Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.

Optimal Unemployment Insurance and International Risk Sharing

Stéphane Moyen, Nikolai Stähler, and Fabian Winkler

2016-054

Please cite this paper as:
Moyen, Stéphane, Nikolai Stähler, and Fabian Winkler (2016). "Optimal Unemployment Insurance and International Risk Sharing," Finance and Economics Discussion Series 2016-054. Washington: Board of Governors of the Federal Reserve System, http://dx.doi.org/10.17016/FEDS.2016.054.

Optimal Unemployment Insurance and International Risk Sharing☆

Stéphane Moyen[a], Nikolai Stähler[a], Fabian Winkler[b]

[a] *Deutsche Bundesbank, Research Department, Wilhelm-Epstein-Str. 14, 60431 Frankfurt, Germany*
[b] *Federal Reserve Board, 20th St and Constitution Ave NW, Washington DC 20551, U.S.A.*

Abstract

We discuss how cross-country unemployment insurance can be used to improve international risk sharing. We use a two-country business cycle model with incomplete financial markets and frictional labor markets where the unemployment insurance scheme operates across both countries. Cross-country insurance through the unemployment insurance system can be achieved without affecting unemployment outcomes. The Ramsey-optimal policy however prescribes a more countercyclical replacement rate when international risk sharing concerns enter the unemployment insurance trade-off. We calibrate our model to Eurozone data and find that optimal stabilizing transfers through the unemployment insurance system are sizable and mainly stabilize consumption in the periphery countries, while optimal replacement rates are countercylical overall. Moreover, we find that debt-financed national policies are a poor substitute for fiscal transfers.

Keywords: Unemployment Insurance, International Business Cycles, Fiscal Union, International Risk Sharing
JEL:, E32, E62, H21, J64

1. Introduction

Europe has seen business cycle movements differ greatly across countries. This development, together with the resulting strains on public budgets, has renewed calls to

☆The views herein are those of the authors and do not represent the views of the Deutsche Bundesbank, the Eurosystem or its staff, the Board of Governors of the Federal Reserve System or the Federal Reserve System. Any errors are ours alone. We would like to thank Klaus Adam, Julien Albertini, Adrien Auclert, Michael Burda, Francesco Caselli, Wouter den Haan, Susanne Ek, Javier Fernandez Blanco, Etienne Gagnon, Sebastian Grisse, Mathias Hoffmann, Josef Hollmayr, Michael Krause, Jochen Mankart, Christian Merkl, Pascal Michaillat, Kurt Mitman, Martin Scheffel, Thepthida Sopraseuth and seminar participants at Bundesbank, Federal Reserve Board, LSE, the EEA annual meeting in Mannheim, the 18th "Theories and Methods in Macroeconomics" conference in Lausanne, IAB-Bundesbank joint conference 2014, Oenb-Bundesbank annual workshop 2015, Banque de France-Bundesbank annual workshop 2014, Humboldt University, University of Cologne, GATE University of Lyon and UAB Barcelona for their helpful comments.

introduce some form of public cross-country risk sharing, sometimes under the name of a "fiscal union". Indeed, a widely held view is that a common currency exacerbates the need for international risk sharing mechanisms, and that fiscal transfers become desirable when the private sector lacks such mechanisms (Mundell (1961), McKinnon (1963) and Kenen (1969)).[1]

At the same time, high unemployment levels in many developed countries have led to renewed interest in the design of unemployment insurance. In the Eurozone in particular, policy makers have argued that the unemployment insurance system is a good and politically viable channel to share risk across countries. The EU Commissioner for Employment, László Andor, states that "*based on the expert work available to date, I consider that the best form of* [...] *a countercyclical fiscal capacity at the EMU level would be a scheme where the participating countries share part of the costs of short-term unemployment insurance*" (Andor, 2014). [2]

Our question is: If a group of countries were to introduce a common unemployment insurance system, what should it look like? We answer it using a two-country business cycle model with search frictions in labor markets. Financial markets are incomplete and labor is immobile across countries, so that country-specific risk and idiosyncratic unemployment risk can only be partially insured privately. The government in each country maintains a mandatory unemployment insurance system. In addition, a supranational unemployment insurance agency is able to administer an additional component of the unemployment insurance system. This component can differ across countries as a function of country-specific shocks.

Starting with a simplified version of our model, we derive two theoretical insights. First, a supranational unemployment insurance system can be used to insure against country-level risk without affecting unemployment levels. The intuition is as follows. Unemployment insurance affects unemployment levels by changing the relative value of employment over unemployment, which determines incentives to search and wage bargaining outcomes. When a country is to receive a fiscal transfer, this relative value can be kept constant by simultaneously increasing the level of benefits and lowering the rate of contributions to the unemployment insurance system. The opposite can be done in the country which is to send the transfer.

Second, the presence of an international risk sharing motive introduces a countercyclical element to the optimal unemployment insurance policy. Here, the intuition is

[1] See Furceri and Zdzienicka (2015) and Kalemli-Ozcan et al. (2014) for recent evidence on the lack of risk sharing mechanisms in the Euro area. Sørensen and Yosha (1998) also provided comparable estimates in the past.

[2] A harmonized unemployment insurance system within the Eurozone as a tool for international risk sharing has also been suggested by the President of the European Council (van Rompuy, 2012), the International Monetary Fund (Blanchard et al., 2014), the German Institute for Economic Research (Bernoth and Engler, 2013), the French Advisory Council (Artus et al., 2013), Dolls et al. (2014), the Banca d'Italia (Brandolini et al., 2015) and Bénassy-Quéré et al. (2016). Brenke (2013) also discusses some of the drawbacks.

as follows. The classic unemployment insurance trade-off for a social planner is between efficiency of employment and insurance. Too much insurance reduces search effort and/or job creation, while too little insurance harms risk-averse workers who cannot insure privately against unemployment risk. When international risk sharing is present, the planner is shielding local consumption from fluctuations in local output. After a negative productivity shock in one country, the planner can then afford to provide more generous insurance and shift employment towards countries where it is more productive. Therefore, insurance becomes more countercyclical.

We then move on to a quantitatively richer model, where we calibrate the two countries to the core and the periphery of the Euro area. We compute the Ramsey-optimal policy and compare it with a policy of constant replacement rates and no international transfers, as such policies are currently in place in most countries. In computing optimal policy, we rule out permanent or perpetual transfers: A country can receive a transfer payment in response to a shock (and does not have to pay it back in the future), but transfers have to average out over time and in expectation, so that no country can expect to be a permanent net contributor or net recipient to the scheme. In our baseline simulation, the optimal unemployment insurance policy is countercyclical even without transfers, and it becomes more generous when a country receives a transfer. The transfers themselves are sizable: The periphery receives 0.70 percent of GDP for every percentage point drop in its own GDP, but has to transfer an almost equal amount to the core for a percentage point reduction in core GDP. However, there is overall very little country-specific business cycle risk in the model, owing to it being calibrated to the high correlation of GDP among Eurozone countries. As a result, transfers are unable to reduce consumption risk everywhere, but rather they distribute it more evenly by shifting some risk from the highly volatile periphery to the more stable core.

A common critique to a supranational unemployment insurance is that its objectives could be achieved just as well by national policies financed with government debt. We show that this argument does not hold: When transfers are replaced by debt as the instrument of the Ramsey planner, the stabilization gains all but disappear. The reason for this is that budget deficits have to be reversed in the future, but international transfers only have to average out over time. This makes them much more effective at stabilizing the economy. Fiscal risk sharing works much like a fairly priced insurance policy: Even though, on average, the premia and expected payments net out to zero, one is still better better off buying the insurance than taking out a bank loan in the event of "damage", because the loan creates a large liability when times are bad.

Our results have one practical limitation, which is that we abstract from the political moral hazard induced by risk sharing (Persson and Tabellini, 1996). It is plausible that a fiscal transfer mechanism reduces incentives for national governments to carry out structural reforms, and this is probably the main political reason for its opposition in the Eurozone. In this paper, we acknowledge this concern insofar as we rule out policies with permanent transfers from one country to another. However, our main focus is on the potential economic benefits rather than the political economy.

The remainder of the paper is organized as follows. We briefly review the related literature in Section 2. In Section 3, we lay out a simplified version of our model with only two periods. This allows us to show our theoretical insights analytically and provide intuition. In Section 4, we lay out the full dynamic model that we use for quantitative analysis and calibrate it to the Euro area. Section 5 contains the numerical results from our calibrated model. Section 6 discusses why government debt is not an effective replacement for the risk sharing achieved by our unemployment insurance scheme. Section 7 concludes.

2. Related literature

Our analysis relates to the literature on international risk sharing and fiscal unions on one hand, and the literature on the design of optimal unemployment insurance on the other hand.

It is well known that the search externality in frictional labor markets can be corrected using unemployment insurance. Because of costly search, employment – and the corresponding fluctuations – may be too low or too high, depending mainly on the relation of the workers' bargaining power to the matching elasticity. In the steady state, this can be resolved by changing the outside option of workers through unemployment benefits (Hosios, 1990). When workers are risk-averse, the correction of the search externality needs to be weighed against the provision of insurance (Baily, 1978). Fredriksson and Holmlund (2006) survey the literature on optimal unemployment insurance in static and steady-state situations. More recently, interest has emerged in unemployment insurance policies that depend on the state of the business cycle. Here, a central point of debate is whether benefits should become more generous in a recession in order to increase insurance (countercyclical policy), or less generous in order to mitigate the fall in employment (procyclical policy). Earlier contributions such as Kiley (2003) and Sanchez (2008) suggest that there is room for countercyclical unemployment benefits. Landais et al. (2015) analyze a model with sticky wages and job rationing and find that a countercyclical policy is optimal as the effect of insurance on equilibrium unemployment is smaller in recessions. Albertini and Poirier (2015) and Kekre (2016) find that more generous unemployment insurance can mitigate the aggregate demand deficiency when the zero lower bound is binding. On the other side, Mitman and Rabinovich (2015) numerically compute optimal dynamic policies and show that the cyclical stance of the unemployment insurance is procyclical in a setting when workers' outside option leads to inefficiently high wages. Moyen and Stähler (2014) analyze the optimal cyclicality of benefits holding their average level fixed. They show that there are situations in which unemployment insurance should be countercyclical even when wages are directly affected and the bargaining power of workers is too high relative to the Hosios condition.[3] Jung and Kuester (2015) analyze first-best policy with sufficiently

[3] Moyen and Stähler (2014) compare the optimal benefit duration policy in Europe and the US. In their European calibration, the bargaining power of workers is larger than the matching efficiency, implying

many fiscal instruments. They find that benefits should rise in recessions if hiring subsidies and layoff taxes also rise at the same time. These two instruments increase the incentives to hire and decrease those to fire workers, which may compensate partly for increased wage costs. However, if hiring subsidies and layoff taxes are not adjusted, they also find procyclical benefits to be optimal.

Importantly, the literature exclusively analyzes closed economies. Our paper instead analyzes optimal policy when unemployment insurance can operate across multiple countries and faces the additional objective of sharing cross-country risk.

Turning to the literature on fiscal unions, Leduc et al. (2009) have shown that, when asset markets are incomplete, country-specific productivity disturbances can have large uninsurable effects on wealth and consumption paths. In a prominent recent paper, Farhi and Werning (2012) find that such uninsurable effects may be especially large in a currency union with nominal rigidities. They suggest forming a transfer union to insure against this risk. Many economists follow their view that, in federal unions, a (fiscal) transfer mechanism to at least compensate for the uninsurable effects due to nominal rigidities may be desirable. However, there is still some debate on how to ideally establish such a transfer mechanism or a fiscal union (see Bargain et al., 2013 and Bordo et al., 2011 for a discussion). Evers (2012) provides a quantitative assessment of federal transfer rules and finds that targeting regional differences in labor income generates highest welfare gains, which primarily stem from reducing the allocative inefficiencies of factor inputs caused by nominal rigidities. Dmitriev and Hoddenbagh (2013) find that a tax union, in which the steady state income tax are harmonized, has to be preferred to cross countries fiscal transfers only if the Armington elasticity is low. Evers (2015) shows that a fiscal revenue equalization system, that shares nominal tax revenues, destabilizes business cycles and worsens welfare while a fully centralized fiscal authority does the opposite. While these papers have mostly focused on symmetric countries, we show, in a model calibrated to Eurozone data, that a transfer mechanism is desirable even without nominal rigidities.

3. Simplified model

The intuition for our results can best be seen in a two-period model that allows us to analytically prove our results and provide a graphical representation. The model is highly simplified: In particular, the real exchange rate is constant and the two countries are in financial autarky. We will relax these assumptions in the quantitative part in the next section.

the optimal benefit to be negative in light of the Hosios condition. However, it is restricted to be positive. Additionally, rule-of-thumb households make average marginal utility of consumption fluctuate relatively much. It can be shown that steady-state benefits above optimum and relatively volatile marginal utility of consumption make optimal benefit policy countercyclical even when the bargaining power of workers is already high.

3.1. Model setup

There are two countries, which we call Home and Foreign. Home is inhabited by a mass $\omega \in (0,1)$ of workers, while Foreign is inhabited by a mass $1-\omega$ of workers. In each country, firms transform labor into a homogenous consumption good (in the quantitative model of Section 4 we will introduce imperfect substitutability between Home and Foreign goods). Firms are owned by workers, but make zero profits in equilibrium. While consumption goods can be traded across countries in competitive markets, labor is immobile across countries and labor markets are subject to search frictions within each country.

In the first period, all workers start out as unemployed. In the second period, firms post vacancies and are matched with workers. Production and consumption take place only in the second period. Expected utility of a worker at Home in the first period is:

$$U = \mathbb{E}\left[nu\left(c_e\right) + (1-n)\,u\left(c_u\right)\right] \tag{1}$$

where c_e is his consumption level if he turns out to be employed, and c_u his consumption level if he turns out to be unemployed. n is the employment level per capita in the second period.[4] We assume logarithmic utility: $u(c) = \log(c)$.

In the second period, the number of vacancies is v and the number of matches is given by

$$n = \kappa_m \theta^{1-\mu}. \tag{2}$$

where θ is the tightness in the labor market. Since the initial stock of employment is zero, employment in the second period equals the number of matches and market tightness equals the number of vacancies.

A firm that posts a vacancy incurs a cost κ_v. The probability that the vacancy is filled is $q = \kappa_m \theta^{-\mu}$. In that case, the match produces a units of output and the worker gets paid a wage w. This wage is determined using Nash bargaining, where the bargaining power of workers is denoted ξ (the bargaining solution is described further below). A zero-profit condition for vacancy creation prescribes

$$\kappa_v = q\,(a-w) \tag{3}$$

We denote by y aggregate output in the Home country net of vacancy costs:

$$y = an - \kappa_v v \tag{4}$$

The productivity a is a random variable which is only revealed in the second period. The Foreign country has a similar structure to the Home country, but with possibly different parameters. We denote Foreign variables with an asterisk, e.g. y^* for foreign output. Home and foreign productivity (a, a^*) are the only sources of aggregate uncertainty.

---

[4] Throughout the paper, quantities will be expressed in per capita terms unless otherwise indicated.

Employed workers receive wages w which are taxed at the rate τ, while unemployed workers receive unemployment benefits b. Payroll taxes τ and benefits b are administered by an unemployment insurance agency. We assume that the two countries are part of an insurance union, such that the agency operates across both countries. It has to run a balanced budget with the constraint:

$$\omega\left[(1-n)\,b-n\tau w\right]+(1-\omega)\left[(1-n^*)\,b^*-n^*\tau^*w^*\right]=0. \tag{5}$$

3.2. Social planner problem

We first look at the social planner problem. A utilitarian social planner maximizes a weighted average of worker utilities subject only to the resource constraint and the search friction by solving the following problem:

$$\max_{\begin{pmatrix} n,\theta,c^e,c^u,\\ n^*,\theta^*,c^{e*},c^{u*}\end{pmatrix}} \begin{array}{c}\tilde{\omega}\mathbb{E}\left[nu\left(c_e\right)+(1-n)\,u\left(c_u\right)\right]\\ +\left(1-\tilde{\omega}\right)\mathbb{E}\left[n^*u\left(c_u^*\right)+(1-n^*)\,u\left(c_u^*\right)\right]\end{array}$$

$$\begin{array}{rcl} \text{s.t. } n &=& \kappa_m\theta^{1-\mu}\\ n^* &=& \kappa_m^*\left(\theta^*\right)^{1-\mu^*}\\ \begin{array}{c}\omega\left(nc_e+(1-n)\,c_u\right)\\ +(1-\omega)\left(n^*c_e^*+(1-n^*)\,c_u^*\right)\end{array} &=& \begin{array}{c}\omega\left(an-\kappa_v\theta\right)\\ +(1-\omega)\left(a^*n^*-\kappa_v^*\theta^*\right)\end{array}\end{array}$$

Here, $\tilde{\omega}$ is the relative weight the planner puts on workers in the Home country, which might be more or less than the size of its population ω. Within a country, all workers are ex-ante homogenous and so weighting of individual workers is inconsequential. The first order conditions of the planner problem are standard:

$$\begin{array}{rcl} \kappa_v &=& \kappa_m\theta^{-\mu}\,(1-\mu)\,a\\ \kappa_v^* &=& \kappa_m^*\theta^{*-\mu^*}\,(1-\mu^*)\,a^*\\ c_u &=& c_e\\ c_u^* &=& c_e^*\\ \dfrac{\omega}{\tilde{\omega}}c_e &=& \dfrac{1-\omega}{1-\tilde{\omega}}c_e^* \end{array}$$

The first two conditions are the Hosios conditions in each country, which determine the number of vacancies that maximize aggregate output net of vacancy costs. The remaining conditions prescribe full risk sharing within and across countries. The consumption levels of employed and unemployed workers within each country should be identical, and each country should consume a constant fraction of union output.

3.3. Optimal policy with private insurance

We now come back to the competitive equilibrium. Throughout this chapter, we assume some form of international market incompleteness, since our focus is on how unemployment insurance can be used to overcome a lack of risk sharing. In this section, we allow workers to only insure domestically against idiosyncratic unemployment risk. Since all workers are ex-ante identical, it is optimal for a Home worker to fully diversify his risk by selling his entire future income in exchange for a diversified portfolio of the income of all other Home workers' income. In this case, the consumption levels of all Home workers are equalized:

$$c_e = c_u = c. \tag{6}$$

The wage w is assumed to be set by Nash bargaining. When workers have bargaining power ξ, the Nash-bargained wage is simply:

$$w(a,\rho) = \frac{\xi a}{\xi + (1-\xi)(1-\rho)} \tag{7}$$

where ρ is the net replacement rate, defined as

$$\rho = \frac{b}{(1-\tau) w}. \tag{8}$$

A higher replacement rate improves workers' outside option and drives up wages. It thereby lowers the incentives for job creation and reduces employment.

We want to know what the optimal unemployment insurance scheme looks like in this situation. Our first result is that *a transfer of resources from one country to another can be implemented through the unemployment insurance system without affecting unemployment levels*.

We first note that the budget constraint of the unemployment insurance agency can be rewritten as

$$0 = \omega(c-y) + (1-\omega)(c^* - y^*). \tag{9}$$

We can therefore choose replacement rates ρ, ρ^* and a transfer from the Foreign to the Home country $T = c - y$ as a policy and back out the necessary benefits b, b^*, τ, τ^* from the budget constraint. We obtain:

$$b = \rho \frac{nw + T}{n + (1-n)\rho} \tag{10}$$

$$\tau = 1 - \frac{1}{w}\frac{nw + T}{n + (1-n)\rho}. \tag{11}$$

The unemployment level n and the wage w depend only on policy through the replacement rate ρ. Therefore, from the formula above we can see that a positive transfer from Foreign to Home ($T > 0$) can be implemented by increasing unemployment benefits b and at the same time lowering payroll taxes τ. This way, all workers get to consume more, but the net replacement rate ρ stays constant and the relative bargaining position of workers is unchanged. Since we generally have $n > 1 - n$, most of this transfer is achieved by reducing the contributions by the employed. Increased benefits of the employed are only a minor part of such a neutral transfer.

While it is possible to make transfers without affecting replacement rates, is this also optimal? With privately insured unemployment risk, the insurance agency has to mitigate three inefficiencies: search externalities in the Home and Foreign countries and lack of international risk sharing. It also has three policy instruments: the Home and Foreign replacement rates and a cross-country transfer. Indeed, there exists a policy that eliminates all three inefficiencies. The replacement rates satisfying the Hosios condition are

$$\rho = \frac{\mu - \xi}{\mu\left(1 - \xi\right)}, \rho^* = \frac{\mu^* - \xi^*}{\mu^*\left(1 - \xi^*\right)}. \tag{12}$$

These rates do not depend on the realizations of a and a^* and are therefore constant. They also do not depend on the planner weight $\tilde{\omega}$. The transfer T however does depend on this weight. In principle, one can choose any value for $\tilde{\omega}$. We determine it by imposing that transfers are zero in expectation:

$$\mathbb{E}\left[T\right] = 0. \tag{13}$$

This condition implicitly defines a value for $\tilde{\omega}$. This choice of the weight implies that neither country expects to be systematically subsidizing the other country through the unemployment insurance system. The value of the weight is

$$\tilde{\omega} = \frac{\mathbb{E}\left[\omega y\right]}{\mathbb{E}\left[\omega y + \left(1 - \omega\right) y^*\right]} \tag{14}$$

and the resulting transfer policy is

$$T = \frac{\mathbb{E}\left[y\right]\mathbb{E}\left[y^*\right]}{\mathbb{E}\left[\frac{\omega}{1-\omega} y + y^*\right]}\left(\frac{y}{\mathbb{E}\left[y^*\right]} - \frac{y}{\mathbb{E}\left[y\right]}\right). \tag{15}$$

The Home country receives a transfer when its output is below average, but has to pay a transfer when output in the Foreign country is below average.

3.4. Optimal policy without private insurance

So far we have abstracted from the most important objective of unemployment insurance, namely to insure against unemployment. We now remove the possibility to privately insure unemployment risk. In this case, the optimal policy becomes genuinely second-best and trade-offs emerge between all three policy objectives: maximizing net output, providing insurance between employed and unemployed, and providing insurance across countries.

We eliminate all asset trade in Period 1 (in the quantitative model of Section 4 we will allow for trade of a non-contingent bond across the border). The consumption levels in Period 2 are simply:

$$c_e = (1-\tau)\, w \tag{16}$$

$$c_u = b = \rho c_u \tag{17}$$

The Nash-bargained wage now takes into account the curvature in the worker's utility function. When workers have bargaining power ξ, the bargained wage is:

$$w\,(a,\rho) = \frac{\xi a}{\xi - (1-\xi)\log\rho}. \tag{18}$$

In this situation, the social planner allocation is no longer feasible. Providing full insurance against idiosyncratic unemployment risk calls for $\rho = 1$, but then the worker captures the whole surplus ($w = a$) and job creation collapses to zero. We therefore have to solve for the (second-best) Ramsey-optimal policy as follows:

$$\max_{\left(\begin{array}{c} n,\theta,c^e,\rho, \\ n^*,\theta^*,c^{e*},\rho^* \end{array}\right)} \begin{array}{rl} \tilde{\omega} & \mathbb{E}\left[nu\,(c_e) + (1-n)\,u\,(\rho c_e)\right] \\ +\,(1-\tilde{\omega}) & \mathbb{E}\left[n^* u\,(c_e^*) + (1-n^*)\,u\,(\rho^* c_e^*)\right] \end{array}$$

$$\begin{array}{rcl} \text{s.t. } n & = & \kappa_m \theta^{1-\mu} \\ n^* & = & \kappa_m^* \,(\theta^*)^{1-\mu^*} \\ \kappa_v & = & \kappa_m \theta^{-\mu}\,(a - w\,(a,\rho)) \\ \kappa_v^* & = & \kappa_m^*\,(\theta^*)^{-\mu^*}\,(a^* - w^*\,(a^*,\rho^*)) \\ \begin{array}{c} \omega\,(nc_e + (1-n)\,\rho c_e) \\ +\,(1-\omega)\,(n^* c_e^* + (1-n^*)\,\rho^* c_e^*) \end{array} & = & \begin{array}{c} \omega\,(an - \kappa_v\theta) \\ +\,(1-\omega)\,(a^* n^* - \kappa_v^* \theta^*) \end{array} \end{array}$$

Here, we have substituted out most equilibrium conditions. In particular, choosing an unemployment insurance policy (b, b^*, τ, τ^*) subject to the insurance agency's budget constraint is equivalent to choosing replacement rates and consumption levels $(\rho, \rho^*, c_e, c_e^*)$ subject to the aggregate resource constraint. The benefit and transfer levels can be inferred from (10)-(11) as before. Again, we choose the social planner weight $\tilde{\omega}$ such that any transfers made across countries net out in expectation: $\mathbb{E}\,[T] = 0$.

The problem has eight choice variables and five constraints, leaving three degrees of freedom. These correspond to the three policy instruments ρ, ρ^* and the cross-country transfer T. The first order condition determining the optimal transfer is derived as:

$$T = \frac{\mathbb{E}\,[y]\,\mathbb{E}\,[y^*]}{\mathbb{E}\left[\frac{\omega}{1-\omega} y + y^*\right]} \left(\frac{y^*}{\mathbb{E}\,[y^*]} - \frac{y}{\mathbb{E}\,[y]}\right). \tag{19}$$

This is the exact same expression as in (15). Each country consumes a constant share of union output. The Home country receives a transfer when its output is below average, but has to pay a transfer when output in the Foreign country is below average. The additional question here is how transfers (and all other resources) are distributed among the employed and the unemployed.

The answer to this question is contained in the first order condition with respect to the replacement rate. For the Home country, it reads as follows:

$$\underbrace{\frac{(1-n)(1-\rho)}{n+(1-n)\rho} - \epsilon_\rho^n \left(\log\rho + \frac{1-\rho}{n+(1-n)\rho}\right)}_{=:I(\rho)} = \underbrace{-\epsilon_\rho^y \frac{1}{n}\frac{y}{y+T}}_{=:H(\rho)} \tag{20}$$

where $\epsilon_\rho^n = \frac{dn}{d\rho}\frac{\rho}{n}$ is the elasticity of Home employment with respect to the Home replacement rate, and $\epsilon_\rho^y = \frac{dy}{d\rho}\frac{\rho}{y}$ is the elasticity of net Home output with respect to the Home replacement rate. A symmetric condition is obtained for the Foreign country. The left-hand side $I(\rho)$ is the marginal benefit of insurance when raising the replacement rate, at a fixed quantity of output available to the country. By raising ρ, the unemployed's marginal utility increases relative to average marginal utility. This is the first term on the left-hand side of Equation (20). At the same time, a higher ρ reduces employment (through higher wages and lower job creation), which shifts the composition of the workforce towards the unemployed. This means that one marginal worker suffers a utility loss, which is the "$\log\rho$" term in the left-hand side of Equation (20). It also implies a composition effect on the insurance budget, captured by the remaining term on the left-hand side. The right-hand side $H(\rho)$ is the marginal cost of raising the replacement rate in terms of net output lost (output minus vacancy costs. This cost is where the level of transfers T enter. A high transfer implies that the planner is relying less on domestic production to support domestic consumption, lowering the marginal cost of raising domestic output.

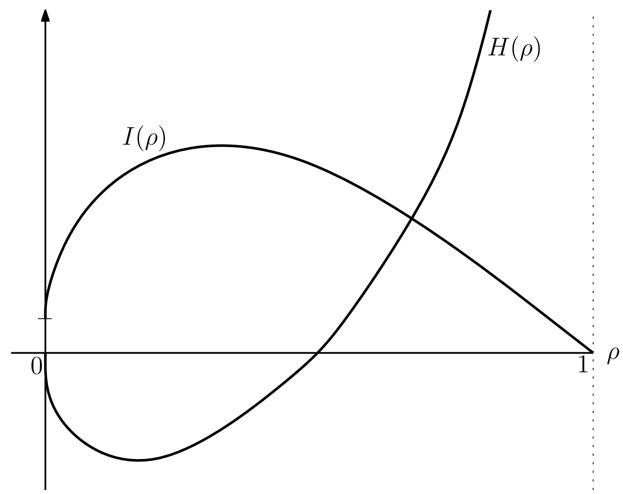
The determination of the optimal replacement rate is graphically depicted in Figure 1, which plots the functions $H(\rho)$ and $I(\rho)$.[5]

We can see that the insurance term $I(\rho)$ is positive and only equals zero at $\rho = 1$. Intuitively, holding output constant, it is always desirable to increase the replacement rate until full insurance is achieved.

The efficiency term $H(\rho)$ is first negative and then turns positive, approaching infinity as $\rho \to 1$. Intuitively, when ρ is too high, there is too little job creation and the amount of resources available for consumption can be increased by lowering replacement rates, thereby lowering bargained wages and increasing job creation. In this case $\epsilon_\rho^y < 0$, and therefore $H(\rho)$ is positive. Conversely, when ρ is too low, there is too much vacancy posting and the amount of resources available for consumption can be increased by raising replacement rates. In this case, $H(\rho)$ is negative.

[5]Proposition 1 in the appendix proves that the shape of the I and H curves are indeed as depicted.

Figure 1: Optimal replacement rate.



The optimal replacement rate lies at the intersection between the two curves. Employment is always lower than in the social planner solution where $H(\rho) = 0$. The marginal benefit of insurance is always positive and so the optimal ρ is higher than what the Hosios condition would call for.

What happens to the replacement rate when shocks to a or a^* hit the economy? We first keep the ratio $y/(y+T)$ constant. This is the case in particular when $T = 0$, as in the limit $\omega \to 1$ of a closed-economy. The left panel of Figure 2 depicts the effects of a recession caused by a reduction in productivity a.

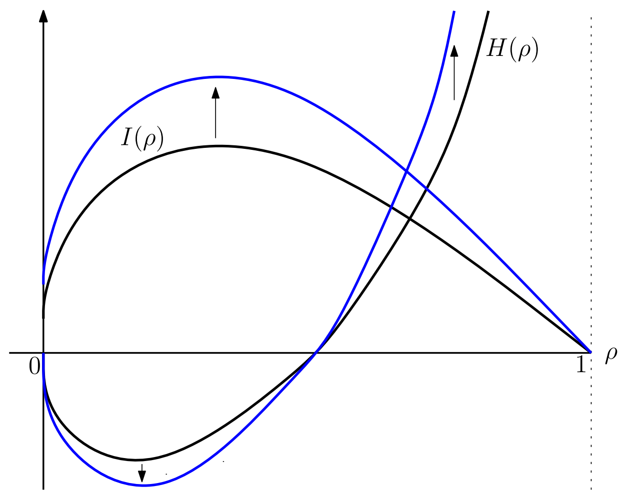
A reduction in a increases the insurance term $I(\rho)$ and scales up the efficiency term $H(\rho)$. The intuition is as follows. A reduction in productivity reduces employment. Holding total resources constant, this translates into an increase in unemployment risk for each worker, raising the social benefit of insurance. $I(\rho)$ shifts up for any value of ρ. At the same time, lower productivity directly reduces net output for any level of employment. Therefore, the average marginal utility of increasing output towards its efficient level increases and $H(\rho)$ is scaled up for any value of ρ. The figure shows that the effects of these two forces on the replacement rate work in opposite directions, so that the change in ρ is ambiguous. This ambiguity reflects the debate in the existing literature about the cyclicality of optimal unemployment insurance in closed economies—it is not clear whether, in a recession, it is more important to keep output from falling or to protect workers from unemployment risk.

In our supranational insurance scheme, however, a drop in productivity a also triggers an incoming transfer T. The risk sharing condition (19) prescribes that T is a decreasing function of y, in order to keep average consumption of the Home country proportional to union output. The increase in T then affects the optimal replacement rate, and it is this risk sharing aspect that is novel to the literature on optimal unemployment insurance.

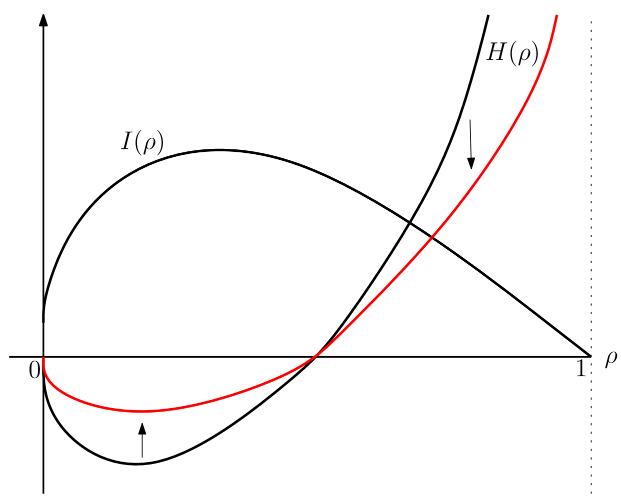
Specifically, *the presence of international risk sharing makes the replacement rate more*

Figure 2: Change of the optimal replacement rate in a recession.

(a) Reduction in a.



(b) Increase in T.



countercyclical. This can be seen from the optimality condition (20). When a falls, Home's output will relatively low compared to union output, and T will increase. The right panel of Figure 2 shows that the optimal replacement rate rises as a response, introducing a countercyclical element to the optimal policy. When transfers increase, the planner is relying less on domestic production to support domestic consumption, lowering the marginal cost of raising domestic output. The efficiency term $H(\rho)$ shrinks and the trade-off between efficiency and insurance shifts towards the latter. The replacement rate ρ rises as a result.[6]

Next, *the Home replacement rate is increasing in Foreign productivity* a^*. Foreign productivity only affects the Home replacement rate because of its effect on Foreign output y and therefore transfers to the Home country. When the Foreign country experiences a drop in productivity, it receives a transfer and T falls. The planner is now relying more on Home output relative to Foreign output, and the trade-off between efficiency and insurance shifts towards the former. The replacement rate ρ falls in order to increase production at Home.

Finally, we can show that *when a and a^* are independent, the optimal Home replacement rate is countercyclical in the limit as* $\omega \rightarrow 0$. When the Home country is small and its shocks are uncorrelated with the rest of the union, its risk can be completely diversified at the supranational level. The appendix shows that in this case, shifts of the H function due to transfers (right panel of Figure 2) dominate those due to movements in productivity (left panel). The Home replacement rate unambiguously rises when productivity falls. It is worth mentioning that even in the case $\omega \rightarrow 0$, the optimal replacement rate is not one. This might seem counterintuitive as it would be costless for the union to perfectly insure all workers in an infinitesimally small country. But this would imply that the country receives positive transfers in all states of the world, which we have ruled out with condition (13). The planner weight $\tilde{\omega}$ shrinks together with ω to ensure that this condition holds. Every country, regardless how small, has to be self-financed in expectation, even as its idiosyncratic business cycle risk can be better insured.

4. Model for quantitative analysis

The simple model of the previous section illustrates the relevant trade-offs involved in supranational unemployment insurance. In this section, extend the model along a number of dimensions and calibrate it to the core and periphery of the Eurozone. We numerically solve for the Ramsey-optimal policies with and without the possibility of transfers. Our simulation results of the optimal policies confirm the insights from the simple model.

We extend the simple model in the following dimensions. First, the model is dynamic with an infinite horizon, where workers transition back and forth between employment and unemployment. Second, we allow for variable search effort, so that unemployment benefits affect the labor market not only through wage bargaining but also

[6] The appendix provides a formal proof.

through their effect on search effort. Third, we add imperfect substitutability between Home and Foreign goods, so that movements in the terms of trade can partly insure against country-specific shocks to production. Fourth, we add price and wage rigidities to help the quantitative fit of the model, in particular the volatility of unemployment. Finally, we allow for a limited degree of trade of intertemporal non-contingent bonds, so that even in the absence of supranational risk sharing mechanisms agents can somewhat smooth business cycle shocks intertemporally.

4.1. Model setup

Time is discrete at $t = 0, 1, 2, ...$ As before, a unit mass of workers and firms populates the economy, where $\omega \in (0, 1)$ workers live in the Home country and $(1 - \omega)$ workers live in the Foreign country. We will only describe the model setup in the Home country. The structure of the Foreign economy is identical up to potentially different parameter values. If we need to show variables and parameters of Foreign, they will be indicated by an asterisk.

4.1.1. Matching

At the beginning of period t, a fraction u_t of workers at Home are unemployed. We assume that labor is immobile across countries, so that workers can only search for jobs domestically. The number of total new hires is determined by the number of searching workers u_t, the search effort e_t, and the number of vacancies v_t. Workers and vacancies are matched according to a standard Cobb-Douglas matching function

$$m_t = \kappa_m \left(e_t u_t\right)^{\mu} v_t^{1-\mu} \tag{21}$$

where κ_m is a matching efficiency parameter and μ is the elasticity of matches with respect to unemployment. Defining labor market tightness as $\theta_t = v_t / e_t u_t$, the probability that a vacancy gets filled, and the probability that a worker putting in one unit of search effort finds a job, are given by $q_t = \kappa_m \theta_t^{-\mu}$ and $f_t = q_t \theta_t$. Separation occurs at the exogenous rate s. Unemployed workers who separate have to wait one period before they can start searching again. Accordingly, the laws of motion for employment and unemployment are given by:

$$n_t = (1 - s)n_{t-1} + q_t v_t \tag{22}$$

$$u_t = 1 - n_{t-1}. \tag{23}$$

4.1.2. Workers

A worker can be employed or unemployed. He maximizes expected lifetime utility, defined recursively as follows:

$$\mathcal{W}_t = u\left(c_{et}\right) + \beta \mathbb{E}_t \left\{(1 - s)\mathcal{W}_{t+1} + s\mathcal{U}_{t+1}\right\} \tag{24}$$

$$\mathcal{U}_t = u\left(c_{ut}\right) + \beta \mathbb{E}_t \left\{(1 - f_{t+1}e_{t+1})\mathcal{U}_{t+1} + f_{t+1}e_{t+1}\mathcal{W}_{t+1} - k\left(e_{t+1}\right)\right\}. \tag{25}$$

where $\mathcal{W}_t$ is the utility of an employed worker with consumption c_{et} and $\mathcal{U}_t$ is the utility of an unemployed worker with consumption c_{ut}. Unemployed workers have to exert search effort e_{t+1} at the beginning of the next period to find a job. We assume the following functional forms:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \gamma \geq 0$$
$$k(e) = \kappa_e \frac{e^{1+\phi}}{1+\phi}, \phi > 0$$

The consumption flow $c_{jt}, j \in \{e, u\}$ denotes expenditure on a consumption basket. This basket consists of goods produced in the Home and Foreign countries and is given by:

$$c_{jt} = \left(\psi \left(c_{jt,H}\right)^{\sigma} + (1-\psi)\left(c_{jt,F}\right)^{\sigma}\right)^{1/\sigma} \tag{26}$$

Here, $c_{jt,H}$ is the amount of goods consumed and produced at Home, while $c_{jt,F}$ is the amount of goods consumed at Home but produced in Foreign. The parameter $\sigma \in (-\infty, 1)$ governs the elasticity of substitution between foreign and domestic goods, which equals $1/(1-\sigma)$, and the parameter ψ represents the relative valuation of Home goods.[7]

We abstract from international trade costs so the law of one price holds for both goods. We take the Home good as the numéraire and let p_t be the relative price of Foreign goods. Thus, p_t also represents the terms of trade of the Foreign country. Next, we define the consumer price index (CPI) at Home by $P_t = \left(c_{jt,H} + p_t c_{jt,F}\right)/c_{jt}$. Utility maximization implies that[8]

$$\frac{c_{jt,H}}{c_{jt,F}} = \left(p_t \frac{\psi}{1-\psi}\right)^{\frac{1}{1-\sigma}} \tag{27}$$
$$P_t = \left(\psi^{\frac{1}{1-\sigma}} + (1-\psi)^{\frac{1}{1-\sigma}} p_t^{-\frac{\sigma}{1-\sigma}}\right)^{-\frac{1-\sigma}{\sigma}} \tag{28}$$

Next, we specify workers' budget constraints and the financial assets to which they have access. We want to capture an incomplete market setting in which workers can neither obtain perfect insurance of their idiosyncratic unemployment risk nor of country-specific risk. We opt for a setting in which workers do not have access to individual

[7]In the case of unitary elasticity of substitution ($\sigma = 0$), the consumption basket is of the Cobb-Douglas form $c_{jt} = \left(c_{jt,H}\right)^{\psi}\left(c_{jt,F}\right)^{1-\psi}$, so that the expenditure share on Home goods is exactly ψ. A situation where $\phi > \omega$ then corresponds to home bias in consumption.

[8]Having normalized the price of the Home good to one, note that P_t is Home CPI relative to Home PPI. It correspondingly holds that P_t^* is Foreign CPI expressed relative to goods produced in Home. We will, hence, express all Foreign variables relative to the price of goods produced in Home.

savings at all, so that they cannot insure their idiosyncratic unemployment risk. However, both unemployed and employed workers will own a fixed amount of shares in firms, and firms are able to access a non-contingent international bond. As will become clear below, this allows us to introduce limited intertemporal trade while escaping the need to keep track of the distribution of assets when solving the model.

Employed workers receive a wage payment w_t (in units of Home goods), of which an amount τ_t of payroll taxes is deducted. The unemployed receive unemployment insurance benefits b_t. All workers receive an equal share of profits Π_t from the firms in their country of residence. Since firms discount profits at a household rate, holding shares in firms effectively gives agents a form of savings through firms' intertemporal decisions, but they cannot save individually and must consume their per period-income, which in real terms reads as follows:

$$c_{et} = \left(w_t - \tau_t + \Pi_t\right) / P_t \tag{29}$$

$$c_{ut} = \left(b_t + \Pi_t\right) / P_t. \tag{30}$$

The only choice variable for workers is the search effort e_t when unemployed. Maximizing the utility of the unemployed with respect to effort leads to the following optimality condition:

$$k'\left(e_t\right) = f_t\left(\mathcal{W}_t - \mathcal{U}_t\right). \tag{31}$$

4.1.3. Firms

We assume that production is divided into a final and an intermediate goods sector, with the latter being subject to search frictions. Each country produces a distinct final good from domestic intermediates. Intermediate producers operate under monopolistic competition and are able to set prices.

More precisely, at Home there is a representative final good producer which purchases a variety of differentiated intermediate goods, bundles these into a final good and sells the latter as a price taker. The price of the Home country's final good is the same in both countries and equal to one (the price for the good produced in Foreign is equal to p_t). The maximization problem of the representative retail firm reads

$$\max_{\{\tilde{y}_t(j):j\in[0,\omega]\}} Y_t - \frac{1}{\omega}\int_0^{\omega} \tilde{p}_t(j) y_t(j) dj, \tag{32}$$

where $\tilde{p}_t(j)$ is the price of a specific variety j, and

$$Y_t = \left(\frac{1}{\omega}\int_0^{\omega} y_t(j)^{(\epsilon-1)/\epsilon} dj\right)^{\epsilon/(\epsilon-1)}, \;\; \epsilon > 1, \tag{33}$$

is the final goods producer's production function. $y_t(j)$ is the final goods producer's demand for the differentiated input $j \in [0, \omega]$. The first-order condition for each input reads:

$$y_t(j) = (\tilde{p}_t(j))^{-\epsilon} Y_t. \quad (34)$$

Combining the latter with (32) and the zero profit condition, we obtain the producer price index in the home country and normalize it to one:

$$1 = \frac{1}{\omega} \int_0^{\omega} \tilde{p}_t(j)^{1-\epsilon} dj. \quad (35)$$

Now, each intermediate producer operates the following technology:

$$y_t(j) = a_t n_t(j) \quad (36)$$

which is linear in labor, and a_t is a productivity shock common to all firms. Employment is subject to search frictions. Firm j needs to post a number of vacancies $v_t(j)$, each of which leads to successful matching with a worker with probability q_t. The vacancy filling rate is taken as given by the firm. Successful matches start production in the next period.

Furthermore, firm j needs to pay its workers a wage $w_t(j)$, and it needs to pay a cost for each vacancy, which takes the form of a constant quantity κ_v of domestically produced goods. Following Arseneau and Chugh (2007), the firm faces a quadratic cost of adjusting real wages.[9]For each of its workers, the real cost of changing wages between period $t-1$ and t is

$$\frac{\kappa_w}{2}\left(\frac{w_t(j)}{w_{t-1}(j)} - 1\right)^2 = \frac{\kappa_w}{2}\left(\pi_{wt}(j) - 1\right)^2,$$

where $\pi_{wt}(j) = w_t(j)/w_{t-1}(j)$ is the gross real wage growth rate. The wage itself is determined through Nash bargaining, as described below. The firm can further save in non-contingent international bonds $d_t(j)$ which pay a gross interest R_t, subject to portfolio adjustment cost $\kappa_d/2d_t^2(j)$, as is standard in the international economics literature (e.g. Benigno, 2009). Given that firms operate in monopolistic competition and the fact that firms also set the price $\tilde{p}_t(j)$ for their goods variety j, we assume that they also face Rotemberg price adjustment costs that are similar to wage adjustment costs described above, with κ_p being the cost parameter. Finally, the firm has to pay a fixed cost F every period. In sum, firm j's profit is given by

$$\Pi_t(j) = \tilde{p}_t(j)y_t(j) - w_t(j)n_t(j) - \kappa_v v_t(j) - d_t(j) - \frac{\kappa_d}{2}d_t^2(j) + R_{t-1}\frac{d_{t-1}(j)}{\pi_{pt}}$$

[9]As pointed out by these authors, we adopted this assumption as cost of nominal wage adjustment do not help increasing significantly the volatility of unemployment. A requirement that will become clear in the calibration section.

$$-\frac{\kappa_w}{2}\left(\pi_{wt}(j)-1\right)^2 n_t(j)-\frac{\kappa_p}{2}\left(\frac{\tilde{p}_t(j)}{\tilde{p}_{t-1}(j)}\pi_{pt}-1\right)^2 y_t(j)-F, \tag{37}$$

where π_t^p is aggregate PPI inflation at Home.

The firm maximizes the discounted sum of profits

$$\mathbb{E}\sum_{t=0}^{\infty} Q_{0,t}\Pi_t(j)$$

where $Q_{s,t}$ is the discount factor between times s and t. Since the firm is owned in part by employed and unemployed workers, it is not obvious what discount factor the firm should use. As in Jung and Kuester (2015) profits are shared equally across households, implying that the firm discount factor is a weighted average of the worker discount factors :

$$Q_{s,t}=\beta^{t-s}\frac{n_t u'\left(c_{et}\right)+\left(1-n_t\right)u'\left(c_{ut}\right)}{n_s u'\left(c_{es}\right)+\left(1-n_s\right)u'\left(c_{us}\right)}\frac{P_s}{P_t}. \tag{38}$$

Maximizing profits with respect to employment while taking into account the employment law-of-motion as well as the demand for each intermediate goods' variety leads to an expression for the value of a filled job $\mathcal{J}_t$:

$$\mathcal{J}_t=mc_t a_t-w_t-\frac{\kappa_w}{2}\left(\frac{w_t}{w_{t-1}}-1\right)^2+(1-s)\,\mathbb{E}_t Q_{t,t+1}\left[\mathcal{J}_{t+1}\right], \tag{39}$$

where mc_t are marginal costs (formally, the Lagrangian parameter on equation (34)). The optimality condition of the firm with respect to vacancy creation takes the familiar form:

$$\kappa_v=q_t\mathcal{J}_t. \tag{40}$$

Finally, the optimality condition with respect to $\tilde{p}_t(j)$ can be written as follows:

$$1-\kappa_p(\pi_{pt}-1)\pi_{pt}+\mathbb{E}_t Q_{t,t+1}\kappa_p(\pi_{pt+1}-1)\pi_{pt+1}\frac{y_{t+1}}{y_t}=(1-mc_t)\epsilon. \tag{41}$$

Note that in the last three equations we made use of the fact that, in equilibrium, all firms will chose the same price and allocation; thus, we dropped the index j due to symmetry and imposed $\tilde{p}_t\left(j\right)=1$ from Equation (35).

4.1.4. Wage determination

The wage paid to workers is determined by Nash bargaining in which workers and firms share the surplus from matching according to

$$\max_{w_t}\left(\mathcal{W}_t-\mathcal{U}_t\right)^{\xi}\mathcal{J}_t^{1-\xi}$$

where ξ is the bargaining power of workers. The wage is determined implicitly by the first-order condition to the above problem:

$$\left(1 + \kappa_w(\pi_t^w - 1)\frac{\pi_{wt}}{w_t} + (1-s)\kappa_w \mathbb{E}_t Q_{t,t+1}(\pi_{wt+1} - 1)\frac{\pi_{wt+1}}{w_t}\right)[\mathcal{W}_t - \mathcal{U}_t] = \frac{\xi}{1-\xi}\frac{u'(c_{et})}{P_t}\mathcal{J}_t. \tag{42}$$

4.1.5. Government

Unlike in the simple model of the previous section, we explicitly spell out national governments as well as a supranational unemployment insurance agency, each of which independently manages its finances.

The government in the Home country gains revenue exclusively from payroll taxes τ_{gt}. These taxes are used to fund benefits for unemployed workers b_{gt} as well as government expenditure g_t. Government expenditure is spent entirely on domestically produced goods.[10] The government has to balance its budget every period. Its budget constraint writes

$$g_t + u_t b_{gt} = \tau_{gt} n_t. \tag{43}$$

The supranational unemployment insurance agency can likewise administer a component of unemployment insurance. This agency also has to balance its budget every period. It collects payroll taxes τ_{xt} and disburses unemployment benefits b_{xt} in the Home country, and collect payroll taxes τ_{xt}^* and disburses unemployment benefits b_{xt}^* in the Foreign country. The agency's budget constraint writes

$$\omega(1-n_t)\, b_{xt} + (1-\omega)(1-n_t^*)\, p_t b_t^* = \omega n_t \tau_{xt} + (1-\omega)\, n_t^* p_t \tau_{xt}^*. \tag{44}$$

Total taxes on employed workers, total benefits received by unemployed workers, and the net replacement rate are then given by:

$$\tau_t = \tau_{gt} + \tau_{xt} \tag{45}$$

$$b_t = b_{gt} + b_{xt} \tag{46}$$

$$\rho_t = \frac{b_t}{w_t - \tau_t} \tag{47}$$

In our benchmark calibration, the supranational agency is inactive ($b_{xt} = b_{xt}^* = \tau_{xt} = \tau_{xt}^* = 0$) and national governments target constant replacement rates $\rho_t = \bar{\rho}$ and $\rho_t^* = \bar{\rho}^*$. Since this situation is close to the current system in place in the Eurozone, we call it the "status quo".

[10]Our setup implicitly assumes that any utility workers receive from government expenditure is separable from market consumption, so that we can ignore it in the utility function.

Monetary policy is set according to a Taylor-type rule

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left[\bar{R} + \varpi_t\left(\phi_\pi \pi_t + \phi_y log\left(\frac{Y_t}{Y_{t-1}}\right)\right)\right.$$
$$\left. +(1-\varpi_t)\left(\phi_\pi \pi_t^* + \phi_y log\left(\frac{p_t Y_t^*}{p_{t-1} Y_{t-1}^*}\right)\right)\right], \quad (48)$$

where $\pi_t = \pi_{pt} P_t / P_{t-1}$ is Home CPI inflation and $\pi_t^* = \pi_{pt} P_t^* / P_{t-1}^*$ is Foreign CPI inflation. The monetary authority reacts to a weighted average of inflation deviations from target and output growth in Home and Foreign with strength ϕ_π and ϕ_y, respectively, where the weight is given by their real GDP $\varpi_t = \frac{\omega Y_t}{\omega Y_t + (1-\omega) p_t Y_t^*}$. The parameter$\rho_R$ is an interest rate smoothing parameter.

4.1.6. Market clearing and shocks

The market clearing conditions for consumption goods produced in each country take the form:

$$\omega\left(Y_t - \kappa^v v_t - g_t\right) = \omega\left(n_t c_{et,H} + (1-n_t) c_{ut,H}\right) + (1-\omega)\left(n_t^* c_{et,H}^* + (1-n_t^*) c_{ut,H}^*\right) \quad (49)$$

$$(1-\omega)\left(Y_t^* - \kappa^{*v} v_t^* - g_t^*\right) = \omega\left(n_t c_{et,F} + (1-n_t) c_{ut,F}\right) + (1-\omega)\left(n_t^* c_{et,F}^* + (1-n_t^*) c_{ut,F}^*\right) \quad (50)$$

The international bond is in zero net supply so that bond market equilibrium prescribes

$$0 = \omega d_t + (1-\omega) p_t d_t^*. \quad (51)$$

Finally, the exogenous shocks in our model are persistent shocks to productivity and government spending. The processes for government spending and productivity in the Home country are as follows:

$$\log a_t = \rho_a \log a_{t-1} + (1-\rho_a) \log \bar{a} + \varepsilon_{at} \quad (52)$$
$$\log g_t = \rho_g \log g_{t-1} + \left(1-\rho_g\right) \log \bar{g} + \varepsilon_{gt} \quad (53)$$

where all shocks are i.i.d. normally distributed with zero mean. Technology and government spending shocks are independent of each other, but we do allow for correlation of the Home and Foreign technology and government spending shocks, respectively.

We require the autoregressive coefficients to be less than one to rule out permanent shocks. This choice is not innocuous in its policy implications. The first best allocation in our model would completely shield domestic consumption from domestic employment and instead tie it to union output. In the presence of permanent shocks that differentially affect the long-run level of GDP in each country, this would effectively prescribe permanent transfers from the country with higher per capita income to the one with lower per capita income, and the Ramsey-optimal policy would then implement this prescription by permanent fiscal transfers. We do not see much practical relevance in such an extreme form of risk sharing and therefore focus exclusively on cyclical shocks. Any cross-country transfers under the Ramsey planner will always fall back to zero in expectation.

4.2. Optimal policy

When we characterize optimal unemployment insurance policies, we define "optimal" to be maximizing a utilitarian welfare function:

$$\mathbb{E}\left[\tilde{\omega}\left(n_t \mathcal{W}_t + (1-n_t)\,\mathcal{U}_t\right) + (1-\tilde{\omega})\left(n_t^* \mathcal{W}_t^* + (1-n_t^*)\,\mathcal{U}_t^*\right)\right] \tag{54}$$

We solve for Ramsey-optimal policies involving the replacement rates ρ_t, ρ_t^* as well as a transfer policy of the supranational agency, defined as

$$T_t = (1-n_t)\,b_t - n_t \tau_t. \tag{55}$$

Unless stated otherwise, we implement transfer policies by setting $b_{xt} = T_t$, $b_{xt}^* = \omega/\,(1-\omega)\,T_t/p_t$, and $\tau_{xt} = \tau_{xt}^* = 0$. This is without loss of generality: For any other set of policies $\left(b_{xt}, b_{gt}, \tau_{xt}, \tau_{gt}, b_{xt}^*, b_{gt}^*, \tau_{xt}^*, \tau_{gt}^*\right)$, one can define T_t as above and set $b_{xt}' = T_t$, $b_{gt}' = b_{gt} + b_{xt} - T_t$, $\tau_{xt}' = 0$, $\tau_{gt}' = \tau_{gt} + \tau_{xt}$ etc. and achieve exactly the same allocation, as the budget constraints of the public sector can be consolidated from a planner perspective.

As in the simple model, we choose the welfare weight on the Home country $\tilde{\omega}$ to rule out permanent transfers from one country to another:

$$\mathbb{E}\left[T_t\right] = 0. \tag{56}$$

We also compute what we call a "no-transfer" Ramsey problem. There, we keep the planner weight $\tilde{\omega}$ at the value imposed by condition (56), but now impose $T_t = 0$, so that the Ramsey planner is left with the Home and Foreign replacement rates as instruments. Comparing the "full" and the "no-transfer" Ramsey solutions will allow us to isolate the interaction effect of transfers and unemployment insurance policies.

4.3. Calibration

We calibrate the model to the Eurozone, where we identify the Home country with six Eurozone core countries (Austria, Belgium, Germany, Finland, France, Luxemburg, Netherlands) and the Foreign country with six periphery countries (Spain, Greece, Ireland, Italy, Portugal). In what follows, we use the term "country" in the model sense and use the words "Home"/"Foreign" and "Core"/"Periphery" interchangeably. Our calibration is summarized in Table 1.

The number of workers in the Home country is set to 60 percent, which corresponds to the relative size of the labor force in the Core. We set the discount factor β in both

Table 1: Calibrated parameter values.

Parameter	Symbol	Core	Periphery
Number of workers	ω	0.601	0.399
Discount factor	β	0.99	
Risk aversion	γ	1.5	
Elasticity param. on Home/Foreign goods	σ	0.744	
Utility weight on Home goods	ψ	0.574	0.479
Inverse elasticity of search effort	ϕ	1.447	0.402
Effort cost scaling parameter	κ_e	0.645	1.47
Matching elasticity	η	0.5	
Worker bargaining power	ξ	0.9	
Matching efficiency	κ_m	0.4583	
Separation rate	s	0.0275	0.0418
Vacancy costs	κ_v	0.0369	0.0686
Firm fixed costs	F	0.214	0.171
Price adjustment cost	κ_p	6.60	
Elasticity of subst. for intermediates	ϵ	4.3	
Wage adjustment cost	κ_w	653	87.4
Portfolio adjustment cost	κ_d	0.01	
Net replacement rate	$\bar{\rho}$	0.725	0.523
Coefficient on inflation	ϕ_π	1.5	
Coefficient on output growth	ϕ_y	0.5	
Interest rate smoothing	ρ_R	0.85	
Steady-state TFP level	$\bar{a}$	1	0.835
Autocorrelation of TFP	ρ_A	0.95	
Std. dev. of TFP shock	$\sigma(\varepsilon_a)$	0.00348	0.00985
Corr. of TFP shock H/F	$\rho(\varepsilon_a, \varepsilon_a^*)$	0.829	
Steady-state level of govt. spending	$\bar{g}$	0.257	0.180
Autocorrelation of spending	ρ_g	0.775	0.855
Std. dev. of spending shock	$\sigma(\varepsilon_g)$	0.00380	0.00443
Corr. of spending shock H/F	$\rho\left(\varepsilon_g, \varepsilon_g^*\right)$	0.293	

Table 2: Targeted moments.

Moment	Core	Periphery	Source
Labor force share	0.601	0.399	OECD
Real GDP share	0.653	0.347	OECD
Government share in GDP	28.1%	24.6%	OECD
S.d. of real GDP (filtered)	0.88%	1.64%	OECD
Mean unemployment rate	8.38%	12.23%	OECD
Sd. of unemployment (filtered)	0.380%	0.646%	OECD
Corr. of unemployment (filtered)	0.510		OECD
Net replacement rate	0.725	0.523	Christoffel et al. (2009)
SS job finding rate	0.3		Balta and Delgado (2009)
SS vacancy filling rate	0.7		Christoffel et al. (2009)
Consumption home bias	0.85		Corbo and Osbat (2013)

OECD data is taken in the range 1984Q1–2014Q4. GDP in this table is defined as the sum of final private and government expenditure. Filtered standard deviations are calculated from Hodrick-Prescott filtered data with smoothing parameter 1600, where GDP is in logarithm.

countries to the standard value of 0.99 which yields an annual real interest rate of 4 per cent. The curvature of consumption γ in both countries is set to 1.5 as reported in Smets and Wouters (2003). The parameter σ in both countries is set to 0.744, implying an elasticity of substitution between Home and Foreign goods of 3.9 matching the European average of estimates reported in Corbo and Osbat (2013). Given that value, we calculate values for the home good preferences ψ, ψ^* to match a GDP-weighted average of domestic expenditure shares of 85 percent as estimated in Balta and Delgado (2009), and ensuring that the relative price of foreign goods in steady state equals $p = 1$. The inverse elasticities of search effort ϕ, ϕ^* are chosen to match the estimate in Meyer (1990) of an elasticity of unemployment duration with respect to the level of benefits of 0.9. The effort scaling parameters κ_e, κ_e^* are set to normalize steady state effort in each country to unity.

We set the matching elasticity μ to the conventional value 0.5 according to estimates by Burda and Wyplosz (1994). The bargaining power of workers ξ is set at 0.9. As observed by Hagedorn and Manovskii (2008), workers need to capture a high share of the match surplus for a search model to reproduce the volatility of unemployment in the data. This holds true even in the presence of the wage rigidities we impose here.[11]

The matching efficiencies κ_m, κ_m^*; separation rates s, s^*; and vacancy costs κ_v, κ_v^* are set in each country to jointly match an average quarterly vacancy-filling probability of

[11]Many ways have been proposed to address the fact that the standard search and matching model fails, for a standard calibration, to account for the cyclical properties of unemployment and vacancies, the so called "Shimer puzzle" (Shimer, 2005). Contributing to this debate is beyond the scope of our paper.

70 percent (Christoffel et al., 2009), a quarterly job finding rate of 30% (Elsby et al., 2013) and unemployment rates of 8.4 percent in the Core and 12.1 percent in the Periphery. The fixed cost F, F^* is set in each country such that profits π_t are zero in the steady state.[12] The replacement rates ρ, ρ^* of the unemployment insurance system are set to match average values reported in Christoffel et al. (2009).

The cost coefficient of price adjustment κ_p is set to 6.06 and the demand elasticity for varieties is set to $\epsilon = 4.3$ in both countries. This leads to a steady-state markup of intermediate good producers of 30 percent and a Phillips curve that has the same slope coefficient on marginal cost as a corresponding Phillips curve with price rigidities á la Calvo with an average price duration of two quarters. This value is lower than what is common in the literature; stronger price rigidities would imply that unemployment rises after positive technology shocks because of the associated fall in markups. This is a common problem in New-Keynesian models with search frictions (see for example Gertler et al., 2008). We do, however, allow for substantial wage rigidity and set the wage adjustment costs κ_w, κ_w^* to match the volatility of (HP-filtered) unemployment rates in each country.

Monetary policy is described by a simple Taylor rule with a coefficient of inflation of $\phi_\pi = 1.5$, a coefficient on output growth of $\phi_y = 0.5$, and an interest rate smoothing coefficient of $\rho_R = 0.85$. The portfolio adjustment costs on the internationally traded non-contingent bond is set to $\kappa_d = 0.01$ following Benigno (2009).

The steady-state level of Home productivity $\bar{a}$ is normalized to one and the Foreign level $\bar{a}^*$ is set to reproduce the ratio of Periphery to Core GDP. We set the technology shock persistence to $\rho_a = 0.95$ in both countries. The standard deviation of the shocks in Home and Foreign are chosen to match the HP-filtered standard deviation of real GDP in the Core and the Periphery. The correlation of the two shocks is chosen to match the correlation between filtered unemployment rates in the Core and Periphery of 51 percent. The resulting correlation is approximately 0.8, which implies that the synchronization of the business cycle across the two countries is quite high. The steady-state levels of government spending $\bar{g}$ are set to match the share of government spending in GDP government spending process is parametrized to match detrended government expenditure data.

The Eurozone-specific moments we target are summarized in Table 2. The Periphery contributes about 40 percent of the EZ-12 labor force, but only about 35 percent of real GDP. It also has a higher volatility of GDP and unemployment. Nevertheless, our calibration features a higher degree of wage rigidity in the core than in the periphery. This is because the relative standard deviation of unemployment to output is actually higher in the core, so that for a shock of a given size unemployment has to rise more

[12]Doing so avoids the problem of ending up with a negative replacement rate in the Ramsey solution. Since we are setting a high bargaining power for workers, the Ramsey planner tends to undo this bargaining power with a lower replacement rate. Without the fixed cost F, it is possible to set a negative replacement rate (i.e. a tax on unemployment instead of an insurance) because unemployed workers can still consume a positive amount of profits.

in the core, which is achieved by setting a higher degree of wage rigidity. Finally, the periphery has a higher average unemployment rate, but a less generous unemployment insurance system as measured by net replacement rates, which in our calibration translates into a higher calibrated value for the separation rate.

5. Results

We now present the results from our numerical simulations. We first confirm that our calibration of the status quo produces second moments that are close to the data. We then compute Ramsey-optimal unemployment insurance policies, and also calculate optimal policy without transfers, i.e. imposing that $T_t = 0$ holds at all times. Even without transfers, we find that optimal replacement rates are countercyclical in the model. With regards to transfers, we confirm our predictions from the simple model: Replacement rates rise when a country receives a transfer, so the generosity of unemployment insurance becomes more countercyclical in the presence of transfers. Nevertheless, the impact of transfers on output and unemployment rates is relatively minor, indicating that transfers can be implemented through the unemployment insurance system without causing large distortions in labor markets. We then explore how our results differ under alternative scenarios.

5.1. Status quo

Table 3 summarizes key second moments of our calibrated model.

The standard deviations of real GDP and unemployment in each country are targeted by the calibration. The standard deviation of consumption is higher than in the data. This is an outcome of the fact that our model does not include capital and has a sizable share of output absorbed by a relatively stable government expenditure process. We see this outcome as an acceptable cost of simplification, but the gains from stabilization might therefore be somewhat overstated. The standard deviation of wages, by contrast, is reasonable. The bottom row also reports statistics on the Home trade balance as a percentage of domestic GDP. The standard deviation is of a realistic magnitude. This implies that even in the absence of cross-country fiscal transfers, households are able to smooth international consumption risk to some extent through savings in the bond market.

The contemporaneous correlations with GDP are close to the data, with the exception of real wages. Our HP-filtered wage series display very little correlation with filtered output. As for the cross-correlations across countries, the cross-correlation of unemployment is targeted by the calibration, and that of output and wages is within reasonable range. But the correlation of real consumption across countries is very close to one. Our model therefore suffers from the Backus et al. (1992) consumption correlation puzzle, overstating the amount of international risk sharing present in the data. Addressing the puzzle will likely strengthen our results.

Table 3: Second order moments in benchmark calibration.

Variable		s.d.	s.d. rel. to dom. GDP	corr. with dom. GDP	cross-corr.
Real GDP	Home	0.88* [0.88]	1.00 [1.00]	1.00 [1.00]	0.86 [0.91]
	Foreign	1.64* [1.64]	1.00 [1.00]	1.00 [1.00]	
Consumption	Home	2.00 [1.16]	2.27 [1.32]	0.98 [0.93]	0.96 [0.58]
	Foreign	2.56 [1.91]	1.56 [1.16]	0.98 [0.98]	
Real wage	Home	0.28 [0.39]	0.32 [0.44]	0.80 [-0.07]	0.81 [0.70]
	Foreign	0.65 [0.70]	0.40 [0.43]	0.62 [0.15]	
Unemployment	Home	0.38* [0.38]	0.43 [0.43]	-0.71 [-0.68]	0.51* [0.51]
	Foreign	0.67* [0.67]	0.41 [0.41]	-0.71 [-0.60]	
Trade bal., %GDP	Home	0.26	0.34	-0.51	

Second moments as obtained from simulating a linear approximation of the model at benchmark calibration. Real GDP, consumption and real wage are in logarithms. All series are HP-filtered with smoothing parameter 1600. Standard deviations are reported in percentage points. Moments targeted by the calibration are marked with an asterisk. Corresponding moment in OECD data in brackets where available (maximal date range 1984Q1–2014Q4), where GDP is taken as the sum of private and government final consumption expenditure, and real wages are taken as manufacturing wages divided by CPI.

5.2. *Ramsey policy*

We now compute Ramsey-optimal policies with and without transfers and confirm our predictions from the simple model about the cyclicality of policy instruments. The results are summarized in Table 4.

The left half of the table documents the standard deviations of optimal replacement rates and transfers and their correlation with output. The standard deviations of replacement rates are relatively large, around three percentage points. Remarkably, all replacement rates are countercyclical. Transfers to the Foreign country as a percentage of Foreign GDP are also large and countercyclical. In the Foreign country, replacement rates are more volatile and more negatively correlated when the planner has access to transfers than without transfers ($T_t = 0$ imposed). However, in the Home country the opposite pattern can be observed.

However, looking at correlation coefficients does not say much about the behavior of policy because it does not say by how much replacement rates change in response to economic conditions. Moreover, GDP across the two countries is highly correlated, and so correlation coefficients mask differences in policy reactions to shocks originating at Home and in Foreign.

For these reasons, our preferred measures of the cyclicality of optimal policy are the coefficients from bivariate regressions of replacement rates and transfers to Home and Foreign GDP. These are shown in the right half of Table 4, and they clearly con-

Table 4: Cyclicality of the Ramsey-optimal unemployment insurance.

	summary statistics:		*bivariate regression:*	
	s.d.	corr. with dom. GDP	coefficient Home GDP	coefficient Foreign GDP
Home replacement rate	2.38	-0.33	-1.37	0.43
(without transfers)	2.45	-0.49	-1.08	0.14
Foreign replacement rate	3.71	-0.61	0.83	-1.16
(without transfers)	3.10	-0.52	0.15	-0.58
Foreign transfer, %GDP	1.18	-0.35	0.69	-0.67

Moments as obtained from simulating a linear approximation of the model at benchmark calibration. Standard deviations are reported in percentage points. Regression coefficients are obtained from regressing each row variable simultaneously on the log of Home and Foreign real GDP.

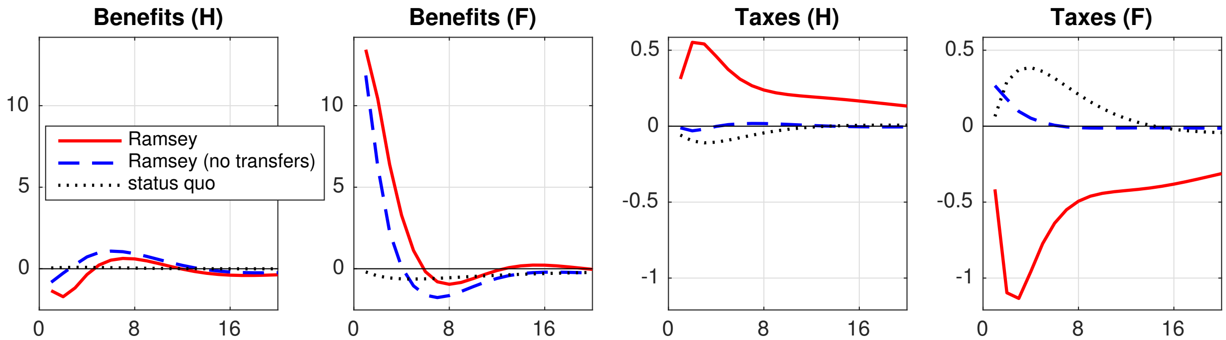
firm our predictions from the simple model. On average, when Home GDP rises by one percent, replacement rates fall by 1.37 percentage points in the Home country with transfers. Without transfers, they only fall by 1.08 percentage points. Foreign replacement rates rise by 0.83 percentage points in the presence of transfers, compared to only 0.15 percentage points without transfers. The Foreign country also receives, on average, a transfer of 0.69 percent of its GDP when GDP at Home rises by one percent. The same patterns appear in response to changes in Foreign GDP: Replacement rates are countercyclical with respect to domestic GDP and procyclical with respect to GDP abroad, and this cyclicality becomes stronger in the presence of transfers.

To further illustrate these results, we compare impulse response functions to a negative productivity shock in the periphery in Figure 3 (the patterns are qualitatively similar for a shock in the core). Specifically, we contrast the optimal Ramsey planner response with the no-transfer Ramsey solution and the status quo policy (no transfers, constant replacement rates).

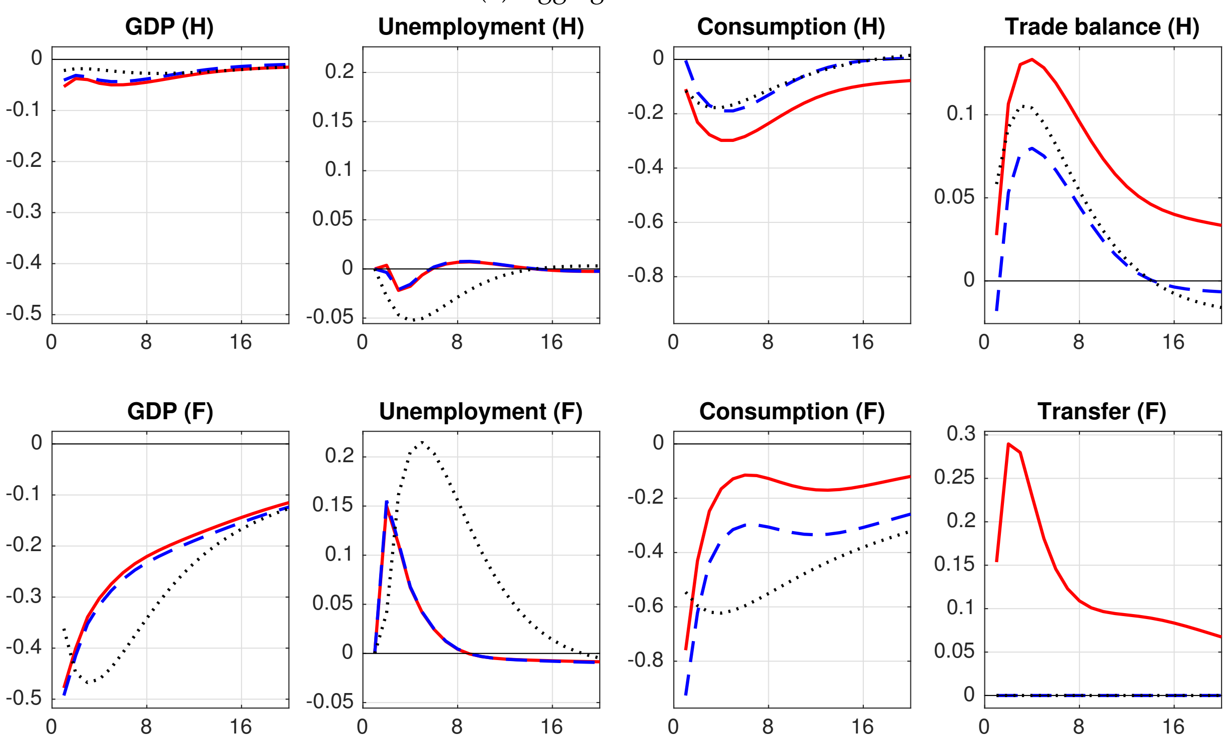
The upper panel of the figure depicts the four direct policy instruments: Home and Foreign benefits b_t, b_t^* and Home and Foreign payroll taxes τ_t, τ_t^*. These can be mapped into the replacement rates and transfer policies, but we want to highlight how variations in these policy variables are implemented by the planner. After a negative productivity shock in the periphery, the Ramsey planner (solid red line) implements a transfer from the core to the periphery. It does so by increasing unemployment benefits in the periphery and at the same time cutting payroll taxes, and effecting the opposite pattern in the core. Benefits drop below their initial level after six quarters in order to make the increase in unemployment dissipate more quickly, while taxes stay low for a long time. A large transfer finds its way to the economy mainly by a reduction in payroll taxes which affect most of the population, rather than benefits which only affect only the relatively few unemployed workers. When the planner does not have access to transfers (blue dashed line), the behavior of the instruments is markedly different. In the core,

Figure 3: Impulse responses, negative Foreign productivity shock.

(a) Policy instruments.



(b) Aggregate outcomes.



Impulse response functions to a one standard deviation negative productivity shock in Foreign. Benefits, taxes, GDP and consumption are in percent deviation from the steady state. Unemployment is in percentage point deviation from the steady state. Trade balance (H) is in percent of Home real GDP and transfer (F) is in percent of Foreign real GDP.

Table 5: Gains from stabilization.

	s.d. %	(1) status quo	(2) Ramsey (no transfers)	(3) Ramsey	(4) Debt
Consumption	Home	4.17	2.91	3.59	2.99
	Foreign	6.59	5.21	3.85	5.05
Unemployment	Home	0.72	0.26	0.27	0.26
	Foreign	1.03	0.54	0.52	0.53

Standard deviations from simulated model data, unfiltered. Consumption is in logarithms.

benefits and taxes stay mostly flat after the shock. The level of benefits still rises in the periphery, but this time the increase in benefits has to be financed by higher taxes. In the status quo calibration with constant replacement rates, taxes also increase even though the level of benefits barely moves. This is because unemployment increases markedly and persistently, and more recipients of benefits and fewer payroll contributors necessitate a higher level of contributions to balance the national unemployment insurance budget.

The lower panel of the figure depicts the aggregate outcomes under the different policies. The responses of GDP and unemployment in the core are barely affected, but there are marked differences in the periphery where the shock hits. The Ramsey planner manages to stabilize GDP and unemployment with respect to the status quo policy. However, the response of these variables is nearly identical whether the planner can make use of cross-country transfers or not. This result obtains despite the fact that the transfer is sizable—0.28 percent of Foreign GDP at its peak—and underscores our prediction that transfers can be channeled through the unemployment insurance system without distorting labor market outcomes.

Of course, transfers have a stabilizing function on consumption. Compared to the status quo, the planner without access to transfers already manages to stabilize consumption in both countries by reducing the size of unemployment fluctuations. Still, consumption drops by more in the periphery where the shock originates. Transfers even out the burden of reducing consumption, raising it in the periphery and reducing it in the core. Finally, the figure also depicts the response of the core trade balance. It is positive under all three policies, reflecting the fact that the periphery runs a current account deficit to privately smooth out the drop in consumption. It does so by taking up debt. However, non-contingent debt is not the same as full risk sharing and the planner can improve on this allocation. The transfer of course results in an even larger trade balance.

How important are the stabilization gains? We measure stabilization in terms of the reduction in the variances of consumption and unemployment , documented in Table 5.

Column (1) shows the standard deviations of unfiltered (log) consumption and unemployment in both countries at the status quo policy. Column (2) shows the same statistics at the Ramsey policy without transfers. The Ramsey planner can achieve a reduction of at least 20 percent in the volatility of consumption and at least 45 percent in the volatility of unemployment by adjusting national replacement rates alone. In Column (3), the planner can additionally pool the unemployment insurance systems at the supranational level. In that case, it achieves additional stabilization in the periphery: unemployment volatility drops three percent, and consumption volatility drops 26 percent. In the core however, the volatility of unemployment and consumption increase by 4 percent and 24 percent, respectively, although they remain lower than under the status quo.

Why is the planner not able to stabilize consumption in both countries? The answer is that business cycles in the model are highly correlated, owing to the calibration. There is relatively little country-specific risk that can be reduced through diversification. Instead, the planner is implementing transfers to shift risk from the periphery to the relatively more stable core. As a result, the standard deviations of consumption in the core and the periphery move closer to each other. The policy distributes risk more evenly across the two countries, but it is not a Pareto-improvement over the no-transfer policy: The core would be better off without transfers. We conjecture that a Pareto improvement could be achieved by a perpetual steady-state payment from the periphery to the core as a compensation for the added risk. But this is ruled out by our requirement that transfers be zero in expectation. We think that this point has not been appreciated in the existing literature: A fiscal risk sharing mechanism in the Eurozone is likely to be detrimental to the more stable economies of the core even when permanent transfers are ruled out by design, simply because risk is perpetually shifted from more volatile to more stable countries. [13]

5.3. Alternative scenarios

How sensitive are our results to the calibration? Our main qualitative result—transfers make optimal unemployment insurance more countercyclical—hold up for any reasonable parametrization of the model, but the quantitative results are sensitive to several parameters. Table 6 computes the cyclicalities of optimal policy under a variety of alternative assumptions on the parameters.

Column (1) repeats the baseline calibration regression coefficients from Table 4. In Column (2), the Home population share is increased to $\omega = 0.8$. This scenario implies that the size of the Foreign country is roughly that of Spain within the Eurozone. Compared to the baseline, the Home replacement rate becomes less countercyclical, Foreign replacement rates more countercyclical, and Foreign transfers become larger. These results are fully in line with the predictions from our simplified model: A smaller country

[13] Our calibration to only two regions might understate the amount of diversifiable business cycle risk. At the single country level, the component of risk that is diversifiable across the union is certainly larger, so that it might be possible to reduce consumption volatility in every country with appropriate risk-sharing policies.

Table 6: Sensitivity of optimal policy to alternative scenarios.

bivariate regression coefficients		(1) Baseline	(2) $\omega = 0.8$	(3) $\sigma_{A^*} = 0$	(4) $\bar{u}^* = \bar{u}$	(5) $\sigma = 0.01$	(6) $\xi = \mu$
Replacement rate, Home	Home GDP	-1.37	-1.06	-2.90	-1.55	0.59	-1.34
	Foreign GDP	0.43	0.25	7.42	0.53	-1.69	0.29
Replacement rate, Foreign	Home GDP	0.83	1.16	2.53	0.45	0.05	0.73
	Foreign GDP	-1.16	-1.44	-9.96	-1.08	-1.12	-1.55
Transfer, Foreign, %GDP	Home GDP	0.69	1.11	1.17	0.71	-0.03	0.65
	Foreign GDP	-0.67	-1.03	-2.69	-0.69	-0.01	-0.66

Regression coefficients are obtained from regressing each row variable simultaneously on the log of Home and Foreign real GDP.

can be better insured by the union, and the countercylical effect of risk sharing on replacement rates becomes stronger.

In Column (3), the variance of Foreign productivity shocks is set to $\sigma_{A^*} = 0$, so that almost all output fluctuations come from shocks originating in the Home country. This scenario isolates the policy reaction to country-specific shocks, which arere not clearly visible in the baseline because of the high correlation of shocks. The cyclicalities of replacement rates and transfers increase everywhere.

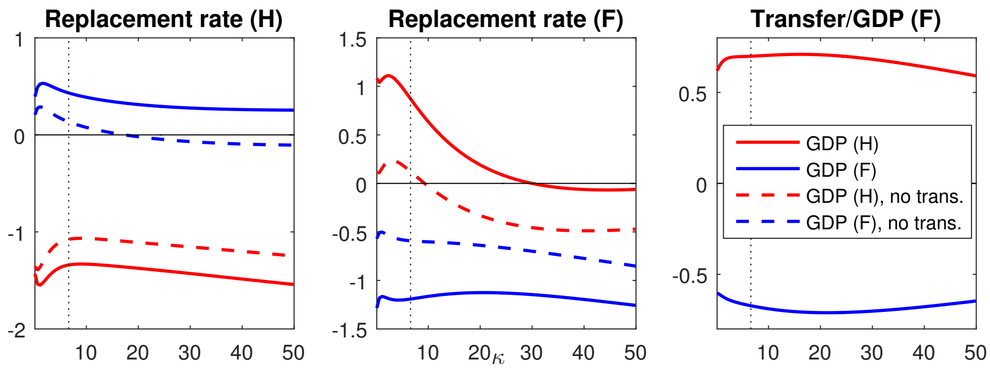
In Column (4), the steady-state unemployment rate in the Foreign country is $\bar{u}^*$ reduced to the value to that in the Home country $\bar{u}$. This scenario can be thought of as the implementation of labor market reforms in the Eurozone periphery that reduce structural unemployment. As a result, replacement rates become more countercyclical in the Home country and less countercyclical in the Foreign country, while the magnitude of transfers is roughly unchanged. Intuitively, when $\bar{u}^*$ falls, Foreign GDP becomes less volatile and its share in total GDP rises, meaning that the core will insure less risk of the periphery. Replacement rates become less countercyclical in the less well-insured Foreign country, while the opposite happens in the Home country.[14]

In Column (5), the elasticity of substitution between Home and Foreign goods is set close to unity. This scenario puts the economy close to the case studied in Cole and Obstfeld (1991) where movements in the terms of trade achieve perfect risk sharing even under financial autarky. In line with their finding, the size of optimal transfers is now close to zero. The replacement rates become more procyclical as a result.

Most other parameters of the model have little impact on the quantitative results. For example, the choice of the bargaining weight ξ is crucial to match the volatility of unemployment in the model, but does not affect optimal policy. In Column (6), we set $\xi = \mu$ so as to bring the model closer to the Hosios (1990) condition. The changes to the

[14] A similar result obtains when the degree of wage rigidity in the Foreign country κ_w^* is reduced.

Figure 4: Sensitivity of optimal policy to price rigidities.



Regression coefficients obtained from regressing each row variable simultaneously on the log of Home and Foreign real GDP, as a function of the parameter κ_p (common across both countries), for the Ramsey-optimal policy with and without transfers. The black dotted line represents the baseline calibration value $\kappa_p = 6.601$.

coefficients in the table are relatively small and there is no clear pattern in the change of cyclicality. Similar results obtain when varying other parameters.

Likewise, the degree of nominal price rigidities has little impact on the optimal policy. Figure 4 plots the bivariate regression coefficients on the optimal policy instruments as a function of the Rotemberg price rigidity parameter κ_p. The dotted line marks the baseline value of $\kappa_p = 6.601$. This is at the lower end of values used in the literature, as discussed above. But a lower or higher value for κ_p does little change to the cyclicality of the Home replacement rate. It has some effect on the response of the Foreign replacement rate to changes in Home GDP (red lines in the middle panel of the figure): Larger price rigidities induce the Foreign replacement rate rise less, on average, when Home GDP rises. Still, the qualitative conclusion of more countercyclical replacement rates in the presence of transfers is unaffected by the size of price rigidities. [15] What's more surprising is that even the transfer policy is little changed as κ_p varies. The inefficiencies caused by the presence of a currency union and nominal rigidities emphasized by Farhi and Werning (2012) seem to be quantitatively unimportant compared with the other financial market imperfections in the model.

6. The difference between debt and insurance

In writing this paper, we frequently encountered the argument that a fiscal risk sharing mechanism without permanent transfers would be no different from using government debt to run countercyclical deficits. Here, we show that this argument does not

[15] This finding is in line with Kekre (2016) who finds that optimal unemployment insurance is affected by nominal rigidities and the ensuing demand externalities only when the zero lower bound on interest rates is binding.

hold. Our fiscal risk sharing mechanism works like a fairly priced insurance policy: Even though on average, the premia and expected payments net out to zero, one is still better better off buying the insurance than taking out a bank loan in the event of damage.

We modify our quantitative model to introduce national government debt. All debt is denominated in nominal terms in the common currency of the two countries. The budget constraint (43) of the Home government is modified as follows:

$$g_t + u_t b_{gt} = \tau_{gt} n_t + d_{gt} - \frac{\kappa_d}{2} d_{gt}^2 - R_{t-1} \frac{d_{gt-1}}{\pi_{pt}}. \tag{57}$$

Note that national governments face the same quadratic portfolio adjustment costs as the private sector. If this were not the case, the primary use of government would be to replace frictional private borrowing with frictionless public borrowing. The Foreign government similarly can similarly issue debt d_{gt}^*. The clearing condition (51) in the bond market is modified to read:

$$0 = \omega \left(d_t - d_{gt} \right) + (1 - \omega) p_t \left(d_t^* - d_{gt}^* \right). \tag{58}$$

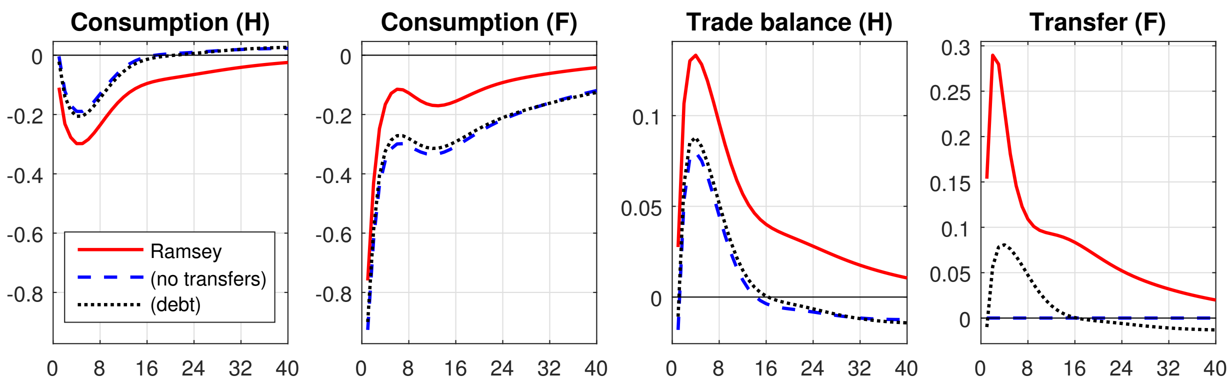
We then compute the Ramsey-optimal policies where the planner does not have access to cross-country transfers T_t but can instead optimally choose the national government debt levels d_{gt} and d_{gt}^*. As before, the planner can also choose the national replacement rates ρ_t, ρ_t^*. From a welfare perspective, the value of the Ramsey problem with debt is bounded from above by that with transfers, and from below by the value of that with nationally balanced budgets.

It turns out that the planner is not able to distribute risk very effectively with debt. This can be seen immediately from Column (4) in Table 5: The values for the volatility of consumption in particular are very close to those without transfers (Column 2).

In Figure 5, we plot impulse responses after a negative country-specific productivity shock in the periphery, comparing the full Ramsey solution, the solution with debt, and the solution with nationally balanced budgets (labeled "no transfers").

It is immediately clear that the solution with debt is very close to the solution with balanced budgets. The drop in consumption is minimally shifted from Foreign to Home, which is reflected in a slightly higher trade balance. It is not advantageous for the planner to move resources from Home to Foreign with debt, and the reason is of course that debt has to be repaid. The rightmost graph in Figure 5, labeled "Transfer (F)", depicts the budget deficit of the Foreign government in the solution with debt. This is the closest correspondence to the supranational transfers in the full Ramsey solution, which are also plotted. The planner does run a budget deficit in the periphery after the shock, but the optimal size of the deficit is only about 0.08 percent of GDP at the peak, since it will have to be repaid eventually (in the graph, starting four years after the shock). By contrast, when the planner has access to transfers the stabilization gains are much larger, despite the fact that transfers are temporary and equal zero in expectation. With

Figure 5: Debt is not the same as insurance.



Impulse responses to a one-standard deviation negative Foreign productivity shock.

debt, any "transfer" has to be repaid with certainty at some point in the future; with fiscal risk sharing, transfers only have to average out over time.

7. Conclusions

In this paper, we have used an international business cycle model with frictional labor markets and incomplete financial markets to discuss optimal unemployment insurance policy operating across multiple countries. Theoretically, the possibility to set different policies across countries augments the classic policy trade-off between efficient employment and insurance of unemployment risk with a concern for international risk-sharing. We have shown that cross-country insurance through the unemployment insurance system can in principle be achieved without affecting unemployment levels; and that the desirability of international risk sharing introduces a countercyclical element to the optimal unemployment insurance policy.

Calibrated to Eurozone core and periphery, our two-country model implied that optimal replacement rates are countercyclical even from a national perspective without transfers. Adding transfers markedly increased this countercyclicality, and the optimal transfers were found to be sizeable (about 0.7 percent of GDP following a one percentage point decrease in GDP). However, we also found that, due to the high correlation of business cycles across the Eurozone, there is very limited scope to diversify risk. Instead, the optimal planner policy mainly implied a shift of consumption risk from the periphery to the core in order to distribute it more evenly.

There is one important direction in which our findings could be extended in further research. The optimal policy we compute here is one in which the planner has perfect knowledge of the structure of the economy. One of the most difficult issues in implementing a policy such as the one in this paper is that the structural rate of unemployment can only be reliably estimated in hindsight, if at all. It would be useful to see whether simple policy rules that are more easily implementable under imperfect information can reasonably approximate the optimal policy.

References

Albertini, Julien and Arthur Poirier, "Unemployment Benefit Extension at the Zero Lower Bound," *Review of Economic Dynamics*, October 2015, *18* (4), 733–751.

Andor, László, "Basic European unemployment insurance: Countering divergences within the Economic and Monetary Union," Speech at the Vienna University of Economics and Business September 2014.

Arseneau, David M. and Sanjay K. Chugh, "Optimal fiscal and monetary policy with costly wage bargaining," International Finance Discussion Papers 893, Board of Governors of the Federal Reserve System (U.S.) 2007.

Artus, P., A. Benassy-Quere, L. Boone, J. Cailloux, J. Delpla, E. Farhi, P. Gourinchas, J. Tirole, and G. Wolff, "Completing the Euro," *Les notes du Conseil d'Analyse Economique*, April 2013.

Backus, David K, Patrick J Kehoe, and Finn E Kydland, "International Real Business Cycles," *Journal of political Economy*, 1992, pp. 745–775.

Baily, Martin Neil, "Some aspects of optimal unemployment insurance," *Journal of Public Economics*, 1978, *10* (3), 379–402.

Balta, Narcissa and Juan Delgado, "Home Bias and Market Integration in the EU," *CESifo Economic Studies*, March 2009, *55* (1), 110–144.

Bargain, Olivier, Mathias Dolls, Clemens Fuest, Dirk Neumann, Andreas Peichl, Nico Pestel, and Sebastian Siegloch, "Fiscal Union in Europe? Redistributive and Stabilising Effects of an EU Tax-Benefit System," *Economic Policy*, 2013, *28* (75), 375–422.

Benigno, Pierpaolo, "Price Stability with Imperfect Financial Integration," *Journal of Money, Credit and Banking*, 2009, *41*, 121–149.

Bernoth, Kerstin and Philipp Engler, "A Transfer Mechanism as a Stabilization Tool in the EMU," *DIW Economic Bulletin*, 2013, *3* (1), 3–8.

Blanchard, Olivier J, Florence Jaumotte, and Prakash Loungani, "Labor market policies and IMF advice in advanced economies during the Great Recession," *IZA Journal of Labor Policy*, 2014, *3* (1), 1–23.

Bordo, Michael, Agnieszka Markiewicz, and Lars Jonung, "A Fiscal Union for the Euro: Some Lessons from History," Working Paper 17380, NBER 2011.

Brandolini, Andrea, Francesca Carta, and Francesco D'Amuri, "A Feasible Unemployment-Based Shock Absorber for the Euro Area," IZA Policy Papers 97, Institute for the Study of Labor (IZA) February 2015.

Brenke, Karl, "A Skeptical View of Mechanisms for Business Cycle Harmonization in the Euro Area," *DIW Economic Bulletin*, 2013, *3* (1), 16–20.

Burda, Michael and Charles Wyplosz, "Gross worker and job flows in Europe," *European Economic Review*, June 1994, *38* (6), 1287–1315.

Bénassy-Quéré, Agnès, Xavier Ragot, and Guntram Wolff, "Which fiscal union for the Euro area?," Policy Note 29, French Council of Economic Analysis February 2016.

Christoffel, Kai, Keith Kuester, and Tobias Linzert, "The role of labor markets for euro area monetary policy," *European Economic Review*, 2009, *53* (8), 908–936.

Cole, Harold L. and Maurice Obstfeld, "Commodity trade and international risk sharing : How much do financial markets matter?," *Journal of Monetary Economics*, August 1991, *28* (1), 3–24.

Corbo, Vesna and Chiara Osbat, "Trade adjustment in the European Union - a structural estimation approach," Working Paper Series 1535, European Central Bank April 2013.

Dmitriev, Mikhail and Jonathan Hoddenbagh, "The Optimal Design of a Fiscal Union," Working paper November 2013.

Dolls, Mathias, Clemens Fuest, Dirk Neumann, and Andreas Peichl, "An Unemployment Insurance Scheme for the Euro Area? A Comparison of Different Alternatives using Micro Data," Discussion Paper 14-095, ZEW 2014.

Elsby, Michael W. L., Bart Hobijn, and Aysegül Sahin, "Unemployment Dynamics in the OECD," *Review of Economics and Statistics*, May 2013, *95* (2), 530–548.

Evers, Michael P., "Federal Fiscal Transfer Rules in Monetary Unions," *European Economic Review*, 2012, *56*, 507–525.

_ , "Fiscal Federalism and Monetary Unions: A Quantitative Assessment," *Journal of International Economics*, 2015, *97*, 59–75.

Farhi, Emmanuel and Ivan Werning, "Fiscal Unions," Working Paper 18280, NBER August 2012.

Fredriksson, Peter and Bertil Holmlund, "Improving incentives in unemployment insurance: A review of recent research," *Journal of Economic Surveys*, 2006, *20* (3), 357–386.

Furceri, Davide and Aleksandra Zdzienicka, "The Euro Area Crisis: Need for a Supranational Fiscal Risk Sharing Mechanism?," *Open Economies Review*, September 2015, *26* (4), 683–710.

Gertler, Mark, Luca Sala, and Antonella Trigari, "An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining," *Journal of Money, Credit and Banking*, December 2008, *40* (8), 1713–1764.

Hagedorn, Marcus and Iourii Manovskii, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review*, 2008, *98* (4), 1692–1706.

Hosios, Arthur J, "On the efficiency of matching and related models of search and unemployment," *The Review of Economic Studies*, 1990, *57* (2), 279–298.

Jung, Philip and Keith Kuester, "Optimal Labor-Market Policy in Recessions," *American Economic Journal: Macroeconomics*, 2015, *7* (2), 124–56.

Kalemli-Ozcan, Sebnem, Emiliano Luttini, and Bent Sørensen, "Debt Crises and Risk-Sharing: The Role of Markets versus Sovereigns," *Scandinavian Journal of Economics*, 01 2014, *116* (1), 253–276.

Kekre, Rohan, "Unemployment Insurance in Macroeconomic Stabilization," Working paper 2016.

Kenen, Peter B., "The Optimum Currency Area: An Eclectic View," in R. A. Mundell and A. Swoboda, eds., *Monetary Problems of the International Economy*, University of Chicago Press, 1969.

Kiley, Michael, "How Should Unemployment Benefits Respond to the Business Cycle?," *The B.E. Journal of Economic Analysis & Policy*, 2003, *3* (1), 1–22.

Landais, Camille, Pascal Michaillat, and Emmanuel Saez, "A Macroeconomic Theory of Optimal Unemployment Insurance," Working paper 2015.

Leduc, Sylvain, Luca Dedola, and Giancarlo Corsetti, "Should Monetary Policy React to Current Account Imbalances?," Meeting Papers 1219, Society for Economic Dynamics 2009.

McKinnon, R. I., "Optimum Currency Areas," *American Economic Review*, 1963, *53*, 717–725.

Meyer, Bruce D., "Unemployment Insurance and Unemployment Spells," *Econometrica*, 1990, *58* (4), 757–782.

Mitman, Kurt and Stanislav Rabinovich, "Optimal unemployment insurance in an equilibrium business-cycle model," *Journal of Monetary Economics*, 2015, *71*, 99–118.

Moyen, Stephane and Nikolai Stähler, "Unemployment Insurance And The Business Cycle: Should Benefit Entitlement Duration React To The Cycle?," *Macroeconomic Dynamics*, April 2014, *18* (3), 497–525.

Mundell, Robert A., "A Theory of Optimum Currency Areas," *American Economic Review*, 1961, *51* (4), 657–665.

Persson, Torsten and Guido Tabellini, "Federal Fiscal Constitutions: Risk Sharing and Moral Hazard," *Econometrica*, May 1996, *64* (3), 623–46.

Sanchez, Juan M, "Optimal state-contingent unemployment insurance," *Economics Letters*, 2008, *98* (3), 348–357.

Shimer, Robert, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, March 2005, *95* (1), 25–49.

Smets, Frank and Raf Wouters, "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, September 2003, *1* (5), 1123–1175.

Sørensen, Bent E. and Oved Yosha, "International risk sharing and European monetary unification," *Journal of International Economics*, 1998, *45* (2), 211 – 238.

van Rompuy, Herman, "Towards a Genuine Economic and Monetary Union," Report by the President of the European Council EUCO 120/12 2012.

Appendix A. Propositions for Section 1

The optimal replacement rate in the absence of private risk sharing satisfies Equation (19) in the main text:

$$\underbrace{\frac{(1-n)(1-\rho)}{n+(1-n)\rho} - \epsilon_{\rho}^{n}\left(\log\rho + \frac{1-\rho}{n+(1-n)\rho}\right)}_{=:I(\rho)} = \underbrace{-\epsilon_{\rho}^{y}\frac{x}{n}}_{=:H(\rho)}$$

where $x = y/(y+T)$ is the income to consumption ratio of the Home country. Here, we prove the properties of the optimal policy as discussed in the text. Throughout, we make the following assumption:

Assumption. *I is strictly concave, H and Hy are strictly convex in ρ given x.*

We numerically verified this assumption for a wide range of parameters. The limit behavior of the functions at the corners is easy to prove and together with our assumption determines the shape of the curves in the main text.

Proposition 1. *$I(0) = \frac{1-\bar{n}}{\bar{n}}$ where $\bar{n} = \kappa_m\left(\frac{\kappa_m}{\kappa_v}a\right)^{(1-\mu)/\mu}$ and $I(1) = 0$. Also, we have $H(0) = 0$, $H\left(exp\left(-\frac{1-\mu}{\mu}\frac{\xi}{1-\xi}\right)\right) = 0$, $\lim_{\rho\to 1} = +\infty$, and $H'(\rho)$ strongly convex given x.*

Proof. We start with the insurance term $I(\rho)$. At the limit when $\rho \to 0$, we have $w \to 0$ and $n = \kappa_m\left(\frac{\kappa_m}{\kappa_v}(a-w)\right)^{(1-\mu)/\mu} \to \bar{n}$. Therefore:

$$\frac{(1-n)(1-\rho)}{n+(1-n)\rho} \overset{\rho\to 0}{\longrightarrow} \frac{1-\bar{n}}{\bar{n}}.$$

The remaining term of $I(\rho)$ must therefore go to zero. Indeed,

$$\begin{aligned}\epsilon_{\rho}^{n} = \frac{dn}{d\rho}\frac{\rho}{n} &= -\frac{a}{a-w}\frac{1-\mu}{\mu}\frac{w^2}{a^2}\frac{1-\xi}{\xi} \\ &= \frac{1}{\log\rho}\frac{1-\mu}{\mu}\frac{\xi}{\xi-(1-\xi)\log\rho}\end{aligned}$$

and therefore

$$\begin{aligned}&\epsilon_{\rho}^{n}\left(\log\rho + \frac{1-\rho}{n+(1-n)\rho}\right) \\ = &\frac{1-\mu}{\mu}\frac{\xi}{\xi-(1-\xi)\log\rho}\left(1+\frac{1-\rho}{n+(1-n)\rho}\frac{1}{\log\rho}\right) \overset{\rho\to 0}{\longrightarrow} 0.\end{aligned}$$

For the case $\rho \to 1$, the first term clearly disappears:

$$\frac{(1-n)(1-\rho)}{n+(1-n)\rho} \overset{\rho\to 1}{\longrightarrow} 0$$

and for the second term, we have:

$$\frac{1-\mu}{\mu}\frac{\xi}{\xi-(1-\xi)\log\rho}\left(1+\frac{1-\rho}{n+(1-n)\rho}\frac{1}{\log\rho}\right) \overset{\rho\to 0}{\longrightarrow} \frac{1-\mu}{\mu}\left(1+\lim_{\rho\to 1}\frac{1-\rho}{\log\rho}\right) = 0.$$

Turn now to the efficiency term $H(\rho)$. As $\rho \to 0$, $n \to \bar{n} > 0$ and and $w \to 0$. Therefore

$$-\epsilon_\rho^y \frac{x}{n} = -\frac{x}{n}\frac{w}{a}\left(\frac{1-\xi}{\xi}+\frac{1-\mu}{\mu}\frac{1}{\log\rho}\right) \overset{\rho\to 0}{\longrightarrow} 0.$$

And as $\rho \to 1$, $w \to 1$ and $n \to 0+$, so that

$$-\frac{x}{n}\frac{w}{a}\left(\frac{1-\xi}{\xi}+\frac{1-\mu}{\mu}\frac{1}{\log\rho}\right) \overset{\rho\to 1}{\longrightarrow} +\infty.$$

□

Proposition 2. *The optimal replacement rate is unique and strictly between* $\exp\left(\frac{\mu}{1-\mu}\frac{1-\xi}{\xi}\right)$ *and one.*

Proof. Since $f(\rho) = H(\rho) - I(\rho)$ is continuous on $[0,1]$ and a strictly concave by our assumption, it crosses zero at most twice. But $f(0) > 0$ and $\lim_{\rho\to} -\infty$, so there is a unique interior solution ρ^* to $f(\rho) = 0$. Since $I(0) > I(1) = 0$ and I is strictly concave, $I(\rho) > 0 \forall \rho \in (0,1)$ and the optimum has $H(\rho^*) > 0$. Since H is a strictly convex function, $H(0) = 0$ and $\lim_{\rho\to 1} H(\rho) = +\infty$ and , $H(\rho_0) = 0$ for exactly one $\rho_0 \in (0,1)$ and $\rho^* > \rho_0$. Finally, $H\left(\exp\left(\frac{\mu}{1-\mu}\frac{1-\xi}{\xi}\right)\right) = 0$. □

Proposition 3. *The optimal replacement rate is strictly decreasing in x.*

Proof. Taking the total derivative of the optimality condition with respect to x, we have

$$0 = \frac{\partial I}{\partial x} - \frac{\partial H}{\partial x} + \frac{\partial I}{\partial \rho}\frac{d\rho}{dx} - \frac{\partial H}{\partial \rho}\frac{d\rho}{dx}$$
$$\Leftrightarrow \frac{d\rho}{dx} = -\frac{\frac{\partial I}{\partial x} - \frac{\partial H}{\partial x}}{\frac{\partial I}{\partial \rho} - \frac{\partial H}{\partial \rho}}.$$

Clearly, $dI/dx = 0$ and at the optimal ρ, we have $dH/dx = H(\rho)/x > 0$. Furthermore, we know that $I(0) > H(0)$ and $I(\rho) = H(\rho)$ for exactly one value of ρ, so it must be the case that $dH/d\rho > dI/d\rho$ at the optimal ρ. Therefore $d\rho/dx < 0$. □

This result shows in particular that the replacement rate is increasing in foreign productivity a^* , since an increase in a^* raises y^* and therefore decreases x.

Finally, we are going to establish countercyclicality of ρ in the limit case when Home country becomes very small, and when a and a^* are independent.

Proposition 4. *In the limit as $\omega \to 0$, the optimal replacement rate is unique, strictly below one, and strictly decreasing in a.*

Proof. As $\rho \to 0$, the risk sharing condition (19) becomes

$$x = \frac{\mathbb{E}[y^*]}{\mathbb{E}[y]} \frac{y}{y^*}.$$

The optimal choice of ρ when x is chosen optimally can now be described as

$$I(\rho) = \tilde{H}(\rho)$$

$$\text{where } \tilde{H}(\rho) = -\epsilon_\rho^y \frac{1}{n} \frac{\mathbb{E}[y^*]}{\mathbb{E}[y]} \frac{y}{y^*}.$$

By our assumption, $\tilde{H}(\rho)$ is a strictly convex function. The value of $\tilde{H}$ at zero is

$$\tilde{H}(0) = H(0) \frac{\mathbb{E}[y^*]}{\mathbb{E}[y]} \frac{\lim_{\rho \to 0} wn}{y^*} = H(0) \cdot 0 = 0.$$

For the limit at one, we note

$$\tilde{H}(\rho) \frac{\mathbb{E}[y]}{\mathbb{E}[y^*]} y^* = -\frac{w^2}{a} \left(\frac{1-\xi}{\xi} + \frac{1-\mu}{\mu} \frac{1}{\log \rho} \right) \xrightarrow{\rho \to 1} +\infty$$

since $w \to a$ as $\rho \to 1$. Therefore, the optimal ρ when y/c is chosen optimally has the same properties that we used before holding x constant. In particular, the optimal replacement rate is unique and strictly below one. Also, we have $d\tilde{H}/d\rho > dI/d\rho$ at the optimal ρ as in Proposition (3). Taking the total derivative again, we have

$$\frac{d\rho}{da} = -\frac{\frac{\partial I}{\partial a} - \frac{\partial \tilde{H}}{\partial a}}{\frac{\partial I}{\partial \rho} - \frac{\partial \tilde{H}}{\partial \rho}}$$

where the denominator of the fraction is negative, so $d\rho/da$ has the same sign as $\frac{\partial I}{\partial a} - \frac{\partial \tilde{H}}{\partial a}$. The derivatives of I and $\tilde{H}$ with respect to productivity a are:

$$\begin{aligned} \frac{\partial I}{\partial a} &= \frac{\partial I}{\partial n} \frac{\partial n}{\partial a} \\ &= \frac{\partial n}{\partial a} \left(\frac{1-\rho}{n + (1-n)\rho} \right)^2 \left(\frac{w}{a} \frac{1-\mu}{\mu} \frac{1}{\log \rho} - \frac{1}{1-\rho} \right) < 0 \end{aligned}$$

and

$$\frac{\partial \tilde{H}}{\partial a} = \frac{\tilde{H}}{a} > 0.$$

Therefore $d\rho/da < 0$. □

The last two propositions combined establish the final result of Section 3. A decline in a raises ρ, and at the same time output y falls. An increase in a^* also raises ρ, and the lower replacement rate causes y to fall. Therefore, the replacement rate is countercyclical conditional on either productivity. Independence of the productivities then implies that it is also countercyclical overall.