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Interpreting Shocks to the Relative Price of Investment with a Two-Sector Model*

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Abstract

Consumption and investment comove over the business cycle in response to shocks that permanently move the price of investment. The interpretation of these shocks has relied on standard one-sector models or on models with two or more sectors that can be aggregated. However, the same interpretation continues to go through in models that cannot be aggregated into a standard one-sector model. Furthermore, such a two-sector model with distinct factor input shares across production sectors and commingling of sectoral outputs in the assembly of final consumption and investment goods, in line with the U.S. Input-Output Tables, has implications for aggregate variables. It yields a closer match to the empirical evidence of positive comovement for consumption and investment.

JEL Classification: E13, E32.

Key Words: DSGE models, multi-sector models, vector auto-regressions, long-run restrictions.

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1. Introduction

One of the striking features of post-WWII U.S. data is that the relative price of investment has a downward trend and displays notable cyclical variation. Exploring these features, Fisher (2006), Smets and Wouters (2007), Justiniano and Primiceri (2008), and Papanikolaou (2011) argued that shocks that affect the relative price of investment can explain a large part of business cycle fluctuations. In particular, building on the long-run identification scheme of Gali (1999), Fisher (2006) used a VAR to show that shocks to the relative price of investment can explain more than 70% of the fluctuations in hours worked over the business cycle. To interpret the permanent shock to the relative price of investment identified from the VAR, Fisher (2006) focused on a one-sector model with investment-specific technology (IST) shocks that increase the efficiency of investment in a capital accumulation equation. This aggregate approach is based on the results of Greenwood, Hercowitz, and Krusell (2000), who showed that, under certain conditions, a two-sector model with a multi-factor productivity (MFP) shock in each sector can be recast as an aggregate model with IST shocks as well as neutral MFP shocks.[1] These conditions include equal factor shares across production sectors, assembly of each final good using the output of a single production sector, and perfect mobility of capital across production sectors. Guerrieri, Henderson, and Kim (2014) showed that the conditions for aggregation are inconsistent with key features of U.S. Input-Output Tables and other data.[2] Nonetheless, much of the literature has proceeded with an aggregate approach.

This paper makes three contributions: 1) We show analytically and numerically that a two-sector model that cannot be aggregated to a one-sector model is still compatible with the long-run identification scheme proposed by Fisher; 2) Extending the VAR estimated by Fisher to include household consumption and investment, we find a positive correlation between consumption and investment—conditional on shocks that move the price of investment permanently; 3) Estimates from our two-sector and aggregate models indicate that the two-sector model is more likely to be consistent with the positive correlation uncovered from the VAR.

Our results indicate that the sectoral sources of technology shocks have implications not just at the sectoral level, but also at the level of commonly scrutinized macroeconomic aggregate series,

[1] Guerrieri, Henderson, and Kim (2014) set out conditions under which this "aggregate equivalence" result holds and–since the conditions are quite restrictive–referred to shocks that influences a capital accumulation equation in a general two-sector model as marginal efficiency of investment (MEI) shocks.

[2] According to the U.S. Input-Output Tables, different production sectors display different intensities of factor inputs and assembly of each final good uses outputs from more than one production sector. Moreover, as shown, for example, by Ramey and Shapiro (1998) it is quite costly to move capital across sectors.

such as consumption and investment. Moreover, our results indicate that relative prices can be informative with regard to sectoral productivity developments. In this respect, our findings stand in contrast with those of Basu, Fernald, Fisher, and Kimball (2013) who argued that the identification scheme of Fisher (2006) does not apply when sectoral production functions display different factor intensities.

We proceed by extending two alternative DSGE models from Guerrieri, Henderson, and Kim (2014): a two-sector model and an aggregate model. The sectoral model has two production sectors, a machinery-producing sector and its complement that is dubbed a non-machinery-producing sector. It also allows for the assembly of consumption and investment goods each of which uses sectoral outputs in different proportions. These two features of the model allow us to reflect key information from the U.S. Input-Output Tables and other sectoral statistics. We estimate the extended models by matching key moments of U.S. data extracted from the same variables included in the VAR. The extensions include a broader set of shocks, habit persistence in consumption, and an endogenous labor supply.

Because the sectoral production functions display different factor intensities, our two-sector model cannot be aggregated into a one-sector model. Nonetheless, we prove that relative prices are still informative about sectoral productivity developments. We proceed in two stages. First, for a simpler version of our two-sector model in which each sectoral output is used in the assembly of one final good, we offer an analytical proof. Second for a fuller, empirically-relevant version of the model, we rely on numerical illustrations that the results in the analytical proof continue to apply.

When the two extended models are estimated to match the same aggregate features, MFP increases in the machinery-producing sector of the two-sector model have effects that are qualitatively different from IST shocks in the aggregate model. One important difference is that, conditional on shocks that move the price of investment permanently, the correlation between consumption and investment is positive in the two-sector model with MFP shocks and negative for the aggregate model with IST shocks. The commingling of sectoral outputs in the assembly of both consumption and investment goods implies that an increase in productivity in one production sector lowers the cost of assembly of both final goods, creating an incentive to increase the assembly of both goods. Allowing for differences in factor intensities across production sectors and restricting capital stocks to be predetermined at the sectoral level both reduce the attractiveness of substituting between consumption and investment.[3]

The imprecision of estimates from long-run identification strategies applied to small samples can make it difficult to discriminate between alternative hypotheses.[4] To investigate the small sample properties of the VAR estimates, we rely on a Monte Carlo experiment. We re-estimate the same VAR used on observed U.S. data on random samples of data generated from the two alternative DSGE models. The cumulative density function for the correlation between consumption and investment for the two-sector model is uniformly closer to that for the VAR estimated on observed data, confirming that the two-sector model is a more plausible candidate data-generating process than the aggregate model.

The rest of the paper proceeds as follows. Section 2 describes the VAR identified with long-run restrictions and documents the positive comovement between consumption and investment in response to shocks that move the price of investment permanently. Section 3 proves that sectoral shocks in a two-sector model are consistent with the identification scheme. Section 4 describes some extensions of the model framework and Section 5 revisits the identification issues in line with these extensions. Section 6 shows that the two-sector model is more likely to be consistent with the positive comovement uncovered by the VAR than the aggregate model.

2. New Empirical Evidence on the Correlation Between Consumption and Investment

A key feature for discriminating between a one-sector model with IST shocks and a two-sector model with MFP shocks is the comovement of consumption and investment conditional on technology shocks. Fisher's seminal work on identifying IST shocks did not include measures of consumption or investment in the VAR, making it impossible to investigate this comovement. We update Fisher's results and extend them to gauge this comovement by including measures of consumption and investment in the VAR.

The VAR that we estimate includes five variables:

1. the growth rate of the relative price of investment, constructed as the log-differenced implicit

[3] Using a calibrated DSGE model, similar to the one considered here, Guerrieri, Henderson, and Kim (2014) showed that allowing for commingling in the assembly is sufficient, by itself, to change the consumption-investment correlation from negative to positive. Furthermore, they showed that incorporating each of these model features by itself makes the consumption-investment correlation less negative.

[4] See, for instance, Faust and Leeper (1997) and Erceg, Guerrieri, and Gust (2005) for an examination of the econometric issues related to long-run restriction schemes.

price deflator for equipment and software from NIPA Table 1.1.9 minus log-difference non-farm business output prices (net of equipment and software using the Laspeyres formula);[5]

2. labor productivity growth, measured as log-differenced labor productivity in the nonfarm business sector from the Bureau of Labor Statistics;

3. hours per capita, constructed as the log of hours worked in the nonfarm business sector minus the log of civilian non-institutional population 16 years and over from the Current Population Survey;

4. the growth rate of real equipment and software per capita, defined as the log-differenced equipment and software (nominal equipment and software divided by its implicit deflator) minus the log-differenced civilian non-institutional population 16 years and over from the Current Population Survey;

5. the growth rate of real consumption per capita, constructed as the log-differenced real personal consumption expenditures from NIPA Table 1.1.6, minus the log-differenced civilian non-institutional population 16 years and over from the Current Population Survey.

Several recent papers have replaced or augmented labor productivity growth in the VAR with the growth of total factor productivity (TFP) measures obtained from growth accounting exercises. See, for instance, Beaudry and Lucke (2010), Schmitt-Grohe and Uribe (2011), and Sims (2011). All those exercises rely, in one form or another, on aggregation of production function across sectors. We continue to use labor productivity growth since the conditions for aggregation underlying those TFP measures do not hold in our model.

We estimate a VAR of order 4. The start date for the estimation sample is 1982:Q3, avoiding the adjustment from the Volcker disinflation. We end the sample in 2008:Q3 to avoid a possible break associated with the zero lower bound on nominal interest rates. In robustness analysis, we also consider a longer sample, spanning all available data.

We follow the long-run identification scheme of Fisher (2006). Building on the idea of Greenwood, Hercowitz, and Krusell (2000) that relative prices are informative about sectoral technological developments, Fisher also focused on relative prices. However, to resolve the problem that, in the short run, in the presence of real rigidities relative prices can be influenced by non-technology shocks, he considered long-run movements in relative prices. Following Fisher's scheme, the identification

[5] Throughout the body of this paper, we take "investment" to mean investment in equipment and software.

Table 1: Historical Variance Decomposition Implied by the VAR

Shock	Growth of Price of Investment	Growth of Labor Productivity	Hours	Growth of Consumption	Growth of Investment
Price of Investment	0.60	0.10	0.71	0.40	0.45
Neutral MFP	0.10	0.56	0.03	0.04	0.19

Variable definitions can be found in Section 2.

scheme we use imposes that only a shock to the relative price of investment can move that price permanently. Moreover, only shocks to the relative price of investment and to labor productivity can move the level of labor productivity permanently. All other shocks are left unidentified.

The thick dashed lines in Figure 1 show the effects of a one-standard-deviation shock estimated by our VAR to reduce the price of investment permanently. The point estimate for the decline in the relative price is close to 3 percent. The areas shaded with vertical dashed lines show 90% confidence intervals following Runkle (1987), and based on 1000 bootstrap replications of the data. While the confidence intervals are strikingly large, they exclude a positive response for the relative price of investment, and negative responses for output, consumption (in all but the first period), and investment. From the point estimates for the impulse responses, it can be correctly inferred that there is conditional comovement between consumption and investment.

Table 1 offers a decomposition of the variance of the variables included in the VAR on average over the estimation sample. Shocks to the price of investment account for 60% of the variation in the growth rate of the relative price of investment and they also account for more than 70% of the variation in hours worked, in line with the results presented by Fisher (2006) and confirmed with estimates from a DSGE model by Justiniano, Primiceri, and Tambalotti (2010). In addition, the same shocks are important for the variation in the growth of consumption and investment, accounting for 40% and 45% of this variation, respectively.

The top panel of Figure 2 shows the cumulative density function (CDF) for the correlation between consumption and investment at business cycle frequencies, conditional on a shock that changes the relative price of investment permanently, as estimated from the VAR on our baseline sample from 1982q3 to 2008q3. The cumulative density function captures the sampling uncertainty for the estimate of the VAR coefficients and is traced from a bootstrap exercise. First, we sample with replacement from the VAR residuals to construct 1000 new synthetic samples of the same length as the original historical sample. Second, we re-estimate the VAR on each synthetic sample. Third, by another bootstrap on the residuals from the VAR estimated on the synthetic samples, we obtain a population estimate for the correlation between consumption and investment at business cycle frequencies, conditional on a shock that changes the relative price of investment permanently.[6] The median correlation is 0.95. The CDF indicates that negative values for the correlation between consumption and investment are an unlikely occurrence.

The lower panel of Figure 2 shows the same CDF based on a longer sample, spanning the period from 1948q2 to 2015q1, which includes all the publicly available data at the point of writing. The results from the smaller sample appear robust. The median estimate of the conditional correlation between consumption and investment at business cycle frequencies is still a high 0.8, and the CDF still indicates that negative values are unlikely, with probability lower than 2%.

In sum, our extensions produce estimates of the correlation between consumption and investment that point to significant comovement over the business cycle conditional on shocks that permanently vary the price of investment. This comovement is robust to alternative sample choices. Moreover, we verified that our extensions do not overturn previously emphasized results on the importance of shocks to the relative price of investment in explaining business cycle fluctuations.

3. The Identification of Technology Shocks in Two-Sector Models: Part I

To interpret his identification scheme, Fisher (2006) wrote down a one-sector model with neutral MFP shocks and IST shocks that enter the capital accumulation equation. The work of Greenwood, Hercowitz, and Krusell (2000) implies that Fisher's identification scheme is consistent with a two-sector model under some restrictive assumptions, including equal factor shares across sectors and complete specialization in the assembly of consumption and investment. These assumptions are at odds with the U.S. Input-Output Tables. We show that Fisher's identification scheme is consistent with our extended two-sector model in which these assumptions are relaxed. Our demonstration has two components.

First, in this this section, we present a baseline version of our two-sector model with factor shares that differ across sectors, with which we can prove analytically that Fisher's identification scheme continues to apply. Specifically, the proof shows that relative prices respond permanently only to sector-specific shocks while labor productivity (aggregated at constant prices or in units of consumption) responds permanently both to equiproportionate sectoral shocks and to sector-

[6] The population estimate of the correlation between consumption and investment is obtained on a bootstrapped sample of 1050 observations, ten times as many as in the original sample. We used a bandpass filter to isolate the oscillations with frequencies between 6 and 32 quarters, typically used to define the business cycle.

specific shocks. Specifically, we derive steady-state relationships for a version of our model with the following features: the model includes only one stock of capital used in both sectors; both capital and labor are perfectly mobile across sectors; there is complete sectoral specialization in the assembly of consumption and investment. Second, in Section 4 we present numerical simulations of a more general version of our model in which important implications of the model for the identification of technology shocks carry through. Accordingly based on the analytical and numerical results, our extended two-sector model is consistent with the identification scheme used in Section 2, despite different factor input shares across sectors and despite the commingling of sectoral outputs in the assembly of final goods.

3.1. The Baseline Model

In period t, the representative household supplies a fixed amount of labor L, and maximizes the intertemporal utility function

$$\max_{C_s, I_s, K_{Ns}, K_{Ms}, B_s} \sum_{s=t}^{\infty} \beta^{s-t} \log C_s, \tag{1}$$

by choosing paths for consumption (C), investment (I), capital for M goods (K_M), capital for N goods (K_N), and for bonds (B) that pay the rate of return ρ after one period. The utility maximization problem is subject to a budget constraint given by

$$W_s L + R_{Ms} K_{Ms} + R_{Ns} K_{Ns} + \rho_{s-1} B_{s-1} = P_{Cs} C_s + P_{Is} I_s + B_s, \tag{2}$$

where W is the wage rate, R_M and R_N are the rental rates for K_M and K_N, respectively, P_C is the price of N goods but also of consumption ($P_C = P_N$), and P_I is the price of M goods but also of investment ($P_I = P_M$). Furthermore, the utility maximization problem is also subject to the following law of motion for the accumulation of capital

$$K_{M,s+1} + K_{N,s+1} = (1 - \delta)(K_{Ms} + K_{Ns}) + I_s, \tag{3}$$

with capital predetermined at the aggregate level and with δ denoting the depreciation rate for capital. There is complete specialization in the assembly of consumption and investment goods. Investment exhausts the output of the M sector ($I = Y_M$), and consumption exhausts the output of the N sector ($C = Y_N$).

In each sector, perfectly competitive firms minimize production costs to meet demand subject to the technology constraint as reflected in the following Lagrangian problems:

$$\min_{K_{Ms}, L_{Ms}, P_{Ms}} R_{Ms}K_{Ms} + W_s L_{Ms} + P_{Ms}(Y_{Ms} - K_{Ms}^{\alpha_M} (A_{Ms}L_{Ms})^{1-\alpha_M}), \tag{4}$$

$$\min_{K_{Ns}, L_{Ns}, P_{Ns}} R_{Ns}K_{Ns} + W_s L_{Ns} + P_{Ns}(Y_{Ns} - K_{Ns}^{\alpha_N} (A_{Ns}L_{Ns})^{1-\alpha_N}), \tag{5}$$

where α_M and α_N determine capital intensities of the production of M and N goods respectively. In addition to satisfying the first-order conditions for the optimization problems of households and firms given above, an equilibrium of the model also requires that all factor and product markets clear.

For the purposes of analyzing the implications of the model in the long run, we focus on the steady-state conditions for an equilibrium, which are summarized in Table 2.

3.2. Proving that the Baseline Model is Consistent with Fisher's Long Run Identification Scheme

In this section we prove analytically that the baseline two-sector model described in Section 3.1 satisfies the restrictions imposed by the identification scheme in Fisher (2006) despite its multi-sector structure with different factor intensities across sectors.

Theorem 1. *In the long run, equiproportionate shocks to technology in the two production sectors M and N affect aggregate labor productivity but do not affect relative prices. Furthermore, shocks to technology in one production sector affect both aggregate labor productivity and relative prices.*

The proof to this theorem is given in two parts below and relies on the steady-state conditions in Table 2. A corollary of this theorem is that the two-sector model of Section 3.1 can be used to interpret the permanent shocks to the relative price of investment and to labor productivity identified in Section 2.

Table 2: Steady State Restrictions

I)	$\frac{R_N}{P_N C} - \frac{P_M}{P_N C} + \beta \frac{P_M}{P_N C}(1-\delta) = 0$	II)	$R_{Nt} = R_{Mt}$
III)	$R_M = P_M \alpha_M \frac{Y_M}{K_M}$	IV)	$W = P_M(1-\alpha_M)\frac{Y_M}{L_M}$
V)	$R_N = P_N \alpha_N \frac{Y_N}{K_N}$	VI)	$W = P_N(1-\alpha_N)\frac{Y_N}{L_N}$
VII)	$Y_M = K_M^{\alpha_M}(A_M L_M)^{1-\alpha_M}$	VIII)	$Y_N = K_N^{\alpha_N}(A_N L_N)^{1-\alpha_N}$
IX)	$Y_M = I$	X)	$Y_N = C$
XI)	$L_M + L_N = L$	XII)	$K_M + K_N = \frac{1}{\delta} Y_M$

3.2.1. The Long-Run Response of Relative Prices

Some quick preliminary manipulations are in order. Notice that the rental rates for the two types of capital will be equalized in steady state, as shown in II) in Table 2, so I) implies

$$R_M = R_N = P_M \left(1 - \beta(1-\delta)\right). \tag{6}$$

Next, from III) and VII), and from V) and VIII) in Table 2, one can relate labor productivity at the sectoral level to the ratio of the sectoral price and the sectoral rate of return for capital:

$$\frac{Y_M}{L_M} = A_M \left(\alpha_M \frac{P_M}{R_M}\right)^{\frac{\alpha_M}{1-\alpha_M}}, \tag{7}$$

$$\frac{Y_N}{L_N} = A_N \left(\alpha_N \frac{P_N}{R_N}\right)^{\frac{\alpha_N}{1-\alpha_N}}. \tag{8}$$

The final preliminary manipulation involves using IV) and VI) in Table 2 to relate the relative price of goods in the two-sectors to the sectoral labor productivities:

$$\frac{P_M}{P_N} = \frac{(1-\alpha_N)}{(1-\alpha_M)} \frac{Y_N}{L_N} \frac{L_M}{Y_M}. \tag{9}$$

Substituting equations 6, 7, and 8 into equation 9, one can solve for $\frac{P_M}{P_N}$ in terms of parameters and the ratio of sector-specific technologies $\frac{A_N}{A_M}$:

$$\frac{P_M}{P_N} = \psi_1 \left(\frac{A_N}{A_M}\right)^{1-\alpha_N}, \quad \text{where } \psi_1 = \left(\frac{(1-\alpha_N)}{(1-\alpha_M)} \frac{\left(\alpha_N \frac{1}{(1-\beta(1-\delta))}\right)^{\frac{\alpha_N}{1-\alpha_N}}}{\left(\alpha_M \frac{1}{(1-\beta(1-\delta))}\right)^{\frac{\alpha_M}{1-\alpha_M}}} \right)^{1-\alpha_N}. \tag{10}$$

Thus, changes in technology in a single production sector will affect relative prices, but equiproportionate changes in technology in the two production sectors, dubbed neutral MFP shocks for the VAR of Section 2, will not affect relative prices. Looking beyond the model at hand with complete specialization, variation in relative prices at the sectoral level is a precondition for variation in relative prices at the level of final goods even in models with incomplete specialization. Accordingly, one can grasp how the result derived here also extends to richer models with incomplete sectoral specialization in the assembly of consumption and investment goods and is reflected in the numerical simulations offered below.

3.2.2. The Long-Run Response of Labor Productivity

Define aggregate labor productivity (at constant prices) as:

$$\frac{Y_{Mt}+Y_{Nt}}{L}=\frac{Y_{Mt}}{L_{Mt}}\frac{L_{Mt}}{L}+\frac{Y_{Nt}}{L_{Nt}}\frac{L_{Nt}}{L}. \tag{11}$$

First work on relating $\frac{L_{Mt}}{L}$ and $\frac{L_{Nt}}{L}$ to the conditions for an equilibrium in Table 2. Using V, VIII, 6 and III, VII, 6 one can obtain, respectively:

$$\frac{K_M}{Y_M}=\frac{\alpha_M}{(1-\beta(1-\delta))}, \tag{12}$$

$$\frac{K_N}{Y_N}=\frac{\alpha_N}{(1-\beta(1-\delta))}\frac{P_N}{P_M}. \tag{13}$$

$\frac{K_N}{Y_N}$ can be related to technology levels through (10). From XII, one has that $\frac{K_N}{Y_N}\frac{Y_N}{Y_M}+\frac{K_M}{Y_M}=\frac{1}{\delta}$, which can be used with to (12) and (13) to solve for $\frac{Y_N}{Y_M}$:

$$\frac{Y_N}{Y_M}=\psi_2\left(\frac{A_N}{A_M}\right)^{1-\alpha_N}, \quad \text{where} \quad \psi_2=\psi_1\left(\frac{(1-\beta(1-\delta))}{\delta\alpha_N}-\frac{\alpha_M}{\alpha_N}\right). \tag{14}$$

Combining IV, VI, and XI, one obtains:

$$\frac{L_M}{L}=\frac{(1-\alpha_M)P_{Mt}Y_{Mt}}{(1-\alpha_N)P_{Nt}Y_{Nt}+(1-\alpha_M)P_{Mt}Y_{Mt}}, \tag{15}$$

which can be expressed as a function of parameters and technology levels as in Equation 16 below, and since $L_N+L_M=L$, Equation 17 also follows:

$$\frac{L_M}{L}=\frac{(1-\alpha_M)\psi_1}{(1-\alpha_M)\psi_1+(1-\alpha_N)\psi_2}, \tag{16}$$

$$\frac{L_N}{L}=\frac{(1-\alpha_N)\psi_2}{(1-\alpha_M)\psi_1+(1-\alpha_N)\psi_2}. \tag{17}$$

Next, work on $\frac{Y_{Mt}}{L_{Mt}}$ and on $\frac{Y_{Nt}}{L_{Nt}}$. Combining equations 7 and 8 with equation 6 yields:

$$\frac{Y_M}{L_M} = A_M \left(\frac{\alpha_M}{(1-\beta(1-\delta))} \right)^{\frac{\alpha_M}{1-\alpha_M}}, \qquad (18) \qquad \frac{Y_N}{L_N} = A_N \left(\frac{\alpha_N}{(1-\beta(1-\delta))} \frac{P_N}{P_M} \right)^{\frac{\alpha_N}{1-\alpha_N}}. \qquad (19)$$

Summing up, remembering that $\frac{P_M}{P_N} = \psi_1 \left(\frac{A_N}{A_M} \right)^{1-\alpha_N}$, one can see that at constant prices:

$$\begin{aligned} \frac{Y_M + Y_N}{L} &= \frac{Y_M}{L_M}\frac{L_M}{L} + \frac{Y_N}{L_N}\frac{L_N}{L} = \\ & A_M \left(\frac{\alpha_M}{(1-\beta(1-\delta))} \right)^{\frac{\alpha_M}{1-\alpha_M}} \frac{(1-\alpha_M)\psi_1}{(1-\alpha_M)\psi_1 + (1-\alpha_N)\psi_2} \\ & + A_M^{\alpha_N} A_N^{(1-\alpha_N)} \left(\frac{\alpha_N}{\psi_1 (1-\beta(1-\delta))} \right)^{\frac{\alpha_N}{1-\alpha_N}} \frac{(1-\alpha_N)\psi_2}{(1-\alpha_M)\psi_1 + (1-\alpha_N)\psi_2}. \end{aligned} \qquad (20)$$

According to Equation 20, in the long run, aggregate labor productivity is a function of constant parameters and of the levels of multi-factor productivity in sectors M and N. Accordingly, labor productivity will vary permanently both in response to sectoral MFP shocks that vary the relative level of A_M and A_N, and in response to neutral MFP shocks that vary the levels of A_M and A_N equiproportionately. In sum, based on equations 10 and 20, our baseline model is consistent with the scheme in Fisher (2006).[7]

4. A Richer Model

To arrive more speedily at our novel results regarding the use of empirical estimates to discriminate between the aggregate and sectoral models, we give here an overview of the salient features of the richer model and relegate a full description to the appendix.

In order to incorporate empirically relevant features, we extend the baseline model along the lines of Guerrieri, Henderson, and Kim (2014). We augment the utility function in Equation 1 to allow for habit persistence in consumption and for endogenous labor supply, using an additively separable function between consumption and leisure. We modify Equation 3 so that the capital stocks are distinct and predetermined across sectors, rather than being predetermined only at the aggregate level, and we introduce investment adjustment costs. We allow for the investment and consumption aggregates to be constant-elasticity functions of machinery and non-machinery outputs.

[7] Notice that Fisher (2006) defined aggregate labor productivity in terms of consumption units, i.e., $\frac{Y_{Mt}}{L_{Mt}}\frac{L_{Mt}}{L}\frac{P_M}{P_N} + \frac{Y_N}{L_N}\frac{L_N}{L}$ using our notation, rather than at constant prices. Even under that alternative aggregation, labor productivity is affected both by equiproportionate shocks across production sectors and by shocks to a single production sector.

In the production functions embedded in Equation 4 and in Equation 5, we distinguish between two types of capital: equipment and structures. This greater degree of flexibility permits differences in factor intensities across sectors and the commingling of sectoral outputs consistent with the U.S. Input-Output Tables. Finally, we augment the stochastic structure of the model with non-technology shocks, namely government spending shocks, consumption preference shocks, and labor supply shocks, which help match key moments of U.S. data.

We estimate two variants of this richer model:

1. *Sectoral Model with MFP shocks* With all the extensions just described that increase the empirical relevance of the model, the resulting model cannot be aggregated to a standard one-sector model. We estimate this richer model capturing the variation in sectoral MFP levels with a neutral shock that varies the levels of MFP in equal ways across sectors and with an MFP shock specific to the machinery sector.

2. *Aggregate Model with IST shocks.* Under special parametric restrictions that impose complete sectoral specialization in the assembly of final goods, equal factor shares across sectors, capital stocks that are predetermined only at the aggregate level, our richer model can still be aggregated to a one-sector model. Moreover, under the same restrictions, sectoral variation in multi-factor productivity can be captured with a neutral MFP shock in the aggregate production function and with IST shocks that vary the efficiency of investment in producing installed capital right in the aggregate capital accumulation equation. We estimate the aggregate variant of the model with IST shocks that are in line with Fisher's original interpretation of the shocks that yield a permanent movement in the relative price of investment.

For each variant, the estimated parameters include the autoregressive coefficients and the standard deviations for all the shock processes. In addition, we estimate the elasticity of substitution between sectoral outputs in the assembly functions for final goods, including consumption and investment in both machinery and structures (for the sectoral model only), the degree of habit persistence in consumption, and the investment adjustment costs. We focus on matching the variances, the covariances, and the first autocorrelations of the same five variables used in the VAR: the growth rate of the relative price of investment, labor productivity growth, hours per capita, the growth rate of equipment and software per capita, and the growth rate of consumption per capita. To weight the various moments we use the diagonal of the simulated method of moments weighting matrix.

5. The Identification of Technology Shocks in Two-Sector Models: Part II

The empirical extension of the aggregate model do not influence its long-run properties. Accordingly, our aggregate model remains in line with the identification scheme described in Section 2. While we do not provide an analytical proof that the empirical extensions considered in the sectoral model are consistent with Fisher's identification scheme, Figure 3 offers a numerical substantiation by showing the response of the relative price of investment and of labor productivity to all the shocks included in the model. Among the shocks included in the model, the only shock that affects the price of investment permanently is an MFP shock in the machinery sector. Moreover, the only two shocks that affect the level of labor productivity permanently are the MFP shock in the machinery sector and the neutral MFP shock (constructed as MFP shocks in both sectors).

6. Discriminating Across Models Based on the VAR Results

Having established that the identification scheme for the VAR estimates is consistent with both variants of our richer model, we proceed by comparing model and VAR estimates. One approach typically used to discriminate across models based on VAR evidence is to check whether the model response to a certain shock is consistent or not with the empirical evidence from the VAR.[8] For our purposes, the problem with this approach is that the VAR confidence intervals for standard significance levels are so wide, as noted above in the description of Figure 1, that we would not be able to tell the models apart.

As noted in Erceg, Guerrieri, and Gust (2005), even imprecise tools such as our VAR can still be useful in discriminating across models. For instance, taking one of the models as the data-generating process, one could check if the VAR implies a bias in the point estimates of the impulse response functions in a certain direction. If that bias is reversed under the alternative model, then even an imprecise tool can offer sharp discriminating evidence. To investigate this possibility, we estimated the same VAR and used the same identification scheme to construct the impulse response functions in Figure 1 based on data generated from the two alternative DSGE models. For this experiment, we used 1000 randomly drawn samples of the same length as the baseline sample. We found that the differential implications of the two alternative models are swamped by the uncertainty associated with our empirical tool and still do not allow us to tell the models apart.[9]

[8] See, for instance, Gali (1999) and Gali and Rabanal (2004).

[9] The results for this experiment are reported in the appendix.

While the estimated impulse response functions do not offer discriminating evidence, a key difference between the two models is the correlation between consumption and investment at business cycle frequencies, conditional on shocks to the price of investment. The population estimate for this correlation is negative and equals -0.74 for the aggregate model with IST shocks and is positive and equal to 0.97 for the two-sector model with MFP shocks. The vertical lines in Figure 4 show these two correlations. Recall that the median population estimate for the conditional correlation between consumption and investment from the VAR is 0.95. For convenience, the red shaded area reproduces the CDF of the same correlation produced from the VAR. The CDF from the VAR indicates that the negative correlation from the aggregate model would be extremely unlikely pointing to the two-sector model as the more plausible candidate to explain the comovement properties extracted from the observed U.S. data.

In addition to the CDF from the VAR, Figure 4 also reports CDFs for the correlation between consumption and investment, obtained through the same Monte Carlo experiment described above for the impulse response functions. These CDFs allow us to gauge how sampling uncertainty affects the estimates for the correlation between consumption and investment when each of the alternative models is taken to be the data-generating process. The solid line shows the CDF for the two-sector model. The dashed line shows the CDF for the aggregate model. As for the case of the impulse response functions, the CDFs indicate that the VAR is an imprecise tool with substantial mass for the density function away from the pseudo-true values for each of the two models. Nonetheless, the CDF for the two-sector model is uniformly closer to the CDF for the VAR estimated on observed U.S. data, indicating that the two-sector model is a more plausible candidate data-generating process even when sampling uncertainty is considered.

7. Conclusion

Consumption and investment comove over the business cycle. Our estimates show that consumption and investment also comove conditional on shocks that change the price of investment permanently. Our finding obtains in our baseline sample, from 1982:Q3 to 2008:Q3, broadly coinciding with the Great Moderation, as well as in our full sample encompassing all publicly available data and spanning the period from 1948:Q2 through 2015:Q1.

We show that this comovement can be used to discriminate between alternative models of the business cycle. Heretofore, the set of models used to interpret permanent movements in the relative price of investment included one-sector models with IST shocks, or multi-sector models that could be aggregated to a one-sector model. We showed that, in fact, the set of admissible models also includes a two-sector model that cannot be aggregated. We found that this two-sector model matches more closely the evidence of a positive correlation between consumption and investment, conditional on shocks that move the price of investment permanently.

In this paper we have examined the connection between empirical evidence from movements in the relative price of investment with sectoral and aggregate treatments of multi-factor productivity changes using DSGE models. A fruitful avenue for further research would be to explore the relationship between sectoral MFP shocks inferred from identified VARs and sectoral measures of MFP levels obtained from growth accounting exercises in the tradition of Solow (1957) and Griliches and Jorgenson (1966). A related direction for further research would be to characterize the general class of DSGE models that is consistent with the restrictions implied by growth accounting exercises.

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Figure 1: VAR Estimates of the Response to a One-Standard Deviation Shock that Lowers the Level of the Relative Price of Investment Permanently

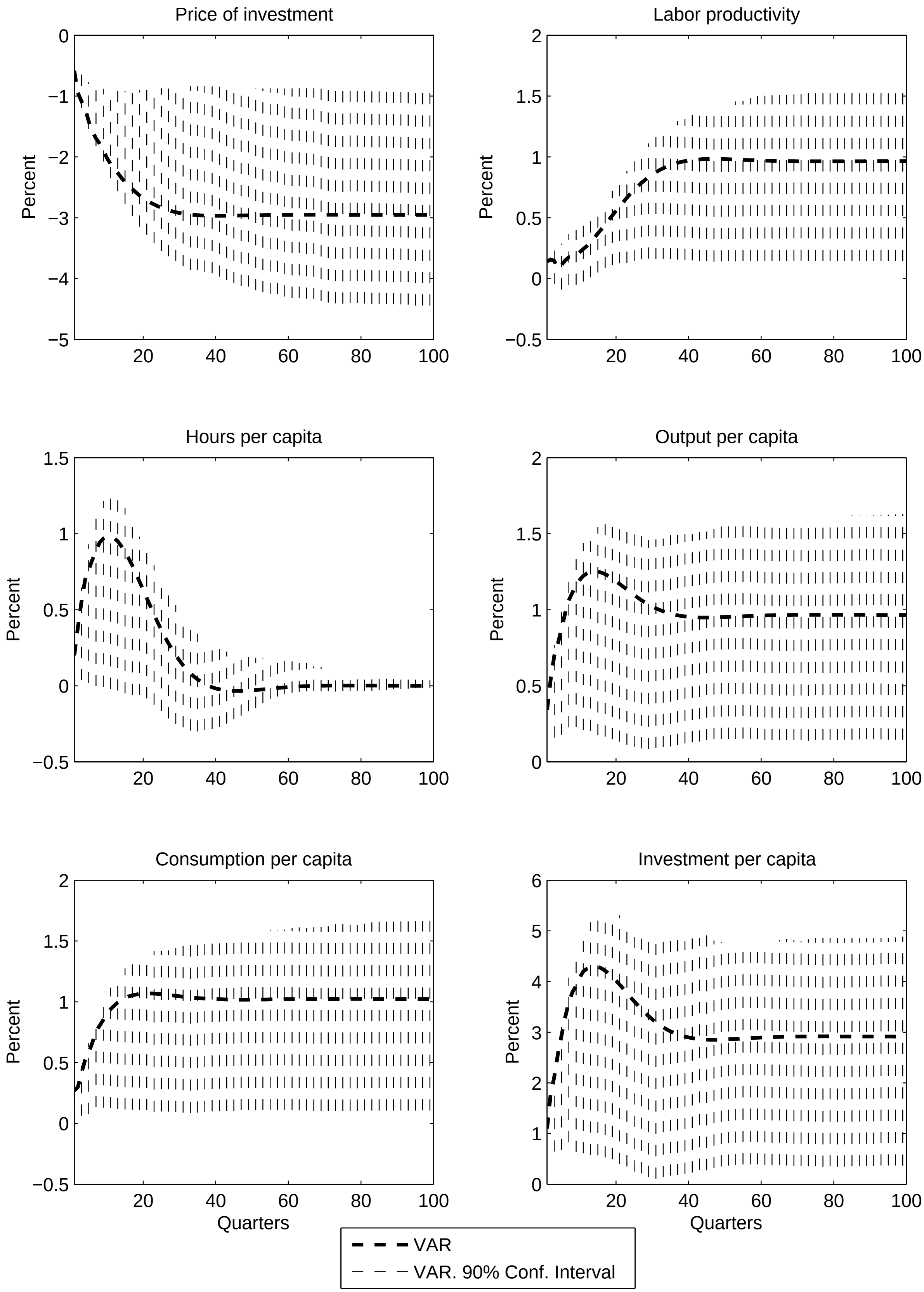
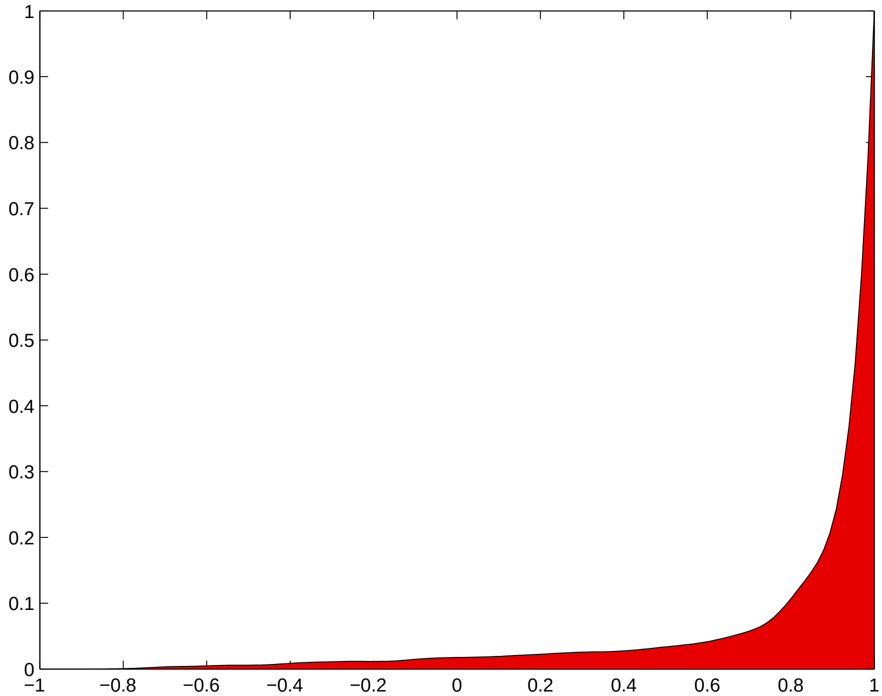


Figure 2: Cumulative Distribution Function for the Estimate of the Long-Run Correlation between Investment and Consumption at Business Cycle Frequencies

Baseline Sample from 1982:Q3 to 2008:Q3



Full Sample from 1948:Q2 to 2015:Q1

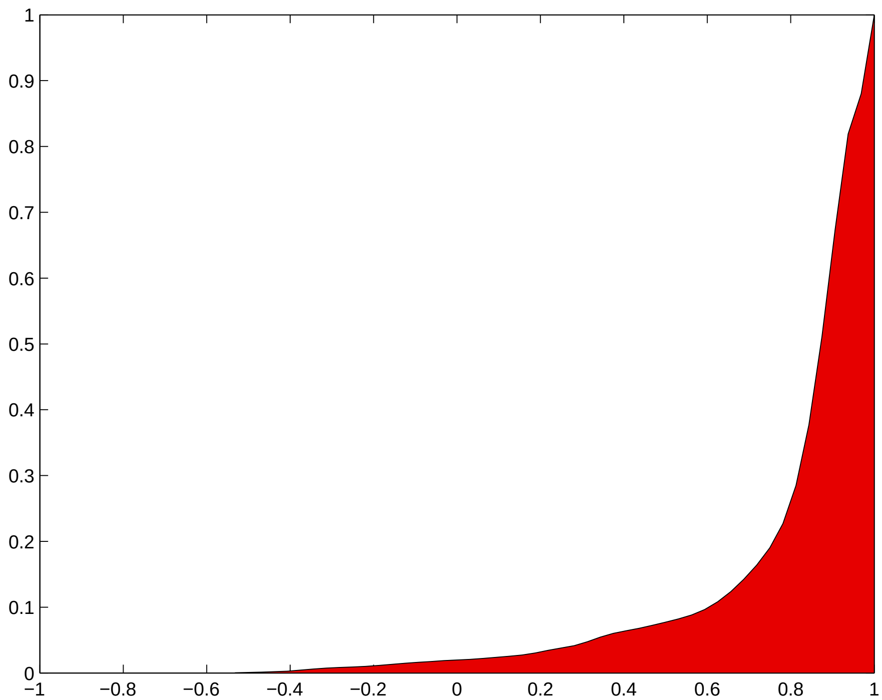


Figure 3: Properties of the Sectoral Model: The Responses of the Relative Price of Investment and of Labor Productivity to Various Shocks

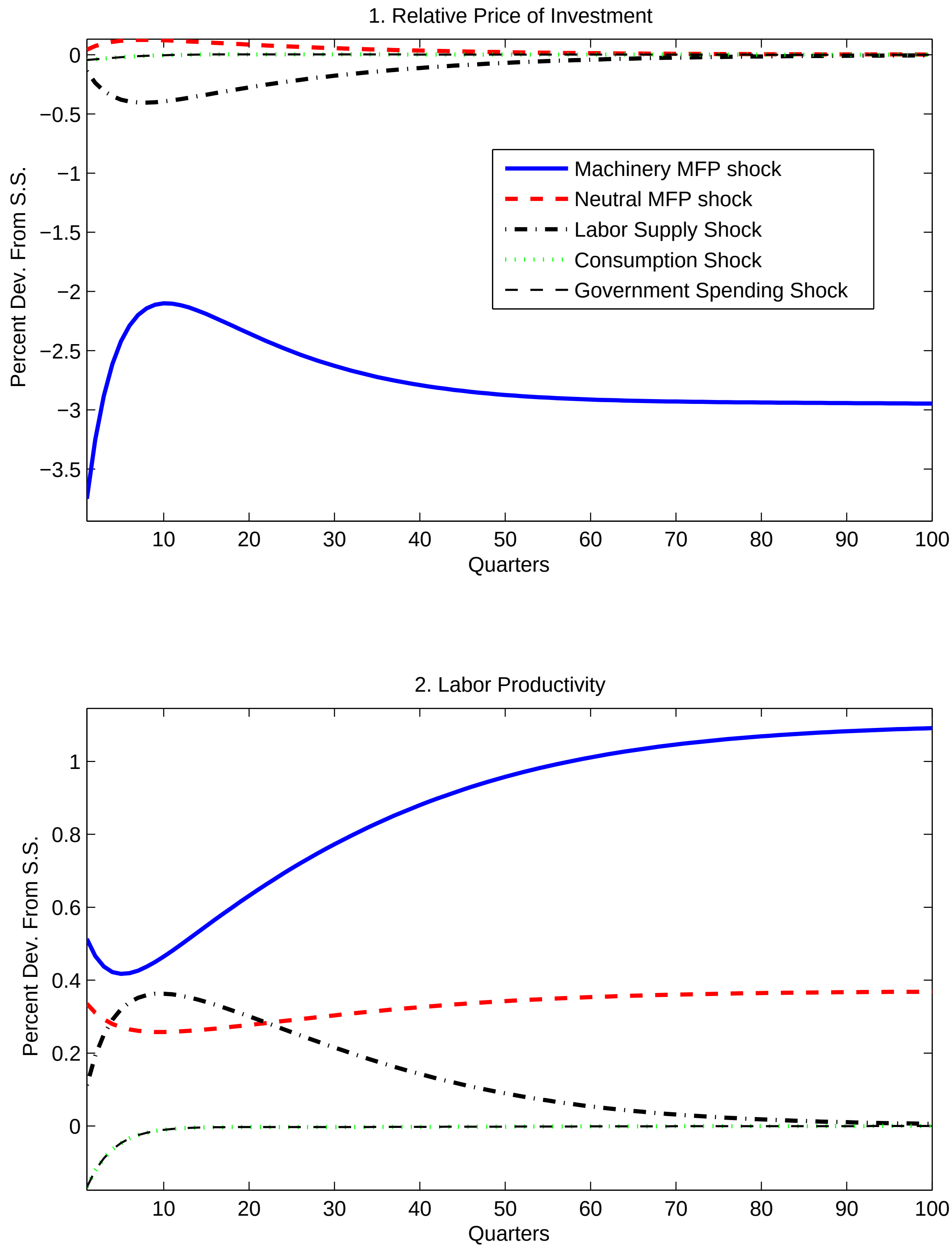
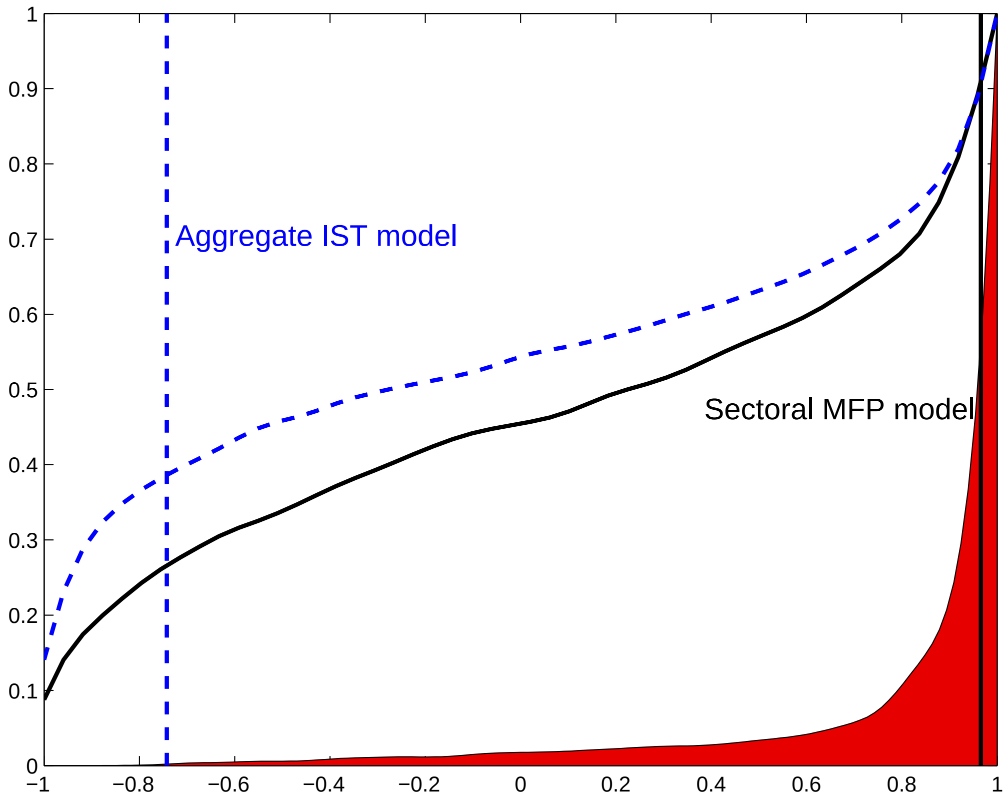


Figure 4: Cumulative Distribution Function for the Estimate of the Correlation Between Consumption and Investment at Business Cycle Frequencies, Conditional on Shocks that Lower the Price of Investment Permanently: VAR and DSGE Model Results

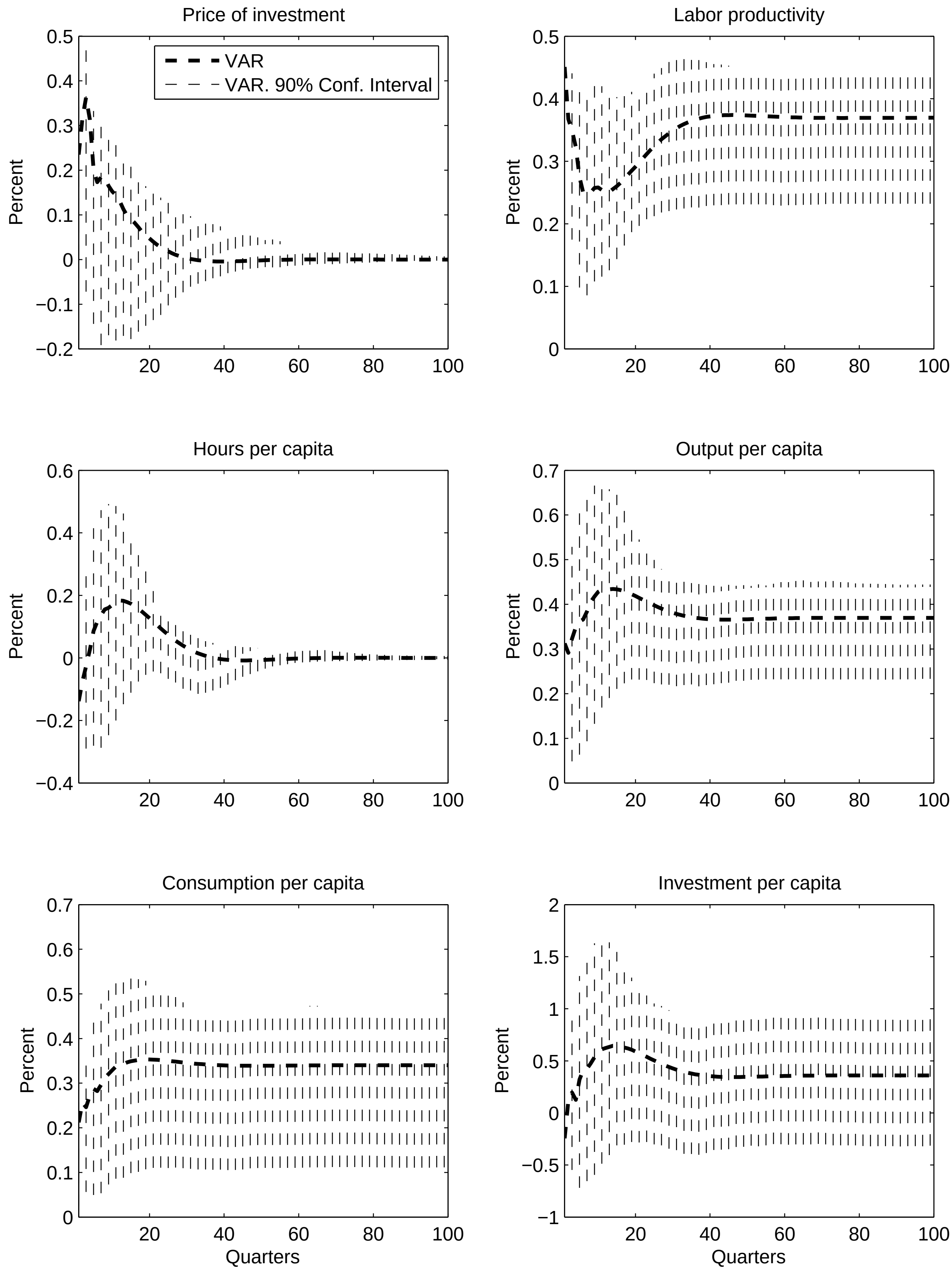


For convenience, the shaded area reports again the CDF for estimates the correlation between consumption and investment conditional on shocks that move the price of investment permanently from a VAR for the baseline sample 1982:q3-2008:Q3. The vertical lines denote estimates conditional on shocks that move the relative price of investment permanently in the aggregate model with IST shocks and in the sectoral model with MFP shocks. The CDF denoted by a dashed line pertains to a Monte Carlo experiment, in which the VAR is estimated on data generated from the aggregate model described in Section 4.The CDF denoted by a solid line pertains to a Monte Carlo experiment, in which the VAR is estimated on data generated from the sectoral model also described in Section 4.

A. Appendix: Additional Results from the VAR

Section 2 provides a description of our VAR, identification strategy, and estimated responses to a shock that moves permanently the relative price of investment. For completeness, Figure 5 shows the estimates of the response from to a one standard deviation shock that increases permanently the level of labor productivity but that does not have a long-run effect on the level of the relative price of investment. Again, for the variables that overlap, our results are close to those in Fisher (2006).

Figure 5: VAR Estimates of the Response to a One-Standard Deviation Shock that Increases the Level of Labor Productivity Permanently



B. Appendix: Full Description of the Extended Models

This appendix describes in detail our extended two-sector model with MFP shocks. Under some parametric restrictions the two-sector model collapses to an aggregate model. Section C reports the estimates of key parameters for both the two-sector model and the aggregate model and the parametric restrictions that allow the two-sector model to nest the aggregate model.

B.1. Production Sectors

Our two production sectors, the M (for Machinery) and N (for Non-machinery) sectors, comprise perfectly competitive firms. Consider the representative firm in sector i (where $i \in \{M, N\}$) in period s. It hires labor (L_{is}) from households at a wage (W_s) that is same for both sectors because labor is perfectly mobile between sectors. It also rents two types of capital from households: equipment capital (K_{is}^E) and structures capital $\left(K_{is}^S\right)$ at rentals (R_{is}^E and R_{is}^S) that are sector-specific when it is costly to reallocate capital. The firm minimizes the unit cost of producing a given number of physical units of its sector's output (Y_{is}) subject to a sector-specific Cobb-Douglas production function:

$$Y_{is} = (L_{is})^{1-\alpha_i^E-\alpha_i^S} \left(K_{is}^E\right)^{\alpha_i^E} \left(K_{is}^S\right)^{\alpha_i^S}. \tag{21}$$

The factor shares for the two types of capital are α_i^E and α_i^S. There is a multi-factor productivity (MFP) process A_is which determines the efficiency units generated by physical machinery output (i.e., $Y_{Ms}^A = A_{Ms}Y_{Ms}$).

Since it is competitive and there are constant returns to scale, the firm ends up selling at a price equal to unit cost. Let P_{is} represent the factor cost of a unit of physical output i. The factor cost of a physical unit of machinery is P_{Ms} and the cost of an efficiency unit of machinery is $P_{Ms}^A = \frac{P_{Ms}}{A_s}$ so that

$$P_{Ms}Y_{Ms} = \left(\frac{P_{Ms}}{A_{Ms}}\right) A_{Ms}Y_{Ms} = P_{Ms}^A Y_{Ms}^A. \tag{22}$$

. Similarly,

$$P_{Ns}Y_{Ns} = \left(\frac{P_{Ns}}{A_{Ns}}\right) A_{Ns}Y_{Ms} = P_{Ns}^A Y_{Ms}^A. \tag{23}$$

.

B.2. Final Goods

There are three final goods: a consumption good (C_s) and two investment goods, one (J_s^E) used for gross investment in E (for Equipment) capital stocks and the other (J_s^S) used for gross investment in S (for Structures) capital stocks. These goods are assembled by perfectly competitive final goods firms that use as inputs the outputs of the two production sectors, and these final goods are measured in efficiency units. When we find it expedient for the exposition, we us an upper bar to denote final goods measured in physical units.

The assembly function for consumption C_s and exogenous government spending G_s are a Cobb-Douglas function of two inputs, efficiency units of M goods along with N goods:

$$C_s = \left[\phi_M^C \left(\frac{A_{Ms}C_{Ms}}{\phi_M^C}\right)^{\frac{\sigma_C-1}{\sigma_C}} + \phi_N^C \left(\frac{A_{Ns}C_{Ns}}{\phi_N^C}\right)^{\frac{\sigma_C-1}{\sigma_C}}\right]^{\frac{\sigma_C}{\sigma_C-1}}, \tag{24}$$

$$G_s = \left[\phi_M^C \left(\frac{A_{Ms}G_{Ms}}{\phi_M^C}\right)^{\frac{\sigma_C-1}{\sigma_C}} + \phi_N^C \left(\frac{A_{Ns}G_{Ns}}{\phi_N^C}\right)^{\frac{\sigma_C-1}{\sigma_C}}\right]^{\frac{\sigma_C}{\sigma_C-1}}, \tag{25}$$

where ϕ_M^C and ϕ_N^C are the weights for M and N goods and σ_C is the elasticity of substitution between M and N goods in the assembly of C_s and of G_s.

The assembly functions for J_s^E and J_s^S are Cobb-Douglas functions of the two investment inputs, efficiency units of M goods along with N goods:

$$J_s^E = \left[\phi_M^E \left(\frac{A_{Ms}I_{Ms}^E}{\phi_M^E}\right)^{\frac{\sigma_E-1}{\sigma_E}} + \phi_N^E \left(\frac{A_{Ns}I_{Ns}^E}{\phi_N^E}\right)^{\frac{\sigma_E-1}{\sigma_E}}\right]^{\frac{\sigma_E}{\sigma_E-1}}, \tag{26}$$

$$J_s^S = \left[\phi_M^S \left(\frac{A_{Ms}I_{Ms}^S}{\phi_M^S}\right)^{\frac{\sigma_S-1}{\sigma_S}} + \phi_N^S \left(\frac{A_{Ns}I_{Ns}^S}{\phi_N^S}\right)^{\frac{\sigma_S-1}{\sigma_S}}\right]^{\frac{\sigma_S}{\sigma_S-1}}, \tag{27}$$

where $\phi_M^E, \phi_N^E, \phi_M^S$ and ϕ_N^S are the weights given to M and N goods, and σ_S and σ_E are the elasticities of substitution between M and N goods.

The assembly firms minimize the unit cost of producing efficiency units of consumption, equipment, and structures. Because they are perfectly competitive, firms end up selling final goods at prices that are equal to these costs and that are indicated by P_s^C, $P_s^{J^E}$, and $P_s^{J^S}$. We assume that the assembly functions for both C_s and J_s^S are intensive in N goods relative to the function for J_s^E.

There is an investment specific technology (IST) shock Z_s which further enhances the efficiency of J_s^E, the efficiency unit of equipment assembled using M and N inputs. The final total amount of equipment efficiency units is given by $Z_s J_s^E$ and the all-in unit cost is $\frac{P_s^{J^E}}{Z_s}$ so that

$$P_s^{J^E} J_s^E = \left(\frac{P_s^{J^E}}{Z_s} \right) Z_s J_s^E. \tag{28}$$

B.3. Tastes and Constraints

In period t, the representative household supplies a fixed amount of labor L and maximizes the following intertemporal utility function[10]

$$\sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{\left(\frac{C_s - \eta C_{s-1} - U_s}{1-\eta} \right)^{1-\gamma} - 1}{1-\gamma} - \sigma_0 V_s L_s \right], \tag{29}$$

where U_s and V_s represent aggregate demand shocks and labor supply shocks. The household also chooses holdings of a single bond (B_s) denominated in the N good (the numeraire good for the model). In addition, for each of the four inherited capital stocks ($D_{Ms}^E, D_{Ns}^E, D_{Ms}^S$, and D_{Ns}^S), the household decides how much to adapt to obtain the four capital stocks rented out for use in production ($K_{Ms}^E, K_{Ns}^E, K_{Ms}^S$, and K_{Ns}^S) as well as the fractions ($j_{Ms}^E, j_{Ns}^E, j_{Ms}^S$, and j_{Ns}^S) of investment of the two types (J_s^E or J_s^S) to be added to the four capital stocks. The distinction between capital inherited from the previous period, the D_{is}^j stocks, and capital used in production, the K_{is}^j stocks, allows us to nest in the same model the case in which capital is predetermined only at the *aggregate* level and the case in which capital is essentially predetermined also at the *sectoral* level.

The household is subject to period budget constraints. In each period, factor income plus income from bonds held in the previous period must be at least enough to cover purchases of final goods (consumption goods and the two types of investment goods), as well as bonds:

$$\begin{aligned} &W_s L + R_{Ms}^E K_{Ms}^E + R_{Ms}^S K_{Ms}^S + R_{Ns}^E K_{Ns}^E + R_{Ns}^S K_{Ns}^S + \rho_{s-1} B_{s-1} \\ &= P_s^C C_s + P_s^{J^E} J_s^E + P_s^{J^S} J_s^S + B_s + T_s, \end{aligned} \tag{30}$$

[10] The assumptions of fixed aggregate labor supply and perfect mobility of labor across sectors were made for simplicity, given our already involved structure with many sectors. Relaxing either of these assumptions matters for the issue of comovement. Katayama and Kim (2012) relax both assumptions.

where R^E_{Ms}, R^S_{Ms}, R^E_{Ns}, R^S_{Ns} are the rental rates for the capital stocks used in production. The term ρ_{s-1} is the gross return on bonds, and T_s represent lump-sum tax.

The household is subject to technological constraints when allocating capital. It inherits four capital stocks from the previous period. Inherited capital suited for one sector can be adapted for use in the other sector before being rented out, but only by incurring increasing marginal costs. For example, inherited equipment capital (D^E_{Ms}) suited for the M sector can be adapted for use in the N sector (K^E_{Ns}). Therefore, the capital of type h actually available for production in sector i in period s depends on how much has been adapted for production in that sector:

$$\begin{aligned} K^h_{Ms} + K^h_{Ns} &= D^h_{Ms}\left[1 - \frac{\omega^h}{2}\left(\frac{K^h_{Ms}}{D^h_{Ms}} - 1\right)^2\right] \\ &+ D^h_{Ns}\left[1 - \frac{\omega^h}{2}\left(\frac{K^h_{Ns}}{D^h_{Ns}} - 1\right)^2\right], \ \ h \in \{E, S\}. \end{aligned} \tag{31}$$

We consider two special cases: the case in which capital can be adapted at no cost ($\omega^h = 0$), so that capital is predetermined only at the aggregate level, and the case in which the marginal cost of adapting capital becomes prohibitive ($\omega^h \to \infty$), so that capital is predetermined at the sectoral level as well.

The household is also subject to technological constraints when accumulating capital. The accumulation equations for structures capital are more straightforward and we consider them first. Let D^S_{is} represent the amount of S capital available for production in sector i in period s without incurring any costs of adaptation:

$$D^S_{is} = \left(1 - \delta^S_i\right) K^S_{is-1} + j^S_{is-1} J^S_{s-1} - \frac{\nu^S_{0i}}{2} j^S_{is-1} J^S_{s-1} \left(\frac{j^S_{is-1} J^S_{s-1}}{j^S_{is-2} J^S_{s-2}} - 1\right)^2, \ \ i \in \{M, N\}, \tag{32}$$

period $s-1$ that is added to the structures capital suitable for sector i in that period. D^S_{is} has three components represented by the three terms on the right hand side of equation (32). The first is the amount of S capital actually used in production in sector i in period $s-1$ remaining after depreciation. The second is the amount of S investment added to structures capital suitable for sector i in period $s-1$. The third represents the adjustment costs incurred if the S investment in a given type of capital in period $s-1$ differs from that in period $s-2$. It is important to note that while the IST shock Z_s does not enter the accumulation equations for structures capital by assumption, the MFP shock A_{Ms} and A_{Ns} do enter through J^S_s.

The accumulation equations for equipment capital are less straightforward because of the distinction between physical units and efficiency units. Let D^E_{is} represent the amount of E capital available for production in sector i in period s without incurring any costs of adaptation:

$$\begin{aligned} D^E_{is} &= \left(1-\delta^E_i\right) K^E_{is-1} + Z_{s-1} j^E_{is-1} J^E_{s-1} \\ &+ \frac{\nu^E_{0i}}{2} Z_{s-1} j^E_{is-1} J^E_{s-1} \left(\frac{Z_{s-1}}{Z_{s-2}} \frac{j^E_{is-1} J^E_{s-1}}{j^E_{is-2} J^E_{s-2}} - 1 \right)^2, \quad i \in \{M, N\}, \end{aligned} \tag{33}$$

where j^E_{is-1} is the proportion of total equipment investment that is devoted to accumulation of structures capital suited for sector i in period $s-1$. Like D^S_{is}, D^E_{is} has three components. The first components of D^S_{is} and D^E_{is} are completely analogous. The second component of D^E_{is} is the amount of investment in equipment capital suited for sector i measured in efficiency units. It reflects the increase in the efficiency of the machinery input resulting from the MFP shocks A_{Ms} or A_{Ns} which are embedded in J^E_s and the increase in efficiency resulting from the IST shock Z_s. The third component represents investment adjustment costs.

The final household constraint is that for each type of investment good the proportions of the total amount added to the two capital stocks of the same type must sum to one:

$$1 = j^E_{Ms} + j^E_{Ns}, \quad 1 = j^S_{Ms} + j^S_{Ns}.$$

B.4. Market Clearing and Stochastic Structure

Market clearing requires that the outputs of the production sectors must be used up in the assembly of final goods:

$$Y_{Ms} = C_{Ms} + I^E_{Ms} + I^S_{Ms} + G_{Ms}, \quad Y_{Ns} = C_{Ns} + I^E_{Ns} + I^S_{Ns} + G_{Ns},$$

that labor demand equal labor supply,

$$L_{Ms} + L_{Ns} = L_s, \tag{34}$$

and that the bond be in zero net supply,

$$B_s = 0, \tag{35}$$

and that lump sum taxes are levied to finance all government spending,

$$T_s = G_s. \tag{36}$$

The conditions that firms' demands for K^E_{Ms}, K^E_{Ns}, K^S_{Ms}, and K^S_{Ns} equal households' supplies are imposed implicitly by using the same symbol for both.

We consider five sources of shocks:

1. The MFP shocks for the M and N sectors are integrated of order 1,

$$A_{Ms} = A_{Ms-1} + \epsilon_{AM} + \epsilon_A, \tag{37}$$

$$A_{Ns} = A_{Ns-1} + \epsilon_A, \tag{38}$$

   with the innovations $\epsilon_A M$, and ϵ_A each normally and independently distributed with mean 0 and standard deviation equal to $\sigma_A M$, σ_A, respectively. Notice that the innovation $\epsilon_A M$ is sector-specific, while the innovation ϵ_A is sector-neutral.

2. The IST shock is integrated of order 1,

$$Z_s = Z_{s-1} + \epsilon_Z, \tag{39}$$

   with the innovation ϵ_Z normally and independently distributed with mean 0 and standard deviation equal to σ_Z.

3. The shock to consumption U_s follows an AR(1) process,

$$U_s = \rho_U U_{s-1} + \epsilon_U, \tag{40}$$

   with the innovation ϵ_U normally and independently distributed with mean 0 and standard deviation equal to σ_U.

4. The shock to labor supply V_s follows an AR(1) process,

$$V_s = \rho_V V_{s-1} + \epsilon_V, \tag{41}$$

with the innovation ϵ_V normally and independently distributed with mean 0 and standard deviation equal to σ_V.

5. And, finally, government spending G_s is governed by an AR(1) process,

$$G_s = \rho_G G_{s-1} + \epsilon_G, \tag{42}$$

with the innovation ϵ_V normally and independently distributed with mean 0 and standard deviation equal to σ_V.

C. Appendix: Parameter Choices

We fix the model parameters with a mix of calibration and estimation. The calibration pertains to steady-state ratios and features that allow the general model described in Section to nest both the aggregate model and the model with sectoral MFP shocks.

C.1. Calibrated Parameters for the Aggregate Model

All calibrated parameters for the aggregate model are reported in Table 3. To facilitate comparisons with previous work on shocks that move the price of investment permanently in an aggregate model, we adhere to the parameter choices of Greenwood et al. (1997) whenever possible.[11] Accordingly, the output share of equipment in both the M and N sectors is 17% and the share of structures is 13%. The parameters governing the assembly functions are set so that there is complete specialization: consumption and structures investment are assembled using inputs from the N sector only, while equipment investment is assembled using inputs from the M sector only.[12] The depreciation rates for equipment and structures capital are 3.1% per quarter and 1.4% per quarter, respectively. The adaptation costs for capital are chosen so that capital is predetermined at the aggregate level and completely flexible in every period at the sectoral level. The discount factor is

[11] For simplicity, we abstract from trend growth as well as capital and labor taxes, while Greenwood et al. (1997) incorporate them in their model.

[12] The substitution elasticities between inputs in assembly become irrelevant under complete specialization.

set at 0.99, consistent with an annualized real interest rate of 4%. The intertemporal substitution elasticity for consumption is set at 1.

C.2. Calibrated Parameters for the Model with Sectoral MFP shocks

All calibrated parameters for the sectoral model are reported in Table 4. We focus here on the parameters that vary relative to the aggregate model.

Sector-specific production functions

To differentiate the intensities of factor inputs across sectors, we used the following restrictions: (a) while allowing variation across sectors, we kept the aggregate factor input intensities the same as in Greenwood et al. (1997); (b) factor payments are equalized across sectors, making the factors' shares of sectoral output proportional to the sectoral stocks of capital (since production functions are Cobb-Douglas)[13]; (c) factor input intensities are equal regardless of where the output of a sector is used.

We combined data for the net capital stock of private nonresidential fixed assets from the U.S. Bureau of Economic Analysis, with data from the Input-Output Bridge Table for Private Equipment and Software. The first data set contains data on the size of equipment and non-equipment capital stocks by sector. The second data set allowed us to ascertain the commodity composition of private equipment and software. Finally, we used BEA data to establish a sector's value added output. We focused on the year 2004, but similar sector-specific production functions would be implied by different vintages of data.

Our calculations show that the machinery-producing sector is less intensive in structures and labor than the aggregate economy, but more intensive in equipment capital. For the machinery sector, the share of structures is 11 percent, the labor share 46 percent, and the share of equipment capital the remaining 43 percent (thus, $\alpha_M^S = 0.11$,$\alpha_M^N = 0.46$, $\alpha_M^E = 0.43$). For the non-machinery sector the share of structures is 13 percent, the share of labor 72 percent, and the share of equipment capital 15 percent. The adaptations costs for capital are fixed at number sufficiently high to imply that capital stocks are predetermined at the sectoral level.

[13] If capital stocks are predetermined at the sectoral level, rentals are equalized only in the long run.

Incomplete specialization

The baseline calibration assumes complete specialization in the assembly of investment and consumption goods. Equipment investment is assembled using output from the M sector only. In contrast, structures investment and consumption goods are assembled using output from the N sector only. This complete specialization does not reflect the composition of final goods revealed in the Input-Output Bridge Tables that link final uses in the NIPA to sectors (industries) in the U.S. Input-Output Tables. For example, according to the data for 2004, wholesale and retail services (part of our non-machinery sector) are important inputs not only for consumption but also for equipment investment, accounting for 15 percent of the total output of private equipment and software.[14] Furthermore, electric and electronic products are used in the assembly of consumption, accounting for 4 percent of the total.[15]

The model captures the commingling implied by the bridge tables through assembly functions that specify how inputs from the M and N sectors are combined to obtain consumption, structures investment, and equipment investment. The share parameters for the assembly functions are set as follows: the shares for equipment investment are $\phi_M^E = 0.85, \phi_N^E = 0.15$ and the shares for consumption and structures investment are $\phi_M^C = \phi_M^S = 0.04, \phi_N^C = \phi_N^S = 0.96$.

[14] There are bridge tables for consumption as well as equipment and software investment but not for structures investment. We assume that the sectoral composition of structures investment is the same as that of consumption.

[15] The machinery sector of our model has two components. The first component is the NIPA definition of "Equipment and Software" Investment, after excluding the Transportation, Wholesale, and Retail Margins from the Input-Output Tables. Most of the industries whose output is used in "Equipment and Software" produce exclusively for "Equipment and Software." The second component of our machinery sector comprises those inputs for consumption assembly from all the industries that produce inputs used in both the NIPA definition of "Equipment and Software" Investment and of "Consumption." These IO Table industries are: (334) Computer and Electronic Products; (335) Electrical Equipment, Appliances, and Components; (513) Broadcasting and Telecommunications; (514) Information and Data Processing Services; and (5412OP) Miscellaneous Professional, Scientific and Technical Services.

Table 3: Calibration for Aggregate Model

Parameter	Determines	Parameter	Determines
	Utility Function		
$\gamma = 1$	Intertemporal consumption elast. $= 1/\gamma$	$\beta = 0.99$	Discount factor
	Depreciation Rates		
$\delta^E = 0.031$	Equipment capital	$\delta^S = 0.014$	Structures capital
	Adaptation Costs		
$\omega^E = 0$	M, N Equipment Capital	$\omega^S = 0$	M, N Structures Capital
	M Goods Production		
$\alpha_M^N = 0.7$	Labor share	$\alpha_M^E = 0.17$	Equipment share
$\alpha_M^S = .13$	Structures share		
	N Goods Production		
$\alpha_N^N = 0.7$	Labor share	$\alpha_N^E = 0.17$	Equipment share
$\alpha_N^S = 0.13$	Structures share		
	Consumption Assembly		
$\phi_M^C = 0$	M goods intensity	$\phi_N^C = 1$	N goods intensity
	Assembly of Equipment Investment		
$\phi_M^E = 1$	M goods intensity	$\phi_N^E = 0$	N goods intensity
	Assembly of Structures Investment		
$\phi_M^S = 0$	M goods intensity	$\phi_N^S = 1$	N goods intensity

Table 4: Calibration for the Model with Sectoral MFP Shocks

Parameter	Determines	Parameter	Determines
	Utility Function		
$\gamma = 1$	Intertemporal consumption elast. $= 1/\gamma$	$\beta = 0.99$	Discount factor
	Depreciation Rates		
$\delta^E = 0.031$	Equipment capital	$\delta^S = 0.014$	Structures capital
	Adaptation Costs		
$\omega^E = 100$	M, N Equipment Capital	$\omega^S = 100$	M, N Structures Capital
	M Goods Production		
$\alpha_M^N = 0.46$	Labor share	$\alpha_M^E = 0.43$	Equipment share
$\alpha_M^S = .11$	Structures share		
	N Goods Production		
$\alpha_N^N = 0.72$	Labor share	$\alpha_N^E = 0.15$	Equipment share
$\alpha_N^S = 0.13$	Structures share		
	Consumption Assembly		
$\phi_M^C = 0.04$	M goods intensity	$\phi_N^C = 0.96$	N goods intensity
	Assembly of Equipment Investment		
$\phi_M^E = 0.85$	M goods intensity	$\phi_N^E = 0.15$	N goods intensity
	Assembly of Structures Investment		
$\phi_M^S = 0.04$	M goods intensity	$\phi_N^S = 0.96$	N goods intensity

C.3. Estimated Parameters

For the estimation, we focus on matching the variance, the covariance, and the first autocorrelation of the same five variables used in the VAR: the growth rate of the relative price of investment, labor productivity growth, hours per capita, the growth rate of equipment and software per capita, and the growth rate of consumption per capita. To weigh the various moments we use the diagonal of the simulated method of moments weighting matrix.

We estimate the parameters governing the shock processes (labor supply, consumption, and government spending shocks). We estimate the parameter η, governing consumption habits, and the parameters ν_{0M} and ν_{0N}, determining the investment adjustment costs. In line with our focus on aggregate data, we restrict the investment adjustment costs to be equal across sectors. Finally, for the sectoral model with MFP shocks, we estimate the elasticity of substitution between factor inputs in the assembly of final goods, governed by the parameters σ_C, σ_E, and σ_S, which are also imposed to equal each other.

We read out the standard deviations for the innovations for the neutral MFP and sectoral MFP or IST shocks from the VAR estimates. The standard deviation of the neutral MFP shock is chosen to match the VAR long-run response of labor productivity to a one-standard-deviation MFP shock. The standard deviation of the sectoral MFP or IST shocks is chosen to match the VAR long-run response of the relative price of investment to a one-standard-deviation shock to the relative price of investment. Under the calibration for the aggregate model, sectoral MFP shocks and IST shocks are equivalent and we drop the sectoral MFP shocks. Under the calibration that maintains the sectoral detail, we drop the IST shocks.

The estimation results are summarized in Tables 5 and 6.

Table 5: Estimated Parameters For the Aggregate Model

Parameter	Determines	Parameter	Determines
Standard Deviations of Shocks			
$\sigma_A = 0.0036$	Neutral MFP	$\sigma_Z = 0.030$	IST
$\sigma_U = 0.022$	Consumption	$\sigma_V = 0.036$	Labor supply
$\sigma_G = 0.11$	Government spending		
Autoregressive Coefficient of Shocks			
$\rho_U = 0.71$	Consumption	$\rho_V = 0.97$	Labor supply
$\rho_G = 0.94$	Government spending		
Other Structural Parameters			
$\eta = 0.40$	Habits	$\nu_0 = 0.25$	Investment adj. costs

Table 6: Estimated Parameters For the Model with Sectoral MFP Shocks

Parameter	Determines	Parameter	Determines
Standard Deviations of Shocks			
$\sigma_A = 0.0037$	Neutral MFP	$\sigma_A M = 0.0576$	Sectoral MFP
$\sigma_U = 0.0055$	Consumption	$\sigma_V = 0.012$	Labor supply
$\sigma_G = 0.062$	Government spending		
Autoregressive Coefficient of Shocks			
$\rho_U = 0.001$	Consumption	$\rho_V = 0.99$	Labor supply
$\rho_G = 0.94$	Government spending		
Other Structural Parameters			
$\eta = 0.77$	Habits	$\nu_0 = 0.14$	Investment adj. costs
$\sigma_C = \sigma_E = \sigma_S = 10.77$	Sub. Elast. between M and N goods		

D. Appendix: Additional Results of Monte Carlo Experiment

The red lines in Figure 6 show the responses to an MFP shock in the machinery sector of our two-sector model. By construction, the long-run response of the relative price of investment matches the response estimated from the VAR, but the short-run response is left unconstrained. The responses of consumption, investment, and hours per capita fall within the 90% confidence intervals estimated from the VAR both in the short and the long run. The most glaring departures from the results of the VAR occur for the relative price of investment and for labor productivity in the short run. However, if we were to match with the model the response of the price of investment from the VAR in every period, the resulting path for labor productivity, as well as all the other variables shown, would fall within the confidence interval of the VAR even in the short run.[16] The areas shaded in solid red show the results of a Monte Carlo experiment in which 1000 samples of the same length as the observed data were drawn using our two-sector model. For each sample we re-estimated the same VAR as for the observed data. The shaded areas are 90% confidence intervals for the response to a shock that lowers the relative price of investment permanently. There is substantial overlap between the areas shaded in solid red and those in dashed black indicating that the VAR results could have been generated from a random sample from our two-sector model.

Figure 7 reports results for the IST model analogous to those described above. For convenience, the VAR results from the observed data are repeated again, as thick dashed and vertical dashed lines. The responses of consumption, investment, and hours per capita to an IST shock in our one-sector model fall within the confidence interval from estimation of the VAR most of the time horizon, except in the short run. Again the most glaring departure concerns the response of the price of investment in the short run—the long-run response for this variable being matched by construction. However, if we were to match with the model the response of the price of investment from the VAR in every period, the resulting paths for all the variables shown would fall within the confidence interval of the VAR even in the short run, in this case, too. Accordingly, based only on the impulse response functions reported in the figure, we would fail to reject the aggregate model with IST shocks.

[16] We confirmed this result by feeding a path of unforeseen shocks for the MFP process of the machinery sector that was devised to replicate the path from the VAR.

Figure 6: The VAR Response to a One-Standard Deviation Shock that Lowers the Relative Price of Investment Permanently, Compared Against the Response to an MFP shock in the Machinery Sector of the Two-Sector Model and Against VAR Estimates Based on a Monte Carlo Experiment

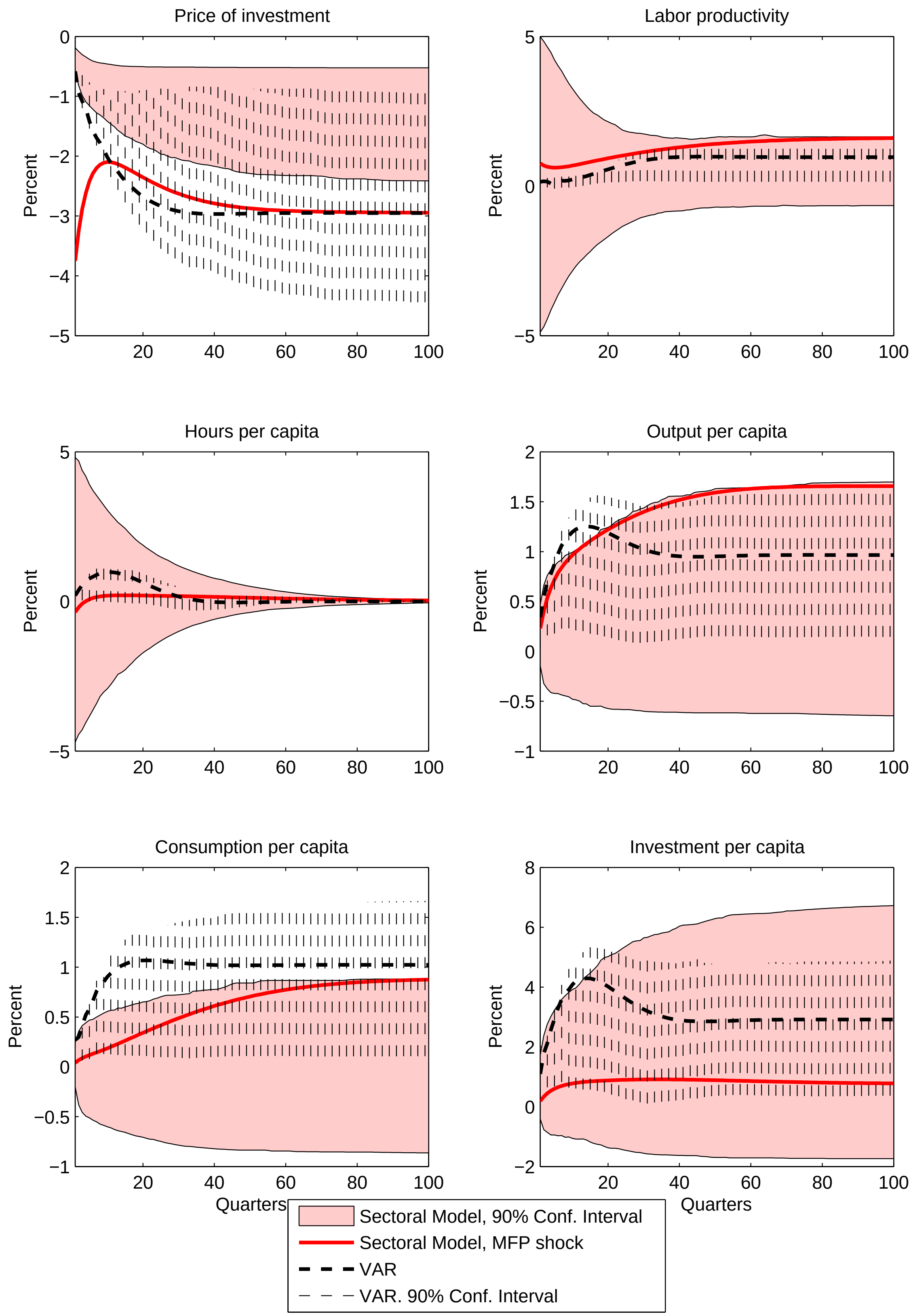


Figure 7: The VAR Response to a One-Standard Deviation Shock that Lowers the Relative Price of Investment Permanently, Compared Against the Response to an IST shock in the Aggregate Model and Against VAR Estimates Based on a Monte Carlo Experiment