\varphi)\, q_{do}(\varphi) = \alpha C_d \frac{(t_{do}\mu w_o \tau_{do})^{1-\sigma} \varphi^{\sigma-1}}{P_d^{1-\sigma}}.$$

Substituting this into Eq. (A56) shows that the integral defining imports in Eq. (A54) depends on the moment of the productivity distribution from Eq. (A45). Substituting this moment into Eq. (A54) and then simplifying gives

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \left[\left(\frac{\sigma - \gamma_{do}}{\sigma t_{do}}\right) \alpha C_d \frac{(t_{do}\mu w_o \tau_{do})^{1-\sigma}}{P_d^{1-\sigma}} \left(\frac{\theta \bar{\varphi}_{do}^{\sigma-1}}{\theta - \sigma + 1}\right) + \gamma_{do} w_o \delta_{do}\right] \bar{\varphi}_{do}^{-\theta}. \tag{A57}$$

We can use $\bar{\varphi}_{do}^{\sigma-1}$ from Eq. (A46) to write

$$\alpha C_d \frac{(t_{do}\mu w_o \tau_{do})^{1-\sigma}}{P_d^{1-\sigma}} \left(\frac{\theta \bar{\varphi}_{do}^{\sigma-1}}{\theta - \sigma + 1}\right) = \sigma t_{do} \left(\frac{\theta}{\theta - \sigma + 1}\right) w_o F(\kappa_{do}).$$

Substituting this into the relevant import equation provides

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \left[(\sigma - \gamma_{do}) \left(\frac{\theta}{\theta - \sigma + 1}\right) + \gamma_{do} \frac{w_o \delta_{do}}{w_o F(\kappa_{do})}\right] w_o F(\kappa_{do})\, \bar{\varphi}_{do}^{-\theta}.$$

Raising the price index from Eq. (22) to θ and substituting that into the threshold Eq. (18) raised to $-\theta$ defined in Eq. (A47) provides

$$\bar{\varphi}_{do}^{-\theta} = \left(\frac{\theta - (\sigma - 1)}{\theta \sigma}\right) \alpha C_d \left(\frac{w_o \tau_{do}}{\rho_d^{\theta}}\right)^{-\theta} (w_o F(\kappa_{do}))^{\frac{-\theta}{\sigma - 1}} t_{do}^{-\mu\theta}.$$

Substitute this expression into $\bar{\varphi}_{do}^{-\theta}$ in the imports equation to get

$$\begin{aligned} IM_{do} &= \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \left[(\sigma - \gamma_{do}) \left(\frac{\theta}{\theta - \sigma + 1}\right) + \gamma_{do} \frac{w_o \delta_{do}}{w_o F(\kappa_{do})}\right] \\ &\quad \times \left(\frac{\theta - (\sigma - 1)}{\theta \sigma}\right) \alpha N_o^x C_d \left(\frac{w_o \tau_{do}}{\rho_d^{\theta}}\right)^{-\theta} (w_o F(\kappa_{do}))^{1 - \frac{\theta}{\sigma - 1}} t_{do}^{-\mu\theta}. \end{aligned}$$

Part of this expression can be written as

$$\left[(\sigma - \gamma_{do}) \left(\frac{\theta}{\theta - \sigma + 1}\right) + \gamma_{do} \frac{w_o \delta_{do}}{w_o F(\kappa_{do})}\right] \left(\frac{\theta - (\sigma - 1)}{\theta \sigma}\right) = 1 - b_{do}$$

in which we use Eqs. (4), (17), (A14), and (A15) to write b_{do} as a function of all parameters and endogenous values,

$$b_{do} = \frac{\gamma_{do}\left(r,\lambda,\beta,\xi,\eta,\kappa_{do}\right)}{\sigma\theta} \times \left(\theta - \frac{w_o\delta_{do}\left(\xi,\eta,f_{do},l_{do},s_{do},\kappa_{do}\right)\left(\theta-\left(\sigma-1\right)\right)}{w_oF\left(r,\lambda,\beta,\xi,\eta,f_{do},l_{do},h_{do},s_{do},\kappa_{do}\right)}\right). \quad \text{(A58)}$$

Importantly, b_{do} is not a function of w_o because $\delta_{do}\left(w_o,\kappa_{do}\right)$ and $F\left(w_o,\kappa_{do}\right)$ are multiplicative in the wage so their ratio cancels out the wage. Eq. (A58) is the import markdown term that captures the aggregate difference between final and negotiated prices. It is the same as the expression in Appendix A.14.1 of KM. Substituting the markdown into the import expression completes the derivation of the gravity equation shown in Eq. (23) of the main text.

A.3.4 Tariff elasticity

A.3.4.1 Tariff elasticity with Pareto and selection

To derive the tariff elasticity in our model use Eq. (23) to form the ratio

$$\frac{IM_{do}}{IM_{dd}} = \frac{\left(1-\frac{u_{do}}{1-i_{do}}\right)\left(1-b_{do}\right)N_o^x\left(w_o\tau_{do}\right)^{-\theta}\left(w_oF_{do}\right)^{-\left(\frac{\theta}{\sigma-1}-1\right)}t_{do}^{-\mu\theta}}{\left(1-\frac{u_{dd}}{1-i_{dd}}\right)\left(1-b_{dd}\right)N_d^x\left(w_d\tau_{dd}\right)^{-\theta}\left(w_dF_{dd}\right)^{-\left(\frac{\theta}{\sigma-1}-1\right)}t_{dd}^{-\mu\theta}}, \quad \text{(A59)}$$

in which α, C_d, and ρ_d are both in the numerator and denominator and therefore cancel in the ratio. Denote the tariff elasticity for any importer d and any pair of exporters $o \neq d$ and $o' \neq d$ as

$$\frac{\partial \ln\left(IM_{do}/IM_{dd}\right)}{\partial \ln\left(t_{do'}\right)} = \zeta_d^{oo'}.$$

This elasticity describes how changing tariffs between destination country d and the origin country o changes relative imports by country d from country o. It also describes how changing tariffs between d and a third origin country o' changes relative imports by country d from country $o \neq o'$.

Take logs and differentiate Eq. (A59) with respect to $\ln\left(t_{do'}\right)$ and use the result from

Eq. (A41) plus definitions from Eqs. (18) and (A58) and that $\theta + \frac{\theta}{\sigma-1} = \mu\theta$ to get:

$$\zeta_d^{oo'} = \begin{cases} -\mu\theta + \left(\frac{u_{do}}{1-i_{do}}\right)(1-\eta)\frac{\partial \ln(\kappa_{do})}{\partial \ln(t_{do'})} - \left(\frac{u_{dd}}{1-i_{dd}}\right)(1-\eta)\frac{\partial \ln(\kappa_{dd})}{\partial \ln(t_{do'})} \\ +\frac{\partial \ln(1-b_{do}(\kappa_{do}))}{\partial \ln(t_{do'})} - \frac{\partial \ln(1-b_{dd}(\kappa_{dd}))}{\partial \ln(t_{do'})} \\ +\frac{\partial \ln(N_o^x/N_d^x)}{\partial \ln(t_{do'})} - (\mu\theta-1)\frac{\partial \ln(w_o/w_d)}{\partial \ln(t_{do'})} \\ -\left(\frac{\theta}{\sigma-1}-1\right)\frac{\partial \ln(F_{do}(\kappa_{do})/F_{dd}(\kappa_{dd}))}{\partial \ln(t_{do'})} & \text{if } o'=o, \\ \\ \left(\frac{u_{do}}{1-i_{do}}\right)(1-\eta)\frac{\partial \ln(\kappa_{do})}{\partial \ln(t_{do'})} - \left(\frac{u_{dd}}{1-i_{dd}}\right)(1-\eta)\frac{\partial \ln(\kappa_{dd})}{\partial \ln(t_{do'})} \\ +\frac{\partial \ln(1-b_{do}(\kappa_{do}))}{\partial \ln(t_{do'})} - \frac{\partial \ln(1-b_{dd}(\kappa_{dd}))}{\partial \ln(t_{do'})} \\ +\frac{\partial \ln(N_o^x/N_d^x)}{\partial \ln(t_{do'})} - (\mu\theta-1)\frac{\partial \ln(w_o/w_d)}{\partial \ln(t_{do'})} \\ -\left(\frac{\theta}{\sigma-1}-1\right)\frac{\partial \ln(F_{do}(\kappa_{do})/F_{dd}(\kappa_{dd}))}{\partial \ln(t_{do'})} & \text{if } o'\neq o. \end{cases} \tag{A60}$$

There are two interesting restrictions for Eq. (A60) that generate the same tariff elasticity. The first case is when search frictions exist but matching rates are exogenous (constant market tightness, $\kappa_{do}\ \forall do$). Because κ_{do} is constant, any derivative of κ_{do} is zero. Exogenous matching rates can be accomplished by setting the matching elasticity, $\eta = 1$. The second case is when there are no search frictions. KM prove that there are no search frictions in the do market if and only if $c_{do} \to 0$, which implies that $\kappa_{do} \to \infty$ and $1 - u_{do}/(1-i_{do}) \to 1$. Under either of these restrictions that $\eta = 1$ or $c_{do} = 0 \forall do$, Eq. (A60) becomes

$$\zeta_d^{oo'} = \begin{cases} -\mu\theta + \frac{\partial \ln(N_o^x/N_d^x)}{\partial \ln(t_{do'})} \\ -(\mu\theta-1)\frac{\partial \ln(w_o/w_d)}{\partial \ln(t_{do'})} & \text{if } o'=o, \\ \\ \frac{\partial \ln(N_o^x/N_d^x)}{\partial \ln(t_{do'})} \\ -(\mu\theta-1)\frac{\partial \ln(w_o/w_d)}{\partial \ln(t_{do'})} & \text{if } o'\neq o. \end{cases} \tag{A61}$$

Importantly, even without the effects of search frictions, the tariff elasticity is a function of the equilibrium wages and the numbers of varieties.

To remove the effect that relative wages have on the tariff elasticity, assume either a homogeneous freely traded good as in Chaney (2008) or that country d has a labor endowment that is small relative to the origin country o, as in Gros (1987). In the latter case, the ratio $L_d/L_o \to 0$ implying that $w_o/w_d \to 0$ and $\partial \ln(w_o/w_d)/\partial \ln(t_{do'}) \to 0$ as $L_o \to \infty$ for fixed w_d, as proved in Appendix A.4.3.6.

Adding the restriction that the relative number of varieties, N_o/N_d, is constant means that Eq. (A61) simplifies to

$$\zeta_d^{oo'} = \begin{cases} -\mu\theta & \text{if } o' = o, \\ \\ 0 & \text{if } o' \neq o. \end{cases} \tag{A62}$$

A.3.4.2 Tariff elasticity with Pareto productivity but without selection

Importantly, homogeneous firms is a sufficient, but not necessary, way to ensure that there is no selection into exporting. A different sufficient condition for no selection is to ensure that the threshold from Eq. (18) is equal to or lower than the minimum of the Pareto productivity distribution given in Section 2.1.3, which is one. More formally, there is no selection when the effective entry cost in Eq. (17) satisfies

$$\mu^{1-\sigma}\left(\frac{\alpha}{\sigma}\right)C_d\left(\frac{w_o\tau_{do}}{P_d}\right)^{1-\sigma}t_{do}^{\mu(1-\sigma)} \geq w_oF\left(\kappa_{do}\right) \Rightarrow 1 \geq \bar{\varphi}_{do}. \tag{A63}$$

Consider an economy that always satisfies Eq. (A63) and has $f_{do} = l_{do} = s_{do} = 0$ so that $\delta_{do} = 0$ from Eq. (A14) and then Eq. (17) becomes

$$F\left(\kappa_{do}\right) = \left(1 + \frac{r+\lambda}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right)h_{do}.$$

With these restrictions, the gravity equation from Eq. (A57) becomes

$$IM_{do} = \left(1 - \frac{u_{do}}{1-i_{do}}\right)N_o^x\left(\frac{\sigma-\gamma_{do}}{\sigma t_{do}}\right)\alpha C_d\frac{\left(t_{do}\mu w_o\tau_{do}\right)^{1-\sigma}}{P_d^{1-\sigma}}\left(\frac{\theta}{\theta-\sigma+1}\right). \tag{A64}$$

Because $\delta_{do} = 0$, Eq. (A58) becomes

$$1 - b\left(\kappa_{do}\right) = \frac{\sigma-\gamma_{do}}{\sigma}.$$

We use this and the other parameter restrictions to write the gravity equation with search frictions and without selection as

$$IM_{do} = \left(1 - \frac{u_{do}}{1-i_{do}}\right)\left(1 - b_{do}\left(\kappa_{do}\right)\right)\alpha N_o^xC_d\frac{\left(\mu w_o\tau_{do}\right)^{1-\sigma}t_{do}^{-\sigma}}{P_d^{1-\sigma}}\left(\frac{\theta}{\theta-\sigma+1}\right). \tag{A65}$$

Using this equation, we form the ratio of gravity equations without selection as

$$\frac{IM_{do}}{IM_{dd}} = \frac{\left(1 - \frac{u_{do}}{1-i_{do}}\right)\left(1 - b_{do}\left(\kappa_{do}\right)\right)N_o^x\left(w_o\tau_{do}\right)^{1-\sigma}t_{do}^{-\sigma}}{\left(1 - \frac{u_{dd}}{1-i_{dd}}\right)\left(1 - b_{dd}\left(\kappa_{dd}\right)\right)N_d^x\left(w_d\tau_{dd}\right)^{1-\sigma}t_{dd}^{-\sigma}}, \tag{A66}$$

in which α, C_d, P_d, and a few parameters are both in the numerator and denominator and therefore cancel in the ratio. Taking logs and differentiating Eq. (A66) with respect to

$\ln(t_{do'})$ gives

$$\zeta_d^{oo'} = \begin{cases} -\sigma + \left(\frac{u_{do}}{1-i_{do}}\right)(1-\eta)\frac{\partial \ln(\kappa_{do})}{\partial \ln(t_{do'})} - \left(\frac{u_{dd}}{1-i_{dd}}\right)(1-\eta)\frac{\partial \ln(\kappa_{dd})}{\partial \ln(t_{do'})} \\ +\frac{\partial \ln(1-b_{do}(\kappa_{do}))}{\partial \ln(t_{do'})} - \frac{\partial \ln(1-b_{dd}(\kappa_{dd}))}{\partial \ln(t_{do'})} \\ +\frac{\partial \ln(N_o^x/N_d^x)}{\partial \ln(t_{do'})} - (\sigma-1)\frac{\partial \ln(w_o/w_d)}{\partial \ln(t_{do'})} & \text{if } o' = o, \\ \\ \left(\frac{u_{do}}{1-i_{do}}\right)(1-\eta)\frac{\partial \ln(\kappa_{do})}{\partial \ln(t_{do'})} - \left(\frac{u_{dd}}{1-i_{dd}}\right)(1-\eta)\frac{\partial \ln(\kappa_{dd})}{\partial \ln(t_{do'})} \\ +\frac{\partial \ln(1-b_{do}(\kappa_{do}))}{\partial \ln(t_{do'})} - \frac{\partial \ln(1-b_{dd}(\kappa_{dd}))}{\partial \ln(t_{do'})} \\ +\frac{\partial \ln(N_o^x/N_d^x)}{\partial \ln(t_{do'})} - (\sigma-1)\frac{\partial \ln(w_o/w_d)}{\partial \ln(t_{do'})} & \text{if } o' \neq o. \end{cases} \tag{A67}$$

As in Eq. (A60), there are two interesting restrictions for Eq. (A67) that generate the same tariff elasticity. The first case is when search frictions exist but matching rates are exogenous (constant market tightness, $\kappa_{do}\ \forall do$). Because κ_{do} is constant, any derivative of κ_{do} is zero. Exogenous matching rates can be accomplished by setting the matching elasticity, $\eta = 1$. The second case is when there are no search frictions. Krolikowski and McCallum (2021) prove that there are no search frictions in the do market if and only if $c_{do} \to 0$, which implies that $\kappa_{do} \to \infty$ and $1 - u_{do}/(1-i_{do}) \to 1$.

Under either of these restrictions on η or $c_{do}\forall do$, Eq. (A67) becomes

$$\zeta_d^{oo'} = \begin{cases} -\sigma + \frac{\partial \ln(N_o^x/N_d^x)}{\partial \ln(t_{do'})} - (\sigma-1)\frac{\partial \ln(w_o/w_d)}{\partial \ln(t_{do'})} & \text{if } o' = o, \\ \\ \frac{\partial \ln(N_o^x/N_d^x)}{\partial \ln(t_{do'})} - (\sigma-1)\frac{\partial \ln(w_o/w_d)}{\partial \ln(t_{do'})} & \text{if } o' \neq o. \end{cases} \tag{A68}$$

Importantly, even with exogenous matching rates or without search frictions, the tariff elasticity is a function of the equilibrium wages and the numbers of varieties. To remove the effect that relative wages have on the tariff elasticity take the same steps as Eq. (A61) and assume that country d has a labor endowment that is small relative to the origin country o, such that the ratio $L_d/L_o \to 0$. Adding the restriction that the relative number of varieties, N_o/N_d, is constant means that Eq. (A67) simplifies to

$$\zeta_d^{oo'} = \begin{cases} -\sigma & \text{if } o' = o, \\ \\ 0 & \text{if } o' \neq o. \end{cases} \tag{A69}$$

A.3.5 Consumption is after-tariff imports evaluated at final sales prices

The value of consumption of the differentiated good in the do market is defined as the integral over all varieties, φ, of the value of $q_{do}(\varphi)$ units evaluated at final sales prices,

$p_{do}(\varphi)$:

$$C_{do} = \left(1 - \frac{u_{do}}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} p_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi). \tag{A70}$$

A shortcut to evaluating the integral in Eq. (A70) uses the fact that the value of imports in Eq. (A54) is a similar expression, but integrates $n_{do}(\varphi)\, q_{do}(\varphi)$ rather than $p_{do}(\varphi)\, q_{do}(\varphi)$. As such, to evaluate the right side of Eq. (A70), set $\gamma_{do} = 0$ and multiply by t_{do} so that the integrand of Eq. (A70) is $p_{do}(\varphi)\, q_{do}(\varphi) = t_{do} n_{do}(\varphi)\, q_{do}(\varphi)$. These two steps imply that consumption takes the same form as imports in Eq. (23), but with two changes. First, setting $\gamma_{do} = 0$ implies that $b_{do} = 0$ in Eq. (A58). Second, we should add one to the exponent on tariffs, t_{do}. This yields the desired result

$$C_{do} = \left(1 - \frac{u_{do}}{1-i_{do}}\right) \alpha N_o^x C_d \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} (w_o F_{do})^{-\left(\frac{\theta}{\sigma-1}-1\right)} t_{do}^{1-\mu\theta}. \tag{A71}$$

Eq. (A71) is analogous to Eq. (26) in Krolikowski and McCallum (2021) but differs in that this new expression includes tariffs whereas the expression in Krolikowski and McCallum (2021) did not. Comparing Eqs. (23) and (A71) implies that

$$C_{do} = \frac{t_{do} IM_{do}}{1-b_{do}}. \tag{A72}$$

We also show that

$$C_{do} = P_{do} Q_{do}. \tag{A73}$$

Plugging in Eq. (2) into Eq. (1b) and using the definition of the price index from Eq. (A3) yields

$$Q_{do} = \alpha C_d \frac{P_{do}^{-\sigma}}{P_d^{1-\sigma}}. \tag{A74}$$

Also from Eq. (2), we obtain

$$p_{do}(\omega)\, q_{do}(\omega) = \alpha C_d \frac{p_{do}(\omega)^{1-\sigma}}{P_d^{1-\sigma}}. \tag{A75}$$

Integrating both sides of this equation over the set $\omega \in \Omega_{do}$ yields

$$C_{do} = \alpha C_d \frac{P_{do}^{1-\sigma}}{P_d^{1-\sigma}} = P_{do} \alpha C_d \frac{P_{do}^{-\sigma}}{P_d^{1-\sigma}} = P_{do} Q_{do}, \tag{A76}$$

in which we use Eq. (A74).

A.3.6 Profits

In this appendix, we present five ownership structures for the profits earned by retailers and producers. Importantly, we assume that these alternative ownership structures do not affect optimal behavior of firms but rather only the apportionment of profits.

First, we discuss ownership of firms by location: Consumers in country d own retailers and producers in country d. Second, we discuss upstream (backward) vertical integration:

Consumers in country d own retailers in country d and they own producers in the potentially many origin countries that produce for country d. Third, we discuss downstream (forward) vertical integration: Consumers in country d own producers in country d and they own the retailers that sell these goods in potentially many countries. Fourth, we discuss that consumers in country d own $w_d L_d$ shares of a global mutual fund that owns all retailers and producers as in Chaney (2008). The mutual fund redistributes profits derived anywhere proportionally to each country in the form of π dividends per share. Fifth, we discuss an inverted ownership structure in which consumers in country d own the producers that source country d and the retailers that sell country d goods. We implement the third ownership structure in this paper to facilitate comparisons with CRW, who take the same approach, but it would be straightforward to implement the other structures.

A.3.6.1 Profits attributed by location

Assume that consumers in country d own retailers and producers in country d. This assumption implies that total profits in country d are profits from retailers in country d selling products from potentially many origin k markets and profits from producers in country d selling to potentially many other markets k markets, in which $k = 1, \ldots, D$.

Retailer profits in country d can then be calculated from the retailer profits from each variety, which are

$$\pi^r_{do}(\varphi) = p_{do}(\varphi)\, q_{do}(\varphi) - t_{do} n_{do}(\varphi)\, q_{do}(\varphi)\,. \tag{A77}$$

Integrating Eq. (A77) over varieties gives retailer profits earned in country d from selling products sourced from origin country o,

$$\Pi^r_{do} = C_{do} - t_{do} IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N^x_o \int_{\bar{\varphi}_{do}} \pi^r_{do}(\varphi)\, dG(\varphi)\,. \tag{A78}$$

Total profits from retailers in country d selling products from potentially many origin k markets is then

$$\Pi^r_d = \sum_k \Pi^r_{dk} = \sum_k C_{dk} - \sum_k t_{dk} IM_{dk}. \tag{A79}$$

Producer profits for each variety are,

$$\pi^p_{do}(\varphi) = n_{do}(\varphi)\, q_{do}(\varphi) - v\left(q_{do}, w_o, \tau_{do}, \varphi\right). \tag{A80}$$

Integrating Eq. (A80) over varieties gives producer profits earned in country d from selling products from origin country o,

$$\Pi^p_{do} = IM_{do} - \Phi^v_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N^x_k \int_{\bar{\varphi}_{do}} \pi^p_{do}(\varphi)\, dG(\varphi)\,, \tag{A81}$$

in which Φ^v_{do} is the variable production costs paid by producers in o that are making products for destination d defined in Eq. (A104d). Total profits from producers in country d selling to potentially many other markets k markets is then

$$\Pi^p_d = \sum_k \Pi^p_{kd} = \sum_k IM_{kd} - \sum_k \Phi^v_{kd} = \sum_k IM_{kd} - \sum_k \frac{C_{kd}}{\mu t_{kd}} = \sum_k IM_{kd} - \Phi^v_d. \tag{A82}$$

in which we use the definition of Φ^{v}_{do} from Eq. (A105). Under this location-based ownership structure, total flow variable profits earned in d are therefore the sum of Eqs. (A79) and (A82)

$$\Pi_d = \Pi^r_d + \Pi^p_d = \sum_k \Pi^r_{dk} + \sum_k \Pi^p_{kd} = \sum_k C_{dk} - \sum_k t_{dk} IM_{dk} + \sum_k IM_{kd} - \sum_k \frac{C_{kd}}{\mu t_{kd}}. \quad \text{(A83)}$$

Notice that Π^r_d and Π^p_d sum over different country indices so further simplification is not possible.

A.3.6.2 Upstream vertical integration

Assume that consumers in country d own retailers in country d and also own all the producers in the potentially many origin k markets that serve market d. This assumption implies that total profits in country d are profits from retailers in country d selling products from potentially many origin k markets and profits that producers in k countries earn from selling to country d but not other countries, in which $k = 1, \ldots, D$.

This upstream vertically-integrated ownership structure and the location-based ownership structure in Appendix A.3.6.1 imply the same retailer profits defined in Eq. (A79).

Producer profits, however, differ in a very simple way between these two approaches. Location-based profits sum across destinations with fixed origin country at d so that $\Pi^p_d = \sum_{k=1}^{D} \Pi^p_{kd}$ using Eq. (A81). In contrast, upstream vertically-integrated profits sum across origin countries with fixed destination country at d so that $\Pi^p_d = \sum_{k=1}^{D} \Pi^p_{dk}$ and also use Eq. (A81). Because the summing index, dk, for retailers' and producers' profits is the same under vertically-integrated ownership, total profits are

$$\Pi_d = \Pi^r_d + \Pi^p_d = \sum_k \Pi^r_{dk} + \sum_k \Pi^p_{dk} = \sum_k C_{dk} - \sum_k t_{dk} IM_{dk} + \sum_k IM_{dk} - \sum_k \frac{C_{dk}}{\mu t_{dk}}. \quad \text{(A84)}$$

which simplifies to

$$\Pi_d = \sum_k \left(1 - \frac{1}{\mu t_{dk}}\right) C_{dk} + \sum_k \left(1 - t_{dk}\right) IM_{dk} = \sum_k \left(1 - \frac{1}{\mu t_{dk}}\right) C_{dk} - G_d, \quad \text{(A85)}$$

in which G_d is defined in Eq. (28).

One could alternatively derive Eq. (A85) by adding Eqs. (A77) and (A80) to get the profits of vertically integrated retailers and producers of each variety,

$$\pi^v_{do}(\varphi) = p_{do}(\varphi)\, q_{do}(\varphi) - t_{do} n_{do}(\varphi)\, q_{do}(\varphi) + n_{do}(\varphi)\, q_{do}(\varphi) - v\left(q_{do}, w_o, \tau_{do}, \varphi\right), \quad \text{(A86)}$$

and then integrating over varieties and summing over countries.

A.3.6.3 Downstream vertical integration

Assume that consumers in country d own producers in country d and that they own the retailers that sell these goods in potentially many countries. This assumption implies that total profits in country d are profits from retailers in country k selling products sourced from country d and profits that producers in country d earn from selling to retailers in country k, in which $k = 1, \ldots, D$.

This downstream vertically-integrated ownership structure and the location-based ownership structure in Appendix A.3.6.1 imply the same producer profits defined in Eq. (A79).

Retailer profits, however, differ in a very simple way between these two approaches. Location-based profits sum across origins with a fixed destination country at d so that $\Pi^r_d = \sum_{k=1}^{D} \Pi^r_{dk}$ using Eq. (A78). In contrast, downstream vertically-integrated profits sum across destination countries with a fixed origin country at d so that $\Pi^r_d = \sum_{k=1}^{D} \Pi^r_{kd}$ and also use Eq. (A78). Because the summing index, kd, for retailers' and producers' profits is the same under downstream vertically-integrated ownership, total profits are

$$\Pi_d = \Pi^r_d + \Pi^p_d = \sum_k \Pi^r_{kd} + \sum_k \Pi^p_{kd} = \sum_k C_{kd} - \sum_k t_{kd} IM_{kd} + \sum_k IM_{kd} - \sum_k \frac{C_{kd}}{\mu t_{kd}}. \quad \text{(A87)}$$

which simplifies to

$$\Pi_d = \sum_k \left(1 - \frac{1}{\mu t_{kd}}\right) C_{kd} + \sum_k \left(1 - t_{kd}\right) IM_{kd}. \quad \text{(A88)}$$

A.3.6.4 Inverted ownership structure

Assume that consumers in country d own producers in country k that source country d and that they own retailers in country k selling products sourced from country d, in which $k = 1, \ldots, D$. As before, we can use Eq. (A78) and sum over kd and Eq. (A81) and sum over dk to obtain

$$\Pi_d = \Pi^r_d + \Pi^p_d = \sum_k \Pi^r_{kd} + \sum_k \Pi^p_{dk} = \sum_k C_{kd} - \sum_k t_{kd} IM_{kd} + \sum_k IM_{dk} - \sum_k \frac{C_{dk}}{\mu t_{dk}}. \quad \text{(A89)}$$

A.3.6.5 The global mutual fund

Assume that all retailers and producers are owned by a global mutual fund that collects all variable profits and rebates them to consumers. Global profits can be expressed in many ways. One way is to sum Eq. (A85) across all countries d to,

$$\Pi = \sum_d \sum_o C_{do} - \sum_d \sum_o \frac{C_{do}}{\mu t_{do}} - \sum_d G_d = \alpha C - \sum_d \sum_o \frac{C_{do}}{\mu t_{do}} - G \quad \text{(A90)}$$

in which $C = \sum_d C_d$ is global consumption and $G = \sum_d G_d$ is global government expenditure. It is useful to apportion Π to each country as a constant share of labor income so that

$$\Pi_d = w_d L_d \frac{\Pi}{\sum_d w_d L_d} = w_d L_d \pi, \quad \text{(A91)}$$

in which

$$\pi = \frac{\Pi}{\sum_d w_d L_d}. \quad \text{(A92)}$$

Notice that the dividend per unit value of labor, π, is proportional to the value of the global labor endowment and constant across countries. This definition matches Chaney (2008) Eq. (6) adjusted to include tariffs.

A.3.6.6 Profits with balanced trade

We show that the various ownership structures described in Appendixes A.3.6.1 to A.3.6.4 are identical in our model if search frictions are removed and trade is balanced.

First, notice that retailer profits in a model without search frictions are zero. In particular, without search, $C_{do} = t_{do} IM_{do}$ so that Eq. (A78) implies $\Pi^r_{do} = 0 \; \forall d, o$.

Second, we show that if search frictions are removed then $\sum_k \Pi^p_{dk} = \sum_k \Pi^p_{kd}$ if and only if trade is balanced. Notice that Eq. (A105) implies that

$$\sum_k \Pi^p_{dk} = \sum_k IM_{dk} - \frac{C_{dk}}{\mu t_{dk}} = \left(1 - \frac{1}{\mu}\right) \sum_k IM_{dk}. \tag{A93}$$

Similarly,

$$\sum_k \Pi^p_{kd} = \left(1 - \frac{1}{\mu}\right) \sum_k IM_{kd}. \tag{A94}$$

Inspection of Eqs. (A93) and (A94) suggests that

$$\sum_k \Pi^p_{dk} = \sum_k \Pi^p_{kd} \iff \sum_k IM_{dk} = \sum_k IM_{kd}, \tag{A95}$$

in which the latter is balanced trade, i.e. $NX_d = 0$.

Finally, we show that the various ownership structures in Appendixes A.3.6.1 to A.3.6.4 are equivalent in our model if search frictions are removed and trade is balanced. Because retailer profits in the model without search frictions are zero, Eq. (A83) in Appendix A.3.6.1 implies that total profits in country d are given by

$$\Pi_d = \sum_k \Pi^p_{dk}. \tag{A96}$$

Similarly, profits in Appendix A.3.6.2, A.3.6.3, and A.3.6.4, are given by Eqs. (A84), (A87), and (A89), and are $\sum_k \Pi^p_{dk}$, $\sum_k \Pi^p_{kd}$, and $\sum_k \Pi^p_{dk}$, respectively. Eq. (A95) implies that all four of these profit expressions are identical if we impose trade balance because summing producer profits over the kd indexes is the same as summing over dk.

A.3.7 Expenditure approach to accounting

In the National Income and Product Accounts (NIPA), gross domestic product (GDP) can be measured in three ways: 1) as the sum of goods and services sold to final users (expenditure approach), 2) as the sum of income payments and other costs incurred in the production of goods and services (income approach), 3) and as the sum of the value added at each stage of production. This appendix, together with Appendices A.3.8 and A.3.9, will explain how we use equating the income and expenditure approaches to define the resource constraint for consumers and solve for the equilibrium in our model.

The expenditure approach sums personal consumption expenditures, C_d, gross private fixed investment, I_d, government consumption expenditures, G_d, net exports of goods and services, NX_d, change in private inventories, and government gross investment. Our model only has the first four of these components. Moreover, in our model, personal consumption expenditure includes government consumption expenditure because final sales prices include the import tariff (Eq. 12). As such,

$$GDP_d = C_d + I_d + NX_d. \tag{A97}$$

Each additive term of the expenditure approach is discussed in detail in the following sections. Because the government runs a balanced budget, government expenditure is exactly equal to government revenue from Appendix A.3.8.2.

A.3.7.1 Personal consumption

Consumption expenditure, C_d, is the total resources devoted to consumption evaluated at final consumer prices, defined in Eq. (26), and can be written as

$$C_d = p_d\left(1\right)q_d\left(1\right) + \sum_k C_{dk}, \tag{A98}$$

in which consumption of the homogeneous good is $p_d\left(1\right)q_d$ and consumption of the differentiated varieties consumed in d but produced in o is given by Eq. (A70).

A.3.7.2 Investment

Investment expenditure, I_d, is the value of resources devoted to creating producers, I_d^e, to creating retailer-producer relationships, I_d^r, and to paying for the per-period fixed costs of production, I_d^f, given by

$$I_d = I_d^e + I_d^r + I_d^f, \tag{A99}$$

in which

$$I_d^e = N_d^x w_d e_d^x, \tag{A100a}$$

$$I_d^r = \sum_k \kappa_{dk} u_{dk} N_k^x w_d c_{dk} + \sum_k u_{kd} N_d^x \left(w_d l_{kd} + w_d s_{kd} \kappa_{kd} \chi\left(\kappa_{kd}\right)\right), \text{ and} \tag{A100b}$$

$$I_d^f = \sum_k \left(1 - u_{kd} - i_{kd}\right) N_d^x w_d f_{kd}. \tag{A100c}$$

We define investment costs as those that must be paid before producing the first unit of output and that do not scale with output.

A.3.7.3 Net exports

Define net exports as

$$NX_d = EX_d - IM_d = \sum_k IM_{kd} - \sum_k IM_{dk}, \tag{A101}$$

in which imports by destination d from origin o are given by Eq. (A54).

A.3.8 Income approach to accounting

The income approach sums compensation of employees, $w_d L_d$, income from taxes or subsidies on production and imports, T_d, net operating surplus, Π_d^{nos}, and consumption of fixed capital (depreciation). Our model only has the first three of these components so that

$$GDI_d = w_d L_d + T_d + \Pi_d^{nos}. \tag{A102}$$

Government tax income equals government expenditure in our model (Eq. A107) because we assume the government runs a balanced budget.

A.3.8.1 Wage income

Wage income, $w_d L_d$, is derived from creating producers, Φ_d^e, the formation of relationships, Φ_d^r, fixed, Φ_d^f, and variable, Φ_d^v, costs of heterogeneous goods production, and the variables costs of homogeneous goods production, $w_d q_d(1)$ given by

$$w_d L_d = \Phi_d^e + \Phi_d^r + \Phi_d^f + \Phi_d^v + w_d q_d(1), \tag{A103}$$

in which

$$\Phi_d^e = N_d^x w_d e_d^x, \tag{A104a}$$

$$\Phi_d^r = \sum_k \kappa_{dk} u_{dk} N_k^x w_d c_{dk} + \sum_k u_{kd} N_d^x \left(w_d l_{kd} + w_d s_{kd} \kappa_{kd} \chi(\kappa_{kd}) \right), \tag{A104b}$$

$$\Phi_d^f = \sum_k (1 - u_{kd} - i_{kd}) N_d^x w_d f_{kd}, \text{ and} \tag{A104c}$$

$$\Phi_d^v = \sum_k \Phi_{kd}^v = \sum_k \left(1 - \frac{u_{kd}}{1 - i_{kd}}\right) N_d^x \int_{\bar{\varphi}_{kd}} v(q_{kd}, w_d, \tau_{kd}, \varphi)\, dG(\varphi). \tag{A104d}$$

With free entry of retailers as we assume, there can be no retailer entry cost in addition to search cost $w_o c_{od}$ as shown in KM. As such, the cost of creating retailers is the same as the cost of forming relationships, $\sum_k \kappa_{dk} u_{dk} N_k^x w_d c_{dk}$.

Using variable costs from Eq. (5), optimal final sales price from Eq. (12) written as $p_{do}(\varphi)/t_{do}\mu = w_o \tau_{do} \varphi^{-1}$, and demand from Eq. (2) we can write

$$\Phi_{do}^v = \frac{C_{do}}{\mu t_{do}} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} v(q_{do}, w_o, \tau_{do}, \varphi)\, dG(\varphi). \tag{A105}$$

Dividing Eq. (A105) by w_d and summing across k destination markets gives

$$\frac{\Phi_d^v}{w_d} = \sum_k \frac{\Phi_{kd}^v}{w_d} = \sum_k \frac{C_{kd}}{\mu w_d t_{kd}} = \sum_k \left(1 - \frac{u_{kd}}{1 - i_{kd}}\right) N_d^x \int_{\bar{\varphi}_{kd}} q_{kd}(\varphi)\, \tau_{kd} \varphi^{-1} dG(\varphi)$$

which is one of the three additive terms in labor demand from Eq. (29).

Comparing investment, I_d, from Eq. (26) to Eq. (A104), we see that, $I_d = \Phi_d^e + \Phi_d^r + \Phi_d^f$ providing another additive term from labor demand. The final term is just the labor used to produce the homogeneous good. As such, Eq. (A103) is just a restatement of equilibrium labor market clearing from Eq. (29).

A.3.8.2 Government income

We assume that retailers in the do market pay a tariff, $t_{do} - 1$, on the value of imported differentiated goods, $n_{do}(\varphi)\, q_{do}(\varphi)$. Integrating over all the products and summing over all the origin countries yields total government tax income

$$T_d = \sum_{k=1}^{D} T_{dk} = \sum_{k=1}^{D} \left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}}^{\infty} (t_{dk} - 1)\, n_{dk}(\varphi)\, q_{dk}(\varphi)\, dG(\varphi). \tag{A106}$$

In Eq. (26) we set

$$T_d = G_d. \tag{A107}$$

Notice that when $t_{dk} = 1 \forall k$ then $T_d = 0$ because there are no import tariffs or subsidies.

We define the value of total imports, IM_{do}, in Appendix A.3.3 (Eq. A54). It is trivial to show that government tax revenue in the do market is given by

$$T_{do} = (t_{do} - 1)\, IM_{do}, \tag{A108}$$

which restates Eq. (A55) with $G_{do} = T_{do}$. Similarly, because $C_{do} = t_{do} IM_{do} / (1 - b_{do})$, as shown in Appendix A.3.5,

$$T_{do} = \frac{(1 - b_{do})(t_{do} - 1)}{t_{do}} C_{do}, \tag{A109}$$

with b_{do} defined in Eq. (A58).

A.3.8.3 Profit income

We present details about profit income in Appendix A.3.6.

A.3.9 Income equals expenditure

In order to solve the model, we equate national output using the income and expenditure approaches:

$$w_d L_d + T_d + \Pi_d^{nos} = C_d + I_d + NX_d. \tag{A110}$$

First, government cancels from each side because of Eq. (A107). Using Eqs. (A103) and (A99) we can write

$$\Phi_d^e + \Phi_d^r + \Phi_d^f + \Phi_d^v + w_d q_d (1) + G_d + \Pi_d^{nos} = C_d + I_d^e + I_d^r + I_d^f + NX_d. \tag{A111}$$

By inspection, $\Phi_d^e = I_d^e$, $\Phi_d^r = I_d^r$, and $\Phi_f^r = I_d^f$, leaving

$$\Phi_d^v + w_d q_d (1) + G_d + \Pi_d^{nos} = C_d + NX_d. \tag{A112}$$

Using the fact that total consumption is the sum of homogeneous and differentiated goods consumption from Eq. (A98) and the fact that $p_d (1) = w_d, \forall d$ from Section 2.1.3,

$$\Phi_d^v + G_d + \Pi_d^{nos} = \sum_k C_{dk} + NX_d. \tag{A113}$$

Using the definition for NX_d from Eq. (A101), moving Φ_d^v to the right-hand side, and adding and subtracting $\sum_k t_{dk} IM_{dk}$ gives

$$G_d + \Pi_d^{nos} = \sum_k C_{dk} + \sum_k IM_{kd} - \sum_k IM_{dk} - \Phi_d^v + \sum_k t_{dk} IM_{dk} - \sum_k t_{dk} IM_{dk}, \tag{A114}$$

which simplifies to

$$G_d + \Pi_d^{nos} = \sum_k C_{dk} - \sum_k t_{dk} IM_{dk} + \sum_k IM_{kd} - \Phi_d^v + \sum_k (t_{dk} - 1)\, IM_{dk}. \tag{A115}$$

Using the definitions of location-based retailer and producer profits from Eqs. (A79) and (A82) and the definition of government revenue from Eq. (28) we can write

$$G_d + \Pi_d^{nos} = \sum_k \Pi_{dk}^r + \sum_k \Pi_{kd}^p + G_d. \tag{A116}$$

This means that $GDP = GDI$ using the location based definition of profits from Appendix A.3.6.1 and when profits include government tariff revenue (because profits are omitted from retailer profits). This makes sense because GDP and GDI are constrained to measure the value of income and expenditure within the geographical borders of a country, which is consistent with the location-based profit accounting but not the vertically-integrated ownership structure of Appendix A.3.6.2.

A.3.10 Labor market clearing country by country

Wages are determined endogenously in each country by setting labor demand equal to labor supply. Labor demand is given by Eq. (29). The cost to create firms, pay fixed costs, and form matches are included in the investment term, I_d. Demand for $q_d(1)$ is given by Eq. (2), in which $p_d(1) = w_d$. Labor demand also includes all labor used to produce the differentiated goods for the domestic and all foreign markets. Rearranging Eq. (A31) gives

$$\frac{p_{od}(\varphi)\, q_{od}(\varphi)}{\mu w_d t_{od}} = q_{od}(\varphi)\, \tau_{od}\varphi^{-1}, \tag{A117}$$

which implies that the labor used to produce the differentiated good can be written as,

$$\left(1 - \frac{u_{od}}{1 - i_{od}}\right) N_d^x \int_{\bar{\varphi}_{od}} q_{od}(\varphi)\, \tau_{od}\varphi^{-1} dG(\varphi) = \frac{C_{od}}{\mu w_d t_{od}}. \tag{A118}$$

Using these facts, Eq. (29) can be written as

$$LD_d = \frac{I_d}{w_d} + \frac{(1-\alpha)\, C_d}{w_d} + \frac{1}{\mu} \sum_o \frac{C_{od}}{w_d t_{od}}.$$

Because labor is immobile and the homogeneous good is not traded, the equilibrium wage in country d is determined by equating labor demand, LD_d, with labor supply, L_d, and then re-arranging slightly:

$$w_d = \frac{I_d + (1-\alpha)\, C_d + \frac{1}{\mu} \sum_o \frac{C_{od}}{t_{od}}}{L_d}. \tag{A119}$$

We choose country o's wage to be the numeraire so that $w_o = 1$.

A.4 Steady-state general equilibrium

A.4.1 Defining the equilibrium

We discuss how we derive the four equilibrium constraints in Eq. (30). First, we derive Eq. (30b) by using the free-entry condition (Eq. 19) and substituting in for retailers' value on the right hand side with Eq. (A22), in which $V_{do} = 0$, and then using the definition of retailer profit in Eq. (A78). Sometimes it is useful to rewrite Eq. (19) as

$$\kappa_{do} = \left(\frac{1}{r+\lambda}\right)\left(\lambda + \kappa_{do}\chi\left(\kappa_{do}\right)\right)\left(\frac{1}{w_d c_{do} N_o^x}\right)\Pi_{do}^r, \tag{A120}$$

in which we used $\chi\left(\kappa_{do}\right) = \xi\kappa_{do}^{-\eta}$ and $\left[1 - u_{do}/\left(1 - i_{do}\right)\right]^{-1} = \left(\lambda + \kappa_{do}\chi\left(\kappa_{do}\right)/\kappa_{do}\chi\left(\kappa_{do}\right)\right)$. Second, Eq. (30c) is simply Eq. (18). Third, Eq. (30d) combines the expenditure approach to national accounting (Eq. 26) with the income approach to national accounting (Eq. 27). Fourth, Eq. (30e) is simply Eq. (A119).

A.4.2 Nesting trade models without search frictions

Our model nests trade models without search frictions if retailers' search costs are zero, $c_{do} = 0$, $\forall do$, among other restrictions. The main difference between our model's equilibrium definition and the definitions in trade models without search is that we introduce market tightness, k_{do}. When search costs are zero, free entry into product vacancies leads to infinite market tightness and instantaneous matching for producers. Instantaneous matching implies that all producers are matched (Eq. 20), as in a standard trade model without search frictions.

In particular, our model exactly reproduces Chaney (2008) if retailers' search costs are zero, we make the same assumptions about the homogeneous good as he does, we assume the same profit redistribution (Appendix A.3.6.5), we assume that the number of producers is proportional to consumption ($N_o^x = C_o/\left(1+\pi\right)$) and that there are no tariffs ($G_d = 0 \forall d$), and we make the same parameter value restrictions that he does ($t_{do} = s_{do} = h_{do} = e_d^x = 0$, $\forall d$, and, $\forall o$). We demonstrate this equivalence by showing that all equilibrium equations are the same. If retailers' search costs are zero, market tightness is infinite and the negotiated price (Eq. 14) attains the final sales price if $t_{do} = 1$, $\forall do$, which is given by Eq. (12). There is, in effect, no intermediate retailer; producers sell their goods directly to the final consumer at price p_{do}. Instant contacts for producers imply that the effective entry cost (Eq. 17) equals the fixed cost of production, $F_{do} = w_o f_{do}$, and our threshold productivity expression (Eq. 18) coincides with Chaney (2008, Eq. 7). With no search costs and $G_d = 0$, Eq. (26) implies that $Y_d = C_d + I_d$ and the only investment expenditure is the fixed cost of production. Using Eq. (27), total income is given by $Y_d = \left(1+\pi\right) w_d L_d$, which also matches Chaney (2008, Eq. 9). Assumptions about the homogeneous good as in Chaney (2008) would imply that $w_d = 1\ \forall d$ and the per-capita dividend is determined by Eq. (A92), as shown in KM. With the same equations defining the equilibrium variables, our ideal price index (Eq. 22) and gravity equation (Eq. 23) would coincide with Eqs. 8 and 10, respectively, in Chaney (2008).

A.4.3 A graphical depiction of the model: Proposition 1

We detail the shape of each line in Fig. 1, described in Section 2.4.2. For each sub-figure, the equilibrium values of other endogenous variables are taken as given. This material borrows heavily from KM, Appendix A.13.

A.4.3.1 Final sales price and negotiated quantity

First, we characterize the marginal cost curve in Fig. 1a. Differentiate the variable cost equation to get marginal cost:

$$\frac{\partial v\left(q_{do}, w_{o}, \tau_{do}, \varphi\right)}{\partial q_{do}} = w_{o}\tau_{do}\varphi^{-1}, \tag{A121}$$

which is independent of q_{do} so is a horizontal line in $\left(q_{do}\left(\varphi\right), p_{do}\left(\varphi\right)\right)$ space.

Second, we characterize the demand curve and the marginal revenue curves in Fig. 1a. Begin with the demand curve for a differentiated variety (Eq. 2) and solve for inverse demand, $p_{do}\left(\varphi\right) = q_{do}\left(\varphi\right)^{-1/\sigma}\left(\alpha C_{d}/P_{d}^{1-\sigma}\right)^{1/\sigma}$, which is downward sloping because

$$\frac{\partial p_{do}\left(\varphi\right)}{\partial q_{do}\left(\varphi\right)} = -\frac{1}{\sigma}q_{do}\left(\varphi\right)^{-\frac{1}{\sigma}-1}\left(\frac{\alpha C_{d}}{P_{d}^{1-\sigma}}\right)^{1/\sigma} < 0.$$

The second derivative is positive, which means the curve is convex and bowed into the origin. Marginal revenue is given by

$$p_{do}\left(q_{do}\right) + \frac{\partial p_{do}\left(q_{do}\right)}{\partial q_{do}}q_{do} = \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\alpha C_{d}}{P_{d}^{1-\sigma}}\right)^{\frac{1}{\sigma}} q_{do}\left(\varphi\right)^{-\frac{1}{\sigma}}, \tag{A122}$$

which is also downward sloping. Furthermore, the marginal revenue curve always lies under the demand curve because $\left(\sigma-1\right)/\sigma < 1$. Bargaining over quantity implies that the equilibrium quantity exchanged *within* matches equates retailers' marginal revenue from consumers (Eq. A122) with the marginal production cost (Eq. A121) times tariffs, as shown in Appendix A.2.2, Eq. (A24). The resulting equilibrium price is Eq. (12) in the main text.

Notice that we are guaranteed existence and uniqueness of $p_{do}^{*}\left(\varphi\right)$ and $q_{do}^{*}\left(\varphi\right)$ in this market. This result follows because the marginal revenue curve is downward sloping with values ranging from ∞ to zero and the marginal cost curve is horizontal at some positive value. Furthermore, for any variety above the threshold productivity, $\bar{\varphi}_{do}$, this figure, along with parameter restrictions such as $\sigma > 1$, $\tau_{do} \geq 1$, and $t_{do} \geq 0$, and the equilibrium values for other endogenous variables, like w_{o}, ensures that $p_{do}^{*}\left(\varphi\right) > 0$ and $q_{do}^{*}\left(\varphi\right) > 0$.

A.4.3.2 Negotiated price for one good

Third, we characterize the negotiated price curve in Fig. 1b. Begin with the negotiated price (Eq. 14) and re-arrange to obtain:

$$n_{do} = \frac{p_{do}}{t_{do}} + \left(r+\lambda\right)\left(1-\beta\right)\left[r+\lambda+\beta\xi\kappa_{do}^{1-\eta}\right]^{-1}\left[\frac{v\left(q_{do}, w_{o}, \tau_{do}, \varphi\right) + w_{o}f_{do} - w_{o}l_{do} - \frac{p_{do}q_{do}}{t_{do}}}{q_{do}}\right], \tag{A123}$$

in which we have assumed that $s_{do} = 0$ and that the retailer contact rate comes from the matching function (Eq. 4). Fig. 1b uses our baseline calibration and does not require that $s_{do} = 0$, but these proofs do. We differentiate this expression with respect to κ_{do}

$$\frac{\partial n_{do}}{\partial \kappa_{do}} = -\frac{(r+\lambda)(1-\beta)}{\left(r+\lambda+\beta\xi\kappa_{do}^{1-\eta}\right)^2}\beta\xi(1-\eta)\kappa_{do}^{-\eta}\left[\frac{v(q_{do},w_o,\tau_{do},\varphi)+w_of_{do}-w_ol_{do}-\frac{p_{do}q_{do}}{t_{do}}}{q_{do}}\right],$$

which is positive. This derivative shows that the negotiated price slopes up in $(\kappa_{do}, n_{do}(\varphi))$ space. As $\kappa_{do} \to 0$ this derivative tends to ∞ and as $\kappa_{do} \to \infty$ this derivative tends to 0. The second derivative is negative so the negotiated price curve is concave.

The intercept of Eq. (A123) if $\kappa_{do} = 0$ is

$$n_{do}(\varphi) \quad = \quad \beta\frac{p_{do}}{t_{do}} + (1-\beta)\left[\frac{v(q_{do},w_o,\tau_{do},\varphi)+w_of_{do}-w_ol_{do}}{q_{do}}\right].$$

The value of Eq. (14) as $\kappa_{do} \to \infty$ is

$$n_{do} \quad = \quad \frac{p_{do}}{t_{do}},$$

which recovers the final sales price in the Melitz (2003) model, but with tariffs.

Fourth, we characterize the vertical market tightness line in Fig. 1b. Inspecting the free entry condition for retailers (Eq. 19) suggests that the negotiated price of a single infinitesimal producer does not affect goods-market tightness. As such, the market tightness is a vertical line in $(\kappa_{do}, n_{do}(\varphi))$ space. The market tightness is taken from Fig. 1d.

Notice that we are guaranteed existence and uniqueness of $n^*_{do}(\varphi)$ in this market. Consider inverting Eq. (A123) so that $\kappa_{do} = \kappa_{do}(n_{do})$. We know that this function will also have positive derivative for all n_{do}. We also know that if $n_{do}(\varphi) = \beta(p_{do}/t_{do}) + (1-\beta)(v(q_{do},w_o,\tau_{do},\varphi)+w_of_{do}-w_ol_{do})/q_{do}$, then $\kappa_{do} = 0$, and if $n_{do} \to p_{do}$ then $\kappa_{do} \to \infty$. As such, this function crosses $\kappa^*_{do} \geq 0$ once.

A.4.3.3 Producer threshold

Fifth, we characterize the threshold productivity curve in Fig. 1c. Differentiate the producer threshold expression (Eq. 18) with respect to consumption to obtain

$$\frac{\partial\bar{\varphi}_{do}}{\partial C_d} \quad = \quad -\frac{\sigma}{(\sigma-1)^2}\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}}\left(\frac{w_o\tau_{do}}{P_d}\right)(w_oF(\kappa_{do}))^{\frac{1}{\sigma-1}}t_{do}^{\mu}C_d^{-\mu},$$

which is negative so this line is sloped down. The second derivative is positive so this line is convex and bowed in toward the origin.

Sixth, equilibrium aggregate consumption is determined in Fig. 1e so we take it as given for this figure and depict it with a vertical line.

Notice that we are guaranteed existence and uniqueness of $\bar{\varphi}^*_{do}$ in this market. Consider inverting Eq. (18) so that $C_d = C_d(\bar{\varphi}_{do})$. We know that this function will also have negative derivative for all $\bar{\varphi}_{do}$. We also know that if $\bar{\varphi}_{do} \to 0$, then $C_d \to \infty$, and if $\bar{\varphi}_{do} \to \infty$, then $C_d \to 0$. As such, this function crosses $C^*_d \geq 0$ once.

A.4.3.4 Retailers' expected negotiated cost and market tightness

Seventh, we characterize mean imports in Fig. 1d. Begin with the negotiated price (Eq. 14), multiply by equilibrium quantity q_{do}, and take an expectation to obtain:

$$\mathbb{E}\left[n_{do}q_{do}\right] = \int_{\bar{\varphi}_{do}} \frac{p_{do}q_{do}}{t_{do}} + \frac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\xi\kappa_{do}^{1-\eta}}\left[v\left(q_{do},w_o,\tau_{do},\varphi\right)+w_of_{do}-w_ol_{do}-\frac{p_{do}q_{do}}{t_{do}}\right]dG\left(\varphi\right), \quad \text{(A124)}$$

in which we have assumed that $s_{do}=0$ and that the retailer contact rate comes from the matching function (Eq. 4) and the expectation is taken over productivity. Fig. 1d uses our baseline calibration and does not require that $s_{do}=0$ but these proofs do. We differentiate this expression with respect to κ_{do},

$$\frac{\partial\mathbb{E}\left[n_{do}q_{do}\right]}{\partial\kappa_{do}} = \int_{\bar{\varphi}_{do}} -\frac{(r+\lambda)(1-\beta)}{\left(r+\lambda+\beta\xi\kappa_{do}^{1-\eta}\right)^2}\beta\xi(1-\eta)\kappa_{do}^{-\eta}\left[v\left(q_{do},w_o,\tau_{do},\varphi\right)+w_of_{do}-w_ol_{do}-\frac{p_{do}q_{do}}{t_{do}}\right]dG\left(\varphi\right),$$

which is positive. As $\kappa_{do}\to 0$ this derivative tends to ∞ and as $\kappa_{do}\to\infty$ this derivative tends to 0. The second derivative is negative so the mean imports curve is concave.

The intercept of Eq. (A124) when $\kappa_{do}=0$ is

$$\mathbb{E}\left[n_{do}q_{do}\right] = \int_{\bar{\varphi}_{do}} \beta\frac{p_{do}q_{do}}{t_{do}} + (1-\beta)\left[v\left(q_{do},w_o,\tau_{do},\varphi\right)+w_of_{do}-w_ol_{do}\right]dG\left(\varphi\right) \quad \text{(A125)}$$

As $\kappa_{do}\to\infty$ this equation converges to

$$\mathbb{E}\left[n_{do}q_{do}\right] = \int_{\bar{\varphi}_{do}} \frac{p_{do}q_{do}}{t_{do}}dG\left(\varphi\right),$$

which recovers the Melitz (2003) model, but with tariffs. Computing the relevant integrals implies that Eq. (A124) can be written as

$$\mathbb{E}\left[n_{do}q_{do}\right] = \frac{C_{do}}{t_{do}\left(1-\frac{u_{do}}{1-i_{do}}\right)N_o^x} + \frac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\xi\kappa_{do}^{1-\eta}}\left[w_of_{do}-w_ol_{do}-\frac{C_{do}}{t_{do}\sigma\left(1-\frac{u_{do}}{1-i_{do}}\right)N_o^x}\right]$$

in which all terms but κ_{do} are taken as given for the graph in $\left(\kappa_{do},\mathbb{E}_\varphi\left[n_{do}q_{do}\right]\right)$ space.

Eighth, we characterize the retailers' free entry curve in Fig. 1d. Begin with the entry condition (Eq. 19), plug in for $M_{do}\left(\varphi\right)$ from Appendix A.4.1, the retailer contact rate from the matching function (Eq. 4), and solve for the expected negotiated cost to get:

$$\mathbb{E}\left[n_{do}q_{do}\right] = \frac{1}{t_{do}}\int_{\bar{\varphi}_{do}} p_{do}\left(\varphi\right)q_{do}\left(\varphi\right)dG\left(\varphi\right) - \frac{1}{t_{do}}(r+\lambda)\frac{\kappa_{do}^{\eta}w_dc_{do}}{\xi}, \quad \text{(A126)}$$

in which all terms but κ_{do} are the equilibrium values for the graph in $\left(\kappa_{do},\mathbb{E}_\varphi\left[n_{do}q_{do}\right]\right)$ space.

Eq. (A126) is negatively sloped with the derivative tending to $-\infty$ as $\kappa_{do}\to 0$ and the derivative tending to 0 as $\kappa_{do}\to\infty$. The second derivative is positive so that the line is convex and bowed in toward the origin. The intercept of Eq. (A126) when $\kappa_{do} = 0$ is

$$\mathbb{E}\left[n_{do}q_{do}\right] = \int_{\bar{\varphi}_{do}} \frac{p_{do}\left(\varphi\right) q_{do}\left(\varphi\right)}{t_{do}} dG\left(\varphi\right), \tag{A127}$$

which is taken as given in this figure. The intercept when $\mathbb{E}\left[n_{do}q_{do}\right] = 0$ is

$$\kappa_{do} = \left[\frac{\xi \int_{\bar{\varphi}_{do}} p_{do}\left(\varphi\right) q_{do}\left(\varphi\right) dG\left(\varphi\right)}{w_d c_{do}\left(r+\lambda\right)}\right]^{\frac{1}{\eta}}.$$

Notice that we are guaranteed existence and uniqueness of $\mathbb{E}\left[n_{do}q_{do}\right]^*$ and κ^*_{do}. This is because the intercept when $\kappa_{do} = 0$ of the entry condition is given by Eq. (A125) and the intercept of the expected cost curve is given by Eq. (A127). We know that the former is larger than the latter and we know that the entry condition slopes down and the expected cost curve slopes up.

A.4.3.5 Goods market clearing

Ninth, we characterize the consumption curve in Fig. 1e. Using the expenditure and income approaches to national accounting (Appendices A.3.7 and A.3.8) and the expression for aggregate investment (Eq. 26) we obtain

$$w_d = \frac{\frac{1}{\mu}\sum_k \frac{C_{kd}}{t_{kd}} - \sum_k C_{dk}}{L_d - \tilde{I}_d} + \frac{C_d}{L_d - \tilde{I}_d}, \tag{A128}$$

in which $\tilde{I}_d = I_d / w_d$ is not a function of w_d because investment can be written as w_d times the labor used for investment. As such, the consumption curve is linear in (C_d, w_d) space. The slope of this curve is positive because labor market clearing (Eq. A119) implies that $L_d \geq \tilde{I}_d$. Notice that $\sum_k C_{dk} = C_d$ if $\alpha = 1$, but that term comes from profits, Π_d, and is determined elsewhere, so should not be susbtituted into this equation.

Tenth, the equilibrium wage is determined in Fig. 1f so we take it as given for this figure and depict it with a horizontal line.

Notice that we are guaranteed existence and uniqueness of C^*_d. Eq. (A129) implies that the intercept of Eq. (A128) is below the equilibrium wage. As such, the consumption and wage curves are guaranteed to cross because the consumption curve has positive slope and the wage curve is horizontal.

A.4.3.6 Labor market clearing

Eleventh, we characterize the labor demand curve in Fig. 1f. Solving Eq. (29) for the wage yields

$$w_d = \frac{\left(1-\alpha\right) C_d + \frac{1}{\mu}\sum_o \frac{C_{od}}{t_{od}}}{LD_d - \tilde{I}_d}, \tag{A129}$$

in which $I_d = w_d \tilde{I}_d$ so that $\tilde{I}_d$ is investment in units of labor

$$\tilde{I}_d = N_d^x e_d^x + \sum_{k=1}^{D} \kappa_{dk} u_{dk} N_k^x c_{dk} + u_{kd} N_d^x \left(l_{kd} + s_{kd}\kappa_{kd}\chi\left(\kappa_{kd}\right)\right) + \left(1 - u_{kd} - i_{kd}\right) N_d^x f_{kd}. \quad \text{(A130)}$$

The derivative of Eq. (A129) with respect to labor demand is

$$\frac{\partial w_d}{\partial LD_d} = -\frac{\left(1-\alpha\right) C_d + \frac{1}{\mu}\sum_o \frac{C_{od}}{t_{od}}}{\left(LD_d - \tilde{I}_d\right)^2} < 0. \quad \text{(A131)}$$

The second derivative of Eq. (A129) is positive, $\partial^2 w_d / \partial LD_d^2 > 0$. Because the wage must always be weakly positive in any equilibrium, $w_d \geq 0$, and because of the signs of the first and second derivatives, $L_d \to \infty$ implies $w_d \to 0$. Finally, labor supply is fixed at each country's labor endowment, and depicted as a vertical line in Fig. 1f.

Notice that we are guaranteed existence and uniqueness of w_d^*. Consider inverting Eq. (A129) so that $LD_d = LD_d\left(w_d\right)$. We know that this function will also have negative derivative for all w_d. We also know that if $w_d \to 0$, then $LD_d \to \infty$, and if $w_d \to \infty$, then $C_d \to \tilde{I}_d$. Eq. (A119) implies that $L_d \geq \tilde{I}_d$. As such, this function crosses L_d^* once.

We also prove that as labor supply in a country tends to infinity, the elasticity of the wage with respect to tariffs tends to zero. This result will be useful when we discuss a small open economy, as in Appendix A.3.4. Eq. (A129) defines a function from $\mathbb{R}^2$ to $\mathbb{R}$, $w_d\left(L_d, t_{do}\right)$. According to the mean value theorem, there exits a point $d \in \left[t_{do}, t'_{do}\right]$ on the line segment from $\left(L_d, t_{do}\right)$ to $\left(L_d, t'_{do}\right)$ such that

$$w_d\left(L_d, t'_{do}\right) - w_d\left(L_d, t_{do}\right) = \frac{\partial w_d\left(L_d, d\right)}{\partial t_{do}}\left(t'_{do} - t_{do}\right).$$

Taking the limit of both sides gives

$$\lim_{L_d \to \infty} w_d\left(L_d, t'_{do}\right) - \lim_{L_d \to \infty} w_d\left(L_d, t_{do}\right) = \lim_{L_d \to \infty} \frac{\partial w_d\left(L_d, d\right)}{\partial t_{do}}\left(t'_{do} - t_{do}\right).$$

The left side of this equation tends to zero and therefore

$$\lim_{L_d \to \infty} \frac{\partial w_d\left(L_d, d\right)}{\partial t_{do}} = 0. \quad \text{(A132)}$$

B Optimal unilateral uniform import tariffs

B.1 Deriving aggregate welfare

B.1.1 Deriving current value aggregate welfare

Here we outline the steps to show that the indirect utility function (welfare) is C_d/Ξ_d, in which C_d is total consumption expenditure, p is the vector of prices for each good, and Ξ_d is the ideal price index. Assume that preferences are homothetic, which is defined in Mas-Colell, Whinston, and Green (1995), Section 3.B.6, page 45. This means that they can be represented by a utility function that is homogeneous of degree one in quantities and that the corresponding indirect utility function is linear in total consumption expenditure. We can begin with the indirect utility function and then manipulate it as follows

$$\begin{aligned} W_d\left(p, C_d\right) &= W_d\left(p, 1\right) C_d \\ W_d\left(p, e\left(p, u\right)\right) &= W_d\left(p, 1\right) e\left(p, u\right) \\ u &= W_d\left(p, 1\right) e\left(p, u\right) \\ 1 &= W_d\left(p, 1\right) e\left(p, 1\right) \\ \frac{1}{e\left(p, 1\right)} &= W_d\left(p, 1\right), \end{aligned}$$

in which the first line comes from homothetic preferences; the second line follows by plugging in for consumption expenditure $C_d = e\left(p, u\right)$; the third line comes from Eq. (3.E.1) in MWG that says $W_d\left(p, e\left(p, u\right)\right) = u$ (also known as duality); and in the fourth line we plug in for utility level $u = 1$. The function $e\left(p, u\right)$ is the consumption expenditure function that solves the expenditure minimization problem. Using this result and the fact that the price index is defined as $e\left(p, 1\right) \equiv \Xi_d$ we can show that

$$W_d\left(p, C_d\right) = W_d\left(p, 1\right) C_d = \frac{1}{e\left(p, 1\right)} C_d = \frac{C_d}{\Xi_d}. \tag{B133}$$

Hence, as long as preferences are homothetic, we will always get welfare equal to consumption expenditure divided by the price index, $W_d\left(p, Y\right) = C_d/\Xi_d$.

We can prove this result directly in our setting. Plugging the optimal quantities in Eq. (2) into the utility function in Eq. (1) yields:

$$U\left(q_d\left(1\right), q_d\left(\varphi\right)\right) = \left(\frac{\left(1-\alpha\right) C_d}{p_d\left(1\right)}\right)^{1-\alpha} \left(\frac{\alpha C_d}{P_d^{1-\sigma}}\right)^{\alpha} \left[\sum_{k=1}^{O} \left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} p_{dk}\left(\varphi\right)^{1-\sigma} dG\left(\varphi\right)\right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)}. \tag{B134}$$

Use Eq. (A42) to obtain

$$P_d^{-\alpha\sigma} = \left[\sum_{k=1}^{O} \left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}}^{\infty} p_{dk}\left(\varphi\right)^{1-\sigma} dG\left(\varphi\right)\right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)}. \tag{B135}$$

Plugging this expression into Eq. (B134) and rearranging yields

$$U\left(q_d\left(1\right), q_d\left(\varphi\right)\right) = \frac{C_d}{\left(\frac{p_d\left(1\right)}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_d}{\alpha}\right)^{\alpha}}. \tag{B136}$$

The right hand side of this expression is C_d/Ξ_d, in which Ξ_d is defined in Eq. (A4).

B.1.2 Deriving present value aggregate welfare

Appendix A.1.1 shows that consumer demand is the same when solving a dynamic discounting or static consumer problem. Because there is no asset to transfer resources across time, writing demand, consumption, price indexes, and other equations as a function of time is superfluous. This fact implies that welfare (indirect utility) for the dynamic problem is

$$\int_0^{\infty} e^{-rt}\frac{C_d}{\Xi_d}dt = \frac{1}{r}\frac{C_d}{\Xi_d}, \tag{B137}$$

which is the discounted welfare for the static problem from Eq. (B133). In other words, Eq. (B137) is present value welfare and Eq. (B133) is current value welfare. Using present versus current value welfare is immaterial for optimal tariffs because r is a constant parameter. As such, the main text uses current value welfare for simplicity. This approach also makes our model easily comparable to static trade models, such as Melitz (2003) and Chaney (2008).

B.2 Optimal unilateral tariffs with passive trade policies

B.2.1 Optimal uniform tariffs with search frictions: Proposition 2

We prove Eq. (34) in Proposition 2 in two steps.

B.2.1.1 The social planner's solution

The goal of the social planner in country d is to maximize utility of the representative consumer in country d subject to that country's labor-supply constraint and the offer curve for exports from country o to country d. Formally, this maximization problem can be written as

$$\max_{Q_{dd},Q_{do},Q_{od}} U_d\left(Q_{dd}, Q_{do}\right) \tag{B138a}$$

subject to:

$$Q_{do} \leq Q_{do}\left(Q_{od}\right) \tag{B138b}$$

$$L_d = L_d\left(Q_{dd}, Q_{od}, \kappa_{dd}, \kappa_{do}\right) \tag{B138c}$$

$$\frac{u_{do}}{1-i_{do}} = \frac{\lambda}{\lambda + \kappa_{do}\chi\left(\kappa_{do}\right)}, o \in \left\{o, d\right\}. \tag{B138d}$$

Computing and rearranging first order conditions from (B138) yields

$$\frac{\left(\frac{\partial L_d\left(Q_{dd}, Q_{od}\right)}{\partial Q_{od}}\right)}{\left(\frac{\partial L_d\left(Q_{dd}, Q_{od}\right)}{\partial Q_{dd}}\right)} = \frac{\left(\frac{\partial U_d\left(Q_{dd}, Q_{do}\right)}{\partial Q_{do}}\right)}{\left(\frac{\partial U_d\left(Q_{dd}, Q_{do}\right)}{\partial Q_{dd}}\right)}\left(\frac{\partial Q_{do}\left(Q_{od}\right)}{\partial Q_{od}}\right). \tag{B139}$$

The marginal rate of transformation, MRT_{od}, and marginal rate of substitution, MRS_{do}, are given by

$$MRT_{od} = \frac{\partial L_d\left(Q_{dd}, Q_{od}\right)/\partial Q_{od}}{\partial L_d\left(Q_{dd}, Q_{od}\right)/\partial Q_{dd}} \quad \text{(B140a)} \qquad MRS_{do} = \frac{\partial U_d\left(Q_{dd}, Q_{do}\right)/\partial Q_{do}}{\partial U_d\left(Q_{dd}, Q_{do}\right)/\partial Q_{dd}}. \quad \text{(B140b)}$$

Substitute these definitions into Eq. (B139) and use the fact that partials of Eq. (B138b) are the same as totals because it is only a function of one variable to get

$$\frac{1}{MRT_{od}/MRS_{do}} = \frac{1}{dQ_{do}\left(Q_{od}\right)/dQ_{od}}. \tag{B141}$$

Next, multiply and divide the right side of Eq. (B141) by Q_{od}/Q_{do} to get

$$\frac{1}{MRT_{od}/MRS_{do}} = \frac{Q_{od}/Q_{do}}{H_{do}}, \tag{B142}$$

in which $H_{do} = d\ln Q_{do}\left(Q_{od}\right)/d\ln Q_{od}$ is the elasticity of the offer curve for exports from o to d. Importantly, none of the steps used to derive Eq. (B142) rely on prices, equilibrium conditions, or any assumptions about country o's behavior other than the fact that the offer curve is given by (B138b).

Define the negotiated price index, $\bar{N}_{do}$, and the aggregate terms of trade in country d (the price of exports from d over the price of imports from o) respectively as

$$\bar{N}_{do}Q_{do} = IM_{do} \quad \text{(B143a)} \qquad T_{do}\left(Q_{do}, Q_{od}\right) = \frac{\bar{N}_{od}\left(Q_{od}\right)}{\bar{N}_{do}\left(Q_{od}, Q_{do}\right)}. \quad \text{(B143b)}$$

Multiply both sides of Eq. (B142) by Eq. (B143b) to get that

$$\frac{T_{do}\left(Q_{do}, Q_{od}\right)}{MRT_{od}/MRS_{do}} = \frac{T_{do}\left(Q_{do}, Q_{od}\right)\frac{Q_{od}}{Q_{do}}}{H_{do}}. \tag{B144}$$

If both countries in this two-country setting maximize utility, then trade between them must be balanced implying that:

$$1 = \frac{IM_{od}}{IM_{do}} = \frac{\bar{N}_{od}\left(Q_{od}\right)Q_{od}}{\bar{N}_{do}\left(Q_{od}, Q_{do}\right)Q_{do}} = T_{do}\left(Q_{do}, Q_{od}\right)\frac{Q_{od}}{Q_{do}}. \tag{B145}$$

Using balanced trade with Eq. (B144) gives

$$\frac{T_{do}\left(Q_{do}, Q_{od}\right)}{MRT_{od}/MRS_{do}} = \frac{1}{H_{do}}, \tag{B146}$$

which defines the optimal condition for the social planner and corresponds to Eq. (15) in Costinot, Rodríguez-Clare, and Werning (2020).

B.2.1.2 Optimal uniform tariffs

Use the fact that because the competitive equilibrium coincides with the social planners solution for consumption, we know that

$$MRS^{d}_{od} = \frac{P_{do}}{P_{dd}}. \tag{B147}$$

Use Eqs. (B147) and (B143b) and multiply and divide the left side of Eq. (B146) by $1/\left(\bar{N}_{od}/\bar{N}_{dd}\right)$ and simplify to write the optimal condition as

$$\frac{\bar{N}_{dd}/\bar{N}_{do}}{\left(\frac{MRT^{d}_{od}}{\bar{N}_{od}/\bar{N}_{dd}}\right)\frac{1}{P_{do}/P_{dd}}} = \frac{1}{H_{do}} \tag{B148}$$

Using Eq. (24) in Eq. (B148) and the fact that tax/tariff policies other than t_{do} are passive so that $t_{dd} = t_{od} = t_{oo} = 1$ implies that Eq. (B148) can be written as

$$t^{u}_{do} = \left(\frac{1 - b^{u}_{do}}{1 - b^{u}_{dd}}\right)\left(\frac{MRT^{d,u}_{od}}{\bar{N}^{u}_{od}/\bar{N}^{u}_{dd}}\right)\frac{1}{H^{u}_{do}}, \tag{B149}$$

which is non-negative because $\left(1 - b^{u}_{do}\right)/\left(1 - b^{u}_{dd}\right) \geq 0$, $H^{u}_{do} > 0$, and $MRT^{d,u}_{od} > 0$.

B.2.2 Parameters for numerical examples

We present the parameter values for the numerical examples in Table A1. The first column shows the parameters in the model with search frictions and the second column has the unit of each parameter. The parametrization assumes that there are 2 symmetric countries. The only difference between the numerical example with and without search frictions is that the model without search has $c_{hf} = 0$.

B.2.3 The inverse of the elasticity of the offer curve: Proposition 3

We prove Eq. (36) in Proposition 3. Because all derivations are done using variables evaluated at the uniform optimal tariff, Eq. (B144) can be used to write the offer curve evaluated at the uniform optimal tariff in equilibrium, which is rewriting trade balance, as

$$\frac{Q_{od}}{Q_{do}\left(Q_{od}\right)} = \frac{1}{T_{d}\left(Q_{do}, Q_{od}\right)}. \tag{B150}$$

Taking logs and the total differential of this expression gives

$$d\ln Q_{od} - d\ln Q_{do}\left(Q_{od}\right) = -d\ln T_{d}\left(Q_{do}, Q_{od}\right) \tag{B151}$$

Dividing both sides by $d\ln Q_{do}\left(Q_{od}\right)$ and simplifying delivers

$$\frac{d\ln Q_{od}}{d\ln Q_{do}\left(Q_{od}\right)} - 1 = -\frac{d\ln T_{d}\left(Q_{do}, Q_{od}\right)}{d\ln Q_{do}\left(Q_{od}\right)} \tag{B152}$$

so that

$$\frac{1}{H_{do}} = \frac{d\ln Q_{od}}{d\ln Q_{do}\left(Q_{od}\right)} = 1 - \frac{d\ln T_{d}\left(Q_{do}, Q_{od}\right)}{d\ln Q_{do}\left(Q_{od}\right)}, \tag{B153}$$

which relies on the definition of H_{do} from Eq. (34b). Using Eqs. (B143b) and (24) gives

$$\frac{d\ln T_d\left(Q_{do},Q_{od}\right)}{d\ln Q_{do}\left(Q_{od}\right)}=-\frac{d\ln\left(1-b_{do}\right)/\left(1-b_{od}\right)}{d\ln Q_{do}\left(Q_{od}\right)}+\frac{d\ln\left(P_{od}/\left(P_{do}/t_{do}\right)\right)}{d\ln Q_{do}\left(Q_{od}\right)}+\frac{d\ln\left(P_{oo}/P_{oo}\right)}{d\ln Q_{do}\left(Q_{od}\right)},\tag{B154}$$

in which we have added the last term (equal to 0) to relate the result to CRW. Rearranging this expression provides

$$\frac{d\ln T_d\left(Q_{do},Q_{od}\right)}{d\ln Q_{do}\left(Q_{od}\right)}=-\frac{d\ln\left(\left(P_{do}/t_{do}\right)/P_{oo}\right)}{d\ln Q_{do}\left(Q_{od}\right)}+\frac{d\ln\left(P_{od}/P_{oo}\right)}{d\ln Q_{do}\left(Q_{od}\right)}-\frac{d\ln\left(1-b_{do}\right)/\left(1-b_{od}\right)}{d\ln Q_{do}\left(Q_{od}\right)},\tag{B155}$$

so

$$\frac{d\ln T_d\left(Q_{do},Q_{od}\right)}{d\ln Q_{do}\left(Q_{od}\right)}=-\epsilon_{\mathcal{P}_{do},Q_{do}}+\epsilon_{\mathcal{P}_{od},Q_{do}}-\epsilon_{\mathcal{B}_{do},Q_{do}},\tag{B156}$$

in which

$$\epsilon^u_{\mathcal{P}_{od},Q_{do}}=\frac{d\ln\left(P^u_{od}/P^u_{oo}\right)}{d\ln Q_{do}\left(Q_{od}\right)}.\tag{B157}$$

Therefore, Eq. (B153) implies that the inverse of the elasticity of the offer curve is

$$\frac{1}{H_{do}}=1+\epsilon_{\mathcal{P}_{do},Q_{do}}-\epsilon_{\mathcal{P}_{od},Q_{do}}+\epsilon_{\mathcal{B}_{do},Q_{do}}.\tag{B158}$$

Notice that

$$\epsilon_{\mathcal{P}_{od},Q_{do}}=\frac{d\ln\left(P_{od}/P_{oo}\right)}{d\ln Q_{do}\left(Q_{od}\right)}=\frac{d\ln\left(MRS^o_{do}\right)}{d\ln Q_{do}\left(Q_{od}\right)}=\frac{d\ln\left(Q_{od}/Q_{oo}\right)^{-\frac{1}{\sigma}}}{d\ln Q_{do}\left(Q_{od}\right)}\tag{B159}$$

because

$$MRS^o_{do}=\frac{\partial U_o\left(Q_{oo},Q_{od}\right)/\partial Q_{od}}{\partial U_o\left(Q_{oo},Q_{od}\right)/\partial Q_{oo}}=\left(\frac{Q_{od}}{Q_{oo}}\right)^{-\frac{1}{\sigma}}\tag{B160}$$

so

$$\epsilon_{\mathcal{P}_{od},Q_{do}}=-\frac{1}{\sigma}\frac{1}{H_{do}}-\frac{1}{\sigma}\left(\frac{1}{x_{oo}}-1\right),\tag{B161}$$

in which we use Eq. (34b) and substituted $d\ln Q_{oo}/d\ln Q_{do}\left(Q_{od}\right)=-\left(\left(1/x_{oo}\right)-1\right)$ from Eq. (36c). Plugging this into Eq. (B158) and simplifying yields Eq. (36).

B.2.4 Recovering the optimal tariff formula in the model without search frictions

In the no search model the inverse of the elasticity of the offer curve and the optimal tariff are equal, as discussed in Section 3.2, and $\epsilon_{\mathcal{B}_{do},Q_{do}}=0$ so that Eq. (36) implies

$$t^u_{do}=\frac{1}{H^u_{do}}=1+\mu\epsilon^u_{\mathcal{P}_{do},Q_{do}}+\frac{1}{\left(\sigma-1\right)x^u_{oo}}=1+\frac{1+\epsilon^u_{\mathcal{P}_{do},Q_{do}}\sigma x^u_{oo}}{\left(\sigma-1\right)x^u_{oo}},\tag{B162}$$

in which $\epsilon_{\mathcal{P}_{do},Q_{do}}$ equals the EoT because

$$\epsilon_{\mathcal{P}_{do},Q_{do}}=\frac{d\ln\left(\left(P_{do}/t_{do}\right)/P_{oo}\right)}{d\ln Q_{do}\left(Q_{od}\right)}=\frac{d\ln MRT^o_{do}}{d\ln Q_{do}\left(Q_{od}\right)}.\tag{B163}$$

See pg. 2760 for the definition of the EoT in CRW and see Eq. S.F.4 for the relationship between MRT^o_{do} and the ratio of prices.

Also notice that

$$\frac{d\ln Q_{oo}\left(Q_{do}\right)}{d\ln Q_{do}\left(Q_{od}\right)} = \left(-MRT^o_{do}\right)\frac{Q_{do}}{Q_{oo}} = \left(-\frac{P_{do}/t_{do}}{P_{oo}}\right)\frac{Q_{do}}{Q_{oo}} = -\frac{C_{do}/t_{do}}{C_{oo}}. \tag{B164}$$

So Eq. (36c) simplifies to

$$x_{oo} = \frac{C_{oo}}{C_{oo}+C_{od}}, \tag{B165}$$

which uses the fact that with no search frictions trade balance implies that $t_{do}C_{od} = C_{do}$ because $IM_{od} = C_{od}$ and $IM_{do} = C_{do}/t_{do}$. CRW define x_{oo} on pg. 2760 under Eq. 22 as the share of expenditure on domestic goods by country o. Finally, notice that using the equation for the EoT in footnote 20 of CRW, Eq. (B162) can also be written as

$$t^u_{do} = 1 + \frac{1}{\left[\mu\theta - 1\right]x^u_{oo}}. \tag{B166}$$

B.2.5 The price elasticity: Proposition 4

We prove Eq. (38) in Proposition 4. Using P_{do} from Eq. (A44), the relevant ratio in Eq. (36b) becomes

$$\frac{P_{do}/t_{do}}{P_{oo}} = \frac{\left[\left(1-\frac{u_{do}}{1-i_{do}}\right)\int_{\bar{\varphi}_{do}}^{\infty}\left(\frac{\tau_{do}}{\varphi}\right)^{1-\sigma}dG\left(\varphi\right)\right]^{\frac{1}{1-\sigma}}}{\left[\left(1-\frac{u_{oo}}{1-i_{oo}}\right)\int_{\bar{\varphi}_{oo}}^{\infty}\left(\frac{\tau_{oo}}{\varphi}\right)^{1-\sigma}dG\left(\varphi\right)\right]^{\frac{1}{1-\sigma}}}, \tag{B167}$$

in which t_{do}, N^x_o, μ, and w_o cancel out. Next, use the Pareto moment from Eq. (A45) to compute the integrals of Eq. (B167)

$$\frac{P_{do}/t_{do}}{P_{oo}} = \frac{\left[\left(1-\frac{u_{do}}{1-i_{do}}\right)\tau_{do}^{1-\sigma}\frac{\theta\bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta-\sigma+1}\right]^{\frac{1}{1-\sigma}}}{\left[\left(1-\frac{u_{oo}}{1-i_{oo}}\right)\tau_{oo}^{1-\sigma}\frac{\theta\bar{\varphi}_{oo}^{\sigma-\theta-1}}{\theta-\sigma+1}\right]^{\frac{1}{1-\sigma}}}. \tag{B168}$$

Eqs. (A74) and (18) imply that

$$\bar{\varphi}_{do}^{\sigma-\theta-1} = \mu^{\sigma-1-\theta}\sigma^{1+\frac{\theta}{1-\sigma}}\left(w_o\tau_{do}\right)^{\sigma-1-\theta}\left(w_oF\left(\kappa_{do}\right)\right)^{1+\frac{\theta}{1-\sigma}}t_{do}^{\mu(\sigma-1-\theta)}\left(\frac{P_{do}^{-\sigma}}{Q_{do}}\right)^{1+\frac{\theta}{1-\sigma}}. \tag{B169}$$

Plug this back into Eq. (B168) and rearrange to obtain

$$\frac{P_{do}/t_{do}}{P_{oo}} = \left(\frac{\tau_{do}}{\tau_{oo}}\right)^{\frac{\theta(\sigma-1)}{\sigma\theta-(\sigma-1)}} \left(\frac{F\left(\kappa_{do}\right)}{F\left(\kappa_{oo}\right)}\right)^{\frac{\theta-(\sigma-1)}{\sigma\theta-(\sigma-1)}} \left(\frac{Q_{oo}}{Q_{do}}\right)^{\frac{\theta-(\sigma-1)}{\sigma\theta-(\sigma-1)}} \left[\frac{\left(1-\frac{u_{oo}}{1-i_{oo}}\right)}{\left(1-\frac{u_{do}}{1-i_{do}}\right)}\right]^{\frac{(\sigma-1)}{\sigma\theta-(\sigma-1)}}. \tag{B170}$$

As such,

$$\begin{aligned}\epsilon_{\mathcal{P}_{do},Q_{do}} &= \left[\frac{\theta-(\sigma-1)}{\sigma\theta-(\sigma-1)}\right]\left[\frac{d\ln Q_{oo}\left(Q_{do}\right)}{d\ln Q_{do}\left(Q_{od}\right)}-1\right]+\left[\frac{\theta-(\sigma-1)}{\sigma\theta-(\sigma-1)}\right]\frac{d\left(\ln\left(F_{do}/F_{oo}\right)\right)}{d\ln Q_{do}\left(Q_{od}\right)}\\ &+ \left[\frac{(\sigma-1)}{\sigma\theta-(\sigma-1)}\right]\frac{d\ln\left(1-\frac{u_{oo}}{1-i_{oo}}\right)}{d\ln Q_{do}\left(Q_{od}\right)}-\left[\frac{(\sigma-1)}{\sigma\theta-(\sigma-1)}\right]\frac{d\ln\left(1-\frac{u_{do}}{1-i_{do}}\right)}{d\ln Q_{do}\left(Q_{od}\right)}\end{aligned}$$

Recall that

$$\frac{d\ln\left(1-\frac{u_{do}}{1-i_{do}}\right)}{d\ln Q_{do}} = \left(\frac{u_{do}}{1-i_{do}}\right)(1-\eta)\frac{d\ln\kappa_{do}}{d\ln Q_{do}} \tag{B171}$$

so that

$$\begin{aligned}\epsilon_{\mathcal{P}_{do},Q_{do}} &= -\frac{\theta-(\sigma-1)}{\left[\sigma\theta-(\sigma-1)\right]x_{oo}}+\left[\frac{\theta-(\sigma-1)}{\sigma\theta-(\sigma-1)}\right]\left[\frac{d\ln F_{do}}{d\ln Q_{do}}-\frac{d\ln F_{oo}}{d\ln Q_{do}}\right] \qquad \text{(B172)}\\ &- (1-\eta)\left[\frac{(\sigma-1)}{\sigma\theta-(\sigma-1)}\right]\left[\left(\frac{u_{do}}{1-i_{do}}\right)\frac{d\ln\kappa_{do}}{d\ln Q_{do}}-\left(\frac{u_{oo}}{1-i_{oo}}\right)\frac{d\ln\kappa_{oo}}{d\ln Q_{do}}\right],\end{aligned}$$

in which we use Eq. (36c). Assuming that $h_{oo}=-l_{oo}$, $h_{do}=-l_{do}$, and $s_{oo}=s_{do}=0$ so that F_{oo} and F_{do} are parameters and $\frac{d\ln F_{do}}{d\ln Q_{do}}=\frac{\partial\ln F_{oo}}{\partial\ln Q_{do}}=0$ yields Eq. (38).

Under additional restrictions we show that $\epsilon^{s}_{\mathcal{P}_{do},Q_{do}}\leq\epsilon^{ns}_{\mathcal{P}_{do},Q_{do}}$, in which the "s" superscript denotes search and "ns" denotes no search. We argue in Section 3.2 that $d\ln\kappa_{do}/d\ln Q_{do}\geq 0$ and $d\ln\kappa_{oo}/d\ln Q_{do}\leq 0$. Now we present an argument that $x^{s}_{oo}\geq x^{ns}_{oo}$. Start with an economy that has no search frictions so that $c_{do}=0\forall do$ and x_{oo} satisfies Eq. (B165). Now increase search costs in the do market only. We know that these search costs reduce imports to country d from country o because of Eq. (23). Trade balance implies that imports to country o from country d also fall. Because country o has passive trade policy, lower imports mean lower consumption of differentiated goods in the od market, C_{od}, reinforced by an import markdown. As such, if effects on domestic consumption, C_{oo}, are relatively small, this increase in search costs, c_{do}, imply an increase in the domestic consumption share in country o.

B.2.6 Offer-curve elasticities in the model with and without search

We provide a sufficient condition so that the inverse of the elasticity of the offer curve in the model with search frictions is below that in the model without search frictions,

$1/H^s_{do} \leq 1/H^{ns}_{do}$. Using Eqs. (36) and (B162) the condition is true if

$$\mu\epsilon^s_{\mathcal{P}_{do},Q_{do}} + \frac{1}{(\sigma-1)\,x^s_{oo}} + \mu\epsilon^s_{\mathcal{B}_{do},Q_{do}} \leq \frac{1}{(\sigma-1)\,x^{ns}_{oo}} + \mu\epsilon^{ns}_{\mathcal{P}_{do},Q_{do}}. \tag{B173}$$

If $\epsilon_{\mathcal{B}_{do},Q_{do}} \approx 0$, the sufficient condition becomes

$$\frac{1}{(\sigma-1)\,x^s_{oo}} + \mu\epsilon^s_{\mathcal{P}_{do},Q_{do}} \leq \frac{1}{(\sigma-1)\,x^{ns}_{oo}} + \mu\epsilon^{ns}_{\mathcal{P}_{do},Q_{do}}. \tag{B174}$$

We discuss that $\epsilon^s_{\mathcal{P}_{do},Q_{do}} \leq \epsilon^{ns}_{\mathcal{P}_{do},Q_{do}}$ in Appendix B.2.5, so that the sufficient condition becomes

$$\frac{1}{(\sigma-1)\,x^{s,u}_{oo}} \leq \frac{1}{(\sigma-1)\,x^{ns,u}_{oo}}, \tag{B175}$$

and yields:

$$x^{s,u}_{oo} \geq x^{ns,u}_{oo}. \tag{B176}$$

Simply put, if x_{oo} at the optimal tariff is larger in the model with search frictions than without them, that reduces the inverse of the offer-cure elasticity in Eq. (36) and provides additional incentives to reduce the optimal tariff in Eq. (34).

B.2.7 Optimal tariffs and the tariff elasticity

Without selection and d having a labor endowment that is small relative to the origin country o, Gros (1987) shows that optimal unilateral tariff is Eq. (B177a), in which $x^u_{oo}\,(t^u_{do})$ is defined in Eq. (B165) for a model without search frictions. When d is not small, Gros (1987) shows the optimal tariff is Eq. (B177b). With selection and θ as the parameter that governs the Pareto distribution of productivity and d small, Demidova and Rodríguez-Clare (2009) shows the optimal tariff is Eq. (B177c). With selection, the same productivity assumptions, and when d is not small, Felbermayr, Jung, and Larch (2013) show the optimal tariff is Eq. (B177d).

$$t^u_{do} = 1 + \frac{1}{(\sigma-1)}, \tag{B177a}$$

$$t^u_{do} = 1 + \frac{1}{(\sigma-1)\,x^u_{oo}\,(t^u_{do})}, \tag{B177b}$$

$$t^u_{do} = 1 + \frac{1}{(\mu\theta-1)}, \tag{B177c}$$

$$t^u_{do} = 1 + \frac{1}{(\mu\theta-1)\,x^u_{oo}\,(t^u_{do})}. \tag{B177d}$$

Tariff elasticities fully determine the optimal tariff only when the destination country d is small. In particular, the tariff elasticity without selection from Eq. (A69), derived in Appendix A.3.4.2, only provides the optimal tariff, Eq. (B177a), when d is small. Likewise, the tariff elasticity with selection from Eq. (A62), derived in Appendix A.3.4.1, only provides the optimal tariff, Eq. (B177c), when d is small. With search frictions, the tariff elasticity (Eq. A60) does not independently determine the optimal tariff in Eq. (34) due to the markdown ratio, among other factors.

B.2.8 Exogenous search frictions and optimal tariffs

We consider a modification to our framework to study optimal tariffs in a model with search frictions that delivers a globally efficient decentralized equilibrium. Specifically, we consider a model in which producers' unmatched rates are taken as given, so that we do not endogenize vacancy creation as in Eq. (19). This assumption appears in a vast literature about labor markets in which unmatched rates are taken as given. Specifically, set the match elasticity in Eq. (4) to one ($\eta = 1$) so that producers' unmatched rates are the same in every market

$$\kappa_{do}\chi\left(\kappa_{do}\right) = \xi, \tag{B178}$$

and the matched rates become

$$1 - \frac{u_{do}}{1 - i_{do}} = \frac{\xi}{\lambda + \xi}. \tag{B179}$$

To obtain allocative efficiency in our model with search frictions, We also assume that idle rates, i_{do}, are fixed so that there is no selection. This need arises because the investment term in Eq. (29) includes the idle rate. With these assumptions, the economy is otherwise the same as the model considered by Costinot, Rodríguez-Clare, and Werning (2020, p. 2741)—namely that consumers have constant elasticity of substitution (CES) preferences—so that each economy attains allocative efficiency and the decentralized equilibrium is also globally efficient (Dhingra and Morrow, 2019).

In any economy, efficient or not, the optimal tariff is given by Eq. (34). It is easy to show that Eq. (B178) implies that b_{do} is a function of only parameters so that

$$\frac{\partial \ln\left(1 - b^u_{do}\right)}{\partial \ln Q^u_{do}} = \frac{\partial \ln\left(1 - b^u_{od}\right)}{\partial \ln Q^u_{do}} = \frac{\partial \ln\left(1 - b^u_{od}\right)}{\partial \ln Q^u_{od}} = \frac{\partial \ln\left(1 - b^u_{do}\right)}{\partial \ln Q^u_{od}} = 0.$$

We can also show that

$$\epsilon_{\mathcal{P}_{do}, Q_{do}} = -\left[\frac{\theta - (\sigma - 1)}{\left[\sigma\theta - (\sigma - 1)\right] x_{oo}}\right]$$

in this case, which is the same expression as in the model without search frictions. Therefore, the optimal tariff with exogenous search frictions is

$$t^u_{do} = \left(\frac{1 - b^u_{do}}{1 - b^u_{dd}}\right)\left(1 + \frac{1}{\left[\mu\theta - 1\right] x^u_{oo}}\right). \tag{B180}$$

For comparison, the optimal tariff without search frictions is determined by Eq. (B166).

Optimal tariffs in the model with exogenous search and without search frictions will differ for at least two reasons. First, the ratio of markdowns in Eq. (B180) will in general not equal to one in the model with exogenous search, but it is equal to one in the model without search frictions. A sufficient condition for the ratio of the markdowns to equal to one in the model with exogenous search frictions is $\delta_{do}/F_{do} = \delta_{od}/F_{od}$, which yields:

$$\frac{f_{do} - l_{do} - \xi s_{do}}{f_{do} + \left(\frac{r+\lambda}{\beta\xi}\right) l_{do} + \left(1 + \frac{r+\lambda}{\beta\xi}\right) h_{do} + \left(\frac{r+\lambda}{\beta}\right) s_{do}} = \frac{f_{od} - l_{od} - \xi s_{od}}{f_{od} + \left(\frac{r+\lambda}{\beta\xi}\right) l_{od} + \left(1 + \frac{r+\lambda}{\beta\xi}\right) h_{od} + \left(\frac{r+\lambda}{\beta}\right) s_{od}} \tag{B181}$$

This condition is satisfied if $h_{ij} = -l_{ij}$, $s_{ij} = 0$ for $ij \in \{do, od\}$, $f_{do} = f_{od}$, and $l_{do} = l_{od}$, or, alternatively, if countries are symmetric. A sufficient condition that is stronger than the former and weaker than the latter is $\delta_{do} = \delta_{od}$ and $F_{do} = F_{od}$.

Second, the domestic consumption share, $x_{oo}\left(t_{do}\right)$, will differ in the two models if we consider large open economies because the equilibria will differ. If country d is a small open economy, equal import markdowns in the do and dd markets ensure that the optimal tariff is the same as in Demidova and Rodríguez-Clare (2009) because $x^u_{oo} = 1$.

In short, the optimal tariff is a fixed point that is a function of general equilibrium variables. These equilibrium variables are affected by the introduction of search frictions, even when the matching rates and idle rates are exogenous, the equilibrium is globally efficient, and each country attains allocative efficiency.

B.2.9 Efficiency conditions

Brancaccio, Kalouptsidi, Papageorgiou, and Rosaia (2023) derive conditions under which the decentralized equilibrium is globally efficient in a related environment with random endogenous search between customers and carriers. They derive several efficiency conditions in Theorem 1 that formalize our discussion in KM, Section 4.6. First, they derive conditions that are like those in Hosios (1990) for carriers and customers that internalize congestion and market thickness externalities on both sides of the market (Eqs. 22 and 27). These conditions require that the elasticity of the matching function with respect to searching carriers (customers) in each location must equal carriers' (customers') surplus share in that location. Second, they derive a condition that internalizes "composition" externalities (Eq. 25), which equalizes the surplus of carriers across different types of locations. They write that, "This logic holds in random search models with heterogeneity and it is not specific to the transport problem studied here" (pg. 2472). This result affirms that some search models—including ours—have participation and output externalities in which Hosios conditions alone will not ensure efficiency. Moreover, we conjecture that conditions similar to the ones in Brancaccio et al. (2023) will guarantee global efficiency of the decentralized equilibrium in our model with search frictions.

If that conjecture is correct, then the decentralized economy's efficiency depends on the bargaining power and the elasticity of the matching function in ways that are well understood. Specifically, if producers' (firms') surplus share, β $(1-\beta)$, is greater (less) than the elasticity of the matching function with respect to unemployment (vacancies), η (1-η), then firms are not compensated enough and the unmatched rate will be inefficiently high in the decentralized equilibrium. The opposite is true if producers' (firms') surplus share is less (greater) than the elasticity of the matching function with respect to unemployment (vacancies). This is the well-known Hosios (1990) result. The condition that internalizes composition effects is, in general, not satisfied if wages are determined by Nash bargain (Mangin and Julien, 2021), regardless of the values for β and η. Alternatively, competitive search models—those in which some agents can post prices and other agents direct their search to the most attractive alternatives—typically yield efficient market tightness in the decentralized economy, both in static and dynamic environments (Moen, 1997; Acemoglu and Shimer, 1999; Rogerson, Shimer, and Wright, 2005).

In summary, four distinct approaches would ensure global efficiency in our model. First, impose search frictions with exogenous unmatched and idle rates. Second, have a global social planner choose market tightnesses and have country social planners choose tariffs. Third, use the results of Brancaccio et al. (2023, Section II.C.3) and introduce various taxes—including taxes on every match and on retailers and producers—to internalize the externalities we mention above. Fourth, change our model from one with random endogenous search and Nash bargaining to one with directed search and price posting (competitive search).

We discuss the first approach in Section 3.3 and Appendix B.2.8. The other approaches are material departures from our current framework and are better suited for future work. That work can also assess the relationship between allocative efficiency in all countries and global efficiency.

B.3 Nash equilibrium and the Nikaidô-Isoda function

We can solve for the Nash equilibrium using the Nikaidô-Isoda (NI) function (Nikaidô and Isoda, 1955) given by

$$\Psi\left(\boldsymbol{t},\boldsymbol{\zeta}\right)=\sum_{d=1}^{D}\left[\mathcal{L}_d\left(\boldsymbol{t},\boldsymbol{\zeta}\right)-\sup_{\hat{\vec{t}}_{d*},\hat{\vec{\zeta}}_{d*}}\mathcal{L}_d\left(\hat{\boldsymbol{t}},\hat{\boldsymbol{\zeta}}\right)\right], \tag{B182}$$

in which the Lagrangian, $\mathcal{L}_d\left(\boldsymbol{t},\boldsymbol{\zeta}\right)$, is written as a function of the exogenous tariffs, $\boldsymbol{t}$, and exogenous matrix of Lagrange multipliers, $\boldsymbol{\zeta}$, corresponding to all of the constraints defined in Eq. (39). The Lagrangian is also a function of the endogenous variables—$\boldsymbol{\kappa},\bar{\boldsymbol{\varphi}},\vec{C},\vec{w}$—that define the economy's equilibrium, but those are determined by satisfying the constraints in Eq. (39) for given values of $\boldsymbol{t}$ and $\boldsymbol{\zeta}$. As such, we do not write out the endogenous variables explicitly in Eq. (B182).

Intuitively, each summand of the NI function in Eq. (B182) can be thought of as the difference in equilibrium welfare for a country d and that country's best response. When the summand for country d is zero, that country has no unilateral incentive to deviate. When the sum for all countries is zero, no country has a unilateral incentive to deviate. Hence, the Nash equilibrium is defined as $\Psi\left(\boldsymbol{t}^n,\boldsymbol{\zeta}^n\right)=0$ because this is when no country can benefit by unilaterally changing their tariffs. It is possible to show that $\Psi\left(\boldsymbol{t}^n,\boldsymbol{\zeta}^n\right)=0$ is a global maximum because $\Psi\left(\boldsymbol{t},\boldsymbol{\zeta}\right)\leq 0$.

C Calibration appendix

We use a calibration strategy similar to that in KM and include the details here. Table 1 summarizes the parameters and main sources of identification and Table 2 presents the data moments and model fit.

C.1 Externally calibrated parameters

We benchmark the producers' bargaining power, β, at 0.5 Drozd and Nosal (2012); Eaton, Eslava, Jinkins, Krizan, and Tybout (2014); Eaton, Jinkins, Tybout, and Xu (2016). We set the annual interest rate, r, to 5 percent. The average duration of a Chinese and U.S. trading relationship is about one year (Monarch and Schmidt-Eisenlohr, 2018, Fig. 9), so we set our separation parameter, $\lambda = 1$, because average match duration in the model is $1/\lambda$. We set the elasticity of substitution between differentiated varieties, σ, to four, consistent with median estimates in Broda and Weinstein (2006), and implying a final sales markup over marginal production cost of 33%. We set the Pareto parameter to be consistent with the firm-size distribution estimate from Axtell (2001) of $1.06 = \theta / (\sigma - 1)$, which implies that θ equals 3.18. We normalize the matching efficiency, ξ, to 1 as in Shimer (2005). Working age populations, L_c and L_u, are taken from total population of ages 15 to 64 from the World Bank (WB, 2016c,d).

Firm endowments, N_u^x N_c^x, are informed by the levels of aggregate consumptions in in China and the U.S., as reported in the national accounts of each country (BEA, 2016a; WB, 2016a; BEA, 2016c; NBSC, 2016b). The normalization value is chosen to express real GDP in trillions of dollars and satisfy purchase power parity of 60 percent between the United States and China. The iceberg origin parameter, $A_{o=u}$, and scale parameter, A, are normalized to one.

We choose domestic taxes, t_{dd}, to match the domestic sales tax in the US and the VAT rate in China. We choose import tariffs for China to match the import VAT rate, and we choose US import tariffs to match the tariff levels plus the domestic sales tax. We choose distance between and within the countries to match the data. Finally, we set the fraction of consumption expenditure on differentiated goods, α to 1 so that the calibration has no homogeneous good.

C.2 Intuition for parameter identification

The search frictions in our model are governed by retailers' flow search cost, c_{do}. If the fraction of matched exporters is low, it implies that there are few searching retailers, market tightness is low, and that international search costs are high. Consequently, we use the fact that 21 percent of Chinese firms export (WB, 2018) and that six percent of U.S. firms export to China (CB, 2016a,b) to identify c_{uc} and c_{cu}, respectively. Eaton et al. (2014) and Eaton et al. (2016) also use the fraction of firms that export to identify search model parameters. We use manufacturing capacity utilization to inform the level of domestic search frictions in goods markets, as in Michaillat and Saez (2015), Petrosky-Nadeau and Wasmer (2017) and Petrosky-Nadeau, Wasmer, and Weil (2018). We target 75 and 74 percent manufacturing capacity utilization in the United States and China in 2016, respectively, to inform c_{uu} and c_{cc} (FRB, 2020; NBSC, 2016a). We also assume that international search costs are larger than domestic search cost so that $c_{uc} \geq c_{uu}$ and $c_{cu} \geq c_{cc}$. We discuss how we calibrate the elasticity of the matching function, η, in Appendix C.3.

This observed expected duration is also broadly consistent with survival probabilities among Colombian-U.S. trading relationships (Eaton et al., 2014).

We use business failure rates to inform the fixed costs of production, f_{do}. Fixed costs inform the productivity thresholds in the model. In turn, these define the idle rates. Business failure rates capture the fraction of firms that cease production, which helps inform the measure of firms in the model that take a productivity draw and do not produce.

Labor share of GDP per working age person informs the wage in the US, and we normalize the Chinese wage to 11. The difference between consumption and GDP as reported in the national accounts of each country (BEA, 2016b; WB, 2016b; BEA, 2016d; NBSC, 2016c) and the exogenous choice of the number of product varieties, N_d^x, together inform the exploration cost e_d^x.

C.3 Calibrating import elasticities with respect to distance and tariffs

To calibrate to import elasticities with respect to distance and tariffs, we must first derive a log-linear estimating equation implied by our model that matches the specifications in the literature. Additionally, iceberg costs τ_{do} are not directly observed but are often modeled as a function of common colonial origin, common language, free or regional trade agreements, and distance D_{do}, for example. As such, we parameterize iceberg costs as

$$\tau_{do} = AA_dA_oD_{do}^{a_1}, \tag{C183}$$

in which A, A_d, A_o, and a_1 are parameters to be calibrated. Substituting Eq. (C183) into Eq. (23) for τ_{do}, taking logs and collecting similar indices of observation gives

$$\begin{aligned} \ln\left(IM_{do}\right) \;=\; & \ln\left(\alpha A^{-\theta}\right) + \ln\left(C_d\left(\frac{A_d}{\rho_d}\right)^{-\theta}\right) + \ln\left(A_o^{-\theta}N_o^x w_o^{-\theta} w_o^{-\left(\frac{\theta}{\sigma-1}-1\right)}\right) \\ +\; & \ln\left(D_{do}^{-\theta a_1}\right) + \ln\left(t_{do}^{-\mu\theta}\right) \\ +\; & \ln\left[\left(1-\frac{u_{do}}{1-i_{do}}\right)\left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}\right]. \end{aligned} \tag{C184}$$

Because the first three terms in Eq. (C184) are either constant or only vary by destination or origin, we can simplify notation by writing

$$\phi = \ln\left(\alpha A^{-\theta}\right), \qquad \phi_d = \ln\left(C_d\left(\frac{A_d}{\rho_d}\right)^{-\theta}\right), \qquad \text{and } \phi_o = \ln\left(A_o^{-\theta}N_o^x w_o^{-\theta} w_o^{-\left(\frac{\theta}{\sigma-1}-1\right)}\right). \tag{C185}$$

Also use Eqs. (20), (17), and (A58) to define the log of the terms that include search frictions as

$$z_{do}\left(\kappa_{do}\right) = \ln\left[\left(1-\frac{u_{do}}{1-i_{do}}\right)\left(1-b_{do}\right)F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}\right]. \tag{C186}$$

Importantly, the only endogenous variable in Eq. (C186) is κ_{do}. Using Eqs. (C185) and (C186), the log-linear gravity equation from our model, Eq. (C184), can be expressed as,

$$\ln\left(IM_{do}\right) = \phi + \phi_d + \phi_o - \theta a_1 \ln\left(D_{do}\right) - \mu\theta\ln\left(t_{do}\right) + z_{do}\left(\kappa_{do}\right), \tag{C187}$$

which is a restatement of imports in our model and therefore has no error term. If our model is true, then Eq. (C187) provides a_1 and fits the data perfectly. Gravity equation estimates in the literature omit search frictions, z_{do}, and instead estimate a specification similar to:

$$\ln(IM_{do}) = \phi + \phi_d + \phi_o + \zeta_1 \ln(D_{do}) + \zeta_2 \ln(t_{do}) + \varepsilon_{do}, \tag{C188}$$

in which ζ is a constant, ζ_d and ζ_o are fixed effects, and ε_{do} is an error term. Omitting z_{do} results in biased estimates of the true parameters. We can characterize that bias by estimating

$$z_{do}(\kappa_{do}) = \phi + \phi_d + \phi_o + \gamma_1 \ln(D_{do}) + \gamma_2 \ln(t_{do}) + \eta_{do} \tag{C189}$$

and then using the omitted variable bias formula (with the true coefficient on the z_{do} in Eq. (C187) being equal to one) to write

$$\mathbb{E}\left[\hat{\zeta}_1 \mid X_{do}\right] = -\theta a_1 + \gamma_1, \qquad \text{and} \qquad \mathbb{E}\left[\hat{\zeta}_2 \mid X_{do}\right] = -\mu\theta + \gamma_2, \tag{C190}$$

in which $X_{do} = (\phi, \phi_d, \phi_o, \ln(D_{do}), \ln(t_{do}))$.

Disdier and Head (2008, Table 1) provide a meta analysis of about 1,500 estimate of the effect of distance on trade. Head and Mayer (2014, Table 3.4) provide a similar analysis of 328 estimates. In both papers, those estimates are about -0.9 and so we set $-0.9 = -\theta a_1 + \hat{\gamma}_1$, which mostly helps to inform a_1. Because $\theta = 3.18$ is externally calibrated in Table 1, Eq. (C190) imposes a linear relationship between a_1 and $\hat{\gamma}_1$.

Fontagné, Guimbard, and Orefice (2022, p. 3) estimate the median elasticity of imports with respect to tariffs is about -5. As such, we set $-5 = -\mu\theta + \hat{\gamma}_2$, which mostly helps to inform η. Because $-\mu\theta = -3.18 \times (4/3) = -4.24$ is externally calibrated in Table 1, Eq. (C190) is effectively targeting $\gamma_2 = -0.76$.

To see why the estimates from Eq. (C189) combined with estimates from the literature help to inform η, notice that we can write

$$\gamma_1 = \mathbb{E}\left[(1-\eta)\left(\frac{u_{do}}{1-i_{do}}\right)\frac{d\ln\kappa_{do}}{d\ln(D_{do})} + \frac{d\ln(1-b_{do}(\kappa_{do}))}{d\ln(D_{do})} - \left(\frac{\theta}{\sigma-1}-1\right)\frac{d\ln F(\kappa_{do})}{d\ln(D_{do})} \mid X_{do}\right],$$

and also,

$$\gamma_2 = \mathbb{E}\left[(1-\eta)\left(\frac{u_{do}}{1-i_{do}}\right)\frac{d\ln\kappa_{do}}{d\ln(t_{do})} + \frac{d\ln(1-b_{do}(\kappa_{do}))}{d\ln(t_{do})} - \left(\frac{\theta}{\sigma-1}-1\right)\frac{d\ln F(\kappa_{do})}{d\ln(t_{do})} \mid X_{do}\right].$$

The A_o parameters are informed by the level of imports and domestic production. For example, raising the origin fixed effect for country d lowers domestic production in country d and lowers trade flows, but affects foreign domestic production little.

The A_d parameters are informed by the price indexes, Ξ_d, in each country but do not otherwise affect the model. For the former, notice that Eq. (A43) implies that for country d, A_d enters the price index in a linear, multiplicative way. As an example of the latter, notice that Eq. (22) implies that ρ_d is a linear multiplicative function of A_d, so that the ratio of τ_{do}/ρ_d in the gravity equation (Eq. 23) is unaffected because A_d cancels in the numerator and denominator. We choose A_d so that $\Xi_d = 1000 \forall d$ in our baseline calibration so that we can treat nominal values in billions.

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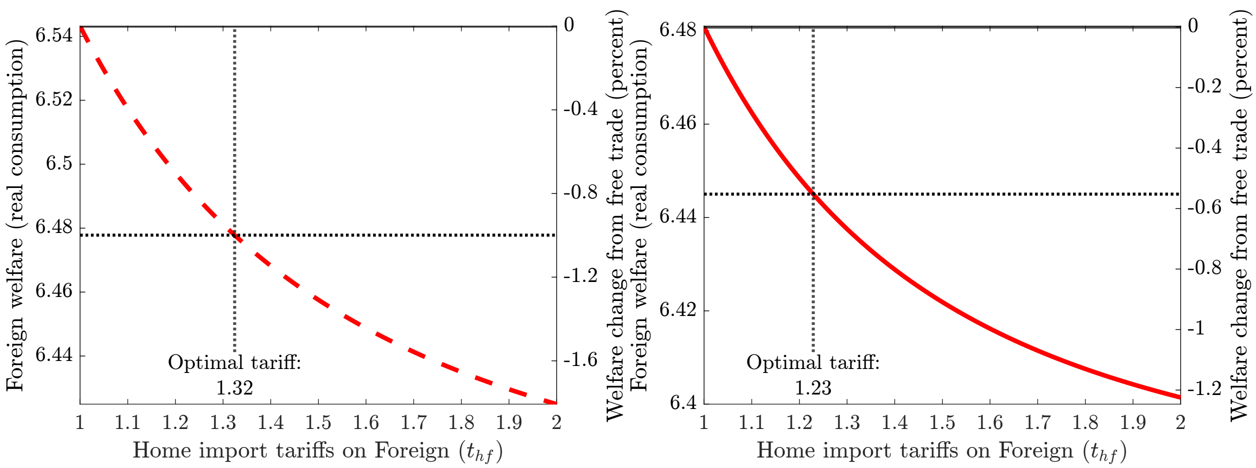
Fig. A1: Foreign welfare as a function of unilateral home tariffs: Numerical example



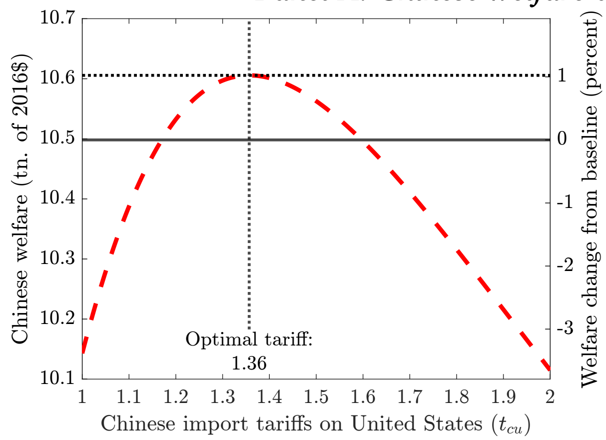
(a) Without search costs ($c_{do} = 0 \; \forall do$)

(b) With search costs ($c_{hf} > 0$)

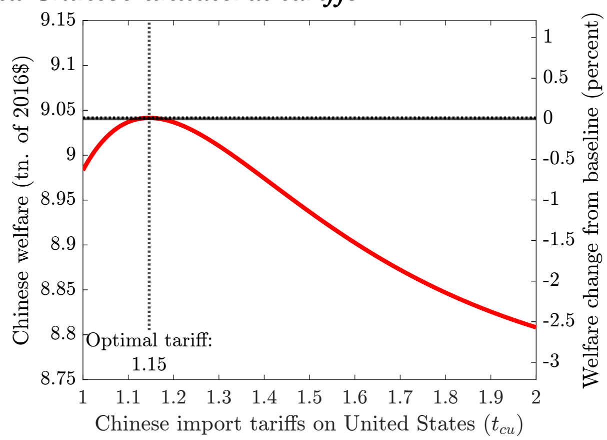
Note: Foreign welfare as a function of Home import tariffs. Fig. A1a plots Foreign welfare as function of the Home import tariff, t_{hf}, in the numerical example without search costs, $c_{do} = 0 \; \forall do$. Fig. A1b plots Foreign welfare as function of the Home import tariff, t_{hf}, in the numerical example with search costs, $c_{hf} > 0$ and $c_{do} = 0 \; \forall do \neq hf$. Similar to Figs. 2a and 2b but depict Foreign welfare instead of Home welfare. See Section 3.2 for further details.

Fig. A2: Chinese unilateral tariffs and Chinese and U.S. welfare for reduced and baseline search frictions

Panel A: Chinese welfare and Chinese unilateral tariffs

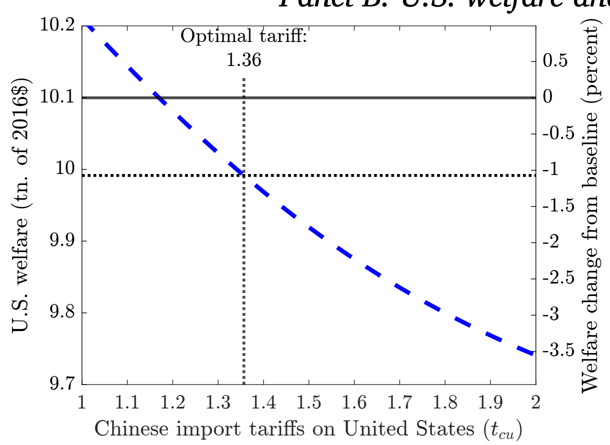


(a) Intl. search costs reduced to domestic levels

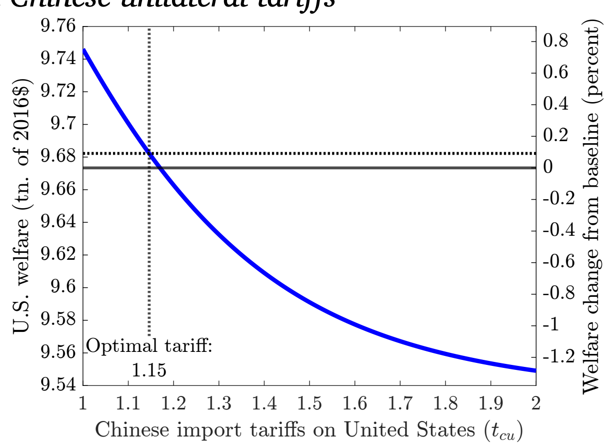


(b) Baseline search costs

Panel B: U.S. welfare and Chinese unilateral tariffs



(c) Intl. search costs reduced to domestic levels

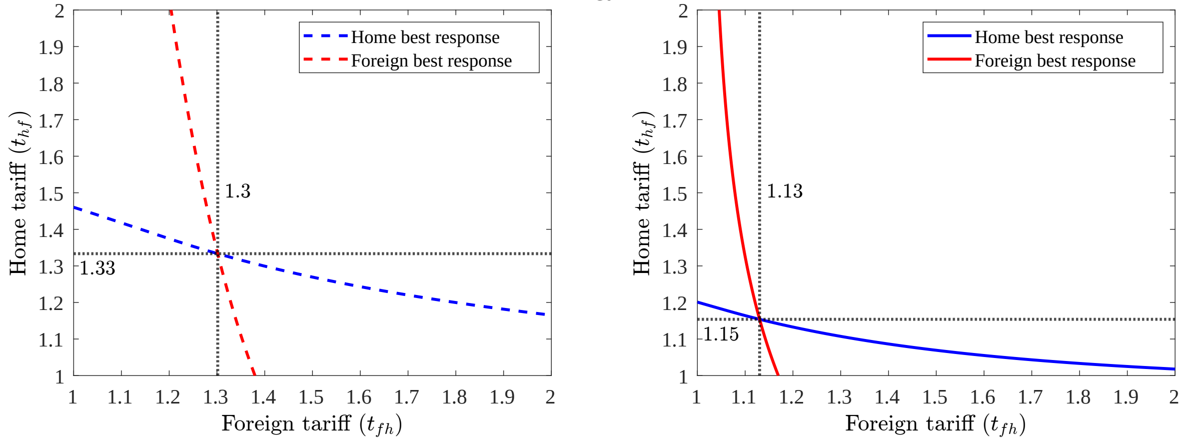


(d) Baseline search costs

Note: Fig. A2a (Fig. A2c) plots Chinese (U.S.) welfare as a function of Chinese import tariffs with all parameters as in the baseline but with international search costs reduced to their domestic levels and given by $c_{uu} \times 10^4 = 10.43$, $c_{cu} \times 10^4 = 15.28$, $c_{uc} \times 10^4 = 10.43$, $c_{cc} \times 10^4 = 15.28$. Fig. A2b (Fig. A2d) plots Chinese (U.S) welfare as a function of Chinese import tariffs in the baseline calibrated model, as in Table 1. The left vertical axis denotes levels and the right vertical axis denotes the welfare change from observed 2016 tariff levels. See Sections 5.1 and 5.2 for further details.

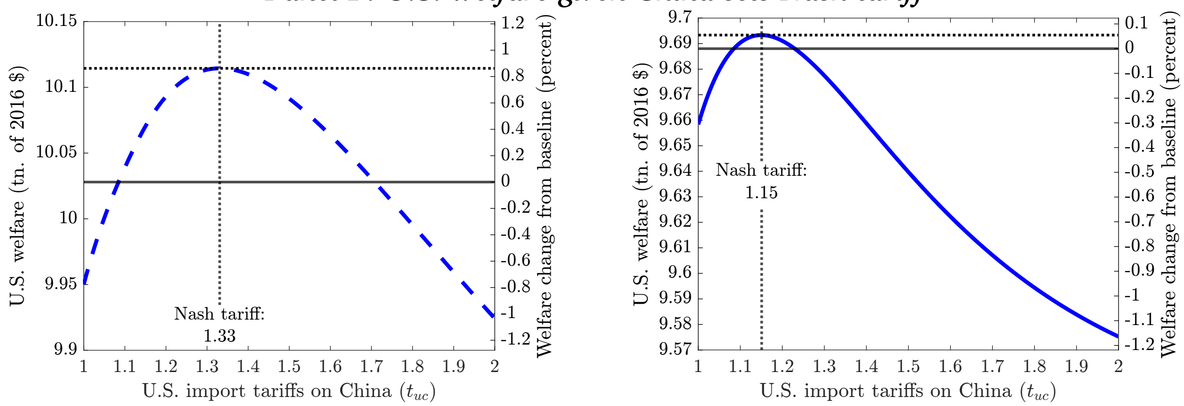
Fig. A3: Nash equilibrium best responses and welfare

Panel A: Pure strategy Nash equilibrium



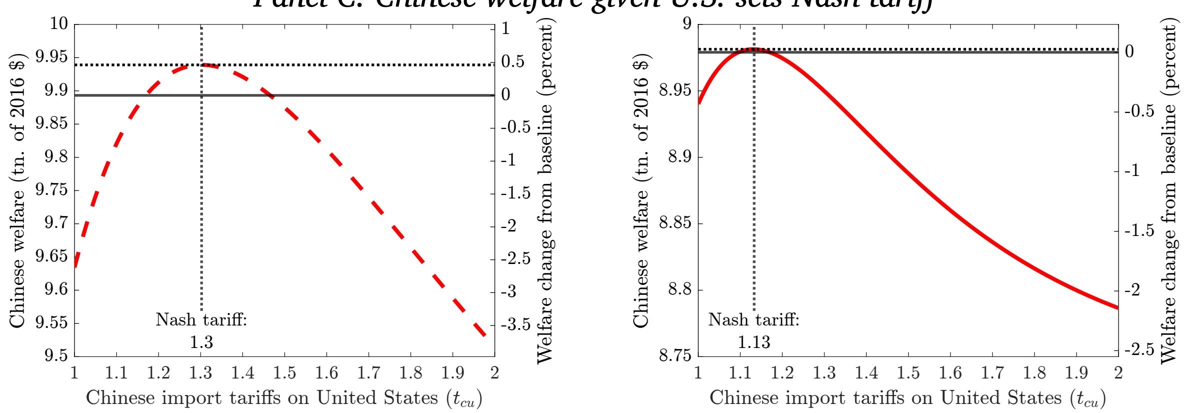
(a) Intl. search costs reduced to domestic levels (b) Baseline search costs

Panel B: U.S. welfare given China sets Nash tariff



(c) Intl. search costs reduced to domestic levels (d) Baseline search costs

Panel C: Chinese welfare given U.S. sets Nash tariff



(e) Intl. search costs reduced to domestic levels (f) Baseline search costs

Note: Figs. A3a, A3c, and A3e show results for a calibration in which all parameters are as in the baseline but with international search costs reduced to their domestic levels and given by $c_{uu} \times 10^4 = 10.43$, $c_{cu} \times 10^4 = 15.28$, $c_{uc} \times 10^4 = 10.43$, $c_{cc} \times 10^4 = 15.28$. Figs. A3b, A3d, and A3f show results for our baseline calibration, as in Table 1. Panel A shows best responses. Panel B shows U.S. welfare as a function of U.S. import tariffs. Panel C shows Chinese welfare as a function of Chinese import tariffs. In Panels B and C the other country's import tariff is set to its respective Nash equilibrium solution. See Sections 5.1 and 5.2 for further details.

Table A1: Parameters for numerical examples

Parameter	Value	Average	Unit
Panel A.			
Producers' bargaining power (β)	0.50		fraction
Risk-free rate (r)	0.05		percent
Separation rate (λ)	1.00		Poisson rate
Elasticity of substitution (σ)	4.00		elasticity
Pareto shape parameter (θ)	3.18		unitless
Efficiency of matching function (ξ)	1.00		elasticity
Labor endowment in US ($L_u \times 10^{-2}$)	10.00		mn. people
Labor endowment in CH ($L_c \times 10^{-2}$)	10.00		mn. people
Firm endowment in US ($N_u^x \times 10^{-2}$)	50.00		mn. varieties
Firm endowment in CH ($N_x^c \times 10^{-2}$)	50.00		mn. varieties
Iceberg origin US ($A_{o=u}$)	1.00		multiple
Iceberg scale (A)	1.00		multiple
US domestic tax (t_{uu})	1.00		multiple
CH import tariff (t_{cu})	1.00		multiple
US import tariff (t_{uc})	1.00		multiple
CH domestic tax (t_{cc})	1.00		multiple
Internal distance US to US (D_{uu})	1.00		kkm
Distance to CH from US (D_{cu})	10.00		kkm
Distance to US from CH (D_{uc})	10.00		kkm
Internal distance CH to CH (D_{cc})	1.00		kkm
Panel B.			
US domestic search cost ($c_{uu} \times 10^4$)	0.00	0.00	labor
CH importers' search cost ($c_{cu} \times 10^4$)	0.00	0.00	labor
US importers' search cost ($c_{uc} \times 10^4$)	5.00	8.27	labor
CH domestic search cost ($c_{cc} \times 10^4$)	0.00	0.00	labor
US domestic fixed cost ($f_{uu} \times 10^4$)	23.00		labor
US export fixed cost ($f_{cu} \times 10^4$)	4.50		labor
CH export fixed cost ($f_{uc} \times 10^4$)	4.50		labor
CH domestic fixed cost ($f_{cc} \times 10^4$)	23.00		labor
US exploration cost (e_u^x)	0.10		labor
CH exploration cost (e_c^x)	0.10		labor
Iceberg destination US ($A_{d=u} \times 10^{-2}$)	34.13		multiple
Iceberg destination CH ($A_{d=c} \times 10^{-2}$)	34.13		multiple
Iceberg origin CH ($A_{o=c}$)	1.00		multiple
Iceberg distance (a_1)	0.35		elasticity
Elasticity of matching function (η)	0.50		elasticity

Note: Parameters for the numerical examples in Section 3. The "Value" column shows the parameters in the model with search frictions except for $l_{do} = s_{do} = h_{do} = 0\ \forall do$. The only difference between parameterizations in the numerical examples with and without search frictions is that the model without search has $c_{hf} = 0$. The levels of the retailer search costs, c_{do}, do not have meaning because they depend on the normalization of the matching efficiency, ξ, as in Shimer (2005). As such, the "Average" column reports average retailer search costs, $c_{do}/\chi(\kappa_{do})$, which have intrinsic meaning. Calibrated parameters of the model are at annual frequency. "H" stands for Home and "F" stands for Foreign. See Appendix B.2.2 for details.