Abstract

This paper examines the importance of productivity shocks in accounting for salient features of the U.S. economy during the second half of the 1990s, including the surge in investment spending, the substantial deterioration of the trade balance, and modest decline in inflation. We calibrate a two-country dynamic general equilibrium model and show that agents’ perceptions regarding the permanence of the shocks that occurred in the late 1990s are crucial in accounting for these developments. Within a signal extraction framework, we attempt to match survey data on long-term projected output growth. Our calibrated model can account for about two-thirds of the rise in the investment share of output, and over half of the deterioration in the trade balance over this period.

Keywords: Technology shocks, international business cycle, trade balance.
1 Introduction

In the late 1990s, the U.S. economy surprised observers by growing briskly with low inflation. This remarkable performance also coincided with an impressive rise in productivity growth. After having averaged 1.4 percent per year from 1973 to 1995, output per hour in the non-farm business sector rose almost 2.5 percent from 1996 to 2001.\(^1\) Surprisingly, other industrial countries did not experience this acceleration in productivity. At the same time, there was a sizeable real appreciation of the U.S. dollar and a sharp deterioration in the U.S. trade deficit, which declined from 1 percent of GDP in 1997 to over 4 percent of GDP in 2000.

A popular explanation for this deterioration in the trade balance is that it merely reflects the acceleration in U.S. productivity relative to other countries. However, based on the existing empirical literature, productivity growth would be an unlikely candidate to explain the magnitude of the deterioration in the trade balance. For example, the estimates of Glick and Rogoff (1995) predict that a 1 percent increase in U.S. productivity unmatched abroad would cause at most a 0.2 percentage point decline in the trade balance as a fraction of GDP. To explain the U.S. trade deterioration that occurred in the late 1990s based on this estimate, one would need an implausibly large rise in U.S. productivity relative to foreign.\(^2\)

In this paper, we argue that estimates based on historical data may be a poor guide to gauging the impact of the rise in productivity growth on the external sector. Based on historical data, there is little evidence of serial correlation in productivity growth and total factor productivity has generally been modelled as a random walk. When shocks to productivity growth are short-lived, one might expect that the impact on the trade balance should be limited. However, survey evidence suggests that while agents may have initially viewed the productivity acceleration of the late 1990s as transient, they eventually became convinced that a more sustained increase in productivity

\(^1\)Part of this discussion follows Gust and Marquez (2000).

\(^2\)Other more recent estimates of this elasticity are of similar magnitude. See Iscan (2000), Marquez (2002), and Gruber (2002).
growth had occurred.\textsuperscript{3}

We use this observation to show how shocks to productivity growth can indeed explain a larger deterioration in the trade balance than suggested by the existing empirical literature. In particular, we calibrate a two-country dynamic general equilibrium (DGE) model to show how the economy’s response to a rise in productivity growth is highly sensitive to agents’ perceptions about the degree of persistence of the shock process. We do this using a signal extraction framework in which agents cannot distinguish between permanent and temporary shocks to the growth rate of productivity.

We show that if agents had continued to regard the productivity acceleration in the late 1990s as transient, the observed rise in productivity growth would have only accounted for about 1/4 of the deterioration in the trade balance that occurred. By contrast, when our model is calibrated to match survey expectations, our model can account for about half of the trade balance deterioration.

We include in our model a number of features that have been emphasized in the closed-economy literature to bolster its empirical realism. As is standard in literature on the dynamic effects of monetary policy shocks, we allow for price and wage rigidities as well as adjustment costs for investment. We also adopt a habit persistence specification of preferences emphasized in the literature on asset market fluctuations.\textsuperscript{4}

On the open economy side, each of the two countries in our setup produces a traded good that is an imperfect substitute for the other country’s good. This allows us to be consistent with estimates from the existing literature on import and export trade elasticities as well as examine the reaction of the real exchange rate to productivity

\textsuperscript{3}See, for example, DeLong (2002) and Sichel and Oliner (2002) for a discussion about estimating trend productivity growth in light of developments in the second half of the 1990s.

\textsuperscript{4}For recent examples of papers emphasizing both nominal rigidities as well as adjustment costs on investment, see Chari, Kehoe, and McGrattan (2000), and Christiano, Eichenbaum, and Evans (2001). For papers with habit persistence, see, for example, Constantinides (1990), Sundaresan (1989), and Christiano and Fisher (1995).
shocks.\textsuperscript{5,6} Since variations in real exchange rates only explain a small fraction of trade flows at short run frequencies, we introduce adjustment costs for imports. One interpretation of these costs is that they capture the distribution costs associated with setting up import and export arrangements. Finally, while there are complete contingent markets within a country, agents can only imperfectly insure themselves against country-specific shocks.\textsuperscript{7}

It is important to emphasize that a number of the above features make it more difficult to explain the large deterioration in the trade balance by way of productivity shocks only. For example, without habit persistence, a positive innovation to productivity would lead to a greater initial response of consumption and hence imports; thereby, magnifying the deterioration in the trade balance. However, we do not exclude these features because our goal is to formulate an empirically realistic model on a wide variety of dimensions, which we can use to examine the dynamic effects that productivity shocks have on key macroeconomic variables.

The rest of this paper proceeds as follows. Section 2 discusses the stylized facts on which this study focuses. Section 3 outlines the model, while section 4 describes the solution method and calibration of model parameters. Section 5 presents the simulation results, and section 6 concludes.

## 2 Stylized Facts

Table 1 replicates a portion of a table published in IMF (2001) and shows the acceleration in labor productivity and total factor productivity in the late 1990s as measured by a number of different authors. As shown there, labor productivity growth picked

\textsuperscript{5}For estimates of multilateral trade elasticities for individual countries, see Hooper, Johnson, and Marquez (2000) and references therein.

\textsuperscript{6}Mendoza (1995) also examines movements in the real exchange rate and terms of trade but in the context of a model with both tradeable and non-tradeable goods.

\textsuperscript{7}As emphasized by Baxter and Crucini (1994) and Backus, Kehoe, and Kydland (1995), international real business cycle models with complete markets fail to account for the observation that there is a negative correlation between output and the trade balance. A feature that would be true in our framework as well if we had complete markets.
up substantially in the late 1990s, while total factor productivity (TFP) growth rose as well but the magnitude of the acceleration varies noticeably across the different estimates. These differences reflect both data and methodological differences. For instance, Gordon (2000), CEA (2001), and Basu, Fernald, and Shapiro (2001) adjust total factor productivity growth for the influence of cyclical factors, while the above estimates of Jorgenson and Stiroh (2000) and Sichel and Oliner (2002) do not.\footnote{Another important difference is that Basu, Fernald, and Shapiro (2001) make no adjustments for changes in the composition of capital and labor, while the other studies make such adjustments.}

While there is less agreement about the magnitude of the acceleration in underlying trend productivity, there is more agreement that that this acceleration reflects developments in the high-tech sector. In particular, as emphasized by a number of the papers listed in Table 1, the increasing use of computers and information technologies (IT) throughout the U.S. economy in the second half of the 1990s resulted in a rise in IT-related capital deepening, boosting labor productivity growth. Furthermore, these new technologies also contributed to faster TFP growth, since the rate of technological progress in producing semiconductors and other high-tech goods rose in the second half of the 1990s.

Evidence that these developments were initially viewed by agents as a temporary surprise is shown in Table 2. This table shows five-year ahead projections of U.S. GDP growth by the Congressional Budget Office and the consensus of Blue Chip forecasters beginning in 1995. Despite the pickup in productivity growth in the second half of the 1990s, 5-year ahead projections by the CBO remained largely unchanged until January of 2000. After that date, the GDP growth projections were revised up gradually, as views became more favorable about how much of the pickup in productivity growth was sustainable.

Figure 1 plots the movements of several key variables during this period of improved productivity growth. As shown there, GDP growth averaged nearly 4 percent throughout the second half of the 1990s (relative to its average of 2.7 percent over the 1980-95 period) before falling sharply in the recession of 2001. Inflation, as measured
Table 1: Acceleration in productivity in the late 1990s with respect to reference period\(^a,b\)

<table>
<thead>
<tr>
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<td>Labor Productivity</td>
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<td>1.0</td>
<td>0.9</td>
<td>1.6</td>
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</table>

Decomposition of Labor Productivity:

<table>
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<th></th>
<th>Capital Deepening</th>
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<th>Cyclical Effect</th>
<th>Price Measurement</th>
<th>Labor Quality</th>
<th>Reallocation</th>
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<tr>
<td></td>
<td>0.3</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
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<tr>
<td></td>
<td>0.5</td>
<td>0.6</td>
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<td></td>
<td>0.7</td>
<td>0.4</td>
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<td>-0.1</td>
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<td></td>
<td>0.4</td>
<td>1.2</td>
<td>0.0</td>
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<tr>
<td></td>
<td>-</td>
<td>1.9</td>
<td>-0.5(^c)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

\(^a\) See sources for data and methodological differences.
\(^b\) Dash indicates study does not make an adjustment for this factor.
\(^c\) Includes both utilization and adjustment cost corrections.
Table 2: 5-Year-Ahead Projections of Real GDP

<table>
<thead>
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<th>Date of Forecast</th>
<th>CBO</th>
<th>Blue Chip</th>
</tr>
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<tbody>
<tr>
<td>1995q1</td>
<td>2.3</td>
<td>2.5</td>
</tr>
<tr>
<td>1996q1</td>
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<td>2001q1</td>
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</tr>
<tr>
<td>2002q1</td>
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</tr>
</tbody>
</table>

Source: Congressional Budget Office.

by core consumer prices, was low in the latter 1990s and declined relative to inflation over the early 1990s.

Figure 1 also plots TFP growth in the United States less TFP growth in the other G7 countries, denoted here as the G6.\textsuperscript{9} As shown by this differential and as argued by Gust and Marquez (2000), the other major industrial countries did not experience a similar pickup in productivity growth in part due to a more subdued pace of adoption of information technologies in the G6 countries. As argued in Glick and Rogoff (1995), such a development in U.S. and foreign productivity growth should imply a deterioration in the U.S. current account. Evidence in support of this view is also shown in Figure 2, as the U.S. trade deficit went from around 1 percent of GDP in 1997 to over 4 percent of GDP in 2000, driven largely by rising imports.\textsuperscript{10}

Finally, the real exchange rate, as measured by a trade-weighted average of major trading partners, rose substantially in the latter 1990s.

\textsuperscript{9}TFP growth is defined here using national data on GDP as well as OECD data on hours per worker and capital stocks. The TFP growth for the G6 is computed as a GDP-weighted average of the TFP growth in the individual countries.

\textsuperscript{10}In Figure 1, we plot real trade, real imports, and real exports by real GDP in chain-weighted dollars instead of the more usual comparison of using nominal quantities.
Figure 1: Productivity Growth and Other Key U.S. Variables in the 1990s
Figure 2: The Trade Balance and Real Exchange Rate in the 1990s

- Trade Balance (percentage of GDP)
- Imports (percentage of GDP)
- Exports (percentage of GDP)
- Real Exchange Rate (1995q1=100)
On the whole, this evidence suggests that there was a noticeable pickup in the rate of technological progress in the late 1990s, which agents eventually viewed as more than a temporary phenomenon, and for the most part has remained unique to the United States. Such an interpretation is also consistent with the high output and low inflation of the latter 1990s as well as the large deterioration in the U.S. trade balance.

3 The Model

Our model consists of two countries that may differ in size, but are otherwise symmetric. Hence, our exposition below focuses on the “home” country. Each country in effect produces a single domestic output good, although we adopt a standard monopolistically competitive framework to rationalize stickiness in the aggregate price level. While household utility depends on consumption of both the domestic output good and imported goods, it is convenient to assume that a competitive distribution sector purchases both inputs, and simply resells a final consumption good to households. Similarly, we assume that competitive distributors combine the domestic output good with imports to produce a final investment good. Given this decentralization, the maximization problem faced by households appears isomorphic to the closed-economy case. We assume that households accumulate capital subject to adjustment costs and exhibit habit persistence in their consumption decisions. Households are regarded as monopolistic competitors in the labor market in order to account for aggregate wage stickiness. Finally, an important feature of our setup is that agents are not able to discern whether shocks to the growth rate of productivity are highly persistent or transitory.


3.1 Firms and Price Setting

Production of Domestic Intermediate Goods There are a continuum of differentiated intermediate goods $Y_t(f) \ (f \in [0, 1])$ produced in the home country. Each domestically-produced good $Y_t(f)$ is produced by a single monopolistically competitive firm. This firm faces a demand function that varies inversely with its output price $P_t(f)$ and directly with aggregate demand $Y_t$:

$$Y_t(f) = \left[ \frac{P_t(f)}{P_t} \right]^{-\frac{(1+\theta_p)}{\theta_p}} Y_t$$

(1)

Each producer utilizes capital services $K_t(f)$ and a labor index $L_t(f)$ (defined below) to produce its respective output good. The production function is assumed to have a constant-elasticity of substitution (CES) form:

$$Y_t(f) = \left( \omega_K^{\frac{\rho}{1+\rho}} K_t(f) \frac{1}{1+\rho} + \omega_L^{\frac{\rho}{1+\rho}} (Z_t L_t(f)) \frac{1}{1+\rho} \right)^{1+\rho}$$

(2)

The production function exhibits constant-returns-to-scale in both inputs, and technological progress $Z_t$ is assumed to be labor-augmenting (as required for balanced growth). Firms face perfectly competitive factor markets for hiring capital and the labor index. Thus, each firm chooses $K_t(f)$ and $L_t(f)$, taking as given both the rental price of capital $R_{Kt}$ and the aggregate wage index $W_t$ (defined below). Firms can costlessly adjust either factor of production. Thus, the standard static first-order conditions for cost minimization imply that all firms have identical marginal cost per unit of output $MC_t$.

We assume that the prices of the intermediate goods are determined by staggered nominal contracts of fixed duration. We assume that each price contract lasts four quarters, and that one-fourth of the firms reset their prices in a given period. As in Yun (1996), we assume that contract prices are indexed to the steady state inflation rate $\pi$; thus, for a firm which resets its price during period $t$, $P_{t+j}(f) = \pi^j P_t(f)$ for $j = 1, 2, 3$. The firm chooses the value of $P_t(f)$ which maximizes the firm’s discounted profits over the life of the price contract, subject to its product demand
curve (1):

\[ \tilde{E}_t \sum_{j=0}^{3} \psi_{t,t+j}(\pi^j P_t(f) Y_{t+j}(f) - MC_{t+j} Y_{t+j}(f)) \]  

(3)

The operator \( \tilde{E}_t \) represents the conditional expectation based the information available to agents at period \( t \).\(^{11}\) The firm discounts profits received at date \( t + j \) by the state-contingent discount factor \( \psi_{t,t+j} \); for notational simplicity, we have suppressed all of the state indices. Let \( \xi_{t,t+j} \) denote the price in period \( t \) of a claim that pays one dollar if the specified state occurs in period \( t + j \); then the corresponding element of \( \psi_{t,t+j} \) equals \( \xi_{t,t+j} \) divided by the probability that the specified state will occur.

The first-order condition for a price-setting firm is

\[ \tilde{E}_t \sum_{j=0}^{3} \psi_{t,t+j} \left( \frac{\pi^j P_t(f)}{(1 + \theta_p)} - MC_{t+j} \right) Y_{t+j}(f) = 0. \]

(4)

Production of the Domestic Output Index. Because households have identical Dixit-Stiglitz preferences, it is convenient to assume that a representative aggregator combines the differentiated intermediate products into a single domestic output index \( Y_t \):

\[ Y_t = \left[ \int_0^1 Y_t(f) \frac{1}{1 + \theta_p} \, df \right]^{1 + \theta_p} \]

(5)

where \( \theta_p > 0 \). The aggregator chooses the bundle of goods that minimizes the cost of producing a given quantity of the output index \( Y_t \), taking the price \( P_t(f) \) of each intermediate good \( Y_t(f) \) as given. The aggregator sells units of each sectoral output index at its unit cost \( P_t \):

\[ P_t = \left[ \int_0^1 P_t(f) \frac{1}{\theta_p} \, df \right]^{-\theta_p} \]

(6)

It is natural to interpret \( P_t \) as the price index of domestically-produced goods (or the GDP deflator).

\(^{11}\) For simplicity, none of the variables is explicitly indexed by the state of nature.
The domestic output index $Y_t$ either is sold to households to be used in the production of final consumption or investment goods, or it is exported:

$$Y_t = C_{D,t} + I_{D,t} + X_t$$

(7)

where $C_{D,t}$ and $I_{D,t}$ are the amount of the domestic index used in producing consumption and investment goods, respectively, and $X_t$ is the level of exports.

Production of Consumption and Investment Goods. Final consumption goods are produced by a representative “consumption good distributor.” This firm combines the domestic output index with imported goods to produce a final consumption good ($C_t$) according to a constant-returns-to-scale CES production function:

$$C_t = \left( \frac{\omega_{1c} \rho_c}{1 + \rho_c} C_{D,t}^{\frac{1}{1 + \rho_c}} + (1 - \omega_{1c})^{\frac{1}{1 + \rho_c}} \left( \varphi_{c,t} M_{c,t} \right)^{\frac{1}{1 + \rho_c}} \right)^{1 + \rho_c}$$

(8)

where $M_{C,t}$ is an index of imported goods, and $\varphi_{c,t}$ reflects costs of adjusting consumption imports. The form of the production function mirrors the preferences of households over consumption of domestically-produced goods and imports. Accordingly, the quasi-share parameter $\omega_{1c,t}$ may be interpreted as determining household preferences for home relative to foreign goods, or equivalently, the degree of home bias in household consumption expenditure. Finally, the adjustment cost term $\varphi_{c,t}$ is assumed to take the quadratic form:

$$\varphi_{c,t} = \left[ 1 - \frac{\varphi M_c}{2} \left( \frac{M_{C,t}}{C_{D,t}} \frac{M_{C,t-1}}{C_{D,t-1}} - 1 \right) \right]^{2^T}$$

(9)

This specification implies that it is costly to change the share of the imported good in total consumption.

Given the presence of adjustment costs, the representative consumption goods distributor chooses (a contingency plan for) $C_{D,t}$ and $M_{c,t}$ to minimize its discounted expected costs of producing the aggregate consumption good:

$$\min_{C_{D,t}, M_{C,t}} \sum_{k=0}^{\infty} \psi_{t,t+k} (P_{t+k} C_{D,t+k} + P_{M,t+k} M_{C,,t+k})$$

(10)
\[ P_{C,t} \left[ C_t - \left( \frac{\varphi_{ct}}{1 + \varphi_{ct}} \frac{1}{C_{D,t}} + (1 - \varphi_{ct}) \frac{1}{1 + \varphi_{ct}} (\varphi_{ct} M_{ct}) \right)^{1 + \rho_{ct}} \right] \]  

The distributor sells the final consumption good to households at a price \( P_{C,t} \), which may be interpreted as the consumption price index (or equivalently, as the shadow cost of producing an additional unit of the consumption good).

We model the production of final investment goods in a symmetric manner. Thus, the representative “investment goods distributor” produces a final investment good by combining the domestic output index with imported goods according to a constant-returns-to-scale CES production function:

\[ I_t = \left( \omega_{II} \frac{1}{I_{D,t}} + (1 - \omega_{II}) \frac{1}{1 + \rho_{II}} (\varphi_{II} M_{II}) \right)^{1 + \rho_{II}} \]

where \( M_{II} \) is an index of imported goods, and \( \varphi_{II} \) reflects costs of adjusting imports of investment goods. As in case of consumption goods, the quasi-share parameter \( \omega_{II} \) may be interpreted as the degree of home bias in the production of final investment goods. Given that the adjustment cost function is of the form:

\[ \varphi_{II} = \left[ 1 - \frac{\varphi_{M_{II}}}{2} \left( \frac{M_{II}}{I_{D,t}} - 1 \right) \right]^2 \]

Investment goods distributors solve an intertemporal cost minimization problem isomorphic to that of consumption goods distributors, c.f., (10). The distributor sells the final investment good to households at a price \( P_{II,t} \), which may be interpreted as the investment price index. Even in the absence of adjustment costs, the price index of consumption and investment goods may differ due to differences in import composition.

### 3.2 Households and Wage Setting

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the intermediate goods-producing sector (the only producers demanding labor services in our
framework). It is convenient to assume that a representative labor aggregator (or “employment agency”) combines households’ labor hours in the same proportions as firms would choose. Thus, the aggregator’s demand for each household’s labor is equal to the sum of firms’ demands. The aggregate labor index $L_t$ has the Dixit-Stiglitz form:

$$L_t = \left[ \int_0^1 (\xi N_t(h))^{\frac{1}{1+\theta_w}} \frac{1}{1+\theta_w} dh \right]^{1+\theta_w}$$  \hspace{1cm} (14)

where $\theta_w > 0$, $N_t(h)$ is hours worked by a typical member of household $h$, and $\xi$ is a constant scale factor determining the size of the average household (effectively, an index of the size of the population). The aggregator minimizes the cost of producing a given amount of the aggregate labor index, taking each household’s wage rate $W_t(h)$ as given, and then sells units of the labor index to the production sector at their unit cost $W_t$:

$$W_t = \left[ \int_0^1 W_t(h) \frac{1}{1+\theta_w} dh \right]^{-\theta_w}$$  \hspace{1cm} (15)

It is natural to interpret $W_t$ as the aggregate wage index. The aggregator’s demand for the labor services of a typical member of household $h$ is given by

$$N_t(h) = \left[ \frac{W_t(h)}{W_t} \right]^{\frac{1+\theta_w}{\theta_w}} L_t$$  \hspace{1cm} (16)

The utility functional of a typical member of household $h$ is

$$\bar{E}_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{1}{1-\sigma} \left( C_{t+j}(h) - \varpi C_{t+j-1}(h) \right)^{1-\sigma} + \right.$$  
$$\left. \frac{\chi_0}{1-\chi} (1-N_{t+j}(h))^{1-\chi} + \frac{\mu_0}{1-\mu} \left( \frac{M_{t+j}(h)}{P_{t+j}} \right)^{1-\mu} \right\}$$  \hspace{1cm} (17)

where the discount factor $\beta$ satisfies $0 < \beta < 1$. The dependence of the period utility function on consumption in both the current and previous period allows for the possibility of habit persistence in consumption spending (e.g., Christiano and Fisher (1995)). In addition, the period utility function depends on current leisure $1 - N_t(h)$, and current real money balances $\frac{M_t(h)}{P_t}$. 

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Each member of household $h$ faces a flow budget constraint in period $t$ which states that its combined expenditure on goods and on the net accumulation of financial assets must equal its disposable income:

$$P_{C,t}C_t(h) + P_{I,t}I_t(h) + MB_{t+1}(h) - MB_t(h) + \int_s \xi_{t,t+1}B_{D,t+1}(h)$$

$$-B_{D,t}(h) + e_tP_{B,t}B_{F,t+1}(h) - e_tB_{F,t}(h) = \bar{W}_t(h)N_t(h) + \Gamma_t(h) + T_t(h)$$

$$+R_{K}K_t(h) - 0.5\phi_KP_tK_t(h)(\frac{I_t(h)}{K_t(h)} - \delta)^2 - \frac{1}{2}\psi_tP_t\frac{(I_t(h) - I_{t-1}(h))^2}{I_{t-1}(h)}$$

$$= W_t(h)N_t(h) + \bar{T}_t(h) + R_{K}K_t(h) - \bar{0},$$

where

$$P_{C,t}, P_{I,t}, B_{D,t}, B_{F,t}, K_t, N_t, W_t, \delta, \psi_t, P_t, I_t, T_t, R_{K}, N_t, \bar{0}, \xi_{t,t+1}$$

are the prices, consumption, investment, capital stock, net worth, depreciation rate, financial assets, disposable income, capital stock, net worth, capital stock, financial assets, depreciation rate, financial assets, and disposable income, respectively.

Final consumption goods are purchased at a price $P_{C,t}$, and final investment goods at a price $P_{I,t}$. Investment in physical capital augments the (end-of-period) capital stock $K_{t+1}(h)$ according to a linear transition law of the form:

$$K_{t+1}(h) = (1 - \delta)K_t(h) + I_t$$

Financial asset accumulation of a typical member of household $h$ consists of increases in nominal money holdings $(MB_{t+1}(h) - MB_t(h))$ and the net acquisition of bonds. We assume that agents within a country can engage in frictionless trading of a complete set of contingent claims, while trade in international assets is restricted to a non-state contingent nominal bond. The term $\int_s \xi_{t,t+1}B_{D,t+1}(h) - B_{D,t}(h)$ represents net purchases of state-contingent domestic bonds. As noted above, $\xi_{t,t+1}$ represents the price of an asset that will pay one unit of domestic currency in a particular state of nature in the subsequent period, while $B_{D,t+1}(h)$ represents the quantity of such claims purchased by a member of household $h$ at time $t$. Thus, the gross outlay on new state-contingent domestic claims is given by integrating over all states at time $t + 1$, while $B_{D,t}(h)$ indicates the value of existing claims given the realized state of nature. The term $e_tP_{B,t}B_{F,t+1}(h) - e_tB_{F,t}(h)$ represents the net accumulation of the non-state contingent bond, measured in units of the home currency. The foreign currency price of a bond that pays one unit of the foreign currency in the subsequent period is $P_{B,t}$, $B_{F,t+1}(h)$ represents the quantity of such claims purchased at time $t$, and $e_t$ is the price of a unit of foreign currency in terms of the home currency (so that a rise in $e_t$ corresponds to a depreciation of the home currency).
Each member of household $h$ earns labor income $W_t(h) N_t(h)$, and receives gross rental income of $R_{K_t} K_t(h)$ from renting its capital stock to firms. Each member also receives an aliquot share $\Gamma_t(h)$ of the profits of all firms and a lump-sum government transfer, $T_t(h)$; we assume that the government’s budget is balanced every period, so that total lump-sum transfers are equal to seignorage revenue. Finally, we assume two types of costs associated with adjusting the capital stock. First, there is a cost associated with changing the net stock of physical capital, as in the standard q-theory literature; specifically, these costs depend on the square of the deviation of the investment-to-capital ratio from its steady state level level of $\delta$. Second, it is also costly to change the level of gross investment from the previous period, so that the acceleration in the capital stock is penalized. The quadratic functional form follows the specification in Christiano, Eichenbaum, and Evans (2001).

In every period $t$, each member of household $h$ maximizes the utility functional (17) with respect to its consumption, investment, (end-of-period) capital stock, money balances, holdings of contingent claims, and holdings of foreign bonds, subject to its labor demand function (16), budget constraint (20), and transition equation for capital (20).

Households set nominal wages in staggered contracts that are analogous to the price contracts described above. In particular, we assume that wage contracts last four periods, and that the households are divided into four cohorts of equal size. In each period, the households in one cohort renegotiate their wage contracts, while the nominal wages of all other households grow at the steady state level of inflation. Thus, for a typical member of household $h$ which resets its contract wage $W_t(h)$ during period $t$, $W_{t+j}(h) = \pi^j W_t(h)$ for $j = 1, 2, 3$. Each member of household $h$ chooses the value of $W_t(h)$ to maximize its utility functional (17), yielding the following first-order condition:

$$
\bar{E}_t \sum_{j=0}^{3} \beta^j \left( \frac{1}{1 + \theta_w^*} \frac{\Lambda_{t+j}^* \pi^j W_t(h) - \chi_0 (1 - N_{t+j}(h))^{-\chi}}{P_{t+j}^*} \right) N_{t+j}(h) = 0 \quad (21)
$$
where $\Lambda_t$ is the marginal value of a unit of consumption. Roughly speaking, equation (21) says that the household chooses its contract wage to equate the present discounted value of working an additional unit of time to the discounted marginal cost.

### 3.3 Monetary Policy

We assume that the central bank follows an interest rate reaction function similar in form to the historical rule estimated by Orphanides and Wieland (1998) over the Volcker-Greenspan period. Thus, the short-term nominal interest rate is adjusted so that the ex post real interest rate rises when inflation exceeds its constant target value, or when output growth rises above some target value. With some allowance for interest rate smoothing, monetary policy is described by the following interest rate reaction function:

$$i_t = \gamma_i i_{t-1} + \bar{r} + \gamma_x (\pi_t^{(4)} - \bar{\pi}) + \gamma_y (y_t - y_{t-1} - g_y) \tag{22}$$

where $\pi_t^{(4)}$ is the four-quarter average inflation rate of the GDP deflator (i.e., $\pi_t^{(4)} = \frac{1}{4} \sum_{j=0}^{3} \pi_{t-j}$), $\bar{r}$ is the steady-state real interest rate, $\bar{\pi}$ is the central bank’s constant inflation target, $y_t - y_{t-1}$ is the (annualized) quarterly growth rate of output, and $g_y$ is the target value of output growth.

### 3.4 Foreign Sector and Aggregate Resource Constraints

We assume that the structure of the foreign economy (the “rest of the world”) is isomorphic to that of the home country. Thus, foreign intermediate goods are combined into a foreign output index $Y_t^*$, and the production of foreign consumption and investment goods require both this foreign output index and imports from the “home” country. Since the imports of consumption and investment goods of the foreign economy ($M_{C,t}^*$ and $M_{I,t}^*$, respectively) must equal the total exports of the
home country \( (X_t) \), the resource constraint of the home country \((7)\) may be rewritten:

\[
Y_t = C_{D,t} + I_{D,t} + M^*_C + M^*_I
\]  

(23)

Moreover, because the “aggregator” selling the domestic output index behaves competitively in the product market, the law of one price holds for traded goods. In particular, the export price index is simply equal to the domestic output price index (i.e., \( P_{Xt} = P^*_t \) for the home country, and \( P^*_X = P^*_t \) for the foreign country), and the import price index of each country is equal to the export price index of its trading partner translated into local currency terms. Thus, in the case of the home country:

\[
P_{Mt} = e_t P^*_{X,t} = e_t P^*_t
\]  

(24)

while an analogous relation holds for the foreign country.

### 3.5 Technology Shocks and the Optimal Filtering Problem

We assume that the growth rate of productivity is the sum of two underlying components:

\[
\Delta Z_t = \Delta Z_{Pt} + \Delta Z_{Tt}
\]  

(25)

where variables are understood to be denoted in logarithms. In particular, shocks to productivity growth may be attributable either to a highly persistent component \( \Delta Z_{Pt} \) that shifts the “trend” level of productivity growth, or to purely transient shocks \( \Delta Z_{Tt} \). The bivariate process determining the evolution of each component may be represented as:

\[
\begin{bmatrix}
\Delta Z_{Pt} \\
\Delta Z_{Tt}
\end{bmatrix} =
\begin{bmatrix}
\rho_p & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta Z_{Pt-1} \\
\Delta Z_{Tt-1}
\end{bmatrix} +
\begin{bmatrix}
\varepsilon_{Pt} \\
\varepsilon_{Tt}
\end{bmatrix}
\]  

(26)

where the persistence parameter \( \rho_p \) is assumed to be less than unity (so that the growth rate of productivity returns to its steady state level in the long-run). For simplicity, it is assumed that the transient shock is i.i.d, so that an innovation to the
growth rate has a permanent effect on the level of productivity, but no effect on the future growth rate. Moreover, the innovations \( \varepsilon_{pt} \) and \( \varepsilon_{Tt} \) are mutually uncorrelated with variances \( v_1 \) and \( v_2 \), respectively.

Agents observe the current level of productivity in the economy, and hence observe \( \Delta Z_t \), but cannot observe the growth rate of the underlying components \( \Delta Z_{Pt} \) and \( \Delta Z_{Tt} \). Thus, agents solve a signal extraction problem to forecast the future level of productivity. Given that agents know the underlying parameters of the bivariate process for productivity growth, the Kalman filter can be used to obtain an optimal solution.

The expected level of productivity at a future date \( k \) periods ahead depends only on the current level of productivity, and on the expected growth rate of the permanent component:

\[
\mathbb{E}_t Z_{t+k} = Z_t + \mathbb{E}_t (Z_{Pt+k} - Z_{Pt}) = Z_t + \sum_{j=1}^{K} \mathbb{E}_t \Delta Z_{Pt+j} \tag{27}
\]

The Kalman filtering algorithm implies that the expected growth rate of the permanent component is updated according to

\[
\mathbb{E}_t \Delta Z_{Pt} = \rho_p \mathbb{E}_{t-1} \Delta Z_{Pt-1} + \frac{k_g}{\rho_p} (\Delta Z_t - \rho_p \mathbb{E}_{t-1} \Delta Z_{Pt-1}) \tag{28}
\]

Thus, agents update their assessment of the persistent component of the productivity growth process by the product of the forecast error innovation and a constant coefficient. This coefficient, which is proportional to the scalar Kalman gain parameter \( k_g \), is an increasing function of the signal-to-noise ratio \( \frac{v_1}{v_2} \) (the ratio of the variances of the persistent and transitory components of the productivity growth process). Finally, given the current estimate \( \mathbb{E}_t \Delta Z_{Pt} \), the optimal forecast of productivity growth \( J \) periods ahead is given by:

\[
\mathbb{E}_t \Delta Z_{Pt+j} = \rho_p^j \mathbb{E}_t \Delta Z_{Pt} \tag{29}
\]
4 Solution Method and Calibration

Because the level of technology is nonstationary, real variables (including output and the expenditure components of GDP) are also nonstationary. Accordingly, prior to solving the model, we scale all “trending” real variables in the home country by the level of home technology \( Z_t \), and all “trending” variables in the foreign country by \( Z_t^* \). Nominal variables are scaled to account both for growth in the corresponding real variable, and for the steady state inflation rate. By construction, the model is stationary in the transformed variables provided that home and foreign technology grow at the same rate.

We solve the model by log-linearizing the equations (specified in terms of the transformed variables) around the steady state associated with a common growth rate of technology in the two countries. To obtain the reduced-form solution of the model, we use the numerical algorithm of Anderson and Moore (1985), which provides an efficient implementation of the method proposed by Blanchard and Kahn (1980); see also Anderson (1997). 12

4.1 Calibration of Parameters

The model is calibrated at a quarterly frequency. Structural parameters are set at identical values for each of the two countries, except for the parameters of the exogenous process for the technology shocks, and the parameters determining population size (as discussed below). Thus, we assume that the discount factor \( \beta = .993 \), consistent with a steady-state annualized real interest rate \( r \) of about 3 percent. The intertemporal elasticity of substitution over consumption services \( \sigma \) is set equal to

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12The steady state around which we linearize depends on the relative level of technology in each country, which we initialize to unity (so that per capita income in each country is identical in the steady state, though GDP may differ across countries due to population differences). We evaluated the robustness of our solution procedure by using a nonlinear Newton-Raphson algorithm that does not rely on linearization around an initial steady state, and found that the results were nearly identical to those reported.
unity, as required for balanced growth. The utility function parameter $\chi = 3$. This implies a Frisch elasticity of labor supply of 1/3, which is considerably lower than if preferences were logarithmic in leisure, but well within the range of most empirical estimates. The utility parameter $\chi_0$ is set so that employment comprises one-third of the household’s time endowment, while the parameter $\mu_0$ on the subutility function for real balances is set an arbitrarily low value (so that variation in real balances has a negligible impact on other variables).

The depreciation rate of capital $\delta = .025$ (consistent with an annual depreciation rate of 10 percent). The price and wage markup parameters $\theta_P = \theta_W = 0.20$, similar to the estimated values obtained by Rotemberg and Woodford (1997) and Amato and Laubach (1999). The parameter $\rho$ in the CES production function of the intermediate goods producers is set to -5/4, implying an elasticity of substitution between capital and labor of 1/5. Thus, capital and labor are considerably less substitutable than the unitary elasticity implied by the Cobb-Douglas specification. The quasi-capital share parameter $\omega_K$ is chosen to imply a steady state investment to output ratio of 25 percent, and consumption to output ratio of 75 percent. We set the steady state inflation rate $\pi$ to yield an annual inflation rate of four percent. We set the cost of adjusting investment parameter $\phi_I = 2$ (in our baseline, we simply set the cost of adjusting capital parameter $\phi_K = 0$).

We parameterize the monetary policy rule based on estimates derived from Orphanides and Wieland over the 1980:1-1996:4 sample period. Hence, we set $\gamma_\pi = 0.625$ in the monetary policy reaction function (22), and the inertia coefficient $\gamma_i = 0.8$. Since these authors estimate a coefficient on the output gap and lagged output gap of roughly one and minus one, respectively, we simplify their rule by imposing a coefficient of unity on the quarterly growth rate of output.

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13Rotemberg and Woodford (1997) found $\theta_P = 0.15$, while Amato and Laubach (1999) obtained $\theta_P = 0.19$ and $\theta_W = 0.13$. Given our assumption that there is perfect capital mobility across firms within a country, the parameter $\theta_P$ only affects the steady state capital-output ratio, and does not otherwise appear in the dynamic equations of the log-linearized model.
The parameter $\omega_{1C}$ is chosen to match the estimated average share of imports in total U.S. consumption of about 7 percent (over the 1983-2000 period); while the parameter $\omega_{1I}$ is chosen to match the average share of imports in total U.S. investment of about 30 percent. Given that trade is balanced in the steady state, this parameterization implies an import or export to GDP ratio for the home country (the United States) of about 13 percent. Since the population parameters $\xi$ and $\xi^*$ are chosen such that the home country constitutes about 25 percent of world output, the implied import (or export) share of output of the foreign country is about 3 percent. We assume that $\rho_{C} = \rho_{I} = 1/2$, consistent with a long-run price elasticity of demand for imported consumption and investment goods of 3. While this is higher than most empirical estimates, we emphasize that the presence of adjustment costs translates into a much lower relative price sensitivity in the short to medium-term. In particular, we set the adjustment cost parameters $\varphi_{MC} = \varphi_{MI} = 10$, implying a price-elasticity of only about unity after $yy$ quarters.

Finally, we discuss below our setting for the parameters of the technology shocks process.

5 Results

In this section, we analyze the behavior of the calibrated model in response to a shock that raises the growth rate of labor-augmenting technical change ($Z_t$) by one percentage point (at an annual rate) over the entire simulation horizon of 20 quarters. As discussed below, this exogenous shock raises labor productivity growth by around one percentage point per year \textit{on average} over the simulation period, and is thus similar in magnitude to the rise in labor productivity observed in the late 1990s.
5.1 Baseline Calibrated Model

Figure 3 compares the responses of key variables to the productivity growth rate shock under alternative assumptions about the information structure. We begin by analyzing the effects of the shock under the assumption that agents have full information, and hence correctly ascertain that the shock will have highly persistent effects on the future growth rate of productivity. The shock raises output growth (here measured by the four quarter change in output) by about 1 percentage point per year over the simulation horizon. Because the shock to the growth rate eventually dies out, expected productivity growth over longer horizons is lower than productivity growth in the current period; we calibrate the persistence parameter $\rho_p = .975$, so that productivity growth five years ahead is projected to rise by slightly over 1/2 percentage point. This setting is roughly in line with the evidence presented in Section 2 showing that agents appeared to adjust their forecasts of long-run output growth by roughly half the magnitude of the pickup in current output growth.

Because the productivity shock is perceived to be heavily “back-loaded” in this case – in the sense that the shock is expected to raise the future level of productivity much more than current productivity – the shock has very strong effects on domestic demand. With a sharply rising projected future income profile, households desire to markedly reduce their savings: in Figure 3, this is reflected in much faster growth in consumption than output in the two years following the shock. The rise in the expected marginal product of capital induces a strong increase in investment spending, with the magnitude of the initial investment response about ten times larger than the output response.

With domestic short-term real interest rates increasing relative to foreign short-term real rates, the terms of trade initially appreciate considerably. The appreciation of the terms of trade induces a substantial fall in the export share of GDP, which falls nearly 1 percentage point by two years after the shock. Moreover, the import share of GDP also ratchets up markedly due to both the boom in domestic demand,
and to the terms of trade appreciation. The shock induces a deterioration in the real trade balance equal to nearly 2 percent of GDP after eight quarters.

We now consider the alternative information assumption that agents perceive that the variance of the permanent innovation to technology is small relative to the variance of the temporary component (i.e., \( \nu_1 \) is small relative to \( \nu_2 \)). In this case, represented by the solid line in Figure 3, agents mistakenly expect productivity growth shocks to be transitory. Accordingly, expected productivity growth at all future horizons is unaffected by the shock (Figure 3 shows only that longer-term growth is unaffected). In this case, domestic demand rises much less sharply than in the case in which productivity growth is expected to rise persistently, in part because the less steeply rising output profile induces a smaller rise in consumption. GDP price inflation actually falls somewhat in this case, as the effect of rising productivity more than offsets upward pressure on unit labor costs due to wage increases. By contrast, inflation rises slightly in the case of perfect information: even with modest nominal wage rigidity, wages increase enough in response to expectations of future productivity growth to push up unit labor costs.

The effects of the productivity shock on the trade balance are also much smaller in this case. In particular, the trade balance deteriorates by only about 0.5 percent of GDP after eight quarters, or one quarter as much as under perfect information. The smaller deficit in part reflects that the import share of GDP rises much less due to the smaller stimulus to domestic demand. In addition, the terms of trade are basically unchanged initially in this case (since short-term real rates rise by less), also serving to cushion the fall in exports, and to reduce the rise in the import share.

Finally, we consider an information structure in which agents attach a non-negligible weight to the innovations of each type. In particular, we set the relative variance of the innovations to each component to technology such that agents gradually raise their assessment of long-run productivity growth over the 20 quarter forecast horizon. In this case, the responses of key variables look like a weighted average of the two
extreme cases: initially, the responses look very similar to the case in which agents perceive shocks to be temporary. Eventually, agents revise upward their assessment of the persistence of the shock to productivity growth, and the responses look similar to the case in which agents perceive the shock as completely permanent. Taken together, this “mixed” case appears to account well for a number of developments that occurred in the late 1990s. In this case, the trade balance deteriorates by about 1-1/4 to 1-1/2 percent of GDP between two and three years after the occurrence of the shock, or roughly half the magnitude of the nearly 3 percent deterioration in the trade balance that occurred between 1997 and 2000. About 2/3 of the deterioration in the trade balance implied by the model is accounted for by a shift in the ratio of investment to output, again reasonably in line with the data. Moreover, the gradual adjustment of both domestic demand and external variables in response to the shock in this case seems more in line with the data than the jumps associated with the case in which agents immediately perceive the true nature of the shock.

5.2 Sensitivity Analysis

The ability of our model to account for a large trade deficit in part reflects our specification of the production function as CES in capital and labor, with a much lower elasticity of substitution between capital and labor than implied by the Cobb-Douglas case. The lower elasticity is important because it has the effect of damping the sensitivity of investment to the real interest rate.

To assess the importance of the CES form, Figure 4 shows alternative parameterizations of the model in which the interest sensitivity of investment is varied by changing the CES parameter. We report the case in which the information structure involves a perceived mix of permanent and temporary shocks, and keep all other parameters at their baseline values (including the monetary policy rule). Interestingly, while investment rises in the Cobb-Douglas case, the investment share of output actually falls, inducing a much smaller expansion of the trade deficit.
Figure 5 analyzes the sensitivity of our results to the long-run price elasticity of import demand, which is set equal to 3 for both imports and exports in our baseline calibration. While our results are only slightly sensitive to a modest increase in the price elasticity, they are very sensitive to a lower elasticity of demand. In particular, with a low price elasticity in the range of unity, our model with differentiated goods would imply an improvement in the trade balance (as the export share rose slightly, and as the import share fell). However, this specification would imply that countries with particularly rapid productivity growth would be expected to experience a sharp secular decline in their ratio of real imports to GDP, and a deterioration of their terms of trade. Hence, we find a specification that imposes a somewhat higher long-run elasticity while forcing a reasonable short-run price elasticity through the incorporation of adjustment costs to be more reasonable.

6 Conclusions

References


Figure 3. Rise in Productivity Growth Rate (1 percent per year)
Different Perceptions about Persistence of Shock

Output Growth

Expected Productivity Growth in 5 Years

GDP Price Inflation (AR)

Terms of Trade

- Perceived as temporary
- Temporary–permanent Mix
- Perceived as Permanent
Figure 3. Rise in Productivity Growth Rate (1 percent per year)
Different Perceptions about Persistence of Shock
Figure 4. Rise in Productivity Growth Rate (1 percent per year)

Alternative Interest Elasticities of Investment

Private Consumption Growth

Fixed Investment Growth

Exports Share of GDP

Import Share of GDP

- Baseline Elasticity (−1/5)
- Higher Elasticity (−1/3)
- Cobb–Douglas (−1)
Figure 5. Rise in Productivity Growth Rate (1 percent per year)
Alternative Long–Run Price Elasticities of Import Demand

Private Consumption Growth

Fixed Investment Growth

Exports Share of GDP

Import Share of GDP

Baseline Elasticity (−3)  
Higher Elasticity (−5)  
Cobb–Douglas (−1)