—— Appendix – For Online Publication ——

A The Mutual Fund's Discount Factor

This appendix gives the exact formula for the mutual fund's discount factor. Notice that in equilibrium knowledge of X and X' implies knowledge of $\widetilde{X}'$. The stochastic discount factor is defined as follows:

$$
\begin{aligned}
&Q(X,X') = \\
&\quad \int_{\{1\}\times S\times B\times A} \sum_{\beta'\in B} \pi_B(\beta,\beta') \sum_{s'\in S} \pi_S(s,s')\, \beta \left(g_a(X,1,s,\beta,a) \frac{\big(1-\lambda+\lambda f(\widetilde{X}')\big) u_c(g_c(X',1,s',\beta',g_a(X,1,s,\beta,a)))}{u_c(g_c(X,1,s,\beta,a))} \right) d\mu(e,s,\beta,a) \\
&+ \int_{\{1\}\times S\times B\times A} \sum_{\beta'\in B} \pi_B(\beta,\beta') \sum_{s'\in S} \pi_S(s,s')\, \beta \left(g_a(X,1,s,\beta,a) \frac{\big(\lambda(1-f(\widetilde{X}'))\big) u_c(g_c(X',0,s',\beta',g_a(X,1,s,\beta,a)))}{u_c(g_c(X,1,s,\beta,a))} \right) d\mu(e,s,\beta,a) \\
&+ \int_{\{0\}\times S\times B\times A} \sum_{\beta'\in B} \pi_B(\beta,\beta') \sum_{s'\in S} \pi_S(s,s')\, \beta \left(g_a(X,0,s,\beta,a) \frac{\big(f(\widetilde{X}')\big) u_c(g_c(X',1,s',\beta',g_a(X,0,s,\beta,a)))}{u_c(g_c(X,0,s,\beta,a))} \right) d\mu(e,s,\beta,a) \\
&+ \int_{\{0\}\times S\times B\times A} \sum_{\beta'\in B} \pi_B(\beta,\beta') \sum_{s'\in S} \pi_S(s,s')\, \beta \left(g_a(X,0,s,\beta,a) \frac{\big((1-f(\widetilde{X}'))\big) u_c(g_c(X',0,s',\beta',g_a(X,0,s,a))}{u_c(g_c(X,0,s,a))} \right) d\mu(e,s,\beta,a),
\end{aligned}
\tag{10}
$$

Above, $u_c(c)$ marks the marginal utility of consumption.

B Definition: Recursive Equilibrium

Definition 1 (Recursive equilibrium) *A recursive equilibrium is a set of functions* $G(X)$, $\widetilde{G}(X)$, $\hat{G}(\widetilde{X})$, $W(X,e,s,\beta,a)$, $g_a(X,e,s,\beta,a)$, $g_c(X,e,s,\beta,a)$, $f(\widetilde{X})$, $p_a(X)$, $d_a(X)$, $w(X)$, $\tau(X)$, $h(X)$, $Q(X,X')$, $J_L(X,s)$, $V(\widetilde{X})$, $r(X)$, $J_K(X,k)$, $i(X)$, $v(X)$, $K'(X)$, $P(X)$, $y_j(X,P_j)$, $J_I(X,P_{j,-1})$, $k_j(X)$, $\ell_j(X)$, $P_j(X)$, $\Pi_j(X)$, $y(X)$, $R(X)$ *such that:*

1. *Given* $\widetilde{G}(X)$, $f(\widetilde{X})$, $w(X)$, $p_a(X)$, $d_a(X)$, $G(X)$, *and* $\tau(X)$, *value function* $W(X,e,s,\beta,a)$ *is a solution to the household's problem.* $g_a(X,e,s,\beta,a)$ *and* $g_c(X,e,s,\beta,a)$ *are the associated optimal decision rules.*

2. *Given* $h(X)$, $w(X)$, $Q(X,X')$, *and* $G(X)$, $J_L(X,s)$ *solves the problem of a labor agency.* $V(\widetilde{X})$ *satisfies the free-entry condition in the labor agency sector.* $f(\widetilde{X})$ *is consistent with* $V(\widetilde{X})$.

3. *Given* $r(X)$, $Q(X,X')$, *and* $G(X)$, $J_K(X,k)$ *solves the problem of a capital-producing firm.* $i(X)$, $v(X)$, *and* $K'(X)$ *are the associated optimal decision rules.*

4. *Given* $r(X)$, $v(X)$, $h(X)$, $P(X)$, $y_j(X,P_j)$, *and* $Q(X,X')$, *value function* $J_I(X,P_{j,-1})$ *solves the problem of an intermediate good producer.* $k_j(X)$, $\ell_j(X)$, $P_j(X)$, *and* $\Pi_j(X)$ *are the associated optimal decision rules.*

5. *Given $P(X)$ and P_j, $y_j(X, P_j)$ and $y(X)$ are the optimal decisions of final good producers.*

6. *The aggregate discount factor $Q(X', X)$ satisfies equation (10).*

7. *$d_a(X)$ satisfies the flow budget constraint of mutual funds (9).*

8. *The wage per efficiency unit of labor is given exogenously by $w(X)$ (5).*

9. *The labor tax $\tau(X)$ satisfies the government budget constraint (8).*

10. *The nominal interest rate $R(X)$ satisfies the Taylor rule (7).*

11. *The aggregate laws of motion $G(X)$, $\widetilde{G}(X)$, and $\hat{G}(\widetilde{X})$ are consistent with the relevant optimal decision rules.*

12. *All market clearing conditions are satisfied.*

C Solution Algorithm

This appendix outlines the solution method of an equilibrium with aggregate uncertainty. The method is a version of the method developed by Krusell and Smith (1998) and is closely related to the solution method based on reference distributions described in Reiter (2002) and Reiter (2010).[15]

1. Following Reiter (2010) we approximate the aggregate state of the economy by $X = (K, N, Z, \zeta_F, \zeta_R)$ and assume that there is a distribution selection function $\hat{\mu}$, a mapping from X into the space of all distributions on the household state variables. We approximate such a distribution following Young (2010) as a histogram on the product of skill state, the discount factor, employment state and a grid on the wealth distribution. All agents use this function to construct their forecasts about the evolution of the economy. We discretize X and interpolate between points using a Smolyak approximation (see Krueger and Kubler 2004).

2. Solve the model without aggregates shocks and follow the steps in Reiter (2010) to construct a first guess for the distribution selection function $\hat{\mu}$.

[15] In earlier versions of the paper we used an approach closer to Krusell and Smith (1998), in which we forecasted the expectation terms in the firms' Euler equations and asset prices. The current method allows for a faster solution of the model, but results were close to each other when we compared both methods.

3. Form an initial guess for the following: the price of the asset, $P_a(X)$, and the terms $\mathbb{E}Q(X,X')\phi_P\Pi(X')(\Pi(X')-\overline{\Pi})$, $\mathbb{E}Q(X,X')\frac{1}{\Pi(X')}$, $\mathbb{E}Q(X,X')\left[r'(X')v' + \frac{1}{\zeta'(i'/K')}\left(1-\delta(v')+\zeta\left(\frac{i'}{K'}\right)\right)-\frac{i'}{K'}\right]$, and $\mathbb{E}\big[Q(X,X')(1-\lambda)J_L(X',s')\big]$ each as functions of X. For a shorthand, we denote these guesses by $\Sigma(X)$.[16]

4. Given these initial guesses, perform the following steps

   (a) Given $\Sigma(X)$ use a numerical equation solver to obtain the solution to the firms' and government's equations on the grid.

   (b) Interpolate the static choices.

   (c) Given the solutions obtained in the previous step, iterate on the value function of the households.

      i. Set a guess for the value function $W(X,e,s,\beta,a)$.

      ii. Use the Bellman equation to update the value function.

      iii. If the updated value function is close to the guess, this step is done. The optimal decision rules $a' = g_a(X,e,s,\beta,a)$ and $c = g_c(X,e,s,\beta,a)$ are obtained. Otherwise, go back with an updated value function.

   (d) Use $\hat{\mu}$ and the solutions to the firms' and government's problems along with the optimal decision rules to compute the discount factor (10) on the grid. Use this to update $\Sigma(X)$ to $\Sigma'(X)$. $P_a(X)$ is updated by solving for the market-clearing price at each grid point using the reference distribution. If $\Sigma(X)$ and $\Sigma'(X)$ are close, go to the next step, otherwise use a weighted average of $\Sigma(X)$ and $\Sigma'(X)$ and start with the firms' and government's equations again.

   (e) Simulate the model. Notice that, for each period a market-clearing p_a has to be found, in the same manner as in Krusell and Smith (1997).

      i. Set the initial state and the initial type distribution. Use the steady-state values as the initial guess.

      ii. At the beginning of period t, draw a new set of shocks. We have the aggregate state in period t, (K_t, N_t, Z_t, D_t).

      iii. Set a guess for the share price $\hat{p}_a$, using the forecasting function with $P_a(X)$.

[16] For the initial guess, we solved the representative-agent version of our model (setting $\beta = 0.99$ so as to match the same real rate). Alternatively, we could have started with the choices in the heterogeneous-agent model in the steady state and guessed $Q(X,X') = 0.99$.

iv. Conditional on $\hat{p}_a$, and the aggregate state variables in period t, solve the problem of the households.

v. Check market clearing. Compute the excess demand for the shares. If it is zero, a market-clearing price in period t, $p_{a,t}$, is obtained for period t. K_{t+1} and N_{t+1} can be computed. Go to the next step. Otherwise, update $\hat{p}_a$ and go back to the previous step.

vi. Update the type distribution and aggregate state variables using $p_{a,t}$ and go to period $t+1$.

vii. Keep simulating until period $T = T_0 + T_1 = 500 + 3000$ periods.

(f) The previous step generates a time series of household distributions $\{\mu\}_{t=0}^{T}$. Drop the first T_0 periods. Using the time series for $t = T_0 + 1, ..., T$ construct a new reference distribution function $\hat{\mu}'$ following Reiter (2002).

(g) Compare $\hat{\mu}$ and $\hat{\mu}'$. If they are close, an equilibrium is obtained. Stop. Otherwise, update $\hat{\mu}$ and return to the firms' and government's problem.

In practice, as ζ_R and ζ_F have the same persistence under our calibration, the only variable affected differently by the two shocks is the nominal interest rate. Therefore, it is possible to solve the model while merging them into one state variable. During simulations of the model we distinguish between the two shocks in order to capture the right movements in the nominal interest rate given our calibration.

D Data and Details on the Calibration

This appendix provides further details on the data used and the calibration.

D.1 Data in Table 3 of the Main Text

The data we compare the model to in Table 3 are either quarterly already or are quarterly averages of monthly data. They are seasonally adjusted, if necessary. Unless noted otherwise, their source is the St. Louis Fed's FRED II database. We start with series that cover the period 1979Q1 to 2013Q3. After filtering them, we drop the first and last 20 quarters to arrive at a sample covering the period 1984Q1 to 2008Q3. Nominal variables are deflated by the GDP deflator, which we also use as our measure of inflation, Π. Personal consumption expenditures, c include total durable and non-durable consumption expenditures as well as services. Investment, i, is gross private domestic investment. Our measure of GDP is the sum of consumption and investment. Capacity utilization, v, is measured by the quarterly average of the Board of Governors' headline index of industrial capacity utilization. We use the civilian unemployment rate, $U(X)$, among those 16 years of age and older. Employment, $N(X)$, is one minus this measure. We measure vacancies V using Barnichon's (2010) composite help-wanted index. The data counterpart to the job-finding rate, f, is the quarterly average of the monthly transition probability from unemployment to employment in the Current Population Survey (CPS). The data are adjusted for time aggregation as in Shimer (2012). The wage, $W(X)$ is computed as wage and salary accruals from the national accounts divided by the GDP deflator. The interest rate, R, is the quarterly average of the effective federal funds rate.

D.2 Details on the Calibration

Figure 12 shows the wealth Lorenz curve for U.S. households aged 21-65 as a red dashed line. The data are from the 2004 Survey of Consumer Finances. For the construction of the wealth variable, we follow Díaz-Giménez et al. (2011). A solid line shows the model's nonstochastic steady-state counterpart.

Figure 12: Wealth Lorenz Curve

E State Dependence of Impulse Responses

This appendix documents the state dependence of the impulse responses and the extent to which heterogeneity amplifies the state dependence.

E.1 TFP Shocks: State Dependence

Figure 15 analyzes the extent to which a TFP shock has different effects depending on the state of the business cycle. We consider three such states: a deep recession (dashed line), a boom

Table 6: "Wall Street's" and "Main Street's" Income Sources

Wealth percentile	0-5	5-20	20-40	40-60	60-80	80-95	95-100
Data: 2004							
Labor income	92	83	91	89	89	81	55
Financial income	1	1	2	5	6	14	41
Transfers	7	16	8	6	5	6	3
Model (steady-state)							
Labor income	96	96	97	97	81	57	32
Financial income	0	0	0.1	2	18	42	68
Transfers	4	4	3	1	1	1	0.3

Notes: Share of income coming from labor and financial income, respectively, by percentile of the wealth distribution. The data are from the Survey of Consumer Finances (2004), for households aged 21-65. "Financial income" includes the categories financial income, business income, and capital gains/loss. Labor income does not include social security or pensions.

Table 7: Second Moments – Comparison HA, RA, and Saver-Spender Variants

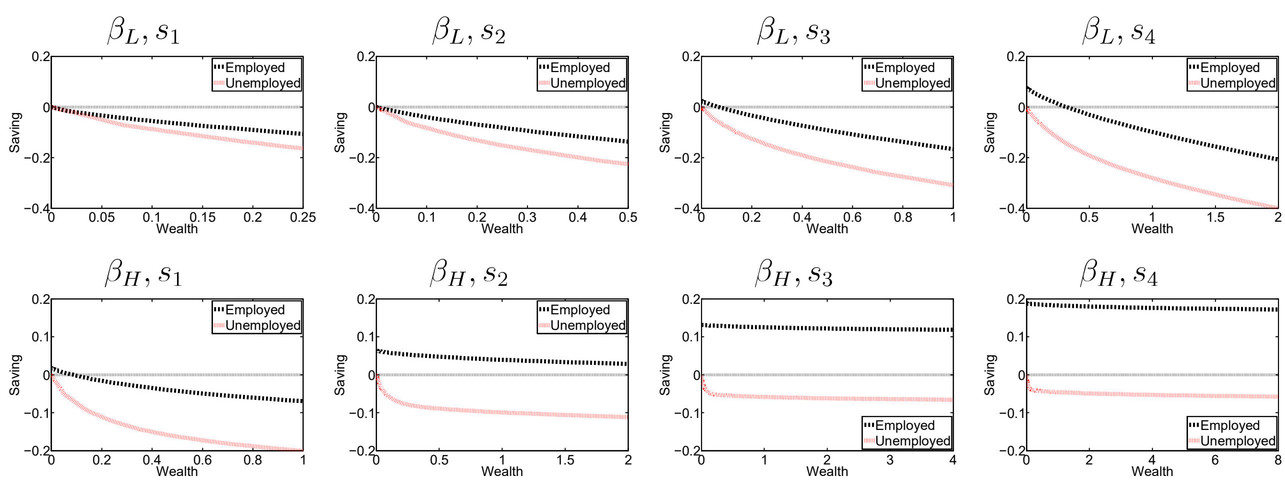
	heterog. hh. (HA)			represent. hh. (RA)			Saver-Spender SP30			SP50		
	Std	Corr	AR(1)	Std	Corr	AR(1)	Std	Corr	AR(1)	Std	Corr	AR(1)
GDP (GDP)	1.69	1.00	0.63	1.62	1.00	0.64	1.66	1.00	0.63	1.68	1.00	0.63
Consumption (c)	1.02	0.99	0.69	0.89	0.98	0.71	0.94	0.98	0.70	0.98	0.99	0.68
Investment (i)	5.28	0.98	0.73	5.86	0.99	0.71	5.66	0.99	0.72	5.51	0.98	0.72
Capacity utilization (v)	0.96	0.78	0.24	0.83	0.75	0.27	0.89	0.75	0.25	0.95	0.76	0.24
Employment $N(X)$	0.65	0.90	0.64	0.62	0.90	0.66	0.64	0.90	0.65	0.66	0.90	0.65
Unemployment $U(X)$	10.9	-0.90	0.65	10.2	-0.89	0.67	10.7	-0.90	0.66	10.9	-0.90	0.65
Vacancies (V)	8.94	0.75	0.07	8.35	0.73	0.10	8.68	0.73	0.08	8.97	0.73	0.07
Job finding rate (f)	5.37	0.88	0.38	5.08	0.87	0.40	5.26	0.87	0.39	5.42	0.87	0.37
$GDP(X)/N(X)$	1.14	0.97	0.62	1.10	0.97	0.63	1.11	0.97	0.63	1.12	0.97	0.62
Wage $W(X)$	0.51	0.97	0.62	0.50	0.97	0.63	0.50	0.97	0.63	0.51	0.97	0.62
Inflation Π[1]	0.67	-0.32	0.62	0.67	-0.40	0.63	0.68	-0.35	0.62	0.69	-0.31	0.61
Nominal rate R[1]	0.97	-0.14	0.58	0.96	-0.25	0.60	0.98	-0.19	0.59	1.00	-0.14	0.58

Notes: Same as Table 3, but listing the saver-spender model variants with 30 percent ("SP30") and 50 percent ("SP50") of spenders instead of the data.

(dotted line), and the stochastic steady state (circles). The deep recession state is obtained as follows. Starting at the stochastic steady state, we feed a sequence of five periods of negative one-standard-deviation TFP shocks, and one-standard-deviation contractionary monetary and financial shocks into the model. The state of the economy after that sequence of shocks is the "deep recession state." The boom state is the result of the same sequence of shocks but with the opposite sign. The first row shows the response of the model economy with a representative

Figure 13: Mass of Households by Idiosyncratic State

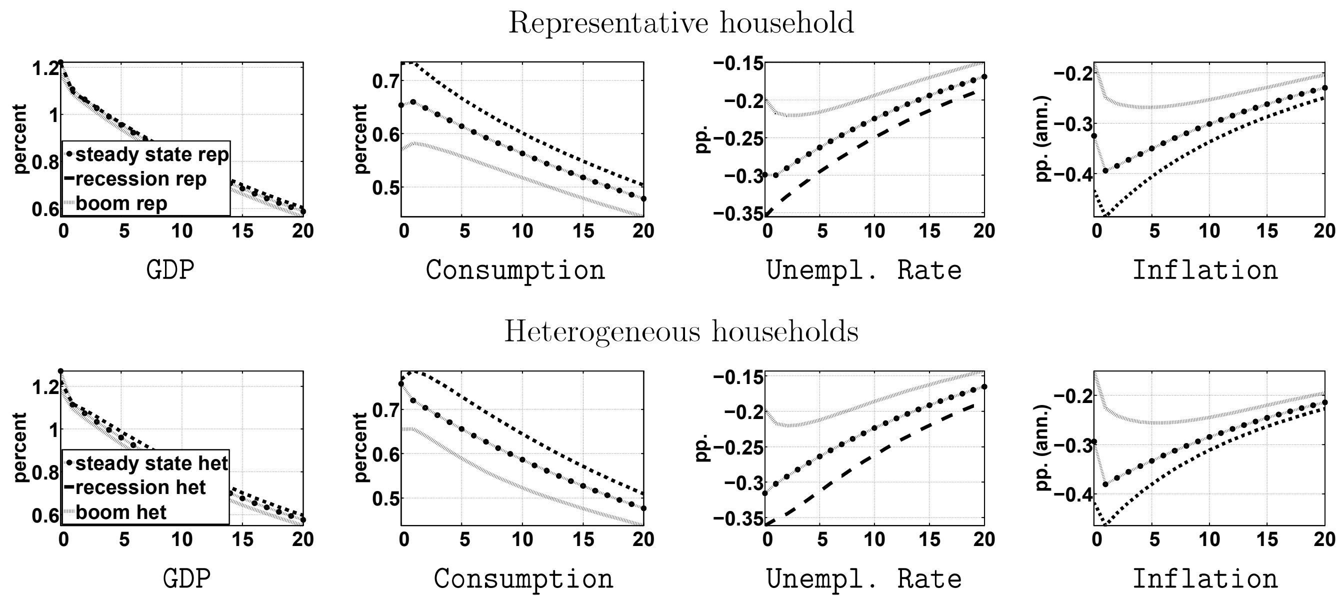
Figure 14: Savings Policy Functions



Notes: Savings policy as a function of current wealth. Policy functions evaluated in the stochastic steady state. x-axis: wealth as a share of annual potential labor income, Wealth:= $p_a \cdot a/(4 \cdot s \cdot w)$, where $s \in S$. y-axis: saving as a share of annual potential labor income, Saving:= $p_a \cdot (a' - a)/(4 \cdot s \cdot w)$. Positive values thus mean accumulation of wealth. Negative values mean dissaving.

agent, the second row the economy with heterogeneous households. We start by discussing the representative-agent economy. A sequence of expansionary shocks means that the unemployment

Figure 15: Effect of State of the Economy on IRFs to TFP Shock



Notes: Impulse response to a 1 standard deviation TFP shock, Z. First row: model economy with representative households. Second row: model economy with heterogenous households. In each of the panels, the response indicated by circles is the response starting from the steady state (as in figure 2) and the response indicated by dashes starts from a deep recession state. The dotted line marks responses starting in a boom. Whenever the figure shows percent responses, it normalizes the response by the steady-state value of the respective variable. This means that larger percent responses also mean larger responses in levels.

rate is lower than in the non-stochastic steady state. This makes hiring more costly for labor-services firms. The costs for labor services increase. As a result, an expansion in TFP has only half the impact on the unemployment rate in a boom as in a recession. Similarly, inflation falls only half as much in a boom as in a recession. Relatively higher costs for labor services mean that, in order to take advantage of the TFP shock, the mutual funds increasingly invest more in physical capital in a boom than they would in a recession (not shown). As a result, consumption responds by less (relative to consumption in the non-stochastic steady state) in the boom than in the recession. Again, the response of output is affected to a lesser extent.

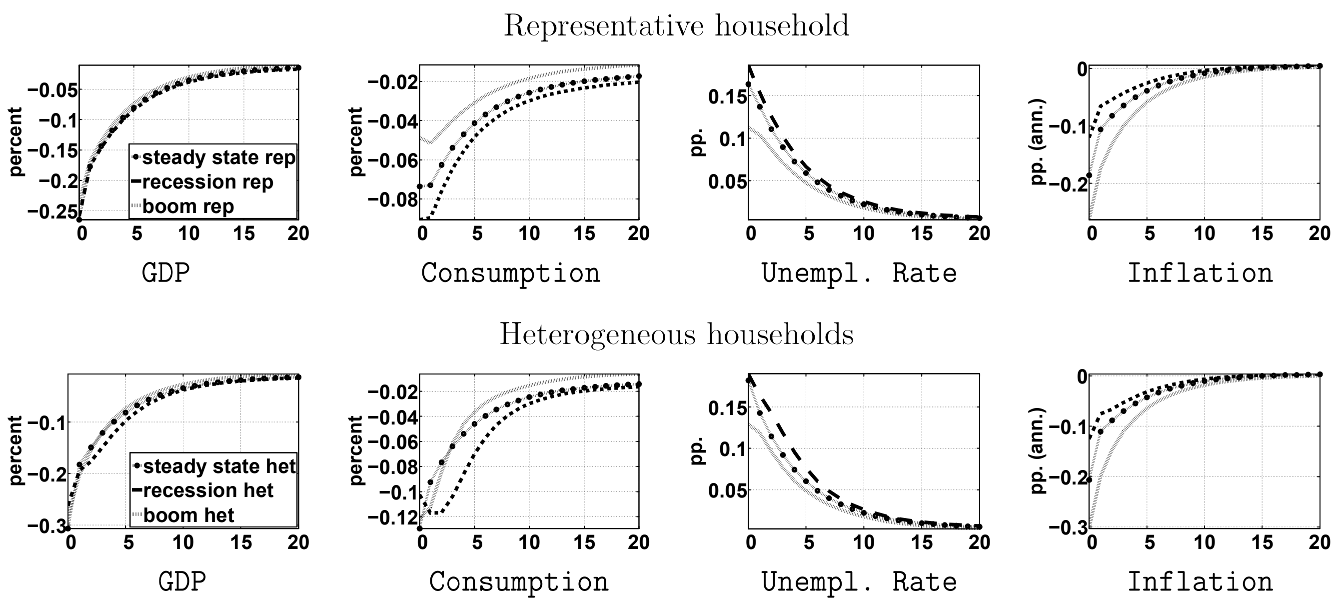
The model economy with heterogeneous households inherits the state-contingence of the responses from the representative-agent economy. What is interesting, however, is that the heterogeneous-agent economy alters the response of consumption in deep recessions. First, relative to the steady state, consumption responds by less on impact in the heterogeneous-agent economy because more households find themselves close to the borrowing constraint in the recession. As a result, they take advantage of the rising incomes that emerge from the expansionary TFP shocks by consuming but also rebuilding their savings. Consumption in a deep recession, therefore, only gradually builds up in response to an expansionary TFP shock.

In sum, in both the RA and HA economies there is notable state dependence of the responses. Responses of consumption, unemployment and inflation to a TFP shock are much larger in a recession than in a boom. The difference between the HA and RA models' responses is state-dependent as well.

E.2 Monetary Policy Shocks: State Dependence

Figure 16 analyzes the extent to which the impulse responses to a contractionary monetary policy shock are state-dependent. Similar to the results we obtained for the TFP shock, the impact of a monetary policy shock is state-dependent, and considerably so. In the heterogeneous-agent economy, the response of GDP to a contractionary monetary policy shock is 20 percent larger (GDP falls by 0.05 percentage points more) in a boom than in a recession. The representative-agent economy shows less (if any) of such state dependence in the response of GDP. Both the RA and HA models have considerable state-dependence in the inflation response, however. Inflation falls almost three times as much in response to a monetary policy shock if that shock hits in a boom than if it hits in a recession. This is so even though the response of the unemployment rate is almost 50 percent larger in a deep recession than in a steep boom. Putting this in slightly different terms, the responses shown here suggest that the "sacrifice ratio" (measured as the rise in unemployment for a given fall in inflation) is considerably lower in boom times than in recessions. Or, putting it still differently, monetary policy (through an expansionary monetary policy shock) can more easily increase output without having to jeopardize price stability in recessions than in booms. Heterogeneity further increases the scope for state-dependent responses to monetary policy shocks. This is most clearly visible again for the consumption response: in a recession, more households will be close to the borrowing constraint or, more generally, farther below their target level of wealth. As a result, those households are less susceptible to the intertemporal substitution that a monetary tightening causes. Actually, on impact, in the HA model consumption falls by less in a recession than in a boom (or in the steady state). Over time, however, in the HA economy, the consumption response is considerably stronger and much more persistent in a recession than in a boom.

Figure 16: Effect of State of the Economy on IRFs to a Monetary Policy Shock



Notes: Impulse response to a one standard deviation monetary policy shock, D. First row: model economy with representative households. Second row: model economy with heterogeneous households. In each of the panels, the response indicated by circles is the response starting from the steady state (as in figure 5) and the response indicated by dashes starts from a deep recession state. The dotted line marks responses starting in a boom. Whenever the figure shows percent responses, it normalizes the response by the steady-state value of the respective variable. This means that larger percent responses also mean larger responses in levels.