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Abstract

Micro- and macro-level evidence indicates that fluctuations in idiosyncratic uncertainty have a large effect on investment; the impact of uncertainty on investment occurs primarily through changes in credit spreads; and innovations in credit spreads have a strong effect on investment, irrespective of the level of uncertainty. These findings raise a question regarding the economic significance of the traditional “wait-and-see” effect of uncertainty shocks and point to financial distortions as the main mechanism through which fluctuations in uncertainty affect macroeconomic outcomes. The relative importance of these two mechanisms is analyzed within a quantitative general equilibrium model, featuring heterogeneous firms that face time-varying idiosyncratic uncertainty, irreversibility, nonconvex capital adjustment costs, and financial frictions. The model successfully replicates the stylized facts concerning the macroeconomic implications of uncertainty and financial shocks. By influencing the effective supply of credit, both types of shocks exert a powerful effect on investment and generate countercyclical credit spreads and procyclical leverage, dynamics consistent with the data and counter to those implied by the technology-driven real business cycle models.

JEL CLASSIFICATION: E22, E32, G31

KEYWORDS: time-varying volatility; asset specificity; capital liquidity shocks; costly external finance; firm heterogeneity; general equilibrium

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1 Introduction

Macroeconomists have made a convincing argument that the well-documented countercyclical behavior of the cross-sectional dispersion of economic variables such as labor income, business profits, productivity, and stock returns reflects fluctuations in the volatility of the underlying economic shocks—that is, swings in economic uncertainty.¹ How such uncertainty fluctuations influence business cycle dynamics has traditionally been analyzed within the framework of irreversible investment (see [Bernanke, 1983](#); [Bertola and Caballero, 1994](#); [Abel and Eberly, 1994, 1996](#); [Caballero and Pindyck, 1996](#); [Bloom, 2009](#); [Bloom et al., 2011](#); [Bachmann and Bayer, 2009, 2013](#)). This approach to corporate resource allocation treats the firm’s future investment opportunities as “real options” and emphasizes the importance of waiting and staging flexibility when making investment decisions—in response to increased uncertainty, firms should wait and see until uncertainty is resolved and the project is more clearly successful.²

Spurred by the surge in asset price volatility and the blowout in credit spreads during the 2007–09 financial crisis, an emergent literature has pointed to financial market frictions as an additional channel through which volatility fluctuations can affect macroeconomic outcomes (see [Arellano et al., 2012](#); [Christiano et al., 2014](#)). According to the canonical framework used to price risky debt, the return on levered equity resembles—under limited liability—the payoff of a call option, whereas the bondholders face the payoff structure that mimics that of an investor writing a put option. An increase in the riskiness of the firm’s assets thus benefits equity holders at the expense of bondholders, implying a rise in credit spreads to compensate bondholders for heightened uncertainty. To the extent that external finance—both through the debt and equity markets—is subject to agency and/or moral hazard problems, an increase in uncertainty will raise the user cost of capital, inducing a decline in investment spending.

Given these two complementary channels of how changes in uncertainty can affect the real economy, this paper seeks to answer the following question: How much of the impact of fluctuations in uncertainty on aggregate investment reflects capital adjustment frictions associated with irreversibility—the traditional wait-and-see effect—and how much of it can be attributed to distortions in financial markets? We answer this question by developing a quantitative business cycle model that allows us to analyze the effect of fluctuations in uncertainty on investment dynamics in a general equilibrium setting featuring partial irreversibility and frictions in both the debt and equity markets. Specifically, we formulate a capital accumulation problem, in which het-

¹See [Campbell and Taksler \(2003\)](#), [Storesletten et al. \(2004\)](#), [Eisfeldt and Rampini \(2006\)](#), [Bloom \(2009\)](#), and [Bloom et al. \(2011\)](#) for arguments and evidence. It is worth emphasizing, however, that there is no objective measure of time-varying economic uncertainty. In addition to the uncertainty proxies noted above, this burgeoning literature has employed indexes based on the frequency of “uncertainty-related” words or phrases that occur across a large number of news sources ([Baker et al., 2013](#)), the cross-sectional dispersion of survey-based business forecasts ([Bachmann et al., 2013b](#)), and the common variation in the unforecastable component of a large number of economic indicators ([Jurado et al., 2013](#)) to infer fluctuations in uncertainty.

²Despite its intuitive appeal, the effect of uncertainty on investment in the presence of irreversibilities can be theoretically ambiguous. As shown, for example, by [Abel \(1983\)](#) and [Veracierto \(2002\)](#), the effect depends importantly on the assumptions regarding the initial accumulation of capital, market structure, and the equilibrium setting.

erogeneous firms employ a decreasing returns-to-scale production technology that is subject to a persistent idiosyncratic shock, the variance of which is allowed to vary over time according to a stochastic law of motion. The capital accumulation problem is subject to partial irreversibility and fixed adjustment costs, two forms of adjustment frictions emphasized in the influential work of [Abel and Eberly \(1994, 1996\)](#), [Caballero et al. \(1995\)](#), [Doms and Dunne \(1998\)](#), [Caballero \(1999\)](#), [Cooper et al. \(1999\)](#), [Cooper and Haltiwanger \(2006\)](#), and [Bachmann et al. \(2013a\)](#).

On the financial side, firms make investment decisions subject to a full range of choices regarding their capital structure—internal funds, debt, and equity financing—in an environment where external funds are costly because of agency problems in financial markets. The partial investment irreversibility plays two roles in our model. First, it creates the option value of waiting. In response to increased uncertainty, the investment inaction region expands, which causes some firms to delay exercising their growth options—that is, they adopt a wait-and-see posture. Second, the greater downside risk limits the firms’ debt capacity because heightened uncertainty—under limited liability—favors the firm’s shareholders over its bondholders, thereby inducing a decline in the market value of debt claims. Thus, the model provides a unified quantitative framework for analyzing the implications of the interaction between the capital and debt overhang problems for business cycle dynamics.³

Our analysis of the interaction between uncertainty and financial conditions is informed importantly by new empirical evidence that highlights the strong relationship between uncertainty, investment, and credit spreads on corporate bonds, a commonly used indicator of the degree of financial market frictions.⁴ Specifically, we construct a new proxy for idiosyncratic uncertainty using high-frequency firm-level stock market data, a measure that arguably reflects exogenous changes in uncertainty, rather than the endogenous effects of informational and contractual frictions that have been theoretically linked to the countercyclical dispersion in economic returns. For a subset of large corporations, we match this estimate of uncertainty to the firms’ income and balance sheet statements and, most importantly, to prices of their individual corporate bonds trading in the secondary market.

Using this novel micro-level data set, we find that conditional on the firm’s leverage, profitability, and other indicators of creditworthiness, idiosyncratic volatility is an important determinant of credit spreads on the firm’s outstanding bonds. We also show that conditional on investment fundamentals—that is, proxies for the marginal product of capital—increases in idiosyncratic volatility are associated with a substantial decline in the rate of capital formation. However, once the information content of credit spreads is taken into account, the impact of uncertainty on business

³Earlier theoretical explorations of this interaction can be found in [Shleifer and Vishny \(1992\)](#), [Eisfeldt \(2004\)](#), and [Manso \(2008\)](#).

⁴The fact that the level of credit spreads provides a useful metric for gauging the tightness of financial conditions in the economy and thus the implied degree of departure from the Modigliani–Miller paradigm of frictionless financial markets is supported by considerable empirical evidence showing that corporate bond credit spreads form the most informative and reliable class of financial indicators for future economic activity and that shocks to credit spreads lead to adverse macroeconomic outcomes (see [Gertler and Lown, 1999](#); [Gilchrist et al., 2009](#); [Gilchrist and Zakrajšek, 2012](#); [Faust et al., 2013](#)).

investment is significantly attenuated. Capital expenditures, by contrast, remain highly responsive to movements in credit spreads.

To gauge the macroeconomic implications of these findings, we use a structural vector autoregression (SVAR) framework to examine the interaction between uncertainty, credit spreads, and economic activity. The results from this macro-level analysis confirm our finding that credit spreads are an important conduit through which fluctuations in volatility are propagated to the real economy. By inducing a significant widening of credit spreads, unanticipated increases in uncertainty lead to a decline in real GDP that is driven primarily by the protracted drop in the investment component of aggregate spending. In contrast, financial disturbances—identified vis-à-vis orthogonalized shocks to credit spreads—have a large effect on economic activity, irrespective of the level of uncertainty. The combination of the micro- and macro-level results is thus consistent with the notion that financial frictions are an important part of the mechanism through which uncertainty shocks affect the economy.

Simulations of our general equilibrium model accord well with the empirical results. By altering the effective supply of credit, financial distortions significantly amplify the response of business investment to volatility shocks. Although the risk-free rate falls in response to increased uncertainty, this easing of financial conditions is more than offset by the sharp and persistent increase in credit spreads. The resulting increase in the user cost of capital leads firms to slash capital outlays and delever, creating a quantitatively important channel through which fluctuations in uncertainty shape business cycle dynamics. In an economy without financial distortions, by contrast, this user cost of capital channel is significantly attenuated, and the response of investment to uncertainty shocks is an amalgam of the wait-and-see decisions of individual firms.

These findings are consistent with those of [Christiano et al. \(2014\)](#), who analyze the macroeconomic implications of volatility shocks in the context of a financial accelerator model adapted from [Bernanke et al. \(1999\)](#), a framework that abstracts from both investment irreversibility and persistent heterogeneity in firm-level outcomes. While [Arellano et al. \(2012\)](#) consider such firm-level heterogeneity, their model does not include capital accumulation and focuses on how increases in uncertainty affect the ability of firms to finance their labor input. In addition, both of these papers consider debt as the sole source of external finance. In contrast, we consider a general equilibrium model with heterogeneous firms facing time-varying idiosyncratic uncertainty, partial investment irreversibility, fixed investment costs, and frictions in both the debt and equity markets. Within the context of our calibrated model, these features allow us to quantify explicitly the extent to which the response of aggregate investment to an uncertainty shock is due to the “real options” effect or distortions in financial markets.

The combination of costly reversible investment and financial market frictions also allows us to identify a potential new source of aggregate disturbances: shocks to the liquidation (or resale) value of capital. With partial irreversibility, an adverse shock to the liquidation value of capital impinges on the debt capacity of firms by reducing the collateral value of capital assets, creating an interaction between the capacity and debt overhang problems. According to our simulations, such

“capital liquidity” shocks have the potential of being an important source of cyclical fluctuations in an economy with financial frictions.⁵ A relatively small amount of variability in the liquidation value of capital generates a plausible degree of volatility in the model’s key endogenous aggregates and delivers realistic comovements between main macroeconomic quantities. Importantly, the simulations show that when the business cycle is driven by either uncertainty or capital liquidity shocks, the level of credit spreads and their cross-sectional dispersion are strongly countercyclical, comovements consistent with the data.

In an economy with financial distortions, uncertainty and capital liquidity shocks induce a deterioration in the quality of borrowers’ balance sheets, which causes the effective supply of credit to shift inward, thus making quantities and spreads move in the opposite direction. Although our model does not include a formal financial intermediary sector, disturbances that affect the collateral value of capital assets can be viewed as a tractable way to model disruptions in the credit-intermediation process. Thus, our model generates macroeconomic dynamics that are consistent with the empirical evidence of [Stock and Watson \(2012\)](#), who argue that the toxic combination of adverse uncertainty and credit supply shocks were the two primary drivers of the drop in economic activity during the “Great Recession.”

2 Empirical Evidence

To infer both the cross-sectional and time-series variation in uncertainty, we look to asset prices, which, in principle, should encompass all aspects of the firm’s environment that the investors view as important. Moreover, by using information from financial markets—specifically from the stock market—we can rely on a standard asset pricing framework to purge our measure of uncertainty of forecastable variation. Using this proxy, we show that financial conditions, as summarized by the level of credit spreads, significantly influence the response of investment to fluctuations in idiosyncratic uncertainty. In fact, our empirical evidence indicates that the impact of uncertainty on investment occurs primarily through changes in credit spreads, a result consistent with the presence of significant financial market frictions.

2.1 Uncertainty, Financial Conditions, and Investment

In this section, we use a novel micro-level data set to document the tight link between corporate bond credit spreads and uncertainty and to shed light on how the interaction of uncertainty and credit spreads affects investment dynamics. First, however, we describe the construction of our benchmark estimate of time-varying idiosyncratic uncertainty. Specifically, from the Center

⁵This result echoes the recent work of [Brunnermeier and Sannikov \(2014\)](#), who show that a deterioration in asset market liquidity—a decline of the worst case liquidation value of capital—can lead to a significant increase in the endogenous risk in the economy. In their framework, the increase in endogenous risk is generated by an adverse feedback loop between the net worth of financially constrained agents and the market value of capital assets. In our model, by contrast, the increase in endogenous risk in response to a capital liquidity shock arises from the link between the liquidation value of capital assets and the firms’ debt capacity and the impact of that interaction on broader financial conditions.

for Research in Security Prices (CRSP) data base, we extracted daily stock returns for all U.S. nonfinancial corporations with at least 1,250 trading days (essentially five years) of data. This selection criterion yielded a panel of 11,303 firms over the period from July 1, 1963 (1963:Q3) to September 30, 2012 (2012:Q3).⁶

Our estimate of uncertainty is based on the following two-step procedure. First, we remove the forecastable variation in daily excess returns using the standard (linear) factor model:

$$(R_{it_d} - r_{t_d}^f) = \alpha_i + \beta_i' \mathbf{f}_{t_d} + u_{it_d}, \quad (1)$$

where i indexes firms and t_d , $d = 1, \dots, D_t$, indexes trading days in quarter t . In equation (1), R_{it_d} denotes the (total) daily return of firm i , $r_{t_d}^f$ is the risk-free rate, and \mathbf{f}_{t_d} is a vector of observable risk factors. In implementing the first step, we employ a 4-factor model—namely, the Fama and French (1992) 3-factor model, augmented with the momentum risk factor proposed by Carhart (1997).

In the second step, we calculate the quarterly firm-specific standard deviation of daily idiosyncratic returns, according to

$$\sigma_{it} = \sqrt{\left[\frac{1}{D_t} \sum_{d=1}^{D_t} (\hat{u}_{it_d} - \tilde{u}_{it})^2 \right]}; \quad t = 1, \dots, T, \quad (2)$$

where \hat{u}_{it_t} denotes the OLS residual—the idiosyncratic return—from equation (1) and $\tilde{u}_{it} = D_t^{-1} \sum_{d=1}^{D_t} \hat{u}_{it_d}$ is the sample mean of daily idiosyncratic returns in quarter t . Thus, σ_{it} in equation (2) is an estimate of time-varying equity volatility for firm i , a measure that is purged of the forecastable variation in expected returns and thus is less likely to reflect the countercyclical nature of contractual and informational frictions.

To analyze how fluctuations in this measure of idiosyncratic uncertainty influence financial conditions, we extracted from the master firm-level data set a subset of more than 1,100 large firms that have a significant portion of outstanding liabilities in the form of long-term bonds actively traded in the secondary market.⁷ Using this data set, we estimate the following credit-spread regression:

$$\log s_{it}[k] = \beta_1 \log \sigma_{it} + \beta_2 R_{it}^E + \beta_3 [\Pi/A]_{it} + \beta_4 \log [D/E]_{i,t-1} + \boldsymbol{\theta}' \mathbf{X}_{it}[k] + \epsilon_{it}[k], \quad (3)$$

where $s_{it}[k]$ denotes the credit spread of a bond issue k in period t , a security that is a liability of firm i . In addition to our estimate of idiosyncratic uncertainty σ_{it} , credit spreads are allowed to depend on the firm's recent financial performance, as measured by the realized quarterly stock return R_{it}^E and the ratio of operating income to assets $[\Pi/A]_{it}$. The ratio of the book value of

⁶To ensure that our results were not driven by a small number of extreme observations, we eliminated all firm/day observations with a daily absolute return in excess of 50 percent.

⁷This data set starts from January 1973, when the bond-level secondary market price quotes were first available; see Section A.1 of the data appendix for the description of the data and the details regarding the construction of the micro-level credit spreads.

Table 1: Idiosyncratic Uncertainty and Credit Spreads

Explanatory Variable	(1)	(2)	(3)	(4)
$\log \sigma_{it}$	0.730 (0.041)	0.459 (0.046)	0.484 (0.049)	0.216 (0.021)
R_{it}^E	-0.095 (0.026)	-0.112 (0.025)	-0.109 (0.024)	-0.053 (0.009)
$[\Pi/A]_{it}$	-4.100 (0.698)	-1.835 (0.502)	-1.500 (0.475)	-1.318 (0.385)
$\log[D/E]_{i,t-1}$	0.212 (0.024)	0.056 (0.013)	0.049 (0.013)	0.078 (0.011)
Adjusted R^2	0.474	0.641	0.648	0.797
Credit Rating Effects ^a	.	0.000	0.000	0.000
Industry Effects ^b	.	.	0.000	0.000
Time Effects ^c	.	.	.	0.000

NOTE: Sample period: 1973:M1–2012:M9 at a quarterly frequency. No. of firms = 1,124; No. of bonds = 6,373; and Obs. = 107,482. The dependent variable in all specifications is $\log(s_{it}[k])$, the logarithm of the credit spread of bond k (issued by firm i) in month t . All specifications include a constant, a vector of bond-specific control variables $\mathbf{X}_{it}[k]$ (not reported), and are estimated by OLS. Robust asymptotic standard errors reported in parentheses are double-clustered in the firm (i) and time (t) dimension (see [Cameron et al., 2011](#)).

^a p -value for the exclusion test of credit rating fixed effects.

^b p -value for the exclusion test of industry (2/3-digit NAICS) fixed effects.

^c p -value for the exclusion test of time fixed effects.

total debt to the market value of the firm’s equity—denoted by $[D/E]_{it}$ —captures the strength of the firm’s balance sheet. By including both equity volatility and debt-to-equity ratio in the specification, we are implicitly capturing the notion of the “distance-to-default,” which, according to [Merton \(1974\)](#), should be a sufficient statistic for default. The vector $\mathbf{X}_{it}[k]$, by contrast, controls for the (pre-determined) bond-specific characteristics that could influence credit spreads through either liquidity or term premiums, including the bond’s duration, the par amount outstanding, the bond’s age, the bond’s (fixed) coupon rate, and an indicator variable for whether the bond is callable.

Table 1 summarizes the estimation results. As shown in column 1, an increase in idiosyncratic uncertainty leads to a significant widening of credit spreads—the elasticity estimate of 0.730 implies that an increase in volatility of 10 percentage points (a move of a bit less than one standard deviation) is associated with an immediate jump in credit spreads of about 50 basis points. The coefficients on the remaining key variables are also economically and statistically highly significant and have their expected signs: Strong profitability performance, as evidenced by a high realized return on equity or an increase in the ratio of operating income to assets, is associated with a narrowing of credit spreads, whereas an increase in leverage leads to a rise in credit spreads.

The inclusion of bond-specific credit rating fixed effects (column 2) substantially improves the fit of the regression—evidently ratings contain important additional information about the borrowers’

credit quality—though it noticeably lowers the coefficient on uncertainty. Nevertheless, the effect remains economically and statistically highly significant, with a 10 percentage point increase in idiosyncratic volatility implying a widening of credit spreads of about 30 basis points. This estimate is robust to the inclusion of industry fixed effects (column 3), which control for any systematic differences in recovery rates across industries.

The specification in column 4 also controls for common macroeconomic shocks by adding a full set of time dummies to the regression. Although the magnitude of the coefficient on uncertainty shrinks further, the impact of uncertainty on credit spreads remains economically and statistically important—an increase of 10 percentage points in idiosyncratic volatility is associated with a rise in credit spreads of 15 basis points. In sum, these results strongly support the notion that fluctuations in uncertainty affect financial conditions by significantly altering the level of credit spreads in the economy.

We now turn to the main question posed by the paper: How does the interaction of uncertainty and financial conditions affect investment dynamics? Following [Leahy and Whited \(1996\)](#) and the more recent work of [Bloom et al. \(2007\)](#) and [Panousi and Papanikolaou \(2012\)](#), our empirical approach involves regressing business fixed investment on the firm-specific measure of idiosyncratic volatility, while controlling for the fundamental determinants of investment spending. In contrast to the earlier research, our regressions also include credit spreads at the level of an individual firm, which allows us to parse out the effect of uncertainty on investment conditional on this important financial indicator.

Specifically, we consider the following empirical investment equation:

$$\log[I/K]_{it} = \beta_1 \log \sigma_{it} + \beta_2 \log s_{it} + \theta \log Z_{it} + \eta_i + \lambda_t + \epsilon_{it}, \quad (4)$$

where $[I/K]_{it}$ denotes the investment rate of firm i in period t ; σ_{it} is our measure of idiosyncratic uncertainty; s_{it} is the credit spread on the portfolio of bonds issued by firm i ; and Z_{it} is a proxy for the marginal product of capital, a variable that measures firm i 's future investment opportunities.⁸ In addition to uncertainty, credit spreads, and investment fundamentals, equation (4) includes a firm fixed effect η_i and a time fixed effect λ_t —the former controls for systematic differences in the average investment rate across firms, while the latter captures a common investment component reflecting macroeconomic factors, which can influence firm-level investment through movements in output or interest rates. The investment fundamentals Z_{it} are measured using either the current sales-to-capital ratio $[Y/K]_{it}$ or the current operating-income-to-capital ratio $[\Pi/K]_{it}$, two widely used proxies for the marginal product of capital (see [Gilchrist and Himmelberg, 1998](#)).⁹ As an alternative

⁸The log-log nature of specification (4) reflects the fact that the firm-level investment rates, uncertainty, and credit spreads are highly positively skewed, a feature of the data that is significantly ameliorated through the use of a logarithmic transformation. Section A.2 of the data appendix contains details regarding the construction of variables used in this exercise.

⁹Taking logs of $[Y/K]_{it}$ is straightforward; but because operating income may be negative, we use $\log(c + [\Pi/K]_{it})$ —where c is chosen so that $(c + [\Pi/K]_{it}) > 0$ for all i and t —when relying on the operating income to measure the firm's investment opportunities. In principle, the estimated elasticities may depend on the constant c . In practice, however, reasonable variation in c had no effect on the estimated elasticities.

Table 2: Idiosyncratic Uncertainty, Credit Spreads, and Investment
(*Static Investment Specification*)

Explanatory Variable	(1)	(2)	(3)	(4)	(5)	(6)
$\log \sigma_{it}$	-0.169 (0.036)	-0.081 (0.034)	-0.157 (0.034)	-0.036 (0.035)	0.022 (0.033)	-0.062 (0.034)
$\log s_{it}$.	.	.	-0.206 (0.021)	-0.172 (0.021)	-0.152 (0.021)
$\log[Y/K]_{it}$	0.558 (0.046)	.	.	0.535 (0.045)	.	.
$\log[\Pi/K]_{it}$.	1.166 (0.086)	.	.	1.075 (0.088)	.
$\log Q_{i,t-1}$.	.	0.715 (0.040)	.	.	0.645 (0.041)
R^2 (within)	0.325	0.307	0.297	0.349	0.323	0.310

NOTE: Sample period: 1973:M1–2012:M9 at an annual frequency; No. of firms = 772; Obs. = 8,557. The dependent variable in all specifications is $\log[I/K]_{it}$, the logarithm of investment rate of firm i in year t . All specifications include time fixed effects (not reported) and firm fixed effects, which are eliminated using the within transformation, with the resulting specification estimated by OLS. Robust asymptotic standard errors reported in parentheses are clustered at the firm level. Parameter estimates for $\log[\Pi/K]_{it}$ and the associated standard errors are adjusted for the fact that $\log[\Pi/K]_{it}$ is computed as $\log(0.5 + [\Pi/K]_{it})$.

forward-looking measure of investment fundamentals, we also consider Tobin’s Q , denoted by Q_{it} .

The result in columns 1–3 of Table 2 indicate a significant role for uncertainty in the capital accumulation process—the coefficient on uncertainty is statistically highly significant, regardless of the measure of investment fundamentals. The estimated elasticities lie in the range between -0.17 and -0.08 , indicating that an increase in idiosyncratic volatility of 10 percentage points is associated with a decline in the investment rate of 0.5 to 1.1 percentage points. However, once the credit spreads are added to the regression (columns 4–6), the marginal effect of uncertainty on investment is virtually eliminated. Credit spreads, in contrast, are statistically and economically highly important determinants of investment spending, with a 100 basis point rise in credit spreads implying a drop in the investment rate of 1.4 to 1.9 percentage points.

A well-documented result from the empirical investment literature is the fact that lagged investment rate is economically an important determinant of current investment spending (see Gilchrist and Himmelberg, 1995; Eberly et al., 2012). Accordingly, we also consider a dynamic specification of the form:

$$\log[I/K]_{it} = \beta_1 \log \sigma_{it} + \beta_2 \log s_{it} + \theta_1 \log Z_{it} + \theta_2 \log[I/K]_{i,t-1} + \eta_i + \lambda_t + \epsilon_{it}. \quad (5)$$

In this case, we eliminate firm fixed effects using the forward orthogonal deviations transformation of Arellano and Bover (1995) and estimate the resulting specification using GMM. Within this dynamic framework, both uncertainty and credit spreads are treated as endogenous and are

Table 3: Idiosyncratic Uncertainty, Credit Spreads, and Investment
(*Dynamic Investment Specification*)

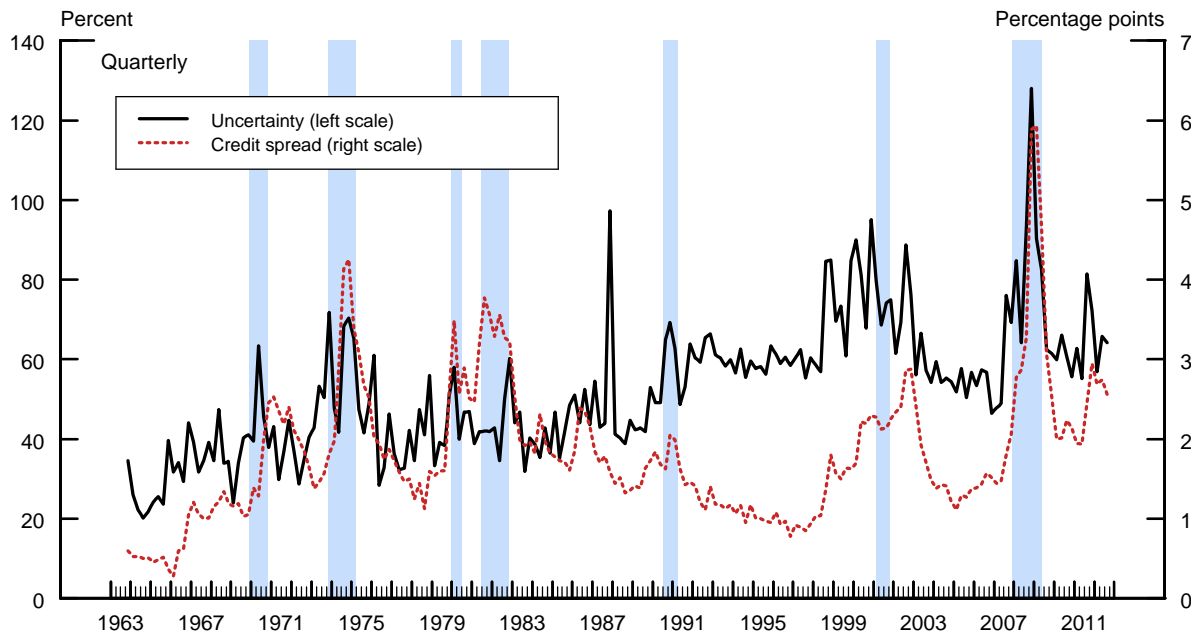
Explanatory Variable	(1)	(2)	(3)	(4)	(5)	(6)
$\log \sigma_{it}$	-0.272 (0.062)	-0.179 (0.059)	-0.199 (0.060)	-0.123 (0.057)	-0.078 (0.054)	-0.106 (0.057)
$\log s_{it}$.	.	.	-0.101 (0.031)	-0.068 (0.031)	-0.080 (0.032)
$\log[I/K]_{i,t-1}$	0.568 (0.028)	0.576 (0.023)	0.538 (0.029)	0.565 (0.027)	0.567 (0.023)	0.535 (0.023)
$\log[Y/K]_{it}$	0.446 (0.056)	.	.	0.452 (0.053)	.	.
$\log[\Pi/K]_{it}$.	0.918 (0.144)	.	.	0.908 (0.135)	.
$\log Q_{i,t-1}$.	.	0.548 (0.045)	.	.	0.507 (0.042)
L-R effect: σ_{it}	-0.630 (0.142)	-0.421 (0.137)	-0.430 (0.125)	-0.282 (0.130)	-0.180 (0.124)	-0.228 (0.124)
L-R effect: s_{it}	.	.	.	-0.233 (0.070)	-0.156 (0.069)	-0.171 (0.066)

NOTE: Sample period: 1973:M1–2012:M9 at an annual frequency; No. of firms = 758; Obs. = 6,615. The dependent variable in all specifications is $\log[I/K]_{it}$, the logarithm of investment rate of firm i in year t . All specifications include time fixed effects (not reported) and firm fixed effects, which are eliminated using the forward orthogonal deviations transformation. The resulting specification is estimated by GMM using a one-step weighting matrix (see [Arellano and Bover, 1995](#)). Robust asymptotic standard errors reported in parentheses are clustered at the firm level. Parameter estimates for $\log[\Pi/K]_{it}$ and the associated standard errors are adjusted for the fact that $\log[\Pi/K]_{it}$ is computed as $\log(0.5 + [\Pi/K]_{it})$.

instrumented with their own lagged values.

As shown in columns 1–3 of Table 3, fluctuations in uncertainty have economically large and statistically significant effects on capital spending. Taking into account the lagged investment dynamics (i.e., the L-R effects), the estimated elasticities imply that an increase in volatility of 10 percentage points depresses the investment rate 2.7 to 4.0 percentage points in the long run. However, the adverse effect of increased uncertainty on investment spending is more than halved once the information content of credit spreads is taken into account—estimates of the long-run elasticities in columns 4–6 imply that the same-sized increase in volatility lowers the investment rate only 1.1 to 1.8 percentage points. In contrast, a jump of 100 basis points in credit spreads is estimated to shave off 1.4 to 2.1 percentage points from the rate of capital formation in the long run. The confluence of results reported in Tables 1–3 is thus consistent with the notion that changes in credit spreads are an important part of the mechanism through which fluctuations in idiosyncratic volatility influence investment dynamics.

Figure 1: Uncertainty and Credit Spreads



NOTE: Sample period: 1963:Q4–2012:Q3. The solid line depicts the estimate of idiosyncratic uncertainty (in annualized percent) based on firm-level equity returns (see the text for details). The dotted line depicts the spread between the 10-year yield on BBB-rated nonfinancial corporate bonds and the 10-year Treasury yield. The shaded vertical bars denote the NBER-dated recessions.

2.2 Macroeconomic Implications

To examine the aggregate implications of the dynamic interaction between uncertainty and financial conditions, we use a SVAR model to trace out the impact of volatility and financial shocks on the macroeconomy. We construct an aggregate proxy for idiosyncratic uncertainty by assuming the firm-specific measure of uncertainty σ_{it} follows an autoregressive process of the form:

$$\log \sigma_{it} = \gamma_i + \delta_i t + \rho \log \sigma_{i,t-1} + v_t + \epsilon_{it}, \quad (6)$$

where γ_i denotes a firm fixed effect intended to control for the cross-sectional heterogeneity in σ_{it} , while the firm-specific term $\delta_i t$ captures secular trends in the idiosyncratic risk of publicly traded U.S. nonfinancial firms documented by [Campbell et al. \(2001\)](#).

At the aggregate level, our uncertainty proxy corresponds to the sequence of estimated time fixed effects \hat{v}_t , $t = 1, \dots, T$, which captures shocks to idiosyncratic volatility that are common to all firms. The presence of this common variation is essential because if fluctuations in σ_{it} were themselves entirely idiosyncratic, the macroeconomic impact of uncertainty shocks should wash out in the aggregate.¹⁰ The solid line in Figure 1 shows our estimate of time-varying uncertainty derived

¹⁰We estimate equation (6) by OLS, using the entire panel of 11,303 firms over the 1963:Q3–2012:Q3 period. Because the average firm is in the panel for more than 60 quarters, the bias of the OLS estimator, owing to the presence of

from the estimated time fixed effects in equation (6). The dotted line is the spread between the 10-year yield on BBB-rated nonfinancial corporate bonds and the 10-year Treasury yield, a barometer of strains in financial markets. Both uncertainty and credit spreads are clearly countercyclical, typically increasing significantly before and during recessions and spiking in periods of widespread financial distress.

Nevertheless, there are also clear exceptions to this pattern, the most obvious one being the end of 1987, a period marked by a surge in idiosyncratic volatility due to the stock market crash early in the fourth quarter that, interestingly, did not elicit a concomitant increase in credit spreads. The fact that this surge in volatility had essentially no effect on the real economy provides prima facie evidence for our hypothesis that changes in financial conditions are an important part of the mechanism through which fluctuations in uncertainty affect the economy.

To quantify the effect of uncertainty and financial shocks on economic activity, we estimate a VAR consisting of the following eight endogenous variables: the log of real business fixed investment (i_t); the log of real personal consumption expenditures (PCE) on durable goods (c_t^D); the log of real PCE on nondurable goods and services (c_t^N); the log of real GDP (y_t); the log of the GDP price deflator (p_t); the nominal effective federal funds rate (m_t)—an indicator of the stance of monetary policy; the 10-year BBB-Treasury credit spread (s_t); and a proxy for idiosyncratic uncertainty at the aggregate level (\hat{v}_t).¹¹ To identify uncertainty shocks, we employ a standard recursive ordering technique, in which innovations in uncertainty have an immediate impact on credit spreads and short-term interest rates, but they affect economic activity and prices with a lag (Identification Scheme I).¹² As a point of comparison, we rely on the same recursive ordering to examine the impact of shocks to credit spreads—that is, “financial shocks”—that are orthogonal to the contemporaneous level uncertainty and other macroeconomic variables. We also consider a specification that reverses this causal ordering, which allows us to examine the implications of uncertainty shocks conditional on the information contained in the current level of credit spreads (Identification Scheme II).

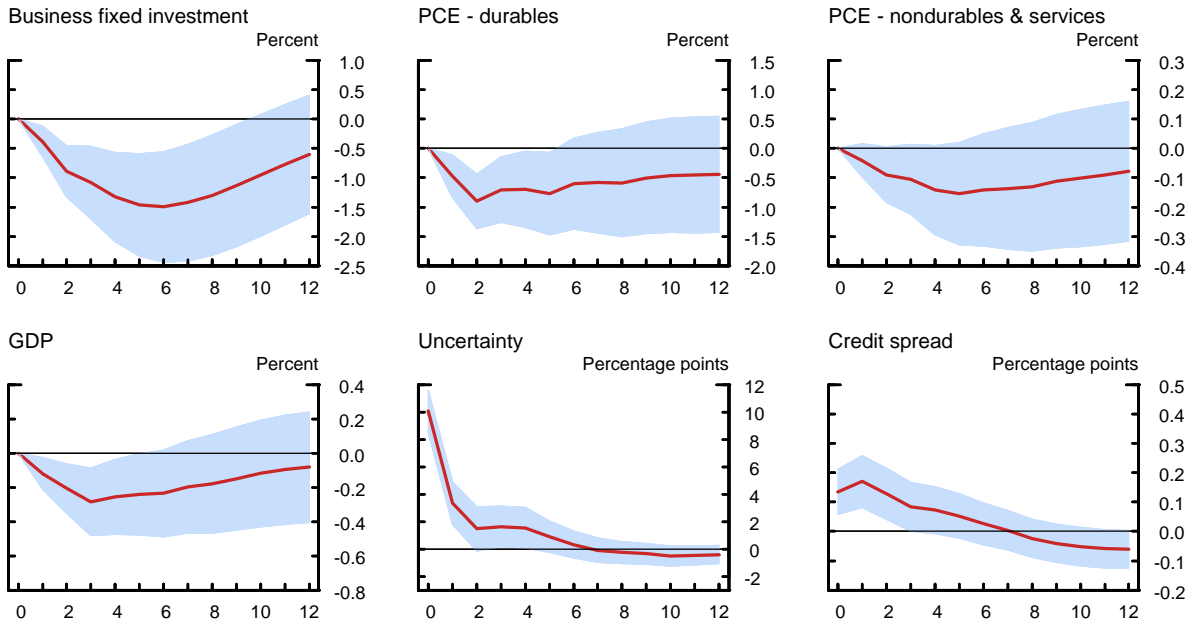
Panel (a) in Figure 2 shows the impulse responses of selected variables to an uncertainty shock, while Panel (b) shows the responses of the same variables to a financial shock, where both shocks are orthogonalized using the first identification scheme. Given these identifying assumptions, an unanticipated increase in volatility is associated with an immediate widening of credit spreads—in fact, the magnitude of the response of credit spreads upon impact is remarkably consistent with the micro-level estimates shown in column 4 of Table 1. The uncertainty shock also has significant

a lagged dependent variable, firm fixed effects, and incidental trends is likely to be negligible (see Phillips and Sul, 2007). The estimation yields an estimate of $\rho = 0.443$, an indication that uncertainty tends to be fairly persistent. The specification also fits the data quite well, explaining more than 75 percent of the variation in the dependent variable.

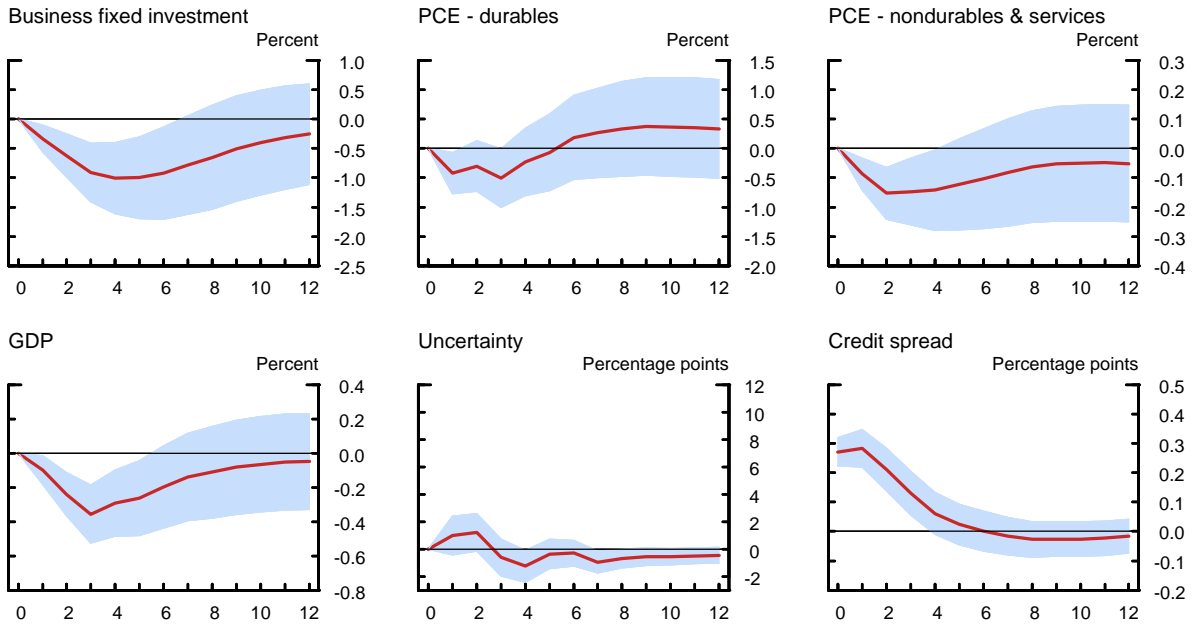
¹¹The VAR is estimated over the 1963:Q3–2012:Q3 period using four lags of each endogenous variable. The very end of the sample (2009:Q1–2012:Q3) includes a period during which the federal funds rate has been very close to zero. However, omitting the zero-rate policy phase from the sample had no discernible effect on the VAR results reported in the paper.

¹²Variants of this approach can be found in Popescu and Smets (2010), Bekaert et al. (2013), Bachmann et al. (2013b), Caldara et al. (2013), and Cesa-Bianchi et al. (2014)

Figure 2: Macroeconomic Implications of Uncertainty and Financial Shocks
(Identification Scheme I)



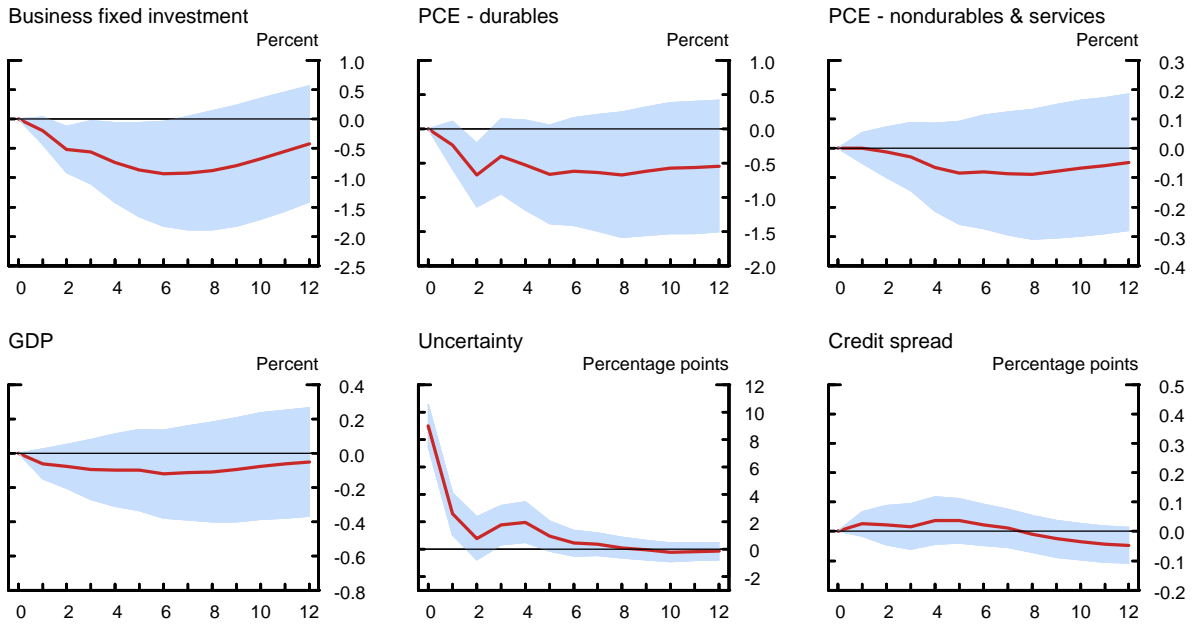
(a) Response of selected macroeconomic variables to an uncertainty shock



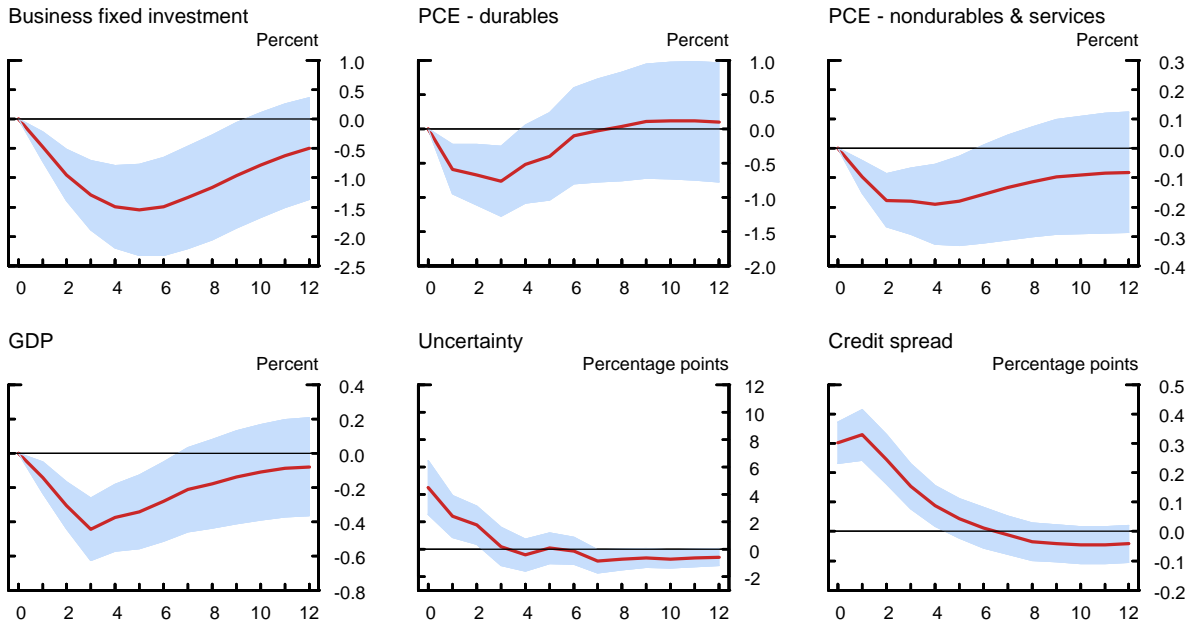
(b) Response of selected macroeconomic variables to a financial shock

NOTE: Panel (a) depicts the impulse responses of selected macroeconomic and financial indicators to an orthogonalized 1 standard deviation shock to our estimate of idiosyncratic uncertainty; Panel (b) depicts the corresponding impulse response to an orthogonalized 1 standard deviation shock to the 10-year BBB-Treasury credit spread. Identification scheme I corresponds to the following recursive ordering of the VAR system: $(i_t, c_t^D, c_t^N, y_t, p_t, v_t, s_t, m_t)$; see the text for details. The shaded bands represent the 95-percent confidence intervals based on 1,000 bootstrap replications.

Figure 3: Macroeconomic Implications of Uncertainty and Financial Shocks
(Identification Scheme II)



(a) Response of selected macroeconomic variables to an uncertainty shock



(b) Response of selected macroeconomic variables to a financial shock

NOTE: Panel (a) depicts the impulse responses of selected macroeconomic and financial indicators to an orthogonalized 1 standard deviation shock to our estimate of idiosyncratic uncertainty; Panel (b) depicts the corresponding impulse response to an orthogonalized 1 standard deviation shock to the 10-year BBB-Treasury credit spread. Identification scheme II corresponds to the following recursive ordering of the VAR system: $(i_t, c_t^D, c_t^N, y_t, p_t, s_t, v_t, m_t)$; see the text for details. The shaded bands represent the 95-percent confidence intervals based on 1,000 bootstrap replications.

adverse consequences for economic growth, as evidenced by the almost immediate decline in output, which bottoms out about a year after the initial spike in volatility.

Consistent with the standard irreversibility argument, the impact of the uncertainty shock falls primarily on the investment component of aggregate spending: Business expenditures on fixed capital fall steadily, bottoming out 1.5 percent below the trend six quarters after the shock, while consumer spending on durable goods also declines significantly. As shown in Panel (b), a financial disturbance, which leads to an increase of almost 30 basis points in the BBB-Treasury spread, is also associated with a significant contraction in economic activity. In addition to depressing the investment component of aggregate output, this shock also induces a significant decline in nondurable goods consumption. Innovations in credit spreads, however, have an economically negligible and statistically insignificant effect on the level of uncertainty in the economy.

Figure 3 shows the implications of these two shocks orthogonalized using an alternative scheme, in which credit spreads are ordered before the uncertainty proxy. As shown in Panel (a), a volatility shock in that case has a noticeably less adverse effect on the macroeconomy. Although both business fixed investment and consumer spending on durable goods fall in response to an increase in uncertainty, the declines are much smaller and less persistent compared with those in Panel (a) of Figure 2; in fact, the response of output to an uncertainty shock is statistically indistinguishable from zero.

Unanticipated increases in credit spreads, in contrast, have significant and long-lasting effects on economic activity, according to Panel (b). A one standard deviation shock to the BBB-Treasury spread is associated with an immediate jump in uncertainty, a substantial fall in real GDP, and a protracted decline in business fixed investment as well as in major categories of consumer spending. Indeed, the magnitude and shape of the responses of output and its major components to credit spread shocks are very similar to those shown in the corresponding panel of Figure 2, indicating that the level of uncertainty in the economy is not very informative for gauging the macroeconomic impact of financial disturbances. The fact that uncertainty responds immediately to the innovation in credit spreads suggests that fluctuations in uncertainty may arise endogenously in response to changes in broader financial conditions (see [Caldara et al., 2013](#)).

All told, the evidence presented above shows that an unanticipated increase in uncertainty leads to an economically and statistically significant widening of credit spreads and a drop in real GDP, where the latter is driven by a protracted decline in the major investment categories of aggregate spending. The VAR analysis is also consistent with our micro-level results, corroborating the fact that changes in financial conditions—as summarized by the movements in credit spreads—appear to be a crucial part of the transmission mechanism propagating uncertainty shocks to the real economy. Indeed, our results indicate that once such disturbances are orthogonalized with respect to the contemporaneous level of credit spreads, the impact of uncertainty shocks on economic activity is significantly attenuated.

3 Model

The key agents in our model are heterogeneous firms facing time-varying idiosyncratic uncertainty, nonconvex capital adjustment frictions—including partial irreversibility and fixed costs—and distortions in both the debt and equity markets. Within this framework, we focus on two nontraditional sources of business cycle fluctuations: Disturbances from the first source affect the volatility of the idiosyncratic technology process, whereas those from the second directly reduce the collateral value of the firms’ capital assets and thus impinge on their borrowing capacity.

In the model, disturbances that alter the dispersion of the idiosyncratic technology shock affect all firms and hence capture “uncertainty” shocks in the aggregate sense. And unlike the “capital efficiency” shocks of [Gertler and Karadi \(2011\)](#), which, in effect, destroy the physical quantity of capital in the economy, “capital liquidity” shocks in our model only affect the resale value of capital assets that can be pledged as collateral. In the presence of irreversibilities and nonconvex capital adjustment costs, uncertainty and capital liquidity shocks have real consequences for macroeconomic outcomes regardless of the structure of financial markets. Distortions in financial markets, however, induce a change in the effective supply of credit in response to both types of disturbances, an effect that significantly amplifies the initial impact of each shock on aggregate investment.

3.1 Production

The production side of the economy contains a continuum of heterogeneous firms that produce goods used for consumption and investment. Firms producing the final good (y) combine labor (h) and capital (k) using a decreasing returns-to-scale (DRS) production technology. The production is subject to aggregate and idiosyncratic technology shocks, denoted by a and z , respectively. It also requires a payment of fixed operating costs that are proportional to firm size as measured by its existing capital stock, with the proportionality factor denoted by $F_o > 0$.

Formally, these assumptions are summarized by a production function

$$y = (az)^{(1-\alpha)\chi} (k^\alpha h^{1-\alpha})^\chi - F_o k; \quad 0 < \alpha < 1 \text{ and } \chi < 1, \quad (7)$$

where α is the value-added share of capital, and χ governs the degree of decreasing returns in production. The normalization factor $(1 - \alpha)\chi$ associated with the exogenous technology shocks ensures that the firm’s profit function is linear in a and z :

$$\pi(a, z, w, k) = \max_{h \geq 0} \{ (az)^{(1-\alpha)\chi} (k^\alpha h^{1-\alpha})^\chi - F_o k - wh \} = az \psi(w) k^\gamma - F_o k, \quad (8)$$

where w denotes the (real) wage and

$$\gamma = \frac{\alpha\chi}{1 - (1 - \alpha)\chi} \quad \text{and} \quad \psi(w) = [1 - (1 - \alpha)\chi] \left[\frac{(1 - \alpha)\chi}{w} \right]^{\frac{(1-\alpha)\chi}{1 - (1-\alpha)\chi}}.$$

The combination of decreasing returns-to-scale and fixed operating costs implies that the firm

can earn strictly positive (or negative) profits in equilibrium. In principle, this can generate non-trivial firm dynamics through entry and exit. To keep the model tractable, however, we do not explicitly model the firm’s endogenous entry/exit decision. As in [Cooley and Quadrini \(2001\)](#) and [Veracierto \(2002\)](#), we assume that a constant fraction $1 - \eta$ of firms exogenously exits the industry in each period and that the exiting firms are replaced by identical new firms within the same period. This stochastic overlapping generation structure provides a convenient way to motivate the use of leverage by firms in the steady state without introducing a corporate income tax shield.

The process governing the evolution of the aggregate technology shock a is standard, as we assume that it follows a continuous Markov process:

$$\log a' = \rho_a \log a + \log \epsilon'_a; \quad \log \epsilon'_a \sim N(-0.5\omega_a^2, \omega_a^2). \quad (9)$$

The idiosyncratic technology shock z , by contrast, evolves according to an N -state Markov chain with *time-varying* volatility. We let $i, j = 1, \dots, N$ index the technology states and let $p_{i,j}$ denote the transition probability of moving from state i in the current period to state j in the subsequent period. Importantly, the Markov chain of the idiosyncratic technology shock is constructed in such a way that its conditional mean is not affected by fluctuations in volatility.¹³ Its conditional variance, however, is a linear function of the realization of the time-varying volatility process, which is given by a continuous Markov process of the form

$$\log \sigma' = (1 - \rho_\sigma) \log \bar{\sigma} + \rho_\sigma \sigma + \log \epsilon'_\sigma; \quad \log \epsilon'_\sigma \sim N(-0.5\omega_\sigma^2, \omega_\sigma^2). \quad (10)$$

3.2 Capital Accumulation

Firms accumulate capital subject to two types of adjustment frictions: fixed costs and partial irreversibility. Formally, the total cost of capital adjustment is given by

$$g(k', k) = F_k k + (p^+ \times \mathbf{1}[k' \geq (1 - \delta)k] + p^- \times \mathbf{1}[k' < (1 - \delta)k]) [k' - (1 - \delta)k], \quad (11)$$

where $\mathbf{1}[\cdot]$ is an indicator function and $0 < \delta < 1$ denotes the depreciation rate. The term $F_k k$ represents the fixed costs associated with capital expenditures, which are assumed to be proportional to the size of a firm.¹⁴

As emphasized by [Cooper and Haltiwanger \(2006\)](#), the fixed costs capture the inherent indivis-

¹³See Section [B.1](#) of the model appendix for technical details. It is worth noting that in our framework, the number of the states of the Markov chain is constant over time. However, because the volatility of the chain is time varying, the nodes in the support of the distribution of the idiosyncratic technology shock change in such a way that firms face a greater dispersion of idiosyncratic technology levels when volatility increases. Hence, the index j in the expression $z'_j(\sigma)$ signifies only the relative position—and not the absolute value—in the support of the realized distribution of z that is associated with the volatility level σ .

¹⁴Note that $g((1 - \delta)k, k) = F_k k$, even when gross investment is equal to zero. This specification of capital adjustment costs, however, does not imply that the firm pays the fixed costs $F_k k$ in every period—that is, irrespective of its investment action/inaction status. As discussed below, the capital adjustment costs $g(k', k)$ enter the firm’s problem multiplied by a decision variable $\nu \in \{0, 1\}$, and when the firm finds it optimal to set $\nu = 0$, it avoids paying the fixed costs of adjustment (see [Abel and Eberly, 1994](#)).

ibility of physical capital and potential increasing returns to both the installation of new capital and restructuring of productive capacity during periods of intensive investment. The second term in equation (11) corresponds to the costly reversible investment framework of [Abel and Eberly \(1996\)](#), whereby the liquidation value of installed capital p^- is a fraction of its purchase price p^+ . The assumption that $p^-/p^+ < 1$ captures the notion of asset specificity and implies that capital in place is less liquid than new capital. To explore the implications of time-varying liquidity of capital assets, we let p^- follow a continuous Markov process:

$$\log p^{-'} = (1 - \rho_{p^-}) \log \tilde{p}^- + \rho_{p^-} \log p^- + \epsilon'_{p^-}; \quad \log \epsilon'_{p^-} \sim N(-0.5\omega_{p^-}^2, \omega_{p^-}^2), \quad (12)$$

where ϵ'_{p^-} denotes a capital liquidity shock that through its effect on the resale value of capital assets affects the amount of pledgeable collateral in the economy.

3.3 The Firm’s Problem

At the beginning of each period, all economic agents observe the realization of the idiosyncratic and aggregate productivity shocks (z and a , respectively). The agents’ information set also includes the level of idiosyncratic uncertainty (σ) and the liquidation value of capital (p^-). The timing assumptions are such that this period’s volatility level σ determines the distribution of $z'(\sigma)$ in the subsequent period, where our notation makes explicit the dependence of the distribution of z' on the realized level of uncertainty. Thus, from a perspective of agents in the model, an increase in σ today represents “news” regarding the distribution of profits tomorrow. To streamline the notation, we define the aggregate state of the economy as a vector $\mathbf{s} = [a, \sigma, p^-, \mu]$, where μ denotes the joint distribution of the idiosyncratic technology, capital, and net liquid asset positions (to be defined below) across heterogeneous firms.

3.3.1 Financing Investment

To finance investment projects, firms use a combination of internal and external funds, where the sources of external funds are debt and equity. Relative to internal funds, external funds command a premium, either because of the direct cost of issuing equity, or in the case of debt, because of the agency costs associated with default.

Bond finance: Debt finance available to the firm consists of one-period, zero-coupon bonds.¹⁵ The debt contract specifies the par value of the issue b' and the price q^B , yielding the total amount of debt financing equal to $q^B b'$ in each period. By combining the proceeds from bond issuance with other sources of funds, the firm purchases capital to be used in production. In the subsequent period—after observing the realization of shocks—the firm decides whether to fulfill its debt obligation. If the firm decides not to default, it pays the face value of the debt b' to the lender and

¹⁵Given the preponderance of callable corporate debt in the United States (see [Faust et al., 2013](#)), this assumption is not as restrictive as it may seem at first glance.

makes its production and financial decisions for the next period. If the firm chooses to default, however, it enters a debt-renegotiation process with bond investors.

The renegotiation process is conducted under limited liability by assuming that there exists a lower bound to the net worth of the firm—denoted by \bar{n} —below which the firm cannot promise to pay back any outstanding liability.¹⁶ The realized net worth next period is defined as the sum of net profits and the market value of undepreciated capital, less the face value of debt:

$$n' = a' z'_j(\sigma) \psi(w(\mathbf{s}')) k'^{\gamma} - F_o k' + p^{-'}(1 - \delta) k' - b'. \quad (13)$$

Note that the value of capital in place is evaluated at the resale value $p^{-'}$, rather than its book value p^+ . By combining the expression for net worth with the default condition $n' \leq \bar{n}$, we can define a level of idiosyncratic technology that triggers default—denoted by z^D —conditional on the tomorrow's aggregate state \mathbf{s}' and individual state (k', b') , as

$$z^D(k', b'; \mathbf{s}') \equiv \frac{\bar{n} + b' + F_o k' - p^{-'}(1 - \delta) k'}{a' \psi(w(\mathbf{s}')) k'^{\gamma}}. \quad (14)$$

Under limited liability, the new level of debt renegotiated by the firm and bond investors—denoted by b^R —cannot exceed the amount of debt $\bar{b}(k', z'_j(\sigma); \mathbf{s}')$ that is consistent with the assumed lower bound on net worth:

$$b^R \leq \bar{b}(k', z'_j(\sigma); \mathbf{s}') \equiv a' z'_j(\sigma) \psi(w(\mathbf{s}')) k'^{\gamma} - F_o k' + p^{-'}(1 - \delta) k'. \quad (15)$$

We assume that the firm does not have any bargaining power during the renegotiation process. This assumption implies that the renegotiated debt is determined by the upper bound on the amount of debt that bond investors can recover in the case of default—that is, $b^R = \bar{b}(k', z'_j(\sigma); \mathbf{s}')$.

The default entails a dead-weight loss, captured by bankruptcy costs that are assumed to be proportional to the amount of liquidated capital. As in [Townsend \(1979\)](#), the bankruptcy costs reflect a loss of resources expended by creditors to prevent managers of a defaulting firm from behaving opportunistically. Thus the actual recovery in the case of default is given by $b^R - \xi(1 - \delta)k'$, where the parameter $0 < \xi < 1$ governs the magnitude of the bankruptcy costs and hence the degree of frictions in the corporate bond market. Therefore, the recovery rate $0 \leq \mathcal{R} < 1$ in the case of default is given by

$$\mathcal{R}(k', b', z'_j(\sigma); \mathbf{s}') = \frac{\bar{b}(k', z'_j(\sigma), \mathbf{s}')}{b'} - \xi(1 - \delta) \frac{k'}{b'}. \quad (16)$$

¹⁶This type of contract is similar to that used by [Cooley and Quadrini \(2001\)](#) and [Hennessy and Whited \(2007\)](#). In our setup, however, default occurs when the net worth of the firm n hits the lower bound \bar{n} , whereas in the aforementioned papers, a firm defaults when the value of its equity v hits the lower bound \bar{v} . If the technology shock follows an i.i.d. process and the analysis is conducted in partial equilibrium, the two assumptions are equivalent. However, if the technology shock is persistent or the firm's value function has other arguments (e.g., aggregate state variables), the two assumptions are no longer equivalent. The decision to use a lower bound for the net worth to determine the default threshold is a simplifying assumption that allows us to avoid the computationally intensive task of inverting the value function to compute the default boundary in each iteration of the dynamic programming routine, which is very costly in our general equilibrium framework. Moreover, it is not clear empirically whether firms declare bankruptcy when the market value of their equity or net asset values becomes negative.

Standard no-arbitrage arguments then imply the following bond pricing formula:

$$q_i^B(k', b'; \mathbf{s}') = \mathbb{E} \left\{ m(\mathbf{s}, \mathbf{s}') \left[1 + \sum_{j \in \mathcal{D}} p_{i,j} [\mathcal{R}(k', b', z'_j(\sigma); \mathbf{s}') - 1] \right] \middle| \mathbf{s} \right\}, \quad (17)$$

where $m(\mathbf{s}, \mathbf{s}')$ is the stochastic discount factor of the representative household and

$$\mathcal{D} = \{j \mid j \in \{1, \dots, N\} \text{ and } z'_j(\sigma) \leq z^D(k', b'; \mathbf{s}')\} \quad (18)$$

is the set of states of the idiosyncratic technology shock z , in which the firm will default on its debt obligations.

Note that the exogenous exit rate $1 - \eta$ does not appear in the bond pricing formula (17). This reflects the assumption that the exit shock is realized *after* the firms make their repayment/default decisions. Consequently, the exit shock does not directly affect the returns of bond investors, and credit spreads that arise in our framework are not a direct result of the exogenous exit risk. However, the exit process does affect credit spreads indirectly by influencing the firms' choice of leverage.

Equity finance: To introduce equity financing in the model, we start with the definition of dividends:

$$d \equiv az_i(\sigma_{-1})\psi(w(\mathbf{s}))k^\gamma - F_o k - \nu g(k', k) - b + q_i^B b' + e, \quad (19)$$

where $\nu \in \{0, 1\}$ is the choice variable indicating whether the firm is in the investment inaction ($\nu = 0$) or action ($\nu = 1$) regime, and e denotes the value of newly issued shares when positive and the value of share repurchases when negative. Consistent with the prevalence of corporate dividend-smoothing policies, we posit that the firms face a minimum dividend constraint,

$$d \geq \bar{d} \geq 0. \quad (20)$$

Thus, when the firm's internal funds and the proceeds raised with the bond issue fall short of its financing needs, the firm must raise outside equity ($e > 0$) to satisfy the dividend constraint.¹⁷

In the absence of financial distortions, the notional amount e of equity issuance reduces the value of existing shares by the same amount. To introduce frictions in the stock market, we assume that equity issuance is costly, in the sense that the value of existing shares is reduced by more than the amount of newly issued shares. We capture this distortion by assuming a constant marginal cost of equity issuance, commonly referred to as equity dilution costs in the corporate finance literature

¹⁷As documented by [Fama and French \(2005\)](#), in an average year between 1973 and 2002, almost 60 percent of dividend paying firms also issued new shares in the same year, on net, evidence that seems difficult to reconcile with the assumption of frictionless financial markets. However, another possibility is that the lower bound on dividends \bar{d} in equation (20) is strictly positive for a fraction of firms that engage in dividend smoothing behavior for reasons unrelated to financial market frictions (see [Leary and Michaely, 2011](#)). Indeed, this is the interpretation we adopt in this paper.

(see Gomes, 2001; Cooley and Quadrini, 2001; Hennessy and Whited, 2007; Bolton et al., 2011). Formally, the loss in the value of existing shares associated with the amount e of newly issued equity is given by

$$\bar{\varphi}(e) = e + \varphi \max\{e, 0\}, \quad (21)$$

where $\varphi > 0$ measures the degree of frictions in the stock market. Share repurchases ($e < 0$), in contrast, are assumed equivalent to dividend payments and thus do not involve any dilution costs.

3.3.2 Financial Policy

To derive the optimal financial and investment policies, we formulate the firm's profit maximization problem recursively. By defining a composite state variable—the net liquid asset position (x)—as

$$x \equiv az_i(\sigma_{-1})\psi(w(\mathbf{s}))k^\gamma - F_0k - b = n - p^-(1 - \delta)k, \quad (22)$$

the firm's dividend can be rewritten as $d = x - \nu g(k', k) + q_i^B b' + e$. This allows us to express the value of equity as $v_i(k, x; \mathbf{s})$, where the subscript i denotes the firm's relative position in the discrete distribution of the idiosyncratic technology level z in the current period. In combination with the realized level of volatility σ_{-1} , this relative position is the only information needed to predict the subsequent values of the idiosyncratic technology shock.¹⁸

The firm's problem can then be formulated recursively as

$$v_i(k, x; \mathbf{s}) = \min_{\phi} \max_{d, e, \nu, k', b'} \left\{ d + \phi(d - \underline{d}) - \bar{\varphi}(e) \right. \\ \left. + \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j=1}^N p_{i,j} \max \{ v_j(k', x'_j(\sigma); \mathbf{s}'), v_j(k', x'_j^R(\sigma); \mathbf{s}') \} \mid \mathbf{s} \right] \right\} \quad (23)$$

subject to (17), (19), and $\mathbf{s}' = \Gamma(\mathbf{s}); \quad i, j = 1, \dots, N$,

where ϕ is the Lagrange multiplier associated with the dividend constraint (20) and $\mathbf{s}' = \Gamma(\mathbf{s})$ is the law of motion governing the evolution of the aggregate state vector, which we describe below. Note that the continuation value of the firm is bounded below by the default/renegotiation value

¹⁸An equivalent way of formulating the problem would involve specifying the value of equity as $v_{i,j}(k, b; \mathbf{s})$, where i denotes the firm's relative position in the distribution of z in the previous period and j denotes the relative position in the current period. In that case, we would need to keep track of both i and j because at any point in time, the realization of the idiosyncratic technology shock depends on i , j , and σ_{-1} . Specifically, as shown in Section B.1 of the model appendix, the realization of the Markov chain process with the time-varying volatility is given by

$$z_{i,j,t}(\sigma_{t-1}) = \bar{z} - \frac{\mu_i}{2}(\sigma_{t-1} - \bar{\sigma}) + \left[2 \left(\frac{j-1}{N-1} \right) - 1 \right] \frac{\sigma_{t-1}}{2},$$

where μ_i is the conditional mean of z for a firm with the productivity level in the i -th bin in period $t-1$. Hence i , j , and σ_{t-1} are needed to infer the exact value of the idiosyncratic technology shock in period t . However, as long as the firm knows its net liquid asset position x , the exact value of the realization of the idiosyncratic shock is irrelevant because only the current position in the distribution and the uncertainty level are needed to construct the conditional expectation of the idiosyncratic technology shock next period.

$v_j(k', x_j^R(\sigma); \mathbf{s}')$, where

$$x_j^R(\sigma) = a' z_j'(\sigma) \psi(w(\mathbf{s}')) k'^\gamma - F k' - b^R.$$

By directly differentiating the Bellman equation (23) with respect to e , we obtain the first-order condition for equity issuance:

$$1 + \phi = 1 + \varphi \times \mathbf{1}[e > 0], \quad (24)$$

an expression with a straightforward interpretation: The firm will issue new shares if and only if the dividend constraint is binding.

Similarly, directly differentiating the Bellman equation with respect to b' yields

$$(1 + \phi) [q_i^B(k', b'; \mathbf{s}) + q_{i,b}^B(k', b'; \mathbf{s}) b'] = \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j \in \mathcal{D}^c} p_{i,j} v_{j,x}(k', x_j'(\sigma); \mathbf{s}') \mid \mathbf{s} \right],$$

where \mathcal{D}^c denotes the complement of the default set defined by equation (18).¹⁹ The associated Benveniste-Scheinkman condition is given by $v_{i,x}(k, x; \mathbf{s}) = 1 + \phi$. In combination with equation (24), this allows us to express the first-order condition for debt issuance as

$$q_i^B(k', b'; \mathbf{s}) + q_{i,b}^B(k', b'; \mathbf{s}) b' = \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j \in \mathcal{D}^c} p_{i,j} \left(\frac{1 + \varphi \times \mathbf{1}[e' > 0]}{1 + \varphi \times \mathbf{1}[e > 0]} \right) \mid \mathbf{s} \right]. \quad (25)$$

The left side of equation (25) represents the marginal benefit of debt finance today, while the expression on the right equals the expected marginal cost of such financing tomorrow, with the ratio of the shadow value of internal funds tomorrow to that of today serving as the effective discounting factor. By increasing its leverage, the firm improves its current cashflow but increases the chance of a future liquidity shortfall, thereby raising the likelihood that it will have to issue costly new shares in the future. Under limited liability, the firm cares about future cashflows only to the extent that it avoids default, an aspect of the firm's financial policy captured by the summation over the states in the non-default set \mathcal{D}^c .

3.3.3 Investment Policy

The presence of irreversibilities and nonconvex capital adjustment costs in the firm's optimization problem complicates the derivation of the conditions characterizing the firm's optimal choice of capital. In this key dimension, however, our problem is similar to that analyzed by [Abel and Eberly \(1994\)](#), who provide a unifying framework for deriving an optimal investment policy under a generalized capital adjustment cost function, encompassing both nonconvex and convex adjustment costs as well as partial irreversibility.

To derive the conditions characterizing the firm's optimal capital choice, we modify their approach to take into account the interaction between the firm's financial and investment policies.

¹⁹The summation over the states in \mathcal{D}^c only reflects the fact that the effect of changes in b' on the default/renewal value $v_j(k', x_j^R(\sigma); \mathbf{s}')$ is zero.

Specifically, let

$$q_i^K(k', b'; \mathbf{s}) = \eta \frac{\partial}{\partial k'} \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j=1}^N p_{i,j} \max \{ v_j(k', x_j^R(\sigma); \mathbf{s}'), v_j(k', x_j^B(\sigma); \mathbf{s}') \} \mid \mathbf{s} \right] \quad (26)$$

denote the marginal value of an additional unit of capital—that is, the Tobin’s marginal q .²⁰ For the sake of argument, assume that the investment efficiency conditions can be obtained directly by differentiating the Bellman equation (23) with respect to k' , thus ignoring any nondifferentiabilities. In addition, assume that action is always optimal—that is, $\nu = 1$. In that case, the first-order condition governing the optimal choice of capital is given by

$$(1 + \phi) [g_{k'}(k', k) - q_{i,k'}^B(k', b'; \mathbf{s})] = q_i^K(k', b'; \mathbf{s}).$$

In practice, however, the above condition may not lead to optimal investment because the firm may find it in its interest to delay (dis)investment. To take into account this possibility, we define the left- and right-hand side derivatives of the marginal q , evaluated at $k' = (1 - \delta)k$, as

$$q_i^{K-} = \lim_{k' \uparrow (1-\delta)k} (1 + \phi) [g_{k'}(k', k) - q_{i,k'}^B(k', b'; \mathbf{s})b']; \quad (27)$$

and

$$q_i^{K+} = \lim_{k' \downarrow (1-\delta)k} (1 + \phi) [g_{k'}(k', k) - q_{i,k'}^B(k', b'; \mathbf{s})b']. \quad (28)$$

Then if $q_i^K((1 - \delta)k, b'; \mathbf{s}) \notin [q_i^{K-}, q_i^{K+}]$, the first-order condition for the optimal choice of capital implies that k' must satisfy

$$g_{k'}(k', k) = q_{i,k'}^B(k', b'; \mathbf{s})b' + \frac{1}{1 + \phi} q_i^K(k', b'; \mathbf{s}), \quad (29)$$

whereas if $q_i^K((1 - \delta)k, b'; \mathbf{s}) \in [q_i^{K-}, q_i^{K+}]$,

$$k' = (1 - \delta)k. \quad (30)$$

Equation (29) characterizes the firm’s optimal choice of capital assuming $\nu = 1$, that is, investment action is always optimal. Intuitively, this first-order condition equates the marginal benefit and cost of an additional unit of capital. In the presence of fixed costs, however, the investment level that equalizes these two margins may not be unique. In that case, the optimal level of investment should not only be consistent with the marginal efficiency condition, but should also justify the fixed costs associated with capital adjustment, an effect that expands the inaction interval $[q_i^{K-}, q_i^{K+}]$.

²⁰Note that the marginal q is well defined despite the kinks in the firm’s value function due to default and non-convexities in the capital adjustment process. This is because q is defined for the expected value of the firm, rather than its realized value, and the kinks correspond to measure-zero events.

To provide sufficient conditions for the action/inaction decision ($\nu \in \{0, 1\}$), we can reformulate the firm's problem (23) as a discrete choice problem:

$$v_i(k, x; \mathbf{s}) = \max \{v_i(k, x; \mathbf{s}|\nu = 1), v_i(k, x; \mathbf{s}|\nu = 0)\}. \quad (31)$$

With this formulation, the sufficient condition for a strictly positive or negative gross investment is then given by $v_i(k, x; \mathbf{s}|\nu = 1) - v_i(k, x; \mathbf{s}|\nu = 0) > 0$. Once this condition is met, the firm's optimal investment policy is fully characterized by equations (29) and (30).²¹

To highlight the role of financial distortion in the model, it is helpful to consider the case of frictionless financial markets. Hence, the firm's capital structure is indeterminate, and $b' = 0$ without loss of generality; furthermore, the shadow value of internal funds—as measured by the Lagrange multiplier on the dividend constraint—is always equal to one. Because the capital adjustment costs do not include a convex cost component (hence, $q_i^{K^-} = p^-$ and $q_i^{K^+} = p^+$), the firm's target stock of capital k' satisfies the following conditions:²²

$$\begin{aligned} p^+ &= q_i^K(k'; \mathbf{s}) \quad \text{if } q_i^K((1 - \delta)k; \mathbf{s}) > p^+; \\ k' &= (1 - \delta)k \quad \text{if } p^- \leq q_i^K((1 - \delta)k; \mathbf{s}) \leq p^+; \\ p^- &= q_i^K(k'; \mathbf{s}) \quad \text{if } q_i^K((1 - \delta)k; \mathbf{s}) < p^-. \end{aligned}$$

The intuition behind these conditions is straightforward. If the Tobin's marginal q evaluated at $k' = (1 - \delta)k$ is strictly greater than the purchase price of capital p^+ , the firm should increase its productive capacity until the marginal value of an additional unit capital is equal to its purchase price. If, on the other hand, the marginal q evaluated at $k' = (1 - \delta)k$ is strictly less than the resale price of capital p^- , the firm should disinvest up to the point where the marginal value of capital rises to the level of the resale price. If neither of these two conditions are met, the firm should do nothing.

Financial frictions modify the above efficiency conditions in two ways. First, investment that expands the firm's productive capacity also increases its debt capacity, which lowers the firm's marginal borrowing costs, an effect captured by the term $q_{i,k'}^B(k', b'; \mathbf{s})b' > 0$ in equation (29). In turn, this reduces the effective marginal cost of investment today. Second, the combination of a potential liquidity shortfall and the associated costly external equity issuance causes the firm to discount the future (marginal) benefit of investment to a greater extent than in the case of a frictionless stock market. The additional discounting factor is given by the inverse of the firm's liquidity condition—the term $(1 + \phi)^{-1}$ in equation (29)—which reflects the fact that from an internal valuation perspective, a dollar within the firm is worth $1 + \phi$ dollars. The firm's liquidity

²¹According to equations (29) and (30), the problem $v_i(k, x; \mathbf{s}|\nu = 1)$ does not preclude the optimality of $k' = (1 - \delta)k$. However, the condition $v_i(k, x; \mathbf{s}|\nu = 1) - v_i(k, x; \mathbf{s}|\nu = 0) > 0$, by construction, ensures that positive or negative gross investment is optimal. To see this, suppose that $k' = (1 - \delta)k$ is optimal in the case of $v_i(k, x; \mathbf{s}|\nu = 1)$. Then $v_i(k, x; \mathbf{s}|\nu = 1) - v_i(k, x; \mathbf{s}|\nu = 0) > 0$ cannot be satisfied because $v_i(k, x; \mathbf{s}|\nu = 1) - v_i(k, x; \mathbf{s}|\nu = 0) = -F_k k < 0$, under the assumption that $k' = (1 - \delta)k$.

²²Note that in this case, the investment inaction interval is given by $[p^-, p^+]$.

condition, therefore, necessitates a higher marginal return on investment, which boosts the required return on equity and induces the firm to scale down its capital outlays.

Within our model, uncertainty and capital liquidity shocks cause fluctuations in aggregate investment by perturbing the tradeoff between the beneficial effects of capital accumulation on the firm's ability to borrow and the increased risk of having to tap costly equity finance to maintain its operations. The essential mechanism is that an increase in uncertainty or a decline in the resale value of capital both damp down the positive effect of investment on the borrowing costs by making corporate bond yields rise more sharply as firms scale up capital expenditures. In the standard framework used to price risky debt, the payoff structure of levered equity resembles the payoff of a call option, while the bondholders face a payoff structure that is equivalent to that of an investor writing a put option. By Jensen's inequality, therefore, an increase in payoff uncertainty benefits equity holders at the expense of bondholders, prompting an adjustment in borrowing terms. If the increase in borrowing costs required to leave the bond investors no worse off is greater than the potential gain to the equity holders, an increase in uncertainty will lead to a decline in aggregate investment. Similarly, a negative shock to the liquidation value of capital depresses aggregate investment because a lower liquidation value directly reduces the debt capacity of firms and thus increases the cost of capital for any given level of borrowing.

3.4 Market Clearing and Aggregation

This section closes the model by specifying conditions required to clear the labor and goods markets. We begin with the problem of the representative household, who solves

$$W(\mathbf{s}) = \max_{b', s', c, h} \{u(c, h) + \beta \mathbb{E}[W(\mathbf{s}') | \mathbf{s}]\}, \quad (32)$$

subject to a budget constraint,

$$c + \int [q^B b' + p_s s'] \mu(dz, dk, dx) = wh + \int [\tilde{R}^B + F_o k + (d + \tilde{p}_s) s] \mu(dz, dk, dx). \quad (33)$$

The period-specific utility function $u(c, h)$ is assumed to be strictly increasing and strictly concave in consumption (c) and strictly decreasing and concave in hours worked (h). In fact, to maintain tractability, we assume a very simple functional form, namely, $u(c, h) = \log c - \zeta h$.

In the budget constraint (33), $\mu(z, k, x)$ denotes the joint distribution of the idiosyncratic technology, capital, and net liquid asset positions across heterogeneous firms. The realized gross return on corporate bonds issued in the previous period is given by $\tilde{R}^B = [1 + \mathbf{1}[z \in \mathcal{D}(k, b, z)]] \times [\mathcal{R}(k, b, z(\sigma_{-1})) - 1]b$, while p_s is the ex-dividend value of equity, \tilde{p}_s is the current market value of equity, and $0 \leq s \leq 1$ is the fraction of outstanding shares owned by the household. The equity valuation terms are linked by the accounting identity $\tilde{p}_s = p_s - \bar{\varphi}(e)$, where $\bar{\varphi}(e) \geq e$ represents the cost of issuing new shares. Note that the fixed costs of operation are rebated to the household in a lump-sum fashion—hence these costs do not affect the economy-wide resource constraint.

The dynamic efficiency conditions associated with the household's problem pin down equity prices, according to

$$p_s(k, x, z; \mathbf{s}) = \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') [d' - \bar{\varphi}(e') + p'_s(k', x', z'; \mathbf{s}')] \mid z, \mathbf{s} \right], \quad (34)$$

where $m(\mathbf{s}, \mathbf{s}') = \beta u_c(c', h')/u_c(c, h)$ is the stochastic discount factor that also determines equilibrium bond prices, according to equation (17). The static optimization condition characterizing the labor supply decision implies that $w = -u_h(c, h)/u_c(c, h)$.

Conditions ensuring that the labor and goods market clear can then be expressed as

$$h^s(\mathbf{s}) = \int h^d(z, k; \mathbf{s}) \mu(dz, dk, dx); \quad (35)$$

$$c(\mathbf{s}) = \int y(z, k; \mathbf{s}) \mu(dz, dk, dx) - \int \nu(k, x, z; \mathbf{s}) g(k'(k, x, z; \mathbf{s}), k) \mu(dz, dk, dx). \quad (36)$$

Note that implicit in the goods market clearing condition (36) is the assumption that the dilution costs associated with seasoned equity issuance take the form of *discount* sales, implying that the gains and losses of new and existing shareholders offset each other—as a result, the dilution costs do not affect the aggregate resource constraint. The stock market clearing condition is simply $s' = s = 1$. The bond market then clears by Walras' law.²³

A key state variable of the model is $\mu(z, k, x)$, $z \in \mathbb{Z} \subset \mathbb{R}$, $k \in \mathbb{K} \subset \mathbb{R}$, and $x \in \mathbb{X} \subset \mathbb{R}$, the joint distribution of the idiosyncratic technology shocks, capital, and net liquid asset positions across heterogeneous firms. For any given aggregate state vector \mathbf{s}' , the law of motion for μ should satisfy

$$\mu'(Z, K, X) = \int \mathbf{1}[(z'_j(\sigma), k'_i(k, x; \mathbf{s}), \tilde{x}_i(k, x, z'_j(\sigma); \mathbf{s}, \mathbf{s}')) \in Z \times K \times X] p_{i,j} \mu(dz, dk, dx), \quad (37)$$

where $Z \in \mathcal{Z}$, $K \in \mathcal{K}$, $X \in \mathcal{X}$, and \mathcal{Z} , \mathcal{K} , and \mathcal{X} are the smallest σ -algebras generated by the subsets of \mathbb{Z} , \mathbb{K} , and \mathbb{X} , respectively. In equation (37), $\tilde{x}_i(k, x, z'_j(\sigma); \mathbf{s}, \mathbf{s}')$ denotes the post-renegotiation value of net liquid asset holdings, which is given by

$$\begin{aligned} \tilde{x}_i(k, x, z'_j(\sigma); \mathbf{s}, \mathbf{s}') &= a' z'_j(\sigma) \psi(w(\mathbf{s}')) k'_i(k, x; \mathbf{s}')^\gamma - F_o k'_i(k, x; \mathbf{s}') \\ &\quad - \min \{ b'_i(k, x; \mathbf{s}), b_i^R(k'_i(k, x; \mathbf{s}'), z'_j(\sigma); \mathbf{s}') \}. \end{aligned}$$

The dependence of μ' on the next period's state \mathbf{s}' , which itself contains μ' as one of its elements, reflects both the use of a composite state variable x in the firm's planning problem and the debt renegotiation process. Unlike the predetermined state variables such as capital stock, the firm's net liquid asset holdings and the post-renegotiation value of debt depend on the realization of all exogenous and endogenous state variables at the aggregate level. As a result, the law of motion for μ cannot be written in closed form, but rather corresponds to a fixed point problem that must be

²³Implicit in these aggregation conditions is also an assumption that firms that exit the market because of an exogenous shock are replaced by entrants that inherit all the technical and financial characteristics of the exiting firms.

solved by the agents in the economy.

Following the literature on computable general equilibrium with heterogeneous agents (see [Krusell and Smith, 1998](#); [Khan and Thomas, 2008](#)), we adopt the assumption of bounded rationality—that is, the agents use only a finite number of moments of the joint distribution to forecast equilibrium prices. Specifically, we assume that the agents use only the first moments of log-linearized laws of motions to predict two prices: the marginal utility of the representative household ($u_c(\mathbf{s})$) and the real wage ($w(\mathbf{s})$). Because our specification for preferences of the representative household specifies an infinitely elastic labor supply, the equilibrium wage can be backed out from the marginal utility of consumption. We also assume that the agents use only the first moment of the distribution of capital (\tilde{k}) to gauge the productive capacity of the economy and only the first moment of the distribution of the post-renegotiation value of debt (\tilde{b}) to infer the indebtedness of the corporate sector.

Formally, the agents in the model use the following system of log-linear equations to forecast equilibrium prices:

$$\begin{bmatrix} \log \tilde{b}' \\ \log \tilde{k}' \\ \log c \end{bmatrix} = \underset{(3 \times 1)}{\mathbf{\Gamma}_0} + \underset{(3 \times 2)}{\mathbf{\Gamma}_1} \begin{bmatrix} \log \tilde{b} \\ \log \tilde{k} \end{bmatrix} + \underset{(3 \times 3)}{\mathbf{\Gamma}_2} \begin{bmatrix} \log \sigma_z \\ \log a \\ \log p^- \end{bmatrix}. \quad (38)$$

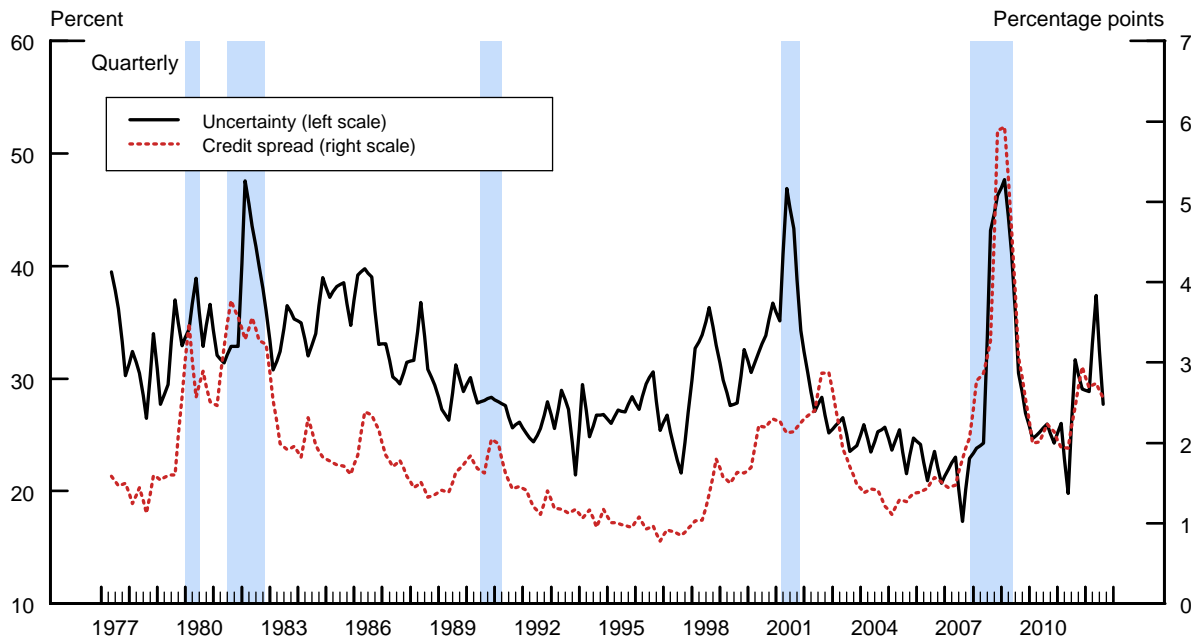
Consistency with the general equilibrium conditions requires that these perceived laws of motion are accurate, in the sense that the forecast errors implied by the system (38) are arbitrarily small. To achieve this consistency, we initialize $\mathbf{\Gamma}_0$, $\mathbf{\Gamma}_1$, and $\mathbf{\Gamma}_2$ with arbitrary values and then simulate the model using Monte Carlo methods with randomly drawn aggregate and idiosyncratic shocks; in the simulation, we let the agents learn from their errors and update the forecasting rules until full convergence. An important aspect of this algorithm involves letting all markets clear, even when the agents' perceived laws of motion are “inaccurate,” that is, before full convergence (see Section B.3 of the model appendix for details).

4 Calibration

For the most part, our calibration of the model relies on parameter values that are standard in the literature. The time period in the model equals one quarter; accordingly, we set the household's rate of time preference $\beta = 0.99$, implying an annualized risk-free rate of 4 percent. We set α , the value-added share of capital in the Cobb-Douglas production function, to 0.3. Together with the estimation procedure described in Section A.4 of the data appendix, this implies an estimate of decreasing returns to scale—the parameter χ —of about 0.85, a value within the range of values used in the literature. The quarterly depreciation rate δ is set equal to 0.025. The quasi-fixed costs of production $F_o = 0.05$, which implies that fixed costs equal about 10 percent of sales.²⁴ With

²⁴According to Compustat data, the median ratio of sales, general, and administrative (SG&A) expenses to sales is about 20 percent. A portion of SG&A expenses is accounted for by investment in intangible capital such as R&D and software expenditures, which is counted as investment by the BEA but is recorded as “expenses” in Compustat. We assume that one-half of this ratio reflects fixed costs of production.

Figure 4: Uncertainty Based on Shocks to the Profit Function



NOTE: Sample period: 1977:Q2–2012:Q3. The solid line depicts the estimate of idiosyncratic uncertainty (in annualized percent) based on shocks to the profit function (see Section A.4 of the data appendix for details). The dotted line depicts the spread between the 10-year yield on BBB-rated nonfinancial corporate bonds and the 10-year Treasury yield. The shaded vertical bars denote the NBER-dated recessions.

these parameter choices, the model generates the ratio of dividends to net income of 50 percent, roughly the same as the average of the (asset-weighted) median for corresponding ratio in the U.S. nonfinancial corporate sector over the 1984–2012 period.

In the model, fluctuations in uncertainty, by changing the underlying distribution of idiosyncratic productivity shocks, have a direct effect on future profits. Accordingly, we use an uncertainty proxy computed directly from the estimated shocks to the firms’ profit function to calibrate the Markov chain for the idiosyncratic technology shock z and the process governing its stochastic volatility (see Section A.4 of the data appendix for details). The solid line in Figure 4 depicts a measure of idiosyncratic uncertainty based on profits, an empirical analogue of the time-varying volatility of the idiosyncratic technology shock in our model. Note that like its counterpart derived from equity prices, this uncertainty measure is highly countercyclical and comoves closely with credit spreads (the dotted line).

In all model simulations, we assume four states for the idiosyncratic technology shock. The resulting four nodes of z are functions of time-varying volatility, such that an increase in volatility generates a greater dispersion in the nodes without changing the conditional expectation of the idiosyncratic technology shock. We use the Markov-chain approximation method of Tauchen (1986) and calibrate ρ_z , the persistence of the idiosyncratic technology process, to be 0.80, somewhat less than that implied by the data (see Table A-4 in the data appendix).

The steady-state level of uncertainty $\tilde{\sigma}_z$ is set to 15 percent (30 percent annualized), which is equal to the sample mean of the uncertainty measure shown in Figure 4. Using this proxy, we also estimate an empirical counterpart to equation (10), which yields $\hat{\rho}_\sigma = 0.75$, with the 95-percent confidence interval of $[0.63, 0.87]$; we set $\hat{\rho}_\sigma = 0.90$, a value at the high end of the estimated range, but in line with the value used by Bloom (2009). To generate fluctuations in uncertainty in the range between 20 and 40 percent (annualized)—a range consistent with the variability of our uncertainty proxy over the 1976–2012 period—we set the standard deviation of uncertainty shocks to 4 percent of the steady-state level of uncertainty (1.2 percent annualized).

Because we are not aware of any data source that tracks the resale value of fixed capital at the macro level, we estimate an AR(1) specification for p^- , using as a proxy the ratio of the price index of used car sales relative to that of new car sales, two components of the monthly CPI published by the BLS. This yields $\hat{\rho}_{p^-} = 0.97$ and $\hat{\omega}_{p^-} = 0.015$, which are the values used in our calibration. The liquidation value of capital in steady state \tilde{p}^- is set equal to 0.5, which implies a steady-state level for the book-value of leverage of one-half, the same as the average leverage calculated from the Compustat data.²⁵ With this calibration, an adverse liquidity shock of one standard deviation reduces the resale value of capital about 3.5 percent. The purchase price of capital p^+ is normalized to one.

The fixed investment adjustment costs $F_k = 0.01$, a value estimated by Cooper and Haltiwanger (2006). Following Prescott (1986), we set the persistence of the aggregate technology shock—the parameter ρ_a in equation (9)—to 0.95; as is usual in the literature, the volatility of innovations of the TFP process—the parameter ω_a —is set to 0.0075 at the quarterly rate. The integration of all exogenous AR(1) processes in the model is approximated by Gaussian quadratures.

Given the calibration of the processes for the idiosyncratic uncertainty and the liquidation value of capital assets, we set the degree of frictions in the corporate bond market—the bankruptcy cost parameter ξ —to generate an average credit spread of 160 basis points, which corresponds to the median of the BBB-Treasury spread shown in Figure 1. Accordingly, we let $\xi = 0.10$, a value consistent with that used by Bernanke et al. (1999) and the micro-level evidence of Levin et al. (2004) and one that implies a relatively modest degree of additional loss for the lender from bankruptcy. Concerning the survival probability, we set $\eta = 0.95$, a value consistent with the survey of the Business Employment Dynamics.

The estimates of the cost of seasoned equity issuance vary substantially in the literature, from a low of 0.08 in Gomes (2001) to a high of 0.30 in Cooley and Quadrini (2001). We make a conservative choice by letting $\varphi = 0.12$. Given this value, we choose the lower bound of dividends \bar{d} such that 15 percent of firms, on average, issue new shares in each quarter, a proportion that is roughly in line with that implied by the Compustat data. Finally, we set the lower bound on net worth $\bar{n} = 0$. (Table B-1 of the model appendix summarizes the calibration of the model.)

²⁵The book value of leverage in the model corresponds to $b/(\pi + p^+k)$. To calculate an empirical analogue using quarterly Compustat data, we let b equal the book-value of total debt; π gross profits; and p^+k the book-value of (gross) property, plant, and equipment. Over the 1976:Q1–2012:Q3 period, the average leverage in the U.S. nonfinancial corporate sector was 0.49, according to this metric.

5 Model Simulations

5.1 Capital Adjustment Dynamics

To illustrate the key aspects of the capital adjustment process, the top panel of Figure 5 displays the contours of the firm’s investment policy function assuming partial irreversibility only, while the policy function in the bottom panel also takes into account the fixed capital adjustment costs.²⁶ Both policy functions feature (S, s) -type adjustment dynamics, where the “S”-shape includes two distinct target capital levels: a lower level associated with the capital expansion problem and an upper level associated with the capital contraction problem, which are separated by an investment inaction region. The presence of financial market frictions implies that the (S, s) rule governing investment spending also depends on the firm’s (net) liquid asset position.

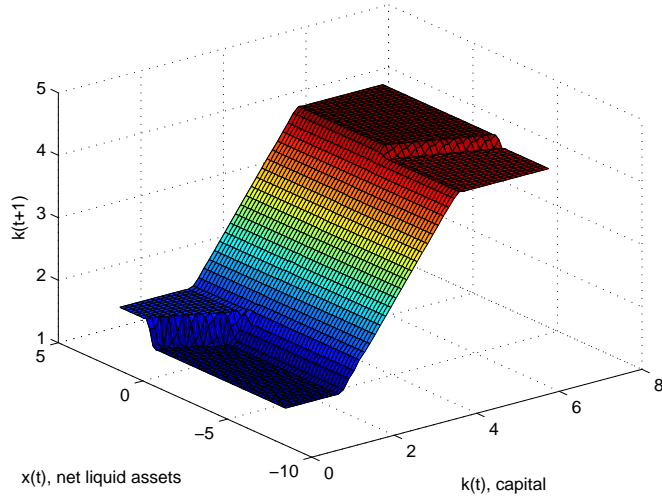
Focusing first on Panel (a), a somewhat simpler case, consider a firm in a strong financial position as reflected by a large value of x . In that case, the firm’s choice of capital tomorrow, as a function of its current production capacity, is well described by the standard (S, s) rule with two target stocks separated by the inaction region. The presence of financial distortion, however, induces two distinct plateaus in both the capital expansion and contraction problems. The upward sloping region between the two plateaus corresponds to the state space where the capital targets respond positively to an improvement in the firms’ financial position. Holding the level of idiosyncratic technology fixed, these regions are in fact characterized by a continuum of targets as summarized by the firm’s overall position (x, k) .

As shown in Panel (b), the introduction of fixed adjustment costs substantially expands the investment inaction region. To economize on these costs, the firm allows its capital stock to depreciate below the level that would, in the absence of fixed costs, trigger a positive investment response; similarly, the firm is willing to tolerate a capital overhang before adjusting its productive capacity. As a result, the adjustment of capital occurs in discrete jumps, which generates lumpy investment dynamics.

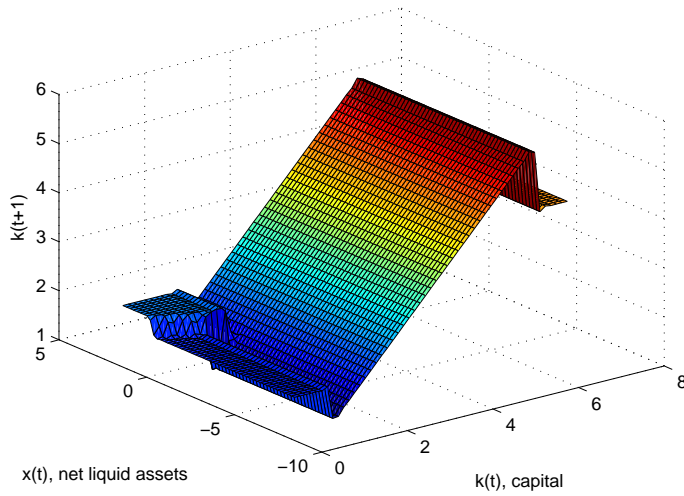
The uneven topography of the policy function in the region associated with the capital expansion problem provides a sense of how costly external finance complicates investment dynamics in that case. Note also that financial conditions have less of an effect on the capital contraction problem. In those states, the firm allows the capital overhang to persist longer, which implies that the resulting reduction in production capacity leaves the firm with sufficient holdings of liquid assets, so that its capital contraction target—and therefore disinvestment—is less sensitive to current financial conditions. It is important to note that for illustrative purposes, we constructed the policy functions in the above two examples over a wide range of capacity levels. In the actual simulations of the model, however, the firms almost never reduce their productive capacity—other than when defaulting—because even a small degree of irreversibility (20 percent in the above two examples) makes the sale of installed capital almost always an unprofitable activity.

²⁶For illustrative purposes, we assume that the liquidation value of capital is 80 percent of the initial purchase price in both examples, which is appreciably above the steady-state value of 50 percent used in our calibration. The fixed investment costs $F_k = 0.01$, the value used in the calibration.

Figure 5: (S, s) Investment Policy Functions



(a) Costly investment reversibility only



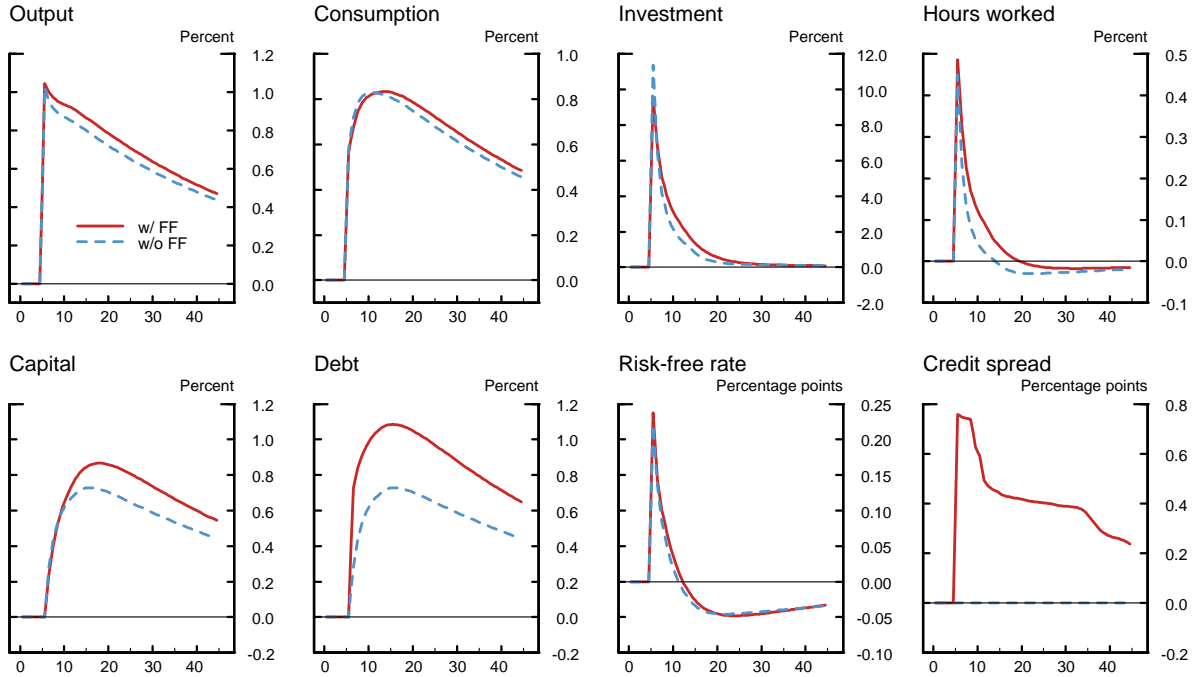
(b) Costly investment reversibility and lumpy adjustment

NOTE: Panel (a) depicts the optimal investment policy in the case of no fixed investment adjustment costs ($F_k = 0$). Panel (b) depicts the optimal investment policy with fixed investment adjustment costs ($F_k = 0.01$). In both examples, the liquidation value of capital $p^- = 0.8$.

5.2 Macroeconomic Implications of Aggregate Shocks

In this section, we analyze the dynamics of the model's key endogenous variables in response to three aggregate shocks: an aggregate technology shock (a); a shock to the volatility of the idiosyncratic technology shock (σ); and a shock affecting the liquidation value of capital assets (p^-). The benchmark model economy features a full set of financial distortions. To assess the quantitative

Figure 6: Impact of an Aggregate Technology Shock



NOTE: The solid lines depict the impulse response functions of the model with financial frictions (w/ FF), while the dashed lines are those of the model without financial frictions (w/o FF). In the experiment, a shock increases aggregate TFP one standard deviation upon impact (period 5); aggregate TFP is then allowed to revert back to its steady-state value following the process in equation (9). The impulse response functions are averages of 50,000 simulations, where each simulation is an aggregation of the impulse responses of 10,000 firms (see Section B.3 of the model appendix).

role of these frictions, we also solve a version of the model without financial distortions. In that case, the firms face the same irreversibility and nonconvex capital adjustment frictions as in the benchmark case, except that investment is financed using only internal funds and equity, where the issuance of the latter is not subject to any dilution costs.²⁷

In computing the model-implied impulse response functions, we take into account the nonlinearities in the firms’ investment and financial policies that arise naturally in an economy with irreversible investment, fixed capital adjustment costs, and financial distortions.²⁸ As described

²⁷In that case, the stock of outstanding corporate debt is no longer an aggregate state variable, and the forecasting rules in equation (38) are modified accordingly (see Section B.3 of the model appendix).

²⁸As shown in Table B-2 in Section B.3 of the model appendix, the linear laws of motion used by the agents to forecast equilibrium prices are very accurate in a statistical sense. In other words, although the agents’ policy functions are highly nonlinear at the micro level, the model’s key endogenous quantities exhibit fairly linear aggregate dynamics. In fact, the existence of such “aggregation smoothing” is typically used to justify the use of an algorithm that uses only a small number of moments to characterize the dynamics of the joint distribution μ (see Khan and Thomas, 2008). In principle, therefore, the response of key endogenous aggregate quantities to aggregate shocks could be constructed using the estimated perceived laws of motion. While computationally straightforward, this approach is limited in scope, however. For example, the response of the average credit spread—an object of great interest in our analysis—cannot be constructed in such a linear fashion.

more fully in Section B.3 of the model appendix, the impulse response functions are based on the following sequence of steps: (1) simulate the model twice—first with the idiosyncratic shocks only and then with an aggregate shock layered on top of the same set of idiosyncratic shocks; (2) for each simulation, aggregate the micro-level impulse responses of endogenous variables of interest; and (3) take the difference between the two sets of aggregated endogenous quantities. To eliminate any sampling bias that may have arisen when drawing idiosyncratic shocks, we repeat these three steps a large number of times and then average the aggregate impulse response functions across replications.

Figure 6 depicts the behavior of the model’s main endogenous variables in response to a positive aggregate technology shock of one standard deviation. The comparison of the two sets of responses reveals that financial distortions have a minimal effect on the magnitude and shape of the responses of economic activity to the aggregate technology shock. In both model economies, the unanticipated increase in aggregate TFP leads to a strong and persistent increase in consumption, investment, and hours worked, dynamics similar to those implied by the canonical real business cycle (RBC) model.

In both cases, the responses of key economic aggregates to the aggregate TFP shock are also consistent with the work of Sim (2006) and Khan and Thomas (2008), who find that the presence of irreversibilities and nonconvex adjustment frictions does not imply significant departures from the dynamics of a standard RBC economy. A novel feature of our analysis is the fact that introducing financial distortions in the model also does not change this conclusion. As evidenced by the responses of debt and capital, the technology-induced boom allows the firms to lever up, though the resulting increase in the demand for credit significantly boosts the risk-free rate and credit spreads. In turn, these changes in financial conditions restrain the expansion of corporate balance sheets and damp the financial accelerator mechanism engendered by financial market frictions.

The top panel of Table 4 compares the business cycle moments of the two economies, conditional on the aggregate technology shocks only. Consistent with the impulse response results, conditional moments of the key endogenous quantities are very similar across the two model specifications. Moreover, in spite of the presence of irreversibilities, nonconvex adjustment frictions, and financial distortions, the benchmark economy exhibits the hallmark features of an RBC model, with the investment about three times more volatile than output and highly correlated with output, consumption, and hours worked.

The macroeconomic impact of an uncertainty shock is shown in Figure 7. The comparison of the investment response across the two models reveals a striking result: More than three-quarters of the impact of the uncertainty shock on investment is due to financial distortions. This result is also mirrored in the dynamics of output and hours worked, both of which exhibit a significantly more muted response to an unanticipated surge in volatility in an economy with perfect financial markets.

To understand how an increase in volatility interacts with the different frictions of the model, we decompose the correlation between the firm-level capital adjustments and idiosyncratic uncertainty

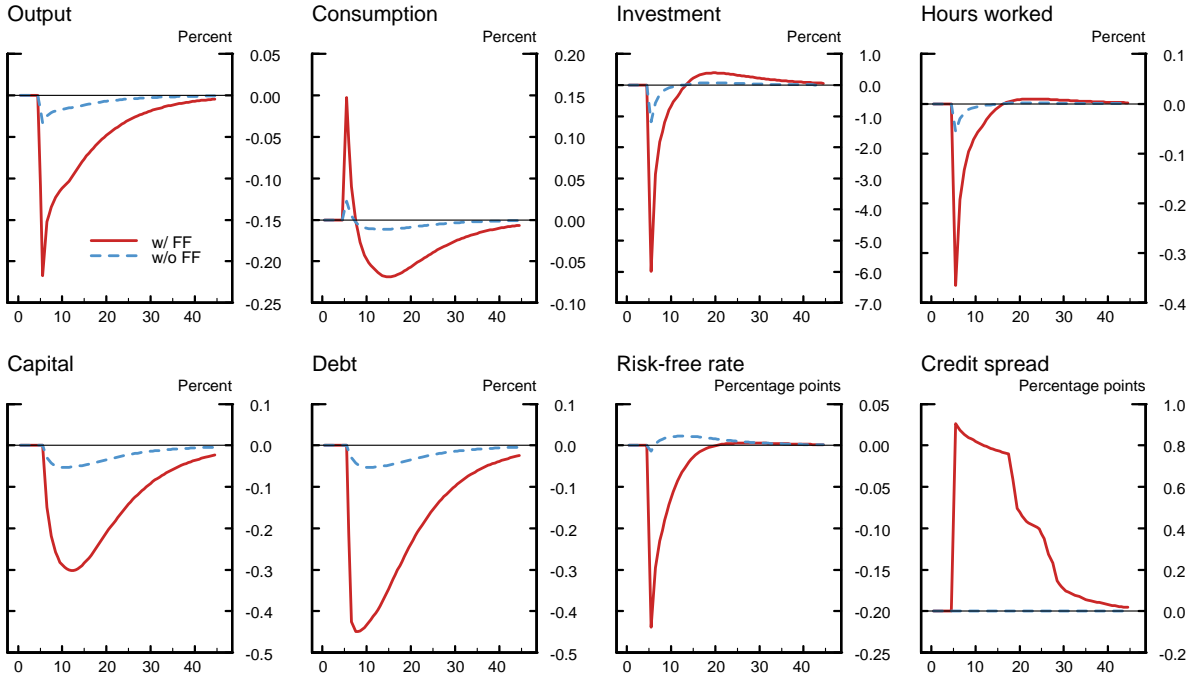
Table 4: Model-Implied Conditional Business Cycle Moments

<i>Conditional on Technology Shocks Only</i>								
	RSTD(I)	RSTD(C)	RSTD(H)	STD(Y)	Corr(I, Y)	Corr(C, Y)	Corr(H, Y)	Corr(C, I)
Model w/ FF	2.60	0.95	0.12	2.47	0.63	0.99	0.46	0.54
Model w/o FF	2.90	0.98	0.12	2.32	0.53	0.99	0.22	0.53
<i>Conditional on Uncertainty Shocks Only</i>								
	RSTD(I)	RSTD(C)	RSTD(H)	STD(Y)	Corr(I, Y)	Corr(C, Y)	Corr(H, Y)	Corr(C, I)
Model w/ FF	13.5	0.63	0.88	0.23	0.65	0.49	0.78	-0.33
Model w/o FF	14.6	0.63	0.71	0.10	0.76	0.71	0.78	0.10
<i>Conditional on Capital Liquidity Shocks Only</i>								
	RSTD(I)	RSTD(C)	RSTD(H)	STD(Y)	Corr(I, Y)	Corr(C, Y)	Corr(H, Y)	Corr(C, I)
Model w/ FF	6.71	0.60	0.55	1.11	0.61	0.88	0.86	0.16
Model w/o FF	14.3	0.63	0.69	0.09	0.78	0.73	0.78	0.14
<i>Memo: Data</i> ^a	2.79	0.42	0.60	1.12	0.63	0.56	0.65	0.43

NOTE: The model-implied conditional moments are based on 2,000 quarters of simulated data for 10,000 firms: Y = output; I = investment; C = consumption; and H = hours worked. RSTD(x) denotes the relative standard deviation of x —that is, the standard deviation of variable $x = I, C, H$, relative to the standard deviation of output (STD(x)/STD(Y)). All model-implied aggregate series are expressed in percent deviations from their respective steady-state values; see the text for the definition of shocks and other details.

^a Sample period: 1962:Q1–2012:Q3. Variable definitions: Y = nonfarm business sector output (c-w \$2005); I = business investment in equipment & software (c-w \$2005); C = PCE on nondurable goods & services (c-w \$2005); and H = index of hours worked for all persons (nonfarm business sector). Actual data are transformed into stationary series by log-differencing.

Figure 7: Impact of an Uncertainty Shock



NOTE: The solid lines depict the impulse response functions of the model with financial frictions (w/ FF), while the dashed lines are those of the model without financial frictions (w/o FF). In the experiment, a shock increases the volatility of the idiosyncratic technology shock (σ) 3 percentage points (annualized) upon impact (period 5), a shock of 2.5 standard deviations; volatility is then allowed to revert back to its steady-state value following the process in equation (10). The impulse response functions are averages of 50,000 simulations, where each simulation is an aggregation of the impulse responses of 10,000 firms (see Section B.3 of the model appendix).

into the adjustment at intensive and extensive margins. We measure the adjustment at the extensive margin by calculating—for each period—the fraction of firms with positive investment expenditures ($\text{Freq}[I^+]$). As another metric, we also consider “lumpy” investment, defined as a proportion of firms with capital expenditures in excess of 10 percent of the book value of installed capital ($\text{Freq}[\text{lumpy-}I^+]$). The intensive margin, by contrast, is defined as the average positive capital expenditures ($\text{Avg}[I^+]$) in each period.²⁹

According to the standard irreversibility theory, aggregate investment dynamics, especially in response to fluctuations in uncertainty, will primarily reflect the firms’ adjustment at the extensive margin—a jump in uncertainty raises the option value of waiting, which increases the proportion of firms in the inactive region. As shown in the top panel of Table 5, this is indeed the case in the model without financial frictions: The frequency of positive investment adjustments is negatively

²⁹In this experiment, the liquidation value of capital is fixed at its steady-state value of 0.5. With this calibration, liquidating capital is almost never optimal, unless the realization of the idiosyncratic technology shock is unusually bad and the firm has a significant capital overhang problem, a combination that generates large losses due to fixed operating costs. As a result, disinvestment at the firm level plays a minor role in the determination of the dynamics of aggregate investment, which allows us to focus on positive investment expenditures only.

Table 5: Extensive and Intensive Capital Adjustment Margins

<i>Conditional on Uncertainty Shocks Only</i>		
Selected Correlations	Model w/o FF	Model w/ FF
Corr(Freq[I^+], σ)	-0.135	0.210
Corr(Freq[lumpy- I^+], σ)	-0.215	0.101
Corr(Avg[I^+], σ)	-0.076	-0.673
<i>Conditional on Capital Liquidity Shocks Only</i>		
Selected Correlations	Model w/o FF	Model w/ FF
Corr(Freq[I^+], p^-)	-0.051	-0.837
Corr(Freq[lumpy- I^+], p^-)	-0.058	0.378
Corr(Avg[I^+], p^-)	-0.038	0.837

NOTE: The model-implied moments are based on 2,000 quarters of simulated data for 10,000 firms. Freq[I^+] = frequency of positive investment expenditures; Freq[lumpy- I^+] = frequency of positive “lumpy” investment expenditures, where lumpy investment is defined as capital expenditures greater than 10 percent of the book value of currently installed capital; and Avg[I^+] = average positive investment expenditure.

correlated with the movements in uncertainty, implying that an increase in volatility pushes more firms in the inaction region. The frequency of lumpy investment episodes is also negatively correlated with fluctuations in volatility. In contrast, the correlation between the average level of capital expenditures and uncertainty is essentially zero, a result consistent with the conventional wisdom that the response of aggregate investment to uncertainty shocks in a model with irreversibilities and nonconvex adjustment frictions reflects primarily adjustment at the extensive margin.

These results change dramatically when the model includes financial distortions. In that case, most of the adjustment in aggregate investment occurs at the intensive margin. The correlation of almost -0.7 between the average size of capital expenditures and uncertainty implies that volatility shocks lead to a large reduction in the target level of capital for active firms, rather than to a reduction in the number of active firms. In fact, both the frequency of positive capital expenditures and lumpy investments are positively correlated with fluctuations in uncertainty, a dynamic that works against generating a large decline in aggregate investment.

As shown in Figure 7, these differences in aggregate investment dynamics are due entirely to financial frictions. While the increase in uncertainty induces an appreciable decline in the risk-free rate, this easing of financial conditions is more than offset by the sharp increase in credit spreads, which jump about 90 basis points upon impact and remain elevated for a considerable period of time. The resulting deterioration in borrowing terms implies a significant increase in the user cost of capital, which leads firms to slash capital expenditures and delever, as evidenced by the response of debt relative to that of capital stock. This interaction between the debt and capital overhang is the key reason why most of the firm-level adjustment occurs at the intensive margin.

The impulse responses from the model with financial frictions are thus consistent with our em-

pirical evidence, which showed that fluctuations in uncertainty are associated with large swings in credit spreads, which, in turn, are an important determinant of investment spending. We interpret the combination of our empirical evidence and model simulations as indicating that general equilibrium models steeped in the Modigliani–Miller paradigm of frictionless financial markets are likely missing a crucial part of the mechanism through which uncertainty shocks are propagated to the real economy.

The middle panel of Table 4 summarizes the conditional business cycle moments of this experiment. The high relative volatility of investment suggests that fluctuations in uncertainty can be an important driver of this cyclically-sensitive component of aggregate output. However, this source of fluctuations generates a relatively modest degree of variability in aggregate output. In both models, a substantial portion of the drop in investment in response to increased uncertainty is offset by an increase in consumption, which attenuates output fluctuations. This negative comovement occurs because the uncertainty shock—unlike the aggregate TFP shock—does not directly affect the resource constraint of the economy.³⁰ As in Christiano et al. (2014), this counterfactual conditional negative correlation between investment and consumption can easily be reversed by introducing nominal price or wage rigidities in the model.

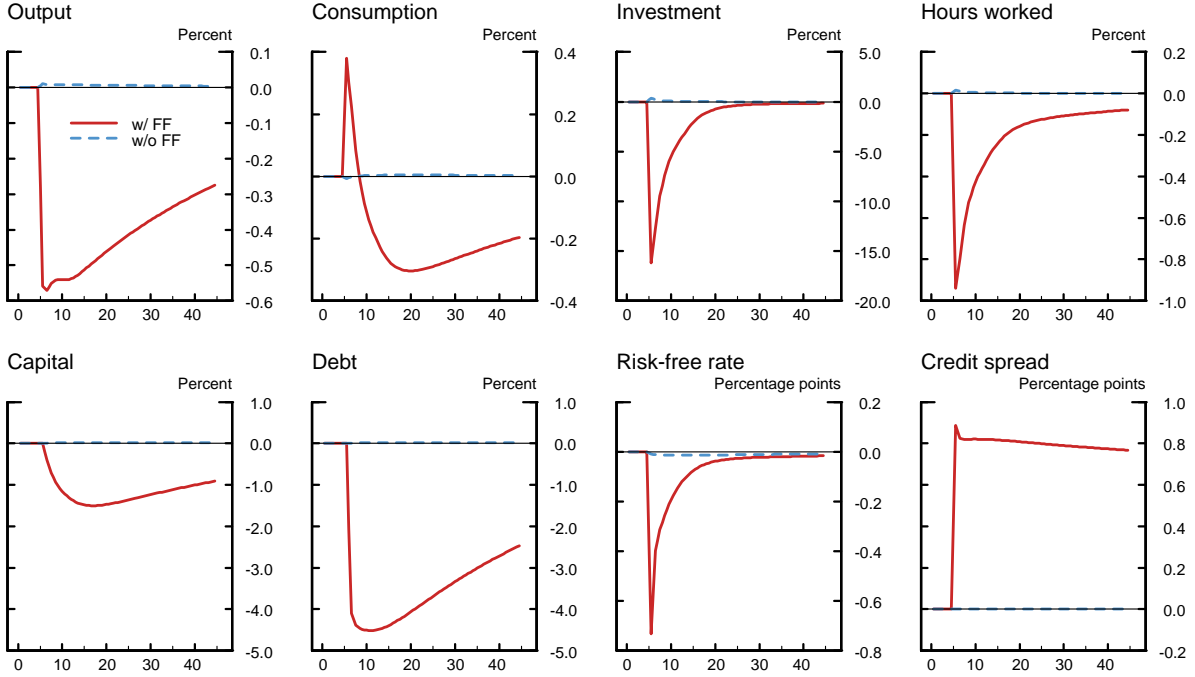
The dynamic behavior of the two economies when hit by an adverse capital liquidity shock is shown in Figure 8. The differences in macroeconomic outcomes between the two models are again striking. In the model without financial frictions, a drop in the liquidation value of capital has essentially no effect on economic activity. The same shock, by contrast, induces a severe recession in the economy with financial distortions. Business investment plunges more than 15 percent upon the impact of the shock, and the hours worked decline sharply and remain below steady state for a prolonged period of time. Even consumption—after the initial increase—declines noticeably and stays subdued over the remainder of the response horizon. These dynamics translate into a decline in aggregate output of more than 0.5 percent upon impact and ensure that the resulting recovery is slow and protracted.

Financial frictions are again the key propagation mechanism of this shock, as the drop in the liquidation value of capital assets immediately curtails the firms’ debt capacity and induces a massive deleveraging of the corporate sector as well as a significant and persistent widening of credit spreads. The acute debt overhang triggered by the capital liquidity shock implies a deterioration in the overall business creditworthiness and leads to a drop in the risk-free rate. The decline in the risk-free rate is of the same magnitude as the increase in credit spreads, which indicates that fluctuations in the credit-risk component of corporate bond prices—as opposed to swings in risk-free base rates—are the source of the information content of the “bond market’s q ” for business fixed investment (see Gilchrist and Zakrajšek, 2007; Philippon, 2009).

The bottom panel of Table 5 shows the correlations of the different capital adjustment margins

³⁰As shown in an earlier version of this paper (available from the authors upon request), financial distortions—in the absence of firm-level capital adjustment costs—cause procyclical capital reallocation. In turn, this generates endogenous procyclical fluctuations in aggregate TFP. However, once we allow for firm-level investment irreversibility, these reallocation-based TFP fluctuations become quantitatively negligible.

Figure 8: Impact of a Capital Liquidity Shock



NOTE: The solid lines depict the impulse response functions of the model with financial frictions (w/ FF), while the dashed lines are those of the model without financial frictions (w/o FF). In the experiment, a shock reduces the liquidation value of the firms' capital assets (p^-) 5 percent upon impact (period 5), a shock of about 1.5 standard deviations; the liquidation value of capital is then allowed to revert back to its steady-state value following the process in equation (12). The impulse response functions are averages of 50,000 simulations, where each simulation is an aggregation of the impulse responses of 10,000 firms (see Section B.3 of the model appendix).

with the liquidation value of capital. Not surprisingly, these correlations are essentially zero in the model with frictionless financial markets. However, when financial distortions are present, the average (positive) capital outlay is strongly positively correlated with fluctuations in the resale value of capital, reflecting the tight link between capital liquidity shocks and the firms' debt capacity. On the extensive margin, the correlation between the liquidation value of capital and the frequency of positive investment expenditures is negative, whereas the correlation with lumpy investment is positive. Evidently, an improvement in the liquidity of the secondary market for capital—an improvement in the sense of higher resale value—makes the nonconvex capital adjustment costs relatively more important for firms that are considering adjusting their production capacity; this induces firms to economize on transaction costs arising from the fixed capital adjustment costs by increasing the average size of investment expenditures and by more frequently making large capital outlays.

As shown in the bottom panel of Table 4, such liquidity shocks have the potential of being an important source of cyclical fluctuations in an economy with imperfect financial markets and capital specificity. Recall that the (quarterly) standard deviation of capital liquidity shocks is only

Table 6: Cyclical Properties of Credit Spreads
(*Model vs. Data*)

Selected Correlations	Conditional on the Type of Shock			
	Technology	Uncertainty	Liquidity	Data ^a
Corr(S, Y)	0.927	-0.811	-0.938	-0.457
Corr(S, I)	0.626	-0.515	-0.577	-0.531
Corr(S, C)	0.916	-0.368	-0.816	-0.498
Corr(STD(S), Y)	0.933	-0.832	-0.950	-0.245

NOTE: The model-implied moments are based on 2,000 quarters of simulated data for 10,000 firms: S = credit spread; Y = output; I = investment; and C = consumption. With the exception of the credit spread, all model-implied aggregate series are expressed in percent deviations from their respective steady-state values; see the text for the definition of shocks and other details.

^a Sample period: 1962:Q1–2012:Q3, unless noted otherwise. Variable definitions: S = 10-year BBB-Treasury corporate bond spread; Y = nonfarm business sector output (c-w \$2005); I = business investment in equipment & software (c-w \$2005); and STD(S) = cross-sectional standard deviation of credit spreads (Sample period: 1973:Q1–2012:Q3; see Section A.1 of the data appendix for details). With the exception of the credit spread (S) and the cross-sectional dispersion of credit spreads (STD(S)), actual data are transformed into stationary series by log-differencing.

1.5 percent, a relatively modest amount of variability in light of, say, historical swings in the value of commercial real estate. Nevertheless, this fairly small amount of volatility in the resale value of capital generates economically realistic amounts of variability in the key endogenous aggregates: The conditional standard deviation of aggregate output implied by the benchmark model closely matches the volatility of U.S. real GDP, and the relative conditional volatilities of consumption and hours worked are also close to their respective empirical counterparts; that said, the model does generate business fixed investment that is too volatile relative to what is observed in the data.

The benchmark model also delivers more realistic comovements between main macroeconomic quantities. The degree of comovement between investment and output is very close to that observed in the data, while the correlation between consumption and output, though still a bit on the high side, is much more reasonable. Importantly, shocks to the liquidation value of capital generate a positive correlation between investment and consumption, though for the same reasons as discussed above, the degree of comovement is still notably below that implied by the actual data.

5.3 Cyclical Properties of Credit Spreads

One of the defining feature of the U.S. business cycle is the strong negative correlation between economic activity and corporate bond credit spreads. Another notable, though less emphasized, feature is the countercyclical behavior of the cross-sectional *dispersion* in credit spreads (see Gilchrist et al., 2013). Table 6 compares the cyclical behavior of credit spreads—both their level and dispersion—implied by our benchmark model with the U.S. data. Assuming that aggregate technology shocks are the sole source of economic fluctuations (column 1) implies that both the average level of credit spreads and their cross-sectional dispersion are strongly procyclical, results that run counter to the

data. The procyclical behavior of credit spreads in the model arises because a positive aggregate TFP shock induces firms to lever up to take advantage of new profitable investment opportunities. The increase in leverage, however, leads to deterioration in the firms' creditworthiness and causes a widening of credit spreads. An adverse technology shock induces a downward shift in the demand for credit, which makes quantities and prices move in the same direction, thereby generating procyclical credit spreads. This counterfactual procyclicality of credit spreads in a technology-driven business cycle is a general feature of general equilibrium models with costly external finance, a fact pointed out by [Gomes et al. \(2003\)](#) in their critique of the financial accelerator mechanism.

When the business cycle is driven by either uncertainty or liquidity shocks (columns 2–3), the level of credit spreads and their dispersion are strongly countercyclical, comovements strongly supported by the data. These two types of economic disturbances share a common feature in that they both impair the borrowers' creditworthiness. In response, the supply curve of loanable funds shifts inward, causing the quantity of credit and its price to move in the opposite direction, which generates the countercyclical behavior in credit spreads.

6 Conclusion

This paper argues that financial distortions due to agency problems between financial market participants are an important part of the transmission mechanism by which fluctuations in volatility affect the economy. We explored the quantitative significance of this mechanism in the context of a general equilibrium model with heterogeneous firms facing time-varying idiosyncratic uncertainty, partial investment irreversibility, fixed investment costs, and frictions in both the debt and equity markets. As in the standard framework, investment irreversibilities cause firms to adopt a wait-and-see attitude in response to an increase in uncertainty. At the same time, the implied illiquidity of capital assets reduces the firms' debt capacity because in the case of costly default, the liquidation of capital lowers the recovery value of corporate debt claims.

Model simulations indicate that financial frictions are a powerful conduit through which uncertainty shocks affect aggregate investment. A jump in uncertainty leads to a sharp and persistent widening of credit spreads, which induces firms to simultaneously slash capital expenditures and delever. This quantitatively important channel is absent in an economy without financial distortions, where the significantly more-attenuated response of investment to uncertainty shocks reflects solely the aggregation of the standard wait-and-see decisions of individual firms. With partial irreversibility, an adverse shock to the resale value of capital—by reducing the collateral value of assets—curtails the debt capacity of firms and creates a powerful interaction between the capital and debt overhang problems. Model simulations indicate that such capital liquidity shocks can be an important source of macroeconomic fluctuations. Importantly, the model-implied dynamics are consistent with our empirical evidence, which shows that movements in corporate bond credit spreads are economically and statistically an important part of the investment-uncertainty nexus.

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Appendices

A Data Appendix

In this appendix, we describe the construction of firm-level credit spreads and other firm-specific variables used in the empirical analysis. Subsection [A.1](#) describes the bond-level data set used to compute firm-specific credit spreads. Subsection [A.2](#) provides the details surrounding the construction of variables used in the estimation of credit spread regressions, while subsection [A.3](#) describes the construction of the panel data set used to estimate the relationship between investment, uncertainty, and credit spreads. And lastly, subsection [A.4](#) describes the estimation procedure—and the associated results—used to estimate the degree of decreasing return-to-scale in U.S. nonfinancial corporate sector, as well as to construct our proxy for idiosyncratic uncertainty based on firm-level profitability shocks.

A.1 Bond-Level Data

For a sample of U.S. nonfinancial firms covered by the S&P’s Compustat and the Center for Research in Security Prices (CRSP), we obtained month-end secondary market prices of their outstanding securities from the Lehman/Warga and Merrill Lynch databases. As discussed by [Gilchrist et al. \(2009\)](#), these two data sources include secondary market prices for a vast majority of dollar-denominated bonds publicly issued in the U.S. corporate cash market. To ensure that we are measuring borrowing costs of different firms at the same point in their capital structure, we limited our sample to only senior unsecured issues with a fixed coupon schedule.

As emphasized by [Gilchrist and Zakrajšek \(2012\)](#), using security-level data allows the construction of credit spreads that are not biased by the maturity/duration mismatch, a problem that plagues credit spread indexes constructed with aggregated data. Specifically, we construct—for each individual bond issue in our sample—a theoretical risk-free security that replicates exactly the promised cash-flows of the corresponding corporate debt instrument. For example, consider a corporate bond k issued by firm i that at time t is promising a sequence of cashflows $\{C(s) : s = 1, 2, \dots, S\}$, which consists of the regular coupon payments and the repayment of the principle at maturity. The price of this bond in period t is given by

$$P_{it}[k] = \sum_{s=1}^S C(s)D(t_s),$$

where $D(t) = \exp(-r_t t)$ is the discount function in period t .

To calculate the price of a corresponding risk-free security—denoted by $P_t^f[k]$ —we discount the promised cashflows $\{C(s) : s = 1, 2, \dots, S\}$ using continuously-compounded zero-coupon Treasury yields in period t , derived from the daily estimates of the U.S. Treasury yield curve estimated by [Gürkaynak et al. \(2007\)](#). The resulting price $P_t^f[k]$ can then be used to calculate the yield—denoted by $y_t^f[k]$ —of a hypothetical Treasury security with identical cashflows as the underlying corporate bond. Consequently, the credit spread $S_{it}[k] = y_{it}[k] - y_t^f[k]$, where $y_{it}[k]$ denotes the yield of the corporate bond k , is thus free of the “duration mismatch” that occurs when the spreads are computed simply by matching the corporate yield to the estimated yield of a zero-coupon Treasury security of the same maturity.

To ensure that our results are not driven by a small number of extreme observations, we eliminated from our data set all observations with credit spreads below 5 basis points and greater than 2,000 basis points. We also eliminated very small corporate issues (par value of less than

\$1 million) and all observations with a remaining term-to-maturity of less than one year or more than 30 years.³¹ These selection criteria yielded a sample of 6,725 individual securities over the 1973:M1–2012:M9 period. These corporate securities were then matched with their issuer’s quarterly income and balance sheet data from Compustat and daily data on equity valuations from CRSP, a procedure that yielded a sample of 1,164 U.S. nonfinancial firms.

Table A-1: Selected Corporate Bond Characteristics

Variable	Mean	StdDev	Min	Median	Max
No. of bonds per firm/month	2.99	3.69	1.00	2.00	76.0
Mkt. value of issue ^a	339.9	338.9	1.22	249.2	5,628
Maturity at issue (years)	12.8	9.2	1.00	10.0	50.0
Term-to-maturity (years)	11.1	8.5	1.00	8.0	30.0
Duration (years)	6.49	3.28	0.91	6.03	18.0
Credit rating (S&P)	.	.	D	BBB1	AAA
Coupon rate (pct.)	7.06	2.14	0.60	6.88	17.5
Nominal effective yield (pct.)	7.23	2.98	0.22	6.92	30.0
Credit spread (pps.)	2.02	2.23	0.05	1.28	20.0

NOTE: Sample period: 1973:M1–2012:M9; Obs. = 385,062; No. of bonds = 6,725; No. of firms = 1,164. Sample statistics are based on trimmed data.

^a Market value of the outstanding issue deflated by the CPI (2005 = 100).

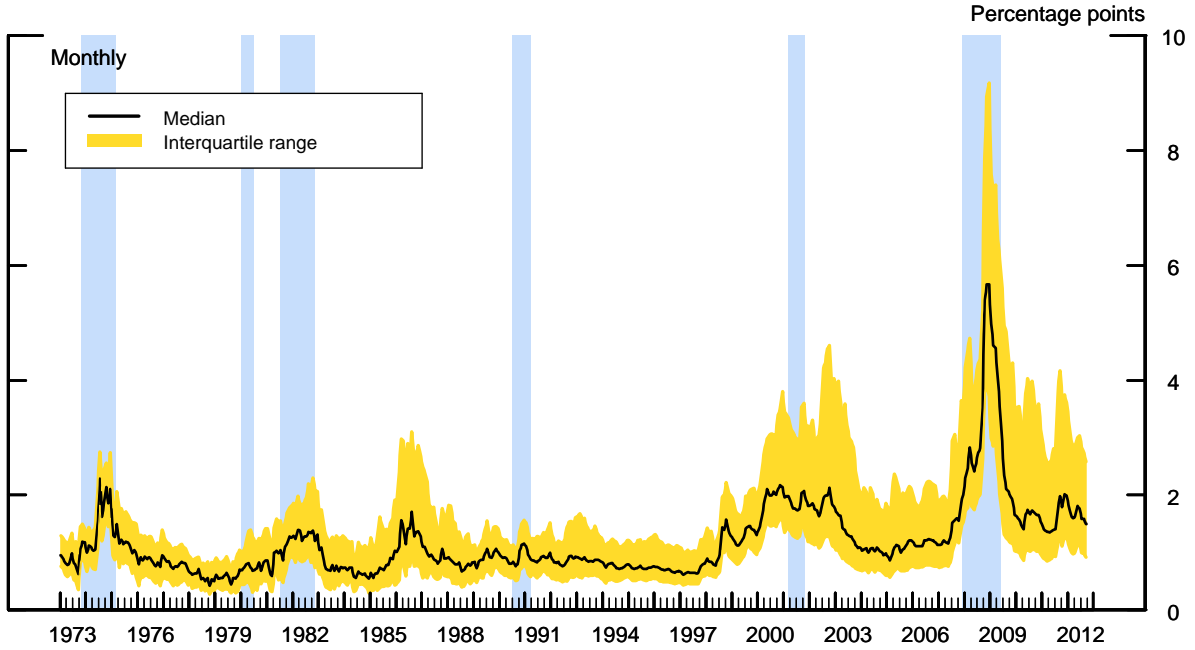
Table A-1 contains summary statistics for the key characteristics of bonds in our sample. Note that a typical firm in our sample has a few senior unsecured issues outstanding at any point in time—the median firm, for example, has three such issues trading in the secondary market in any given month. This distribution, however, is skewed to the right and has a heavy right tail, as some firms can have many more issues trading in the secondary market at a point in time. The size distribution of these issues—as measured by their (real) market value—runs from \$1.2 million to more than \$5.6 billion and has a very similar shape.

Given our focus on the corporate bond market, the maturity of these debt instruments is fairly long—the average maturity at issue of almost 13 years. The average remaining term-to-maturity, by contrast, is about 11 years. In terms of default risk—at least as measured by the S&P credit ratings—our sample spans the entire spectrum of credit quality, from “single D” to “triple A.” At “BBB1,” however, the median observation is still in the investment-grade category. Note that the central tendencies of our sample—in both the maturity and credit-risk dimensions—closely match those of the 10-year BBB-Treasury spread used in the VAR analysis. Lastly, an average bond has an expected return of 202 basis points above the comparable risk-free rate, while the standard deviation of 223 basis points is again indicative of the wide range of credit quality in the sample.

Figure A-1 depicts the time-series evolution of the selected moments of the cross-sectional distribution of credit spreads in our sample. As shown by the solid line, the median credit spread is countercyclical and typically starts to increase before the official onset of recessions. Importantly, the dispersion of credit spreads—as measured by the interquartile range—also tends to lead the business cycle and increases markedly during periods of financial market distress, a pattern consistent with the well-documented countercyclical heterogeneity in firm stock returns, profits, and productivity.

³¹Calculating credit spreads for maturities of less than one year and more than 30 years would involve extrapolating the Treasury yield curve beyond its support.

Figure A-1: Credit Spreads in U.S. Nonfinancial Corporate Sector



NOTE: Sample period: 1973:M1–2012:M9. The solid line depicts the median spread on senior unsecured bonds in our sample, while the shaded band depicts the corresponding interquartile range. The shaded vertical bars denote the NBER-dated recessions.

A.2 Credit Spread Regressions

While our micro-level data on credit spreads reflect month-end values, the requisite firm-level income and balance sheet items from Compustat are available only quarterly; in addition, our measure of firm-level uncertainty is estimated at a quarterly frequency. The time-series frequency of the panel data set used in the estimation of our credit spread regressions is, therefore, quarterly. In constructing the data set used to estimate the credit spread regressions reported in Table 1 in Section 2.1, we also took into account the fact that the firms’ fiscal years end at different months of the year. As a result, observations in the data set occur at different months of the year but are spaced at regular quarterly (i.e., three-month) intervals.

Letting $v\#$ denote the quarterly Compustat data item number, the key explanatory variables used in the credit spread regressions are defined as follows:

- σ_{it} : *quarterly uncertainty proxy*, constructed using daily idiosyncratic returns over the three months of the firm’s fiscal quarter;
- R_{it}^E : *quarterly stock return*, constructed from daily (total) log returns from the CRSP database over the three months of the firm’s fiscal quarter;
- $[\Pi/A]_{it}$: *ratio of operating income to assets*, defined as operating income before depreciation and amortization ($v21$) in quarter t and scaled by total assets ($v44$) in quarter $t - 1$;
- $[D/E]_{it}$: *ratio of debt to equity*, defined as the book value of debt in current liabilities ($v45$) plus the book value of long-term debt ($v51$)—both in quarter t —and scaled by the market

value of common equity in quarter t as computed from the CRSP database.

Table A-2: Summary Statistic of Selected Firm Characteristics

Variable	Mean	StdDev	Min	Median	Max
σ_{it} (pct.)	29.9	16.7	3.96	25.7	211.4
R_{it}^E (pct.)	7.03	75.2	-635.3	11.5	598.2
$100 \times [\Pi/A]_{it}$	3.67	1.88	-2.00	3.51	9.99
$100 \times [D/E]_{it}$	77.8	102.5	< .01	46.2	1,182.3

NOTE: Sample period: 1973:M1-2012:M9 at a quarterly frequency; Obs. = 39,723; No. of firms = 1,124. Sample statistics are based on trimmed data. Uncertainty (σ_{it}) and quarterly stock returns (R_{it}^E) are annualized.

After deleting observations with missing data items and applying standard filters to remove a small number of outliers, we were left with 1,124 firms for a total of 39,723 firm/quarter observations. Summary statistics of these key firm characteristics are summarized in Table A-2.

A.3 Investment Regressions

In constructing the data set used in the estimation of the investment regressions reported in Tables 2-3 in Section 2.1 of the paper, the Compustat data on capital expenditures are available only at an annual frequency. In constructing this data set, we also take into account the fact that the firms' fiscal years end at different months of the year. As a result, observations in the data set occur at different months of the year but are spaced at regular yearly (i.e., twelve-month) intervals.

Letting $v\#$ denote the annual Compustat data item number, the key variables used in the investment regressions are defined as follows:

- $[I/K]_{it}$: *ratio of investment to capital*, defined as capital expenditures ($v128$) in year t and scaled by (net) property, plant, and equipment ($v8$) in year $t - 1$;
- σ_{it} : *yearly uncertainty proxy*, constructed using daily idiosyncratic returns over the twelve months of the firm's fiscal year;
- s_{it} : *yearly average portfolio credit spread*, constructed using month-end credit spreads over the twelve months of the firm's fiscal year;³²
- $[Y/K]_{it}$: *ratio of sales to capital*, defined as (net) sales ($v12$) in year t and scaled by (net) property, plant, and equipment ($v8$) in year $t - 1$;
- $[\Pi/K]_{it}$: *ratio of operating income to capital*, defined as operating income before depreciation and amortization ($v178$) in year t and scaled by (net) property, plant, and equipment ($v8$) in year $t - 1$;
- Q_{it} : *Tobin's Q*, constructed as the book value of total liabilities ($v181$) plus the market value of common equity from the CRSP database—both in year t —and scaled by the book value of total assets ($v6$) in year t .

³²For the firms that have more than one bond issue trading in the secondary market in a given period, we calculate the portfolio spread in that period by computing an average of credit spreads on all of the firm's outstanding bonds.

Table A-3: Summary Statistic of Selected Firm Characteristics

Variable	Mean	StdDev	Min	Median	Max
σ_{it} (pct.)	30.1	14.6	8.86	26.5	143.8
s_{it} (pps.)	2.11	2.07	0.13	1.38	17.6
$[I/K]_{it}$	0.19	0.12	0.01	0.17	1.00
$[Y/K]_{it}$	3.52	2.83	0.07	2.78	15.0
$[\Pi/K]_{it}$	0.49	0.39	-0.48	0.37	2.50
Q_{it}	1.49	0.76	0.45	1.28	10.7

NOTE: Sample period: 1973:M1–2012:M9 at an annual frequency; Obs. = 8,557; No. of firms = 772. Sample statistics are based on trimmed data.

After deleting observations with missing data items, applying standard filters to remove outliers, and imposing a restriction that a firm needed to be in the panel for a minimum of three years, we were left with 772 firms for a total of 8,557 firm/year observations. Summary statistics of these key firm characteristics are summarized in Table A-3.

A.4 Uncertainty Proxy Based on Profit Shocks

In this section, we describe the procedure used to calibrate the curvature of the profit function and the parameters governing the stochastic volatility process of the idiosyncratic technology shock. Under the assumption of a Cobb-Douglas production function, gross profits (profits before fixed operation costs) differ from sales only up to a constant. Hence, one can estimate the returns to scale using data on either sales or gross profits. We chose sales in order to avoid the occasionally negative gross profit observations.

Letting $v\#$ denote the quarterly data item number, we selected from the Compustat database all U.S. nonfinancial firms with at least 20 quarters of non-missing data on net sales ($v2$) and net property, plant, and equipment ($v42$) over the 1976:Q1–2012:Q3 period, a procedure yielding an unbalanced panel of 9,411 firms for a total of 549,946 observations.³³ To ensure that our results were not driven by a small number of extreme observations, we dropped from the sample all observations with the sales-to-capital ratio below 0.01 and above 20.0 and observations with quarterly growth rates of sales and capital above and below 100 percent.

Our empirical counterpart of the profit function in equation (8) in Section 3.1 of the main text is given by

$$\log Y_{it} = c_{it} + \gamma_s \log K_{i,t-1} + \lambda_{st} + u_{it}, \quad (\text{A-1})$$

where Y_{it} denotes the sales of firm i in quarter t and $K_{i,t-1}$ is the capital stock at the end of quarter $t - 1$. The regression disturbance term u_{it} corresponds to the idiosyncratic technology shock $\log z_{it}$ in our model, while the coefficient γ_s determines—in conjunction with the relative share of capital—the degree of decreasing returns to scale in production, according to

$$\chi_s = \frac{\gamma_s}{\alpha + (1 - \alpha)\gamma_s}.$$

As indicated by the subscript s , we allow the curvature of the profit function to differ across production sectors as defined by the 2/3-digit NAICS codes.

³³Prior to 1976, most firms in Compustat did not report their capital stock data on the quarterly basis.

Because quarterly firm-level sales are characterized by a strong seasonal pattern, we specify that the firm-specific term c_{it} in regression (A-1) satisfies:

$$c_{it} = \sum_{n=1}^4 \phi_n \times \mathbf{1}[\text{QTR}_{it} = n],$$

where $\mathbf{1}[\text{QTR}_{it} = n]$ is an indicator function that equals one if firm i 's observation in period t falls in quarter n and zero otherwise—that is, the regression includes a full set of *firm-specific* quarterly dummies. And lastly, industry-specific (3-digit NAICS) time fixed effects—denoted by λ_{st} —are included in the regression to control for the persistent nature of cyclical profitability shocks within an industry (see McGahan and Porter, 1999).

We use the residuals from the estimation of equation (A-1) to calibrate the process for the idiosyncratic technology shock. First, the persistence of the process is obtained by estimating the following pooled OLS regression:

$$\hat{u}_{it} = \rho_z \hat{u}_{i,t-1} + \epsilon_{it}, \tag{A-2}$$

Second, if the error term ϵ_{it} in regression (A-2) is distributed normally, then an unbiased estimator of the true standard deviation of ϵ_{it} is given by

$$\hat{\sigma}_{\epsilon,it} = \sqrt{\frac{\pi}{2}} |\hat{\epsilon}_{it}|,$$

which yields an estimate of the volatility of the idiosyncratic technology shock for each firm i in every quarter t .³⁴ To obtain a proxy for the time-varying uncertainty of productivity shocks, the last step involves estimating a panel regression of the form:

$$\log \hat{\sigma}_{\epsilon,it} = \sum_{k=1}^4 \beta_k \log \hat{\sigma}_{\epsilon,i,t-1} + \eta_i + v_t + \zeta_{it},$$

where η_i denotes a fixed firm effect. In keeping with our earlier approach, a measure of uncertainty based on profit shocks shown in Figure 4 corresponds to the sequence of estimated time fixed effects \hat{v}_t , $t = 1, \dots, T$, which captures common fluctuations in the idiosyncratic uncertainty regarding the profitability prospects in the nonfinancial corporate sector.³⁵

The results of the first two steps of our estimation procedure are summarized in Table A-4. According to the entries in the table, the sector-specific estimates of the curvature of the profit function yield economically sensible degree of decreasing returns to scale in each broad production sector of the U.S. nonfinancial corporate sector. The estimates of χ lie in a relatively narrow interval, ranging from a low of about 0.81 in agriculture and arts, entertainment, and related services to a high of 0.90 in the mining sector. The average estimate of the decreasing return to scale is about 0.85, which is the value used in our calibration. As indicated by the second memo item in the table, our approach implies the persistence of the idiosyncratic technology shock, the coefficient ρ_z in equation (A-2), to be about 0.90.

³⁴This approach is similar to that of Kim and Nelson (1999) and McConnell and Perez Quiros (2000), who estimate the volatility of aggregate output growth in each quarter from a single observation.

³⁵To ease the interpretation, the sequence \hat{v}_t , $t = 1, \dots, T$, has been re-scaled and expressed in annualized percent. In addition, the residual seasonality has been removed by using the X11 filter.

Table A-4: Estimated Returns to Scale in U.S. Nonfinancial Corporate Sector

Sector (2/3-digit NAICS)	Estimate	[95% Conf. Interval]	Firms	Obs.
Agriculture	0.806	[0.697, 0.882]	42	1,551
Mining	0.906	[0.893, 0.916]	641	31,902
Utilities	0.831	[0.768, 0.873]	250	23,902
Construction	0.813	[0.781, 0.842]	176	7,784
Manufacturing – Durables	0.851	[0.845, 0.857]	3,257	187,213
Manufacturing – Nondurables	0.867	[0.858, 0.875]	1,455	92,547
Wholesale Trade	0.856	[0.839, 0.873]	371	20,848
Retail Trade	0.882	[0.865, 0.898]	577	35,174
Transportation	0.859	[0.837, 0.881]	295	18,014
Information Services	0.838	[0.825, 0.849]	1,095	52,272
Real Estate Services	0.823	[0.796, 0.848]	188	11,481
Professional Services	0.844	[0.828, 0.858]	400	25,433
Administrative Services	0.844	[0.809, 0.866]	205	12,285
Health Care	0.816	[0.782, 0.849]	199	11,776
Arts & Entertainment	0.803	[0.766, 0.856]	66	3,798
Accommodation Services	0.862	[0.840, 0.885]	224	13,965
<i>Memo: Average</i>	0.844	[0.833, 0.852]	.	.
<i>Memo: AR(1)^a</i>	0.898	[0.895, 0.899]	.	.

NOTE: Sample period: 1976:Q1–2012:Q3; No. of firms = 9,441; Obs. = 549,946. Entries in the table denote the estimates of the sector-specific parameter χ_s , measuring the degree of decreasing returns-to-scale, as implied by the OLS estimates of the parameter γ_s from regression (A-1) and the share of capital $\alpha = 0.3$ in a Cobb-Douglas production function. The 95-percent confidence intervals are based on a cluster bootstrap with 500 replications.

^a Estimate of ρ_z , the parameter governing the persistence of the idiosyncratic technology shock; see equation (A-2) and the main text for details.

B Model Appendix

In this appendix, we provide details regarding the key elements of our model and its solution method. Section B.1 describes the construction of the Markov chain with time-varying volatility, which governs the evolution of the idiosyncratic technology shock. Section B.2 summarizes the calibration of the model, while Section B.3 elaborates on the computational details.

B.1 Markov Chain with Time-Varying Volatility

Consider an N -state Markov chain with a transition matrix,

$$\mathbf{P} = \begin{bmatrix} p_{1,1} & \cdots & p_{1,N} \\ \vdots & \ddots & \vdots \\ p_{N,1} & \cdots & p_{N,N} \end{bmatrix}; \quad \text{with} \quad \sum_{j=1}^N p_{i,j} = 1, \quad (\text{B-1})$$

and let p_i , $i = 1, \dots, N$, denote its ergodic distribution. We assume, without loss of generality, that N is an even number. We also assume that the ergodic distribution is symmetric, in the sense that $p_i = p_{N-(i-1)}$ for all $i = 1, \dots, N$.

The N states of the chain are given by

$$\bar{z} - \frac{\bar{\sigma}}{2}, \dots, \bar{z} - \frac{3\bar{\sigma}}{2(N-1)}, \bar{z} - \frac{\bar{\sigma}}{2(N-1)}, \bar{z} + \frac{\bar{\sigma}}{2(N-1)}, \bar{z} + \frac{3\bar{\sigma}}{2(N-1)}, \dots, \bar{z} + \frac{\bar{\sigma}}{2};$$

where \bar{z} and $\bar{\sigma}^2$ denote the unconditional mean and unconditional variance of the process, respectively. Because the N states are equispaced, they can be expressed as

$$z_i = \bar{z} + \left[2 \left(\frac{i-1}{N-1} \right) - 1 \right] \frac{\bar{\sigma}}{2}; \quad i = 1, \dots, N. \quad (\text{B-2})$$

The first two conditional moments of this process are then given by

$$\mathbb{E}[z_{t+1} \mid z_t = z_i] = \bar{z} + \frac{\mu_i}{2} \bar{\sigma}_z; \quad (\text{B-3})$$

$$\text{Var}[z_{t+1} \mid z_t = z_i] = \Xi_i \bar{\sigma}_z^2, \quad (\text{B-4})$$

where

$$\mu_i = 2 \sum_{j=1}^N p_{i,j} \left(\frac{j-1}{N-1} \right) - 1 \quad \text{and} \quad \Xi_i = \sum_{j=1}^N p_{i,j} \left[\frac{j-1}{N-1} - \sum_{j=1}^N p_{i,j} \left(\frac{j-1}{N-1} \right) \right]^2.$$

Now consider the same discrete-time Markov chain, except suppose that its volatility follows a stationary process denoted by $\{\sigma_t\}$. In this case, we assume that conditional on observing σ_t in period t , the N equispaced states in period $t+1$ are given by

$$z_{j,t+1} = \bar{z} - \frac{\mu_i}{2} (\sigma_t - \bar{\sigma}) + \left[2 \left(\frac{j-1}{N-1} \right) - 1 \right] \frac{\sigma_t}{2}; \quad j = 1, \dots, N. \quad (\text{B-5})$$

The conditional mean of this modified Markov chain is then given by

$$\begin{aligned} \mathbb{E}(z_{t+1} \mid z_t = z_i) &= \bar{z} + \frac{\mu_i}{2} \bar{\sigma} + \frac{\sigma_t}{2} \sum_{j=1}^N p_{i,j} \left[2 \left(\frac{j-1}{N-1} \right) - 1 - \mu_i \right] \\ &= \bar{z} + \frac{\mu_i}{2} \bar{\sigma} + \sigma_t \sum_{j=1}^N p_{i,j} \left[\frac{j-1}{N-1} - \sum_{k=1}^N p_{i,k} \left(\frac{k-1}{N-1} \right) \right]. \end{aligned}$$

Note that

$$\begin{aligned}
\sum_{j=1}^N p_{i,j} \left[\frac{j-1}{N-1} - \sum_{k=1}^N p_{i,k} \left(\frac{k-1}{N-1} \right) \right] &= \sum_{j=1}^N p_{i,j} \left(\frac{j-1}{N-1} \right) - \sum_{j=1}^N p_{i,j} \left[\sum_{k=1}^N p_{i,k} \left(\frac{k-1}{N-1} \right) \right] \\
&= \sum_{j=1}^N p_{i,j} \left(\frac{j-1}{N-1} \right) - \left[\sum_{k=1}^N p_{i,k} \left(\frac{k-1}{N-1} \right) \right] \sum_{j=1}^N p_{i,j} \\
&= \sum_{j=1}^N p_{i,j} \left(\frac{j-1}{N-1} \right) - \sum_{k=1}^N p_{i,k} \left(\frac{k-1}{N-1} \right) \\
&= 0,
\end{aligned}$$

which implies that

$$E(z_{t+1} \mid z_t = z) = \bar{z} + \frac{\mu_i}{2} \bar{\sigma}. \quad (\text{B-6})$$

Thus, the conditional mean of the modified Markov chain with stochastic volatility is identical to that of the conventional Markov chain with time-invariant volatility (see equation [B-3](#)). Hence, an increase in volatility represents a mean-preserving-spread (MPS) of z , a property reflecting the presence of the mean-correction term $-0.5\mu_i(\sigma_t - \bar{\sigma})$ in equation [\(B-5\)](#).

In contrast, the conditional variance of the process is given by

$$\begin{aligned}
\text{Var}(z_{t+1} \mid z_t = z_i) &= \sigma_t^2 \sum_{j=1}^N p_{i,j} \left[\left(\frac{j-1}{N-1} \right) - \sum_{j=1}^N p_{i,j} \frac{j-1}{N-1} \right]^2 \\
&= \Xi_i \sigma_t^2.
\end{aligned} \quad (\text{B-7})$$

Thus the conditional volatility of this process depends linearly on the realization of the stochastic process $\{\sigma_t\}$. In this formulation, the support of the distribution of the idiosyncratic technology shock z is evolving stochastically over time, with an increase in σ today inducing a greater dispersion in z tomorrow and vice versa. Under the assumption of a symmetric ergodic distribution ($p_i = p_{N-(i-1)}$, $i = 1, \dots, N$), it is straightforward to show that the realization of the volatility process $\{\sigma_t\}$ does not alter the unconditional mean and variance of z .

B.2 Model Calibration Summary

Table B-1 summarizes the parameter values used in the calibration of the model.

Table B-1: Calibrated Model Parameters

Model Parameter	Value
<i>Production and capital accumulation</i>	
Value-added share of capital (α)	0.30
Decreasing returns-to-scale (χ)	0.85
Fixed costs of production (F_o)	0.05
Fixed costs of investment (F_k)	0.01
Depreciation rate (δ)	0.025
Purchase price of capital (p^+)	1.00
<i>Financial markets</i>	
Lower bound on net worth (\bar{n})	0.00
Survival probability (η) ^a	0.95
Bankruptcy costs (ξ)	0.10
Equity issuance costs (φ)	0.12
<i>Representative household</i>	
Discount factor (β)	0.99
Disutility of labor (ζ) ^b	.
<i>Exogenous shocks</i>	
Persistence of the idiosyncratic technology shock process ^c	0.80
Steady-state level of idiosyncratic uncertainty ($\tilde{\sigma}_z$)	0.15
Persistence of the idiosyncratic uncertainty process (ρ_σ)	0.90
Volatility of innovations of the idiosyncratic uncertainty process (ω_σ)	0.04
Persistence of the aggregate technology shock process (ρ_a)	0.98
Volatility of innovations of the aggregate technology shock process (ω_a)	0.0075
Steady-state liquidation value of capital (\bar{p}^-)	0.50
Persistence of the liquidation value of capital process (ρ_{p^-}) ^d	0.98
Volatility of innovations of the liquidation value of capital process (ω_{p^-}) ^d	0.015

NOTE: Period in the model equals one quarter. Values of parameters with a time dimension are expressed at quarterly rates. See the main text for details.

^a According to the survey of Business Employment Dynamics, the average yearly survival rate for the establishments that were established between 1994 and 2009 is 0.784, which implies a quarterly survival rate of 0.942.

^b The weight of the (linear) disutility of hours worked is chosen so that the real wage in the steady state is equal to one.

^c The idiosyncratic technology shock follows a 4-state Markov chain process with time-varying volatility; see Section B.1 of the model appendix for details.

^d These estimates have been converted from monthly estimates based on the 1998:M2–2011:M11 sample period.

B.3 Computational Details

The model is solved using the *inner-and-outer-loop* algorithm developed by Krusell and Smith (1998). In the inner loop, the agents predict market-clearing prices using log-linear laws of motion for the aggregate moments. Under this bounded rationality assumption, we solve the inner-loop problem using a value function iteration routine, which allows for a fully nonlinear global solution under several occasionally binding constraints; in our case, these constraints are associated with

the dividend constraint, partial irreversibility, and nonconvex capital adjustment costs. Once the inner-loop problem is solved, we use Monte Carlo methods to simulate the model economy for $T = 2,000$ (quarters) and $N = 10,000$ (firms).

The dimension of the inner-loop problem is quite large, compared with most models of this type. As described in the main text, the set of states in our model is given by $\{z, k, x, a, p^-, \sigma_{-1}, \tilde{k}, \tilde{b}\}$. In the numerical implementation, we allow for 40 equispaced grid points for the endogenous state variables k and x ; for the decision variables k' and b' , however, we use 200 equispaced grid points.

For the state variables that are exogenous from the perspective of the firms in the economy—that is, $\{a, p^-, \sigma_{-1}, \tilde{k}, \tilde{b}\}$ —we use 3 grid points. For the exogenous aggregate state variables a , p^- , and σ , we use a Gauss-Hermite quadrature method, so that the value function and policy variables can be computed for continuous variation in these state variables. We use two points for the Gauss-Hermite quadrature integration for each shock associated with the exogenous aggregate state variables. The continuation values off the grid points for $\{z, k, x, a, p^-, \sigma_{-1}, \tilde{k}, \tilde{b}\}$ then need to be evaluated using a tensor product spline approximation. Because we specified a 4-state Markov Chain for the idiosyncratic technology shock z , the dimension of the problem $\{z, k, x, a, p^-, \sigma_{-1}, \tilde{k}, \tilde{b}, k', b', z', a', \sigma\}$ is given by $4 \times 40 \times 40 \times 3 \times 3 \times 3 \times 3 \times 3 \times 200 \times 200 \times 4 \times 2 \times 2 \times 2 \approx 1.99e + 12$.

In the outer loop, we update the aggregate laws of motion using a Monte Carlo simulation. At this stage, it is important to ensure that all markets clear, even when the perceived aggregate laws of motion are “inaccurate.” To this end, we solve for the marginal utility of consumption $u_c(\mathbf{s}_t)$ that is consistent with the market clearing conditions, using a nonlinear root finder for each $t = 1, \dots, T$ of the Monte Carlo simulation. This step substantially slows down computations in the outer loop but allows for a maximum amount of learning by the agents in the economy. Once the economy is simulated, we use OLS to update the aggregate laws of motion.

Within this framework, it is important to check how well does the aggregation methodology used to compute the solution of the model approximate the model’s true rational expectations equilibrium. The top panel of Table B-2 shows the estimates of the parameters governing the aggregate laws of motion implied by the system of equations (38) that are used by the agents to predict future prices in the model with financial frictions. As evidenced by the high R^2 values, the agents’ perceived aggregate laws of motion are highly accurate. According to this commonly used metric, our solution of the model is thus likely a good approximation of the model’s true rational expectations equilibrium.

The coefficients of the forecasting rules used by the agents to forecast equilibrium prices also have a number of intuitive properties. For example, the negative coefficient on the stock of debt (\tilde{b}) in the law of motion for the aggregate capital stock reflects the effect of debt overhang on macroeconomic outcomes: All else equal, a high level of corporate debt implies less capital accumulation going forward. In contrast, the aggregate debt stock has a positive effect on consumption, a result reflecting the fact that outstanding corporate debt is a part of the representative household’s wealth. However, the coefficient on the stock of debt (\tilde{b}) in the law of motion for consumption is very small compared with that of the aggregate capital stock (\tilde{k}), which suggests that at the general equilibrium level, the drag from the debt overhang in the corporate sector reduces the marginal propensity to consume out of claims on corporate debt. As evidenced by the relatively large positive coefficients on the liquidation value of capital (p^-) in the law of motion for both the aggregate capital and debt stocks, capital liquidity shocks importantly influence the dynamics of capital and debt accumulation in the model.

We also solved a version of the model without financial distortions—in that case, the firms face the same capital adjustment frictions as before, except that they finance their investment expenditures using only internal funds and equity, where the issuance of the latter is not subject to any dilution costs. The bottom panel of Table B-2 shows the aggregate laws of motion for the

Table B-2: Agents' Perceived Aggregate Laws of Motion

<i>Model With Financial Frictions</i> ^a						
Explanatory Variables						
Dep. Variable	$\log \tilde{k}$	$\log \tilde{b}$	$\log a$	$\log \sigma$	$\log p^-$	R^2
$\log \tilde{k}'$	0.8545	-0.0093	0.0980	-0.0114	0.0708	0.9986
$\log \tilde{b}'$	0.6361	0.0350	0.3324	-0.0348	0.7989	0.9989
$\log c$	0.3475	0.0125	0.3318	0.0113	-0.0746	0.9955
<i>Model Without Financial Frictions</i> ^b						
Explanatory Variables						
Dep. Variable	$\log \tilde{k}$	$\log \tilde{b}$	$\log a$	$\log \sigma$	$\log p^-$	R^2
$\log \tilde{k}'$	0.8044	.	0.0954	-0.0005	0.0012	0.9958
$\log c$	0.3803	.	0.3445	-0.0000	-0.0009	0.9975

NOTE: Both simulations assume that there are 10,000 heterogeneous firms at any point in time and are simulated for 2,100 quarters by feeding into the specified model randomly drawn aggregate and idiosyncratic shocks. In updating the agents' perceived aggregate laws of motion, the initial 100 quarters are dropped and the remaining observations are used to estimate the aggregate laws of motion (all specifications include a constant, not reported). The updated laws of motion are then used to update the individual policy rules in a numerical dynamic programming problem. The algorithm stops when the changes in the aggregate laws of motion in the subsequent iteration are smaller than the pre-specified tolerance criterion.

^a In this version of the model, the firm finances investment expenditure with internal funds, equity, and bonds. Financial frictions imply that equity issuance is subject to dilution costs, while bankruptcy costs imply an additional loss for bond investors in the case of default.

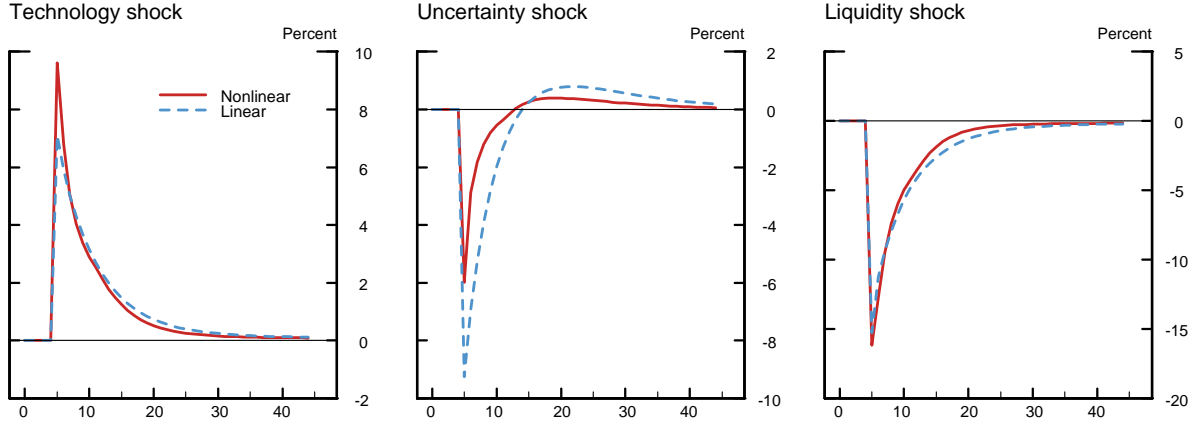
^b In this version of the model, the firm finances investment expenditures with internal funds and equity only, with the issuance of the latter not being subject to any dilution costs. As a result, the aggregate debt \tilde{b} is not a state variable and does not appear in the agents' perceived laws of motion.

model without financial frictions. According to the goodness-of-fit criteria, the agents' perceived aggregate laws of motion are again highly accurate. Note that in this case, the coefficient on the liquidation value of capital (p^-) in the law of motion for the aggregate capital stock is tiny, which implies that capital liquidity shocks have a negligible effect on the evolution of the aggregate capital stock in the absence of financial distortions.

To compute impulse responses for a wider set of endogenous aggregate variables than those implied by the system of equations (38) and to fully take into account the nonlinearities at play at the micro level—all while maintaining the assumption of bounded rationality—we use the following algorithm: Let $i = 1, \dots, N$ index the N heterogeneous firms in the economy, $t = 1, \dots, T$ index the T periods of the impulse response horizon, and let $\mathcal{Z} = \{z_{it} | i = 1, \dots, N \text{ and } t = 1, \dots, T\}$ denote the associated set of idiosyncratic technology states implied by the Markov chain with time-varying volatility. Using the set \mathcal{Z} , we construct two model simulations over the T periods: a simulation perturbed by an aggregate shock and a simulation without an aggregate shock. Specifically, let x_{it} denote a generic model variable (e.g., capital stock k_{it}) and index these two simulations by x_{it}^1 and x_{it}^0 , respectively. Note that the same set of idiosyncratic technology states \mathcal{Z} underlies the construction of x_{it}^1 and x_{it}^0 , for all $i = 1, \dots, N$ and $t = 1, \dots, T$.³⁶ The only difference between

³⁶Note that all aggregate shocks, with the exception of the uncertainty shock, have no effect on \mathcal{Z} , the set of

Figure B-1: Aggregate Investment Dynamics
(Linear vs. Nonlinear Impulse Response Functions)



NOTE: The solid lines depict the impulse response functions of aggregate investment to the specified shock implied by the benchmark model with financial frictions, which are computed taking into account the nonlinearities of the firms' investment and financial policies; the red dash-dotted lines are the corresponding impulse response functions based solely on the agents' perceived (log-linear) aggregate laws of motion (see the notes to Figures 6-8 and the main text for details).

these two simulations is that we introduce an aggregate shock at t^* in the first simulation, which is then allowed to die out according to its specified law of motion over the remainder of the impulse response horizon.

To remove the effects of sampling variation associated with the simulation of the idiosyncratic technology shock, we repeat the above procedure M times. The model-implied impulse response function of the aggregate variable x in response to an aggregate shock—denoted by \hat{x}_h —is then calculated according to

$$\hat{x}_t = 100 \times \log \left[\frac{\sum_{m=1}^M \sum_{i=1}^N x_{m,it}^1}{\sum_{m=1}^M \sum_{i=1}^N x_{m,it}^0} \right]; \quad t = 1, \dots, T. \quad (\text{B-8})$$

In implementing this procedure, we set $N = 10,000$, $T = 45$, $M = 50,000$, and introduce aggregate shocks at $t^* = 5$.

This computationally intensive approach was chosen, in part, because the response of the model to various aggregate shocks may not be reflected fully by the log-linear specification of the agents' perceived aggregate laws of motion. The quantitative significance of failing to account for the model's inherent nonlinearities is illustrated in Figure B-1. The solid lines show the response of aggregate investment to the specified shock constructed according to equation (B-8), while the dotted lines depict the corresponding responses based solely on the agents' perceived aggregate laws of motion. The latter method clearly overstates the sensitivity of aggregate investment to the aggregate TFP and uncertainty shocks. These differences in impulse responses suggest that the agents' perceived laws of motion may be misspecified in spite of the very high R^2 values reported in

idiosyncratic technology states. By design, however, the impact of the uncertainty shock today will have an effect on the dispersion of the idiosyncratic technology shocks in the future. Hence, $z_{m,it}^1 \neq z_{m,it}^0$ for this type of aggregate shock. However, the *relative* position of each individual firm in the distribution of the idiosyncratic technology shock will be the same as in the case when the economy is not perturbed by an uncertainty shock. For all other aggregate shocks, $z_{m,it}^1 = z_{m,it}^0$, for all m, i , and t .

Table B-2. However, exploring nonlinear forms for the laws of motion—for example, by including higher-order moments—is from a computational perspective prohibitively expensive, and we leave this issue for future research.