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Accounting for Productivity Dispersion over the Business Cycle *

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Abstract

This paper presents accounting decompositions of changes in aggregate labor and capital productivity. Our simplest decomposition breaks changes in an aggregate productivity ratio into two components: A mean component, which captures common changes to firm factor productivity ratios, and a dispersion component, which captures changes in the variance and higher order moments of their distribution. In standard models with heterogeneous firms and frictions to firm input decisions, the dispersion component is a function of changes in the second and higher moments of the log of marginal revenue factor productivities and reflects changes in the extent of distortions to firm factor input allocations across firms. We apply our decomposition to public firm data from the United States and Japan. We find that the mean component is responsible for most of the variation in aggregate productivity over the business cycle, while the dispersion component plays a modest role.

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1 Introduction

What drives changes in aggregate productivity? One explanation that has been widely used to explain the variation of aggregate productivity over the business cycle or over time more generally is that frictions to the allocation of labor and capital between firms are time-varying: Greater frictions to the distribution of capital and labor between firms reduce the amount of output produced with a given amount of capital and labor and reduce measures of aggregate productivity.[1] This paper presents accounting decompositions of changes in aggregate labor, capital, and total factor productivity that addresses this economic mechanism, and can help to quantify the extent to which the changing distribution of labor and capital drive fluctuations in aggregate productivity over time.

The accounting decompositions in this paper rely on the property that aggregate factor productivity ratios can be expressed as the weighted sum of firm-level productivity ratios. Our first decomposition splits changes in measures of aggregate factor productivities into a mean component, changes in the weighted average of log productivities across firms, and a dispersion component, which captures changes in the higher order moments of the distribution of productivities across firms.[2] The two components add up to the change in a given aggregate factor productivity ratio. We compute the decomposition separately for both aggregate labor and capital productivity. Crucially, for the decomposition of aggregate labor productivity, we require only firm-level panel data on value added and labor, and for the decomposition of aggregate capital productivity, we require firm-level panel data on value added and capital.

The allocation of labor and capital may vary across firms not only due to distortions but for technological reasons as well; the second decomposition allows us to group firms (by industry or other categorical groups) to address this point. We implement our first decomposition on each sector, resulting in sectoral mean and dispersion components. We can then weight each sector's mean and dispersion components by sectoral factor shares to obtain aggregate mean and dispersion components. Thus, by an accounting property, the change in aggregate factor productivities can be decomposed into three components: First,

[1] This economic mechanism plays a role in driving the dynamics of productivity and other macroeconomic aggregates in a number of recent influential papers, including Arellano et al. (2012), Bloom et al. (2014), Gilchrist et al. (2014), Khan and Thomas (2013), Midrigan and Xu (2014), and Moll (2014), as examples.

[2] To be precise, the dispersion component can be expressed as a function of the second and higher order cumulants of the distribution of firm productivity measures, while the mean component is only a function of the first cumulant.

an aggregated mean component which captures changes in the weighted average of log factor productivities *within* sectors. Second, an aggregated dispersion component which captures changes in the dispersion of log factor productivities across firms *within* sectors. Third, a sectoral-share component, which captures the changes in the distribution of inputs *between* sectors.

Our decompositions, when applied to aggregate labor or capital productivity, are purely accounting identities. To combine aggregate capital and labor productivity into a measure of total factor productivity, we rely on the standard model assumptions that allow us to compute the Solow residual. We then show that the Solow residual has the nice property that we can express it as the weighted average of the mean, dispersion, and sectoral-share components of capital and labor productivity.

Our decompositions are useful tools for researchers testing whether models where frictions to the allocation of labor or capital across firms play a meaningful role in driving aggregates are consistent with firm level behavior. We present a series of results to demonstrate this point. In the model of Hsieh and Klenow (2009), we demonstrate how our decomposition captures changes in the distribution of the log of marginal revenue factor productivities. We prove that changes in the expected value of the log of marginal revenue factor productivities, as well as changes in production function coefficients, drive changes in the mean component of our decompositions. We prove that changes in the second central moment and all higher order moments of the log of marginal revenue factor productivities drive changes in the dispersion component of our decompositions.

We then use a more general model of production by heterogeneous firms to demonstrate how distortions to firm capital and labor decisions are captured in our decomposition. We demonstrate analytically that the dispersion component of our decomposition captures changes in productivity due to heterogeneous distortions to firm-level input allocation. The mean component of our decompositions captures changes in technology or common distortions to firm capital or labor choices. We prove that this general model of production has a mapping to a large number of macroeconomic models in the literature that utilize frictions to the allocation of labor or capital across firms to help drive aggregate dynamics.

We compute our decompositions for aggregate labor productivity, capital productivity, and TFP using firm-level data on U.S. nonfinancial public firms. To see if the results are consistent for another large, developed nation, we perform a similar analysis for nonfinancial public firms from Japan. The results for the United States and Japan from the second decomposition applied to labor productivity show that the mean component is highly correlated with movements in aggregate labor productivity and are essentially solely responsible for its cyclical variation. The magnitude of movements in the dispersion component are small, and the dispersion component has a weak negative correlation with changes in aggregate labor productivity. Our results are different for aggregate capital productivity. The dispersion component moves much more closely with changes in aggregate capital productivity, and does play a role in contributing to cyclical variation in aggregate capital productivity. Our decomposition, when applied to TFP, yields the result that the mean component is responsible for the vast majority of its cyclical variation, because much of the cyclical movements in TFP are driven by changes in aggregate labor productivity.

Related Literature The contribution of this paper is to provide accounting decompositions of aggregate labor and capital productivity, which can be implemented without structural estimation, and can guide the specification of firm-level frictions to capital and labor allocation in business cycle models. The fact that our decompositions only require measures of firm-level value added, labor, and capital, and do not require estimation to be computed is an attractive property, as it implies the use of our decomposition not only avoids potential biases from estimation, but also means that our decomposition can be computed in both data and heterogeneous firm models with relative ease. A large number of papers in the literature work with production environments that map into the class of production environments that we rely on to prove how our decomposition maps into models in Section 3. The general class of models to which our theoretical results apply include the influential models of Arellano, Bai, and Kehoe (2012), Bloom et al. (2014), Kehrig (2015), and Khan and Thomas (2013), as only a few recent examples. Thus, the dispersion component of our decomposition reflects changes in the distribution of distortions to firm input allocation in such papers. Hence, the role of frictions to firm labor and capital allocation in a large number of models can be compared to the data through the use of our decomposition. Our empirical results alone can also help to guide model selection in standard, widely-used production environments. In this sense, our decomposition is similar in spirit to Chari, Kehoe, and McGrattan (2007).

Our paper is also related to a number of recent studies which examine the role of reallocation or allocative efficiency in driving aggregate productivity dynamics. One group of papers estimate production function coefficients and firm-level total factor productivities to assess the role of allocative frictions in driving productivity over the business cycle, such as Oberfield (2013), Osotimehin (2013), and Sandleris and Wright (2014). Our approach differs from this set of the literature in that our decompositions are accounting identities requiring only measures of firm-level value added, capital, and labor, and thus we do not require the estimation of production function coefficients. Our method therefore avoids the potential econometric biases in these estimation procedures (which are discussed in Appendix C) and can be implemented immediately on a wide array of models and data. The magnitude of the dispersion component of our decompositions can be viewed as an approximation to the extent to which allocative efficiency affects aggregate productivity in such models (we show this in Appendix C). Thus, our decomposition, if applied to the respective datasets used in these papers, could be used to complement the paper's structural approaches and potentially address concerns regarding the assumptions required for estimation. Another group of papers examine the role of resource reallocation through the use of aggregate productivity decompositions, such as Foster, Haltiwanger, and Krizan (2001) and Basu and Fernald (2002). The sectoral share component of our second decomposition also speaks to the role resource reallocation between sectors can play in driving productivity dynamics. Differently from these papers, however, the dispersion component of our decomposition captures the role allocative efficiency plays in driving productivity dynamics.[3] Additionally, our decomposition does not require the estimation of firm-level TFP.

The rest of the paper proceeds as follows. Section 2 defines the components of our decompositions for aggregate labor productivity, aggregate capital productivity, and TFP. Section 3 discusses how shocks to firm-level wedges map into the components of our decomposition. Section 4 applies our decomposition to data from U.S. and Japanese nonfinancial public firms. Section 5 concludes.

2 Productivity Decompositions

In this section, we first present our decompositions of changes in aggregate labor and capital productivity, and then we present how to combine these decompositions to perform decompositions of changes in TFP. Decomposition I breaks changes in the log of each aggregate productivity ratio into a mean and a dispersion component to help identify whether it is changes in the mean or dispersion of the log of firm-level productivity ratios that are driving changes in aggregate productivity. Decomposition II allows for groupings of firms

[3] Alternatively, adjustment costs could generate a dynamically efficient allocation that observationally is consistent with static misallocation; this point is made in Asker et al. (2014), e.g.

(sectors) to each have a mean and a dispersion component, and for the allocation of inputs between each grouping of firms to change over time. In turn, when analyzing changes in aggregate productivity, there is also a sectoral-share component, which reflects how input shares are changing across sectors over time.

2.1 Decomposition I: Mean and Dispersion Components

We start with a static decomposition of aggregate labor productivity. We define L as aggregate labor and l as firm-level labor. Aggregate labor is the sum of all firm-level labor. We define K as the aggregate capital stock and k as the firm-level capital stock, where the aggregate capital stock is the sum of all firm capital stocks. The decomposition below holds for capital productivity as well, if we substitute K for L and k for l.

We define Y as aggregate output and v as firm value added, where aggregate output is the sum of all firm-level value added. We have the following identity, which holds at each time t:

$$\frac{L_t}{Y_t} \equiv \sum_i \frac{l_{i,t}}{v_{i,t}} \frac{v_{i,t}}{Y_t}, \tag{1}$$

where i indexes the set of firms in the economy.

Building on (1), we can now perform a static version of our first decomposition:

$$\log\left(\frac{L_t}{Y_t}\right) = \underbrace{\sum_i \log\left(\frac{l_{i,t}}{v_{i,t}}\right)\frac{v_{i,t}}{Y_t}}_{\text{static mean component}} + \underbrace{\left(\log\left(\sum_i \frac{l_{i,t}}{v_{i,t}}\frac{v_{i,t}}{Y_t}\right) - \sum_i \log\left(\frac{l_{i,t}}{v_{i,t}}\right)\frac{v_{i,t}}{Y_t}\right)}_{\text{static dispersion component}}. \tag{2}$$

Aggregate labor to output at each time t is now broken into a "mean component," which is the weighted average of the log of labor to value added, and a "dispersion component." If we treat labor to value added as a random variable with a probability density function (reflecting the number and size of firms with a given productivity ratio), the dispersion component takes the form of the log of the expectation of firm-level labor to value-added ratios less the expectation of the log of firm-level labor to value-added ratios. This term is always non-negative due to Jensen's inequality. This measure has useful statistical properties related to the measure of entropy in Backus, Chernov, and Zin (2014). Assuming some regularity conditions on the distribution of firm labor to value-added ratios such that the cumulant generating function exists, the dispersion component captures all higher-order cumulants of the distribution of firm-level labor to value-added ratios.[4] This can be interpreted as the following: The dispersion component captures the effect of all second and higher order moments of the distribution of firm labor productivity on aggregate labor productivity.

We are interested in *changes* in labor productivity. We can recover changes in labor productivity as:

$$\Delta \log\left(\frac{Y_t}{L_t}\right) = \underbrace{-\Delta \sum_i \log\left(\frac{l_{i,t}}{v_{i,t}}\right)\frac{v_{i,t}}{Y_t}}_{\text{mean component}} \underbrace{-\Delta\left(\log\left(\sum_i \frac{l_{i,t}}{v_{i,t}}\frac{v_{i,t}}{Y_t}\right) - \sum_i \log\left(\frac{l_{i,t}}{v_{i,t}}\right)\frac{v_{i,t}}{Y_t}\right)}_{\text{dispersion component}}. \tag{3}$$

An increase in dispersion in firm-level labor to value-added ratios decreases aggregate labor productivity. Similarly, an increase in the weighted average of firm-level labor to value-added ratios decreases aggregate labor productivity. Our mean/dispersion decomposition allows us to determine whether it is changes in the mean or the dispersion in the log of firm-level labor to value added which is driving changes in aggregate labor productivity. We present our second decomposition below, which allows each sector to have a mean and dispersion component. Hence, changes in aggregate labor productivity can be driven by changes in the mean of log firm-level labor to value-added ratios *within* sectors, changes in their dispersion *within* sectors, or changes in the allocation of inputs *between* sectors.

2.2 Decomposition II: Mean, Dispersion, and Sectoral Share Components

For a sector (or any given grouping of firms), the identity in (1) holds. Hence, if j indexes a given sector, we have the following identity for aggregate labor to added value ratio within that sector at time t:

$$\frac{L_t^j}{Y_t^j} \equiv \sum_i \frac{l_{i,t}^j}{v_{i,t}^j}\frac{v_{i,t}^j}{Y_t^j}. \tag{4}$$

[4] Cumulants summarize the distribution of a random variable, as we explain in more detail in Section 3. Backus, Chernov, and Zin (2014) also provide an excellent discussion of why functions of this form capture all higher-order cumulants.

In turn, for each sector at time t, we can decompose the aggregate labor to added value ratio within a sector into a mean and dispersion component:

$$\log\left(\frac{L_t^j}{Y_t^j}\right) = \underbrace{\sum_i \log\left(\frac{l_{i,t}^j}{v_{i,t}^j}\right)\frac{v_{i,t}^j}{Y_t^j}}_{M_t^j} + \underbrace{\left(\log\left(\sum_i \frac{l_{i,t}^j}{v_{i,t}^j}\frac{v_{i,t}^j}{Y_t^j}\right) - \sum_i \log\left(\frac{l_{i,t}^j}{v_{i,t}^j}\right)\frac{v_{i,t}^j}{Y_t^j}\right)}_{D_t^j}, \quad (5)$$

where M_t^j is the static mean component in sector j and D_t^j is the static dispersion component in sector j.

By an identity, aggregate labor productivity is equivalent to:

$$\frac{Y_t}{L_t} = \sum_j e^{-M_t^j - D_t^j}\frac{L_t^j}{L_t}. \quad (6)$$

This implies that aggregate labor productivity can be expressed as an aggregate of sectoral mean and dispersion components, weighted by the share of labor allocated to each sector. Hence, when we look at *changes* in aggregate labor productivity, we have to account for the fact that input shares of different sectors can be changing over time. In turn, we have a third component, which reflects changes in the input share of a given sector, which we call the sectoral-share component:

$$\begin{aligned}\log\left(\frac{\frac{Y_t}{L_t}}{\frac{Y_{t-1}}{L_{t-1}}}\right) &= \underbrace{\log\left(\frac{\sum_j \left(e^{-M_t^j}\right)\frac{L_{t-1}^j}{L_{t-1}}}{\sum_j \left(e^{-M_{t-1}^j}\right)\frac{L_{t-1}^j}{L_{t-1}}}\right)}_{\text{mean component}} + \underbrace{\log\left(\frac{\sum_j e^{-M_t^j - D_t^j}\frac{L_t^j}{L_t}}{\sum_j e^{-M_t^j - D_t^j}\frac{L_{t-1}^j}{L_{t-1}}}\right)}_{\text{sectoral share}} \\ &+ \underbrace{\log\left(\frac{\sum_j e^{-M_t^j - D_t^j}\frac{L_{t-1}^j}{L_{t-1}}}{\sum_j e^{-M_{t-1}^j - D_{t-1}^j}\frac{L_{t-1}^j}{L_{t-1}}}\right) - \log\left(\frac{\sum_j \left(e^{-M_t^j}\right)\frac{L_{t-1}^j}{L_{t-1}}}{\sum_j \left(e^{-M_{t-1}^j}\right)\frac{L_{t-1}^j}{L_{t-1}}}\right)}_{\text{dispersion component}}. \quad (7)\end{aligned}$$

In this decomposition, changes in aggregate labor productivity are broken into three components. First, a mean component which captures changes in an aggregation of sectoral mean log labor productivities. Second, a sectoral share component which captures the effect of the changing allocation of labor between sectors. This second component will be positive if labor is flowing from low labor productivity sectors to high labor productivities onces. Third, a dispersion component, which captures changes in the dispersion of firm log labor productivities *within* sectors.

2.3 Decomposing Changes in TFP using Decomposition II

We measure TFP, A_t, as:

$$A_t = \frac{Y_t}{K_t^{\alpha} L_t^{1-\alpha}}. \tag{8}$$

We assume that capital's share of output, α, is positive. We can thus rewrite (8) as:

$$\log(A_t) = \alpha \log(\frac{Y_t}{K_t}) + (1-\alpha)\log(\frac{Y_t}{L_t}). \tag{9}$$

Taking changes in (9),

$$\Delta \log(A_t) = \alpha \Delta \log(\frac{Y_t}{K_t}) + (1-\alpha)\Delta \log(\frac{Y_t}{L_t}). \tag{10}$$

In (7), we showed that changes in $\log(\frac{Y_t}{L_t})$ can be broken into mean, dispersion, and sectoral-share components. Denote these components for labor as M_t^L, D_t^L, and S_t^L, respectively. Denote these components for capital as M_t^K, D_t^K, and S_t^K, respectively. Hence,

$$\Delta \log(\frac{Y_t}{K_t}) = M_t^K + D_t^K + S_t^K. \tag{11}$$

In turn, we can rewrite changes in log TFP from (10) as changes in the weighted sum of the mean components for capital and labor, the dispersion components for capital and labor, and the sectoral-share components for capital and labor:

$$\Delta \log(A_t) = (\alpha M_t^K + (1-\alpha)M_t^L) + (\alpha D_t^K + (1-\alpha)D_t^L) + (\alpha S_t^K + (1-\alpha)S_t^L). \tag{12}$$

3 Decomposition Applied to Models

In this section, we demonstrate the economics of our decomposition in standard production environments. First, in the production environment described by Hsieh and Klenow (2009), we demonstrate that changes in the production technology, prices, or the expected value of the log of marginal revenue products of capital will manifest themselves in the mean component of our decomposition. Second, changes in the variance or higher order moments of the log of marginal revenue products of capital will be reflected in the dispersion component of our decomposition.

Building on the above results, we demonstrate in a standard production environment how the components of our decomposition capture changes in the distribution of distortions to firm labor and capital choices. We find that common changes to the frictions to input choices facing firms are reflected in movements in the mean component. We also derive conditions under which distributional changes in such frictions are reflected in the dispersion component of our decomposition. Such results are derived in a more general framework than that of Hsieh and Klenow (2009), and we identify a number of relevant papers that can be mapped into our environment.

Our results are particularly relevant to the literature that studies the role financial frictions play in amplifying movements in aggregates over the business cycle. We analytically demonstrate how a change in a financial friction in a simple model of production will present itself as a distortion. We then demonstrate that an increase in the extent to which this financial friction affects firms will increase the dispersion in wedges.

3.1 Hsieh and Klenow (2009) Production Environment

The model consists of heterogeneous firms that produce differentiated goods. There are S industries, and the outputs of each industry, Y_S, are aggregated into a final good (total output), Y, using Cobb-Douglas technology in a perfectly competitive market. Hence, aggregate output can be defined as:

$$Y = \Pi_{s=1}^{S} Y_s^{\theta_s}, \text{ where } \sum_{s=1}^{S} \theta_s = 1. \tag{13}$$

From standard arguments: $P_S Y_S = \theta_s PY$, where the price of industry output is P_s and P is the price of the final good, which is set to be the numeraire.

There are M_s firms in a sector s. Industry output, Y_s is produced using CES technology:

$$Y_s = \left(\sum_{i=1}^{M_s} Y_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{14}$$

Within an industry, firms are heterogeneous in a few dimensions. First, they vary in aspects of their physical productivity. Second, they vary in the magnitude of frictions to their labor and capital choices. One can write these two distortions as distortions that affect the marginal products of labor and capital evenly, which one can write as an output distortion τ_Y, and distortions that affect the marginal product of capital relative to labor, which one can write as a capital distortion, τ_K. Firm i within sector s produces output, Y_{si}, from its firm TFP, A_{si}, capital stock K_{si}, and labor L_{si}, using the following Cobb-Douglas technology:

$$Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}. \tag{15}$$

Profits of firm i in sector S are thus:

$$\pi_{si} = (1 - \tau_{Y_{si}}) P_{si} Y_{si} - w L_{si} - (1 + \tau_{K_{si}}) R K_{si}. \tag{16}$$

From standard arguments, in this setup, the marginal revenue product of capital for a firm, $MRPK_{si} \triangleq \frac{\partial P_{si} Y_{si}}{\partial K_{si}}$, is a function of the rental rate of capital and firm level wedges:

$$MRPK_{si} = R \frac{1 + \tau_{K_{si}}}{1 - \tau_{Y_{si}}}. \tag{17}$$

As in Hsieh and Klenow (2009), it is useful to define the marginal product of capital (in total) for a sector as the following:[5]

$$\overline{MRPK_s} \triangleq \frac{R}{\sum_{i=1}^{M_S} \frac{1-\tau_{Y_{si}}}{1+\tau_{K_{si}}} \frac{K_{si}}{K_S}}. \tag{18}$$

For our decomposition, it is also useful to define the weighted average of the log marginal product of capital for firms in a sector, which is:

$$\overline{LMRPK_s} \triangleq \sum_{i=1}^{M_S} log(R \frac{1 + \tau_{K_{si}}}{1 - \tau_{Y_{si}}}) \frac{P_{si} Y_{si}}{P_S Y_S}. \tag{19}$$

[5]There was an error in the specification of this object in the original paper of Hsieh and Klenow (2009). The specification here corresponds to the form in the published corrections.

3.1.1 Our Decomposition in this Production Environment

In the environment above, from (18) and (19) and the definition of the marginal revenue product of capital, our decomposition applied to capital productivity for sector s can be expressed as the following:

$$\begin{aligned}\Delta\, log\left(\frac{P_t^s Y_t^s}{K_t^s}\right) = \quad & \underbrace{\Delta\, log\left(\frac{1}{\alpha_s}\frac{\sigma}{1-\sigma}\right) + \Delta\,\overline{LMRPK_s}}_{\text{mean component}} \\ & + \underbrace{\Delta\, log\left(\overline{MRPK_s}\right) - \Delta\,\overline{LMRPK_s}}_{\text{dispersion component}}. \end{aligned} \tag{20}$$

We now demonstrate how the changing distribution of marginal productivities are reflected in our decomposition by demonstrating which cumulants of the distribution of log marginal revenue productivities show up in which components of our decomposition. Cumulants are similar to moments; the cumulant-generating function of a random variable is an alternative specification of a probability distribution, similar to a moment-generating function. The first cumulant is the expected value of the variable, the second cumulant is its variance, and the higher order cumulants are polynomial combinations of centralized moments. Consider the distribution of firm log marginal revenue products of capital, reflecting both the mass of firms at a given productivity and their relative output shares (to be precise, the CDF would be written $G_s(X) = \int_{i\in s} \mathbb{1}\left(MRPK_{si} \leq X\right) \frac{P_{si}Y_{si}}{P_S Y_S} di$). Denote the cumulants of this distribution as $\kappa_1^s, \kappa_2^s, ...$ Using properties of the cumulant generating function, our decomposition can be expressed as the following function of the cumulants of the distribution of log marginal revenue productivities of capital:[6]

$$\Delta\, log\left(\frac{P_t^s Y_t^s}{K_t^s}\right) = \underbrace{\Delta \log\left(\frac{1}{\alpha_s}\frac{\sigma}{1-\sigma}\right) + \Delta\kappa_1^s}_{\text{mean component}} + \underbrace{-\frac{\Delta\kappa_2^s}{2!} + \frac{\Delta\kappa_3^s}{3!} - \frac{\Delta\kappa_4^s}{4!} + \dots}_{\text{dispersion component}}. \tag{21}$$

Note that $\kappa_1^s = \overline{LMRPK_s}$ is the weighted average of the log of firm marginal revenue products of capital, while κ_2^s is the variance of the log of firm marginal revenue products of capital. The mean component captures only changes in technology or $\overline{LMRPK_s}$.

[6] See Appendix A for details of this derivation.

Changes in the second cumulant (and thus second central moment), or higher order cumulants (and thus all of the remaining higher order moments) of the distribution of the log of firm marginal revenue products of capital are reflected in the dispersion component of our decomposition.

Note that if firm marginal revenue products of capital are lognormally distributed, then only the first two cumulants are non-zero. In that case, our decomposition is isomorphic to a mean-variance decomposition. Increases in the expected value of the log of firm marginal revenue productivities are reflected in the mean component of our decomposition, while the negative effect of the greater variance of the log of firm marginal revenue products is reflected in the dispersion component of our decomposition. This is apparent if in (21) the dispersion component is further broken into variance and higher order terms as below:

$$\Delta \, log\left(\frac{P_t^s Y_t^s}{K_t^s}\right) = \underbrace{\Delta \log\left(\frac{1}{\alpha_s}\frac{\sigma}{1-\sigma}\right) + \Delta\kappa_1^s}_{\text{mean component}} + \underbrace{\overbrace{-\frac{\Delta\kappa_2^s}{2!}}^{\text{variance}} + \overbrace{\sum_{n=3}^{\infty}(-1)^{n-1}\frac{\Delta\kappa_n^s}{n!}}^{\text{higher-order terms}}}_{\text{dispersion component}} . \quad (22)$$

Notice, if we define $\kappa_{L,n}$ as the $n'th$ cumulant of the log of the marginal revenue product of labor and $\kappa_{K,n}$ as the $n'th$ cumulant of the log of the marginal revenue product of capital, we can apply our decomposition to changes in TFP using (12):

$$\begin{aligned} \Delta \, log\left(TFPR_s\right) \quad = \quad & \underbrace{\Delta \log\left(\frac{\sigma}{1-\sigma}\alpha_s^{-\alpha_s}\left(1-\alpha_s\right)^{\alpha_s-1}\right) + (1-\alpha_s)\Delta\kappa_{L,1}^s + \alpha_s\Delta\kappa_{K,1}^s}_{\text{mean component}} \\ & \underbrace{\overbrace{-\frac{(1-\alpha_s)\Delta\kappa_{L,2}^s + \alpha_s\Delta\kappa_{K,2}^s}{2!}}^{\text{variance}} + \overbrace{\sum_{n=3}^{\infty}\frac{(1-\alpha_s)\Delta\kappa_{L,n}^s + \alpha_s\Delta\kappa_{K,n}^s}{\left(-1\right)^{n-1}n!}}^{\text{higher-order terms}}}_{\text{dispersion component}} . \end{aligned} \quad (23)$$

3.2 Simple Model of Production and Allocation

Given an increase in the dispersion of wedges likely results in a change in value added shares, we demonstrate under what conditions we can analytically demonstrate that an increase in the dispersion of wedges leads to an increase in the dispersion component of our decomposition. Similarly, we demonstrate under what conditions we can analytically demonstrate that an increase in the mean of wedges will increase the mean component of our decomposition, all else equal. We present our results within a similar environment to Hsieh and Klenow (2009), but more general technology.

As in Hsieh and Klenow (2009), the model consists of heterogeneous firms who produce differentiated goods, which are aggregated into a final good (total output), but now with a more general aggregation technology to be described below. Firms are heterogeneous in the following dimensions: They vary in aspects of their physical productivity, z_{it}, and they vary in the magnitude of frictions to their labor and capital choices.

3.2.1 Intermediate Good Firm Technology

Firms are indexed by i and time by t. Firm i at time t produces $y_{i,t}$ units of an intermediate good using $l_{i,t}$ units of homogeneous labor and $k_{i,t}$ units of capital with the production function $y_{i,t} = z_{i,t} l_{i,t}^{\gamma} k_{i,t}^{\nu}$. Labor and capital are homogeneous, therefore aggregate labor and capital clearing imply that $\int_i l_{i,t} di = L_t$ and $\int_i k_{i,t} di = K_t$, where L_t and K_t denote aggregate labor and capital.

3.2.2 Aggregation Technology and Value Added

Total output, Y_t, is aggregated from firm output with technology $Y_t = \left(\int_i y_{i,t}^{\varphi} di\right)^{\phi}$. Note that this general form nests the two most common final good technologies considered in the literature as special cases: The CES aggregator and heterogeneous firms producing a single good. The final good sector is competitive and cost minimizing. Standard arguments imply the price of each intermediate good, $p_{i,t}$, is $p_{i,t} = Y^{\frac{\phi-1}{\phi}} P_t \left(y_{i,t}\right)^{\varphi-1}$, where P_t is the price of the final good, which we set to be the numeraire. Therefore value added in real terms, $v_{i,t} = \frac{p_{i,t}}{P_t} y_{i,t}$, can be expressed as a function of prices and firm output:

$$v_{i,t} = Y_t^{\frac{\phi-1}{\phi}} \left(y_{i,t}\right)^{\varphi}. \tag{24}$$

To compute our decomposition, one requires firm-level productivity ratios and firm-level value-added shares. For a given firm, we can compute a firm-level value-added share as: $\frac{v_{i,t}}{Y_t}$.

3.3 The Optimal Allocation of Inputs and the Role of Firm-level Wedges

In this subsection, we solve the optimal allocation of resources in the planner's problem, and we demonstrate how firm-specific wedges can distort the allocation of labor and capital between firms from this allocation.

We show that the optimal allocation of resources in the planner's problem is such that all firms have the same productivity ratios. This choice is unique and can be characterized as a function of the distribution of firm-level TFP, $F_t^z(z)$. We then show that any allocation of capital and labor between firms can be expressed as a function of the optimal input choice and firm-specific wedges. We utilize this final result in the following subsection to evaluate how shocks to firm-level wedges show up in our decomposition.

3.3.1 Optimal Allocation

We now present a proposition that highlights the known result that for any fixed amount of total capital and labor, the optimal allocation of resources (to maximize static output) is such that all firms with identical production function coefficients have the same factor productivity ratios.

Proposition 1.

(i) Given a fixed amount of total labor and capital, L_t and K_t, the allocation of capital and labor across firms that maximizes output is such that there are unique optimal labor and capital productivity ratios, $\frac{v_t^*}{l_t^*}$ and $\frac{v_t^*}{k_t^*}$, which are common among all firms and only depend on the CDF of firm productivity, $F_t^z(z)$.

Proof. See Appendix A. □

A full (static) planner's problem maximizing current welfare could be split into two parts: First, solve for the optimal allocation rule of capital and labor between firms for any fixed amount of both capital and labor; and second, choose the total amount of capital and labor to maximize current period utility. Therefore Proposition 1 implies that the allocation which maximizes static utility is one where firms have constant productivity ratios. We do not place any restrictions on the level of statically optimal total labor and capital.

3.3.2 Firm-Level Wedges

We then use the optimal labor and capital productivity ratios to define firm-level wedges, defined as the firm productivity ratio, $\frac{v_{i,t}}{l_{i,t}}$ or $\frac{v_{i,t}}{k_{i,t}}$, over the optimal productivity ratio, $\frac{v_t^*}{l_t^*}$ or $\frac{v_t^*}{k_t^*}$. We formally define firm level wedges as:

$$\omega_{l,i,t} \triangleq \frac{v_{i,t}}{l_{i,t}} \frac{l_t^*}{v_t^*}, \tag{25}$$

and

$$\omega_{k,i,t} \triangleq \frac{v_{i,t}}{k_{i,t}} \frac{k_t^*}{v_t^*}, \tag{26}$$

for labor and capital, respectively.

These wedges capture how far a firm's productivity ratio is from the one that maximizes welfare in the social planner's static optimization problem. They also capture aggregate distortions, which distort every firm's input decision and change aggregate labor or capital, as well as changes in the relative distribution of resources between firms.

In the model of Hsieh and Klenow (2009), which is a special case of this production environment, these wedges can be expressed as functions of the firm-level distortions in their model, τ_{Ysi} and τ_{Ksi}. The firm-level wedges are proportional to these distortions: $\omega_{l,i,t} \propto \frac{1}{1-\tau_{Ysi}}$ and $\omega_{k,i,t} \propto \frac{1+\tau_{Ksi}}{1-\tau_{Ysi}}$.

3.4 Shocks to Firm-level Wedges in our Decompositions

In this subsection, we illustrate how changes to the distribution of firm-level wedges are captured in our decomposition and how such changes affect aggregates. The model of production and allocation (from subsection 3.2) we consider only has a single sector of production with identical production function coefficients, so we can perform our analysis using Decomposition I. However, Decomposition II first applies Decomposition I individually to each sector and then aggregates up the sectoral mean and dispersion components. Therefore, the way in which our components capture firm-level wedges will be similar for a multi-sector version of our model with production function coefficients varying across sectors. We perform our analysis of shocks to firm-level wedges only for Decomposition I due to the greater tractability and cleaner demonstration of the economics of our decomposition.

We begin *without* making parametric assumptions on the distribution of wedges. Let

$\frac{Y_t^*}{K_t^*}$ denote the undistorted (absent any idiosyncratic or common distortions) aggregate capital factor productivity ratio. Let $F_{\omega,k}$ denote the density function of wedges to firm capital choices, which reflects both the mass of firms and their relative value added shares. Formally, this can be expressed as an integral over firms (denoted by i): $F_{\omega,k}(x) = \int_i 1_{(\omega_{k,i}\leq x)} \frac{v_i}{Y} di$. The cumulants of the log of firm-level wedges are denoted as: $\kappa_{k,1,t}$, $\kappa_{k,2,t}$, $\kappa_{k,3,t}$, et cetera. Cumulants are similar to moments; we discuss their statistical properties in subsubsection 3.1.1. Our decomposition of changes in aggregate capital productivity can be expressed as:[7]

$$\Delta\, log\left(\frac{Y_t}{K_t}\right) = \underbrace{\Delta \log\left(\frac{Y_t^*}{K_t^*}\right) + \Delta\kappa_{k,1,t}}_{\text{mean component}} + \underbrace{\overbrace{-\frac{\Delta\kappa_{k,2,t}}{2!}}^{\text{variance}} + \overbrace{\sum_{n=3}^{\infty}(-1)^{n-1}\frac{\Delta\kappa_{k,n,t}}{n!}}^{\text{higher-order terms}}}_{\text{dispersion component}}. \tag{27}$$

Equation (27) shows that the mean component captures changes in the undistorted productivity ratio (capturing changes unrelated to distortions) and changes in the first cumulant (which capture common changes in distortions). Changes in the variance of firm log wedges (the second cumulant), or any higher moments of their distribution, are reflected in the dispersion component. Such results easily carry over for labor productivity by replacing capital for labor in (27).

We can then use (12) to express total factor productivity as a function of undistorted TFP, TFP_t^*, and the cumulants of wedges to capital and labor:[8]

$$\begin{aligned}\Delta\, log\left(TFP_t\right) &= \underbrace{\Delta \log\left(TFP_t^*\right) + \alpha\Delta\kappa_{k,1,t} + (1-\alpha)\,\Delta\kappa_{l,1,t}}_{\text{mean component}} \\ &\quad \underbrace{\overbrace{-\frac{\alpha\Delta\kappa_{k,2,t} + (1-\alpha)\,\Delta\kappa_{l,2,t}}{2!}}^{\text{variance}} + \overbrace{\sum_{n=3}^{\infty}\frac{\alpha\Delta\kappa_{k,n,t} + (1-\alpha)\,\Delta\kappa_{l,n,t}}{(-1)^{n-1}\, n!}}^{\text{higher-order terms}}}_{\text{dispersion component}}.\end{aligned} \tag{28}$$

[7] See Appendix A for details of this derivation.

[8] For this exercise, we make the standard assumption that TFP is measured as $TFP_t = \frac{Y_t}{K_t^{\alpha} L_t^{1-\alpha}}$, where α is constant over time.

Equation (28) shows that the mean component captures changes in undistorted TFP and changes in the first cumulants of log wedges (corresponding to the mean of log wedges). The dispersion component captures changes in the variance or higher order moments of log wedges.

In the remainder of this subsection, we present special cases of the results in (27) and (28). We show how common changes in wedges and changes in technology are reflected in the mean component of our decomposition. Finally, we show that changes in the variance and higher-order moments of log wedges are captured in the dispersion component our decomposition.[9]

3.4.1 Common Shocks to Distortions

We first show that common changes to distortions are reflected *only* in the mean component of our decomposition. First, consider a shock, ξ_{t+1}, that affects firms evenly such that $\omega_{k,t+1} = \xi_{l,t+1}\omega_{k,t}$. This is the sort of shock that would arise, for example, from a distortion to the rental rate of capital faced by all firms. Below, we show the effect of this shock on aggregate capital factor productivity and TFP is reflected only in the mean component:

$$\Delta log(\frac{Y_t}{K_t}) = \underbrace{log(\xi_{k,t})}_{\text{mean component}} ,$$

and

$$\Delta log(TFP_t) = \underbrace{\alpha\, log(\xi_{k,t})}_{\text{mean component}} .$$

Only the mean component will change if the economy is hit by no other shocks. Such results extend to the decomposition of labor productivity when we replace labor for capital.

3.4.2 Shocks to the Variance and Higher-order Moments of Distortions

Note that the second cumulant is the variance of log wedges, while all of the higher order cumulants can be expressed as polynomial combinations of the second and higher-order central moments. Therefore (27) and (28) imply that changes in the variance and any higher-order moments of log wedges are reflected in the dispersion component, without having to make any parametric assumptions.

[9]These results follow directly from our decomposition and properties of cumulant generating functions; an outline of their derivation are found in Appendix A.

To provide further intuition, we now demonstrate how shocks to the distribution of wedges are realized in our decomposition under some standard parametric assumptions. For example, if wedges to capital are lognormally distributed with mean $\mu_{\omega,k,t}$ and variance $\sigma^2_{\omega,k,t}$, then changes in aggregate capital productivity can be decomposed as:

$$\Delta\, log\left(\frac{Y_t}{K_t}\right) = \underbrace{\Delta \log\left(\frac{Y_t^*}{K_t^*}\right) + \Delta\mu_{\omega,k,t}}_{\text{mean component}} + \underbrace{-\frac{\Delta\sigma^2_{\omega,k,t}}{2}}_{\text{dispersion component}}.$$

With the lognormal assumption, only the first and second cumulant exist. A typical way of adding variation in higher-order central moments (and thus higher-order cumulants) is to create a mixture of lognormals. If wedges to capital are modeled as a mixture of lognormals, with weights $\lambda_{k,n,t}$ on lognormal distributions with means $\mu_{k,n,t}$ and variances $\sigma^2_{k,n,t}$, then we can express our decomposition as follows:

$$\Delta\, log\left(\frac{Y_t}{K_t}\right) = \underbrace{\Delta \log\left(\frac{Y_t^*}{K_t^*}\right) + \Delta\sum_n \lambda_{k,n,t}\mu_{k,n,t}}_{\text{mean component}}$$
$$\underbrace{-\frac{\Delta\sum_n \lambda_{k,n,t}\sigma^2_{k,n,t}}{2} - \Delta\, log\left(\sum_n \lambda_{k,n,t} e^{\left(\mu_{k,n,t}-\sum_j \lambda_{k,j,t}\mu_{k,j,t}\right)} e^{\frac{1}{2}\left(\sigma^2_{k,n,t}-\sum_j \lambda_{k,j,t}\sigma^2_{k,j,t}\right)}\right)}_{\text{dispersion component}}.$$

The mean component captures changes in the weighted means of the lognormal distributions or in the undistorted factor productivity ratio. All other changes in the distributions will be reflected in the dispersion component. Changes in the variances of the lognormal distributions which make up the mixture will be reflected here, as will higher order moments. For example, a skewed distribution is often parameterized as a mixture of lognormals with different means, which will be reflected, via the term $e^{\left(\mu_{k,n,t}-\sum_j \lambda_{k,j,t}\mu_{k,j,t}\right)}$, in the dispersion component. Kurtosis is often often parameterized as a mixture of lognormals with different variances, which will be reflected, via the term $e^{\frac{1}{2}\left(\sigma^2_{k,n,t}-\sum_j \lambda_{k,j,t}\sigma^2_{k,j,t}\right)}$, in the dispersion component.

3.5 Mapping to Other Models

In this subsection, we show that our model has a mapping to several models of frictions to the allocation of labor and capital between firms. We then demonstrate how frictions in a simple model of financial frictions would be reflected in wedges, and then discuss how our decomposition would capture changes in such frictions.

3.5.1 Mapping to Models of Labor or Capital Allocation

Our simple model consists only of a production environment with wedges representing frictions to the allocation of labor and capital. Therefore, there is a mapping to any model with a production environment consistent with ours. This includes the models of Khan and Thomas (2013), Bloom et al. (2014), and Arellano et al. (2012), as well as numerous other heterogeneous agent models considered in the macroeconomics literature.

We formally show this correspondence by proving that given the production environment in our model, aggregate output, employment, capital, and the full distribution of output, labor, capital, and technology across firms can be characterized using wedges and either firm-level technology or output shares for any allocation of labor, capital, and technological productivity across firms. We denote $G_t\left(z, \omega_l, \omega_k\right)$ as the joint distribution of firm technological productivity and firm-level wedges to labor and capital, $J_t\left(\frac{v}{Y}, \omega_l, \omega_k\right)$ as the joint distribution of firm output shares and firm-level wedges to labor and capital, and $Z_t = \left(\int_i \left(z_{i,t}^{\varphi\left(\frac{1}{1-\nu\varphi-\gamma\varphi}\right)}\right) di\right)^{\phi(1-\nu\varphi-\gamma\varphi)}$ as an index of aggregate productivity. The following proposition states that this representation can map any resulting allocation of resources and aggregates using these firm-level wedges:

Proposition 2. *The full distribution of labor, capital, and productivity across firms, $F_t\left(z, l, k\right)$, and aggregate output, employment, and capital have a 1-1 mapping with any of the following:*

(i) $G_t\left(z, \omega_l, \omega_k\right)$.

(ii) $J_t\left(\frac{v}{Y}, \omega_l, \omega_k\right)$ *and a measure of aggregate productivity* Z_t.[10]

Proof. See Appendix A. □

3.5.2 Mapping to a Model of Financial Frictions

In this subsection, we show that a financial friction in a simple model map can be redefined as a firm-level wedge. We then discuss how a tightening of the friction in the model can generate a greater variance of wedges.

[10]More generally, it can be shown that $J_t\left(\frac{v}{Y}, \omega_l, \omega_k\right)$ together with $\left(\int z^\tau dF^z(z)\right)$, for any $\tau \neq 0$, is sufficient to characterize output, employment, and capital at both the aggregate and firm level.

Consider a simple model where heterogeneous firms produce a homogeneous consumption good with technology $y_{i,t} = z_{i,t} l_{i,t}^b$, where $b < 1$. In this setting, value added is equivalent to firm output, $v_{i,t} = z_{i,t} l_{i,t}^b$. Households have utility function $U(C, L) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{L^{1+\nu}}{1+\nu}$ and discount the future at rate β. Production in this simple model is a special case of the production environment introduced in Subsection 3.2; thus, the planner's problem states all firms optimally have the same labor productivity ratios.

The friction in this model is a simple borrowing constraint: Firms have wealth $a_{i,t}$, which we consider exogenous for our analysis. Firms must pay their workers at the beginning of the period but only receive cash flows from production at the end. The borrowing constraint, $l_{i,t} W \leq a_{i,t} \rho$, where W is the wage and ρ is a positive constant, restricts the labor decisions of firms when it binds. Assume firms may also exogenously exit each period with probability δ.

The optimization problem of firms can be expressed as the following Lagrangian:

$$\mathcal{L} = \max_{l_{i,t}} z_{i,t} l_{i,t}^b - l_{i,t} W + \lambda_{i,t} \left(a_{i,t} \rho - l_{i,t} W_t \right), \tag{29}$$

where $\lambda_{i,t}$ is the Lagrange multiplier on the borrowing constraint. The multiplier is 0 if the borrowing constraint does not bind, and positive otherwise. Taking the first-order conditions of (29) and manipulating the labor-leisure condition allows us to express the firm's labor choice as the following:

$$l_{i,t} = z_{i,t}^{\frac{1}{1-b}} b^{\frac{1}{1-b}} Y_t^{-\frac{\sigma}{1-b}} L_t^{-\frac{\nu}{1-b}} \left(1 + \lambda_{i,t}\right)^{-\frac{1}{1-b}}. \tag{30}$$

From (30) we can derive firm-level wedges, $\omega_{l,i,t} = \frac{v_{i,t}}{l_{i,t}} \frac{l_t^*}{v_t^*}$, as a function of aggregates and the Lagrange multiplier faced by the firm:

$$log\left(\omega_{l,i,t}\right) = \underbrace{\sigma log\left(\frac{Y_t}{Y_t^*}\right) + \nu log\left(\frac{L_t}{L_t^*}\right)}_{Aggregate} + \underbrace{log\left(1 + \lambda_{i,t}\right)}_{Firm\,Specific}. \tag{31}$$

Note that the wedge can be expressed as a function of the distortion of aggregates from their optimal value (which affect the wage rate) as well as the firm-specific distortion captured by the Lagrange multiplier in the firm's problem. We can express the Lagrange multiplier as the following function of aggregates and each firm's $z_{i,t}$ and $a_{i,t}$:

$$log\left(1+\lambda_{i,t}\right)=\begin{cases}0 & z_{i,t}bY_t^{-b\sigma}L_t^{-b\nu}a_{i,t}^{b-1}\rho^{b-1}\le 1\\ log\left(bY_t^{-b\sigma}L_t^{-b\nu}z_{i,t}a_{i,t}^{b-1}\rho^{b-1}\right) & z_{i,t}bY_t^{-b\sigma}L_t^{-b\nu}a_{i,t}^{b-1}\rho^{b-1}>1\end{cases}. \tag{32}$$

Note that the only way that there is no heterogeneity in Lagrangian multipliers is either if (a) the borrowing constraint never binds, or (b) it binds for all firms, but wealth is proportional to productivity ($z_{i,t} = a_{i,t}^{1-b}$). This second condition implies no inefficiencies in the distribution of resources between firms; all inefficiencies arise from the reduced aggregate demand for labor.

Now consider what a shock to borrowing constraints does. Assume that at time $t = 0$, ρ is high enough such that the constraint binds for no firms. Then the mean and variance of $log\left(1+\lambda_{i,0}\right)$ is 0. A decrease in ρ to the point where the constraint binds for some but not all firms leads to a rise in both the mean and variance of $log\left(1+\lambda_{i,t}\right)$. At the same time, the borrowing constraint reduces Y_t and L_t, as some firms cannot hire the amount of labor they would prefer were they unconstrained. Therefore the change to the mean of firm wedges, $log\left(\omega_{l,i,t}\right)$, as a result of this shock is ambiguous, as the aggregate component and the mean of the firm-specific component move in opposite directions. However, the direction of the change in variance of the firm-level labor wedge is unambiguous, increasing in response to such a shock.

4 Decomposition Applied to Data

In this section, we apply our decompositions of aggregate productivity described in Section 2 to data on U.S. public firms and Japanese public firms. In our discussion of the results applied to U.S. public firms, we also include a comparison of our measures of labor productivity, capital productivity, and TFP to those from the national income and product accounts (NIPA). In Appendix B, we describe how we clean our data on U.S. nonfinancial public firms, and measure the objects of interest.

4.1 Discussion of results — Data from the United States

Figures 1 and 2 display results of year-over-year changes in aggregate labor productivity and its components from Decompositions I and II, respectively. From the eye test alone it should be clear that in the recent recession, aggregate labor productivity and its dispersion component have a negative correlation, and the mean component is highly correlated with aggregate labor productivity. Figures 3 and 4 further demonstrate this point: Over four recession periods, the mean component moves closely with aggregate labor productivity. In Decomposition II, the dispersion component has very little cumulative change over any of the four episodes in our sample.

Figures 5 and 6 display results from Decompositions I and II of year-over-year changes in aggregate capital productivity, and tell a different story. The dispersion component is positively correlated with aggregate capital productivity over the past two business-cycle episodes. For previous episodes, the mean component moves more closely with aggregate capital productivity. These results are more starkly apparent in Figures 7 and 8, which show cumulative changes in aggregate capital productivity and its components from Decompositions I and II. In the recent episodes, for either decomposition, the dispersion component moves much more closely with aggregate capital productivity.

As we describe in the previous subsection on measurement, to compute TFP, by the nature of our assumptions on the production function and values for its coefficients, changes in labor productivity get more weight (65 percent) than changes in capital productivity (35 percent). Hence, as should be expected, we see that the results for TFP are much more qualitatively consistent with the results from what drives changes in labor productivity. This is apparent in the year-over-year changes charts from Decompositions I and II in Figures 9 and 10, as well as the cumulative changes charts from Decompositions I and II in Figures 11 and 12.

Table 1 displays correlations between the components of Decomposition II and their respective aggregates. The results from the figures are further codified in this table. The correlation between the dispersion component and sectoral share components and labor productivity are especially striking. Movements in labor productivity are much more correlated with the mean of firm-level log labor to value-added ratios than with their dispersion. These results are dampened when looking at TFP because capital productivity has a positive correlation with its dispersion component. However, the relationship between TFP and its dispersion component is ultimately close to zero.

Our sample represents a significant slice of the U.S. economy; in 2011, it accounted for over 15 percent of GDP and over 17 million employees. To understand the extent to which our sample reflects the mechanisms responsible for driving aggregate productivity changes, we compare the time-series behavior of each aggregate productivity ratio as aggregated from Compustat to that of the respective productivity ratio computed from NIPA. Figures 16 and 17 show that the time-series properties of TFP computed from both Compustat and NIPA are similar both in their cyclical dynamics and long-term trends. These figures suggest that some of the key forces driving TFP over time are likely present in Compustat data. If there were significant factors driving TFP over the business cycle that existed only in small, private firms, we would expect systematic differences in the behavior of TFP and our measure computed from publicly listed firms over time. However, there are some differences in the measures. TFP from Compustat is more volatile, which is unsurprising given the documented greater volatility of corporate profits measured with generally accepted accounting principles (GAAP) than corporate profits as measured in NIPA.[11] There are also some slight timing differences, particularly in the timing of the trough (of TFP) of the 2007—2009 recession. These timing differences may be due to the reporting dates of firms in Compustat. However, the measure of TFP for the United States in the Penn World Tables (8.0) has the trough in 2009, so the timing differences may also be due to some technical adjustments made in the NIPA aggregation. In Figures 18 and 19, we look at changes in each productivity ratio and its NIPA equivalent. We see the timing and volatility issues are present for each productivity ratio separately.

4.2 Discussion of results - Data from Japan

In Figures 13 and 14, we display results from Decompositions I and II of year-over-year changes in aggregate TFP in Japan. The results from the second decomposition are more consistent with those from the United States for the recent recession in that the dispersion component is not correlated with movements in TFP. The results from the first decomposition, however, show the dispersion component to be more highly correlated with aggregate TFP over the recent episode. This result is true for labor productivity as well.

5 Conclusion

This paper presents decompositions of changes in aggregate labor productivity, capital productivity, and TFP. We demonstrate how the dispersion component of our decompositions reflects changes in the degree to which frictions affect firms in many heterogeneous firm models that attempt to explain the nature of the business cycle. In turn, computing the components of our decomposition in data and comparing them to the same metrics in a given model of the class we consider will help to assess whether such a model is consistent

[11] Hodge (2011) compares the properties of corporate profits computed from the GAAP accounting statements of firms in the S&P 500 index with the corresponding measure from NIPA, finding significantly greater volatility in the S&P measure.

with firm-level behavior. As we demonstrate in this paper, it is not only useful to compute our decompositions on models that have already been solved; one can also compute our metrics in the data *before* writing down a model to help motivate which mechanisms should be key in driving patterns over the business cycle.

Appendix A

Proof for Proposition 1

Given the model of production in Subsection 3.2, we can define the following Lagrangian for the social planner to solve supposing she gets to allocate a fixed amount of labor and capital across firms, which are indexed by i:[12]

$$\mathcal{L} = \max_{l_i, k_i} \left(\int_i \left(z_i l_i^{\gamma} k_i^{\nu} \right)^{\varphi} di \right)^{\phi} + \lambda_1 \left(K - \int k_i di \right) + \lambda_2 \left(L - \int l_i di \right). \tag{33}$$

We want to show that there exists optimal labor and capital productivity ratios that are shared by all firms that share the same production function coefficients. From the first-order conditions of (33):

$$\nu \varphi \phi Y^{\frac{1-\phi}{\phi}} \frac{\left(z_i l_i^{\gamma} k_i^{\nu} \right)^{\varphi}}{k} = \lambda_1, \tag{34}$$

and

$$\gamma \varphi \phi Y^{\frac{1-\phi}{\phi}} \frac{\left(z_i l_i^{\gamma} k_i^{\nu} \right)^{\varphi}}{l} = \lambda_2. \tag{35}$$

Also, the planner will fully allocate labor and capital to all firms, so:

$$K = \int k_i di, \tag{36}$$

and

$$L = \int l_i di. \tag{37}$$

[12]To economize on notation, time subscripts are omitted.

With some algebra, it can be shown that:

$$v_i = k_i \frac{\lambda_1}{\nu\varphi\phi}, \tag{38}$$

and

$$v_i = l_i \frac{\lambda_2}{\gamma\varphi\phi}. \tag{39}$$

Hence, summing over i in (38) and (39):

$$Y = K \frac{\lambda_1}{\nu\varphi\phi}, \tag{40}$$

and

$$Y = L \frac{\lambda_2}{\gamma\varphi\phi}. \tag{41}$$

In turn, from (38) and (40):

$$\frac{Y}{K} = \frac{v_i}{k_i}. \tag{42}$$

Also, from (39) and (41):

$$\frac{Y}{L} = \frac{v_i}{l_i}. \tag{43}$$

In turn, all firms will optimally have the same firm-level capital and labor productivity ratios.

We can now express optimal productivity ratios $\frac{v^*}{l^*}$ and $\frac{v^*}{k^*}$ as a function of L, K, and Y^*.

From the production technology, (42), and (43):

$$k_i = Y^{\frac{-1}{\phi}} z_i^{\varphi} l^{\varphi\gamma} k_i^{\varphi\nu} K^* \left(F^z(z)\right), \tag{44}$$

and

$$l_i = Y^{\frac{-1}{\phi}} z_i^{\varphi} l^{\varphi\gamma} k_i^{\varphi\nu} K^* \left(F^z(z)\right). \tag{45}$$

Combining the production technology with (44) and (45), along with some algebra, yields:

$$Y^* = L^{\varphi\phi\gamma} K^{\phi\varphi\nu} \left(\int_i \left(z_i^{\varphi\left(\frac{1}{1-\nu\varphi-\gamma\varphi}\right)} \right) di \right)^{\phi(1-\nu\varphi-\gamma\varphi)}. \tag{46}$$

Note that this optimal output is just a function of the distribution of productivity, $F^z(z)$, and total labor and capital.

Thus, we can express the optimal productivity ratios as:

$$\frac{v^*}{l^*} = \frac{Y^*}{L}, \tag{47}$$

and

$$\frac{v^*}{k^*} = \frac{Y^*}{K}. \tag{48}$$

Proof for Proposition 2

This proof is done in the following parts:

(i) $F(z,l,k)$ fully characterizes output, employment, and capital.

(ii) $F(z,l,k)$ has a 1-1 mapping with $G(z,\omega_l,\omega_k)$.

(iii) $G(z,\omega_l,\omega_k)$ has a 1-1 mapping with $J(\frac{v}{Y},\omega_l,\omega_k)$ and a measure of aggregate productivity $Z = \left(\int_i \left(z_i^{\varphi\left(\frac{1}{1-\nu\varphi-\gamma\varphi}\right)} \right) di \right)^{\phi(1-\nu\varphi-\gamma\varphi)}$.

Part (i): $F(z,l,k)$ **fully characterizes output, employment, and capital.**

This must be true, by the definition of production technology and the clearing conditions $L = \int l dF(z,l,k)$ and $K = \int k dF(z,l,k)$. Thus, $F(z,l,k)$ fully characterize aggregate output, employment, and capital.

Part (ii): $F(z,l,k)$ **has a 1-1 mapping with** $G(z,\omega_l,\omega_k)$**.**

The portion of the proof has the following parts:

(a) $F(z,l,k)$ has a unique mapping χ_1 into $G(z,\omega_l,\omega_k)$.

(b) $G(z,\omega_l,\omega_k)$ has a unique mapping χ_2 into $F(z,l,k)$.

(c) $\chi_2 = \chi_1^{-1}$.

$F(z,l,k)$ has a unique mapping χ_1 into $G(z,\omega_l,\omega_k)$:
(24) combined with the production technology gives us v_i as a function of only z, l, k:

$$v_i = \left(\int z^{\varphi} l^{\gamma\varphi} k^{\nu\varphi} dF\left(z,l,k\right)\right)^{\phi-1} z_i^{\varphi} l_i^{\gamma\varphi} k_i^{\nu\varphi}. \tag{49}$$

Combining (25), (26), and (49) yields:

$$\omega_k\left(z_i,l_i,k_i,F\right) = \frac{\left(\int z^{\varphi} l^{\gamma\varphi} k^{\nu\varphi} dF\left(z,l,k\right)\right)^{\phi-1} z_i^{\varphi} l_i^{\gamma\varphi} k_i^{\nu\varphi}}{k_i} \frac{k^*}{v^*}, \tag{50}$$

and

$$\omega_l\left(z_i,l_i,k_i,F\right) = \frac{\left(\int z^{\varphi} l^{\gamma\varphi} k^{\nu\varphi} dF\left(z,l,k\right)\right)^{\phi-1} z_i^{\varphi} l_i^{\gamma\varphi} k_i^{\nu\varphi}}{l_i} \frac{l^*}{v^*}. \tag{51}$$

These equations characterize the wedges implied by a given distribution of capital, labor, and productivity. We can rearrange (50) and (51) to solve for labor and capital as a function of wedges:

$$k\left(z_i,\omega_{l,i},\omega_{k,i},F\right) = \left(\frac{\left(\int z^{\varphi} l^{\gamma\varphi} k^{\nu\varphi} dF\left(z,l,k\right)\right)^{\phi-1} z_i^{\varphi} \left(\frac{l^*}{v^*}\right)^{\gamma\varphi} \left(\frac{k^*}{v^*}\right)^{1-\gamma\varphi}}{\left(\omega_{l.i}\right)^{\gamma\varphi} \left(\omega_{k,i}\right)^{1-\gamma\varphi}}\right)^{\frac{1}{1-\gamma\varphi-\nu\varphi}}, \tag{52}$$

and

$$l\left(z_i,\omega_{l,i},\omega_{k,i},F\right) = \left(\frac{\left(\int z^{\varphi} l^{\gamma\varphi} k^{\nu\varphi} dF\left(z,l,k\right)\right)^{\phi-1} z_i^{\varphi} \left(\frac{l^*}{v^*}\right)^{1-\nu\varphi} \left(\frac{k^*}{v^*}\right)^{\nu\varphi}}{\left(\omega_{l,i}\right)^{1-\nu\varphi} \left(\omega_{k,i}\right)^{\nu\varphi}}\right)^{\frac{1}{1-\gamma\varphi-\nu\varphi}}. \tag{53}$$

(52) and (53) allow us to obtain the unique mapping from F to G:

$$G\left(\bar{z},\bar{\omega_l},\bar{\omega_k}\right) = \int_{z,\omega_l,\omega_k=0}^{\bar{z},\bar{\omega_l},\bar{\omega_k}} dF\left(z, l\left(z,\omega_l,\omega_k,F\right), k\left(z,\omega_l,\omega_k,F\right)\right). \tag{54}$$

$G(z,\omega_l,\omega_k)$ has a unique mapping χ_2 into $F(z,l,k)$:

Combining (50) and (51) allows us to express $F(z,l,k)$ as the following:

$$F\left(\bar{z},\bar{l},\bar{k}\right)=\int_{z,l,k=0}^{\bar{z},\bar{l},\bar{k}}dG\left(z,\omega_l\left(z,l,k,F\right),\omega_k\left(z,l,k,F\right)\right). \tag{55}$$

This expression is not sufficient to characterize $F(z,l,k)$ as a function of $G(z,\omega_l,\omega_k)$, as the functions $\omega_l()$ and $\omega_k()$ on the right-hand side depend on the term $\left(\int z^{\varphi}l^{\gamma\varphi}k^{\nu\varphi}dF\left(z,l,k\right)\right)^{\frac{\phi-1}{\phi}}$. Note that $\left(\int z^{\varphi}l^{\gamma\varphi}k^{\nu\varphi}dF\left(z,l,k\right)\right)^{\frac{\phi-1}{\phi}}=Y^{\frac{\phi-1}{\phi}}$. All we have to do now is express Y as a function of G. Plugging (52) and (53) into the aggregate production function yields:

$$Y=\left(\left(\frac{k^*}{v^*}\right)^{\varphi\phi\nu}\left(\frac{l^*}{v^*}\right)^{\varphi\phi\gamma}\left(\int\frac{z^{\frac{\varphi}{1-\gamma\varphi-\nu\varphi}}}{\omega_l^{\frac{\gamma\varphi}{1-\gamma\varphi-\nu\varphi}}\omega_k^{\frac{\nu\varphi}{1-\gamma\varphi-\nu\varphi}}}dG\left(z,\omega_l,\omega_k\right)\right)^{\phi}\right)^{\frac{1-\gamma\varphi-\nu\varphi}{1-\phi(\gamma\varphi+\nu\varphi)}}. \tag{56}$$

$Y\left(G\left(z,\omega_l,\omega_k\right)\right)$ can thus be defined as a function of z and wedges.

(55) and (56) can be combined to obtain the functions $\omega_l\left(z,l,k,G\right)$ and $\omega_k\left(z,l,k,G\right)$. Thus, we can obtain the unique mapping:

$$F\left(\bar{z},\bar{l},\bar{k}\right)=\int_{z,l,k=0}^{\bar{z},\bar{l},\bar{k}}dG\left(z,\omega_l\left(z,l,k,G\right),\omega_k\left(z,l,k,G\right)\right). \tag{57}$$

$\chi_2=\chi_1^{-1}$:
Combining (54) and (57) yields the result that $F\left(z,l,k\right)=\chi_2\left(\chi_1\left(F\left(z,l,k\right)\right)\right)$ for any $F\left(z,l,k\right)$. It follows that $\chi_2=\chi_1^{-1}$.

Part (iii): Claim: $G(z,\omega_l,\omega_k)$ has a 1-1 mapping with $J(\frac{v}{Y},\omega_l,\omega_k)$ and a measure of aggregate productivity $Z=\left(\int_i\left(z_i^{\varphi\left(\frac{1}{1-\nu\varphi-\gamma\varphi}\right)}\right)di\right)^{\phi(1-\nu\varphi-\gamma\varphi)}$.

The portion of the proof has the following parts:

(a) $G(z,\omega_l,\omega_k)$ has a unique mapping χ_3 into $J(\frac{v}{Y},\omega_l,\omega_k)$ and pins down Z.

(b) $J(\frac{v}{Y},\omega_l,\omega_k)$ and Z has a unique mapping χ_4 into $G(z,\omega_l,\omega_k)$.

(c) $\chi_3=\chi_4^{-1}$.

$G(z,\omega_l,\omega_k)$ has a unique mapping χ_3 into $J(\frac{v}{Y},\omega_l,\omega_k)$ and pins down Z: (49), (52), (53), and (56) can be combined to characterize $\frac{v}{Y}$:

$$\frac{v_i}{Y} = \frac{z_i^{\frac{\varphi}{1-\gamma\varphi-\nu\varphi}} \left(\omega_{l,i}\right)^{\frac{-\gamma\varphi}{1-\gamma\varphi-\nu\varphi}} \left(\omega_{k,i}\right)^{\frac{-\nu\varphi}{1-\gamma\varphi-\nu\varphi}}}{\int z^{\frac{\varphi}{1-\gamma\varphi-\nu\varphi}} \left(\omega_l\right)^{\frac{-\gamma\varphi}{1-\gamma\varphi-\nu\varphi}} \left(\omega_k\right)^{\frac{-\nu\varphi}{1-\gamma\varphi-\nu\varphi}} dG(z,\omega_l,\omega_k)}, \tag{58}$$

and thus express z as a function of $\frac{v}{Y}$, ω_l, ω_k, and G:

$$z\left(\frac{v_i}{Y},\omega_{l,i},\omega_{k,i},G\right) = \omega_{l,i}^{\gamma}\omega_{k,i}^{\nu}\left(\frac{v_i}{Y}\int \frac{z^{\frac{\varphi}{1-\gamma\varphi-\nu\varphi}}}{\omega_l^{\frac{\gamma\varphi}{1-\gamma\varphi-\nu\varphi}}\omega_k^{\frac{\nu\varphi}{1-\gamma\varphi-\nu\varphi}}}dG\left(z,\omega_l,\omega_k\right)\right)^{\frac{1-\gamma\varphi-\nu\varphi}{\varphi}}. \tag{59}$$

(59) can be used to characterize $J(\frac{v}{Y},\omega_l,\omega_k)$:

$$J\left(\frac{\bar{v}}{Y},\bar{\omega_l},\bar{\omega_k}\right) = \int_{\frac{v}{Y},\omega_l,\omega_k=0}^{\frac{\bar{v}}{Y},\bar{\omega_l},\bar{\omega_k}} dG\left(z\left(\frac{v}{Y},\omega_l,\omega_k,G\right),\omega_l,\omega_k\right). \tag{60}$$

$G\left(z,\omega_l,\omega_k\right)$ trivially maps into a unique Z.

$J(\frac{v}{Y},\omega_l,\omega_k)$ and Z has a unique mapping χ_4 into $G(z,\omega_l,\omega_k)$:

(59), rearranged and integrated, yields:

$$\int \frac{z^{\frac{\varphi}{1-\gamma\varphi-\nu\varphi}}}{\omega_l^{\frac{\gamma\varphi}{1-\gamma\varphi-\nu\varphi}}\omega_k^{\frac{\nu\varphi}{1-\gamma\varphi-\nu\varphi}}}dG\left(z,\omega_l,\omega_k\right) = \frac{Z^{\frac{1}{\phi(1-\nu\varphi-\gamma\varphi)}}}{\int\left(\frac{v}{Y}\right)\omega_l^{\frac{\gamma\varphi}{1-\nu\varphi-\gamma\varphi}}\omega_k^{\frac{\nu\varphi}{1-\nu\varphi-\gamma\varphi}}dJ\left(\frac{v}{Y},\omega_l,\omega_k\right)}. \tag{61}$$

Combining (59) and (61) yields:

$$z\left(\frac{v_i}{Y},\omega_{l,i},\omega_{k,i},J,Z\right) = \omega_{l,i}^{\gamma}\omega_{k,i}^{\nu}\left(\frac{\frac{v_i}{Y}Z^{\frac{1}{\phi(1-\nu\varphi-\gamma\varphi)}}}{\int\left(\frac{v}{Y}\right)\omega_l^{\frac{\gamma\varphi}{1-\nu\varphi-\gamma\varphi}}\omega_k^{\frac{\nu\varphi}{1-\nu\varphi-\gamma\varphi}}dJ\left(\frac{v}{Y},\omega_l,\omega_k\right)}\right)^{\frac{1-\gamma\varphi-\nu\varphi}{\varphi}}. \tag{62}$$

(62) implies that we can express $G(z,\omega_l,\omega_k)$ as a function of $J(\frac{v}{Y},\omega_l,\omega_k)$ and Z:

$$G\left(\bar{z},\bar{\omega_l},\bar{\omega_k}\right) = \int_{z,\omega_l,\omega_k=0}^{\bar{z},\bar{\omega_l},\bar{\omega_k}} dJ\left(\frac{v}{Y}\left(z,\omega_l,\omega_k,J,Z\right),\omega_l,\omega_k\right). \tag{63}$$

$\chi_3 = \chi_4^{-1}$:
These two mappings are trivially inverses of each other.

Other Derivations

Decomposition as a Function of Cumulants

Consider the cumulative density function of firm log capital productivity, weighted by output shares, $G(X) = \int_i \mathbb{1}\left(log\left(\frac{v_i}{k_i}\right) \leq X\right) \frac{v_i}{Y}$. The mean component expressed as a function of this distribution is: $-\int_x -xdG(x)$, while the dispersion component is: $-\left(\int e^{-x}dG(x) - \int -xdG(x)\right)$.

We know, by definition, that the first cumulant can be written as: $\int_x xdG(x)$. A property of the cumulant generating function is that $E\left[e^{tx}\right] = \sum_{t=1}^{n} t^n \frac{\kappa_n}{n!}$, which yields:

$$\int e^{-x}dG(x) = \sum_{t=1}^{n} (-1)^n \frac{\kappa_n}{n!} = -\kappa_1 + \sum_{t=2}^{n} (-1)^n \frac{\kappa_n}{n!}.$$

Therefore the mean component can be written as:

$$-\int_x -xdG(x) = \kappa_1.$$

While the dispersion component is:

$$\begin{aligned} -\left(\int e^{-x}dG(x) - \int -xdG(x)\right) &= -\left(-\kappa_1 + \sum_{t=2}^{n} (-1)^n \frac{\kappa_n}{n!} + \kappa_1\right) \\ &= -\sum_{t=2}^{n} (-1)^n \frac{\kappa_n}{n!} = \sum_{t=2}^{n} (-1)^{n+1} \frac{\kappa_n}{n!}. \end{aligned}$$

This means our decomposition can be expressed as:

$$log\left(\frac{Y}{K}\right) = \underbrace{\kappa_1}_{\text{Mean Component}} + \underbrace{-\kappa_2 + \sum_{t=3}^{n} (-1)^{n+1} \frac{\kappa_n}{n!}}_{\text{Dispersion Component}}.$$

Now note that for any variable of the form $z_i = c\frac{v_i}{k_i}$ (such as capital wedges or marginal products in a Hsieh and Klenow case) yields $log(z_i) = log(c) + log\left(\frac{v_i}{k_i}\right)$. Standard properties of cumulants imply that $\kappa_{1,z} = \kappa_{1,\frac{v}{k}} + log(c)$, and $\kappa_{n,z} = \kappa_{n,\frac{v}{k}}$ for all $n > 1$.

Therefore for such variables, our decomposition implies

$$log\left(\frac{Y}{K}\right) = \underbrace{\kappa_{1,z} - c}_{\text{Mean Component}} + \underbrace{-\kappa_{2,z} + \sum_{t=3}^{n} (-1)^{n+1} \frac{\kappa_{n,z}}{n!}}_{\text{Dispersion Component}}.$$

(21) and (27) follow immediately from this derivation.

Our Decomposition for Different Models and Shocks

Common changes in firm revenue products, whether driven by technology or distortions, are reflected only in the mean component of our decomposition. Consider a change in revenue products such that $\frac{v_{i,t+1}}{k_{i,t+1}} = x\frac{v_{i,t}}{k_{i,t}}$. Then, our decomposition implies that:

$$\begin{aligned} log\left(\frac{Y_{t+1}}{K_{t+1}}\right) &= \underbrace{-\int log\left(\frac{k_{i,t}}{v_{i,t}}\frac{1}{x}\right)\frac{v_{i,t}}{Y_t}di}_{\text{mean component}} \underbrace{-\left(log\left(\int \frac{k_{i,t}}{v_{i,t}}\frac{1}{x}\frac{v_t}{Y_{i,t}}di\right) - \int log\left(\frac{k_{i,t}}{v_{i,t}}\frac{1}{x}\right)\frac{v_t}{Y_t}di\right)}_{\text{dispersion component}} \\ &= \underbrace{log(x) - \int log\left(\frac{k_{i,t}}{v_{i,t}}\right)\frac{v_{i,t}}{Y_t}di}_{\text{mean component}} \underbrace{-\left(log\left(\int \frac{k_{i,t}}{v_{i,t}}\frac{v_t}{Y_{i,t}}di\right) - \int log\left(\frac{k_{i,t}}{v_{i,t}}\right)\frac{v_t}{Y_t}di\right)}_{\text{dispersion component}}. \end{aligned}$$

The results in subsubsections 3.4.1 immediately follow from the above. The results in subsubsection 3.4.2 can be derived by using standard formulas for the expectation of log-normally distributed variables.

Specifically, consider the case where the output-share weighted distribution of wedges is a mixture of lognormals. The pdf of wedges is thus $g\left(log\left(\omega\right)\right) = \sum_{n=1}^{N} \lambda_n \phi_n\left(log\left(\omega\right)\right)$, where ϕ_n, where ϕ_n are normal pdfs and $\sum_n \lambda_n = 1$. Note that lognormal wedges is the special case of this with $N = 1$. Standard formulas for expectations over lognormal distributions imply that $\int log\left(\omega\right) g\left(log\left(\omega\right)\right) d\omega = \sum_n \lambda_n \mu_n$ and $\int \omega^t g\left(log\left(\omega\right)\right) d\omega = \sum_n \lambda_n e^{t\mu_n + \frac{1}{2}t^2\sigma_n^2}$. The results in subsubsection 3.4.2 immediately follow.

Appendix B

Measurement of Objects — Data from the United States

For the empirical analysis in Section 4 on U.S. firms, we use annual data on firms that exist in the Compustat database. We take the following steps, in order. First, firms must be headquartered in the United States and have a U.S. currency code. We then keep only firms with December fiscal year-ends. We then drop firms if their employment, property, plant, and equipment — net of depreciation, sales, or our measure of firm-value added — are missing or negative. We then exclude firms with 4-digit SIC codes between 4000 and 4999, between 6000 and 6999, or greater than 9000, as our model is not representative of regulated, financial, or public service firms. We then clean the data by winsorizing each series at the 1st percentile over the entire sample. For our analysis, we lastly only keep data from 1971 to 2011.

Firm-level value added, firm-level capital stock, and firm-level employment are the only firm-level objects we need for our decomposition. When computing year-over-year changes in the components of our decomposition, we also adjust for entry and exit by only keeping data on firms that exist in consecutive years. In the second decomposition, firms are grouped into sectors by two-digit SIC codes.

We measure labor as the number of employees reported in Compustat. We measure capital as the firm's plant, property, and equipment, adjusted for accumulated depreciation. The aggregate capital stock is annual, taken from the Penn World Tables. To adjust for potential changes in the valuation of capital over time, we construct a perpetual inventory measure of the aggregate capital stock and use the ratio of this measure to the value of the aggregate capital stock to deflate the firm-level measure of capital. The investment measure used in the perpetual inventory method is annual gross private domestic investment from the Bureau of Economic Analysis (BEA). To construct our measure of capital using the perpetual inventory method (starting from 1959), we use a depreciation rate of 4.64 percent and growth rate of technology of 1.6 percent, following Chari, Kehoe, and McGrattan (2007). Our measure is then deflated by the December value of the monthly CPI, which is CPI for All Urban Consumers, seasonally adjusted, from the Bureau of Labor Statistics.

We create a measure of value added in public firms using income accounting. GDP has an income equivalent, GDI, which has similar time-series properties. The major components of this measure have equivalents to income statement measures that are required on 10-K forms for U.S. public firms. In order of magnitude, GDI is made up of the following components: compensation of employees, net operating surplus, consumption of fixed capital (depreciation), and taxes on production and imports less subsidies. While we do not observe the taxes or subsidies on production and imports firms pay in our dataset, we do observe measures of the other three components, all of which make up over 90 percent of GDI for all years in our sample. We observe labor compensation in Compustat annually.

If labor compensation is missing, we replace it with selling, general, and administrative expenses. We also observe net operating profits before depreciation, which is the sum of a firm's net operating surplus and its capital consumption. We define a firm's contribution to output as the sum of labor compensation and operating profits before depreciation. In practice, the BEA uses a similar, more detailed approach, where they use firm tax data to aggregate up the components of domestic income and make adjustments for differences between accounting and economic treatment of factors such as capital consumption and inventory valuation.

To compute TFP, following Chari, Kehoe, and McGrattan (2007), we set capital's share of income, $\alpha = .35$ and back it out from (9). When we compare our measure against the NIPA-equivalent, we require a NIPA equivalent measure of our value-added measure, a measure of aggregate labor, and a measure of aggregate capital. To compute our NIPA equivalent of our pseudo-GDI measure, we use data from NIPA table 1.12 on National Income by Type of Account. We take compensation of employees (line 2) and subtract government (line 4), then add to this measure corporate profits with inventory valuation adjustment and capital consumption adjustment less taxes on corporate income (line 43). Finally, we add to this measure consumption of fixed capital, which comes from the BEA. All measures are quarterly, and we only use the fourth-quarter values of these measures. We put this measure in per-capita terms using population including armed forces overseas. This measure is mid-period and monthly. We only keep its December value. We then put this measure in real terms using the CPI measure described in this subsection. Our measure of the real aggregate capital stock was already described in this subsection. This measure is also put in per-capita terms. Our measure of aggregate labor is total non-farm employment and is monthly. We only use the December observation of this variable.

Measurement — Data from Japan

Our data on Japanese public firms comes from the Compustat global database, and our firm-level variables are measured annually. We clean the data as we do for data from the United States, except we only keep firms with currency codes corresponding to the Japanese Yen and country headquarter codes corresponding to Japan. Also, the years of our sample are different: They only cover 2001 to 2011. Consistent with our application to U.S. data, when computing year-over-year changes in the components of our decomposition, we also adjust for entry and exit by only keeping data on firms that exist in consecutive years. In Decomposition II, firms are grouped into sectors by two digit SIC codes

As for the U.S. data, we measure firm-level labor as the number of employees reported and firm-level capital as the firm's plant, property, and equipment, adjusted for accumulated depreciation. We deflate the firm-level Japanese capital stock by the U.S. capital deflater. To put the capital stock in real terms, we deflate it by the OECD's measure of the quarterly CPI in Japan. We only keep the fourth-quarter value of this measure.

In a manner consistent with our application to U.S. data, we create a measure of value added in public firms using income accounting, which is the sum of labor compensation and operating profits before depreciation. As for the U.S. data, if labor compensation is missing, we replace it with selling, general, and administrative expenses. We eventually deflate by the same Japanese CPI measure as for capital. To compute TFP, we again set $\alpha = .35$ and back it out from (9).

Appendix C

Our Decomposition in the Context of Other Methodologies

To apply our decomposition to data, one does not need to estimate firm-level TFP or sectoral production function coefficients. There is already potential for measurement issues biasing the results from our decompositions, as measures of labor, value added, and capital can all be measured incorrectly. Further, we could be incorrectly grouping firms with our sectoral definitions. However, it is easy enough to check different measures of labor, capital, or value added, if available, and see if the results change. Also, one could add measurement error to firm variables and test the extent to which the results change. Similarly, one can check the results from our second decomposition on different definitions of "groupings" or sectors. However, to compute sectoral production function coefficients, as is commonly done in papers assessing the role of labor and capital allocation on productivity over the business cycle, some issues cannot be "checked." Data from 30 years prior can be crucial in providing "correct" estimates of sectoral production function coefficients. But what if such data are unavailable to the researcher? In addressing the role of resource reallocation in productivity dynamics over the business cycle, the literature has relied on the estimation of these technological measures for all sectors in the economy. In this section, we will demonstrate how some of the econometric biases associated with such an approach can lead one to produce quantitatively and qualitatively different results on the role of allocative efficiency over the business cycle.

We first demonstrate the most difficult-to-correct econometric bias associated with measuring production function coefficients, which is the fact that data for the entire sector over the entire sample are needed to estimate them. We also show that different definitions of factor prices can crucially affect one's results. Second, we show that our decomposition can help to assess the role of resource reallocation in productivity dynamics over the business cycle. In particular, in a relatively general setting, we demonstrate that the within-industry component of our decomposition is reflective of within-sector allocative efficiency. Ultimately, this section is meant to demonstrate that our decomposition can, at the very least, be a useful check on such attempts at measuring the role of resource allocation over the business cycle that require estimates of sectoral production function coefficients and firm-level TFP.

Illustrating issues with identification

In Figure 15, we demonstrate one possible issue with identification that can severely change the interpretation of the qualitative and quantitative importance of the role of reallocation over the business cycle. We show that our results change substantially when we follow a standard procedure and only slightly vary the estimation procedure for production function coefficients.[13] Figure 15 shows the cumulative change in the contribution of allocative efficiency to TFP over the recent recession for three different standard "versions" of estimating production function coefficients.[14] We estimate production function coefficients as the average of the ratio of capital expenditures to labor expenditures, $\frac{rk}{wl}$, across firms within a sector over time, where r is the rental rate, k is the capital stock in the firm, w is the wage, and l is labor utilization in the sector. In Version 1, the baseline version, we drop all observations before 1972, use capital and labor utilization from our firm-level data, and estimate the rental rate and wage following Chari, Kehoe, and McGrattan (2007).[15]

We see that in Version 1 of our estimation of production function coefficients, there seems to be a decrease in allocative efficiency from pre-recession levels to the trough in

[13] Specifically, we implement the approach of Oberfeld (2013) to measure changes in allocative efficiency in our sample of U.S. publicly listed firms. This model of production and aggregation is identical to that in Hsieh and Klenow (2009). Details of our dataset construction and measurement can be found in section 4 and Appendix B. As in Oberfeld (2013) and Hsieh and Klenow (2009), we set the elasticity of substitution within sectors to 3.

[14] Positive changes indicate an increase in the extent of allocative efficiency.

[15] Following Chari, Kehoe, and McGrattan (2007) entails setting $r = \alpha * \frac{Y}{K}$ and $w = (1-\alpha) * \frac{Y}{L}$, where α is defined in the measurement subsection above and come directly from Chari, Kehoe, and McGrattan (2007). Our measures and Y, L, and K are all computed as describe in subsection 5. These measures are computed differently from how Y, L, and K are computed in Chari, Kehoe, and McGrattan (2007).

2008 to 2009. In Version 2, we take capital, k, and labor expenditures, wl, from the firm-level data, but still estimate r following Chari, Kehoe, and McGrattan (2007). Estimating labor expenditures using firm-level data changes the year-over-year behavior of the contribution of allocative efficiency to TFP. In Version 3, we follow the same procedure as in Version 1 but drop all observations before 1976. In this version, which is only different from Version 1 in that we assume there is slightly less data available decades prior to the recession we are examining, we find an increase in allocative efficiency from 2006 to the trough of the recession. These results demonstrate just one of the potential problems with identification to which the standard model-based approaches are susceptible, a problem that can substantially change the qualitative and quantitative implications of the role of reallocation over the business cycle. Other issues with identification remain; another example is that we find that when we vary the elasticity of substitution across firms between 3 and 10 (standard values used in the literature as noted in Hsieh and Klenow (2009)), the qualitative and quantitative nature of our results change substantially.

Relation to Models of Allocative Efficiency

This appendix demonstrates how the dispersion component of our decomposition relates to the aggregate productivity loss suffered from misallocation in a standard static model of allocative efficiency. We show that the dispersion component directly enters this loss and discuss the relative magnitude of the the social losses and the dispersion components.

We consider the one-sector economy introduced in subsection 3.2, and derive two statistics commonly used as measures of productivity loss due to distortions. First, we derive the difference between efficient and observed log TFP. This difference may be driven by misallocation or frictions to the total amount of capital/labor used. Second, we compute the difference between the log TFP implied by the output-maximizing allocation of the observed total labor and capital and observed TFP. This difference will be driven only by misallocation.

Difference from Optimal Allocation Note that an analogue (in levels instead of differences) to (28) yields:

$$
\begin{aligned}
log\left(TFP_t\right) &= \underbrace{\log\left(TFP_t^{*}\right)+\alpha\kappa_{k,1,t}+\left(1-\alpha\right)\kappa_{l,1,t}}_{\text{static mean component}} \\
&+ \underbrace{\overbrace{-\frac{\alpha\kappa_{k,2,t}+\left(1-\alpha\right)\kappa_{l,2,t}}{2!}}^{\text{variance}}+\overbrace{\sum_{n=3}^{\infty}\left(-1\right)^{n-1}\frac{\alpha\kappa_{k,n,t}+\left(1-\alpha\right)\kappa_{l,n,t}}{n!}}^{\text{higher-order terms}}}_{\text{static dispersion component}},
\end{aligned}
\tag{64}
$$

where $\kappa_{k,n,t}$, $\kappa_{l,n,t}$ are the cumulants of log wedges in capital and labor to firms productivity ratios, as defined in subsection 3.3. This implies that we can express the difference between efficient and realized log TFP as the following:

$$
log\left(TFP_t^{*}\right)-log\left(TFP_t\right)=-\alpha\kappa_{k,1,t}-\left(1-\alpha\right)\kappa_{l,1,t}-\alpha\,D_t^{K}-\left(1-\alpha\right)\,D_t^{L}.
$$

Where D_t^K and D_t^L are the dispersion components of labor and capital in our decomposition. The first cumulants are the output-share weighted averages of log wedges, and are the only terms other than the dispersion components to enter these losses.

Total Labor and Capital Fixed Consider a firm optimization problem, with fixed stock of total capital and labor within the production environment defined in subsection 3.2. Firms take the wage and rental rate as given, and clearing conditions $K_t = \sum_i k_{i,t}$ and $L_t = \sum_i l_{i,t}$ are satisfied. Firm labor and capital choices are distorted from their optimal allocation by $\theta_{i,t}^l$ and $\theta_{i,t}^k$, respectively.

$$
\underset{k_{i,t},l_{i,t}}{max}\;p_{i,t}y_{i,t}-k_{i,t}\theta_{i,t}^{k}r_t-l_{i,t}\theta_{i,t}^{l}w_t. \tag{65}
$$

The equilibrium decision rules imply that firm marginal revenue products are the following:

$$
\frac{v_{i,t}}{k_{i,t}}=\theta_{i,t}^{k}\frac{r_t}{\varphi\gamma}, \tag{66}
$$

and

$$
\frac{v_{i,t}}{l_{i,t}}=\theta_{i,t}^{l}\frac{w_t}{\varphi\phi}. \tag{67}
$$

We can then express the losses due to misallocation in this fixed environment as a function of the rental rate, efficient rental rate, and cumulants of firm-level distortions $\theta^k_{i,t}$ and $\theta^l_{i,t}$.[16] We derive the following expression for $log\left(T\hat{F}P_t\right) - log\left(TFP_t\right)$:

$$log\left(\frac{T\hat{F}P_t}{TFP_t}\right) = \alpha\left(log\left(\frac{\hat{r}_t}{r_t}\right) - \kappa_{k,1,t}\right) - D^K_t) + (1-\alpha)\left(log\left(\frac{\hat{w}_t}{w_t}\right) - \kappa_{l,1,t} - D^L_t\right),$$

where $T\hat{F}P_t$ is TFP if labor and capital are allocated efficiently but total labor and capital are fixed at the observed levels, and $\hat{w}_t$ and $\hat{r}_t$ are the wage and rental rate required to keep total capital and wages fixed if distortions are removed.

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[16]These distortions differ only from the wedges defined before in that they are distortions from the input choices implied by rental rates and wages instead of the efficient allocation.

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Tables

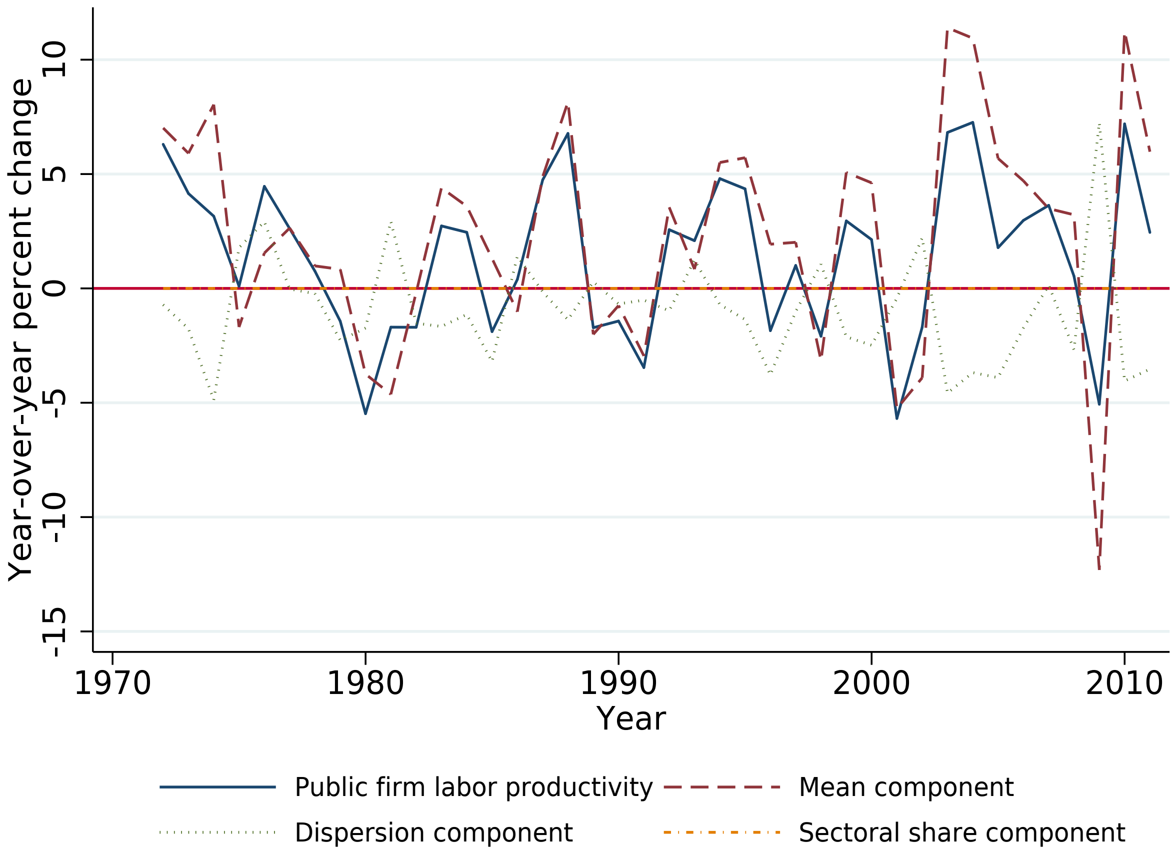
Table 1: Correlations between Changes in Aggregates and Changes in their Components from Decomposition II — U.S. Data

	TFP	Labor productivity	Capital productivity
Mean component	0.974	0.955	0.955
Dispersion component	0.042	-0.346	0.399
Sectoral share component	-0.245	-0.329	0.259

Notes: Sample period is from 1972 to 2011. Data are from U.S. nonfinancial public firms. Firms are grouped by two digit SIC codes. Changes in aggregate measures (TFP, labor productivity, and capital productivity) and components of our decompositions are measured year over year.

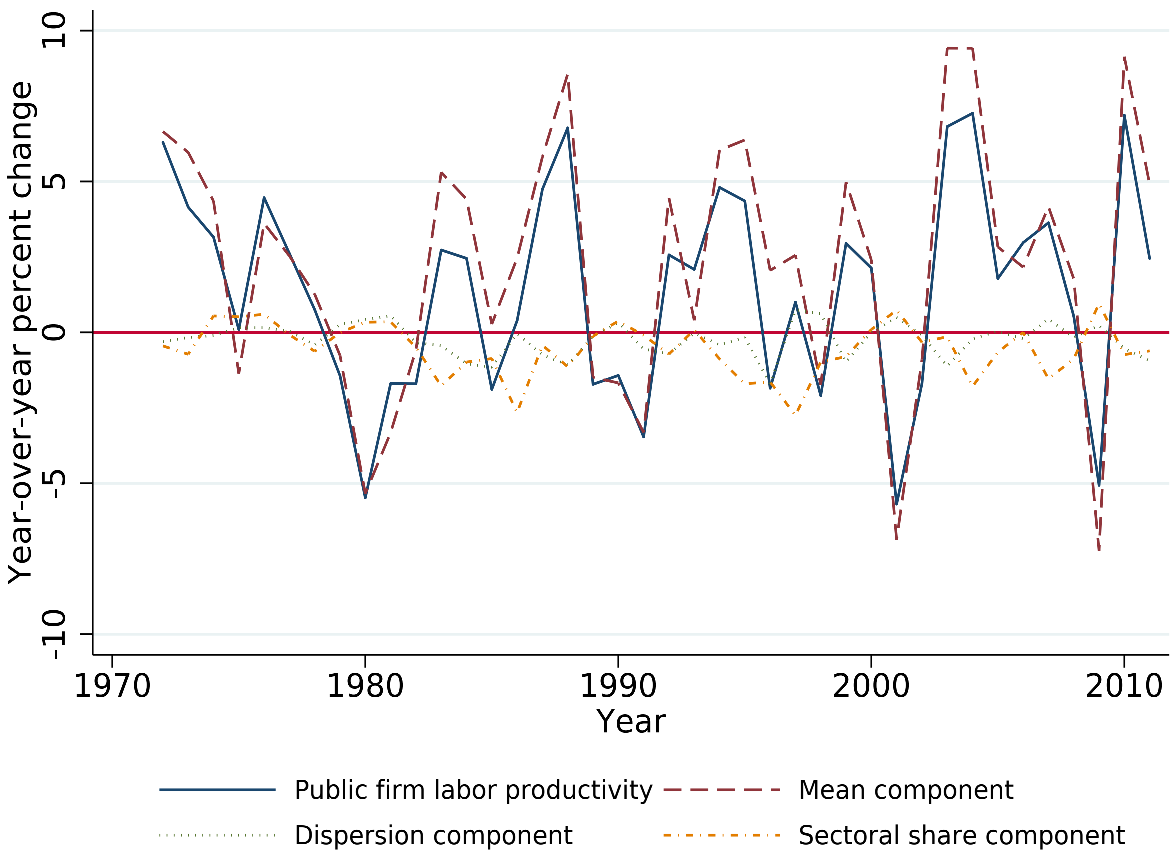
Figures

Figure 1: Decomposition I Applied to Aggregate Labor Productivity: Year-over-Year Changes



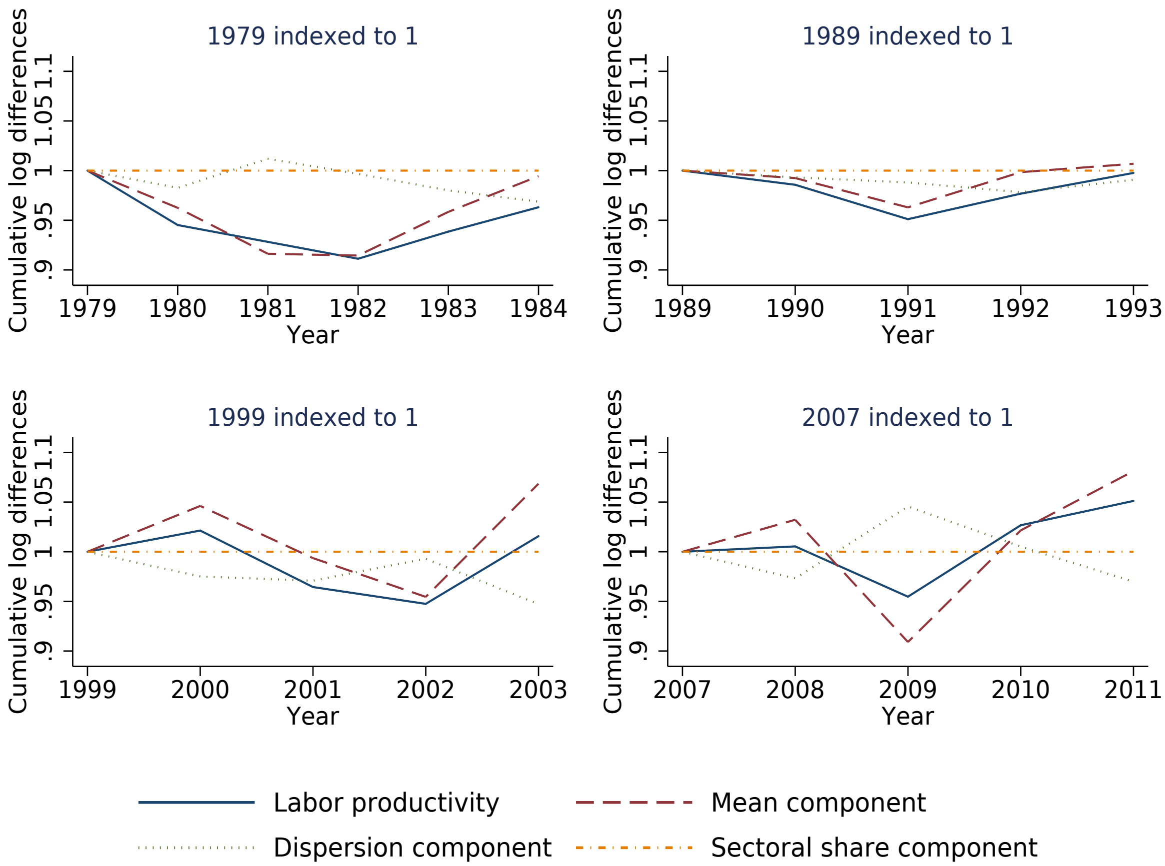
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate labor productivity and its components from Decomposition I using data from U.S. nonfinancial public firms. Because there is no grouping by sector for Decomposition I, the sectoral share component (the yellow line) is, in turn, flat.

Figure 2: Decomposition II Applied to Aggregate Labor Productivity: Year-over-Year Changes



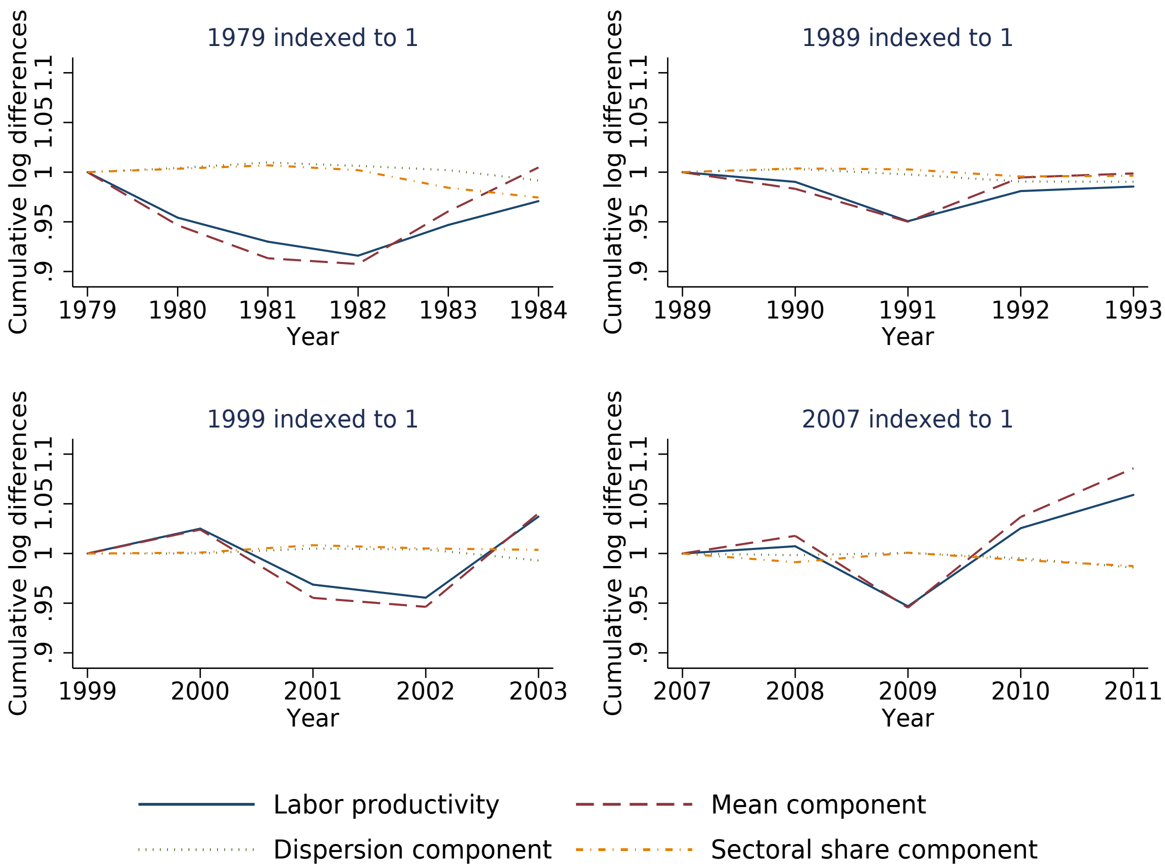
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate labor productivity and its components from Decomposition II using data from U.S. nonfinancial public firms. Firms are grouped by two digit SIC codes.

Figure 3: Decomposition I Applied to Aggregate Labor Productivity: Cumulative Changes over Four Business Cycle Episodes



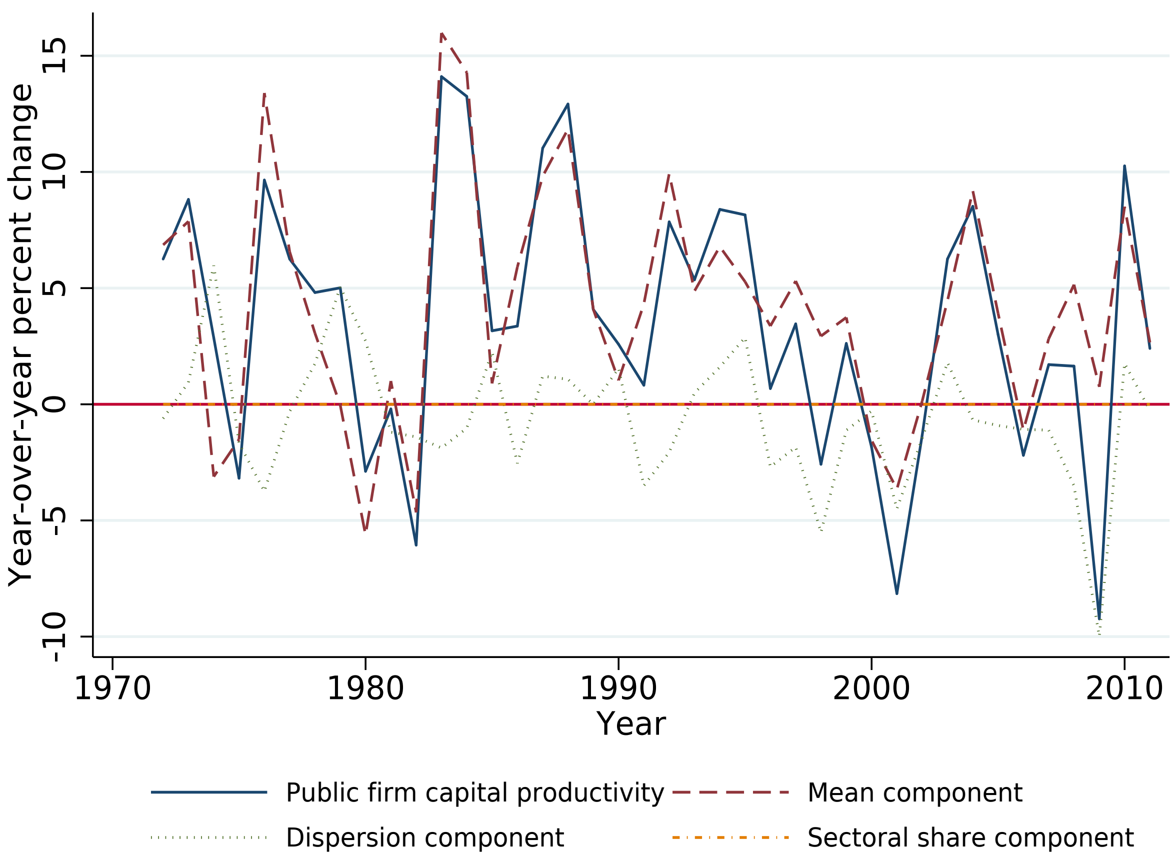
Note: Sample period is from 1972 to 2011. We compute cumulative changes in aggregate labor productivity and its components from Decomposition I using data from U.S. nonfinancial public firms over four business cycle episodes, with the pre-recession index year in the title of the plot. Because there is no grouping by sector for Decomposition I, the sectoral share component (the yellow line) is, in turn, flat.

Figure 4: Decomposition II Applied to Aggregate Labor Productivity: Cumulative Changes over Four Business Cycle Episodes



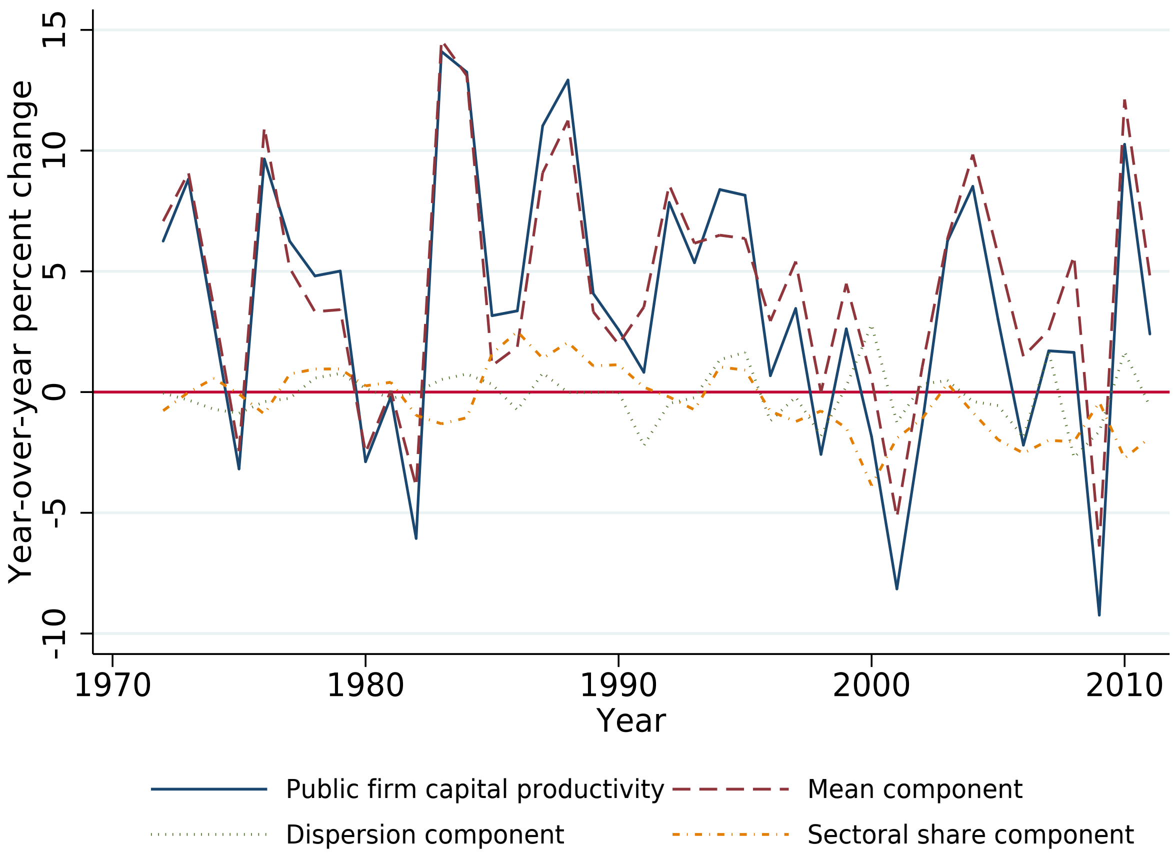
Note: Sample period is from 1972 to 2011. We compute cumulative changes in aggregate labor productivity and its components from Decomposition I using data from U.S. nonfinancial public firms over four business cycle episodes, with the pre-recession index year in the title of the plot. Firms are grouped by two digit SIC codes.

Figure 5: Decomposition I Applied to Aggregate Capital Productivity: Year-over-Year Changes



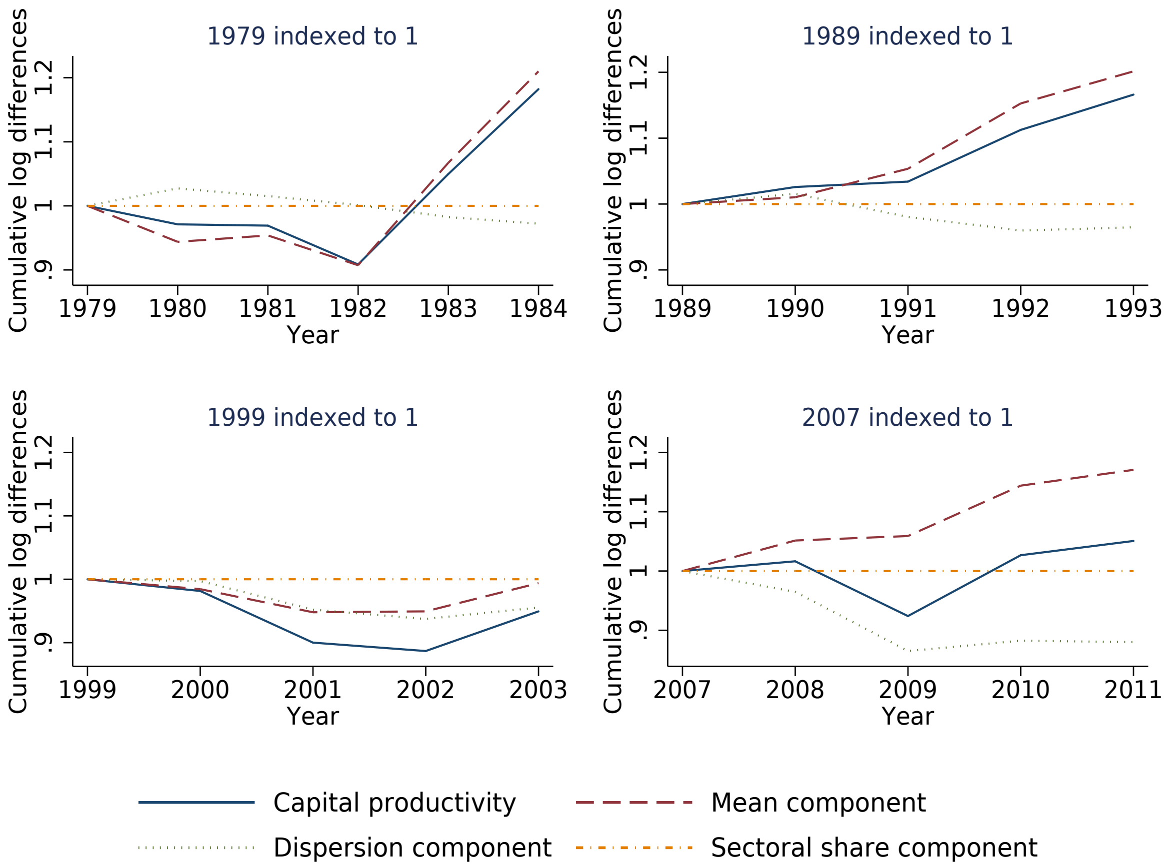
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate capital productivity and its components from Decomposition I using data from U.S. nonfinancial public firms. Because there is no grouping by sector for Decomposition I, the sectoral share component (the yellow line) is, in turn, flat.

Figure 6: Decomposition II Applied to Aggregate Capital Productivity: Year-over-Year Changes



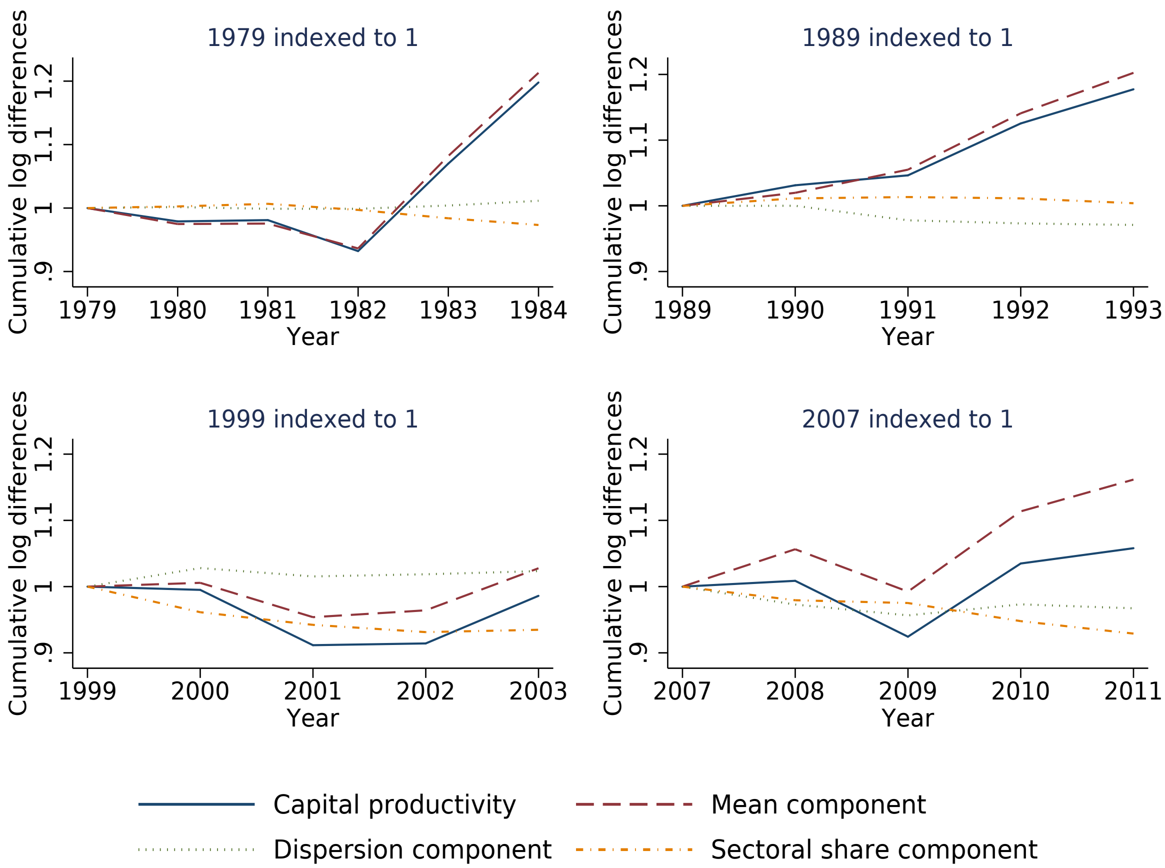
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate capital productivity and its components from Decomposition II using data from U.S. nonfinancial public firms. Firms are grouped by two digit SIC codes.

Figure 7: Decomposition I Applied to Aggregate Capital Productivity: Cumulative Changes over Four Business Cycle Episodes



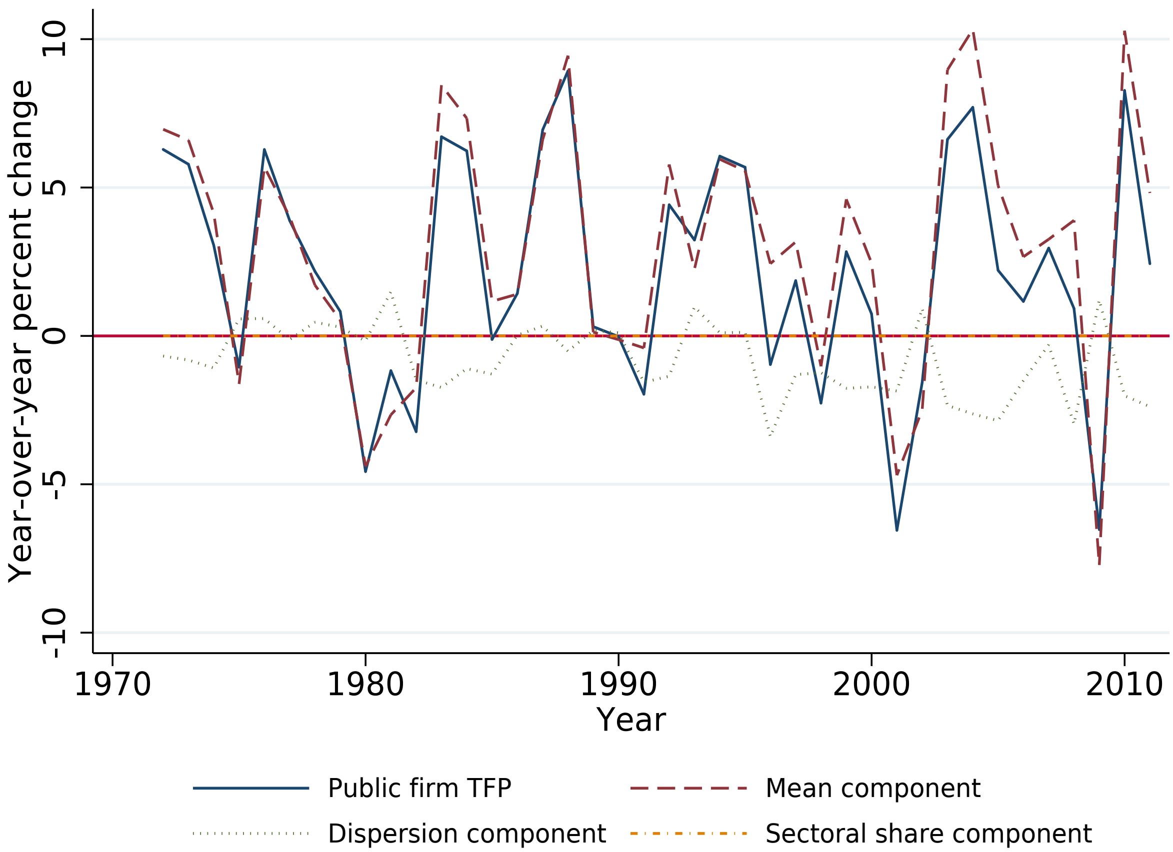
Note: Sample period is from 1972 to 2011. We compute cumulative changes in aggregate capital productivity and its components from Decomposition I using data from U.S. nonfinancial public firms over four business cycle episodes, with the pre-recession index year in the title of the plot. Because there is no grouping by sector for Decomposition I, the sectoral share component (the yellow line) is, in turn, flat.

Figure 8: Decomposition II Applied to Aggregate Capital Productivity: Cumulative Changes over Four Business Cycle Episodes



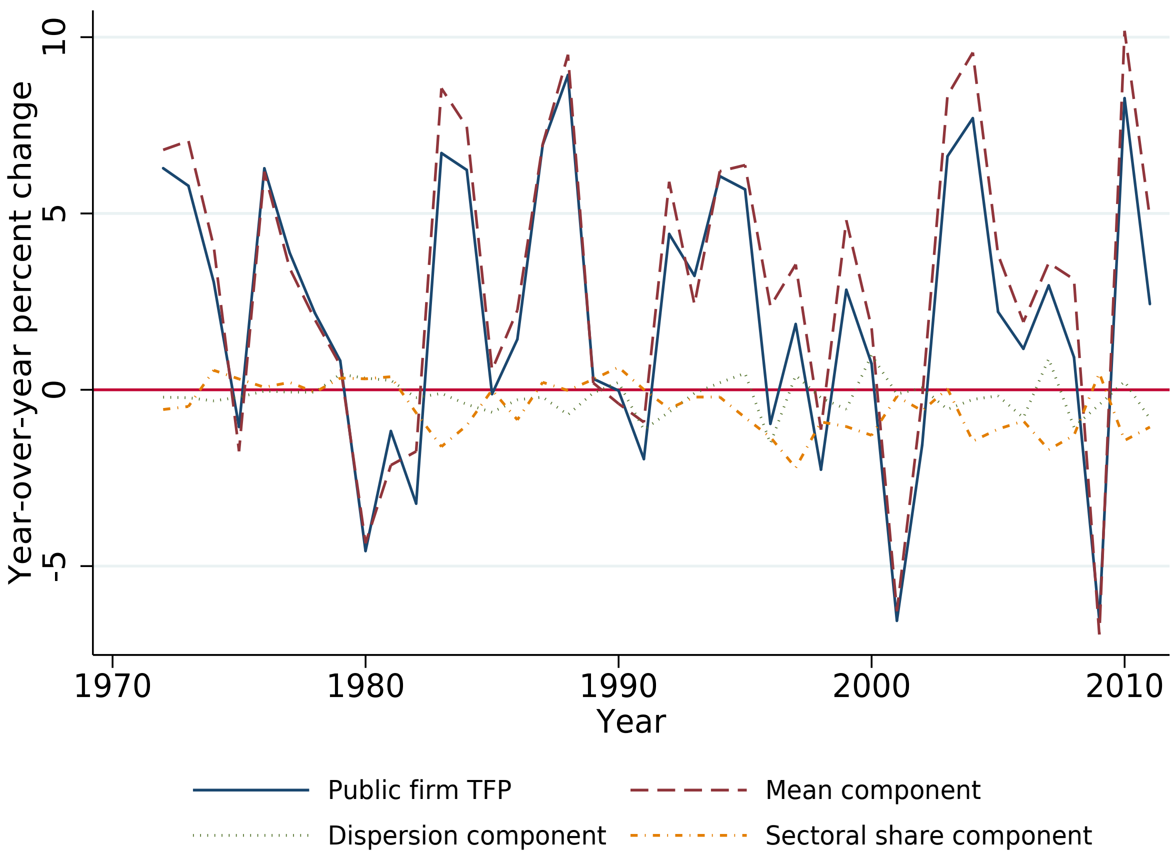
Note: Sample period is from 1972 to 2011. We compute cumulative changes in aggregate capital productivity and its components from Decomposition I using data from U.S. nonfinancial public firms over four business cycle episodes, with the pre-recession index year in the title of the plot. Firms are grouped by two digit SIC codes.

Figure 9: Decomposition I Applied to Aggregate TFP: Year-over-Year Changes



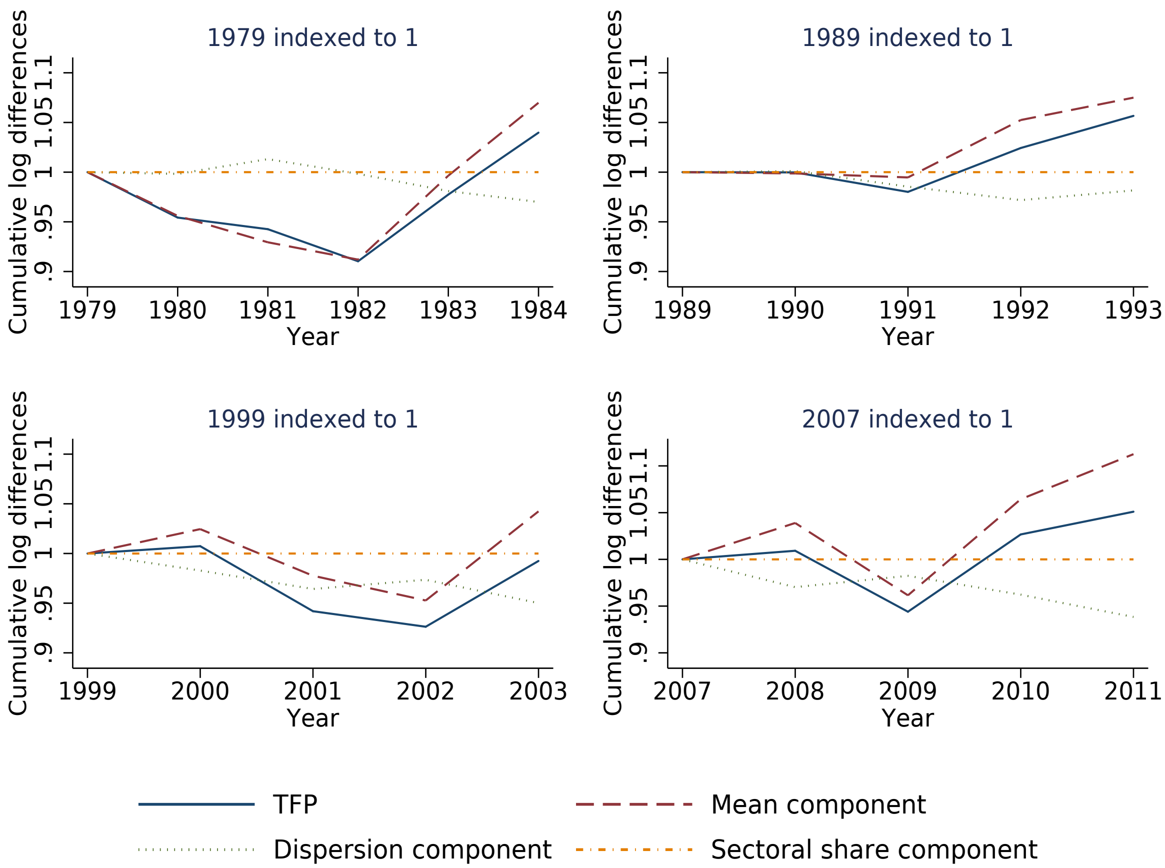
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate TFP and its components from Decomposition I using data from U.S. nonfinancial public firms. Because there is no grouping by sector for Decomposition I, the sectoral share component (the yellow line) is, in turn, flat.

Figure 10: Decomposition II Applied to Aggregate TFP: Year-over-Year Changes



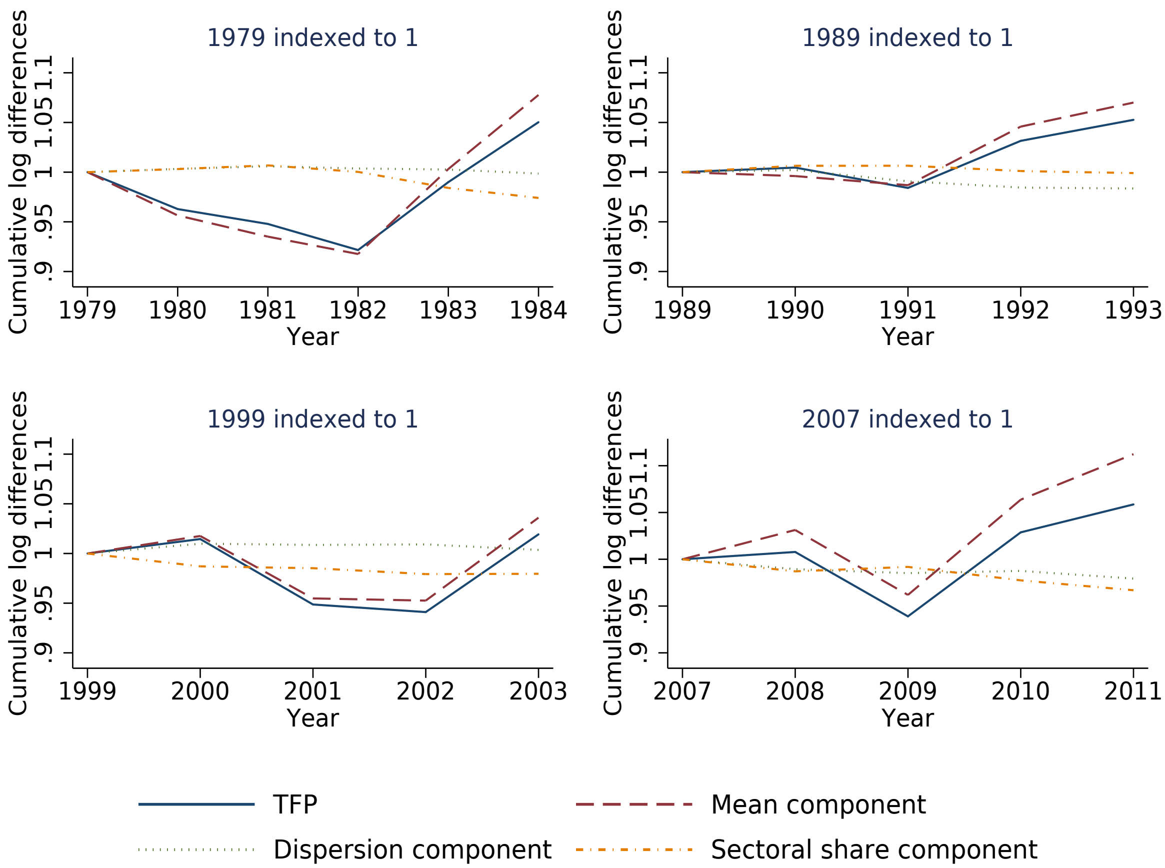
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate TFP and its components from Decomposition II using data from U.S. nonfinancial public firms. Firms are grouped by two digit SIC codes.

Figure 11: Decomposition I Applied to Aggregate TFP: Cumulative Changes over Four Business Cycle Episodes



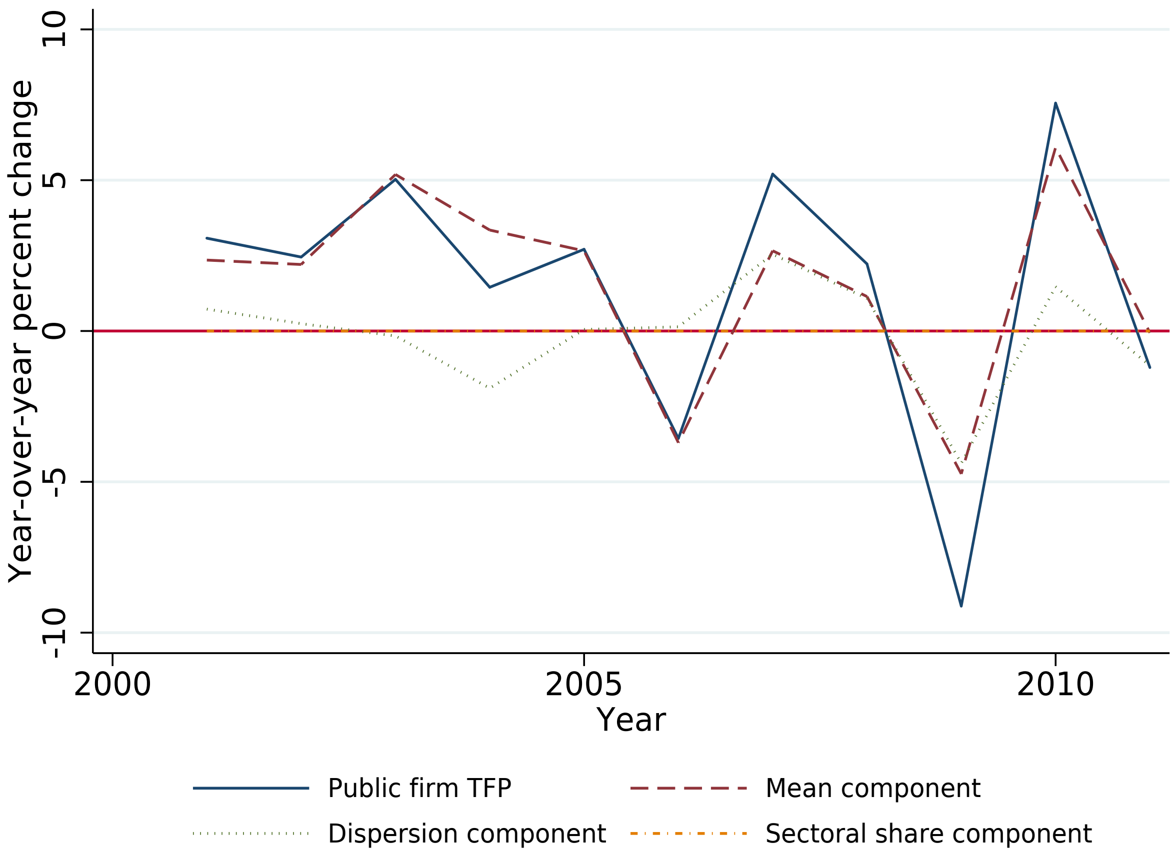
Note: Sample period is from 1972 to 2011. We compute cumulative changes in aggregate TFP and its components from Decomposition I using data from U.S. nonfinancial public firms over four business cycle episodes, with the pre-recession index year in the title of the plot. Because there is no grouping by sector for Decomposition I, the sectoral share component (the yellow line) is, in turn, flat.

Figure 12: Decomposition II Applied to Aggregate TFP: Cumulative Changes over Four Business Cycle Episodes



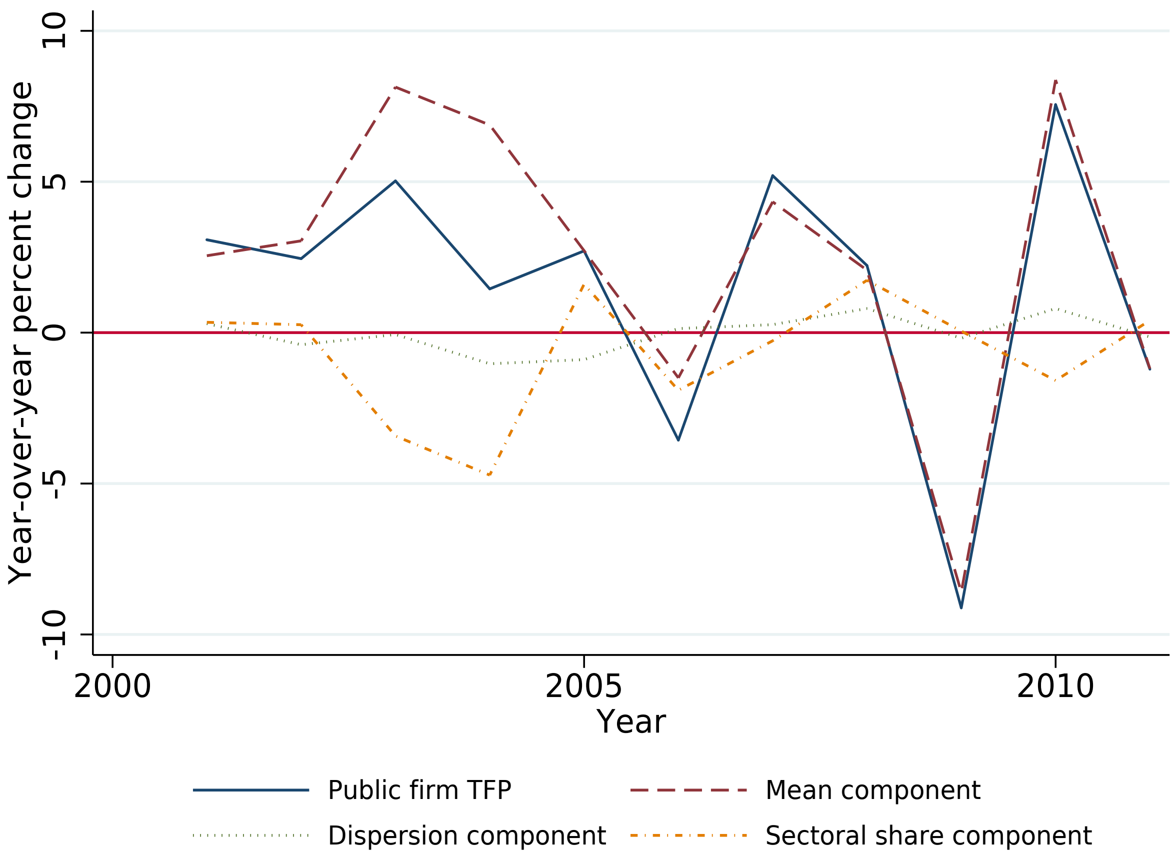
Note: Sample period is from 1972 to 2011. We compute cumulative changes in aggregate TFP and its components from Decomposition I using data from U.S. nonfinancial public firms over four business cycle episodes, with the pre-recession index year in the title of the plot. Firms are grouped by two digit SIC codes.

Figure 13: Decomposition I Applied to Aggregate TFP: Year-over-Year Changes



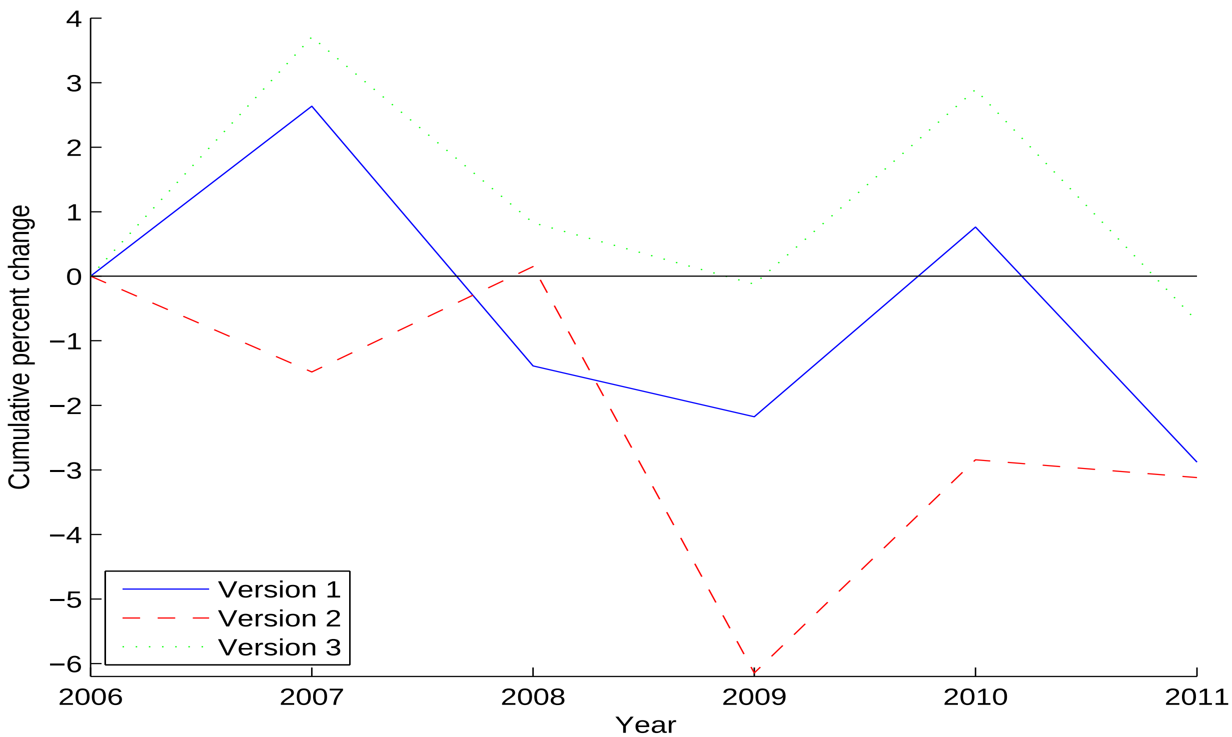
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate TFP and its components from Decomposition I using data from Japanese nonfinancial public firms. Because there is no grouping by sector for Decomposition I, the sectoral share component (the yellow line) is, in turn, flat.

Figure 14: Decomposition II Applied to Aggregate TFP: Year-over-Year Changes



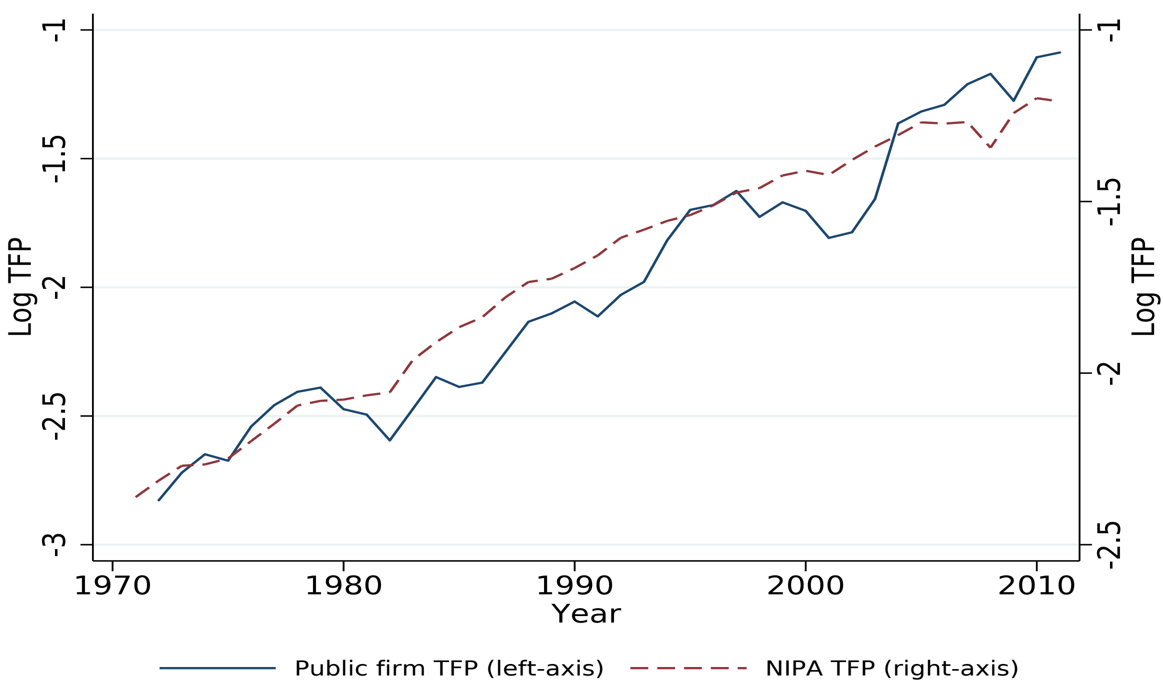
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate TFP and its components from Decomposition II using data from Japanese nonfinancial public firms. Firms are grouped by two digit SIC codes.

Figure 15: Three Estimation Techniques for the Contribution of Allocative Efficiency to TFP



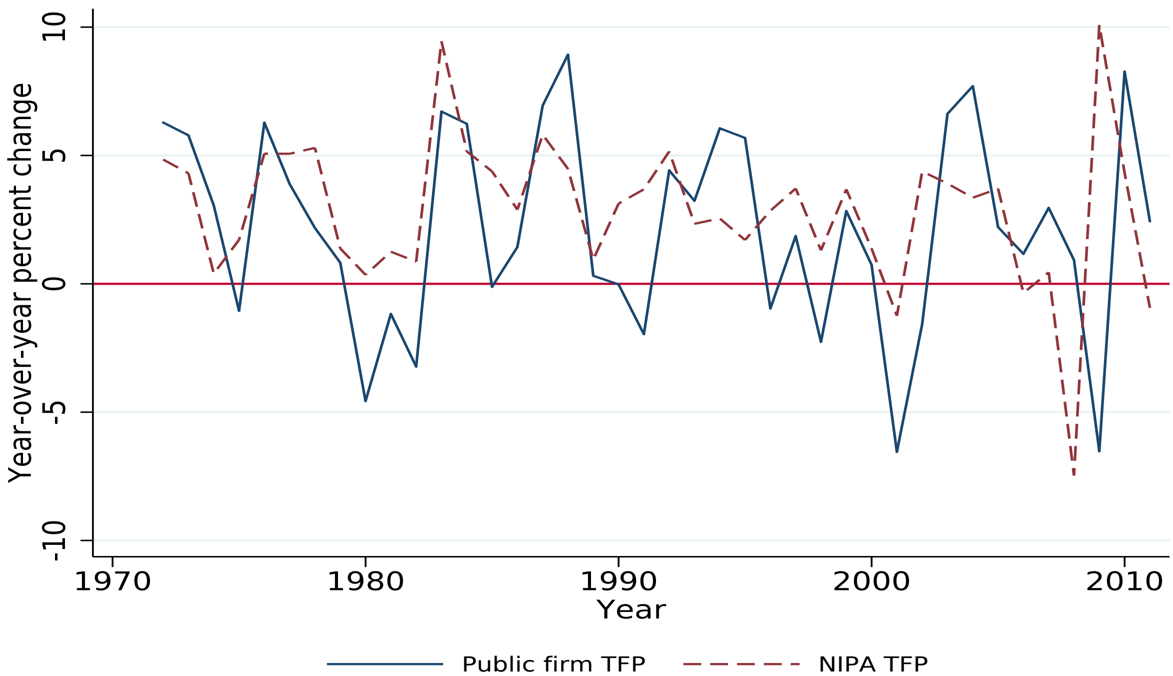
Note: We can express TFP as a function of the hypothetical efficient productivity, TFP_t^{eff}, at time t, and the allocative efficiency of resources a_t such that $TFP_t = a_t TFP_t^{eff}$. The figure shows $log(a_t) - log(a_{2006})$. The lines can thus be interpreted as the cumulative percent change in TFP over 2006 levels due to changes in allocative efficiency. The estimation of the different "versions" only differs in the estimation of production function coefficients as follows: (1) Version 1 is the baseline version, (2) Version 2 uses wage data from Compustat rather than from NIPA, and (3) Version 3 uses data back to 1976 instead of 1972.

Figure 16: Trend in Aggregate TFP: NIPA vs. Compustat



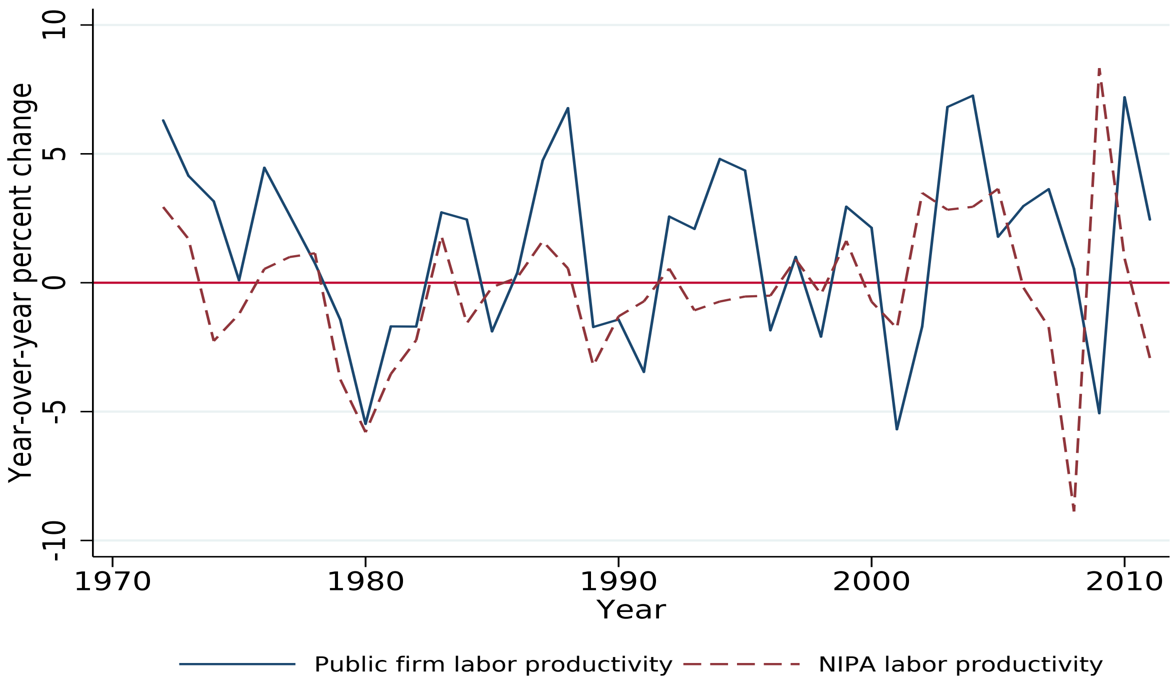
Note: Sample period is from 1972 to 2011. This figure depicts the trend in the natural logarithm of aggregate TFP computed from two different samples. The blue line corresponds to a measure computed from public firm data. The red line corresponds to a measure computed from NIPA.

Figure 17: Changes in Aggregate TFP: NIPA vs. Compustat



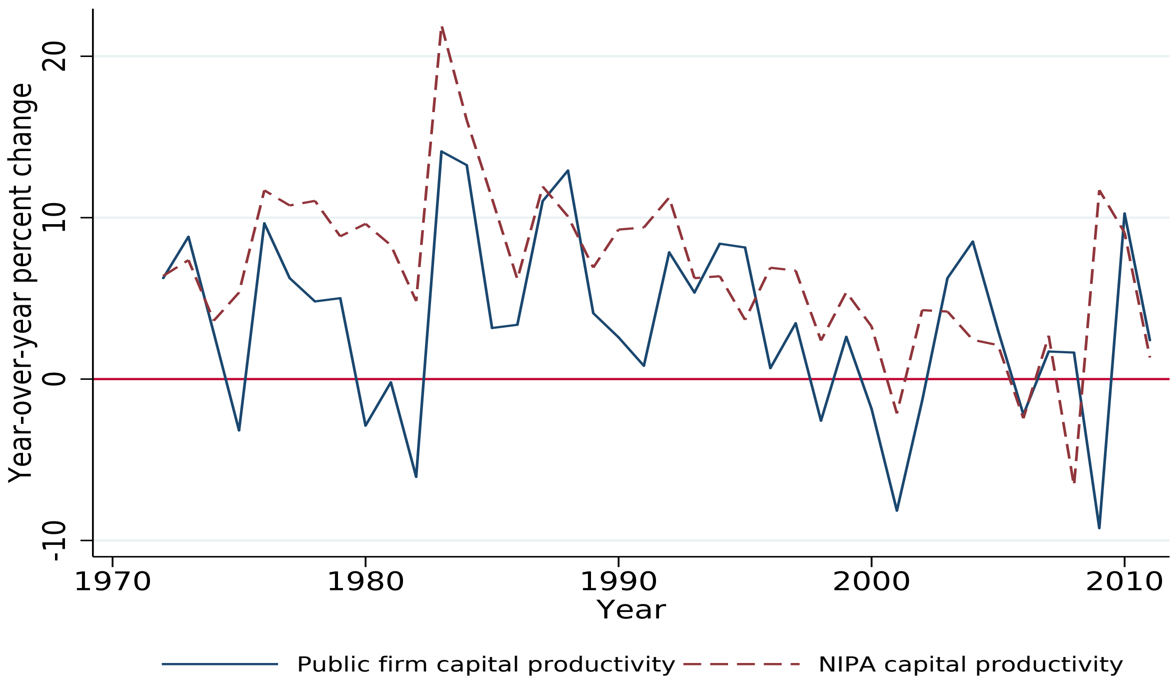
Note: Sample period is from 1972 to 2011. This figure depicts changes in the natural logarithm of aggregate TFP computed from two different samples. The blue line corresponds to a measure computed from public firm data. The red line corresponds to a measure computed from NIPA.

Figure 18: Changes in Aggregate Labor Productivity: NIPA vs. Compustat



Note: Sample period is from 1972 to 2011. This figure depicts changes in the natural logarithm of aggregate labor productivity computed from two different samples. The blue line corresponds to a measure computed from public firm data. The red line corresponds to a measure computed from NIPA.

Figure 19: Changes in Aggregate Capital Productivity: NIPA vs. Compustat



Note: Sample period is from 1972 to 2011. This figure depicts changes in the natural logarithm of aggregate capital productivity computed from two different samples. The blue line corresponds to a measure computed from public firm data. The red line corresponds to a measure computed from NIPA.