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Sina T. Ates
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Fewer but Better:

Sudden Stops, Firm Entry, and Financial Selection

Sînâ T. Ateş*

Division of International Finance
Federal Reserve Board

Felipe E. Saffie†

Department of Economics
University of Maryland

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Abstract

We incorporate endogenous technical change into a real business cycle small open economy framework to study the productivity costs of sudden stops. In this economy, productivity growth is determined by the entry of new firms and the expansion decisions of incumbent firms. New firms are created after the implementation of business ideas, yet the quality of ideas is heterogeneous and good ideas are scarce. Selection of the most promising ideas gives rise to a trade-off between mass (quantity) and composition (quality) in the entrant cohort. Chilean plant-level data from the sudden stop triggered by the Russian sovereign default in 1998 confirm the main mechanism of the model, as firms born during the credit shortage are *fewer, but better.* The quantitative analysis shows that four years after the crisis, 12.5% of the output deviation from trend is due to permanent productivity losses. Distortions in the entry margin account for 40% of the loss, and the remainder is due to distortion in the expansion decisions of incumbents.

Keywords: Selection, Sudden Stop, Endogenous Growth, Firm Dynamics.
JEL Code: F40, F41, F43, O11, O16

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*The author is a staff economist in the Division of International Finance, Board of Governors of the Federal Reserve System, Washington, D.C. 20551 U.S.A. The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. Contact: `sina.t.ates@frb.gov`

†Assistant Professor, Department of Economics, University of Maryland, 4118E Tydings Hall, College Park, MD 20742. Phone: +1-301-405-3490, Fax:+1-301-405-3542, Email: `saffie@econ.umd.edu`; Corresponding author.

1 Introduction

In August 1998, the Russian sovereign default triggered a violent sudden stop in the developing world.[1] Interest rate spreads for the seven biggest Latin American economies tripled in the weeks after this crisis, decreasing the availability of external funding by 40% between 1998 and 2002. Most of the economic analysis of sudden stops focuses on the short-run detrimental effects that they impose on the real economy. However, the empirical studies of Cerra and Saxena (2008) and Reinhart and Rogoff (2014) have documented persistent output losses associated with large economic downturns, pointing to permanent losses in total factor productivity (TFP). Because firm entry is an important driver of productivity growth and because start-ups depend on external funding, distortions in firm entry are likely to contribute to this long-run cost. This paper develops a framework that links short-run financial crises with long-run output losses through distortions in firm entry and firm dynamics.

Three aspects are key for a meaningful study of the entry margin. First, behind every firm lies an entrepreneur's idea, and ideas are heterogeneous in quality. Drastic innovations are a scarce resource. Second, the financial system does not allocate funding randomly, and not every idea has the same chance of being funded. Not surprisingly, when resources are scarce, banks adopt higher lending standards, and fund only the most promising projects. Third, entrants become incumbents with heterogeneous life cycles, with expansions and, contractions, and, finally, exit. In this sense, it is critical to incorporate firm dynamics in the analysis, because distortions in the entry margin can trigger persistent effects in productivity by distorting the life cycle of firms.

The main novelty of this study is the recognition that the scarcity of good ideas and the presence of financial selection induce a trade-off between the size of the entrant cohort and the average contribution of each firm within that cohort to aggregate productivity. Moreover, this mass-composition trade-off has persistent effects because of the post-entry decisions of firms. Thus, the ability of the financial system to allocate resources between heterogeneous projects and the subsequent dynamic decisions of incumbent firms need to be taken into account when answering the main question of this paper: What is the long-run productivity cost of a sudden stop?

To answer this question, we generalize the real business cycle small open economy model of Neumeyer and Perri (2005) to include firm-driven endogenous growth as in Klette and Kortum (2004).[2] We extend this hybrid framework by modeling business plan hetero-

[1] A sudden stop in capital flows is a large and abrupt decrease in capital inflows, characterized by jumps in sovereign spreads and quick reversals of current accounts deficits. See Calvo and Talvi (2005) for details of the Russian sovereign default episode.

[2] This combination also provides a micro foundation for the stochastic trend dynamics that Aguiar and

geneity and scarcity. In particular, a financial intermediary has a portfolio of business plans (or projects) that (if funded) can generate either high- or low-type firms. High-type firms give rise to drastic productivity improvement in the production technology of intermediate varieties when they expand, while low-type firm give rise to only marginal improvements in the production technology of varieties. Every project is characterized by its idiosyncratic probability of giving rise to a high-type firm. Hence, projects are *ex-post* heterogeneous in terms of the productivity advantage that the new firm enjoys after entering the industry, and they are also *ex-ante* heterogeneous with respect to their idiosyncratic probability of generating a high-type firm. Scarcity arises from the fact that only a handful of ideas have a strong chance of generating high-type firms. The optimal allocation of funding follows a cut-off rule based on the idiosyncratic probability of becoming a high-type firm, which introduces a linkage between the size of the entrant cohort and the average efficiency gains generated by its members. The endogenous decision of incumbents to acquire new products gives rise to a non-degenerate size distribution.

Because of heterogeneity, selection, and firm dynamics, interest rate shocks trigger firm and productivity dynamics that are absent in a traditional open economy framework. First, a mass-composition trade-off arises at the entrant cohort level: Periods of high interest rates are characterized by high credit standards that give rise to smaller cohorts with higher expected average productivity. Second, incumbents' innovation decisions are distorted. On the one hand, lower profits during the crisis decrease the benefits of expanding; on the other hand, lower entry rates decrease the threat of replacement, which increases the value of incumbents and promotes expansion. The overall effect on current incumbents is therefore a quantitative matter. The model economy also allows for meaningful cleansing effects in the exit margin. Because high-type firms expand endogenously at higher rates than low-type firms, high-type firms are less likely to exit than low-type firms. Third, because of the future expansion decisions of the cohorts born during the crisis, composition dynamics persist long after the crisis has vanished. Note that ignoring the three forces that we have described would imply that discarded projects are just as productive as actual entrants and that good firms exit with the same frequency as mediocre firms. The model developed in this paper provides a novel framework to study firm dynamics and endogenous productivity in discrete time models with aggregate risk.

The empirical section studies the Chilean sudden stop of 1998 to validate the trade-off between mass and composition at the core of the model. We focus on Chile for three reasons: (i) it is a small open economy; (ii) plant-level data for Chilean manufacturing firms are publicly available, and these data allow us to directly study entrant cohorts; and (iii) as argued by Calvo et al. (2006), the sudden stop after the Russian sovereign default was essentially an exogenous shock to the Chilean economy. We show that firm entry in

Gopinath (2007) use to explain business cycles in small open economies.

Chile decreased by 45% during the sudden stop. However, firms born in crisis are not just fewer, they are also better. In fact, the econometric analysis in Section 4 shows that, after controlling for individual characteristics, firms born during normal times are, on average, 30% less profitable and 10% less productive (measured by revenue total factor productivity) during their life span than firms born during the sudden stop.

In the quantitative section of the paper, we calibrate the model to the pre-crisis Chilean economy. The balanced growth path of the calibrated model matches non-targeted moments of the firm life cycle and dynamics. In particular, the model mimics the size distribution, the age composition, and the hazard rates observed in the data. The stochastic solution of the model also matches non-targeted moments of the Chilean business cycle.

We then use the Chilean sudden stop to assess the performance of the model during the crisis. Because the model economy has two exogenous disturbances to interest rates and stationary productivity only two series can be targeted. Therefore, we filter the stationary productivity component and the interest rate shocks using interest rate and output data. The model is able to capture not only the dynamics of consumption, investment, labor and measured total factor productivity, but also the dynamics of firm entry, firm exit, and the dynamic selection effects observed in the data during the crisis. Given the empirical success of the model, we use it to quantify the productivity loss due to the sudden stop. By 2002 Chilean output is still 4% below trend in the data. Without a general equilibrium model there is no hope of disentangling how much of that loss is due to permanent productivity distortions and how much is purely stationary, or evaluating the welfare cost of the episode. The model measures the accumulated endogenous productivity loss at 0.5% in 2002.

In order to understand the importance of heterogeneity, selection, and firm dynamics, we calibrate alternative models that lack these features. Note that by construction, a model with exogenous growth like that of Neumeyer and Perri (2005) attributes all of the gap to stationary fluctuations. Interestingly, a model with no heterogeneity and no firm dynamics implies an accumulated productivity loss of 2.5%, five times the effects implied by the baseline model. This difference is a large economic magnitude that can bias public policy. There are two main forces behind this difference. First, heterogeneity and selection imply that most of the decrease in entry is concentrated among low-type firms; therefore, the contribution of entry to productivity decreases less than in a model with no heterogeneity. Second, heterogeneity and firm dynamics imply that low-type firms are more likely to exit and that high-type firms are more likely to expand. This dynamic composition effect increases the fraction of high-type firms in the economy for several years after the crisis, boosting the contribution of incumbents to productivity after the crisis and fueling the recovery.

The structure of the paper is as follows. Section 2 reviews the related literature. Section 3 introduces our model. Section 4 presents the analysis of the Chilean economy as a *pseudo* natural experiment for the model, exploring at the macro and micro levels the consequences of the sudden stop for the Chilean economy. Section 5 presents the calibration of the model and the quantification of the long-run cost of a sudden stop. Finally, Section 6 concludes and suggests avenues for future research.

2 Related Literature

This paper belongs to the intersection between the literatures on endogenous technical change and international finance. This is not the only paper introducing endogenous growth into the small open economy real business cycle framework of Mendoza (1991). For example, Queraltó (2013) studies the permanent productivity effects of a financial crisis. In his model, an interest rate shock triggers a balance sheet channel, which harms the processes of invention and implementation. Therefore, fewer firms enter the market and fewer ideas are developed for future use. The endogenous growth model at the core of that paper is the framework that Comin and Gertler (2006) build around Romer (1990). Guerrón-Quintana and Jinnai (2014) use a similar framework to study the effect of the 2008-09 liquidity crash on U.S. economic growth. Gornemann (2014) combines the endogenous default model of Mendoza and Yue (2012) with the variety model of Romer (1990) to study how endogenous growth affects the decision of the sovereign to default. Because default increases the price of imported intermediate goods in his model, it decreases the expected profits of potential entrants and, hence, depresses productivity growth.

This paper makes three contributions to the literature. First, by introducing an endogenous growth framework based on Grossman and Helpman (1991) and Aghion and Howitt (1992) instead of Romer (1990), it recognizes the dual effect of firm entry: Newcomers are a creative force because they are more productive, but they are also a destructive force that replaces former producers. Second, it develops a tractable framework that introduces heterogeneity in the firm dynamic set up of Klette and Kortum (2004) and nests it in a discrete time model with aggregate risk. Heterogeneity and firm dynamics have proven to be essential in separately analyzing the short-run and the long-run behavior of an economy. As noted by Clementi and Palazzo (2016), heterogeneous firm dynamics significantly affects the short-run fluctuations of an economy.[3] Moreover, the quantitative literature on innovation also shows that firm heterogeneity is crucial for understanding the long-run effects of policies. Salient examples are Akcigit and Kerr (2010) and Lentz and Mortensen (2008).[4] Therefore, includ-

[3]Other studies using models with exogenous productivity and a short-run focus that study entry dynamics during crises include Lee and Mukoyama (2015) and Siemer (2014).

[4]Atkeson and Burstein (2010) show that heterogeneous firms' decisions are immaterial when evaluating the welfare effect of opening a country to trade. Their model abstracts from intertemporal spillovers that generate endogenous growth and product expansion by incumbents. In our quantitative analysis, we show that ignoring heterogeneous firms' decisions in our baseline model implies a two times larger welfare cost in

ing heterogeneity and firm dynamics when studying the link between short-run crises and long-run productivity is a natural extension. The third contribution is the use of firm-level data to provide evidence of the main driving force in the model and to discipline the quantitative experiment. This class of models, where the main driving force is micro-founded, should be compared not only to macro aggregates, but also to firm-level data. This paper takes a step in that direction by taking seriously the firm dynamics implications of the model and comparing them directly to the micro data. Therefore, this paper is also related to the empirical literature that uses firm-level data to study financial crises. For instance, Schnabl (2012) uses the sudden stop triggered by the Russian default to document how international banks reduced lending to Peruvian banks and how Peruvian banks diminished lending to Peruvian firms during the crisis. Hallward-Driemeier and Rijkers (2013) evaluate the effects of the Asian crisis of 1997 using firm-level data from Indonesia. Their dynamic productivity decomposition points to a strong selection component at the entry margin. We add to this literature by documenting that firms born during a sudden stop are fewer, but better.

3 A Stochastic Open Economy with Firm Dynamics

In this section, we introduce a small open economy model augmented with endogenous technical change, firm heterogeneity, financial selection and firm dynamics, subject to exogenous interest rate and stationary productivity shocks. Endogenous productivity in this economy is modeled following the Klette and Kortum (2004) extension of the Grossman and Helpman (1991) and Aghion and Howitt (1992) framework.[5] This framework builds on a Schumpeterian concept of growth where productivity increases with the innovation performed by new firms and incumbents. In particular, innovation improves the technology available for the production of a particular variety. Because the latest innovator is more productive than the established producer in a given product line, Bertrand monopolistic competition implies that the newcomer sets a price that enables her to steal the product line from the previous incumbent. Therefore, the engine of growth in this economy is Schumpeterian creative destruction. The international aspect of the model follows the small open economy real business cycle framework of Mendoza (1991). The two exogenous stochastic components of the model are an aggregate productivity shock and a stochastic interest rate. This feature introduces economic dynamics into an otherwise deterministic model. Figure 1 presents a diagram of the model economy.

There are four major economic agents in the economy. First, the representative household consumes the final tradable good and provides labor and capital services to the economy.

response to interest rate shocks.

[5] A detailed review of this literature can be found in Aghion et al. (2014).

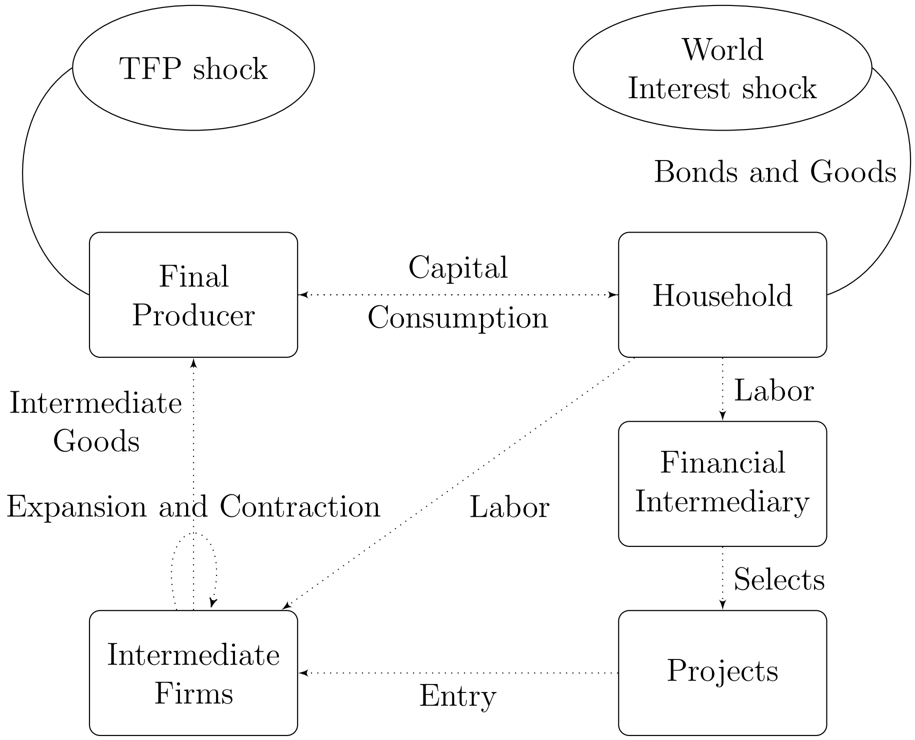


Figure 1: Model Economy

She trades bonds and goods with the rest of the world. The interest rate she faces on the bond follows an exogenous stochastic process. Second, the final good producer combines capital and non-tradable intermediate varieties to produce the unique tradable good in the economy. The production of the final good is subject to a stationary and exogenous aggregate productivity shock. Third, intermediate good producers use labor to produce varieties. There is a fixed mass of varieties dominated by incumbent firms, and incumbents can own several varieties. Incumbents increase their number of varieties by optimally choosing their expansion effort. Fourth, a representative financial intermediary buys a portfolio of projects from the household and funds the most promising ones. Funded projects become entrants, i.e., new firms that start with a single variety. Therefore, innovation in this economy can be due to the expansion of incumbents or the entry of new firms. When a firm innovates, it increases the productivity of a variety, and, because of Bertrand competition, it becomes the new leader and producer of that variety. Despite the mass of varieties being fixed, the mass of firms in the economy is endogenously determined by the expansion and contraction of incumbents, the entry of new firms, and the exit of firms that have lost all of their varieties.

Heterogeneity and selection are at the core of the model. First, firms are *ex-post* heterogeneous in the productivity advantage that they enjoy when expanding. This layer of heterogeneity is discrete because the advantage can be either high or low. Second, firms are ex-ante heterogeneous at the project stage. Projects differ in their idiosyncratic probability of giving rise to a high-type firm. This ex ante heterogeneity is probabilistic in a continuous space. The selection performed by the financial intermediary aims to enact projects with a higher ex ante probability of becoming high-type firms. There is also dynamic selection due to high-type firms endogenously expanding at higher rates than low-type firms. In this section we explain in detail each component of the model and how the firm-level dynamics are related to the evolution of aggregate productivity. We also derive model predictions to guide the empirical analysis of the paper.

3.1 The Representative Household

Time is discrete. We denote a history $(s_0, s_1, ..., s_t)$ by s^t, where s^t contains all the relevant past information that agents need to make decisions in period t. There is a representative consumer modeled following the open economy literature that builds on Mendoza (1991). We include both capital adjustment costs and a bond holding cost.[6] The household chooses state-contingent sequences of consumption $C(s^t)$, labor $l(s^t)$, bond holding $B(s^t)$, and investment $I(s^t)$, given sequences of interest rates $R(s^t)$, wages $W(s^t)$, capital rental rates $r(s^t)$, and initial bond and capital positions, in order to solve the following:

$$\max_{\{B(s^t),C(s^t),l(s^t),I(s^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[u(C(s^t), l(s^t))|s_0\right] \tag{1}$$

subject to

$$C(s^t) \leq W(s^t)l(s^t) + r(s^t)K(s^{t-1}) + B(s^{t-1})R(s^{t-1}) + T(s^t) - I(s^t) - B(s^t) - \Psi(\bullet) \tag{2}$$

$$I(s^t) = K(s^t) - (1-\delta)K(s^{t-1}) + \Phi(\bullet) \tag{3}$$

where $E\left[\bullet|s_0\right]$ is the expectation over history s^t, conditional on the information at $t = 0$; $0 < \beta < 1$ is the constant discount factor; investment is subject to convex adjustment costs $\Phi(\bullet)$; and bond holdings are subject to the convex cost function $\Psi(\bullet)$. In every period the household receives a lump sum transfer $T(s^t)$ from the ownership of the representative financial intermediary and the firms. In this small open economy, the interest rate is completely

[6]Capital adjustment costs are particularly important in an open economy setup with an exogenous interest rate. Without them, moderate fluctuations in the interest rate generate implausible variations in investment. Bond holding costs are even more important in this literature because a fundamental indeterminacy arises between consumption and bond holdings. Schmitt-Grohé and Uribe (2003) discuss several alternatives to solve this issue. Bond holding costs can be thought to capture legal and bureaucratic costs related unusual levels of debt.

exogenous, and we use the following AR(1) process to model it:[7]

$$\ln\left(\frac{R(s^t)}{\bar{R}}\right) = \rho_R \ln\left(\frac{R(s^{t-1})}{\bar{R}}\right) + \sigma_R \epsilon_{R,t} \quad \text{where} \quad \epsilon_{R,t} \overset{iid}{\sim} N(0,1), \tag{4}$$

where $\bar{R}$ is the long-run interest rate in the economy. As shown in the sequences of budget constraints defined by equation (2), the price of consumption is set to unity because we use the final good as the numeraire. The problem also requires transversality conditions on capital and bond holdings.

We modify Greenwood et al. (1988) preferences (GHH) to allow for a balanced growth path equilibrium. Note that, in our setup, because aggregate labor productivity ($A(s^t)$) grows at an endogenous rate, the scaling of labor disutility is time-variant.[8] We follow Neumeyer and Perri (2005) when choosing the functional forms for Ψ and Φ:

$$u(C(s^t), l(s^t)) = \frac{1}{1-\gamma}\left(C(s^t) - \Theta A(s^t)\left(l(s^t)\right)^{\chi}\right)^{1-\gamma} \tag{5}$$

$$\Psi(B(s^t), Y(s^t)) = \frac{\psi}{2} Y(s^t)\left(\frac{B(s^t)}{Y(s^t)} - \bar{b}\,(1+\bar{g})\right)^2 \tag{6}$$

$$\Phi(K(s^{t-1}), K(s^t)) = \frac{\phi}{2} K(s^{t-1})\left[\frac{K(s^t)}{K(s^{t-1})} - (1+\bar{g})\right]^2. \tag{7}$$

where $\Theta > 0$ is the labor dis-utility level, $\chi > 1$ determines the Frisch elasticity of labor supply $\left(\frac{1}{\chi-1}\right)$, γ is the utility curvature, and $\phi > 0$ and $\psi > 0$ determine the convexity of the cost functions. Note that, because $\bar{b}$ is the long-run household debt-output ratio and $\bar{g}$ is the long-run growth of the economy, the household pays neither adjustment nor bond holding costs along the balanced growth path. Because the household is the ultimate owner of firms, we need to define the stochastic discount factor of the household ($m(s^t, s_{t+1})$) in order to characterize the value of a firm:

$$m(s^t, s_{t+1}) = \beta \frac{\dfrac{\partial u(C(s^t, s_{t+1}), l(s^t, s_{t+1}))}{\partial C(s^t, s_{t+1})}}{\dfrac{\partial u(C(s^t), l(s^t))}{\partial C(s^t)}}.$$

[7] Neumeyer and Perri (2005) use two uncorrelated autoregressive processes: one for the spread and one for the international interest rate. Uribe and Yue (2006) use a VAR to estimate the determinants of the domestic interest rate, and then feed it into their model. The purpose of this paper is not to decompose the effects of interest rate shocks into parts due to world interest rates and spreads. We therefore use a single stochastic process.

[8] The usual economic intuition used to justify the scaling of labor disutility by labor productivity is that the opportunity cost of labor consists mostly of home production. Therefore, if nonmarket labor productivity grows at the same rate as market labor productivity, the disutility of labor must be scaled by it. Benhabib et al. (1991) study how home production shapes participation in the formal labor market and how that intuition can be represented by the preferences used in this model.

3.2 Final Good Producer

There is a representative final good producer that combines intermediate inputs ($\{X_j(s^t)\}_{j\in[0,\Lambda]}$), indexed by $j \in [0,\Lambda]$, with capital ($K^D(s^t)$), to produce the only final good of this economy ($Y(s^t)$). The parameter $\Lambda > 0$ determines the mass of varieties in the economy and is time invariant. The constant return to scale production function is given by

$$\ln Y(s^t) = z(s^t) + \frac{\alpha}{\Lambda}\int_0^\Lambda \ln X_j(s^t)dj + (1-\alpha)\ln K^D(s^t), \tag{8}$$

where $z(s^t)$ is the exogenous component of aggregate productivity and is characterized by the following AR(1) process:

$$z(s^t) = \rho_z z(s^{t-1}) + \sigma_z \epsilon_z(s^t) \quad \epsilon_z(s^t) \overset{iid}{\sim} N(0,1). \tag{9}$$

Equation (8) is an extension of a standard unit elastic production function where α determines the production share of the mass Λ of intermediate varieties. In particular, given input prices ($p_j(s^t)$) and the rental rate of capital ($r(s^t)$), the final good producer demands intermediate goods and capital in every period in order to solve

$$\max_{\{X_j(s^t)\}_{j\in[0,1]},K^D(s^t)} \left\{ Y(s^t) - \int_0^1 X_j(s^t)p_j(s^t)dj - K^D(s^t)r(s^t) \right\}. \tag{10}$$

The solution to (10) is characterized by the following set of demands for varieties and capital:

$$X_j(s^t) = \frac{\frac{\alpha}{\Lambda}Y(s^t)}{p_j(s^t)} \quad \forall j \quad , \tag{11}$$

$$K^D(s^t) = \frac{(1-\alpha)Y(s^t)}{r(s^t)}. \tag{12}$$

Because of the unit elastic demand, a monopolist in variety j facing the demand in equation (11) would choose $p_j(s^t) \to \infty$ and, hence, $X_j(s^t) \to 0$. Only the existence of a potential competitor can force the intermediate producer to set a finite price in a given product line. The next subsection introduces Bertrand monopolistic competition within each variety, providing a rationale for limit pricing.

3.3 Intermediate Goods Sector

There is a measure Λ of intermediate goods indexed by j. Each intermediate good is owned by a firm, and a firm can own several intermediate goods. The measure of firms is denoted by $\Omega(s^t) \in (0,\Lambda]$, which is an endogenous object. Firms are indexed by f. A firm is defined by a collection of varieties $\mathbb{J}_f = \{j : j \text{ is owned by firm } f\}$. Each intermediate good is produced using labor ($l_j(s^t)$) as input. The production of variety j is given by

$$X_j(s^t) = l_j(s^t) q_j(s^t). \tag{13}$$

The efficiency of labor ($q_j(s^t)$) in the production of intermediate goods evolves with each technological improvement generated by successful innovations. Innovations are heterogeneous in their capacity to improve the existing technology. Drastic innovations are generated by type H firms and they improve the efficiency level by a factor of $1+\sigma^H$, while marginal innovations, performed by type L firms, generate improvements with a smaller factor $1+\sigma^L$. Firm types are determined at the entry stage and remain fixed thereafter. We can define the indicator functions $I_j^d(s^{t-1}, s_t)$ taking the value 1 if product line j receives an innovation of type $d \in \{L, H\}$ under $s^t = (s^{t-1}, s_t)$ and 0 otherwise. We can summarize the evolution of the productivity in product line j as follows:

$$q_j(s^t) = \left(1 + I_j^H(s^{t-1}, s_t) \cdot \sigma^H + I_j^L(s^{t-1}, s_t) \cdot \sigma^L\right) \cdot q_j(s^{t-1}). \tag{14}$$

The most recent innovator in intermediate good j owns that product line. Hence, productivity in intermediate good j remains unchanged next period if, and only if, no innovation takes place in j. In this case, the same firm continues producing that intermediate good. In line with the endogenous technical change literature, we assume Bertrand monopolistic competition. This setup implies that the competitor with the lower marginal cost dominates the market by following a limit pricing rule—i.e., she sets her price ($p_j(s^t)$) equal to the marginal cost of the closest follower. We can denote the efficiency of the closest follower by $\tilde{q}_j(s^t)$. Then we have:

$$p_j(s^t) = \frac{W(s^t)}{\tilde{q}_j(s^t)} \left(1 + \underbrace{\eta(R(s^{t-1}) - 1)}_{\text{Cost wedge}}\right). \tag{15}$$

Labor input is subject to a working capital constraint. In particular, the intermediate good producer needs to hold a proportion $\eta > 0$ of the wage bill before production takes place. To do so, she borrows at the interest rate at the beginning of the period and pays back just after production takes place.[9] Because this is an intra-period loan, we follow Neumeyer and Perri (2005) and use the interest rate of the previous period.[10] Note that (14) implies that a leader with type d has productivity $q_j(s^t) = (1+\sigma^d) \cdot \tilde{q}_j(s^t)$. Then, using the demand for varieties of the final good producer from (11) we find the following expression for the

[9]This modeling assumption is standard in the open economy literature. It is mostly used to amplify interest rate shocks using a labor channel.

[10] Uribe and Yue (2006) show that this constraint can be summarized as a wedge in the cost of the input when interest rates are positive.

profits ($\Pi_j^d(s^t)$) of the leader in product line j with productivity advantage d:

$$\Pi_j^d(s^t) = X_j(s^t)\left(p_j(s^t) - \frac{W(s^t)}{q_j(s^t)}\left(1+\eta(R(s^{t-1})-1)\right)\right) = \frac{\alpha}{\Lambda}\frac{\sigma^d}{(1+\sigma^d)}Y(s^t). \tag{16}$$

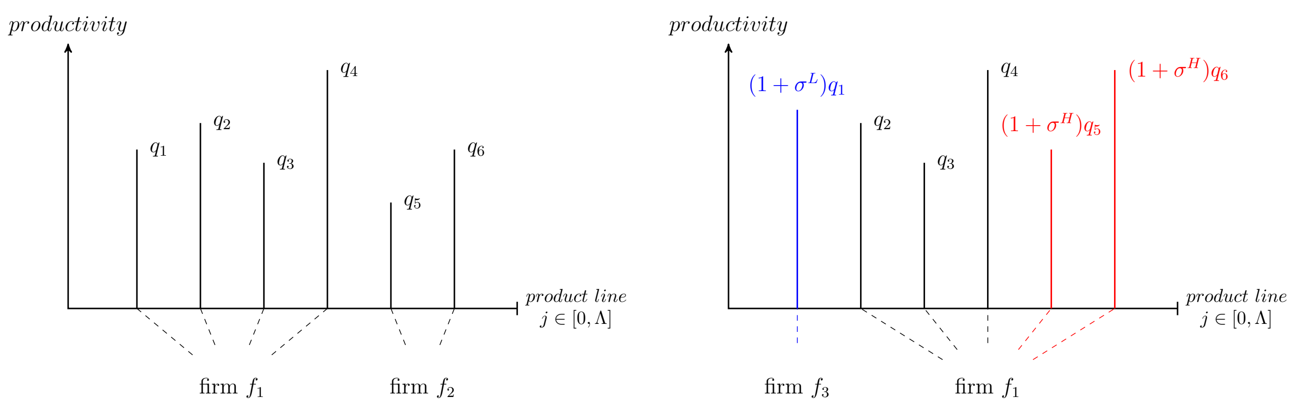
Note that profits only depend on the type of the current leader and the state of the economy, not on the productivity level of the variety ($q_j(s^t)$). Moreover, type H leaders always enjoy higher profits per product line than type L ones do. The labor employed in the production of each variety is also independent of the productivity level:

$$l_j^d(s^t) = \frac{\frac{\alpha}{\Lambda}Y(s^t)}{W(s^t)\left(1+\eta(R(s^{t-1})-1)\right)(1+\sigma^d)}. \tag{17}$$

Having characterized the pricing and productive labor decision of incumbent firms we turn to their optimal expansion decisions.

3.4 Innovation and Firm Dynamics

In this economy, innovations arise as a result of both the expansion decision of incumbents and the entry decision of firms. Therefore, the endogenous evolution of the productivity of each variety is determined by the firm dynamics of the economy. To fix ideas before going into the details of these two sources of innovation, Figure 2 illustrates how the expansion of incumbents and the entry of new firms governs both the number of varieties owned by a firm and the evolution of the productivity of each variety.



(a) Intermediate Good Firms

(b) Innovation and Evolution of Firms

Figure 2: Evolution of firm varieties and their productivity levels

Figure 2a shows six product lines with different productivity levels and two firms. Firm f_1 owns four product lines, while firm f_2 owns only two. Note that initially firm f_2 produces variety 6 at the productivity level at which f_1 produces variety 1. Figure 2b portrays the same varieties in the following period. Three elements are important to understand the dynamics of the model. First, firm f_1 has successfully innovated in the two products formerly owned by firm f_2, forcing f_2 to exit the market. Second, firm f_3 is a new entrant that has performed an innovation on variety 1. Therefore, firm f_1 has lost her dominance over the first variety. Nevertheless, by acquiring two new varieties, firm f_1 has expanded from five to six varieties on net. Third, note that f_1 is a high-type firm whereas f_3 is of low type. This implies that variety 1 and variety 6 no longer have the same productivity level as they were subject to improvements of different scale. Next we characterize innovation by incumbents and then we study innovation by entrants.

Innovation by Incumbent Firms: Expansion and Contraction

A type d firm owning n product lines can engage in innovation to acquire technological leadership over other intermediate varieties by hiring labor. In particular, a type d firm with n product lines hires a total of $l^d_{r(tot)}(s^t, n)$ workers for expansion purposes. We assume that a firm acquires new product lines according to a binomial process with success probability $\iota^d(s^t, n)$ and n trials, where $\iota^d(s^t, n)$ is given by

$$\iota^d(s^t, n) = \left(\frac{l^d_{r(tot)}(s^t, n)}{\varphi n}\right)^{\frac{1}{\xi}}, \quad \text{where} \quad \xi > 1 \quad \text{and} \quad \varphi > 0. \tag{18}$$

This setup can be interpreted as ideas for improving products coming from the existing varieties of the firm. In this sense, for every product line that a firm owns, a potential new application or a spinoff product arises with probability $\iota^d(s^t, n)$. Note that when a firm with n product lines hires $l^d_{r(tot)}(s^t, n)$ workers, it generates in expectation $\mathbb{N}^d \equiv n \cdot \iota^d(s^t, n)$ new varieties. Therefore,

$$\mathbb{N}^d = \left(\frac{1}{\varphi}\right)^{\frac{1}{\xi}} \left(l_{r(tot)}(s^t, n)\right)^{\frac{1}{\xi}} n^{1-\frac{1}{\xi}}. \tag{19}$$

Note that this Cobb-Douglas expression is isomorphic to the one used in the Klette and Kortum (2004) framework. In their continuous time model they use Poisson arrival rates to avoid multiple acquisitions or losses of products in an instant. Intuitively, the continuous time limit of a binomial process is a Poisson process. However, multiple acquisitions cannot be avoided when working in discrete time. Therefore, this paper proposes a discrete time mapping of the continuous time endogenous technical change literature with firm dynamics that can handle multiple acquisitions of varieties. This mapping is the key to introducing firm dynamics and endogenous technical change into a fully stochastic business cycle model in a tractable way. The cost of generating in expectation $\mathbb{N}$ new varieties for a firm with n

varieties is given by

$$cost(\mathbb{N}) = \varphi \frac{W(s^t)\mathbb{N}^{\xi}}{n^{\xi-1}} \left(1 + \eta(R(s^{t-1}) - 1)\right) \quad \text{for} \quad \mathbb{N} \leq n. \tag{20}$$

Intuitively, the more product lines a firm has, the cheaper it is to acquire new products, and the higher the wage or the interest rate in the economy is, the more costly it is to acquire new products.

Denote the endogenous replacement probability of this economy by $\Delta(s^t)$. Because replacement is undirected, every product line faces the same type-independent probability $\Delta(s^t)$ of receiving an innovation and being dominated by a new firm starting next period. Define $\mathbb{P}(k, n, p)$ as the probability of observing k events in a binomial process with n trials and success probability p:

$$\mathbb{P}\left(k, n, p\right) = \binom{n}{k} \left(p\right)^k \left(1 - p\right)^{n-k}.$$

Define the productive value of a firm as the present discounted value of the stream of profits generated by its current set of n product lines with technological leadership d as

$$Q^d(s^t, n) = n \cdot \Pi^d(s^t) + \mathbb{E}\left[m(s^{t+1}) \cdot \sum_{k=0}^{n} \left[\mathbb{P}\left(k, n, \Delta(s^t)\right) Q^d(s^{t+1}, n-k)\right] |s^t\right]$$

where $\mathbb{E}\left[\bullet|s^t\right]$ denotes the conditional expectation over every possible s_{t+1} event after history s^t. The first component captures the profits of its n product lines at time t and the second component reflects the expectation over k of the discounted productive value of having $n-k$ product lines at time $t+1$.

We can also define the innovation value of a firm owning n product lines with technological leadership d as

$$\begin{aligned} U^d(s^t, n) = \max_{\iota(s^t)} \Bigg\{ & -n \cdot W(s^t) \cdot \left(1 + \eta(R(s^{t-1}) - 1)\right) \cdot \varphi \cdot \iota(s^t)^{\xi} \\ & + \mathbb{E}\left[m(s^t, s_{t+1}) \cdot \left(\sum_{k=0}^{n} \left[\mathbb{P}\left(k, n, \iota^d(s^t, n)\right) \left(Q^d(s^{t+1}, n+k) - Q^d(s^{t+1}, n)\right)\right] \right.\right. \\ & \left.\left. + \sum_{\tilde{k}=0}^{n} \mathbb{P}\left(\tilde{k}, n, \Delta(s^t)\right) \left[\sum_{k=0}^{n} \left[\mathbb{P}\left(k, n, \iota^d(s^t, n)\right) U^d(s^{t+1}, n - \tilde{k} + k)\right]\right] |s^t \right)\right] \Bigg\}. \end{aligned} \tag{21}$$

The first component reflects the innovation cost when generating an arrival rate $\iota^d(s^t, n)$. The summation in the second line represents the expectation over k of the increase in the discounted productive value of having $n+k$ product lines at time $t+1$ instead of just n. The third component captures the expected innovation value in the next period after losing $\tilde{k}$ products and winning k products. Note that because there is a continuum of firms, each firm takes the replacement process as given. Therefore, the binomial process governing innovation and the binomial process governing the destruction of product lines are independent when characterizing the expected innovative value.

Using $Q^d(s^t, n)$ and $U^d(s^t, n)$ we can characterize the value of a type d firm with n product lines by

$$V^d(s^t, n) = Q^d(s^t, n) + U^d(s^t, n).$$

To solve for the innovation rate, we guess and verify that

$$Q^d(s^t, n) = n \cdot \bar{Q}^d(s^t) \quad \text{and} \quad U^d(s^t, n) = n \cdot \bar{U}^d(s^t),$$

where $\bar{Q}(s^t)$ and $\bar{U}(s^t)$ are independent of n. Guessing the former implies that the production value per product line is given by

$$\bar{Q}^d(s^t) = \Pi^d(s^t) + \mathbb{E}\left[m(s^{t+1})\left(1 - \Delta(s^t)\right)\bar{Q}^d(s^{t+1})|s^t\right],$$

and the innovation value per product line can be written as

$$\begin{aligned}\bar{U}^d(s^t) = \max_{\iota^d(s^t,n)} &\left\{-W(s^t)\left(1 + \eta(R(s^{t-1}) - 1)\right) \cdot \varphi \cdot \iota^d(s^t, n)^\xi\right. \\ &\left.+\mathbb{E}\left[m(s^{t+1})\left[\iota(s^t)\bar{Q}^d(s^{t+1}) + \left(1 + \iota^d(s^t, n) - \Delta(s^t)\right)\bar{U}^d(s^{t+1})\right]|s^t\right]\right\}.\end{aligned}$$

The optimal innovation rate is given by

$$\iota^d(s^t, n) = \left(\frac{\mathbb{E}\left[m(s^{t+1})\bar{V}^d(s^{t+1})|s^t\right]}{\varphi\xi W(s^t)\left(1 + \eta(R(s^{t-1}) - 1)\right)}\right)^{\frac{1}{\xi-1}} = \iota^d(s^t). \tag{22}$$

Note that innovation intensity is increasing in the value of new varieties and decreasing in the wage and interest rate. The optimal number of research workers per product line is given by:

$$l_r^d(s^t) \equiv \frac{l_{r(tot)}(s^t, n)}{n} = \varphi\iota^d(s^t)^\xi \tag{23}$$

Thus, the value of a product line is recursively defined by

$$\begin{aligned}\bar{V}^d(s^t) = \Pi^d(s^t) &- W(s^t)\left(1 + \eta(R(s^{t-1}) - 1)\right)\varphi\iota^d(s^t)^\xi \\ &+ \mathbb{E}\left[m(s^{t+1})\left[\left(1 - \Delta(s^t) + \iota^d(s^t)\right)\bar{V}^d(s^{t+1})\right]|s^t\right].\end{aligned} \tag{24}$$

Finally, it is straightforward to verify that $Q^d(s^t, n) = n \cdot \bar{Q}^d(s^t)$ and $U^d(s^t, n) = n \cdot \bar{U}^d(s^t)$ is a solution; therefore, the linearity in the number of product lines holds. Note that $\bar{V}^d(s^t)$ is the value of a type d firm with one product line and $\bar{V}^H(s^t) > \bar{V}^L(s^t)$. Conveniently, *ex-post* firm heterogeneity can be summarized by $d \in \{L, H\}$ because every type d firm charges the same price, hires the same number of workers, earns the same profits, and innovates at the same rate per product line. Therefore, there is no need to keep track of the distribution of labor productivity across product lines. Moreover, the innovation rate is also independent of n, but type H firms innovate and expand at a higher rate than type L firms do.

Innovation by Entrants: Financial Intermediary and Selection

The entry of new firms is determined by the funding of projects every period. A continuum of identical risk-neutral financial intermediaries buy from the household a unit mass of heterogeneous projects every period. Because of perfect competition between financial intermediaries, the price of the portfolio is given by the expected profits arising from managing it. The portfolio is a continuum of projects indexed by h and uniformly spread on the unit interval ($h \in [0, 1]$). The fixed cost of starting (enacting) a project is κ units of labor. Conditional on paying the fixed cost, a project generates a new firm with one variety. Projects are heterogeneous in their expected step size; every project has an idiosyncratic probability $P^H(h) = h^\nu$ ($\nu > 0$) of generating a type H firm characterized by step size $\sigma^H > \sigma^L$. The higher the index h, the more likely that project h will generate a type-H firm.[11] In this sense, h is more than an index; it is a ranking among projects based on their idiosyncratic $P^H(h)$. Note that ν governs the scarcity of good ideas in this economy. Therefore, the implied probability distribution of P^H is given by

$$f(P^H) = \frac{1}{\nu}\left(\frac{1}{P^H}\right)^{1-\frac{1}{\nu}}.$$

The mean of this distribution reflects the expected proportion of type H entrants when projects are enacted randomly. In particular, the fraction of high-type firms when enacting a set of projects randomly is given by $\underline{\tilde{\mu}} = \frac{1}{\nu+1}$. Interestingly, the skewness of $f(P^H)$ is

$$Sk(\nu) = \frac{2(\nu - 1)\sqrt{1 + 2\nu}}{1 + 3\nu}.$$

$Sk(\nu)$ is fully determined by ν, and it is positive and increasing for every $\nu > 1$. A positive skewness indicates that the left tail concentrates most of the probability density. This means that only a few ideas have strong chances of generating drastic improvements in productivity. Thus, ν summarizes the underlying scarcity of promising ideas in the economy.

[11] Heterogeneity and scarcity of ideas is one explanation for the high skewness in firm-level variables. See, for instance, Scherer (1998) and Silverberg and Verspagen (2007).

Because projects are heterogeneous and good ideas are scarce, selection plays a critical role in this economy. The representative financial intermediary performs that task. In particular, it borrows funds to cover working capital needs and selects projects to fund according to their expected present value. Because $\bar{V}^H(s^t) > \bar{V}^L(s^t)$, the representative financial intermediary strictly prefers to enact projects with higher h. Therefore, the optimal strategy for a financial intermediary financing $M(s^t)$ projects at time t is to set a cutoff $\bar{h}(s^t) = 1 - M(s^t)$, and to enact projects only with $h \geq \bar{h}(s^t)$.[12] When the financial intermediary selects a mass $M(s^t)$ of projects, the proportion $\tilde{\mu}(\bar{h}(s^t))$ of high-type firms in the enacted projects is given by

$$\tilde{\mu}(M(s^t)) = \frac{1}{M(s^t)} \int_{1-M(s^t)}^{1} P^H(h)\, dh = \underbrace{\frac{1}{\nu+1}}_{\underline{\tilde{\mu}}} \times \underbrace{\left[\frac{1 - [1 - M(s^t)]^{\nu+1}}{M(s^t)} \right]}_{\geq 1}. \tag{25}$$

For any mass $(M(s^t))$, the fraction of high-type entrants $(\tilde{\mu}(M(s^t)))$ decreases with the scarcity of high type projects (ν). Moreover, in terms of the resulting composition, financial selection performs at least as well as random selection. Then, given $\left\{\bar{V}^H(s^t),\, \bar{V}^L(s^t),\, R(s^{t-1}))\right\}$, the representative financial intermediary chooses $M(s^t)$ in order to

$$\max_{M(s^t)\in(0,1)} \left\{ \underbrace{M(s^t)}_{\text{Cohort's mass}} \left[\underbrace{\mathbb{E}\left[m(s^{t+1}) \left\{ \tilde{\mu}(M(s^t))\bar{V}^H(s^{t+1}) + (1 - \tilde{\mu}(M(s^t))\bar{V}^L(s^{t+1}) \right\} | s^t \right]}_{\text{Cohort's expected value}} \right] \right.$$
$$\left. - \underbrace{M(s^t)\left(1 + \eta(R(s^{t-1}) - 1)\right) W(s^t)\kappa}_{\text{Total cost of enaction}} \right\}. \tag{26}$$

The bracketed term is the expected return of the portfolio with composition $\tilde{\mu}\left(M(s^t)\right)$. The intermediary needs to pay back the borrowed amount plus the interest. As equation (25) shows, the financial intermediary faces a trade-off between mass and composition of the enacted pool: A higher $M(s^t)$ increases the mass of new firms, but it also decreases the average value of the entrant cohort. If an interior solution $(M(s^t) \in (0,1))$ exists, it is unique and characterized by

$$M(s^t) = 1 - \left(\frac{(1 + \eta(R(s^{t-1}) - 1))\, W(s^t)\kappa - \mathbb{E}\left[m(s^{t+1})\bar{V}^L(s^{t+1}) | s^t \right]}{\mathbb{E}\left[m(s^{t+1})\left(\bar{V}^H(s^{t+1}) - \bar{V}^L(s^{t+1}) \right) | s^t \right]} \right)^{\frac{1}{\nu}}. \tag{27}$$

From a partial equilibrium perspective, the optimal mass $(M(s^t))$ decreases with the interest rate. Therefore, a higher interest rate implies a smaller cohort with a higher fraction of type-H firms.[13] The main partial equilibrium predictions of the model that motivate the

[12]Because the expected value is strictly increasing in the idiosyncratic probability of becoming an H-type firm and the enacting cost is fixed, the cutoff strategy is optimal and unique.

[13]Interestingly, equation (27) can also be the outcome of a different setup where each entrepreneur seeks

empirical analysis of the next section can be seen clearly in equation (27). We summarize these predictions in the following proposition:

Proposition 1. *Firms born under higher interest rates:*

1. *Belong to smaller cohorts,*
2. *are more profitable, and*
3. *have higher survival rates.*

It is interesting to note that none of the decisions of the firms or the financial intermediary depend on the distribution of productivity across varieties or the distribution of products across firms. In fact, because incumbents' decisions only depend on the relative productivity between the leader and the follower summarized by σ^d, and because the value functions are linear in the number of product lines, the only relevant state variable is the fraction of product lines dominated by type H firms. The next subsection characterizes the law of motion of that fraction and derives the main equation that links incumbent and entrant innovation to aggregate productivity.

3.5 Endogenous Replacement and Productivity Growth

To characterize the evolution of the fraction of products dominated by H-type firms, it is useful to specify the timing of the model. Figure 3 summarizes the flow of events in a given period of time.

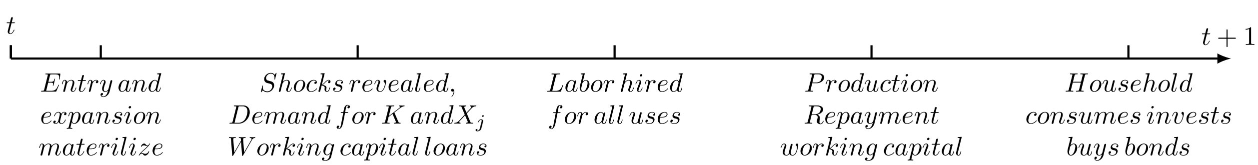


Figure 3: Timing Convention

Figure 3 divides a period into five consecutive intervals. First, the entry and the expansion chosen last period take place simultaneously. New entrants become single-product

funding for business plans independently. Under that decentralization, the marginal entrepreneur would be indifferent between borrowing and starting a firm or not enacting her project:

$$\left(1+\eta(R(s^{t-1})-1)\right)W(s^t)\kappa = \mathbb{E}\left[m(s^{t+1})\left(\bar{h}(s^t)^{\nu}\bar{V}^H(s^{t+1}) + (1-\bar{h}(s^t)^{\nu})\bar{V}^L(s^{t+1})\right)|s^t\right]. \tag{28}$$

Simple algebra shows that equation (28) and equation (27) are the same mathematical object. In this sense, the model is silent about the nature of the financial selection process. Selection can arise because the best entrepreneurs apply for funding or because the financial intermediary rejects bad applicants. Only direct data on application and rejection of credit can tell these stories apart. For instance, the second explanation implies large variations of the rejection to application ratio during crises, while the first one does not. Although these data are not available for Chile, Jiménez et al. (2014)'s loan-level data for Spain are consistent with large variations in the rejection to application ratio.

incumbents, and some of the former incumbents exit while others expand or contract. Second, the shocks are revealed, the final good producer demands capital and varieties, and the intermediate producers and the financial intermediary prepare to hire labor with intra-period borrowing in order to meet working capital requirements. Third, incumbents hire labor for production and expansion, and the financial intermediary hires labor to pay the entry cost of the selected projects. Fourth, production takes place and intra-period loans are repaid. Fifth, the representative household consumes, invests, repays last-period debt, and optimally chooses her bond holding for next period.

Having detailed the timing of the model we can characterize the endogenous replacement of varieties ($\Delta(s^t)$) and the fraction of product lines operated by type-H firms ($\mu(s^t)$) needed to close the model and solve for the endogenous productivity growth of the economy. Note that $\iota^H(s^t)$, $\iota^L(s^t)$, and $M(s^t)$ are determined in period t but materialize at the beginning of period $t+1$. Because the product space is continuous and because replacements by entrants and incumbents occur simultaneously, the same product line cannot be acquired by two firms in the same period and the probability of an incumbent innovating on its own product is zero. Therefore, we can define the aggregate replacement rate of the economy as

$$\Delta(s^t) \equiv \underbrace{\frac{M(s^t)}{\Lambda}}_{\text{Replacement by Entrants}} + \underbrace{\mu(s^t)\iota^H(s^t) + \left(1-\mu(s^t)\right)\iota^L(s^t)}_{\text{Replacement by Incumbents}}. \tag{29}$$

We can also derive the law of motion for the composition of product lines:

$$\mu(s^{t+1}) = \mu(s^t) + \underbrace{\frac{M(s^t)}{\Lambda}\left[\tilde{\mu}(M(s^t)) - \mu(s^t)\right]}_{\text{Changes due to entrants}} + \underbrace{\mu(s^t)\left(1-\mu(s^t)\right)\left(\iota^H(s^t) - \iota^L(s^t)\right)}_{\text{Changes due to incumbents}}. \tag{30}$$

A higher fraction of high types in the entrant cohort ($\tilde{\mu}(M(s^t))$) implies a higher fraction of product lines dominated by high-type incumbents. Also, larger gaps between the innovation rate of high and low types ($\iota^H(s^t) - \iota^L(s^t)$) trigger increases in the fraction of products dominated by H-type firms.

We can now derive an expression for the endogenous productivity process of this economy. Combining the production function from equation (8) and equation (13), and recognizing that intermediate labor used in a product line depends only on the step size of the incumbent that controls it, we obtain:

$$Y(s^t) = \exp\left(z(s^t)\right)\left(\underbrace{A(s^t)}_{\text{Endogenous Productivity}}\right)^{\alpha}\left[\left(l^H(s^t)\right)^{\mu(s^t)}\left(l^L(s^t)\right)^{1-\mu(s^t)}\right]^{\alpha}\left(K(s^{t-1})\right)^{1-\alpha} \tag{31}$$

where $A(s^t)$ is defined as

$$\ln(A(s^t)) \equiv \frac{1}{\Lambda}\int_0^{\Lambda} \ln q_j(s^t)dj.$$

$A(s^t)$ is endogenous, and we can characterize it using the evolution of firm level labor productivity in equation (14) together with the entry rate and the innovation decisions of incumbents. In particular, the growth rate of $A(s^t)$ is given by

$$\begin{aligned} 1+a(s_t) &\equiv \frac{A(s^t,s_{t+1})}{A(s^t)} \\ &= \underbrace{\left[\left(1+\sigma^H\right)^{\tilde{\mu}(s^t)}\left(1+\sigma^L\right)^{1-\tilde{\mu}(s^t)}\right]^{\frac{M(s^t)}{\Lambda}}}_{\text{Entrants}}\underbrace{\left(1+\sigma^H\right)^{\mu(s^t)\iota^H(s^t)}\left(1+\sigma^L\right)^{\left(1-\mu(s^t)\right)\iota^L(s^t)}}_{\text{Incumbents}}. \end{aligned} \tag{32}$$

Equation (32) is the core of the model; it shows that crises have permanent productivity effects because they distort the productivity accumulation process. Endogenous technical change provides a link between stationary fluctuations and productivity growth. The crisis affects productivity via the entry of new firms and innovation of incumbent firms. Note that the contribution of entrants to productivity growth boils down to a scaled geometric weighted average of the step sizes, where the weights are given by the fraction of each type in the entrant cohort (composition) and the scale is given by the fraction of varieties that the cohort improves (mass). The innovation of incumbent firms is driven by two forces. First, equation (22) shows how fluctuations of the value of products during the crisis trigger changes in the expansion efforts of incumbents ($\iota^d(s^t)$). Second, because today's entrants are tomorrow's incumbents, changes ni the composition of entrants have dynamic effects on the fraction of product lines dominated by H-type firms in future periods. This dynamic effect of the composition of the entrant cohort can be seen clearly in equation (30).

Note that the size distribution of firms and the productivity distribution are not needed to solve for the growth rate or any other macro aggregate. In particular, because firm's decisions are i) independent of the specific productivity level and ii) scaled linearly on their number of product lines, the only distribution needed is the type composition of product lines ownership, perfectly defined by $\mu^d(s^t)$. Therefore, from a pure macro perspective, this model has only one additional state variable compared with a traditional small open economy real business cycle model. Nevertheless, from a firm-level perspective, the model has a well-defined size distribution that can be compared to micro data. Recovering the unique path of firm dynamics associated with the macro dynamics is critical in order to assess the firm-level performance of the model. Appendix A explains how to calculate the steady-state size distribution and how to recover its stochastic evolution. It also defines an equilibrium, a balanced growth path, and presents the normalized dynamic system of equations that characterizes this economy. Before moving to the quantitative analysis, the next section provides empirical evidence supporting the mass and composition tradeoff that lies at the center of the model.

4 The Chilean Case: Fewer, but Better

This section explores Chilean microeconomic data to assess empirically the main mechanism of the model, i.e., the existence of a mass-composition tradeoff on the entry margin. We focus the analysis on Chile for three reasons. First, it is a small open economy with detailed macroeconomic data. Second, the violent sudden stop triggered by the Russian default provides an ideal natural experiment to test our mechanism. Third, we have access to detailed plant-level panel data that can be used to directly study firm entry. We first introduce the plant-level data set, and then we show that firms born in crisis are not just *fewer*, they are also *better*.

4.1 The Sudden Stop

In August 1998, the Russian government declared a moratorium on its debt obligations to foreign creditors. This default triggered a sudden and radical increase in the interest rates faced by emerging markets, including those in Latin America. Calvo and Talvi (2005) present a detailed analysis of the effect of the Russian default on the seven biggest economies of the region. One of the most successful economies of Latin America, Chile, also suffered the consequences of the Russian default. The real interest rate peaked in 1998:III, increasing 5 percentage points in a quarter. The interest rate spread, as reported by Calvo and Talvi (2005), increased from 120 basis points before the crisis to 390 basis points in October 1998, triggering a 47% decrease in cumulative external financial flows between 1998 and 2002. The macroeconomic consequences of a sudden stop in emerging markets have been widely studied, but the effects of the firm entry dynamics triggered by these episodes have not. From a Schumpeterian point of view, those changes in entry are harmful even in the long run, when the well-studied short-run effects have long vanished. This section presents empirical support for the composition effect, contributing to the empirical literature on the microeconomic consequences of a sudden stop.

4.2 Mass and Composition during a Sudden Stop

There was no change in the domestic fundamentals of Chile that could have caused or predicted an increase in the interest rate as sudden and substantial as the one observed in the data. The average annualized real GDP growth of Chile between 1990:IV and 1997:IV was 8.6%, its fiscal policy was steady and sober, and the monetary policy of its autonomous central bank was not expansionary. Moreover, as argued by Calvo et al. (2006), the generalized and synchronized nature of the increase in spreads charged in emerging markets also points to an exogenous and common origin for this episode. Thus, taking the Russian crisis as an exogenous shock, unrelated to Chilean fundamentals, and completely unforeseen by firms and authorities, we perform a *pseudo* natural experiment in order to test the most novel mechanism of the model: Cohorts born during the sudden stop window should be smaller but more profitable.

Chile's National Institute of Statistics (INE) performs a manufacturing census (ENIA) every year, collecting plant-level data from every unit with more than 10 employees.[14] The survey contains yearly plant information on sales, costs, value added, number of workers, energy consumption, and other variables. For the empirical analysis in this section, we use the information in the surveys between 1995 and 2007 to build a panel.[15] We take the first appearance in the data as the entry year and the last appearance as the exit date.[16] The sample contains $9,224$ plants and $56,665$ observations.[17]

We first calculate entry rates at year t at the industry level for each cohort, dividing the number of new plants in year t by the average of the total plants in years t and $t-1$. Table 8 in Appendix (B) presents two-year average entry rates for every industry in the sample. Figure 4 plots two-year average entry rates by industry for the two years preceding the crisis and the first two years of the sudden stop. Every industry below the 45^o line decreased its two-year average entry rate during the crisis.

For all industries but two (355 (rubber-based products) and 369 (other non-metallic products)), the average entry rate of 1998 through 1999 is lower than in 1996 through 1997. Moreover, Table 8 shows that, for practically every industry, entry rates remain low until $2002-03$. Entry dropped dramatically at the industry level during the Chilean sudden stop.

[14] Although firms can have multiple plants, in this paper we assume that every firm is a single plant. According to Pavcnik (2002) more than 90% of the Chilean manufacturing firms are single-plant firms.

[15] We restrict attention to this period because the questionnaire and the identification number of each firm are practically invariant.

[16] Note that a small firm might appear in the panel after passing the threshold of 10 employees, and it should not be counted as an entry. To minimize this issue, we focus on plants with more than 11 workers. The results are also robust to a threshold of 15 workers. Because of lack of entry in some industries, we restrict our attention to 20 of the 29 industries. For example, the tobacco industry is characterized by only one or two plants, and we observe positive entry in only two years.

[17] Appendix (B) shows the details of the data construction and a summary of the variables used in the analysis.

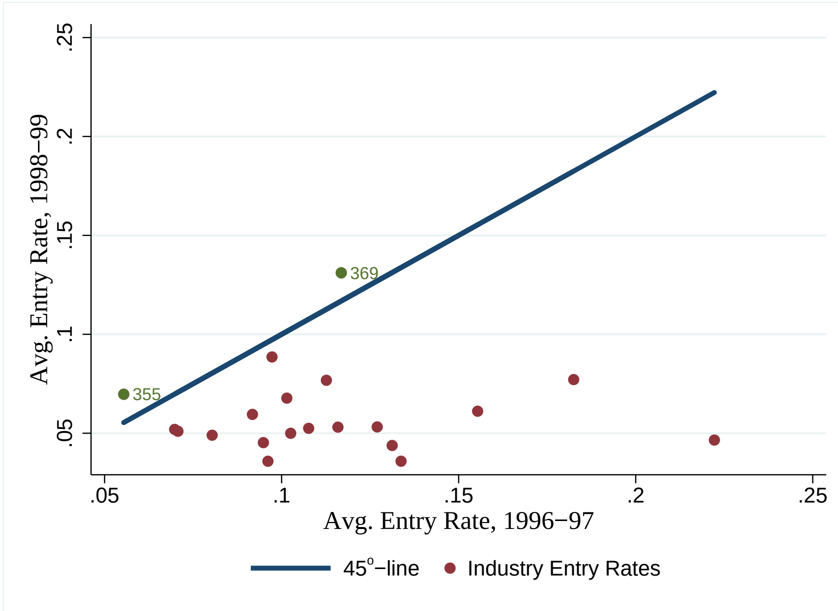


Figure 4: Mass (quantity)

The average percentage change in the entry rate is -45% between 1996 and 1997 and 1998 and 1999.

Although it is clear that *fewer* firms are born during the crisis, we still have to analyze whether they are *better*. In this sense, we want to show that firms born during the sudden stop are intrinsically more profitable. To capture the profitability of each plant every year, we build the following measure:

$$P_{i,t} = \frac{Revenue_{i,t} - Cost_{i,t}}{Revenue_{i,t}}.$$

We define a firm that is one standard deviation above the mean profitability of its industry in its first year of life (second observation) as a *superstar* entrant. The mean and standard deviation are calculated using every plant operating in a given year.[18] We estimate the probability of being a superstar firm using the following logit specification:

$$Pr(\text{Superstar} = 1|\text{age} = 1) = \frac{e^{x_i'\beta}}{1 + e^{x_i'\beta}} \quad \text{where} \quad x_i'\beta = \alpha + \alpha_j + \alpha_r + \beta \ln(L_{i,0}) + \gamma_{\text{cohort}} + u_{i,t}, \tag{33}$$

where α_j is an industry control, α_r is a geographical control, and $L_{i,0}$ uses workers at entry to control for size. In the baseline specification, the cohort coefficient indicates whether a firm was born during the sudden stop window. Table 1 presents the results for five alternative regressions.

The first regression compares cohorts born during the crisis (1998 to 2000) against

[18]In particular, we do not drop the firms born before 1995 from the sample to calculate these moments.

	Superstar at age 1			Superstar at age 0	Superstar at age 2	Superstar at age 1		
	$P_{i,t}$	$P_{i,t}$	$P_{i,t}$	$P_{i,t}$	$P_{i,t}$	$A_{i,t}$	$A_{i,t}$	$A_{i,t}$
main								
Crisis Born	0.540***			0.309***	0.312**	0.412***		
	(0.112)			(0.0901)	(0.124)	(0.120)		
During Crisis		0.697***					0.565***	
		(0.130)					(0.140)	
After Crisis		0.240*					0.234*	
		(0.126)					(0.133)	
Avg entry$_{j,0}$			-1.575*					-1.440
			(0.862)					(0.903)
$\ln(L_{i,0})$	0.222***	0.216***	0.209***	0.141***	0.153**	0.838***	0.833***	0.824***
	(0.0490)	(0.0496)	(0.0486)	(0.0439)	(0.0606)	(0.0532)	(0.0537)	(0.0533)
Ind. Control (α_j)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Region Control (α_r)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Constant	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	3197	3197	3197	4089	2618	3197	3197	3197

Logit regression on the probability of becoming a superstar firm. A superstar firm at age h is a firm that is one standard deviation above the mean profitability (productivity) of its industry, in its h^{th} year of life. The dependent variable in the first five specifications is calculated based on profitability of firm i in year t ($P_{i,t}$) while the last three regressions are based on firm-level productivity $A_{i,t}$ (TFPR). The "Crisis Born" takes the value 1 for firm i if i has started the business in a crisis year, and it measures the cohort effect on the probability of becoming a superstar firm. "During Crisis" and "After Crisis" dummies distinguish firms born during (from 1998 through 2000) and after crisis years (after 2000). "Avg entry$_{j,0}$" is the average entry rate in the specific industry, in which firm i operates, in the year of i's entry. In all regressions we include initial firms size ($\ln(L_{i,0})$) and controls for industry and region of operation. Standard errors are presented in parentheses (bootstrapped using 250 samples). $\{^{*}, ^{**}, ^{***}\}$ denote significance at $p < 0.10$, $p < 0.05$, $p < 0.01$, respectively.

Table 1: Probability of a superstar firm

every other cohort. Firms born during the crisis are statistically more likely to become superstars in their industries. In fact, evaluating the regression at the mean for the most populated region (central) and the most common two-digit industry code (31), we find that the probability of being a superstar is 21% for firms born during the episode, while the probability for a firm born outside this window is 13.4%. The second specification shows that allowing cohorts born before and after the episode to differ does not change the results. In line with the "fewer but better" hypothesis, the third specification shows that larger cohorts at the industry level are associated with a lower probability of being a superstar. The fourth and fifth specifications show that the results do not change when the probability of being a superstar is evaluated at the year of entry or two years after entering.[19]

We use profitability for our baseline analysis because in the model only the relative productivity between a leader and a follower is well defined and it has a direct mapping to the step size. Nevertheless, the main results are robust when we use the estimated firm level productivity instead of the profitability measure. The last three columns of Table 1 replicate the first three specifications using firm productivity instead of profitability. We calculate firm productivity applying the Wooldridge (2009) extension of Levinsohn and Petrin (2003)'s methodology. The industry factor elasticities are reported in Appendix B.2. Note that because of the lack of price variables, every empirical productivity measure in this paper is revenue TFP.[20] This last set of experiments show that our results are robust to using productivity instead of profitability.

Although this exercise using superstar firms is suggestive, the predictions of the model are stronger. The model predicts that firms born during crises are on average more profitable during their entire life, even after controlling for post-entry decisions. In this context, we explore both the continuous nature of the profitability variable and the panel dimension of the data. In general, we would like to estimate the following equation:

$$P_{i,t} = \alpha + \beta_1 X^1_{i,t} + \beta_2 X^2_{i,t} + \gamma_1 Z^1_i + \gamma_2 Z^2_i + \mu_i + u_{i,t} \tag{34}$$

where $X^1_{i,t}$ represents exogenous time-varying variables (e.g., vacancy index of the economy), $X^2_{i,t}$ refers to endogenous time-variant variables (e.g., number of workers), Z^1_i correspond to exogenous time-invariant variables (e.g., region of the country), and Z^2_i are endogenous time-invariant variables (e.g., workers in the entry year). Note that variables with a superscript 2 are endogenous in the sense that they are likely to be correlated with the unobserved fixed effect μ_i. The main challenge of this panel estimation is that the variable of interest, *being born in crisis*, is not only time-invariant, but also endogenous. On the one hand, coefficients

[19] If a cohort dummy is introduced year by year, beside the three crisis years, only firms born in 2006 have a significant coefficient (but of lower magnitude than the crisis years). Controlling by initial capital instead of initial workers also does not change the results. Results are available upon request.

[20] See Foster et al. (2016) for a useful discussion of revenue-based measures.

on time-invariant variables can be consistently and efficiently estimated by random effects regression, but the estimation is not consistent when the variable is also endogenous. On the other hand, fixed effects panel regression can consistently estimate every coefficient associated with the time-variant variables, but it cannot identify the coefficients of the time-invariant variables. In this situation, the Hausman and Taylor (1981) procedure delivers consistent and efficient estimators for every coefficient in equation (34).[21]

Table 2 presents the results for six different specifications.[22] In the first three regressions, the dependent variable is $P_{i,t}$. The only difference in the first three specifications is the coefficient of interest. In the first regression, we use a single dummy to determine whether the cohorts born in 1998 through 2000 perform better than every other cohort. In the second regression, we use two dummies in order to allow a differential effect for cohorts *pre-* and *post-* crisis. The third specification studies the effect of the three-digit industry entry rate at the moment of entry. This industry-level entry rate is a continuous variable common to every firm in the same industry born in the same year and is also time-invariant for a particular firm. Note that all the coefficients of interest are associated with time-invariant endogenous variables because better firms (with a higher unobserved fixed effect μ_i) are expected to enter in years of crisis. In the case of the third specification, when *fewer* firms enter, we expect them to be *better*. The next three specifications replicate the first set using the estimated firm productivity as the dependent variable.

Back to our main question: Are those *fewer* firms born in crisis *better*? The first specification shows that firms born in crisis are significantly more profitable than firms born in normal times. In fact, after controlling for initial size, macroeconomic conditions, and post-entry decisions, firms born during the sudden stop have, on average, a profitability index 8.8 percentage points higher. This coefficient is robust to allowing post-crisis cohorts to differ from before-crisis cohorts (specification 2). Table 2 also shows the relative effect evaluated at the means, i.e., the predicted profitability of a firm born during normal times divided by the predicted profitability of a firm born during the crisis, minus one. The baseline regressions suggest that if we focus on a fictitious firm, setting every observable at its mean and changing only the period of entry, we find that being born in normal times implies 31% lower profitability. The third specification is more general in the sense that it aims to directly unveil a mass-composition tradeoff at the entry level. The coefficient suggests that firms born in smaller cohorts have a permanently positive effect in their profitability

[21] See Appendix B.7 for a succinct explanation. Intuitively, this procedure aims to remove the endogenous component from the original regression in order to meet the main assumption of random effects. More details on this method can be found in Wooldridge (2010), Chapter 11. STATA software has built-in routines for both procedures; see Schaffer and Stillman (2011). After every estimation, we perform the Sargan-Hansen test to assess the validity of the instrumental variables procedure at the core of Hausman and Taylor (1981). The null hypothesis is that the instruments are valid, so the higher the p-value, the better.

[22] A detailed set of variables used and the regression results with alternative specifications can be found in Appendix B.7.

	(1)	(2)	(3)	(4)	(5)	(6)
	$\mathrm{P}_{i,t}$	$\mathrm{P}_{i,t}$	$\mathrm{P}_{i,t}$	$\mathrm{A}_{i,t}$	$\mathrm{A}_{i,t}$	$\mathrm{A}_{i,t}$
Crisis Born	0.0888*			0.591**		
	(0.0466)			(0.236)		
During Crisis		0.0866*			0.554***	
		(0.0466)			(0.199)	
After Crisis		0.0104			0.196*	
		(0.0232)			(0.110)	
Avg entry$_{j,0}$			-0.689**			-5.847***
			(0.325)			(1.547)
Ind. Control	Yes	Yes	Yes	Yes	Yes	Yes
Region Control	Yes	Yes	Yes	Yes	Yes	Yes
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Observations	16827	16827	16827	16814	16814	16814
Relative effect at means	-31.58	-31.61		-9.959	-9.583	
Sargan-Hansen (p)	0.416	0.201	0.109	0.134	0.126	0.106

The dependent variable in the first three specifications is calculated based on profitability $P_{i,t}$ of firm i in year t while the last three regressions are based on firm-level productivity $A_{i,t}$ (TFPR). The "Crisis Born" takes the value 1 for firm i if i has started the business in a crisis year, and it measures the cohort effect on the probability of becoming a superstar firm. "During Crisis" and "After Crisis" dummies distinguish firms born during (from 1998 through 2000) and after crisis years (after 2000). "Avg entry$_{j,0}$" is the average entry rate in the specific industry, in which firm i operates, in the year of i's entry. In all regressions we include controls for industry and region of operation (for the description of additional explanatory variables, see Appendix B.7). Relative effects are calculated at the mean values of the variables for the region and the industry with most observations. The statistic measures the percentage deviation in the dependent variable calculated at the means relative to the average cohort born during crisis years (a negative value implies a larger value of the dependent variable for the crisis cohort). Sargan-Hansen statistic tests the validity of overidentifying restrictions in Hausman and Taylor procedure. The null hypothesis is that the restrictions are valid. Standard errors are presented in parentheses (bootstrapped using 250 samples and clustered by firm). $\{^{*}, ^{**}, ^{***}\}$ denote significance at $p < 0.10$, $p < 0.05$, $p < 0.01$, respectively.

Table 2: Hausman and Taylor

measure. In particular, every extra percentage point in entry decreases the profitability of the firm 0.69%. Note that the main predictions are robust to the alternative specifications when using firm productivity as our performance measure. In fact, being born in normal times implies 9% lower productivity.

One caveat related to *post-entry* selection can be added to the preceding results. If firms born during crisis are more likely to die early, then those cohorts would seem more profitable after that initial selection. Moreover, the model predicts that firms born during the crisis episode should be more resilient. Appendix B.8 estimates a proportional hazard model in order to evaluate this concern. The main empirical question in the Appendix is whether firms born during the crisis window are more likely to exit. The answer is negative; if anything, firms born during the crisis have lower hazard rates in each of their first six years of life.

in summary, the Chilean sudden stop had strong macroeconomic consequences. At the firm level, the effect is relatively more complex. Cohorts born during the crisis and in its aftermath are 45% smaller; nevertheless, firms born in normal times are at least 30% less profitable and 10% less productive after controlling for observables. Hence, taking the average quality of the entrant cohort as a reference to evaluate the losses from forgone entry is extremely misleading, as unborn firms are substantially *worse* than the observed ones. As these unborn firms are often the excuse for policy interventions, such as indiscriminate government credit, it is crucial to correctly assess the economic cost of that forgone entry. For this reason, we proceed to calibrate our model and quantify the long-run cost imposed by a sudden stop.

5 Quantitative Analysis

In this section we explore the quantitative behavior of the model economy. First we calibrate the model to the Chilean plant- and macro-level data. Second, we test the calibrated model using non-targeted firm dynamics and business cycle moments. The calibrated model delivers firm dynamics and macroeconomic aggregates that closely mimic their data counterparts. Third, we use the model to disentangle the permanent productivity effect of the Chilean sudden stop from mean-reverting stationary deviations.

5.1 Calibration

Externally Calibrated Parameters

The 22 parameters of the model are calibrated to Chilean data on a quarterly basis. A first group of 12 parameters is externally calibrated according to the literature and features of the Chilean data. Table 3 presents the values for every externally calibrated parameter.

The capital share $(1-\alpha)$, the intertemporal elasticity of substitution $(1/\gamma)$, and the Frisch elasticity of labor supply $(1/(\chi-1))$ are set in accordance with Mendoza (1991). The curvature of the expansion cost function of incumbents (χ) is taken from Akcigit and Kerr (2010) and their discussion of the empirical literature on endogenous technical change. We set the persistence of the stationary TFP process (ρ_z) to the value used by Neumeyer and Perri (2005) for Argentina and estimated by Aguiar and Gopinath (2007) for the stationary component of TFP in Mexico. The depreciation rate of capital (δ) is set at 8% annually, consistent with the study by Bergoeing et al. (2002) of the Chilean economy. The parameter governing the debt adjustment cost (ψ) is set to a low value that assures stationary behavior. We use interest payments and production costs from the Chilean micro data together with the series for the country interest rate to calculate the working capital requirement (η). Our calculations imply that 60% of the wage bill has to be kept as working capital.[23] We set $\bar{b}$ to match the average quarterly debt-to-GDP ratio of Chile. We follow Uribe and Yue (2006) and use the Chilean EMBI spreads along with U.S. data on inflation and interest rates to build the international real interest rates series and estimate the parameters of the interest rate process for the period from $1996 : I$ through $2011 : II$.[24]

Internally Calibrated Parameters

A second group of nine parameters is calibrated to salient features of both macroeconomic and firm-level data. The first seven parameters in Table 4 are calibrated to the balanced growth path of the model. Although every long-run moment is related to the first seven parameters, we can point to some strong relationships between targets and parameters that identify the model. The mass of varieties (Λ) is used to normalize the mass of firms in the economy to unity. The disutility of labor (Θ) is set to match a long-run labor supply of 33%. The average cost of starting a firm (κ) is related to the long-run entry rate; we set that target to a level consistent with the average entry of the pre-crisis years in our sample.[25]

[23] Appendix B.4 shows how this number is calculated. Note that it is substantially lower than the 100% used by Neumeyer and Perri (2005) and the 125% used by Uribe and Yue (2006). Appendix C.4 explores other values for robustness purposes.

[24] Appendix B.3 describes how the original EMBI series is extrapolated for the earlier period and how the interest rate data are built.

[25] To be consistent with the annual frequency of ENIA we measure entry annually in the model.

Parameter	Symbol	Value	Source
Capital share	$1-\alpha$	0.32	Mendoza (1991)
Elasticity of substitution $(1/\gamma)$	γ	2	Mendoza (1991)
Frisch elasticity $(1/(\chi-1))$	χ	1.455	Mendoza (1991)
Expansion cost curvature	ξ	2	Akcigit and Kerr (2015)
AC stationary TFP	ρ_z	0.95	Neumeyer and Perri (2005)
Depreciation rate	δ	1.94%	Bergoeing et al (2002)
Debt Adjustment Cost	ψ	10^{-4}	Stationarity
Working capital	η	0.6	Data
Long-run debt to GDP ratio	$\bar{b}$	$4*(-0.44)$	Data
Long-run interest rate	$\bar{R}$	1.015	Data
AC interest rate	ρ_r	0.88	Data
Stdev interest rate	σ_r	0.24%	Data

Table 3: Externally Calibrated Parameters

In order to understand the identification of the parameters governing heterogeneity and firm dynamics, note that, without heterogeneity and in continuous time, this economy collapses to a version of Klette and Kortum (2004) where the analytic size distribution is logarithmic.[26] Recall that in a logarithmic distribution, one parameter governs all the moments of the distribution. Introducing heterogeneity allows the model to target more than one moment of the size distribution. Intuitively, the size distribution of the model with two types can be thought as the combination of two logarithmic distributions. Thus, there are three degrees of freedom: the two parameters governing the distributions and the weights of each distribution.[27] Given these degrees of freedom, we identify the parameter governing the scarcity of good projects in the economy (ν) with the standard deviation of the firm's size distribution.[28] Because most of the firms are small ν governs the composition of low and high-type firms at small sizes. This role of ν is the key determinant of the standard deviation of the size distribution. The productivity improvement that characterizes H-type firms (σ^H) determines the annual growth rate of the economy. Because type H firms expand faster and live longer, their step size is key for the long-run growth of the economy. The step size of L-type firms (σ^L) is related to the mean of size distribution. In fact, given σ^H, we adjust σ^L to match the average number of workers per firm in the data. The scale parameter of the cost of expanding (φ) determines the labor share of the largest 10% of firms, as φ governs the shape of the right tail of the size distribution.

The last two calibrated parameters are set to match business cycle moments of the Chilean economy. The standard deviation of the aggregate productivity disturbance (σ_z) and the parameter governing the capital adjustment cost (ϕ) are set to match the volatility of the HP filtered series of output and investment between $1996 : I$ and $2011 : II$, respectively. The model is able to match the targets successfully. Table 4 presents the performance of the model regarding the nine targets and the corresponding values for each parameter.[29] Finally, the patience parameter (β) is set so that there is no bond holding cost paid along the balanced growth path.[30]

The scarcity of good ideas implies that, under random selection, only $\underline{\tilde{\mu}} = 2.1\%$ of ideas would generate a high-type firm. Nevertheless, because the financial intermediary sets its credit standards to accept only the top 3% of projects, financial selection implies an ex-post fraction of high types of $\tilde{\mu} = 53\%$ at entry. The value of Λ implies that the average firm has seven products. Among the unit mass of firms, 64% are high-type firms and they

[26]See Appendix A for details.

[27]The first two are related to the endogenous expansion rates of firms, and the third one is related to the endogenous share of high-type firms in the economy.

[28]The size distribution is measured in terms of workers per firm and normalized such that the smallest firm has size 1. The same normalization is applied to the model.

[29]To calibrate we minimize the sum of the absolute value of the deviations between model and targets.

[30]From the bond holding first-order condition we obtain $\beta = \frac{(1+a)^{\gamma}}{R}$.

Parameter	Symbol	Value	Main identification	Target	Model
Mass of Varieties	λ	6.82	Mass of Firms	1.00	0.98
Labor disutility level	Θ	30.32%	Working time	33.00%	34.01%
Entry Cost	κ	5.15%	Entry rate	11.30%	11.00%
Scarcity	ν	46.82	Stdev of firm employment distribution	12.76	12.47
Step Size H	σ^H	6.80%	Annual GDP Growth	2.50%	2.56%
Step Size L	σ^L	6.58%	Mean of firm employment distribution	6.69	6.94
Expansion Cost scale	φ	30.14%	Share of employment of 10% largest firms	50.00%	51.32%
Stdev TFP	σ_z	0.79%	Quarterly output volatility (HP filtered)	1.98%	1.99%
Capital adjustment cost	ϕ	8.11	Quarterly investment volatility (HP filtered)	9.56%	9.62%

Table 4: Internally Calibrated Parameters

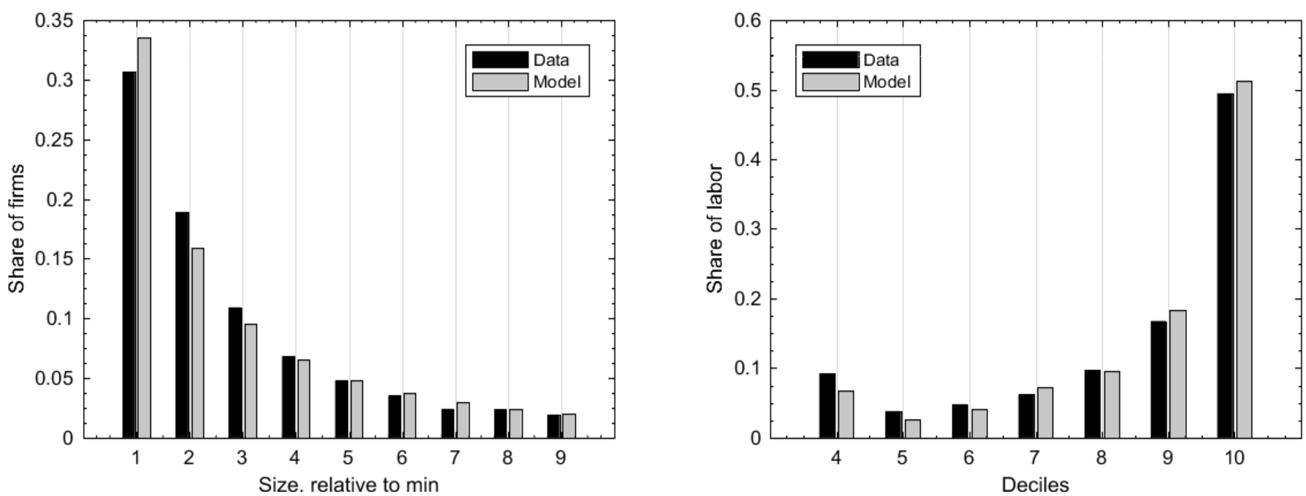
dominate $\mu = 84\%$ of the products. Although the two step sizes seem to be very similar, they imply extremely different firm dynamics. In fact, low-type firms expand at a rate $\iota^L = 8.3\%$ achieving an average size of only three product lines, while high-type firms expand at a rate $\iota^H = 9.4\%$ achieving an average size of nine product lines. Because every product line is lost with probability $\Delta = 9.6\%$, H-type firms, being larger on average, survive longer. The next subsection evaluates the performance of the model using non-targeted moments.

5.2 The Micro and Macro Performance of the Model

Before using the model to evaluate the productivity cost of a sudden stop, we evaluate the calibration by assessing the ability of the model to match firm-level and business cycle moments that were not used in the calibration procedure.

Firm Dynamics and Long-Run Moments

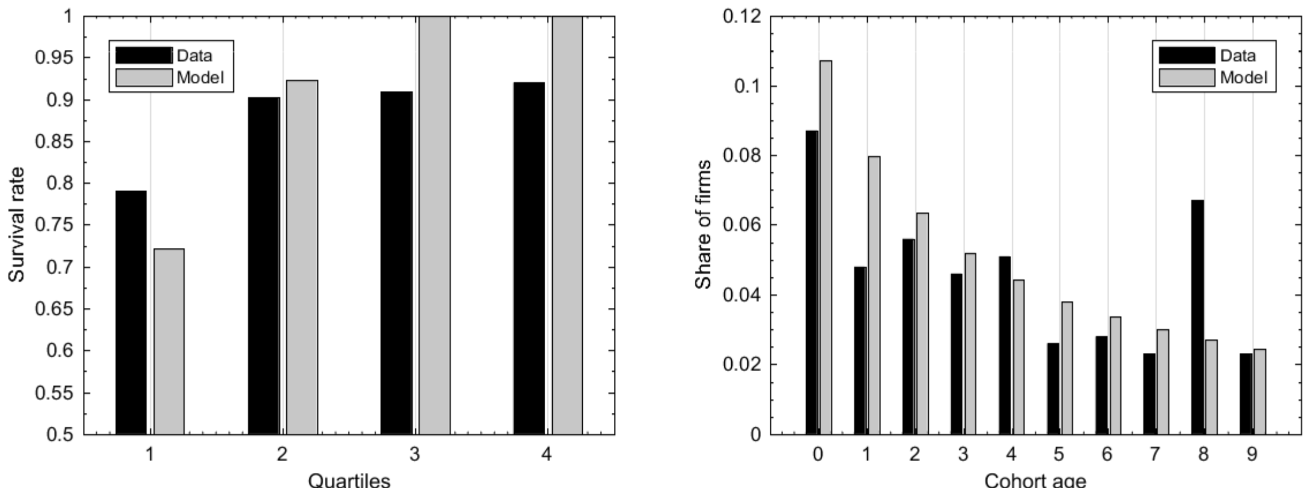
We start by evaluating the long-run calibration of the model using micro data, a dimension about which a standard small open economy model would be silent. First, Figure 5 evaluates the performance of the model with respect to the whole size distribution. Note that the calibration procedure only targets three moments related to the firm size distribution: i) mean employment, ii) the standard deviation of employment, and iii) the share of employment at the 10% largest firms.



(a) Size (workers) (b) Employment Share

Figure 5: Firm-level validation: Firm size and employment shares

Figure 5a shows that the model tracks the complete size distribution of firms in 1995. Figure 5b shows that the model also replicates the distribution of employment shares. Each bar in Figure 5b represents the share of workers employed by firms in that size range. Although only the last bar is targeted, the model accounts for the other six non-targeted bins.



(a) Survival rates by firm size

(b) Age

Figure 6: Cohort level validation: Employment growth and share of firms

Figure 6a shows the one-year survival rates for firms in different size quartiles. The model underestimates the survival of small firms and overestimates the survival of larger firms, although the general concave shape is consistent with the data. Figure 6b extends the analysis to the cohort dimension by plotting the shares of firms in each age bin in 1995 and comparing them with the stationary distribution of the model.[31] Note that nothing related to age was used in the calibration, and despite the noisy nature of the data, the model delivers the general trend of the empirical age distribution.

Table 5 shows a sample of non-targeted moments. Note that the model delivers an entry cost to income ratio consistent with the earliest available value in the World Bank data. The share of employment accounted for by firms that are one year old or less is of the same order of magnitude as its empirical counterpart. Although the model misses the true value in the data, it does not overstate the importance of entrants in the economy. The next three moments show the slope of linear regressions related to the average dynamics of a cohort. We denote by $\beta(y, x)$ the coefficient associated to x when y is regressed on a constant and x. All coefficients are significant in the model and data regressions. The first coefficient shows that the average fraction of active firms in a given cohort decreases with age with a similar slope in model and data. The second regression shows that correlation of firm size and age is consistent with the data. The third regression shows that average firm

[31] We thank Veronika Penciakova for using the 1986 − 96 panel to provide us with the age for the firms in 1995 and labor growth between 1994 and 1995.

Variables	Data	Model	Source
Entry cost to income ratio	11.6%	10.1%	World Bank (2004)
Employment share entry cohort	7.2%	1.8%	ENIA (1995)
$\beta(\text{survival}, \text{age})$	-3.1%	-3.9%	ENIA (1995)
$\beta(\log(\text{average labor}), \text{age})$	2.6%	2.7%	ENIA (1995)
$\beta(\text{average labor growth}, \text{age})$	-0.4%	-1.6%	ENIA (1994-1995)

Table 5: Non-targeted moments on entrant firms

growth slows down with age at a similar pace in the model and the data. We conclude that the model is able to replicate non-targeted characteristics of entrants and the patterns of size, age, and survival of the Chilean plant-level data.

Business Cycle Behavior

Having established the ability of the model to generate a stationary distribution aligned with the micro data, we evaluate the business cycle behavior of the model by studying the sudden stop of the Chilean economy in 1998 triggered by the Asian crisis and the subsequent Russian default. In particular, we evaluate the ability of the model to replicate not only the macro aggregates but also the entry, exit, and composition dynamics observed during the crisis. Because the model has only two exogenous shocks, only two series can be perfectly targeted. We assume that the model is in steady state in 1996:I, and we use the output and interest rate deviations in the data to filter through the model productivity and interest rate innovations.[32] To inform the model we use the demeaned series of log differences of output and the demeaned series of the log of $R(s^t)$. Appendix C.1 shows the data and the filtered shocks. Feeding the filtered innovations into the model allows us to use the Chilean crisis to evaluate the business cycle behavior of the model. Figure 7 compares the model-implied path for the log differences of consumption, investment, hours, and measured aggregate productivity with their empirical counterparts.[33]

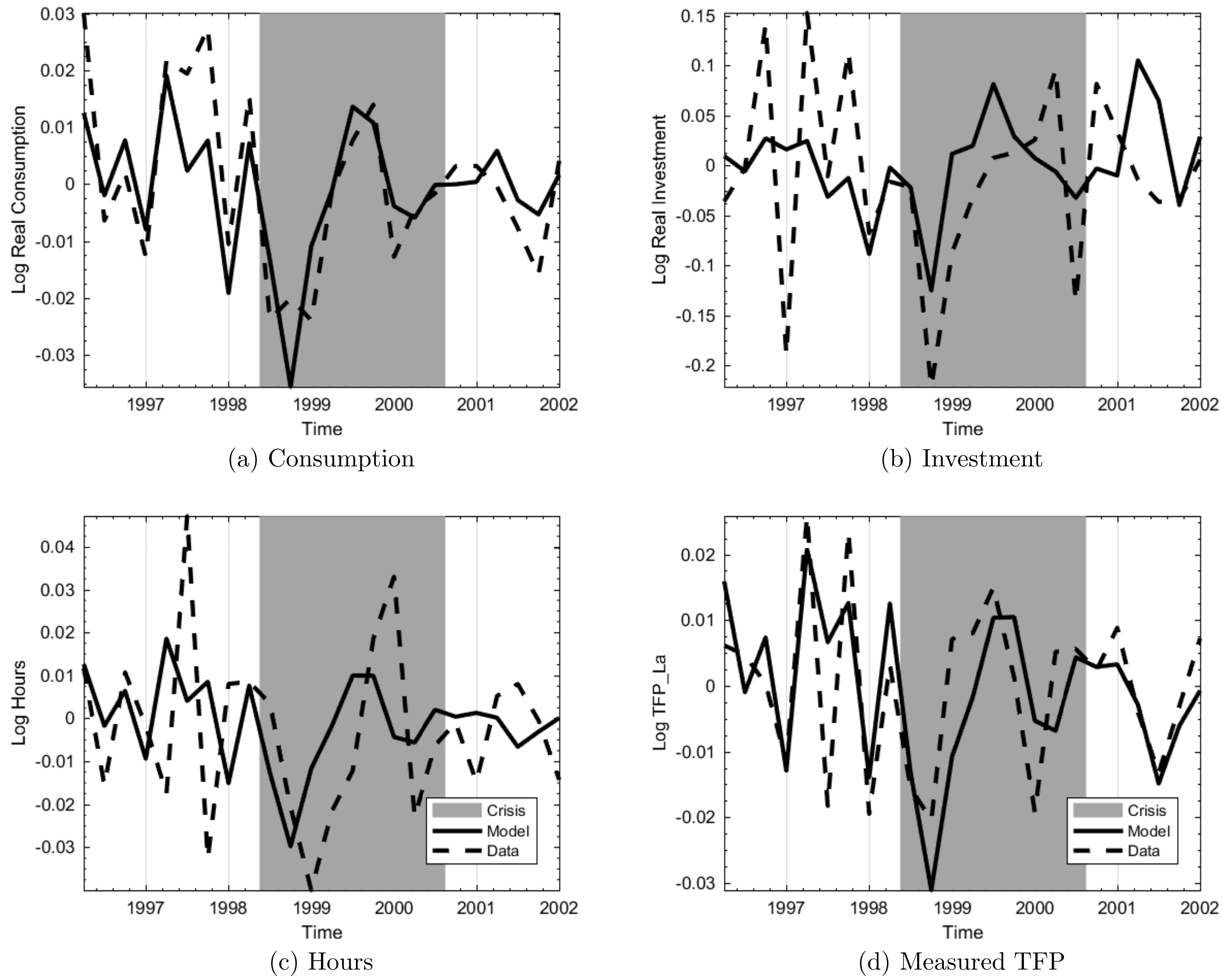
The model tracks well the behavior of the macro aggregates during the period.[34] With respect to the crisis episode, the model overstates the decrease in consumption and measured productivity, and it understates the decrease in labor and investment. Appendix C.2 further explores the business cycle properties of the model and shows that the model is consistent with the main business cycle moments of the Chilean economy. More interestingly, the model presented in this paper has novel predictions for the entry, exit, and composition of firms in the economy. Figure 8 evaluates the performance of the calibrated model along those dimensions.

Figure 8a shows that the annual entry behavior in the model is aligned with the U-shape behavior observed in the data during the crisis. Moreover the model explains more than two-thirds of the drop in entry during the Asian crisis in 1997 and the Russian default in 1998. Figure 8b shows that the exit rate in the model is also consistent with the data.

[32]The model is solved by second-order perturbations using Dynare. Note that the model has no kinks in value or policy functions. As discussed in Aruoba et al. (2006), higher order perturbation methods are appropriate for smooth systems with strong nonliearity subject to large shocks.

[33]Measured aggregate productivity is defined as $MTFP = \frac{Y_t}{K_t^{1-\alpha} L_t^{\alpha}}$.

[34]Investment is less volatile in the model during this period, and hours exhibit a delay of a quarter that can be attributed to the underlying survey data used to generate the series. This delay carries over to measured TFP.



(a) Consumption (b) Investment

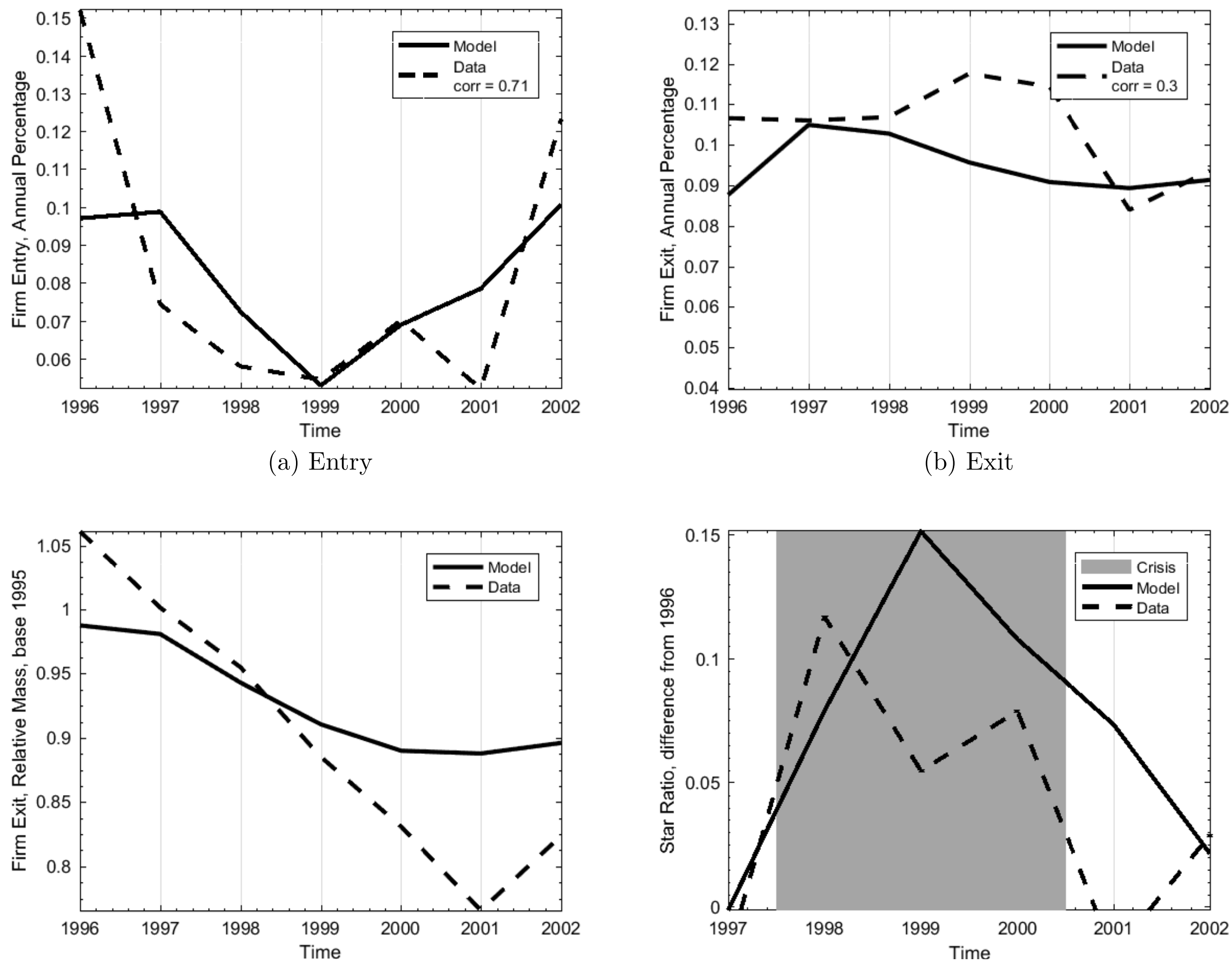
(c) Hours (d) Measured TFP

Figure 7: Non-targeted Crisis

Note that the exit rate is flatter and less volatile than the entry rate.[35] Figure 8c shows the number of firms relative to the 1995 value. The path generated by the model mimics the behavior of its empirical counterpart. Finally, to assess the evolution of the composition of entrants, we estimate equation (33), replacing the crisis dummy with a cohort fixed effect, and we compare it to the fraction of H-type entrants in the model.[36] Figure 8d shows that the estimated cohort fixed effects, which capture the cohort specific probability of a superstar firm arising relative to the 1996 probability, closely follow the model behavior of the fraction of high-type firms in the entrant cohort in excess of the 1996 value. In particular, the model predicts that in 1999, there is an increase of 0.15 in the probability of an entrant to be high type with respect to 1996. In the empirical estimation, the probability of finding a superstar

[35] Incumbent dynamics are critical for the model to deliver this asymmetric behavior. Without incumbent expansion, the entry and exit rates are necessarily the same, even outside the balanced growth path.

[36] Appendix B.9 shows the details of this estimation.



(a) Entry

(b) Exit

(c) Relative Mass

(d) H-type Ratio

Figure 8: Non-Targeted Entry and Exit Dynamics

firm reaches its maximum in 1998, when it is 0.11 higher than in 1996.[37]

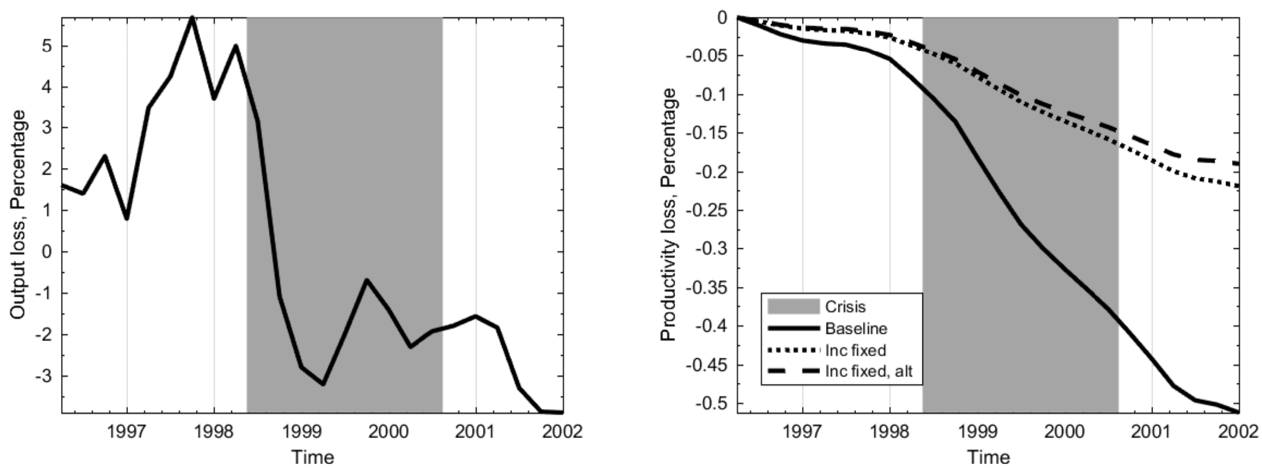
In sum, the calibrated model is able to mimic the firm dynamic patterns of Chilean firms, the macro responses of the economy to a sudden stop, and the mass and composition stylized facts documented in Section 4. Therefore, we proceed to use the model for quantifying the permanent productivity cost associated with the Chilean crisis.

5.3 The Permanent Productivity Loss of a Sudden Stop

Having validated the calibrated model, we use the model economy to estimate the permanent productivity loss due to the sudden stop. Figure 9a shows the deviations of output

[37]Beside this productivity analysis, Appendix C.5 shows that the behavior of labor growth of the crisis cohort is consistent with what Moreira (2015) documents for the U.S. economy.

from its trend during the crisis period. Note that by 2002, output is still 4% below trend.[38] A standard small open economy model would attribute all that difference to stationary productivity shocks. Therefore, in the long-run, a complete recovery would be expected. Because in the baseline model productivity is endogenous, part of this distance can be attributed to the slowdown of productivity growth triggered by the changes of firm dynamics during the crisis. The solid line in Figure 9b shows that the endogenous productivity level $A(s^t)$ is 0.5% below what it would have been in the absence of shocks.[39] This difference implies that 12.5% of the deviation of output in 2002 is due to the endogenous permanent productivity loss triggered by the crisis. To highlight the stochastic forces behind the permanent loss in productivity, Figure 10 shows the dynamic importance of stationary TFP and interest rate shocks in explaining the fluctuations in output growth and endogenous trend dynamics.



(a) Output deviation

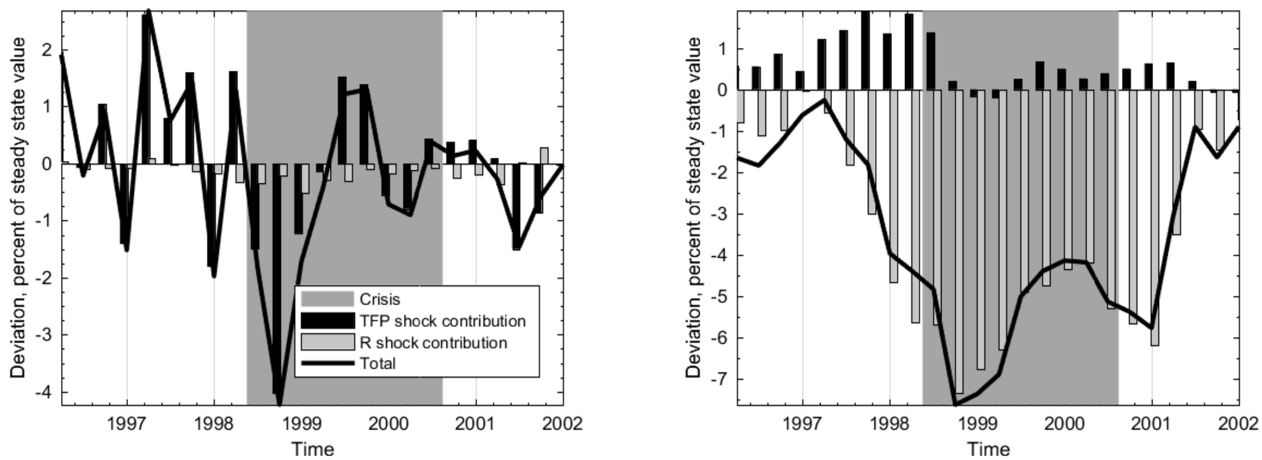
(b) Endogenous TFP deviation

Figure 9: Output growth and endogenous productivity

In line with the variance decomposition analysis in Appendix C.2, most of the short-run effect on output growth comes from the stationary TFP shock. Nevertheless, the majority of the deviations in endogenous productivity are accounted for by the interest rate shock. In this sense, the short-run effects of a crisis are mostly related to stationary productivity movements but the medium and long-run effects are mostly related to the interest rate shock. In this application, stationary productivity shocks are isomorphic to demand shocks that can arise as a result of weaker external conditions or negative terms of trade associated with the crisis. However, practically all the permanent loss in productivity is due to the behavior of interest rate during the crisis. Therefore, the calibrated model supports the view that financial shocks are special in terms of productivity dynamics.

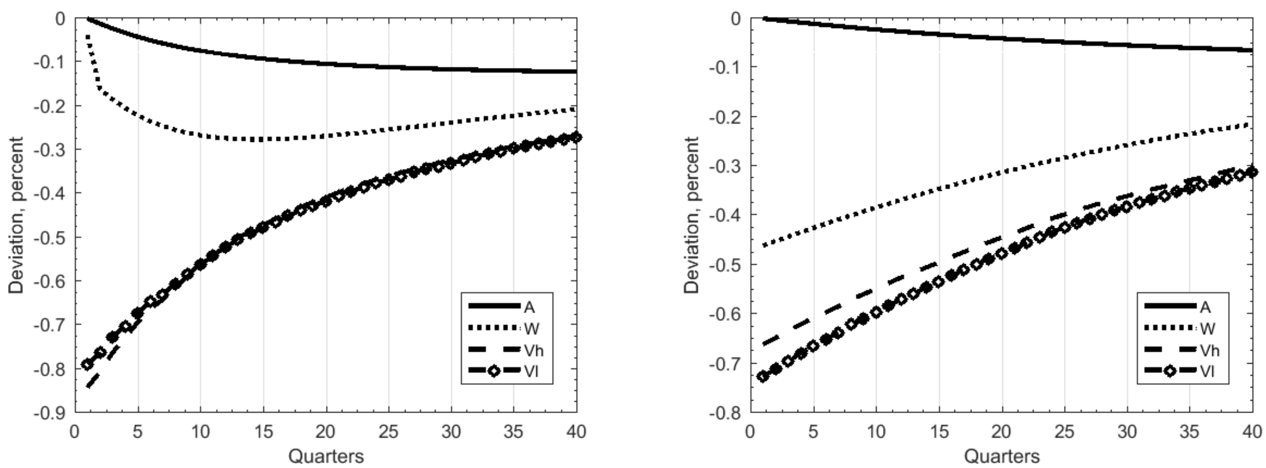
[38]Because this series was used in the filtering process, the data and model coincide in the predicted path of output.

[39]See Appendix A.3 for a detailed explanation of the difference between endogenous and exogenous growth and how it relates to the permanent productivity loss.



(a) Output deviation (b) Endogenous TFP growth

Figure 10: Output growth and endogenous component



(a) Impulse Response to R (b) Impulse Response to Z

Figure 11: IRFs to R (left panel) and TFP (right panel) shocks

The differential effect of stationary productivity shocks and interest rate shocks on endogenous productivity is depicted in Figure 11. Figures 11a and 11b show the impulse response functions of firm values, wages and endogenous productivity to one standard deviation shocks to stationary productivity and the interest rate. Note that interest rate shocks have a two times larger effect on endogenous productivity than stationary productivity shocks. To understand the source of this difference, compare the relative responses of firm value and wages in the two cases. Negative interest rate shocks decrease firm values much more than wages. Because costs of entry and expansion are in terms of labor and because the benefits of entry and expansion are linked to firm values, technological innovations decrease more with adverse interest rate shocks than with negative stationary productivity shocks. The differential effect of interest rate shocks is driven by its differential impact on the stochastic discount factor of the household that determines the value of varieties. Figure 12 shows the impulse response functions of the expected stochastic discount factor to one standard deviation stationary productivity and interest rate shocks. The differential response of $\mathbb{E}\left[m(s^{t+1})|s^t\right]$ shows that future payoffs are strongly discounted after an increase in interest rates but not after a negative productivity shock.[40] In this sense, the larger decrease of values with respect to wages that explains the dominant role of interest rate shocks on the long-run productivity cost of a crisis is due to the endogenous response of the stochastic discount factor.

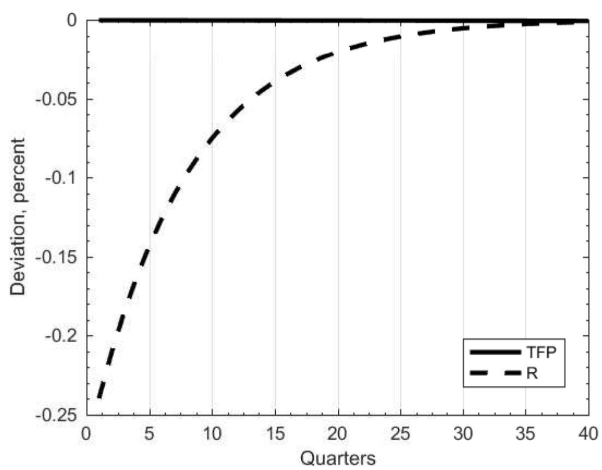
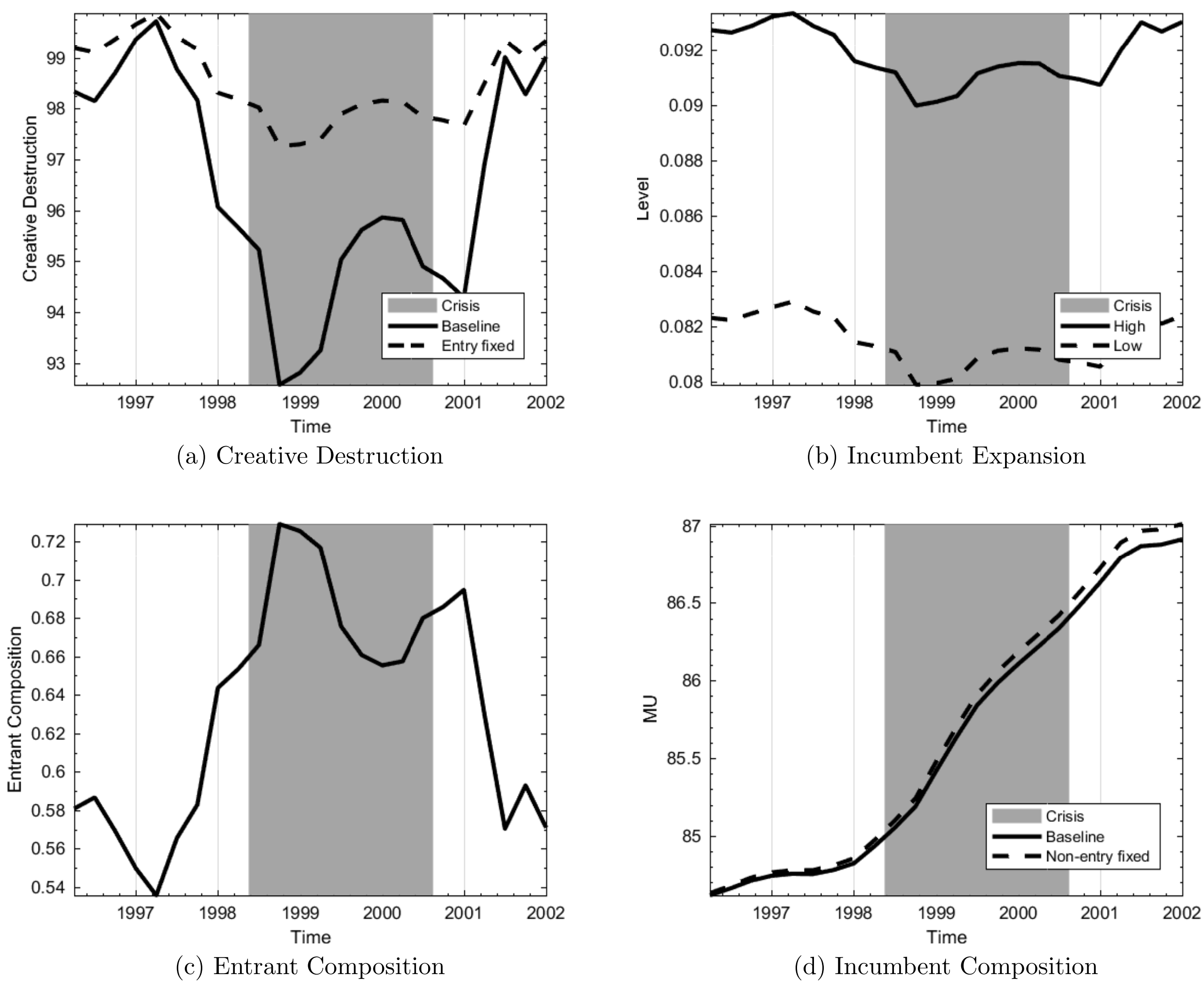


Figure 12: IRF of the Stochastic Discount Factor

To further decompose the permanent productivity loss, we focus on the different variables that determine endogenous productivity growth in equation (32). First, the solid line in Figure 13a shows the decrease in creative destruction (Δ) during the crisis. Fixing the entry related component in equation (29) to its balanced growth value (dashed line), we see that most of this decrease is due to the fall in the mass of entrants. All other conditions equal, this decline in the rate of creative destruction should promote expansion by incumbents because the threat of replacement diminishes. Nevertheless, Figure 13b shows a decrease in the expansion rate (ι^d) for both type of incumbents. This result indicates that the fall in profits due to the recession and the increase in the stochastic discount factor due to the interest rate shock dominate the overall effect on the value of varieties, ultimately decreasing the expansion rate. From a composition perspective, Figure 13c shows that the sharp decrease in entry triggers an increase in the fraction of high-type firms entering the economy ($\tilde{\mu}$). The solid line in Figure 13d shows that the fraction of products dominated

[40]The other potential difference is given by the working capital constraint that links labor costs and interest rate shocks. Appendix C.4 shows that eliminating the working capital channel does not affect the quantification of the permanent productivity loss of the crisis. This result is not surprising, given that the pass-through of interest shocks due to the working capital constraint is isomorphic to stationary productivity fluctuations. The magnitude of the working capital constraint only affects the short-run behavior of the economy.

by high-type firms (μ) increases steadily during the period. The increase in the fraction of products dominated by high-type firms can be due to either the inflow of more high-type entrants or the differential expansion between high- and low- type firms during the period. Fixing the expansion rates of incumbents in equation (30) to their balanced growth value (dashed line), we see that practically all of the the increase on the share of product lines dominated by high-type firms after the crisis is due to the dynamic effects of the increase in the composition of entrants and not due to variations in the differential expansion rate of incumbents during the crisis. This result is consistent with Figure 13b, in which expansion rates move practically in tandem for both types. In this sense, the benign effect of the composition of the entrant cohort persists well after the crisis, by shaping the fraction of products dominated by high-type firms.



(a) Creative Destruction (b) Incumbent Expansion

(c) Entrant Composition (d) Incumbent Composition

Figure 13: Productivity Related Variables during the Crisis.

We use the decomposition in equation (32) to separate the productivity loss between entry and incumbent distortions. In particular, the dotted line in Figure 9b compares the total productivity loss to an alternative path where the entry component in equation (32) is fixed to its balanced growth path level. According to this decomposition, entry and incumbent distortions account for similar fractions of the long-run cost. Nevertheless, by fixing the incumbent term, we also attribute to incumbents the dynamic composition effects of entrants. To remove this distortion, the dashed line in Figure 9b fixes the expansion rate of incumbents in equations (30) and (32) but allows the composition of incumbents to evolve according to the evolution of the composition of entrants (dashed line in Figure 13d). The magnitude of the dynamic composition effect can be seen by comparing the dashed and dotted line in Figure 9b. In particular, the share of the productivity cost due to entrants diminishes to 40% as a result of the dynamic effects of the cohorts born during the crisis.

In summary, only 12.5% of the output deviation observed after the crisis is due to permanent productivity losses triggered by distortions to firm dynamics. To better understand of the importance of heterogeneity and firm dynamics when evaluating the productivity cost of a crisis, the next subsection compares the baseline model with alternative economies that lack those dimensions.

5.4 The Role of Heterogeneity and Firm Dynamics

In this last subsection we highlight the role of heterogeneity and firm dynamics when quantifying the productivity cost of a financial crisis. In particular, we compare the baseline model to two alternative economies that feature endogenous growth: An economy with no incumbent dynamics and no heterogeneity (NDNH) and an economy with incumbent dynamics and no heterogeneity (NH). NH differs from the baseline version in that it has a single step size, removing heterogeneity and selection from the economy. NDNH goes one step further by removing the expansion decision of incumbent firms. Therefore, in this version incumbent firms do not expand or shrink, holding one variety until they are replaced. Each economy is calibrated to the same period of data. Appendix C.3 shows the details of these alternative models and their calibrated parameters. Figure 14 shows the deviations of the endogenous TFP for each model.

First, note that the model with no heterogeneity (NH) estimates a permanent productivity loss 40% larger than the baseline. The main reason behind this amplification is that, in the NH economy, enacted projects are just as good as the discarded ones. This homogeneity implies that the marginal contribution of projects is flat with respect to the entry rate. Therefore, entry declines in NH more than in the baseline. Second, the loss estimated by the model that lacks both incumbent dynamics and heterogeneity (NDNH) is five times the loss estimated by the baseline model. Thus, heterogeneity and firm dynamics are important elements when evaluating the productivity loss of a sudden stop. Although the importance of heterogeneity is large, the effect of firm dynamics is an order of magnitude larger. To

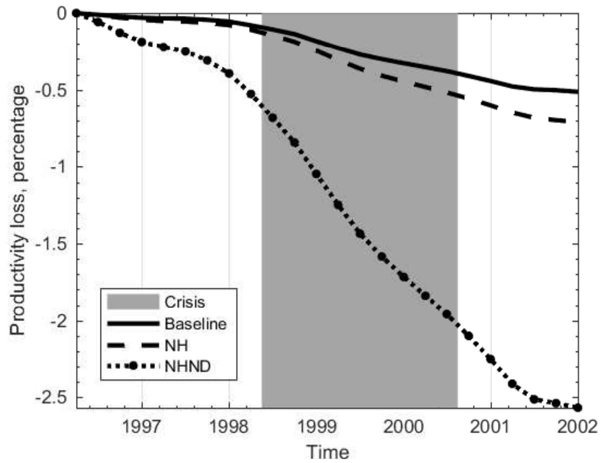
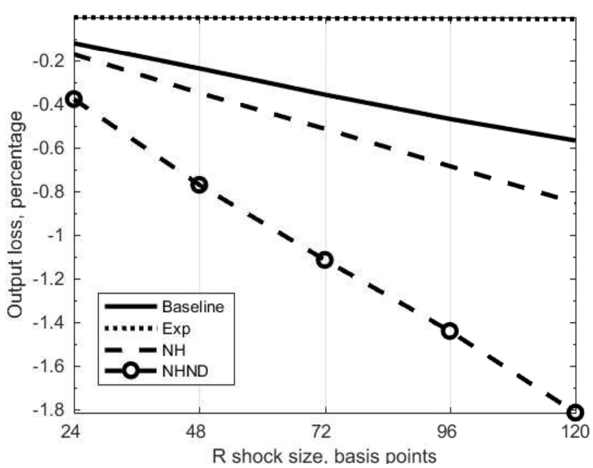


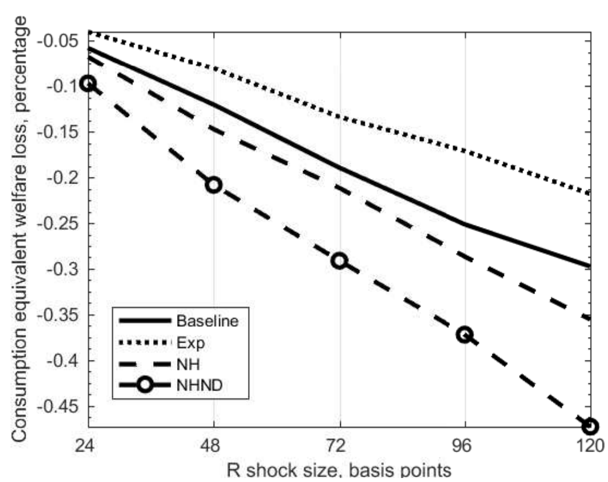
Figure 14: Model Comparison: Sudden Stop

understand this large effect, note that, by construction, in a model with no firm dynamics, entrants account for all the productivity growth in the economy. Because the crisis moves the entry margin violently, endogenous productivity is largely affected.[41]

To highlight the importance of heterogeneity and firm dynamics, Figure 15 shows the permanent productivity loss and the consumption equivalent welfare cost across models when the economy faces mean-reverting interest rate shocks of different magnitudes. For comparison purposes we include a fourth economy with exogenous growth (Exo).



(a) Productivity loss



(b) Consumption equivalent welfare loss

Figure 15: Model Comparison: Interest Rate Shock

Figure 15a shows the permanent productivity loss associated with mean-reverting interest rate shocks of different sizes. Note that, in a given economy, every growing endogenous

[41]These extreme reactions are behind the large cost of business cycles found by Barlevy (2004) when introducing endogenous growth in an RBC model with no incumbent dynamics and no heterogeneity.

variable is subject to the same permanent level drop because of the productivity loss. When productivity is exogenous (dotted line), there is no permanent loss. Ignoring heterogeneity (NH) implies a 30% larger productivity loss, and when heterogeneity and firm dynamics are omitted (NHND), the loss is more than four times the one in the baseline.

To understand the economic importance of these differences, Figure 15b calculates the consumption equivalent welfare loss associated with each interest rate shock. First, note that the agent in the baseline economy is willing to forgo more than 40% more relative consumption than the agent in the exogenous productivity model (Exo) in order to avoid the shock. Therefore, the endogenous permanent productivity losses have a similar importance to the standard short-lived effects of a crisis. In this sense, by ignoring the medium and long-run effects of a crisis, the standard small open economy model significantly underestimates the welfare cost of a crisis. Second, abstracting from incumbents and firm heterogeneity increases the consumption equivalent welfare cost by at least 50% when compared to the baseline economy. Thus, introducing endogenous growth without modeling heterogeneity and firm dynamics overestimates the welfare costs of a crisis, potentially providing misleading guidance to public policy.

6 Concluding Remarks

In this paper, we revisit the effects of sudden stops by considering the effect of a crisis on productivity growth. With that aim, we present an open economy endogenous growth model subject to interest rate and stationary productivity shocks. The engine of growth in this economy is the creative destruction induced by new entrants and by the expansion of incumbents. Because potential entrants are heterogeneous and promising entrants are scarce, financial selection introduces a tradeoff between the mass (quantity) and the composition (quality) of the entrants. In particular, a crisis triggered by an interest rate shock increases credit standards, giving rise to a smaller cohort of entrants with higher productivity. We use the Chilean sudden stop to test the main mechanism of the model. Our empirical analysis confirms that although fewer firms are born during the crisis, they are better in that they contribute more to aggregate productivity. This composition effect has persistent consequences in the economy as entrants become incumbents and make expansion decisions. The calibrated model successfully reproduces non-targeted features of the firm-level dynamics and the business cycle behavior of the Chilean economy.

The model reveals some interesting insights about the role of firm dynamics during a financial crisis. For instance, in the quantitative section, we explore the long-run cost of a sudden stop driven by the endogenous changes in TFP growth that the crisis triggers. An increase in the interest rate has a permanent effect on output, investment and consumption.

Not accounting for heterogeneity and firm dynamics multiplies by five the estimate of the permanent productivity loss of the sudden stop. In terms of welfare, an interest rate shock triggers a consumption equivalent welfare loss 50% larger in a model with no heterogeneity and no firm dynamics. Because governments often use forgone entry as a foundation for policy interventions, a correct assessment of the cost of foregone entry is critical. This model provides a tractable framework that future studies can use to evaluate those policies.

The scope of this model is far beyond sudden stop episodes or the particular Chilean experience. We develop a framework that allows researchers to introduce firm heterogeneity, firm dynamics, and endogenous growth in any dynamic general equilibrium model with aggregate risk with only one extra state variable and no approximation in distributions. We encourage future research to continue closing the gap between the quantitative firm dynamics-innovation literature and the DSGE literature. Moreover, this class of models provides a natural bridge to reconcile firm-level micro data and macro dynamics. Finally, the pass-through of stationary fluctuations into permanent productivity distortions is not only relevant for developing countries, where the distinction between short-run fluctuations and medium to long-run trends seems rather arbitrary, but also for developed economies. Indeed, the Great Recession challenged traditional macroeconomic models by exhibiting persistent effects in aggregate productivity, diminishing potential output even at long horizons. This paper suggests that financial frictions and their distortions of the stochastic discount factor, as opposed to stationary productivity shocks, could trigger permanent productivity losses and slow recoveries that are consistent with the data.

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References

P. Aghion and P. Howitt. "A Model of Growth through Creative Destruction". *Econometrica*, 60(2):323–51, 1992.

P. Aghion, U. Akcigit, and P. Howitt. "Chapter 1 - What Do We Learn From Schumpeterian Growth Theory?". In P. Aghion and S. N. Durlauf, editors, *Handbook of Economic Growth*, volume 2 of *Handbook of Economic Growth*, pages 515 – 563. Elsevier, 2014.

M. Aguiar and G. Gopinath. "Emerging Market Business Cycles: The Cycle is the Trend". *Journal of Political Economy*, 115:69–102, 2007.

U. Akcigit and W. R. Kerr. "Growth Through Heterogeneous Innovations". NBER Working Papers 16443, National Bureau of Economic Research, 2010.

S. B. Aruoba, J. Fernandez-Villaverde, and J. F. Rubio-Ramirez. "Comparing Solution Methods for Dynamic Equilibrium Economies". *Journal of Economic Dynamics and Control*, 30(12):2477–2508, 2006.

A. Atkeson and A. T. Burstein. "Innovation, Firm Dynamics, and international Trade". *Journal of Political Economy*, 118(3):433–484, 2010.

G. Barlevy. "The Cost of Business Cycles Under Endogenous Growth". *American Economic Review*, 94(4):964–990, 2004.

J. Benhabib, R. Rogerson, and R. Wright. "Homework in Macroeconomics: Household Production and Aggregate Fluctuations". *Journal of Political Economy*, 99(6):1166–87, 1991.

R. Bergoeing, P. J. Kehoe, T. J. Kehoe, and R. Soto. "A Decade Lost and Found: Mexico and Chile in the 1980s". *Review of Economic Dynamics*, 5(1):166 – 205, 2002.

G. A. Calvo and E. Talvi. "Sudden Stop, Financial Factors and Economic Collapse in Latin America: Learning from Argentina and Chile". NBER Working Papers 11153, National Bureau of Economic Research, 2005.

G. A. Calvo, A. Izquierdo, and E. Talvi. "Sudden Stops and Phoenix Miracles in Emerging Markets". *American Economic Review*, 96(2):405–410, 2006.

V. Cerra and S. C. Saxena. "Growth Dynamics: The Myth of Economic Recovery". *American Economic Review*, 98(1):439–57, 2008.

G. L. Clementi and B. Palazzo. "Entry, Exit, Firm Dynamics, and Aggregate Fluctuations". *American Economic Journal: Macroeconomics*, 8(3), 2016.

D. Comin and M. Gertler. "Medium-Term Business Cycles". *American Economic Review*, 96(3):523–551, 2006.

L. Foster, C. Grim, J. Haltiwanger, and Z. Wolf. "Productivity, Regulation, and Allocation:

Macro, Industry-and Firm-Level Evidence Firm-Level Dispersion in Productivity: is the Devil in the Details?". *The American Economic Review*, 106(5):95–98, 2016.

N. M. Gornemann. "Sovereign Default, Private Investment and Economic Growth". *manuscript*, 2014.

J. Greenwood, Z. Hercowitz, and G. W. Huffman. "Investment, Capacity Utilization, and the Real Business Cycle". *American Economic Review*, 78(3):402–17, 1988.

G. M. Grossman and E. Helpman. "Quality Ladders in the Theory of Growth". *Review of Economic Studies*, 58(1):43–61, 1991.

P. A. Guerrón-Quintana and R. Jinnai. "Liquidity, Trends, and the Great Recession". *manuscript*, 2014.

M. Hallward-Driemeier and B. Rijkers. "Do Crises Catalyze Creative Destruction? Firm-level Evidence from Indonesia". *Review of Economics and Statistics*, 95(1):1788–1810, 2013.

J. A. Hausman and W. E. Taylor. "Panel Data and Unobservable Individual Effects". *Econometrica*, 49(6):1377–98, 1981.

G. Jiménez, S. Ongena, J.-L. Peydró, and J. Saurina. "Hazardous Times for Monetary Policy: What Do Twenty-Three Million Bank Loans Say About the Effects of Monetary Policy on Credit Risk-Taking?". *Econometrica*, 82(2):463–505, 2014.

T. J. Klette and S. Kortum. "Innovating Firms and Aggregate Innovation". *Journal of Political Economy*, 112(5):986–1018, 2004.

Y. Lee and T. Mukoyama. "Entry and Exit of Manufacturing Plants over the Business Cycle". *European Economic Review*, 77:20–27, 2015.

R. Lentz and D. T. Mortensen. "An Empirical Model of Growth through Product Innovation". *Econometrica*, 76(6):1317–1373, 2008.

J. Levinsohn and A. Petrin. "Estimating Production Functions Using Inputs to Control for Unobservables". *The Review of Economic Studies*, 70(2):317–341, 2003.

E. G. Mendoza. "Real Business Cycles in a Small Open Economy". *American Economic Review*, 81(4):797–818, 1991.

E. G. Mendoza and V. Z. Yue. "A General Equilibrium Model of Sovereign Default and Business Cycles". *The Quarterly Journal of Economics*, 127(2):889–946, 2012.

S. Moreira. "Firm Dynamics, Persistent Effects of Entry Conditions, and Business Cycles". *manuscript*, 2015.

P. A. Neumeyer and F. Perri. "Business Cycles in Emerging Economies: the Role of Interest Rates". *Journal of Monetary Economics*, 52(2):345–380, 2005.

N. Pavcnik. “Trade Liberalization, Exit, and Productivity Improvements: Evidence from Chilean Plants”. *The Review of Economic Studies*, 69(1):245–276, 2002.

A. Queraltó. “A Model of Slow Recoveries from Financial Crises”. International Finance Discussion Papers 1097, Board of Governors of the Federal Reserve System (U.S.), 2013.

C. M. Reinhart and K. S. Rogoff. “Recovery from Financial Crises: Evidence from 100 Episodes”. *American Economic Review*, 104(5):50–55, 2014.

P. M. Romer. “Endogenous Technological Change”. *Journal of Political Economy*, 98(5): 71–102, 1990.

M. E. Schaffer and S. Stillman. “XTOVERID: Stata Module to Calculate Tests of Overidentifying Restrictions after xtreg, xtivreg, xtivreg2, xthtaylor”. *Statistical Software Components*, 2011.

F. M. Scherer. “The Size Distribution of Profits from Innovation”. *Annals of Economics and Statistics / Annales d'Économie et de Statistique*, (49/50):495–516, 1998.

S. Schmitt-Grohé and M. Uribe. “Closing Small Open Economy Models”. *Journal of International Economics*, 61(1):163–185, 2003.

P. Schnabl. “The International Transmission of Bank Liquidity Shocks: Evidence from an Emerging Market”. *The Journal of Finance*, 67(3):897–932, 2012.

M. Siemer. “Firm Entry and Employment Dynamics in the Great Recession”. Finance and Economics Discussion Series 2014-56, Board of Governors of the Federal Reserve System (U.S.), 2014.

G. Silverberg and B. Verspagen. “The Size Distribution of Innovations Revisited: An Application of Extreme Value Statistics to Citation and Value Measures of Patent Significance”. *Journal of Econometrics*, 139(2):318–339, 2007.

M. Uribe and V. Z. Yue. “Country Spreads and Emerging Countries: Who Drives Whom?”. *Journal of International Economics*, 69(1):6–36, 2006.

J. M. Wooldridge. “On Estimating Firm-Level Production Functions Using Proxy Variables to Control for Unobservables”. *Economics Letters*, 104(3):112–114, 2009.

J. M. Wooldridge. *“Econometric Analysis of Cross Section and Panel Data”*, volume 1 of *MIT Press Books*. The MIT Press, 2010.

Appendices

A Firm Dynamics and Equilibrium Definition

The distribution of firms is determined by the mass of type d firms with n product lines at every aggregate state. We denote this mass by $\Omega_n^d(s^t)$; note that the $\Omega_n^d(s^t)$ firms in this category control $\Omega_n^d(s^t) \cdot n$ product lines. Because time is discrete in this model, the changes in the number of product lines of each firm are described by a binomial process. The firms in the $\Omega_n^d(s^t)$ category might end up with any number of product lines in $[0, 2 \cdot n]$ depending on the interaction between their innovation effort and the replacement rate of the economy. For example, a firm with five product lines that successfully generates spinoffs in four of them but also loses two of its former products, will end up with seven product lines. Therefore, we can use the law of large numbers to write the law of motion of each size class as

$$\begin{aligned}
\Omega_1^H(s^t) &= M(s^{t-1})\tilde{\mu}(s^{t-1}) \\
&+ \Omega_1^H(s^{t-1}) \sum_{k=0}^{1} \mathbb{P}\left(k, 1, \iota^H(s^{t-1})\right) \cdot \mathbb{P}\left(k, 1, \Delta(s^{t-1})\right) \\
&+ \sum_{n=2}^{\infty} \Omega_n^H(s^{t-1}) \cdot \sum_{k=0}^{1} \mathbb{P}\left(k, n, \iota^H(s^{t-1})\right) \cdot \mathbb{P}\left(k+n-1, n, \Delta(s^{t-1})\right)
\end{aligned} \tag{35}$$

$$\begin{aligned}
\Omega_1^L(s^t) &= M(s^{t-1})\left(1 - \tilde{\mu}(s^{t-1})\right) \\
&+ \Omega_1^L(s^{t-1}) \sum_{k=0}^{1} \mathbb{P}\left(k, 1, \iota^L(s^{t-1})\right) \cdot \mathbb{P}\left(k, 1, \Delta(s^{t-1})\right) \\
&+ \sum_{n=2}^{\infty} \Omega_n^L(s^{t-1}) \cdot \sum_{k=0}^{1} \mathbb{P}\left(k, n, \iota^L(s^{t-1})\right) \cdot \mathbb{P}\left(k+n-1, n, \Delta(s^{t-1})\right)
\end{aligned} \tag{36}$$

$$\begin{aligned}
\Omega_{\tilde{n}>1}^d(s^t) &= \sum_{n=\mathbb{I}^+\left(\frac{\tilde{n}}{2}\right)}^{\tilde{n}-1} \Omega_n^d(s^{t-1}) \cdot \sum_{k=0}^{2n-\tilde{n}} \mathbb{P}\left(\tilde{n}-n+k, n, \iota^d(s^{t-1})\right) \cdot \mathbb{P}\left(k, n, \Delta(s^{t-1})\right) \\
&+ \Omega_{\tilde{n}}^d(s^{t-1}) \sum_{k=0}^{\tilde{n}} \mathbb{P}\left(k, \tilde{n}, \iota^d(s^{t-1})\right) \cdot \mathbb{P}\left(k, \tilde{n}, \Delta(s^{t-1})\right) \\
&+ \sum_{n=\tilde{n}+1}^{\infty} \Omega_n^d(s^{t-1}) \cdot \sum_{k=0}^{\tilde{n}} \mathbb{P}\left(k, n, \iota^d(s^{t-1})\right) \cdot \mathbb{P}\left(k+n-\tilde{n}, n, \Delta(s^{t-1})\right)
\end{aligned}$$

where $\mathbb{I}^{+}(x)$ refers to the integer closest to x such that $\mathbb{I}^{+}(x) \geq x$. To understand the intuition of these expressions, we will first focus on the general expression for $\Omega^d_{\tilde{n}>1}(s^t)$. The first line represents the successful innovators of lower size classes that achieve size $\tilde{n}$, the second term represents the firms that keep their $\tilde{n}$ products, and the third term shows the formerly larger firms that shrink to exactly $\tilde{n}$ products. Further simplifications lead to

$$\begin{aligned}\Omega^d_{\tilde{n}>1}(s^t) = & \sum_{n=\mathbb{I}^{+}\left(\frac{\tilde{n}}{2}\right)}^{\tilde{n}} \Omega^d_n(s^{t-1}) \cdot \sum_{k=\tilde{n}-n}^{n} \mathbb{P}\left(k, n, \iota^d(s^{t-1})\right) \cdot \mathbb{P}\left(k-(\tilde{n}-n), n, \Delta(s^{t-1})\right) \\ & + \sum_{n=\tilde{n}+1}^{\infty} \Omega^d_n(s^{t-1}) \cdot \sum_{k=0}^{\tilde{n}} \mathbb{P}\left(k, n, \iota^d(s^{t-1})\right) \cdot \mathbb{P}\left(k-(\tilde{n}-n), n, \Delta(s^{t-1})\right) \end{aligned} \tag{37}$$

Because every product line belongs to a firm, we have

$$\sum_{n=1}^{\infty} \left(\Omega^H_n(s^t) + \Omega^L_n(s^t)\right) \cdot n = \Lambda. \tag{38}$$

The total mass of firms in the economy is

$$\Omega(s^t) = \sum_{n=1}^{\infty} \left(\Omega^H_n(s^t) + \Omega^L_n(s^t)\right).$$

In line with its empirical counterpart, the quarterly entry rate is defined as

$$\text{Entry rate}\,(s^t) = \frac{2 \cdot M(s^{t-1})}{\Omega(s^t) + \Omega(s^{t-1})}.$$

Analogously, the quarterly exit rate is given by

$$\begin{aligned}\text{Exit rate}\,(s^t) &= \frac{2 \cdot \left[\sum_{d\in\{H,L\}} \sum_{n=1}^{\infty} \Omega^d_n(s^t) \cdot \mathbb{P}\left(0, n, \iota^d(s^{t-1})\right) \cdot \mathbb{P}\left(n, n, \Delta(s^{t-1})\right)\right]}{\Omega(s^t) + \Omega(s^{t-1})} \\ &= \frac{2 \cdot \sum_{d\in\{H,L\}} \sum_{n=1}^{\infty} \Omega^d_n(s^t) \cdot \left[(1-\iota^d(s^{t-1}))\Delta(s^{t-1})\right]^n}{\Omega(s^t) + \Omega(s^{t-1})}.\end{aligned}$$

Note that given an initial distribution and a sequences of innovation intensities and replacement rates, we can uniquely pin down the evolution of the size distribution in the economy. When comparing with data, we measure entry and exit annually.

A.1 Equilibrium Definition

To render the model stationary, we adopt the following convention: Any lowercase variable represents the productivity scaled version of its uppercase counterpart; for instance, the stationary transformation of output is given by $y(s^t) = \frac{Y(s^t)}{A(s^t)}$. In the case of capital and bonds, because of the timing convention we have $k = \frac{K(s^{t-1})}{A(s^t)}$ and $b = \frac{B(s^{t-1})}{A(s^t)}$. This transformation is performed for consumption, bond holdings, capital, wages, intermediate goods production, investment, and output. With this transformation, we define a stationary competitive equilibrium for this economy:

Definition 1. *A competitive equilibrium for this small open economy, given an initial efficiency level $q_j(0)$ for every product line, an initial fraction of type H incumbents, and initial levels of bond holding and capital for the household, is given by the following:*

1. *Household optimally chooses $\{c(s^t), b(s^t), k(s^t), l(s^t)\}$ given prices and transfers to solve* (1) *subject to* (2) *and* (3).
2. *Final good producer optimally chooses $\left\{\{x_j(s^t)\}_{j\in[0,\Lambda]}, k(s^{t-1})\right\}$ given prices to solve* (10).
3. *Intermediate firm f with n product lines of type d optimally chooses its price $\{p_j(s^t)\}_{j\in[0,\Lambda]}$ and its production and expansion labor usage $\left\{l_f^d(s^t,n) \equiv n \cdot l^d(s^t), l_{r,f}^d(s^t,n) \equiv n \cdot l_r^d(s^t)\right\}$ given wages and their type according to* (15), (17), *and* (23).
4. *Financial intermediary optimally chooses $\{M(s^t)\}$ given values and prices in order to satisfy* (26).
5. *Capital markets clear in every history, and intermediate good markets clear in every history for every product line.*
6. *Labor, asset, and final good markets clear in every history:*

$$l(s^t) = \Lambda\mu(s^t)\left(l^H(s^t) + l_r^H(s^t)\right) + \Lambda(1-\mu(s^t))\left(l^L(s^t) + l_r^L(s^t)\right) + \kappa M(s^t) \quad (39)$$

$$d(s^t) = b(s^{t-1}) - \eta\frac{\alpha y(s^t)}{1+\eta(R(s^{t-1})-1)} - M(s^t)\kappa w(s^t)\eta \quad (40)$$

$$nx(s^t) = y(s^t) - c(s^t) - i(s^t) - \frac{\psi}{2}y(s^t)\left(\frac{b(s^t)}{y(s^t)}\left(1+a(s^t)\right) - \bar{b}\left(1+\bar{g}\right)\right)^2 \quad (41)$$

7. *$\left\{v_j^d(n,s^t) = n \cdot \bar{v}^d(s^t), q_j(s^t)\right\}_{j\in[0,\Lambda], d\in\{L,H\}}$ and $\mu(s^t)$ evolve according to* (24), (14), *and* (30).
8. *The mass of firms of type d with n product lines evolves according to* (35), (36), *and* (37).
9. *Every product belongs to a firm, so that* (38) *holds.*
10. *Transversality and non-negativity conditions are met.*

We can also define a balanced growth path (BGP) for this economy as follows:

Definition 2. *A BGP is a non-stochastic ($\sigma_R = \sigma_z = 0$) equilibrium where $\{M(s^t)\}$ is constant, and consumption, bond holdings, capital, wages, intermediate goods production,* *investment, net exports, and output grow at a constant rate. Along the BGP, Ω_n^d is constant for every n and d.*

A.2 Normalized System of Equations

Representative Household

$$m(s^{t+1}) = \frac{\beta}{(1+a(s^t))^\gamma} \frac{\left(c(s^{t+1}) - \Theta\left(l(s^{t+1})\right)^\chi\right)^{-\gamma}}{\left(c(s^t) - \Theta\left(l(s^t)\right)^\chi\right)^{-\gamma}} \tag{42}$$

$$b(s^t) = \left\{\frac{\mathbb{E}\left[m(s^t, s_{t+1})|s^t\right] R(s^t) - 1}{\psi} + \bar{b}\,(1+\bar{g})\right\} \frac{y(s^t)}{(1+a(s^t))} \tag{43}$$

$$1 = \mathbb{E}\left[m(s^{t+1}) \frac{r(s^{t+1}) + (1-\delta) - \frac{\phi}{2}\left([1+\bar{g}]^2 - \left[\frac{k(s^{t+1})}{k(s^t)}\left(1+a(s^{t+1})\right)\right]^2\right)}{1+\phi\left[\frac{k(s^t)}{k(s^{t-1})}\left(1+a(s^t)\right) - (1+\bar{g})\right]} |s^t\right] \tag{44}$$

$$l(s^t) = \left(\frac{w(s^t)}{\Theta\chi}\right)^{\frac{1}{\chi-1}} \tag{45}$$

$$i(s^t) = k(s^t)\left(1+a(s^t)\right) - (1-\delta)k(s^{t-1}) + \frac{\phi}{2}k(s^{t-1})\left(\frac{k(s^t)}{k(s^{t-1})}(1+a(s^t)) - (1+\bar{g})\right)^2 \tag{46}$$

$$\begin{aligned} c(s^t) = {} & w(s^t)l(s^t) + r(s^t)k(s^{t-1}) + b(s^{t-1})R(s^{t-1}) + t(s^t) - i(s^t) - b(s^t)\left(1+a(s^t)\right) \\ & - \frac{\psi}{2}y(s^t)\left(\frac{b(s^t)}{y(s^t)}(1+a(s^t)) - \bar{b}(1+\bar{g})\right)^2 \end{aligned} \tag{47}$$

Final Good Producer

$$y(s^t) = \exp\left(z(s^t)\right) \cdot \left(\left(l^H(s^t)\right)^{\mu(s^t)} \left(l^L(s^t)\right)^{1-\mu(s^t)}\right)^\alpha \left(k(s^{t-1})\right)^{1-\alpha} \tag{48}$$

$$k(s^{t-1}) = \frac{(1-\alpha)y(s^t)}{r(s^t)} \tag{49}$$

Intermediate Good Producers

$$l^d(s^t) = \frac{\frac{\alpha}{\Lambda} y(s^t)}{w(s^t)(1+\sigma^d)\left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)} \tag{50}$$

$$\pi_j^d(s^t) = \frac{\alpha}{\Lambda} \frac{\sigma^d}{(1+\sigma^d)} y(s^t) \tag{51}$$

$$\begin{aligned} \bar{v}^d(s^t) &= \pi^d(s^t) - w(s^t)\left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)\varphi l^d(s^t)^\xi \\ &\quad + \mathbb{E}\left[m(s^{t+1})\left(1+a(s^t)\right)\left(1-\Delta(s^t)+\iota^d(s^t)\right)\bar{v}^d(s^{t+1})|s^t\right] \end{aligned} \tag{52}$$

$$\iota^d(s^t) = \left(\frac{\mathbb{E}\left[m(s^{t+1})\left(1+a(s^t)\right)\bar{v}^d(s^{t+1})|s^t\right]}{\varphi\xi w(s^t)\left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)}\right)^{\frac{1}{\xi-1}} \tag{53}$$

$$l_r^d(s^t) = \varphi\left(\iota^d(s^t)\right)^\xi \tag{54}$$

Financial Intermediary

$$M(s^t) = 1 - \left[\frac{\left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)w(s^t)\kappa - \mathbb{E}\left[m(s^{t+1})\left(1+a(s^t)\right)\bar{v}^L(s^{t+1})|s^t\right]}{\mathbb{E}\left[m(s^{t+1})\left(1+a(s^t)\right)\left(\bar{v}^H(s^{t+1})-\bar{v}^L(s^{t+1})\right)|s^t\right]}\right]^{\frac{1}{\nu}} \tag{55}$$

$$\tilde{\mu}(s^t) = \frac{1}{\nu+1}\left[\frac{1-\left[1-M(s^t)\right]^{\nu+1}}{M(s^t)}\right] \tag{56}$$

Aggregate Variables

$$a(s^t) = \left[\left(1+\sigma^H\right)^{\tilde{\mu}(s^t)}\left(1+\sigma^L\right)^{1-\tilde{\mu}(s^t)}\right]^{\frac{M(s^t)}{\Lambda}}\left(1+\sigma^H\right)^{\mu(s^t)\iota^H(s^t)}\left(1+\sigma^L\right)^{\left(1-\mu(s^t)\right)\iota^L(s^t)} - 1 \tag{57}$$

$$\mu(s^t) = \mu(s^{t-1}) + \frac{M(s^{t-1})}{\Lambda}\left[\tilde{\mu}(M(s^{t-1})) - \mu(s^{t-1})\right] + \mu(s^{t-1})\left(1-\mu(s^{t-1})\right)\left(\iota^H(s^{t-1}) - \iota^L(s^{t-1})\right) \tag{58}$$

$$\Delta(s^t) = \frac{M(s^t)}{\Lambda} + \mu(s^t)\iota^H(s^t) + \left(1-\mu(s^t)\right)\iota^L(s^t) \tag{59}$$

$$\begin{aligned} t(s^t) &= \mu(s^t)\Lambda\left[\pi^H(s^t) - \left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)w(s^t)l_r^H(s^t)\right] \\ &+ \left(1-\mu(s^t)\right)\Lambda\left[\pi^L(s^t) - \left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)w(s^t)l_r^L(s^t)\right] \\ &- \left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)M(s^t)\kappa w(s^t) \end{aligned} \tag{60}$$

$$nx(s^t) = y(s^t) - c(s^t) - i(s^t) - \frac{\psi}{2}y(s^t)\left(\frac{b(s^t)}{y(s^t)}(1+a(s^t)) - \bar{b}(1+\bar{g})\right)^2 \tag{61}$$

$$d(s^t) = b(s^{t-1}) - \eta w(s^t)l(s^t) \tag{62}$$

$$l(s^t) = \Lambda\mu(s^t)\left(l^H(s^t) + l_r^H(s^t)\right) + \Lambda(1-\mu(s^t))\left(l^L(s^t) + l_r^L(s^t)\right) + \kappa M(s^t) \tag{63}$$

Exogenous Shocks

$$\ln\left(\frac{R(s^t)}{\bar{R}}\right) = \rho_R \ln\left(\frac{R(s^{t-1})}{\bar{R}}\right) + \sigma_R\epsilon_{R,t} \quad \text{where} \quad \epsilon_{R,t} \overset{iid}{\sim} N(0,1), \tag{64}$$

$$z(s^t) = \rho_z z(s^{t-1}) + \sigma_z\epsilon_z(s^t) \quad \text{where} \quad \epsilon_z(s^t) \overset{iid}{\sim} N(0,1) \tag{65}$$

Solving for Balanced Growth Path

Consider a system with three equations and three unknowns $(\iota^H, \iota^L, \bar{z})$ that characterizes the BGP of this economy. We start with some auxiliary equations. To start, note that, after imposing BGP, the composition of entrants and incumbents are given by

$$\tilde{\mu} = \frac{1}{\nu+1}\left[\frac{1-(1-M)^{\nu+1}}{M}\right]$$

$$\mu = \frac{\iota^H - \iota^L - \frac{M}{\Lambda} + \sqrt{\left(\iota^H - \iota^L - \frac{M}{\Lambda}\right)^2 + 4\tilde{\mu}\frac{M}{\Lambda}\left(\iota^H - \iota^L\right)}}{2\left(\iota^H - \iota^L\right)}.$$

Therefore, the replacement rate of the economy is given by

$$\Delta = \frac{M}{\Lambda} + \mu \iota^H + (1-\mu)\, \iota^L.$$

The long-run growth rate of the economy can be characterized as

$$a = \left[\left(1+\sigma^H\right)^{\tilde{\mu}} \left(1+\sigma^L\right)^{1-\tilde{\mu}} \right]^{\frac{M}{\Lambda}} \left(1+\sigma^H\right)^{\mu\iota^H} \left(1+\sigma^L\right)^{(1-\mu)\iota^L} - 1.$$

From (42) and (43), in order to have $\frac{b}{y} = \bar{b}$ so that no bond holding costs are paid in the long-run, there is a unique value for the internal calibration of β:

$$\beta = \frac{(1+\bar{g})^\gamma}{\bar{R}}.$$

The normalized long-run level of capital is given by

$$k = \frac{1-\alpha}{\bar{R}-1+\delta} y.$$

The demand for labor at the product line level is given by

$$l^d = \frac{\frac{\alpha}{\Lambda} y}{w(1+\sigma^d)\left(1+\eta\left(\bar{R}-1\right)\right)}.$$

Replacing the capital demand for the final good producer and the demand for labor of the intermediate good producer in equation (48), we get the equilibrium wage:

$$w = \left(\frac{1-\alpha}{\bar{R}-(1-\delta)}\right)^{\frac{1-\alpha}{\alpha}} \frac{\frac{\alpha}{\Lambda}}{(1+\sigma^H)^\mu \left(1+\sigma^L\right)^{1-\mu} \left(1+\eta\left(\bar{R}-1\right)\right)}.$$

We can characterize output using the labor market clearing condition from equation (63):

$$y = w \left(\frac{1+\eta\left(\bar{R}-1\right)}{\alpha}\right) \left(\frac{\left(\frac{w}{\Theta\chi}\right)^{\frac{1}{\chi-1}} - \Lambda\varphi \left[\mu \left(\iota^H\right)^\xi + (1-\mu)\left(\iota^L\right)^\xi\right] - \kappa M}{\frac{\mu}{1+\sigma^H} + \frac{1-\mu}{1+\sigma^L}} \right).$$

We now characterize the profits associated to product lines and the value of firms:

$$\pi^d = \frac{\alpha}{\Lambda}\left(\frac{\sigma^d}{1+\sigma^d}\right) y$$

$$\bar{v}^d = \frac{\pi^d - w\left(1+\eta\left(\bar{R}-1\right)\right)\varphi\left(\iota^d\right)^\xi}{1-\frac{1+\bar{g}}{\bar{R}}\left(1+\iota^d-\Delta\right)}.$$

The BGP is the solution of the following nonlinear system of three equations and three unknowns:

$$\iota^d = \left(\frac{(1+\bar{g})\bar{v}^d}{\bar{R}\varphi\xi w \left(1+\eta\left(\bar{R}-1\right)\right)} \right)^{\frac{1}{\xi-1}}$$

$$(1-M)^\nu = \frac{(1+\eta(\bar{R}-1))w\kappa - \frac{1+\bar{g}}{\bar{R}}\bar{v}^L}{\frac{1+\bar{g}}{\bar{R}}(\bar{v}^H - \bar{v}^L)}.$$

Note that with the solution of this system, we can characterize every variable of the BGP. In particular, the household budget constraint pins down c, and nx is determined by the final good market clearing. Note that all of the above derivations are independent from the size distribution of firms. Nevertheless, the next section shows that the firm size distribution is unique and well defined.

Long-Run Distribution: Poisson Case

In continuous time, with Poisson processes, we can find an analytic expression for the distribution of firms. Because we will use this distribution as a guess in the algorithm for the binomial case, we first characterize this distribution. Note that entry rate equals exit rate along the BGP. Therefore, the total mass of firms with one product line is the following:

$$\Omega_1^H + \Omega_1^L = \frac{M}{\Delta}.$$

Along the BGP we have

$$\Omega_2^H = \frac{\Omega_1^H\left(\Delta + \iota^H\right) - M\tilde{\mu}}{2\Delta}$$

$$\Omega_2^L = \frac{\Omega_1^L\left(\Delta + \iota^L\right) - M\left(1-\tilde{\mu}\right)}{2\Delta}$$

$$\Omega_{n+1}^d = \frac{n\Omega_n^d\left(\Delta + \iota^d\right) - (n-1)\iota^d\Omega_{n-1}^d}{(n+1)\Delta}, \quad \forall d \in \{H, L\}, n > 2.$$

We also have

$$
\begin{aligned}
\mu &= \frac{1}{\Lambda}\sum_{n=1}^{\infty} n\cdot\Omega_n^H \\
&= \frac{1}{\Lambda}\left[\Omega_1^H + \frac{\Omega_1^H\left(\Delta+\iota^H\right) - M\tilde{\mu} + \sum_{n=2}^{\infty}\left[n\Omega_n^H\left(\Delta+\iota^H\right) - (n-1)\iota^H\Omega_{n-1}^H\right]}{\Delta}\right] \\
&= \frac{1}{\Lambda}\left[\Omega_1^H + \frac{\Omega_1^H\Delta - M\tilde{\mu} + \sum_{n=2}^{\infty} n\Omega_n^H\Delta}{\Delta}\right] \\
&= \frac{1}{\Lambda}\left[\Omega_1^H - \frac{M\tilde{\mu}}{\Delta} + \sum_{n=1}^{\infty} n\Omega_n^H\right] \quad \Rightarrow \quad \Omega_1^H = \frac{M\tilde{\mu}}{\Delta} \\
&\Rightarrow \Omega_1^L = \frac{M\left(1-\tilde{\mu}\right)}{\Delta}.
\end{aligned}
$$

Thus, we can solve for all the shares as

$$
\begin{aligned}
\Omega_2^H &= \frac{\Omega_1^H}{2}\left(\frac{\iota^H}{\Delta}\right) \quad \text{and} \quad \Omega_2^L = \frac{\Omega_1^L}{2}\left(\frac{\iota^L}{\Delta}\right) \\
\Omega_3^H &= \frac{\Omega_1^H}{3}\left(\frac{\iota^H}{\Delta}\right)^2 \quad \text{and} \quad \Omega_3^L = \frac{\Omega_1^L}{3}\left(\frac{\iota^L}{\Delta}\right)^2 \\
&\dots \\
\Omega_n^d &= \frac{\Omega_1^d}{n}\left(\frac{\iota^d}{\Delta}\right)^{n-1}.
\end{aligned}
$$

Note that the number of firms in each size class decreases with the number of product lines. Nevertheless, the number of high types decreases at a slower rate. Therefore, the share of type-H firms increases with the number of product lines:

$$
\frac{\Omega_n^H}{\Omega_n^L} = \frac{\tilde{\mu}}{1-\tilde{\mu}}\left(\frac{\iota^H}{\iota^L}\right)^{n-1}.
$$

Note that the total mass of firms in the economy of each type is given by

$$
\Omega^d = \Omega_1^d\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{\iota^d}{\Delta}\right)^{n-1} = \frac{\Delta}{\iota^d}\ln\left(\frac{\Delta}{\Delta-\iota^d}\right)\Omega_1^d.
$$

Therefore, the long-run size distribution by type is logarithmic:

$$
P_n^d = \frac{\left(\frac{\iota^d}{\Delta}\right)^n}{n\ln\left(\frac{\Delta}{\Delta-\iota^d}\right)},
$$

The unconditional size distribution is characterized by

$$P_n = \frac{\frac{\Omega_1^H}{n}\left(\frac{\iota^H}{\Delta}\right)^{n-1} + \frac{\Omega_1^L}{n}\left(\frac{\iota^L}{\Delta}\right)^{n-1}}{\frac{\Delta}{\iota^H}\ln\left(\frac{\Delta}{\Delta-\iota^H}\right)\Omega_1^H + \frac{\Delta}{\iota^L}\ln\left(\frac{\Delta}{\Delta-\iota^L}\right)\Omega_1^L} = \frac{\tilde{\mu}\left(\frac{\iota^H}{\Delta}\right)^n + (1-\tilde{\mu})\left(\frac{\iota^L}{\Delta}\right)^n \frac{\iota^H}{\iota^L}}{n\left[\tilde{\mu}\ln\left(\frac{\Delta}{\Delta-\iota^H}\right) + (1-\tilde{\mu})\ln\left(\frac{\Delta}{\Delta-\iota^L}\right)\frac{\iota^H}{\iota^L}\right]}.$$

Long-Run Distribution: Binomial Case

Algorithm:

1. Use Poisson as initial guess for Ω_n^d for $n = 1...\bar{n}$.
2. Iterate using the law of motion and the BGP values until $\max\left(\Omega_n^d(s^t) - \Omega_n^d(s^{t+1})\right) < tol$.

A.3 Long-Run Cost and Endogenous Growth

Note that growing variables, such as output or investment, are normalized by A_t. Denoting log-deviations of a variable H from its last period value by a hat ($\hat{H}_t = \ln(H_t/H_{t-1})$), we will now focus on output to highlight the source of the long-run cost:

$$y_t = \frac{Y_t}{A_t} \Rightarrow \hat{Y}_t \approx \hat{y}_t + \hat{A}_t. \tag{66}$$

In the absence of a shock, because y_t is constant, we get $\hat{Y}_t = \hat{A}_t \approx a_{ss}$. Hence, for scaled variables, we can define the distance at time t between the nonshocked economy and the one subject to the shock as $\tilde{x}_t^Y$:

$$\tilde{x}_t^Y \approx \sum_{i=1}^{i=t}\left\{\hat{y}_i + \hat{A}_i\right\} - t * a_{ss}. \tag{67}$$

The main difference between models with exogenous growth and models with endogenous growth is that, because growth is exogenous, $\hat{A}_t \approx a_{ss}$, and then $\tilde{x}_t^Y = \sum_{i=1}^{i=t}\hat{y}_i$. Because y_t is stationary, this term converges to zero when time goes to infinity. This illustrates why there is no long-run cost of a sudden stop for a model with exogenous growth. But a model with endogenous growth has a long-run cost (LRC), in any normalized variable, approximately equal to

$$LRC \approx \lim_{t\to\infty}\left\{t * a_{ss} - \sum_{i=1}^{i=t}\left\{\hat{A}_i\right\}\right\} < \infty. \tag{68}$$

Note that, because $\hat{A}_t$ converges to a_{ss}, this long-run cost is finite. Moreover, as is clear from equation (67), this long-run cost arises only for variables that exhibit long-run growth.

B Empirical Analysis

B.1 ENIA: Data Cleaning

The Encuesta Nacional Industrial Anual (ENIA, Annual National Industrial Survey) conducted the by the INE covers all manufacturing plants in Chile with more than 10 workers. Our version extends from 1995 to 2007.

We eliminate observations with one or more of the following inconsistencies, with original variable names provided in parenthesis: negative electricity consumption (*elecons*), worked days less than or equal to 0 (*diatra*), gross value of the production less than value added (*vpn*<*va*), value added less than 0 (*va*), remuneration of workers equal to 0 (*rem-pag*), size equal to 0 (*tamano*), ISIC code less than 3000 (bad coding in *sector*), and sales income less than income from exports (*ingtot*<*ingexp*). Finally, as mentioned in the text, we dropped industries 314 (Tobacco), 323 (Leather), 353 (Oil and Gas 1), 354 (Oil and Gas 2), 361 (Pottery), 362 (Glass), 371 (Metals 1), 372 (Metals 2), and 385 (other) because of an insufficient number of observations or inadequate entry dynamics. To minimize problems due to the 10 workers threshold, we count as the first observation of a firm the first time it appears in the data with 11 or more workers. The restricted sample contains 80% of the original observations and 89% of all workers in the sample.

B.2 Variable Construction and Other Controls

We calculate entry rates at year t at the industry level for each cohort, dividing the number of new plants in year t by the average of the total plants in years t and $t-1$. The revenue (*ingtot-revval-reviva*) used to calculate the profitability measures and the Herfindahl-Hirschman concentration index (HHI) excludes nonmanufactured products (reselling products and their tax shield); the costs include wages and exclude the costs and taxes associated with nonmanufactured products (*costot-mrevval-mreviva+rempag*). The variable used to build the productivity used in Table 2 is value added. We define capital as the end-of-period value of land, machinery, buildings and vehicles (*salter+salmaq+saledi+salveh*). We deflate monetary variables using three-digit industry level deflators provided by the INE. The index of manufacturing production (22866EY.ZF...), the unemployment rate (22867R..ZF...), and the producer price and wholesale price index (PPI/WPI, 22863...ZF...) are taken from the IFS database. The labor cost index is from the Chilean central bank.

For each three-digit industry (denoted by s) we separately estimate the following production function:

$$\log y_{it} = d_t^s + \beta^{sl}\log l_{it} + \beta^{sk}\log k_{it} + \log z_{it} + \varepsilon_{it},$$

where y_{it} is real value added for firm i in year t, d_t^s is a time fixed effect, l_{it} is total workers and k_{it} is real capital stock. The coefficient β^{sl} denotes the industry-specific elasticity of value added with respect to labor and β^{sk} denotes the elasticity of value added with respect to capital. We estimate these elasticities using the methodology described in Wooldridge (2009). Using the estimated elasticities $\hat{\beta}^{sl}$ and $\hat{\beta}^{sk}$, we calculate firm productivity as:

$$\log z_{it} = \log y_{it} - \hat{\beta}^{sl}\log l_{it} - \hat{\beta}^{sk}\log k_{it}$$

Table 6 shows the estimated elasticities. Note that the sum of the elasticities is always less than one.

B.3 Macro Data

In this subsection, we present the sources of the macroeconomic data used in this paper and the behavior of the aggregated time series during the crisis. To start, note that Chile is a small economy both in terms of population and aggregate output. It has also experienced spectacular growth, which led it to be the first OECD member in South America (2010). Its trade and debt ratio justify the small open economy framework adopted in this paper. In particular, while its trade to GDP ratio is quite high, according to the *World Trade Organization* database, in 2011 Chile had 0.45% of the world's exports and 0.41% of the world's imports. Chile is also the 7^{th} freest economy in the world (2013 *International Economic Freedom Ranking*).

The main source of data for the macroeconomic analysis in Section 4 is the International Financial Statistics (IFS) database from the International Monetary Fund (IMF). From that source, we use the following series between 1996:I and 2011:II: GDP volume index (22899BVPZF...), nominal GDP (22899B..ZF...), gross fixed capital formation (22893E..ZF...), changes in inventory (22893I..ZF...), exchange rate (228..RF.ZF...), exports (22890C..ZF...), imports (22898C..ZF...), financial accounts (22878BJ DZF...), direct investment abroad (22878BDDZF...), direct investment in Chile (22878BEDZF...), net errors and omissions (22878CADZF...), household consumption (22896F..ZF...), and government consumption (22891F..ZF...). We use employment data from the Instituto Nacional de Estadística (INE, National institute of Statistics) of Chile and hours worked per week from the *Encuesta de Ocupación y Desocupación* from the Economics Department of *Universidad de Chile*.

The EMBI+ spread data for Chile starts only in May 1999. To obtain an interest rate

Industry	$\hat{\beta}^l$	$\hat{\beta}^k$	$\hat{\beta}^l + \hat{\beta}^k$
311	0.48	0.12	0.59
312	0.69	0.10	0.79
313	0.30	0.10	0.40
321	0.71	0.06	0.77
322	0.67	0.10	0.77
324	0.69	0.18	0.88
331	0.53	0.16	0.70
332	0.65	0.14	0.79
341	0.46	0.12	0.57
342	0.51	0.14	0.65
351	0.47	0.18	0.65
352	0.61	0.04	0.65
355	0.72	0.03	0.76
356	0.42	0.13	0.54
369	0.63	0.14	0.78
381	0.69	0.11	0.80
382	0.66	0.06	0.72
383	0.59	0.12	0.70
384	0.62	0.06	0.67
390	0.71	0.11	0.82

For each three-digit industry (denoted by s) we separately estimate the following production function:

$$\log y_{it} = d_t^s + \beta^{sl} \log l_{it} + \beta^{sk} \log k_{it} + \log z_{it} + \varepsilon_{it},$$

where y_{it} is real value added for firm i in year t, d_t^s is a time fixed effect, l_{it} is total workers and k_{it} is real capital stock. The coefficient β^{sl} denotes the industry-specific elasticity of value added with respect to labor and β^{sk} denotes the elasticity of value added with respect to capital.We estimate these elasticities using the methodology described in Wooldridge (2009), an extension of Levinsohn and Petrin (2003).

Table 6: Estimated Elasticities by Industry

series that goes back long enough to match our firm-level data, we augment the EMBI+ spread series in the following way. First, we find a country whose spreads have a high correlation with the Chilean series. The spread for South Africa, which starts in 1994, has a correlation higher than 90% with Chilean spreads. As metals and mineral products constitute a large export share in both countries, this correlation is not surprising. Second, we estimate a linear OLS regression of log South African spreads on log Chilean series over the concurrent period using average monthly levels. With the regression coefficients, we compute the fitted Chilean monthly spreads and the implied month-over-month growth rate in this fitted series. Third, using these growth rates, we go backward to fill in the missing earlier months for Chile, starting from the first available month. Figure 16 shows the resulting Chilean spread augmented for the period before 1999 and the South African spread. The vertical line denotes the first available month for Chile in the original database, and the part to the left of this line in the Chilean series is extrapolated.

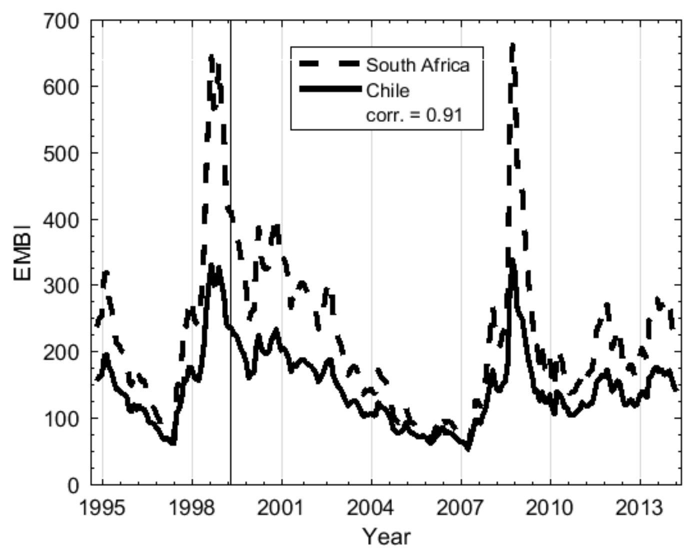


Figure 16: EMBI+ spreads

Finally, following Neumeyer and Perri (2005), we measure log Chilean real interest rate as the sum of the log U.S. real interest rate and the log of J. P. Morgan's EMBI+ spread for Chile. The U.S. real interest rate is defined as the 90-day T-bill rate divided by the average gross inflation over the current and the previous three quarters, which proxies for the expected inflation. Figure 19a shows the resulting interest rate for Chile.

B.4 Working Capital in the Data

In order to discipline the working capital parameter in the model, we use firm-level information on interest payments (*intgas*) and total cost of production (*totcost*) from ENIA.

We link these variables to their model counterparts using the following relationship:

$$\eta\left(R(s^{t-1})-1\right)(\text{production cost}) = \text{interest spending} \Rightarrow$$
$$\eta = \frac{\text{interest spending}}{(\text{production cost})\left(R(s^{t-1})-1\right)},$$

where R is the Chilean real interest rate. We derive this ratio at the firm level. The value of η is roughly 50% before the crisis period when calculated as the simple average across firms. When firm-specific values for η are weighted by the employment size of firms, the average value increases to 70% for the same period. Taking an average value of these two estimates, we use $\eta = 60\%$ in our baseline calibration. Appendix C.4 presents a robustness analysis for different values of η.

B.5 Data Summary

Table 7 presents the mean, standard deviation, number of observations, and the 25^{th}, 50^{th}, and 75^{th} percentiles of the key variables used in the empirical analysis and for calibration purposes. For firm-level observations, the top and bottom 1% have been removed to control for outliers. Firms born prior to 1996 are excluded from the tables and regressions. Firms born in 2007 are also excluded because we observe them only at age 0.

B.6 Entry by Industry

Table 8 presents two-year average entry rates by industry across time.

Variable	*Mean*	*SD*	*P25*	*P50*	*P75*
Profitability ($P_{i,t}$)	0.23	0.21	0.13	0.24	0.36
TFPR ($A_{i,t}$)	5.33	1.04	4.67	5.24	5.91
Real capital stock ($\log K_{i,t}$)	7.05	2.03	5.77	7.02	8.29
Real electricity consumption ($\log ElecCon_{i,t}$)	-0.00	1.78	-1.18	-0.20	0.99
Number of workers ($\log L_{i,t}$)	3.45	0.91	2.77	3.18	3.89
Number of workers at entry ($\log L_{i,0}$)	3.38	0.87	2.71	3.14	3.81
Firm age ($\log age_{i,t}$)	1.10	0.76	0.69	1.10	1.79

Table 7: Summary Statistics of Main Variables

Industry	*1996-1997*	*1998-1999*	*2000-2001*	*2002-2003*	*2004-2005*
311	11.59	5.31	4.22	10.23	7.33
312	13.37	3.59	6.26	9.96	8.55
313	11.27	7.68	9.07	12.63	19.71
321	9.61	3.59	4.10	6.90	6.30
322	15.54	6.11	5.49	12.86	5.67
324	7.07	5.10	4.14	5.73	3.01
331	10.15	6.77	6.25	13.67	6.89
332	18.25	7.71	11.18	13.06	12.29
341	12.70	5.32	6.87	9.03	8.19
342	6.98	5.19	8.13	20.24	6.44
351	9.73	8.86	7.05	7.38	14.84
352	10.26	5.00	5.10	10.69	5.82
355	5.54	6.97	2.50	8.94	8.75
356	8.04	4.90	5.22	12.82	8.64
369	11.69	13.11	9.86	7.12	8.41
381	13.12	4.38	8.76	13.60	9.84
382	10.76	5.25	7.83	17.31	11.42
383	9.18	5.95	8.75	13.89	7.02
384	9.49	4.53	3.64	9.66	10.40
390	22.22	4.66	10.54	7.51	8.89

Table 8: Entry Rate

B.7 Hausman and Taylor (1981)

The method can be summarized as a four-step procedure. First, a fixed effects regression delivers consistent estimators $\hat{\beta}_1$ and $\hat{\beta}_2$ that are used to retrieve estimators $\hat{u}_{i,t}$ and $\hat{\sigma}_u$. The second step is an instrumental variables (IV) regression with $\hat{u}_{i,t}$ as dependent variable, Z^1 and Z^2 as independent variables, and Z^1 and X^1 as instruments; this delivers a consistent estimator for $\tilde{\sigma}$ (the dispersion of the residual). Third, an estimator for the variance of the unobserved fixed effect component can be built as $\hat{\sigma}^2_\mu = \tilde{\sigma}^2 - \frac{\hat{\sigma}^2_u}{T}$, in order to form the usual generalized least squares (GLS) correction. Finally, the GLS correction is used to transform the original equation and estimate all the coefficients simultaneously in equation (34), using an IV procedure where the instruments are given by Z^1, the mean of X^1, and the deviations from the mean of X^1 and X^2. After every estimation we perform the Sargan-Hansen test to assess the validity of the instrumental variables procedure.

Table 9 presents the details of the regression results from the main text. In our regressions we use as time-variant exogenous variables ($X^1_{i,t}$) four macroeconomic aggregates: an index of manufacturing production, the unemployment rate, an index of wholesale producer prices, and an index of the cost of labor.[42] The coefficients associated with these variables are stable across the profitability regressions. The signs of the significant coefficients suggest that profitability is higher when production is high, labor costs are low, and inflation in producer prices is also low. There are four endogenous time-variant variables ($X^2_{i,t}$),: electricity consumption, number of workers, capital stock, and the age of the plant. We use five geographic regions and two-digit industry controls as time-invariant exogenous variables (Z^1_i). Besides the coefficients of interest, we include the initial size of the plant specified as the initial number of workers.[43] To control for competition at the moment of entry, we also include the Herfindahl-Hirschman concentration index of the industry at the particular region in the year of entry among the time-invariant endogenous variables (Z^2_i). In line with the firm dynamics literature, larger entrants are more profitable and more productive than smaller entrants. Finally, firms that enter into more concentrated industries are more profitable and more productive than firms facing more competition.

[42] Because this method relies on $X^1_{i,t}$ to build instruments, and because they are all aggregate variables, we cannot include year dummies, which are perfectly correlated with our instruments.

[43] The results do not change if the initial capital is used as the size measure.

	(1) $P_{i,t}$	(2) $P_{i,t}$	(3) $P_{i,t}$	(4) $A_{i,t}$	(5) $A_{i,t}$	(6) $A_{i,t}$
Crisis Born	0.0888*			0.591**		
	(0.0466)			(0.236)		
During Crisis		0.0866*			0.554***	
		(0.0466)			(0.199)	
After Crisis		0.0104			0.196*	
		(0.0232)			(0.110)	
Avg entry$_{j,0}$			-0.689**			-5.847***
			(0.325)			(1.547)
log Manu Prod$_t$	0.124***	0.120***	0.119***	0.0108	-0.00794	0.00487
	(0.0401)	(0.0400)	(0.0400)	(0.116)	(0.117)	(0.116)
Unemp Rate$_t$	0.205	0.191	0.204	0.0318	-0.0249	0.0241
	(0.156)	(0.157)	(0.157)	(0.435)	(0.436)	(0.434)
log PPI/WPI$_t$	-0.0857*	-0.0825*	-0.0856*	0.233*	0.246*	0.231*
	(0.0490)	(0.0490)	(0.0491)	(0.138)	(0.137)	(0.138)
log L Cost$_t$	-0.353*	-0.379**	-0.403**	-0.203	-0.335	-0.317
	(0.187)	(0.193)	(0.184)	(0.472)	(0.485)	(0.475)
log Elec Con$_{i,t}$	-0.00114	-0.00112	-0.00106	0.0623***	0.0625***	0.0624***
	(0.00243)	(0.00243)	(0.00242)	(0.00705)	(0.00707)	(0.00705)
log L$_{i,t}$	-0.0108	-0.0108	-0.0108	0.0289	0.0290	0.0288
	(0.00737)	(0.00738)	(0.00737)	(0.0207)	(0.0207)	(0.0207)
log K$_{i,t}$	0.00692***	0.00686***	0.00684***	-0.0571***	-0.0573***	-0.0573***
	(0.00265)	(0.00265)	(0.00265)	(0.00789)	(0.00790)	(0.00790)
log age$_{i,t}$	0.0107*	0.0123*	0.0148***	0.0134	0.0220	0.0222
	(0.00559)	(0.00637)	(0.00525)	(0.0190)	(0.0198)	(0.0182)
HHI$_{j,0}$	0.453***	0.425**	0.322***	2.608***	1.832*	1.476**
	(0.159)	(0.206)	(0.108)	(0.994)	(0.944)	(0.606)
log L$_{i,0}$	0.464***	0.437***	0.356***	3.119***	2.488***	2.274***
	(0.130)	(0.163)	(0.0789)	(0.754)	(0.753)	(0.429)
Ind. Control	Yes	Yes	Yes	Yes	Yes	Yes
Region Control	Yes	Yes	Yes	Yes	Yes	Yes
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Observations	16827	16827	16827	16814	16814	16814
Relative effect at means	-31.58	-31.61		-9.959	-9.583	
Sargan-Hansen (p)	0.416	0.201	0.109	0.134	0.126	0.106

The dependent variable in the first three specifications is calculated based on profitability $P_{i,t}$ of firm i in year t while the last three regressions are based on firm-level productivity $A_{i,t}$ (TFPR). The "Crisis Born" takes the value 1 for firm i if i has started the business in a crisis year, and it measures the cohort effect on the probability of becoming a superstar firm. "During Crisis" and "After Crisis" dummies distinguish firms born during (from 1998 through 2000) and after crisis years (after 2000). "Avg entry$_{j,0}$" is the average entry rate in the specific industry, in which firm i operates, in the year of i's entry. We include yearly controls for manufacturing production (log Manu Prod$_t$), unemployment rate (Unemp Rate$_t$), aggregate price index (log PPI/WPI$_t$), and labor cost (log L Cost$_t$). We also include time-varying firm-level variables for real electric consumption (log Elec Con$_{i,t}$), employment size (log L$_{i,t}$), real capital stock (log K$_{i,t}$), and firm age (log age$_{i,t}$). For each firm, we also add initial employment size (log L$_{i,0}$) and the Herfindahl-Hirschman index (HHI$_{j,0}$) in the year of the firm's entry for the industry in which the firm operates. In all regressions we include controls for the region and the industry. Relative effects are calculated at the mean values of the variables for the region and the industry with most observations. The statistic measures the percentage deviation in the dependent variable calculated at the means relative to the average cohort born during crisis years (a negative value implies a larger value of the dependent variable for the crisis cohort). Sargan-Hansen statistic tests the validity of overidentifying restrictions in Hausman and Taylor procedure. The null hypothesis is that the restrictions are valid. Standard errors are presented in parentheses (bootstrapped using 250 samples and clustered by firm). $\{^{*},^{**},^{***}\}$ denote significance at $p < 0.10$, $p < 0.05$, $p < 0.01$, respectively.

Table 9: Hausman and Taylor

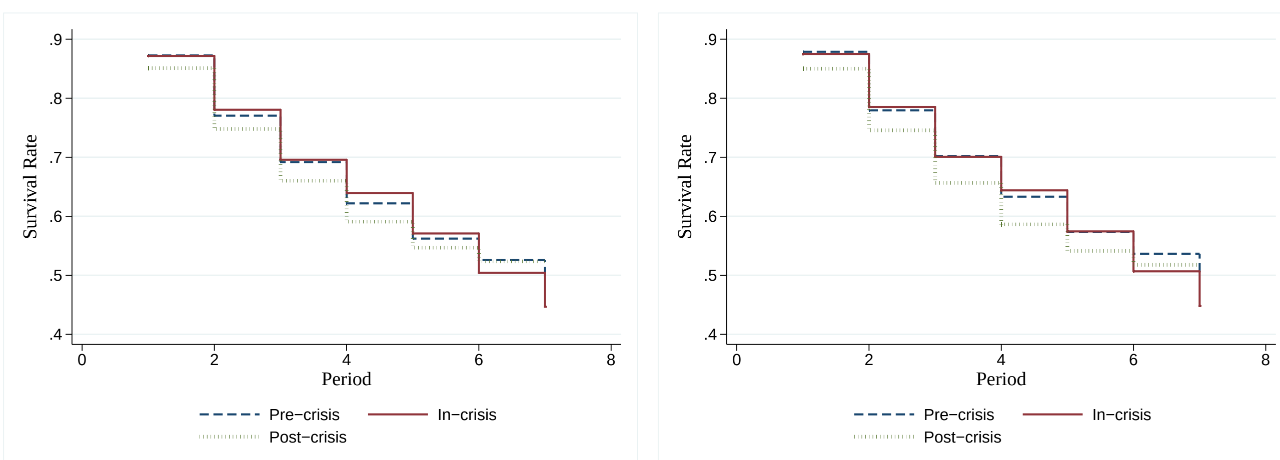
B.8 Cox Estimation

This section shows that the higher profitability of the cohorts born during the sudden stop is not due to *ex-post* selection. In particular, we perform the following stratified proportional hazard estimation to show that firms born during the crisis are not more likely to die at any horizon:

$$h_{r,c}\left(t|\boldsymbol{X}_i\right) \quad = \quad h_{0,r,c}\left(t\right)\exp\left[\boldsymbol{X}_i \cdot \boldsymbol{\beta}\right].$$

The two strata are geographical region (r) and time period (c). This means that the baseline hazard $h_{r,c}$ varies across these two dimensions. We divide Chile into five geographical regions. The time periods correspond to the *pre-crisis*, *crisis*, and *post-crisis* period of the second specification in the Hausman and Taylor estimation of Section 4. The Cox-Snell test cannot reject the proportional hazard structure with 95% confidence. Sub-index t refers to time, while i refers to a plant, and j to an industry. The following table shows the estimates of the common covariates.

Note that bigger plants have less probability of exiting (for both electricity consumption and number of workers), while the initial size increases the probability of exiting (for number of workers and electricity consumption). The specification controls for the industry cycle (using the average profitability of the industry $\bar{P}_{j,t}$ or the average productivity $\bar{A}_{j,t}$) and industry-specific effects. Figure 17 plots the survival rates at different horizons for cohorts born during the three different time periods in the central zone of Chile. We pick this zone because it concentrates most of the plants in the sample; the main message does not change when considering the other four regions.



(a) Profitability

(b) Productivity

Figure 17: Survival Rates, Cox Proportional Hazard Model

Note that firms born during the crisis do not exit more than other cohorts. Moreover,

	(1)	(2)
$\ln(L_{i,t})$	-0.546***	-0.548***
	(0.0708)	(0.0710)
$\ln(L_{i,0})$	0.451***	0.450***
	(0.0710)	(0.0711)
$\ln(elec_{i,t})$	-0.0758***	-0.0725***
	(0.0263)	(0.0264)
$\ln(elec_{i,0})$	0.0498**	0.0496**
	(0.0252)	(0.0253)
$\ln(K_{i,t})$	-0.0273	-0.0262
	(0.0247)	(0.0248)
$\ln(K_{i,0})$	-0.0356	-0.0332
	(0.0237)	(0.0238)
$\mathrm{P}_{j,t}$	0.0514	
	(0.188)	
$\mathrm{A}_{j,t}$		-0.138**
		(0.0578)
$\mathrm{HHI}_{j,t}$	-0.0942	-0.109
	(0.356)	(0.356)
Ind. Control	Yes	Yes
Observations	16546	16546
Plants	3780	3780
Exits	2026	2026
Hazard assumption test (p)	0.568	0.541

Stratified proportional hazard estimation using the following proportional hazard specification:

$$h_{r,c}\left(t|\boldsymbol{X}_i\right) \quad = \quad h_{0,r,c}\left(t\right)\exp\left[\boldsymbol{X}_i \cdot \boldsymbol{\beta}\right]$$

where the two strata are geographical region (r) and time period (c). This means that the baseline hazard $h_{r,c}$ varies across these two dimensions. We divide Chile into five geographical regions. The time periods correspond to the *pre-crisis* (before 1998), *crisis* (from 1998 through 2000), and *post-crisis* (after 2000). Independent variables include time-varying firm-level variables for real electric consumption ($\ln(elec_{i,t})$), employment size ($\ln(L_{i,t})$), real capital stock ($\ln(K_{i,t})$), and their respective initial values at the time of entry. We control for market concentration by including Herfindahl-Hirschman index ($\mathrm{HHI}_{j,t}$) for each industry across time. Along with industry fixed effects, we also include the average profitability ($\bar{P}_{j,t}$) or the average revenue productivity ($\bar{A}_{j,t}$) of the industry to control for industry-specific cycles. Hazard assumption test cannot reject the proportional hazard assumption.

Table 10: Proportional Hazard

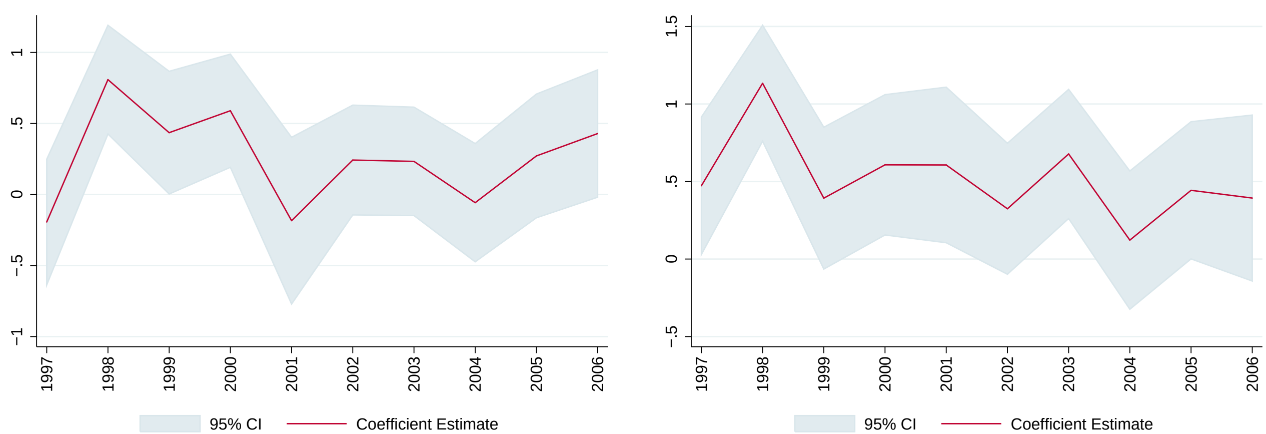
they even seem stronger in this dimension, in that until year 6, they have a higher predicted survival probability than firms born either before or after the episode. Hence, *ex-post* selection does not explain the higher profitability of cohorts born during the sudden stop.

B.9 Cohort Superstar Analysis

We estimate the probability of being a superstar firm using the following logit specification:

$$Pr(\text{Superstar} = 1|\text{age} = 1) = \frac{e^{x_i'\beta}}{1+e^{x_i'\beta}} \quad \text{where} \quad x_i'\beta = \alpha + \alpha_j + \alpha_r + \beta \ln(L_{i,0}) + \gamma_{\text{cohort}} + u_{i,t}, \tag{69}$$

where α_j is an industry control, α_r is a geographical control, and $L_{i,0}$ uses workers at entry to control for size. The cohort coefficient indicates the year that each firm was born. Figure 18 presents the results for the cohort coefficient.



(a) Profitability

(b) Productivity

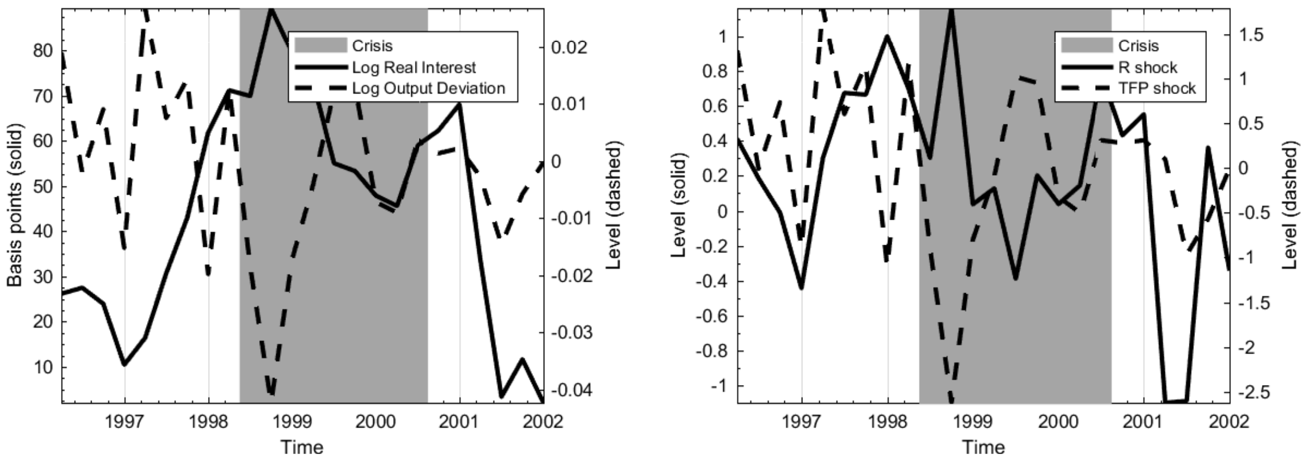
Figure 18: Logit by Cohort

Using profitability or productivity does not change the main result: More superstar firms are born during crisis.

C Quantitative Appendix

C.1 Data and Filtered Shocks

To back up shocks to TFP and the interest rate, we feed into the model two series obtained from the data. For the TFP process we use logarithmic difference of quarterly real output series over 1996:I-2011:II. We subtract the mean from the differenced log-series to remove the trend. The second series we use is the demeaned logarithm of quarterly real interest rate over the same period. Figure 19a shows the data used for the filtering procedure. The crisis is characterized by an increase of 80 basis points in the quarterly interest rate between the beginning of the Asian crisis and the Russian default and as well as a 4.5% drop in quarterly output. Figure 19b shows the filtered series for the interest and productivity innovations. Note that the sudden stop is explained by a negative productivity shock and a simultaneous positive interest rate shock.



(a) Data

(b) Filtered Shocks

Figure 19: Series used for shock decomposition and filtered shocks

C.2 Business Cycle Analysis

In this subsection we study the business cycle dynamics of the model. First, Table 11 shows the standard deviation and autocorrelation of the hp-filtered series of log-output, log-labor, log-consumption, and log-investment.[44] Note that only the standard deviation of output and investment are targeted in the calibration. The business cycle moments are broadly consistent with the behavior of the Chilean economy; the only exceptions are the excess smoothness in consumption and the lower persistence of labor. Second, because interest rate fluctuations play a fundamental role in this paper, we also compare the correlation of the main macro variables with the lagged interest rate. The final two columns of Table 12 shows that the model is consistent with the counter-cyclical interest rate of the Chilean economy. Table 12 also presents contemporaneous correlations with output. The signs are consistent with the data. Our model can easily be enriched with more stochastic forces and other preferences to improve on this dimension.

[44]The hp parameter is set to 1600, and growing variables are normalized in the model by the productivity level $A(s^t)$.

	AC data	AC model	STD data	STD model
y	0.770	0.714	0.020	0.020
L	0.590	0.721	0.017	0.014
c	0.750	0.734	0.027	0.017
inv	0.620	0.667	0.096	0.096

Table 11: Autocorrelation and Standard Deviations

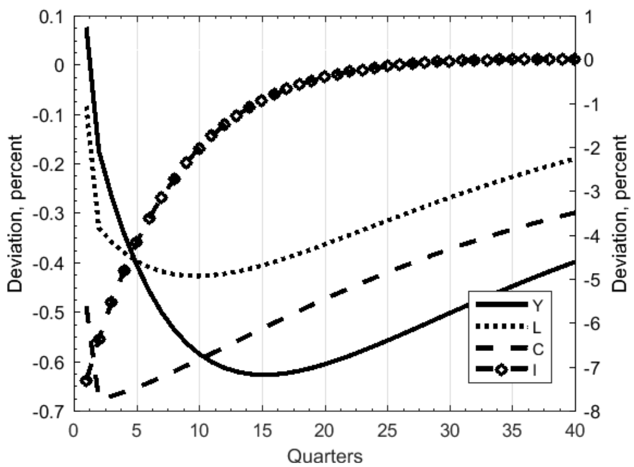
	corr(x,y) data	corr(x,y) model	corr(x,R(-1)) data	corr(x,R(-1)) model
y	1.000	1.000	-0.027	-0.114
L	0.592	0.980	-0.301	-0.279
c	0.201	0.886	-0.318	-0.429
inv	0.652	0.304	-0.452	-0.619

Table 12: Correlations with output and interest rate

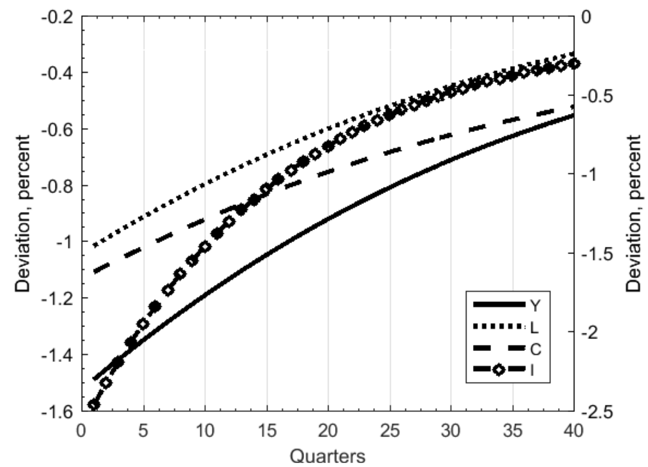
	TFP	R
c	0.836	0.164
y	0.835	0.165
L	0.795	0.205
inv	0.211	0.789
a	0.142	0.858
entry	0.192	0.808
ih	0.153	0.847
il	0.207	0.793
vh	0.574	0.426
vl	0.616	0.384

Table 13: Variance Decomposition

Table 13 shows the variance decomposition of the macro aggregates. The calibrated model is consistent with the evidence in Neumeyer and Perri (2005) and Uribe and Yue (2006) where interest rate fluctuations explain one-third of the fluctuations in Argentinian output. Because Chilean spreads are less volatile than Argentinian spreads, it is natural that interest rate fluctuations play a lower role with respect to Chilean output. Figure 20 shows the impulse response functions to a one standard deviation shock for the main macro variables.



(a) Macro Aggregates Response to R



(b) Macro Aggregates Response to Z

Figure 20: IRFs to R (left panel) and TFP (rigt panel) shocks

Figures 20a and 20b show that the responses of output, labor, consumption and investment (right axis) are aligned with the literature. Note that consumption responds more on impact to interest rate shocks than output but output responds more than consumption to stationary productivity shocks. In this sense, with a more volatile interest rate, the model would generate less smoothing in consumption. Note that none of the variables will return to its original long-run trend. In fact, for the case of the interest rate shock, the new path for these variables is permanently 0.1% lower. This hysteresis arises because of the permanent lost in the level of productivity pictured in Figure 11a.

C.3 Alternative Models

C.3.1 Model without Heterogeneity (NH)

The model with no heterogeneity eliminates firm types keeping the expansion decision of firms. This transformation is equivalent to setting $\sigma = \sigma^L = \sigma^H$ in the original model. The following two changes convert the baseline set of equations to the set of equations needed to characterize NH:

1. Any generic variable $\boldsymbol{x}^d$ has a single value; nad
2. composition variables in the economy are set to unity, i.e. $\mu = \tilde{\mu} = 1$.

Note that the problem of the financial intermediary is linear, and simplifies to a zero expected profit condition:

$$\mathbb{E}\left[m(s^{t+1})\left(1+a(s^t)\right)\bar{v}^L(s^{t+1})|s^t\right] = \left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)w(s^t)\kappa. \tag{70}$$

C.3.1.1 Calibration

We want to assess the permanent productivity loss estimated by a model with no heterogeneity. Therefore, we recalibrate the model to match a subset of the original moments. Note that NH has only one step size and no scarcity parameter (ν) therefore, we drop the mean and the standard deviation of the size distribution from the targets and re-calibrate the model. The measure of firms is fixed to the calibrated value in the baseline model. Table 14 shows the results of this exercise:

C.3.2 Model without Heterogeneity and Firm Dynamics (NDNH)

The NDNH economy goes one step further and eliminates the expansion decision of firms. In this sense, every firm has only one product, and firms remain in operation until they are replaced by an entrant. Therefore, NDNH is equivalent to NH without ι decision.

C.3.2.1 Normalized System of Equations

The following set of equations represents all the equations of NDNH that differ from the baseline economy.

Final Good Producer

Parameter	Symbol	Value	Main identification	Target	Model
Labor disutility level	Θ	31.62%	Working time	33.00%	34.01%
Entry Cost	κ	4.30%	Entry rate	11.30%	11.00%
Step Size	σ	5.86%	Annual GDP Growth	2.50%	2.56%
Expansion Cost scale	φ	20.76%	Share of labor of 10% larger firms	50.00%	51.32%
Stdev TFP	σ_z	0.79%	Quarterly output volatility (HP filtered)	1.98%	1.99%
Capital adjustment cost	ϕ	8.17	Quarterly investment volatility (HP filtered)	9.56%	9.62%

Table 14: Internally Calibrated Parameters

$$y(s^t) = \exp\left(z(s^t)\right) \cdot \left(l_p(s^t)\right)^{\alpha} \left(k(s^{t-1})\right)^{1-\alpha} \tag{71}$$

$$k(s^{t-1}) = \frac{(1-\alpha)y(s^t)}{r(s^t)} \tag{72}$$

Intermediate Good Producers

$$l_p(s^t) = \frac{\frac{\alpha}{\Lambda} y(s^t)}{w(s^t)(1+\sigma)\left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)} \tag{73}$$

$$\pi_j(s^t) = \frac{\alpha}{\Lambda}\frac{\sigma}{(1+\sigma)} y(s^t) \tag{74}$$

$$\bar{v}(s^t) = \pi(s^t) + \mathbb{E}\left[m(s^{t+1})\left(1+a(s^t)\right)\left(1-\Delta(s^t)\right)\bar{v}(s^{t+1})|s^t\right] \tag{75}$$

Financial Intermediary

$$\left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right) w(s^t)\kappa = \mathbb{E}\left[m(s^{t+1})\left(1+a(s^t)\right)\bar{v}(s^{t+1})|s^t\right] \tag{76}$$

$$\tilde{\mu}(s^t) = 1 \tag{77}$$

Aggregate Variables

$$a(s^t) = (1+\sigma)^{\frac{M(s^t)}{\Lambda}} - 1 \tag{78}$$

$$\mu(s^t) = 1 \tag{79}$$

$$\Delta(s^t) = \frac{M(s^t)}{\Lambda} \tag{80}$$

$$t(s^t) = \pi(s^t) - \left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right) M(s^t)\kappa w(s^t) \tag{81}$$

$$nx(s^t) = y(s^t) - c(s^t) - i(s^t) - \frac{\psi}{2} y(s^t)\left(\frac{b(s^t)}{y(s^t)}(1+a(s^t)) - \bar{b}(1+\bar{g})\right)^2 \tag{82}$$

$$d(s^t) = b(s^{t-1}) - \eta w(s^t) l(s^t) \tag{83}$$

$$l(s^t) = l_p(s^t) + \kappa M(s^t) \tag{84}$$

Parameter	Symbol	Value	Main identification	Target	Model
Labor disutility level	Θ	26.14%	Working time	33.00%	34.01%
Entry Cost	κ	29.91%	Entry rate	11.30%	11.00%
Step Size	σ	25.55%	Annual GDP Growth	2.50%	2.56%
Stdev TFP	σ_z	0.72%	Quarterly output volatility (HP filtered)	1.98%	1.99%
Capital adjustment cost	ϕ	7.85	Quarterly investment volatility (HP filtered)	9.56%	9.62%

Table 15: Internally Calibrated Parameters

C.3.2.2 Calibration

Compared with NH we drop φ and the share of labor of the 10% larger firms. Table 15 presents the result.

Note that compared with NH, the unique step size is five times larger. This result is due to the fact that the same entry rate needs to trigger the same growth rate but without incumbent dynamics. We can think of the step size in NDNH as a summary of all the innovations that an average entrant on NH would perform during its life cycle.

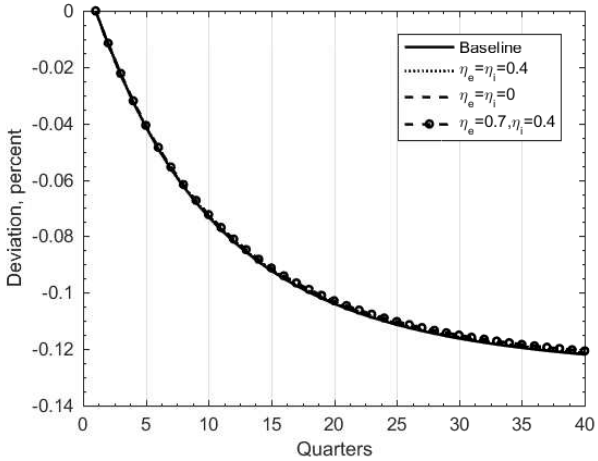
C.3.3 Model with Exogenous Growth

The economy with exogenous growth is characterized by the same set of equations as the baseline. However, expansion rates (ι^d) and entry mass (M) are taken as parameters and they are set to the balanced growth path. Thus, the equations that correspond to those variables are dropped from the system. Therefore, by construction, the parameters of Exo are the same as the baseline calibration. This model is practically analogous to the economy of Neumeyer and Perri (2005).

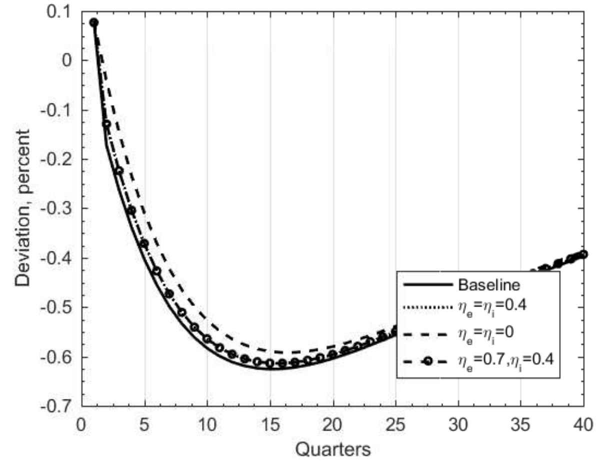
C.4 Robustness: Working Capital

To explore the role of the working capital constraint Figure 21 compares the baseline calibration of $\eta = 0.6$ to several alternatives. In particular, the dotted line represents an economy with a slightly lower level of working capital needs ($\eta = 0.4$), the dashed line is an economy with no working capital constraint ($\eta = 0$), and the dashed and dotted line represents an economy where the financial intermediary (entrants) face a tighter working capital constraint than the one faced by incumbents ($\eta_e = 0.7$ and $\eta_i = 0.4$).

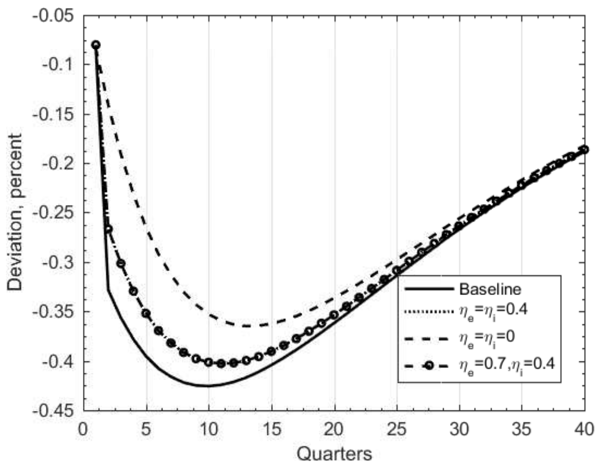
Figure 21a shows that the endogenous productivity component reacts very similarly in every economy to interest rate shocks. Because interest rates are the main driver of endogenous productivity, this similarity implies that our quantification of the permanent productivity loss of the Chilean sudden stop does not depends on the value of η. In fact, Figure 21d shows that every economy predicts the same long-run productivity loss. In contrast, because the working capital channel makes stationary interest rate shocks behave like productivity shocks we do see a difference in the short-run behavior of output in Figure 21b and employment in Figure 21c. In line with Neumeyer and Perri (2005), the larger the working capital channel is, the stronger the real short-run effects of interest rate shocks are. Interestingly, the economy where entrants are more constrained than incumbents behave very similarly to the economy where entrants and incumbents are equally constrained. This



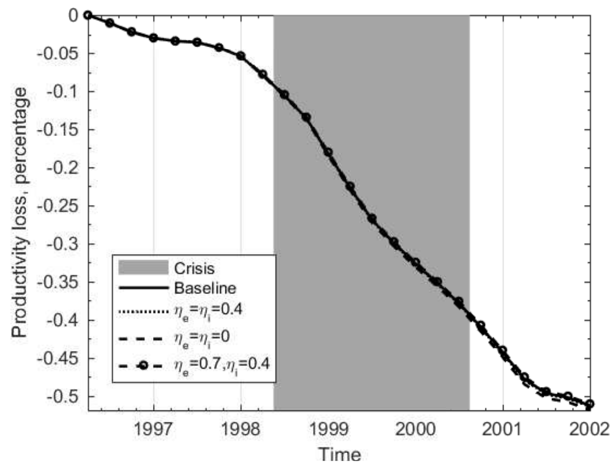
(a) Productivity Response to R



(b) Output Response to R



(c) Labor Response to R



(d) Prodcutivity loss

Figure 21: Impulse Response Functions to R and long-run productivity cost of the crisis

similarity is driven by the fact that because entrants have only one product, they therefore account for a very small portion of the economy wide labor. Finally, this outcome illustrates that the permanent productivity loss of sudden stop is not driven by the working capital constraint but by the effect that the interest rate has on innovation. This effect is driven by the pass-through of interest rate shocks to the value of varieties triggered by fluctuations in the stochastic discount factor.

C.5 Firm Size and Crises

Moreira (2015) uses US census data to document the following: i) fewer firms are born during downturns, ii) firms born during downturns are more productive, and iii) firms born during downturns grow more slowly than other cohorts. Because of the partial equilibrium result in Proposition 1, we might expect that this model is inconsistent with iii). To shed light on this point, Figure 22 shows the average size implied by the model for firms born before, during, and after the crisis at age five.

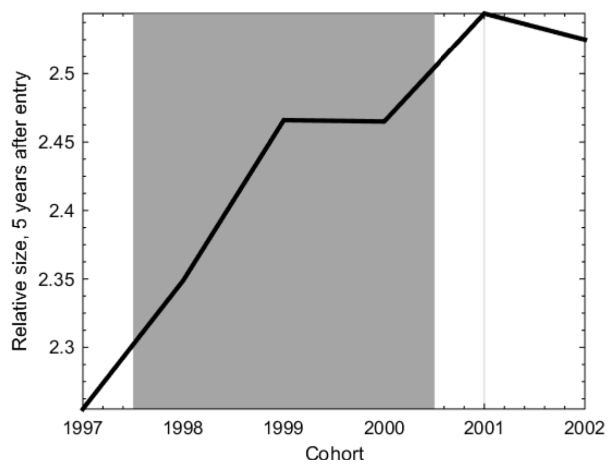


Figure 22: Average cohort employment 5 years after entry, relative to first year

Our baseline economy predicts that firms born during the peak of the crisis (1998) are smaller than almost all of the other cohorts at age five.[45] Note that expansion rates (ι^d) are common to every firm of type d regardless of its size or age. Moreover, ι^d is procyclical. Therefore, a d type firm with age T will be larger on expectation if most of those T years were expansions. For this reason, firms born in 1997 are, according to the model, the smallest at age 5. Note that the composition effect could be strong enough to overcome this force. In fact, high-type firms always expand faster than low-type firms; then, if cohorts born during crises have more high-type firms they could end up larger on average. However, this analysis considers older firms (five years old in this example), implying that most of the cohort is composed by high-type firms, because those are the ones that reach that age. Thus, the composition of cohorts born during booms and downturns is different at young ages, but the older a cohort gets, the higher the proportion of high types it has. Therefore, this model can generate all the facts documented by Moreira (2015) for the U.S. economy. Future research should explore a closed economy version of our economy and compare it to the U.S. firm-level dynamics.

[45] When we perform the empirical analysis on equation (34) using labor growth rate as the dependent variable we see that firms born during crises do not grow more quickly. Interestingly, when we use physical investment as a measure of growth, we see that firms born during the crisis accumulate capital more quickly. Because this analysis is beyond the scope of this paper, those regressions are available upon request.