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Risky Mortgages, Bank Leverage and Credit Policy *

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Abstract

Two key channels that allowed the 2007-2009 mortgage crisis to severely impact the real economy were: a *housing net worth channel*, as defined by Mian and Sufi (2014), which affected the wealth of leveraged households; and a *bank net worth channel*, which reduced the ability of financial intermediaries to provide credit. To capture these features of the Great Recession, I develop a DSGE model with balance-sheet constrained banks financing both risky mortgages and productive capital. Mortgages are provided to agents facing idiosyncratic housing depreciation risk, implying an endogenous default decision and a link between their borrowing capacity and house prices. The interaction among the *housing net worth channel*, the *bank net worth channel* and endogenous foreclosures generates novel amplification mechanisms. I analyze the quantitative implications of these new channels by considering two different shocks linked to the supply of mortgage credit: an increase in the variance of housing risk and a deterioration in the collateral value of mortgages for bank funding. Both shocks are able to produce co-movements in house prices, business investment, consumption and output. Finally, I study two types of policy interventions that are able to reduce the severity of a mortgage crisis: debt relief for borrowing households and central bank credit intermediation.

Keywords: Financial frictions, Housing, Mortgages, Banking, Unconventional Monetary Policy

JEL Classification: E32, E44, E58, G21

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1 Introduction

Looking back at the recession of 2007-2009, we can identify two key channels that allowed a housing market collapse to impact the real economy: a *housing net worth channel*, as defined by Mian and Sufi (2014), which affected borrowing households; and a *bank net worth channel*, which reduced the ability of financial intermediaries to provide credit.

The first channel operated by decreasing the value of housing wealth, which was a key determinant of consumption and borrowing for a specific set of constrained households, because real estate served as an important source of collateral for these agents. In particular, Mian et al. (2013) show substantial heterogeneity in the effects of this channel, with poorer and more leveraged households experiencing a larger marginal propensity to consume with respect to housing wealth. While the housing net worth channel can explain how lower house prices can affect consumption, it does not provide a clear mechanism for the dramatic drop in non-residential investment that also occurred in 2007-2009. This can be accomplished through the second channel, which operated through the balance sheet of highly leveraged financial institutions. One critical element linking house prices to the value of assets held by financial intermediaries were defaultable mortgages and costly house repossessions. In fact, the high exposure of U.S. banks to mortgage-backed securities (MBS), whose value was closely related to house prices, made them particularly vulnerable to the turmoil in the subprime mortgage market. By negatively affecting their net worth, this channel reduced banks' ability to provide loans to other sectors of the economy, including to firms financing investment.

The main contribution of this paper is to develop a macroeconomic model in which both the *housing net worth channel* and the *bank net worth channel* are active, and, most importantly, to study the interaction between the two channels that results from the presence of endogenous mortgage foreclosures.

In particular, I build a New Keynesian DSGE model characterized by financial intermediaries facing an endogenous leverage constraint, similar to the one proposed by Gertler and Karadi (2011), and lending to two non-financial sectors: firms needing to finance the purchase of capital and "impatient" households requiring funds to purchase a house. A key innovation of this work is that I model the funding problem for homeowners by using mortgages with endogenous default. This is done by assuming that houses are subject to idiosyncratic depreciation risk, implying a default decision depending on borrowers' leverage and on house prices. The specification of the mortgage contract allows for simple aggregation, making the problem tractable even in a medium-scale DSGE framework.

In this model, a drop in the value of houses will have several implications for the real sector through the two channels described above and through their interaction.

First, a decline in house prices will have a "demand effect" through the housing net worth channel. The optimization problem of borrowing households implies that their demand for consumption will be proportional to their wealth, which is mainly composed by the value of their house. In addition, their demand for housing and their debt capacity depend on housing wealth, so that a second round of house prices declines exacerbates the decline in consumption. As a result of mortgagers' higher marginal propensity to consume, lower house price will imply lower aggregate demand for the final good, putting downward pressure on output and wages, especially if nominal rigidities are present. This housing collateral channel can generate comovements between house prices and aggregate consumption, but by itself it is not enough to obtain also lower business investment.

The drop in real investment can be obtained with a "spillover effect," taking place through the bank net worth channel. Lower house prices imply higher default rates because the value of dwellings becomes lower than the outstanding mortgages. This causes losses for financial intermediaries, both because of their missed mortgage payment and because of the depressed value of the foreclosed houses they acquire. As their net worth is eroded, leveraged banks experience a tighter borrowing constraint that forces them to deleverage by selling assets. As a result, financial intermediaries will also decrease their supply of business loans, implying a rise in the spread they charge on these assets and a drop in the price of capital, which further affects bank net worth through the well-known *financial accelerator* mechanism.

Finally, a key contribution of this paper resides in modeling the link between the net worths of banks and homeowners, which results in an "interaction effect" as a housing crisis unfolds. In particular, a tightening in their borrowing constraint also causes a decrease in banks' supply of mortgages and a consequent increase in the interest rate charged to borrowing households. This reduces households' demand for houses, depressing house prices further and increasing defaults even more. As a result of this interaction, both the housing net worth channel and the bank net worth channel are further amplified, causing a more pronounced downturn.

This process generates significant comovements among house prices, business investment, consumption and output, a feature that has been documented empirically and that characterized the recent recession.[1]

A key goal of the model is to reproduce a housing crisis in line with the data reported in figure 1. The first panels reports the FHFA house price index, which dropped more than 10% from the peak to the trough of the recession. The second panel shows how this house price decline was accompanied by a rapid increase in the foreclosure rate, which had been quite stable at around 1% before 2006 and reached 4.5% by the end of the recession. Delinquency rates, whose movements lead actual foreclosures, faced an even steeper increase. The third panel contains the path of the Asset-Backed Commercial Paper (ABCP) spread, an important measure of banks' cost of funding for mortgage-backed securities. This spread went from about 25 basis points in early 2007 to almost 230 basis points at the height of the financial crisis in the end of 2008, when the collapse of Lehman Brothers occurred.

In order to account for these movements in housing-related variables, I focus on two shocks which directly affect the supply of mortgage credit and that I define as "housing financial shocks." These shocks provide a realistic narrative of the financial crisis and are in line with the evidence found by Justiniano et al. (2015a, 2015b) that points to changes in the supply of credit as a more likely explaination of the housing boom and bust, compared to changes in the demand for credit.

[1] See, for example, Liu, Wang and Zha (2013) and Mian and Sufi (2010b).

The first shock that I consider is a "housing risk shock", modeled as an increase in the variance of the idiosyncratic housing depreciation distribution. The motivation behind this shock is to simulate an initial disturbance increasing the default rate on mortgages and their perceived riskiness, similar to what happened in the subprime market at the beginning of the Great Recession, when an increasing share of riskier borrowers pushed up the aggregate foreclosure rate. The increase in defaults interacts with all of the channels described above, and it produces a more severe downturn compared to a model with no financially constrained intermediaries. In fact, without leveraged banks, such a shock would not affect real investment and would imply smaller movements in consumption and output.

The second shock is specific to the financial sector and consists in the exogenous deterioration of the collateral value of mortgages for financial intermediaries. This event can be thought of as replicating the collapse of the market for MBS, which represented an important external credit channel for banks, and whose demise can be inferred from the third panel of figure 1. For this reason, I refer to this shock as an "MBS collateral shock." Such a shock directly tightens banks' leverage constraint, causing fire sales both in mortgage securities and in business loans, generating a crisis through the same channels affected by a housing risk shock. It is important to stress that both these shocks, unlike a capital quality shock or a productivity shock, would not have a real impact in a frictionless setup.

I show that both shocks, when calibrated to reproduce a drop in house prices similar to the one occurred in the recent recession, produce realistic increases in foreclosures and substantial declines in consumption, hours and investment. However, the MBS shock generates a much larger decline in investment, since it produces a stronger "spillover effect". This finding suggests that an MBS shock might have played a more important role during the U.S. financial crisis. This intuition is supported also by an experiment in which a series of MBS shocks, obtained by matching the path of house prices over the Great Recession, create paths for aggregate variables that are reasonably in line with the ones seen in the data.

In the last part of the paper, I use this model as a natural laboratory to evaluate the effects of different types of credit policies put in place by the Federal Reserve and by the U.S. government during the financial crisis, and which were focused on improving the conditions in the mortgage market. In particular, I study the impact of large-scale asset purchases (LSAP) in the mortgage market performed by the Federal Reserve, and of the Housing Affordable Modification Program (HAMP) launched by the U.S. government. The former consisted in direct purchases of mortgage-backed securities by the central bank, whereas the latter was a debt relief program, which included principal forebearance for leveraged households. I show that both policies, calibrated to replicate the size of actual interventions, are able to reduce the severity of the crisis, and they even imply welfare improvements for both lenders and borrowers when mortgage defaults entail some real costs.

The rest of the paper is organized as follows: section 2 reviews the literature, section 3 presents the baseline model, section 4 contains the quantitative excercises performed to simulate a crisis, section 5 introduces credit policy and analyzes its effects and section 6 concludes.

2 Related Literature

This paper is linked to the growing literature on macroeconomic models with financial frictions, which was initiated with the seminal works of Kiyotaki and Moore (1997) and Bernanke et al. (1999), and that has more recently been extended to introduce the role of financial intermediaries by, for example, Gertler and Karadi (2011), Gertler and Kiyotaki (2010), Brunnermeier and Sannikov (2011) and He and Krishnamurty (2013). In particular, my paper builds on the framework of Gertler and Karadi (2011), who first analyzed constrained banks and unconventional monetary policy in a DSGE model, and extends their work by introducing a set of borrowing households and defaultable mortgages. Compared to their paper, this model presents a more realistic characterization of the shocks initiating the financial crisis, which originated in the housing sector, and it also allows us to study the interaction between banks' "financial accelerator" and the wealth of borrowing households. In addition, my paper presents a more accurate description of the response enacted by the Federal Reserve during the crisis, which was focused on the market for MBS.

Abstracting from financial intermediation, a related strand of literature is the one studying the effects of shocks linked to the value of housing in models *á* la Kiyotaki and Moore (1997). For example, Iacoviello (2005), studies a model where also nominal contracts are present, while Iacoviello and Neri (2010) introduce a multi-sector structure and a richer set of shocks. However, in general, these models are not able to produce meaningful comovements between house prices and business investment, because capital financing is not subject to any agency problem. One way to overcome this issue is presented in Liu, Wang and Zha (2013). In their paper this correlation is obtained by assuming that houses also serve as collateral for credit-constrained entrepreneurs, so that a housing preference shock, together with a collateral shock, can explain investment fluctuations. My model can be interpreted as an attempt to obtain a more realistic foundation for this mechanism, and to provide a more accurate narrative of the events of the 2007-2009 crisis, by modeling the interaction between mortgage defaults and banks balance sheets and by focusing on residential land rather than commercial real estate. In addition, the focus of my work is on shocks affecting the supply of credit rather than the demand of housing or borrowed funds.

Another paper providing a DSGE model for the relationship between housing and the financial sector over the recent crisis is Iacoviello (2014), which extends the setup of Liu, Wang and Zha (2013) by having financial intermediaries lending both to entrepreneurs, who also have to finance a share of their wage bill in advance, and to impatient households, who use housing as collateral as well. In addition, banks are subject to a regulatory capital requirement. In this model, exogenous loan loss shocks, transferring resources from banks to borrowing households and entrepreneurs, generate a downturn and a comovement between house prices and non-residential investment. My paper endogenizes loan losses by explicitly modeling mortgage foreclosures, so that I can show how these default events can also be a consequence of credit supply shocks, a channel that is not present in the model by Iacoviello (2014).

As regards the modeling of mortgage defaults in a DSGE framework, the first paper to analyze this issue has been Forlati and Lambertini (2011). Their model extends the contractual framework of Bernanke et al. (1999) to the housing market but does not have a role for banks' net worth. As a result, a housing risk shock or a housing demand shock only has a modest impact on business investment. Lambertini et al. (2015) estimate a model based on Forlati and Lambertini (2011) and Iacoviello and Neri (2010), using U.S. data and perform a policy experiment similar to the HAMP exercise that I present in section 5. Compared to their analysis, I can also study the interaction of mortgage debt relief with the bank net worth channel, and consequently with real investment. Rabanal and Taheri Sanjani (2015) adapt the model of Forlati and Lambertini (2011) in order to provide a new measure of the output gap in a monetary union with financial frictions, but in their model there is no role for capital or constrained intermediaries.

The structure of the mortgage contract used in my model is based on Jeske et al. (2013), who study the welfare implications of the bailout guarantees provided by the U.S. government-sponsored enterprises in a model with heterogeneous agents. A simple modification of their setup allows me to obtain policy functions that are linear in homeowners' wealth, implying an easy aggregation that facilitates the analysis of the dynamic properties of the model.

Finally, among the papers introducing housing in incomplete markets models with heterogeneous agents, two relevant works are Favilukis, Ludvigson and Van Nieuwerburg (2011) and Kiyotaki, Michaelides and Nikolov (2008), both studying the implications of financial liberalization in a framework without banks but with two productive sectors and housing as a collateral for household finance.

3 The Model

I develop a medium-scale monetary DSGE model, based on Gertler and Karadi (2011). To their framework I add a second set of "impatient agents," who derive utility from housing services and purchase houses that are subject to idiosyncratic depreciation shocks. These agents can only borrow by issuing defaultable mortgages collateralized by their house. The specification of the housing market and of the mortgage contract will imply that borrowers' consumption and mortgage demand will be linked to the value of their dwelling, which represents the main component of their net worth.

Mortgages are financed by banks that also provide funds to non-financial goods producers for investment in capital. For simplicity, I assume that bank loans to firms are non-defaultable, resembling an equity security as in Gertler and Karadi (2011).[2] Financial intermediaries face an agency problem when raising funds from patient households, which implies an endogenous leverage constraint. As a result, bank net worth plays a crucial role in determining the supply of credit for firms and homeowners.

In addition, capital producers and monopolistically competitive retailers are also present in the model, where the latter serve only to obtain nominal price rigidities. In this section the central bank

[2] For a paper that focuses on the interaction between banks' balance sheet and firms' defaults see, for example, Navarro (2014).

only conducts conventional monetary policy, whereas unconvetional monetary policy is discussed in section 5.

3.1 Patient Households

There is a continuum of patient households that consume, save in the form of deposits, and provide labor.[3] As in Gertler and Karadi (2011), I assume that a fraction g of these agents are "workers," whereas a fraction $(1-g)$ are "bankers." Workers provide labor to the consumption-good sector and return their wages to their household. Bankers manage a financial intermediary that returns its profits to the family at the end of every period. In order to avoid that bankers save their way out the financial constraint, I assume that with probability $1-\sigma$ they exit the financial sector and become workers; at the same time a fraction $(1-g)(1-\sigma)$ of workers replaces them and keeps the proportion of types unchanged. New bankers will be endowed with some start-up funds, which I will explain in detail later. Bankers are the only agents that are able to lend funds to goods producers and impatient households. Within the household there is perfect consumption insurance. As a result, each patient household effectively owns a bank, but I assume that he invests in the deposits of an intermediary he does not own.

Whenever confusion is possible, I will use hatted variables to refer to patient households as opposed to impatient ones. Patient households gain utility from consumption $\hat{C}_t$, and have disutility from labor $\hat{N}_t$, according to the following preference structure:

$$\max \ E_t \sum_{i=0}^{\infty} \hat{\beta}^i U\left(\hat{C}_{t+i}, \hat{N}_{t+i}\right) = \max \ E_t \sum_{i=0}^{\infty} \hat{\beta}^i \left[log(\hat{C}_{t+i}) - \chi \frac{\hat{N}_{t+i}^{\gamma_n+1}}{\gamma_n + 1} \right] \tag{1}$$

In addition, patient households can save by using one-period risk-free debt issued by financial intermediaries that we can define deposits, D_t.[4] As a result, households maximize their discounted utility by choosing $\hat{C}_t, \hat{N}_t$, and D_t, subject to the following budget constraint

$$\hat{C}_t = \hat{w}_t \hat{N}_t + \Pi_t - D_t + R_t D_{t-1} \tag{2}$$

where $\hat{w}_t$ is the wage paid to patient agents, R_t is the risk-free rate and Π_t are profits from the ownership of banks and capital producing firms.

If we define $\Lambda_{t,t+1} = \hat{C}_t / \hat{C}_{t+1}$, we obtain the following first order conditions for labor and deposits

$$\chi \hat{N}_t^{\gamma_n} = \hat{w}_t / \hat{C}_t \tag{3}$$

$$1 = E_t \hat{\beta} \Lambda_{t,t+1} R_{t+1} \tag{4}$$

[3] I will refer to patient households also as lenders or depositors.

[4] As it will be clear in section 4, when government unconventional monetary policy is active, D_t represents both deposits and government debt.

3.2 Impatient Households

Impatient households discount the future with a discount factor $\beta < \hat{\beta}$.[5] Because in equilibrium impatient households borrow from banks, I will also refer to them as "borrowers." They derive utility from consumption c_t and housing services x_t, which can be obtained by renting a house at price $r_{x,t}$. I assume that the borrower's utility function is given by

$$U(c_t, x_t) = \rho_t \log c_t + (1 - \rho_t) \log x_t$$

where ρ_t represents a housing preference shock following an AR1 process.

Borrowers have access to two types of assets: a one-period mortgage m_t and houses h_t. For both assets I assume that short-selling is not possible. If a house is purchased at time t at a price q_t^h, next period a fraction ι_x can be used to produce housing services that can be sold for $r_{x,t}$.[6] The remaining fraction $1 - \iota_x$ can be rented to goods producing-firms, which employ housing in their Cobb-Douglas production function and pay a rental rate $r_{y,t}$.[7] I generically define the convex combination of the two rental rates, $r_t = \iota_x r_{x,t} + (1 - \iota_x) r_{y,t}$, as rent.

The assumption that only impatient households invest in houses, and derive utility from it, is meant to capture the segmentation in the US housing market, where there is little trading of houses between rich agents (lenders) and poor ones (borrowers), indicating that the two types of houses can probably be considered as two different types of goods.[8] A similar assumption is also used also by Justiniano et al. (2015), and has the important implication of having houses priced by borrowing agents. This stylized framework produces a set of richer lending agents whose wealth is mainly composed of capital, and a set of borrowing agents whose wealth crucially depends on house prices. This result is consistent with the finding of Mian et al. (2013), who showed that poorer and more levered households had a significantly higher marginal propensity to consume out of housing wealth during the 2007-2009 recession, whereas wealthier households' consumption did not react as much or even increased in response to the drop of house prices. As shown in the quantitative experiments, this assumption implies movements in consumption and house prices consistent with the experience of the Great Recession.

As in Jeske et al. (2013), houses are subject to idiosyncratic depreciation shocks ξ_t, so that in period t, after having rented the house, the owner is left with $\xi_t h_{t-1}$ units of housing.[9] The shock

[5] I will refer to impatient households also as "borrowers" or "homeowners". The lower discount factor guarantees that in the steady state of the model they are willing to borrow by issuing mortgages.

[6] The presence of two distinct markets for housing services and houses simplifies aggregation for impatient households because the rental rate equalizes the marginal utility from housing across agents.

[7] The assumption that houses are used in the production function is needed for the impatient agents to have sufficient income for consumption, after they have paid for housing rent and house purchases. In a previous version of the paper, instead of assuming that houses could be used in the goods production function, I assumed that impatient agents provided labor inelastically. The two frameworks produce similar quantitative results, but the current one is analytically more tractable. The modeling of a variable labor supply by impatient agents with an endogenous default decision poses challanges to aggregation, and is the subject of future research.

[8] For evidence of housing market segmentation, see, for example, Landvoigt et al. (2013).

[9] I assume that the shock affects the value of the house after it has been rented, but this assumption has no important effects on the analytical and quantitative results of the paper.

ξ_t follows a cdf $F(\xi_t, \lambda_t)$ where λ_t is an exogenous disturbance, following an AR1 process, that I define as "housing risk," affecting the variance of the distribution but not the mean. In particular $E_t(\xi_t) = 1$ for any λ_t, so that houses are in aggregate fixed supply $\bar{H}$.[10]

The only way for impatient households to borrow is to use a one-period defaultable mortgage m_t, collateralized by the house they purchase. After renting their house, and after observing the realization of their idiosyncratic shock ξ_t, borrowers can decide to default on their outstanding debt m_{t-1} at the only cost of losing their collateral, whose value is $\xi_t q_t^h h_{t-1}$. There is no other cost for defaulting households, and they can immediately purchase new housing with their available wealth. Such an assumption implies that borrowers will default whenever the value of their house is lower than the face value of their mortgage, that is if $\xi_t q_t^h h_{t-1} < m_{t-1}$. This specification of the default decision is similar to the one used in Jeske et al. (2013). As a result, a borrower will default whenever he is hit by an idiosyncratic housing shock that is below a certain threshold $\bar{\xi}_t$, given by

$$\bar{\xi}_t(\eta_{t-1}) = \frac{m_{t-1}}{q_t^h h_{t-1}} = \frac{\eta_{t-1}}{q_t^h} \tag{5}$$

where $\eta_t = \frac{m_t}{h_t}$ represents the impatient household's leverage. Hence, the borrower will be more likely to default the higher is his outstanding leverage η_{t-1}, and the lower are current house prices q_t^h.

As I will show in the following sections, this simple characterization of the default decision implies that the only individual variable affecting the price of the mortgage, Q_t, will be η_t, so that in the household problem we can use the notation $Q_t(\eta_t)$.

3.2.1 Recursive Formulation of the Impatient Agent Problem

It is useful to separate the problem of the impatient household between a static decision on the expenditures' allocation between consumption and housing services, and a dynamic consumption-saving decision. In particular, if we define $\tilde{c}_t$ as the total expenditures in consumption and housing services, then we can write the static problem as

$$u(\tilde{c}_t, r_{x,t}) = \max_{c_t, x_t} U(c_t, x_t) \text{ s.t.}$$

$$c_t + r_{x,t} x_t = \tilde{c}_t$$

Given the logarithmic form of the utility function, it is easy to show that[11]

$$u(\tilde{c}_t, r_{x,t}) = \log(\tilde{c}_t) + \Theta(\rho_t, r_{x,t})$$

[10] The modeling of housing in fixed supply follows Liu, Wang and Zha (2013) and is justified by the finding that most of the fluctuations in house prices are driven by land prices rather than by the cost of reproducible structures (Davis and Heathcote, 2007).

[11] The formula for $\Theta(\rho_t, r_{x,t})$ can be found in the appendix.

In addition, it can be showed that consumption and housing services are a fraction ρ_t and $(1-\rho_t)$ of total expenditures respectively

$$c_t = \rho_t \tilde{c}_t \tag{6}$$

$$r_{x,t} x_t = (1-\rho_t)\, \tilde{c}_t \tag{7}$$

so that once we have characterized the choice of $\tilde{c}_t$ we can easily obtain also c_t and x_t.

Define ω_t as the wealth for the borrower in period t after the default decision has taken place. This represents the individual state variable and it includes the income from renting the house the borrower owns and, if defaulted has not occurred , the difference between the value of the house and the value of the mortgage, so that $\omega_t = \max\left\{h_{t-1}\left[\left(q_t^h \xi_t + r_t\right) - \eta_{t-1}\right], h_{t-1} r_t\right\}$.[12]

The problem of the borrower will then be to choose total expenditures $\tilde{c}_t$, houses h_t and leverage η_t in order to solve

$$V_t(\omega_t) = \max_{\tilde{c}_t, h_t, \eta_t} \left\{u\left(\tilde{c}_t, r_{x,t}\right) + \beta E_t V_{t+1}(\omega_{t+1})\right\}$$

$$\text{s.t.} \quad \tilde{c}_t + h_t\left[q_t^h - Q_t(\eta_t)\,\eta_t\right] \leq \omega_t \tag{8}$$

$$\omega_{t+1} = \begin{cases} h_t\left[\left(q_{t+1}^h \xi_{t+1} + r_{t+1}\right) - \eta_t\right] & \text{if} \quad \xi_{t+1} \geq \bar{\xi}_{t+1}(\eta_t) \\ h_t r_{t+1} \quad \text{if } \xi_{t+1} < \bar{\xi}_{t+1}(\eta_t) \end{cases} \tag{9}$$

Equation (8) represents the budget constraint, where $\left[q_t^h - Q_t(\eta_t)\,\eta_t\right]$ is the down payment needed to purchase a house that is financed with a mortgage equal to a fraction η_t of the housing good. Equation (9) is the evolution of financial wealth, which depends on whether default occurs or not. As mentioned in the previous section, the default threshold $\bar{\xi}_t$ can be written as a function of last period's leverage η_{t-1}. It is important to notice that the borrower internalizes how its leverage choice affects his default probability next period, and hence the interest rate that the lender will charge on the mortgage, $1/Q_t(\eta_t)$.

We can define the return at time t+1 on a house financed with leverage of η_t as

$$R_{t+1}^h(\eta_t, \xi_{t+1}) = \frac{\max\left(r_{t+1}, \left(q_{t+1}^h \xi_{t+1} + r_{t+1}\right) - \eta_t\right)}{\left[q_t^h - Q_t(\eta_t)\,\eta_t\right]} \tag{10}$$

In particular, in case of default the return is given only by r_{t+1}, otherwise it also includes the difference between the residual value of the house and the face value of the mortgage.

The non-standard features of the impatient agent's problem are the possibility of default and the fact that he internalizes how his leverage decision affects the price of his debt. However, given the preference structure and the simple characterization of default, which does not require us to keep track of the default history, this problem has a simple solution as described in the following proposition.[13]

[12] Again, the rent used in this formula is the combination of residential rent and commercial rent $r_t = \iota_x r_{x,t} + (1-\iota_x)\, r_{y,t}$.

[13] The proof of Proposition 1 can be found in the appendix.

Proposition 1 *Given prices, the borrower's optimal choices for consumption, housing services, housing, and mortgage debt are linear in wealth:*

$$c_t = \rho_t (1-\beta)\omega_t \tag{11}$$

$$r_{x,t}x_t = (1-\rho_t)(1-\beta)\omega_t \tag{12}$$

$$h_t = \frac{1}{\left[q_t^h - Q_t(\eta_t)\eta_t\right]}\beta\omega_t \tag{13}$$

$$m_t = \eta_t h_t \tag{14}$$

where η_t *is determined by*

$$\frac{d\left[Q_t(\eta_t)\eta_t\right]}{d\eta_t} = E_t\left\{\frac{1}{R_{t+1}^h(\eta_t,\xi_{t+1})}1\left\{\xi_{t+1} \geq \frac{\eta_t}{q_{t+1}^h}\right\}\right\} \tag{15}$$

and the evolution of wealth follows

$$\omega_{t+1} = \beta\omega_t R_{t+1}^h(\eta_t,\xi_{t+1}) \tag{16}$$

The policy functions for consumption and housing services expenditures, eq (11) and (12), simply follow from the fact that given log-utility, consumption expenditures $\tilde{c}_t$ will be a constant fraction $(1-\beta)$ of wealth.[14] Combining this fact with equations (6) and (7) delivers equations (11) and (12).

The leverage decision of the impatient agent is described by equation (15). The left-hand side represents the benefits of issuing a mortgage equal to a fraction η_t of the housing that the borrower is purchasing. In particular, this quantity can be rewritten as

$$\frac{d\left[Q_t(\eta_t)\eta_t\right]}{d\eta_t} = Q_t(\eta_t) + \eta_t Q_t'(\eta_t)$$

where the first term represents the amount received per unit of mortgage, whereas the second term takes into account how a marginal increase in η_t will affect the pricing of the mortgage. As I will explain in the following section, $Q_t'(\eta_t) < 0$, due to the presence of foreclosure costs and to fact that a higher leverage increases the probability of default next period. The right-hand side of equation (15) represents the expected mortgage costs next period, which are given by the repayment of the face value of debt, but only in the non-default states.[15]

An important result is that equation (15), determining η_t, only depends on aggregate variables. This implies that this variable will be the same for every impatient household, so that all borrowers

[14] The linearity of the policy functions would still be present as long as we focus on homothetic utility functions with homogeneous budget constraints. CRRA utility would satisfy this requirement, but it would imply a time varying saving rate instead of a constant one.

[15] The term $1\left\{\xi_{t+1} \geq \frac{\eta_t}{q_{t+1}^h}\right\}$ represents an indicator function that takes the value of 1 when $\xi_{t+1} \geq \frac{\eta_t}{q_{t+1}^h}$. As shown

will have the same leverage and consequently only one type of risky mortgage will be traded in equilibrium.

Given η_t, equations (13) and (14) simply follow from the budget constraint and the definition of η_t. Finally, equation (16) is obtained from (9) together with (13).

As I will show in the next subsection, the linearity of the policy functions, together with the fact that η_t only depends on aggregate variables, will allow for a simple aggregation of the choices of impatient households, without having to keep track of the wealth distribution of this type of agent.[16]

3.2.2 Aggregation for Impatient Agents

If we define H_t as the aggregate amount of houses for impatient agents, we can write the evolution of their aggregate net worth for impatient agents, NW_t^{imp}, as

$$NW_t^{imp} = \int \omega_t^i di = H_{t-1} \left\{ r_t + q_t^h \int_{\bar{\xi}_t(\eta_{t-1})}^{\infty} \xi_t f\left(\xi_t, \lambda_t\right) d\xi_t - \left(1 - F\left(\bar{\xi}_t\left(\eta_{t-1}\right), \lambda_t\right)\right) \eta_{t-1} \right\} \quad (17)$$

where I have used the result that η_t is the same for all borrowers together with (9). Therefore, in addition to the value of rents, the aggregate wealth for impatient agents will be increasing in the value of the houses of non-defaulting agents and decreasing in their outstanding debt.

In addition, the linearity of the policy functions implies that the borrowers' aggregate demand for consumption goods C_t, housing services X_t and houses will follow

$$C_t = \rho_t \left(1 - \beta\right) NW_t^{imp} \quad (18)$$

$$r_{x,t} X_t = \left(1 - \rho_t\right)\left(1 - \beta\right) NW_t^{imp} \quad (19)$$

$$H_t = \frac{1}{\left[q_t^h - Q_t\left(\eta_t\right)\eta_t\right]} \beta NW_t^{imp} \quad (20)$$

Equation (18) together with (17) show how the consumption of impatient agents is affected by the value of houses, since it can be shown that NW_t^{imp} is increasing in q_t^h. Therefore, (18) captures the housing net worth channel that has been emphasized by Mian et al. (2013), and which allows the model to produce a drop in consumption of leveraged agents when house prices decline. In addition, from equation (20) we can see that the aggregate demand for housing is linear in net worth and increasing in the amount of dollars raised from mortgages per unit of housing, $Q_t\left(\eta_t\right)\eta_t$.

in the appendix, this implies that we can rewrite the right-hand side of equation (15) as

$$E_t \left[\int_{\bar{\xi}_{t+1}(\eta_t)}^{\infty} \frac{\left[q_t^h - Q_t\left(\eta_t\right)\eta_t\right]}{\left[\left(q_{t+1}^h \xi_{t+1} + r_{t+1}\right) - \eta_t\right]} f\left(\xi_{t+1}\right) d\xi_{t+1} \right]$$

[16] This constrasts, for example, with the model of Forlati and Lambertini (2011), in which perfect consumption sharing, and perfect housing sharing, need to be assumed in order to keep the model tractable.

As a result, *ceteris paribus,* a lower supply of mortgages, in the form of a lower $Q_t(\eta_t)$, can put downward pressure on house prices, igniting the housing net worth channel.

3.3 The Banker's Problem

The role of banks is to transfer funds from patient households to intermediate goods producers to finance capital purchases, and to impatient households, to finance house purchases. I will refer to the first type of assets as loans, z_t, and to the second one as mortgages m_t.

As in Gertler and Karadi (2011), I assume that there is no friction between bankers and non-financial firms, so that goods producers can issue a state contingent security, which can be thought of as equity, whose price will be equal to the price of capital q_t^k, and which will provide a return R_t^k.[17]

As described above, the relationship between banks and homeowners is characterized by defaultable debt. In particular, each bank can potentially invest in a continuum of mortgages, each indexed by the leverage of the borrowing household, $m_t(\eta_t)$, and for which the banker will pay a price $Q_t(\eta_t)$. The expected return per unit of a mortgage with leverage η_t and price $Q_t(\eta_t)$ will be

$$E_t R_{t+1}^m(\eta_t) = E_t \frac{\left\{ \left[1 - F\left(\bar{\xi}_{t+1}(\eta_t), \lambda_{t+1}\right)\right] + \gamma \frac{q_{t+1}^h}{\eta_t} \int_0^{\bar{\xi}_{t+1}(\eta_t)} \xi_{t+1} dF\left(\xi_{t+1}, \lambda_{t+1}\right) \right\}}{Q_t(\eta_t)} \tag{21}$$

$$= E_t \frac{\varrho_{t+1}\left(\eta_t, \lambda_{t+1}, \xi_{t+1}\right)}{Q_t(\eta_t)} \tag{22}$$

Equation (21) is important to understand the expected return of a bank financing a mortgage. The term $\varrho_{t+1}\left(\eta_t, \lambda_{t+1}, \xi_{t+1}\right)$ represents the payoff on a mortgage with a loan-to-value ratio equal to η_t. With probability $1 - F\left(\bar{\xi}_{t+1}(\eta_t), \lambda_{t+1}\right)$ the debt is repaid, and the bank receives the face value of the mortgage. Otherwise, when $\xi_{t+1} < \eta_t / q_{t+1}^h$, the household defaults and walks away and the bank can repossess an amount of housing whose value before depreciation is $q_{t+1}^h h_t = q_{t+1}^h m_t / \eta_t$. In addition, I assume that there are also default costs, equal to a fraction $(1-\gamma)$ of the value of the house, which are incurred in the foreclosure process.

Each bank finances itself with retained earnings n_t, and by issuing risk-free deposits d_t to patient households. As a result, we can write the budget constraint for a bank as

$$q_t^k z_t + \int Q_t(\eta_t) m_t(\eta_t) d\eta_t = n_t + d_t$$

[17] At the cost of additional complexity it would be possible to model also defaultable loans to non-financial firms, by assuming some idiosyncratic disturbance to the firm return and a default decision similar to the one of impatient households. For an example see Navarro (2014).

We can then characterize the evolution of the net worth of an individual bank as

$$n_{t+1} = q_t^k z_t R_{t+1}^k + \int \left\{ Q_t(\eta_t) m_t(\eta_t) R_{t+1}^m(\eta_t) \right\} d\eta_t - R_{t+1} d_t \tag{23}$$

$$= q_t^k z_t \left(R_{t+1}^k - R_{t+1} \right) + \int Q_t(\eta_t) m_t(\eta_t) \left[R_{t+1}^m(\eta_t) - R_{t+1} \right] d\eta_t + n_t R_{t+1} \tag{24}$$

As long as the banker makes an expected return on his assets greater than or equal to R_{t+1}, he will choose z_t, m_t and d_t in order to maximize the accumulated value of his net worth before it has to exit and become a worker. Hence, his value function at the end of time t, before knowing the realization of the exit random variable, is given by

$$V_t^{bank} = E_t \sum_{i=0}^{\infty} (1-\sigma) \sigma^i \hat{\beta}^{i+1} \Lambda_{t,t+1+i} n_{t+1+i} \tag{25}$$

where σ is the probability of staying in the market. As I described above, banks are owned by patient households, and for this reason their stochastic discount factor enters the value function in (25). In addition, as in Gertler and Karadi (2011), I introduce an agency problem between the bank and the depositors in order to limit the amount of risky assets that the financial sector can hold and generate accordingly a gap between the rate of returns on assets and liabilities. In particular, I assume that after raising deposits, the banker can default and divert back to his own household a fraction θ^k of his loans and a fraction θ_t^m of his mortgages. If the banker does so, depositors can force him to bankruptcy and consequently to leave the banking sector forever, while recovering the remaining fractions of the assets.

As a result, the banker's problem entails the following incentive constraint, needed for patient households to provide deposits to the bank

$$V_t^{bank} \geq \theta_t^m \left[\int Q_t(\eta_t) m_t(\eta_t) d\eta_t \right] + \theta^k q_t^k z_t \tag{26}$$

This constraint guarantees that the value from continuing to operate the bank, the left-hand side, is larger than the value of running away with the diverted assets. In addition, I assume that θ_t^m is subject to exogenous shocks according to

$$\log \theta_t^m = (1 - \rho_{\theta^m}) \log \theta_{ss}^m + \rho_{\theta^m} \log \theta_{t-1}^m + \varepsilon_{\theta^m,t}$$

The idea is that such disturbances should capture changes in the tightness of funding markets for mortgage-backed securities that are not related to fundamental shocks in the model. In particular, $\varepsilon_{\theta^m,t}$ is a shock specific to the financing of mortgages. In the numerical experiments, I will focus on a shock affecting θ_t^m as a stylized way to capture the collapse of the market for mortgage-backed securities and of securitization, which followed run episodes in several non-traditional banking markets.[18]

[18] For models studying the effects of bank runs in a macroeconomic framework see, for example, Gertler and Kiyotaki

We can write the banker's value function recursively as follows

$$V_t^{bank}(n_t) = \max_{z_t, \{m_t(\eta_t)\}_{\eta_t}} E_t \hat{\beta} \Lambda_{t,t+1} \left\{ (1-\sigma) n_{t+1} + \sigma V_{t+1}^{bank}(n_{t+1}) \right\}$$

where the maximization is subject to (26) and (23).

It can be showed that the value function for the banker is linear in net worth and can be rewritten as $V_t(n_t) = \nu_t n_t$.[19] If we define μ_t as the multiplier on the incentive constraint, the implied first order conditions for z_t and m_t are

$$E_t \hat{\beta} \Lambda_{t,t+1} \Omega_{t+1} \left(R_{t+1}^k - R_{t+1} \right) = \mu_t \theta^k$$

$$E_t \hat{\beta} \Lambda_{t,t+1} \Omega_{t+1} \left(R_{t+1}^m(\eta_t) - R_{t+1} \right) = \mu_t \theta_t^m \quad \forall \eta_t$$

where $\Omega_t = \{(1-\sigma) + \sigma \nu_t\}$ represents the adjusted marginal value of net worth. As a result, if the constraint does not bind, $(\mu_t = 0, \Omega_t = 1)$, the expected discounted return on both bank assets should be equal to the risk-free rate. However, when the constraint binds, loans and MBS will imply an excess return on the risk-free rate.

In addition, the equations above imply the following no-arbitrage relationship

$$E_t \hat{\beta} \Lambda_{t,t+1} \Omega_{t+1} \left(R_{t+1}^m - R_{t+1} \right) = \frac{\theta_t^m}{\theta^k} E_t \hat{\beta} \Lambda_{t,t+1} \Omega_{t+1} \left(R_{t+1}^k - R_{t+1} \right) \tag{27}$$

Equation (27) establishes a link between the expected returns on capital and houses, which is also going to depend on the tighteness of the leverage constraint, as measured by μ_t. In particular, in steady state, if $\theta^m < \theta^k$, the excess return on MBS will be lower than the one on loans to the productive sector.

Given the linear form of the value function, it can be showed that, when the constraint is binding, the following endogenous constraint on bank's adjusted leverage will be in place

$$\left[q_t^k z_t + \frac{\theta_t^m}{\theta_t^k} \int Q_t(\eta_t) m_t(\eta_t) d\eta_t \right] \leq \phi_t n_t \tag{28}$$

where

$$\phi_t = \frac{E_t \hat{\beta} \tilde{\Lambda}_{t,t+1} R_{t+1}}{\theta^k - E_t \hat{\beta} \tilde{\Lambda}_{t,t+1} \left(R_{t+1}^k - R_{t+1} \right)} \tag{29}$$

and $\tilde{\Lambda}_{t,t+1} = \Lambda_{t,t+1} \Omega_{t+1}$.

The constraint in (28) sets the value of the bank portfolio at a point such that the incentive constraint is exactly satisfied. In particular, if $\theta_t^m < \theta^k$, this implies a slacker limit on the bank's investment in mortgages. Also, the maximum leverage ratio will be inversely related to θ^k and positively related to the spread in expected returns. Equation (28) is at the heart of the standard

(2015), Gertler, Kiyotaki and Prestipino (2015), and Ferrante (2015).

[19] See the appendix for a detailed solution of the problem of the financial intermediary.

bank financial accelerator by linking banks asset demand to their net worth.

In addition, we can rewrite equation (27) in order to obtain the mortgage pricing equation that impatient agents will internalize when choosing their optimal leverage, that is

$$\begin{aligned} Q_t(\eta_t) &= \frac{E_t \hat{\beta} \tilde{\Lambda}_{t,t+1}}{E_t \hat{\beta} \tilde{\Lambda}_{t,t+1} R_{t+1} + \theta_t^m \mu_t} \varrho_{t+1}(\eta_t, \lambda_{t+1}, \xi_{t+1}) \quad (30) \\ &= E_t \tilde{\Omega}_{t+1} \varrho_{t+1}(\eta_t, \lambda_{t+1}, \xi_{t+1}) \quad (31) \end{aligned}$$

This relationship will be crucial for the additional amplification mechanism present in this paper. In fact, $\tilde{\Omega}_{t+1}$ is the stochastic discount factor that bankers use to price risky mortgages. During a crisis, the incentive constraint on financial intermediaries becomes tighter. As a result, *ceteris paribus*, μ_t increases, putting downward pressure on $Q_t(\eta_t)$ and increasing the spread charged on mortgages. As I will show in the quantitative exercises, by reducing borrowers' demand for housing and consequently depressing house prices, this mechanism will be key in reinforcing the effects of the housing net worth channel and the bank net worth channel.

Equation (30) also shows how the costly default of mortgages introduces an additional spread between the cost of funding for banks and the one for impatient households. In fact, since the term $\varrho_{t+1}(\eta_t, \lambda_{t+1}, \xi_{t+1})$ in eq. (30) is smaller than one, this implies that

$$E_t \tilde{\Omega}_{t+1} \frac{1}{Q_t(\eta_t)} > 1 = E_t \tilde{\Omega}_{t+1} R_{t+1}^m \quad (32)$$

where the right-hand side can be interpreted as the required rate of return for bankers. Therefore the price of a mortgage will include an additional default-premium that compensates financial intermediaries for the possibility of costly foreclosure.

In addition, we can use (30) to compute the derivative of the mortgage price with respect to leverage. In particular, we obtain

$$Q_t'(\eta_t) = -E_t \tilde{\Omega}_{t+1} \frac{1}{\eta_t} \left\{ f\left(\bar{\xi}_{t+1}(\eta_t), \lambda_{t+1}\right) \frac{\eta_t}{q_{t+1}^h} (1-\gamma) + \gamma \frac{q_{t+1}^h}{\eta_t} \int_0^{\bar{\xi}_{t+1}} \xi_{t+1} dF\left(\xi_{t+1}, \lambda_{t+1}\right) \right\} < 0 \quad (33)$$

The negative relationship between mortgage prices and leverage is intuitive, since a higher leverage implies a higher probability of default. Furthermore, it can be showed that

$$\frac{d\left[Q_t(\eta_t)\eta_t\right]}{d\eta_t} = Q_t(\eta_t) + \eta_t Q_t'(\eta_t) = E_t \tilde{\Omega}_{t,t+1} \left\{ \left[1 - F\left(\bar{\xi}_{t+1}(\eta_t), \lambda_{t+1}\right)\right] - (1-\gamma) f\left(\bar{\xi}_{t+1}(\eta_t), \lambda_{t+1}\right) \frac{\eta_t}{q_{t+1}^h} \right\} \quad (34)$$

a quantity that is needed to determine the optimal η_t in (15).

Finally, it has to be noted that if the constraint does not bind, then $\mu_t = 0$, $\tilde{\Lambda}_{t,t+1} = \Lambda_{t,t+1}$ and $E_t \hat{\beta} \tilde{\Lambda}_{t,t+1} R_{t+1} = 1$ so that

$$Q_t(\eta_t) = E_t \beta \Lambda_{t,t+1} \varrho_{t+1}^m(\eta_t, \xi_{t+1}) \quad (35)$$

When the incentive problem does not play a role, banks will be just a veil and the mortgages will be priced with the stochastic discount factor of patient households. Equation (35) will be used instead of equation (30) to simulate the model without financially constrained banks, and to evaluate the amplification that ensues from the bankers' agency problem.

3.4 Aggregation in the Banking Sector

Given the linearity of the incentive constraint in (28), the fact that ϕ_t only depends on aggregate quantities, and that in equilibrium all mortgages will have the same leverage, we can obtain the following aggregate version of the constraint on the bank portolio

$$\left[q_t^k Z_t + \frac{\theta_t^m}{\theta_t^k} Q_t\left(\eta_t\right) M_t^b\right] \leq \phi_t NW_t^b \tag{36}$$

where M_t^b and Z_t represent banks' aggregate holdings of mortgages and loans, whereas NW_t^b is the aggregate net worth of the financial system. Importantly, equation (36) relates the value of assets held by intermediaries to the aggregate level of their net worth, so that any shock negatively affecting this variable will put downward pressure on Q_t and q_t^K.

The evolution of aggregate net worth will be given by the wealth of the surviving bankers plus a transfer that patient households will provide to the new bankers, equal to a fraction $\varpi/(1-\sigma)$ of the value of the assets of exiting bankers

$$NW_t^b = \sigma[R_t^m Q_{t-1} M_{t-1}^b + R_{t+1}^k q_t^k Z_t - R_{t+1} D_t] + NW_t^e \tag{37}$$

where

$$NW_t^e = \varpi(Q_t M_{t-1}^b + q_t^k Z_{t-1}) \tag{38}$$

From equation (37) we see how any shock affecting the realized return of the two types of assets will directly impact aggregate net worth. This effect will be larger for the asset representing a larger share of the aggregate portfolio.

3.5 Intermediate Goods Producers

Consumption good producers are competitive and produce output to be sold to retailers at the real price P_t^m. They operate a standard Cobb-Douglas technology using capital, labor and non-residential real estate $H_{y,t}$.

$$Y_t = A_t K_{t-1}^{\alpha_k} \hat{N}_t^{\alpha_n} H_{y,t-1}^{1-\alpha_k-\alpha_n}$$

where A_t represents an aggregate productivity shock that follows an AR1 process.

The first order conditions with respect to labor and housing will imply

$$\hat{w}_t = A_t \alpha_n \frac{P_t^m Y_t}{\hat{N}_t} \tag{39}$$

$$r_{y,t} = A_t \left(1 - \alpha_k - \alpha_n\right) \frac{P_t^m Y_t}{H_{y,t-1}} \tag{40}$$

where $\hat{w}_t$ is the real wage and $r_{y,t}$ is the rental rate of non-residential housing.

The firm has no initial endowment and needs to fund the purchase of capital by issuing state contingent debt claims Z_t equal to the amount of new capital acquired K_t. By no-arbitrage, these claims will have a price equal to the price of capital q_t^K. In particular, given that the firm will make zero profits state by state, we have that the one-period return on capital, obtained by the bank, will be given by:

$$R_{t+1}^K = \frac{P_{t+1}^m \alpha Y_{t+1}/K_t + (1-\delta_k) q_{t+1}^k}{q_t^k} \tag{41}$$

where δ_k is the depreciation rate of capital.

3.6 Capital Producers

Capital good producers create new capital by combining final good input I_t with aggregate capital K_{t-1} according to the technology $\Phi\left(\frac{I_t}{K_{t-1}}\right) K_{t-1}$. They operate competitively and sell the new capital at the price q_t^k. Their problem will be given by

$$\max_{I_t,} \left[q_t^k \Phi\left(\frac{I_t}{K_{t-1}}\right) K_{t-1} - I_t \right]$$

As a result the price of capital will satisfy

$$q_t^k = \left[\Phi'\left(\frac{I_t}{K_{t-1}}\right)\right]^{-1}$$

Following Bocola (2015), I use $\Phi(x) = a_1 x^{1-\gamma_i} + a_2$ where γ_i will measure price elasticity with respect to investments and a_1 and a_2 are normalizing parameters used to obtain a steady state price of unity.

3.7 Final Goods Producers

The final output Y_t is a CES composite of a continuum of varieties produced by retail firms, owned by patient households, that employ intermediate output as input. The final good composite is

$$Y_t = \left[\int_0^1 Y_t(z)^{(\varepsilon-1)/\varepsilon} dz\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{42}$$

where $Y_t(z)$ is the output produced by firm f. Each retailer (f) faces the demand function

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} Y_t \tag{43}$$

where the aggregate price level P_t is given by

$$P_t = \left[\int (P_t(z))^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}} \tag{44}$$

In addition, I introduce nominal rigidities by assuming that each period a firm is able to adjust its prices only with probability $(1-\zeta)$. As a result, the problem for the firm-setting firm is to select P_t^* to maximixe

$$E_t \sum_{i=0}^{\infty} \zeta^i \hat{\beta}^i \Lambda_{t,t+1} \left[\frac{P_t^*}{P_{t+i}} - P_{t+i}^m \right] Y_{t+i}^*(z) \tag{45}$$

so that the first order condition will be given by

$$E_t \sum_{i=0}^{\infty} \zeta^i \hat{\beta}^i \Lambda_{t,t+1} \left[\frac{P_t^*}{P_{t+i}} - \frac{\varepsilon}{\varepsilon - 1} P_{t+i}^m \right] Y_{t+i}^*(z) \tag{46}$$

Finally, aggregating over (44) we obtain the following evolution for P_t

$$P_t = \left[(1-\zeta)\left(P_t^*\right)^{(1-\varepsilon)} + \zeta \left(P_{t-1}\right)^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}$$

3.8 Market Clearing and Resource Constraint

The equilibrium in the capital market requires that the value of the loans held by financial intermediaries equals the value of capital in place at time t

$$Z_t = K_t$$

In addition, the equlibrium in the housing market will be given by

$$H_t = \bar{H} \tag{47}$$

implying also the following conditions for the clearing of the housing services market and the non-residential real estate market

$$X_t = \iota_x \bar{H}$$

$$H_{y,t} = (1-\iota_x)\bar{H}$$

The evolution of aggregate capital will be given by

$$K_t = (1-\delta_k) K_{t-1} + \Phi\left(\frac{I_t}{K_{t-1}}\right) K_{t-1}$$

As in Gomes et al. (2014), I allow for the possibility of real costs of bankruptcy, and I define output net of default costs as

$$\bar{Y}_t = Y_t - I_{DefCost} * (1-\gamma)\, q_t^h H_{t-1} \int_0^{\bar{\xi}_t} \xi_t dF(\xi_t, \lambda_t)$$

where the parameter $I_{DefCost} \in \{0, 1\}$. If $I_{DefCost} = 1$, the value used in the baseline calibration, defaults imply also a cost in terms of the final good, representing, for example, the legal fees linked to the lengthy foreclosure process. In case of $I_{DefCost} = 0$, mortgage defaults entail no real cost and the share of foreclosed houses not repossessed by the bank is simply redistributed to borrowing agents. As I will show in the following section, most of the quantitative results do not depend on the value of this parameter.

As a result, we can write the aggregate resource constraint as

$$\bar{Y}_t = C_t + \hat{C}_t + I_t \tag{48}$$

and aggregate consumption as

$$\bar{C}_t = C_t + \hat{C}_t$$

3.9 Monetary Policy

Monetary policy is characterized by the following Taylor rule

$$(i_t) = (i_{t-1})^{(\rho_i)} \left[(i_{ss}) (\pi_t)^{\kappa_\pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{\kappa_Y} \right]^{(1-\rho_i)} \left(\varepsilon_t^i \right) \tag{49}$$

where changes in output are used as a proxy for the output gap, ρ_i is a smoothing parameter on interest rates, ε_t^i is a monetary policy shock and the gross nominal rate i_t is given by the Fisher equation

$$i_t = R_{t+1} E_t \pi_{t+1} \tag{50}$$

I delay the description of unconventional monetary policy to section 5.

4 Quantitative Analysis

4.1 Calibration

The model is solved by perturbation methods after log-linearization, and it is calibrated to have a steady state in which the bank incentive constraint is always binding.[20] Table 1 summarizes the parameter values used for the numerical simulations. The time horizon is quarterly and the calibration is aimed at matching some aggregate quantities as a share of GDP, where this variable does not include the value of rents coming from the housing sector.

[20] For the solution of a model with bankers a la Gertler and Karadi (2011) and an occasionally binding incentive constraint see Bocola (2015) or Prestipino (2014).

The preference parameters for patient households are standard: I use a discount factor $\hat{\beta}$ equal to .99 and the parameter determining the Frish elasticity of labor supply, γ_n, is the same as the one used in Gertler and Karadi (2011).

For impatient households, β,ρ and ι_x are calibrated to match the following quantities: an average loan-to-value ratio η equal to 0.8, which is a conservative estimate relative to the high household leverage experienced in the years leading to the crisis; a value of residential rents equal to about 15% of output, close to the value used by Jeske et al. (2013); and a ratio of residential rents over house prices equal to 3.5%, close to the number for 2006 reported by Morris et al. (2008).[21] The implied parameters are $\beta = .81, \rho = .28$ and $\iota_x = .99$, so that only a very small share of the housing stock is used for non-residential purposes.[22]

I set γ equal to 0.8, in line with the average foreclosure losses reported in Jeske et al. (2013). I use a log-normal distribution for the depreciation shock, $\ln(\xi_t) \sim N\left(-\frac{\lambda^2}{2}, \lambda^2\right)$ and I calibrate the steady state value of λ in order to have a default rate of 1%, a number in line with the foreclosure rate before the crisis as reported by the Mortgage Bankers Association (see also figure 1).

The parameters of the financial sector $\theta^k_{ss}, \theta^h_{ss}$, σ and $\bar{\omega}$ are calibrated to hit the moments of specific financial variables. In particular, it is useful to define the following annualized spreads

$$spread_{k,t} = 4\left(E_t R^k_{t+1} - R_{t+1}\right)$$

$$spread^{Bank}_{m,t} = 4\left(E_t R^m_{t+1} - R_{t+1}\right)$$

$$spread^{Borr}_{m,t} = 4\left(1/Q_t - R_{t+1}\right)$$

The first two variables represent the difference between the expected return that the bank is making on its assets and its cost of funding. I refer to $spread_{k,t}$ as the business loan spread and $spread^{Bank}_{m,t}$ as the MBS spread. The mortgage spread $spread^{Borr}_{m,t}$ represents the spread between the mortgage rate faced by impatient households and the risk-free rate; this variable will be affected both by the default premium $(1/Q_t - E_t R^m_{t+1})$ and by the MBS spread, and it will be an important indicator of the availability of mortgage credit.

The values of the banking sector's parameters imply a bank leverage ratio of 10, an annual spread on business loans of 100 basis points, a value of the MBS spread of 20 basis points and an average life for the bankers of 5 years. The leverage ratio should represent an average of the leverages of commercial banks, investment banks and firms. The spread on R^k is chosen to correspond to the spread on a BAA corporate bond as used by Gertler and Karadi (2011), whereas the one on R^m should capture the lower funding cost on MBS securities before the crisis, as represented by the ABCP spread in figure 1. Asset-backed commercial paper was in fact one of the main funding

[21] The complete dataset can be found at http://www.lincolninst.edu/resources/

[22] A larger value for β could be probably obtained by calibrating a distribution with 3 parameters, instead of only 2. For example Jeske et al. (2013) use a generalized Pareto. However with that distribution it is not obvious how to structure the same type of risk shock which I consider in the paper. Alternatively, if we assumed that impatient agents also have access to other assets, or supply labor, this would help to obtain a larger value for the discount factor.

channels for mortgage-backed securities, and the collapse of the ABCP market in 2007-2009 can be interpreted as a clear sign of the deterioration in the liquidity of MBS.[23] The lower spread on MBS is achieved in steady state by setting $\theta^m_{ss} < \theta^k_{ss}$, implying that it is more difficult for the banker to divert mortgage-related assets than corporate loans. What I try to capture is the idea that, before the crisis, because of several factors not directly modeled here, including financial innovation and securitization, mortgage-backed securities were perceived as a safer and more liquid type of asset. In addition, such calibration implies a spread on mortgages, $(1/Q - R)$, of 110 basis points annually.

I use $\delta_k = .025$, which implies that investment accounts for 17% of GDP. I assume $\alpha_k = .35$ and $\alpha_n = .5$, which results in a value of houses and capital approximately equal to 1.2 and 1.8 of annual GDP respectively, and a ratio of capital to housing of 1.5, similar to the one used by Iacoviello and Neri (2010). The fixed supply of housing is normalized to one. In addition, this calibration results in borrowers' consumption accounting for about 10% of total consumption in steady state.

The CES parameter for the retailers is set at 4.167 as in Gertler and Karadi (2011) and the Calvo parameter ζ is set to .83, the value estimated by Iacoviello and Neri (2010). As regards monetary policy, I use conventional parameters for the Taylor rule, with a feedback coefficient on inflation of 1.5, a 0.5 coefficient on the change in the output gap and a smoothing parameter of 0.8.

Finally, in the baseline calibration I set $I_{DefCost} = 1$, but I present also the impulse responses for the case $I_{DefCost} = 0$ in the appendix.

4.2 Experiments

As a first set of experiments, I study the quantitative performance of the calibrated model with respect to standard shocks. I then focus on two "housing financial shocks" affecting the supply of mortgage credit and aimed at replicating some of the features of the recent recession. In this section I assume that credit policy is not active.

4.2.1 Model Behavior

I begin by considering a set of conventional shocks to illustrate the model behavior in comparison to a corresponding model without financial frictions in the banking sector. The objective of this exercise is to show how constrained financial intermediaries can influence the way in which shocks affect the real economy and the role played by housing variables. In this alternative model, bankers raise funds from depositors without any incentive problem, so that their net worth and leverage play no role for aggregate fluctuations. This framework is equivalent to one in which patient households can frictionlessly invest in mortgages or capital. For this reason, we can label this model as the "no-banks" model. As a result, in the no-banks model the following no-arbitrage relationship for

[23] Another spread that was informative of the liquidity of mortgage backed securities during the crisis was the one on the ABX index. This index tracked the performance of securities backed by pools of subprime mortgages, and it witnessed a much more dramatic rise than the ABCP spread during the crisis. For example, the spread over the Libor of the ABX index on the AAA tranche (2006 Issue 2) went from 15 basis points in mid-2006 to 1400 basis points in June 2009.

the returns on both types of assets will hold

$$E_t\hat{\beta}\Lambda_{t,t+1}\left(R^j_{t+1} - R_{t+1}\right) = 0 \text{ for } j = m, k \tag{51}$$

implying that both $spread_{k,t}$ and $spread^{Bank}_{m,t}$ are zero in steady state.

Figure 2 shows the response of the baseline model to three negative shocks to productivity, housing preferences and nominal interest rates. The solid blue line represents the baseline model, whereas the red dotted line is the no-banks model.

The TFP shock is a 1% drop in A_t with a persistence of 0.9. The baseline model delivers a more severe decline in output, investment and consumption, compared to the no-banks model. One driver behind the amplification in the output drop is the fact that real investment declines three times more in the baseline model. This is due to the well known "financial accelerator" mechanism that operates through a drop in bankers' net worth and a consequent increase in the cost of capital, which results in a lower capital demand by non-financial firms. However, in this first experiment, we can already see an additional channel that increases the correlation between house prices and business investment, and causes a larger drop both in house prices and aggregate consumption. Such correlation is the result of the interaction between the bank net worth channel and the housing net worth channel. In fact, in this model, financial intermediaries react to the deterioration of their balance sheet by reducing both the supply of business loans and the supply of mortgages. Consequently, the lower demand for houses, due to the higher mortgage rate charged by banks, causes house prices to drop about 2% more in the baseline model. The lower value of dwellings has two negative implications: first, it increases the default rate, amplifying the negative effect of the bank net worth channel; second, it activates the housing net worth channel, by decreasing the net worth of borrowing households. As a result, the declines in output and consumption are 1% and .5% larger, respectively, in the baseline model.

The monetary shock is a 10 basis point increase in the short-term nominal interest rate. The amplification in this case comes from the fact that this shock directly tightens banks incentive constraint by increasing the real interest rate, which leads to downward pressure on asset prices, and, through the consequent drop in bank net worth, the multi-sector financial accelerator of the previous experiment causes similar movements in investment, house prices and consumption.

Finally, the housing preference shock is a 1% increase in ρ_t with a persistence of 0.9. In this case, a drop in the demand for houses decreases the value of dwellings and consequently of borrowers' net worth in both the baseline model and in the no-banks model. As a result both models imply a decline in borrowers' consumption.[24] However, only the baseline model is also able to generate a comovent between house prices and investment, because of the effect of higher mortgage defaults on bank net worth and on the total supply of credit. This transmission chain will also be important for the financial shocks that I consider below.

[24]It has to be noted that the partial equilibrium effect of the housing preference shock is that of increasing the borrowers' demand for the consumption good, as indicated by equation (11). However the effect of lower house prices and lower net worth dominates and results in lower consumption for borrowers.

4.2.2 Housing Financial Shocks

The main experiments of this paper analyze the response of the model to two types of financial shocks that affect the supply of mortgage credit. In particular, each of these shocks will affect a different component of the total spread charged by banks to mortgagers, that is $(1/Q_t - R_{t+1})$. A justification for focusing on credit supply shocks can be found, for example, in Justiniano et al. (2015), who present evidence on how changes in the funding constraint of financial intermediaries provide a better explaination of the housing boom and bust, compared to shocks to the collateral constraint of borrowers. A credit supply shock is also studied in the model of Chen and Zha (2015).

The first type of shock that I consider is a "housing risk shock," which consists in an increase in the variance of the idiosyncratic depreciation distribution, λ_t. This shock increases the probability of foreclosures and hence raises the default premium, $(1/Q_t - E_t R^m_{t+1})$. For example, a higher default probability could have been due to the larger share of subprime lenders entering the housing market in the period leading to the financial crisis.

The second shock is what I call an "MBS collateral shock," modeled as an increase in the value of θ^m_t. This shock pushes up the spread faced by banks when raising funds to finance mortgage securities, $(E_t R^m_{t+1} - R_{t+1})$, and is meant to capture the turmoil in the markets that were crucial for the funding of mortgage-backed securities (like ABCP, MMMF shares, repos, etc), and the collapse of most of the securitization process.

In the next two experiments I assume that both shocks have an autoregressive factor of 0.9 and I calibrate each shock to generate a 10% drop in house prices, a conservative estimate of the decline suffered by house prices from the peak to the trough of the recent recession.

A Housing Risk Shock In figure 3, I show the model response to an 8.5% increase in λ_t, calibrated to bring about a 10% decline in q^h_t. Such shock is aimed at capturing the impact of the increase in subprime delinquencies on house prices during the financial crisis. The first effect of such disturbance is to increase mortgage foreclosures on impact, since it increases the mass of agents below the threshold $\bar{\xi}_t$. Therefore, everything else equal, the unexpected component of this shock implies a positive transfer from bankers to borrowers at time t.

However, the most important effect of this shock results from the persistent component of λ_t. In fact, it is the *expected* dispersion of ξ_{t+1} that is priced by mortgage lenders and that affects the spread charged to home owners when financing new house purchases at time t. In particular, the downturn generated in the no-banks model is entirely due to this effect: the higher mortgage rate implies a lower housing demand and the resulting lower house prices depress consumption through the housing net worth channel.[25] The relevance of the expected disturbances to λ_t is in line with the finding of Christiano et al. (2014), who stress how the "news" component of a risk shock on firms investment opportunities is the driving force of business-cycle fluctuations in a framework á la Bernanke, Gertler and Gilchrist (1999).

[25] In an experiment not reported in the paper, it can be seen that if the persistence of the shock to λ_t is set to zero, the no-banks model actually generates an increase in house prices and aggregate consumption.

In the baseline model of this paper, the presence of constrained financial intermediaries makes both components of the housing risk shock relevant and amplifies their effect relative to the no-banks model. In fact, on impact, financial intermediaries suffer losses because of the higher mortgage defaults and, because of their leverage constraint, once their net worth is negatively affected they begin divesting from both mortgages and loans. These fire sales imply two additional negative feedback mechanisms, compared with a model with unconstrained intermediaries. First, there is a spillover effect that depresses investments and consequently output, as we can see from the fact that investment drops by more than 3% in the baseline model whereas it barely moves in the no-banks model. Furthermore, the tightening of the banks' leverage constraint also causes a stronger impact of the housing net worth channel. This is due to the higher value of the incentive constraint multiplier, μ_t, and the consequent decrease in the stochastic discount factor of the bank $\tilde{\Omega}_{t,t+1}$, which results in additional downward pressure on the price for mortgages Q_t, as we can see from (30). This effect entails a lower demand for houses by impatient agents, as we can see from the policy function in (20) so that q_t^h drops as well. As we can see from figure 3, the prices of houses and mortgages experience a drop that is more than 20% larger than in the no-banks model, which results in a decline of almost 50% in borrowers' net worth and in a 3.5% drop in consumption in the baseline model. Because of nominal rigidities, lower aggregate demand also entails a drop in wages and labor, due to the increase in mark-ups. While this demand channel is also present in the model without banks, in that case, because of the absence of banks deleveraging, it implies a drop in consumption and output that is approximately 20% and 30% smaller, respectively.

Finally, a lower q_t^h also means an increase in the default threshold $\bar{\xi}_t = \eta_{t-1}/q_t^h$, so that the initial increase in defaults is reinforced, and the level of foreclosures is about 1% higher in the baseline model compared to the no-banks model.

A positive indicator of the quantitative performance of this model comes from the fact that this calibrated shock delivers a total increase in defaults of about 3.5%, a number very close to the increase reported in figure 1.

In order to show how these results do not depend on the presence of real foreclosure costs, in figure A1 I report the impulse responses with respect to the same shock, in a calibration in which $I_{DefCost} = 0$. As we can see in this figure, aggregate quantities move by very similar amounts, as the absence of real default cost in the available output is compensated by a sharper decline in labor.

An MBS Collateral Shock In the previous section I analyzed a crisis generated by an increase in mortgage riskiness. In figure 4, I study the effect of a tightening in bank funding conditions, specific to the financing of mortgage securities.

The idea behind this exercise is that of capturing the turmoil in the market for asset-backed securities that began in mid-2007. As documented by Covitz et al. (2013), asset-backed commercial paper (ABCP) programs faced several bank-run episodes during the summer of 2007, implying a lower demand for AAA-rated tranches of mortgage-backed securities. Subsequently, it was the turn of repo markets, which heavily employed securitized mortgages as collateral, to experience runs (see Gorton (2010)). As a result of these events, the collateral value of MBS deteriorated consistently and almost permanently.

To reproduce in a stylized way a decrease in the liquidity of mortgage-backed securities, I consider an 80% increase in θ_t^m, calibrated again to generate a 10% drop in house prices on impact. Such a shock does not have a counterpart in the no-banks specification, because it affects an agency problem that is absent in this second model.

The initial effect of an increase in θ_t^m is that of negatively affecting banks' supply of mortgage credit as it is clear from equation (30). In particular, this shock pushes down Q_t by increasing the MBS spread, $E_t R_{t+1}^m - R_{t+1}$. The higher mortgage rate, $1/Q_t$, causes a lower housing demand and a decline in house prices. Again, the drop in q_t^h is amplified by the two net worth channels. Through the housing net worth channel, lower house prices imply a drop in borrowers' consumption, which reduces aggregate consumption by a similar amount compared to the housing risk shock experiment. This is not surprising since both shocks are calibrated to produce the same drop in house prices, which are the main driver of borrowers net worth. However, the bank net worth channel produces stronger effects in this experiment. The amplification is initiated by lower house prices, which cause higher default rates. As expected, this shock produces an increase in foreclosures that is slightly smaller than in figure 3. The consequent drop in NW_t^b causes a sell-off of mortgages and capital that reinforces the decline in q_t^h and also affects business investment.

It has to be noted that in this case the MBS spread, $spread_{m,t}^{Bank}$, is the main driver of the increase in $spread_{m,t}^{Borr}$, whereas in figure 3 the increase in mortgage rates was mainly due to a higher default premium. In addition, since banks are the only type of agent able to intermediate capital, for them to keep providing loans to goods producers, $spread_t^k$ has to increase proportionally as well, as indicated by (27). As a result, in this experiment, investment drops considerably more, by about 15% on impact, so that the total decline in the aggregate net worth of financial intermediaries is more than twice as large as the one occurring with the housing risk shock. This is a consequence of the much larger decrease in the price of capital, which seriously affects NW_t^b. The lower level of capital also causes a deeper and more prolonged decline in output, which drops by about 6% on impact.

Finally, in figure A2 I report the impulse responses to the same shock of a model in which $I_{DefCost} = 0$. Also in this case, the behavior of aggregate variables is very similar.

4.2.3 Housing Financial Shocks and the Great Recession

In the previous two experiments we have seen that both housing risk shocks and MBS collateral shocks are able to produce comovements in house prices, business investment, consumption and output. However, for the same drop in q_t^h, the MBS shock causes a much larger drop in investment and output, while also producing plausible movements in foreclosures and spreads.

As a further investigation of the quantitative properties of this model, and of its ability to explain the 2007-2009 recession, I perform the following exercise. For each of the two housing financial shocks, I extract the sequence of shocks that would reproduce the path of house prices from 2006 to 2010, and I compare the path of other aggregate variables, implied by the model conditional on this set of shocks, to the data.[26]

Figure 5 presents the results of this experiment. The dotted black line represents the data, the thin red line represents the model behavior obtained by using only the housing risk shock to match house prices, and the thick blue line represents the same experiment performed only with MBS shocks. All variables, apart from foreclosures and spreads, are normalized to zero in 2007Q1, which marked the beginning of the decline in house prices. Shaded areas indicate the NBER recession period.

From the central panel, we see that both shocks closely match the drop in consumption over the recession period. However, the magnitudes of the responses of other aggregate variables are quite different between the two shocks, with the MBS shock being more effective at replicating a downturn similar to the one that occurred in the period encompassing the Great Recession.

The main difference between the two shocks is in the implied path of investment. In fact, the MBS shock generates a drop in business investment of about 23% between 2007Q1 and 2009Q2, which is more than half of the size of what occurred in the data. On the other hand, in line with the results from the previous excercises, the housing risk shock produces a decline in investment that is only about 8%.This difference also results in a smaller decrease in output for the housing risk shock, whereas also in this case the MBS shock does a good job at reproducing the decline in GDP over the recession. In addition, the behavior of hours is also replicated much more closely by the MBS shock.

As regards the financial spreads, the MBS shock produces reasonable estimates of $spread_m^{Bank}$ and $spread_k$ until their peak in late 2008, coinciding with the collapse of Lehman Brothers. After that, the model predicts spreads that continue to rise in order to bring about the continuous decline in house prices. On the one hand, the path of the MBS spread produced by the model could be similar to the one for the ABX index, whose spread skyrocketed during the financial turmoil without ever returning to its pre-crisis level. On the other hand, several factors or shocks, not present in this simple experiment or in this model, could explain the inability of the data on financial spreads to account for a slow recovery. First of all, unconventional monetary policy played a role in containing the increase in credit costs; I will try to study the effects of such credit interventions in the next section. In addition, credit rationing, both in bank lending to firms and to households, could imply that credit was provided at lower rates but to fewer borrowers.

Finally, this experiment implies also a steeper increase in foreclosures than what occurred in

[26] As regards the aggregate real variables, I use real GDP expressed in 2009 chained dollars. Investment corresponds to fixed private investment. Hours are from all persons in the nonfarm business sector. Consumption represents personal consumption expenditures. I transform all these series in per capita terms by dividing them by working age population. In addition I track their changes in real terms by using the GDP deflator. House prices are from the FHFA price index for the US (NSA). Foreclosure rates are from the National Delinquency Survey of the Mortgage Bankers Association. The variable $spread_m^{Bank}$ is compared with the ABCP spread on the 3-months AA commercial paper. The variable $spread_k$ is compared with the credit spread on non-financial firms constructed by Gilchrist and Zakrajsek (2012). For most of the series I use US quarterly data from 1980Q1 to 2012Q1 and, apart from foreclosures and spreads, I report the detrended logarithm of each variable, by using a linear trend.

the data, expecially for the housing risk shock. In this case, debt-relief programs for leveraged households, implemented by the US government since 2009, might be responsible for the deceleration in mortgage defaults; in the next section I will focus on the effects of a specific policy aimed at containing the foreclosure crisis by through mortgage principal forebearance.

5 Credit Policy

In this section, I explain how we can use the model to study the effects of two types of credit policies targeted at the mortgage market. In particular, I study the effect of MBS purchases performed by the Federal Reserve and of a mortgage debt relief program launched by the U.S. government.

As a response to the unprecedented turmoil in financial markets, the Federal Reserve began to employ unconventional monetary policy tools. For the purpose of this paper, the most relevant intervention took place in the market for agency mortgage-backed securities, where the Fed purchased assets starting in 2009. In addition, in order to deal directly with the foreclosure crisis, at the beginning of 2009 the U.S. government started the Making Home Affordable initiative, which included several programs aimed at reducing the impact of mortgage defaults.[27] In this paper, I focus on the Home Affordable Modification Program (HAMP), which consisted in permanently modifying some features of outstanding mortgages in order to make foreclosures less likely.

5.1 MBS Purchases

The purchase of agency mortgage-backed securities was the largest unconventional monetary policy program employed by the Federal Reserve. The asset purchases started in January 2009 and the stock of MBS held by the Federal Reserve topped \$1.1 trn by mid-2010, reaching approximately 15% of outstanding agency MBS securities. The main aim of this program was to reduce mortgage interest rates on the primary and secondary market, in order to "support housing markets and foster improved conditions in financial markets more generally."[28]

In order to capture the effects of such a policy in this stylized model, I assume that the central bank is able to purchase mortgages M_t^g directly from the financial sector, even if, in reality, the Federal Reserve only acquired agency MBS securities.[29] In particular, I assume that the Fed buys its mortgages from financial intermediaries, right after they are originated, at the origination price $Q_t(\eta_t)$. Importantly, these mortgages are not subject to the agency problem between depositors

[27] In particular MHA included the Home Affordable Modification Program (HAMP), the Home Affordable Refinance Program (HARP) and the Home Affordable Foreclosure Alternatives (HAFA) programs. More information on the different programs can be found at www.makinghomeaffordable.gov

[28] Federal Reserve press release from November 25, 2008.

[29] The most relevant paper modeling unconventional monetary policy in a DSGE framework is Gertler and Karadi (2011), in which, however, there is no role for a housing sector. Compared to Gertler and Karadi (2011), the two-sector framework of this model allows for a more realistic representation of the Fed's asset purchase program, which was mainly targeted at mortgage securities.

and bankers. Therefore, the aggregate amount of mortgages at time t will be given by

$$M_t = M_t^b + M_t^g \tag{52}$$

As in Gertler and Karadi (2011), I assume that the central bank can finance this credit policy by issuing risk-free government debt to patient households, not subject to any agency problem. Given their risk-free nature, bank deposits are perfect substitutes for government debt. Importantly, the profits arising from this activity are rebated to patient agents with lump sum transfers.

To characterize this credit policy, I consider a central bank intermediating a fraction Ψ_t^M of total assets, that is

$$M_t^g = \Psi_t^M M_t \tag{53}$$

As a result, the total amount of mortgages financed at time t can be also written as

$$M_t = \frac{M_t^b}{1 - \Psi_t^M} \tag{54}$$

so that we see how an increase in Ψ_t^M will imply that the constraint on the leverage of intermediaries will have a smaller impact on the aggregate amount of mortgages intermediated.

Following the apprach of Gertler and Karadi (2011), I assume that this type of credit policy is active only at the onset of a housing-related crisis. Therefore, in order to model Ψ_t^M, I assume that the Fed intervenes when the total spread on mortgages $(1/Q_t - R_{t+1})$ increases over its steady state value. As discussed above, this spread includes both the MBS spread and the default premium, which both rise during a mortgage market turmoil.

In particular, I consider the following policy rule

$$\Psi_t^{MBS} = \begin{cases} \Psi_1^{MBS} \{[\log(1/Q_t) - \log(R_{t+1})] - [\log(1/Q_{ss}) - \log(R_{ss})]\} & \text{if } (1/Q_t - R_{t+1}) > (1/Q_{ss} - R_{ss}) \\ 0 & \text{otherwise} \end{cases} \tag{55}$$

Thus, the central bank will start intermediating assets when there is a positive excess expected return. The parameter ψ_1^M will determine the intensity of the intervention.

Finally, as in Gertler and Karadi (2011), I assume that Federal Reserve intermediation entails efficiency costs τ per dollar of asset purchased. These costs counterbalance the profits that the central bank makes during a crisis with this policy.

5.2 Housing Affordable Modification Program (HAMP)

In early 2009 the US government launched the Housing Affordable Modification Program (HAMP), in an attempt to tackle the dramatic increase in mortgage defaults. The program provided substantial financial rewards to lenders and servicers as an incentive for them to offer permanent loan modifications that would avoid costly foreclosures. Such modifications also included the permanent reduction of mortgage principal.[30] By July 2012, the HAMP program had achieved an estimated \$11 billion in savings in mortgage payments, of which about \$6.7 billion in principal forebearance, corresponding to approximately 0.1% of agency debt outstanding at the time.

I model a simplified version of this policy by assuming that during a crisis the government is willing to reduce the outstanding value of borrowers' mortgages by a fraction Ψ_t^{Hamp}. As a result, the default threshold at time t will be given by

$$\bar{\xi}_t = \frac{\eta_{t-1}\left(1 - \Psi_t^{Hamp}\right)}{q_t^h}$$

Importantly, I assume that, in line with the incentive structure of HAMP, the cost of debt restructuring is not borne by financial intermediaries and does not affect the principal payment obtained by the bank. As a result, banks price mortgages in the same way as before but now internalize how the policy affects the threshold $\bar{\xi}_t$. The amount needed to implement the policy in every period, $\Psi_t^{Hamp} M_{t-1}$, is financed by levying lump sum taxes on patient households.

Also in this case, I assume that the policy reacts to changes in the total spread on mortgages by following the rule[31]

$$\Psi_t^{Hamp} = \begin{cases} \Psi_1^{Hamp}\left\{\left[\log\left(1/Q_t\right) - \log(R_{t+1})\right] - \left[\log\left(1/Q_{ss}\right) - \log(R_{ss})\right]\right\} & \text{if } \left(1/Q_t - R_{t+1}\right) > \left(1/Q_{ss} - R_{ss}\right) \\ 0 & \text{otherwise} \end{cases} \tag{56}$$

In this case I assume that this policy does not cause any additional deadweight loss, because the cost of HAMP is of a purely redistributive nature. In fact, unlike MBS purchases, patient agents do not receive any return from this intervention, and their benefits are only due to general equilibrium effects.

5.3 Crisis Experiments with Credit Policy

In figures 6 and 7, I compare the response of the baseline model to the housing risk shock and the MBS shock, to two alternative versions of the model in which each of the two types of credit policies are active. For each shock I calibrate the parameter Ψ_1^{MBS} to have the central bank intermediating 15% of mortgages on impact, and the parameter Ψ_1^{Hamp} to bring about a 0.1% mortgage principal reduction.[32] These numbers are roughly in line with the magnitudes of the interventions. As regards the efficiency cost for MBS purchases, τ, I set it equal to about 0.1%, a number in the range of what Gertler and Karadi (2011) consider a reasonable value for securitized mortgage-backed securities.

[30] Other types of modifications consisted in interest rate reductions or term extensions. For a detailed description of the program and an estimate of its effectiveness see Agarwal, Amromin, Ben-David, Chomsisengphet, Piskorski and Seru (2012).

[31] An alternative might be to have the MBS purchases policy to react to the MBS spread $(E_t R_{m,t+1}^{Bank} - R_{t+1})$ and the HAMP policy to react to the default premium $(1/Q_t - E_t R_{m,t+1}^{Bank})$. However, I chose to have both policies reacting to the same spread in order to have a more sensible comparison between the two.

[32] For the housing risk shock, this implies values of Ψ_1^{MBS} =40 and Ψ_1^{Hamp} =.215. For the MBS collateral shock Ψ_1^{MBS} =23 and Ψ_1^{Hamp} =.135.

The green dashed line depicts the reaction of the model when only MBS purchases are in place. For both types of shocks, such credit policy is effective in moderating the contraction, mostly because central bank intermediation reduces the spread that banks face when financing mortgages, $spread_{m,t}^{Bank}$. This action generates positive spillover effects on several aggregate variables by reversing the negative amplification mechanisms described above. First, a lower MBS spread translates also into a lower spread on capital, $spread_{k,t}$, which implies higher investment and a positive feedback on the balance sheet of financial intermediaries because of the higher price of capital. This effect is similar to the result obtained by Gertler and Karadi (2011). In addition, in this model MBS purchases have a novel type of positive externality because of their implications for house prices: a higher supply of mortgages implies higher housing demand and higher house prices, which benefit banks through lower default rates and help leveraged households through a higher value of their wealth.

The black dotted line shows the impact of the HAMP intervention. This policy mitigates the drop in house prices through two channels. First, on impact, HAMP causes a transfer of resources from lenders to borrowers, and thus a higher demand for housing and consumption goods. Second, the anticipated component of HAMP acts as a mortgage subsidy by reducing the expected foreclosures faced by bankers and hence by decreasing the default premium. As a result, mortgage rates decline, further stimulating the demand for housing. Also in this case, higher house prices have a positive effect on banks' balance sheets by reducing foreclosures; it is interesting to notice that this policy has some positive impact on investment as well, especially in the case of the housing risk shock.

In figures A3 and A4, I report the results for the case when $I_{DefCost} = 0$, in order to show that the responses of the model are quantitatively very similar.

By comparing the effects of the two policies in figures 6 and 7, we notice that, on impact, MBS purchases have a larger positive effect on investment than HAMP. This is a consequence of the fact that the former directly targets the MBS spread, by lessening the agency problem affecting financial intermediaries, who also provide funds for capital investment in this model. Because of this positive spillover effect on the price of capital, this policy is able to hamper considerably the drop in banks' net worth on impact. On the other hand, HAMP is more effective in reducing the default premium and in reducing the decline in borrowers' net worth, but it affects banks' balance sheets only indirectly through a reduction in mortgage defaults.

In order to evaluate the redistributive costs of these two policies, I take a second order approximation of the expected utility of the two types of agents, $\hat{\Omega}_t$ for patient agents and Ω_t for impatient ones, where

$$\hat{\Omega}_t = U_t\left(\hat{C}_t, \hat{L}_t\right) + \hat{\beta} E_t \hat{\Omega}_{t+1}$$

$$\Omega_t = U_t\left(C_t, X_t\right) + \beta E_t \Omega_{t+1}$$

and compute the conditional welfare gains that each policy delivers, in terms of consumption equivalent annuities. In each experiment, this exercise is meant to capture the welfare impact of the policies with respect to a specific set of shocks, rather than the global welfare implications of each policy.

The results are reported in table 2, in which I repeat the exercise with and without real default costs. We see that both policies provide considerable welfare gains to borrowing agents, with HAMP interventions outperforming MBS purchases. As regards patient agents, both policies entail a very small cost, compared to the size of the benefit of impatient agents, when $I_{DefCost} = 0$. When $I_{DefCost} = 1$, also the lender gains from both types of intervention, providing additional support for credit policies during financial crises.

As a final *caveat*, it has to be noted that the results presented in this section are not meant to represent a comprehensive comparison of the two types of policies, because the welfare gains crucially depend on the size of the intervention and on the specific policy function used. The main point of this section is rather that of showing how both policies are beneficial for aggregate variables during a housing crisis, and how they can provide gains for both borrowers and lenders.

6 Concluding Remarks

This paper presents a new framework to study the interaction among mortgage defaults, house prices, and banks' balance sheets in a macroeconomic model. All these elements have been important ingredients for the Great Recession. In particular, the presence of constrained intermediaries, heterogeneous households and endogenous defaults can create novel negative feedback mechanisms that amplify the response of business investment, output and house prices during a financial crisis. When these episodes occur, unconventional monetary policy in the form of central bank asset purchases, or a government debt relief program, can be particularly beneficial.

Several elements can be added to this model to improve its realism and quantitative performance. For example, the introduction of long-term mortgages might considerably strenghten the amplification mechanism, through the movements in the value of outstanding mortgages present on banks' balance sheets. In addition, to obtain a more comprehensive welfare analysis of the policies described in this paper, it would be interesting to study the global solution of the model, in order to capture the effects of occasionally binding leverage constraints for bankers and the nonlinearities arising from the mortgage contract. With this type of analysis it would also be possible to study the moral hazard implications of each policy.[33] All these are interesting topics for future research.

[33] For an analysis of credit policies in a model with occasionally binding leverage constraints see Prestipino (2014).

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8 Appendix

In this appendix I provide the details for the solution of the optimization problems of impatient households and bankers.

8.1 Solution of the Impatient Household Problem

Define the total rental income obtained from a house as

$$r_t = \lambda_x r_{x,t} + (1-\lambda_x) r_{y,t}$$

Then the original problem to be solved is

$$V_t(\omega_t) = \max_{\tilde{c}_t, h_t, \eta_t} \{U(c_t, x_t) + \beta E_t V_{t+1}(\omega_{t+1})\}$$

$$c_t + r_{x,t} x_t + h_t \left[q_t^h - Q_t(\eta_t)\eta_t\right] \leq \omega_t$$

$$\omega_{t+1} = \begin{cases} h_t\left[\left(q_{t+1}^h \xi_{t+1} + r_{t+1}\right) - \eta_t\right] & \text{if} \quad \xi_{t+1} \geq \bar{\xi}_{t+1}(\eta_t) = \eta_t / q_{t+1}^h \\ h_t r_{t+1} \quad \text{if } \xi_{t+1} < \bar{\xi}_{t+1}(\eta_t) = \eta_t / q_{t+1}^h \end{cases}$$

As in the main text, I begin by solving the static expenditures problem, that is

$$u(\tilde{c}_t, r_t) = \max \{\rho_t \log(c_t) + (1-\rho_t)\log(x_t)\} \text{ s.t.}$$

$$c_t + r_{x,t} x_t = \tilde{c}_t$$

The first order conditions imply

$$\frac{c_t}{r_{x,t} x_t} = \frac{\rho_t}{(1-\rho_t)}$$

and using this together with the constraint implies

$$c_t = \rho_t \tilde{c}_t \tag{57}$$

$$r_{x,t} x_t = (1-\rho_t)\tilde{c}_t \tag{58}$$

Then substituting these two equations in the objective function we obtain

$$\begin{aligned} u(\tilde{c}_t, r_{x,t}) &= \log(c_t) + \{\rho_t \log(\rho_t) + (1-\rho_t)[\log(1-\rho_t) - \log(r_{x,t})]\} \\ &= \log(\tilde{c}) + \Theta(\rho_t, r_{x,t}) \end{aligned}$$

At this point we can rewrite the problem as

$$V_t(\omega_t) = \max_{\tilde{c}_t, h_t, \eta_t} \{u(\tilde{c}_t, r_t) + \beta E_t V_{t+1}(\omega_{t+1})\}$$

$$\tilde{c}_t + h_t\left[q_t^h - Q_t(\eta_t)\eta_t\right] \leq \omega_t$$

$$\omega_{t+1} = \begin{cases} \omega_{t+1}^{nd} = h_t\left[\left(q_{t+1}^h\xi_{t+1} + r_{t+1}\ \right) - \eta_t\right] & \text{if} \quad \xi_{t+1} \geq \bar{\xi}_{t+1}(\eta_t) \\ \omega_{t+1}^d = h_t r_{t+1} \quad \text{if } \xi_{t+1} < \bar{\xi}_{t+1}(\eta_t) \end{cases}$$

The FOC for h_t and η_t are

$$u_{c,t}\left[q_t - Q_t(\eta_t)\eta_t\right] = \beta E_t\left\{V_{t+1}'(\omega_{t+1})\frac{d\omega_{t+1}}{dh_t}\right\}$$

$$u_{c,t}\frac{d\left[Q_t(\eta_t)\eta_t\right]}{\eta_t} = \beta E_t\left\{V_{t+1}'(\omega_{t+1})\,1\left\{\xi_{t+1} > \bar{\xi}_{t+1}(\eta_t)\right\}\right\}$$

and using the evolution of housing wealth and the relationship $\frac{d[Q_t(\eta_t)\eta_t]}{\eta_t} = \left[Q_t(\eta_t) + Q_t'(\eta_t)\eta_t\right]$, these can be rewritten as

$$u_{c,t}\left[q_t - Q_t(\eta_t)\eta_t\right] =$$

$$= \beta E_t\left[r_{t+1}F\left(\bar{\xi}_{t+1}(\eta_t)\right)V_{t+1}'\left(\omega_{t+1}^d\right) + \int_{\bar{\xi}_{t+1}(\eta_t)}^{\infty}\left[\left(q_{t+1}^h\xi_{t+1} + r_{t+1}\right) - \eta_t\right]V_{t+1}'\left(\omega_{t+1}^{nd}\right)f\left(\xi_{t+1}\right)d\xi_{t+1}\right]$$

and

$$U'(\tilde{c})\left[Q_t(\eta_t) + Q_t'(\eta_t)\eta_t\right] = \beta E_t\left[\int_{\bar{\xi}_{t+1}(\eta_t)}^{\infty}V_{t+1}'\left(\omega_{t+1}^{nd}\right)f\left(\xi_{t+1}\right)d\xi_{t+1}\right]$$

Then we guess the policy function $\tilde{c}_t = (1-\chi)\omega_t$ so that from the budget constraint we obtain the policy function for housing purchases

$$h_t\left[q_t^h - Q_t(\eta_t)\eta_t\right] = \chi\omega_t$$

In addition, the evolution of wealth becomes

$$\omega_{t+1} = \frac{\chi\omega_t}{\left[q_t^h - Q_t(\eta_t)\eta_t\right]}\begin{cases} h_t\left[\left(q_{t+1}^h\xi_{t+1} + r_{t+1}\right) - \eta_t\right] & \text{if} \quad \xi_{t+1} \geq \bar{\xi}_{t+1}(\eta_t) \\ \omega_{t+1}^d = h_t r_{t+1} \quad \text{if } \xi_{t+1} < \bar{\xi}_{t+1}(\eta_t) \end{cases}$$

which can be rewritten as

$$\omega_{t+1} = \chi\omega_t R_{t+1}^h(\eta_t, \xi_{t+1})$$

where

$$R_t^h(\eta_{t-1}, \xi_t) = \frac{\max\left(r_t, \left(q_{t+1}^h\xi_{t+1} + r_{t+1}\right) - \eta_t\right)}{\left[q_t^h - Q_t(\eta_t)\eta_t\right]}$$

As a next step, we can guess the value function form as $V_t(\omega_t) = A_t + B\log(\omega_t)$. From the envelope theorem this implies

$$V_t'(\omega_t) = u_{c,t}$$

$$\Longrightarrow B = \frac{1}{(1-\chi)}$$

At this point, by substituting our guesses into the FOC for h_t we obtain

$$\frac{1}{(1-\chi)\,\omega_t}\left[q_t^h - Q_t(\eta_t)\,\eta_t\right] =$$

$$= \beta B E_t\left[r_{t+1}F\left(\bar{\xi}_{t+1}(\eta_t)\right)\frac{1}{\omega_{t+1}^d} + \int_{\bar{\xi}_{t+1}(\eta_t)}^{\infty}\left[\left(q_{t+1}^h\xi_{t+1} + r_{t+1}\right) - \eta_t\right]\frac{1}{\omega_{t+1}^{nd}}f\left(\xi_{t+1}\right)d\xi_{t+1}\right]$$

which implies

$$\frac{1}{(1-\chi)\,\omega_t}\left[q_t^h - Q_t(\eta_t)\,\eta_t\right] =$$

$$= \beta\frac{B}{\chi\omega_t}E_t\left[\begin{array}{c} r_{t+1}F\left(\bar{\xi}_{t+1}(\eta_t)\right)\frac{\left[q_t^h - Q_t(\eta_t)\eta_t\right]}{r_{t+1}} \\ + \int_{\bar{\xi}_{t+1}(\eta_t)}^{\infty}\left[\left(q_{t+1}^h\xi_{t+1} + r_{t+1}\right) - \eta_t\right]\frac{\left[q_t^h - Q_t(\eta_t)\eta_t\right]}{\left[\left(q_{t+1}^h\xi_{t+1} + r_{t+1}\right) - \eta_t\right]}f\left(\xi_{t+1}\right)d\xi_{t+1}\end{array}\right]$$

which gives the following value for the policy function of $\tilde{c}_t$

$$\chi = \beta$$

Finally we can rewrite the FOC for η_t as

$$\frac{1}{(1-\chi)\,\omega_t}\left[Q_t(\eta_t) + Q_t'(\eta_t)\,\eta_t\right] = \beta\frac{B}{\chi\omega_t}E_t\left[\int_{\bar{\xi}_{t+1}(\eta_t)}^{\infty}\frac{\left[q_t^h - Q_t(\eta_t)\,\eta_t\right]}{\left[\left(q_{t+1}^h\xi_{t+1} + \tilde{r}_{t+1}\right) - \eta_t\right]}f\left(\xi_{t+1}\right)d\xi_{t+1}\right]$$

which can be rewritten as

$$\begin{aligned}\left[Q_t(\eta_t) + Q_t'(\eta_t)\,\eta_t\right] &= E_t\left[\int_{\bar{\xi}_{t+1}(\eta_t)}^{\infty}\frac{\left[q_t^h - Q_t(\eta_t)\,\eta_t\right]}{\left[\left(q_{t+1}^h\xi_{t+1} + r_{t+1}\right) - \eta_t\right]}f\left(\xi_{t+1}\right)d\xi_{t+1}\right] \\ &= E_t\left\{\frac{1}{R_t^h\left(\eta_{t-1},\xi_t\right)}1(\xi_{t+1} > \frac{\eta_t}{q_{t+1}})\right\}\end{aligned}$$

As a result, the system of equations solving the impatient agent problem is

$$\frac{d\left[Q_t(\eta_t)\,\eta_t\right]}{d\eta} = E_t\left\{\frac{1}{R_t^h\left(\eta_{t-1},\xi_t\right)}1(\xi_{t+1} > \frac{\eta_t}{q_{t+1}})\right\}$$

$$\tilde{c}_t = (1-\beta)\,\omega_t$$

$$h_t\left[q_t^h - Q_t(\eta_t)\,\eta_t\right] = \chi\omega_t$$

Finally, if we use the policies for c_t and x_t from the static problem we obtain the equations from Proposition 1

$$\frac{d\left[Q_t(\eta_t)\,\eta_t\right]}{d\eta} = E_t\left\{\frac{1}{R_t^h\left(\eta_{t-1},\xi_t\right)}1(\xi_{t+1} > \frac{\eta_t}{q_{t+1}})\right\}$$

$$c_t = \rho_t(1-\beta)\,\omega_t$$

$$r_{x,t}x_t = (1-\rho_t)(1-\beta)\,\omega_t$$
$$h_t\left[q_t^h - Q_t(\eta_t)\,\eta_t\right] = \chi\omega_t$$
$$\omega_{t+1} = \chi\omega_t R_{t+1}^h(\eta_t, \xi_{t+1})$$

8.2 Solution to the Banker's Problem

The banker's problem can be written as

$$V_t^{bank}(n_t) = \max_{k_t, \{m_t(\eta_t)\}_{\eta_t}} E_t\hat{\beta}\Lambda_{t,t+1}\left\{(1-\sigma)\,n_{t+1} + \sigma V_{t+1}(n_{t+1})\right\} \text{ s.t.}$$

$$q_t^k z_t + \int Q_t(\eta_t)\, m_t(\eta_t)\, d\eta_t = n_t + d_t$$
$$n_{t+1} = q_t^k z_t R_{t+1}^k + \int \left\{Q_t(\eta_t)\, m_t(\eta_t)\, R_{t+1}^m(\eta_t)\right\} d\eta_t - R_{t+1}d_t \tag{59}$$
$$V_t^{bank}(n_t) \geq \theta_t^m\left[\int Q_t(\eta_t)\, m_t(\eta_t)\, d\eta_t\right] + \theta_t^k q_t^k z_t$$

If we define μ_t as the multiplier on the incentive constraint, and guess a value function of the form $V_t(n_t) = \varphi_t n_t$. Then the FOCs for k_t, $m_t(\eta_t)$ and μ_t are

$$E_t\hat{\beta}\Lambda_{t,t+1}\left\{\left[(1-\sigma) + \sigma\varphi_{t+1}\right]\left(R_{t+1}^k - R_{t+1}\right)\right\} = \mu_t\theta^k$$
$$E_t\hat{\beta}\Lambda_{t,t+1}\left\{\left[(1-\sigma) + \sigma\varphi_{t+1}\right]\left(R_{t+1}^m(\eta_t) - R_{t+1}\right)\right\} = \mu_t\theta^m \quad \forall\eta_t$$
$$\mu_t\left\{\varphi_t n_t - \left[\theta^m\left(\int Q(\eta_t, s_t)\, m_t(\eta_t)\, d\eta_t\right) + \theta^k q_t^k k_t\right]\right\} = 0$$

where the first two equations imply that

$$\frac{E_t\hat{\beta}\Lambda_{t,t+1}\left\{\left[(1-\sigma) + \sigma\varphi_{t+1}\right]\left(R_{t+1}^k - R_{t+1}\right)\right\}}{\theta_t^k} = \frac{E_t\hat{\beta}\Lambda_{t,t+1}\left\{\left[(1-\sigma) + \sigma\varphi_{t+1}\right]\left(R_{t+1}^m(\eta_t) - R_{t+1}\right)\right\}}{\theta_t^m} \quad \forall\eta_t$$

Plugging the guess into the value function we obtain

$$\begin{aligned} V_t^{bank}(n_t) &= \varphi_t n_t \\ &= E_t\hat{\beta}\Lambda_{t,t+1}\left\{\begin{array}{c} \left[1-\sigma+\sigma\varphi_{t+1}\right]\left[q_t^k z_t\left(R_{t+1}^k - R_{t+1}\right) + \int Q_t(\eta_t)\, m_t(\eta_t)\left(R_{t+1}^m(\eta_t) - R_{t+1}\right) d\eta_t\right] \\ +R_{t+1}n_t \end{array}\right\} \end{aligned}$$

and using the relationship between the spreads, this becomes

$$\varphi_t n_t = E_t\hat{\beta}\Lambda_{t,t+1}\left\{\left[1-\sigma+\sigma\varphi_{t+1}\right]\left[\left(R_{t+1}^k - R_{t+1}\right)\left(q_t^k k_t + \frac{\theta^m}{\theta^k}\int Q(\eta_t, s_t)\, m_t(\eta_t)\, d\eta_t\right)\right] + R_{t+1}n_t\right\}$$

As a result, the marginal value of net-worth will have to satisfy

$$\varphi_t = E_t \hat{\beta} \Lambda_{t,t+1} \left\{ \left[1 - \sigma + \sigma \varphi_{t+1} \right] \left[\left(R^k_{t+1} - R_{t+1} \right) \phi_t + R_{t+1} \right] \right\}$$

where

$$\phi_t = \left[q^k_t k_t + \frac{\theta^m_t}{\theta^k_t} \int Q \left(\eta_t, s_t \right) m_t \left(\eta_t \right) d\eta_t \right] / n_t$$

In addition, if the constraint binds

$$\varphi_t n_t = \left\{ \theta^m_t \left[\int Q \left(\eta_t, s_t \right) m_t \left(\eta_t \right) d\eta_t \right] + \theta^k_t q^k_t z_t \right\}$$

$$\Longrightarrow \varphi_t = \phi_t \theta^k$$

that implies

$$\phi_t \theta^k = E_t \hat{\beta} \Lambda_{t,t+1} \left\{ \left[1 - \sigma + \sigma \phi_{t+1} \theta^k \right] \left[\left(R^k_{t+1} - R_{t+1} \right) \phi_t + R_{t+1} \right] \right\}$$

and consequently a value for leverage

$$\phi_t = \frac{E_t \hat{\beta} \Lambda_{t,t+1} \left[1 - \sigma + \sigma \phi_{t+1} \theta^k \right] R_{t+1}}{\theta^k_t - E_t \hat{\beta} \Lambda_{t,t+1} \left[1 - \sigma + \sigma \phi_{t+1} \theta^k \right] \left(R^k_{t+1} - R_{t+1} \right)}$$

In addition, by rewriting the FOC for m_t we obtain the mortgage pricing equation

$$Q_t \left(\eta_t, s_t \right) = \frac{E_t \hat{\beta} \tilde{\Lambda}_{t,t+1}}{E_t \hat{\beta} \tilde{\Lambda}_{t,t+1} R_{t+1} + \theta^m \mu_t} \varrho_{t+1} \left(\eta_t, \xi_{t+1} \right)$$

9 Tables and Figures

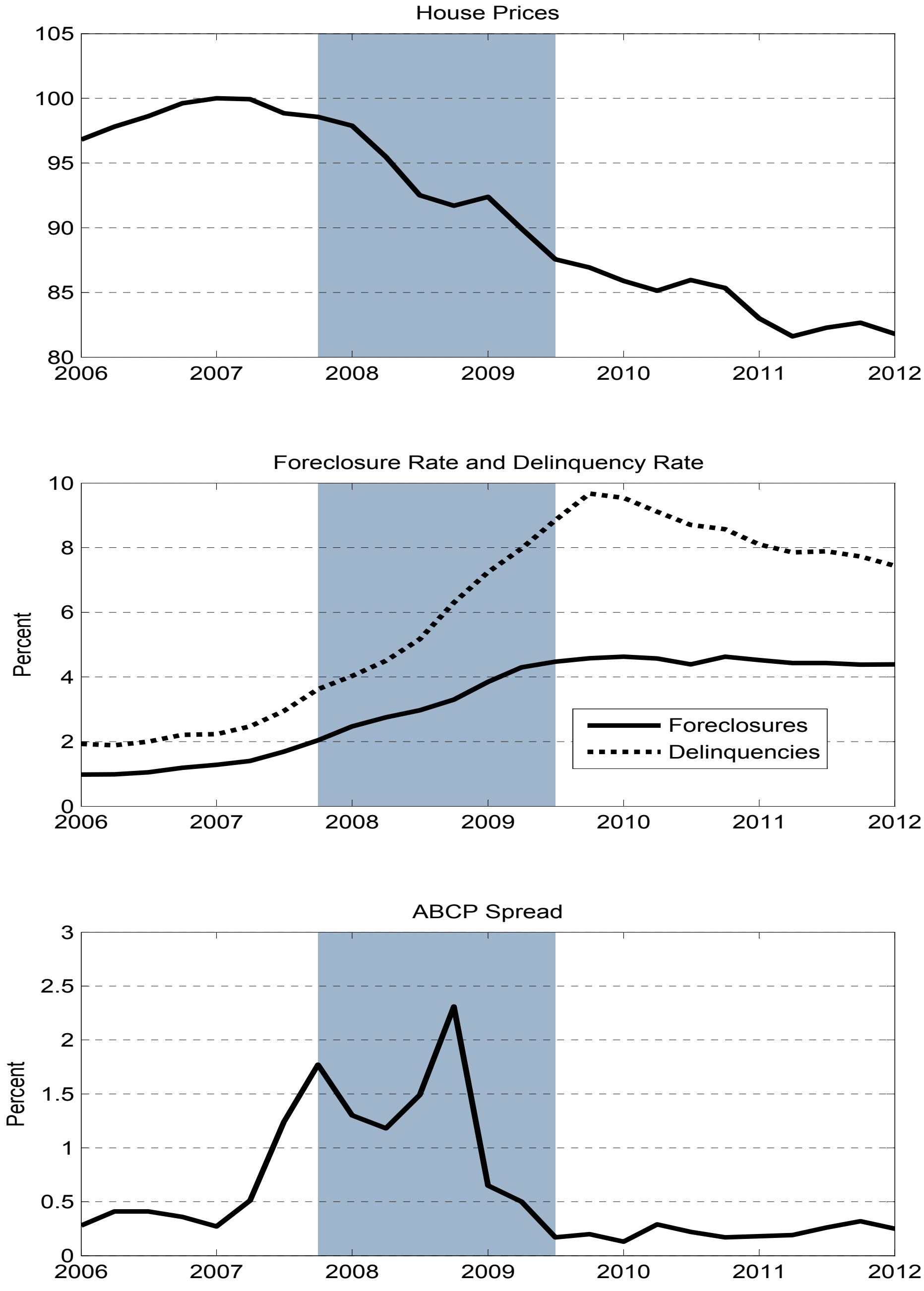
Table 1:Calibration

Parameter	Value	Description
Patient Household		
$\hat{\beta}$	0.99	Discount rate patient HH
γ_n	.276	Inverse Frisch Elasticity
Impatient Household		
β	.81	Discount rate impatient HH
ρ	.27	Housing Preference Parameter
ι_x	.99	Share of Residential Housing
Intermediate Good Firms		
α_k	.35	Capital Share in Production
α_n	.5	Labor Share in Production
δ_k	.025	Capital Depreciation Rate
Capital Producing Firms		
γ_i	.25	Elasticity of Price to Investments
Retail Firms		
ϵ	4,167	Elasticity of Substitution
ξ	.83	Calvo Probability of Fixed Price
Bankers		
θ_{ss}^m	0.038	Divertable Mortgage Share
θ_{ss}^k	0.193	Divertable Capital Share
$\bar{\omega}$	.001	Transfer to Entering Bankers
σ	.95	Bankers survival probability
Mortgages		
$1-\gamma$	.2	Default Cost
λ	0.094	Housing Risk Variance
Monetary Policy		
ρ_i	.8	Smoothing parameter
κ_π	1.5	Inflation Coefficient
κ_y	.50/4	Output Coefficient

Table 2: Welfare Gains from Credit Policies

	Housing Risk Shock		**MBS Collateral Shock**	
$I_{DefCost} = 1$				
	Gain Savers	Gain Borrowers	Gain Savers	Gain Borrowers
MBS Purchases	.014%	15%	.0004%	25%
HAMP	.024%	19%	.018%	27.5%
$I_{DefCost} = 0$				
	Gain Savers	Gain Borrowers	Gain Savers	Gain Borrowers
MBS Purchases	-.055%	14%	-.097%	24%
HAMP	-.06%	19%	-.098%	27%

Figure 1: **The 2007-2009 Mortgage Crisis**



Notes: the first panel reports the FHFA house price index, normalized at 100 in 2007Q1. The second panel reports the foreclosures rate and the serious delinquency rate from the National Delinquency Survey reported by the Mortgage Bankers Association. The third panel reports the Asset Backed Commercial Paper (ABCP) spread, measured as the difference between the AA 90-Day Commercial Paper (RIFSPPAAAD90NB) and the 3-month Treasury Bill

Figure 2: **Model Behavior**

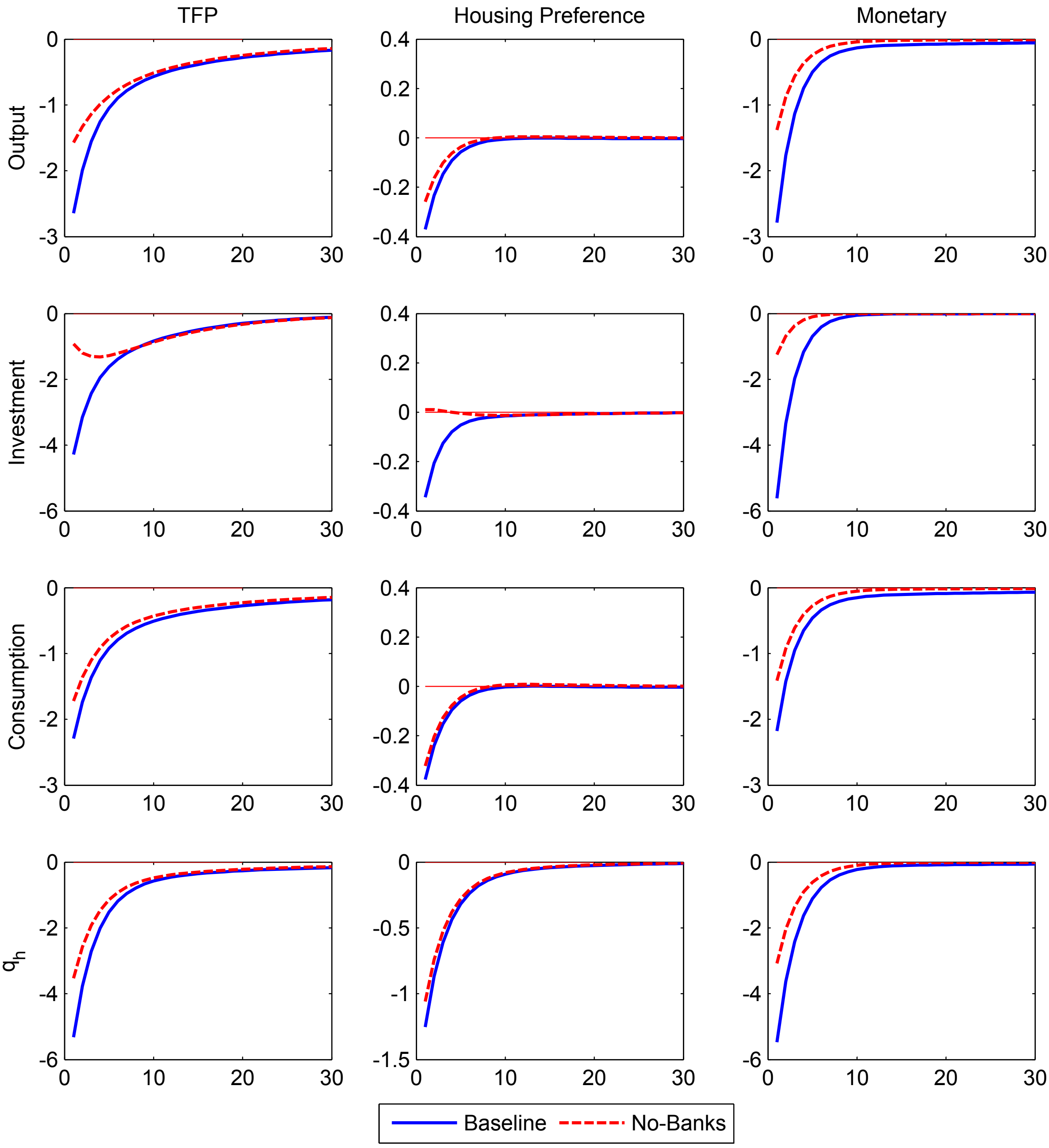


Figure 3: **Housing Risk Shock**

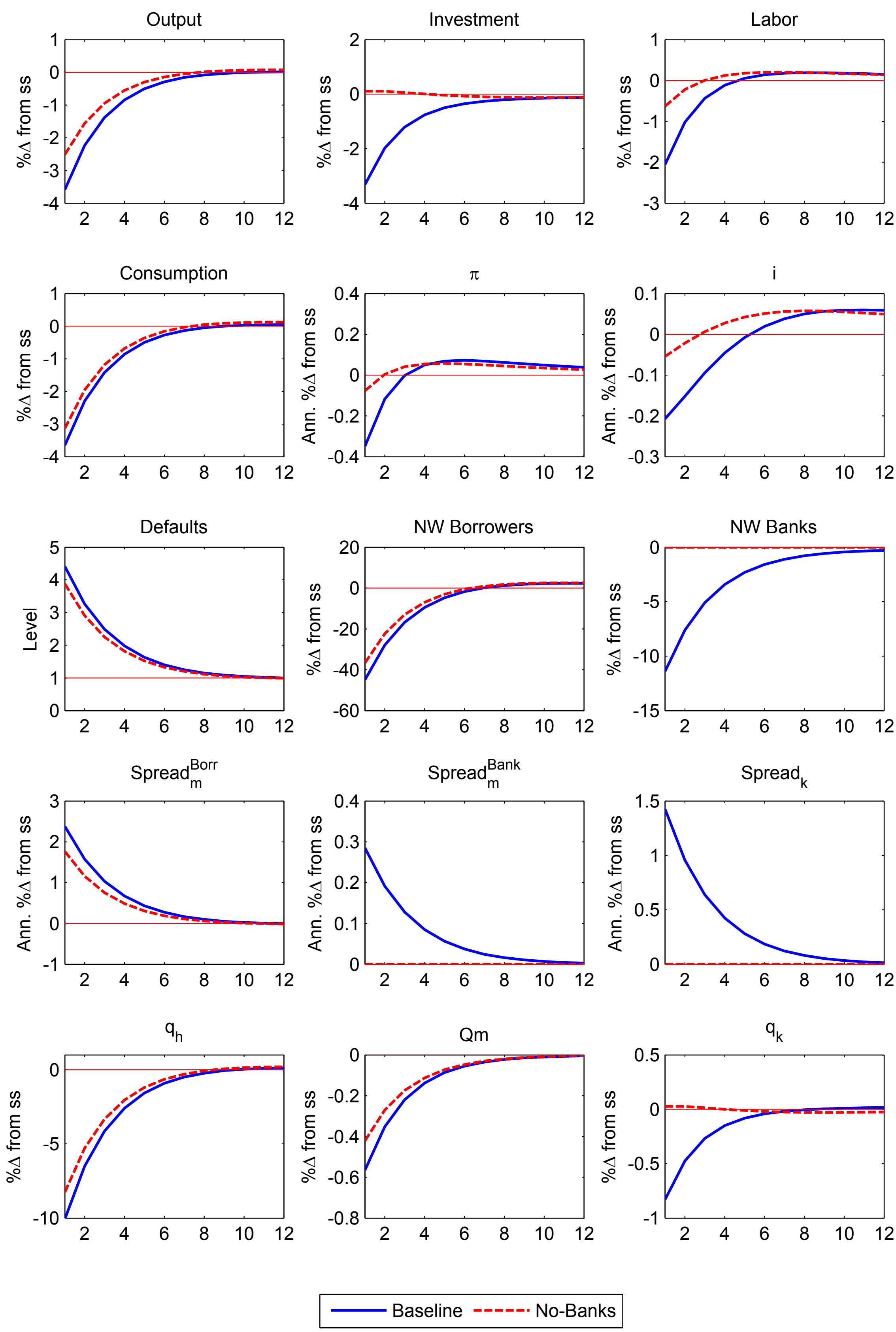


Figure 4: **MBS Collateral Shock**

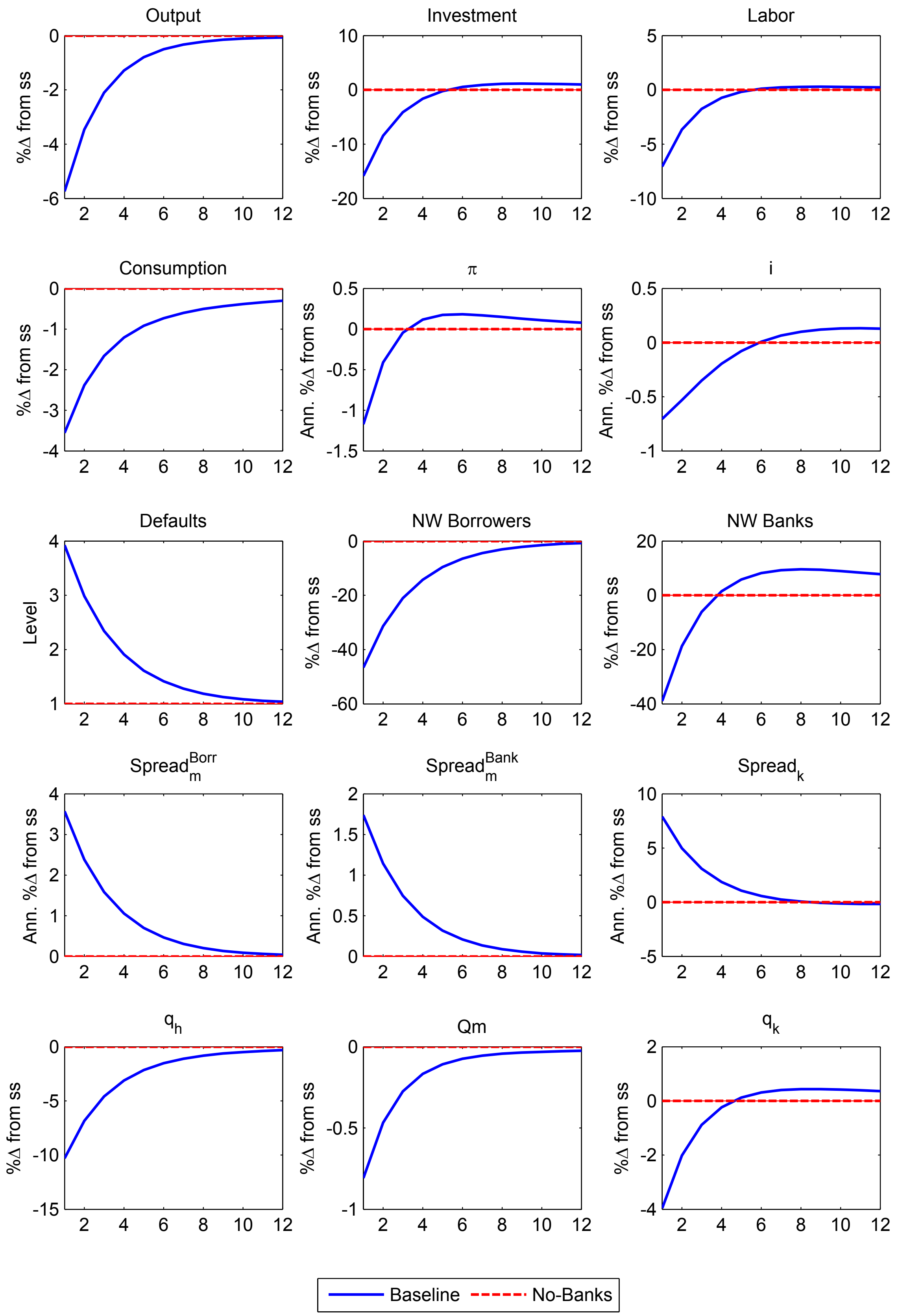
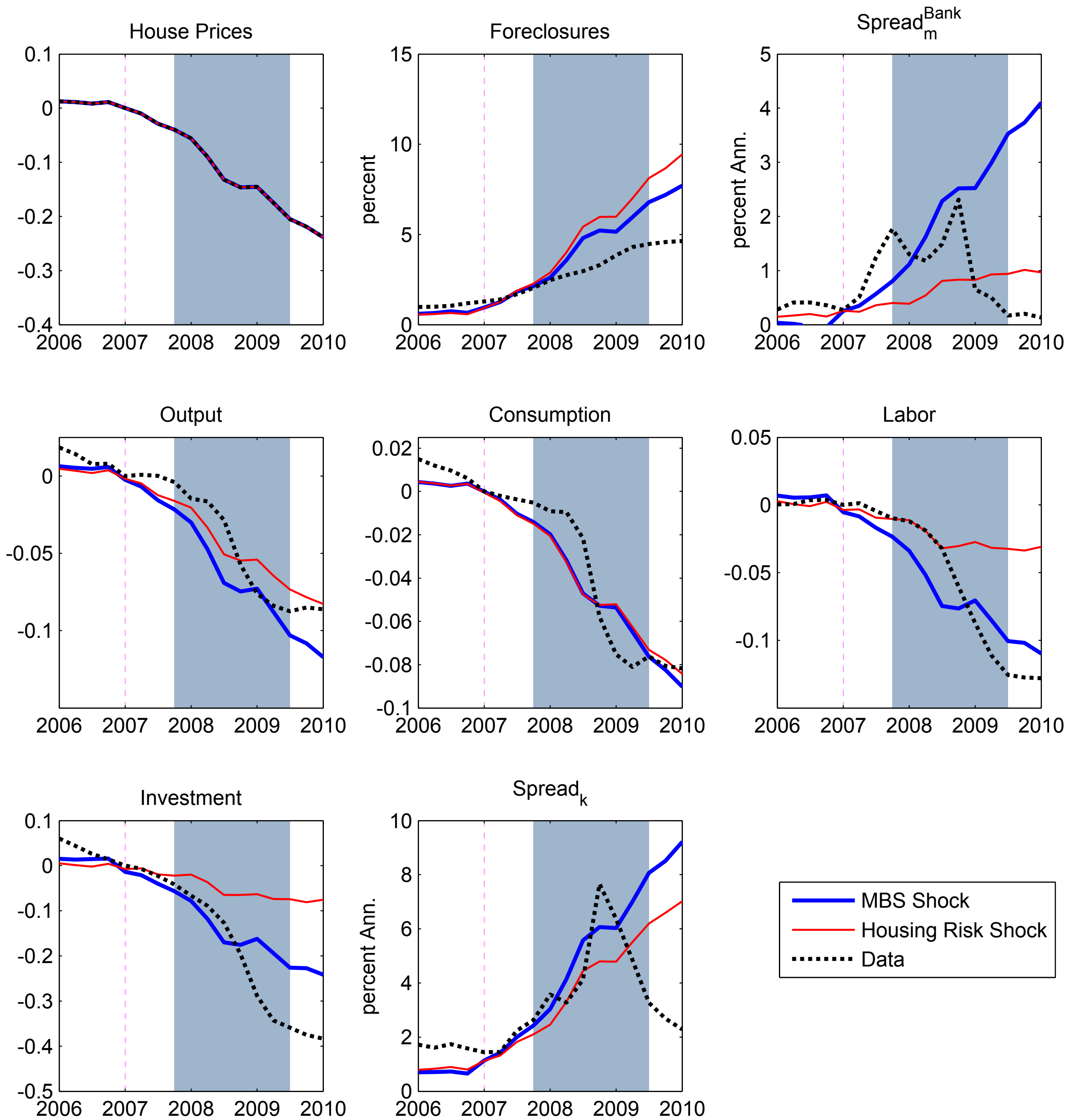


Figure 5: **Housing Financial Shocks and the 2007-2009 Crisis**



Notes: The thick blue line represents the model behavior when the MBS shock is used to match the path of house prices in the top left panel. The thin red line shows the same experiment performed using only housing risk shocks. The dotted black line represents the following data. Output is GDP expressed in 2009 chained dollars. Investment corresponds to fixed private investment. Hours are from all persons in the nonfarm business sector. Consumption represents personal consumption expenditures. I transform all these series in per capita terms by dividing them by working age population. In addition I track their changes in real terms by using the GDP deflator. House prices are from the FHFA price index for the US (NSA). Foreclosure rates are from the National Delinquency Survey of the Mortgage Bankers Association. The variable $spread_m^{Bank}$ is compared with the ABCP spread on the 3-months AA commercial paper. The variable $spread_k$ is compared with the credit spread on non-financial firms constructed by Gilchrist and Zakrajsek (2012). I use US quarterly data from 1980Q1 to 2012Q1, when they are available, and, apart from foreclosures and spreads, I report the detrended logarithm of each variable, by using a linear trend. The time series for house prices, output, consumption, labor and investment are normalized to zero in 2007Q1.

Figure 6: **Housing Risk Shock and Credit Policy**

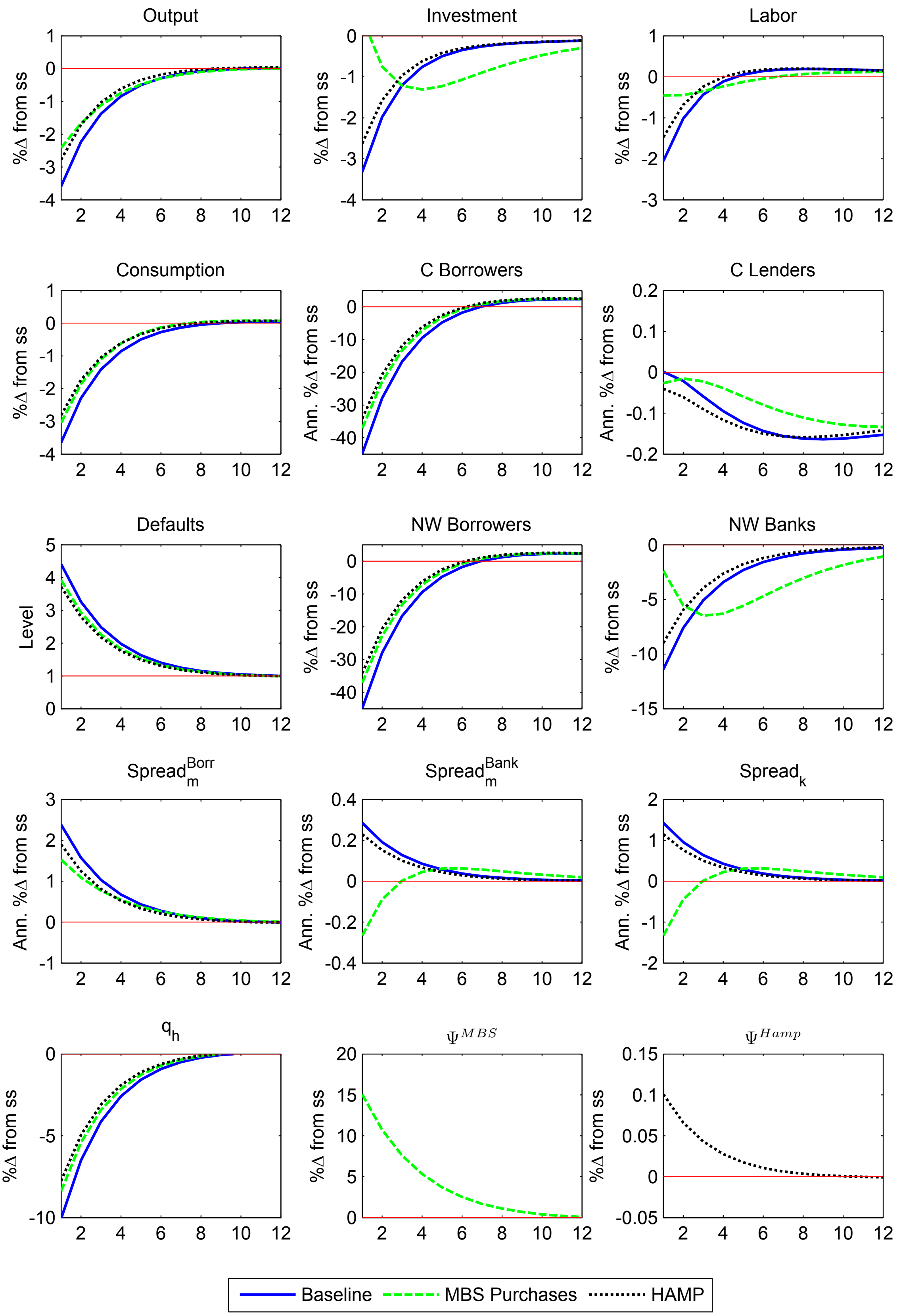


Figure 7: **MBS Collateral Shock and Credit Policy**

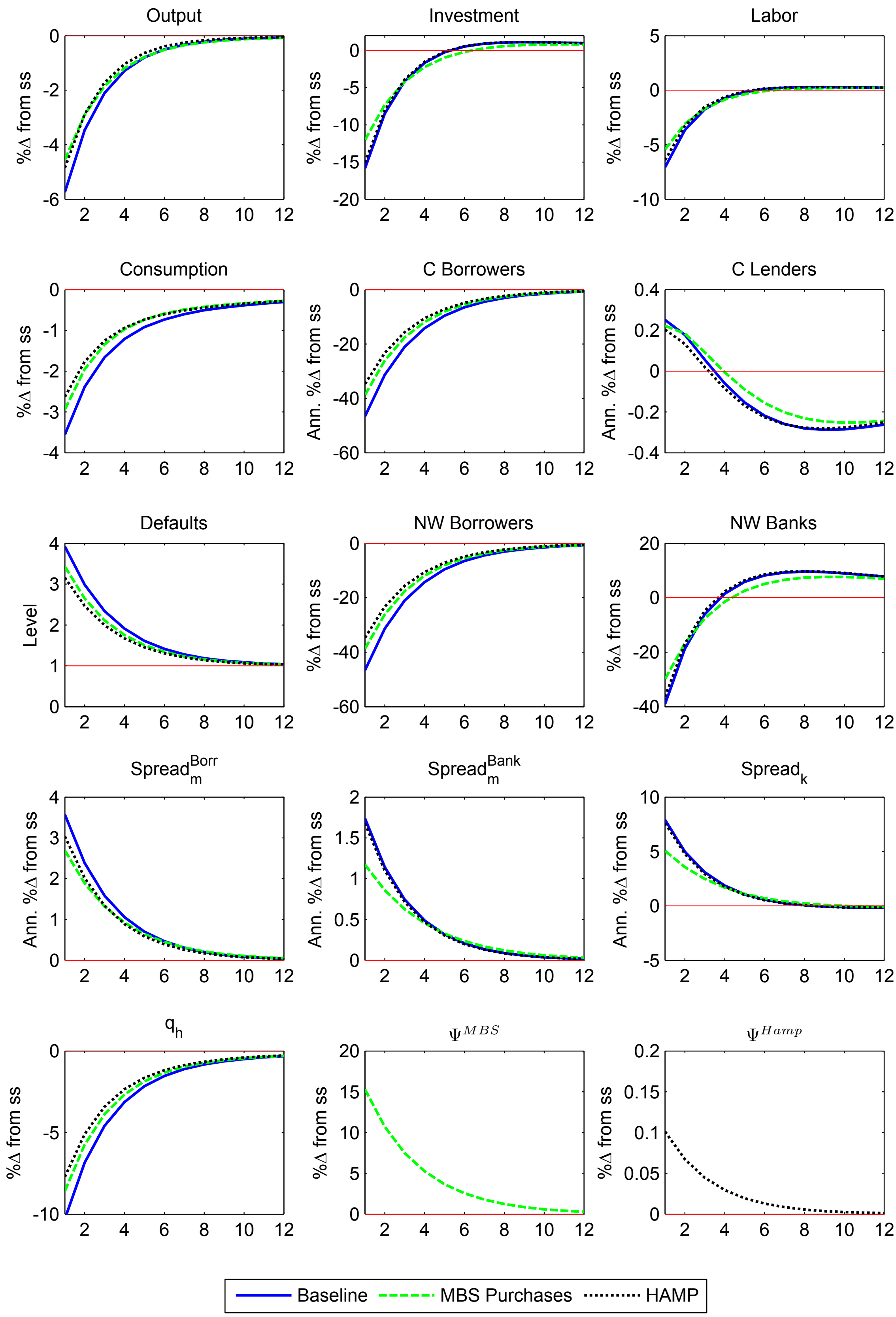


Figure A 1: **Housing Risk Shock** ($I_{DefCost} = 0$)

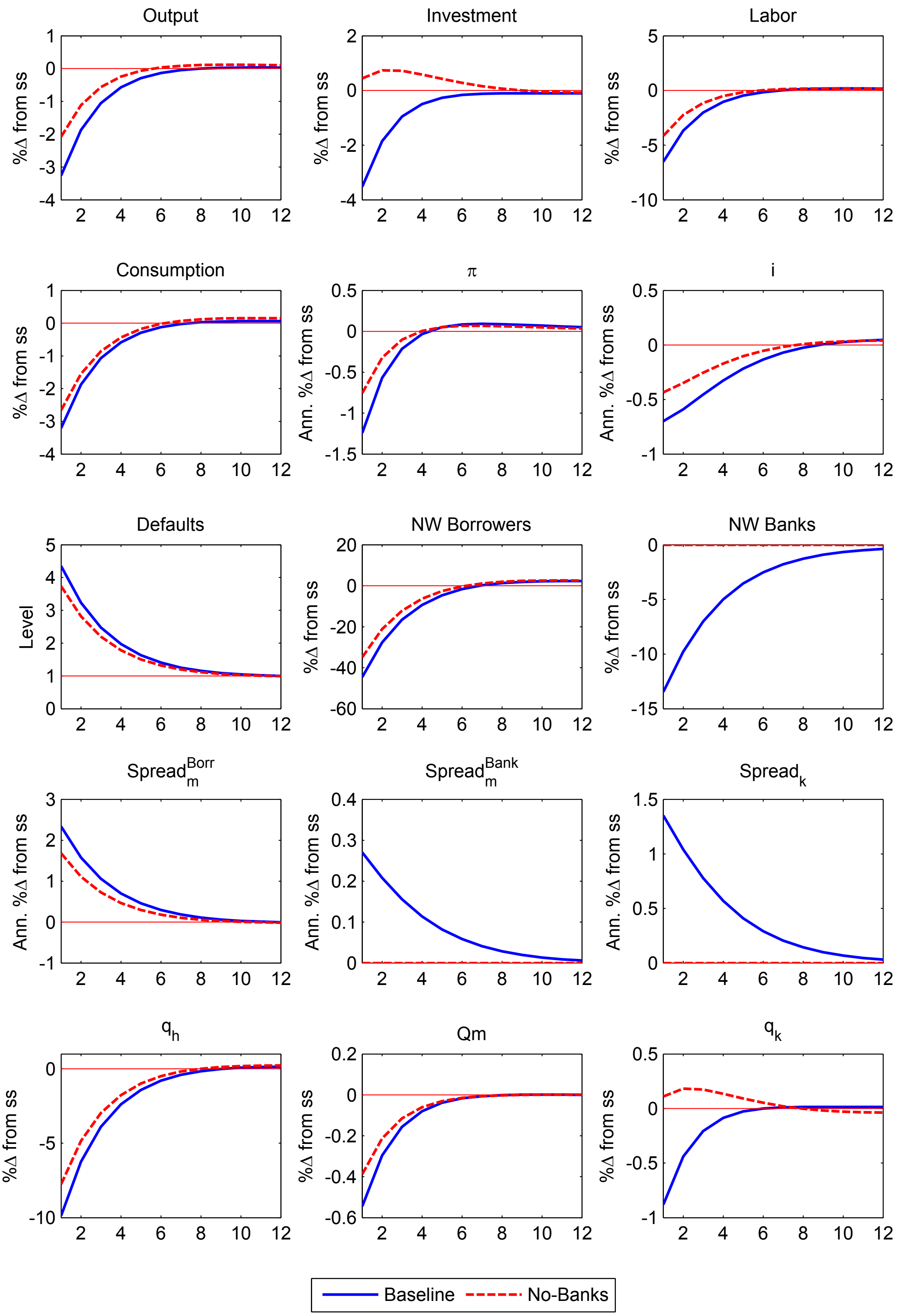


Figure A 2: **MBS Collateral Shock** ($I_{DefCost} = 0$)

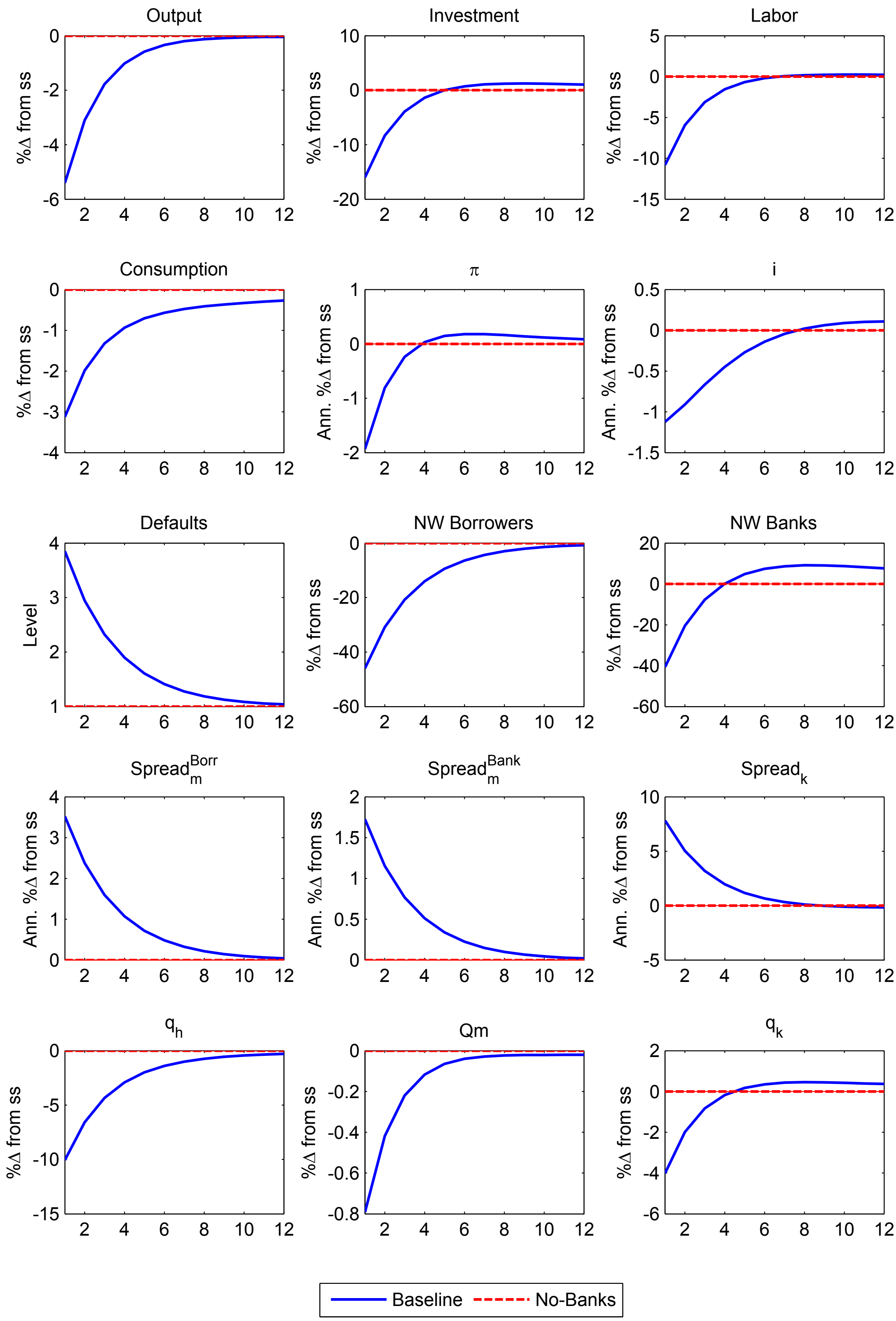


Figure A 3: **Housing Risk Shock and Credit Policy** ($I_{DefCost} = 0$)

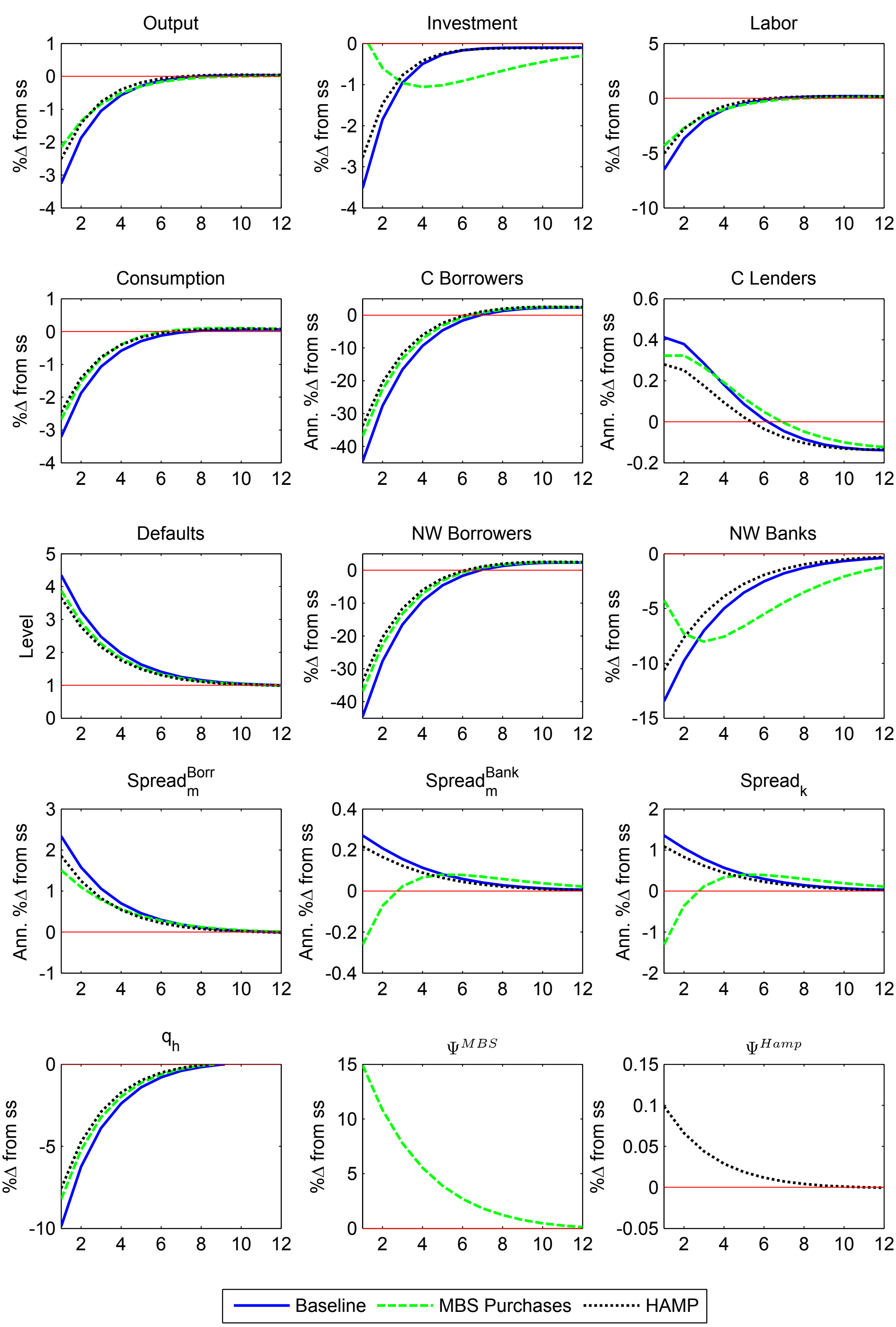


Figure A 4: **MBS Collateral Shock and Credit Policy** ($I_{DefCost} = 0$)