Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.

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2016-069

Please cite this paper as:
Carey, Mark, and Michael B. Gordy (2016). "The Bank as Grim Reaper: Debt Composition and Bankruptcy Thresholds," Finance and Economics Discussion Series 2016-069. Washington: Board of Governors of the Federal Reserve System, http://dx.doi.org/10.17016/FEDS.2016.069.

The Bank as Grim Reaper: Debt Composition and Bankruptcy Thresholds

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July 6, 2016

Abstract

We offer a model and evidence that private debtholders play a key role in setting the endogenous asset value threshold below which corporations declare bankruptcy. The model, in the spirit of Black and Cox (1976), implies that the recovery rate at emergence from bankruptcy on all of the firm's debt taken together is increasing in the pre-bankruptcy share of private debt in all debt. Empirical evidence supports this and other implications of the model. Indeed, debt composition has a more economically material empirical influence on recovery than all other variables we try taken together.

Keywords: credit risk, recovery rates, bankruptcy, debt default
JEL Codes: G12, G33, G32

*This paper represents the authors' opinions and not necessarily those of the Board of Governors, the Federal Reserve System, or other members of its staff. We thank Viral Acharya, Jim Booth, Jean-Marc Bourgeon, Max Bruche, Richard Cantor, Jens Christensen, Michel Crouhy, Magnus Dahlquist, Sanjiv Das, Sergei Davydenko, Georges Dionne, Darrell Duffie, Klaus Duellmann, Mark Flannery, Pascal François, Edie Hotchkiss, Victoria Ivashina, David Keisman, Mark Leary, Hayne Leland, Jim O'Brien, Kasper Roszbach, Per Stromberg, Suresh Sundaresan, Hao Zhou and participants at several seminars and conference sessions for encouragement and suggestions, and Bradley Howells, Matthew Cox, Salil Gadgil and Hossein Alidaee for excellent research assistance. Much of this research was completed while Michael Gordy was a visitor at the Indian School of Business.
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1 Introduction

We offer a model and empirical evidence showing that the composition of corporate debt strongly influences corporate bankruptcy decisions and recovery rates on debt of bankrupt corporations. Our work is in the spirit of structural and strategic models of default, but differs in the locus of strategic behavior. In such models, the firm defaults when the value of its assets falls below a threshold. Implicitly or explicitly, debtholders recover the threshold value of assets, perhaps less a haircut for deadweight costs of bankruptcy. In early structural models of default, such as Merton (1974) or Longstaff and Schwartz (1995), the threshold is exogenous. In models of strategic default, such as Leland (1994) or Fan and Sundaresan (2000), equityholders choose the threshold endogenously to maximize the value of their claims.

Our model is a generalization of the first-passage model of Black and Cox (1976). In our model, a firm's private debtholders ("banks" for simplicity) endogenously choose the bankruptcy threshold value of assets. Private debt has covenants that give the bank the right to force a distressed firm into bankruptcy, even if the firm has made all debt payments. The firm's public debt is junior and has no material covenants. Because private debt is also senior, the bank has an incentive to foreclose only when the borrower's asset value drops to the neighborhood of the loan's face value, which can be well below the insolvency value of assets and can leave public debtholders with a low recovery. Therefore, the lower the bank loan share in total debt, the lower the asset value of the borrower at bankruptcy and the lower the recovery to debt as a whole.[1]

The locus of strategic behavior in structural models depends on the interpretation of the asset-sale restrictions that are invariably attached to debt contracts.[2] The branch of the literature in the spirit of Leland (1994) takes a strict view of these restrictions: Coupon payments must be financed out-of-pocket by equityholders (or via new equity issues), so equityholders default when the value of continuation of their call option on assets is below the required "new money" payment. A looser interpretation of asset-sale restrictions would constrain only attempts to divert assets. Even when (net) asset returns are negative, firms typically generate substantial (gross) cashflows. So long as enough cashflows can be used to make required debt payments, equityholders may be able to retain control well past the point of insolvency without having to make payments out-of-pocket. If asset-sale restrictions *never* bind on coupon payments, equityholders will never voluntarily default—all bankruptcies will be forced by banks. We follow the latter, extreme interpretation mainly for simplicity and to complement the well-developed literature that flows from the opposite view. An extension of the model to allow for active equityholders and explicit transfer of control rights, along the lines of Broadie et al. (2007), is left for future work.

Empirically, we find a robust, economically and statistically significant relationship between recovery and bank debt share of total debt at default. A marginal one percentage point increase

[1] Non-bankruptcy defaults and renegotiations of debt contract terms are a material source of credit losses. Indeed, a number of models of strategic default, such as Mella-Barral and Perraudin (1997) and Hackbarth et al. (2007), focus on such events. We do not consider them because we believe understanding of bankruptcy payoffs is an important step in understanding non-bankruptcy defaults. Bargaining out of bankruptcy is likely to be influenced by expectations about bankruptcy timing and outcomes.

[2] Lando (2004, §2.13.2) discusses the fundamental role of assumptions on asset-sale restrictions in structural models of credit risk.

in bank debt share improves recovery at emergence from bankruptcy ("ultimate recovery") on all the firm's debt taken together ("total" or "firm-level" recovery) by about one-quarter percentage point or more. That is, an increase from a small amount of bank debt to all bank debt would be associated with an increase in recovery rate of about 25 percentage points, other things equal, which is large relative to the sample mean recovery of about 50 percent. We find evidence for our model predictions concerning the effect of loan coupon interest rate on firm-level and instrument-level recovery. We demonstrate that our empirical findings cannot easily be explained by maturity effects or variations in deadweight costs of bankruptcy.

Our model predicts very high recovery rates on bank loans. In our sample, mean and median recoveries on loans are 84.5 and 99 percent, respectively, and over 60 percent of bank debt receives approximately a full recovery. Furthermore, our model explains why average bond recoveries are relatively low. In structural models of default with an exogenous threshold, it is often assumed that the threshold is the boundary between solvency and insolvency, in which case firm-level recovery should be not far from 100 percent.[3] Jumps in asset value (Zhou, 2001), accounting uncertainty (Duffie and Lando, 2001), liquidation costs (Fan and Sundaresan, 2000; Mella-Barral, 1999) or (closely-related) asset specificities that imply a large reduction in value when assets are transferred to new owners (Baird and Jackson, 1988) no doubt play a role, but the magnitudes required to produce an average recovery rate on all debt near 50 percent seem implausibly large. In contrast, for reasonable parameter values, such a recovery rate is broadly consistent with our model, given that the empirical mean bank debt share is near one-third.

A potential objection to our empirical evidence is that the bank debt share of total debt predicts firm-level recovery not because banks choose the bankruptcy threshold, but instead because a higher bank debt share causes deadweight costs of bankruptcy to be lower and thus recoveries to be higher. Our evidence does not support such an explanation. Only the share of bank debt with covenants predicts firm-level recovery, not bank debt without covenants, and the powers granted by covenants expire when bankruptcy is declared. Moreover, the presence or absence of common proxies for deadweight costs in the empirical specication has no material impact on the estimated coefficient on bank debt share.

About 20 percent of our sample of bankrupt firms have no bank debt. This fact, and evidence already in the literature, suggest that the timing of default and bankruptcy may be influenced by multiple stakeholders, most notably by equityholders and managers as well as by banks. Each of these actors could choose its own threshold strategically, with the threshold of highest value determining the default value of assets. We do not claim that our findings exclude any extant model of the bankruptcy decision, but only that our model describes a large subset of all bankruptcies.

Our paper has implications for related areas of research, such as the capital structure decisions of firms. Our focus throughout this paper is on the endgame phase of the firm's life (bankruptcy and recovery). Upon the onset of severe financial distress, the costs of altering debt composition or raising new equity are likely to be high, and so it is reasonable to take the firm's capital structure as fixed. For firms with assets well above the insolvency value, however, debt composition should be a material endogenous decision for the firm's owners. For example, by choosing bank debt share,

[3] Practitioner implementations, such as Moody's Analytics' KMV model (Crosbie and Bohn, 2003), typically assume an exogeneous boundary of this sort.

the firm can influence the states of the world in which deadweight costs of bankruptcy are incurred, just as the choice of leverage influences the incidence of such costs in the existing capital structure literature. As our model takes equityholders as passive, it is not well-suited to analysis of the firm's ex-ante choice of capital structure. The model of Park (2000) is complementary to our own in that it offers an optimal contracting explanation of the seniority and covenant protection of bank claims in the presence of borrower moral hazard.

Another implication is that debt composition should matter for debt pricing and credit risk management. Credit spreads and economic capital charges are roughly linear in expected loss-given-default (one minus the recovery rate), so errors in the specification of recovery are potentially costly.[4] And yet, in models and empirical studies of debt pricing, recovery is almost invariably treated as an afterthought. Expected recovery rates are typically assumed to be homogeneous within very broad debt classes (e.g., "senior unsecured bonds"). Our results indicate that expected recovery rates on individual debt instruments ought to be conditioned on debt composition and on more sophisticated treatments of seniority than the traditional debt classes.

Our assumptions that covenants give creditors rights to call loans to distressed borrowers and that such rights are attached to loans, not bonds, are realistic. Nash et al. (2003) characterize *bond* covenants as restricting financing, investing and restructuring activities. A common feature of such covenants is that they are violated only when the borrower takes a forbidden action, such as selling a large share of its assets.[5] An increase in the borrower's probability of bankruptcy does not by itself trigger a violation. Chava et al. (2010) find that only 4 percent of nonfinancial corporate bonds have a leverage or net worth covenant. In contrast, Carey (1996) finds that around 70 percent of bank loan agreements contain financial ratio covenants, such as interest-coverage, debt-to-cash-flow, and leverage ratios, and Carey (1996) offers evidence that such covenants are more likely to appear in loans to riskier borrowers. Sufi (2009) finds that 72 percent of bank lines of credit feature financial covenants, and Roberts and Sufi (2009) observe at least one financial covenant in 97 percent of a sample of private credit agreements. Berlin et al. (2016) find that even when some of a borrower's loans do not have covenants, it is very likely that other of the borrower's loans do have covenants.

Dichev and Skinner (2002) and Chava and Roberts (2008) offer evidence that such covenants are customized to be relatively tight, that is, trigger values of ratios are close to those reported by the borrower at the time the loan is made. Nini et al. (2012) document that 10–20 percent of public U.S. nonfinancial firms report violations of loan covenants in any given year. Violations are likely to accompany an increase in the probability of borrower default. Dichev and Skinner report that borrower financial performance is much worse than average in quarters when a net worth covenant is violated. Gârleanu and Zwiebel (2009) propose a theoretical explanation for the ubiquity of tight covenants in private debt as a consequence of asymmetric information between managers and creditors, and the absence of covenants in public debt as a consequence of costly renegotiation.

[4]For example, in Basel II, capital charges under the Internal Ratings-Based approach are proportional to expected loss-given-default (Basel Committee on Bank Supervision, 2006, ¶272). This formula has been retained under Basel III (Basel Committee on Bank Supervision, 2011).

[5]Some papers that use "covenants" to motivate model assumptions, such as Fan and Sundaresan (2000), focus on the borrower's promise to pay interest and principal on schedule. Legally this is a covenant, but it appears in all U.S. corporate debt contracts and is not viewed as a customizable contract-design feature by practitioners, as are other covenants.

There is evidence as well that loan covenants provide banks with significant control rights over weak borrowers well before default. Beneish and Press (1993) and Chava and Roberts (2008) report that resolution of covenant violations commonly includes fees paid to the lenders, increases in interest rates, and incorporation of additional covenants into the credit agreement. Sufi (2009) find that violations are associated with a drop of 15–25 percent in availability of total and unused lines of credit. Roberts and Sufi (2009) show that average net debt issuance drops from 80 basis points per quarter in the year before a violation to -25 basis points per quarter in the six months following a violation. Nini et al. (2012) show that violations are followed by a reduction in firm leverage and risk-taking, as well as by an increase in CEO turnover. Chen and Wei (1993) find that measures correlated with distance-to-default predict whether a covenant violation is resolved by a waiver or by the lender calling the loan.

Some papers in the literature on structural models of default have considered debt composition. Hackbarth et al. (2007) examine optimal capital structure in a model in which firms can issue bank debt, public bonds, and equity. The special quality of bank debt in their model is the ability to renegotiate outside formal bankruptcy. Bank debt offers a better tradeoff between tax shields and bankruptcy costs, whereas non-renegotiable public debt offers higher debt capacity. In our model, the special role of bank debt derives instead from the strong covenants that typically are attached to loans but not to publicly issued bonds. So far as we are aware, ours is the first structural model to explore the implications for bankruptcy thresholds and recovery rates of this ubiquitous feature of private debt.[6]

The existing recovery literature is largely empirical and has related debt characteristics to recoveries (e.g., Altman and Kishore, 1996; Qi and Zhao, 2013) or has examined sources of systematic variation in recoveries (e.g., Frye, 2000; Altman et al., 2005; Acharya et al., 2007). Nearly all of this literature studies recovery at the level of the individual debt instrument. From the perspective of our paper, an individual defaulted instrument is a collar option on the underlying firm-level recovery with strike prices that depend on the instrument's position in the firm's capital structure. Linear regression models of instrument recovery do not account for the nonlinearity of option returns. More importantly, seniority and collateral status are only rough proxies for the strike prices because different firms' debt structures have different patterns of seniority and collateral. By focusing mainly on firm-level recovery, we avoid these specification issues.

Few previous studies have examined firm-level recovery. Hamilton and Carty (1999) split their sample into firms with and without publicly issued debt and find that the former have smaller firm-level recoveries on average, which is broadly consistent with our findings. They attribute the difference to larger deadweight costs of bankruptcy due to bargaining frictions associated with more complex capital structures, which is quite different from our explanation. Suo et al. (2013) show that the recovery predicted by the Leland and Toft (1996) model is a significant predictor of the

[6]As noted previously, we use the terms "banks," "bank debt" and "bank loans" as convenient shorthand for senior debt with strong covenants. Such terminology does not perfectly represent historical patterns of debt structure in the U.S. Loans frequently were most senior in firms' debt structures and had the strongest covenants, but lenders included both banks and nonbanks, and contract forms included both loans and privately placed bonds. Publicly issued bonds rarely had such features. In recent years, "covenant-lite" loans have been issued more frequently, but usually as part of a package of loans with strong covenants and equal seniority, so the holders of loans with covenants bargain as if the whole loan package has strong covenants.

realized firm-level recovery one month after Chapter 11 filing, although the estimated coefficient is significantly below one. Davydenko et al. (2012) measure the deadweight cost of firm default using the change in the firm's approximate market value of equity and debt liabilities from the month before to the month after default.

Our model and some comparative statics are presented in Section 2. Section 3 summarizes testable implications, and describes the data and measures we use in empirical analysis. We contrast our model predictions with the predictions of three stylized alternative models. Results are presented in Section 4. Concluding remarks follow.

2 Model

We model loan contracts in which covenants permit the bank to foreclose on the borrower and force repayment through the bankruptcy process. In the simplest version of the model, we assume that the bank is effectively able to foreclose at will, and derive the bank's optimal choice of "foreclosure threshold." So long as the borrower's asset value remains above this threshold, the borrower is permitted to continue. Upon first-passage across this threshold, the bank forecloses. In an extended version of the model, we recognize that covenant violation is needed for foreclosure. We introduce a contractually-specified "covenant threshold" that serves as an upper bound on the foreclosure threshold and also triggers payment of penalty fees by the borrower to the bank in exchange for forbearance.

Our model is an extension of a model in Black and Cox (1976) for perpetual corporate debt with continuous coupons. These assumptions remove time-dependence in the value of debt, which simplifies both the solution of the model and analysis of comparative statics. We also assume there is no restriction on asset sales. When asset sales are restricted, we are led to strategic bankruptcy by equityholders as in Leland (1994) and Leland and Toft (1996). The focus of our model is on the bank's role in initiating bankruptcy, so we therefore assume that assets may be sold freely for the purpose of paying debt coupons. To avoid diversion of assets to equityholders, we assume that debt contracts specify a fixed dividend rule. The borrower's capital structure is assumed fixed with no possibility of raising new equity or debt.

The baseline model is presented in Section 2.1. This model is identical to the model of Black and Cox (1976) except that the foreclosure boundary is chosen endogeneously by the bank. Comparative statics for the baseline model are explored in Section 2.2. The primary interest here is how the share of bank debt in total firm debt influences the distribution of recoveries at the bankruptcy estate level. In Section 2.3, we extend the model to allow for a stochastic shock to firm value upon bankruptcy. The full model is developed in Section 2.4. This allows for a firm-value boundary above which the bank cannot foreclose and below which the bank receives a waiver fee so long as the bank forbears.

2.1 Baseline model

The firm is financed by debt and equity. Without loss of generality, we assume that the total face value of debt is 1. This unit of debt is divided into a single loan with face value λ and a single class of bonds with face value $1-\lambda$. The bond is junior to the loan, and (for simplicity) only the loan has covenants that permit foreclosure. The loan receives continuous coupon c and the bond receives continuous coupon γ. Equity receives a continuous dividend of $\delta + \rho V_t$, where V_t is the firm's asset value at time t. We take these parameters as nonnegative constants. For notation convenience, let $\mathcal{C}$ be the rate of fixed cash outflows per unit time, i.e.,

$$\mathcal{C} = c\lambda + \gamma(1-\lambda) + \delta.$$

Notation for the model is summarized in Table 1.

To keep the focus on credit risk, we assume riskfree interest rates are fixed at r and that $0 \leq \rho < r \leq c$. The asset value (*cum* coupons and dividends) follows a geometric Brownian motion with fixed variance σ^2. Under the risk-neutral measure, we have

$$dV_t = V_t((r-\rho)dt + \sigma\, dZ_t) - \mathcal{C}dt \tag{1}$$

In the event of bankruptcy at time t, coupon and dividend payments are frozen, i.e., ρ and δ are reset to zero. We assume that the legal claims of debtholders accrue at the riskfree rate during court proceedings.[7] Settlement occurs after a fixed length of time τ, and the bank receives $\min\{\exp(r\tau)\lambda, V_{t+\tau}\}$. As Chapter 11 bankruptcy implies a change in control over the firm's assets, we allow for a change in the level of asset volatility (to $\tilde{\sigma}$) at bankruptcy. The standard fixed-maturity, zero coupon Merton (1974) formula can be used to price the recovery value at bankruptcy, which is given by

$$B(V) = M(V, \lambda, \sqrt{\tau\tilde{\sigma}^2}) \tag{2}$$

where

$$M(V, D, s) \equiv V\Phi\left(-\frac{1}{s}\log(V/D) - \frac{s}{2}\right) + D\Phi\left(\frac{1}{s}\log(V/D) - \frac{s}{2}\right). \tag{3}$$

Our depiction of the bankruptcy process generalizes the treatment in the existing literature (e.g., in Leland, 1994), where it is assumed that settlement is immediate (i.e., that $\tau = 0$).

Applying Black and Cox (1976, eq. 18), the valuation equation for the loan satisfies the second-order ordinary differential equation

$$\frac{1}{2}\sigma^2 V^2 F'' + ((r-\rho)V - \mathcal{C})F' - rF + c\lambda = 0, \tag{4}$$

for which the general solution is

$$F(V) = \frac{c\lambda}{r} - A_1 \cdot \psi(V; \alpha, \beta, \zeta) - A_2 \cdot \psi(V; 1-\beta, 1-\alpha, \zeta) \tag{5}$$

where A_1, A_2 are constants that are determined by boundary value conditions. The function ψ is given by

$$\psi(V; a, b, \zeta) = \begin{cases} (\zeta V)^{-a} \cdot {}_1F_1\left(a, a+b, -1/(\zeta V)\right) & \text{for } V > 0 \\ \Gamma(a+b)/\Gamma(b) & \text{for } V = 0 \end{cases} \tag{6}$$

[7] It is more typically (but not universally) observed in practice that the claim on a defaulted loan accrues interest at its *contractual* rate while in bankruptcy, whereas the accrued interest on a defaulted bond is a very junior claim. We impose accrual at r as it simplifies the analysis somewhat. Results are unaffected over the empirically plausible range of parameter values.

Table 1: **Model notation**
We summarize parameters appearing in the baseline model and stochastic bankruptcy cost extension.

State-independent parameters		
Share of bank debt in total debt, by face value		λ
Time from foreclosure to resolution		τ
Riskfree rate		r
Mean lognormal shock at foreclosure		χ
Volatility of lognormal shock at foreclosure		η
Foreclosure threshold		κ
State-dependent parameters	Pre-bankruptcy	Post-bankruptcy
Continuous coupon rate on bank debt	c	r
Continuous coupon rate on bonds	γ	r
Continuous dividend received by equity	$\delta + \rho V_t$	0
Volatility per unit time of asset value process	σ	$\tilde{\sigma}$
Drift in asset value process under physical measure	μ	$\tilde{\mu}$

where ${}_1F_1$ is the confluent hypergeometric function and Γ is Euler's gamma function; see Appendix A on the special case for $V = 0$. The constants α, β and ζ are given by

$$\begin{aligned}
\alpha &= \sqrt{\left(\frac{1}{2} - \frac{r-\rho}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} - \left(\frac{1}{2} - \frac{r-\rho}{\sigma^2}\right) \\
\beta &= \alpha + 2 - \frac{2(r-\rho)}{\sigma^2} \\
\zeta &= \frac{\sigma^2}{2}\frac{1}{\mathcal{C}}
\end{aligned}$$

By re-writing the equation for α as

$$\alpha - \frac{2(r-\rho)}{\sigma^2} = \sqrt{\left(\frac{1}{2} + \frac{r-\rho}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}} - \left(\frac{1}{2} + \frac{r-\rho}{\sigma^2}\right) \geq 0,$$

we can bound parameters $\alpha \geq 2(r-\rho)/\sigma^2 > 0$ and $\beta \geq 2$.

Let κ denote the foreclosure threshold. Given a choice of κ, the boundary conditions to equation (4) are $F(\kappa) = B(\kappa)$ and $F(\infty) = \lambda c/r$. Given bounds on α and β, it is straightforward to show that $\psi(V; 1-\beta, 1-\alpha, \zeta)$ increases without bound as $V \to \infty$. Therefore, to satisfy the boundary conditions on $F(V)$, we must have $A_2 = 0$. For $\kappa > 0$, the solution to A_1 is

$$A_1 = \left(\lambda\frac{c}{r} - B(\kappa)\right)\frac{1}{\psi(\kappa; \alpha, \beta, \zeta)}$$

which implies

$$F(V;\kappa) = \lambda\frac{c}{r} - \left(\lambda\frac{c}{r} - B(\kappa)\right) \cdot \frac{\psi(V; \alpha, \beta, \zeta)}{\psi(\kappa; \alpha, \beta, \zeta)} \tag{7}$$

where we have written $F(V;\kappa)$ to emphasize the dependence on κ. The ratio $\psi(V)/\psi(\kappa)$ can be interpreted as the present value of receiving \$1 contingent on future bankruptcy (see Leland, 1994, p. 1219 for the corresponding expression in the strategic default model).

We now allow the bank to choose κ. For simplicity, we assume in the baseline model that the bank can foreclose at will, and that renegotiation is prohibited. As the bank's right to foreclose is a perpetual American option, the optimal foreclosure threshold sets the marginal exercise value equal to the marginal continuation value, i.e.,

$$B'(\kappa) = \left.\frac{\partial F(V;\kappa)}{\partial V}\right|_{V=\kappa} \tag{8}$$

Substituting equation (7), we obtain the first order condition

$$\mathcal{F}(\kappa) = B'(\kappa) - \left(\lambda\frac{c}{r} - B(\kappa)\right)\Xi(\kappa;\alpha,\beta,\zeta) \tag{9}$$

where

$$\Xi(\kappa;\alpha,\beta,\zeta) \equiv \frac{-\psi'(\kappa;\alpha,\beta,\zeta)}{\psi(\kappa;\alpha,\beta,\zeta)} \tag{10}$$

and will henceforth usually be abbreviated as $\Xi(\kappa)$. The first and second order conditions on the optimal threshold κ^* are $\mathcal{F}(\kappa^*) = 0$ and $\mathcal{F}'(\kappa^*) < 0$.

The first order condition is evaluated as follows. The derivative of B is $M_1(y,\lambda,\sqrt{\tau\tilde{\sigma}^2})$, where M_i denotes the partial derivative of M with respect to its i^{th} parameter, which simplifies to

$$M_1(V,D,s) = \Phi\left(-\frac{1}{s}\log(V/D) - \frac{s}{2}\right). \tag{11}$$

The derivative of ψ also simplifies:

$$\psi'(y;a,b,\zeta) = -a\zeta(\zeta y)^{-(a+1)} \cdot {}_1F_1\left(a+1, a+b, -1/(\zeta y)\right) = -a\zeta\psi(y;a+1,b-1,\zeta) \tag{12}$$

where the last equality follows from FWC 07.20.20.0024.02.[8] The optimal κ^* does not have closed-form solution in general, but numerical solution using standard routines for one-dimensional nonlinear roots is straightforward. For the limiting case of deterministic recovery, we prove in Appendix B that

Proposition 1 *When $\tau = 0$ or $\tilde{\sigma} = 0$, the optimal foreclosure threshold is $\kappa^* = \lambda$.*

That is, when there is no uncertainty in recovery upon foreclosure, the bank chooses the threshold to protect fully its own claim, but leaves bondholders with zero recovery.

There always exists a finite solution to the first order condition, and κ^* is strictly positive so long as there are positive fixed cashflows to investors other than the bank. In Appendix A, we use the Intermediate Value Theorem to prove:

Proposition 2 *There exists $\kappa^* \geq 0$ such that $\mathcal{F}(\kappa^*) = 0$ and $\mathcal{F}'(\kappa^*) < 0$. When $\gamma(1-\lambda)+\delta > 0$, κ^* is strictly positive.*

[8] FWC refers to the website `functions.wolfram.com`.

Our main variable of interest in this paper is total recovery as a share of total debt claims. Measured by post-default market price, recovery rates for all debtholders, the bank and the bondholders are given by

$$R = M(\kappa^*, 1, \sqrt{\tau\tilde{\sigma}^2}) \tag{13a}$$

$$R_\ell = \frac{1}{\lambda}M(\kappa^*, \lambda, \sqrt{\tau\tilde{\sigma}^2}) \tag{13b}$$

$$R_b = (R - \lambda R_\ell)/(1-\lambda) \tag{13c}$$

respectively. Note that we express recovery rates as a share of the present discounted value of the legal claim. This definition of recovery is known as the Recovery of Face Value (RFV) convention (see Lando, 2004, §5.7), and cleaves most closely to practice in bankruptcy court and accounting treatment (Guha, 2003).

In our data, recovery is measured at emergence. As this recovery is obtained under the physical measure, the geometric Brownian motion for V_t contains a risk-premium, so we introduce the the post-foreclosure drift $\tilde{\mu}$ in place of the riskfree rate r. Expected recovery rates at the date of emergence are

$$R^e = M(\exp(\tau(\tilde{\mu} - r))\kappa^*, 1, \sqrt{\tau\tilde{\sigma}^2}) \tag{14a}$$

$$R^e_\ell = \frac{1}{\lambda}M(\exp(\tau(\tilde{\mu} - r))\kappa^*, \lambda, \sqrt{\tau\tilde{\sigma}^2}) \tag{14b}$$

$$R^e_b = (R^e - \lambda R^e_\ell)/(1-\lambda) \tag{14c}$$

for all debtholders, the bank and the bondholders, respectively.

2.2 Comparative statics for the baseline model

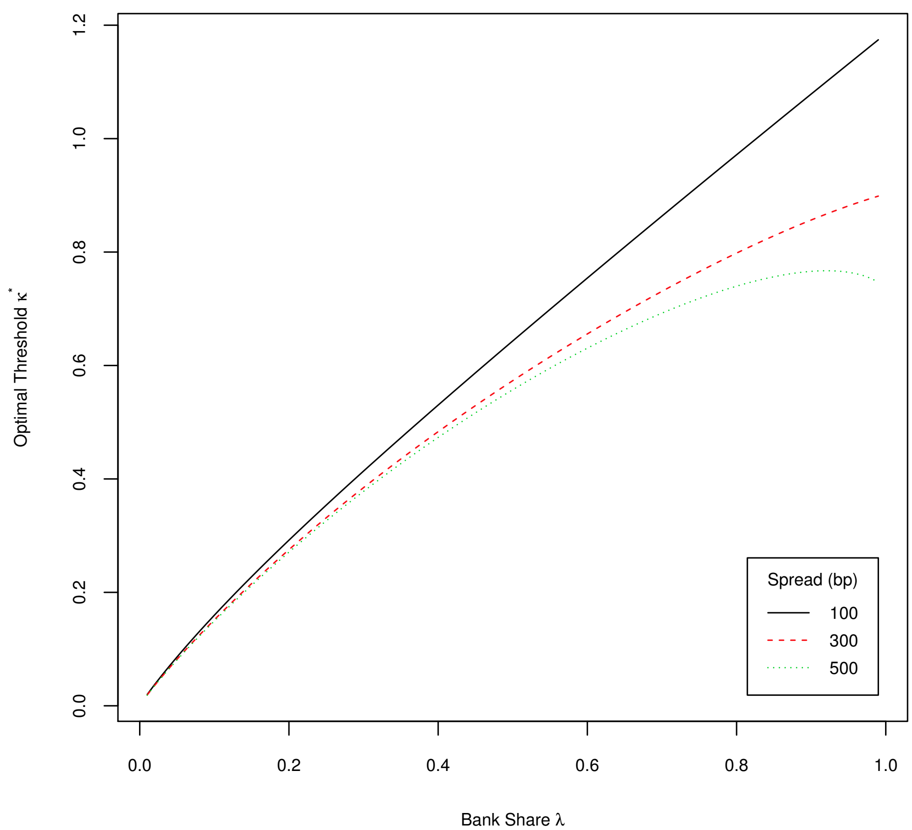
In this section, we examine how the optimal foreclosure threshold and recovery rates vary with changes in model parameters. We recognize that some of these parameters were determined endogenously at the time of contracting as functions of other parameters. The loan coupon rate, for example, would surely have depended on the loan share λ and the asset volatility σ. At the time of contracting, therefore, the total derivative of κ^* with respect to, say, σ would need to account both for the direct effect of σ on bank incentives and indirect effects through the effect of σ on coupon c and other endogeneous variables. By contrast, this paper takes the perspective of a firm in severe distress. The firm's asset value has changed since the time of contracting–fallen substantially, most likely–and therefore the endogeneous relationships that at one time bound c to λ and σ no longer pertain. Empirically, one can observe in data on defaulted firms a wide range of combinations of these variables. It is for this reason that we examine each of our comparative statics as partial derivatives, i.e., with all other parameters held fixed.

Comparative statics for the parameters of main interest are by no means straightforward even in parsimonious versions of the model, and so we resort to numerical exercises. We begin with the influence of bank share λ and loan coupon c on the optimal foreclosure threshold. We expect the bank's choice of κ^* to increase with its share of total debt, as the bank forecloses to protect its own stake. When there is no uncertainty on recovery in bankruptcy, as in Proposition 1, $\kappa^* = \lambda$.

For modest degrees of recovery risk, we might expect a roughly linear relationship. The influence of loan coupon works through two channels. A higher loan coupon increases the cashflow of the loan, but also drains the firm's assets at a higher rate. Intuition suggests that the positive effect of the first channel on the marginal continuation value of the loan (which pushes κ^* down) should dominate the second channel.

Both of these predicted relationships are supported by our numerical results. Figure 1 shows a roughly linear (slightly concave) relationship between λ and κ^*, and that κ^* decreases with c. This pattern is robust over a wide range of parameter values. The negative relationship between

Figure 1: Effect of debt composition on foreclosure threshold



Spread is $c-r$, measured in basis points. Parameters: $r = 0.03$, $\gamma = 2(c-r)+r$, $\delta = \rho = 0$, $\sigma = \tilde{\sigma} = 0.25$, $\tau = 1$.

bankruptcy threshold and coupon rate stands in contrast to the positive relationship predicted by Leland (1994) and other strategic default models.

As shown in the figure, κ^* may decrease in λ at high values of λ when the loan coupon is high and the dividend rate δ is low. For $\delta = 0$, the bank's share of the cashflow drained from assets $(c\lambda/\mathcal{C})$ approaches 1 as $\lambda \to 1$, so the cost of forbearance falls towards zero. When c is high, the marginal continuation value can then increase more quickly in λ than the marginal exercise value of foreclosure, in which case κ^* falls.

As shown in the bottom panel of Figure 2, total recovery displays similar comparative statics. Somewhat more complicated is the influence of bank share on recoveries at the instrument level. In absolute dollar terms, loan recovery increases with λ. However, as depicted in the upper left panel, the loan recovery rate (i.e., as a share of λ) in general is decreasing with bank share. The bond recovery rate (upper right panel) is generally increasing with λ when c is small, but may be humped-shaped in λ when c is large.

Intuition suggests that the bank's optimal κ^* should increase with pre-bankruptcy volatility σ. For any fixed foreclosure threshold, higher σ reduces the expected first-passage time to the threshold, which lowers the marginal continuation value. The effect of post-bankruptcy volatility $\tilde{\sigma}$ is ambiguous. For any fixed κ, the loan's recovery value $B(\kappa)$ is decreasing in $\tilde{\sigma}$. At low levels of post-bankruptcy volatility, an increase in $\tilde{\sigma}$ should cause the bank to raise the foreclosure threshold in order to protect its recovery. At very high levels of post-bankruptcy volatility, however, protection of recovery becomes too expensive in terms of forgone loan coupons, in which case κ^* falls with $\tilde{\sigma}$.

These relationships are displayed in the top panel of Figure 3. The foreclosure threshold is everywhere non-decreasing in σ, but the effect is small over the empirically plausible range of $\sigma \in [0.1, 0.3]$ and for $\tilde{\sigma} < 0.3$. As was observed for the loan coupon, the "survival time" channel appears to have only a second-order effect on the optimal foreclosure rule. The effect of $\tilde{\sigma}$ on κ^* is much larger in magnitude, but ambiguous in sign. As shown in the bottom panel, this non-monotonicity is even more apparent at higher values of λ. Similar patterns are observed in the comparative statics for total recovery with respect to the volatility parameters.

Comparative statics for bond coupon (γ) and the fixed rate of equity dividends (δ) can be signed analytically. These two parameters enter the first order condition for the foreclosure threshold through the ζ parameter of the ψ function, so we begin with the lemma:

Lemma 1

$$\zeta \cdot \frac{\partial}{\partial \zeta} \Xi(V; \alpha, \beta, \zeta) = \Xi(V; \alpha, \beta, \zeta) + V \cdot \Xi'(V; \alpha, \beta, \zeta) > 0.$$

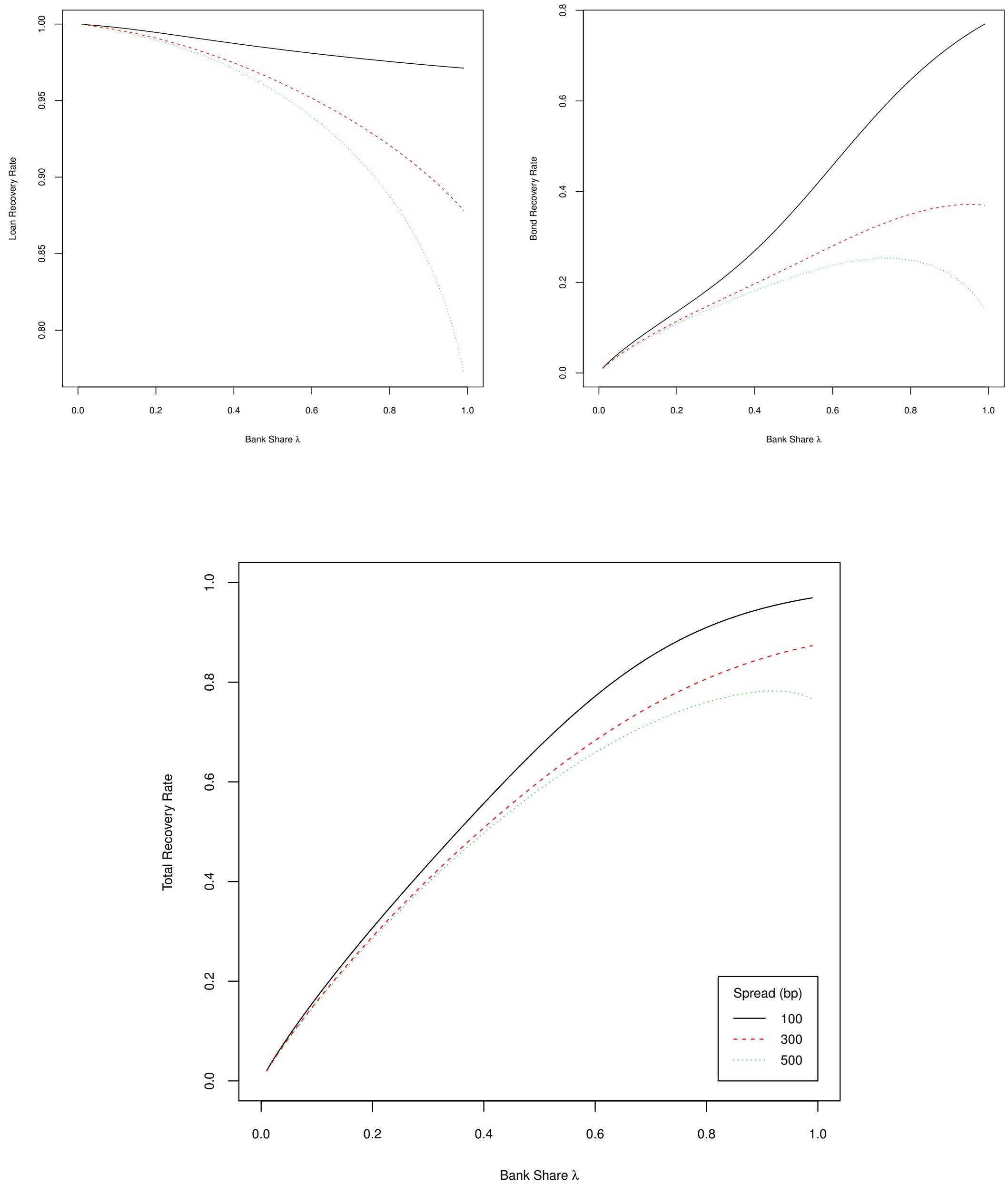
This is proved in Appendix C. It follows that

$$\frac{\partial}{\partial \gamma} \Xi(V) = \frac{\partial \zeta}{\partial \gamma} \cdot \frac{\partial}{\partial \zeta} \Xi(V) = -\frac{\sigma^2}{2} \frac{(1-\lambda)}{(c\lambda + \gamma(1-\lambda) + \delta)^2} \cdot \frac{\partial}{\partial \zeta} \Xi(V) = -\frac{(1-\lambda)}{\mathcal{C}} \zeta \frac{\partial}{\partial \zeta} \Xi(V) < 0.$$

Writing the first order condition for κ^* as $\mathcal{F}(\kappa; \gamma)$ to emphasize its dependence on γ, we have

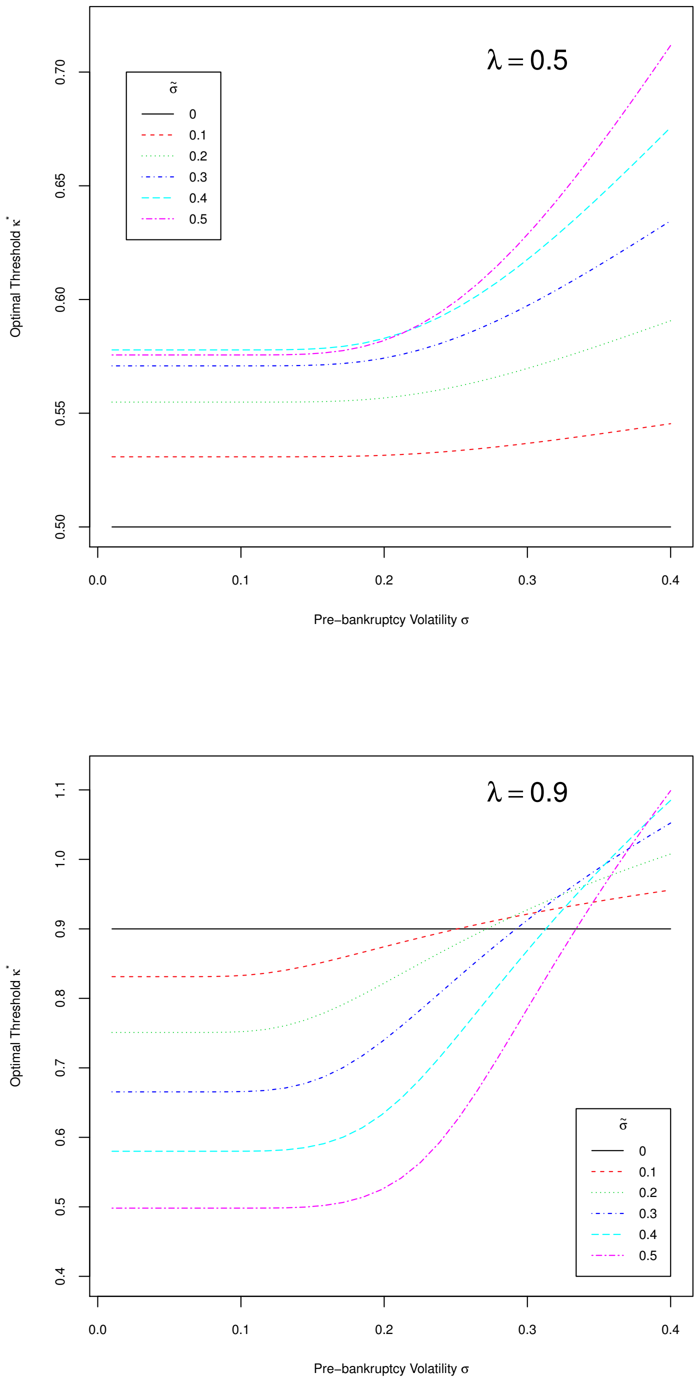
$$\frac{\partial \kappa^*}{\partial \gamma} = -\frac{\partial \mathcal{F}(\kappa; \gamma)/\partial \gamma}{\partial \mathcal{F}(\kappa; \gamma)/\partial \kappa}\bigg|_{\kappa = \kappa^*}$$

Figure 2: Effect of debt composition on recovery at emergence



Spread is $c - r$, measured in basis points. Parameters: $r = 0.03$, $\tilde{\mu} = r + 0.05$, $\gamma = 2(c - r) + r$, $\delta = \rho = 0$, $\sigma = \tilde{\sigma} = 0.25$, $\tau = 1$.

Figure 3: Effect of volatility on foreclosure threshold



Parameters: $r = 0.03$, $c = r + 0.03$, $\gamma = 2(c - r) + r$, $\delta = \rho = 0$, $\tau = 1$.

The numerator is

$$\frac{\partial \mathcal{F}(\kappa;\gamma)}{\partial \gamma} = -\left(\lambda \frac{c}{r} - B(\kappa)\right) \frac{\partial}{\partial \gamma} \Xi(\kappa) > 0,$$

as $B(\kappa) \leq \lambda$ for all κ. Since $\partial \mathcal{F}/\partial \kappa < 0$ at the optimum, we must have κ^* increasing in γ. Parallel arguments show that κ^* is increasing in δ. The intuition for these results is straightforward and based on the "survival time" channel discussed above. An increase in γ or δ increases the rate at which firm assets are drained by subordinated claimants, and so reduces the present discounted value of future cashflows to the bank.

2.3 Extension: Stochastic bankruptcy cost

The event of foreclosure can often impart a shock to asset value. Besides the legal costs associated with bankruptcy, franchise value might be sacrificed and certain contracts might be invalidated at foreclosure. In some cases, bankruptcy can help the firm escape an unfavorable labor contract or pension liability, so the shock need not be negative. We extend the model of the previous section to allow for a foreclosure shock.

We model bankruptcy costs as a multiplicative shock to asset value that is realized immediately following foreclosure by the bank. We assume that the shock is distributed logNormal(χ, η^2). The recovery value $B(V)$ is now

$$B(V) = M(\exp(\chi + \eta^2/2)V, \lambda, \sqrt{\tau \tilde{\sigma}^2 + \eta^2}) \tag{15}$$

It is only through altering the recovery value that χ and η affect the optimal choice of κ.

For this extended model, Proposition 1 generalizes to:

Proposition 1' *There exists* $\underline{\chi} < 0$ *such that the optimal foreclosure threshold is* $\kappa^* = e^{-\chi}\lambda$ *when* $\eta^2 = \tau\tilde{\sigma}^2 = 0$ *and* $\chi \geq \underline{\chi}$.

The proof is a straightforward but tedious extension of the proof in Appendix B. The intuition remains the same as for the baseline case: for bankruptcy shocks that are not too negative, the bank forecloses when the borrower's asset value is just sufficient for the bank to recover fully its own principal.

When the mean shock size is not too negative ($\chi \geq -\eta^2/2$), Proposition 2 holds without modification. A corner solution $\kappa^* = 0$ for the optimal threshold may arise otherwise; see Appendix A. When χ is negative and large in magnitude, it may become too costly (in terms of forgone interest revenue) to protect recoveries, and so κ^* goes to zero. When χ is positive and large, the bank is able to obtain full recovery at a low foreclosure threshold, and so the foreclosure threshold tends to zero in this case too. More formally, in Appendix D we show

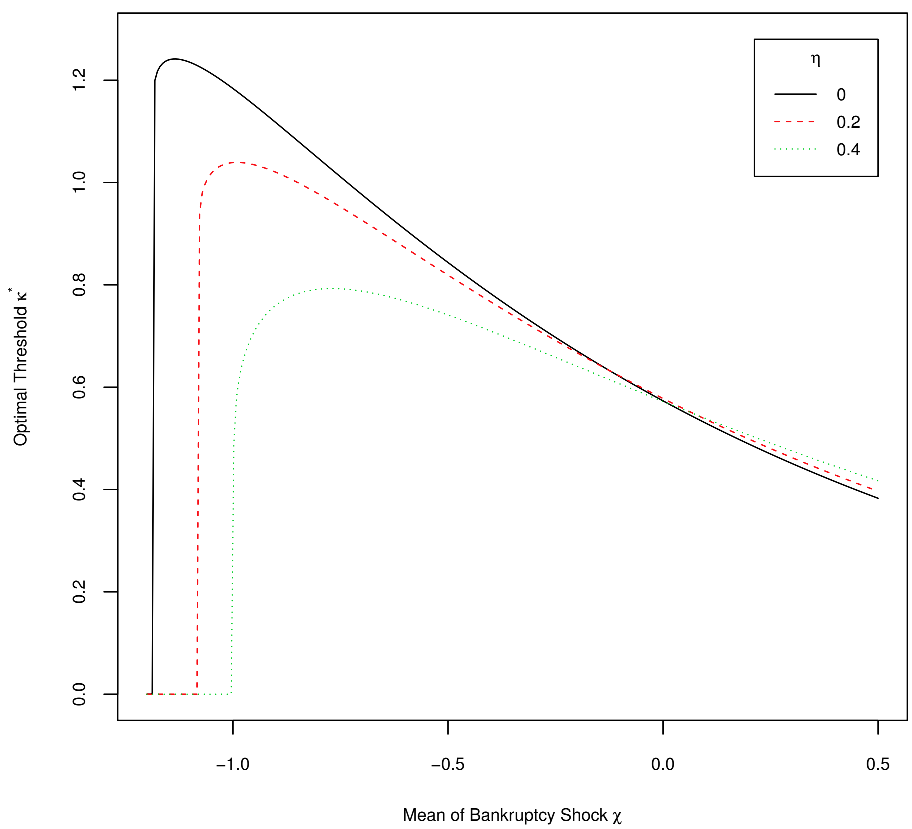
Proposition 3

$$\lim_{\chi \to -\infty} \kappa^* = \lim_{\chi \to \infty} \kappa^* = 0.$$

Thus, κ^* is non-monotonic in χ.

The effects of both shock parameters on κ^* are explored numerically in Figure 4. We see that κ^* increases smoothly as χ decreases, and then drops rapidly to zero at extreme values of χ. Similar to $\tilde{\sigma}$, the effect of shock volatility η is non-monotonic. All else equal, higher η reduces the bank's recovery and so compounds the effect of a large negative χ. Therefore, larger η increases the "turning point" in the relationship between χ and κ^*.

Figure 4: Effect of bankruptcy shock on foreclosure threshold



Parameters: $\lambda = 0.5$, $r = 0.03$, $c = r + 0.03$, $\gamma = 2(c - r) + r$, $\delta = \rho = 0$, $\sigma = \tilde{\sigma} = 0.25$, $\tau = 1$.

The effect on recoveries at emergence is seen in Figure 5. The loan recovery rate (upper left panel) is increasing in χ and decreasing in η. Comparing the loan recovery panel of Figure 5 with that of Figure 2, we see that this extension to the baseline model allows for materially lower loan recovery rates. The total recovery rate (bottom panel) is also increasing in χ, but the effect of η is non-monotonic. The bond recovery rate (upper panel) is qualitatively similar to the total recovery rate.

2.4 Covenant boundary and waiver fees

In Appendix E, we introduce a finite covenant boundary ν. Whenever $V_t \leq \nu$, the borrower is considered to be in violation of covenants and the bank has an option to foreclose at will. Whenever $V > \nu$, covenants are satisfied and the bank cannot foreclose. Loan contracts may specify a fee to be paid to the bank when a covenant violation is waived, and in other cases something similar might be achieved by renegotiation at the time of covenant violation. For simplicity, we assume that a waiver penalty of w is added to the coupon rate c whenever $\kappa < V \leq \nu$.

These changes to the setup add realism and richness at the cost of increased complexity. The model's broad implications are not altered. Similar to coupon rate effects, the recovery rates for both debt classes are decreasing in the waiver penalty. The effect on the loan's loss given default of varying w can be quite large on a relative basis, even if not terribly large on an absolute basis.

3 Empirical Strategy, Data and Measures

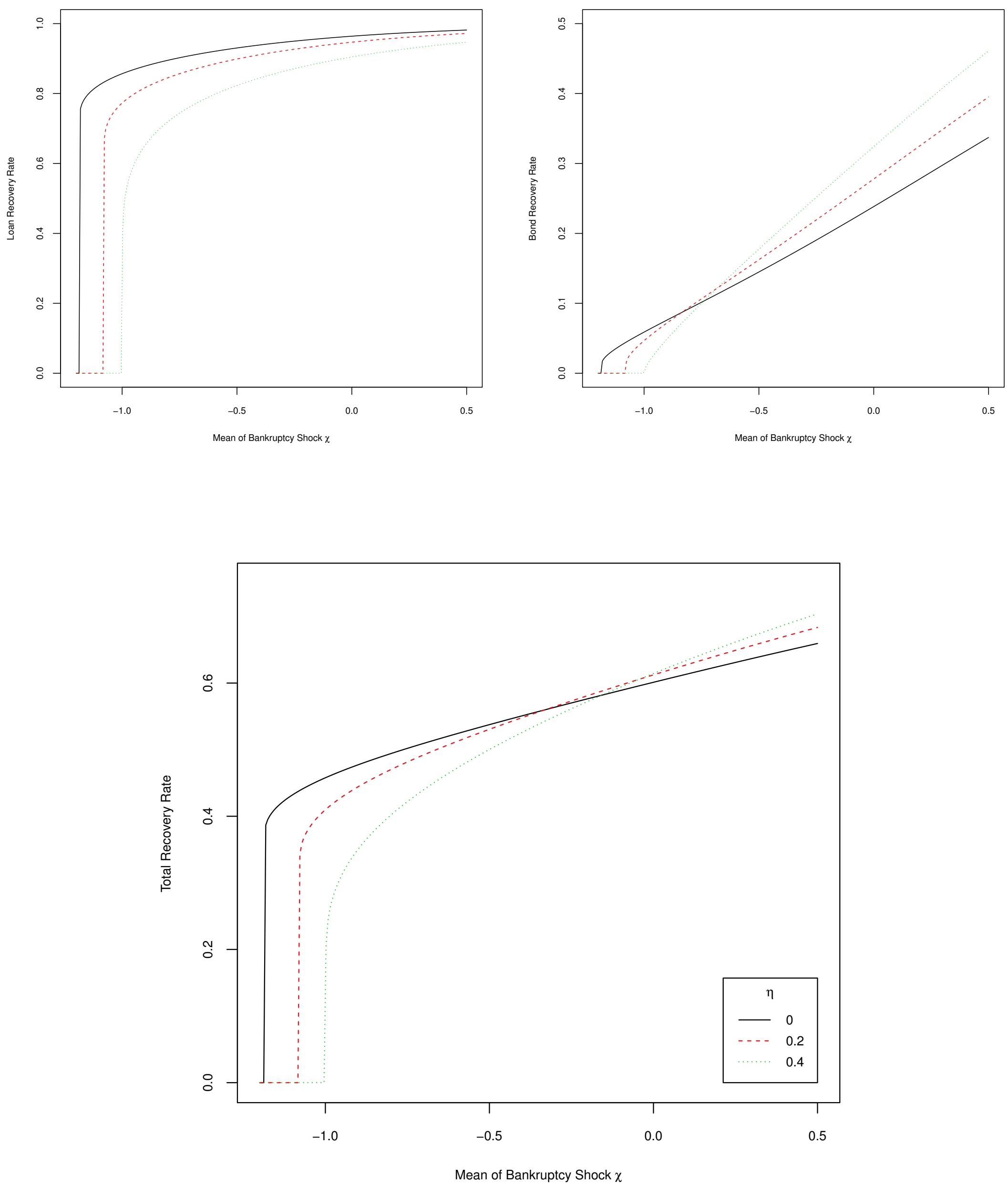
We offer evidence that bankruptcy outcomes are consistent with our model. Prominent alternatives cannot easily explain all of the evidence.

3.1 Testable implications and empirical strategy

Our model has seven testable implications. Total (firm-level) recovery rates are 1) increasing in the share of bank debt in total debt (λ), but 2) the slope of the relationship is decreasing in the coupon interest rate on bank debt (c). The presence of financial covenants determine whether an instrument conveys control over the bankruptcy threshold, not whether it is a loan or the investor is a bank. Thus, 3) only the ratio of loans-with-covenants to total debt should be positively related to total recovery rates, not the ratio of loans-without-covenants to total debt. Our model also has implications for recovery rates at the instrument-level: 4) A large fraction of firms should display high recovery rates on bank debt because banks act to protect themselves, whereas recovery rates on bonds should be lower because bondholders are passive and junior. 5) Bank debt recovery rates are slowly decreasing in bank debt share, and 6) bond recovery rates are increasing. Finally, 7) both relationships are conditional on bank debt's coupon spread over the risk-free rate.

Our model's implications for the relationship between recovery and the firm's asset value volatility are not readily tested. The predicted relationship between pre-bankruptcy volatility and firm-level recovery is positive, but weak and nonlinear. Recovery rates are more sensitive to post-bankruptcy volatility, but the relationship is nonmonotonic (see Figure 3). We cannot observe post-bankruptcy volatility, for which we would need time series of market prices of the firm's equity and all of its debt, which are rarely observable after bankruptcy is declared. Moreover, in our model, commonly used measures of pre-bankruptcy volatility are measured with an error that is

Figure 5: Effect of bankruptcy shock on recovery at emergence



Parameters: $\lambda = 0.5$, $r = 0.03$, $\tilde{\mu} = r + 0.05$, $c = r + 0.03$, $\gamma = 2(c - r) + r$, $\delta = \rho = 0$, $\sigma = \tilde{\sigma} = 0.25$, $\tau = 1$.

correlated with bank debt share.[9] Overall, asset volatility is not a useful variable for testing the realism of our model or for distinguishing it from other models.[10]

Our model's testable implications hold when other parameters of the model are held fixed. The impact of sample variation in such parameters on point estimates and confidence intervals is particularly likely to be material in regressions on loan and bond recovery rates (less so for total recovery rates; see Figures 2 and 5). Moreover, our model cannot explain all bankruptcies because some involve firms with no bank debt. As noted previously, different models may describe different bankruptcies.

To test the first three implications of our model, we examine parameter estimates from variants of a regression of the form

$$R = a_0 + a_1\lambda + a_2\lambda c + b \cdot \text{Controls} + \epsilon \tag{16}$$

where R is the recovery rate at emergence on all debt of the firm taken together ("firm-level" recovery), λ is the share of bank debt in total debt, c is the spread over the risk-free rate that the firm pays on bank debt, and Controls is a vector of control variables and other variables of interest taken from the empirical literature on recoveries. If our model explains a substantial share of bankruptcies, we expect $a_1 > 0$ and $a_2 < 0$. If our model explains all bankruptcy decisions and recoveries, we would expect $a_1 \approx 1$, especially for firms with relatively low bank debt share.

To test implications 4 through 7, we provide summary statistics about loan and bond recovery rates and we examine the relationship of such recovery rates to bank debt share using separate regressions for loans and bonds that are otherwise similar to the one above.

3.2 Alternatives

Our model, and the central idea of this paper, are new. However, the existing literature offers alternative stories of default, bankruptcy, recovery rates, and debt pricing that may coincide with our model in some of its empirical predictions. The number of extant models is too large for each to be considered in detail here. To give some structure to the discussion, we consider two stylized alternatives as well as variants of Leland (1994). These and other alternatives might describe some bankruptcies, but the evidence shows that our model describes many.

Random threshold: A stylized alternative in the spirit of Merton (1974) or Longstaff and Schwartz (1995) has a threshold that is exogenous, known *ex ante*, and that is an independent variable on the unit interval. Draws represent a multiplier on the solvency-threshold value of assets. By construction, firm-level recovery rates are unrelated to bank debt share. As in our model,

[9] Common measures implicitly assume a fixed bankruptcy threshold value of assets, such as the insolvency threshold. If the threshold varies across firms, volatility will be measured with error, and in our model, the error will be correlated with bank debt share and thus with recovery rates. If we regress recovery rates on measured volatility, any empirical relationship may simply reflect the endogenous error.

[10] To satisfy curiosity, we added Moody's Analytics' measure of asset volatility as an explanatory variable to our primary specification (for a reduced sample that was matchable to Moody's database). For several variants of the measure and of the specification (e.g., linear, spline with various knots, quadratic, various measurement dates), the coefficient on volatility was generally statistically insignificant and its sign was not robust. The presence or absence of volatility in the specification did not affect other results.

this alternative implies a negative relationship between loan coupon rates and recovery (but the reason is different: firms with low exogenous threshold values are more likely to have low loan recoveries, which should be priced *ex ante*). Unlike our model, this alternative has no implication for the impact of the presence of covenants on recovery rates.

Strategic default: In the large class of models of strategic default building upon Leland (1994), all debt service payments must be financed by outside equity. As the burden of the debt-service payment increases with the coupon rate, bankruptcy thresholds (and thus firm-level recovery rates) should be increasing in the interaction between the loan coupon spread and the bank debt share. This prediction is opposite to that of our own model. In Hackbarth et al. (2007), the capital structure includes both renegotiable bank debt and non-renegotiable bonds. In this setting, a higher share of bank debt lowers the equityholder's default threshold *ex post*, and so reduces recoveries. This prediction too is opposite to that of our model.[11] In Leland and Toft (1996), firms with shorter-maturity debt have larger required debt-service payments because the amount of principal coming due in each period is larger, so the bankruptcy threshold and recovery rates should decrease in the maturity of the debt (our model is silent about maturity effects). Thus, if bank debt maturities are shorter than bond maturities on average, a positive empirical relationship between total recovery rates and bank debt share might be due to maturity effects in the absence of controls for maturity. We include loan coupon spread and measures of maturity in some specifications.

Deadweight costs are driven by bank debt share: Deadweight costs of bankruptcy drive a wedge between recoveries and threshold values. Thus, if deadweight costs fall as bank debt share increases, a positive empirical relationship between firm-level recovery rates and bank debt share might be observed even if bank debt share is unrelated to the bankruptcy threshold. Banks' deadweight-cost-reducing actions might occur either *ex ante* or *ex post*. An *ex ante* example is that banks might know deadweight costs for each firm and lend no more than the shareholders' bankruptcy threshold value of assets less such costs. An *ex post* example is that banks might act more strongly to reduce bargaining frictions as banks' share of the firm's debt increases, thus reducing deadweight costs.[12] We do not associate such deadweight-cost stories with any single fully fleshed-out model because they have the potential to fit with several. We offer several pieces of evidence that are relevant. One is to include in the empirical specification common proxies for bargaining frictions and circumstances of the bankruptcy, which are commonly assumed to be related to deadweight costs.[13] We are interested in whether inclusion or exclusion of such proxies has a material effect on the estimated coefficient on bank debt share, which we would expect if bank

[11] Bourgeon and Dionne (2013) introduce *bank* strategic behavior to the Hackbarth et al. (2007) model. Here the bank faces a repeated game, and so can commit to a mixed strategy in which renegotiation is sometimes refused. In this case, recovery need not fall with bank debt share.

[12] Deadweight costs might be priced at loan issuance, but only if banks are unable to insulate themselves from reduced recoveries due to such costs. In the ex ante story they would insulate themselves with loan size limits.

[13] For our purposes, it is reasonable to ignore bargaining by equityholders after bankruptcy is declared because, in the U.S. during our sample period, equityholders of bankrupt firms in effect lose control of the firm. Their claim is deeply subordinated by the court, so their threat against debtholders is weak once bankruptcy is declared.

debt share is standing in for deadweight costs.[14] Another is to decompose bank debt share into bank debt with covenants and without covenants; if the deadweight cost story is driving our results then both types of bank debt should be similarly predictive of recovery rates (because covenants no longer give banks power once bankruptcy is declared), whereas our model implies only bank debt with covenants should be predictive. Another piece of evidence comes from the fact that, under the deadweight costs alternative, the bankruptcy threshold would be determined by a model other than ours. If it is by a model in the spirit of Leland (1994), the relationship between recovery rates and loan coupons should be positive, as described above. Finally, and more informally, the average size and the cross-firm variation in deadweight costs due to bank actions would both have to be implausibly large to be the sole explanation for the statistics reported below.

3.3 Data

For bankrupt firms, we measure recovery rates, the debt structure of firms, and firm and debt characteristics. Recovery rates and debt structure are from Standard and Poor's LossStats database, which tracks debt structure and ultimate recovery for each debt instrument outstanding at default for each firm in the database.[15] For example, suppose a firm defaulted and declared bankruptcy on a given date, that it emerged from bankruptcy exactly one year later, and that the firm's debt on the bankruptcy date consisted of a single bank loan and a single bond issue. Suppose that at emergence, the holders of the loan and bond received a mixture of cash and debt obligations of the emerging firm in compensation for their claims. For such a firm, LossStats records:

- The market value of such compensation at the time of emergence, separately for each pre-bankruptcy debt instrument.[16]
- The identity and some characteristics of the firm and of its experience in bankruptcy, such as the court which handled the case.
- Some characteristics of each debt instrument, such as original-issue amount, amount of principal outstanding at default, coupon interest rate, whether the instrument is subordinated or secured, and the priority class to which the instrument is assigned by the bankruptcy court.

We use data only for bankruptcies, not for distressed restructurings.[17] Almost all the firms are U.S. firms. Many had publicly issued debt or equity outstanding at default. We also matched our

[14] In terms of the model of Section 2.3, any mechanism that causes variation across firms in χ to be negatively correlated with variation in λ would be relevant. We focus on deadweight costs as the most intuitive specific alternative.

[15] Most available recovery databases do not support measurement of debt structure at bankruptcy or of recoveries to the firm's debt taken as a whole because they have data for only a subset of debt instruments. S&P obtains LossStats data primarily by analyzing SEC filings and bankruptcy court documents. Values of compensation received at emergence are gathered from a variety of sources.

[16] LossStats has information for the complete debt structure of each firm, but not about equity or preferred stock claims and their recoveries, nor about accounts-payable or other liabilities (discussed further below).

[17] In pre-bankruptcy bargaining and contracting, agents' expectations about bankruptcy outcomes and how it will work are likely to influence what they do. We focus on bankruptcy as a step toward better understanding of contracting and bargaining.

dataset to Moodys Ultimate Recovery Database, which like LossStats includes measures of ultimate recovery. When we use Moodys measures, results are very similar.

The LossStats release that we use is dated mid-2013 and contains bankruptcies declared between 1987 and 2012, which is our main sample period. Bankruptcies appear in the database only after they are resolved, because only then can ultimate recovery be determined. This raises the possibility of bias: Firms that take a long time to emerge from bankruptcy may be more likely to be omitted from our analysis. A common supposition is that the debt of such firms tends to have smaller recovery. Rather than complicating estimation by including corrections for censoring, we check robustness by also producing results for a subsample that includes only bankruptcies declared by end-2006. Apart from the asbestos-related bankruptcy of W.R. Grace, which spent twelve years in bankruptcy and emerged in 2014, we are confident that every bankruptcy declared by end-2006 had been resolved in time to be included in our database.[18] Ending in 2006 also avoids the financial crisis.

Mean (median) time in bankruptcy is 14 (12) months, regardless of how many trailing bankruptcies are dropped, and the longest bankruptcy apart from W.R. Grace took a bit less than six years to resolve.

We matched LossStats observations to entries in Compustat (to obtain financial statement variables and ratings) and to Loan Pricing Corporation's Dealscan database (primarily to obtain information about loan covenants). In creating and cleaning variables, we used a variety of sources to learn about details of bankruptcies or debt structure, especially SEC filings and Moody's Bond Record. The date of Compustat balance-sheet and income-statement variables is the latest fiscal year-end date that precedes the bankruptcy date. Where the available fiscal year-end date is more than 1.1 years before the bankruptcy date, we eliminate the firm from the Compustat-matched subsample.

3.4 Recovery measures

We normalize recovery cashflows by amount-owed (bankruptcy claim amount) in order to work with recovery rates. Our instrument-level measure of recovery is the total dollar value of the recovery received by holders of the debt instrument as a fraction of the total amount owed according to the terms of the debt contract and the rules of bankruptcy. Our firm-level recovery measure is the sum of dollar recoveries on all of the firm's debt instruments as a fraction of the sum of amounts owed. Thus, firm-level recovery rates are weighted averages of the recovery rates on the firm's individual obligations. For some of our examinations of recoveries on loans and bonds, we compute similar weighted averages of all individual bank debt (bond) recoveries so that we have a single representative bank debt (bond) recovery for the firm. Details on our construction of recovery measures are found in Appendix F.

We examine firm-level recovery (and representative loan and bond recoveries for each firm) partly because our model suggests it and partly because it seems likely to be a cleaner measure

[18] We spot-checked a later release of LossStats for appearance of bankruptcies declared in 2006 or earlier but not in the 2013 release and found none apart from W.R. Grace. We found that bankruptcies in our sample with very long lags between filing and emergence were asbestos-related, so we searched for asbestos-related bankruptcies not in the sample (other than W.R. Grace) and found none.

than individual debt instrument recoveries for examining how characteristics of the firm and the economic environment affect bankruptcy thresholds. Some firms have many bonds or loans for small amounts and some have a few large ones, so instrument-level regressions would implicitly weight the experiences of firms differently.

Figure 6 illustrates the different properties of instrument- and firm-level recovery. The hump-shaped line plots the kernel-smoothed empirical distribution of firm-level recovery at emergence for our data. The U-shaped line plots the empirical distribution of recovery for individual debt instruments (not combined into a single representative loan and bond for each firm). The instruments resemble collar options so it is unsurprising that their distribution is bimodal with peaks at or near out-of-the-money (zero recovery) and full recovery.[19] As noted previously, the firm-level data in the figure were constructed from the instrument-level data, so differences in sample selection do not drive shapes of the curves.

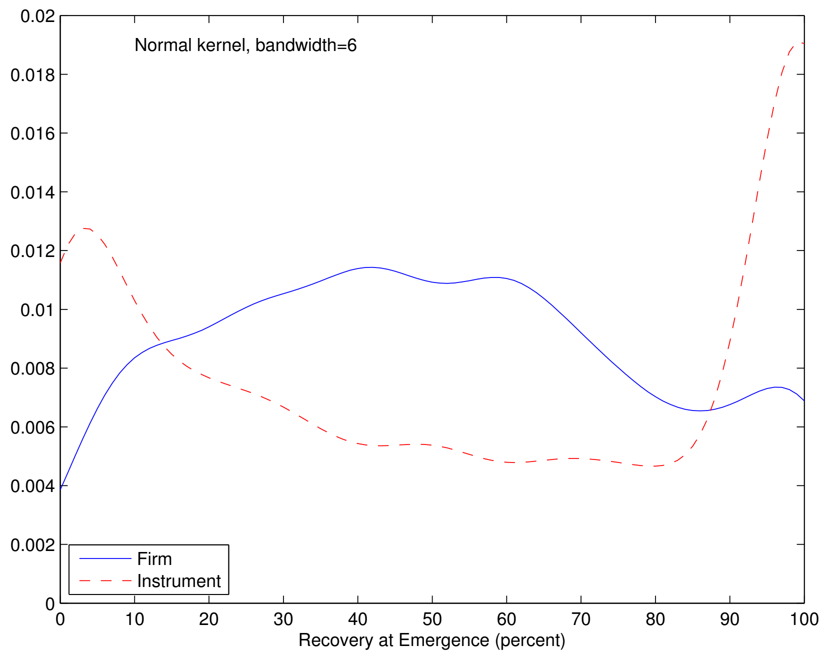
The small fraction of firms with high total recovery rates (illustrated by the rather low level of the hump-shaped line at recovery rates above 80 percent) is striking, as is the wide spread of recovery rates. Such a distribution either implies that most firms are deeply insolvent on the bankruptcy filing date or that deadweights costs (or other jumps in firm value) are large. Deep insolvency arises naturally in our model when bank debt share is near its mean value in our data, but is difficult for many structural models of default to explain.

We examine ultimate recovery, defined as recovery received at emergence from bankruptcy, because data limitations are such that it is the best available measure of the payoff on debt of a bankrupt firm. Much of the empirical literature has examined recovery-at-default, proxied by the secondary market trading price of defaulted debt instruments approximately 30 days after default. One problem is that post-default price data are not available for many instruments, making it impossible to construct firm-level measures of recovery-at-default for most bankruptcies. Another problem is that the trading price soon after default embeds expectations of the seniority class to which the instrument will be assigned by the court as well as market estimates of the present value of the firm at emergence.[20]

[19]Using pools of individual instrument-level recoveries would require adequate controls for the nonlinear payoff properties of instruments, which are difficult to develop. Individual debt instruments of a bankrupt firm, like any corporate debt, are contingent claims on the value of the firm at emergence from bankruptcy. However, payoff properties of the claims are specified by bankruptcy law rather than the pre-bankruptcy contractual terms of the instrument. The court uses the rules of absolute priority to rank-order debt instruments into classes. Assets are allocated to each class in order of priority until assets are exhausted. Thus, a debt instrument of a bankrupt firm is similar to a collar option written on the value of the firm: It can be out-of-the-money, receive part of its claim, or all of its claim, depending on court's determination of the value of the firm. A number of practical problems in measuring the strike prices or attachment points for each instrument make satisfactory specification of the relationship between instrument-level recovery and firm characteristics a subject for future research.

[20]Our model allows for variation in firm value during the time between bankruptcy and emergence but assumes a constant time in bankruptcy. To examine robustness to the effect of variations in time in bankruptcy purely due to the time value of money, we discounted values at emergence back to the bankruptcy date using a variety of assumptions about discount rates, as discussed in more detail in Appendix F. Our results are robust to use of any of the discounted measures. The cross-sectional variation of discount factors due to time in bankruptcy is quite small relative to the cross-sectional variation in recovery rates at emergence.

Figure 6: Distribution of recovery at firm-level and instrument-level



Kernel estimate of density of recovery at emergence. The "firm" curve is for total recovery on all debt, whereas the "instrument" curve is for all individual debt instruments observed in our data.

3.5 Bank debt share

We define a loan as any debt that LossStats' broad debt type variable describes as a "Line of Credit," "Revolving Credit," or "Term Loan" (examples of other common classifications are "Subordinated Bonds," "Senior Unsecured Bonds," etc.). We examined LossStats' more detailed description of each instrument (and other sources as necessary) and removed from the loan category any instrument that does not appear to be arms-length debt owed to bank-like lenders (for example, loans from suppliers or parents) or is not senior. Results are robust to leaving such instruments as loans.

In our model, loans without financial covenants are not "bank debt" and should not be included in measures of bank debt share. However, in some models, it is the nature of the lender that matters. Thus, identification of loans with covenants is potentially useful. We matched firms and loans with LPC Dealscan and consulted other sources in order to identify loans with financial covenants. Of 621 sample firms with any loan amount outstanding at bankruptcy, we found positive evidence of financial covenants in loans for 499 and only 21 firms with loans having no financial covenants. For the remaining 101 firms, insufficient information is available.

For most of our empirical exercises, we include in measures of bank debt share all loans except those at the 21 firms identified as having no covenants. In some exercises, we employ separate measures of the shares of debt with covenants, without covenants, and with unknown covenant status.

3.6 Non-debt claims

At emergence from bankruptcy, firm value is allocated not only to holders of pre-bankruptcy debt claims, but also to pay administrative costs of the bankruptcy, to pay taxes, and to other claims such as accounts payable.[21] As noted, our data report only recoveries on debt, so our firm-level recoveries represent a lower-bound estimate of the value of the firm's assets at emergence. For example, accounts payable are usually treated as "general unsecured claims." Other things equal, a larger share of accounts payable in total liabilities should reduce the dollar amount of our measure of firm-level recovery. We check robustness by using the shares of different types of non-debt and non-equity claims in total liabilities as predictors and find that only accounts payable predicts our measure of recovery, as discussed further below.[22]

[21] Many bankrupt firms obtain superpriority debtor-in-possession (DIP) loan commitments, usually from banks, which almost always are repaid in full if any balances are outstanding at emergence. However, funds are generally not drawn under such facilities. The facilities help the firm to continue operations by assuring trade creditors that the firm will not experience a liquidity problem while in bankruptcy. Only a few DIP loans appear in our data (which includes only loan commitments for which balances were outstanding at bankruptcy or emergence), and in only one case were new balances added after the bankruptcy was filed. Thus, our results are not affected by the superpriority status of DIP loans.

[22] Deviations from absolute priority that involve transfers from holders of one debt instrument in our sample to another are immaterial to our analysis of firm-level recovery. However, deviations that involve payoffs to equityholders will reduce firm-level debt recovery. Bharath et al. (2007) offer evidence that such deviations are usually small during our sample period.

3.7 Summary statistics

Table 2 presents mean, median, minimum and maximum values for many of the variables that appear in the analysis below, for the full sample (773 usable observations) and for the Compustat-matched subsample (448 observations). Average firm-level recovery is near 50 percent, but individual-firm recoveries range widely, with the best outcome being a gain of 67 percent of the amount of the claim and the worst being a total loss.[23] The total amount of debt claims is $315 million for the median firm. The median firm had four debt instruments outstanding, and the median firm in the Compustat-matched subsample had approximately a zero net worth at the fiscal year-end before filing (Book Leverage Ratio ≈ 1, computed as book liabilities/assets).

On average, bank debt represents about one-third of all firm debt, and ranges from none to all. 23 percent of sample firms had no bank debt as of the filing date.

About 64 percent of Compustat subsample firms had an S&P rating at the fiscal year-end before filing. Only about 10 percent of such firms were rated BB or better, and very few of these were rated investment-grade.

4 Empirical Results

Table 3 reports estimates from ordinary least-squares models of firm-level recovery for a base-case specification, which is estimable for the full sample. We omit λc from this specification because data on loan coupon rates are unavailable for a significant number of observations. Table 4 reports results for the base case and several alternatives but, to save space, omits coefficients for the year, industry and court dummy variables reported in Table 3 (all such dummies are included in all specifications). Parameter estimates and statistical significance are similar when produced by Tobit estimation or when all observations with recovery rates above 100 percent are dropped (not tabulated). Reported p-values are from standard OLS variance-covariance matrices, as the White (1980) specification test does not reject homoskedasticity for any of the regressions we run. Use of robust standard errors, or clustering standard errors by year, does not materially change statistical significance of estimated coefficients, except that clustering increases the significance of some year dummies.

[23] The dollar value received by debtholders at emergence sometimes exceeds the amount of debtholders' claims. Some such cases may arise because our measure of claims is imperfect, but many occur because time elapses between filing of the firm's plan of reorganization and emergence from bankruptcy. If the value of the firm increases sharply during this interval, or if the court underestimates firm value as embodied in the plan of reorganization, debtholders may receive some value that would have gone to equityholders (or other deeply subordinated claimants) in a world of instantaneous action and perfect information. Our results are robust to dropping all observations with firm level recovery greater than 100 percent, which comprise less than 5 percent of observations in our sample.

Table 2: **Sample summary statistics**

Data are for all bankruptcies of U.S. firms in the mid-2013 release of the LossStats database. Debt claim amounts and assets of the firm are in millions of dollars. Compustat data are as of the most recent fiscal year-end date preceding the bankruptcy filing date, except that data for year-end dates more than 1.1 years prior to the filing date are eliminated. Number of debt instruments is the number of separate debt obligations of the firm at the time bankruptcy is filed, whereas number of priority classes is the number of different class labels assigned by the court that are shown for debt instruments in LossStats.

	Full Sample				Compustat Sample			
Variable	Mean	Median	Min	Max	Mean	Median	Min	Max
Recovery rate in percent	53	50	0	167	53	52	0	141
Amount of claims $mil	854	315	12	43441	1000	371	12	43441
Loan coupon spread	3.06	2.88	0.32	9.50	2.96	2.83	0.32	7.00
Loan maturity (years left)	1.64	1.24	0.00	7.18	1.64	1.23	0.00	7.18
Bond maturity (years left)	5.14	4.95	0.00	22.81	5.64	5.47	0.00	22.81
Debt structure controls								
Share bank debt	0.37	0.32	0	1	0.35	0.32	0	1
No bank debt dummy	0.23	0.00	0	1	0.22	0.00	0	1
All bank debt dummy	0.10	0.00	0	1	0.08	0.00	0	1
Share secured debt	0.48	0.46	0	1	0.44	0.43	0	1
All sub debt dummy	0.05	0.00	0	1	0.05	0.00	0	1
No sub debt dummy	0.42	0.00	0	1	0.38	0.00	0	1
Share sub debt	0.28	0.15	0	1	0.30	0.20	0	1
Deadweight-loss proxies								
Time in bankruptcy	1.20	0.95	0.05	6.24	1.22	0.92	0.08	6.24
Time from plan to emerge	0.49	0.33	0.01	3.82	0.47	0.33	0.02	3.82
Time in default pre-filing	0.28	0.05	0.00	3.15	0.25	0.04	0.00	3.06
Prepackaged bankruptcy	0.28	0.00	0	1	0.27	0.00	0	1
Number debt instruments	4.54	3.00	1.00	55.00	5.06	4.00	1.00	55.00
Number priority classes	2.20	2.00	1.00	16.00	2.33	2.00	1.00	16.00
Fraud dummy	0.04	0.00	0	1	0.05	0.00	0	1
Filed again within 5 yrs dum	0.03	0.00	0	1	0.05	0.00	0	1
Court dummies								
California	0.05	0.00	0	1	0.06	0.00	0	1
New York	0.21	0.00	0	1	0.22	0.00	0	1
Delaware	0.37	0.00	0	1	0.37	0.00	0	1
Illinois	0.02	0.00	0	1	0.02	0.00	0	1
Texas	0.08	0.00	0	1	0.08	0.00	0	1
Firm characteristics								
Log total assets					6.34	6.16	0.83	11.55
Non-intang. assets/assets					0.87	0.96	0.19	1.00
Book liabs./assets					1.19	1.01	0.25	5.50
Operating income/assets					0.03	0.05	-0.25	0.25
Accts payable/tot liabilities					0.09	0.07	0.00	0.30
PPE/assets					0.38	0.36	0.00	0.96
BB or safer					0.10	0.00	0	1
B					0.30	0.00	0	1
CCC					0.16	0.00	0	1
CC or worse					0.08	0.00	0	1
Selected industry dummies								
Asbestos dummy	0.01	0.00	0	1	0.01	0.00	0	1
Bubble-firm dummy	0.07	0.00	0	1	0.08	0.00	0	1
Utilities	0.03	0.00	0	1	0.02	0.00	0	1
Number Of Bankruptcies			773				448	

Table 3: **Base-case firm-level recovery rate regression**

The dependent variable in the OLS regression, with p-values based on conventional standard errors, is the firm-level recovery rate at emergence in percentage points. The shares of bank, subordinated and secured debt are the fractions of each type of debt outstanding at bankruptcy. The utility dummy indicates regulated public utilities, such as natural gas delivery companies. The asbestos dummy indicates firms that filed for bankruptcy as part of their asbestos litigation strategy. The bubble dummy indicates firms in the internet, telecom, or energy trading sectors that filed for bankruptcy during 2000–2004. Court dummies identify the location of the court that supervised the bankruptcy. The omitted court is "all others." Industry dummies are based on a judgmental collapsing of industry codes provided by S&P into sixteen categories, all of which are included in the regression, but only utility, telecom, computer, and airline are shown to save space. Others are statistically insignificant.

	Coeff.	p-value
Intercept	58.64	0.000
Variable of most interest		
Share bank debt	23.90	0.000
Debt structure controls		
No bank debt dummy	-5.57	0.175
All bank debt dummy	6.83	0.182
Share secured debt	-1.18	0.758
All sub debt dummy	-9.60	0.129
No sub debt dummy	2.61	0.475
Share sub debt	-7.66	0.234
Deadweight-cost proxies		
Time in bankruptcy	0.34	0.817
Time from plan to emergence	-1.78	0.480
Time in default pre-filing	-0.43	0.854
Prepackaged bankruptcy	7.26	0.006
Number debt instruments	0.44	0.113
Number priority classes	-0.35	0.736
Fraud dummy	1.10	0.820
Filed again within 5 yrs dummy	-7.27	0.190
Court dummies		
California	-1.26	0.801
New York	-5.45	0.070
Delaware	-5.63	0.039
Illinois	-2.76	0.681
Texas	1.66	0.689
Selected industry dummies		
Asbestos dummy	26.57	0.013
Bubble-firm dummy	-20.39	0.001
Utilities	13.86	0.042
Telecom	-2.41	0.669
Computer	-6.20	0.163
Airline	-7.94	0.334

(continued)	Coeff.	p-value
Bankruptcy year dummies		
1987–88	-1.62	0.860
1989	-16.45	0.061
1990	-13.93	0.050
1991	-0.00	1.000
1992	-0.12	0.985
1994	-6.34	0.419
1995	0.94	0.896
1996	-0.97	0.897
1997	-8.50	0.290
1998	-13.96	0.073
1999	-6.35	0.313
2000	-14.89	0.015
2001	-17.21	0.003
2002	-15.29	0.009
2003	-1.47	0.813
2004	-4.95	0.480
2005	5.87	0.501
2006	12.03	0.172
2007	12.20	0.348
2008	-29.74	0.001
2009	-11.68	0.086
2010	-22.75	0.009
2011	-11.72	0.215
2012	-20.76	0.090
Number observations	773	
Adjusted R-squared	0.27	

Table 4: **Additional firm-level recovery rate regressions**
Details are the same as in Table 3, with the following exceptions. Year, industry and court dummies are included in regressions but not shown. In column 2, bank debt share and the loan coupon rate (measured as the mean spread on the borrower's loans over LIBOR in percentage points as recorded in the LossStats database) are interacted. For maturity, where a borrower has more than one loan or bond outstanding, the mean maturity is used.

	(1) Base case		(2) Interact λ, c		(3) No deadweight		(4) Add maturity		(5) Separate covs	
	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value
Intercept	58.64	0.000	63.03	0.000	61.74	0.000	64.80	0.000	58.70	0.000
Variables of most interest										
Share bank debt	23.90	0.000	41.11	0.000	21.27	0.000	31.77	0.000		
Share bank debt with cov									25.15	0.000
Share bank debt no cov									-5.10	0.687
Share bank debt no info on cov									19.26	0.009
Share bank debt * coupon			-7.03	0.000						
Loan maturity (years left)							-2.21	0.018		
Bond maturity (years left)							-1.00	0.012		
Debt structure controls										
No bank debt dummy	-5.57	0.175			-7.11	0.044	-9.28	0.071	-5.52	0.182
All bank debt dummy	6.83	0.182	8.95	0.145	6.93	0.173	-2.91	0.631	6.97	0.171
Share secured debt	-1.18	0.758	-2.44	0.691	-1.02	0.788	-4.17	0.357	-1.51	0.697
All sub debt dummy	-9.60	0.129			-8.56	0.173	-6.22	0.391	-9.78	0.124
No sub debt dummy	2.61	0.475	5.09	0.325	1.72	0.618	5.53	0.191	2.42	0.508
Share sub debt	-7.66	0.234	-0.71	0.940	-9.55	0.121	1.13	0.881	-8.15	0.204
Deadweight-cost proxies										
Time in bankruptcy	0.34	0.817	1.42	0.493			-0.49	0.777	0.27	0.856
Time from plan to emergence	-1.78	0.480	-7.80	0.056			-0.75	0.799	-1.63	0.518
Time in default pre-filing	-0.43	0.854	-1.07	0.793			0.49	0.864	-0.27	0.909
Prepackaged bankruptcy	7.26	0.006	2.44	0.502			6.37	0.031	7.12	0.007
Number debt instruments	0.44	0.113	1.02	0.002			0.91	0.008	0.44	0.112
Number priority classes	-0.35	0.736	-1.39	0.262			-1.10	0.465	-0.37	0.725
Fraud dummy	1.10	0.820	1.09	0.867			3.57	0.500	1.11	0.820
Filed again within 5 yrs dummy	-7.27	0.190	-6.70	0.343			-11.45	0.096	-7.50	0.177
Number observations	773		412		773		607		773	
Adjusted R-squared	0.27		0.26		0.26		0.28		0.27	

4.1 Bank debt share

Bank debt share is an economically and statistically significant predictor of firm-level recovery rates, with a 1 percentage point increase in share associated with about a one-quarter percentage point increase in recovery rate, other things equal (bank debt share is measured as a fraction, so the estimated coefficient is the change in percentage points of recovery for a change in share from 0 to 1, or 23.90). If all debt structure variables are dropped from the regression, the adjusted R^2 drops by more than half, roughly implying that debt structure is a more important predictor of recovery than all the other variables taken together (not tabulated). These results are consistent with the alternatives we consider only if bank debt share is proxying for something not in our model.[24] The economic size of the relationship seems too large to be due to variation in deadweight costs driven by bank presence in negotiations: It is difficult to believe that going from, say, a bank debt share of 25 percent to 75 percent would have an impact of one-quarter of the average recovery. That would imply a very large impact of moderate changes in bank presence among creditors on negotiating frictions or operating efficiency of the firm while in bankruptcy.

4.2 Loan coupon rate

We measure the loan coupon as the mean spread over LIBOR (in percentage points) paid by the borrower on loans outstanding at the time of bankruptcy, as reported in LossStats (spreads are missing for some loans in LossStats, so the sample is smaller for this exercise). When the interaction term λc is included in the specification, as in column 2 of Table 4, the coefficient is negative, statistically significantly different from zero, and with a magnitude roughly consistent with our model. (The much larger coefficient on bank debt share in this specification is due to the addition of the interaction term, not to restricted sample.) The result is consistent with our model and with the aforementioned alternatives except for Leland (1994) and related models. When we simply include the coupon in place of λc, we find a negative and significant coefficient (coefficient -2.8, p-value 0.02; not tabulated).

4.3 Proxies for bargaining frictions and deadweight costs

In most specifications, we include several proxies for bargaining frictions among creditors and deadweight losses. These include:

- The time in bankruptcy (measured in years from filing to emergence).
- The time between the filing of the first plan of reorganization to emergence. Median time is 4 months. Much longer times are likely due to the first plan being voted down and thus may indicate substantial bargaining problems among creditors.

[24] In a two-threshold world, where the higher of the bank's and the equityholders' threshold determines the bankruptcy value of assets, and if a low bank debt share makes it more likely that the equityholders' threshold controls, then the estimated coefficient on bank debt share will be biased toward zero because recovery will tend to be higher than implied by low bank debt share alone. As a very crude indicator of such bias, if we drop from the sample all bankruptcies with no bank debt, the estimated coefficient on bank debt share increases to roughly 27.

- The time between the borrower's first default on a debt payment and the bankruptcy filing date.
- A dummy for pre-packaged and pre-arranged bankruptcies, in which the firm has negotiated a tentative plan of reorganization with creditors before filing, implying lesser bargaining frictions.
- The number of debt instruments outstanding at filing.
- The number of priority classes into which the court aggregated the debt instruments.
- A dummy for bankruptcies involving pre-bankruptcy fraud problems at the borrower.
- A dummy for firms that filed for bankruptcy again within five years of emergence.[25]

Only one proxy is a statistically significant predictor of recovery rates: prepackaged bankruptcies are associated with larger recovery rates. Importantly, when we omit all bargaining friction proxy variables from the specification, the coefficient on bank debt share remains statistically significant and does not change much, as shown in column 3 of Table 4.[26]

We compute a proxy for deadweight costs in the spirit of the measure in Davydenko et al. (2012): the change in the combined market value of the firm's loans and bonds from about one month before the bankruptcy date to about one month after. We were able to find market prices before and after bankrutpcy for at least one of the firm's bonds and one of its loans for 108 of the bankruptcies in our sample (we use averages to value bonds and loans). When we regress recovery rates on bank debt share for this small subsample, the coefficient on bank debt share is slightly smaller than in the baseline specification of Table 3 but remains statistically significant. Adding the measure of deadweight costs to the regression does not much affect the size or significance of the coefficient on bank debt share, and the deadweight-cost measure has a statistically significant coefficient of about 33 (not tabulated). This, and the results of omission of other proxies from the base specification, is evidence against the alternative in which bank debt share is merely serving as a proxy for variation in deadweight costs.[27]

[25] Wang (2007) offers evidence that recovery rates are lower for bankruptcies precipitated at least in part by fraud, and for bankruptcies managed by certain courts. We construct a fraud variable in a manner similar to Wang (2007) (by examining Lynn Lopucki's Bankruptcy Research Database, supplemented by some additional frauds we noticed while cleaning the data). We also used similar sources to identify firms that experienced more than one bankruptcy within five years of that recorded in any given observation (often called "Chapter 22" bankruptcies).

[26] The finding that total time in bankruptcy does not matter is a bit of a surprise. It is conventional wisdom that deadweight costs of bankruptcy increase with duration of the bankruptcy. However, as noted previously, average time in bankruptcy in our sample is relatively short. It is possible that the conventional wisdom comes from experience in the 1970s or early 1980s and that bankruptcy practice has changed. Moreover, most empirical studies to date have examined recovery at the individual debt-instrument level, which implicitly overweights bankruptcies of firms with many instruments. We ran regressions similar to those in Table 3 using instrument-level data and found that results for variables such as time in bankruptcy and court dummies are sensitive to details of the specification. We view this as a potential subject for future research.

[27] Asquith et al. (1990) and Gilson et al. (1990) examine whether distress is resolved in or out of bankruptcy and find that those with a smaller share of private debt in total debt are more likely to declare bankruptcy. Our results are not inconsistent with theirs; we study only bankruptcies.

A number of authors, such as Lopucki (2005) and Wang (2007), have suggested that the efficiency of bankruptcy courts varies and that firms that are deeply insolvent may be more likely to file in some venues than others. We include dummy variables for each bankruptcy court that handles a substantial volume of bankruptcies in our sample (the omitted category is all other courts). Although coefficient values are negative for most of the court dummies and are weakly statistically significant for the Delaware and New York courts in the base specification reported in Table 3, none are very large and significance varies across specifications. Moreover, other results are not materially affected if we omit the court dummies (not tabulated).

4.4 Role of covenants

In column 5 of Table 4, we interact bank debt share with dummy variables for three types of firms: those having loans with covenants; those with loans without covenants; and those for which we are unable to identify covenant status even after thorough searching by hand. Consistent with our model, the coefficient on the share of loans without covenants is not statistically significantly different from zero and, further, that coefficient and the one on loans with covenants are statistically significantly different from each other. While bearing in mind that our subsample of firms with no covenants is small, we view these results as inconsistent with the deadweight costs alternative because it is not obvious why financial-ratio covenants that only give banks power outside of bankruptcy would affect deadweight costs in bankruptcy. The estimated coefficient for firms for which covenant status is unknown is similar to that for firms with loans with covenants. This is not surprising because the unknown-status firms are those for which information is not available. The proportions of such firms with and without covenants are likely similar to the proportions for those for which status is known.[28]

4.5 Debt maturity

The model of Leland and Toft (1996) suggests a negative relationship between debt maturity and recovery. When we include a measure of overall average maturity (pooling loans and bonds) in our firm-level regression, its estimated coefficient is around -6 and is statistically significant (not tabulated). We find that the estimated coefficient on bank debt share changes little, implying that debt structure variables are not proxying for the firm's debt maturity profile.

We experimented with other measures of maturity, and in no case is the effect on the estimated coefficient on bank debt share material. One example is displayed in column 4 of Table 4, which includes mean time to maturity separately for bank debt and bonds.[29] In the particular specification shown, both coefficients are negative and statistically significant, but signs and statistical significance are not very robust to variations in the definitions of measures nor to use of robust

[28] Many borrowers had multiple loans outstanding at bankruptcy. We measure variables as if all a borrower's loans have covenants if any do. This is reasonable because the loans are usually similar in bankruptcy priority. In our model, the choice of foreclosure threshold by a lender with covenants is invariant to the covenant rights of other *pari passu* creditors.

[29] Sample size is smaller because maturity is missing for some instruments in LossStats. The increase in the magnitude of the coefficient on bank debt share from 24 to 32 is a feature of the smaller sample, not the addition of maturity variables.

errors. It is worth noting that loan maturities are not very short: Mean remaining loan time to maturity at bankruptcy is 1.6 years, whereas for bonds the mean time is 5.1 years. Typical term to maturity at origination for loans is 3 to 5 years; for bonds it is 7 to 10 years.[30]

4.6 Instrument-level recovery rates

Our model implies that bank debt should usually receive a high expected recovery rate, although some lower recoveries are to be expected due to asset volatility during the bankruptcy period. The median and mean loan recovery rate are 99 and 84.5 percent, respectively. Figure 7 displays a histogram of loan recovery rates for all loans.[31] The recovery rate on bank debt (senior, secured, first-lien loans with covenants) is 95 percent or more for about 62 percent of sample firms. For the remaining firms, bank debt recovery rates are roughly uniformly distributed between 0 and 95 percent.

We inspected some of the bankruptcies with very low bank debt recovery rates and found that some had collateral that turned out to be of little value and some bank debt ended up being classified as equal in priority to the firm's bonds. Some are cases where the asset value of the firm may have experienced a big negative jump (such as some telecom and tech firms during 2001–2003). While the possibility of low recovery on bank debt arises even in our baseline model (specifically, when time to settlement (τ) is long or post-bankruptcy volatility ($\tilde{\sigma}$) is high), we suspect that banks sometimes make idiosyncratic mistakes. Such mistakes are outside the scope of our model but are not inconsistent with it.

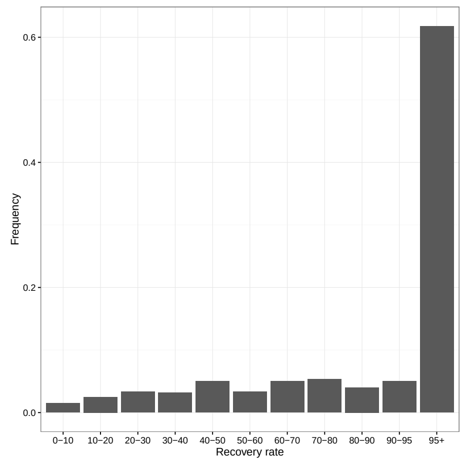
As depicted in the top panels of Figure 2, our model implies that the empirical relationship between recovery rates on loans alone (bonds alone) and bank debt share is negative (positive) conditional on the coupon spread, and that the slope of the relationship falls in rough proportion to the loan coupon spread. Table 5 reports results for specifications which include both bank debt share and the interaction of the bank debt share with coupon spread.[32] Point estimates of coefficients of interest have the signs predicted by our model. Apart from bank debt share in the bond regression, the relationships are statistically significant. Contrary to our findings, Leland (1994) and other models of strategic default predict that instrument recoveries should be invariant to bank debt share and should be increasing in the interaction of the bank debt share with coupon spread.

[30] In the specification shown, remaining loan maturity is measured as the log of the mean number of years remaining to maturity on the bankruptcy date for all outstanding loans of the borrower, and similarly for bond maturity. If time is measured in levels rather than log of levels, significance patterns remain similar and the estimated coefficients are near -1, implying economically modest effects of maturity on recovery rates. If time is measured as minimum time to maturity statistical significance is marginal at best and coefficients are economically small.

[31] For firm's with multiple loans outstanding, loans were aggregated to one representative loan per bankruptcy; results are similar for the distribution of individual loans.

[32] We limit the sample to bankruptcies of firms with loans outstanding and with coupon spread data available, so dummies for no-bank-debt and all-sub-debt are not included in regressions. For each such bankruptcy, we aggregated all loans (bonds) into a single representative loan (bond) with the recovery rate being the rate for all the loans (bonds) taken together (in essence, a claim-weighted average, like our firm-level recovery measure). Such aggregation allows us to ignore differences in priority across individual instruments, which are very difficult to measure well. Our model's implications are for representative loans and bonds.

Figure 7: Frequency distribution of bank debt recovery rates



4.7 Other debt structure variables

Firms with no bank debt are not inconsistent with our model but it seems likely that their bankruptcy thresholds are determined by mechanisms outside our model. To account for this and for the possibility of nonlinearities at corners and for unmodeled aspects of debt structure, most regression specifications include dummies for firms with no bank debt at filing and firms with all bank debt, for the share of debt that is secured, for the share that is contractually subordinated, and dummies for firms with all subordinated debt and no subordinated debt. None of the coefficients are statistically significantly different from zero in Table 3 and coefficients are rarely statistically significant in regressions reported in other tables. When we omit the additional debt structure variables entirely, as in column 3 of Table 6, the estimated coefficient on bank debt share is substantially larger than in the base specification. These results make us believe that the share of loans with covenants is a key determinant of recovery, not other aspects of debt structure.[33]

[33]That the secured debt share variable is not significant is not particularly surprising given that collateral merely gives one class of claimants priority over other classes. However, liens might protect assets from dissipation by the firm prior to bankruptcy. Perhaps such protection is not very material to recovery rates because banks take into account the degree of such protection on a case-by-case basis in setting the foreclosure threshold.

Table 5: **Debt instrument-level recovery rate regressions**

Details are the same as in Table 3, with the following exceptions. Observations are for a single representative loan and a single representative bond per bankruptcy in columns 1 and 2, respectively, rather than at the firm level as in other tables. Where a firm has more than one loan or one bond, the recovery rates on individual instruments are aggregated to a representative instrument with weights being each instruments share of the total loan or bond claims. Year, industry and court dummies are included in regressions but not shown. The coupon rate on a borrower's loans is the mean spread over LIBOR in percentage points as recorded in the LossStats database. Where a firm has multiple bank loans outstanding, the mean spread is used.

	(1) Loans		(2) Bonds	
	Coeff.	*p*-value	Coeff.	*p*-value
Intercept	111.71	0.000	47.04	0.002
Variables of most interest				
Share bank debt	-17.74	0.090	14.39	0.308
Share bank debt * coupon	-5.17	0.010	-7.83	0.019
All bank debt dummy	9.82	0.116		
Share secured debt	-10.23	0.102	-1.46	0.849
No sub debt dummy	6.97	0.185	16.25	0.006
Share sub debt	6.48	0.498	4.03	0.708
Deadweight-cost proxies				
Time in bankruptcy	2.41	0.252	-2.53	0.313
Time from plan to emergence	-2.70	0.514	-3.03	0.535
Time in default pre-filing	2.18	0.599	2.23	0.649
Prepackaged bankruptcy	0.62	0.866	0.42	0.924
Number debt instruments	-0.20	0.557	1.38	0.000
Number priority classes	-0.82	0.513	-1.14	0.417
Fraud dummy	-6.86	0.298	21.08	0.014
Filed again within 5 yrs dummy	-1.00	0.889	-5.24	0.516
Number observations	412		355	
Adjusted R-squared	0.19		0.28	

4.8 Year and industry dummies

Returning to Table 3, dummies for the year in which bankruptcy was declared also are included in all regression specifications (1993 is the omitted year; 1987 and 1988 are combined because the number of observations for those years is small). With conventional standard errors, most coefficients are not significantly different from zero and no trend is evident, but point estimates are often large, negative and statistically significant in bad years (e.g., 1989–90, 1998–2002, 2008–10). With standard errors clustered by year, coefficients are statistically significantly different from zero for a few more years (1994, 1997, and all years 2006–2012). Future work on systematic risk in recoveries using firm-level recovery measures might be useful.

All regressions include a full set of industry dummies that we created by boiling down S&P's more than 100 industry designations appearing in LossStats to 17 categories (retail is the omitted category in regressions). Coefficients on most of these dummies are never statistically significant. Only those industries that sometimes have significant coefficients are tabulated in Table 3. Industry effects are very large only for Utilities (like prior researchers, we speculate that the regulated nature of utilities in the United States is responsible) and for bubble firms and those involved in asbestos litigation. "Bubble" firms are those in the telecom, internet, or energy trading sectors that filed for bankruptcy in the years 2000–2004. We classified bubble firms by inspection, as S&P's industry classifications are not always indicative. Such firms have economically and statistically significantly smaller recovery rates than other firms, which we regard as consistent with the finding of Acharya et al. (2007) that recoveries are lower for firms whose industry is deeply distressed when bankruptcy is filed. For firms involved in asbestos litigation, bankruptcy is declared as part of the firm's litigation strategy and the firm appears otherwise to be in better condition than the average bankrupt firm, as implied by the statistically significant coefficient of about 27.

4.9 Robustness checks

For realistic parameter values and for values of bank debt share well below 1, our model implies a nearly monotonic relationship between bank debt share and firm-level recovery rates. For higher values of bank debt share, the slope is flatter (see Figure 2). Moreover, the relationship depends on the loan coupon spread. We choose a linear base specification for our regressions because the nonlinearities are modest and because the coupon spread is available only for a subset of our data. However, the nonlinearities are of some interest because it is not obvious they are implied by the alternative models.

The first column of Table 6 reports results when we spline the bank debt share variable with a single knot at 0.3, which is near the mean and median values of bank debt share. We find evidence of the flatter slope shown in the bottom panel of Figure 2 for high values of bank debt share: The coefficient on the low-share segment is 57, whereas the coefficient on the high-share segment is 19. The coefficients are statistically significantly different from zero. However, significance of the difference in estimated coefficients is sensitive to the value of bank debt share at which the knot is located.

Some may believe that the burst of the tech bubble in the early 2000s may have caused bankruptcies with unusual features, and similarly for the financial crisis that began in 2007. Moreover, as

Table 6: **Alternative specifications of firm-level recovery rate regressions**
Details are the same as in Table 3 with the following exceptions. In Column 1, bank debt share has a spline representation with a single knot at 0.3. The "dummy for 0.3+ segment" coefficient allows the intercept for the second segment of the spline to differ. The last two columns show results when the sample is restricted to bankruptcies filed in the years 1987–97 and 1987–2006, respectively.

	(1) Spline λ		(2) Only λ		(3) 1987–97		(4) 1987–2006	
	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value
Intercept	54.30	0.000	49.13	0.000	56.69	0.000	54.22	0.000
Variables of most interest								
Share bank debt			35.04	0.000	33.83	0.004	23.15	0.001
Share bank debt range 0–0.3	57.29	0.016						
Share bank debt range 0.3+	18.71	0.076						
Intercept for 0.3+ segment	7.58	0.274						
Debt structure controls								
No bank debt dummy	-0.62	0.906			-3.55	0.623	-4.29	0.324
All bank debt dummy	8.94	0.133			-3.72	0.742	9.21	0.110
Share secured debt	-0.54	0.888			-2.19	0.740	0.21	0.960
All sub debt dummy	-8.89	0.162			-5.09	0.553	-10.18	0.116
No sub debt dummy	2.13	0.564			-1.19	0.866	4.65	0.245
Share sub debt	-8.49	0.193			-25.08	0.018	-6.14	0.365
Deadweight-cost proxies								
Time in bankruptcy	0.21	0.885	0.47	0.750	0.75	0.748	0.04	0.981
Time from plan to emergence	-1.65	0.512	-1.24	0.625	-4.73	0.243	-2.16	0.401
Time in default pre-filing	-0.48	0.838	-1.04	0.658	-0.93	0.775	-0.07	0.976
Prepackaged bankruptcy	7.16	0.007	7.85	0.003	5.82	0.271	7.45	0.010
Number debt instruments	0.43	0.123	0.50	0.065	-0.48	0.538	0.32	0.282
Number priority classes	-0.40	0.705	-0.11	0.895	2.72	0.170	0.11	0.918
Fraud dummy	1.29	0.790	-1.08	0.825	-18.96	0.155	1.36	0.793
Filed again within 5 yrs dummy	-6.65	0.232	-8.45	0.129	-1.58	0.844	-6.48	0.247
Number observations	773		773		257		682	
Adjusted R-squared	0.27		0.25		0.30		0.27	

mentioned previously, sample selection is a concern for the full sample in that bankruptcies declared late in the sample period that have a long time between filing and emergence are omitted. We address all of these concerns by reporting results in columns 3 and 4 of Table 6 for two subsamples, 1987–97 and 1987–2006. Results are generally similar to those for the full sample. As noted previously, we are confident that no bankruptcies filed by 2006 appear in LossStats after mid-2013 apart from W.R. Grace.

Table 7 reports results of regressions for a subsample of firms for which we were able to find usable data in Compustat. The Compustat subsample affords an opportunity to examine whether other characteristics of the firm are associated with recovery, such as the nature of its assets, its size, or its operating cashflow not long before filing. Estimates imply that the majority of such characteristics are not predictive of firm-level recovery, whether debt structure variables are included or not. The borrower's S&P rating at the fiscal year-end before filing also is not significant. Moreover, the significance of the bank debt share variable is maintained in the smaller Compustat subsample, regardless of what other variables are included.

Table 7: **Firm-level recovery rate regressions, Compustat-matched subsample**
Details are the same as in Table 3, with the following exceptions: Balance sheet and income statement variables, as well as rating dummies, are from Compustat and are dated as of the firm's last fiscal year-end date before filing bankruptcy for which data are available.

	(1) Base case		(2) Add firm vars		(3) No debt struc		(4) Ratings	
	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value
Intercept	63.82	0.000	64.69	0.000	57.00	0.000	67.71	0.000
Variable of most interest								
Share bank debt	23.06	0.006	22.63	0.008			24.08	0.005
Debt structure controls								
No bank debt dummy	-2.94	0.588	-2.35	0.670			-3.09	0.574
All bank debt dummy	11.72	0.086	12.72	0.065			12.02	0.087
Share secured debt	-5.34	0.302	-4.53	0.380			-5.36	0.310
All sub debt dummy	-17.10	0.028	-18.53	0.018			-17.06	0.031
No sub debt dummy	1.89	0.670	3.64	0.414			2.33	0.601
Share sub debt	-13.19	0.086	-8.79	0.261			-12.36	0.111
Firm characteristics								
Log total assets			1.63	0.266	2.17	0.167		
Non-intangible assets/assets			5.13	0.552	11.49	0.220		
Book liabilities/assets			-4.96	0.040	-7.02	0.007		
Operating income/assets			15.58	0.220	23.39	0.087		
Accts payable/total liabilities			-43.71	0.031	-21.45	0.323	-22.49	0.225
PPE/assets			-12.69	0.089	-14.39	0.078		
BB or safer							3.53	0.470
B							-1.27	0.702
CCC							2.92	0.488
CC or worse							2.23	0.660
Deadweight-cost proxies								
Time in bankruptcy	-1.41	0.428	-1.83	0.305	-2.44	0.209	-1.59	0.376
Time from plan to emergence	1.36	0.673	1.75	0.584	1.70	0.628	1.63	0.614
Time in default pre-filing	-0.38	0.905	1.31	0.692	1.31	0.716	-1.30	0.703
Prepackaged bankruptcy	8.75	0.009	7.82	0.019	4.26	0.243	8.79	0.010
Number debt instruments	0.12	0.707	-0.02	0.961	-0.07	0.868	0.10	0.768
Number priority classes	0.78	0.579	0.33	0.816	0.62	0.614	0.67	0.639
Fraud dummy	-0.08	0.989	-5.33	0.363	-4.82	0.447	-1.10	0.851
Filed again within 5 yrs dummy	-8.38	0.154	-9.10	0.123	-8.13	0.204	-8.02	0.177
Number observations	436		436		436		436	
Adjusted R-squared	0.34		0.36		0.22		0.34	

The variable of most interest for purposes of checking robustness is the share of total liabilities that is accounts payable. This category includes trade credit extended to the firm, which is likely to be treated by the court as a senior unsecured claim and, especially in the case of small accounts payable, may be paid in full during Chapter 11 bankruptcies in order to reduce the number of creditors and to permit the firm to continue operating with normal trade relationships. Because accounts payable are not measured in LossStats, a marginal additional dollar of payables represents an additional dollar of claims not measured in LossStats. Payments to such claims reduce our measured firm-level recovery rate by reducing assets available to pay debtholders. The estimated coefficient on the accounts payable variable in column 2 of Table 7, at -47, implies a reduction of about half a percentage point of our measured recovery rate for each additional percentage point of total liabilities that are accounts payable, which is a sensible magnitude. Inclusion of the variable does not materially affect the estimated coefficient on bank debt share.

Another exception is leverage. Measured as the ratio of book total liabilities to total assets, the coefficient estimate implies a moderate reduction in firm-level recovery rate of about five percentage points if book leverage increases from its median ratio of 1 to a value of 2.

In the spirit of our model, perhaps it is unsurprising that most observable firm characteristics are not strongly associated with recovery. They might be in a world with an exogenous default boundary, but banks can observe such variables and thus can be expected to take such variables into account in setting the default boundary in a manner likely to erase any correlation.[34]

4.10 Summing up the evidence

We regard the totality of the empirical evidence as providing strong support for our model. None of our model's predictions are rejected, and some of its predictions that are supported are novel. We find a consistent and enormously robust role of bank debt share in predicting firm-level recovery that is difficult to pass off as being due to maturity effects or deadweight costs of bankruptcy. Debt structure variables contribute more than half of the explanatory power of the base case regression, which along with the large size of coefficients implies debt structure is economically important. Bank debt with covenants matters but not bank debt without covenants, implying that it is the control rights granted by the covenants that matters, not the lender's status as a bank or bank-like entity. Loan coupon rates predict firm-level recovery in a manner predicted by our model, and we find predicted nonlinearities and interaction effects of coupon rates and bank debt share that are not obvious implications of alternatives. Results are robust to inclusion or exclusion of a wide array of auxiliary and control variables and to use of subsamples that effectively control for sample-selection bias. The great majority of loan recovery rates are near 100 percent as predicted.

We specify and discuss alternatives in order to make a case that the empirical relationships we find are not simply due to alternative mechanisms. We do not propose to interpret the evidence as rejecting the alternatives. For example, we do find a negative relationship between recovery rates and maturity in some specifications as predicted by Leland and Toft (1996). We believe it is likely that bankruptcies are generated by a diversity of mechanisms. This is conceptually consistent with a first-passage model: If different mechanisms generate bankruptcy filings at different asset values for any given firm, the mechanism associated with the highest threshold will describe the decision for that firm. The evidence strongly supports our model as capturing the determinants of many bankruptcies, but it does not imply our model describes all bankruptcies. Given the large number of extant models, sorting out which models describe which bankruptcies is a subject for future research.

[34]We are grateful to Richard Cantor for this point.

5 Discussion

This paper offers a model and evidence supportive of a hypothesis that private debtholders play an important role in determining the value of assets at which firms declare bankruptcy. In order to protect the recovery they receive, and using the control rights granted by loan covenants, private lenders set a threshold that is higher the larger is their share of the firm's debt. Because asset value at bankruptcy strongly influences the value distributed to claimants at emergence, a higher private debt share is associated with higher ultimate firm-level recovery rates. Our model also sheds light on the long-standing puzzle of relatively low average recoveries on defaulted corporate bonds.

We do not claim that private lenders *always* set the default boundary–casual inspection of the news reveals obvious cases of strategic default by equityholders–but their role appears to be of substantial empirical importance. Nor do we claim that our empirical evidence applies in all jurisdictions. Rather, an implication of our paper is that recovery rates may be sensitive to the enforceability of the conditional control rights that covenants give creditors and to details of bankruptcy law and practice.

In closing, we offer some suggestions for future research. First, our results suggest that literature on the capital structure decision might be enriched by further analysis of the choice of the private debt share of total debt. We assume the share is exogenous, which is reasonable for firms near the bankruptcy threshold, but it is clearly a choice variable for very solvent firms. Given that debt structure influences the states of the world in which bankruptcy occurs, the debt composition decision may interact with the leverage decision. More research is needed to reveal the nature and relevance of such interactions.

Second, modeling of recoveries on individual debt instruments, which has been the focus of most empirical work on recovery to date, might be revisited. Combining a model of firm-level recovery with nonlinear modeling of the impact of debt instrument seniority might provide more insight than models suggested to date, which are usually linear and ignore debt composition.

Third, our examination of firm-level measures of ultimate recovery differs from almost all prior studies. Most have examined samples of recoveries to individual debt instruments and some have interpreted results as revealing information about the relationship between firm characteristics and recovery rates. To the extent that our results differ for similar variables, more research may be needed because we expect that firm-level explanations would be revealed most clearly in firm-level regressions.

Finally, our paper may help point the way toward resolution of some puzzles implicit in existing literature. Faulkender and Petersen (2006) find that firms with bonds outstanding are considerably more leveraged on average than firms with only private debt in their capital structure. We do not examine capital structure decisions, but in our model, a firm that wished to increase leverage while holding its bankruptcy probability fixed could do so by issuing more bonds and no more loans. Davydenko (2005) finds that while fixed boundary models of default do reasonably well in predicting default rates on average, such models do not perform so well in the cross section. Cross sectional variation in the absence of controls for debt structure is natural in our framework because the boundary varies with bank debt share.

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A Existence of optimal foreclosure threshold

We prove Proposition 2, and show how it must be modified under the extended model of Section 2.3. We also provide boundary values for the ψ and Ξ functions.

The result for $\psi(0)$ in equation (6) follows directly from the asymptotic limit (FWC 07.20.06.0009.01)

$$\lim_{z\to\infty} z^a {}_1F_1(a, a+b, -z) = \frac{\Gamma(a+b)}{\Gamma(b)}$$

This limiting case also arises in Black and Cox (1976, eq. 19), as Black and Cox impose $\kappa = 0$ exogeneously. From equations 10 and 12, we can write $\Xi(\kappa)$ as

$$\Xi(\kappa; \alpha, \beta, \zeta) = \alpha\zeta \frac{\psi(\kappa; \alpha+1, \beta-1, \zeta)}{\psi(\kappa; \alpha, \beta, \zeta)}$$

From the boundary value for ψ, we have

$$\Xi(0; \alpha, \beta, \zeta) = \alpha\zeta \frac{\gamma(\alpha+\beta)/\gamma(\beta-1)}{\gamma(\alpha+\beta)/\gamma(\beta)} = \alpha(\beta-1)\zeta$$

Noting that $\alpha(\beta-1) = 2r/\sigma^2$, we arrive at

$$\lim_{\kappa\to 0} \Xi(\kappa; \alpha, \beta, \zeta) = r/\mathcal{C}. \tag{17}$$

It is easily verified that $M(0, D, s^2) = 0$ and $M_1(0, D, s^2) = 1$, which implies that $B'(0) = \exp(\chi + \eta^2/2)$ and $B(0) = 0$. We arrive at the boundary value

$$\mathcal{F}(0) = \exp(\chi + \eta^2/2) - \frac{c\lambda}{\mathcal{C}} \tag{18}$$

Next consider the behavior of $\mathcal{F}(\kappa)$ as κ grows large. Applying the asymptotic expansion of the standard normal cdf (FWC 06.27.06.0017.01) to equation (11), we obtain $\kappa B'(\kappa) \to 0$ as $\kappa \to \infty$. It is straightforward to show that $\kappa\Xi(\kappa) \to \alpha$ and $B(\kappa) \to \lambda$ as $\kappa \to \infty$, so

$$\lim_{\kappa\to\infty} \kappa\mathcal{F}(\kappa) = -\alpha\lambda\frac{c-r}{r} < 0$$

This implies that $\mathcal{F}(\kappa)$ converges to zero from below as κ increases towards infinity.

In the setting of Proposition 2, $\mathcal{F}(0)$ is nonnegative. The Intermediate Value Theorem implies that there must be a finite nonnegative κ^* such that $\mathcal{F}(\kappa^*) = 0$ and $\mathcal{F}'(\kappa^*) < 0$. When there are positive cashflows to investors other than the bank ($\gamma(1-\lambda) > 0$ or $\delta > 0$), $\mathcal{F}(0)$ is strictly positive. In this case, κ^* must be strictly positive as well.

In the extended model, $\mathcal{F}(0)$ remains nonnegative if the mean of the shock to asset value is not too negative ($\chi \geq -\eta^2/2$), in which case the proposition holds without modification. In the event that $\chi < -\eta^2/2$, we can have $\mathcal{F}(0) < 0$, in which case we have the corner solution $\kappa^* = 0$.

B Optimal foreclosure when recovery is deterministic

We require the following intermediate result:

Lemma 2
The function $\Xi(\kappa)$ is decreasing in σ and is bounded from above by

$$\lim_{\sigma\to 0}\Xi(\kappa)=\frac{r}{\mathcal{C}-\kappa(r-\rho)}$$

for all $\kappa<\mathcal{C}/(r-\rho)$.

The limit as $\sigma\to 0$ can be derived from the asymptotic formula for the ${}_1F_1$ function in FWC 07.20.06.0008.01. To see that $\Xi(\kappa)$ must be decreasing in σ, observe that

$$\Xi(\kappa)=\frac{\partial}{\partial\kappa}\left(\frac{\psi(V;\alpha,\beta,\zeta)}{\psi(\kappa;\alpha,\beta,\zeta)}\right)\Bigg|_{V=\kappa}$$

and so

$$\frac{\partial}{\partial\sigma}\Xi(\kappa)=\frac{\partial}{\partial\kappa}\frac{\partial}{\partial\sigma}\left(\frac{\psi(V;\alpha,\beta,\zeta)}{\psi(\kappa;\alpha,\beta,\zeta)}\right)\Bigg|_{V=\kappa}$$

The ratio $\psi(V)/\psi(\kappa)$ gives the present value of receiving \$1 contingent on future bankruptcy. As the value of this option must be increasing in σ whenever the option is out of the money, we have

$$\frac{\partial}{\partial\sigma}\left(\frac{\psi(V;\alpha,\beta,\zeta)}{\psi(\kappa;\alpha,\beta,\zeta)}\right)>0$$

for all $\kappa<V$. At $V=\kappa$, the option is worth exactly \$1, regardless of σ, so $\partial/\partial\sigma(\psi(V)/\psi(\kappa))$ must be decreasing in κ for V in the neighborhood of κ. This implies that $\partial\Xi(\kappa)/\partial\sigma$ must be negative.

When recovery is deterministic ($\tau=0$ and/or $\tilde{\sigma}=0$), recovery is given by $B(\kappa)=\min\{\lambda,\kappa\}$. For all $\kappa>\lambda$, $B(\kappa)=\lambda$ and $B'(\kappa)=0$, so

$$\mathcal{F}(\kappa)=-\left(\lambda\frac{c}{r}-\lambda\right)\Xi(\kappa)<0.$$

For all $\kappa<\lambda$, $B(\kappa)=\kappa$ and $B'(\kappa)=1$, so

$$\begin{aligned}\mathcal{F}(\kappa)&=1-\left(\lambda\frac{c}{r}-\kappa\right)\Xi(\kappa)\\&>1-\left(\lambda\frac{c}{r}-\kappa\right)\frac{r}{\mathcal{C}-\kappa(r-\rho)}=\frac{\gamma(1-\lambda)+\delta+\kappa\rho}{\mathcal{C}-\kappa(r-\rho)}\geq 0\end{aligned}$$

Note that the application of Lemma 2 is valid because $\kappa<\lambda\leq 1\leq\mathcal{C}/(r-\rho)$. It follows that the cusp point at $\kappa=\lambda$ is the optimal foreclosure threshold.

C Proof of Lemma 1

Define the function $\tilde{\Xi}(V,\zeta) \equiv \Xi(V;\alpha,\beta,\zeta)/\zeta$ for α,β fixed. It is easily checked that $\tilde{\Xi}$ depends only on the product ζV and not on V and ζ individually, which implies

$$V \cdot \frac{\partial}{\partial V}\tilde{\Xi}(V,\zeta) = \zeta \cdot \frac{\partial}{\partial \zeta}\tilde{\Xi}(V,\zeta).$$

We substitute back to get

$$\zeta \cdot \frac{\partial}{\partial \zeta}\Xi(V;\alpha,\beta,\zeta) = \Xi(V;\alpha,\beta,\zeta) + V \cdot \Xi'(V;\alpha,\beta,\zeta).$$

To sign the right hand side, observe that

$$\begin{aligned}
\Xi'(V) &= \Xi(V)^2 - \frac{\alpha\zeta\psi'(V;\alpha+1,\beta-1,\zeta)}{\psi(V;\alpha,\beta,\zeta)} \\
&= \Xi(V)\left(\frac{\alpha}{V}\frac{{}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))}{{}_1F_1(\alpha,\alpha+\beta,-1/(\zeta V))} - \frac{\alpha+1}{V}\frac{{}_1F_1(\alpha+2,\alpha+\beta,-1/(\zeta V))}{{}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))}\right).
\end{aligned}$$

This implies

$$\begin{aligned}
\zeta\cdot\frac{\partial}{\partial \zeta}\Xi(V;\alpha,\beta,\zeta) &= \Xi(V)\cdot\left(1+\alpha\frac{{}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))}{{}_1F_1(\alpha,\alpha+\beta,-1/(\zeta V))} - (\alpha+1)\frac{{}_1F_1(\alpha+2,\alpha+\beta,-1/(\zeta V))}{{}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))}\right) \\
&= \Xi(V)\cdot\left(1 - \frac{{}_1F_1(\alpha+2,\alpha+\beta,-1/(\zeta V))}{{}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))}\right) \\
&+ \alpha\Xi(V)\cdot\left(\frac{{}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))}{{}_1F_1(\alpha,\alpha+\beta,-1/(\zeta V))} - \frac{{}_1F_1(\alpha+2,\alpha+\beta,-1/(\zeta V))}{{}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))}\right). \quad (19)
\end{aligned}$$

The ${}_1F_1$ function is decreasing in its first parameter when the argument is negative, so the first term in the last line of equation (19) is positive for all finite $V > 0$. The main theorem in Barnard et al. (2009) guarantees that

$${}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))^2 > {}_1F_1(\alpha+2,\alpha+\beta,-1/(\zeta V)) \cdot {}_1F_1(\alpha,\alpha+\beta,-1/(\zeta V))$$

for all finite V, so the second term in equation (19) is positive as well.

D Asymptotic analysis for large expected bankruptcy shocks

This appendix shows that $\kappa^* \to 0$ whenever χ is very large in magnitude. When $\chi \to -\infty$, we have

$$B(\kappa) = \mathrm{E}\left[\min\{\lambda, \exp(\chi+\eta^2/2)V_{t+\tau}\}|V_t = \kappa\right] \to 0$$

and

$$B'(\kappa) = \exp(\chi+\eta^2/2)M_1(\exp(\chi+\eta^2/2)\kappa, \lambda, \sqrt{\tau\sigma^2+\eta^2}) \to 0$$

for any fixed κ. Therefore, for χ sufficiently large and negative, $\mathcal{F}(\kappa)$ is dominated by the term $-\lambda\frac{c}{r}\Xi(\kappa)$ which is negative. This pushes us to the corner solution $\kappa^* = 0$.

When $\chi \to \infty$, we have

$$B(\kappa) = \mathrm{E}\left[\min\{\lambda, \exp(\chi + \eta^2/2)V_{t+\tau}\}|V_t = \kappa\right] \to \lambda$$

for any $\kappa \to 0$, so again $B'(\kappa) \to 0$. Therefore, for χ sufficiently large and positive, $\mathcal{F}(\kappa)$ is dominated by the term $-(\lambda\frac{c}{r} - \lambda)\Xi(\kappa) < 0$. This pushes us towards $\kappa^* = 0$, though the corner solution will not be reached for any finite χ.

E Covenant boundary and waiver fees

In this Appendix, we introduce a finite covenant boundary ν. Whenever $V_t \leq \nu$, the borrower is considered to be in violation of covenants and the bank has an option to foreclose at will. Whenever $V > \nu$, covenants are satisfied and the bank cannot foreclose. Loan contracts may specify a fee to be paid to the bank when a covenant violation is waived, and in other cases something similar might be achieved by renegotiation at the time of covenant violation. For simplicity, we assume that a waiver penalty of w is added to the coupon rate c whenever $\kappa < V \leq \nu$.

To maintain clarity in notation, we mark with a check any parameter that pertains under $V > \nu$, and mark with a hat any parameter that pertains under $V \leq \nu$. Thus, $\check{c}$ is the normal coupon rate, and $\hat{c} = \check{c} + w$ is the penalty coupon rate. (Think "smile" for the normal state and "frown" for the violation state.) We allow for the possibility that the contract requires lower dividend payments to equityholders when $V \leq \nu$, so similarly distinguish $\hat{\delta} \leq \check{\delta}$ and $\hat{\rho} \leq \check{\rho}$. All other fundamental parameters are fixed across the two regimes, but derived parameters such as α, β and ζ vary and so are marked with checks and hats. For fixed κ, the loan price is

$$F(V) = \begin{cases} \hat{F}(V) & \text{if } V \leq \nu, \\ \check{F}(V) & \text{if } V > \nu. \end{cases} \tag{20}$$

where $\hat{F}(V)$ and $\check{F}(V)$ are solutions to equation (4) under the two parameter regimes. It is important to recognize here that the $\hat{F}$ and $\check{F}$ functions differ from equation (7) because the relevant boundary conditions are not the same.

For the moment, take default boundary κ as fixed. The lower boundary value for $\hat{F}$ is $\hat{F}(\kappa) = B(\kappa)$. The upper boundary value for $\check{F}$ is $\check{F}(\infty) = \lambda\check{c}/r$. Two additional boundary restrictions are required to provide the upper boundary of $\hat{F}(V)$ and lower boundary of $\check{F}(V)$ where they join at covenant threshold $V = \nu$. These are given by the smooth pasting conditions, $\hat{F}(\nu) = \check{F}(\nu)$ and $\hat{F}'(\nu) = \check{F}'(\nu)$. As V is driven by a diffusion, passage across the threshold at ν is an accessible event, which implies that F must be continuous at $V = \nu$. Dixit (1993, §3.8) provides a no-arbitrage argument for continuity in the first derivatives.

Let f_ν be the value of the loan at ν. Solution to $\check{F}(V)$ proceeds exactly as for the baseline model, except that the lower boundary is $\check{F}(\nu) = f_\nu$. This implies

$$\check{A}_1 = \left(\lambda\frac{\check{c}}{r} - f_\nu\right)\frac{1}{\psi(\nu; \check{\alpha}, \check{\beta}, \check{\zeta})} = \left(\lambda\frac{\check{c}}{r} - f_\nu\right)\frac{1}{\check{\psi}_1(\nu)}$$

where for convenience we define

$$\check{\psi}_1(y) = \psi(y; \check{\alpha}, \check{\beta}, \check{\zeta}).$$

We similarly define for the violation state

$$\begin{aligned}\hat{\psi}_1(y) &= \psi(y; \hat{\alpha}, \hat{\beta}, \hat{\zeta}) \\ \hat{\psi}_2(y) &= \psi(y; 1-\hat{\beta}, 1-\hat{\alpha}, \hat{\zeta})\end{aligned}$$

The boundary conditions for $\hat{F}(V)$ lead to simultaneous linear equations

$$\begin{aligned}\hat{A}_1 \cdot \hat{\psi}_1(\kappa) + \hat{A}_2 \cdot \hat{\psi}_2(\kappa) &= \lambda\frac{\hat{c}}{r} - B(\kappa) \\ \hat{A}_1 \cdot \hat{\psi}_1(\nu) + \hat{A}_2 \cdot \hat{\psi}_2(\nu) &= \lambda\frac{\hat{c}}{r} - f_\nu.\end{aligned}$$

which has solution

$$\begin{aligned}\hat{A}_1 &= \frac{1}{\hat{\Delta}}\left(\hat{\psi}_2(\nu)\left(\lambda\frac{\hat{c}}{r} - B(\kappa)\right) - \hat{\psi}_2(\kappa)\left(\lambda\frac{\hat{c}}{r} - f_\nu\right)\right) \\ \hat{A}_2 &= \frac{1}{\hat{\Delta}}\left(-\hat{\psi}_1(\nu)\left(\lambda\frac{\hat{c}}{r} - B(\kappa)\right) + \hat{\psi}_1(\kappa)\left(\lambda\frac{\hat{c}}{r} - f_\nu\right)\right)\end{aligned}$$

where $\hat{\Delta}$ is the determinant

$$\hat{\Delta} \equiv \hat{\psi}_1(\kappa)\hat{\psi}_2(\nu) - \hat{\psi}_1(\nu)\hat{\psi}_2(\kappa)$$

Finally, we impose $\hat{F}'(\nu) = \check{F}'(\nu)$ to pin down f_ν as

$$f_\nu = \frac{\lambda\frac{\check{c}}{r}\check{\Xi}(\nu) + \lambda\frac{\hat{c}}{r}\hat{\Xi}(\kappa,\nu) - \left(\lambda\frac{\hat{c}}{r} - B(\kappa)\right)\hat{\Xi}(\nu,\nu)}{\check{\Xi}(\nu) + \hat{\Xi}(\kappa,\nu)} \tag{21}$$

where

$$\hat{\Xi}(a,b) \equiv \frac{1}{\hat{\Delta}}\left(\hat{\psi}_1(a)\hat{\psi}_2'(b) - \hat{\psi}_2(a)\hat{\psi}_1'(b)\right).$$

The two-variable Ξ function extends the one-variable function in the sense that

$$\lim_{\nu\to\infty} \Xi(\nu,\kappa) = \Xi(\kappa).$$

Two examples are shown in Figure 8. The solid curve is $F(V)$. The points $(\kappa^*, B(\kappa^*))$ and $(\nu, F(\nu))$ are marked with circles. Observe that F need not be monotonic in V. If the waiver fee is high enough, then the loan is most valuable when covenants are in violation while V is still not too close to the default boundary. In this case, F peaks between κ and ν, and $\check{F}$ converges to its asymptotic value from above rather than from below.

The dashed curves are lower and upper bounds derived from the baseline model. The value of the loan must be no less than the value of a loan in which parameters are held fixed at $c = \check{c}$, $\delta = \check{\delta}$, and $\rho = \check{\rho}$, and where the initial condition is a value of $B(\kappa)$ at $V = \kappa$. Similarly, $F(V)$ can be no greater than the value of a loan in which parameters are held fixed at $c = \hat{c}$, $\delta = \hat{\delta}$, and $\rho = \hat{\rho}$, for the same initial condition. Therefore,

$$F^{lower}(V) \leq F(V) \leq F^{upper}(V)$$

where

$$\begin{aligned} F^{lower}(V) &= \lambda\frac{\check{c}}{r} - \left(\lambda\frac{\check{c}}{r} - B(\kappa)\right) \cdot \frac{\check{\psi}_1(V)}{\check{\psi}_1(\kappa)} \\ F^{upper}(V) &= \lambda\frac{\hat{c}}{r} - \left(\lambda\frac{\hat{c}}{r} - B(\kappa)\right) \cdot \frac{\hat{\psi}_1(V)}{\hat{\psi}_1(\kappa)} \end{aligned}$$

Observe that F clings to its upper bound at very low V (where the violation state parameters are the dominant influence), and converges to its lower bound as V tends to infinity (where the normal state parameters dominate).

To complete the solution of our model, we solve for the optimal κ^* satisfying equation (8), and obtain the first order condition

$$\mathcal{F}(\kappa) = B'(\kappa) - \left(\lambda\frac{\hat{c}}{r} - B(\kappa)\right)\hat{\Xi}(\nu,\kappa) + \left(\lambda\frac{\hat{c}}{r} - f_\nu(\kappa)\right)\hat{\Xi}(\kappa,\kappa) \tag{22}$$

where we have written $f_\nu(\kappa)$ to emphasize the dependence of f_ν on κ. As κ^* is constrained to the interval $[0, \nu]$, corner solutions must be checked. Otherwise, numerical solution for κ^* is straightforward.

We can rearrange equation (22) to emphasize its relationship to the FOC for the baseline model. We substitute in equation (21) to arrive at

$$\mathcal{F}(\kappa) = B'(\kappa) - \left(\lambda\frac{\hat{c}}{r} - B(\kappa)\right)\left(\hat{\Xi}(\nu,\kappa) - \frac{\hat{\Xi}(\nu,\nu)\hat{\Xi}(\kappa,\kappa)}{\check{\Xi}(\nu) + \hat{\Xi}(\kappa,\nu)}\right) + \lambda\frac{w}{r}\frac{\check{\Xi}(\nu)\hat{\Xi}(\kappa,\kappa)}{\check{\Xi}(\nu) + \hat{\Xi}(\kappa,\nu)} \tag{23}$$

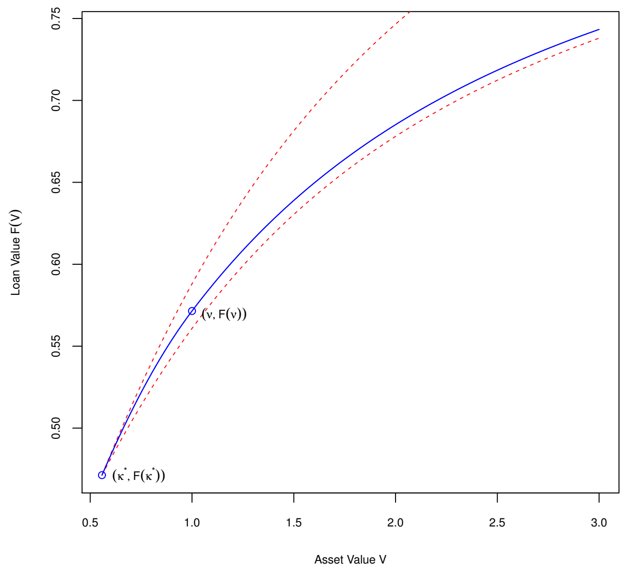
When the penalty state does not alter contractual parameters (i.e., $w = 0$, $\hat{\delta} = \check{\delta}$, and $\hat{\rho} = \check{\rho}$), then $\check{\Xi}(\nu) = \hat{\Xi}(\nu)$. Some tedious algebra can verify that

$$\hat{\Xi}(\nu,\kappa) - \frac{\hat{\Xi}(\nu,\nu)\hat{\Xi}(\kappa,\kappa)}{\hat{\Xi}(\nu) + \hat{\Xi}(\kappa,\nu)} = \hat{\Xi}(\kappa)$$

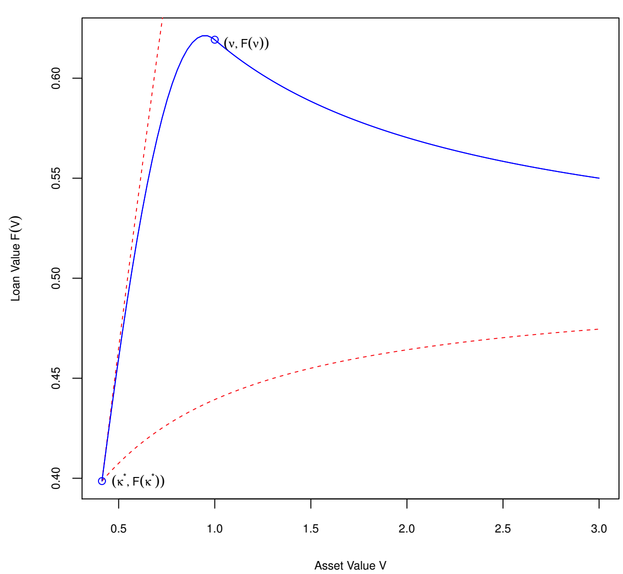
in which case equation (23) reduces to equation (9).

Figure 9 explores the dependence of the optimal foreclosure boundary on the waiver fee $w = \hat{c} - \check{c}$ and the normal state spread $\check{c} - r$. We find that κ^* decreases with w over this range of parameters. As the waiver fee is received by the bank only until foreclosure (or a return to the "normal" state $V > \nu$), an increase in the waiver fee increases the bank's incentive to forbear. Finally, in Figure 10, we explore the effect on recovery. As we would expect, the higher is w, the lower is the recovery rate for both debt classes. The effect shown on the loan's loss given default can be quite large on a relative basis, even if not terribly large on an absolute basis.

Figure 8: Loan value and bounding functions



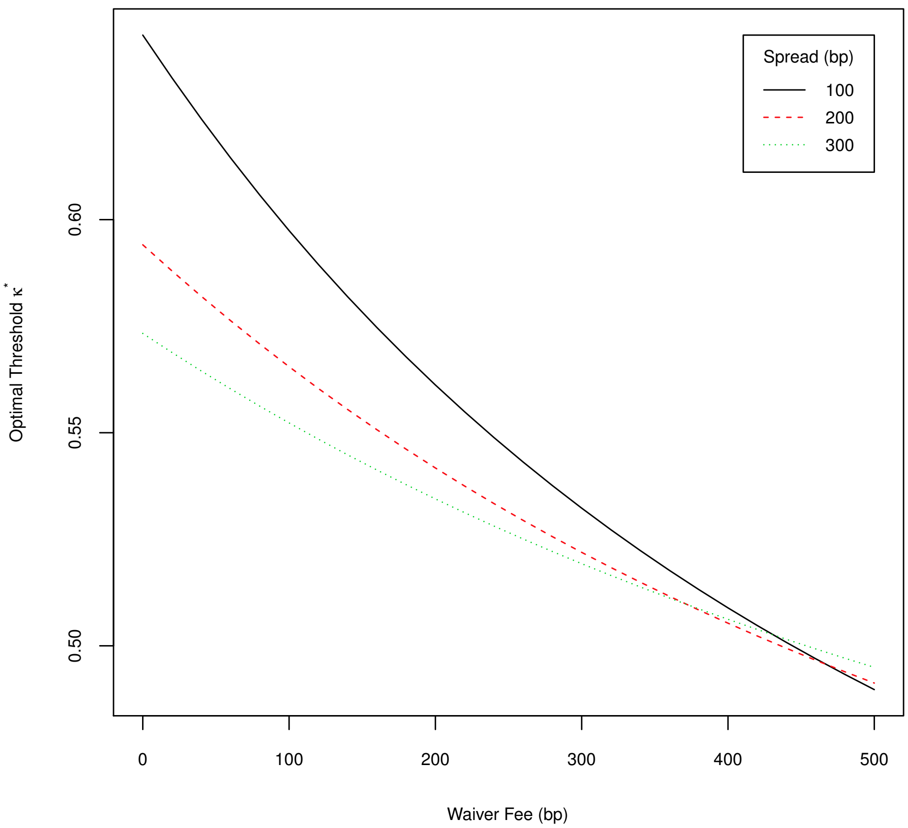
(a) Small waiver fee ($\check{c} = r + 0.025, \hat{c} = \check{c} + 0.01$)



(b) Large waiver fee ($\check{c} = r, \hat{c} = \check{c} + 0.10$)

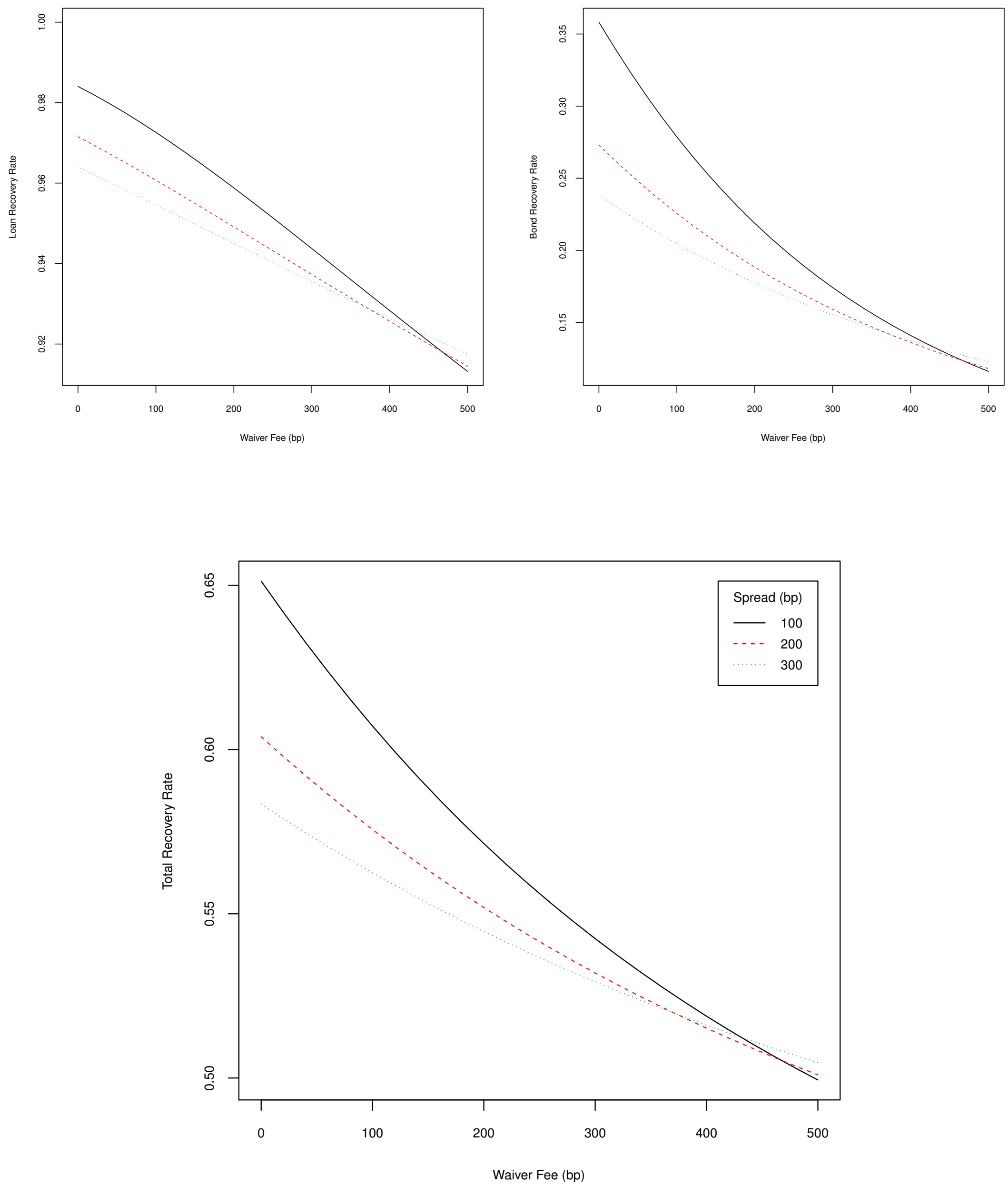
Solid blue line is $F(V)$, dashed red lines are upper and lower bounds from baseline model. Parameters: $r = 0.03$, $\gamma = 2(\check{c} - r) + r$, $\delta = \rho = 0$, $\sigma = \tilde{\sigma} = 0.25$, $\chi = \eta = 0$, $\nu = 1$, $\tau = 1$.

Figure 9: Effect of waiver fee on foreclosure threshold



Spread is $\check{c} - r$, expressed in basis points. Parameters: $\lambda = 0.5$, $r = 0.03$, $\gamma = 2(\check{c} - r) + r$, $\delta = \rho = 0$, $\sigma = \tilde{\sigma} = 0.25$, $\chi = \eta = 0$, $\nu = 1$, $\tau = 1$.

Figure 10: Effect of waiver fee on recovery at emergence



Spread is $\check{c} - r$, expressed in basis points. Parameters: $\lambda = 0.5$, $r = 0.03$, $\tilde{\mu} = r + 0.05$, $\gamma = 2(\check{c} - r) + r$, $\delta = \rho = 0$, $\sigma = \tilde{\sigma} = 0.25$, $\chi = \eta = 0$, $\nu = 1$, $\tau = 1$.

F Miscellaneous details about the data

Recovery measures: LossStats measures recovery in three ways: 1) The market value of the prepetition instruments near the date of emergence from bankruptcy; 2) The market value of the instruments given by the court to holders of each prepetition instrument, again near the date of emergence; and 3) where market values are not available near emergence, the value of instruments received at a later "liquidity" event. For example, if holders of prepetition loans are given new loans to satisfy their claims, and the new loans are refinanced a year after emergence, the new loans are assumed to be worth par at emergence. Often values are available from more than one method, with S&P designating a preferred method. We accept their preferred method in all but a few cases where the preferred–method valuation is for a valuation date much farther from the emergence date than other methods. We restricted the sample to cases using only the first two methods and obtained similar results, though of course the sample was smaller.

Valuation dates often differ somewhat from the emergence date. For the results reported in this paper, we did not discount reported values to the emergence date because in many cases claimants received new debt instruments and we presume the coupons on those instruments are set by the court to provide appropriate compensation for risk. However, we obtained untabulated results where we discounted to the emergence date using an appropriate Treasury zero-coupon interest rate plus 250 basis points, and also using spreads appropriate to the instrument received by the claimant (for example, the discount rate was higher for equity than for cash). Discounting and discount rates have no material impact on results, perhaps because valuation dates are generally not far from emergence dates.

We also produced recovery measures at bankruptcy by discounting recovery at emergence back to the bankruptcy date using a variety of discount rate assumptions. Again there is no material effect on our results, perhaps because the time between filing and emergence is generally not far from one year.

Measurement of recovery rates requires not only measures of value received by claimants, but also measures of the amount of their claims. Prepetition principal and interest are valid claims, but postpetition interest due to lenders is a deeply subordinated claim except for secured lenders whose collateral is valuable enough to cover both prepetition and postpetition claims. We cannot observe the value of collateral, so we used three different measures of claims. Results are not materially affected by the choice of measure. Our preferred measure allow postpetition interest in the claim only if the lender is secured and to the extent that value received by the lender is greater than the prepetition claim.

We produced and examined a large number of alternative recovery measures using a variety of assumptions about discounting and claims. The measures are highly correlated and are highly correlated with those of S&P (Pearson correlations are between 0.97 and 0.99), and our results are robust to choice of measure.

Structural subordination: In the U.S., most subordination is contractual. Structural subordination refers to cases where debt is a claim on a holding company and the debt is not guaranteed by subsidiary operating companies. Holding company debtholders are not legal claimants in the operating company bankruptcies and will receive a recovery only if the holding company's equity interest in the subs is worth something at emergence (or if the holding company has other assets). Thus, structurally subordinated debtholders often lose everything or almost everything. Because we are interested in recovery to the firm as a whole, without regard to the structure of the firm, we have identified cases of related-company bankruptcies in LossStats and have combined each set of related entities into a single simulated entity. There are six such cases. Results are robust to use of uncombined data.