Monetary Policy and the Equity Premium*

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Abstract

We develop a DSGE model in which monetary policy generates endogenous movements in risk. The key feature of our model is that households rebalance their financial portfolio allocations infrequently, as they face a fixed cost of transferring cash across accounts. We show that the model can account for the mean returns on equity and the risk-free rate, and generates countercyclical movements in the equity premium that help explain the response of stock prices to monetary shocks. While stimulative monetary policy can lower risk in equity markets, it is also associated with higher inflation risk premia in our model. The model gives rise to periods in which the zero lower bound constraint on the nominal interest rate binds and demand for liquidity surges. Although this constraint binds only occasionally, the mere possibility of its occurrence gives rise to a precautionary demand for money, leading to procyclical movements in velocity.

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1 Introduction

Monetary policy primarily affects the macroeconomy through its effect on financial markets. In standard monetary models, this interaction between the financial and real sides of the economy occurs through short-term interest rates, as changes in monetary policy affect the conditional mean of the short-term interest rate which in turn influences macroeconomic variables such as output, employment, and inflation. These models, however, abstract from another channel through which monetary policy affects financial markets and the macroeconomy. In these models, there is little or no role for monetary policy to influence the conditional variances of variables or the perceived riskiness of the economy.[1] In contrast, Bernanke and Kuttner (2005) provide evidence that monetary policy does affect risk, suggesting that standard monetary models are potentially missing an important channel through which monetary shocks propagate from the financial to the real economy. They show that, while an unanticipated easing of monetary policy lowers real short-term interest rates, it also has a significant effect on equity returns occurring through a reduction in the equity premium.

In this paper, we develop a DSGE model in which monetary policy affects the economy through the standard interest rate channel and through its effect on economic risk. The key feature of our model is that asset and goods markets are segmented, because it is costly for households to transfer funds between these markets. Accordingly, they may only infrequently update their desired allocation of cash between a checking account devoted to purchasing goods and a brokerage account used for financial transactions. The optimal decision by an individual household to rebalance their cash holdings is a state-dependent one, reflecting that doing so involves paying a fixed cost in the presence of uncertainty. Households are heterogenous in this fixed cost, and only those households that rebalance their portfolios during the current period matter for determining asset prices. Because the fraction of these household changes over time

[1] See Alvarez, Atkeson, and Kehoe (2007a) for an extended discussion of this point. In a companion paper, Alvarez, Atkeson, and Kehoe (2007b) argue that the evidence on the relationship between interest rates and exchange rates is consistent with movements in these variables that are driven mainly by changes in conditional variances.

in response to both real and monetary shocks, risk in the economy is both time-varying and endogenous.

We show that our model, unlike standard monetary models, generates countercyclical movements in the equity premium and in particular, a reduction in the equity premium resulting from a monetary easing. For reasonable calibrations of the model, the reduction in the equity premium is an important determinant of the response of stock prices to a monetary policy shock. While stimulative monetary policy can lower risk in equity markets, it is also associated with higher expected inflation and a small increase in inflation risk premia.

The model also gives rise to periods in which the demand for liquidity surges. In very bad states of the world, risk spreads rise, inducing to a "flight to safety" by investors. This drives down the economy's risk-free real rate so that the nominal interest rate hits its zero lower bound constraint, creating an incentive to carry excess cash. This extra liquidity translates into a fall in velocity. Although the ZLB binds only occasionally, the mere possibility of its occurrence gives rise to a precautionary motive for money, leading to procyclical movements in velocity.

We also examine the model's ability to account for the mean returns on equity and the risk-free rate. As shown by Mehra and Prescott (1985), standard representative agent models with power utility have difficulty quantitatively accounting for these moments. For reasonable calibrations of monetary and technology shocks, our model is able to match the observed means on equity and risk-free rates with a power utility function that implies constant relative risk aversion equal to two.[2] We show that to match these moments, the average fraction of households that reallocates funds across markets can not be too large. For our benchmark calibration, about 20 percent of households, on average, rebalance their portfolios in a quarter. Underlying this average fraction of rebalancing, there is a considerable degree of heterogeneity across households in our model, with some rebalancing every period and another fraction rarely rebalancing away from their initial allocation.

Recent microdata on household finance provides strong support for infrequent portfolio re-

[2]In a companion paper, Gust and López-Salido (2009), we focus on the asset pricing implications of the endogenous rebalancing model.

balancing. For instance, in two recent papers, Calvet, Campbell, and Sodini (2008) and Calvet, Campbell, and Sodini (2009) document that, while there is little rebalancing of the financial portfolios of stockholders by the average household, there is a great deal of heterogeneity at the micro level with some households rebalancing these portfolios very frequently. In addition, using information on asset holdings from the PSID, Bilias, Georgarakos, and Haliassos (2008) and Brunnermeier and Nagel (2008) provide evidence that household portfolio allocation display substantial inertia. Surveys conducted by the Investment Company Institute (ICI) and the Securities Industry Association (SIA) also suggest that households rebalance their portfolios infrequently. For instance, in 2004, the median number of total equity transactions for an individual was four. In addition, sixty percent of equity investors did not conduct any equity transactions during 2004. Finally, in a 2005 survey, the ICI reports that more than two-thirds of the time the proceeds from the sales of stocks by households are fully reinvested.[3]

Our model is most closely related to and builds on the analysis of Alvarez, Atkeson, and Kehoe (2007b). They introduce endogenously segmented markets into an otherwise standard cash-in-advance economy and show how changes in monetary policy can induce fluctuations in risk. However, our model differs from theirs in two important respects. First, we incorporate production and equity returns. Second and more importantly, in their model, risk is endogenous, because the fraction of households that participates in financial markets is state-dependent. In our model, all households participate in financial markets, but it is costly to reallocate cash between the asset and goods markets from a household's initial allocation. In other words, endogenous asset segmentation occurs along an intensive margin in our model rather than an extensive margin. This distinction is important, because we show that for reasonable calibrations this model can not match the average equity premium and monetary policy shocks have no effect on the equity premium.[4]

[3]See, Figure F.7, "Disposition of Proceeds from most recent sale of individual stocks", in ICI 2005.

[4]Polkovnichenko (2004), Vissing-Jorgensen (2002), Vissing-Jorgensen (2003), and Gomes and Michaelides (2006) also show that it is difficult to match the equity premium in a model with endogenous stock market participation. Guvenen (2005) considers a model in which stock market participation is fixed exogenously and shows that this feature in addition to heterogeneity in preferences can help account for the average equity

Our paper is also related to portfolio choice models that emphasize infrequent adjustment. In this literature, the paper most closely related to ours is Abel, Eberly, and Panageas (2007).[5] They also model infrequent adjustment of cash between a transaction account used to purchase goods and another account used to purchase financial assets. Their framework differs from ours, since they do not consider the role of monetary policy and use a partial equilibrium framework in which returns are exogenous. However, they show that infrequent portfolio adjustment can arise due to rational inattention on the part of households.

The rest of this paper proceeds as follows. The next section describes the model and its calibration. Section 3 presents the results, emphasizing the model's ability to match unconditional moments such as the mean returns on equity and a risk-free asset as well as the conditional responses of these variables to a monetary policy shock. Section 4 concludes and discusses directions for future research.

2 The Model

The model builds on the cash-in-advance economy of Alvarez, Atkeson, and Kehoe (2007b), which we extend to incorporate equity prices. Our approach differs from theirs, since we emphasize that time-varying risk is driven by costly portfolio rebalancing of financial accounts rather than limited participation in financial markets.

The economy is populated by a large number of households, firms, and a government sector. Trade occurs in financial and goods markets in separate locations so that they are segmented from each other. After choosing an initial non-state contingent plan that allocates funds across asset and goods markets, households must pay a fixed cost to make state contingent transfers between these markets. This fixed cost is constant over time but varies across households. We refer to a household's cash balances in the goods market as his checking account, and his cash balances in the asset market as his brokerage account. An active household is one that pays his

premium.

[5] Also, see Lynch (1996), Marshall and Parekh (1999), Gabaix and Laibson (2001), and more recently, Bacchetta and van Wincoop (2009).

fixed cost and rebalances cash across these two accounts, and an inactive household does not.

There are two sources of uncertainty in our economy — aggregate shocks to technology, θ_t, and money growth, μ_t. We let $s_t = (\theta_t, \mu_t)$ index the aggregate event in period t, and $s^t = (s_1, ..., s_t)$ denote the state, which consists of the aggregate shocks that have occurred through period t.

2.1 Firms

There is large number of perfectly competitive firms, which each have access to the following technology for converting capital, $K(s^{t-1})$, and labor, $L(s^t)$, into output, $Y(s^t)$ at dates $t \geq 1$:

$$Y(s^t) = \exp(\theta_t) K(s^{t-1})^{\alpha} L(s^t)^{1-\alpha}. \tag{1}$$

We assume that the technology shock, θ_t follows a first-order Markov process. Capital does not depreciate, and there exists no technology for increasing or decreasing its magnitude. We adopt the normalization that the aggregate stock of capital is equal to one. Labor is supplied inelastically by households, and its supply is normalized to one.

Firm production begins at date 1. Following Boldrin, Christiano, and Fisher (1997), we assume that firms have a one-period planning horizon. To operate capital in period $t+1$, a firm must purchase it at the end of period t from those firms operating during period t. To do so, a firm issues equity $S(s^t)$ at dates $t \geq 0$, and purchases capital subject to its financing constraint,

$$P_k(s^t) K(s^t) \leq S(s^t), \tag{2}$$

where $P_k(s^t)$ denotes the price of capital and $K(s^0)$ is given.

A firm derives revenue from its sale of output, $P(s^{t+1})Y(s^{t+1})$, and the sale of its capital stock, $P_k(s^{t+1})K(s^t)$, at the end of period $t+1$. A firm's expenses include its obligations on equity, $(1 + R^e(s^{t+1}))S(s^t)$, and payments to to labor, $W(s^{t+1})L(s^{t+1})$. A firm's net revenues, $V(s^{t+1})$, including its expenses, must be greater than zero in each state so that:

$$V(s^{t+1}) = P(s^{t+1})Y(s^{t+1}) + P_k(s^{t+1})K(s^t) - (1 + R^e(s^{t+1}))S(s^t) - W(s^{t+1})L(s^{t+1}) \geq 0. \tag{3}$$

The firm's problem at date $t+1$ is to maximize $V(s^{t+1})$ across states of nature by choice of $K(s^t)$ and $L(s^{t+1})$ subject to (1) and (2). This problem implies that the financing constraint (2) is satisfied as a strict equality in equilibrium. The equilibrium real wage, $w(s^{t+1})$ is given by:

$$w(s^{t+1}) = \frac{W(s^{t+1})}{P(s^{t+1})} = (1-\alpha)\frac{Y(s^{t+1})}{L(s^{t+1})}, \tag{4}$$

Linear homogeneity of the firm's objective, together with the weak inequality in equation (3) imply that $V(s^{t+1}) = 0$ for all s^{t+1} so that:

$$1 + r^e(s^{t+1}) = \frac{1 + R^e(s^{t+1})}{\pi(s^{t+1})} = \frac{\left[\alpha\frac{Y(s^{t+1})}{K(s^t)} + p_k(s^{t+1})\right]}{p_k(s^t)}. \tag{5}$$

In the above, $p_k(s^t) = \frac{P_k(s^t)}{P(s^t)}$ denotes the real price of capital and $\pi(s^{t+1}) = \frac{P(s^{t+1})}{P(s^t)}$ is the economy's inflation rate.

2.2 Households

There are a large number of households of type γ, which denotes a household's fixed cost of making state contingent transfers from a brokerage account to a checking account. This cost is constant across time but differs across household types according to the probability density function $f(\gamma)$.

Brokerage Account. At date 0, a household learns her type and engages in an initial round of trade in the asset market, as goods markets do not open until date 1. With initial asset holdings, $\overline{B}(\gamma)$ in her brokerage account at date 0, the household purchases equity, $S(s^0, \gamma)$, issued by the firms, a complete set of one-period contingent claims, $B(s^1, \gamma)$, issued by the government, or can opt to keep some of her portfolio as cash, $N(s^0, \gamma)$, which earns no interest. In addition, the household sets up a non-state contingent plan, $A(\gamma)$, allocating cash between his checking and brokerage accounts in future periods. Accordingly, the flow of funds in a household's brokerage account at date 0 is given by:

$$\overline{B}(\gamma) = S(s^0, \gamma) + \sum_{s_1} q(s^1)B(s^1)ds_1 + N(s^0, \gamma) + P_A A(\gamma), \tag{6}$$

where $q(s^1)$ is the price of the bond in state, s^1.

The initial non-state contingent plan, $A(\gamma)$, can only be altered in the future by a paying the fixed cost, γ. We view this fixed cost as reflecting cognitive costs associated with collecting and processing information necesssary to recompute the optimal portfolio allocation in response to shocks. Our approach is similar to Gabaix and Laibson (2001) and Bacchetta and van Wincoop (2009); however, we emphasize that the decision to reoptimize portfolio holdings is state dependent rather than time dependent.

The key assumption we make about a household's initial allocation scheme, $A(\gamma)$, is that it is non-state contingent. In principle, a household could choose a time-varying, non-state contingent plan and would do so if the model allowed for life-cycle considerations. In that case, a household born with low initial assets would save some of her wage income by setting up a plan that at first transferred a fixed amount of funds from her checking account to her brokerage account. However, the household would also set up her transfer scheme to reverse this flow at her expected retirement date, when cash transfers from her brokerage to her checking account become her primary source of cash for consumption. While allowing households to specify their financial allocations over their life cycles would add realism to the model, we abstract from such considerations to keep the analysis tractable and simply focus on the non-state contingent nature of $A(\gamma)$. By incorporating this initial portfolio decision and a fixed cost of altering it in response to shocks, our model is broadly consistent with the micro evidence that many households adjust their portfolio decisions very infrequently.

For dates $t \geq 1$, a household's brokerage account evolves according to:

$$\begin{aligned} B(s^t,\gamma) + (1+R^e(s^t))S(s^{t-1},\gamma) + N(s^{t-1},\gamma) = \sum_{s_{t+1}} q(s^t,s_{t+1})B(s^t,s_{t+1},\gamma)ds_{t+1} + \\ S(s^t,\gamma) + N(s^t,\gamma) + P(s^t)[x(s^t,\gamma)+\gamma]z(s^t,\gamma), \end{aligned} \tag{7}$$

where $x(s^t,\gamma)$ denotes a state contingent transfer of funds from a household's brokerage account to checking account at date t and $z(s^t,\gamma)$ is an indicator variable equal to one if a household opts to pay her fixed cost and make this transfer and equal to zero if a household chooses not to transfer $x(s^t,\gamma)$.

Checking Account. For $t \geq 1$, a household purchases goods for consumption, $c(s^t, \gamma)$, and works in the labor market. To purchase goods in period t, a household uses cash in her checking account, which evolves according to:

$$P(s^t)c(s^t, \gamma) = M(s^{t-1}, \gamma) + P(s^t)x(s^t, \gamma)z(s^t, \gamma) + P(s^t)A(\gamma) - P(s^t)a(s^t, \gamma). \tag{8}$$

At the beginning of period t, a household has $M(s^{t-1}, \gamma)$ dollars in its checking account with which to purchase goods. A household also receives cash from her non-state contingent transfer plan and $P(s^t)x(s^t, \gamma)$ dollars from her brokerage account, if she chooses to incur the fixed cost and transfer additional funds.[6] In addition, a household can save extra cash, $a(s^t, \gamma)$, in her checking account for future consumption rather than using it for consumption today.

Each household inelastically supplies her labor to the economy's firms. With a household's labor supply normalized to one, a household earns real wage income, $w(s^t)$. This wage income is received at the end of the period so it can not be used for current consumption. Accordingly, a household cash in its checking account at the end of period t is given by:

$$M(s^t, \gamma) = P(s^t)\left[w(s^t) + a(s^t, \gamma)\right]. \tag{9}$$

A household's problem is to choose $A(\gamma)$ and $\{c(s^t, \gamma), x(s^t, \gamma), z(s^t, \gamma), M(s^t, \gamma), a(s^t, \gamma), N(s^{t-1}, \gamma), B(s^t, \gamma), S(s^{t-1}, \gamma)\}_{t=1}^{\infty}$ to maximize:

$$\sum_{t=1}^{\infty} \sum_{s^t} \beta^t U(c(s^t, \gamma))g(s^t)ds^t \tag{10}$$

subject to equations (6)-(9), taking prices and initial holdings of money, bonds, and stocks as given. In equation (10), the function $g(s^t)$ denotes the probability distribution over history s^t.

Endogenous Participation. If households are not able to set up the initial, non-state contingent transfer plan, the model is similar to the endogenous participation framework of Alvarez, Atkeson, and Kehoe (2007b). In this case, asset markets are completely segmented

[6] A household can reoptimize by setting $x(s^t, \gamma) < 0$, thereby transferring additional cash from his checking to brokerage account. Similarly, a household is free to choose $A(\gamma) < 0$.

from goods markets for agents who do not transfer $x(s^t, \gamma)$ between them. We call this version of the model, the "endogenous participation model", because the decision to transfer funds from asset to the goods market determines whether a household participates in financial markets. In contrast, in the "endogenous rebalancing" model, all households participate in financial markets. Instead, their decision to transfer funds $x(s^t, \gamma)$ at date t amounts to a rebalancing of cash from their initial allocation determined at date 0.

2.3 The Government

The government issues the economy's one-period state-contingent bonds and controls the economy's money stock, M_t. At date 0, the government also issues an annuity, A_0, at price, P_A, which has a constant payoff in units of consumption. Its budget constraints at date 0 is given by:

$$\bar{B} = \int_{s_1} q(s_1)B(s_1)ds_1 + P_A A_0, \tag{11}$$

where $\bar{B}$ is given. At dates $t \geq 1$, the government's budget constraint is given by:

$$B(s^t) + M_{t-1} + P(s^t)A_0 = M_t + \int_{s_{t+1}} q(s^t, s_{t+1})B(s^t, s_{t+1})ds_{t+1}, \tag{12}$$

with $M_0 > 0$ given. Finally, the government injects cash into the economy via a first-order Markov process for money growth, $\mu_t = \frac{M_t}{M_{t-1}}$.

2.4 Market Clearing

We assume that the distribution of household types is discrete so that $\gamma \in [\gamma_1, \gamma_2, ..., \gamma_J]$. The economy's resource constraint is:

$$Y(s^t) = \sum_{j=1}^{J} \left[c(s^t, \gamma_j) + \gamma_j z(s^t, \gamma_j)\right] f(\gamma_j), \tag{13}$$

while market clearing in factor markets requires $K(s^t) = 1$ and $L(s^t) = 1$. In asset markets, for stock and bond markets to clear at dates $t \geq 0$ we have:

$$S(s^t) = \sum_{j=1}^{J} S(s^t, \gamma_j)f(\gamma_j) \qquad\qquad B(s^{t+1}) = \sum_{j=1}^{J} B(s^{t+1}, \gamma_j)f(\gamma_j).$$

At date 0, we also have $\bar{B} = \sum_{j=1}^{J} \bar{B}(\gamma_j) f(\gamma_j)$ and $A_0 = \sum_{j=1}^{J} A(\gamma_j) f(\gamma_j)$. For $t \geq 1$, beginning of the period cash holdings satisfy:

$$\sum_{j=1}^{J} \left\{ M(s^{t-1}, \gamma_j) + P(s^t)[x(s^t, \gamma_j) + \gamma_j] z(s^t, \gamma_j) + P(s^t) A(\gamma_j) + N(s^t, \gamma_j) \right\} f(\gamma_j) = M_t, \tag{14}$$

where the presence of the fixed cost reflects it is paid using cash from the brokerage account.

An equilibrium is a collection of asset prices, $\{P_A, q(s^t), R^e(s^t), P_k(s^t)\}$, wages, and goods prices, which together with money, bonds, stocks, and allocations, $\{c(s^t, \gamma), x(s^t, \gamma), N(s^{t-1}, \gamma), a(s^t, \gamma), z(s^t, \gamma)\}$ satisfying the household's optimization problem. These prices together with the allocations $\{S(s^{t-1}), K(s^{t-1}), Y(s^t), L(s^t)\}$ satisfies the firm's optimization problem. Finally, the government budget constraint holds and the resource constraint along with the market clearing conditions for capital, labor, bonds, stocks, and money are all satisfied.

2.5 Equilibrium Characterization

We now characterize equilibrium consumption and transfers of households, and describe when a household decides to hold extra cash in her checking and/or brokerage account (i.e, choose $a(s^t, \gamma) > 0$ and/or $N(s^{t-1}, \gamma) > 0$). We then show how this excess cash holdings gives rise to non-constant velocity holdings and affects inflation. Finally, we discuss the link between the consumption of households that actively rebalance their portfolios (i.e., choose $z(s^t, \gamma) = 1$) and equity returns.

2.5.1 Consumption and Transfers

We begin by characterizing a household's consumption conditional on their choice of paying the fixed cost of making the state dependent transfer. To do so, we use the fact that in equilibrium $Y(s^t) = \exp(\theta_t)$ and define the economy's inflation rate as $\pi(s^t) = \frac{P(s^t)}{P(s^{t-1}}$. With these expressions, we can combine equations (8) and (9) using equation (4) to substitute out the equilibrium real wage to write the consumption of an inactive household (i.e., one that sets $z(s^t, \gamma) = 0$) as

$$c_I(s^t, \gamma) = \frac{(1-\alpha)\exp(\theta_{t-1}) + a(s^{t-1}, \gamma)}{\pi(s^t)} + A(\gamma) - a(s^t, \gamma). \tag{15}$$

From this expression, we can see that inflation is distortionary, since, all else equal, it reduces the consumption of inactive households. Accordingly, an unanticipated increase in money that raises inflation will induce the marginal household to pay her fixed cost and become active. Although the consumption of inactive households rises due to an increase in wages following an unexpected technological improvement, the benefits of being active are even greater, reflecting that active consumption is also boosted by higher capital income. Thus, a technology shock will also boost the number of active households.

There is perfect risk-sharing amongst active households, and as discussed in the appendix, we assume that the initial asset holdings, $\bar{B}(\gamma)$, of the households implies:

$$c_A(s^t, \gamma) = c_A(s^t). \tag{16}$$

Accordingly, the consumption of active households is independent of γ. To further characterize, the consumption of active and inactive households, we need to determine $A(\gamma)$. In the appendix, we show that the price of the annuity is given by

$$P_A = \sum_{t=1}^{\infty} \sum_{s^t} Q(s^t) P(s^t) ds^t, \tag{17}$$

where $Q(s^t) = \Pi_{j=1}^{t} q(s^j)$ and a household's choice of $A(\gamma)$ must satisfy:

$$\sum_{t=1}^{\infty} \sum_{s^t} \beta^t \left[U'(c_A(s^t)) - U'(c_I(s^t, \gamma)) \right] (1 - z(s^t, \gamma)) g(s^t) ds^t = 0. \tag{18}$$

This latter condition states that in the states of the world in which a household is inactive (i.e., $z(s^t, \gamma) = 0$), the household chooses $A(\gamma)$ to equate the expected discounted value of marginal utility of its consumption to the expected discounted value of the marginal utility of the consumption of the active households in those states of the world. Accordingly, the non-state contingent transfer plan provides some consumption insurance to households with relatively large fixed costs of rebalancing their portfolio allocation. Such households will choose $A(\gamma) > 0$ to compensate for their infrequent access to capital income derived from equity markets.

While equation (18) places restrictions on the choice of $A(\gamma)$ for households that are inactive in at least one state of the world, this condition is irrelevant for a household that is active in each state of the world. In this case, a household's choice of $A(\gamma)$ is irrelevant, since she can use $x(s^t,\gamma)$ to achieve her desired level of consumption.

We now characterize a household's decision to actively rebalance or not given optimal decisions for $c(s^t,\gamma)$, $x(s^t,\gamma)$, $a(s^t,\gamma)$, and $A(\gamma)$. As discussed in the appendix, a household will choose to be active if:

$$U\left(c_A(s^t)\right) - U\left(c_I(s^t,\gamma)\right) - U'\left(c_A(s^t)\right)\left[c_A(s^t) - c_I(s^t,\gamma) + \gamma\right] \geq 0, \tag{19}$$

and inactive otherwise. Equation (19) states that the net gain for a household that rebalances must be greater or equal to the cost of transferring funds across the two markets. The net gain, $U\left(c_A(s^t)\right) - U\left(c_I(s^t,\gamma)\right)$, is simply the difference in the level of utility from being active as opposed to inactive, while the cost comprises the transaction fee γ and the amount transferred by the household, since $x(s^t,\gamma) = c_A(s^t) - c_I(s^t,\gamma)$.

For households with a small γ, equation (19) will be positive, while it will be negative for households with a large value of γ. Accordingly, we can use equation (19) to define a marginal household, $\bar{\gamma}(s^t) \in [\gamma_1, \gamma_2, ..., \gamma_J]$ such that there is a cutoff rule in which households with $\gamma \leq \bar{\gamma}(s^t)$ will choose $z(s^t,\gamma) = 1$ or otherwise $z(s^t,\gamma) = 0$. In our context, the fixed cost of rebalancing leads to a state-dependent rule determining the fraction of households that reoptimize their portfolio allocations. With this rule, the resource constraint can be rewritten as:

$$c_A(s^t) \sum_{\gamma \leq \bar{\gamma}(s^t)} f(\gamma) + \sum_{\gamma > \bar{\gamma}(s^t)} c_I(s^t,\gamma) f(\gamma) = \exp\left(\theta_t\right) - \sum_{\gamma \leq \bar{\gamma}(s^t)} \gamma f(\gamma). \tag{20}$$

2.5.2 Consumption of Rebalancers and the Equity Premium

The asset pricing kernel in the economy depends on the consumption of the rebalancers and is given by:

$$m(s^t, s_{t+1}) = \beta \frac{U'[c_A(s^{t+1})]}{U'[c_A(s^t)]}. \tag{21}$$

This pricing kernel is the state-contingent price of a security expressed in consumption units and normalized by the probabilities of the state. This pricing kernel can be used to determine the real risk-free rate (r^f) as well as the real return on equity (r^e). These returns are given by:

$$[1 + r^f(s^t)]^{-1} = \sum_{s_{t+1}} m(s^t, s_{t+1}) g(s_{t+1}|s^t), \tag{22}$$

$$1 = \sum_{s_{t+1}} m(s^t, s_{t+1})[1 + r^e(s^t, s_{t+1})] g(s_{t+1}|s^t), \tag{23}$$

where $g(s_{t+1}|s^t) = \frac{g(s^{t+1})}{g(s^t)}$ denotes the probability of state s_{t+1} conditional on state s^t. From equation (24) in the firm's problem, the equilibrium real return on equity is given by:

$$1 + r^e(s^{t+1}) = \frac{[\alpha \exp(\theta_{t+1}) + p_k(s^{t+1})]}{p_k(s^t)}. \tag{24}$$

Using these two equations, we can then define the equity premium in our economy as:

$$\frac{\mathrm{E}_t[1 + r^e_{t+1}]}{1 + r^f_t} = 1 - \mathrm{cov}_t\left(m_{t+1}, 1 + r^e_{t+1}\right), \tag{25}$$

where for convenience we have switched notation to express both the expected return on equity and the covariance between the pricing kernel and the return on equity, which are both conditional on the state of the world at date t.

2.5.3 Velocity, Inflation, and Money Demand

To define velocity in the model, we substitute equations (8) and (13) into equation (14) and rewrite it as:

$$V(s^t) \equiv \frac{P(s^t)Y(s^t)}{M_t} = \frac{Y(s^t)}{Y(s^t) + \sum_{j=1}^{J}\left[a(s^t, \gamma_j) + N(s^t, \gamma_j)\right] f(\gamma_j)}. \tag{26}$$

From this expression, we can see that if the households decide not to hold excess cash in their checking and brokerage accounts (i.e., $a(s^t, \gamma) = 0$ and $N(s^t, \gamma) = 0$ for all γ), then velocity is constant. Using this definition of velocity, the economy's inflation rate is given by:

$$\pi(s^t) = \mu_t \frac{\mu_v(s^t)}{\mu_y(s^t)}, \tag{27}$$

where $\mu_y(s^t) = \frac{Y(s^t)}{Y(s^{t-1})}$ and $\mu_v(s^t) = \frac{V(s^t)}{V(s^{t-1})}$.

Each household will choose not to store excess cash in its brokerage account if the nominal interest rate, $i(s^t)$, is positive, where the nominal interest rate is given by:

$$1 + i(s^t) = \frac{U'\left(c_A(s^t)\right)}{\beta \sum_{s_{t+1}} U'\left(c_A(s^{t+1})\right) \frac{g(s_{t+1}|s^t)}{\pi(s^{t+1})}}. \tag{28}$$

If the nominal interest rate reaches its zero lower bound (ZLB), then households may carry excess cash in their brokerage accounts. In this case, the ZLB constraint can be used to determine an equilibrium value for $N(s^t) = N(s^t, \gamma)$, $\forall \gamma$. Accordingly, we focus on an equilibrium in which all households choose the same amount of excess cash to carry in their brokerage account, as each household faces the same first order condition regarding its choice of $N(s^t, \gamma)$.

A household may also choose to forego consumption today and save some cash in their checking account for future consumption. A household will not carry excess cash in its checking account if:

$$\frac{U'\left(c_A(s^t)\right) z(s^t, \gamma) + U'\left(c_I(s^t, \gamma)\right)\left(1 - z(s^t, \gamma)\right)}{\beta \sum_{s_{t+1}} \left[U'\left(c_A(s^{t+1})\right) z(s^{t+1}, \gamma) + U'\left(c_I(s^{t+1}, \gamma)\right)\left(1 - z(s^{t+1}, \gamma)\right)\right] \frac{g(s_{t+1}|s^t)}{\pi(s^{t+1})}} > 1. \tag{29}$$

There can be times when this condition is violated so that a household has a precautionary motive to store cash in her checking account. If current consumption is relatively high for a household that rebalances infrequently, she may hold extra cash for the future when she expects her consumption to be low. For a household that always rebalances (i.e., $z(s^t, \gamma) = 1$ $\forall s^t$), excess cash in their checking account is irrelevant, and condition (29) becomes $i(s^t) > 0$, which is the condition guaranteeing that households do not carry excess cash in their brokerage account.

2.6 Parameter Values and Numerical Solution

A household's per-period preferences are given by:

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \tag{30}$$

where σ is the coefficient of constant relative risk aversion, which is set equal to 2 based on the survey of the literature in Hall (2008). The discount factor, $\beta = 0.99$, is chosen to be consistent with a quarterly model, while the economy's capital share, α, is 0.36.

For the technology and monetary shocks, we follow Tauchen (1986) and Flodén (2008), who show how to construct the transition probabilities of a first-order Markov process to well approximate an AR(1) process. In particular, the technology shock, θ_t follows a nine-state, symmetric Markov chain in which the associated transition probabilities imply that technology shocks are highly persistent with a first-order autocorrelation of nearly 0.99. The nodes for the technology shock are equally-spaced between [-0.225,0.225] and imply that the annualized standard deviation of aggregate consumption growth in the model is just over 3 percent, consistent with U.S. annual consumption growth over 1889-2004 period.

The monetary shock follows a three-state, symmetric Markov chain with equally-spaced nodes between [1.0092, 1.015] and transition probabilities implying a first-order autocorrelation of money growth close to 0.9. This autocorrelation is higher than for M2 in the post-war data. Accordingly, we also report results for alternative transition probabilities which imply a first-order autocorrelation of 0.68 in line with growth in M2.

The number of types of households, J, is set to 3 to help avoid the "curse of dimensionality", as there are as many state variables in the model as the number of household types.[7] With $J = 3$, there is one household type that always rebalances ($\gamma_1 = 0$), another that frequently rebalances ($\gamma_2 = 0.02$), and a type that rarely rebalances ($\gamma_3 = 0.04$). These three types are distributed so that there are a large mass of households who rarely rebalance ($f(\gamma_3) = 0.712$), a moderate mass of frequent rebalancers ($f(\gamma_2) = 0.22$), and a smaller mass of households that always rebalance ($f(\gamma_1) = 0.068$). This calibration implies that 20 percent of households rebalance their portfolios in a quarter and is broadly in line with the micro evidence from the household finance literature that provides strong support for infrequent portfolio rebalancing.[8]

If we use the same values for the fixed costs in the endogenous participation model (i.e., $A(\gamma) = 0\ \forall\gamma$), then the three types of households will always participate in financial markets, leading to implications for this model that are very similar to a representative agent model.

[7] An additional complication is that the zero lower bound constraint and constraints associated with expression (29) may occassionally bind, which implies that the decision rules for $N(s^t)$ and $a(s^t, \gamma_j)\ forj = 2, 3, ..., J$ are determined from one of J^2 cases.

[8] See, for example, Souleles (2003) and Ameriks and Zeldes (2004).

Accordingly, we choose much larger values of the fixed costs for this version of the model so that on average slightly less than 30 percent of the households participate in financial markets.

Solving the Model. With three types of households, there are 3 state variables — excess cash holdings in the brokerage account and excess cash holdings of types γ_2 and γ_3 in their checking accounts — and 3 inequality constraints that may be occasionally binding — the ZLB constraint and the two constraints associated with the possibility of households carrying excess cash in their checking account.

We obtain an approximate solution to our model using the global method of Christiano and Fisher (2000). Their parameterized expectation algorithm (PEA) involves obtaining the policy functions for $N(s^t)$, $a(s^t, \gamma_2)$, and $a(s^t, \gamma_3)$ indirectly by approximating the conditional expectations in equations (28) and (29). These expectations are approximated using collocation and Chebychev polynomials. For details of this procedure, see the appendix.

3 Results

Before discussing the model's implications for monetary policy and the equity premium, it is helpful to characterize the model's non-stochastic steady state.

3.1 Deterministic Steady State

In a deterministic environment with average money growth equal to 1.25% on a quarterly basis and output equal to unity, the endogenous rebalancing model reduces to a representative agent economy. This model only becomes interesting in the presence of uncertainty; nevertheless, it is useful to compare its deterministic steady state with the version of the model with endogenous participation.

In the non-stochastic steady state of the endogenous rebalancing model, all households will have the same level of consumption. According to equation (18), a household that chooses to be inactive obtains the same level of consumption as an active household. Households with

γ_2 and γ_3 will choose to be inactive and will choose their non-state contingent transfer plan so that $c_A = c_I = \frac{1-\alpha}{\mu} + A$, where $A = A(\gamma_2) = A(\gamma_3)$. With consumption the same across households, type γ_2 and γ_3 will never rebalance their portfolios, and households with $\gamma_1 = 0$ will be indifferent between rebalancing or using the non-state contingent transfer plan, $A(\gamma_1)$.

In the non-stochastic steady state of the endogenous participation model (i.e., $A(\gamma) = 0$ for all γ), type γ_1 and γ_2 households choose to be active and their consumption exceeds the consumption of γ_3 households who are inactive households and set $c_I = \frac{1-\alpha}{\mu}$. By choosing to be inactive, these households do not receive the capital income associated with participating in the stock market. Accordingly, without the much larger fixed costs used to calibrate this version of the model, there is a strong incentive for all households to participate in financial markets.

3.2 Endogenous Rebalancing and the Equity Premium

Table 1 compares selected statistics from different versions of the model. The first column shows the results from the endogenous rebalancing model in which the average equity premium and average risk-free rate are 5.4% and 0.7% percent, respectively, at an annual rate. These values are modestly lower than the average equity premium and risk-free rate for U.S. data but well within a 95% confidence region (see Figure 1). In contrast, both the endogenous participation and representative agent models yield a much smaller average equity premium — only 0.5 percent — well outside the 95 percent confidence region.

This small equity premium in the endogenous participation model occurs despite having very large fixed costs. In the endogenous participation model, the average fixed cost incurred by households is more than 8 percent of GDP. In contrast, in the endogenous rebalancing model, the average fixed cost is about 0.25 percent of GDP or more than 30 times smaller than in the endogenous participation model.

The only difference between the rebalancing and participation version of the model is that in the endogenous rebalancing model, households have access to the initial, non-state contingent transfer plan allocating cash across their brokerage and checking accounts. Accordingly, Table

1 demonstrates the important role this choice variable has in accounting for the equity premium puzzle. Table 1 also shows that the demand for this transfer plan is increasing in a household's fixed cost, with $A(\gamma_2) = 0.25$ and $A(\gamma_3) = 0.35$. ($A(\gamma_1)$ is indeterminate since γ_1 households always rebalance). The function $A(\gamma)$ is increasing, because a household with a higher fixed cost anticipates that she will rebalance her portfolio allocations less frequently and therefore demand a larger value of $A(\gamma)$ to help ensure against consumption losses.

The transfer plan helps account for the level of the equity premium by driving up the volatility of active consumption. The fifth row of Table 1 labeled $\frac{\sigma_{\Delta c_A}}{\sigma_{\Delta c}}$ shows that the standard deviation of consumption growth of active households is 3.8 times greater than aggregate consumption in the endogenous rebalancing model, while active consumption growth is only 10% higher in the endogenous participation model. This higher volatility of consumption growth for active households leads to a higher positive covariance between the return on equity and active consumption growth, which then translates into a higher average equity premium via expression (25).

The fourth column of Table 1 labeled $\frac{Ec_A}{Ec}$ indicates that active households have a higher level of consumption than inactive households. In effect, households that rebalance more frequently are trading off higher consumption volatility against a higher level of consumption. The consumption of active households is more volatile than the consumption of inactive households, because the two aggregate shocks only affect the consumption of the latter type of household through changes in labor income, while active households experience fluctuations in both labor and capital income. Later, we develop the intuition of this result more formally using a version of the model without occasionally binding inequality constraints.

The implication of the model regarding the consumption volatility of different household types appears to be in line with recent evidence provided by Parker and Vissing-Jorgensen (2009). Using data from the Consumer Expenditure (CEX) Survey, they found that the consumption of 'high-consumption' households is more exposed to fluctuations in aggregate consumption (and income) than that of low-consumption households. In particular, they found that the exposure to changes in aggregate consumption growth of households in the top 10 percent of the consumption distribution is about five times that of households in the bottom 80 percent.

As indicated in Table 1, households that rarely rebalance never choose to hold excess cash in their checking accounts (i.e., choose $a(s^t, \gamma_3) > 0$), while households that frequently rebalance only rarely hold excess cash in their checking accounts. In contrast, the ZLB constraint binds about 15% of the time, resulting in households occasionally carrying excess cash in their brokerage accounts. Later, we examine how the zero lower bound constraint affects the propagation of monetary policy shocks.

To investigate how changes in the level of the fixed cost affect the average equity premium in the endogenous rebalancing model, the solid blue line in Figure 1 shows the model's average equity premium and risk-free rate for different values of the fixed cost. In the figure, we also report the sample averages for the risk-free rate and the equity premium (see the red dot labeled "U.S. Data") from Cecchetti, Lam, and Mark (1993) and the 5% confidence ellipse based on their estimates. A proportional increase (decrease) in γ_2 and γ_3 reduces (raises) the average frequency of rebalancing, and the average risk premium increases (decreases), while the average risk-free rate falls (rises). Thus, the model is capable of generating larger equity premiums when there is more portfolio inertia.

3.3 Monetary Policy and Equity Prices

In the endogenous rebalancing model, technology shocks account for the bulk of the mean excess return on equity. Still, monetary policy shocks can induce important fluctuations in equity prices and the equity premium.[9] In this section, we investigate this relationship and compare our model's implications to the estimates of Bernanke and Kuttner (2005).

Using high-frequency data on the federal funds rate, Bernanke and Kuttner (2005) construct a measure of unanticipated changes in monetary policy. They find that a broad index of stock prices registers a one-day gain of 1 percent in reaction to a 25 basis point easing of the federal

[9] In Gust and López-Salido (2009) we provide further analysis on the role of technology shocks in accounting for asset pricing puzzles.

funds rate. Bernanke and Kuttner (2005) then use monthly data with a structural VAR to decompose the response of stock prices into three components: changes in current and expected future dividends, changes in current and expected future real interest rates, and changes in expected future excess equity returns or equity premia. While an unanticipated easing lowers interest rates, they conclude that an important channel in which stock prices increase occurs through changes in the equity premia or the perceived riskiness of stocks.

To compare our model's implications to these stylized facts, we compute impulse response functions from the endogenous rebalancing model. Since our model is nonlinear, it is important to define how we construct these impulse responses. Follow the discussion in Hamilton (1994), we define the impulse response of variable, $y(s^t)$, at date t to a monetary innovation that occurs at date 1 as:

$$E\left[log\left(y(s^t)\right) \mid \mu_1 = \mu^H, \Omega_0\right] - E\left[log\left(y(s^t)\right) \mid \Omega_0\right], \forall t \geq 1, \tag{31}$$

where μ^H denotes money growth in the high state and $\Omega_0 = \{N(s^0), a(s^0, \gamma_j), \theta_0, \mu_0\}$ for $j = 2, 3$.[10] Thus, an impulse response to a monetary shock is defined as the revision in expectations in response to a shock occurring at date 1. For log-linear models, equation (31) simplifies to the usual analytical representation in which (up to a scaling factor) the model's linear coefficients characterize the impulse response function. Since in our context evaluating the expectations in equation (31) involves multidimensional integrals, we use Monte Carlo integration to compute the impulse response functions.

Figure 2 shows the impulse response of different economic variables to a monetary innovation that raises money growth rate a 100 basis points at date 1. Starting with the endogenous rebalancing model (the solid blue line), the nominal interest rate falls about 15 basis points on impact, and the real rate falls about 20 basis points. Thus, as in the limited participation models of Lucas (1990), Fuerst (1992), and Alvarez, Atkeson, and Kehoe (2002), the economy displays a liquidity effect. Moreover, the effect is persistent, as interest rates gradually rise

[10]We set $N(s^0) = a(s^0, \gamma_j) = 0 \ forj = 2, 3$. For the date 0 technology and monetary shocks, we used the ergodic distribution for θ_t and μ_t to sum over the states of the world at date 0. See Flodén (2008) for a discussion on how to construct the ergodic distribution using the transition matrix of the Markov process.

back to their pre-shocked level. Equity prices rise more than 1 percent on impact, implying a multiplier of about 7 from a 100 basis point change in the nominal rate. Such a multiplier is in line with Bernanke and Kuttner (2005), who estimate multipliers between 3 and 6. The model is broadly consistent with their evidence, as much of the rise in equity prices reflects a decline in the equity premium. On impact, the equity premium moves down more than 30 basis points, and its response mirrors that of equity prices.[11]

To understand why the model generates a fall in the equity premium, the bottom left panel of Figure 2 shows the response of consumption of active households (rebalancers). The monetary injection has no effect on output but has an important redistributive effect. It reduces the consumption of non-active households, whose real money balances available for consumption fall, and raises the consumption of those that choose to rebalance. This redistributive effect provides the households with a greater incentive to rebalance their portfolios, which in turn helps reduce the equity premium. The intuition for this result is developed more formally below.

The fall in the nominal interest rate associated with the increase in money growth slightly increases the probability of hitting the zero lower bound constraint. Accordingly, households' expectations regarding the probability of carrying excess cash in their brokerage account increase. These higher expectations underlie the small reduction in velocity associated with the higher level of the equity premium.

The red dashed line shows the effects of the same increase in money growth in the endogenous participation model of Alvarez, Atkeson, and Kehoe (2007b). This model does not generate a liquidity effect and the increase in equity prices is much smaller than in the endogenous rebalancing model. In the endogenous participation model, there is a similar initial decline in the real rate, while the equity premium remains unchanged, as does velocity. Accordingly, variation along the intensive margin (i.e., the frequency of rebalancing between checking and brokerage accounts) appears to be a more fruitful way of generating movements in the equity premium than the extensive margin (i.e., the number of financial market participants).

[11] The increase in stock prices also reflects the decline in real rates. Dividends in our model are simply a function of technology and do not change in response to the monetary innovation.

Figure 3 compares the effects of the monetary shock for two different values of the autocorrelation of money growth. The solid blue line shows the results for an autocorrelation close to 0.9, the value used in the previous figure and used by Alvarez, Atkeson, and Kehoe (2007b). The dashed red line shows the results for an autocorrelation of 0.68. With a less persistent increase in money growth, there is a larger liquidity effect. The real rate falls about 0.8 percentage point on impact, while the equity premium declines 0.2 percentage point.

3.4 Monetary Policy at the Zero Lower Bound

We now examine the effect of stimulative monetary policy at the zero lower bound. To do so, as the baseline scenario, we consider a very large contraction in technology that results in a sharp decline in output and the real rate, pushing the economy to the zero lower bound constraint. The solid blue line in Figure 4 shows the effects of a technology shock that induces output to plunge more than 15 percent below its pre-shocked level and only gradually begin to recover. The consumption of rebalancers (not shown) and the number of rebalancers plummets, and the real interest rate drops dramatically. This drop in real rates mainly reflects greater precautionary savings by the remaining active households, whose conditional standard deviation of consumption growth jumps more than 1 percentage point above its pre-shocked level (of roughly 13 percent). Households decide to hold excess cash in their brokerage accounts, as the nominal interest reaches its ZLB constraint in the period of the shock. In future periods, the probability of staying at the ZLB declines slightly and remains above 70 percent 2 years after the shock.

A key result of our model is that the equity premium is countercyclical. In particular, the negative technology shock results in a substantial increase in the equity premium. The countercyclical nature of the equity premium is true regardless of whether the economy reaches the ZLB constraint or not. A unique feature of obtaining the ZLB constraint is that there is a surge in the demand for cash as risk in the equity market spikes. In effect, investor "flight to safety" is so pronounced after the large drop in technology that money and bonds become perfect substitutes, creating excess liquidity. This gives rise to procyclical movements in velocity, which declines persistently in response to the negative technology shock.

Figure 5 shows the effects of an increase in money growth that occurs at the same time as the technology shock, with the variables plotted in deviation from the scenario with just a bad technology shock (i.e., the baseline scenario). The monetary shock is small and therefore the technology shock is the dominant source behind fluctuations in the variables. As in Figure 2, the monetary easing leads to a decline in equity premium and reduces the degree of precautionary savings by active households. However, expected inflation jumps and there is a higher inflation risk premium. Thus, while the easing of monetary policy helps reduce risk in equity markets, it increases the level of inflation risk.[12]

3.5 Understanding the Mechanism

To understand how endogenous changes in the number of rebalancers can induce time-varying movements in risk, it is helpful to abstract from the occasionally binding inequality constraints by assuming that $N(s^t, \gamma) = a(s^t, \gamma) = 0$, $\forall \gamma$. We also assume, for illustrative purposes, that $A(\gamma) = A$ $\forall \gamma$ and that γ is distributed according to a continuous, uniform distribution with $\gamma \in [0, \gamma_J]$. Under these conditions, $\bar{\gamma}(s^t)$ satisfies equation (19) as an equality and $c_I = \frac{(1-\alpha)exp(\theta)}{\mu}$.[13] In addition, with $\sigma = 2$, equation (19) can be rewritten as:

$$(c_A - c_I)^2 = c_I \bar{\gamma}, \tag{33}$$

and the resource constraint can be rewritten as:

$$\frac{\bar{\gamma}}{\gamma_J} c_A + (1 - \frac{\bar{\gamma}}{\gamma_J}) c_I = (1-\alpha)\exp(\theta) - \frac{\bar{\gamma}^2}{2\gamma_J}, \tag{34}$$

[12]We define the inflation risk premium using a version of the Fisher equation that takes into account uncertainty about inflation:

$$\frac{(1+i_t)E_t(\pi_{t+1}^{-1})}{(1+r_t^f)} - 1 = -(1+i_t)\text{cov}_t(m_{t+1}, \pi_{t+1}^{-1}). \tag{32}$$

The inflation risk premium is then measured as the covariance term on the left-hand side of the equality.

[13]Given our assumptions, the equations determining the equilibrium are static, and we simplify our notation, ignoring that these variables depend on s^t.

With these two equations, it is possible to obtain a unique and closed form solution for c_A and $\bar{\gamma}$. The first equation characterizes the marginal household's decision to rebalance its portfolio (i.e., the type $\bar{\gamma}$ household). We call the schedule of the combination of values of $\bar{\gamma}$ and c_A the MR curve in reference to the marginal rebalancer. The top panel of Figure 6 shows that this curve is a parabola with a minimum occurring at c_I. For $c_A > c_I$, the state-contingent transfer of an active household (i.e., $x = c_A - c_I$) is positive, as is the cost of transferring funds. A rise in the consumption of active households, all else equal, makes it more attractive to make the state contingent transfer, and thus for $c_A > c_I$, the MR schedule is increasing.

The second equation represents the combination of values of c_A and $\bar{\gamma}$ that satisfy goods market clearing, and can be used to express the fraction of rebalancers (i.e., $\frac{\bar{\gamma}}{\gamma_J}$) as a function of their consumption. We call this schedule the GM curve for goods market clearing. For $c_A > c_I$ where $c_I = \frac{1-\alpha}{\mu} + A$, the top panel of Figure 6 shows that this curve is downward sloping. This reflects that an increase in γ raises the average level of transaction costs in the economy, reducing real resources, and lowering the consumption of rebalancers.

Figure 6 displays the equilibrium in the endogenous rebalancing model as the intersection of these two curves. In the endogenous rebalancing model, this intersection occurs at a point in which $c_A > c_I$, though consumption of a rebalancer is not much higher than the consumption of a non-rebalancer, reflecting $A > 0$. In contrast, the bottom panel of Figure 6 shows that the equilibrium in the endogenous participation model ($A = 0$) occurs at a point in which active consumption is much higher than inactive consumption (given by the vertex of the parabola). The equilibrium financial market participation rate shown in the bottom panel is also much higher than the the rate of rebalancing that occurs in equilibrium in the top panel.

3.5.1 Technology Shocks

This difference in the equilibrium positions of the two economies has important implications for the volatility of the consumption of active households and therefore the equity premium. To demonstrate this using our illustrative example, Figure 7 displays the effects of a deterministic increase in technology on the equilibrium allocations implied by equations (33) and (34). An increase in technology shifts the GM curve upward and to the right, as the economy's resources expand. The MR curve shifts to the right, as the wage income of non-rebalancers rises. This boosts the consumption of non-rebalancers from c_{I0} to c_{I1}.

With the initial equilibrium occurring near the minimum of the parabola, there is a large increase in the consumption of non-rebalancers that exceeds both the increase in non-rebalancer consumption and technology. This large increase reflects that the technology shock, by raising the return on equity, also involves a redistribution away from non-rebalancers to rebalancers. While this redistribution occurs in the endogenous participation model, its effects on active consumption are modest, given that the initial equilibrium is at a point at which the MR curve is relatively steep. Accordingly, Figure 7 demonstrates that active consumption in the endogenous rebalancing model will be considerably more volatile than the active consumption in the participation model.

3.5.2 Monetary Policy Shocks

To understand why a monetary easing induces a decline in the equity premium, it is helpful to consider the effect of a deterministic change in money growth on the GM and MR schedules. The top panel of Figure 8 shows the effects of a small positive increase in the money growth. This increase shifts the GM curve to the right, since the consumption of rebalancers rises for a fixed $\bar{\gamma}$. In addition, the lower consumption of the non-rebalancers (which occurs at the minimum of the parabola) shifts the MR curve upward and to the left, implying that the benefit to making the state-contingent transfer has gone up. Hence, the deterministic increase in money growth leads to an equilibrium with both higher consumption of active rebalancers and a higher fraction of rebalancers.

With the increase in money growth occurring from an initial equilibrium close to the minimum of the parabola, a small monetary expansion may induce a relatively large increase in the consumption of rebalancers. However, this effect diminishes as the monetary shock becomes larger, reflecting the concavity of the MR schedule. Near the initial equilibrium, small increases in monetary policy induce relatively large increases consumption, while larger shocks lead to smaller effects on consumption. Intuitively, as the shock becomes larger, a greater fraction of households rebalance their portfolios, so that the nominal shock begins to have smaller and smaller real effects.

Building on this analysis, the top two panels of Figure 9 show the first and second derivatives of the logarithm of active consumption with respect to the logarithm of money growth. The top panel shows that the first derivative (i.e., $\frac{\partial \log c_A(\mu_t)}{\partial \log \mu_t}$) is positive and decreasing, reflecting that higher money growth boosts active consumption but by progressively less. The middle panel shows that the second derivative is increasing, reflecting the high degree of concavity of active consumption in the neighborhood of the unconditional mean rate of money growth.

This nonlinearity drives the endogenous fluctuations in risk in our model. An increase in money growth reduces the sensitivity of active consumption to expected future changes in money growth, as the fraction of active rebalancers increases. Thus, for higher rates of money growth, active consumption growth becomes less volatile and its covariance with the return on equity diminishes, leading to a decline in the equity premium. The middle panel of Figure 8 also suggests that this logic holds in reverse for small monetary contractions. In particular, a monetary contraction induces a relatively large decline in active consumption, with active consumption becoming more volatile. With the return on equity also falling in response to a monetary contraction, active rebalancers demand a higher risk premium on equity.

Another nonlinearity in our model is that the fraction of rebalancers may actually rise for a large enough contraction in money growth. This possibility is illustrated in the bottom panel of Figure 8 which shows that the GM curve becomes an upward sloping function that intersects the MR curve to the left of its minimum value. For large monetary contractions, households real money balances are high and the cost of making the state-contingent transfer becomes negative, since $x = c_A - c_I < 0$. Thus, more households choose to become active and transfer funds into their brokerage accounts.

4 Conclusions

We have developed a dynamic stochastic general equilibrium model in which monetary policy affects the economy through movements in risk. Our model extends the neoclassical framework by incorporating segmentation between asset and goods markets. We view our model as the next link in a chain beginning with Lucas (1990) and more recently extended by Alvarez, Atkeson, and Kehoe (2002). However, we depart from the former in two important respects. First, we explicitly model a production economy with equity returns. Second and more importantly, all households participate in financial markets by deciding on an initial, non-state contingent allocation of funds between a checking and brokerage account. It is then costly to make state contingent transfers between the two accounts. In this way, endogenous asset segmentation occurs along an intensive margin (i.e., portfolio rebalancing) rather than along the extensive margin (i.e., participation). With this modification, we are able to account for the average excess returns on equity and endogenous movements in risk following a monetary shock. In line with the evidence of Bernanke and Kuttner (2005), a monetary easing leads to a decline in equity premium, as more households choose to rebalance their portfolios. In ongoing work, we plan to incorporate endogenous capital and labor supply and examine whether the model can account for key features of both asset prices and business cycles.

References

Abel, A. B., J. C. Eberly, and S. Panageas (2007). Optimal Inattention to the Stock Market and Information Costs and Transaction Costs. mimeo, Wharton School of the University of Pennsylvania.

Alvarez, F., A. Atkeson, and P. Kehoe (2007a). If Exchange Rates are Ramdom Walks, Then Almost Everything we Say About Monetary Policy is Wrong. Federal Reserve Bank of Minneapolis, Research Dept. Staff Report 388,.

Alvarez, F., A. Atkeson, and P. Kehoe (2007b). Time-Varying Risk, Interest Rates, and Exchange Rates in General Equilibrium. Federal Reserve Bank of Minneapolis, Research Dept. Staff Report 371,.

Alvarez, F., A. Atkeson, and P. J. Kehoe (2002). Money, Interest Rates, and Exchange Rates with Endogenously Segmented Markets. *Journal of Political Economy 110*, 73–112.

Ameriks, J. and S. P. Zeldes (2004). How Do Household Portfolio Shares Vary with Age? mimeo, Columbia University.

Bacchetta, P. and E. van Wincoop (2009). Infrequent Portfolio Decisions: A Solution to the Forward Discount Puzzle. *American Economic Review*. Forthcoming.

Bernanke, B. and K. Kuttner (2005). What Explains the Stock Markets Reaction to Federal Reserve Policy? *Journal of Finance LX*, 1221–1257.

Bilias, Y., D. Georgarakos, and M. Haliassos (2008). Portfolio Inertia and Stock Market Fluctuations. CFS Working Paper 2006/4. Revised July 2008.

Boldrin, M., L. J. Christiano, and J. D. Fisher (1997). Habit Persistence and Asset Returns in an Exchange Economy. *Macroeconomic Dynamics 1*, 312–332.

Brunnermeier, M. and S. Nagel (2008). Do Wealth Fluctuations Generate Time-Varying Risk Aversion? Micro-Evidence from Individuals Asset Allocation. *American Economic Review 98*, 713–736.

Calvet, L. E., J. Y. Campbell, and P. Sodini (2008). Fight or Flight? Portfolio Rebalancing by Individual Investors. *Quarterly Journal of Economics*. forthcoming.

Calvet, L. E., J. Y. Campbell, and P. Sodini (2009). Measuring the Financial Sophistication of Households. *American Economic Review, Papes and Proceedings*. forthcoming.

Cecchetti, S., P.-S. Lam, and N. Mark (1993). The Equity Premium and the Risk-Free Rate: Matching the Moments. *Journal of Monetary Economics 31*, 21–45.

Christiano, L. J. and J. D. Fisher (2000). Algorithms for Solving Dynamic Models with Occasionally Binding Constraints. *Journal of Economic Dynamics and Control 24*, 1179–1232.

Flodén, M. (2008). A Note on the Accuracy of Markov-Chain Approximations to Highly Persistent AR(1) Processes. *Economic Letters 99*, 516–520.

Fuerst, T. S. (1992). Liquidity, Loanable Funds, and Real Activity. *Journal of Monetary Economics 29*, 3–24.

Gabaix, X. and D. Laibson (2001). The 6D Bias and the Equity-Premium Puzzle. In B. Bernanke and J. Rotemberg (Eds.), *NBER Macroeconomics Annual*. MIT Press.

Gomes, F. and A. Michaelides (2006). Asset Pricing with Limited Risk Sharing and Heterogeneous Agents. mimeo, London School of Economics.

Gust, C. and D. López-Salido (2009). Portfolio Inertia and Asset Pricing. mimeo Federal Reserve Board.

Guvenen, F. (2005). A Parsimonious Macroeconomic Model for Asset Pricing: Habit Formation or Cross-sectional Heterogeneity? mimeo, University of Rochester.

Hall, R. (2008). Sources and Mechanisms of Cyclical Fluctuations in the Labor Market. mimeo, Stanford University.

Hamilton, J. D. (1994). *Time Series Analysis*. Princeton, NJ: Princeton University Press.

Lucas, R. E. (1990). Liquidity and Interest Rates. *Journal of Economic Theory 50*, 237–264.

Lynch, A. (1996). Decision Frequency and Synchronization Across Agents: Implications for Aggregate Consumption and Equity Returns. *Journal of Finance LI*, 1479–1497.

Marshall, D. and N. Parekh (1999). Can Costs of Consumption Adjustment Explain Asset Pricing Puzzles? *Journal of Finance LIV*, 623–654.

Mehra, R. and E. Prescott (1985). The Equity Premium Puzzle. *Journal of Monetary Economics 15*, 145–166.

Parker, J. and A. Vissing-Jorgensen (2009). Who Bears Aggrgate Fluctuations and How? *American Economic Review, Papes and Proceedings*. forthcoming.

Polkovnichenko, V. (2004). Limited Stock Market Participation and the Equity Premium. *Finance Research Letters 1*, 24–34.

Souleles, N. S. (2003). Household Portfolio Choice, Transactions Costs, and Hedging Motives. mimeo, University of Pennsylvania.

Tauchen, G. (1986). Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions. *Economic Letters 20*, 177–181.

Vissing-Jorgensen, A. (2002). Towards an Explanation of Household Portfolio Choice Heterogeneity: Non-Financial Income Participation Costs Structures. National Bureau of Economic Research Working Paper 8884.

Vissing-Jorgensen, A. (2003). Perspectives on Behavioral Finance: Does Irrationality Disappear with Wealth? Evidence from Expectations and Actions. In M. Gertler and K. Rogoff (Eds.), *NBER Macroeconomics Annual*. MIT Press.

Table 1: Statistics from Model Economies

	Endogenous Rebalancing	Endogenous Participation	Representative Agent
$E(r^e - r^f)$	5.4	0.5	0.4
$E(r^f)$	0.7	3.7	3.7
$\sigma_{\Delta c}$	3.4	3.8	3.5
$\frac{E(c_A)}{E(c)}$	1.08	1.75	1
$\frac{\sigma_{\Delta c_A}}{\sigma_{\Delta c}}$	3.8	1.09	1
Percent of Active Types	20.0	28.8	100
Average Fixed Cost (% of GDP)	0.26	8.8	0
Percent of Time at ZLB	15.7	0	0
Percent of Time $a(\gamma_2, s^t) > 0$	0.2	0	0
Percent of Time $a(\gamma_3, s^t) > 0$	0.0	0	0
$A(\gamma_2)$	0.25	0	0
$A(\gamma_3)$	0.35	0	0

Note: The equity premium, the real rate, and the standard deviation of consumption are expressed at an annual percentage rate.

Figure 1: Endogenous Rebalancing and the Equity Premium

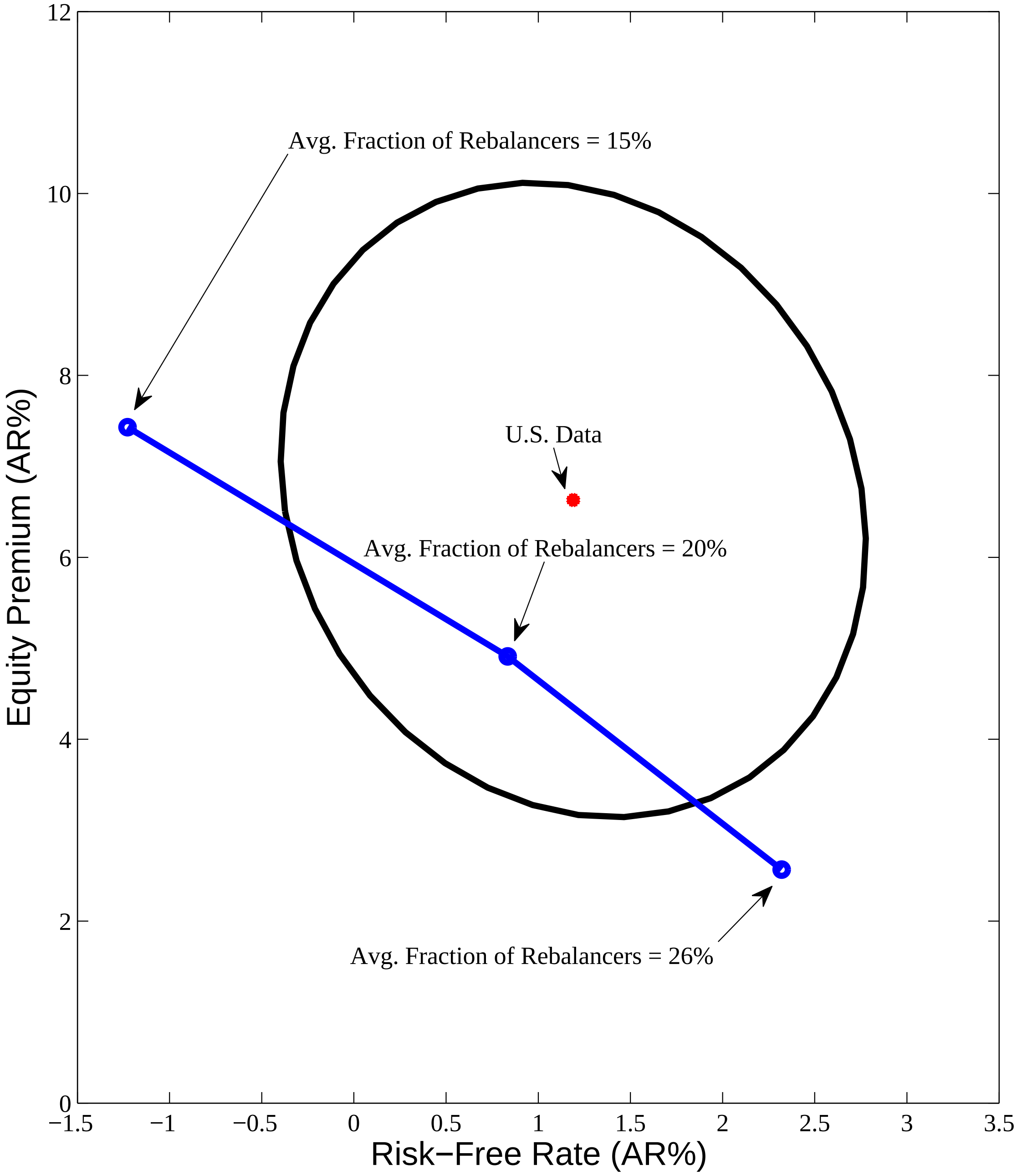


Figure 2: Impulse Response to an Expansionary Monetary Shock: Model Comparison
(Deviation from Date 0 Expectation of a Variable)

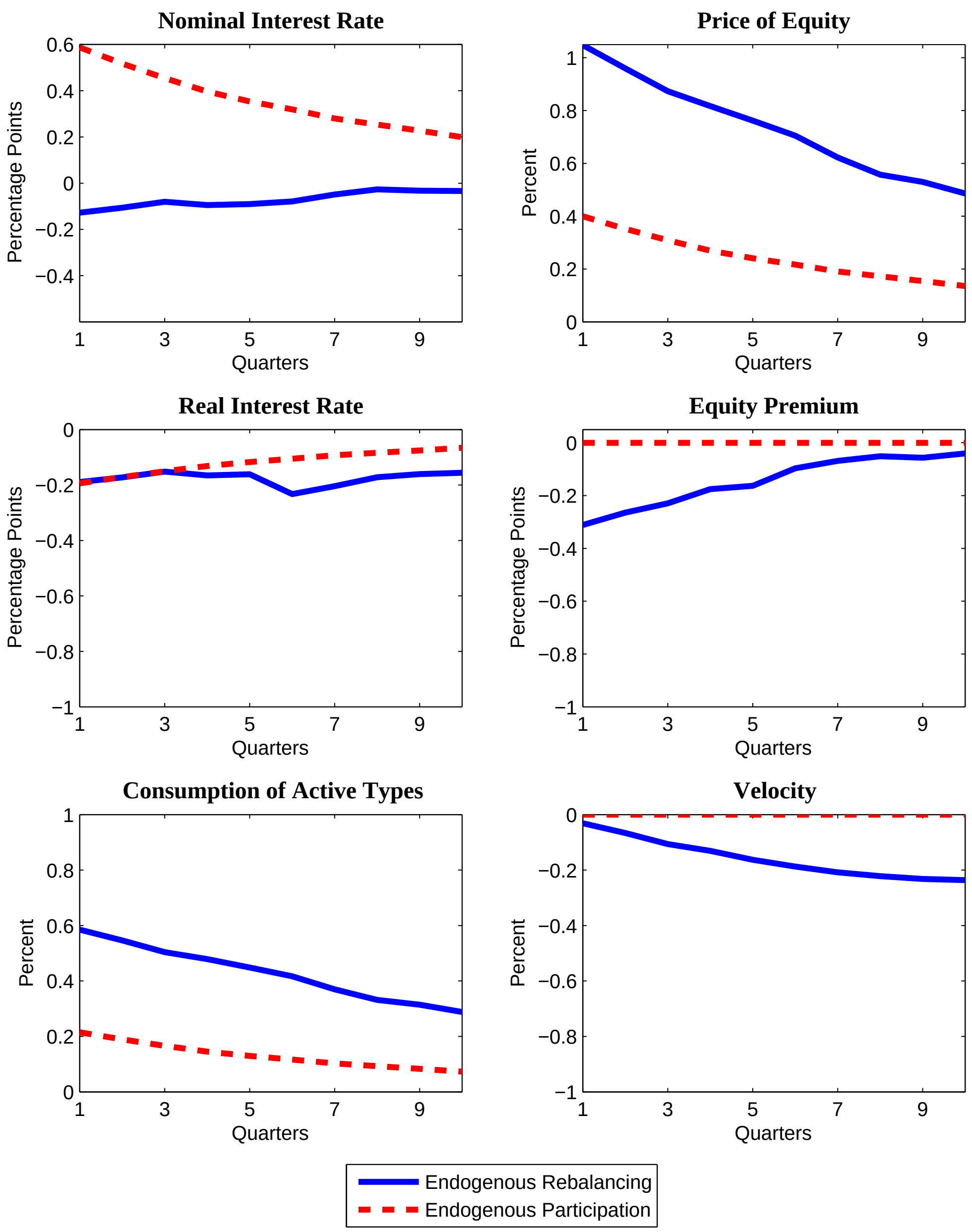


Figure 3: Impulse Response to an Expansionary Monetary Shock: Alternative Persistence (Deviation from Date 0 Expectation of a Variable)

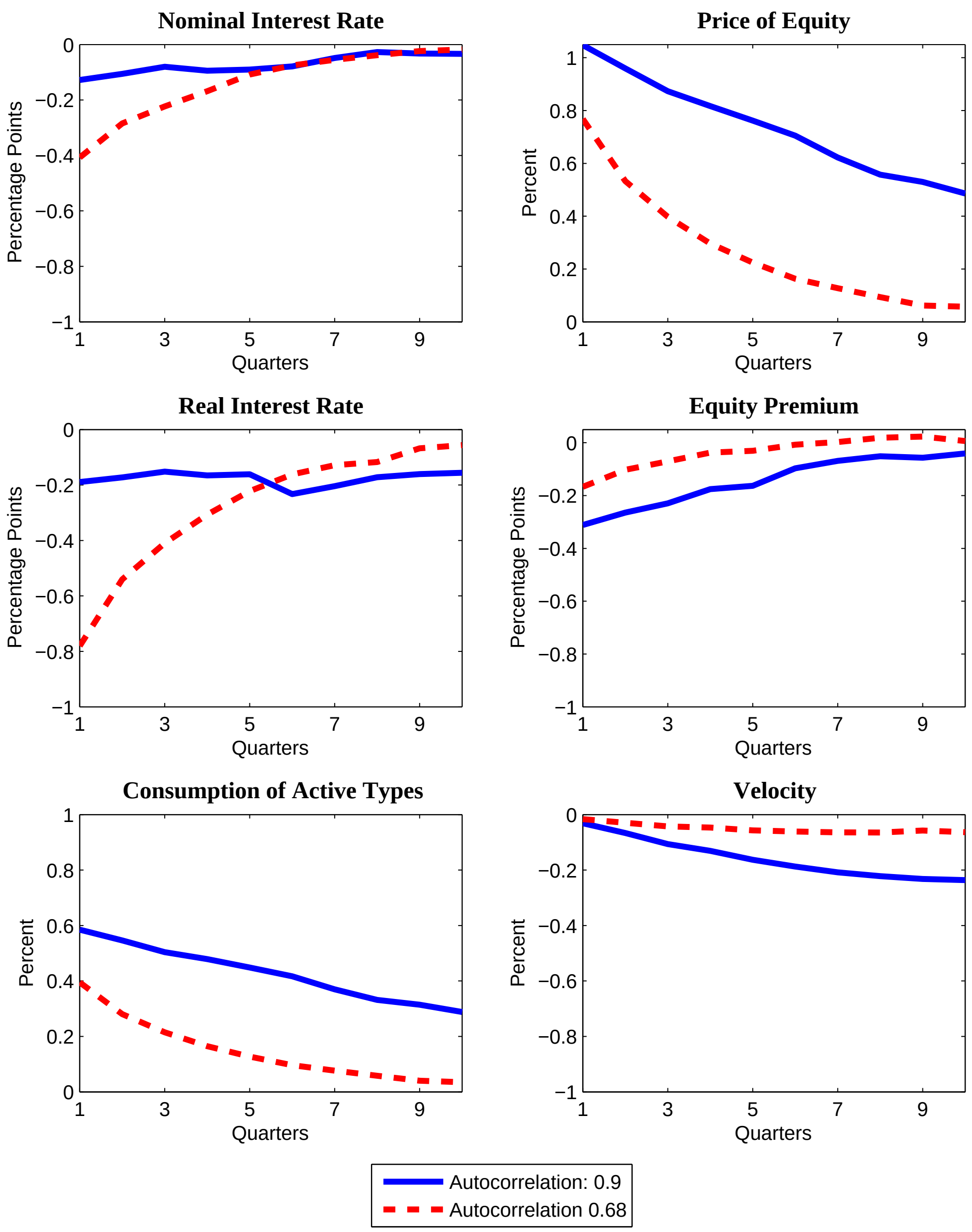


Figure 4: The Effect of a 15 Percent Decline in Technology

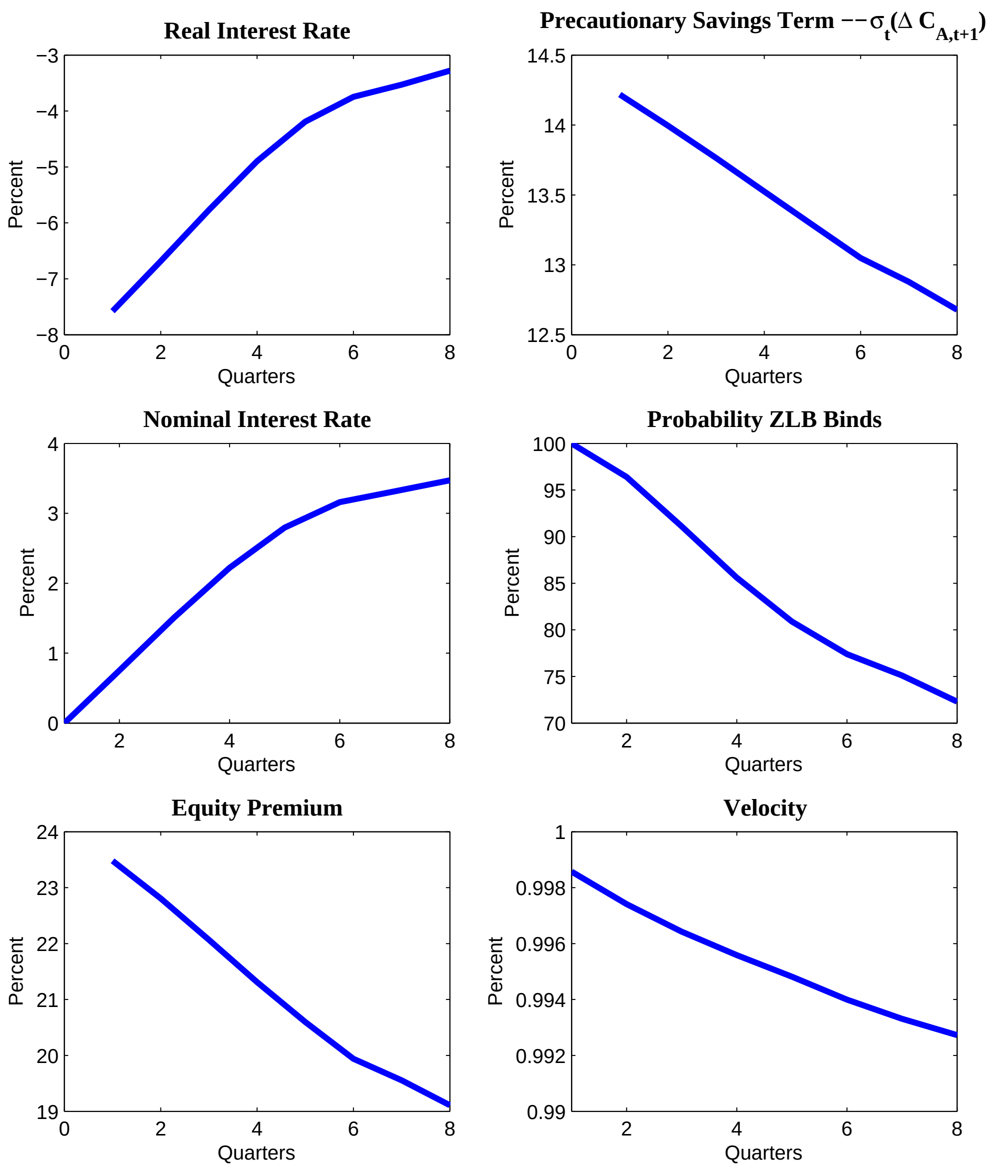


Figure 5: The Effect of Monetary Easing at the ZLB

(Deviation from Baseline Scenario)

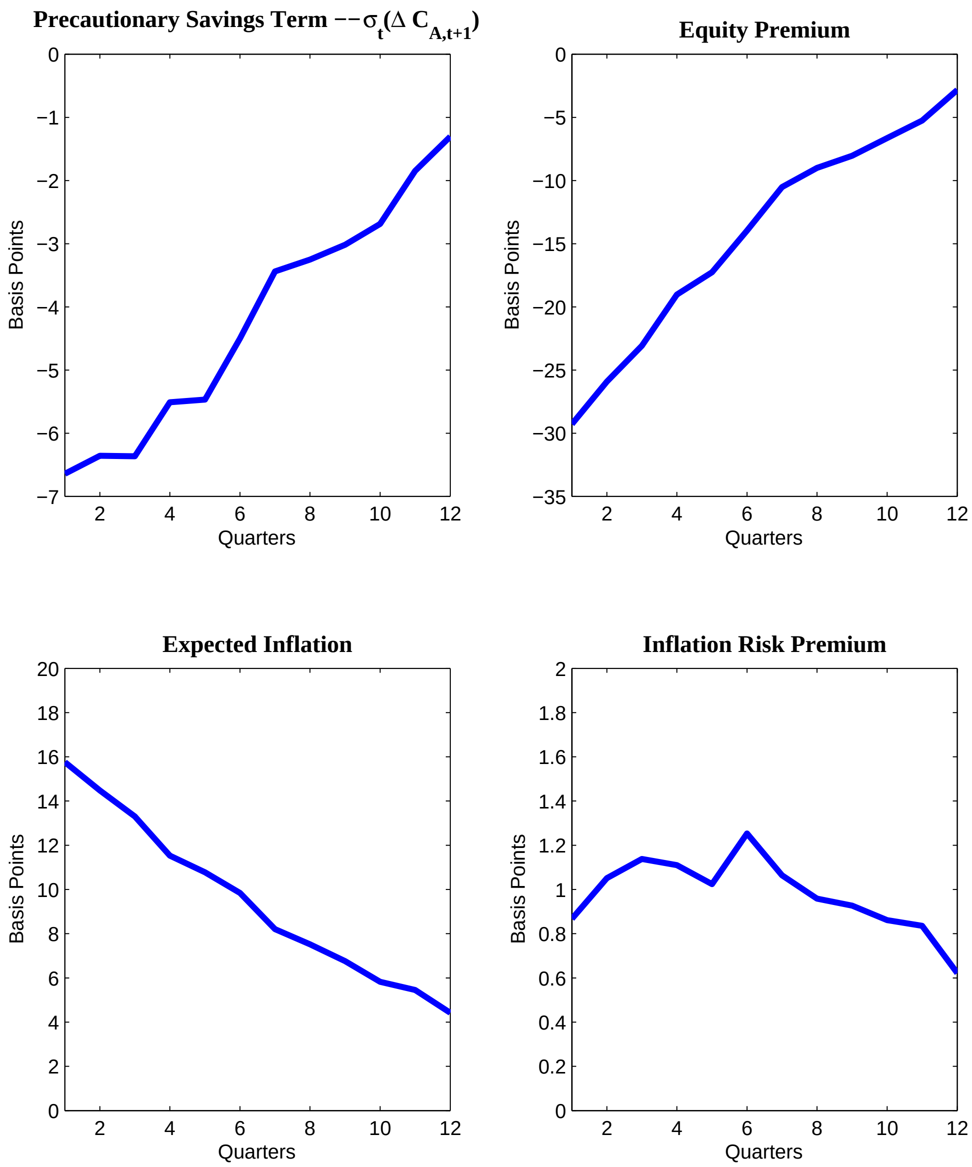


Figure 6: Equilibrium in the Two Models

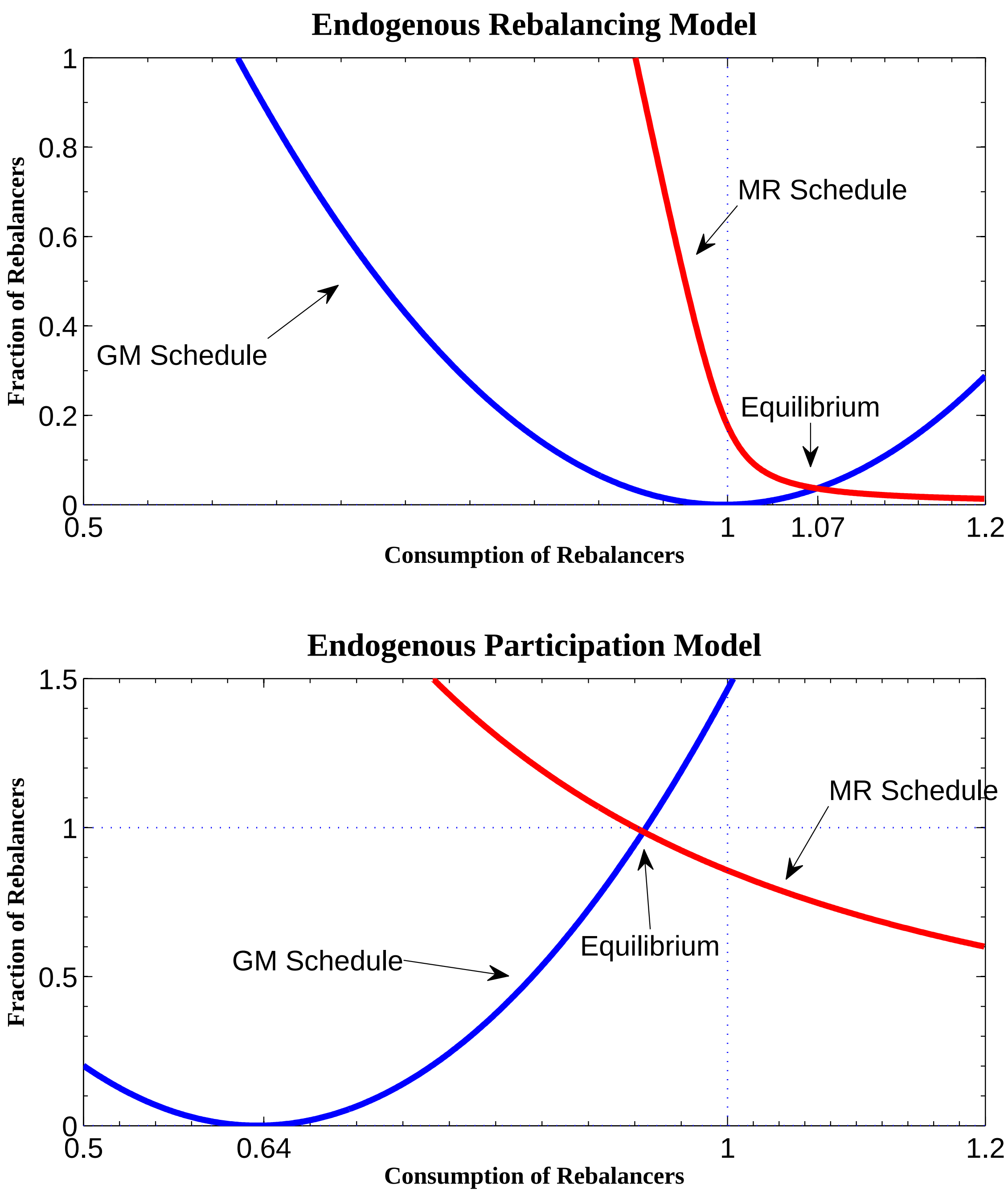


Figure 7: A Deterministic Increase in Technology in the Rebalancing Model

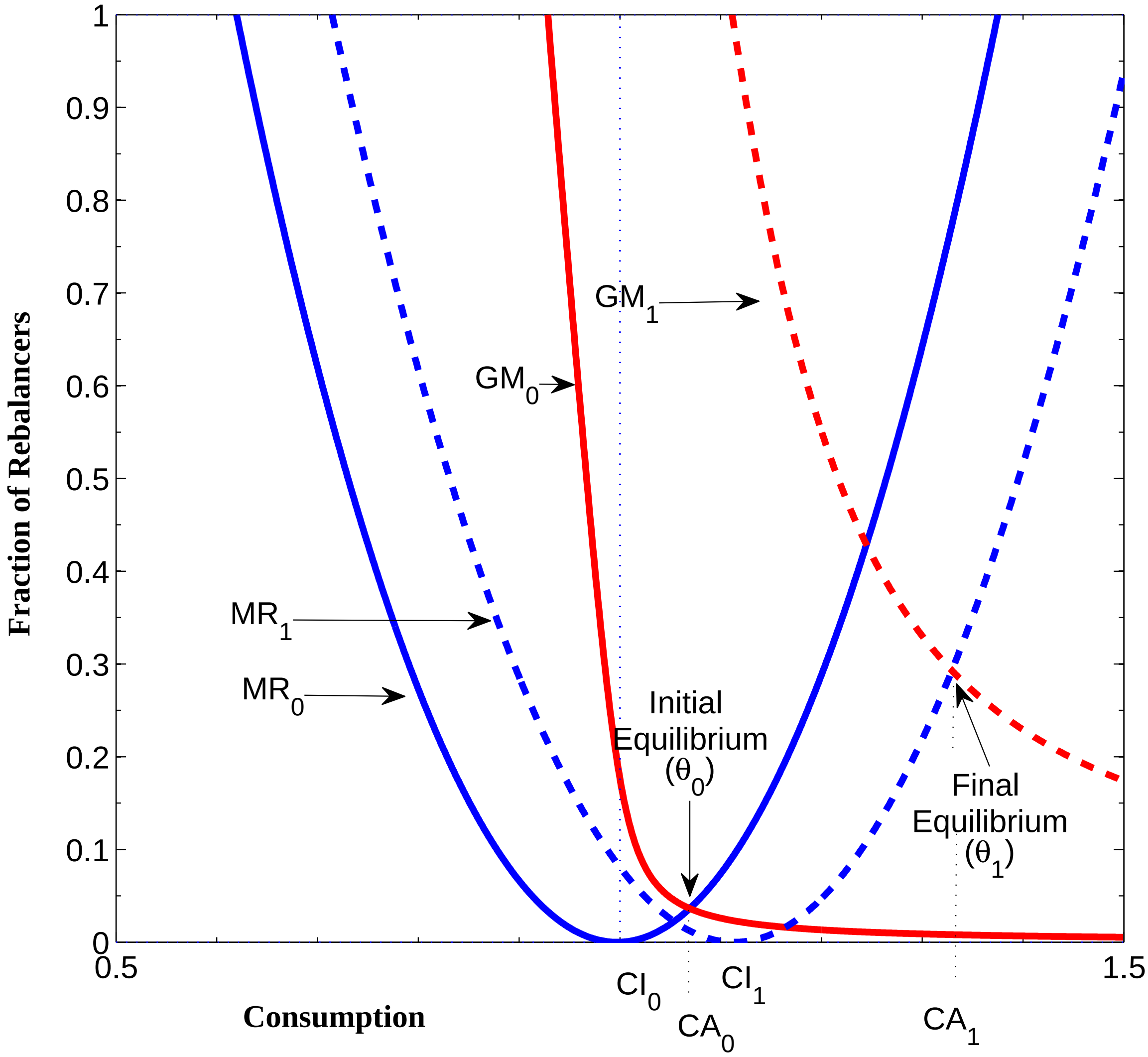


Figure 8: A Deterministic Change in Money Growth in the Rebalancing Model

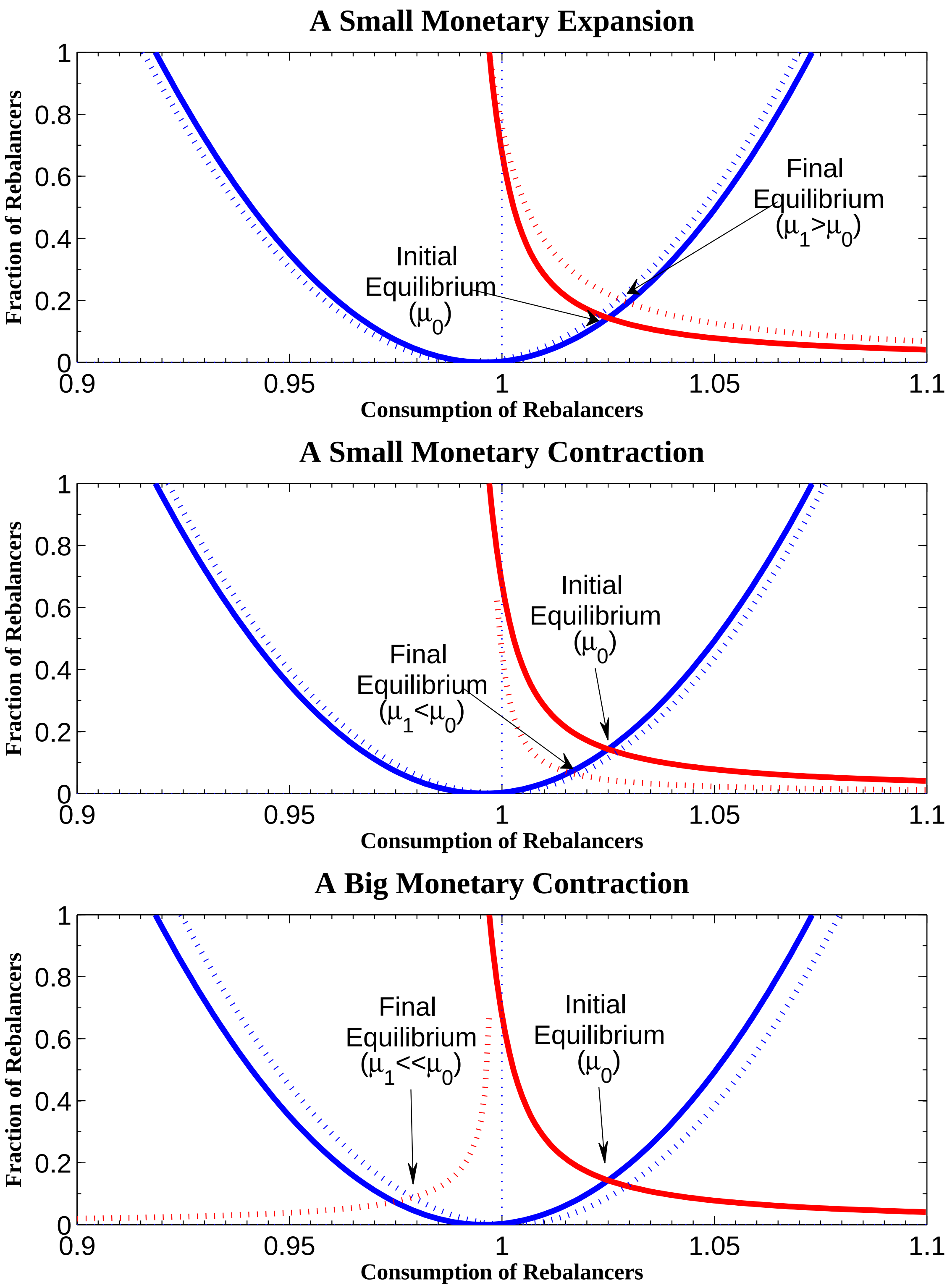
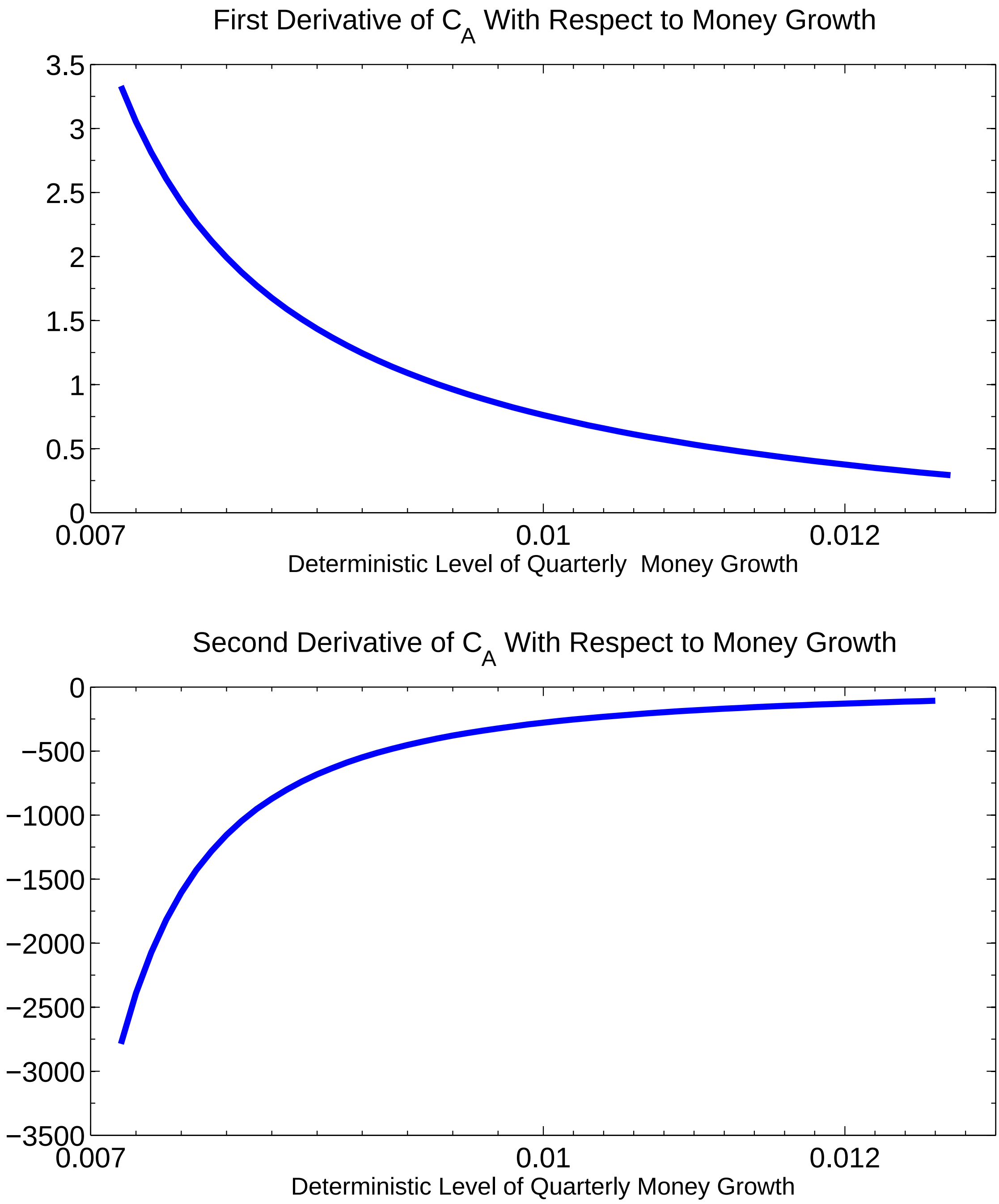


Figure 9: The Shape of Active Consumption



5 Technical Appendix

5.1 Derivation of Equilibrium Conditions

To be completed.

5.2 Numerical Solution

To be completed.