Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.

The Risk-Adjusted Monetary Policy Rule

Taisuke Nakata and Sebastian Schmidt

2016-061

Please cite this paper as:
Nakata, Taisuke, and Sebastian Schmidt (2016). "The Risk-Adjusted Monetary Policy Rule," Finance and Economics Discussion Series 2016-061. Washington: Board of Governors of the Federal Reserve System, http://dx.doi.org/10.17016/FEDS.2016.061.

The Risk-Adjusted Monetary Policy Rule*

Taisuke Nakata† Federal Reserve Board

Sebastian Schmidt‡ European Central Bank

First Draft: July 2016

Abstract

Macroeconomists are increasingly using nonlinear models to account for the effects of risk in the analysis of business cycles. In the monetary business cycle models widely used at central banks, an explicit recognition of risk generates a wedge between the inflation-target parameter in the monetary policy rule and the risky steady state (RSS) of inflation—the rate to which inflation will eventually converge—which can be undesirable in some practical applications. We propose a simple modification to the standard monetary policy rule to eliminate the wedge. In the proposed *risk-adjusted* policy rule, the intercept of the rule is modified so that the RSS of inflation equals the inflation-target parameter in the policy rule.

JEL: E32, E52

Keywords: Effective Lower Bound, Inflation Targeting, Monetary Policy Rule, Risk, Risky Steady State.

*We would like to thank Mike Kiley, Matthias Paustian, John Roberts, Hiroatsu Tanaka, Bob Tetlow, and seminar participants at the Board of Governors of the Federal Reserve System for helpful discussions. Paul Yoo provided excellent research assistance. The views expressed in this note, and all errors and omissions, should be regarded as those solely of the author and are not necessarily those of the Federal Reserve Board of Governors, the Federal Reserve System, or the European Central Bank.

†Division of Research and Statistics, Federal Reserve Board, 20th Street and Constitution Avenue N.W. Washington, DC 20551; Email: taisuke.nakata@frb.gov

‡European Central Bank, Monetary Policy Research Division, 60640 Frankfurt, Germany; Email: sebastian.schmidt@ecb.int.

1 Introduction

Macroeconomists are increasingly using nonlinear stochastic models to account for the effects of risk in the analyses of business cycles. In nonlinear models, an explicit recognition of risk creates a discrepancy between the deterministic steady state (DSS) and risky steady state (RSS).[1] In monetary business cycle models, this discrepancy implies a wedge between the inflation-target parameter in the monetary policy rule and the RSS of inflation—the rate to which inflation will eventually converge once all headwinds and tailwinds dissipate. Unless it is explicitly taken account of, this wedge means that inflation persistently deviates from the central bank's inflation objective, which can be undesirable depending on the purpose of the analysis.

In this paper, we propose a simple modification to the standard policy rule—the *risk-adjusted* policy rule—to eliminate the wedge. In the proposed modification, the intercept of the policy rule is adjusted so that the RSS of inflation equals the inflation-target parameter in the policy rule. We show that this modification is mathematically equivalent to adjusting the value assigned to the inflation-target parameter in the standard policy rule, but has the advantage that the inflation-target parameter can be interpreted structurally as the central bank's inflation objective.

This paper is organized as follows. In section 2, we illustrate the wedge between the inflation-target parameter in the policy rule and the RSS of inflation that arises in nonlinear stochastic economies with standard monetary policy rules, using as a laboratory a model with an occasionally binding effective lower bound (ELB) constraint on nominal interest rates. In section 3, we describe a risk-adjusted policy rule that eliminates this wedge with an appropriate intercept adjustment. Section 4 puts the paper into broader context by relating it to the discussion of how to interpret the central bank's inflation objective and by highlighting other applications where our risk-adjusted policy rule is useful. Section 5 concludes.

2 A wedge

We use an empirical model considered in Hills, Nakata, and Schmidt (2016b) to illustrate the wedge between the RSS of inflation and the inflation-target parameter in the policy rule on the one hand and our proposed approach to eliminating it on the other. Our model is a standard sticky-price model with an occasionally binding ELB constraint on nominal interest rates, consumption habits, sticky wages, and a non-stationary TFP process. While the model is more complicated than necessary to illustrate the wedge, as well as our proposed approach to eliminating it, we believe that the use of an empirically plausible model is useful in highlighting the quantitative relevance of the wedge. The details of the model and the calibration are relegated to the Appendix.[2]

The policy rule in the model is given by a standard Taylor rule truncated below at the ELB:

[1] The DSS of a model is the steady state associated with the version of the model without uncertainty. The RSS of a model is a point to which the economy eventually converges when all headwinds and tailwinds dissipate. For a formal definition, see Coeurdacier, Rey, and Winant (2011) and Hills, Nakata, and Schmidt (2016b). In linear models, the DSS and RSS coincide because certainty equivalence holds.

[2] We use a version of the baseline empirical model augmented with total factor productivity (TFP) shocks that is discussed in Appendix E of Hills, Nakata, and Schmidt (2016b). The same model is used in Hills, Nakata, and Schmidt (2016a), a nontechnical summary of Hills, Nakata, and Schmidt (2016b).

$$R_t = \max\left[R_{ELB}, R_t^*\right],$$

where R_t is the short-term nominal interest rate, R_{ELB} is the ELB, and R_t^* is the notional short-term nominal interest rate. The notional rate is given by

$$R_t^* = R_{DSS}\left(\frac{R_{t-1}^*}{R_{DSS}}\right)^{\rho_R}\left(\frac{\Pi_t^p}{\Pi^{targ}}\right)^{(1-\rho_R)\phi_\pi}\left(\frac{\tilde{Y}_t}{\tilde{Y}_{DSS}}\right)^{(1-\rho_R)\phi_y}, \tag{1}$$

where Π_t^p is the rate of price inflation, $\tilde{Y}_t$ is detrended output (i.e., output divided by the level of TFP), and Π^{targ} is the inflation-target parameter. The DSS of inflation equals Π^{targ}.

R_{DSS} is the deterministic steady state of R_t and is a function of structural parameters:

$$R_{DSS} = \frac{a^{\chi_c}\Pi^{targ}}{\beta}, \tag{2}$$

where a is the trend growth rate of technology, β is the discount rate of the household, and χ_c is the inverse intertemporal elasticity of substitution for consumption goods. We will refer to R_{DSS} as the intercept of the standard policy rule, as this term shows up as the intercept in the linearized version of the standard policy rule. The intercept term is given by equation (2) in the monetary policy rule in almost all DSGE (Dynamic Stochastic General Equilibrium) models.[3]

$\tilde{Y}_{DSS}$ is the deterministic steady state of $\tilde{Y}_t$ and is a function of structural parameters:

$$\tilde{Y}_{DSS} = \left(\frac{(\theta_p - 1)(\theta_w - 1)}{\theta_p\theta_w}\right)^{\frac{1}{\chi_c+\chi_n}},$$

where θ_p is the elasticity of substitution among intermediate goods, θ_w is the elasticity of substitution among intermediate labor inputs, and χ_n is the inverse labor supply elasticity. Note that $\tilde{Y}_{DSS}$ does not depend on the inflation-target parameter because firms' price-adjustment costs are assumed to depend on the deviation of inflation from the inflation-target parameter in our model.

Hills, Nakata, and Schmidt (2016b) show that the RSS inflation of this model is non-trivially below Π^{targ}, as the tail risk in future marginal costs induced by the possibility of hitting the ELB constraint exerts downward pressures on inflation. Table 1a shows the risky and deterministic steady state values of inflation, the output gap, and the policy rate from their empirical model in which Π^{targ} is set to 2 percent, the target rate of inflation in many advanced economies. Inflation falls 29 basis points below the target rate of inflation at the RSS. Note that the wedge between the RSS of inflation and the inflation-target parameter is non-trivial even in the model without the ELB, indicating the pervasiveness of the wedge in nonlinear economies.

To provide an alternative look at this undershooting result, we plot the modal projections of the policy rate, inflation, and the output gap from the model after a combination of demand and TFP shocks push down the policy rate to the ELB. The magnitudes of the two shocks are chosen so that inflation is 50 basis points and the output gap is minus 7 percent at time one. The model projects inflation to return to a level below 2 percent. If an economist wants

[3]To our knowledge, all published papers on monetary DSGE models specify the intercept of the monetary policy rule in this way.

Table 1: **The Risky Steady State in a Model with ELB**

(a) **With the Standard Policy Rule**

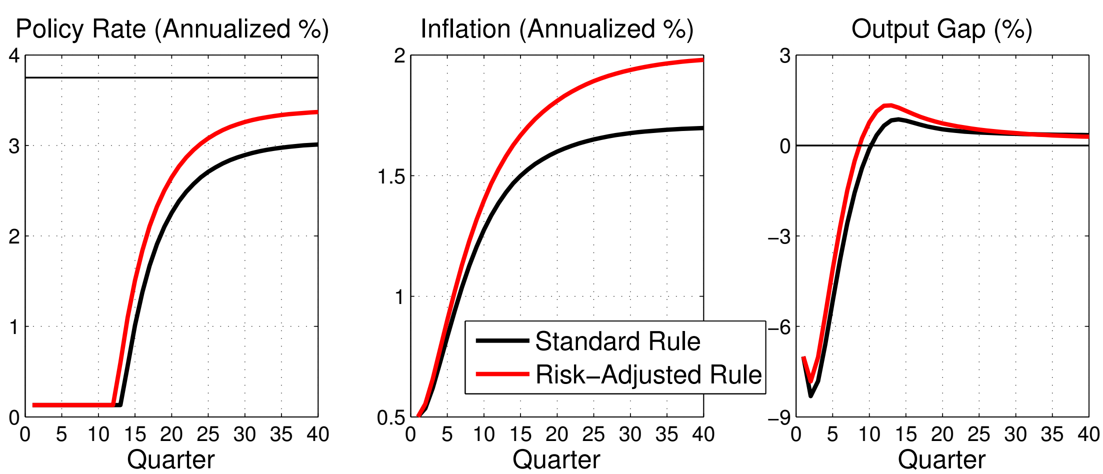
	Inflation	Output Gap	Policy Rate
Deterministic steady state	2	0	3.75
Risky steady state	1.71	0.32	3.04
(Wedge)	(−0.29)	(0.32)	(−0.71)
$E[\cdot]$	1.66	0.29	3.17
Prob($R_t = R_{ELB}$)	16%		
Risky steady state w/o the ELB	1.88	0.04	3.37
(Wedge)	(−0.12)	(0.04)	(−0.38)

(b) **With the Risk-Adjusted Policy Rule**

	Inflation	Output Gap	Policy Rate
Risky steady state	2	0.22	3.39
$E[\cdot]$	1.97	0.24	3.50
Adjusted Intercept			3.17
Prob($R_t = R_{ELB}$)	11.9%		

to conduct policy analyses using a model in which inflation eventually returns to the central bank's inflation objective, she or he needs to find a sensible way to modify the policy rule to avoid this permanent undershooting of the inflation objective.

Figure 1: **Modal Forecasts from a Model with ELB**



[†]The thin black horizontal line in the left panel at 3.75 is the deterministic steady state of the policy rate.

3 The risk-adjusted policy rule

3.1 The proposed rule

We propose to eliminate the wedge between the inflation-target parameter and the RSS of inflation by adjusting the intercept of the policy rule. Let the *intercept-adjusted* policy rule be given by

$$R_t^* = S_R \frac{a^{\chi_c}\Pi^{targ}}{\beta} \left(\frac{R_{t-1}^*}{S_R \frac{a^{\chi_c}\Pi^{targ}}{\beta}} \right)^{\rho_R} \left(\frac{\Pi_t^p}{\Pi^{targ}} \right)^{(1-\rho_R)\phi_\pi} \left(\frac{\tilde{Y}_t}{\tilde{Y}_{DSS}} \right)^{(1-\rho_R)\phi_y}, \tag{3}$$

where S_R that appears in front of $\frac{a^{\chi_c}\Pi^{targ}}{\beta}$ is the intercept adjustment term and $S_R \frac{a^{\chi_c}\Pi^{targ}}{\beta}$ is the adjusted intercept.[4] We call S_R an intercept-adjustment term because this term would show up as the intercept in the linearized version of the policy rule.

A key feature of equation (3) is that the presence of the intercept-adjustment parameter breaks the standard link between the intercept and the model's structural parameters. When the standard policy rule is specified in the context of structural models, the intercept is a function of the model's structural parameters—a, β, χ_c, and Π^{targ} in our model—as seen in equations (1) and (2). Thus, under the standard policy rule, one needs to adjust at least one of the structural parameters of the model to change the intercept. Under the intercept-adjusted policy rule, the intercept of the rule is a free parameter of the model.

Our proposal is to use the intercept-adjusted policy rule with the value for S_R chosen so that the RSS inflation and the inflation-target parameter coincide. We will refer to the intercept-adjusted policy rule with S_R so chosen as the *risk-adjusted* policy rule. The size of the intercept adjustment that equates the RSS of inflation to the inflation-target parameter depends on the specifics of the model and needs to be found numerically. Figure 2 shows how the RSS of inflation changes with the adjusted intercept in the model of Hills, Nakata, and Schmidt (2016b). Since the RSS of inflation is below 2 percent under the standard policy rule, the necessary adjustment would be to lower the intercept, as a lower intercept implies, all else being equal, a more accommodative policy stance, generating upward pressures on inflation. According to the figure, lowering the intercept by about 60 basis point (from 3.75 to 3.17 percent) leads to the RSS inflation of 2 percent.

Table 1b reports the RSS of the model with ELB and the risk-adjusted policy rule. By construction, under the risk-adjusted policy rule, the RSS of inflation is 2 percent. Even though the intercept in the policy rule is lower in the risk-adjusted policy rule than in the standard policy rule, the RSS policy rate is higher—3.04 versus 3.39 percent—reflecting a higher RSS inflation. The RSS output gap is positive, but it is a bit lower under the risk-adjusted policy rule than under the standard policy rule. The lower RSS output gap is due to the lower frequency of being at the ELB (11.9 percent under the risk-adjusted rule versus 16 percent under the standard rule), which in turn is due to the higher RSS policy rate. Note that the RSS policy rate is higher than the intercept of the policy rule (3.39 percent versus 3.17 percent) because the output gap is positive at the RSS. To visualize the difference in

[4] An equivalent policy rule under which the policy rate responds to the deviation of the detrended output from its risky steady state counterpart is

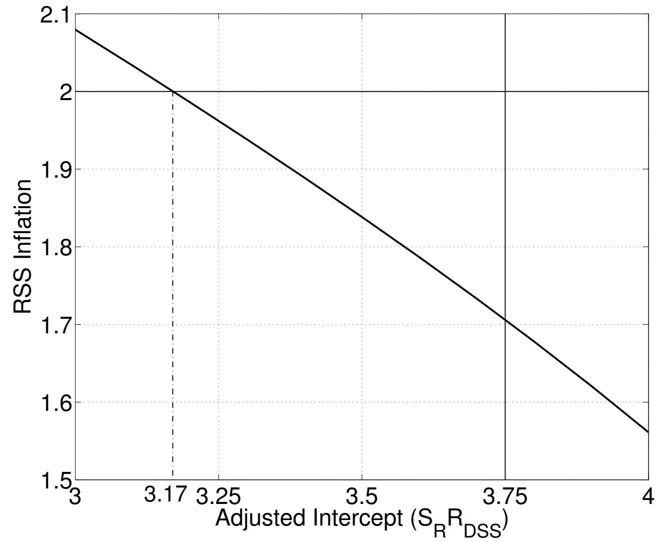
$$R_t^* = \hat{S}_R \frac{a^{\chi_c}\Pi^{targ}}{\beta} \left(\frac{R_{t-1}^*}{\hat{S}_R \frac{a^{\chi_c}\Pi^{targ}}{\beta}} \right)^{\rho_R} \left(\frac{\Pi_t^p}{\Pi^{targ}} \right)^{(1-\rho_R)\phi_\pi} \left(\frac{\tilde{Y}_t}{\tilde{Y}_{RSS}} \right)^{(1-\rho_R)\phi_y},$$

where

$$\hat{S}_R = S_R \left(\frac{\tilde{Y}_{DSS}}{\tilde{Y}_{RSS}} \right)^{-\phi_y}.$$

This representation of the policy rule may first seem appealing as the output deviation term is zero at the RSS by construction. However, this representation is not practically useful for modelers as the RSS output level is not known to the modelers before the model is solved.

Figure 2: **The Risky Steady State Inflation and the Adjusted Intercept**



Notes: The figure shows how the RSS inflation varies with the adjusted intercept. Both the RSS inflation and the adjusted intercept are expressed in annualized percent.

the RSS between the models with standard and risk-adjusted rules, solid red lines in figure 1 show the modal projections of the policy rate, inflation, and the output gap from the model with the risk-adjusted policy rule where the magnitudes of the shocks are chosen so that inflation is 50 basis points at time one and the output gap is minus 7 percent.

To better understand how our proposed method works, it is useful to graphically illustrate how the intercept adjustment affects the RSS of inflation and the policy rate. Figure 3 plots the standard Fisher relation, the risk-adjusted Fisher relation, the standard policy rule, and the risk-adjusted policy rule. The risky (deterministic) steady state of the model with the standard policy rule is given by the intersection of the risk-adjusted (standard) Fisher relation and the standard policy rule. The RSS of the model with the risk-adjusted policy rule is given by the intersection of the risk-adjusted Fisher relation and the risk-adjusted policy rule.

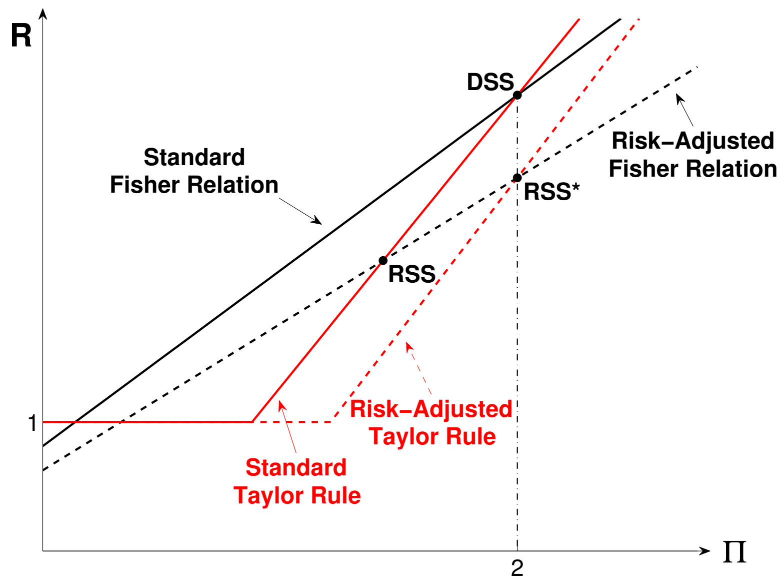
According to the figure, and as discussed in detail in Hills, Nakata, and Schmidt (2016b), the presence of risk lowers the line representing the Fisher relation, pushing down the steady-state inflation rate in the absence of an adjustment in the policy rule. The risk adjustment in the policy rule lowers the policy rate corresponding to a given inflation rate, pushing up RSS inflation back to the level consistent with the inflation-target parameter.

3.2 Comparison with the inflation-target parameter adjustment

The intercept adjustment in equation (3) is mathematically equivalent to an adjustment in the value assigned to the inflation-target parameter in the standard policy rule. Let Π^{CB} be the value of the central bank's inflation objective. The standard policy rule with an adjusted value for the inflation-target parameter is given by

$$R_t^* = \frac{a^{\chi_c} S_\Pi \Pi^{CB}}{\beta} \left(\frac{R_{t-1}^*}{\frac{a^{\chi_c} S_\Pi \Pi^{CB}}{\beta}} \right)^{\rho_R} \left(\frac{\Pi_t^p}{S_\Pi \Pi^{CB}} \right)^{(1-\rho_R)\phi_\pi} \left(\frac{\tilde{Y}_t}{\tilde{Y}_{DSS}} \right)^{(1-\rho_R)\phi_y},$$

Figure 3: **The Risk-Adjusted Fisher Relation and the Risk-Adjusted Taylor Rule**



†DSS stands for "deterministic steady state," and RSS stands for "risky steady state."

where S_Π is the inflation-target adjustment term and $S_\Pi \Pi^{CB}$ is the adjusted value assigned to the inflation-target parameter. This approach has a precedent. With the interpretation that the model's unconditional average of inflation corresponds to the central bank's inflation objective, Reifschneider and Williams (2000) proposed an upward adjustment in the value assigned to the inflation-target parameter in the standard policy rule so that the model's unconditional average of inflation is 2 percent, which is the Federal Reserve's inflation objective.

To see the equivalence between the intercept-adjusted policy rule with $\Pi^{targ} = \Pi^{CB}$ and the standard policy rule with an adjusted value assigned to the inflation-target parameter ($\Pi^{targ} = S_\Pi \Pi^{CB}$), notice that there is one-to-one mapping between the intercept-adjustment term and the inflation-target adjustment term:[5]

$$S_R = S_\Pi^{1-\phi_\pi}. \tag{4}$$

When the Taylor principle is satisfied (i.e., $\phi_\pi > 1$), this relationship implies that an adjustment to lower the intercept (i.e., a lower S_R) is equivalent to an adjustment to increase the inflation-target parameter (i.e., a higher S_Π).[6] Figure 4 shows this mapping between the

[5]Note that this mapping holds when price-adjustment costs depend on the deviation of inflation from the inflation-target parameter and $\tilde{Y}_{DSS}$ does not depend on the inflation-target parameter, $S_\Pi \Pi^{targ}$. This condition holds true for our model. In models in which $\tilde{Y}_{DSS}$ depends on the value assigned to the inflation-target parameter, the mapping is given by $S_R = S_\Pi^{1-\phi_\pi} \left(\frac{\tilde{Y}_{DSS}(S_\Pi \Pi^{CB})}{\tilde{Y}_{DSS}(\Pi^{CB})} \right)^{-\phi_y}$, where $\tilde{Y}_{DSS}(\cdot)$ is a function mapping the inflation-target parameter to the steady state output.

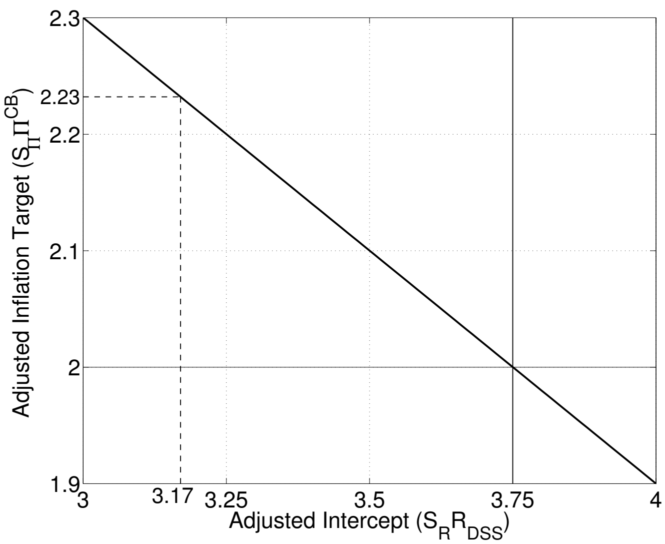
[6]In a log-linearized policy rule, this mapping becomes

$$s_r \cong (1-\phi_\pi) s_\pi,$$

where $s_r := \log(S_R)$ and $s_\pi := \log(S_\Pi)$. See the appendix for more details on the risk-adjusted policy rule in linear form.

adjusted intercept and the adjusted inflation-target parameter in our model.[7]

Figure 4: **Mapping Between the Adjusted Intercept and the Adjusted Inflation-Target Parameter**



Note: The figure shows the mapping between the adjusted intercept and the adjusted inflation-target parameter. Both the adjusted intercept and the adjusted inflation-target parameter are expressed in annualized percent.

As a result of the mathematical equivalence between the intercept-adjusted policy rule and the standard policy rule with an adjusted value assigned to the inflation-target parameter, there are two ways in which modelers can arrive at the right size of the intercept adjustment for the risk-adjusted policy rule. One is to solve the model with the intercept-adjusted policy rule and iterate on S_R until the RSS of inflation equals the desirable inflation objective. The other is to solve the model with the standard policy rule, iterate on S_Π until the RSS of inflation equals the desirable inflation objective, and translate the inflation-target adjustment into the intercept adjustment using equation (4).

An advantage of the risk-adjusted policy rule over the standard policy rule with an adjusted value assigned to the inflation-target parameter is that the former allows for a simple structural interpretation of the inflation-target parameter. In the risk-adjusted policy rule, the inflation-target parameter can be naturally interpreted as the central bank's inflation objective because the RSS of inflation coincides with the value of the inflation-target parameter. In the standard policy rule, the RSS of inflation is different from the value assigned to the inflation-target parameter unless the model is linear, complicating the interpretation of the inflation-target parameter. In our model, assigning the value of 2.23 percent to the inflation-target parameter achieves the RSS inflation of 2 percent, as seen in figure 4. However, in this setup, the value assigned to the inflation-target parameter is simply a number that allows the model to generate the RSS of inflation so that it is consistent with the central bank's inflation objective and does not have any structural interpretation.

The desirability of being able to interpret the inflation-target parameter as the central bank's inflation objective depends on the purpose of the analysis. In many estimation exer-

[7] In our model, $\phi_\pi = 3.5$.

cises using U.S. data, the estimated value for the inflation-target parameter can substantially differ from 2 percent, depending on the sample used for estimation. The discrepancy between the estimated value and the Federal Reserve's target rate of 2 percent may not pose any issue if the goal of estimation is only to fit the data and understand the past.

However, if the goal of the analysis is to think about the implications of alternative policy rules for the economic outlook and if such analyses are used to inform policymakers, it is typically useful—from the perspective of communication between modelers and policymakers—if the inflation-target parameter in the policy rule can be simply interpreted as the central bank's inflation objective. For example, in the EDO Model—short for Estimated Dynamic Optimization-based Model—which is used at the Board of Governors of the Federal Reserve System for various policy analyses, the inflation-target parameter in the policy rule is set to 2 percent, the Federal Reserve's target rate of inflation, to allow for a simple interpretation of the inflation-target parameter (Chung, Kiley, and Laforte (2010)).[8] Similarly, in the New Area-Wide Model of the Euro Area, the DSGE model used for policy analyses at the European Central Bank, the inflation-target parameter is interpreted as the monetary authority's long-run inflation objective and is set to 1.9 percent, which is "consistent with the ECB's quantitative definition of price stability of inflation below, but close to 2 percent" (Christoffel, Coenen, and Warne (2008)).[9]

4 Discussion

4.1 Interpreting the central bank's inflation objective

Throughout the paper, we interpret the central bank's inflation objective as specifying the desired level of the RSS inflation. Under this interpretation, the dynamics of inflation are consistent with the central bank's inflation objective of x percent if the RSS of inflation is x percent. Accordingly, our focus in Section 3 was on how to modify the policy rule to eliminate the wedge between the RSS inflation and the central bank's inflation objective. However, some have interpreted the central bank's inflation objective as specifying the average rate of inflation over a long period of time. Under this alternative interpretation, the dynamics of inflation are consistent with the central bank's inflation objective of x percent if the unconditional average of inflation is x percent. The paper by Reifschneider and Williams (2000) is a prominent example adopting this interpretation. In our model, the unconditional average of inflation is also below the target rate of 2 percent, as shown in table 1a. Thus, the need for adjusting the policy rule can also arise under this alternative interpretation of the central bank's inflation objective.

Our intercept-adjustment procedure can be easily modified if modelers take this alternative interpretation and want to set the unconditional average of inflation, as opposed to

[8] Chung, Kiley, and Laforte (2010) explain their calibration choice by saying, "some important determinants of steady-state behavior were calibrated to yield growth rates of GDP and associated price indexes that corresponded to "conventional" wisdom in policy circles, even though slight deviations from such values would have been preferred (in a "statistically significant" way) to our calibrated values."

[9] These models are used for policy analyses in the context of the ELB on nominal interest rates (see, for instance, Chung, Laforte, Reifschneider, and Williams (2012) and Coenen and Warne (2014)). Due to the computational difficulty of globally solving large-scale DSGE models, the models are currently solved and simulated using solution methods that rely on the assumption that certainty equivalence holds ("perfect-foresight" assumption). As a result of this assumption, the risky steady state coincides with the deterministic steady state in these applications.

the RSS of inflation, to the central bank's inflation objective. Specifically, modelers would need to search for the size of the intercept adjustment such that the unconditional average of inflation, instead of the RSS inflation, equals the central bank's inflation objective. In our example, the intercept that achieves the unconditional average of inflation of 2 percent is lower than the intercept that achieves the RSS inflation of 2 percent, because the unconditional average of inflation is lower than the RSS inflation in the model with the standard policy rule, as shown in table 1a.

While our intercept-adjustment procedure is useful regardless of the interpretation of the central bank's inflation objective adopted by modelers, it is perhaps appropriate to explain why it is more plausible to interpret the central bank's inflation objective as specifying the desired level of the RSS inflation than as specifying the desired unconditional average of inflation.

The interpretation that the central bank's inflation objective specifies the desired unconditional average of inflation is inconsistent with the inflation projections by U.S. policymakers in the following sense. In the United States, both core and overall PCE inflation rates averaged non-trivially below 2 percent over the last decade or so. Thus, to achieve the average inflation rate of 2 percent over a long period of time, policymakers will need to overshoot the target rate non-trivially and persistently in the future. However, according to recent releases of the Summary of Economic Projections (SEP), U.S. policymakers expect inflation to return to 2 percent in the long-run without any overshooting.[10] As shown in figure 1, in the aftermath of a recession, the model's inflation projection monotonically increases with forecast horizon and eventually converges to the RSS inflation. Thus, the eventual return of the inflation projection to 2 percent without any overshooting in the SPE is consistent with the interpretation that the central bank's inflation objective specifies the desired level of the RSS inflation.

In a similar vein, Draghi (2016) states that "In the ECB's case, our aim is to keep inflation below but close to 2 percent over the medium term. Today, this means raising inflation back towards 2 percent." Thus, even though inflation in the euro area, as measured by the HICP, averaged below 2 percent over recent years, ECB policymakers do not seem to interpret their mandate to be consistent with aiming for a transitory overshooting of inflation rates close to 2 percent.

4.2 Other applications

While we used a model with an occasionally binding ELB constraint to illustrate our proposed modification to the monetary policy rule, our proposal is useful in a wide variety of nonlinear models.

As briefly discussed in section 2, the difference between the DSS and RSS are nontrivial in our model even without the ELB constraint. In our experience, the difference between the DSS and RSS can be quite large in New Keynesian models with recursive preferences often used to analyze the dynamics of term premiums or the effects of uncertainty shocks. Our proposed intercept-adjustment will be useful in such applications even if the model abstracts from the ELB constraint.

In addition to the ELB constraint on nominal interest rates, researchers are increasingly interested in the implications of other inequality constraints in macroeconomic models. For

[10]See, for example, the Summary of Economic Projections released on June 15, 2016 (available at `www.federalreserve.gov/monetarypolicy/files/fomcprojtabl20160615.pdf`).

example, since the Great Recession, the literature on financial frictions has been developing rapidly, examining the implications of occasionally binding borrowing constraints on the household or leverage constraints on banks. Our risk-adjusted policy rule is likely to be useful in models featuring these other inequality constraints.

4.3 Contrast with linear models

The issue described in this paper has not received much attention because linear monetary DSGE models have been predominant tools for model-based analyses of monetary policy. In linear monetary DSGE models, the inflation-target parameter in the policy rule coincides with the RSS of inflation and the unconditional average of inflation. Researchers typically set the inflation-target parameter to the central bank's inflation objective, and because of this coincidence, there is less need to think about conceptual differences among these objects. Our analysis highlights the importance and difficulty of understanding conceptual differences among the central bank's objective, the inflation-target parameter, and the RSS and the unconditional average of inflation, as nonlinear models become more widely used in the analyses of monetary policy.

5 Conclusion

We have proposed a simple method to close the wedge between the inflation-target parameter in the policy rule and the RSS of inflation in nonlinear stochastic monetary DSGE models. The proposed method is to numerically find the right adjustment in the intercept of the risk-adjusted policy rule. While this method is mathematically equivalent to adjusting the inflation-target parameter in the standard policy rule, it has the advantage that it allows modelers to align the inflation-target parameter in the policy rule with the central bank's inflation objective.

The Great Recession has provided macroeconomists with a new set of challenging questions. Properly addressing some of these questions requires macroeconomists to go beyond linear models. While we use a model with an occasionally binding ELB constraint as a laboratory to illustrate our proposed method, we expect our method to be useful in a broad set of nonlinear models.

References

Christoffel, K., G. Coenen, and A. Warne (2008): "The New Area-Wide Model of the Euro Area: A Micro-Founded Open-Economy Model for Forecasting and Policy Analysis," ECB Working Paper Series 944, European Central Bank.

Chung, H., M. Kiley, and J.-P. Laforte (2010): "Documentation of the Estimated, Dynamic, Optimization-based (EDO) Model of the U.S. Economy: 2010 Version," Finance and economics discussion series, Board of Governors of the Federal Reserve System (U.S.).

Chung, H., J.-P. Laforte, D. Reifschneider, and J. C. Williams (2012): "Have We Underestimated the Likelihood and Severity of Zero Lower Bound Events?," *Journal of Money, Credit and Banking*, 44(s1), 47–82.

Coenen, G., and A. Warne (2014): "Risks to Price Stability, the Zero Lower Bound and Forward Guidance: A Real-Time Assessment," *International Journal of Central Banking*, 10(2), 7–54.

Coeurdacier, N., H. Rey, and P. Winant (2011): "The Risky Steady State," *American Economic Review*, 101(3), 398–401.

Draghi, M. (2016): "Delivering a Symmetric Mandate with Asymmetric Tools: Monetary Policy in a Context of Low Interest Rates," *Remarks at the Ceremony to Mark the 200th Anniversary of the Oesterreichische Nationalbank, Vienna.*

Hills, T. S., T. Nakata, and S. Schmidt (2016a): "The Risky of Returning to the Effective Lower Bound: An Implication for Inflation Dynamics after Lift-Off," Feds notes, Board of Governors of the Federal Reserve System (U.S.).

——— (2016b): "The Risky Steady State and the Interest Rate Lower Bound," Finance and Economics Discussion Series 2016-009, Board of Governors of the Federal Reserve System (U.S.).

Reifschneider, D., and J. C. Williams (2000): "Three Lessons for Monetary Policy in a Low-Inflation Era," *Journal of Money, Credit and Banking*, 32(4), 936–966.

For Online Publication: Technical Appendix

A The Risk-Adjusted Policy Rule in Linear Form

By taking the log of the standard policy rule, we obtain the following linear policy rule:

$$\begin{aligned} r_t^* = &\left[\chi_c \log(a) + \pi^{targ} - \log(\beta)\right] \\ &+ \rho_R \left(r_{t-1}^* - \left[\chi_c \log(a) + \pi^{targ} - \log(\beta)\right]\right) \\ &+ (1-\rho_R)\phi_\pi(\pi_t^p - \pi^{targ}) \\ &+ (1-\rho_R)\phi_y \tilde{y}_t, \end{aligned}$$

where $x := \log(X)$ for $x \in [r_t^*, \pi_t, \pi^{targ}]$ and $\tilde{y}_t := \log(\tilde{Y}_t) - \log(\tilde{Y}_{DSS})$.

The intercept-adjusted policy rule in linear form is given by

$$\begin{aligned} r_t^* = &\left[s_r + \chi_c \log(a) + \pi^{targ} - \log(\beta)\right] \\ &+ \rho_R \left(r_{t-1}^* - \left[s_R + \chi_c \log(a) + \pi^{targ} - \log(\beta)\right]\right) \\ &+ (1-\rho_R)\phi_\pi(\pi_t^p - \pi^{targ}) \\ &+ (1-\rho_R)\phi_y \tilde{y}_t, \end{aligned}$$

where $s_r := \log(S_R)$.

The standard policy rule with an adjusted value assigned to the inflation-target parameter in linear form is given by

$$\begin{aligned} r_t^* = &\left[\chi_c \log(a) + s_\pi + \pi^{CB} - \log(\beta)\right] \\ &+ \rho_R \left(r_{t-1}^* - \left[s_R + \chi_c \log(a) + s_\pi + \pi^{CB} - \log(\beta)\right]\right) \\ &+ (1-\rho_R)\phi_\pi(\pi_t^p - (s_\pi + \pi^{CB})) \\ &+ (1-\rho_R)\phi_y \tilde{y}_t, \end{aligned}$$

where Π^{CB} is the central bank's inflation objective, $\pi^{CB} := \log(\Pi^{CB})$, and $s_\pi := \log(S_\Pi)$.

The mapping between s_r and s_π is given by

$$s_r = (1 - \phi_\pi) s_\pi.$$

Let Π^{adj} (and $\pi^{adj} := \log(\Pi^{adj})$) denote the adjusted value assigned to the inflation-target parameter. That is,

$$\Pi^{adj} := S_\Pi \Pi^{CB}.$$

Then,

$$s_\pi = \pi^{adj} - \pi^{CB}$$

and the mapping of the intercept adjustment and the inflation-target adjustment can be written as

$$s_r = (1 - \phi_\pi)(\pi^{adj} - \pi^{CB})$$

B Details of the Model

This section describes an extension of the stylized model with a representative household, a final good producer, a continuum of intermediate goods producers with unit measure, and the government.

B.1 Household markets

B.1.1 Labor packer

The labor packer buys labor $N_{h,t}$ from households at their monopolistic wage $W_{h,t}$ and resells the packaged labor N_t to intermediate goods producers at W_t. The problem can be written as

$$\max_{N_{h,t},h\in[0,1]} W_t N_t - \int_0^1 W_{h,t} N_{h,t} df$$

subject to the following CES technology

$$N_t = \left[\int_0^1 N_{h,t}^{\frac{\theta^w-1}{\theta^w}} dh\right]^{\frac{\theta^w}{\theta^w-1}}.$$

The first order condition implies a labor demand schedule

$$N_{h,t} = \left[\frac{W_{h,t}}{W_t}\right]^{-\theta^w} N_t.^{11}$$

θ^w is the wage markup parameter.

B.1.2 Household

The representative household chooses its consumption level, amount of labor, and bond holdings so as to maximize the expected discounted sum of utility in future periods. As is common in the literature, the household enjoys consumption and dislikes labor. Assuming that period utility is separable, the household problem can be defined by

$$\max_{C_{h,t},w_{h,t},B_{h,t}} \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[\prod_{s=0}^{t-1} \delta_s\right] \left[\frac{(C_{h,t} - \zeta C_{t-1}^a)^{1-\chi_c}}{1-\chi_c} - A_t^{1-\chi_c} \frac{N_{h,t}^{1+\chi_n}}{1+\chi_n}\right]$$

subject to the budget constraint

$$P_t C_{h,t} + R_t^{-1} B_{h,t} \leq W_{h,t} N_{h,t} - W_t \frac{\varphi_w}{2} \left[\frac{W_{h,t}}{a W_{h,t-1} \left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_{t-1}^w\right)^{\iota_w}} - 1\right]^2 N_t + B_{h,t-1} + P_t \Phi_t - P_t T_t$$

or equivalently

$$C_{h,t} + \frac{B_{h,t}}{R_t P_t} \leq w_{h,t} N_{h,t} - w_t \frac{\varphi_w}{2} \left[\frac{w_{h,t}}{a w_{h,t-1}} \frac{\Pi_t^p}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_{t-1}^w\right)^{\iota_w}} - 1\right]^2 N_t + \frac{B_{h,t-1}}{P_t} + \Phi_t - T_t$$

[11] This implies that the labor packer will set the wage of the packaged labor to $W_t = \left[\int_0^1 W_{h,t}^{1-\theta^w} dh\right]^{\frac{1}{1-\theta^w}}$.

and subject to the labor demand schedule

$$N_{h,t} = \left[\frac{W_{h,t}}{W_t}\right]^{-\theta^w} N_t.$$

–or equivalently

$$N_{h,t} = \left[\frac{w_{h,t}}{w_t}\right]^{-\theta^w} N_t.$$

where $C_{h,t}$ is the household's consumption, $N_{h,t}$ is the labor supplied by the household, P_t is the price of the consumption good, $W_{h,t}$ ($w_{h,t}$) is the nominal (real) wage set by the household, W_t (w_t) is the market nominal (real) wage, Φ_t is the profit share (dividends) of the household from the intermediate goods producers, $B_{h,t}$ is a one-period risk free bond that pays one unit of money at period t+1, T_t are lump-sum taxes or transfers, and R_t^{-1} is the price of the bond. C_{t-1}^a represents the aggregate consumption level from the previous period that the household takes as given. The parameter $0 \leq \zeta < 1$ measures how important these external habits are to the household. Because we are including wage indexation, measured by the parameter ι_w, we assume the household takes as given the previous period wage inflation, Π_{t-1}^w, where $\Pi_t^w = \frac{W_t}{aW_{t-1}} = \frac{w_t P_t}{aw_{t-1}P_{t-1}} = \frac{w_t}{aw_{t-1}}\Pi_t^p$. $\bar{\Pi}^w$ is the target rate of wage inflation, which is set to equal to $\bar{\Pi}^p$, the inflation-target parameter in the policy rule.

The discount rate at time t is given by $\beta\delta_t$ where δ_t is the discount factor shock altering the weight of future utility at time t+1 relative to the period utility at time t. δ_t is assumed to follow an AR(1) process:

$$\delta_t = (1 - \rho_\delta) + \rho_\delta \delta_{t-1} + \epsilon_t^\delta \quad \forall t \geq 2$$

and δ_1 is given. The innovation ϵ_t^δ is normally distributed with mean zero and standard deviation σ_δ. It may therefore be interpreted that an increase in δ_t is a preference imposed by the household to increase the relative valuation of the future utility flows, resulting in decreased consumption today (when considered in the absence of changes in the nominal interest rate).

A_t is a non-stationary total factor productivity shock that also augments labor in the utility function in order to accommodate the necessary stationarization of the model later on. See the next section for more details on this process.

B.2 Producers

B.2.1 Final good producer

The final good producer purchases the intermediate goods $Y_{f,t}$ at the intermediate price $P_{f,t}$ and aggregates them using CES technology to produce and sell the final good Y_t to the household and government at price P_t. Its problem is then summarized as

$$\max_{Y_{f,t}, f\in[0,1]} P_t Y_t - \int_0^1 P_{f,t} Y_{f,t} di$$

subject to the CES production function

$$Y_t = \left[\int_0^1 Y_{f,t}^{\frac{\theta^p - 1}{\theta^p}} di\right]^{\frac{\theta^p}{\theta^p - 1}}.$$

θ^p is the price markup parameter.

B.2.2 Intermediate goods producers

There is a continuum of intermediate goods producers indexed by $f \in [0,1]$. Intermediate goods producers use labor to produce the imperfectly substitutable intermediate goods according to a linear production function ($Y_{f,t} = A_t N_{f,t}$) and then sell the product to the final good producer. Each firm maximizes its expected discounted sum of future profits[12] by setting the price of its own good. Any price changes are subject to quadratic adjustment costs. φ_p will represent an obstruction of price adjustment, the firm indexes for prices—measured by ι_p—and takes as given previous period inflation Π_{t-1}^p, and $\bar{\Pi}^p$ is the inflation-target parameter.

$$\max_{P_{f,t}} \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[\prod_{s=0}^{t-1} \delta_s\right] \lambda_t \left[P_{f,t} Y_{f,t} - W_t N_{f,t} - P_t \frac{\varphi_p}{2} [\frac{P_{f,t}}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_{t-1}^p\right)^{\iota_p} P_{f,t-1}} - 1]^2 Y_t\right]$$

such that

$$Y_{f,t} = \left[\frac{P_{f,t}}{P_t}\right]^{-\theta^p} Y_t.$$ [13]

λ_t is the Lagrange multiplier on the household's budget constraint at time t and $\beta^{t-1} \left[\prod_{s=0}^{t-1} \delta_s\right] \lambda_t$ is the marginal value of an additional profit to the household. The positive time zero price is the same across firms (i.e. $P_{i,0} = P_0 > 0$).

A_t represents total factor productivity which follows a random walk with drift:

$$\ln(A_t) = \ln(a) + \ln(A_{t-1}) + a_t.$$

a is the unconditional rate of growth of productivity. a_t is a productivity shock following an AR(1) process:

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \epsilon_t^A.$$

where ϵ_t^A is normally distributed with mean zero and standard deviation σ_A. This growth factor will imply that some of the variables will acquire a unit root, meaning the model will have to be stationarized. Monetary policy will also have to accommodate this growth factor as well.

---

[12] NOTE: Each period, as it is written below, is in *nominal* terms. However, we want each period's profits in *real* terms so the profits in each period must be divided by that period's price level P_t which we take care of further along in the document.

[13] This expression is derived from the profit maximizing input demand schedule when solving for the final good producer's problem above. Plugging this expression back into the CES production function implies that the final good producer will set the price of the final good $P_t = \left[\int_0^1 P_{f,t}^{1-\theta^p} di\right]^{\frac{1}{1-\theta^p}}$.

B.3 Government policies

It is assumed that the monetary authority determines nominal interest rates according to a truncated notional inertial Taylor rule augmented by a speed limit component.

$$R_t = \max\left[1, R_t^*\right]$$

where

$$\frac{R_t^*}{R_{DSS}} = \left(\frac{R_{t-1}^*}{R_{DSS}}\right)^{\rho_R} \left(\frac{\Pi_t^p}{\Pi^{targ}}\right)^{(1-\rho_R)\phi_\pi} \left(\frac{Y_t}{A_t \tilde{Y}_{DSS}}\right)^{(1-\rho_R)\phi_y}$$

where $\Pi_t^p = \frac{P_t}{P_{t-1}}$ is the inflation rate between periods $t-1$ and t and $R_{DSS} = \frac{\Pi^{targ} a^{\chi_c}}{\beta}$ (see the section on stationarization to see why). $\tilde{Y}_{DSS}$ is the deterministic steady state of stationarized output, and Π^{targ} is the inflation-target parameter.

B.4 Market clearing conditions

The market clearing conditions for the final good, labor and government bond are given by

$$Y_t = C_t + \int_0^1 \frac{\varphi_p}{2}\left[\frac{P_{f,t}}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi_{t-1}^p\right)^{\iota_p} P_{f,t-1}} - 1\right]^2 Y_t df + ...$$

$$... + \int_0^1 w_t \frac{\varphi_w}{2}\left[\frac{w_{h,t}}{a w_{h,t-1}} \frac{\Pi_t^p}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_{t-1}^w\right)^{\iota_w}} - 1\right]^2 N_t dh$$

$$N_t = \int_0^1 N_{f,t} di$$

$$C_t^a = C_t = \int_0^1 C_{h,t} dh$$

and

$$B_t = \int_0^1 B_{h,t} dh = 0.$$

B.5 An equilibrium

Given P_0 and stochastic processes for δ_t, an equilibrium consists of allocations $\{C_t$, N_t, $N_{f,t}$, Y_t, $Y_{f,t}$, $G_t\}_{t=1}^\infty$, prices $\{W_t, P_t, P_{f,t}\}_{t=1}^\infty$, and a policy instrument $\{R_t\}_{t=1}^\infty$ such that
(i) allocations solve the problem of the household given prices and policies

$$\partial C_{h,t} : \left(C_{h,t} - \zeta C_{t-1}^a\right)^{-\chi_c} - \lambda_t = 0$$

$$\partial w_{h,t} : \theta^w A_t^{1-\chi_c} \frac{N_t^{1+\chi_n}}{w_t} \left(\frac{w_{h,t}}{w_t}\right)^{-\theta^w(1+\chi_n)-1}$$

$$+(1-\theta^w)\lambda_t \left(\frac{w_{h,t}}{w_t}\right)^{-\theta^w} N_t$$

$$-\lambda_t w_t \varphi_w \left(\frac{w_{h,t}}{a w_{h,t-1}} \frac{\Pi_t^p}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_{t-1}^w\right)^{\iota_w}} - 1\right) N_t \frac{\Pi_t^p}{a w_{h,t-1} \left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_{t-1}^w\right)^{\iota_w}}$$

$$+\beta\delta_t \mathrm{E}_t \lambda_{t+1} w_{t+1} \varphi_w \left(\frac{w_{h,t+1}}{a w_{h,t}} \frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_t^w\right)^{\iota_w}} - 1\right) N_{t+1} \frac{w_{h,t+1}}{a w_{h,t}^2} \frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_t^w\right)^{\iota_w}} = 0$$

$$\partial B_{h,t} : -\frac{\lambda_t}{R_t P_t} + \beta\delta_t \mathrm{E_t} \frac{\lambda_{t+1}}{P_{t+1}} = 0$$

(ii) $P_{f,t}$ solves the problem of firm i

By making the appropriate substitution (the intermediate goods producer's constraints in place of $Y_{f,t}$ and subsequently in for $N_{f,t}$) and by dividing each period's profits by that period's price level P_t so as to put profits in real terms (and thus make profits across periods comparable) we get the following:

$$\partial P_{f,t} : \lambda_t \frac{Y_t}{P_t} \Big[\frac{P_t}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_{t-1}^p\right)^{\iota_p} P_{f,t-1}} \varphi_p \left(\frac{P_{f,t}}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_{t-1}^p\right)^{\iota_p} P_{f,t-1}} - 1\right) - (1-\theta^p) \left(\frac{P_{f,t}}{P_t}\right)^{-\theta^p}$$

$$-\theta^p \frac{w_t}{A_t} \left(\frac{P_t}{P_{f,t}}\right)^{1+\theta^p} \Big] = \beta\delta_t \mathrm{E_t} \frac{\lambda_{t+1} Y_{t+1}}{P_{t+1}} \left[P_{t+1} \varphi_p \left(\frac{P_{f,t+1}}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_t^p\right)^{\iota_p} P_{f,t}} - 1\right) \frac{P_{f,t+1}}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_t^p\right)^{\iota_p} P_{f,t}^2} \right]$$

(iii) $P_{f,t} = P_{j,t} \quad \forall i \neq j$

$$\frac{Y_t}{\lambda_t^{-1}} \left[\varphi_p \left(\frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_{t-1}^p\right)^{\iota_p}} - 1\right) \frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_{t-1}^p\right)^{\iota_p}} - (1-\theta^p) - \theta^p \frac{w_t}{A_t} \right] = \ldots$$

$$\ldots = \beta\delta_t \mathrm{E_t} \frac{Y_{t+1}}{\lambda_{t+1}^{-1}} \varphi_p \left(\frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_t^p\right)^{\iota_p}} - 1\right) \frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_t^p\right)^{\iota_p}}$$

(iv) R_t follows a specified rule
and
(v) all markets clear.

Combining all of the results derived from the conditions and exercises in (i)-(v), a symmetric equilibrium can be characterized recursively by $\{C_t, N_t, Y_t, w_t, \Pi_t^p, R_t\}_{t=1}^{\infty}$ satisfying the following equilibrium conditions:

$$\lambda_t = \beta\delta_t R_t \mathrm{E_t} \lambda_{t+1} \left(\Pi_{t+1}^p\right)^{-1}$$

$$\lambda_t = (C_t - \zeta C_{t-1})^{-\chi_c}$$

$$\frac{N_t}{\lambda_t^{-1}}\left[\varphi_w\left(\frac{\Pi_t^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_{t-1}^w\right)^{\iota_w}}-1\right)\frac{\Pi_t^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_{t-1}^w\right)^{\iota_w}}-(1-\theta^w)-\theta^w\frac{A_t^{1-\chi_c}N_t^{\chi_n}}{\lambda_t w_t}\right]=...$$

$$...=\beta\delta_t\mathrm{E}_t\frac{N_{t+1}}{\lambda_{t+1}^{-1}}\varphi_w\left(\frac{\Pi_{t+1}^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_t^w\right)^{\iota_w}}-1\right)\frac{\Pi_{t+1}^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_t^w\right)^{\iota_w}}\frac{w_{t+1}}{w_t}$$

$$\Pi_t^w=\frac{w_t}{a w_{t-1}}\Pi_t^p$$

$$\frac{Y_t}{\lambda_t^{-1}}\left[\varphi_p\left(\frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi_{t-1}^p\right)^{\iota_p}}-1\right)\frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi_{t-1}^p\right)^{\iota_p}}-(1-\theta^p)-\theta^p\frac{w_t}{A_t}\right]=...$$

$$...=\beta\delta_t\mathrm{E_t}\frac{Y_{t+1}}{\lambda_{t+1}^{-1}}\varphi_p\left(\frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi_t^p\right)^{\iota_p}}-1\right)\frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi_t^p\right)^{\iota_p}}$$

$$Y_t=C_t+\frac{\varphi_p}{2}\left[\frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi_{t-1}^p\right)^{\iota_p}}-1\right]^2 Y_t+\frac{\varphi_w}{2}\left[\frac{\Pi_t^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_{t-1}^w\right)^{\iota_w}}-1\right]^2 w_t N_t$$

$$Y_t=A_t N_t$$

$$R_t=\max\left[1,R_t^*\right]$$

where

$$\frac{R_t^*}{\bar{R}}=\left(\frac{R_{t-1}^*}{\bar{R}}\right)^{\rho_R}\left(\frac{\Pi_t^p}{\bar{\Pi}^p}\right)^{(1-\rho_R)\phi_\pi}\left(\frac{Y_t}{A_t\bar{Y}}\right)^{(1-\rho_R)\phi_y}$$

and given the following processes ($\forall t\geq 2$):

$$\delta_t=(1-\rho_\delta)+\rho_\delta\delta_{t-1}+\epsilon_t^\delta$$

and

$$\ln(A_t)=\ln(a)+\ln(A_{t-1})+a_t.$$

$$\ln(a_t)=\rho_a\ln(a_{t-1})+\epsilon_t^A.$$

B.6 A stationary equilibrium

Let $\tilde{Y}_t=\frac{Y_t}{A_t}$, $\tilde{C}_t=\frac{C_t}{A_t}$, $\tilde{w}_t=\frac{w_t}{A_t}$, and $\tilde{\lambda}_t=\frac{\lambda_t}{A_t^{-\chi_c}}$ be the stationary representations of output, consumption, real wage, and marginal utility of consumption respectively. The stationary symmetric equilibrium can now be characterized by the following system of equations.

$$\tilde{\lambda}_t=\frac{\beta}{a^{\chi_c}}\delta_t R_t\mathrm{E_t}\tilde{\lambda}_{t+1}\left(\Pi_{t+1}^p\right)^{-1}\exp(-\chi_c\epsilon_{t+1}^A)$$

$$\tilde{\lambda}_t=(\tilde{C}_t-\tilde{\zeta}\tilde{C}_{t-1}\exp(-\epsilon_t^A))^{-\chi_c},\quad\tilde{\zeta}=\frac{\zeta}{a}$$

$$\frac{N_t\tilde{w}_t}{\tilde{\lambda}_t^{-1}}\left[\varphi_w\left(\frac{\Pi_t^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_{t-1}^w\right)^{\iota_w}}-1\right)\frac{\Pi_t^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_{t-1}^w\right)^{\iota_w}}-(1-\theta^w)-\theta^w\frac{N_t^{\chi_n}}{\tilde{\lambda}_t\tilde{w}_t}\right]=...$$

$$...=\frac{\beta\varphi_w}{a^{\chi_c-1}}\delta_t\mathrm{E}_t\frac{N_{t+1}\tilde{w}_{t+1}}{\lambda_{t+1}^{-1}}\left(\frac{\Pi_{t+1}^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_t^w\right)^{\iota_w}}-1\right)\frac{\Pi_{t+1}^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_t^w\right)^{\iota_w}}\exp\left((1-\chi_c)\,\epsilon_{t+1}^A\right)$$

$$\Pi_t^w = \frac{\tilde{w}_t}{\tilde{w}_{t-1}} \Pi_t^p \exp\left(\epsilon_t^A\right)$$

$$\frac{\tilde{Y}_t}{\tilde{\lambda}_t^{-1}} \left[\varphi_p \left(\frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_{t-1}^p\right)^{\iota_p}} - 1 \right) \frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_{t-1}^p\right)^{\iota_p}} - (1-\theta^p) - \theta^p \tilde{w}_t \right] = ...$$

$$... = \frac{\beta \varphi_p}{a^{\chi_c - 1}} \delta_t \mathrm{E}_t \frac{\tilde{Y}_{t+1}}{\tilde{\lambda}_{t+1}^{-1}} \left(\frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_t^p\right)^{\iota_p}} - 1 \right) \frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_t^p\right)^{\iota_p}} \exp\left((1-\chi_c)\, \epsilon_{t+1}^A\right)$$

$$\tilde{Y}_t = \tilde{C}_t + \frac{\varphi_p}{2} \left[\frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p} \left(\Pi_{t-1}^p\right)^{\iota_p}} - 1 \right]^2 \tilde{Y}_t + \frac{\varphi_w}{2} \left[\frac{\Pi_t^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_{t-1}^w\right)^{\iota_w}} - 1 \right]^2 \tilde{w}_t N_t$$

$$\tilde{Y}_t = N_t$$

and

$$R_t = \max\left[1, R_t^*\right]$$

where

$$\frac{R_t^*}{R_{DSS}} = \left(\frac{R_{t-1}^*}{R_{DSS}} \right)^{\rho_R} \left(\frac{\Pi_t^p}{\Pi^{targ}} \right)^{(1-\rho_R)\phi_\pi} \left(\frac{Y_t}{A_t \tilde{Y}_{DSS}} \right)^{(1-\rho_R)\phi_y}$$

and given the following processes ($\forall t \geq 2$):

$$\delta_t = (1-\rho_\delta) + \delta_{t-1} + \epsilon_t^\delta$$

and

B.7 Stationary deterministic steady-state values

For each variable, X_t, we denote its corresponding *stationary* deterministic steady-state value as $\bar{X}$. The following is a list of analytical expressions for the stationary steady states for each of the variables of the model.

$$\Pi_{DSS}^p = \Pi^{targ}$$

$$\Pi_{DSS}^w = \Pi_{DSS}^p$$

$$R_{DSS} = \frac{a^{\chi_c} \Pi_{DSS}^p}{\beta}$$

$$\tilde{w}_{DSS} = \frac{\theta_p - 1}{\theta_p}$$

$$\tilde{C}_{DSS} = \left(\frac{\tilde{w}_{DSS} (\theta_w - 1)}{\theta_w \left(1 - \tilde{\zeta}\right)^{\chi_c}} \right)^{\frac{1}{\chi_c + \chi_n}}$$

$$\tilde{\lambda}_{DSS} = \left[\left(1 - \tilde{\zeta}\right) \tilde{C}_{DSS} \right]^{-\chi_c}$$

$$\tilde{N}_{DSS} = \tilde{Y}_{DSS} = \tilde{C}_{DSS}$$

B.8 Parameter values

Table 2: **Parameter Values for the Empirical Model**

Parameter	Description	Parameter Value
β	Discount rate	0.99875
a	Trend growth rate of productivity	$\frac{1.25}{400}$
χ_c	Inverse intertemporal elasticity of substitution for C_t	1.0
ζ	Degree of consumption habits	0.5
χ_n	Inverse labor supply elasticity	0.5
θ_p	Elasticity of substitution among intermediate goods	11
θ_w	Elasticity of substitution among intermediate labor inputs	4
φ_p	Price adjustment cost	700
φ_w	Wage adjustment cost	300
ι_p	Price indexation parameter	0
ι_w	Wage indexation parameter	0
Interest-rate feedback rule		
$400(\overline{\Pi}^p - 1)$	(Annualized) target rate of inflation	2.0
ρ_R	Interest-rate smoothing parameter in the Taylor rule	0.8
ϕ_π	Coefficient on inflation in the Taylor rule	3.5
ϕ_y	Coefficient on the output gap in the Taylor rule	0.25
$400(R_{ELB} - 1)$	(Annualized) effective lower bound	0.13
Shocks		
ρ_d	AR(1) coefficient for the discount factor shock	0.85
$\sigma_{\epsilon,\delta}$	The standard deviation of shocks to the discount factor	$\frac{0.66}{100}$
$\sigma_{\epsilon,a}$	The standard deviation of TFP shocks	$\frac{0.16}{100}$