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**Modeling Money Market Spreads: What Do We Learn about
Refinancing Risk?**

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Modeling Money Market Spreads: What Do We Learn about Refinancing Risk?*

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Abstract

We quantify the effect of refinancing risk on euro area money market spreads, a major factor driving spreads during the financing crisis. With the advent of the crisis, market participants' perception of their ability to refinance over a given period of time changed radically. As a result, borrowers preferred to obtain funding for longer tenors and lenders were willing to provide funding for shorter tenors. This discrepancy resulted in a need to refinance more frequently in order to borrow over a given horizon, thus increasing refinancing risk. We measure refinancing risk by quantifying the sensitivity of the spread to the refinancing frequency. In order to do so we introduce a model to price EURIBOR-based money market spreads vis-à-vis the overnight index swap. We adopt a methodology akin to a factor model in which the parameters determining the spreads are the intensity of the crisis, its expected half-life, and the sensitivity of spreads to the refinancing frequency. Results suggest that refinancing risk affects the spread significantly across time, albeit in a largely varying manner. Central bank interventions have reduced the spreads as well as the effect of refinancing risk on them.

JEL classification: E58, G12, G21

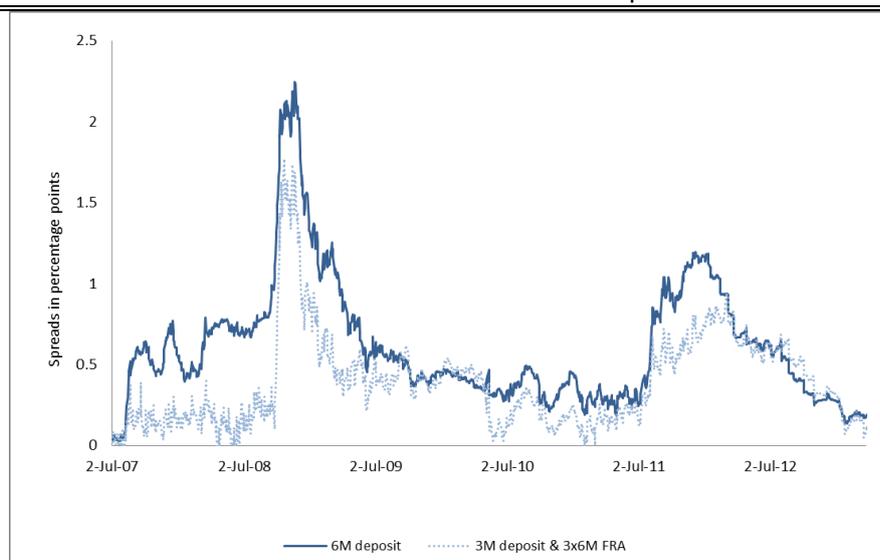
Keywords: money markets, money market spread, refinancing risk, liquidity risk, financial crisis

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1 Introduction

Following August 2007, money market rates changed their behavior dramatically.¹ A positive wedge appeared among money market rates with the same maturity but different floating-leg frequencies (Morini, 2009; Ametrano and Bianchetti, 2009; Mercurio, 2010). For example, a three-month deposit rate shot up compared with a three-month overnight index swap (OIS) rate, creating a spread that became the main policy reference for the intensity of the financial crisis. Similarly, a six-month deposit rate rose higher compared with alternative strategies for borrowing money for six months, such as a combination of a three-month deposit rate and a forward rate agreement (FRA) for three months, three months ahead (see chart 1). In circles of policymakers, academics, and central bankers, the question of what was driving the spreads sprang up vigorously in debates over how to tackle the rising spreads.

Chart 1: Six-month deposit spread versus a combination of a three-month deposit spread and a three-month on three-month FRA spread



Note: The chart presents the evolution of the six-month EURIBOR-OIS spread and a combination of the three-month EURIBOR-OIS spread and a three-month FRA-OIS spread, three months ahead. Source: Reuters and authors' calculations.

¹ The term “money market rates” in this paper defines the following instrument types: overnight index swap rates, forward rate agreement rates, EURIBOR and EONIA rates, interest rate swaps, and forward rates.

This paper argues that the main driver of the spreads was the risk of not being able to access the market to refinance at some future point in time—or, more simply, the risk of refinancing. In other words, what changed after the crisis was the perception of money market participants about their future ability to re-access the market. This change created a preference among lenders to refinance at shorter horizons and among borrowers to refinance at longer horizons. As a result, in order to obtain financing for a given period, borrowers needed to either access the markets more frequently or face a premium to avoid it. We argue that the differences in preferences between lenders and borrowers regarding refinancing frequency became a major driver of the spreads. We then measure the sensitivity of the spread to the refinancing frequency: The higher the sensitivity is, the higher the spread is, as the premium to avoid refinancing rises. The measure of sensitivity becomes our measure of refinancing risk.

In order to demonstrate the importance of the refinancing risk, a deeper understanding of the money market's functioning before and after the crisis is useful. Consider the spreads (EURIBOR-based rates versus the OIS) in chart 1. They represent two alternative strategies for borrowing money for six months: One would borrow money upfront for six months at the six-month deposit rate, and the other would borrow money for three months at the three-month deposit rate and lock in the rate for the remaining three months by entering into a FRA. The second strategy would entail entering the market *again* in three months for a second three-month period. This is because the FRA contract, being merely an agreement on rates and therefore an unfunded contract, would not commit the lender to lending the actual amount of money to the borrower for the second three-month period. The borrower would need to re-enter the market in three months and refinance for another three

months. The FRA contract would hedge interest rate risk away completely for the second three-month period, but it would not hedge away the risk of not being able to re-enter the market in three months. Overall, in this simple strategy the refinancing need arises once, after the first three-month period has elapsed, and it creates a refinancing risk that is not hedged away.

Before the crisis, such access would have been taken for granted, and the existence of different implicit refinancing legs would not have been priced; that is, the refinancing risk would have been zero. However, the crisis introduced a change in the risk perceptions of market access in the two strategies. Borrowers were willing to pay a premium in order to obtain liquidity for a given horizon with as few refinancing legs as possible, thus avoiding the risk of not being able to access the market at a later point; that is, borrowers wanted to avoid refinancing risk. Consequently, once refinancing risk was priced, the six-month deposit, which does not involve refinancing, shot up compared with the alternative strategy, which involves refinancing once.

The effect of different refinancing legs on the spreads, although important in the crisis, has not yet been measured. This paper fills this gap by introducing a pricing model for money market spreads that also measures the effect of the refinancing frequency on the spread, thus measuring the risk of refinancing.

The building block of the proposed pricing model is the “instantaneous forward spread,” which represents the expectation at the current time for the price of a commitment at a future time to lend money for an infinitesimally short period at some time further ahead. The price of this commitment is expressed, in our case, in terms of a spread from a baseline rate. This definition is flexible because any observed spread (in our case, a EURIBOR-based spread over the OIS rate) can then

be calculated as a sum of the elementary building blocks over the relevant period and over the different refinancing tenors that this period may involve.

The functional form of the instantaneous forward spread is akin to a factor model. It is parsimonious, yet it reflects empirical regularities of the spread and also captures the fact that higher refinancing tenors result in smaller spreads (for example, the spread between a one-year and a six-month swap rate is higher than that between a one-year and a one-month swap rate). In order to do that, the functional form allows the refinancing tenors to form distinct blocks. These blocks together determine the shape and size of the spread. To the extent that the refinancing blocks remain distinct, the frequency of refinancing affects the spread.

The flexibility of the functional form relies on three parameters, which relate to each other in a multiplicative manner. The first, α , captures the intensity of the crisis and can be seen as a broad measure of marketwide, systemic tensions. The multiplicative relationship among implies that the remaining parameters, β and γ , only matter when α is positive—in other words, this model is for a crisis.

The second, β , determines the extent to which the different refinancing legs form distinct blocks, thus capturing the sensitivity of the spread to the refinancing frequency. As the sensitivity rises, refinancing increasingly affects the spread. Therefore, the higher the sensitivity, the higher both the risk of refinancing and the spread would become.

Finally, the third parameter, γ , captures the expected length of the crisis in terms of half-lives. This is the first attempt, to the best of our knowledge, to produce an endogenously determined measure for the expected length of the crisis and the identification of its most intense periods. Up until now, crisis periods were mainly

identified based on the calendar days of relevant events. As the crisis period stretched over time, particularly in the euro area with the advent of the sovereign crisis, there was no mechanical way to establish the relative importance of the various events on money market sentiment. Our parameter, γ fills this gap.

The intuition and methodology of this paper, while novel, do relate to previous literature. The idea of using the instantaneous forward spread as a building block for our model relates to standard yield-pricing models, in which the zero-coupon yield is an equally weighted average of forward rates. Given the forward curve, any coupon bond can be priced as the sum of the present values of the future coupon and principal. The difference in this paper lies in the functional form, which essentially maps a three-dimensional space (spread, time of entry into the contract, and time to maturity), whereas the typical yield curve maps a two-dimensional space (rate and time to maturity). The proposed formula also reveals parameters with characteristics that are different from those in standard yield curve pricing.

Furthermore, a number of models have attempted to rationalize the existence of the spreads with reference to a credit premium, a liquidity premium, or both using regression analysis. Evidence in favor of one or the other type of risk is mixed. Most models in the literature conclude that both credit and liquidity factors were behind the increase in risk premiums in the interbank money market during the financial crisis. Some papers find a stronger role for credit factors (Morini, 2009; Taylor and Williams, 2009; Gorton and Metrick, 2012; Filipovic and Trolle, 2013). However, these conclusions have been challenged by other papers, which stress the importance of liquidity factors in determining the spreads (Michaud and Upper, 2008; Wu, 2008; McAndrews, Sarkar, and Wang, 2008; Schwartz, 2010). Soon it became obvious that separating the two effects is a daunting if not impossible task due to the endogeneity

between liquidity and credit risk (He and Milbrandt, 2013; Heider et al., 2010), especially in times of crises and while using proxies for both liquidity risk and credit risk, which complicate identification issues.

Indeed, in this paper we abstract from identifying the relative effect of liquidity risk versus credit risk and focus instead on refinancing risk, which we consider the major driver of the spreads. We measure refinancing risk directly from the underlying data based on characteristics specific to money markets. The results are time varying, thus providing information about the evolution of refinancing risk in the various phases of the crisis and the role of the central banks in denting it.

Moreover, in our case, refinancing risk relates to both liquidity and credit risk. More precisely, refinancing risk is typically directly related to liquidity risk, but under certain circumstances it may also be affected by counterparty credit risk (see section 3). We elaborate on these circumstances: In summary, we suggest that when credit risk affects the EURIBOR-panel banks in an asymmetric manner (that is, when some banks are more likely than others to drop from the panel at some point in the future), there is an option value for the lender to extending lending piecewise, and credit risk affects the frequency of refinancing. On the other hand, when credit risk affects the EURIBOR-panel banks in a relatively symmetric manner, there is a parallel increase in the spreads across instruments while the frequency of refinancing remains unaffected by the increase in the spreads.

Our definition of refinancing risk is linked to the theoretical literature on roll-over risk. He and Xiong (2012a) analyze the interaction between liquidity risk and credit risk through roll-over risk in a model of endogenous default in the corporate bond market. In their model, the effect of roll-over risk (and liquidity risk) on credit risk comes from a conflict between equity holders and bond holders over roll-over

losses, which affects the time to default.² A similar “conflict” of perceptions between borrowers and lenders occurs in our case over future refinancing access, which affects the refinancing frequency. However, in money markets, counterparty credit risk may also play a role.³ Moreover, in our case, we show that refinancing risk matters even when interest rate risk, a potentially important roll-over consideration, is hedged away. Regarding runs on financial firms, He and Xiong (2012b) suggest that roll-over risk relates to credit risk through the risk of a possible coordination failure among future maturing creditors while rolling over. In our context such a coordination risk could be relevant for the ability of the borrower to refinance at a future point in time. In this case, as we suggest, refinancing risk could be affected by counterparty credit risk as long as the other borrowers are not equally affected by it. If all borrowers are equally affected, refinancing does not matter because either it is consistently priced across refinancing strategies or because markets have frozen, so it does not occur. This reasoning is different than the one presented for secured money markets by Acharya et al. (2011), in which roll-over risk is at a maximum and leads to market freezes when the debt capacity (the collateral value) of the asset is a small fraction of its fundamental value.

Finally, the idea of refinancing risk as an insurance premium is also adopted by Drehmann and Nikolaou (2013) for funding liquidity risk, and is in line with the asset-pricing literature in which market liquidity risk can demand a premium (Holmstrom and Tirole, 2001; Acharya and Pedersen, 2005; Fontaine and Garcia, 2012).

² In the theoretical model of He and Xiong (2012) equity holders fully bear the roll-over losses when liquidity is low, whereas bond-holders are paid in full. This conflict implies that equity holders may choose to default earlier, as then bond holders can only recover their debt by liquidating the firm’s assets at a discount.

³ Note that the borrower’s own perception about his or her credit risk is not equally relevant, as the Euribor is an offer-rate and effectively the lender chooses the refinancing frequency of the borrower.

Results suggest that the proposed model prices market spreads very closely. Parameter α appears to broadly track the intensity of the crisis, recording its largest spikes at the beginning of the crisis, when Lehman Brothers fell, and during the euro-area crisis period. Parameter β suggests that, throughout the crisis, the frequency of refinancing had a strong effect on the spreads. However, the effect almost disappears at the period of the Lehman collapse. We argue that the collapse of Lehman led to an increase in all observed money market spreads due to increased overall credit risk in a manner that effectively muted the effect of the refinancing frequency on the spread. In other words, in a situation where markets freeze due to overall heightened credit risk, it is of little relevance whether lending is extended piecewise or not. On the contrary, our results suggest that during the European crisis, the effect of the refinancing frequency was higher, probably because the credit risk of certain institutions was affected more than other institutions, thus increasing the scope for lenders to extend piecewise lending. In terms of market sentiment, parameter γ identifies two plausible periods of low sentiment, one ranging from the beginning of the US money market crisis in August 2007 to one year after the Lehman collapse and the other spanning the euro area sovereign crisis (September 2011 to September 2012).

Results also suggest that central bank interventions are effective in lowering both α and β . Central bank liquidity interventions tend to lower the spreads (α), a result that is in line with previous research. In addition, central bank policies have been successful in lowering β by a modest amount, effectively lowering the sensitivity of the spread to the refinancing frequency.

The remainder of the paper is organized as follows. Section 2 describes the motivation of the paper, recording the arbitrage failures in money markets during the

crisis (section 2.1) and discussing the links between refinancing risk, liquidity, and credit risk (section 2.2). Section 3 presents the methodology of our pricing model. Section 4 discusses data sources and data manipulation. Section 5 presents the results. And section 6 concludes.

2 Motivation

2.1 The importance of the refinancing frequency in money markets

Consider an illustrative example: A borrower (a generic prime bank) would like to borrow money for a certain period of time—say, six months. Such borrowing could be undertaken in various ways. In the interest of simplicity, consider only two: 1) the borrower could borrow the money unsecured for six months (at the six-month EURIBOR), or 2) the borrower could borrow the money unsecured for only three months (at the three-month EURIBOR) and, at the same time, lock in the rate for the remaining three months of the six-month period by entering into a forward rate agreement.

In the second strategy, after the first three months elapsed, the borrower would repay the three-month deposit to the original lender. It would then need to enter the market again and borrow money for the three remaining months (from a potentially different lender) at the then-prevailing EURIBOR rate. It is important to clarify that the FRA is an unfunded contract that does not represent a commitment to lend money; it is for hedging and not funding purposes. Once the borrower refinanced in the market, the FRA would hedge away the risk of the EURIBOR rate for the second three-month period. This is because the FRA would give the borrower the right to

receive the EURIBOR rate (which the borrower would use to pay the lender for the second three-month period) but instead pay the agreed-upon FRA rate.⁴

If no risk is priced into re-borrowing the amount of money needed in three months, the two strategies would be equivalent, and arbitrage would ensure that they are consistently priced; that is, the six-month rate would be the same as the compounding of the three-month rate and the three-month FRA. In the opposite case, in which there is a risk that the borrower would not be able to refinance the loan in the market in three months, the strategy involving exchanging money upfront for the whole period of six months (only one refinancing leg) would be more costly. Therefore, the risk of being unable to refinance could be mitigated by avoiding refinancing legs for the borrower and by inducing refinancing legs for the lender. As a result, refinancing risk would drive up the price of strategies that involve fewer refinancing legs.

Turning from theory to observation, after August 2007, a wedge appeared between the two strategies, making the six-month deposit strategy relatively more expensive. As shown in chart 1, the spread (to the OIS rate) of the six-month deposit was consistently higher than the compounding of the three-month deposit spread and the three-month FRA spread.

Such spreads presented consistent characteristics. Notably, they remained positive for a significant period ahead, suggesting that spreads were pricing actual underlying risks. Moreover, this observation was consistent across a broad spectrum of money market instruments. For example, a one-year deposit rate was higher than

⁴ In practice FRA contracts are cash settled: Payments related to the FRA contract are calculated for a notional amount over a certain period and then netted. In other words, only the differential is paid when the FRA contract expires, not the principal.

a combination of a six-month deposit rate and a six-month FRA. The latter combination was in turn higher than a one-year swap with three-month refinancing frequencies and even higher than a one-year swap with a one-month refinancing frequency.⁵ In general, the lower the refinancing frequency was, the higher the spread became.

The illustrative example shows that the frequency of refinancing matters for the spread. It is therefore important to be able to quantify its effect on the spread. In order to do that, we need to model the euro area spreads.

2.2 How does the risk of refinancing relate to liquidity risk and credit risk?

Let us consider first how the frequency of refinancing affects liquidity risk. The link is direct: In line with definitions proposed by the literature (Diamond, 1991; Drehmann and Nikolaou, 2013), liquidity risk relates to the risk of being unable to access the market when money is needed or, equivalently, to the risk of refinancing. From the point of view of the borrower, liquidity risk may arise due to the inability to refinance at some point in the future, resulting for example from a market freeze, market illiquidity, or changes in the credit-worthiness of the borrower.⁶

⁵ In general, the FRA contract can be seen as a special case of an interest rate swap (IRS) contract, and therefore a similar logic would hold: Any IRS can be reduced to a combination of a deposit and FRAs. For example, a three-month swap for one year would be priced consistently with a three-month deposit and three three-month FRAs for each consecutive three-month period. Therefore, such an IRS contract would involve four legs and would be less expensive compared with a one-year deposit, as the latter would price the fact that money has been exchanged for one year.

Overall, OIS, FRA, and IRS contracts are agreements on rates only, useful to lock in the rates when combined with matching refinancing strategies aimed at obtaining longer-term financing while hedging away interest rate risk.

⁶ From the point of view of the lender, liquidity risk could arise if the lender needs the money already lent out to the borrower or if the borrower defaults on his payment to the lender at the end of the period, which would push the lender to the position of the borrower. We therefore consider liquidity risk from the point of view of the borrower.

Overall, in view of liquidity risk, a lender would prefer to lend piecewise; that is, to extend a loan with shorter refinancing legs. He or she would then assign higher prices to loans with less frequent refinancing legs. The reverse would happen for a borrower: The borrower would prefer to get the money for the whole period needed, rather than having to refinance within the period. This preference would create a spread between lending strategies of different refinancing legs, like the ones observed during the crisis. Therefore, liquidity risk and the risk of refinancing could be seen as synonymous.

However, as already hinted at by the example, refinancing risk can be affected by counterparty credit risk in our setting, albeit in a more subtle manner. Consider the following setting from the point of view of the lender: A lender faces a pool of banks and updates information about their creditworthiness over time. The lender knows that a fraction of them will default after a certain period ahead—for example, between three and six months from now—but not which ones will be in the defaulting fraction. The lender expects to learn more within the first three-month period. A generic bank of this group asks the lender for a loan of six months. Would the lender prefer to offer a loan for three months instead?

In this setting, the answer would be yes. At the time when the request for lending is made, the three-month deposit would reflect the probability of default of the underlying instrument (the EURIBOR in our case) given the original pool of banks; the FRA would reflect the probability of default given the pool of yet-unknown surviving banks; and the six-month deposit would reflect the probability of default given the original pool of banks over the six-month period. Therefore, in that setting, the six-month deposit would cost more than the alternative combination strategy if only credit risk was driving the spread. Overall, there is an implicit option

value for the lender to extend lending piecewise in order to reassess the creditworthiness of the borrower over time.

The crucial feature of the example above is the uncertainty over the number of banks that survive in the second three-month period. As the FRA is merely an unfunded bet on the EURIBOR rate in that period, it would be the surviving banks that would determine the rate. In other words, the FRA rates reflect the intensity of default of the banks in the EURIBOR panel at a future rate, while the current expectation of a three-month deposit contract three months ahead (the implied forward rate three months on three months) reflects the intensity of default of the banks that are currently (at the time the loan is requested) in the EURIBOR panel.

Consider now a different setting: A lender faces a pool of banks and updates information about their creditworthiness over time. The lender expects that after a certain period ahead—in this example, three months—*all* banks may suffer a similar credit shock (for example, due to a market freeze following the collapse of a systemically relevant bank), but all would survive (for example, because all of them are the highest-quality banks). In other words, the average or generic default risk of all banks increases in the second three-month period.

In this setting, the average EURIBOR rate would increase in the second three-month period to reflect the increased risk of default for the surviving prime banks. The increase would be such that a combination strategy of a three-month deposit and a three-month FRA three months ahead would cost the same as a six-month deposit. Seeing it differently, default risk would lead to a parallel increase of all rates across instruments, so that the underlying increase of the EURIBOR default risk would cancel out in the two alternative borrowing strategies and the frequency of refinancing would be irrelevant in that case. There is no option value for the lender

to extend lending piecewise in this setting.

The two examples above suggest that across time, depending on whether credit risk affects the pool of prime banks in an asymmetric manner, our measure of refinancing risk may or may not be affected by credit risk, but it is always affected by liquidity risk.

In practice, the pool of EURIBOR panel banks and the EURIBOR-based instruments present certain characteristics that could mitigate the (asymmetric) effect of credit risk in our measure of refinancing risk. To begin with, money market rates based on the EURIBOR are offer rates for generic prime banks, implying that individual credit risk should not enter into the spreads, whereas average credit risk is relatively low.⁷ If the borrower is (perceived by the lender to be) a prime bank—that is, a bank of the highest credit quality—the borrower would receive the observed prime rate. Moreover, in practice, the observed FRA has a maximum maturity of six months. In that sense, the asymmetry effect relies on the perceived probability that a bank which is currently prime will cease to be prime over the next 6 months. When such a concern becomes relevant for the pricing, the bank may not be considered as prime in the first place.

3 Methodology

This section defines the instantaneous forward spread, which represents the fundamental building block for the generic formula describing the spreads to the OIS

⁷ The EURIBOR is an “asked” rate, based on truncated average quotes that prime banks offer when asked at which rate a prime bank would extend a loan of a specific maturity to another prime bank. According to the Euribor code of Contact (EBF, 2013) “a Prime Bank should be understood as a credit institution of high creditworthiness for short-term liabilities, which lends at competitive market related interest rates and is recognized as active in Euro-denominated money market instruments while having access to the Eurosystem’s open market operations.”

rate of various instruments at specific maturities and refinancing frequencies (section 3.1). Section 3.2 assigns a functional form to this building block in order to understand the underlying driving factors. Section 3.3 describes the fitting methodology of the actual data to the theoretical functional form.

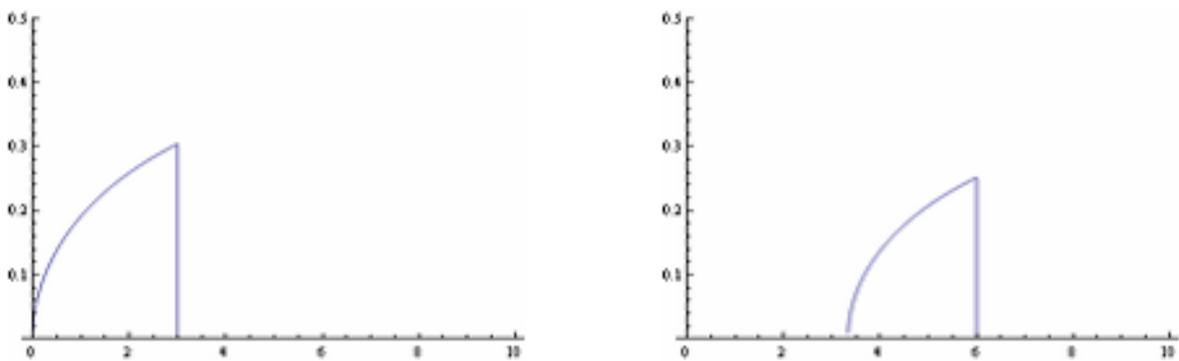
3.1 The generalized formula of spreads to the OIS rate

The building block of a generic formula of the spreads should have a convenient, flexible, and practical representation. The instantaneous forward spread serves this purpose. It represents the expectation at the current time, t , for the price (rate) of a commitment at time t_1 to lend money unsecured for an infinitesimal amount of time at a later time t_2 , where $t \leq t_1 \leq t_2$, at a rate that is the EONIA plus a spread η . The instantaneous forward spread is denoted as $\eta_t(\tau_1, \tau_2)$, where $\tau_2 = t_2 - t$ is the time to maturity of the contract and $\tau_1 = t_1 - t$ is the duration until the date of the commitment. This representation gives us the shortest possible maturity for any spread and is the building block from which any other spread in the market can be derived by integrating the instantaneous forward rate over the period $t_2 - t_1$. Indeed, the later sections present the cases of a deposit, an FRA, a forward, and a swap.

Moreover, the instantaneous forward spread has certain properties. Namely, it is bound by zero in the sense that $\eta_t(\tau_1, \tau_1) = \eta_t(\tau_2, \tau_2) = 0$, because immediate settlements do not entail a premium. It is positive because it can be considered an insurance premium. It is increasing with t_2 , as uncertainty increases with time to maturity. Moreover, the shape of the function is similar for the various times to settlement (τ_1). Overall, it is dimensionally homogeneous to an interest rate or a spread and can therefore be measured in percentage points or in basis points.

Figure 1 presents some graphical representations of the instantaneous forward spread, integrated over different periods of time. Appendix A describes in detail the various ways to integrate the instantaneous forward spread, which result in the generic formulas adopted for each type of money market instrument.

Figure 1: Graphical representations of $\eta_i(\tau_1, \tau_2)$



Note: Figure 1 presents a plot of an instantaneous forward spread for different realizations of τ_1 and τ_2 . The vertical axis depicts the spread, the horizontal axis depicts time (τ). In the left-hand panel $\tau_1=0$ and $\tau_2=3$, suggesting the spread of a three-month deposit. In the right-hand panel, $\tau_1=3$ and $\tau_2=6$, suggesting a six-month on three-month FRA. Source: Authors' calculations.

3.2 The functional form of the generic formula

Our chosen functional form summarizes a large body of information in a parsimonious three-dimensional space. It is therefore akin to a “factor analysis.” It does not derive from a theoretical model but simply assumes a level of consistency in the pricing of derivative instruments that respects the constraints imposed by the main properties of the η function, as described earlier. In addition to those properties, the function is concave, which ensures that it is subject to the law of diminishing returns and declines exponentially to allow the expectation that the turmoil, and therefore the positivity of the spreads, is temporary. A simple functional

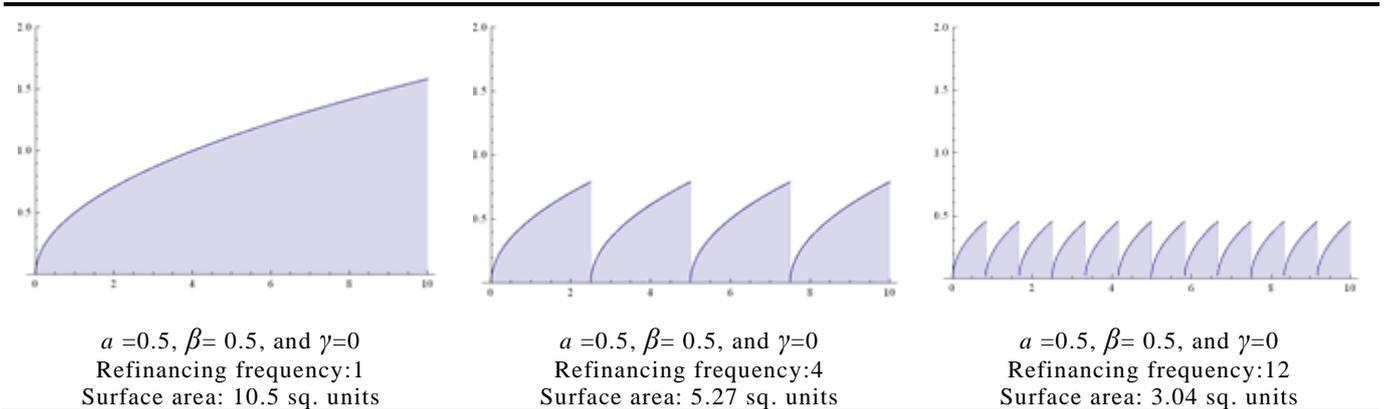
form with all the above properties is the following:

$$\eta(\tau_1, \tau_2) = \alpha(\tau_2 - \tau_1)^\beta \exp(-\gamma\tau_2),$$

where α, γ are positive, $0 \leq \beta \leq 1$, and $0 \leq \tau_1 \leq \tau_2$. Positive α and β render the function increasing. Positive γ renders the function integrable. With $0 \leq \beta \leq 1$, the function is concave and the exponential ensures that spreads approach zero asymptotically.

Notably, the functional form captures the fact that, as the refinancing frequency declines (that is, as refinancing becomes more infrequent), the spreads of money market rates over the OIS rate increase, suggesting that borrowers are willing to pay a premium in order to avoid more frequent refinancing. It also explains why, in overnight segments, the premium is minimal. Figure 2 presents this behavior graphically for given values of parameters α , β , and γ , and for various refinancing frequencies.

Figure 2: Instantaneous forward spread function for various frequencies of refinancing and constant parameters β (α and γ).



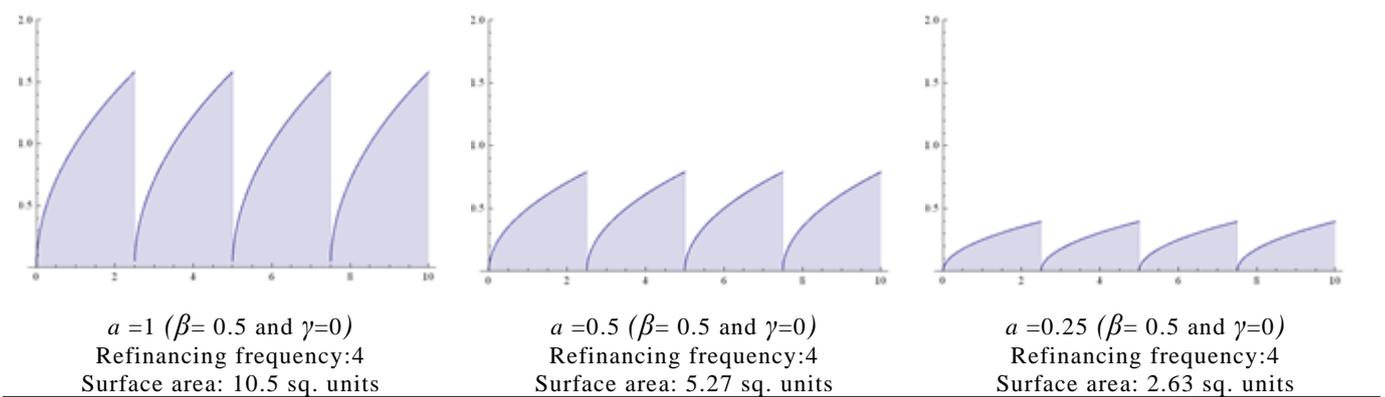
Regarding the three parameters, each of them has a clear interpretation.

Parameter α

Parameter α provides a rough measure of the level shift which affects all levels in a similar manner and thus the *intensity of the marketwide tension*. It can be seen

as a broad measure of the systemic, overall market tensions in the sense that, as α rises, the premia also rise across the cross section of our chosen money market instruments. It is therefore a measure of market distress. Figure 3 depicts spreads for different values of the parameter α (1, 0.5, and 0.25). As expected, the bigger the spreads, the higher the intensity of the market distress and thus the higher the parameter α .

Figure 3: Instantaneous forward spread function for various values of parameter α and constant parameters β and γ



Parameter β

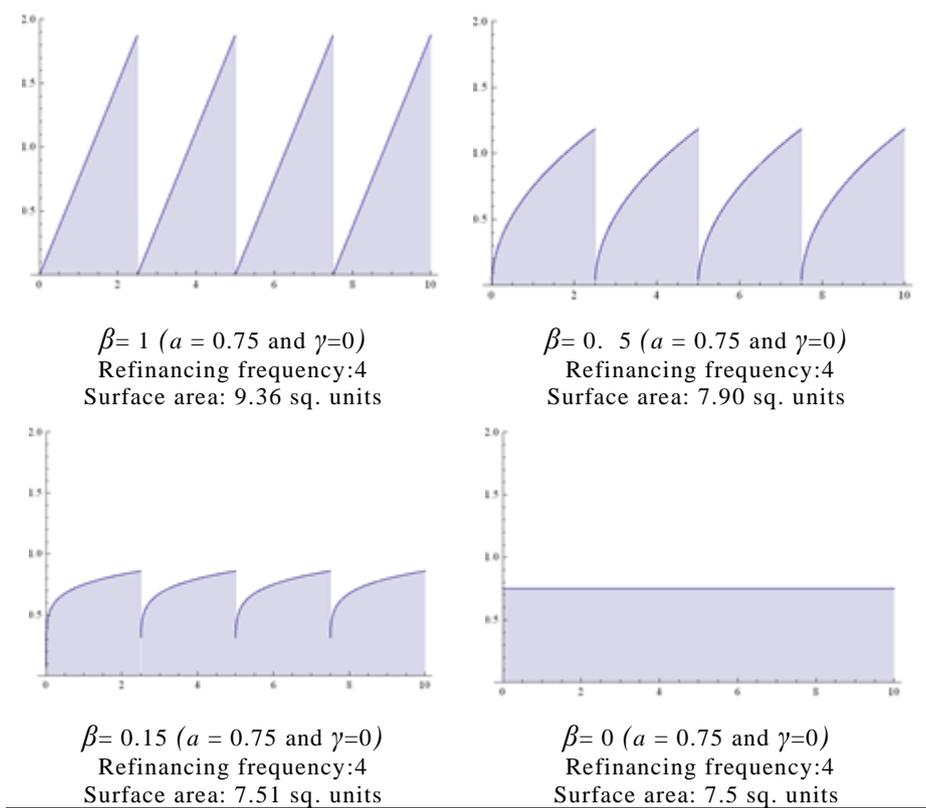
Parameter β governs the *effect of the refinancing frequency* on the spreads. In practice, β governs the concavity of the function and therefore the degree to which the various refinancing legs appear distinctly and the extent to which they affect the spread.

When $\beta = 1$, the refinancing frequency fully determines the shape of the spread. As shown in the top-left panel of figure 4, for $\beta = 1$ and for a refinancing frequency equal to 4, the total premium is the sum of four distinct segments.

As β approaches zero (*ceteris paribus*), these segments gradually decline and

eventually morph into a rectangular shape when $\beta = 0$. In the extreme case, in which the spread is the same irrespective of the refinancing frequency, the spread is constant over time, which is akin to a constant hazard rate of default. If α is positive, credit risk alone should be driving the spread, irrespective of the refinancing frequency. This is the visualization of a credit risk increase affecting all prime banks in a broadly symmetric manner, as explained in section 2.2. Overall, as parameter β declines from 1 to 0, the effect of the refinancing frequency on the spread fades.

Figure 4: Instantaneous forward spread function for various values of parameter β and constant parameters α and γ



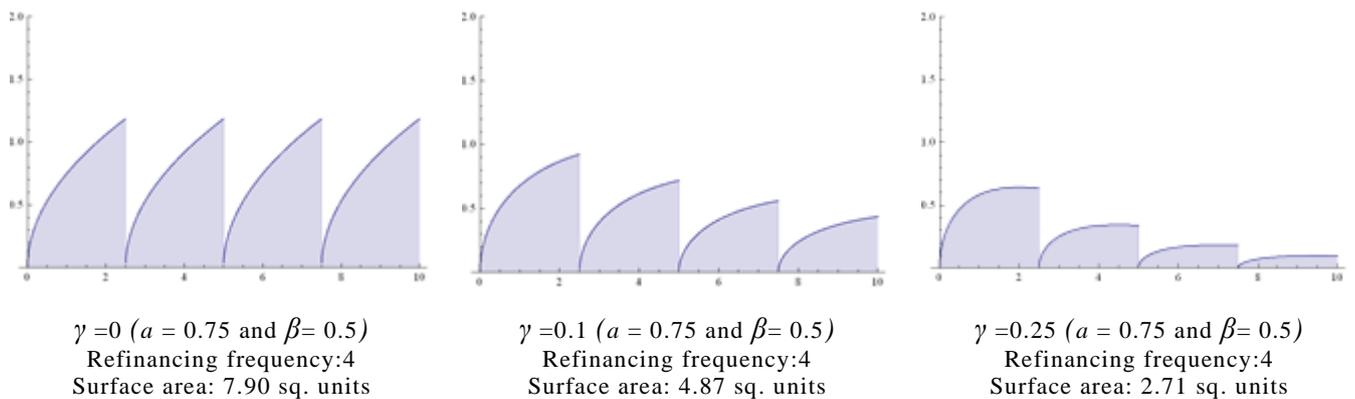
Parameter γ

Parameter γ captures the decay of the intensity of the turmoil in the period leading up to the maturity of the contract, or the “*implied expected length*” of the turmoil. As parameter γ increases, the implied length of the turmoil declines, as

spreads are expected to revert to zero faster. Parameter γ , when transformed into half-lives, gives a real-time approximation of the expected length of the crisis. Note that parameter γ was modeled with built-in expectations that the crisis would fade over time. We assume that markets are optimistic and consider that good times are long lasting, whereas shocks are transient, in line with the literature on financial crises (Brusco and Castiglionesi, 2007). In this sense, lower half-lives suggest that the stress will dissipate faster and indicate a period of stress.

Figure 5 shows the behavior of the spread for different values of parameter γ . As parameter γ increases, the spread declines faster (that is, it converges faster to the OIS rate).

Figure 5: Instantaneous forward spread function for various values of parameter γ and constant parameters α and β



The model has been developed to analyze the behavior of spreads during the financial turmoil. Before the turmoil, these spreads were negligible, and therefore the variation in the data was not sufficient to identify our parameters. Indeed, in normal circumstances (that is, when α is almost zero), the refinancing frequency is not an issue because liquidity risk and the decay of the intensity of the turmoil are essentially irrelevant. Therefore, the values of the parameters β and γ are meaningful only when α is not close or equal to zero. Namely, for $\alpha = 0$, $\eta_t(\tau_1, \tau_2) = 0$ for any

value of the β and γ parameters.

3.3 Fitting the data to the functional form

In order to fit the empirical data in the theoretical functional form, we minimized the sum of the squared differences between the actual data and the theoretical data. The actual data are the market-observed spreads, as discussed in section 5, while the theoretical data are the values derived from our functional form. The fitting uses the cross-section of different instruments' spreads at each point in time. The downhill simplex algorithm was used to estimate the values of a , β , and γ , which minimize the following objective function, F :

$$F = \sqrt{\frac{\sum weight(i) * (actual_spread_i - theoretical_spread_i)^2}{\sum weight(i)}}$$

where i refers to a specific instrument spanning the whole range of observable instruments in the markets—in our case, deposits, swaps, and FRAs. (The list of instruments used in the estimation is shown in table 1 and briefly described in Section 4.) The weights assigned to each instrument or group of instruments in this minimization problem were chosen on the basis of (inter alia) their market liquidity and maturity. In this exercise, we tried assigning both an equal weight of unity and weights based on liquidity considerations.⁸

⁸ There are good reasons to weight instruments in an asymmetric manner. Apart from the fact that some instruments may convey more accurate information than others because they are more liquid, assigning weights adds flexibility in our model. For example we may wish to restrict the experiment to some subfamily of EURIBOR-linked instruments that do not include futures contracts, as the latter are not traded over the counter, or that exclude deposit rates, on the basis that they are not derivatives. This restriction is easily implemented by the respective weighting of the available EURIBOR instruments.

In order to ascertain that the results, however, were not driven by the choice of weights, we relied on the notion of Shannon's entropy. Any weighting scheme can be changed proportionally without changing the difference between the actual and theoretical values given by formula F , and, by making the scheme sum to unity, it can appear as a probability distribution on the discrete set of available instruments. Such a probability

The value of F following the solution to this minimization problem provides a measure of the goodness of fit of our model. It is measured in basis points, as it refers to spreads. For example, a value of 7 basis points indicates the typical deviation in basis points of the theoretically constructed spread from the actual one. We used this value to assess the goodness of fit of our model.

Following the estimation of a , β , and γ parameters, a number of transformations were undertaken in order to express a and γ in percentage points. For the case of a , the standardized a (a^*) that results from the transformation can be obtained through the following formula: $a^* = (a \cdot 100^\beta)^{1/(1 + \beta)}$. For the case of γ , the percentage point transformation is straightforward—namely, $\gamma' = \gamma \cdot 100$. A second transformation took place in order to calculate the half-lives, where *half-lives* = $\ln(2)/\gamma$.

4 Data and data manipulations

The data set comprises money market rates on EURIBOR- and EONIA-linked instruments. In the family of the EURIBOR-linked instruments, we also included the unsecured deposit rates of a maturity longer than one day, as they are the underlying of the EURIBOR fixing—that is, the instruments that are supposedly represented by that EURIBOR fixing. The other instruments included the derivative instruments that are cash settled on the EURIBOR—namely, the FRAs; the swap against one-month, three-month, and six-month EURIBOR; and the London International Financial Futures and Options Exchange (or LIFFE) futures contracts on three-month EURIBOR. Within the EONIA-linked instruments, we included the

distribution has an entropy, which, in the case of this paper, should be higher than 3.

one-day deposit rate and EONIA swaps that range from one week to 30 years. Overall, the data comprise 78 different instruments and range from one-day to 10-year maturities. We used data from every instrument that was recorded in the market for the period from 4 July 2007 to 30 May 2013. In that sense, the data set is a holistic representation of market activity in these instruments. With the exception of the futures, all instruments considered are traded over the counter. A list of the instruments and their weights (as described in section 3.3) is provided in appendix B.1.

In order to construct our final data set, we calculated the spreads for each instrument from the EONIA swap of the respective maturity (see appendix B.2 and B.3). The data source is Reuters, which provides these data according to standard market conventions. The latter are, however, different for each instrument. Knowledge of these market conventions is therefore needed in order to use such data simultaneously in a consistent manner. Such conventions include the effect of the TARGET (or Trans-European Automated Real-time Gross settlement Express Transfer) calendar of working days on the actual cash flows, the compounding rule, and the day-count rule of the quoted interest rate.⁹ To our knowledge, such peculiarities of the money market instruments are rarely documented, much less in a referenced manner. The extent and quality of our data set strongly support the quality of our results.

Finally, our sample contains a number of significant dates that lead to

⁹ For example, repayments of money market loans falling on a weekend are shifted to the following Monday, unless the following Monday falls in the following calendar month, in which case the repayment dates are shifted to the previous Friday.

In the particular case of EURIBOR swaps, it is important to know that the interest rates are paid annually, whereby the variable leg consists of the compounding of two six-month EURIBOR or of four three-month EURIBOR, exchanged against fixed rate, which is not compounded at all.

turbulence in the money markets. Some of the more important dates include 9 August 2007, the beginning of the financial market turmoil; 15 September 2008, the fall of Lehman; 21 July 2012, the beginning of sovereign contagion risk in Europe; 27 June 2012, the request of financial assistance from the European Union by Spain; and 26 July 2012, the speech of European Central Bank (ECB) President Draghi, in which he vowed to do “whatever it takes” to save the euro.¹⁰

5. Results

5.1 Descriptive statistics and goodness of fit

The functional form chosen provides a very good fit of the observed spreads. Chart C.1 (appendix C) presents a measure of the fit, which contains the measurement error in percentage points. The pricing of the entire set of 78 money market instruments in our data can be reconstructed with an average error of only 0.08 percentage points, which is particularly small, especially with regard to the values assumed by a^* .

Descriptive statistics (see appendix C, chart C.2 and table C.1) suggest skewed distributions of our parameters. Parameter a^* was 7.43 percentage points on average during the crisis, being generally skewed toward higher spreads and reaching a maximum of 30.5 percentage points. Parameter β lies on average around 0.6 but ranges from as low as 0.2 to 1. The average expected half-life of the crisis appears to be around 1.6 years; however, there is skewness toward longer half-lives, the maximum amounting to 12 years.

¹⁰ For more information on the speech, see Draghi (2012).

It is notable that the vast majority of positive outliers in our parameters appears at the outbreak of the turmoil in August 2007. Higher volatility is also largely attributed to the beginning (approximately the first few weeks) of our data sample. This is a period confound with confusion by practitioners over the correct modelling procedure. Our results reflect the difficulty prevailing in the market at that time of pricing the instruments involved, given the new, unprecedented situation: There were inconsistencies in market pricing, which enhanced high volatility in the explanatory parameters beyond the effect of the turmoil.

5.2 Behavior of parameters

Looking at the behavior of our estimated α , β , and γ period by period, we gain a plausible intuition on the crisis (chart 2, panels a, b, and c). Overall, the evolution of α appears to track the intensity of the crisis quite well. Parameter α recorded relatively small values before the turmoil, in line with low market tensions.¹¹ However, at the outset of the turmoil (9 August 2007) its value spiked and subsequently dropped gradually, with some obvious spikes on events such as the Northern Rock support (in September 2008), the year-end, and the Bear Stearns collapse (in March 2008). It then rose again significantly following the Lehman failure in mid-September. At that point, what had previously been characterized as “turmoil” intensified into a full-blown crisis. In the most intense period of crisis after the Lehman failure, α increased dramatically. It then declined and remained relatively low, until it rose again following a series of events that all pointed to the

¹¹ Parameter α is around 4 basis points before the turmoil. This spread is a “structural” one that has always existed and remained broadly constant, and it most likely reflects the fact that the EURIBOR is an “asked” rate, whereas the OIS is a “mid” rate.

underlying fear of the euro breakup.¹² It is interesting that the value of the parameter remains positive, suggesting a “new normal” situation for money markets after 2007.

Three features in the behavior of β deserve attention. First, the parameter always remains above zero, hovering around 0.6 on average. This result clearly suggests that there is considerable concavity in the $\eta_t(\tau_1, \tau_2)$ function; therefore, the effect of refinancing frequency on the spreads is always present and is quite strong. Second, following the default of Lehman, β drops to its lowest point, close to zero. This result is consistent with a market freeze prevailing in that period due to increased credit risk across markets. Our interpretation is that, following the collapse of Lehman, the average credit risk for all Euribor panel banks increased in a broadly symmetric manner, leading to an almost parallel shift in spreads across instruments without affecting the refinancing frequency. This is represented in figure 4 as the rise in the rectangular part when β is low. Third, β recorded a couple of significant drops during the euro-area crisis, but on average, its values were significantly higher compared with the beginning of the turmoil and the Lehman collapse. This could be viewed as the result of credit risk asymmetrically affecting the Euribor panel banks during that period, given that certain panel banks were more exposed to the crisis than others (due to the exposure of their sovereigns). The asymmetry would increase the option value of the lender for piecewise lending and therefore increase the effect of the refinancing frequency on the spread.

Finally, the half-lives series follows a seemingly inverse pattern compared with a . The series (the inverse of parameter γ) reflects the expected length of the

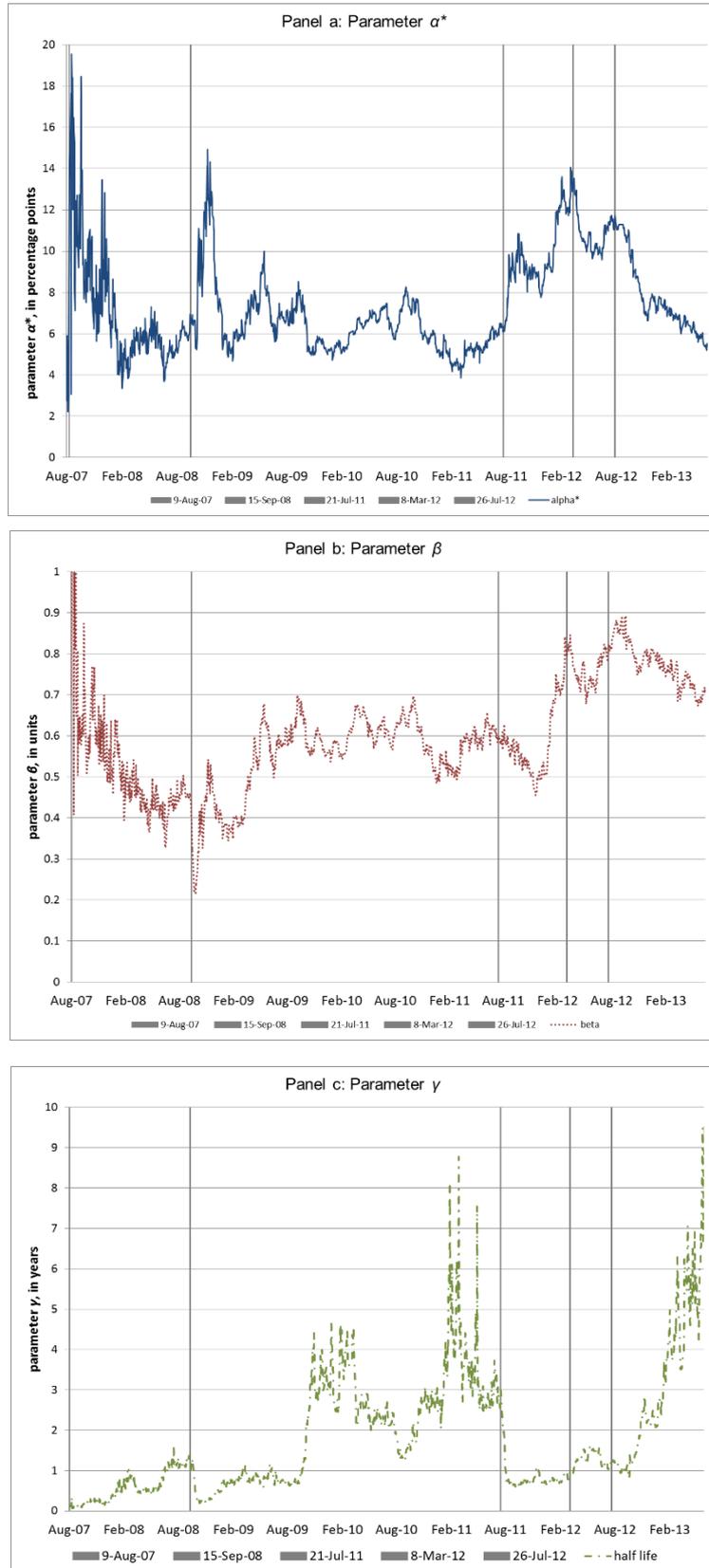
¹² These events involved the realization of Greece’s selective default on its debt at a time (21 July 2011) concurrent with subsequent contagion of the crisis to Italy and Spain; the Greek private-sector involvement agreement (concluded on 8 March 2012); the Spanish appeal for European Union financial support (27 June 2012); and, finally, the speech of ECB President Draghi (26 July 2012), which stabilized markets significantly.

turmoil in half-lives—that is, the time it takes (in years) for the spread to dissipate by one-half. As mentioned, lower half-lives suggest that markets are more stressed, as they perceive that state as a transient shock. Indeed, results suggest that half-lives are low during periods of high market tensions. Overall, γ indicates that there were two periods of low market spirit in the sample. The first period ranges from the beginning of the crisis to one year after the Lehman collapse, and the second begins with the discussions of the Greek exit from the euro and continues until the ECB's suggested interventions to preserve the integrity of the euro (September 2011 to September 2012).

This is the first attempt, to the best of our knowledge, to produce an endogenously determined measure for the duration of the crisis and the identification of its most intense periods. Up until now, crisis periods were mainly identified based on the calendar days of events considered relevant for the crisis. However, as the crisis period stretched over time, particularly in the euro area with the advent of the sovereign crisis, reliance on calendar events does not offer insights into the relevant importance of such dates.

Our parameter γ offers important insights on the crisis. It establishes, as expected, that the beginning of the crisis in August 2007 and the period immediately following the collapse of Lehman has been the most detrimental to money market sentiment. However, the parameter reveals that the euro area crisis affected money market in an indirect, but similarly grave manner. Moreover, the worse period for market sentiment relates to the discussions of the Greek exit from the euro (from September 2011 to March 2012), despite the unprecedented monetary policy apparatus already in place by that time and, in fact, leading to the use of further unconventional monetary policy measures by the ECB.

Chart 2: Parameters α , β , and γ



Note: Sample period from 7 August 2007 to 31 March 2013. Vertical lines denote events as described in section 4.

5.2 Central bank provision of liquidity and the behavior of parameters

Were central bank interventions effective in denting money market spreads and refinancing risk? And, were some measures more efficient compared to others? In this section, we analyze the effect of central bank liquidity-providing operations on a^* and β . During the years of the crisis, the ECB provided vast amounts of liquidity well above the benchmark (that is, the amount of liquidity that would, in theory, satisfy demand created by reserve requirements and autonomous factors).

Starting on 30 October 2008, the traditionally competitive liquidity-providing operations were replaced with more frequent liquidity operations under a fixed rate with full allotment (FRFA operations). Apart from the weekly and monthly operations (following the duration of the maintenance period), there were also standard three-month operations. Operations longer than three months were introduced as extraordinary measures and were much fewer in number. Eighteen six-month operations occurred in our sample period, mostly in 2008 and 2009; four one-year operations occurred in 2009 and 2011; and two three-year operations took place in late 2011 and early 2012, following the double dip in money market sentiment.

Relevant data issues are the following: The empirical analysis takes place from the fall of Lehman on 15 September 2008 to 31 March 2013 so as to capture the period of the FRFA liquidity provision. The data consist of daily liquidity conditions provided by the ECB in this period. The relevant variables include the liquidity injected by each type of operation (measured as the total allotment in each operation) divided by the outstanding excess liquidity (EL)¹³. On the days where no operations took place, the value of these variables is zero. Four variables are

¹³ EL is calculated by summing the liquidity provided (via open market operations and the Securities Markets Program) and subtracting the liquidity withdrawn (via autonomous factors and reserve requirements).

constructed in this manner, two for the three- and six-month operations and another two for the one- and three-year operations. We look at specific operations and standardize them by the level of outstanding EL so as to maintain the relative order of magnitude for the operations conducted, as some of them (for example, the first one-year LTRO and the first three-year LTRO) were considered more important than others because of their large participation. The end-of-maintenance-period dates are deleted from the sample so as to “clean” the EL series from the end-of-period effect. We also construct a dummy taking the value of 1 for every FRFA operation conducted in that period, including the one-month operations but not the weekly liquidity-providing operations. The chosen regression models separately regress parameters a^* and β on their first lag and relevant variables.

Table 1: Regression results					
Dependent variable: β					
	A	B	C	D	E
$\beta(-1)$	0.994 [0.000]	0.993 [0.000]	0.991 [0.000]	1.000 [0.000]	0.992 [0.000]
D_LTRO	-0.010 [0.048]				
D_3mLTRO		0.001 [0.861]			
D_6mLTRO			-0.006 [0.000]		
D_1yLTRO				-0.022 [0.000]	
D_3yMLTRO					-0.021 [0.086]
C	0.004 [0.077]	0.004 [0.092]		0.004 [0.070]	
R-squared	0.976	0.986	0.986	0.986	0.985

Notes: Table presents regression results from regressing parameter β on its lagged value, a constant and various (dummy) variables. Specification A includes a dummy for all Long-term Refinancing Operations (LTROs) conducted by the ECB. Specification B includes a variable with the amount of liquidity injected by the 3-month LTROs on the days when such LTROs were conducted, divided by the prevailing excess liquidity on those days and zero otherwise (D_3mLTRO). Specification C includes a variable with the amount of liquidity injected by the 6-month LTROs on the days when such LTROs were conducted, divided by the prevailing excess liquidity on those days and zero otherwise (D_6mLTRO). Specification D includes a variable with the amount of liquidity injected by the 1-year LTROs on the days when such LTROs were conducted, divided by the prevailing excess liquidity on those days and zero otherwise (D_1yLTRO). Specification E includes a variable with the amount of liquidity injected by the 3-year LTROs on the days when such LTROs were conducted, divided by the prevailing excess liquidity on those days and zero otherwise (D_3yLTRO). EL is the sum of liquidity provided (via open market operations and the SMP programme) minus the total liquidity withdrawn (via autonomous factors and reserve requirements). It is measured in millions of euros. Amounts of liquidity injected by the various operations are also measured in millions of euros. Sample period runs from 15 September 2008 to 31 March 2013 for all but specification E, when the sample starts on 15 September 2011. Standard errors are corrected for heteroscedasticity and autocorrelation.

Overall, the regression results (table 1) suggest that liquidity-providing operations tend to have a negative effect on parameter β . In other words, all things being equal, liquidity provision tends to lower the effect of the refinancing frequency on the spreads. This could reflect the fact that central bank operations reduced liquidity risk. However, given the endogeneity between liquidity and credit risk, reduced liquidity risk could also relate to the fact that central bank interventions essentially reduced the credit-risk disparities among banks, thus reducing overall counterparty credit risk and the option value of extending piecemeal lending for the lender. Credit risk shocks would then affect banks in a more uniform, symmetric manner, effectively muting the impact of the refinancing frequency on the spread.

Although the effect is, in the vast majority of cases, statistically significant, in economic terms it is not large. On average, LTROs appear to push parameter β down by 0.01 (specification A). Extraordinary liquidity operations (longer than 3 months) have been more effective compared to standard, three-month ones, which have a negligible effect (specification B). Among the extraordinary ones, the one- and

three- year operations appear to have a larger effect compared to the six-month ones, however, their impact remains modest. For example, should the one-year or three-year operations be large enough to double the EL outstanding, the parameter β would drop on average by 0.02 (specifications D and E). Overall, although central bank operations affected refinancing risk in the desired direction, the impact was relatively small and refinancing risk remained a significant determinant of the spread throughout the crisis.

Given the multiplicative relationship between a^* and β , the effect of central bank interventions would achieve its purpose if overall market tensions also declined. Otherwise, should parameter a^* increase while parameter β declines, it would essentially suggest that credit risk is rising. Alternatively, both parameters declining would suggest that spreads decline via lowering refinancing risk.

The regression results (table 2) suggest that central bank interventions negatively affect parameter a^* . On average, LTROs appear to push parameter a^* down by 9 basis points (specification A). The impact increases with the length of the operation. Three-month operations appear to have a negligible effect (specification B), whereas the three-year operation has the largest effect. Namely, should the three-year operation be large enough to double the EL outstanding, parameter a^* would decline on average by 33 bps (specification E).

Looking jointly at the results, central bank interventions affect the spreads negatively via lowering refinancing risk. Although previous research has confirmed the negative effect of central bank operations on the spreads in the euro area (see Carpenter et al., 2013), this study is the first to look into the effect of such operations on refinancing risk as well as on the spreads.

Table 2: Regression results					
Dependent variable: α^*					
	A	B	C	D	E
$\alpha(-1)$	0.986 [0.000]	0.987 [0.000]	0.989 [0.000]	0.990 [0.000]	0.989 [0.000]
D_LTRO	-0.089 [0.038]				
D_3mLTRO		-0.072 [0.116]			
D_6mLTRO			-0.060 [0.092]		
D_1yLTRO				-0.201 [0.004]	
D_3yMLTRO					-0.327 [0.037]
C	0.109 [0.008]	0.087 [0.026]	0.088 [0.024]	0.083 [0.034]	0.088 [0.024]
R-squared	0.973	0.986	0.973	0.972	0.972

Notes: Table presents regression results from regressing parameter α^* on its lagged value, a constant and various (dummy) variables. Specification A includes a dummy for all Long-term Refinancing Operations (LTROs) conducted by the ECB. Specification B includes a variable with the amount of liquidity injected by the 3-month LTROs on the days when such LTROs were conducted, divided by the prevailing excess liquidity on those days and zero otherwise (D_3mLTRO). Specification C includes a variable with the amount of liquidity injected by the 6-month LTROs on the days when such LTROs were conducted, divided by the prevailing excess liquidity on those days and zero otherwise (D_6mLTRO). Specification D includes a variable with the amount of liquidity injected by the 1-year LTROs on the days when such LTROs were conducted, divided by the prevailing excess liquidity on those days and zero otherwise (D_1yLTRO). Specification E includes a variable with the amount of liquidity injected by the 3-year LTROs on the days when such LTROs were conducted, divided by the prevailing excess liquidity on those days and zero otherwise (D_3yLTRO). EL is the sum of liquidity provided (via open market operations and the SMP programme) minus the total liquidity withdrawn (via autonomous factors and reserve requirements). It is measured in millions of euros. Amounts of liquidity injected by the various operations are also measured in millions of euros. Sample period runs from 15 September 2008 to 31 March 2013 for all but specification E, when the sample starts on 15 September 2011. Standard errors are corrected for heteroscedasticity and autocorrelation.

6 Conclusion

This paper is the first to quantify the effect of the refinancing frequency on the spread, a major factor behind the money market spreads during the financial crisis. What changed with the advent of the crisis was essentially the perceptions about the ability to refinance. Therefore, money market rates of the same maturity, but with different refinancing tenors, were priced differently for the first time. Borrowers were willing to pay a premium to obtain financing for an extended period rather than having to refinance several times.

This paper proposes a model that measures the effect of the frequency of refinancing on the spread. This is done by modeling the whole surface of observed Euribor-based money market spreads over all maturities and commitment periods ahead. The information contained in this very rich money market data set is summarized into three parameters, which evolve over time. In this respect, the proposed model is akin to a factor model. The three parameters reflect the market-wide intensity of the crisis, the effect of the refinancing frequency on the spread, and the implied duration of the turmoil expressed in half-lives. As the effect of the refinancing frequency on the spread increases, refinancing risk becomes more important in driving the spread. The importance of refinancing risk can vary over time.

This paper also elaborates on the link between refinancing risk, liquidity, and credit risk. Refinancing risk is affected by liquidity risk and, under certain circumstances, also by (counterparty) credit risk. Crucially, when credit risk is expected to affect certain market participants more than others, there is an option

value for the lender in extending loans for shorter horizons in order to assess the creditworthiness of the counterpart across time. In that case, the refinancing frequency is affected by credit risk. On the contrary, when credit risk is expected to increase across instruments and panel banks in a symmetric manner, the refinancing frequency is not affected by credit risk.

The featured model produces a number of interesting results. First, the proposed simple and parsimonious model achieves a good fit of the observed data. Second, it provides evidence that refinancing risk was always present during the crisis. However, in the period following Lehman's collapse, the sensitivity of the spread to the refinancing frequency drops considerably, consistent with a market freeze. On the other hand, refinancing risk appears relatively higher during the European crisis compared with previous periods, as certain panel banks were affected more than others. Third, the model offers a novel, endogenously determined measure for the expected length of the crisis and the identification of its most intense periods. According to this measure, the euro area crisis and the fear of a break-up of the euro affected euro area money markets in a similarly grave manner as the money market crisis stemming from the US in 2007 and 2008. Finally, it appears that central bank interventions have a negative effect on the spreads by lowering refinancing risk.

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A.1 FRA-OIS spread—The generic formula

We start with the forward rate agreement (FRA) spread, given that it can be used as the generic formula from which spreads of other instruments can be derived. The spread between an FRA with a time to maturity of $\tau_2 = t_2 - t_1$ and the respective overnight index swap (OIS) will be the sum of all of the instantaneous forward rates in between that period. In continuous time, this spread is expressed as follows:

$$FRA - OIS \text{ spread}_t(\tau_1, \tau_2) = \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \eta_\tau(\tau_1, \zeta) d\zeta,$$

where the integral is divided by the difference between the maturity of the contract, τ_2 and the duration until the date of the commitment, τ_1 . The time variable of integration is ζ . For example, for the FRA three months on three months ($FRA_t^{3,3}$), $\tau_1 = 3$ months and $\tau_2 = 6$ months; therefore, the spread would be $\frac{1}{3} \int_{3m}^{6m} \eta(3m, \zeta) d\zeta$. The graphical representation of this spread appears in figure 1, in the right-hand-side panel.

A.2 Deposit-OIS spread

A deposit can be seen as a special case of an FRA, in which the commitment to borrow or lend money unsecured is made at time t and not at some future date. Therefore, t_1 is always equal to t , and, consequently, the duration $\tau_1 = 0$ and the time to maturity is simply t_2 . In that sense, the deposit-OIS spread with maturity $\tau_2 = t_2$ can be described as

$$Depo - OIS \text{ spread}_t(t_2) = \frac{1}{\tau_2} \int_0^{\tau_2} \eta_\tau(0, \zeta) d\zeta.$$

For example, the spread of the deposit for three months (dep_t^3) would be $\frac{1}{3m} \int_0^{3m} \eta(0, \zeta) d\zeta$.

The graphical representation of this spread appears in figure 1, in the left-hand-side panel.

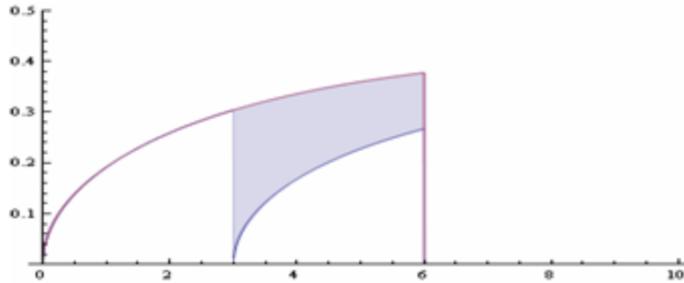
For the simple borrowing example presented in section 2.1, chart 1, the spread would be

equal to the difference between the spread of the six-month deposit ($6m*dep_t^6$) minus that of the strategy involving borrowing at the deposit for three months ($3m*dep_t^3$) and then entering into an FRA ($3*FRA_t^{3,3}$). That is, the spread of our above example would equal

$$\left\{ (6*dep_t^6) - \left[(3*dep_t^3) + (3*FRA_t(3,3)) \right] \right\} = \left[\int_0^6 \eta(0,\zeta) d\zeta - \left(\int_0^3 \eta(0,\zeta) d\zeta + \int_3^6 \eta(3,\zeta) d\zeta \right) \right].$$

Figure A1 offers a perhaps more intuitive understanding of this example. In this figure, a spread for a deposit of three and six months is plotted, together with the spread for the FRA. The shaded area is the difference between the two strategies. Before the turmoil, this area was practically zero.

Figure A1: Spread of a six-month deposit versus a combination of a three-month deposit and a three-month on three-month FRA



Note: The shaded area of this chart represents the spread between a six-month deposit and a combination of a three-month deposit and a three-month on three-month FRA. The vertical axis measures spreads in percentage points. The horizontal axis measures time in months.

A.3 Forward-OIS spread

Any (synthetic) forward rate can be derived by a combination of deposit rates. In general, the spread of a forward contract with settlement at t_1 and maturity at t_2 would be the following:

$$Forward - OIS - spread_t(\tau_1, \tau_2) = \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \eta_\tau(\tau_1, \zeta) d\zeta.$$

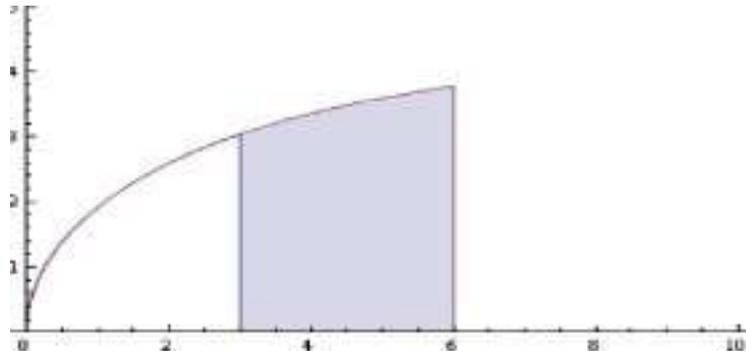
Note that this spread is different from an FRA spread in that t_1 is 0, as the commitment is

imminent, the same as for the underlying deposit contracts. The implication is that the forward spread is always bigger than the FRA spread. For example, the forward rate three months in three months, ($fwd_t^{3,3}$), can be derived by taking the difference between the deposit six months and the deposit three months—that is, in our convention, the spread of the ($fwd_t^{3,3}$) would be as follows:

$$\begin{aligned}
 \text{Forward} - \text{OIS}_{\text{spread}}_t(3,3) &= \\
 &= \frac{1}{3} \left[(6 * dep_t^6) - (3 * dep_t^3) \right] = \\
 &= \frac{1}{3} \left(\int_0^6 \eta_\tau(0, \varsigma) d\varsigma - \int_0^3 \eta_\tau(0, \varsigma) d\varsigma \right) = \frac{1}{3} \int_3^6 \eta_\tau(0, \varsigma) d\varsigma.
 \end{aligned}$$

Figure A2 presents a graphical representation of the implied forward spread (the shaded area) as the difference between the six- and the three-month spread.

Figure A2: Implied forward rate three-months on three-months



Note: The shaded area of this chart represents the implied forward rate three-months on three-months. It resulted from subtracting a three-month from a six-month deposit spread. The vertical axis measures spreads in percentage points. The horizontal axis measures time in months.

A.4 Swap-OIS spread

Finally, a swap is effectively a combination of FRAs, which are entered into consecutively according to a certain frequency. The notation of a swap would be $swap(T_{\tau^*N}, \tau)$, where T_{τ^*N} is the maturity of the swap, τ is the duration of the underlying FRAs, and N would be the number of underlying cash flows. The generic formula can be understood more easily through an example:

The swap one year against three months would be $swap(12m, 3m)$, or $swap(T_{3m*4}, \tau)$ is effectively a number of consecutive FRAs of a duration (τ) of three months, which are entered into every three months for a period of one year (that is, generating $N = 4$ notional cash flows). In terms of spreads, this formula could be expressed as

$$Swap - OIS_spread_t(12,3) =$$

$$\frac{1}{12} \left[(3 * dep_t^3) + (3 * FRA_t^{3,3}) + (3 * FRA_t^{6,3}) + (3 * FRA_t^{9,3}) \right] =$$

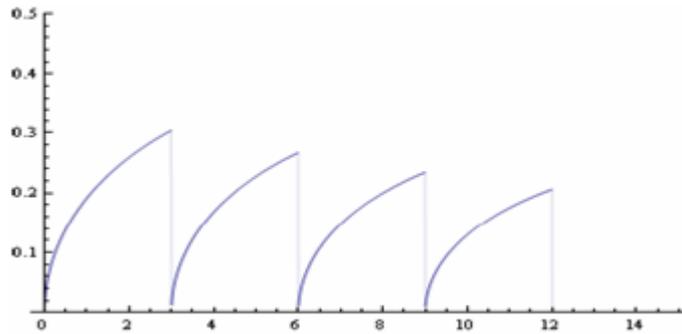
$$\frac{1}{12} \left(\int_0^3 \eta_\tau(0, \zeta) d\zeta + \int_3^6 \eta_\tau(3, \zeta) d\zeta + \int_6^9 \eta_\tau(6, \zeta) d\zeta + \int_9^{12} \eta_\tau(9, \zeta) d\zeta \right).$$

In short, the generic formula of the Swap-OIS spread could be written as

$$Swap - OIS_t(\tau_1, \tau_2) = \frac{1}{\tau * N} \left[\sum_{i=0}^{i=N-1} \int_{\tau*i}^{\tau*(i+1)} \eta(\tau*i, \zeta) d\zeta \right],$$

where i is an index ranging from 0 to $N - 1$. The graphical representation of a swap (12m, 3m) versus the OIS is presented in figure A3:

Figure A3: Swap spread twelve-months on three-months



Note: This chart represents the swap spread between a swap extending over a year with quarterly flows and the OIS rate. The vertical axis measures spreads in percentage points. The horizontal axis measures time in months.

B.1. List of instruments and their weights

Table B.1: List of instruments and their weights

Deposit	Weight	FRA	Weight	IRS	Weight	Futures	Weight
EURSWD	1.163	EUR1X4F	1	EURAB1E1Y	5.000	FEIH5	1
EUR2WD	0.732	EUR2X5F	1	EURAB1E2Y	2.121	FEIM5	1
EUR3WD	0.559	EUR3X6F	1	EURAB1E3Y	1.153	FEIU5	1
EUR1MD	0.431	EUR1X7F	1	EURAB1E4Y	0.749	FEIU7	1
EUR2MD	0.271	EUR4X7F	1	EURAB1E5Y	0.536	FEIZ7	1
EUR3MD	0.208	EUR2X8F	1	EURAB1E6Y	0.407	FEIH8	1
EUR4MD	0.173	EUR5X8F	1	EURAB1E7Y	0.323	FEIM8	1
EUR5MD	0.148	EUR3X9F	1	EURAB1E8Y	0.264	FEIU8	1
EUR6MD	0.132	EUR6X9F	1	EURAB1E9Y	0.222	FEIZ8	1
EUR7MD	0.119	EUR4X10F	1	EURAB1E10Y	0.189	FEIH9	1
EUR8MD	0.108	EUR7X10F	1	EURAB3E1Y	5.000	FEIM9	1
EUR9MD	0.101	EUR2X11F	1	EURAB3E18M	3.270	FEIU9	1
EUR10MD	0.094	EUR5X11F	1	EURAB3E2Y	2.121	FEIZ9	1
EUR11MD	0.088	EUR8X11F	1	EURAB3E3Y	1.153	FEIH0	1
EUR1YD	0.083	EUR6X12F	1	EURAB3E4Y	0.749	FEIM0	1
EUR2YD	0.052	EUR9X12F	1	EURAB3E5Y	0.536	FEIU0	1
EUR3YD	0.040	EUR1X13F	1	EURAB3E6Y	0.407	FEIZ0	1
EUR4YD	0.033	EUR7X13F	1	EURAB3E7Y	0.323	FEIH1	1
EUR5YD	0.028	EUR10X13F	1	EURAB3E8Y	0.264	FEIM1	1
EUR7YD	0.022	EUR2X14F	1	EURAB3E9Y	0.222	FEIU1	1
EUR10YD	0.017	EUR8X14F	1	EURAB3E10Y	0.189	FEIZ1	1
		EUR11X14F	1	EURAB6E1Y	5.000	FEIH2	1
		EUR3X15F	1	EURAB6E15M	4.282	FEIM2	1
		EUR9X15F	1	EURAB6E18M	3.270	FEIU2	1
		EUR12X15F	1	EURAB6E21M	2.596	FEIZ2	1
		EUR4X16F	1	EURAB6E2Y	2.121	FEIH3	1
		EUR10X16F	1	EURAB6E3Y	1.153	FEIM3	1
		EUR5X17F	1	EURAB6E4Y	0.749	FEIU3	1
		EUR11X17F	1	EURAB6E5Y	0.536	FEIZ3	1
		EUR12X18F	1	EURAB6E6Y	0.407	FEIH4	1
		EUR7X19F	1	EURAB6E7Y	0.323	FEIM4	1
		EUR8X20F	1	EURAB6E8Y	0.264	FEIU4	1
		EUR9X21F	1	EURAB6E9Y	0.222	FEIZ4	1
		EUR10X22F	1	EURAB6E10Y	0.189		
		EUR11X23F	1	EURAM1E3M	5.000		
		EUR12X24F	1	EURAM1E6M	5.000		
		EUR18X24F	1	EURAM1E9M	5.000		
				EURAM1E1Y	5.000		

Notes: The table contains the instruments used in the paper for the estimation of the spread and the weights assigned on each instruments in the fitting process.

B.2 Construction of the reference EONIA swap curve

As mentioned, we analyze the spreads of the aforementioned market instruments with respect to a reference yield curve. We define the reference yield curve, constructed for each day in the sample, as the euro overnight index average

(EONIA) swap curve. The EONIA fixing represents the overnight unsecured deposit. The data used to construct the curve are as follows: for the short maturities, the three deposits of one-day maturity traded on the money market—that is, the overnight, the “tom next,” and the “spot next”—and, for the longer maturities, EONIA swap contracts from one week up to maturities of 10 years.¹⁴

In order to construct a smooth EONIA swap curve from the existing individual data points, we use a bootstrap method.¹⁵ This method amounts to assuming that the instantaneous forward rate is piecewise constant. This method gives reasonably good results in terms of smoothness of the rates (at least of the zero-coupon rates) and in terms of low oscillations. We follow the bootstrap method used in Brousseau and Durré (2014).

B.3 Construction of the spread data

After the construction of the reference yield curve, the next step is to calculate the spread for each of the available EURIBOR-linked instruments for each day in the sample. We use the parallel-shift spread (in the terminology used by Reuters, the z-spread). A parallel shift is a transformation of the curve by which all zero-coupon rates—and, consequently, all of the instantaneous forward rates—are moved by the same amount. The z-spread of an instrument is defined as the extent to which the reference curve needs to be moved (in the form of a parallel shift) to reprice the instrument at its original price or rate. In practice, we consider spreads from

¹⁴ The tom-next and spot-next are overnight transactions agreed one working day and two working days, respectively, in advance.

¹⁵ This bootstrap method is not to be confused with its statistical counterpart. The construction procedure is recursive—that is, it should determine the values of the rates of increasing maturities using the rates already determined in order to calculate the following ones. The resulting bootstrapped yield curve is entirely described by a finite (though variable) number of existing rates and dates, as well as by the interpolating rule allowing us to reconstruct rates for the maturity dates that are not explicitly specified.

EURIBOR instruments only, as spreads from EONIA-linked instruments would coincide with our reference curve and would therefore be zero by construction.

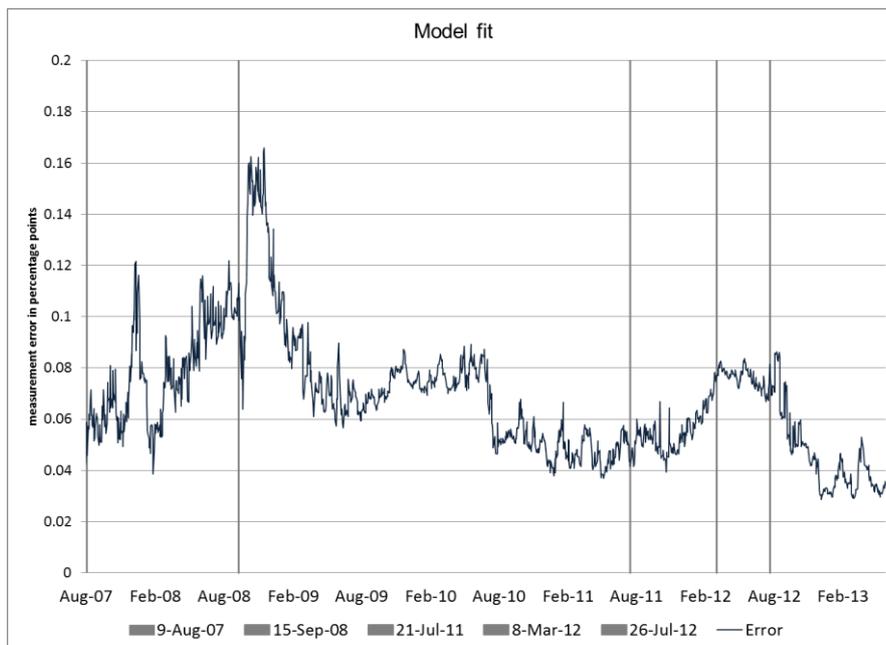
The choice of this methodology to construct the spread is motivated by several reasons. First, we can construct spreads for instruments at any maturity, even when EONIA swap contracts at that maturity are not available in our original data. Second, spreads of two instruments can be meaningfully compared even if the market conventions of those two instruments are different, as there is a common reference yield curve. Third, the mathematical interpretation is simpler than for any other concurrent choices of the definition of the spread.

Table C1: Descriptive Statistics

	α^*	β	γ (half-life)
Mean	7.43	0.59	1.58
Median	6.74	0.58	1.11
Maximum	30.56	1.00	12.09
Minimum	1.90	0.21	0.06
Std. Dev.	2.62	0.14	1.30
Skewness	1.79	0.44	1.88
Kurtosis	10.99	3.37	9.87
Jarque-Bera Probability	4,504.30 0.00	53.89 0.00	3,603.99 0.00
Observations	1,412	1,412	1,412

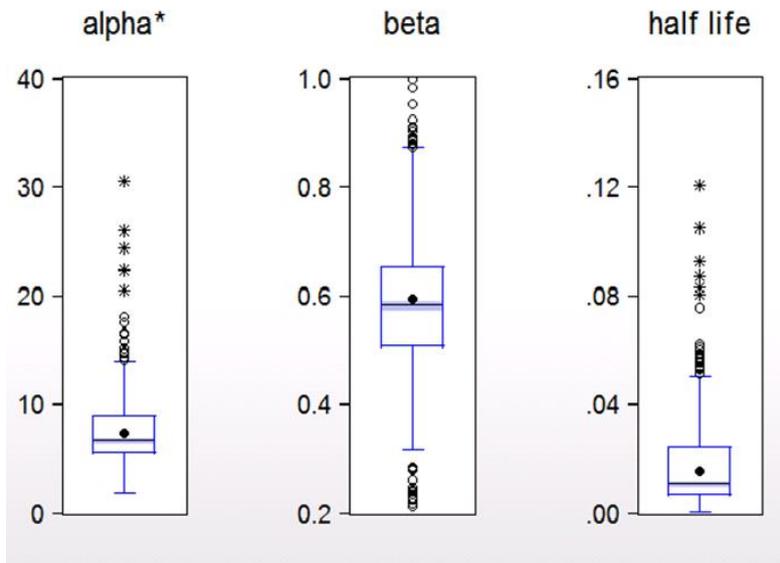
Notes: Table presents descriptive statistics of parameters α^* , β and γ , constructed as outlined in section 5.1.

Chart C1: Model fit



Note: The chart depicts the evolution of parameter a^* (RHS), and the fit of the model (error). The chart also depicts a number of interesting dates in the sample period.

Chart C2: Box plots



Note: Box plots summarize the distributions of parameters α^* , β , and γ . The box portion represents the first and third quartiles, the median is represented by the dark-blue line within the box, and the mean is represented by the rectangular symbol. The T-shaped blue lines indicate the 5 percent and the 95 percent quartile. The rectangular dots represent near outliers (up to the 1 percent and 99 percent quartiles), and the stars denote outer outliers.
