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Reconciling Full-Cost and Marginal-Cost Pricing

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Abstract

Despite the clear prescription from economic theory that a firm should set price based only on variable costs, firms routinely factor fixed costs into pricing decisions. We show that full-cost pricing (FCP) can help firms uncover their optimal price from economic theory. FCP marks up variable cost with the contribution margin per unit, which in equilibrium includes the fixed cost. This requires some knowledge of the firm's equilibrium return, though this is arguably easier a lower informational burden than knowing one's demand curve, which is required for optimal economic pricing. We characterize when FCP can implement the optimal price in a static game, a dynamic game, with multiple products, and under a satisficing objective.

Keywords: Optimal Pricing, Full Cost Pricing, Marginal Cost Pricing

*The views expressed in this paper are those of the authors and do not necessarily reflect the view of the Board of Governors or its staff.

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1 Introduction

Economic theory provides an unambiguous prediction on how a profit maximizing firm should set its price for a product: the firm includes only marginal (or variable) costs. It should not factor fixed costs into the pricing decision. Yet a wealth of survey evidence (Shim and Sudit [1995]; Noreen and Burgstahler [1997]; Shim [1993]; Govindarajan and Anthony [1983]) indicates that more than 60 percent of American manufacturing companies do include fixed costs in prices, a practice called full-cost pricing (FCP). We seek to resolve this puzzle between economic theory and real accounting practices with the following observation: optimal pricing by a firm requires accurate knowledge of their residual demand curve, even though firms rarely have such knowledge, and certainly not to the extent and precision needed for ascertaining the optimal price. FCP, on the other hand, may be based on a less stringent informational requirement, such as some knowledge of equilibrium income, and still lead to the optimal price. This lessens the apparent paradox, and full-cost pricing may simply be a concrete way to implement optimal pricing.

We provide a specific algorithm by which the firm can formulate its full-cost price and achieve the optimal price. This may be achieved even when knowledge of equilibrium income is imperfect, such as for a multiproduct firm with complementarities between products, as well as in a firm that chooses to satisfice rather than optimize.

In the classical economic textbook solution, the firm maximizes profits by equalizing marginal revenue and marginal cost, thereby marking up variable cost in a manner related to the elasticity of demand. Fixed costs vanish from the firm's optimization problem and therefore do not factor into the optimal price. Instead, they only affect the firm's decision on whether to enter the market. For the firm to pick the price correctly, it must know its demand curve, since the optimal price (via marginal revenue) is a function of the demand curve's shape and parameters. But even a slight bit of uncertainty regarding demand parameters can lead the firm to misprice its product. The economic model of the profit-maximizing firm works if the firm does, in fact, have all the necessary information on its market environment, including the demand coefficients. However, if these strong informational requirements are not met, pricing resulting from erroneous estimates of demand will be suboptimal, which may explain why firms don't actually price according to economic theory.[1]

[1] The classic defense of the economic model comes from Milton Friedman, who argued that firms behave "as if" they are solving an explicit optimization problem (Friedman [1953]). We are explicitly concerned with the *actual* pricing practice of firms, i.e. a "what exactly" explanation, rather than an "as if" explanation. Indeed, one can interpret our results as supporting Friedman's argument: full-cost pricing may be the practical method by which firms achieve optimal pricing. Friedman's "as if" argument begs the question of what the actual method is, and full-cost pricing may provide the answer.

In contrast, the firm could also set a "full-cost" price that includes its fixed costs. Rather than maximizing profits explicitly and arriving at optimal price through first order conditions, the full-cost price emerges from an equilibrium condition. Income of the firm at the equilibrium price equals, by definition, equilibrium income. Rearranging this identity, equilibrium price equals variable cost plus equilibrium contribution margin per unit (the latter term being the fixed cost plus equilibrium income, all divided by quantity). Therefore, if the firm can measure its equilibrium income, it can "back out" an estimate of equilibrium price. This yields a candidate full-cost price, which can then be used to estimate, or iterate to, the optimal price.

In order to achieve the optimal price using FCP, the firm must have some knowledge or estimate of optimal (equilibrium) income. We perceive this to be a less burdensome informational requirement than knowing their entire demand curve. Knowledge of equilibrium income may be based on accounting measures such as return on investment (ROI) or return on assets (ROA). A variety of market data and studies suggest firms have better knowledge of an equilibrium rate of return than of the demand curve necessary for optimal pricing through economic theory. In surveys, firms often base price on some measure of "desired" profit or return. All of this suggests firms use some measure of equilibrium income (or return) to set price. Our formulation of the full-cost price formalizes this heuristic.

We consider full-cost pricing in both static and dynamic contexts. In the static model, the firm sets a single price and FCP achieves the optimal price if and only if the firm's estimate of equilibrium income is exact, and it knows equilibrium quantity.[2] This latter condition shows that the information requirements for FCP to achieve optimal price in a static environment are stringent. Indeed, conditional on knowing equilibrium income, knowing equilibrium quantity is informationally equivalent to knowing optimal price. However, full-cost pricing, as we will show, can achieve the optimal pricing in a dynamic setting when the firm has no knowledge of its equilibrium quantity, and only has an estimate of equilibrium income. We provide an algorithm by which the firm prices over time, setting the initial price equal to its variable cost and revising the price upwards by the equilibrium contribution margin per unit, taking the number of units to be the quantity the market purchases at the price in the prior stage. In this algorithm, the full-cost price converges to the optimal price. Variable cost is a natural starting value since the competitive benchmark sets price equal to the marginal (variable) cost, however firms may use other starting values with slight modification to the algorithm as we discuss later. This convergence works both when the firm knows equilibrium income exactly, and when the firm does not know income exactly, but has a consistent estimate of it. This form of price experimentation may seem ad hoc, but it is documented in surveys of company practice has theoretic foundations. We provide several examples based on linear demand curves

[2]Its estimate of optimal price is unbiased if the estimate of equilibrium income is unbiased.

and constant elasticity demand curves to illustrate convergence.

The model is robust to a number of extensions. First, FCP achieves the optimal price even in a multiproduct firm. Such firms often have complementarities between products, such as an auto manufacturer who makes multiple cars and trucks. With such complementarities, the price of one product may affect the demand for another. But because the full-cost price is based on equilibrium income and not on any first order condition, these cross-price externalities do not interfere with the convergence properties of the full-cost price. The algorithm is more complex since the full-cost price at each stage is a vector rather than a scalar, but it converges nonetheless. A second extension is that FCP still "converges" if the firm decides not to optimize but to satisfice, i.e. achieve a minimum level of profit. We provide conditions under which the full-cost price meets this more relaxed criteria: namely that its forecast of equilibrium quantity must be biased downward.

Accounting research has developed a large volume of survey evidence to show that full-cost pricing dominates managerial practices (Shim and Sudit [1995]; Emore and Ness [1991]; Cooper [1990]; Govindarajan and Anthony [1983]; Gordon et al. [1981]; Drury and Tayles [2000]; Bright et al. [1992]). This debate between real accounting practices and economic theory has persisted for quite some time (as early as Friedman [1953]; and as recently as Lucas [2003]). Yet formal theoretical models of this debate have appeared only recently. Al-Najjar et al. [2005] consider a case of a Bertrand oligopoly and find that firms which follow a naive adaptive learning process to adjust prices will eventually include fixed cost into their pricing (a phenomenon they call the sunk-cost bias). Their adaptive learning algorithm is similar to ours in the sense that the firm prices dynamically over time, though their emphasis is slightly different, focusing on strategic interaction between firms.[3] The sunk-cost bias (FCP) persists because firms do not optimize in their models, but instead adopt a behavioral strategy of adaptive learning. In contrast, we do not assume at the outset that firms fail to optimize, but rather examine the informational environment in which they do so. Nonetheless, Al-Najjar et al. [2005] is a recent paper in economic literature that investigates the same puzzle we do, even though they arrive at different conclusions and justifications for full-cost pricing.

Netzer and Thepot [2008] also seek to understand the paradox, but work within a leader-follower game of a two-tier organization, namely an upstream unit that produces capacity and a downstream unit that chooses output level. They find full-cost pricing dominates other pricing rules in a Cournot oligopoly case, stemming from the two-tiered structure of their organization. Finally, Noreen and Burgstahler [1997] arrive at a negative result of full-cost pricing, namely that in a multiproduct firm with fixed costs, full-cost pricing does not necessarily lead to a

[3] Our model is general enough to handle such interactions, though the interactions add little insight into our central question so we abstract from them.

satisfactory profit. They consider firms that satisfice rather than optimize. However, they take a more narrow definition of FCP and do not base it on any measure of equilibrium income as we do. This is why our FCP does achieve satisfactory profits, while their FCP does not. Other papers within accounting and economic literature examine absorption costing practices (such as full-cost pricing) in a strategic environment (Alles and Datar [1998]; Narayanan and Smith [2000]; and Hughes and Kao [1998]). We focus on a non-strategic environment - a given residual demand curve for the firm - in order to distill the mechanism in which we are most interested.

The paper proceeds as follows. Section 2 reviews the benchmark of the economic model, delivering the optimal price and discussing impediments to achieving the optimal price. Section 3 introduces full-cost pricing and gives necessary and sufficient conditions for FCP to achieve optimal pricing in a static model. Section 4 explores dynamic FCP both when equilibrium income is known and unknown. Section 5 provides extensions of the model to a multiproduct firm and a satisficing firm. Section 6 concludes.

2 First Best Benchmark: The Optimal Price

Consider a firm who sells a single product in a market. The firm sets the price $p > 0$ and demand is $q(p) > 0$. Assume throughout that demand is decreasing, so $q'(p) < 0$ for all p.[4] The firm's cost of producing q units of the product is $C(q) = F + vq$. Observe that the marginal cost $C'(q) = v$ is the constant variable cost of production.[5] The fixed cost F includes fixed manufacturing overhead, fixed SG&A, and any other fixed costs of the firm.

The classical textbook analysis solves the inverse problem of a firm choosing quantity against inverse demand $p(q)$. Because our focus is on pricing, we will consider the more intuitive (though slightly more algebraically involved) formulation of the firm choosing price directly.[6] The firm chooses price to maximize income $y(p)$, where

$$y(p) = pq(p) - C(q(p)) = (p - v)q(p) - F$$

We further assume the regularity condition that the firm's income function is strictly concave,

[4] Throughout the paper, "demand" means the residual demand curve facing the firm. It may be a market wide-demand curve in the case of a monopolist, or a residual demand curve in an oligopolistic setting, but it must be downward sloping. Perfectly competitive firms (with perfectly elastic demand curves) do not face a pricing decision, so the central question of this paper is ill-posed.

[5] Our exposition uses constant marginal cost, but results hold with any cost structure that results in a concave income function, $y''(p) < 0$.

[6] All results on price maximization can be reformulated to results on quantity to more easily compare the results of the economic model with those from accounting practices.

so $y''(p) < 0$ for all p.[7] This second order sufficient condition guarantees a unique solution p^* to the firm's problem. The first order condition for the firm's problem gives the optimal price:

$$p^* = v - \frac{q(p^*)}{q'(p^*)} \tag{1}$$

This condition implicitly delivers the optimal price as a function of the demand curve. Optimal quantity is simply $q^* = q(p^*)$.[8] Since demand slopes down, $q'(p) < 0$, so the optimal price is a positive markup from the firm's variable cost. In particular, this markup is a function of the elasticity of demand. Elasticity of demand is $\epsilon = pq'(p)/q(p)$, and so rewriting this into the optimal price, $p^* = v - p^*/\epsilon$, or $p^* = v/(1 + 1/\epsilon)$, a rearrangement of the Lerner equation. The firm bases its markup purely on conditions of demand, charging higher markups when consumers are willing to pay more (less elastic demand).[9] Importantly, the optimal price is *not* a function of F.[10]

The benchmark model does not consider fixed costs in the pricing decision because it falls out of the firm's optimization problem. The firm chooses its price on the margin, considering the marginal cost of production against the marginal revenue from selling additional products in the market, which the demand curve determines. The only decision which is affected by fixed costs is the entry decision. Firms with sufficiently low fixed costs to make positive economic profit will enter the market, namely, those with fixed costs less than the equilibrium contribution margin over all units ($F < (p^* - v)q^*$). For those firms that do enter the market, they will make non-negative profits in equilibrium, and fixed cost only affects the entry decision, not the pricing decision of the firm.[11]

[7] This second order sufficient condition is equivalent to assuming that the firm's demand is not "too convex." A more intuitive and primitive condition would be that the firm's demand is weakly concave, $q''(p) \leq 0$, meaning a demand curve that is linear or more concave than linear. However, that would be a more restrictive assumption than the one we make, which allows for some convexity in the demand curve, such as the function $q(p) = \frac{1}{p} - a$.

[8] In the textbook model under linear inverse demand $p(q) = a - bq$, demand is $q(p) = \frac{a-p}{b}$, yielding the familiar $p^* = \frac{a+v}{2}$ and $q^* = \frac{a-v}{2b}$. At the optimal quantity, marginal revenue equals marginal cost.

[9] ϵ is a negative number.

[10] To see the relation to FCP, let y^* be the equilibrium income (or profit) for the firm at the optimal price p^*, so $y^* = y(p^*)$. A useful identity is that $y^* + F = (p^* - v) \cdot q^*$. Thus, the equilibrium profit plus the fixed cost equals the equilibrium contribution margin. Note that while this relation holds at every price, it is most relevant at the optimal price p^*.

[11] These results on entry and economic profit are true whether the environment is monopoly, oligopoly, or perfect competition. While it is often assumed that monopolists earn more than oligopolists (who, in turn, earn more than perfectly competitive firms) this is not always the case in equilibrium. Monopolists may earn just enough to just cover F so they earn zero economic profit, while oligopolistic firms may earn greater than zero economic profit due to indivisibilities in firm entry or product differentiation.

2.1 Impediments to First Best

It is clear from (1) above that the firm must know the demand curve in order to correctly set the optimal price. Without this knowledge, the firm cannot properly markup from variable cost.

To gain traction on this issue, consider linear demand, $q(p) = a - bp$,[12] where $a > 0$ is the intercept of the demand curve, and $b > 0$ is a coefficient on the slope. Under linear demand, the optimal price and quantity are:

$$p^* = \frac{a + bv}{2b} \text{ and } q^* = \frac{a - bv}{2} \tag{2}$$

Even if the firm does not know the demand parameters a and b precisely, it can use signals of these parameters to approximate the price and quantity. The firm may use the signals to estimate p^* directly, or to estimate p^* through an estimate of q^*. The latter case is perhaps less likely, so we consider it first to dispense with it. If the firm has available unbiased estimators $\hat{a}$ and $\hat{b}$ of a and b, respectively, then it can approximate $\hat{q} = (\hat{a} - \hat{b}v)/2$ as a forecast for the quantity consumers will purchase. Because the estimators are unbiased and the expectation operator is linear, expected quantity is also unbiased:

$$E[\hat{q}] = E[\frac{\hat{a} - \hat{b}v}{2}] = \frac{a - bv}{2} = q^*, \tag{3}$$

Even if the firm has $\hat{a}$ and $\hat{b}$, leading to an unbiased estimate of q^*, p^* is more elusive. The firm cannot construct an unbiased estimate of p^*, regardless of whether it uses its unbiased estimator $\hat{q}$ of q^*, or the demand primitive estimates ($\hat{a}$ and $\hat{b}$) themselves.

Let $\tilde{p} = \hat{a} - \hat{b}\hat{q}$ be the estimated price based on the forecasted quantity and forecasted demand. Observe that the random variables $\hat{b}$ and $\hat{q}$ are not independent. Their covariance is non-zero (it is negative, see (3)). Therefore, $\tilde{p}$ is a biased estimate of p^*, as

$$E[\tilde{p}] = E[(\hat{a} - \hat{b}\hat{q})] \neq a - bq^* = p^*, \tag{4}$$

Thus, even if the firm has independent unbiased estimates of the parameters of the demand function, this does not lead to an unbiased estimate of the optimal price through an estimate of (unbiased) q^*.

Alternatively, the firm may try to estimate the optimal price directly using its signals on the parameters of the demand function. A natural estimator would be $\hat{p} = (\hat{a} + \hat{b}v)/2\hat{b}$. Again, even though the signals are unbiased estimates of their true parameters, this composite estimator of p^*, is not, since:

[12] Note, these are slightly inverted parameters relative to footnote 8. This eases exposition here.

$$E[\hat{p}] = E\left[\frac{\hat{a} + \hat{b}v}{2\hat{b}}\right] > \frac{a + bv}{2b} = p^*, \quad (5)$$

where the inequality follows from Jensen's inequality.[13] Thus, the firm will overestimate its price if it seeks to estimate price directly. Once again, the firm fails to set the optimal price based on signals of the parameters of the demand function.

There is an even larger problem at stake, however. In the above discussion, we have assumed that the firm knows that demand is linear (but does not know the parameters of that linear function). In reality, the firm likely does not even know the shape of the demand curve. In particular, suppose that the actual demand is CES, so $q(p) = ap^{-b}$. In that case, the FOC for constant elasticity demand delivers an optimal price and quantity

$$p^* = \frac{v}{1 - 1/b} \text{ and } q^* = a\left(\frac{v}{1 - 1/b}\right)^{-b}.$$

Optimal prices under differently shaped demand curves can be quite different, so if even the "type," or shape, of the demand curve is unknown, the firm will have more difficulty establishing optimal p^*. Estimating the parameters of a misspecified model will only produce erroneous estimates.[14]

In sum, the textbook model of optimal pricing provides a clear and unambiguous solution to the optimal price. However, this clarity comes at a cost: to avoid blind experimentation, the firm must have a tremendous amount of detail about the demand it faces. It must know the "shape" of its demand curve, as well as unbiased estimates of the parameters of the the shape, which are both exceedingly strong and unrealistic assumptions, and still lead to biased estimates of optimal price. Because firms having nothing like this information in real environments, it is no wonder that firms may rely on accounting practices. These do not require estimates of a demand curve, and instead require only knowledge of returns, a metric about which firms are more likely to have some information.

3 Static FCP

The survey evidence reviewed in the introduction suggests firms routinely include fixed costs into their pricing decisions. This method of full-cost pricing directly conflicts with the benchmark model above, where the optimal price relies only on variable cost and demand. There

[13] For a random variable Y, the function $1/Y$ is convex, so $E[1/Y] > 1/E[Y]$ through Jensen's inequality. If X and Y are independent, then $E[X/Y] = E(X \cdot \frac{1}{Y}) = E(X)E(\frac{1}{Y}) > E(X)/E(Y)$.

[14] Nor are real demand curves not likely to even conform to specific functional forms, or "types", in the first place.

are informational requirements (discussed below) for FCP to achieve optimal pricing, but FCP does not require knowledge of the demand curve. It only requires knowledge (or an estimate) of equilibrium returns, and can still lead to the optimal price (albeit in a different way than the textbook model does).

First, we show a limited case for illustration: if the firm knows (or can estimate) its equilibrium income, then FCP immediately achieves (or estimates) optimal pricing if and only if the firm knows its equilibrium quantity. This necessary and sufficient condition entirely characterizes when static full price costing can immediately - without experimentation - achieve (or estimate) optimal pricing. In the next sections, we will discuss dynamic FCP, which requires firms to only know returns, not quantities.

To fix ideas, recall that the firm faces demand $q(p)$ and optimizes income $y(p)$ over price, delivering an optimal price p^*, an optimal quantity $q^* = q(p^*)$, and optimal (equilibrium) income $y^* = y(p^*)$. The optimal price, quantity, and income all follow from the demand curve, which the firm takes as given. The firm does not know the demand curve, but instead has estimates of income and quantity with unbiased estimators $\hat{y}$ and $\hat{q}$ respectively (so $E\hat{y} = y^*$ and $E\hat{q} = q^*$). The signal $\hat{q}$ is a forecast of quantity, which the firm uses to plan production and sales, while the signal $\hat{y}$ is the firm's estimate of its equilibrium income, based on historical information or other priors. Observe that, by construction, $y(p^*) = y^*$. Solving for p^* at this y^* delivers:

$$p^* = v + \frac{F + y^*}{q^*}.$$

This is simply the price that maximizes income. It is also a rewriting of the identity that price equals variable cost plus contribution margin per unit, since contribution margin per unit is precisely price minus variable cost. Thinking of the pricing function in this formula - as the sum of variable cost and contribution margin per unit - will be a constant theme throughout the paper. Since the income function is strictly concave in price, this price p^* is unique. Of course, the firm cannot simply pick this p^* since it does not know $q^* = q(p^*)$. But it does have an estimate of q^*, as well as an estimate of y^*. Therefore, consider the candidate estimator of optimal price

$$\hat{p} = v + \frac{F + \hat{y}}{\hat{q}}, \tag{6}$$

where the estimators $\hat{q}$ and $\hat{y}$ replace their equilibrium counterparts.

Observe that $\hat{p}$ is a markup on variable cost, but unlike the markup from the benchmark model, which is based on the elasticity of demand, this markup contains the fixed cost plus equilibrium income and is thus FCP. More specifically, the price is a function of the unit cost under absorption costing, $v + \frac{F}{\hat{q}}$. Absorption costing includes fixed manufacturing overhead per unit in the unit cost, and is both required by U.S. Generally Accepted Accounting Principles (GAAP, a standard set by the Financial Accounting Standards Board) and also taught extensively in managerial accounting textbooks. This full-cost price $\hat{p}$ immediately achieves the optimal price p^* under the following condition (all proofs in Appendix):

Proposition 1. *The static full cost price immediately achieves (or unbiasedly estimates) the optimal price ($E\hat{p} = p^*$) if and only if the firm knows equilibrium quantity exactly ($\hat{q} = q^*$), and has an unbiased estimator of equilibrium income ($E\hat{y} = y^*$).*

This proposition shows that the firm's knowledge of its equilibrium quantity completely characterizes whether FCP can immediately achieve (or unbiasedly estimate) optimal pricing. Of course, if the firm knows its equilibrium quantity q^*, it's clear from the equations for $\hat{p}$ and p^* above that the two will coincide on average, since $\hat{y}$ is unbiased. More surprising is that *anytime* FCP achieves optimal pricing ($E(\hat{p}) = p^*$), it must be the case that the firm knows its equilibrium quantity, $\hat{q} = q^*$. The proof relies on a simple application of Jensen's inequality.

3.1 Discussion

The analysis so far rests on two assumptions: (1) the firm can estimate its equilibrium profits and (2) the firm knows its equilibrium quantity. We discuss each of these assumptions in turn.

The first assumption is that the firm knows or can estimate its equilibrium income. An equivalent and slightly more palatable version of this assumption is that the firm knows the equilibrium return on investment. Return on investment $r = y/A$, where y is income and A is a measure of the assets of the firm. Then $y^* = r^*A$. It is conceivable that the firm knows the equilibrium r^* for similarly situated firms within an industry. For example, a railroad company can consider past data on the railroad industry to learn that railroad firms earn on average a 7% return (on income, assets, or net assets). This could be obtained through historical data on the market performance of past firms in this industry, or some *a priori* knowledge of the equilibrium rate of return.

The second assumption, that the firm knows its equilibrium quantity, is much stronger. For FCP to *immediately* achieve optimal pricing, it is not enough that the firm has an unbiased estimator $\hat{q}$. Instead, this estimator must be exact at every realization $\hat{q}$, so $\hat{q} = q^*$ everywhere. Indeed, knowing the equilibrium quantity at any informational level, even an estimate, is almost as stringent an informational requirement as knowing the full demand curve, which would allow the firm to simply do textbook optimization. Thus, at an information level, the requirements for *immediate* FCP in the static model are no weaker than the requirements for optimal pricing in the benchmark model.

Accounting textbooks have noted this stringent requirement for static optimality. For instance Garrison et al. [2010], discussing the very same candidate optimal price, $\hat{p}$, in equation (6), recognize the informational demands: "some managers believe the absorption costing approach to pricing is safe. This is an illusion. The absorption costing approach is safe only if customers choose to buy as many units as managers forecasted they would buy." (page 769). The authors are careful to note that the optimality of the full-cost price $\hat{p}$ rests on accurate estimates of equilibrium income ($\hat{y}$) and equilibrium quantity ($\hat{q}$). They go further and argue that the full-cost price $\hat{p}$ looks deceptively simple, but will misestimate the optimal price p^* if either of the estimates of $\hat{q}$ or $\hat{y}$ are incorrect. Proposition 1 refines this claim, showing it is not enough for the firm to only have an estimate of its equilibrium quantity, but it must know its equilibrium quantity precisely. Thus, the informational requirements for FCP are even *more* stringent than these textbook authors characterize.

This is precisely why we discuss FCP with "experimentation" in the next section. This dynamic process allows FCP to converge to p^* with only an estimate ($\hat{y}$) of y^* and no information on q^*. If the firm seeks to achieve the optimal price in a single stage, either by the canonical economics model or by FCP, the information requirements are quite strong. But if the firm is willing to experiment with prices over time, then FCP converges to the optimal price even when the firm does not know its equilibrium quantity. Furthermore, FCP provides information and structure for directing a firm's experimentation. We now turn to this dynamic case.

4 Dynamic FCP

While static FCP requires strong informational assumptions to achieve optimal pricing, this is not the case for dynamic pricing. Dynamic pricing refers to a process of experimentation in which firms may engage to search for the optimal price. Indeed, firms often change prices over time, and while some initial changes may occur in focus groups and labs with sample populations, many of the changes occur in real markets over time. Here, we show that if the firm sets full-cost prices over time, then these prices will converge towards the optimal price. This is a defense of the practice of FCP, namely, that it allows firms to eventually reach their optimal price. And the firm can do this without any knowledge of its equilibrium quantity q^*, using only knowledge of y^*.

To gain traction on the dynamic pricing problem, we first suppose that the firm knows its equilibrium income. This will lay bare the intuition behind the convergence algorithm. After that, we consider the more general case, when the firm does not know its equilibrium income precisely, but has an estimate of it. We view this later case as a realistic and plausible scenario.

4.1 Convergence When Income Is Known

Suppose the firm knows its equilibrium income y^*. Recall that at the optimal price $y(p^*) = y^*$, or $(p^* - v^*)q^* - F = y^*$. This is simply the identity for equilibrium income. In other words, equilibrium income is simply income evaluated at the optimal price p^*. Solving this equation for price shows that

$$p^* = v + \frac{F + y^*}{q^*} \tag{7}$$

Consider a candidate convergence algorithm that simply replaces q^* with $q(p_t)$ and p^* with p_{t+1}. It is natural to suppose this price might converge to the optimal price, since the optimal price is the unique price that satisfies the identity in equation (7) above. Below we define this algorithm and explore its convergence properties.

The convergence algorithm operates as follows. First, let $p_0 = v$ be the starting price.[15] Then, set

$$p_{t+1} = v + \frac{F + y^*}{q(p_t)} \quad \text{for each } t \geq 0. \tag{8}$$

This defines an algorithm that delivers a price path. The initial value is the firm's marginal cost (variable cost), which the firm knows. The more competition, the closer equilibrium price is to v. Also, since demand slopes down, the price path is an increasing sequence. Starting below optimal price ($p_0 = v < p^*$) assumes convergence from below.[16] We can rewrite the equation above as

$$\text{Price} = \text{Marginal Cost} + \text{Equilibrium Contribution Margin per Unit} \tag{9}$$

where the "per unit" is evaluated at $q(p_t)$, the quantity demand based on the price in the prior stage.

Observe that each price in the price path in (8) is a full-cost price. The "new" price p_{t+1} is a function of the "old" price, p_t. In particular, it is a markup on variable cost, but the markup itself, unlike the classical model, relies not on the elasticity of demand, but instead on the fixed cost and equilibrium income. Furthermore, $v + F/q(p_t)$ is the unit cost under absorption costing. The pricing algorithm sets the new price equal to the unit cost plus unit profit, where the number of units is determined by demand at the old price, p_t. The market then demands $q(p_t)$ and the firm sets its new price p_{t+1}, based on the formula for p_{t+1} above.

[15] We discuss other starting values in the proof, and the footnote below.

[16] In the proof of the next proposition, we show it is straightforward to "correct from above," as well. Iterative prices will accelerate away (upwards) from p^* if firms (outside of equilibrium strategy) iterate above p^*. This indicates to firms that they have experimented with prices that are too high, and they can correct downwards.

The firm then iterates in this manner in every subsequent stage. The next proposition shows that this converges to the optimal price.

Proposition 2. *If firms know y^*, the full cost price p_t converges to the optimal price p^*.*

The pricing algorithm thus has the attractive property that it converges to p^*. It is also based on real accounting practice: GAAP accounting standards require absorption costing. The proof of the proposition gives conditions under which the full-cost price p_t converges to the optimal price p^*. The price path is a nonlinear first order difference equation. It is nonlinear because of the nonlinear demand curve $q(p_t)$, and it is first order because it connects p_{t+1} to p_t.

4.2 Example with CES Demand

To see an example of convergence, consider constant elasticity demand, given by $q(p) = ap^{-b}$. Observe that the elasticity of demand $\epsilon = pq'(p)/q(p) = -b$. Using this demand curve,

$$p^* = \frac{v}{1 - 1/b} \quad \text{and } q^* = a(p^*)^{-b}. \tag{10}$$

Observe that price is a constant markup over marginal (variable) cost, with the amount of the markup depending on the elasticity of demand. As demand becomes more inelastic, the firm optimally charges a higher price, extracting more rent from the consumer. The convergence algorithm starts with an initial price p_0, and for each of the following stages $t \geq 1$, the price path is given by

$$p_{t+1} = v + \frac{F + y^*}{q(p_t)} = v + \left(\frac{F + y^*}{a}\right) p_t^b \tag{11}$$

In steady-state $p_{t+1} = p_t = p$. Inserting into (11) and rearranging,

$$(y^* + F)p^b - ap + av = 0. \tag{12}$$

The steady-state equilibrium solves this equation. Note this is a polynomial of order b, with coefficients consisting of $y^* + F$, a, v. Observe $y^* = y(p^*) = (p^* - v)q^* - F$. Combining this with (10) gives

$$y^* + F = (p^* - v)q^* = \frac{a}{4v}. \tag{13}$$

This allows us to write the equilibrium profit plus the fixed cost (and the solution to the polynomial in (12)) in terms of the exogenous parameters of the model, a and v. In general, high-order polynomials have difficult solutions. For tractability, consider a quadratic, so $b = 2$. Merging (12) and (13) and simplifying,

$$p^2 - 4vp + 4v^2 = 0.$$

Apply the quadratic formula, so

$$p = \frac{4v \pm \sqrt{16v^2 - 4 \cdot 4v^2}}{2v} = 2v = p^*.$$

Indeed, the determinant of this polynomial conveniently equals 0. While polynomials of order two have two solutions, the zero determinant shows that the polynomial in (13) has only a single solution. That solution is exactly the optimal price p^*. Using (11) and (13), observe that

$$p_{t+1} = f(p_t) = v + p_t^2/4v.$$

Figure 1 plots convergence, both with a simulated price path (left figure) and the phase diagram (right figure). We simulated convergence under CES demand and plot the evolution of price as a function of t in the lefthand figure. The convergence is quite fast, as the full-cost price achieves 95% of the optimal price in five iterations. In the phase diagram on the right, the price path graphs the new price p_{t+1} as a function of the old price p_t. To see convergence, apply the usual phase diagram logic: start at $p_0 = v$ and evaluate the function at p_0 to generate $p_1 = f(p_0)$. Then draw a line horizontally to the 45° line to determine where p_1 sits on the x-axis. Then, locate $p_2 = f(p_1)$ on the graph to find the new price. Draw another line horizontal to the 45° line to find where p_2 sits on the x-axis. Then, evaluate the function to discover $p_3 = f(p_2)$. Continuing in this fashion shows convergence to the equilibrium point $p^* = 2v$. As the figure shows, this is exactly the tangency point where the hyperbola given by $f(p_t)$ is tangent to the 45° line. This is no accident. For any choice of parameters a, b, and v, the price path will always be tangent at the unique point p^*. This follows because there is a single solution to the polynomial given in (12), which determines the steady-state equilibrium. This coincides exactly with the optimal price p^*.

4.3 Convergence When Income Is Unknown (Consistency)

Suppose the firm does not know its equilibrium income precisely, but must estimate it based on historical market data and internal cost information. Assume the firm has an estimate $\hat{y}$ of its equilibrium quantity y^*. We consider the case where this estimator is consistent for y^*.

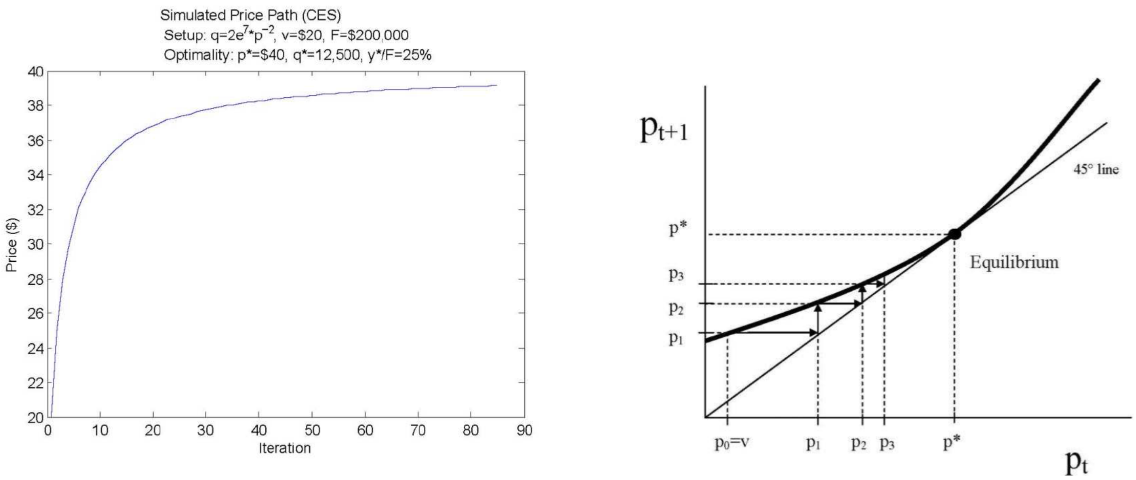


Figure 1: Convergence under CES demand. The price path is $p_{t+1} = v + p_t^2/4v$. The left figure plots the price path and right figure plots the phase diagram.

We will treat the case of unbiasedness next.[17]

The convergence algorithm will be similar, though not identical, to before. Rather than using equilibrium income y^*, the firm will use its estimate $\hat{y}$ in its place. The convergence algorithm works as follows. First, the firm picks price $p_0 = v$. Then, for each $t \geq 0$, the firm selects the new price as

$$p_{t+1} = v + \frac{F + \hat{y}}{q(p_t)}$$

Just as before, this is FCP because it includes the fixed cost in the pricing algorithm. Also, as before, the price is simply the unit price under absorption costing plus the estimated profit per unit. The firm need not know its equilibrium quantity q^*, since it endogenously generates demand through the prices p_t it offers in the market place. This price path now has additional uncertainty because it relies on an estimate of income, rather than its true value. Nonetheless, provided that this estimate is consistent, we still arrive at convergence.

[17] $\hat{y}$ being consistent for y^* means $\operatorname{plim} \hat{y} = y$, where the probability limit is taken over many IID draws of the estimator $\hat{y}$. This is opposed to the natural case where $\hat{y}$ would be unbiased for y^* ($E(\hat{y}) = y^*$). It may be more intuitive to think of each firm as having a single, unbiased estimate of y^*, rather than every firm having an estimate that incorporates enough IID estimates as to converge to y^*. However, as we'll argue, there is much information accessible to firms regarding y^*. Furthermore, if there were a wedge between $\hat{y}$ and y^* (whether $\hat{y}$ were biased or consistent), the firms would detect this experimentation and be able to correct. The consistent case is more tractable than the unbiased case, though both will converge under even more general assumptions with extra (but uninstructive) iterative machinery.

Proposition 3. *If $\hat{y}$ is a consistent estimator of y^*, then the full-cost price p_t converges to the optimal price p^*.*

The proof of Proposition 3 mirrors the one from the earlier section, with the only difference in the nature of convergence. Instead of relying on the true value y^*, the proof uses convergence in probability, since the estimator $\hat{y}$ is a consistent estimator of y^*. The logic is similar. The firm forms the sample price path as before, now using the estimator $\hat{y}$ in place of equilibrium income y^*. The steady-state equilibrium of the dynamic model satisfies $y(p) = \hat{y}$, which converges in probability to the optimal income y^*. And because the income function is strictly concave and has a unique solution, the steady-state converges to the optimal price as the number of draws of the estimator tends towards infinity. Note there are two relevant dimensions here. The first is time, or pricing iterations, which indexes the price path as the firm experiments with different prices. The second is the number of draws of the estimator $\hat{y}$. The probability limit refers to the behavior of the estimator as the number of draws becomes large. Proposition 2 shows that the price path p_t converges to the optimal price p^* as the number of draws tends towards infinity. This is a theoretical property describing the accuracy of the price path p_t based on the accuracy (consistency) of the estimator $\hat{y}$.

Thus, the firm need not know its equilibrium income exactly, but can estimate it based on market data or other priors. As discussed earlier, this estimate of equilibrium income is equivalent to an estimate of equilibrium return on investment. Having an informed estimate of your expected income is more plausible than knowing it before it's realization.

4.4 Non-Convergence when Income is Unknown (Unbiasedness)

The criteria chosen for the estimator above, consistency, was deliberately strong. A weaker criteria does not lead to convergence. If $\hat{y}$ is unbiased so ($E(\hat{y}) = y^*$), then the full-cost price will not converge to the optimal price.

Proposition 4. *If $\hat{y}$ is an unbiased estimator of y^*, the full-cost price p_t does not converge to the optimal price.*

Lack of bias is too weak of a criteria because expectation is a linear operator, whereas the profit function is strictly concave. Because of this, the expectations operator cannot pass inside of the profit function, and this disrupts the convergence proof. The strict concavity of the profit function combined with Jensen's inequality delivers a contradiction, which prevents convergence. This was not a problem when the criteria for the estimator was consistency because probability limits can pass seamlessly between non-linear functions. Below, we provide an explicit calculation under linear demand, which shows convergence when the estimator is consistent, but not if it is unbiased.

4.5 Example with Linear Demand

To gain traction on convergence, consider linear demand $q(p) = a - bp$ for $a, b > 0$. The optimal price and quantity pair are

$$p^* = \frac{a + bv}{2b} \quad \text{and } q^* = q(p^*) = \frac{a - bv}{2}.$$

Start with some initial estimate of price p_0. Form the candidate price p_{t+1} for each $t \geq 0$ as

$$p_{t+1} = v + \frac{F + \hat{y}}{a - bp_t}.$$

In the steady state, $p_{t+1} = p_t = p$. Replacing p_{t+1} and p_t with the steady state value p in the equation above and rearranging gives the quadratic equation

$$bp^2 - p(a + bv) + av + (F + \hat{y}) = 0$$

By the quadratic formula, the solution to this polynomial is

$$p = \frac{a + bv \pm \sqrt{D}}{2b} \tag{14}$$

where the determinant

$$D = (a + bv)^2 - 4b(av + F + \hat{y}). \tag{15}$$

Recall $y(p^*) = y^*$, so $y^* + F = (p^* - v)q^*$. Plugging in the optimal prices p^* and q^* from above,

$$y^* + F = \frac{(a - bv)^2}{bv} \tag{16}$$

Since $\hat{y}$ is a consistent estimator of y^*, $\operatorname{plim} \hat{y} = y^*$, where the probability limit is taken over many draws of $\hat{y}$. To ease exposition, we suppress notation that indexes the actual draws.[18] Then

$$\operatorname{plim} D = (a + bv)^2 - 4b(av + F + y^*) = 0$$

by inserting (16) and simplifying. Combining with (14),

[18] An estimator $\hat{y_n}$ is consistent if $\operatorname{plim}_{n\to\infty} \hat{y_n} = y^*$. The subscript n is implicit, and hence, we suppress it. Note that the limit is not taken with respect to t, the stages of convergence, but rather with respect to n, the draws of the estimator.

$$\operatorname{plim} p = \operatorname{plim} \frac{a + bv \pm \sqrt{D}}{2b} = \frac{a + bv}{2b} = p^*$$

by the laws of the probability limit operator. Thus, the estimated price p_t converges to a quantity that is a consistent estimator of the optimal price p^*. Thus, even if the firm does not know equilibrium income y^* exactly, as long as its estimator is consistent, FCP will still converge to the optimal price in probability. Indeed, the flexibility of the probability limit is what guarantees that if $\hat{y}$ converges to y^*, p_t converges to p^*.

Consistency is a stronger criteria than lack of bias. Indeed, lack of bias is too weak of a criteria for the signal to guarantee convergence. To see this, suppose $\hat{y}$ is unbiased, so $E\hat{y} = y^*$. The firm will still run its convergence algorithm as before, and the steady-state solution is given by (14). The steady-state p is the solution to a quadratic polynomial, which has two roots. By the quadratic formula, this p contains the determinant D given in (15). The expectation of this determinant is

$$ED = (a + bv)^2 - 4b(av + F + y^*) = 0$$

by inserting (16) and simplifying, just as before. Indeed, the determinant is a linear function of the estimator $\hat{y}$, and because the expectation operates linearly (as do probability limits), the expected determinant is zero.

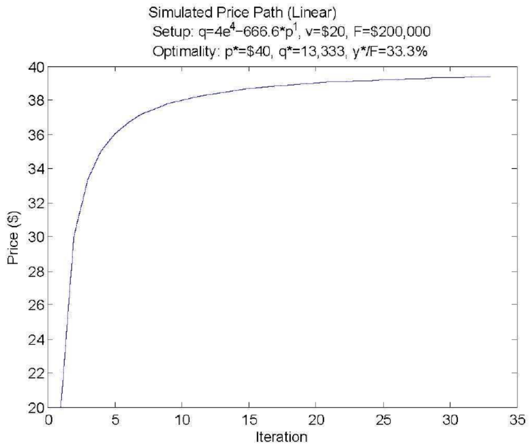
Let p_1^* and p_2^* be the two roots of the steady-state equation, given in (14). Since the function $f(x) = \sqrt{x}$ is concave, by Jensen's inequality $E[\sqrt{D}] \leq \sqrt{E[D]} = 0$. Therefore, writing out the root of the polynomial given in (14) explicitly and taking expectations,

$$Ep_1^* = E\left(\frac{a + bv + \sqrt{D}}{2b}\right) \leq \frac{a + bv}{2b} = p^*$$

and

$$Ep_2^* = E\left(\frac{a + bv - \sqrt{D}}{2b}\right) \geq \frac{a + bv}{2b} = p^*.$$

Jensen's inequality shows each of the steady-states will be bounded away from the optimal price p^*. The two roots are equidistant from p^*. This occurs precisely because the steady-state equation (14) is a non-linear function of the estimator $\hat{y}$ (because of the square root of the determinant). This nonlinearity prevents the expectations operator from passing inside the square root function, which leads to our application of Jensen's inequality. This is not the case with probability limits, since a plim can pass easily between nonlinear functions, whereas expectation is a linear operator. In words, this means the full-cost price p_t will converge to either p_1^* or p_2^*, but both will be bounded away from p^*. Their distance from p^* is precisely



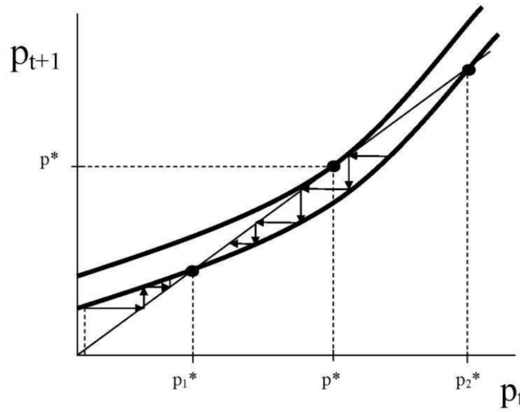


Figure 2: Convergence under linear demand. The left figure plots the simulated price path and right figure plots the phase diagram. The full-cost price p_t converges to p^* in probability, but to p_1^* in expectation.

$\sqrt{D/2b}$. Expanding this expression generates a comparative static with respect to D, the slope of the demand curve. We state this comparative static as a corollary.

Corollary 1. *As the demand curve becomes steeper, the steady state price p_1^* is closer to the optimal price p^*.*

Figure 2 shows a simulated price path and the phase diagram under linear demand. We simulated convergence under linear demand and plot the evolution of price as a function of t in the lefthand figure. The convergence is quite fast, as the full-cost price achieves 90% of the optimal price in five iterations. The upper curve in the righthand figure is the phase diagram that the full cost price p_t converges to in probability, while the lower curve is the phase diagram that the full cost price converges to in expectation. Observe that the upper curve has a single tangency at p^*, and the price path converges to this unique p^* when starting at an initial price $p_0 = v$. This is identical logic to the earlier case when equilibrium income was known exactly. The lower curve, however, lies strictly below, and is the phase diagram to which the full-cost converges in expectation after many draws of the estimator $\hat{y}$. This phase diagram has two roots, p_1^* and p_2^*, which intersect the 45° line. Furthermore, it is clear from the shape of the phase diagram that the price path p_t converges to $p_1^* < p^*$. Thus, averaging over all possible signals $\hat{y}$, the price path will not converge to p^*, but will instead converge to a price below the optimal price. As the demand curve becomes more steep, the lower curve moves upward towards the upper curve, and therefore, p_1^* approaches p^*. The slope of the demand curve determines the distance between the two curves. Thus, as demand becomes more elastic, not only does the optimal price rise (since price is a markup over variable cost depending on the elasticity of demand), but the two steady-states p_1^* and p^* also move closer together.

In conclusion, dynamic FCP has informational benefits relative to economic pricing, and converges to optimal pricing, but does require a strong criteria for the estimator of income (consistency rather than lack of bias).

5 Extensions

5.1 The Multiproduct Firm

The dynamic full-cost pricing algorithm in the prior section generalizes and applies to a multiproduct firm. This shows full-cost pricing is a robust pricing method.[19] The results hold even when the demand between different products is dependent, i.e. that there are complementarities between products. Indeed, any complementarity inside the firm does not affect the convergence property of full-cost pricing. Moreover, the firm must include a cost allocation method in order to share its common fixed cost across multiple products. As long as the cost allocation rule allocates all the fixed costs, the firm can implement optimal pricing through full-cost pricing.

Suppose the firm produces n products. It charges a price p_i for each product $i = 1 \dots n$. Let $p = (p_i)$ be the price vector, and let $q_i(p)$ be the demand for product i. Observe that each product's demand is a function of the entire price vector, not only of its own price. This allows for complementarity across products.[20] Each product has a variable cost v_i and the firm has a common fixed cost, F. The income function is

$$y(p) = \sum_{i=1}^{n} (p_i - v_i) q_i(p) - F$$

The firm maximizes income over the price vector.[21] The first order condition from this problem delivers the optimal price vector, p^*, which satisfies for each i

$$\sum_{j=1}^{n} (p_j^* - v_j) \frac{\partial q_j(p^*)}{\partial p_i} + q_i(p^*) = 0 \tag{17}$$

[19] We show the case when the firm knows its equilibrium income y^*. The results when the firm estimates y^* with $\hat{y}$ are analogous and we omit them for brevity.

[20] Products i and j are substitutes if the cross-partial $\frac{\partial q_i(p)}{\partial p_j} > 0$, and they are complements if this cross-partial is negative.

[21] We assume prices are set centrally rather than allowing the individual divisions to set prices. This is a reasonable assumption if there are significant complementarities between products, since divisional income may not capture the relevant externalities from other divisions.

Assume the second order conditions hold, so the Hessian matrix of the income function $y(p)$ is negative semi-definite.[22] This guarantees solution p^* is unique.

Because multiple divisions in the firms share a common fixed cost, define a cost allocation rule that distributes this fixed cost throughout the firm. Let the vector $s = (s_i)$ represent such an allocation, where each $0 < s_i < 1$ is product i's share of fixed cost F. Assume the allocation is "tidy" in that $\sum s_i = 1$. A tidy allocation rule allocates all of the fixed cost to the multiple products.[23] There are many ways to allocate costs. One common method (Garrison et al. [2010]) is to pick a cost allocation basis b_i (such as direct labor) that drives overhead costs, and to allocate the fixed cost based on the proportional consumption of the allocation basis, so $s_i = b_i / \sum b_j$. The central office of the firm can use the cost allocation rule to define divisional income and to write contracts for divisional managers based on divisional income.[24]

A full-cost price, as before, includes the fixed cost. A dynamic full-cost price is a price sequence p_t, where each p_t is a full-cost price vector in stage t. Let $p_{i,t}$ denote the i^{th} component of the vector p_t. As before, select the initial starting value as $p_{i,0} = v_i$ for each product. For a given cost-allocation rule $s = (s_i)$, the full-cost pricing algorithm sets

$$p_{i,t+1} = v_i + \frac{s_i(y^* + F)}{q(p_t)}$$

This full-cost price has several features. First, it is a mark-up on variable cost based on *allocated* fixed cost, where the allocation share is given by s_i. Second, because demand is a function of the entire price vector p_t, the new price for product i ($p_{i,t+1}$) is a function of the entire old price vector p_t. Because the price path consists of a sequence of vectors, rather than a simple sequence of numbers, the convergence is more intricate than before, but full-cost pricing still does converge.

Proposition 5. *The full-cost price vector p_t converges to the optimal price p^*.*

While the nature of the convergence is slightly more complex because of the multi-dimensional

[22] Recall that the Hessian of the income function $y(p)$ is the second derivative matrix consisting of the cross partials $\frac{\partial^2 y}{\partial p_i \partial p_j}$. The second order condition is that the matrix is semi-definite, so the n principle minors are positive and they alternate in sign.

[23] A "tidy" cost allocation has a long history in both the accounting and economic literatures. It is the same requirement that the transfers in a mechanism design problem be budget balanced. Most cost allocation rules used in practice and taught in textbooks are tidy, such as the proportional usage of some cost-driver. See Ray and Goldmanis [2012] for an extensive examination of the incentive effects of tidy cost allocation rules, and in particular under what conditions they can achieve efficiency.

[24] To focus the analysis on pricing, we do not consider the incentive or agency effects of different cost allocation rules. See Zimmerman [1979] and Ray and Goldmanis [2012] for extensive treatment of the incentive effects of cost allocation rules. Of course, different rules will create different incentives for divisional managers, which will affect production. Including an elaborate agency model would only distract from the pricing problem here.

price vector, the logic is similar to before. The algorithm starts by setting every initial price equal to each product's variable cost, and then updates the price by marking up variable cost by each division's share of target income plus fixed-cost per unit. The markup is simply the contribution margin per unit, since each product's contribution margin is its share of target income plus fixed costs. Each iteration of the algorithm defines a price path for each product, which then enters the demand curve to generate the new price for each product. The steady-state vector p sets the total contribution margin equal to the allocated profit plus allocated cost: $(p_i - v_i)q(p) = s_i(y^* + F)$. Since the shares sum to 1, aggregating these contribution margins across all products shows that the income at the steady-state equals income at the optimal price. And because the optimal price is unique, the steady-state value p must coincide exactly with the optimal price p^*. Showing that the price path p_t actually converges to p^* involves the same argument as before on the shape of the phase diagram. Please see the proof for further details.

Note that the demand curve allows for dependence between products, and this does not affect the convergence. Indeed, this occurs precisely because the full-cost price does not emerge from any first order condition. The FOC in Proposition 5 can be quite complex since there may be many cross-partial terms $\frac{\partial q_j}{\partial p_i}$. But this is irrelevant for the full-cost pricing algorithm, since each iteration of the algorithm simply evaluates demand at the price vector without needing to calculate any marginal effects. This shows yet another benefit of full-cost pricing, namely, that it need not require any knowledge of the shape of the demand curve or its rate of change, but simply requires the firm to set a price and observe the quantity demanded at that price. The proposition extends to the case when the firm doesn't know y^* exactly but estimates it with a consistent estimator $\hat{y}$. The logic and framework is similar. And finally, this result is robust to a wide variety of cost allocation rules, so long as they allocate F completely.

Our results stand in contrast to Noreen and Burgstahler [1997], who find that full-cost pricing in a multiproduct firm prevents the firm from reaching "satisfactory" profits. However, they do not look at optimal profits, but rather, at profits that clear a minimal threshold. They also look at a more narrow class of full-cost pricing schemes, and in particular do not base the dynamic full-cost price on an estimate of equilibrium income as we do. This is why their full-cost pricing line never intersects their satisficing region. In the next section, we will discuss satisficing as an alternative criteria to optimality.

5.2 Satisficing

The focus of this paper is on optimality, where optimal is defined as the solution to the classical economic model. An alternative criteria is satisficing, first introduced by economist Herbert Simon. Satisficing is the practice of choosing the least costly or most readily available alternative that satisfies a minimal payoff. Here, firms may not know or even be able to estimate equilibrium income y^*, but simply seek some minimum income threshold $\bar{y}$. Surveys of company practice (Shim and Sudit [1995]; Govindarajan and Anthony [1983]) find that managers routinely refer to some "desired" profit level. We can interpret this either as equilibrium profit, or more loosely as a minimum profit.

To fix ideas, suppose the firm has in mind a minimum income level $\bar{y}$. A satisfactory price is any price, which generates income that clears this threshold:

Definition 1. A price p is satisfactory at $\bar{y}$ if $y(p) > \bar{y}$.

Since the income function y is concave, the set of satisfactory prices will be a closed interval. Rather than aiming to pick the exact optimal price, the firm now seeks to pick a satisfactory price, which can be any price within that set.[25] Form the static full-cost price as:

$$\hat{p} = v + \frac{F + \bar{y}}{\hat{q}}$$

This is a variation on the static full-cost price where $\hat{q}$ is an estimate of quantity and the firm sets its income forecast to $\hat{y} = \bar{y}$. We make no assumption that the firm knows or can estimate y^*, but instead assume it only knows $\bar{y}$. If so, then:

Proposition 6. *The full-cost price $\hat{p}$ is satisfactory at $\bar{y}$ if and only if $q(\hat{p}) > \hat{q}$.*

The proposition gives a condition on the forecasted quantity $\hat{q}$ to achieve satisficing: the forecast must be biased downward. The firm first estimates $\hat{q}$ and then selects $\hat{p}$ based on this $\hat{q}$. When the firm offers $\hat{p}$ to the market, the market demand is $q(\hat{p})$, which must exceed $\hat{q}$ in order to achieve satisficing. The firm effectively sets a low forecast, which inflates the fixed-cost per unit in the full-cost price. This raises the full-cost price, and because demand slopes down, lowers demand $q(\hat{p})$. As long as the generated demand based on the full-cost price $q(\hat{p})$ exceeds the initial forecast, the full-cost price is indeed satisfactory.

5.3 Satisficing: Example With Linear Demand

Suppose that $q(p) = a - bp$ for demand coefficients a and b. Then

$$q(\hat{p}) = a - b(v + \frac{F + \bar{y}}{\hat{q}})$$

This is a function of $\hat{q}$. We seek to find the set of forecasted quantities $\hat{q}$ such that $q(\hat{p}) > \hat{q}$. Using the expression from $q(\hat{p})$ above and comparing it to $\hat{q}$, we see that $q(\hat{p}) > \hat{q}$ iff

[25] Of course, the optimal price is satisfactory, though every satisfactory price is not optimal. Hence, satisficing is a much weaker criteria than optimality.

$$\hat{q}^2 + \hat{q}(bv - a) + b(F + \bar{y}) < 0$$

The set of all $\hat{q}$, which satisfy this inequality, is given by $[q_1, q_2]$, where q_1 and q_2 are the two roots of the quadratic polynomial above. By the quadratic formula:

$$q_i = \left((a - bv) \pm \sqrt{(a - bv)^2 - 4b(F + \bar{y})} \right) /2 \tag{18}$$

This interval relies on the demand coefficients a and b, which the firm may not know. Therefore, it my replace these coefficients with consistent estimates $\hat{a}$ and $\hat{b}$ to generate an estimate of the interval $[q_1, q_2]$. To see a concrete example, suppose $a = 5$, $b = v = F = 1$. Then the optimal price is $p^* = \frac{a+b}{2} = 3$, and the optimal quantity is $q^* = \frac{a-v}{2b} = 2$. The equilibrium income is $y^* = (p^* - v)q^* - F = 3$. If the firm doesn't know equilibrium income, but only seeks income above $\bar{y} = 2$, inserting this into the expression for q_i above gives $q_1 = 1$ and $q_2 = 3$. Thus, provided for any estimate $\hat{q}$ in [1,3], the full-cost price $\hat{p} = 1 + 3/\hat{q}$ will generate income above $\bar{y} = 2$.

Satisficing is a natural relaxation of optimality, both in terms of its benefits as well as its requirements. Rather than requiring knowledge of equilibrium income, it only requires knowledge of a minimal threshold $\bar{y}$, and this allows for a range of possible prices that are satisfactory. And rather than requiring an accurate point estimate of equilibrium quantity, Proposition 6 shows it only requires a conservative forecast of quantity. Under linear demand, this condition takes the form of an interval, so the forecasted quantity must lie within some range $[q_1, q_2]$.

It remains an open question as to whether firms actually estimate equilibrium income, or whether they simply satisfice according to some minimum income level. Indeed, when firms talk of "desired profit," they could be talking about either one. But the main message is that full-cost pricing behaves well in either case. The analysis gives precise conditions when the full-cost price can achieve this income level, whether that level is equilibrium income or some other lower target. The firm can therefore choose to satisfice or optimize, and the results here provide the conditions for the full-cost price to achieve whichever objective it chooses.

6 Conclusion

Economics and accounting offer different views of optimal pricing. In the canonical economic model, fixed costs need not and should not be involved in pricing decisions. In accounting texts, as well as in standard industry practice, fixed costs are incorporated into pricing decisions. We attempt to resolve this tension by noting that firms are unlikely to have the information about their demand curves that is required for optimal economic pricing. They are more likely to have information regarding their equilibrium income which may be used, along with full cost pricing techniques, to find their optimal price. We propose a full-cost pricing algorithm which converges to the optimal price. This may provide guidance for managers or insight for researchers regarding the correspondence between the two approaches.

In a static setting, the informational requirements for FCP and economic pricing to yield the optimal price are identical: the firm needs to know its demand curve. But in a dynamic setting, with iteration of full cost prices, FCP can converge to optimal pricing with only an estimate of equilibrium income. Experimentation towards the optimal price is directed by the ROA estimate, and will converge to the optimal price if the estimate of ROA is consistent (not just unbiased). Our results on satisficing indicate that less stringent informational requirements may also allow the firm to achieve some level of optimality.

Future research in this area may further explore theories of actual, rather than "as if," firm behavior, drawing on observations of industry practice and using both accounting and economic quantities.

7 Appendix

Proof of Proposition 1. The firm knows its equilibrium quantity if the estimator $\hat{q} = q^*$ for all possible realizations of $\hat{q}$. The proposition states that $\hat{q} = q^*$ is a necessary and sufficient condition for $E(\hat{p}) = p^*$.

Sufficiency: Suppose $\hat{q} = q^*$. Then the static full-cost price is $\hat{p} = v + \frac{F+\hat{y}}{q^*}$. Taking expectations over both sides,

$$E(\hat{p}) = v + \frac{F + E\hat{y}}{q^*} = v + \frac{F + y^*}{q^*} = p^*.$$

Necessity: We prove the contrapositive. Suppose $\hat{q} \neq q^*$ for some realization of the random variable $\hat{q}$. So $\hat{q}$ has a nontrivial distribution. Therefore,

$$E(\hat{p}) = v + E[\frac{F + \hat{y}}{\hat{q}}] > v + \frac{F + y^*}{E(\hat{q})} = p^*,$$

since $\hat{q}$ is unbiased ($E(\hat{q}) = q^*$). The inequality follows from Jensen's inequality since the function $f(x) = \frac{1}{x}$ is convex. Thus, $E(\hat{p}) \neq p^*$. ■

Proof of Proposition 2. Here, we prove the more general claim that p_t converges to p^* iff $p_0 < p^*$. This stronger statement implies Proposition 2, since $p_0 = v < p^*$ by definition of p^*. The convergence algorithm is given by

$$f(p_t) \equiv p_{t+1} = v + \frac{F + y^*}{q(p_t)} \quad \text{for} \quad \text{each } t \geq 0 \tag{19}$$

Observe that

$$f'(p_t) = \frac{-(F + y^*)q'(p_t)}{q(p_t)^2} > 0$$

For each p_t, since $q'(p_t) < 0$. Furthermore, observe that $f(0) > v > 0$. Thus, the function $f(p_t) = p_{t+1}$ is the phase diagram for the price path, which starts at a constant above v and increases. In a steady-state equilibrium, $f(p) = p$. Plugging this into (19) gives

$$f(p) = v + \frac{y^* + F}{q(p)} = p$$

Rearranging this equation,

$$(p - v)q(p) - F = y(p) = y^* = y(p^*)$$

Since the income function y is strictly concave, it has a unique maximizer. Therefore, $p = p^*$. So the optimal price p^* is the unique steady-state of the price path. Next, we need to show convergence.

Suppose $p_0 < p^*$. Let $g(x) = x$ be the 45° line. Observe that $g(p^*) = p^* = f(p^*)$. Because f is increasing, continuous, and because $f(0) > v > 0 = g(0)$, $f(p) > g(p)$ for each $p < p^*$. Therefore, for each $t \geq 0$,

$$p_{t+1} = f(p_t) > g(p_t) = p_t.$$

So the sequence p_t is increasing. Recall that $f(0) > v > 0 = g(0)$ and $f(p^*) = p^* = g(p^*)$. Since f and g are both continuous, for any p_t and p_{t-1},

$$f(p_t) - f(p_{t-1}) < g(p_t) - g(p_{t-1}).$$

Substituting in the definitions for f and g, this inequality reduces to

$$p_{t+1} - p_t < p_t - p_{t-1}.$$

Thus, the differences in the sequence shrinks in t. Hence, p_t converges to p^*.

Now suppose $p_0 > p^*$. Observe that

$$f''(p) = \frac{F + y^*}{q(p)^2} \left(\frac{2q'(p)^2}{q(p)} - q''(p) \right)$$

Thus,

$$f''(p) > 0 \ \text{ iff } 2q'(p)^2 > q''(p)q(p) \tag{20}$$

since $q'(p) < 0$. The first order condition for p^* is

$$p^* = v - \frac{q(p^*)}{q'(p^*)}. \tag{21}$$

Recall that $y(p) = (p - v)q(p) - F$. The second derivative is

$$y''(p) = (p - v)q''(p) + 2q'(p)$$

Plugging in (21), the second derivative evaluated at p^* is

$$y''(p^*) = -\frac{q(p^*)}{q'(p^*)} q''(p^*) + 2q'(p^*).$$

Income is concave, so $y''(p^*) < 0$. Rewriting, this means

$$2q'(p^*)^2 > q(p^*)q''(p^*).$$

Referencing (20), this means $f''(p^*) > 0$. Hence, f is convex at p^*, and p^* is the unique fixed point of f. Therefore, f does not cross g at any other point, and $f(p) > g(p)$ for all $p > p^*$. Therefore,

$$p_{t+1} = f(p_t) > g(p_t) = p_t.$$

Thus, p_t is an increasing sequence. Now, because f is convex, and $f(p^*) = g(p^*)$, $f'(p) > g'(p)$ for all $p > p^*$. So, for any p_t and p_{t-1},

$$p_{t+1} - p_t = f(p_t) - f(p_{t-1}) > g(p_t) - g(p_{t-1}) = p_t - p_{t-1}.$$

Thus, the differences in p_t increase in t. Therefore, this increasing sequence of increasing differences diverges. ■

Proof of Proposition 3. Consider the following convergence algorithm. Pick some initial value p_0. For all $t \geq 0$, let

$$p_{t+1} = v + \frac{F + \hat{y}}{q(p_t)}.$$

In a steady state solution, $p_{t+1} = p_t = p^*$ for all t and therefore the equation above becomes

$$p^* = v + \frac{F + \hat{y}}{q(p^*)}$$

Rearranging, $y(p^*) = \hat{y}$. Taking the limit as the number of draws of $\hat{y}$ goes to infinity,

$$y(\operatorname{plim} p) = \operatorname{plim} y(p) = \operatorname{plim}(\hat{y}) = y^* = y(p^*).$$

Because the income function is strictly concave, it has a unique solution. Therefore, $\operatorname{plim} p = p^*$. The rest of the argument for convergence is identical to the proof of Proposition 2. ■

Proof of Proposition 4. Take some initial p_0. For each $t \geq 0$, let $p_{t+1} = v + \frac{F+\hat{y}}{q(p_t)}$.

Suppose the price p_t does converge to some p. In the steady-state equilibrium, $p_{t+1} = p_t = p$. Plugging this into the equation above and rearranging we have $y(p) = \hat{y}$. Now take expectations over all realizations of $\hat{y}$,

$$y(E(p)) > E(y(p)) = E(\hat{y}) = y^* = y(p^*),$$

where the inequality follows from Jensen's inequality since the firm's income function is strictly concave. But p^* maximizes y, so $y(p^*) \geq y(z)$ for all z, and in particular for $z = Ep$. Contradiction.

■

Proof of Corollary 1. Let p_i^* be the root given in (14) for $i = 1, 2$. Observe that

$$|p^* - p_i^*| = \sqrt{D}/2b$$

Rewriting D and simplifying,

$$|p^* - p_i^*|^2 = (\frac{a}{2b} + \frac{v}{2})^2 - \frac{av + F + \hat{y}}{b},$$

which is decreasing in b. ■

Proof of Proposition 5. The full-cost pricing sets $p_{i,0} = v_i$ for each i. Let $p_t = (p_{i,t})$ be the price vector in each stage. Then,

$$f_i(p_t) = p_{i,t+1} = v_i + \frac{s_i(y^* + F)}{q_i(p_t)}$$

This generates a phase diagram $f_i(p_t)$ for each product i. In steady state, $p_{i,t} = p_i$ for each i and t. Therefore, plugging in the steady state into the equation above gives

$$p_i = v_i + \frac{s_i(y^* + F)}{q_i(p)} \quad \text{for each } i.$$

Rearranging this gives $(p_i - v_i)q_i(p) = s_i(y^* + F)$, for each i. Summing up both sides over i and observing that $\sum s_i = 1$ gives

$$\sum_{i=1}^{n}(p_i - v_i)q_i(p) = \sum_{i=1}^{n} s_i(y^* + F) = y^* + F \tag{22}$$

Subtracting F from both sides and recalling the definition $y(p)$, we have $y(p) = y^* = y(p^*)$. By the second order sufficient condition, p^* is the unique maximum, therefore $p = p^*$. Thus, the steady state is the optimal price.

To show convergence, let $g(x) = x$ be the identity function for a vector x. Let $g_i(x) = x_i$ be the i^{th} component of x. Since the steady-state is p^*, we have $f_i(p^*) = p_i^* = g_i(p^*)$. And

$f_i(0) > v_i > 0 = g_i(0)$. Because f is strictly increasing and continuous, the sequence $p_{i,t}$ increases in t. Now, for any p_t and p_{t-1},

$$p_{i,t} - p_{i,t-1} = f_i(p_t) - f_i(p_{t-1}) < g_i(p_t) - g_i(p_{t-1}) = p_{i,t} - p_{i,t-1} \tag{23}$$

Hence the differences in the sequences $p_{i,t}$ decrease in t for each i. So, $p_{i,t}$ converges to p_i^* for each i, and p_t converges to p^*.

■

Proof of Proposition 6. Let $P = \{p : y(p) > \bar{y}\}$ be the set of all satisfactory prices at $\bar{y}$. Recall that $\hat{p} = v + \frac{F+\bar{y}}{\hat{q}}$. Thus,

$$y(\hat{p}) = (\hat{p} - v)q(\hat{p}) - F = \frac{F+\bar{y}}{\hat{q}}q(\hat{p}) - F.$$

Now $\hat{p} \in P$ if $y(\hat{p}) > \bar{y}$. Plugging into the equation above and rearranging, this occurs if and only if $q(\hat{p}) > \hat{q}$.

■

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