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Non-Linear Phillips Curves with Inflation Regime-Switching

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Abstract

Building on the results in Nalewaik (FEDS 2015-93), this work models wage growth and core PCE price inflation as regime-switching processes, whose characteristics in the 1970s, 1980s and early 1990s differ fundamentally from their characteristics in the 1960s and from the mid-1990s to present. The key innovation here is the addition to the models of fundamental driving variables like labor-market slack, and the evidence strongly suggests a non-linear effect of slack on wage growth and core PCE price inflation that becomes much larger after labor markets tighten beyond a certain point. The results are informative for assessing the likelihood and risks of meeting certain inflation targets on a sustained basis.

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1 Introduction

> When the demand for a commodity or service is high relatively to the supply of it we expect the price to rise, the rate of rise being greater the greater the excess demand.[1]

Above is the first sentence of the paper that ushered in its namesake, the Phillips curve. The sentence could have been taken from any introductory economics textbook discussing price theory. Phillips (1958) then modestly suggest that it: "seems plausible that this principle should operate as one of the factors determining the rate of change of money wage rates, which are the price of labour services." Two sentences later Phillips (1958) describes a notion that sounds much like downward nominal wage rigidity—see Akerlof, Dickens and Perry (1996) and Daly and Hobijn (2014): "it appears that workers are reluctant to offer their services at less than the prevailing rates when the demand for labour is low and unemployment is high so that wage rates fall only very slowly." Phillips (1958) then concludes the last sentence of his first paragraph with: "The relation between unemployment and the rate of change of wage rates is therefore likely to be highly non-linear." Of the subsequent papers critical of Phillips (1958), one of the most prominent, Phelps (1967), found the non-linearity assumption sensible, positing that the slope of the Phillips curve should approach minus infinity as the unemployment rate approaches zero. So, it is somewhat odd that this paper needs to qualify the term Phillips curve with "non-linear," since non-linearity was central to the Phillips curve from the beginning. But that qualifier is indeed necessary, since most recent applied work using Phillips curves assumes linearity.

This paper embeds non-linear Phillips curves into regime-switching processes for wage

[1] Phillips (1958), page 283.

growth and core PCE price inflation, building on the work of Nalewaik (2015).[2] Modern time series models of the inflation process now routinely embed time-varying volatility and persistence; examples include Stock and Watson (2007), Cogley, Primiceri and Sargent (2010), Mertens (2011), and Clark and Doh (2014). The Markov-switching approach of Nalewaik (2015) does as well, and results here show one advantage of that approach is that a rich set of control variables—labor-market slack, a non-linear function of labor-market slack, the real dollar exchange rate, and bank lending—can be included into the model easily without the imposition of strong prior assumptions.

In the Markov-switching models, the characteristics of wage growth and core PCE price inflation in the 1970s, 1980s and early 1990s differ fundamentally from their characteristics in the 1960s and from the mid-1990s to present. In the 1970s, 1980s and early 1990s, the effect of variation in the control variables on the inflation measures is cumulative, carrying over from one period to the next because the processes are non-stationary. In the 1960s and from the mid-1990s to present, the processes are stationary, so the effect of the control variables on the inflation measures does not carry over from one period to the next.

In the stationary world, the terms natural rate of unemployment or NAIRU are meaningless, since level shifts up or down in the labor market slack time series are simply absorbed into the constant term of the equation, leaving model fit and forecasts unchanged. However, based on widely used estimates of the natural rate from the Congressional Budget Office (CBO), the paper shows a sharp steepening of the Phillips curve after labor-market slack becomes sufficiently negative, so the effect of slack on inflation becomes much larger after labor markets tighten beyond a certain point. These results suggest an alternative definition

[2]The core PCE price index is the price index for personal consumption expenditures (PCE) excluding food and energy.

of the natural rate of unemployment: the rate below which non-linearities in the Phillips curve begin to appear.

With their rich set of control variables, the models here are designed to be useful for both forecasting and risk assessments.[3] The estimated probabilities of the non-stationary inflation regime should be useful for assessing the likelihood that inflation expectations have become "unanchored," or *adaptive-causal* to be more precise, meaning inflation expectations both respond strongly to past inflation and have a strong causal effect on subsequent inflation, thereby imparting non-stationarity to the inflation processes.

Such a transition occurred once in the sample studied here, after the relatively high inflation in the second half of the 1960s. With the unemployment rate running well below the natural rate throughout that period, the effect of the non-linear Phillips curve likely caused that high inflation initially. However, that high inflation then appears to have entrenched elevated inflation expectations and institutional mechanisms for dealing with it like indexation of wage growth to past price changes, forces that probably kept inflation high even after labor markets slackened from 1969-70. While the models here show the regime transition occurred after that slackening of the labor market, some results suggest that an earlier slackening in 1967 or 1968 would have resulted in the same regime transition outcome. In other words, five years of elevated inflation generated by tight labor markets were enough to "unanchor" inflation expectations, but a shorter period of just a couple such years might have been sufficient to produce that outcome as well. That would be consistent with results from the models without conditioning variables in Nalewaik (2015), which show the transition into the non-stationary regime likely occurred in 1967.

To the author's knowledge, this paper is the first to embed a non-linear Phillips curve

[3] Out-of-sample forecast evaluation of the models, both pseudo and real, is left for future work.

into a Markov-switching model of the inflation process. Other recent papers on non-linear Phillips curves include Daly and Hobijn (2014), Kumar and Orrenius (2014), Fisher and Koenig (2014), and Speigner (2014). Other regime-switching models of the inflation process include Kim (1993), Evans and Wachtel (1993), Lanne (2006), and Davig and Doh (2014).

2 Model Structure

The log difference of the core PCE price index from the fourth quarter of year $t-1$ to the fourth quarter of year t, π_t, and the annual average change in log wages, π_t^w, both follow a two regime or two state ($S_t = 1$ or $S_t = 2$) Markov-switching process (see Hamilton (1989)), where:

$$\left[\begin{array}{c|c} \pi_t & \\ \pi_t^w & \end{array} S_t = 1, X_{t-1}\right] \sim N\left(\begin{bmatrix} \mu + \Theta_1 X_{t-1} \\ \mu^w + \Theta_1^w X_{t-1} \end{bmatrix}, \begin{bmatrix} (\sigma_1)^2 & \rho_1\sigma_1\sigma_1^w \\ \rho_1\sigma_1\sigma_1^w & (\sigma_1^w)^2 \end{bmatrix}\right)$$

$$\left[\begin{array}{c|c} \pi_t & S_t = 2, X_{t-1}, \\ \pi_t^w & \pi_{t-1}, \pi_{t-1}^w, \varepsilon_{t-1}, \varepsilon_{t-1}^w \end{array}\right] \sim N\left(\begin{bmatrix} \pi_{t-1} + \Theta_2 X_{t-1} + \theta\varepsilon_{t-1} \\ \pi_{t-1}^w + \Theta_2^w X_{t-1} + \theta^w \varepsilon_{t-1}^w \end{bmatrix}, \begin{bmatrix} (\sigma_2)^2 & \rho_2\sigma_2\sigma_2^w \\ \rho_2\sigma_2\sigma_2^w & (\sigma_2^w)^2 \end{bmatrix}\right)$$

The specification of the quarterly-frequency analog to this model is described in Appendix A. After conditioning on the explanatory variables X_{t-1}, the inflation measures are stationary in regime 1—hence the means μ and μ^w, while the measures follow a non-stationary process in regime 2, an MA(1) in differences with parameters θ and θ^w. The motivation for modeling inflation as this type of regime-switching process can be found in Nalewaik (2015), where total PCE price inflation follows an MA(2) in differences in its non-stationary regime. For parsimony, the work here uses the more traditional MA(1), the equivalent of modeling each of the inflation processes as the sum of a permanent and a transitory component—see Stock and Watson (2007). The explanatory variables X_{t-1} include slack and a non-linear function of slack discussed in the next section.

Let $\mathcal{H}_{t-1} = \{1, \pi_{t-1}, \pi^w_{t-1}, X_{t-1}, \pi_{t-2}, \pi^w_{t-2}, X_{t-2}, \ldots, \pi_1, \pi^w_1, X_1, \varepsilon_0 = \varepsilon_{-1} = \varepsilon^w_0 = \varepsilon^w_{-1} = 0\}$ denote the history of the inflation and explanatory variables, $\text{prob}\,(S_{t-1} = 1 \quad | \quad \mathcal{H}_{t-1}) = p_{t-1|t-1}$, and: $\text{prob}\,(S_{t-1} = 2 \quad | \quad \mathcal{H}_{t-1}) = 1 - p_{t-1|t-1}$. The probabilities and likelihood function, a weighted average of the state-contingent likelihood functions with these probabilities as weights, is computed as in Kim (1994), with the probability of each state persisting from one period to the next governed by the transition matrix:

$$\begin{pmatrix} p_{t+1|t} \\ 1 - p_{t+1|t} \end{pmatrix} = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix} \begin{pmatrix} p_{t|t} \\ 1 - p_{t|t} \end{pmatrix}.$$

The initial probability p_0 is treated as an estimated parameter.

The key characteristics of regime 2 are:

- Inflation is non-stationary, so the effect of X_t on inflation is cumulative. For example, assume core PCE inflation is 2 percent in year t and slack imparts one percentage point of upward pressure, moving inflation to 3 percent in year $t + 1$. Through the lagged inflation term, the effect of that past pressure persists into year $t + 2$, and if slack continues to impart one percentage point of upward pressure each year, inflation then moves up to 4 percent in year $t + 2$, and 5 percent in year $t + 3$, and 6 percent in year $t + 4$, etc. This cumulative effect provides the rationale for the term NAIRU (non-accelerating inflation rate of unemployment): as long as the unemployment rate remains below (above) the NAIRU in the non-stationary regime 2, inflation moves ever higher (lower).

- This persistence in the inflation process likely works, in large part, through inflation expectations. Nalewaik (2016) shows that expectations appear *adaptive-causal* in periods classified as belonging to the non-stationary regime 2, meaning that inflation expectations both (1) respond strongly to lagged inflation, and (2) appear to cause subsequent inflation. The evidence for the second condition is the strong approximately one-for-one predictive power of expectations for subsequent inflation. Widespread awareness of high inflation by the general public and institutional mechanisms for dealing with it like indexation of wage growth to past price change likely explain the adaptive-causal features of inflation expectations in the non-stationary regime 2. In other words, high price inflation leads to high wage demands by the general public, which then facilitates higher price inflation as businesses pass on higher wage costs—i.e., a wage-price spiral.[4]

- The most extreme tail risks to the inflation process, like the steep price declines in the U.S. Great Depression, or the hyperinflations seen in other counties, are possible in the non-stationary regime 2.

The key characteristics of regime 1 are:

- Inflation is stationary, so the effect of X_t on inflation is not cumulative. In the above example, assume core PCE inflation is at its hypothetical mean of 2 percent in year t, and slack then imparts one percentage point of upward pressure, moving inflation to 3 percent in year $t+1$. If slack continues to impart the same degree of upward pressure each year, inflation simply remains at 3 percent in year $t+2$ and all subsequent years.

[4] Such feedback effects could operate through the common lagged explanatory variables X_{t-1} in the model, particularly slack. Specifications allowing wage inflation to depend on lagged price inflation as well as lagged wage inflation show statistically significant evidence for such dependence, as well as the reverse, with price inflation depending on lagged wage inflation. However, allowing for these effects produced only modest improvements in model fit, so the paper reports results from the more parsimonious version of the model.

Absent non-linearities in the Phillips curve, the term NAIRU is meaningless in the stationary regime 1, as shifting $UGAP$ up or down by any arbitrary constant has no effect on model fit or forecasts, since those shifts are simply offset one-for-one in μ.

- Nalewaik (2016) shows that inflation expectations appear less adaptive and non-causal in periods classified as belonging to the stationary regime 1. So inflation expectations respond less strongly to lagged inflation in those periods, and, importantly, there is no evidence inflation expectations cause subsequent inflation. This lack of pass-though of inflation to inflation expectations and lack of a causal effect of inflation expectations on subsequent inflation likely explains the relatively-low inflation persistence in the stationary regime 1. Indifference of the general public to inflation when inflation is low and stable—see Akerlof, Dickens, and Perry (2000)—is one explanation for these results.

- The probability of the most extreme tail risks to the inflation process materializing in the stationary regime 1 is negligible, vanishingly small.

3 Data

The model in this paper is reduced form rather than structural, but theoretical considerations guide the choice of explanatory variables X_{t-1}, which are labor-market slack in the fourth quarter of the prior year $UGAP_{t-1}$, a quadratic function of slack when the gap turns negative $-(min(UGAP_{t-1}, 0))^2$, the two-year change in the real dollar exchange rate lagged one year $\Delta XR_{t-1,t-3}$ (i.e. the log change in the real exchange rate from the fourth quarter of year t-3 to the fourth quarter of year t-1), and the fourth quarter to fourth quarter (Q4/Q4) log difference in bank lending lagged one year ΔB_{t-1}, normalized by its 10-year moving average.[5] The exchange rate explains only price inflation, not wage inflation, and bank lending drives inflation only in the non-stationary regime 2:

$$\begin{aligned}
\Theta_1 X_{t-1} = &\quad \beta_{d,1}\Delta XR_{t-1,t-3} + \beta_{u,1}UGAP_{t-1} - \gamma_u(min(UGAP_{t-1},0))^2 \\
\Theta_2 X_{t-1} = &\quad \beta_{b,2}\Delta B_{t-1} + \beta_{d,2}\Delta XR_{t-1,t-3} + \beta_{u,2}UGAP_{t-1} - \gamma_u(min(UGAP_{t-1},0))^2 \\
\Theta_1^w X_{t-1} = &\quad \beta_{u,1}^w UGAP_{t-1} - \gamma_u^w(min(UGAP_{t-1},0))^2 \\
\Theta_2^w X_{t-1} = &\quad \beta_{b,2}^w \Delta B_{t-1} + \beta_{u,2}^w UGAP_{t-1} - \gamma_u^w(min(UGAP_{t-1},0))^2.
\end{aligned}$$

The coefficients are free to vary across states except the coefficient on the quadratic function of slack, which is common across states. The remainder of this section discusses these choices.

3.1 Price and Wage Inflation

Working with the four-quarter log change in the core PCE price index has the benefit of averaging away some of the higher-frequency noise in the monthly or quarterly changes, driven by residual seasonality, sampling error, and other reasons.[6] After taking lags, the available sample extends from 1961Q1 to 2016Q1. The wage growth measure is compensation per hour in the non-farm business sector, the only well-known, publicly available U.S. wage

[5]These explanatory variables are lagged to facilitate the usefulness of the model for forecasting, but also because wage and price inflation tend to lag many other economic indicators. The lagging nature of inflation helps motivate the choice of frequency and forecast horizon; with inflation, looking a year ahead or more is prudent.

[6]U.S. Bureau of Economic Analysis, Personal Consumption Expenditures Excluding Food and Energy (Chain-Type Price Index) [PCEPILFE], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PCEPILFE.

growth measure that is benchmarked to administrative tax records.[7] Irregular payments like bonuses add considerable volatility to compensation per hour, and this paper works with annual average growth rates to smooth through some of this volatility. The initial estimates of both of these variables are subject to potentially important subsequent revisions. This paper uses the vintage of both time series available in June 2016.[8]

These annual growth rates are shown in Figures 9 and 10. The paper estimates the quarterly analog to the annual Markov-switching model on the annualized quarterly log-changes in core PCE price inflation, shown in Figure 12.

3.2 Labor-Market Slack

The measure of slack $UGAP_t$ used in the paper is the difference between the quarterly unemployment rate from the Bureau of Labor Statistics and estimates of the natural rate of unemployment available from the CBO in March 2016. These two components, along with the non-linear term $-(min(UGAP_t, 0))^2$ are plotted in Figure 1. The lack of variation in the non-linear term over most of the sample motivates the decision to estimate γ_u using the full sample, although the γ_u estimates in Tables 4-5 use only periods classified as belonging to the stationary regime 1.

Of course, the natural rate of unemployment is unobserved, so the CBO estimates are just that, estimates. As emphasized by Orphanides (2002, 2003), gap estimates available in real time can be revised in important ways later. For example, the natural rate estimates in Figure 1 range from $5\frac{1}{2}$ to 6 percent from the early 1960s to early 1970s, but in real time,

[7] The 2015 wage and salary estimates have been benchmarked to tax records but are still subject to further adjustments, as are the supplements to wages and salaries in prior years.

[8] Examination of how the results in the paper might change if the data available in real time were used for estimation is left for future research.

economic policy advisers thought that "full employment" was 4 or $4\frac{1}{2}$ percent; see Barber (1975) and De Marchi (1975). This is important to keep in mind when applying the results in the paper to real time forecasting and policy analysis, but Tables 4 and 5 show that the main results in the paper hold when the raw unemployment rate U_t is substituted for $UGAP_t$.

Providing a complete theoretical justification for the relation between slack and inflation is beyond the scope of the paper, but in applied empirical work, the wage Phillips curve is often motivated by the same type of supply/demand considerations as discussed by Phillips (1958). Speaking loosely, the assumption is that "tight" labor markets with low unemployment rates typically coincide with relatively strong labor demand that bids up wage growth. The price Phillips curve is typically motivated either by pass through of wage inflation to price inflation, appealing to the large share of labor compensation in production costs, or by making similar arguments about product markets "tightening" at low unemployment rates. Again speaking loosely, the assumption is that tight labor markets boost demand for some products, leading to scarcity and price inflation. For some markets like those for apartment and house rentals, direct measures of "tightness" such as rental vacancy rates are observable.

3.3 Bank Lending

This variable is loans and leases in bank credit, all commercial banks, seasonally adjusted.[9] The four quarter log change in this variable, lagged one year and normalized by its 10-year moving average to make it approximated mean zero, is plotted in Figures 2 and 3 with four-

[9]Board of Governors of the Federal Reserve System (US), Loans and Leases in Bank Credit, All Commercial Banks [LOANS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/LOANS.

quarter core PCE price inflation and annual-average compensation-per-hour growth.[10] Even lagged one year, it is clear that bank lending growth had some predictive power for wage and price inflation from the mid-1970s to early 1980s, when both measures surged, came back down, and then surged again.

The idea that rapid bank lending growth might be inflationary is not new. For example, struggling to bring down inflation in the aftermath of World War II, president Truman and his economic advisers sent a "Special Message to the Congress" in November 1947, and its first policy proposal was: "1. To restore consumer credit controls and to restrain the creation of inflationary bank credit."[11] The effect of bank lending on inflation could work through a number of potential theoretical channels; a few are listed below:

- Bank lending could be a proxy for effects of the rate of change in real economic activity on inflation, sometimes called "speed" effects. As discussed in the original Phillips (1958) and also Gordon (1980), these effects are distinct from the effect of slack on inflation, since slack measures the level of economic activity (relative to its trend) instead of its rate of change. Romer (1996) suggests "speed" effects stem largely from the behavior of raw materials prices, which could move with the rate of change of economic activity because their supply is relatively inelastic.

- In the "debt-deflation" theory of Fisher (1933), private sector attempts to liquidate aggregate debt put downward pressure on the aggregate price level. Movements in bank lending should reflect this dynamic, which could provide another explanation for rate of change or "speed" effects. Indeed, these effects appeared prominently in the

[10]The approximate mean zero property is important in the non-stationary regime 2, given the cumulative property of the X_{t-1} variables on π_t.

[11]See Goodwin and Herren (1975), page 31.

dynamics of the Great Depression, when PCE price inflation showing remarkably high correlation with real GDP growth—see Figure 4.

- Bank lending could be a proxy for the effect of money growth on price inflation, due to the importance of bank lending for the money multiplier. Meltzer (2005) proposes money growth as an explanation for the behavior of inflation in the 1970s and early 1980s. More recently, during the first of several quantitative easing programs, members of the Federal Open Market Committee (FOMC) discussed the role of bank lending in determining whether the large increase in bank reserves generated by the program would result in inflationary pressures:

> CHAIRMAN BERNANKE. Thank you. Just a couple of comments. Saying that reserves facilitate deposits implies that the banks have to lend the money to the consumers.
>
> MR. KOCHERLAKOTA. Yes.
>
> CHAIRMAN BERNANKE. So they are at the moment not lending, as you know, and if they were to lend, it would show up in inflation but also show up in aggregate demand.[12]

After conditioning on the other explanatory variables, the annual-frequency Markov-switching model estimates showed no significant positive effect of lagged bank lending growth on price or wage inflation in the stationary regime 1, so the model restricts the lagged effect of bank lending to appear only in the non-stationary regime 2. However, bank lending growth has remained contemporaneously positively correlated with both price and wage inflation over the past 20 year (see Figures 5 and 6).

[12]Transcripts of November 11th, 2009 meeting of the Federal Open Market Committee,

3.4 The Real Dollar Exchange Rate

Figure 7 plots the two-year change in the quarterly real broad dollar exchange rate, lagged one year and on an inverted scale, against core PCE price inflation.[13] Some predictive power for price inflation appears from the mid-1970s to mid-1980s. Figure 8 examines the last twenty years in isolation, showing the continued predictive power of the exchange rate for inflation fluctuations. Clearly, an increase in the exchange value of the dollar tends to hold down the dollar price of imported consumer goods and thus core PCE price inflation. While much of this effect may be quite rapid, as in the model in Yellen (2015), the long lagged effect shown here suggests some of the pass through of exchange rate movements to consumer prices is delayed. Or the lagged effect may represent a combination of other factors:

- In recent times, movements in the dollar exchange rate often have been negatively correlated with movements in the price of crude oil, an important input into the production of many goods and services. These movements in production costs may take time to pass through to consumer prices.

- Dollar exchange rate movements have been similarly negatively correlated with movements in a range of other commodity prices that could have lagged effects on production costs and consumer prices.

- One explanation for the relatively low core PCE inflation in the second half of the 1990s was rapid productivity growth, which may have put downward pressure on price inflation even though wage increases were quite robust. At the same time, that surpris-

[13] Board of Governors of the Federal Reserve System (US), Real Trade Weighted U.S. Dollar Index, Broad [TWEXBPA], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/TWEXBPA. Prior to 1973, the exchange rate change is set to zero, reflecting adherence to the Bretton-Woods system of fixed exchange rates.

ingly fast productivity growth may have produced upward pressure on the exchange value of the dollar, contributing to the correlation shown in Figure 8.

Due in part to that experience in the second half of the 1990s, the Markov switching models found no strong relation between lagged exchange rate movements and wage growth, so the exchange rate was not included in the set of explanatory variables for wage growth.[14]

4 Empirical Results

4.1 Prologue: Interpreting the 1960s

The 1960s provide much (but not all) of the identification of the Phillips curve non-linearities in the 1961Q1 to 2016Q1 sample studied here. A vast body of work has studied why inflation took off in the 1960s, both contemporaneously and retrospectively—see Cochrane (1975), De Marchi (1975), Meltzer (2005), and Levin and Taylor (2010) for reviews. Much of it examines monetary policy, fiscal policy, and their interaction, showing how policy errors may have led to the 1960s inflation. But those are secondary causes, and this paper focuses on the list of possible primary causes of inflation—i.e. the channels through which the policy errors had their effect on inflation. The list includes:

1. Real activity effects, which work through slack and perhaps the other variables capturing speed effects.

2. Expectations effects, which are likely operable through lagged inflation in the non-stationary inflation regime.

[14]The effect of imported goods price inflation on wage growth was discussed in the original Phillips (1958), which concluded its effect could be ignored except under exceptional circumstances.

3. Money supply effects, which are potentially operable through bank lending in the non-stationary inflation regime.

For example, if government spending increases were the secondary cause of the 1960s inflation, they may have operated through the primary causes of either real activity effects driving down the unemployment rate or perhaps higher inflation expectations that had a positive causal effect on subsequent inflation. Similarly, a secondary cause of excessively loose monetary policy could have operated through any of the above three primary causes, so disentangling them is useful.

Some other potential explanations for the rise of inflation in the 1960s, like the power of labor unions to secure outsized increases in compensation per hour, might have operated outside of the three channels listed above. But as shown by the leftmost column of data in Table 1, compensation per hour grew a relatively modest 3 to 4 percent per year through 1965. The "moral suasion" of government officials urging adherence to "guideposts" for wage and price setting might have restrained wage growth over this period, but that restraint largely disappeared in 1966 after the unemployment rate fell below 4 percent, which surely improved the bargaining position of labor.[15] Those are precisely the type of developments that are supposed to be captured by Phillips curves, even if the mechanics of how labor contracts are set may have changed over time.[16]

Regarding expectations effects, Table 1 shows a proxy for "inflation expectations", forecasts from the semi-annual Livingston survey of professional forecasters of CPI inflation over

[15]See Barber (1975) and Cochrane (1975).

[16]As discussed below, since the Phillips curve model residuals for compensation-per-hour growth are not systematically positive in the 1960s, they do not suggest extra-model factors led to excessive wage gains.

the next 12 months.[17] As discussed in Nalewaik (2016), movements in inflation expectations are most likely to have a strong positive causal effect on subsequent inflation when they are widespread throughout the general public, driving demands that wage growth keep up with price inflation and allowing businesses to pass along price increases expected by their customers. As such, data on the inflation expectations of the general public would be more informative for whether inflation expectations drove inflation in the 1960s, but such data are unavailable. With surveys of professional forecasters, any positive correlation between inflation forecasts and inflation realizations may simply reflect predictability in the inflation process known to the forecasters, rather than a causal effect of their forecasts on subsequent inflation.

For example, Table 1 shows the CPI inflation and unemployment rate readings that prevailed at the time each Livingston inflation forecast was made; the upward drift in the inflation forecasts might have been a passive response to prior downward movements in the unemployment rate, as many forecasters were doubtless aware of the results in Phillips (1958). Indeed, the timing of those unemployment rate declines lines up well with the wage and price accelerations from 1966 to 1968, suggesting a tightening labor market may have been the key driving factor. The $RVAC_t$ column shows that the rental vacancy rate followed the unemployment rate down over this period, lending credence to the Phillips curve mechanism of tightening labor markets boosting demand for some products, leading to scarcity and price inflation. In contrast, inflation expectations appear to have lagged behind CPI inflation through most of the 1960s. For example, CPI inflation moved up to 1.6 percent and then well above 2 percent six months before the Livingston forecasts reached

[17] Federal Reserve Bank of Philadelphia, Livingston Survey, https://www.philadelphiafed.org/research-and-data/real-time-center/livingston-survey.

those levels in December 1965 and December 1966, not suggestive of a strong causal effect of inflation expectations on subsequent inflation.

When did inflation expectations begin to have a strong causal effect on subsequent inflation? The estimated probabilities of the non-stationary inflation regime provide an answer, assuming the non-stationarity is driven by adaptive-causal inflation expectations. The one-sided probabilities in the last column of Table 1 (from the unrestricted model estimates in Table 3 below) show these effects appear after the 1969-70 recession. Before then, the model explains and predicts inflation with just the non-linear Phillips curve, but after, something else is necessary to explain the difference between 1 to 2 percent inflation in June 1963 and 4 to 5 percent inflation in June 1971 with similar unemployment rates. That additional ingredient is non-stationarity in the inflation process. By the early 1970s, multiple years of high inflation likely entrenched high inflation expectations, consistent with the Livingston forecasts of continued elevated inflation, and those expectations were likely an important causal determinant of subsequent inflation by that point.[18]

Core PCE price inflation was between 3 and 5 percent from 1966 to 1970, so five years of core inflation in that range certainly appears to have been sufficient to entrench high inflation expectations and produce a transition to the non-stationary inflation regime. Would a shorter period of relatively high inflation have produced the same outcome? A period as short as a single year of core inflation around 3 percent seems unlikely to produce a regime transition, since the institutional mechanisms that make expectations causal for actual subsequent inflation, mechanisms like indexation of wage growth to past price changes, probably

[18]Some contemporaneous narrative evidence corroborates that notion. For example, economists at the Council of Economic Advisers (CEA) were forecasting inflation with a stationary (and non-linear, of course) Phillips curve in 1970, but by 1971, they concluded that the model had broken down because inflation expectations had become a more important driving force in the inflation process—see De Marchi (1975).

take time to develop and become widespread. Further, the regime probabilities from the annual models estimated below tend to smooth through transitory fluctuations. However, it would be too sanguine to conclude that core PCE inflation of 3 percent or higher for a two-to-four-year period does not increase the risk of a regime transition substantially, for at least three reasons.

First, the unrestricted model attributes the pick up in core PCE price inflation from 3 percent from 1966-7 to between 4 and 5 percent from 1968-9 to the decline in the unemployment rate from just under 4 percent in 1967 to around $3\frac{1}{2}$ percent in 1968. While the statistical evidence favors that interpretation, it is possible that the Phillips curve is not so non-linear. In that case, the step up in core inflation in 1968 may have reflected slack starting to have a cumulative effect on inflation after a transition to the non-stationary regime—see section 2. The Markov switching models estimated with constraints below do have Phillips curves that are more linear, and the one-sided probabilities from such a model shown in Table 1 do interpret the move up in inflation in 1968 as marking a regime transition. So, it is possible that the regime transition occurred after just two years of 3 percent core PCE inflation.[19]

Second, while the unrestricted, highly non-linear effect of slack on inflation does not appear to be cumulative in the 1960s, that cumulative effect may have been present but masked by offsetting shocks or movements in other conditioning variables. An obvious candidate is the 1969-1970 recession itself, as the unemployment rate began to edge up in the second half of 1969 and rose throughout 1970. The rate of change or "speed" effects in the models are lagged one year rather than contemporaneous, but contemporaneous "speed"

[19]The two-sided probabilities from this model show the parameters are estimated restricting the entire 1960s to be governed by the non-stationary regime. However, the one-sided probabilities show the stationary regime fits the data better up until 1968.

effects could have been present, offsetting the cumulative effect of slack after a transition to the non-stationary inflation regime in 1969 or 1970. If that is the case, three to four years of relatively high inflation may have been sufficient to cause a regime transition.[20]

Third, had the 1969-1970 increase in the unemployment rate occurred earlier, core inflation still might have remained high rather than settling back down between 1 and 2 percent. While they are occurring, multiple years of high inflation do not increase the probability of a regime transition in the models if they are well explained by a Phillips curve with a low unemployment rate and a stable mean. However, they may markedly increase the probability of a regime transition later by entrenching high inflation expectations that keep inflation elevated even after the unemployment rate moves up, detaching inflation from its stable mean. Formally, this can be modeled by making the transition matrix parameters functions of the length and intensity of non-linear Phillips curves effects over the recent past, interacted with a variable measuring whether slack is still putting upward pressure on inflation. One quarterly model below includes these time-varying transition probabilities, and finds that after the unemployment rate fell sufficiently far below the natural rate in the mid-1960s, a regime transition may have been inevitable when the upward pressure from slack abated.

4.2 Annual-Frequency Markov-Switching Model Estimates

The paper considers two different sets of model parameter estimates. To force the non-linear Phillips curve to compete with expectations effects and money supply effects in explaining the 1960s, the first set of estimates imposes non-stationarity throughout that period. By constraining $p_{11} = 1$ and $p_0 = 0$, inflation in this version of the model must start in the non-

[20]Contemporaneous "speed" effects are not included in the models because they are designed to be useful for forecasting, but the negative residuals from the model in recession years like 2008 and 2009 suggest they do exist.

stationary regime in 1961 and cannot return to the non-stationary regime after a transition out of it; the model then imposes non-stationarity on the 1960s because it is necessary to adequately fit subsequent inflation variation in the 1970s and 1980s. The top panels of Table 2 report parameter estimates for Q4/Q4 core PCE price inflation and annual average compensation per hour growth. The two-sided probabilities from this version of the model, not shown, show the single possible transition to the stationary regime occurs in 1996.

The bottom panels show the unrestricted parameter estimates of the model. As can be seen in the rightmost column, dropping the restrictions increases the maximized likelihood function substantially, easily passing a likelihood ratio test.[21] The largest change is in the γ_u and γ_u^w estimates, showing the size of the non-linearity in the Phillips curve approximately triples when moving from the restricted to the unrestricted specification. Standard errors computed using numerical derivative outer product estimates are reported in parentheses below the parameters. The standard errors around some parameter estimates are quite small and might overstate the statistical significance of some of the unconstrained parameter estimates (see the previous footnote), but the non-linearities are also highly statistically

[21]On a computational note, the negative of these log likelihood functions were minimized using the interior point algorithm of the fmincon function in Matlab R2016a. For the unrestricted version of the model (but not the restricted), the minimization results depended on the vector of parameters chosen to initialize the algorithm, and the results reported in table 2 are the global minimum across minima from a large array of initializations. The differences across these many minima were relatively minor. For example, considering 253 sets of initializations, over 90 percent of the minima yielded negative likelihoods less than 60, relatively close to the 55.4 shown in table 2, and of these, the mean value of γ_u was -0.51 with a standard deviation across the set of minima of 0.02. The main results of the paper would not change materially if any of these local minima were reported instead of the global minimum in table 2. However, these computational issues suggest two points. First, it is always possible that examination of an even wider array of initializations could have produced a lower global minimum, in which case the gap between the restricted and unrestricted likelihoods would be larger than in Table 2. Second, small changes in data or rounding issues may cause jumps across these relatively close local minima, and that possibility suggests the standard deviations across the global minima might give a better sense of the uncertainty around the parameter estimates than some of the standard errors in table 2. Example include μ, where the standard deviation across the set of relatively-close minima was 0.04 around a mean of 1.66; μ^w, with a standard deviation of 0.07 around a mean of 3.86, and γ_u^w, with a standard deviation of 0.04 around a mean of -0.28.

significant in the OLS regression results reported in the next subsection. The relatively low core PCE inflation in the 1960s holds down the unrestricted estimate of μ, at 1.61, compared to the restricted 1.74 estimate fit to 1996-2015 only. The restricted estimate is probably closer to the current long-run mean of core PCE price inflation in the stationary regime with zero slack and a stable real dollar exchange rate.[22] The two estimates of the mean growth rate of compensation per hour, μ^w, are close to identical, not suggestive of special factors producing outsized wage growth in the 1960s.

One- and two-sided probabilities from the unrestricted model, with data and year-ahead forecasts, are shown in Figures 9 and 10. As discussed in the previous subsection, the non-stationary regime probabilities are low through the 1960s as the non-linear Phillips curve explains the pickup in both core PCE price inflation and compensation-per-hour growth. Given the low non-stationary regime probability, the model predicts a sharp drop in both inflation measures in 1971 after the increase in the unemployment rate in 1970; when that does not occur, the probability of the non-stationary regime shoots up close to 100 percent.

In the non-stationary regime, the model predicts well the disinflations in the early 1980s and early 1990s, aided by the addition of lagged bank lending growth to the model. When the two inflation measures stabilize around their stationary regime means in the mid-1990s, the probabilities show the model then infers a transition to that regime. Nalewaik (2016) discusses potential explanations for this change in the inflation regime in the 1990s, includ-

[22] The slightly lower mean in the 1960s could have been the result of a number of factors, including the efforts by the Kennedy and Johnson administrations to actively hold price inflation close to zero. For example, president Kennedy used threats of antitrust action to force major steel companies to roll back price increases in 1962—see Barber (1975). Other than holding down μ, it is not clear what effect these interventions had on model parameters like the slope of the Phillips curve. Such interventions are probably rarer today, but some major sectors of the economy such as health care are subject to more government controls than in the 1960s, and the relatively small increases in Medicare and Medicaid reimbursement rates to doctors and hospitals were an important factor holding down core PCE price inflation from 2011 to 2015—see Clemens, Gottlieb, and Shapiro (2016).

ing (1) the stabilization of inflation expectations and (2) the disappearance of a one-for-one causal effect of inflation expectations on subsequent inflation. Supporting the second explanation, available measures of inflation expectations became either uncorrelated with or *negatively correlated with* subsequent inflation after the 1990s. This may have been the result of economic decision-makers becoming "rationally inattentive" to aggregate inflation after it stabilized at low levels—see Akerlof, Dickens, and Perry (2000).

In the stationary inflation regime starting in the mid-1990s, most of the variation identifying the Phillips curve non-linearity occurs from 1998 to 2001, when the unemployment rate fell about a percentage point below the current natural rate estimates. Figure 10 shows compensation-per-hour growth was actually *higher* than predicted by the non-linear Phillips curve over that period, by an average of $\frac{3}{4}$ percentage point. In contrast, core PCE price inflation was a little below the model forecasts, but not by much, as the lagged effect of dollar exchange rate appreciation—see Figure 8—roughly offsets the effect of the non-linear Phillips curve over much of that period. The experience of the late 1990s, then, is broadly consistent with non-linear Phillips curve parameters estimated assuming the 1960s and 1996-2015 were governed by the same stationary inflation process.

For core PCE price inflation, the two standard deviation range around the mean in the stationary regime is quite narrow, extending from 0.9 to 2.3 percent conditional on zero slack and a stable real dollar exchange rate. That range is considerably tighter than the 0.4 to 3.3 percent range Nalewaik (2015) found for total PCE price inflation using a longer sample and no conditioning variables. Core PCE price inflation occupied a narrow range from 1996-2015, accounting for part of this difference; total PCE price inflation moved more due to large swings in energy prices.

Conditioning on explanatory variables explains part of the difference between these ranges as well, and the non-linear Phillips curve in Figure 11 illustrates how core PCE inflation varies more widely with slack in the stationary inflation regime. With the unrestricted mean of 1.61 percent for core PCE price inflation, an unemployment rate 0.8 percentage point below the natural rate generates 2 percent core PCE price inflation. The curve then steepens sharply as the unemployment rate falls further below the natural rate, producing a wide range of inflation outcomes over a relatively small range of unemployment gap values. For example, unemployment rates 1.5, 2.0, and 2.5 percentage points below the natural rate generate 3, 4, and 5 percent core PCE price inflation, respectively.[23] For reference, the CBO estimates show the unemployment rate was 2.0 percentage points below the natural rate at the end of 1966 and 2.5 percentage points below it at the end of 1969. The next subsection section examines a specification with time-varying transition probabilities where years of high inflation generated by such large negative unemployment gaps entrench high inflation expectations and increase the probability that inflation does not come back down after the gap normalizes.

4.3 Quarterly-Frequency Markov-Switching Model Estimates

The first two sets of results in Table 3 show restricted and unrestricted parameter estimates from the quarterly version of the model described in Appendix A. The purpose of this model is not to forecast inflation one quarter ahead, but rather to allow the model to be updated more frequently than once per year. The model is structured to predict annualized quarterly core PCE price inflation four quarters ahead, and these realizations and four-quarter ahead forecasts are shown with one- and two-sided probabilities from the unrestricted model in

[23]With the milder non-linearities from the restricted set of parameter estimates in the top panels of table 2, a unemployment rate 1.0 percentage point below the natural rate generates 2 percent core PCE price inflation, while a rate 2.5 percentage points below generates 3 percent inflation.

Figure 12. This quarterly version of the model is univariate; the annualized quarterly growth rates of compensation per hour are too volatile to include in the model. Further, this quarterly version of the model should be used more cautiously than the annual version for forecasting and risk assessments, since it is estimated on latest-vintage data that could be quite different from that observed in real time because of revisions to seasonal factors.

As in Table 2, the likelihood improves substantially when the restrictions on P_{11} and P_0 are relaxed, showing the stationary regime fits much of the 1960s better than the non-stationary regime. The γ estimates governing the non-linearity in the Phillips curve are virtually identical to those in the annual-frequency specification, with the largest differences in the non-stationary regime coefficients.[24] The timing of the regime shifts is similar: the one- and two-sided probabilities of the non-stationary regime jump from negligible to close to 100 percent in 1971Q1 and 1970Q4, respectively. The non-stationary regime probabilities then remain close to 100 percent until 1994-1995, when they begin to move down just a bit sooner than in the annual-frequency version of the model.

The two-standard deviation range around the mean of the annualized quarterly growth rates in the stationary regime extends from 0.6 to 2.6 percent, conditional on zero slack and a stable real dollar exchange rate. For these quarterly estimates, this range could be wider for the initial estimates available in real time if subsequent data revisions smooth some volatility out of them. Further, inflation readings must be far outside of that range or persist outside of that range for multiple periods for the non-stationary regime probability to rise appreciably, given the very high regime persistence probability p_{11}. A single transitory

[24] The step-down in inflation from 1971Q3 to 1973Q1 was likely due to the price controls imposed by the Nixon administration in late 1971. These controls had little effect on the parameter estimates reported in table 3; replacing inflation in those quarters with its average over the prior year (1970Q3 to 1971Q2) produced similar estimates.

reading or two just outside the range would only increase the estimated standard deviation in the stationary regime, widening the range.

Given the similarity of the μ, $\beta_{u,1}$ and γ_u parameters across the annual- and quarterly-frequency specifications, the non-linear Phillips curve derived from the quarterly-frequency parameters is very similar to the one plotted in Figure 11, and shows that sufficiently negative values for slack can generate core PCE price inflation well above 2.6 percent in the stationary regime. However, when such high inflation readings appeared in the 1960s, they may have entrenched high inflation expectations and increased the probability of a regime transition once the upward pressure from slack abated by making it unlikely that inflation would move back down to μ. The last specification of Table 3 examines this possibility by allowing for time variation in the stationary regime persistence probability, modeling $p_{11,t}$ as a logistic function of the average non-linear Phillips curves effect over the past five years multiplied by a term measuring the extent to which the non-linear effect has diminished relative to its peak value over that period. The details of the specification are in Appendix A.

With only one transition out of the stationary inflation regime in the sample, identifying time variation in the transition probability is somewhat tenuous. However, Table 3 shows allowing for such time variation does improve the maximized value of the likelihood. Given the model parameters, $p_{11,t}$ falls from very close to 1.0 to around 0.2 in the second half of 1970 when the upward pressure on inflation from slack begins to diminish. What if that upward pressure had diminished sooner? By the end of 1966, the unemployment rate had fallen two percentage points below the natural rate, and had been more than a percentage point below the natural rate for a year and a half. In this specification, the upward pressure on inflation in 1966 and 1967 generated by those low unemployment rates is enough to guarantee a transition out of the stationary inflation regime, as $p_{11,t}$ would have been close to zero had the non-linear Phillips curve pressure disappeared in any quarter after 1967Q3. So the inevitability of the regime transition may have been decided in 1967, a result that agrees with the smoothed probabilities from the unconditional model of Nalewaik (2015), which show the transition out of the stationary inflation regime occurred that year.

4.4 Regression-Based Tests

Tables 4 and 5 examine the robustness of the non-linearities in the Phillips curve using OLS regressions estimated on annual-frequency data over periods classified by the Markov-switching models as belonging to the stationary inflation regime, 1961-1970 and 1995-2015.[25] The last specification of the top panel of each table shows regression results explaining inflation with slack, the quadratic function of slack when the gap turns negative, a lagged dependent variable, and the dollar exchange rate for core PCE price inflation. Newey-West standard errors with two lags are below each coefficient in parentheses. The estimates are similar to the stationary-regime coefficients in Tables 2 and 3, with the non-linear slack coefficient a bit smaller for core PCE price inflation and a bit larger for compensation-per-hour growth.

The remainder of the specifications in the top panels of Tables 4 and 5 run linear regressions of the inflation measures on slack, with or without controls, either allowing for breaks in the slack coefficient or not. The break points κ are estimated as in Hansen and Seo (2002), and the heteroskedasticity-robust sup-LM statistic from that paper tests the significance of the breaks, computing asymptotic p-values with their fixed regressor bootstrap. This alternative approach to modeling non-linearity in the Phillips curve produces a fit very similar to

[25] The two-sided probabilities from the quarterly-frequency model include 1994 and 1995 in the stationary regime, while those from the annual-frequency model do not; the sample chosen here splits the difference.

that produced by the quadratic, with break points for core PCE price inflation mimicking the quadratic which steepens sharply when the slack is less than minus one percent. The break points for compensation-per-hour growth are higher, showing no negative relation between slack and compensation growth when slack is above two percent. For slack below that cutoff, the Phillips curve slope is steeper than minus one. The non-linearities are all statistically significant according to the sup-LM tests.

A major issue with unemployment gap measures is that the natural rate of unemployment is unobserved. However, the bottom panels of Tables 4 and 5 show that similar non-linearities are evident in the relation between the inflation measures and the raw unemployment rate. The specification explaining core PCE price inflation with controls shows a Phillips curve slope about ten times the slope from the linear specification when the unemployment rate is below 4.8 percent. For compensation-per-hour growth, when the unemployment rate is below 5.0 percent, the Phillips curve slope is about four to five times the slope from the linear specification. These non-linearities are statistically significant.

The original Phillips (1958) curve used a function similar to the inverse of the unemployment rate to explain wage growth, and the regressions labeled $(U-\kappa)^{-1}$ explore a similar specification subtracting a constant from the unemployment rate before the inverse is taken.[26] This approach fits the data about as well as the approach allowing for discrete breaks in the coefficients, especially for compensation per hour growth. These results show the non-linearity in the Phillips curve can be modeled in a number of ways that yield broadly similar results, suggesting the precise form of the non-linearity is not particularly important.

The last specifications of table 5 show results for compensation per hour growth using

[26] These specifications are obviously not defined for unemployment rates below these κ values, 2.6 percent for core PCE price inflation and 1.6 percent for compensation-per-hour growth.

only 1995 to 2015. Statistically significant Phillips curve non-linearities exist even in this recent short sample period. The form resembles the top panel of the table, with a flat Phillips curve when the unemployment rate is above 6.9 percent, and a slope steeper than minus one when the unemployment rate is below that cutoff.

4.5 Informal "Out-of-Sample" Evaluation

While the model samples used here are constrained by the quarterly core PCE price index extending back to only 1959, other price indexes extend back farther at the annual frequency. Annual estimates of the unemployment rate from Lebergott (1957) extend back quite far as well, showing very low values at certain points in U.S. history. It is instructive to examine, at least informally, to what extent those periods corroborate non-linearities in the Phillips curve.

Moving backwards in time, the total PCE price inflation of close to 3 percent and the compensation-per-hour growth of 6 percent observed in 1956 and 1957 are about what Figure 11 would predict from the observed unemployment rates around 4 percent and CBO natural rate estimates around 5.4 percent. Very low unemployment rates in the Korean War, below 3 percent, were accompanied by low inflation, but this was likely due to wartime wage and price controls, anticipation of which appears to have led to the surge in inflation in late 1950 and early 1951—see Goodwin and Herren (1975).[27]

From 1946-1948, the unemployment rate was low, just below 4 percent, and total PCE price inflation was quite high, consistent with a steeper non-linear Phillips curve than shown in Figure 11. The probabilities in Nalewaik (2015) suggest this period was governed by the

[27] Around the Korean war, the negative PCE price inflation rates observed in 1949 and 1954 were probably the result of strong contemporaneous "speed" effects, as the unemployment rate spiked up in both years.

non-stationary inflation regime, but this evidence is broadly consistent with a non-linear Phillips curve. Wartime wage and price controls also confound the interpretation of the data during World War II, but annual total PCE price inflation was generally quite high, averaging over 7 percent, and the unemployment rate was very low, often below 2 percent, again broadly consistent with a non-linear Phillips curve.

Examination of the international evidence for Phillips curve non-linearities is beyond the scope of this paper, but, of course, the original Phillips (1958) provided supportive evidence from a sample of data on the United Kingdom extending back 100 years. It is also noteworthy that the unemployment rate in Japan has been relatively low through most of modern history, and its price Phillips curve has been quite steep, with a coefficient close to minus one—see Nalewaik (2016).

More broadly still, through the addition of the bank lending conditioning variable in the non-stationary inflation regime, the Markov-switching models here might be capable of anticipating the most extreme inflation outcomes like the deflation in the United States in the Great Depression or the hyperinflations experienced at certain points in time by other countries. On the former, Friedman and Schwartz (1963) report that bank lending contracted by about half from 1929 to 1933, so the bank lending conditioning variable might go a long way towards explaining the 27 percent drop in the PCE price index over that time. For a model designed to gauge risks to the inflation process, it is important to have these channels through which the most extreme tail risks can appear under some circumstances.

5 Conclusion

The generally low inflation since the end of the 2007-2009 recession has raised questions about the ability of central banks to hold inflation around their stated targets, 2 percent for total PCE price inflation in the case of the U.S. Federal Reserve.[28] The Markov-switching models estimated in this paper have little trouble explaining that recent experience: the declines in labor-market slack through most of the recovery have occurred on the flat region of the Phillips curve, and so have had little effect on core PCE price inflation. The Phillips curve for compensation per hour is generally less flat, but the modest acceleration in this measure in recent years has been obscured by the volatility of the time series, some resulting from changes in tax laws in 2013.

However, the models here suggest it would be unwise to assume the Phillips curve remains so flat at all levels of the unemployment rate. The experience of the 1960s strongly suggests a sharp increase in the effect of slack on wage growth and core PCE price inflation after labor markets tighten beyond a certain point. Further, evidence for non-linearity in the Phillips curve is far from confined to the 1960s, and, for wage growth, includes the relatively recent experience of the late 1990s. Core PCE price inflation remained relatively subdued over that period, but likely only because the upward pressure from the non-linear Phillips curve was masked by other factors including downward pressure from a strongly appreciating dollar exchange rate.

In discussing the Phillips curve, Stock and Watson (2009) ask hypothetical forecasters: "... suppose you are told that next quarter the economy would plunge into recession, with the unemployment rate jumping by 2 percentage points. Would you change your inflation

[28]From July 2009 to April 2016, total and core PCE price inflation in the U.S. averaged 1.4 and 1.5 percent, respectively, modestly below target.

forecast?" Those skeptical of a non-linear Phillips curve should consider a related question: if the unemployment rate were to fall below 4 percent on a sustained basis, as in the late 1960s, would you expect wage inflation to remain low (say, around 2 percent) and core PCE price inflation to remain below 2 percent? What if the unemployment rate were to fall below 3 percent, as in the Korean War, or below 2 percent, as may have occurred in World War II (see Lebergott, 1957)? In other words, is there no unemployment rate below which wage and price Phillips curves begin to steepen sharply? To believe not seems tantamount to denying that the forces of supply and demand, to which the original Phillips (1958) appealed, are ever operable to any appreciable degree in the labor market.

The other reason core PCE price inflation has been mostly below the FOMC's 2 percent target for total PCE price inflation since 2009 is that its mean in the stationary regime governing its behavior since the mid-1990s appears to be somewhat below that target. If price and wage inflation were certain to remain governed by that stationary regime, a lower mean simply implies that the FOMC's 2 percent inflation target is likely to be met on a sustained basis at a lower unemployment rate (assuming total PCE price inflation runs about in line with core PCE price inflation). For example, with a mean of 1.6 percent and a stable dollar exchange rate, the results here suggest that an unemployment rate $\frac{3}{4}$ percentage point below the natural rate produces 2 percent core PCE price inflation. However, reaching that point necessitates moving up the steep portion of the Phillips curve, so if labor-market slack falls just an additional $\frac{3}{4}$ percentage point, the result is sustained 3 percent core PCE price inflation. Little in U.S. history suggests a regime of stationary, stable inflation can be maintained for long at an average level of inflation much above 3 percent.

Indeed, the evidence here suggests that it was sustained inflation at or above 3 percent in the 1960s, probably caused by unemployment rates running well below the natural rate, that entrenched the inflation expectations that kept inflation high even after the unemployment rate moved back up to the natural rate in 1970. At that point, inflation expectations likely had become highly adaptive and an important driving force for subsequent inflation, meaning the regime of stationary, stable inflation was over. Results here suggest that an earlier slackening of the labor market very well may have resulted in the same regime transition outcome, so an end to the era of stationary, stable inflation may have been all but inevitable by 1967 or 1968.

Appendix A: Quarterly Model Structure

Let t index quarters in this Appendix, so the four-quarter change in PCE price inflation is $\pi_t = 100 * (\ln(P_t) - \ln(P_{t-4}))$, and let $\pi_t^q = 400 * (\ln(P_t) - \ln(P_{t-1}))$ be the annualized quarterly log difference. Then, for $S_t = 2$, the annual model with t indexing quarters is $\pi_t = \pi_{t-4} + \Theta_2 X_{t-4} + \varepsilon_t + \theta\varepsilon_{t-4}$, estimated on each 4th-quarter observation, $T, T-4, T-8, \ldots$. Here X_{t-4} is the vector of controls: quarterly labor-market slack and its non-linear function lagged four quarters, the eight-quarter change in the exchange rate lagged four quarters, and the four-quarter change in bank lending lagged four quarters. Then the quarterly analog to this annual-frequency model for $S_t = 2$, estimated on the full set of quarterly observations, is:

$$\begin{aligned}
\sum_{i=1}^{4} \pi_{t+1-i}^q &= \sum_{i=1}^{4} \pi_{t-3-i}^q + \Theta_2 X_{t-4} + \sum_{i=1}^{4} \varepsilon_{t+1-i}^q + \theta \sum_{i=1}^{4} \varepsilon_{t-3-i}^q, \quad \text{or:} \\
\pi_t^q &= -\sum_{i=1}^{3} \pi_{t-i}^q + \sum_{i=1}^{4} \pi_{t-3-i}^q + \Theta_2 X_{t-4} + \sum_{i=1}^{4} \varepsilon_{t+1-i}^q + \theta \sum_{i=1}^{4} \varepsilon_{t-3-i}^q.
\end{aligned}$$

The $-\sum_{i=1}^{3} \pi_{t-i}^q$ terms difference the quarterly-model innovations ε_{t-1}^q, ε_{t-2}^q, and ε_{t-3}^q out of the right-hand side of this equation, and all the explanatory variables are dated $t-4$ or earlier, so this equation uses only information from four quarters prior to forecast π_t^q. In other words, the equation is essentially designed to forecast the annualized quarterly inflation rate four quarters ahead. The analog for $S_t = 1$ is similar. The transition matrix and variance-covariance matrices are as in section 2 but at the quarterly rather than the annual frequency.

In the model with the time-varying transition probability, $p_{11,t}$ is a logistic function of the average non-linear Phillips curve effect over the last five years multiplied by a term measuring the extent to which the non-linear effect has diminished relative to its peak value over the past five years, so:

$$
\begin{aligned}
p_{11,t} &= \frac{\exp(\xi_0^{11} + \xi_u^{11} Z_{t-4})}{\exp(\xi_0^{11} + \xi_u^{11} Z_{t-4}) + 1} \quad \text{where:} \\
Z_{t-4} &= \frac{\kappa_{t-4}}{20} \sum_{i=1}^{20} -min(UGAP_{t-4-(i+1)}, 0)^2 \quad \text{and:} \\
\kappa_{t-4} &= -\left(min(UGAP_{t-4}, 0)^2 - min(UGAP_{t-4}, UGAP_{t-5}, \ldots, UGAP_{t-23}, 0)^2 \right).
\end{aligned}
$$

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Table 1: Inflation Measures, Livingston Inflation Forecasts, Unemployment, Vacancy Rates, and Non-Stationary Regime Probabilities in the 1960s

Date	$\pi_t^{w,CPH}$	π_t	π_t^{CPI}	$E_t^{LIV}\left(\pi_{t+1}^{CPI}\right)$	U_t	$RVAC_t$	Regime Probabilities (% p.p.) Constrained	Unconstrained
1961Q2	3.5	1.2	1.0	1.0	7.1	8.8	98	1
1961Q4	3.3	1.2	0.7	1.1	6.1	8.5	78	0
1962Q2	3.8	1.5	1.3	1.1	5.5	8.1	18	0
1962Q4	3.9	1.2	1.3	1.05	5.7	8.1	14	0
1963Q2	3.5	1.2	1.0	1.05	5.9	8.2	2	0
1963Q4	3.4	1.5	1.3	1.0	5.7	8.3	0	0
1964Q2	3.2	1.6	1.3	1.3	5.1	8.1	0	0
1964Q4	3.1	1.3	1.3	1.2	4.8	8.3	0	0
1965Q2	3.3	1.2	1.6	0.9	4.6	8.2	0	0
1965Q4	3.3	1.3	1.6	1.6	4.1	8.5	0	0
1966Q2	4.3	2.0	2.9	1.9	3.9	7.4	0	0
1966Q4	5.7	3.0	3.8	2.2	3.6	7.7	0	0
1967Q2	5.9	2.9	2.8	2.4	3.8	6.9	0	0
1967Q4	5.7	3.1	2.7	2.8	3.9	6.2	0	0
1968Q2	6.3	4.2	3.9	2.9	3.5	6.2	100	0
1968Q4	7.3	4.5	4.7	2.85	3.4	5.4	100	0
1969Q2	7.0	4.6	5.5	3.45	3.4	5.7	100	0
1969Q4	6.6	4.6	5.9	3.4	3.5	5.1	100	0
1970Q2	6.9	4.5	6.0	3.95	4.8	5.4	100	0
1970Q4	6.8	4.7	5.6	3.8	5.9	5.2	100	0
1971Q2	6.3	4.9	4.4	4.1	5.9	5.3	100	100

Notes: The table is semi-annual to align with the timing of the Livingston survey of professional forecasters. $\pi_t^{w,CPH}$ is the annual average growth rate of compensation per hour in the non-farm business sector and π_t is the four-quarter growth rate of the core PCE price index. $E_t^{LIV}\left(\pi_{t+1}^{CPI}\right)$ is the Livingston survey forecast of CPI inflation over the the next 12 months, made in the last month of the quarter (June and December). π_t^{CPI} is 12-month CPI inflation and U_t is the unemployment rates that prevailed at the time the forecasts were made (i.e. the May and November values known at the time of the June and December forecasts.) The timing of the Livingston Survey is conveniently aligned with the timing of the CPI; it is sent to participants the day after the release of the CPI data on the previous month—see Croushore (1997). $RVAC_t$ is the quarterly U.S. average vacancy rate on rental units, from the U.S. census bureau. The regime probabilities are from the quarterly model estimates in Table 3.

Table 2: Markov Switching Models explaining Q4/Q4 Core PCE Price Inflation and Annual Average Compensation-Per-Hour Growth

Constraining Markov Transition Matrix, Stationary State Absorbing

$$\begin{pmatrix} \pi_{t|t-1} \\ 1-\pi_{t|t-1} \end{pmatrix} = \begin{bmatrix} 1.000 & \begin{matrix} 0.029 \\ (0.037) \end{matrix} \\ 0 & \begin{matrix} 0.971 \\ (0.037) \end{matrix} \end{bmatrix} \begin{pmatrix} \pi_{t-1|t-1} \\ 1-\pi_{t-1|t-1} \end{pmatrix}. \quad \widehat{\pi}_0 = \; 0.000 \; .$$

	Stationary Regime Parameters				Non-stationary Regime Parameters					
γ_u	μ	$\beta_{u,1}$	$\beta_{d,1}$	σ_1	θ	$\beta_{u,2}$	$\beta_{d,2}$	$\beta_{b,2}$	σ_2	$\mathcal{L}$
-0.16	**1.74**	**-0.12**	**-0.07**	0.23	**-0.96**	**-0.05**	**-0.06**	**0.16**	0.83	-73.1
(0.02)	(0.08)	(0.04)	(0.03)	(0.08)	(0.33)	(0.06)	(0.04)	(0.05)	(0.23)	
γ_u^w	μ^w	$\beta_{u,1}^w$	ρ_1	σ_1^w	θ^w	$\beta_{u,2}^w$	ρ_2	$\beta_{b,2}^w$	σ_2^w	
-0.09	**3.90**	**-0.65**	0.00	1.02	**-1.03**	**-0.17**	0.51	**0.14**	0.98	
(0.03)	(0.51)	(0.25)	(1.32	(0.34)	(0.47)	(0.09)	(0.28)	(0.08)	(0.40)	

Unconstrained Markov Transition Matrix

$$\begin{pmatrix} \pi_{t|t-1} \\ 1-\pi_{t|t-1} \end{pmatrix} = \begin{bmatrix} \begin{matrix} 0.9597 \\ (0.0415) \end{matrix} & \begin{matrix} 0.0418 \\ (0.0491) \end{matrix} \\ \begin{matrix} 0.0403 \\ (0.0415) \end{matrix} & \begin{matrix} 0.9582 \\ (0.0491) \end{matrix} \end{bmatrix} \begin{pmatrix} \pi_{t-1|t-1} \\ 1-\pi_{t-1|t-1} \end{pmatrix}. \quad \widehat{\pi}_0 = \begin{matrix} 0.90 \\ (1.40) \end{matrix} \; .$$

	Stationary Regime Parameters				Non-stationary Regime Parameters					
γ_u	μ	$\beta_{u,1}$	$\beta_{d,1}$	σ_1	θ	$\beta_{u,2}$	$\beta_{d,2}$	$\beta_{b,2}$	σ_2	$\mathcal{L}$
-0.53	**1.61**	**-0.08**	**-0.08**	0.37	**-1.47**	**-0.09**	**-0.07**	**0.14**	0.59	-55.4
(0.01)	(0.01)	(0.004)	(0.03)	(0.07)	(0.04)	(0.06)	(0.06)	(0.04)	(0.20)	
γ_u^w	μ^w	$\beta_{u,1}^w$	ρ_1	σ_1^w	θ^w	$\beta_{u,2}^w$	ρ_2	$\beta_{b,2}^w$	σ_2^w	
-0.26	**3.93**	**-0.64**	0.50	0.94	**-1.57**	**-0.30**	0.39	**0.11**	0.70	
(0.01)	(0.003)	(0.004)	(0.68)	(0.18)	(0.06)	(0.12)	(0.88)	(0.01)	(0.16)	

Table 3: Markov Switching Models Explaining
Annualized Quarterly Core PCE Price Inflation One Year Ahead

Constraining Markov Transition Matrix, Stationary State Absorbing

	Stationary Regime Parameters				Non-stationary Regime Parameters					
γ_u	μ	$\beta_{u,1}$	$\beta_{d,1}$	σ_1	θ	$\beta_{u,2}$	$\beta_{d,2}$	$\beta_{b,2}$	σ_2	$\mathcal{L}$
-0.17	**1.74**	**-0.06**	**-0.04**	0.50	**-1.76**	**-0.71**	**-0.12**	**0.42**	0.73	-296.8
(0.04)	(0.05)	(0.04)	(0.02)	(0.03)	(0.25)	(0.19)	(0.06)	(0.10)	(0.10)	
	Transition-Matrix Parameters									
		p_{11}	p_{22}	p_0						
		1.000	0.9924	0.000						
		—	(0.0093)	—						

Unconstrained Markov Transition Matrix

	Stationary Regime Parameters				Non-stationary Regime Parameters					
γ_u	μ	$\beta_{u,1}$	$\beta_{d,1}$	σ_1	θ	$\beta_{u,2}$	$\beta_{d,2}$	$\beta_{b,2}$	σ_2	$\mathcal{L}$
-0.47	**1.63**	**-0.04**	**-0.05**	0.50	**-2.96**	**-1.19**	**-0.13**	**0.21**	0.52	-262.1
(0.01)	(0.04)	(0.03)	(0.02)	(0.03)	(0.22)	(0.21)	(0.08)	(0.01)	(0.06)	
	Transition-Matrix Parameters									
		p_{11}	p_{22}	p_0						
		0.9993	0.9979	0.92						
		(0.0008)	(0.0048)	(1.53)						

Unconstrained Markov Transition Matrix, Time-Varying Transition Probabilities

	Stationary Regime Parameters				Non-stationary Regime Parameters					
γ_u	μ	$\beta_{u,1}$	$\beta_{d,1}$	σ_1	θ	$\beta_{u,2}$	$\beta_{d,2}$	$\beta_{b,2}$	σ_2	$\mathcal{L}$
-0.46	**1.67**	**-0.06**	**-0.05**	0.52	**-2.43**	**-0.67**	**-0.19**	**0.18**	0.61	-254.1
(0.005)	(0.03)	(0.02)	(0.02)	(0.03)	(0.19)	(0.21)	(0.06)	(0.01)	(0.06)	
	Transition-Matrix Parameters									
	ξ_0^{11}	ξ_u^{11}	p_{22}	p_0						
	13.93	4.60	0.9982	0.90						
	(7.68)	(2.44)	(0.0036)	(1.53)						

Table 4: OLS Regressions Explaining Q4/Q4 Core PCE Price Inflation 1961-1970, 1995-2015

$$\pi_t = \alpha + \beta_{\pi,1}\pi_{t-1} + \beta_{d,1}\Delta XR_{t-1,t-3} + \beta_{U,1}UGAP_{t-1} + \gamma_{u,1}\left(min(UGAP_{t-1}, \kappa\right) + u_t$$

	α	$\beta_{\pi,1}$	$\beta_{d,1}$	$\beta_{u,1}$	$\gamma_{u,1}$	κ	Adj. R^2	SUP-LM p-val.
	2.09			**-0.39**			0.39	
	(0.22)			(0.17)				
	-0.45			**-0.06**	**-2.10**	-1.00	0.83	0.000
	(0.27)			(0.05)	(0.20)			
	0.50	**0.80**	**-0.05**	**-0.18**			0.77	
	(0.23)	(0.10)	(0.03)	(0.08)				
	0.39	**0.35**	**-0.06**	**-0.06**	**-1.11**	-0.60	0.88	0.023
	(0.10)	(0.07)	(0.02)	(0.03)	(0.15)			
$-(min(UGAP, 0))^2$	**1.11**	**0.31**	**-0.06**	**-0.07**	**-0.38**		0.87	
	(0.13)	(0.08)	(0.02)	(0.03)	(0.06)			

$$\pi_t = \alpha + \beta_{\pi,1}\pi_{t-1} + \beta_{d,1}\Delta XR_{t-1,t-3} + \beta_{U,1}U_{t-1} + \gamma_{u,1}\left(min(U_{t-1}, \kappa\right) + u_t$$

	α	$\beta_{\pi,1}$	$\beta_{d,1}$	$\beta_{u,1}$	$\gamma_{u,1}$	κ	Adj. R^2	SUP-LM p-val.
	3.81			**-0.33**			0.26	
	(1.06)			(0.17)				
	16.38			**-0.06**	**-3.36**	4.30	0.71	0.006
	(1.61)			(0.06)	(0.39)			
	1.00	**0.87**	**-0.05**	**-0.12**			0.75	
	(0.57)	(0.10)	(0.04)	(0.07)				
	6.38	**0.53**	**-0.07**	**-0.03**	**-1.15**	4.80	0.84	0.020
	(2.01)	(0.16)	(0.03)	(0.03)	(0.36)			
$(U-\kappa)^{-1}$	**0.11**	**0.55**	**-0.07**		**1.84**	2.60	0.81	
	(0.12)	(0.15)	(0.03)		(0.56)			

Table 5: OLS Regressions Explaining Annual Average Compensation-Per-Hour Growth 1961-1970, 1995-2015

$$\pi^w_t = \alpha^w + \beta^w_{\pi,1}\pi^w_{t-1} + \beta^w_{U,1}UGAP_{t-1} + \gamma^w_{u,1}\left(min(UGAP_{t-1}, \kappa^w\right) + u^w_t$$

	α^w	$\beta^w_{\pi,1}$	$\beta^w_{u,1}$	$\gamma^w_{u,1}$	κ^w	Adj. R^2	SUP-LM p-val.
	4.04		**-0.81**			0.60	
	(0.18)		(0.15)				
	3.89		**0.27**	**-1.40**	2.00	0.70	0.004
	(0.16)		(0.14)	(0.22)			
	3.61	**0.11**	**-0.72**			0.59	
	(0.83)	(0.21)	(0.22)				
	4.06	**-0.05**	**0.14**	**-1.34**	1.80	0.69	0.001
	(0.85)	(0.20)	(0.15)	(0.27)			
$-(min(UGAP,0))^2$	**3.83**	**-0.04**	**-0.58**	**-0.38**		0.65	
	(0.67)	(0.15)	(0.19)	(0.09)			

$$\pi^w_t = \alpha^w + \beta^w_{\pi,1}\pi^w_{t-1} + \beta^w_{U,1}U_{t-1} + \gamma^w_{u,1}\left(min(U_{t-1}, \kappa^w\right) + u^w_t$$

	α^w	$\beta^w_{\pi,1}$	$\beta^w_{u,1}$	$\gamma^w_{u,1}$	κ^w	Adj. R^2	SUP-LM p-val.
	8.03		**-0.75**			0.53	
	(1.02)		(0.17)				
	14.09		**-0.35**	**-1.76**	5.10	0.70	0.001
	(0.85)		(0.07)	(0.23)			
	6.27	**0.23**	**-0.58**			0.53	
	(1.90)	(0.22)	(0.22)				
	17.22	**-0.20**	**-0.45**	**-2.17**	5.00	0.70	0.000
	(3.02)	(0.22)	(0.12)	(0.42)			
$(U-\kappa)^{-1}$	**0.17**	**-0.20**		**15.18**	1.60	0.70	
	(0.32)	(0.23)		(3.13)			
1995-2015 only	**6.63**		**-0.55**			0.39	
	(0.94)		(0.14)				
	9.76		**0.14**	**-1.29**	6.90	0.52	0.034
	(1.19)		(0.14)	(0.34)			

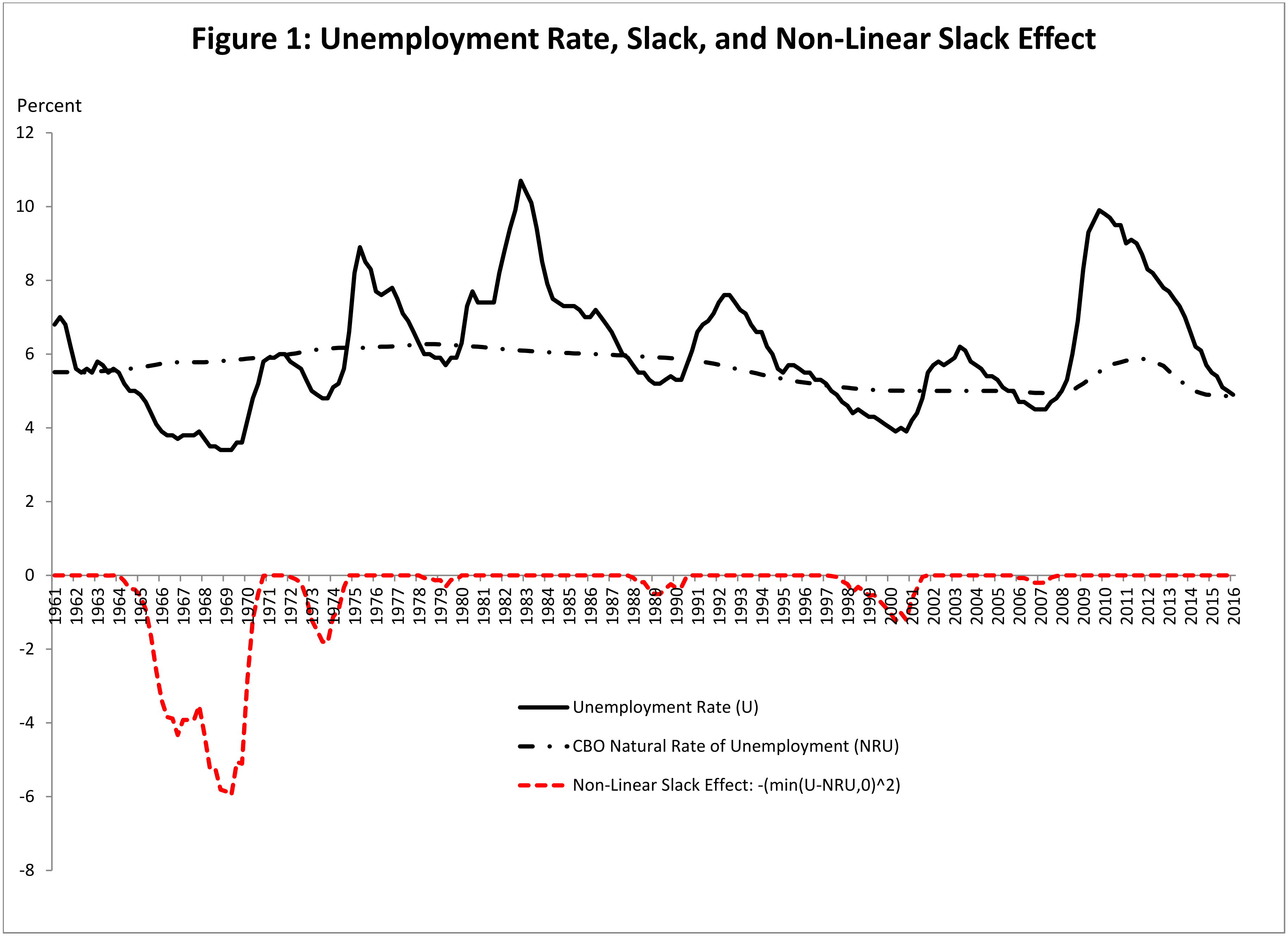
Figure 1: Unemployment Rate, Slack, and Non-Linear Slack Effect
Percent
12
10
8
6
4
2
0
-2
-4
-6
-8
1961
1962
1963
1964
1965
1966
1967
1968
1969
1970
1971
1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
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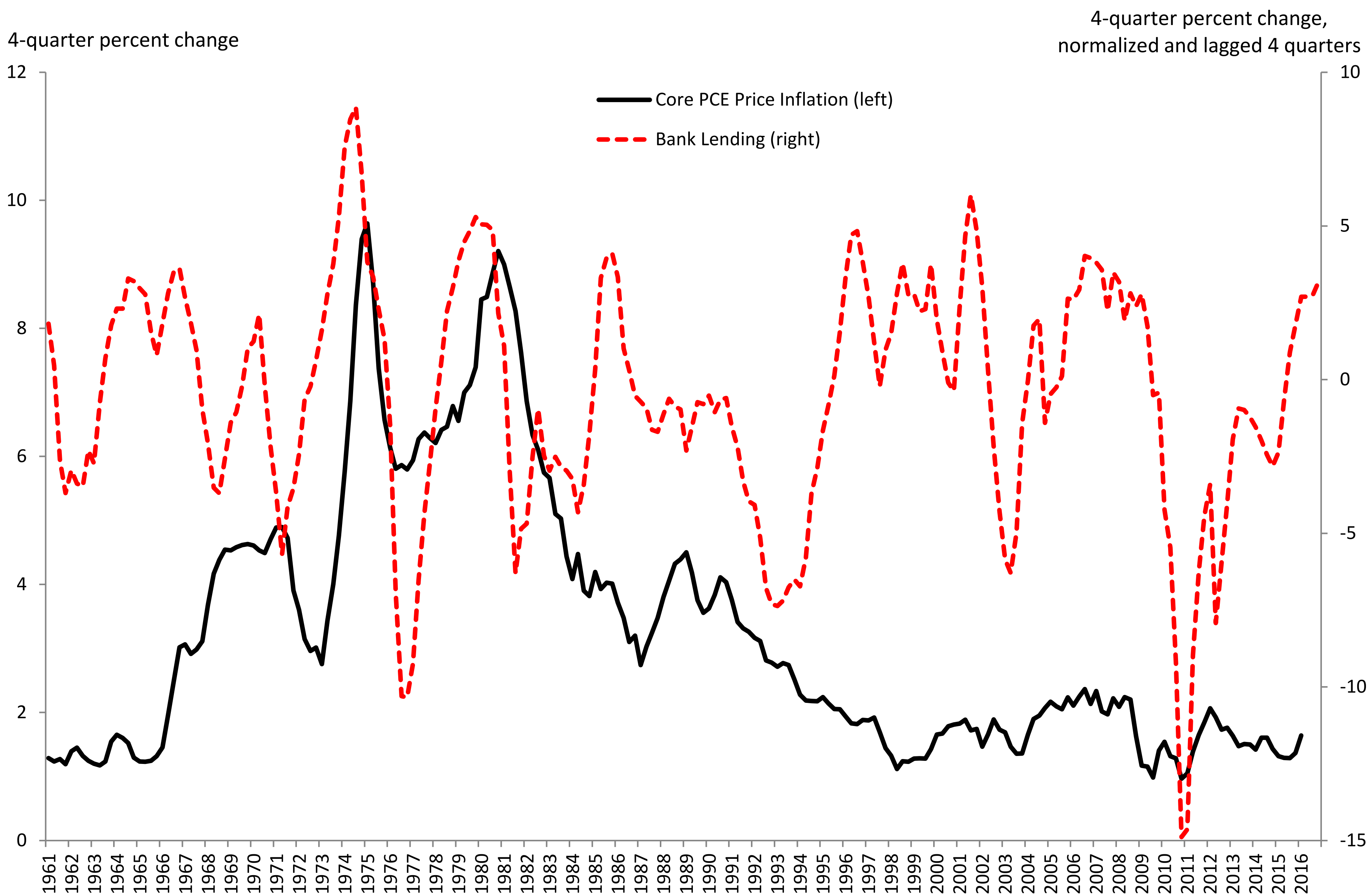
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
Unemployment Rate (U)
CBO Natural Rate of Unemployment (NRU)
Non-Linear Slack Effect: -(min(U-NRU,0)^2)

Figure 2: Core PCE Price Inflation, with Lagged Bank Lending



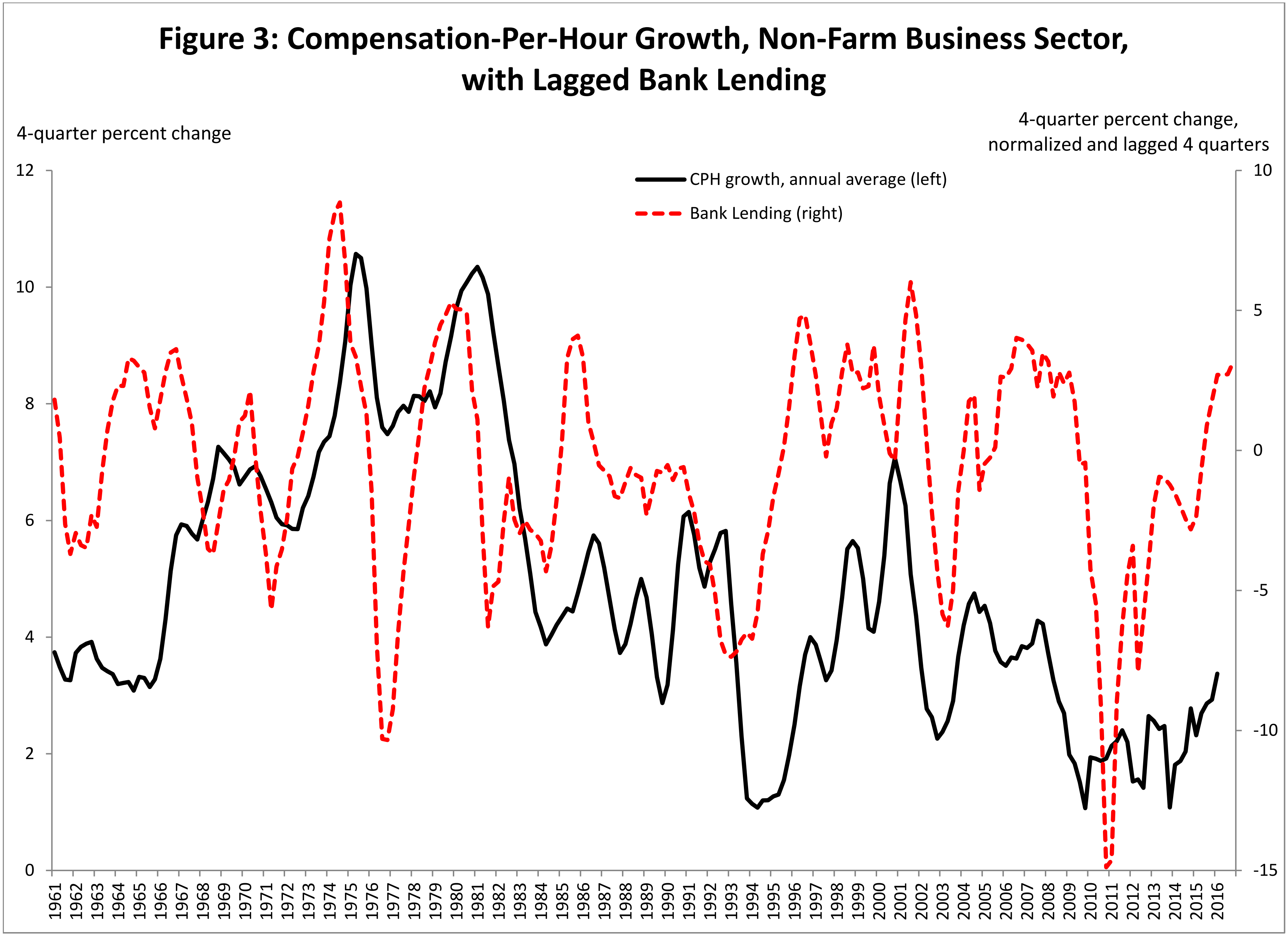


Figure 3: Compensation-Per-Hour Growth, Non-Farm Business Sector, with Lagged Bank Lending
4-quarter percent change
4-quarter percent change, normalized and lagged 4 quarters
CPH growth, annual average (left)
Bank Lending (right)
12
10
8
6
4
2
0
10
5
0
-5
-10
-15
1961
1962
1963
1964
1965
1966
1967
1968
1969
1970
1971
1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
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1998
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2011
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2014
2015
2016

Figure 4: Rate of Change or "Speed" Effects in the Great Depression

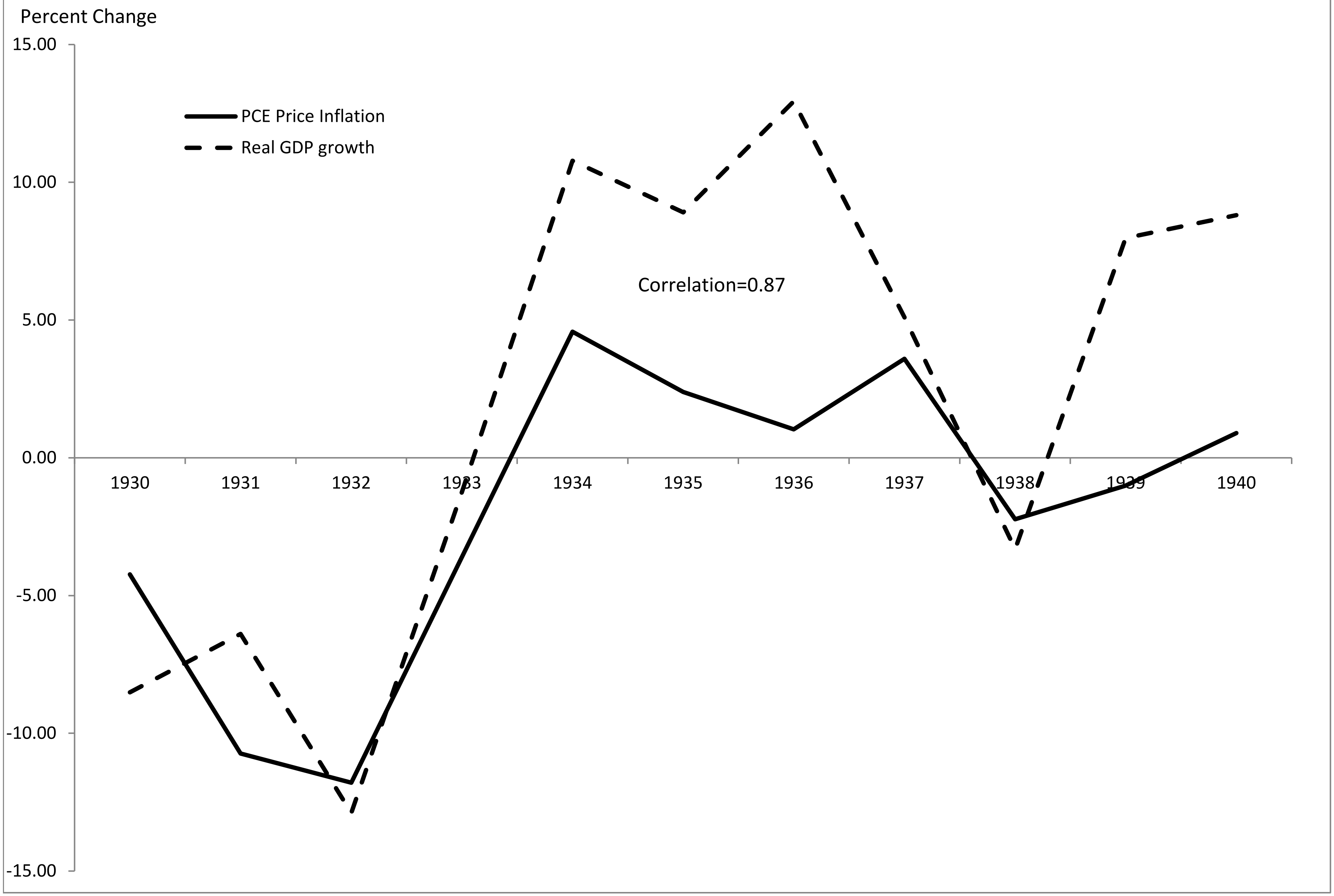


Figure 5: Core PCE Price Inflation, with Bank Lending

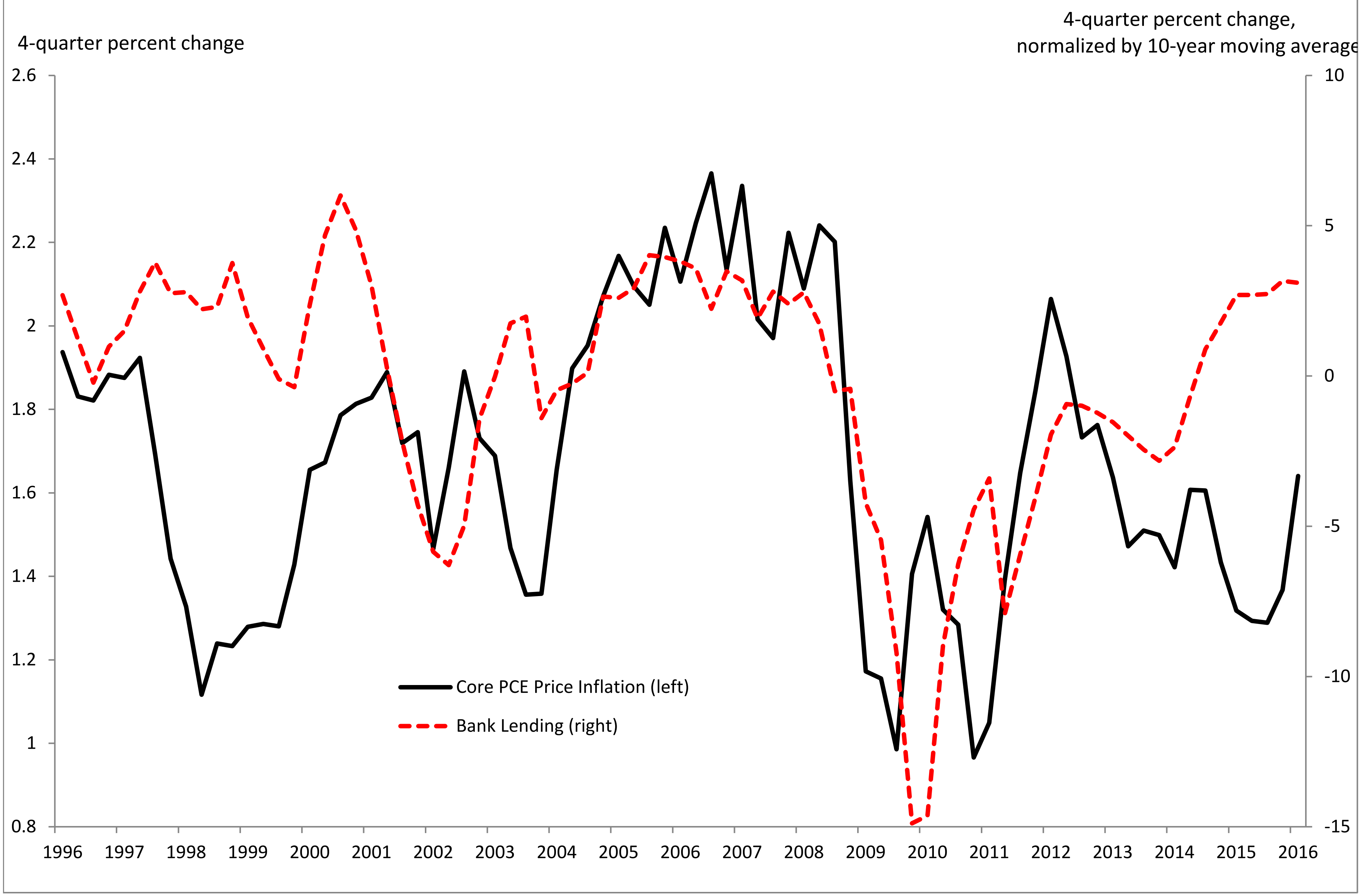
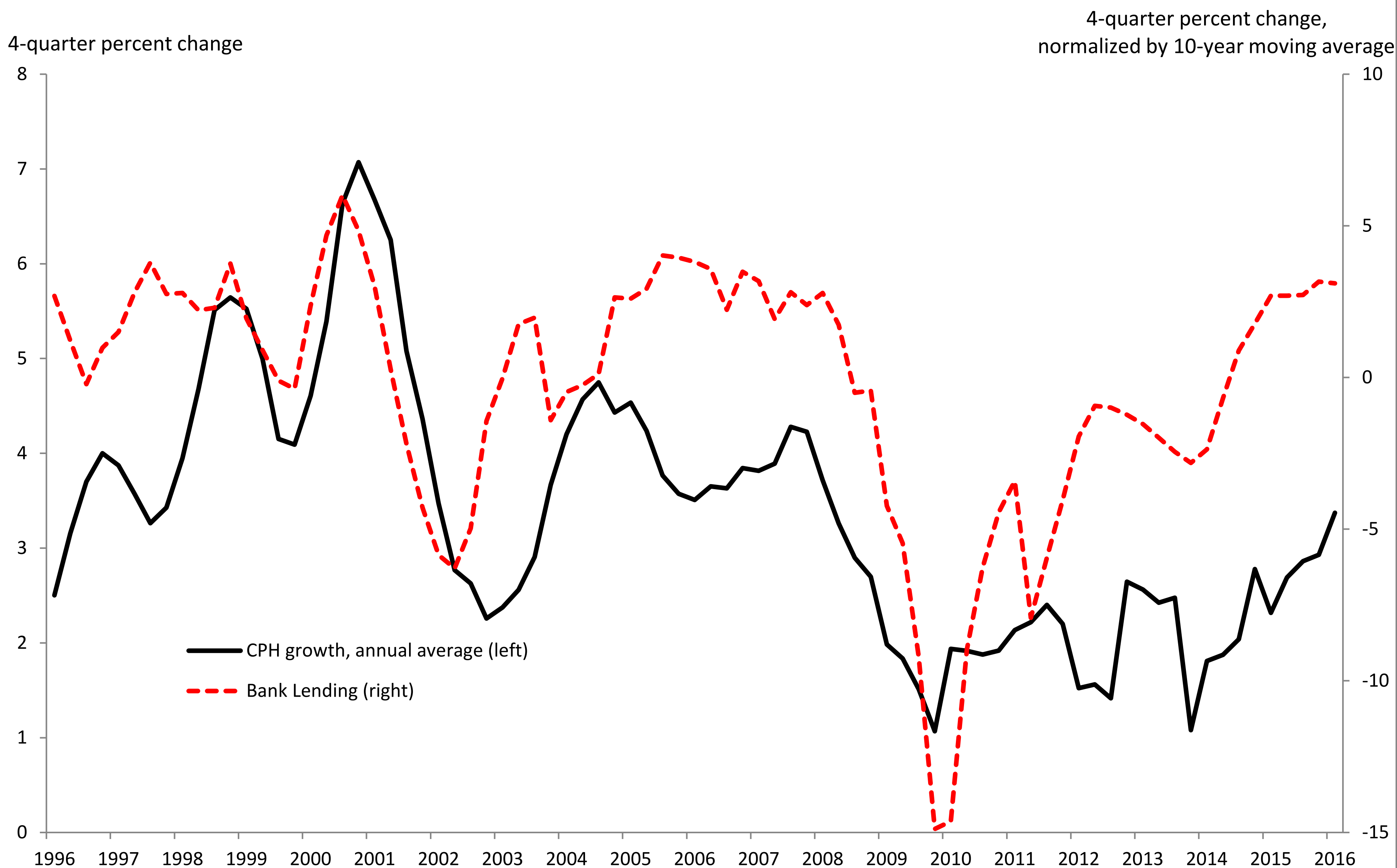


Figure 6: Compensation-Per-Hour Growth, Non-Farm Business Sector, with Bank Lending



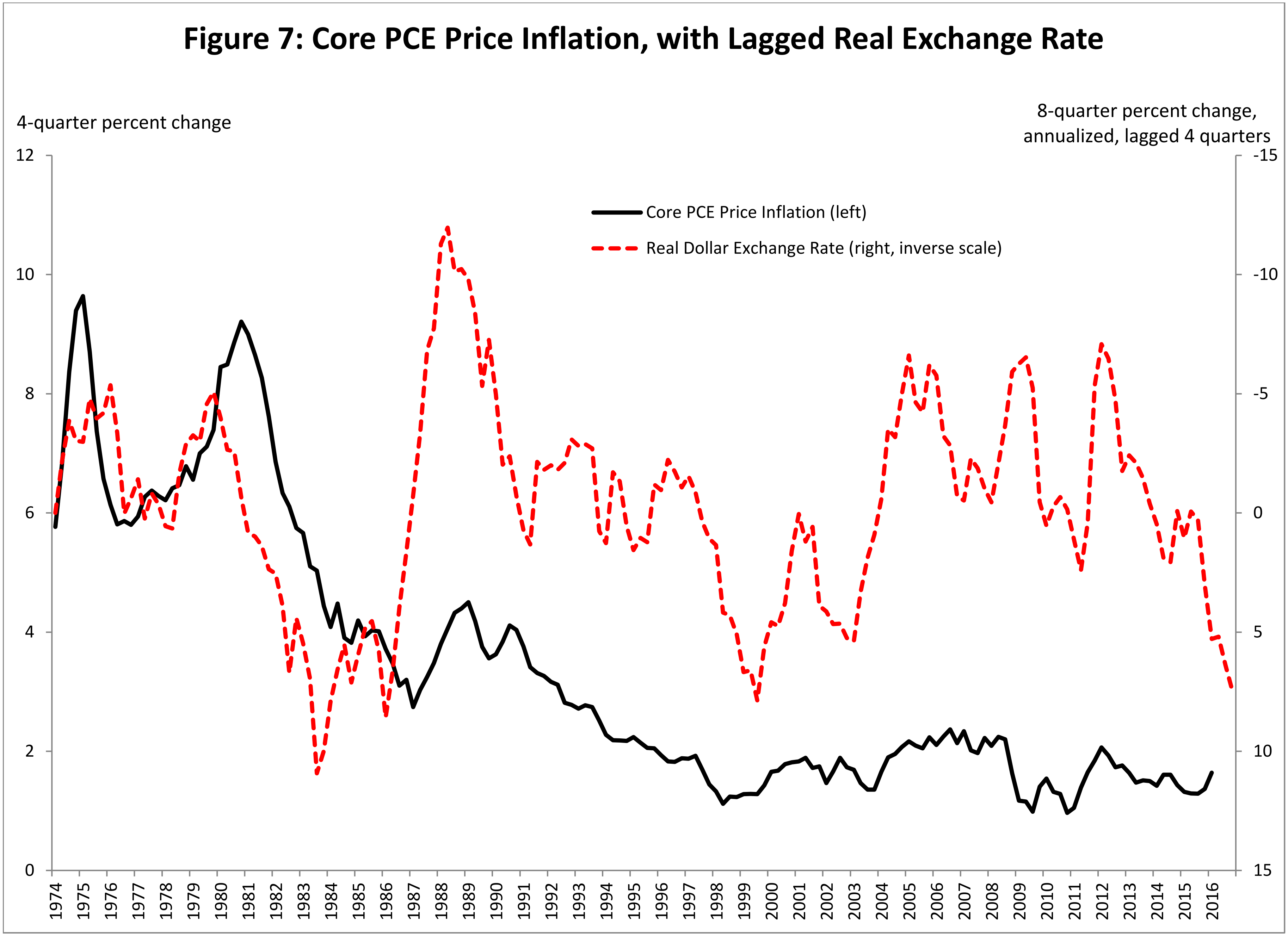


Figure 7: Core PCE Price Inflation, with Lagged Real Exchange Rate
4-quarter percent change
8-quarter percent change, annualized, lagged 4 quarters
Core PCE Price Inflation (left)
Real Dollar Exchange Rate (right, inverse scale)
12
10
8
6
4
2
0
-15
-10
-5
0
5
10
15
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
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2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016

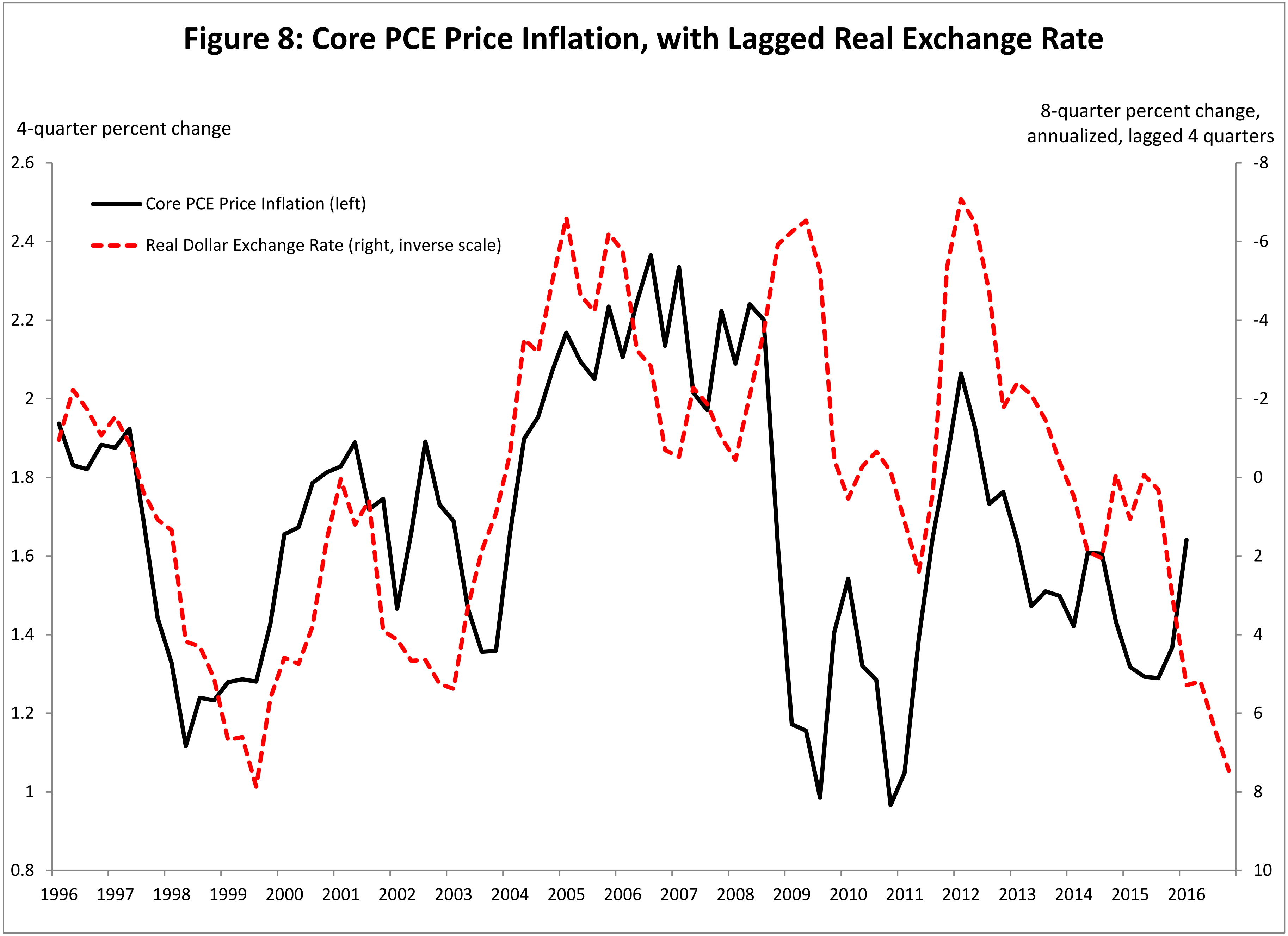


Figure 8: Core PCE Price Inflation, with Lagged Real Exchange Rate
4-quarter percent change
8-quarter percent change, annualized, lagged 4 quarters
Core PCE Price Inflation (left)
Real Dollar Exchange Rate (right, inverse scale)
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
-8
-6
-4
-2
0
2
4
6
8
10
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016

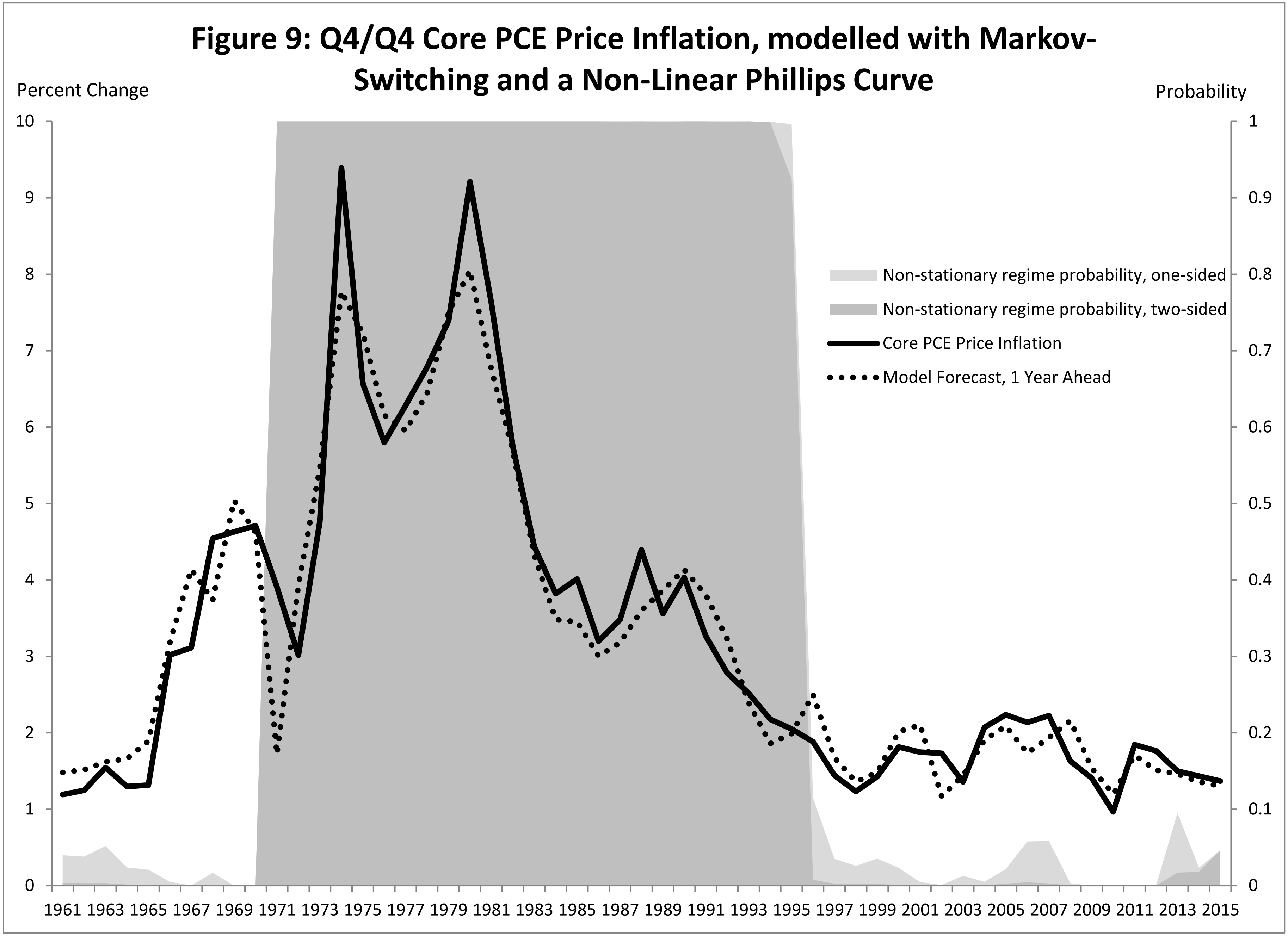


Figure 9: Q4/Q4 Core PCE Price Inflation, modelled with Markov-Switching and a Non-Linear Phillips Curve
Percent Change
Probability
10
9
8
7
6
5
4
3
2
1
0
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Non-stationary regime probability, one-sided
Non-stationary regime probability, two-sided
Core PCE Price Inflation
Model Forecast, 1 Year Ahead
1961 1963 1965 1967 1969 1971 1973 1975 1977 1979 1981 1983 1985 1987 1989 1991 1993 1995 1997 1999 2001 2003 2005 2007 2009 2011 2013 2015

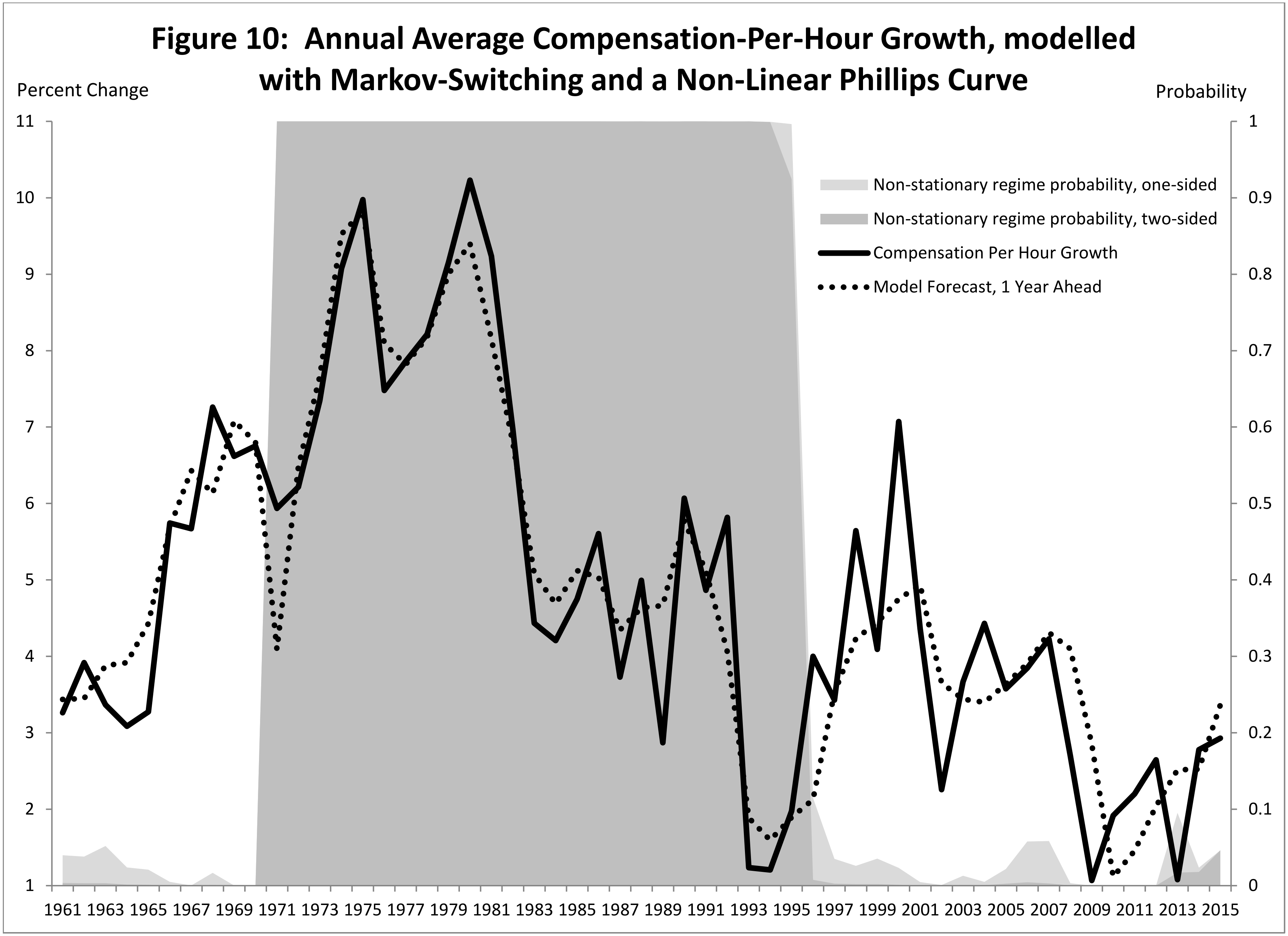
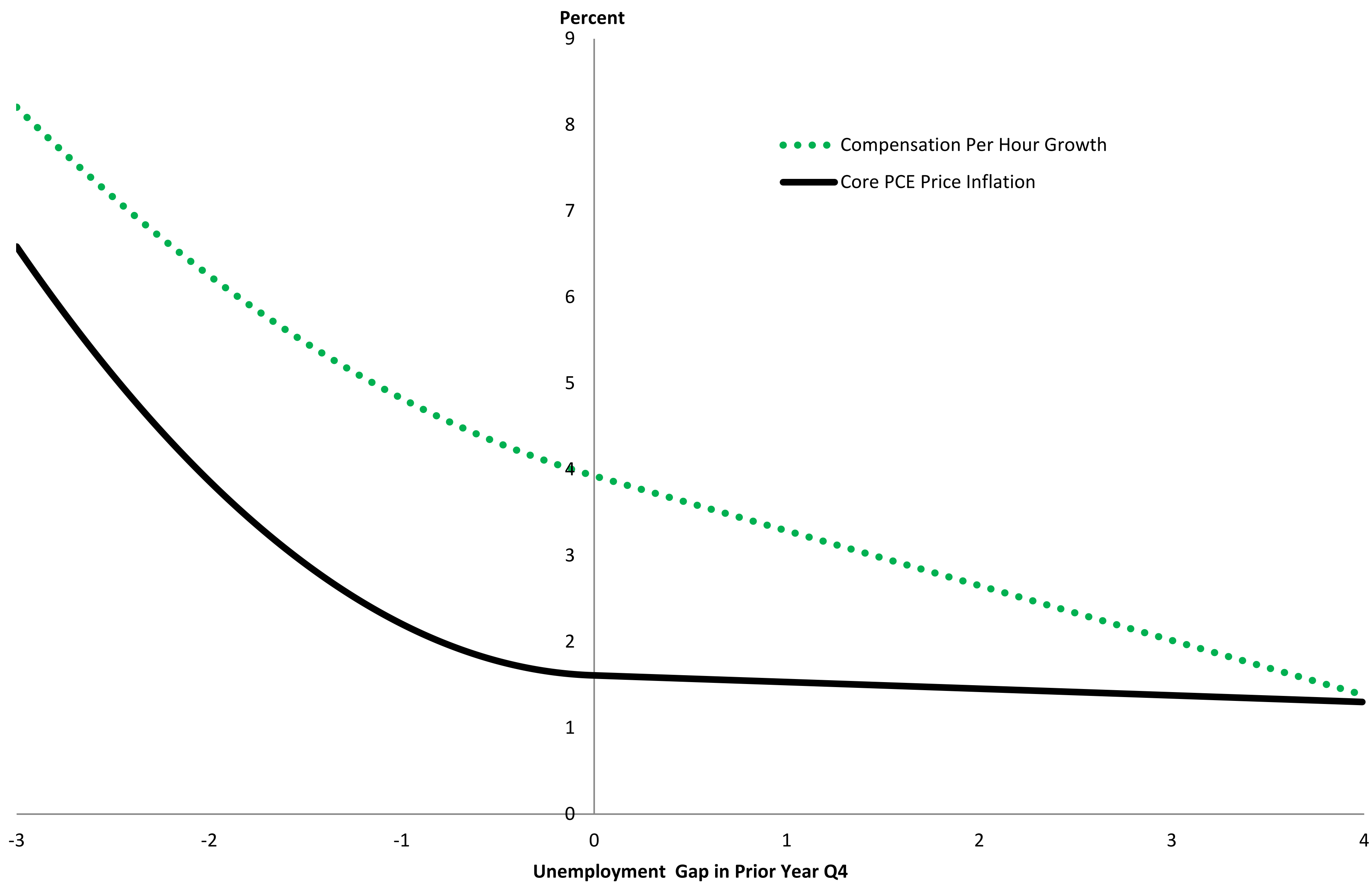


Figure 10: Annual Average Compensation-Per-Hour Growth, modelled with Markov-Switching and a Non-Linear Phillips Curve
Percent Change
Probability
Non-stationary regime probability, one-sided
Non-stationary regime probability, two-sided
Compensation Per Hour Growth
Model Forecast, 1 Year Ahead
11
10
9
8
7
6
5
4
3
2
1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1961 1963 1965 1967 1969 1971 1973 1975 1977 1979 1981 1983 1985 1987 1989 1991 1993 1995 1997 1999 2001 2003 2005 2007 2009 2011 2013 2015

Figure 11: Non-Linear Phillips Curves in Stationary Inflation Regime, Annual-Frequency Model



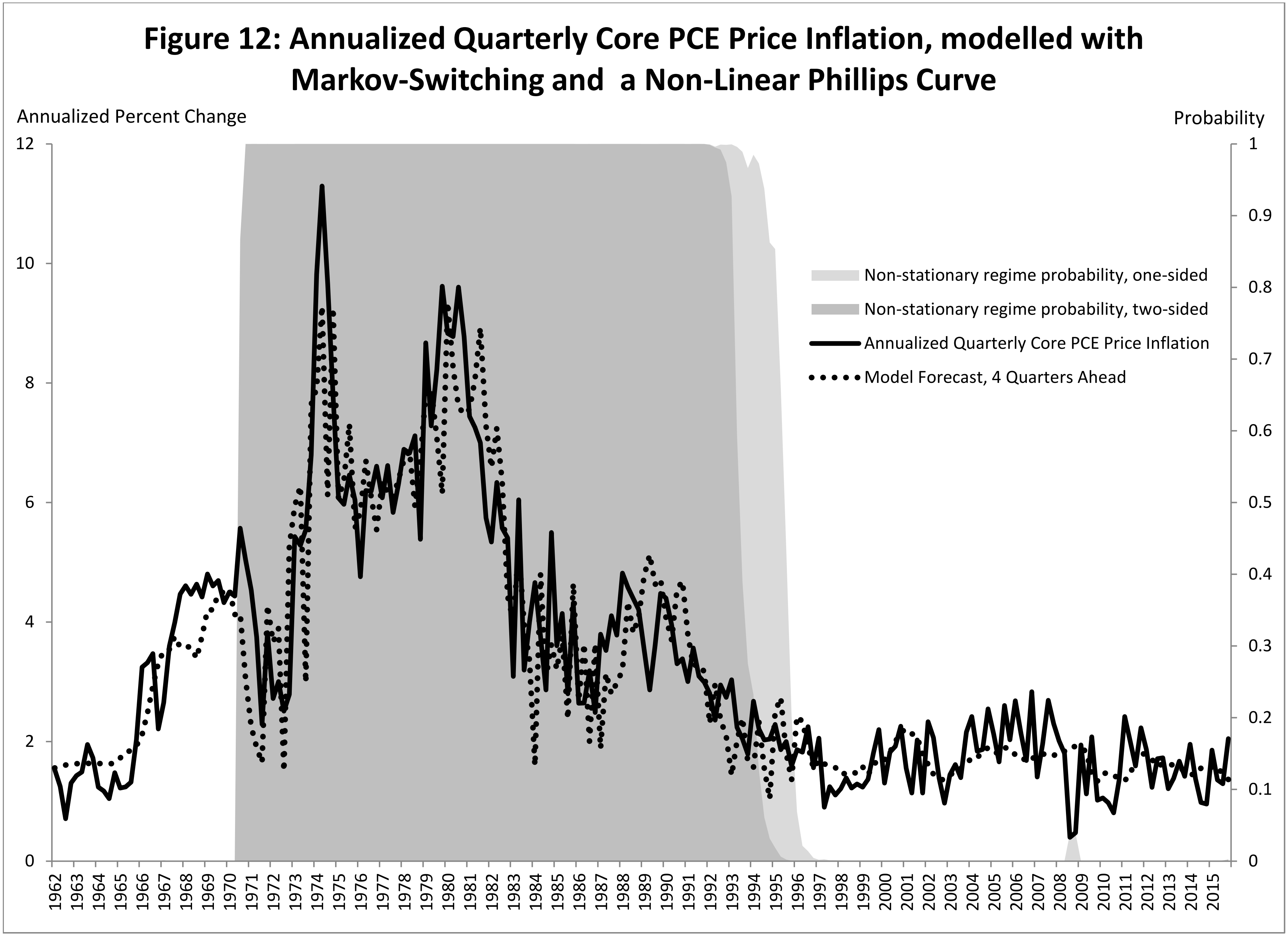


Figure 12: Annualized Quarterly Core PCE Price Inflation, modelled with Markov-Switching and a Non-Linear Phillips Curve
Annualized Percent Change
Probability
Non-stationary regime probability, one-sided
Non-stationary regime probability, two-sided
Annualized Quarterly Core PCE Price Inflation
Model Forecast, 4 Quarters Ahead