Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.

An Empirical Test of Auction Efficiency: Evidence from MBS Auctions of the Federal Reserve

Pietro Bonaldi, Ali Hortacsu, and Zhaogang Song

2015-082

Please cite this paper as:
Bonaldi, Pietro, Ali Hortacsu, and Zhaogang Song (2015). "An Empirical Test of Auction Efficiency: Evidence from MBS Auctions of the Federal Reserve," Finance and Economics Discussion Series 2015-082. Washington: Board of Governors of the Federal Reserve System, http://dx.doi.org/10.17016/FEDS.2015.082.

An Empirical Test of Auction Efficiency: Evidence from MBS Auctions of the Federal Reserve*

Pietro Bonaldi† Ali Hortaçsu‡ Zhaogang Song§

First version: February 2015

This version: August 2015

PRELIMINARY DRAFT

Abstract

Auction theory has ambiguous implications regarding the relative efficiency of three formats of multiunit auctions: uniform-price, discriminatory-price, and Vickrey auctions. We empirically evaluate the performance of these three auction formats using the bid-level data of the Federal Reserve's purchase auctions of agency mortgage-backed securities (MBS) from June 1, 2014 through November 17, 2014. We estimate marginal cost curves for all dealers, at each auction, based on structural models of the multiunit discriminatory-price auction. Our preliminary results suggest that neither uniform-price nor Vickrey auctions outperform discriminatory-price auctions in terms of the total expenditure. However, they do outperform it in terms of efficiency, with efficiency gains around 0.74% of the surplus that dealers extract. We caution that our empirical estimation and analysis involve technical assumptions made about the specific auction mechanism the Federal Reserve uses and how auction participants perceive the auction mechanism, both of which may be distinct from practice and may alter the conclusions substantively.

---

*The analysis and conclusions set forth are those of the authors and do not indicate concurrence by the Federal Reserve System. In particular, discussions of the institutional setup of the Federal Reserve's auction mechanism are only based on the information published on the website of the Federal Reserve Bank of New York. We are grateful to Michelle Ezer, Linsey Molloy, and Min Wei for helpful discussions.

†Department of Economics, University of Chicago

‡Department of Economics, University of Chicago and NBER

§Federal Reserve Board

1 Introduction

For multiunit auctions, multiple Bayesian-Nash equilibria can exist so that no definitive theoretical predictions can be made about the equilibrium bidding strategies and auction outcomes. Consequently, auction theory has ambiguous implications regarding the relative efficiency of three formats of multiunit auctions: uniform-price, discriminatory-price, and Vickrey auctions (see, for example, Bikhchandani & Huang (1993), Back & Zender (1993), and Ausubel, Cramton, Pycia, Rostek & Weretka (2013)). In this paper, we empirically evaluate the performance of these three auction formats using the bid-level data of the Federal Reserve's purchase auctions of agency mortgage-backed securities (MBS), which will shed light on the theory development of multiunit auctions.

The Federal Reserve's purchase auctions of agency MBS provide a fertile ground for empirically investigating multiunit auctions. The large scale asset purchases of agency MBS since the recent financial crisis are one of the most important events in the history of U.S. monetary policies. These MBS purchases meant to put downward pressure on longer-term interest rates, support mortgage and housing markets, and make broader financial conditions more accommodative, when the federal funds rate is stuck at the zero lower bound (Bernanke (2012)). The amount purchased is huge. As of November 2014, the Federal Reserve (Fed) had accumulated \$1.74 trillion of agency MBS on its balance sheet (after principal payments), around 30% of the total outstanding amount of all agency MBS. [1] In the securities with certain coupon rates, the Fed has actually accumulated about two-thirds of the total outstanding amount of their issues by the end of 2009 (Sack (2009)).

While the objective of the MBS purchases is to provide monetary policy stimulus to the economy, the implementation of the purchase programs need be designed to obtain the securities at competitive and appropriate prices, for the sake of the U.S. taxpayers (Potter (2013); Sack (2011)). Since April 2014, the Fed has been employing a multiunit discriminatory-price purchase auction on its proprietary FedTrade system to execute the purchases of agency MBS. In this paper, we conduct an empirical analysis of the three auction formats–uniform price, discriminatory price, and Vickrey–using the bid-level data of the Fed's agency MBS purchase auctions from June 1, 2014 through November 17, 2014.

In particular, we base our analysis on the share auction model of Wilson (1979). More precisely, we use an extension of such model proposed by Kastl (2012) and Kastl (2011),

[1] The purchases include \$1.25 trillion in the QE1 program from January 2009 to March 2010, about \$20 billion a month in the Reinvestment program of principal payments of agency debt and MBS holdings starting from September 2011, and about \$40 billion a month in the QE3 program staring from September 2012 till December 2013 - when the Fed started to taper after which the monthly purchase pace decreased steadily until termination in October 2014. Through November 2014, the end of sample period in our study, the Reinvestment program is still ongoing.

that accounts for restrictions in the number of bids that a dealer is allowed to submit. We estimate the model, following Cassola, Hortaçsu & Kastl (2013), which allows us to recover marginal cost functions for all dealers participating in all the auctions in our sample. Using the estimated marginal cost, we empirically evaluate the performance of the discriminatory auction, in terms of efficiency and total expenditure by the Fed. In particular, we compare the mechanism assumed to be in place to both a uniform price and a Vickrey auction. Our preliminary results suggest that neither uniform-price nor Vickrey auctions outperform discriminatory-price auctions in terms of the total expenditure. However, they do outperform them in terms of efficiency, with efficiency gains around 0.74% of the surplus that dealers extract from participating in the MBS auctions.

To the best of our knowledge, this paper provides the first analysis of the Fed's purchase auctions of agency MBS. Studying these MBS auctions is not only important for the United States, but also helpful for a growing number of countries that are considering or have implemented large scale asset purchases of their own as unconventional monetary policies, such as Japan, United Kingdom, and the European Central Bank. We caution that our empirical estimation and analysis involve technical assumptions made about the specific auction mechanism the Federal Reserve uses and how auction participants perceive the auction mechanism, both of which may be distinct from practice and may alter the conclusions substantively.

The paper is organized as follows. Section 2 provides the institutional details of the Fed's agency MBS purchase auctions, while Section 3 discusses the data. The structural model is in Section 4 and empirical results are presented in Section 5. Section 6 concludes.

2 Institutional Background

2.1 Agency MBS Market

Agency mortgage-backed-securities (MBS) guaranteed by Ginnie Mae (GNMA), Fannie Mae (FNMA), and Freddie Mac (FHLM) form a major component of the U.S. fixed-income market.[2] The combined market value of outstanding securities is about $6.01 trillion as of November 2014, around half of the outstanding $12.50 trillion of U.S. Treasury securities, according to the Securities Industry and Financial Markets Association (SIFMA).

Over 90% of Agency MBS trading occurs in a unique liquid forward market, known as the to-be-announced (TBA) market. In a TBA trade, the buyer and seller decide general trade parameters, such as agency, coupon, original mortgage term, par amount, price, and settlement date, but the buyer does not know which MBS the seller will deliver until two

[2] These securities are residential mortgage-backed-securities, not those backed by commercial mortgages.

days before the settlement. Hence, sellers have an incentive to deliver the cheapest possible mortgage backed securities that meet the TBA specifications - and buyers rationally expect they will. The delivered MBS is cheap mainly because it has inferior prepayment characteristics not specified in the TBA contract, such as the loan-to-value ratio, FICO score, past prepayment behavior, and location of the mortgage, relative to other MBS eligible to be delivered into the same TBA cohort.

This pool trading design allows both buyers and sellers to only price the cheapest MBS under a TBA cohort rather than analyzing specific mortgage characteristics in determining the MBS values. As a result, investors can trade thousands of different MBS backed by millions of individual mortgages using only a few TBA contracts. This dramatically increases the set of deliverable MBS and substantially improves market liquidity, which is particularly important to the Fed in executing its huge purchase programs. In fact, the average daily trading volume of agency MBS is 20 times larger than that of corporate bonds, and close to 60% of that for Treasury securities in 2010, according to Vickery & Wright (2011). Moreover, the TBA market settlements are up to three months out, with contracts for the next two months particularly active.

2.2 The Fed's MBS Purchase Auctions

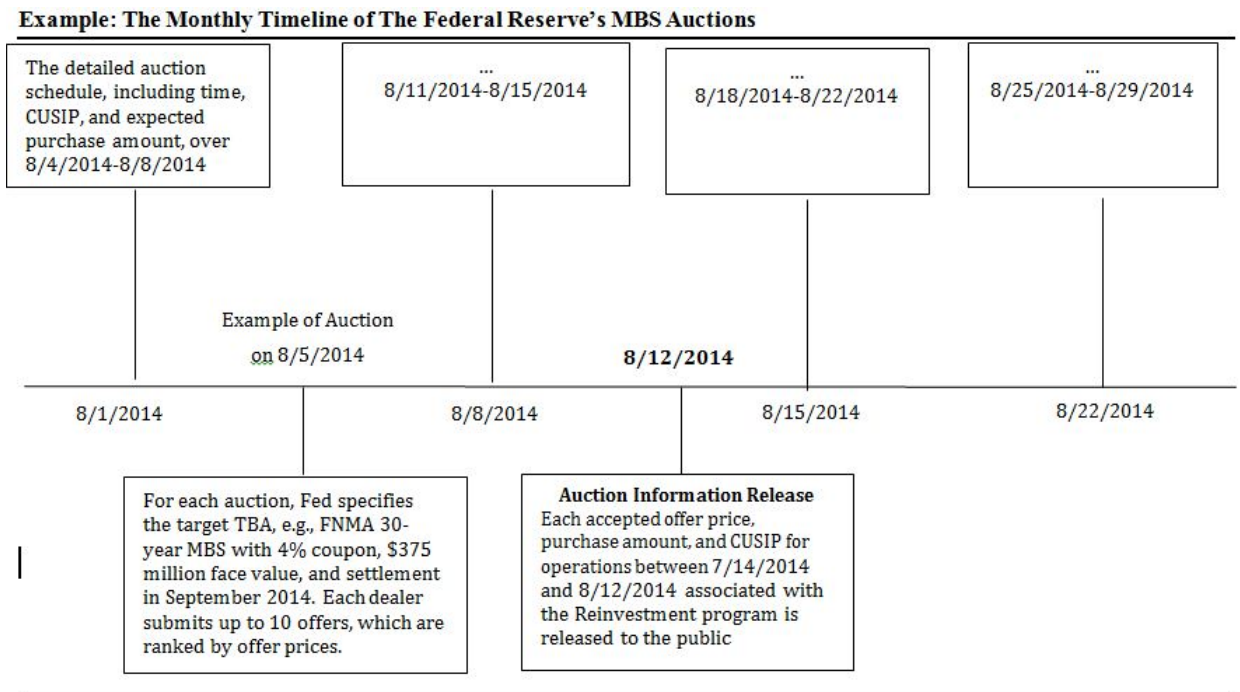
The Federal Reserve's purchases of agency MBS are conducted in the TBA market, concentrated in newly-issued fixed-rate agency MBS guaranteed by Fannie Mae, Freddie Mac, and Ginnie Mae.[3] In general, the amount of Fed's MBS purchases in the TBA market across agencies, loan terms, and coupon rates roughly follows the anticipated composition of new issuances of these securities, which varies over time depending on the level of interest rates and the refinancing and home purchase activity. In particular, the Fed focuses its purchases in "production coupons", the TBA contracts covering the bulk of newly issued agency MBS with the most intensive trading and delivery activities. These "production coupons" are the most liquid segment of the TBA market with their yields closely tied to the primary mortgage rates. To acquire the desired amount of agency MBS, Fed employs mainly an outright buy and hold strategy.[4]

The Fed's MBS auctions are designed as a series of sealed-bid, multiunit, and discriminatory-

[3]MBS securities such as CMOs, REMICs, Trust IOs/Trust POs and other mortgage derivatives or cash equivalents are ineligible assets for MBS auctions, as announced on the website of the Federal Reserve Bank of New York.

[4]The Fed also conducts dollar roll transactions that consist of a simultaneous purchase and sale of TBA contracts with different settlement months but same other characteristics, and coupon swap transaction that involve a simultaneous sale and purchase of agency MBS with only different coupons to facilitate settlements. The dollar roll transactions do not affect the amount of agency MBS on the Fed's balance sheet.

Figure 1: Example of the Timeline of the Fed's MBS Auctions



price (reverse) auctions, conducted on the FedTrade system. Direct participants of MBS auctions only include the primary dealers with MBS businesses recognized by the Federal Reserve Bank of New York during our sample period, whereas other investors can indirectly participate through the primary dealers. The list of primary dealers is stable, consisting of 22 dealers, during our sample period.[5]

Figure 1 describes the timeline of MBS auctions. During the time period of this analysis, the Fed published a detailed schedule of the auctions to be conducted at the end of each week for the subsequent week, which contained the auction time, TBA contract, and maximum purchase amount in terms of par amount. [6] For example, the weekly schedule included an auction from 2:00 to 2:30 pm on August 5, 2014 to purchase a TBA contract of FNMA 30-year MBS with a 3% coupon and $375 million maximum purchase par amount.

Each of the primary dealers can submit up to 10 offers, i.e., quantity-price pairs. The

[5]See `http://www.newyorkfed.org/markets/pridealers_current.html\#tabs-2` for details.

[6]The amount of agency MBS to be purchased each month associated with the QE3 program was determined by the Federal Open Market Committee (FOMC) and announced in FOMC meeting statements and associated Desk statements. Moreover, on or around the eighth business day of each month, the Fed published a tentative amount of reinvestment-related purchases expected to take place between the middle of the current month and the middle of the following month. This amount was approximately equal to the amount of principal payments from agency debt and agency MBS expected to be received over that period, adjusted for any variations from prior periods.

minimum offer size is $1 million, with a minimum increment of $1 million. The price tick size is 1/32 of a dollar. The Fed will select the offers starting from the lowest prices. Participating dealers will receive the operation results, including their accepted offers, via FedTrade, immediately following the close of the auction. The winning dealers are expected to deliver the Fed the specified amount of MBS at the specified price in their offers.[7]

Immediately after each auction, aggregate auction quantities are released on the website of the Federal Reserve Bank of New York, including the total amount submitted and accepted. In the middle of each month, the Fed discloses the auction results for the purchase operations executed from the middle of last month until the current day. The disclosed information, public on the Federal Reserve Bank of New York website, includes the price, par amount, and CUSIP (that specifies the agency, coupon, loan term, and settlement month) for winning offers.[8]

3 Data and Summary Statistics

3.1 Data

We use the bid-level data of the MBS purchase auctions from June 1, 2014 through November 30, 2014, covering both the Reinvestment program that was started on October 3, 2011, at a pace of about $20 billion per month, and the QE3 program that was started on September 13, 2013 and ended on October 29, 2014, at a pace of about $40 billion per month before December 2013 and a steadily declining pace after that. Over this period, the Federal Reserve conducted a series of 596 purchase auctions on its FedTrade system. The auction data include the security (TBA) identifiers such as the CUSIP, agency, loan term, coupon rate, and settlement day, the face value, all (winning and losing) offers submitted, and an indicator of whether an offer is accepted, rejected, or partially accepted. It is worth noting that the Fed auction announcement usually involves several different securities at the same time. However, no information is available to tell whether these securities are purchased in separate auctions or one joint auction.[9] Throughout the paper, we assume the former case for simplicity, which gives us the 596 auctions mentioned above.

In addition, we obtain the security-level outstanding balance and new issuance at the monthly frequency from eMBS, and dollar roll financing rates and option-adjusted spreads from J.P. Morgan. We also obtain weekly observations of primary mortgage rates (PMMS)

[7]See `http://www.newyorkfed.org/markets/ambs-treasury-faq.html` for details.

[8]Detailed auction results including the winning dealer identity will be released two years after each quarterly auction period, in accordance with the Dodd-Frank Act.

[9]See `http://www.newyorkfed.org/markets/ambs/AMBS_Schedule_090814.pdf` for an example.

for 30-year and 15-year fixed-rate mortgage loans from the Freddie Mac primary mortgage market survey.

3.2 Summary statistics of MBS auctions

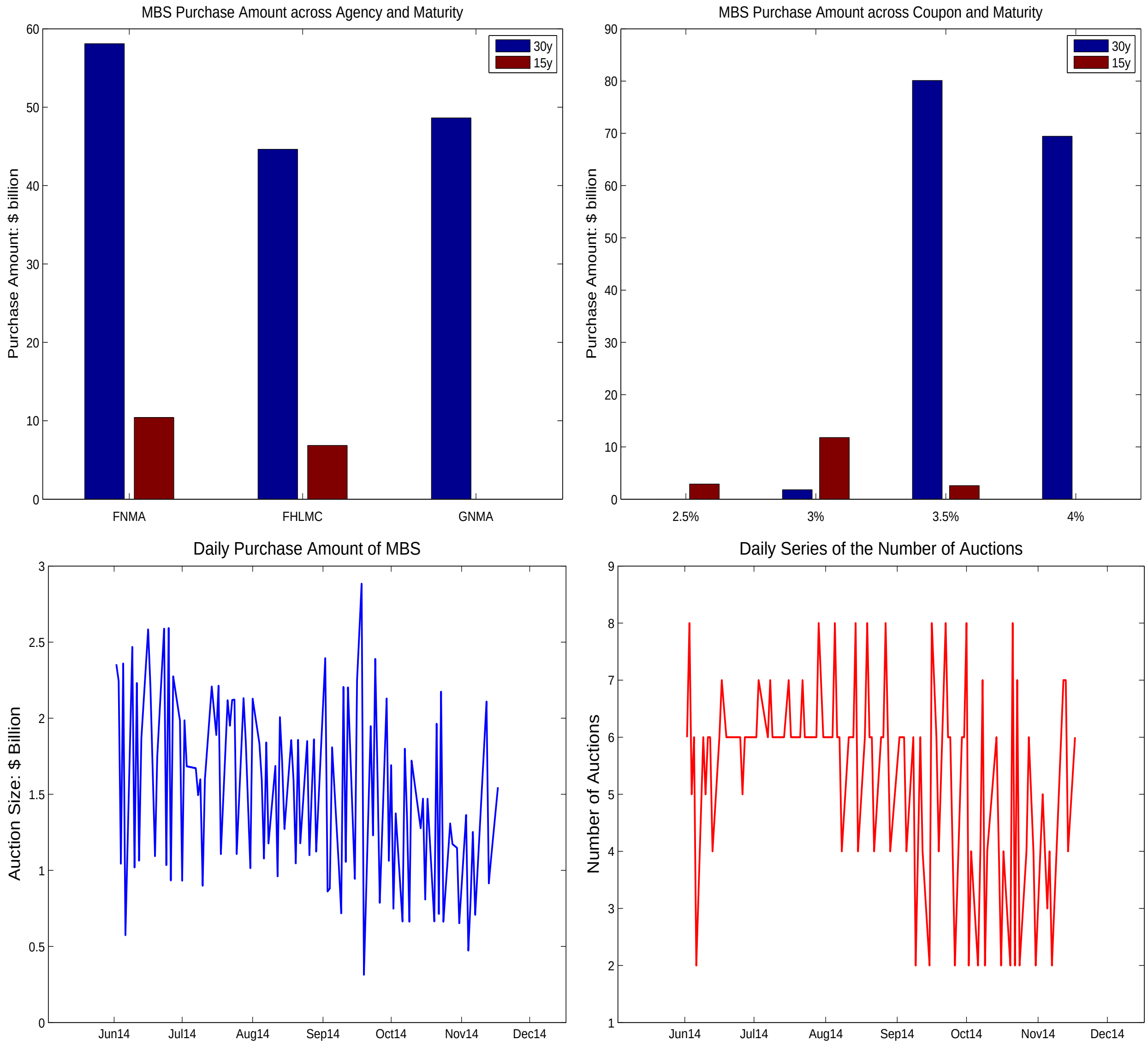
Figure 2 reports the summary statistics of the Federal Reserve's MBS purchases from June 1, 2014 through November 17, 2014. From the top panels, we observe that the purchases were concentrated in 30-year MBS with 3.5% and 4% coupon rates, spread roughly evenly among FNMA, FHLMC, and GNMA securities. The bottom panels show that a par amount of $1.5 billion is purchased with about 5 auctions on average each day.

Figure 3 plots the series of certain auction variables of the 596 MBS auctions, whereas corresponding summary statistics are provided in Table 1. On average, each auction has a purchase size of $280 million, with an offer-to-cover ratio of around 7.5, implying that these MBS auctions are generally well received.

Table 1 reports summary statistics of the MBS auctions of different securities. There are five combinations of agencies and maturities, including FNMA 30-year, FNMA 15-year, FGLMC 30-year, FGLMC 15-year, and GNMA 30-year MBS. The 30-year MBS have coupon rates of 3%, 3.5%, and 4%, while the 15-year MBS have coupon rates of 2.5%, 3%, and 3.5%. The numbers reported are the mean values of the corresponding variables across auctions.

We observe that the MBS auctions of 30-year MBS were concentrated in coupon rates of 3.5% and 4%, while those of 15-year MBS were concentrated in coupon rates of 2.5% and 3%. The offer-to-cover ratio, defined as par amount offered/par amount accepted, is lower for 30-year MBS than that for 15-year MBS, probably because the former has a higher purchase size. Also reported are the average auction price (the par amount–weighted average price of accepted offers), the average offer price (the par amount-weighted price of submitted offers), and the variance of offers (calculated using the formula $\sum_{i=1}^{N} (p_i - p_{avg})^2 q_i / \sum_{i=1}^{N} q_i$, where p_{avg} is the average offer price) to capture the offer dispersion. We observe that the average offer price is higher than the average auction price, by default, but only slightly. Interestingly, the variance of offers is increasing significantly across FNMA, FGLMC, and GNMA, and is also higher for 15-year MBS than for 30-year MBS. The heterogeneity patterns of these auction variables imply that there are important differences in dealers' supply curves of various agency MBS. Understanding these differences will be valuable for policy makers to design appropriate policies, such as the asset purchases by central banks and the reforms of the MBS markets.

Figure 2: Summary of the MBS Purchases



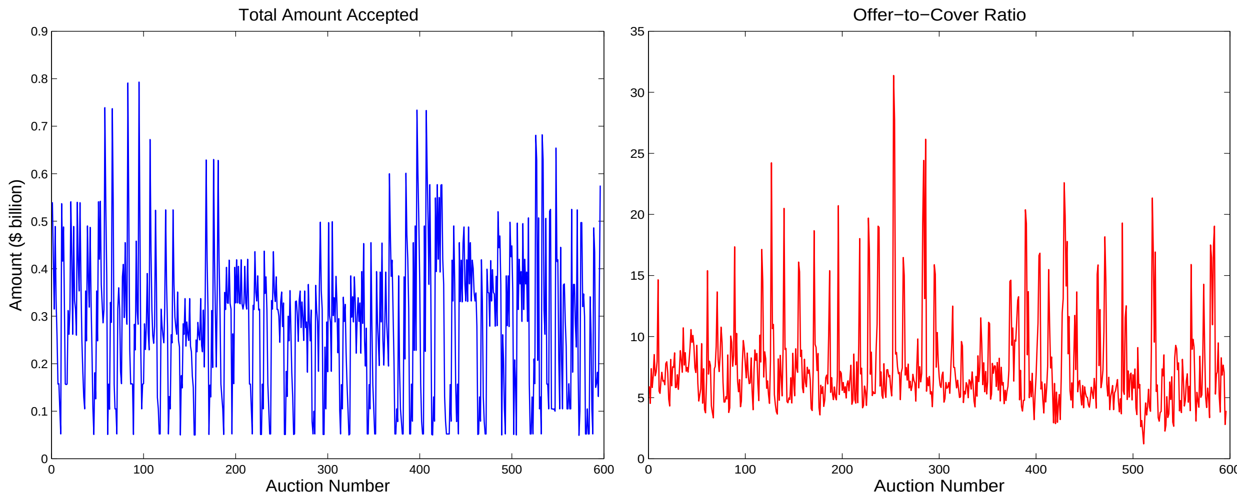
Note: This figure plots the summary statistics of the Fed's MBS purchases from June 1, 2014 through November 17, 2014.

Table 1: Summary Statistics of MBS Auctions of Different Securities

A: FNMA	30y			15y		
	3	3.5	4	2.5	3	3.5
# of Auctions	5	69	66	26	44	21
Size ($ billion)	0.13	0.41	0.44	0.07	0.16	0.08
Offer-to-Cover Ratio	7.62	5.87	5.72	15.3	8.24	13.13
Average Auction Price	99.95	102.22	105.28	100.9	103.19	105.27
Average Offer Price	99.98	102.24	105.3	100.94	103.22	105.31
Variance of Offers	0.03	0.07	0.16	0.1	1.02	0.23
B: FGLMC	30y			15y		
	3	3.5	4	2.5	3	3.5
# of Auctions	3	69	66	16	44	18
Size ($ billion)	0.11	0.34	0.32	0.06	0.11	0.05
Offer-to-Cover Ratio	7.41	5.33	5	14.29	9.25	15.55
Average Auction Price	100.16	102.07	105.14	100.88	103	105
Average Offer Price	100.2	102.1	105.18	100.93	103.05	105.05
Variance of Offers	0.08	0.13	0.63	0.16	1.72	0.16
C: GNMA	30y					
	3	3.5	4			
# of Auctions	5	74	70			
Size ($ billion)	0.17	0.38	0.28			
Offer-to-Cover Ratio	5.8	6.03	7.34			
Average Auction Price	101.8	103.44	106.14			
Average Offer Price	101.83	103.47	106.17			
Variance of Offers	0.07	1.18	1.13			

Note: This table reports summary statistics of the MBS auctions of different securities, conducted by the Federal Reserve from June 1, 2014 through November 17, 2014. The numbers reported are the mean values of the corresponding variables across auctions. The offer-to-cover ratio is par amount offered/par amount accepted. The average auction price is the par amount–weighted average price of accepted offers, whereas the average offer price is the par amount-weighted price of submitted offers. The variance of offers is calculated using the formula $\sum_{i=1}^{N} (p_i - p_{avg})^2 q_i / \sum_{i=1}^{N} q_i$, where p_{avg} is the average offer price.

Figure 3: MBS Auction Variables



Note: This figure plots summary statistics of the 596 MBS auctions by the Federal Reserve from June 1, 2014 through November 17, 2014.

4 The Model

We base our analysis on the share auction model of Wilson (1979). More precisely, we use an extension of such model proposed by Kastl (2012) and Kastl (2011), suitable for the case where bidders are restricted to submit no more than a given number of steps as their bids. Consequently, we follow Cassola et al. (2013), and introduce a set of assumptions under which we can estimate the model using their estimation procedure.

Assumption 1. *There are N potential bidders that participate in each auction.*

Only the primary dealers recognized by the Federal Reserve Bank of New York can participate in the auctions directly during our sample period.

Assumption 2. *Bidder i's information at auction t is described by a signal (ω_t, θ_{it}). The vector ω_t is observed by all bidders but not necessarily by the econometrician. θ_{it} is a vector of private signals, observed only by i.*

In our context, ω_t contains information available to all bidders before they submit their bids. For instance, it might include other rates or yields, as those on Treasuries, current coupon MBS, or residential mortgages as well as bid and ask quotes on the TBA market. θ_{it} could include private determinants of i's opportunity costs, like the terms on specific MBS deals that it might have exclusive access to.

Assumption 3. *Conditional on ω_t, θ_{it} and θ_{jt} are independent for all pairs of different bidders i and j.*

The θ_{it}'s are idiosyncratic, any common cost component is assumed to be already included in ω_t.

Assumption 4. *Bank i's opportunity cost of selling MBS to the FED in auction t is given by a marginal cost function of the form $c_{it}(x, \theta_{it}, \omega_t)$. c_{it} is weakly increasing in x.*

This implies that, conditional on ω_t, the marginal cost of dealer i does not depend on the marginal costs of other dealers.

Assumption 5. *From the perspective of the bidders, the total par amount purchased at auction t, Q_t, is a random variable with distribution $M(Q_t|\omega_t)$. Such distribution is common knowledge and has a strictly positive density with support on $[\underline{Q}, \bar{Q}]$. Conditional on ω_t, Q_t is independent of θ_{it}.*

Note that the assumption of Q_t being a random variable may be distinct from the practice of the Federal Reserve, which is not verifiable based on the public information from the Federal Reserve Bank of New York. We make this assumption for two reasons. First, it facilitates the structural estimation. Second, from the perspective of the auction participants, this assumption may well describe the situation that no public information is available from the Federal Reserve about how Q_t is determined precisely so that the auction size appears random from the perspective of auction participants. In fact, before each auction the Fed announces a maximum purchase amount, but the actual amount purchased is usually distinct from the announced one.

We define bidder's pure strategies as mapping from the set of private signals to the set of allowed bid functions. Since the Fed does not allow bidders to submit more that 10 offers (quantity-price pairs), we impose the following restriction on the set of auctions available to each bidder.

Assumption 6. *Bidder i's action set is:*

$$A_i = \left\{ \begin{array}{c} (b_i, q_i, K_i) : \dim(b) = \dim(q) = K_i \in \{1, ..., 10\}, \\ b_{ik} \in [0, \infty], \; q_{ik} \in [0, \bar{Q}], \; b_{ik} < b_{ik+1}, \; q_{ik} < q_{ik+1} \end{array} \right\}$$

where $(0, 0, 1)$ denotes non-participation.

Under Assumption 6, an action available to bidder i can also be described by a bid function, that is, a non-decreasing step function with at most 10 steps, $y_i : \mathbb{R}_+ \to [0, \bar{Q}]$, where $y_i(p) = \sum_{k=1}^{K_i} q_{ik} I(p \in (b_{ik-1}, b_{ik}])$ and I is an indicator function. For each type θ_i, the bid function $y_i(\cdot|\theta_i)$ specifies, for each price p, the share $y_i(p|\theta_i)$ of total demand that (type θ_i of) bidder i offers.[10]

All bidders submit their bids simultaneously and the auctioneer fixes the amount Q to be purchased. The auctioneer purchases those units that are being offered at the lowest prices, usually not all of them from the same bidder, until it reaches the desired amount Q. Given that the bids are step functions, rationing will happen with positive probability in equilibrium. Given that there is no public information available regarding whether or how the Federal Reserve take partial offers, we assume rationing pro-rata on-the-margin for simplicity, under which the auctioneer proportionally adjusts the marginal bids so as to equate supply and demand[11].

4.1 Equilibrium

In this section we will focus on a specific auction and hence we will drop the index t, and the shared information ω_t. At a given auction, a Bayes Nash Equilibrium is a collection of strategies $y = \{y_i(\cdot|\cdot) : i \in \{1, ..., N\}\}$, such that for almost every type θ_i, $y_i(\cdot|\theta_i)$ maximizes i's expected utility, given its information and all other dealers' strategies $\{y_j(\cdot|\cdot) : j \neq i\}$. Such expected utility is given by:

$$EU_i(\theta_i) = E_{Q,\theta_{-i}|\theta_i} \left[\begin{array}{c} \sum_{k=1}^{K} I(q_i^c(Q,\theta,y) > q_k)(q_k - q_{k-1}) b_k \\ + \sum_{k=1}^{K} I(q_k \geq q_i^c(Q,\theta,y) > q_{k-1})(q_i^c(Q,\theta,y) - q_{k-1}) b_k \\ - \int_0^{q_i^c(Q,\theta,y)} c_i(x,\theta_i)\, \mathrm{d}x \end{array} \right] \quad (1)$$

where $q_i^c(Q,\theta,y)$ is the amount that i sells in equilibrium, if the state is (θ, Q) and dealers' bid functions are $\{y_i(\cdot|\theta_i) : i \in \{1, ..., N\}\}$. The first term inside the brackets is the revenue that the dealer receives from selling all units corresponding to steps in its bid function that are not affected by rationing (steps such that the largest amount offered its accepted). The second term reflects that rationing might occur with positive probability. The third term is the total opportunity cost of selling $q_i^c(Q,\theta,y)$ units.

Under the additional assumption that marginal cost functions are continuous in x, except maybe in the last step, Kastl (2011) and Cassola et al. (2013) provide necessary conditions

[10] Assumption 6 only restricts the amount of bids as bounded, rather than exceeding certain specific amount, e.g., the maximum purchase amount announced by the Fed.

[11] Under rationing pro-rata on the margin the rationing coefficient at the market clearing price P^c satisfies $R(P^c) = \frac{Q - TS_+(P^c)}{TS(P^c) - TS_+(P^c)}$, where $TS(P^c)$ denotes total supply at price P^c, and $TS_+(P^c) = \lim_{p \uparrow P^c} TS(p)$. Also, since bidders use step functions, a situation may arise in which multiple prices would clear the market. If that is the case, we assume consistently with our application that the auctioneer selects the most favorable price from his perspective, i.e., the lowest price.

for equilibrium in discriminatory auctions with private costs (values). Proposition 1 below states such conditions in terms of $P^c(Q, \theta, y)$, the market clearing price, given the state (θ, Q) and all dealers' strategies y.

Proposition 1. *Under assumptions 1-6 in any Bayes Nash Equilibrium of a discrimiatory auction, for almost all θ_i, every step k in the equilibrium bid function $y_i(\cdot|\theta_i)$ satisfies the following necessary conditions for optimality.*

(i) If $k < K_i(\theta_i)$ and $c_i(x, \theta_i)$ is continuous in a neighborhood of q_k then

$$c_i(q_k, \theta_i) = b_k + \frac{\Pr(b_{k+1} \leq P^c)}{\Pr(b_k < P^c < b_{k+1})}(b_{k+1} - b_k) \tag{2}$$

and at the final step,$k = K_i(\theta_i)$,

$$b_k = c_i(\bar{q}, \theta_i) \tag{3}$$

where $\bar{q} = \sup_{(Q,\theta_{-i})} q_i^c(Q, \theta, y)$ is the largest quantity bought from type θ_i in equilibrium.

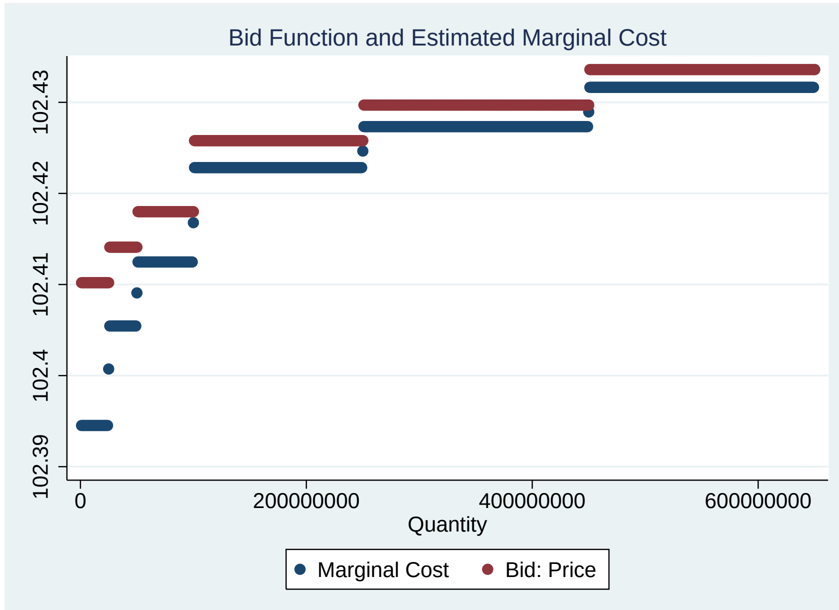
(ii) If $k < K_i(\theta_i)$ and $c_i(x, \theta_i)$ is a step function in x at a step k where $c_i(x, \theta_i) = c_k$, then

$$c_k = b_k + \frac{\Pr(b_k \leq P^c)}{\frac{\partial \Pr(b_k > P^c)}{\partial b_k}} \tag{4}$$

Equilibrium existence is guaranteed by Proposition 2 in Kastl (2012). Our goal is to estimate dealers' marginal costs. Identification is provided by equations (2) - (4) from wich we recover point estimates of the marginal costs for each quantity-price pair submitted. The estimation is done non-parametrically, using the resampling method proposed in Hortaçsu & McAdams (2010), Hortaçsu & Kastl (2012) and Cassola et al. (2013). As in the latter, to control for shared information unobserved by the econometrician, ω_t, we only use data from auction t to estimate the marginal costs of type θ_{it}, for all dealers i participating in auction t.

When all N bidders are ex ante symmetric, private information is independent across bidders, and the data are generated by a symmetric Bayesian Nash equilibrium, the resampling method operates as follows. First fix a bidder. Then, from all the observed data (all auctions and all bids), draw randomly (with replacement) N − 1 actual bid functions submitted by bidders. This simulates one possible state of the world from the perspective of the fixed bidder, a possible vector of private information, and thus results in one potential realization of the residual demand. Intersecting this residual demand with the fixed bidder's bid, we obtain a market clearing price. Repeating this procedure a large number of times, we obtain an estimate of the full distribution of the market clearing price conditional on the fixed bid. Using this estimated distribution of market clearing price, we can obtain our estimates of the marginal cost at each step submitted by the bidder whose bid we fixed using equations

Figure 4: Estimation Results. We estimate the marginal cost at each step in each dealers bid function. The figure shows the result of such estimation for a particular dealer in a given auction.



(2) - (4). Cassola et al. (2013) prove consistency of the resulting estimator as the number of bidders N goes to infinity.

5 Results

All what follows applies only to auctions of FNMA securities with 30y maturity and 3.5 coupon rates. In a future version of the paper we will extend our analysis to all other securities purchased by the Fed through its MBS auctions.

For every auction in our sample, and every dealer bidding at each auction, we estimate the dealer's marginal cost at each step in its biding function. Figure 4 illustrates the results of such estimation, for a specific bidder at a particular auction.

Following Hortaçsu & McAdams (2010), we compare the three auction formats – uniform-price, discriminatory-price, and Vickrey – of multiunit auctions. Such comparison is done under two different criteria, net expenditure by the auctioneer and efficiency of the resulting allocation.

5.1 Net Revenue

Using the estimated marginal costs, we can compute the counterfactual expenditure on a hypothetical uniform price auction with truthful bidding (UPATB). The UPATB expenditure provides a lower bound on the expenditures of both the uniform price auction with strategic bidding and the Vickrey auction. Table 2 shows the Ex post expenditure reduction from switching from the discriminatory auction to the UPATB. The highest reduction across all auctions is 0.0078% of total expenditure in the discriminatory auction. For several auctions, ex post expenditure is even slightly larger in the UPATB. In fact, on average, the difference in ex post expenditure between both types of auctions is negligible. Even though we are not reporting standard errors yet in this version of the paper, our conjecture is that most of the differences in ex post expenditure reported in Table 2 are not statistically significant. That is, we cannot reject the null hypothesis that both auction types lead to the same ex post expenditure by the auctioneer.

5.2 Efficiency

We compute the ex post efficiency loss as in Hortaçsu & McAdams (2010). That is, we observe that there must be an efficiency loss whenever the marginal cost of a unit that was sold in the auction is higher than the cost of a unit that was not sold. Thus, we can compute the efficiency gain of a Vickrey auction where bidders rationally bid their marginal costs (or to the hypothetical UPATB), relative to the discriminatory-price auction. We do so by computing the difference between the sum of the marginal costs of all units sold in the auction and the sum of the marginal costs of all units that would have been sold in an auction with truthful bidding. In Table 2, the efficiency gain is reported as a percent of dealers' surplus in the discriminatory auction. For most auctions, the efficiency gain is less that 1% of dealers' surplus, however for auctions 24, 29 and 61 the gains are around 2%, 3% and 15%. On average, the ex post efficiency gain is 0.74% of dealers' surplus.

5.3 Bid Mark-up

For all auctions, and all dealers, we compute the quantity weighted bid mark-ups as a measure of market power. For dealear i submitting K_i steps at given auction, we define this mark up as: $\mu_i = {\sum_{k=1}^{K_i}(q_k - q_{k-1})b_k}/{\sum_{k=1}^{K_i}(q_k - q_{k-1})c_k} - 1$. The average mark-ups per dealer, across all auctions, are reported in Table 3. While some dealers sime to have no market power, others are able to sell MBS to Fed at prices almost 0.5% higher than their opportunity costs. On average, across all dealers and auctions, the mark-up is lower than 0.1%.

Table 2: Ex Post Expenditure Reduction and Efficiency Gains with Truthful Bidding: Uniform vs Discriminatory

Auction	Expenditure (Million USD)	Expenditure reduction	Efficiency Gain	Auction	Expenditure (Million USD)	Expenditure reduction	Efficiency Gain
1	552.5	0.0074%	0.049%	36	295.4	-0.0000%	0.987%
2	548.1	-0.0004%	0.020%	37	295.0	-0.0012%	0.000%
3	550.7	0.0024%	0.011%	38	347.8	-0.0009%	1.222%
4	550.3	0.0030%	0.156%	39	347.4	0.0034%	0.155%
5	499.3	-0.0000%	0.000%	40	348.2	0.0014%	0.057%
6	549.7	-0.0024%	0.000%	41	346.6	0.0078%	1.521%
7	334.1	0.0017%	0.002%	42	403.4	0.0040%	0.775%
8	333.9	-0.0023%	0.210%	43	404.0	-0.0013%	3.834%
9	185.7	-0.0006%	0.172%	44	404.5	0.0012%	0.000%
10	464.4	0.0007%	0.007%	45	403.4	-0.0033%	0.015%
11	468.6	-0.0019%	0.067%	46	614.4	0.0015%	3.636%
12	398.8	0.0039%	0.101%	47	614.8	0.0038%	0.601%
13	265.9	-0.0002%	1.430%	48	745.9	-0.0038%	0.042%
14	263.2	-0.0012%	0.826%	49	742.1	-0.0006%	4.073%
15	264.9	0.0028%	0.019%	50	556.5	-0.0015%	2.184%
16	302.5	-0.0031%	0.082%	51	556.2	-0.0052%	0.001%
17	289.0	0.0009%	0.088%	52	426.5	-0.0029%	0.022%
18	292.7	-0.0001%	0.014%	53	425.5	-0.0007%	0.000%
19	293.4	-0.0003%	0.053%	54	426.9	-0.0013%	0.404%
20	319.3	0.0003%	0.001%	55	409.2	-0.0011%	0.044%
21	317.5	0.0021%	0.286%	56	408.6	-0.0032%	0.025%
22	319.6	-0.0019%	0.117%	57	407.3	-0.0018%	0.014%
23	412.4	-0.0031%	0.003%	58	534.3	0.0028%	0.117%
24	410.9	0.0003%	2.030%	59	513.2	-0.0014%	0.268%
25	414.3	-0.0007%	0.095%	60	437.8	-0.0075%	0.638%
26	411.2	-0.0001%	0.032%	61	433.7	0.0030%	15.848%
27	443.6	-0.0057%	0.250%	62	704.8	-0.0038%	0.036%
28	445.2	-0.0012%	0.092%	63	704.1	-0.0051%	0.490%
29	443.9	0.0012%	3.003%	64	538.8	0.0020%	0.434%
30	306.6	0.0038%	1.199%	65	428.3	-0.0008%	0.251%
31	305.3	-0.0005%	1.034%	66	376.7	-0.0000%	0.119%
32	306.2	-0.0011%	0.084%	67	377.8	0.0008%	0.000%
33	305.5	-0.0019%	0.060%	68	512.7	-0.0003%	0.139%
34	293.8	0.0047%	0.848%	69	502.2	-0.0008%	0.014%
35	295.1	0.0035%	0.518%	Average	422.5	-0.0001%	0.7380%

Note: Ex post expenditure difference is computed as a percentage of the expenditure in the discriminatory auction.

Table 3: Average quantity weighted bid mark-ups.

Dealer	1	2	3	4	5	6	7	8	9
Mark-up	0.122%	0.004%	0.001%	0.011%	0.003%	0.028%	0.043%	0.004%	0.004%

Dealer	10	11	12	13	14	15	16	17	Average
Mark-up	0.225%	0.466%	0.034%	0.011%	0.151%	0.007%	0.003%	0.471%	0.094%

6 Conclusion

Auction theory has ambiguous implications regarding the relative efficiency of three formats of multiunit auctions: uniform-price, discriminatory-price, and Vickrey auctions. We empirically evaluate the performance of these three auction formats using the bid-level data of the Federal Reserve's purchase auctions of agency mortgage-backed securities (MBS) from June 1, 2014 through November 17, 2014. We estimate marginal cost curves for all dealers, at each auction, based on structural models of the multiunit discriminatory-price auction. Our preliminary results suggest that neither uniform-price nor Vickrey auctions outperform discriminatory-price auctions in terms of the total expenditure. However, they do outperform in terms of efficiency, with efficiency gains around 0.74% of the surplus that dealers extract.[12]

[12] We caution that our empirical estimation and analysis involve technical assumptions made on the specific auction mechanism the Federal Reserve uses and how auction participants may perceive the specific auction mechanism, which may be distinct from the practice. These assumptions make it difficult to draw concrete conclusions from our analysis about the performance of the Federal Reserves purchases of agency MBS. A general auction model to accommodate the imprecise information of auction participants about the auction mechanism will be an important future extension.

References

Ausubel, L., Cramton, P., Pycia, M., Rostek, M. & Weretka, M. (2013), Demand reduction and inefficiency in multi-unit auctions, Working paper.

Back, K. & Zender, J. (1993), 'Auctions of divisible goods: On the rationale for the treasury experiment', *Review of Financial Studies* **6**, 733–764.

Bernanke, B. (2012), Monetary policy since the onset of the crisis. remarks at the federal reserve bank of kansas city economic symposium.

Bikhchandani, S. & Huang, C. (1993), 'The economics of treasury securities markets', *Journal of Economic Perspectives* **7**, 117–134.

Cassola, N., Hortaçsu, A. & Kastl, J. (2013), 'The 2007 subprime market crisis in the euro area through the lens of ecb repo auctions', *Econometrica* **81**(4), pp. 1309–1345.

Hortaçsu, A. & Kastl, J. (2012), 'Valuing dealers' informational advantage: A study of canadian treasury auctions', *Econometrica* **80**(6), pp.2511–2542.

Hortaçsu, A. & McAdams, D. (2010), 'Mechanism choice and strategic bidding in divisible good auctions: An empirical analysis of the turkish treasury auction market', *Journal of Political Economy* **118**(5), pp. 833–865.

Kastl, J. (2011), 'Discrete bids and empirical inference in divisible good auctions', *Review of Economic Studies* **78**, pp. 978–1014.

Kastl, J. (2012), 'On the properties of equilibria in private value divisible good auctions with constrained bidding', *Journal of Mathematical Economics* **48**(6), pp. 339–352.

Potter, S. (2013), The implementation of current asset purchases. remarks at the annual meeting with primary dealers, new york city.

Sack, B. (2009), The fed's expanded balance sheet. remarks at the money marketeers of new york university, new york city.

Sack, B. (2011), Implementing the federal reserve's asset purchase program. remarks at global interdependence center central banking series event, federal reserve bank of philadelphia.

Vickery, J. & Wright, J. (2011), Tba trading and liquidity in the agency mbs market. Federal Reserve Bank of New York Economic Policy Review 19.

Wilson, R. (1979), 'Auctions of shares', *The Quarterly Journal of Economics* **93**(4), pp. 675–689.