Credit Spreads and Monetary Policy: Technical Appendix*

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1 Overview

This technical appendix describes in its next three sections the different versions of the credit frictions model:

FF model The full model, with heterogeneous agents and credit frictions.

NoFF model The model with heterogeneous agents but no credit frictions.

RepHH model The model with representative agent and no credit frictions, equivalent to the standard New Keynesian model discussed in Woodford (2003).

2 FF model: heterogeneous agents and credit frictions

This section describes complete model of credit frictions, also refered in the text as the "FF" model. It includes both heterogeneous agents and credit frictions.[1] The first subsection contains all the non-linear equations and objective welfare function, the second presents the steady state, the third the log-linearized equations, and the fourth presents a detailed description of the parameter values used for the numberical exercises.

*The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

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[1] For details on the derivations please refer to Cúrdia and Woodford (2009) and its technical appendix.

2.1 All equations

The objective:

$$\tilde{U}_t = \pi_b \frac{\left(\lambda_t^b\right)^{1-\sigma_b} \bar{C}_t^b}{1-\sigma_b^{-1}} + (1-\pi_b) \frac{\left(\lambda_t^s\right)^{1-\sigma_s} \bar{C}_t^s}{1-\sigma_s^{-1}} - \frac{\psi}{1+\nu} \left(\frac{\tilde{\lambda}_t}{\tilde{\Lambda}_t}\right)^{-\frac{1+\nu}{\nu}} \bar{H}_t^{-\nu} \left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} \Delta_t \tag{2.1}$$

The equations describing the economy are summarized below:

$$0 = \left(1+i_t^d\right)(1+\omega_t)\beta E_t \left[\left[\delta + (1-\delta)\pi_b\right] \frac{\lambda_{t+1}^b}{\Pi_{t+1}} + (1-\delta)(1-\pi_b) \frac{\lambda_{t+1}^s}{\Pi_{t+1}}\right] - \lambda_t^b \tag{2.2}$$

$$0 = \left(1+i_t^d\right)\beta E_t \left[(1-\delta)\pi_b \frac{\lambda_{t+1}^b}{\Pi_{t+1}} + \left[\delta + (1-\delta)(1-\pi_b)\right] \frac{\lambda_{t+1}^s}{\Pi_{t+1}}\right] - \lambda_t^s \tag{2.3}$$

$$0 = \Lambda\left(\lambda_t^b, \lambda_t^s\right) \mu^p (1+\omega_y) \psi \mu_t^w \tilde{\lambda}\left(\lambda_t^b, \lambda_t^s\right)^{-1} \bar{H}_t^{-\nu} \left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} + \alpha\beta E_t \left[\Pi_{t+1}^{\theta(1+\omega_y)} K_{t+1}\right] - K_t \tag{2.4}$$

$$0 = \Lambda\left(\lambda_t^b, \lambda_t^s\right)(1-\tau_t) Y_t + \alpha\beta E_t \left[\Pi_{t+1}^{\theta-1} F_{t+1}\right] - F_t \tag{2.5}$$

$$\begin{aligned} 0 \;=\; & \pi_b (1-\pi_b) B\left(\lambda_t^b, \lambda_t^s, Y_t, \Delta_t; \xi_t\right) - \pi_b b_t^g \\ & + \delta \left[b_{t-1}(1+\omega_{t-1}) + \pi_b b_{t-1}^g\right] \frac{1+i_{t-1}^d}{\Pi_t} - (1+\pi_b\omega_t) b_t \end{aligned} \tag{2.6}$$

$$0 = \pi_b \bar{C}_t^b \left(\lambda_t^b\right)^{-\sigma_b} + (1-\pi_b) \bar{C}_t^s \left(\lambda_t^s\right)^{-\sigma_s} + \tilde{\Xi}_t b_t^\eta + G_t - Y_t \tag{2.7}$$

$$0 = \alpha \Delta_{t-1} \Pi_t^{\theta(1+\omega_y)} + (1-\alpha) \left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta(1+\omega_y)}{\theta-1}} - \Delta_t \tag{2.8}$$

$$0 = \frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha} - \left(\frac{F_t}{K_t}\right)^{\frac{\theta-1}{1+\omega_y\theta}} \tag{2.9}$$

$$0 = 1 + (1+\varkappa)\tilde{\chi}_t b_t^\varkappa + \eta \tilde{\Xi}_t b_t^{\eta-1} - (1+\omega_t) \tag{2.10}$$

Auxiliary:

$$\begin{aligned} B\left(\lambda_t^b, \lambda_t^s, Y_t, \Delta_t; \xi_t\right) \;\equiv\; & \bar{C}_t^b \left(\lambda_t^b\right)^{-\sigma_b} - \bar{C}_t^s \left(\lambda_t^s\right)^{-\sigma_s} \\ & - \left[\left(\frac{\lambda_t^b}{\psi_b}\right)^{\frac{1}{\nu}} - \left(\frac{\lambda_t^s}{\psi_s}\right)^{\frac{1}{\nu}}\right] \left(\frac{\tilde{\lambda}_t}{\psi}\right)^{-\frac{1+\nu}{\nu}} \mu_t^w \bar{H}_t^{-\nu} \left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} \Delta_t \end{aligned} \tag{2.11}$$

$$\Lambda\left(\lambda_t^b, \lambda_t^s\right) \equiv \pi_b \lambda_t^b + (1-\pi_b)\lambda_t^s \tag{2.12}$$

$$\tilde{\lambda}\left(\lambda_t^b, \lambda_t^s\right) \equiv \psi \left[\pi_b \left(\frac{\lambda_t^b}{\psi_b}\right)^{\frac{1}{\nu}} + (1-\pi_b)\left(\frac{\lambda_t^s}{\psi_s}\right)^{\frac{1}{\nu}}\right]^\nu \tag{2.13}$$

$$\tilde{\Lambda}\left(\lambda_t^b, \lambda_t^s\right) \equiv \psi^{\frac{1}{1+\nu}} \left[\pi_b \psi_b^{-\frac{1}{\nu}} \left(\lambda_t^b\right)^{\frac{1+\nu}{\nu}} + (1-\pi_b)\, \psi_s^{-\frac{1}{\nu}} \left(\lambda_t^s\right)^{\frac{1+\nu}{\nu}}\right]^{\frac{\nu}{1+\nu}} \tag{2.14}$$

$$c_t^b = \bar{C}_t^b \left(\lambda_t^b\right)^{-\sigma_b} \tag{2.15}$$

$$c_t^s = \bar{C}_t^s \left(\lambda_t^s\right)^{-\sigma_s} \tag{2.16}$$

2.2 Zero inflation steady state

We consider the solution to steady state in which we simply assume zero inflation. For simplification of the analysis consider the following definitions

$$\begin{aligned}
s_c &\equiv \pi_b s_b + (1-\pi_b)\, s_s, \\
s_b &\equiv \bar{c}^b / \bar{Y}, \\
s_s &\equiv \bar{c}^s / \bar{Y}, \\
s_c^{bs} &\equiv s_b / s_s, \\
\bar{\sigma} &\equiv \pi_b s_b \sigma_b + (1-\pi_b)\, s_s \sigma_s, \\
\sigma_{bs} &\equiv \sigma_b / \sigma_s, \\
\rho_b &\equiv \bar{b} / \bar{Y}, \\
s_\Xi &\equiv \bar{\Xi}\left(\bar{b}\right) / \bar{Y}, \\
\rho_b^g &\equiv \bar{b}^g / \bar{Y}, \\
s_g &\equiv \bar{G} / \bar{Y}, \\
\psi_{bs} &\equiv \psi_b / \psi_s.
\end{aligned}$$

Without any loss of generality we calibrate the following values:

$$\begin{aligned}
\bar{Y} &= 1, \\
\psi &= 1.
\end{aligned}$$

The values of s_c, s_b/s_s and σ_b/σ_s are set according to the calibration described in the paper.

For the interest rate we have:

$$1 + \bar{r}^d = \beta^{-1} \frac{(\delta+1) + \bar{\omega}\left[\delta + (1-\delta)\,\pi_b\right] - \sqrt{\left\{(\delta+1) + \bar{\omega}\left[\delta + (1-\delta)\,\pi_b\right]\right\}^2 - 4\delta\,(1+\bar{\omega})}}{2\delta\,(1+\bar{\omega})}. \tag{2.17}$$

(Note that if $\bar{\omega} = 0$, this reduces to $1 + \bar{r}^d = \beta^{-1}$.) We use this steady-state relation to calibrate β, given assumed values for δ, π_b, $\bar{\omega}$ and $\bar{r}^d$.

The nominal deposit rate is equal to the real rate:

$$1 + \bar{\imath}^d = 1 + \bar{r}^d. \tag{2.18}$$

The markup is calibrated so that $\bar{\chi}$ and $\bar{\Xi}$ insure that the equation defining $\bar{\omega}$ is satisfied. We consider two cases:

- Exogenous, takes resources: set $\eta = 1$, $\varkappa = 0$ and $\overline{\tilde{\chi}} = 0$, implying that $\overline{\tilde{\Xi}} = \bar{\omega}$ and $s_\Xi = \bar{\omega}\rho_b$.
- Endogenous, takes resources: set $\eta > 1$, $\varkappa > 0$ and $\bar{\chi} = 0$, implying that $\bar{\Xi}'\left(\bar{b}\right) = \bar{\omega}$, $s_\Xi = \frac{\bar{\omega}}{\eta}\rho_b$ and $\overline{\tilde{\Xi}} = \bar{\omega}/\left(\eta\bar{b}^{\eta-1}\right)$.

The two marginal utilities of consumption are related through:

$$\bar{\lambda}^b = \bar{\Omega}\bar{\lambda}^s, \tag{2.19}$$

with

$$\bar{\Omega} \equiv \frac{1 - \left(1 + \bar{r}^d\right)\beta\left[\delta + (1-\delta)(1-\pi_b)\right]}{\left(1 + \bar{r}^d\right)\beta(1-\delta)\pi_b}. \tag{2.20}$$

Given the assumption that we calibrate ψ_{bs} and ψ, we can then write

$$\psi_s = \left[\pi_b\psi_{bs}^{-\frac{1}{\nu}} + (1-\pi_b)\right]^{-1}, \tag{2.21}$$

and $\psi_b = \psi_{bs}\psi_s$. This implies that, given ψ_b and ψ_s, we get

$$\Lambda\left(\bar{\lambda}^b, \bar{\lambda}^s\right) = \left[\pi_b\bar{\Omega} + (1-\pi_b)\right]\bar{\lambda}^s, \tag{2.22}$$

$$\tilde{\lambda}\left(\bar{\lambda}^b, \bar{\lambda}^s\right) = \psi\left[\pi_b\bar{\Omega}^{\frac{1}{\nu}}\psi_b^{-\frac{1}{\nu}} + (1-\pi_b)\psi_s^{-\frac{1}{\nu}}\right]^{\nu}\bar{\lambda}^s, \tag{2.23}$$

$$\tilde{\Lambda}\left(\bar{\lambda}^b, \bar{\lambda}^s\right) = \psi^{\frac{1}{1+\nu}}\left[\pi_b\psi_b^{-\frac{1}{\nu}}\bar{\Omega}^{\frac{1+\nu}{\nu}} + (1-\pi_b)\psi_s^{-\frac{1}{\nu}}\right]^{\frac{\nu}{1+\nu}}\bar{\lambda}^s. \tag{2.24}$$

Using $\bar{F} = \bar{K}$,

$$(1-\bar{\tau}) = \mu^p(1+\omega_y)\psi\mu_t^w\tilde{\lambda}\left(\bar{\lambda}^b, \bar{\lambda}^s\right)^{-1}\frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}},$$

hence

$$\bar{\lambda}^s = \frac{\mu^p(1+\omega_y)\mu_t^w\frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}}}{(1-\bar{\tau})\left[\pi_b\bar{\Omega}^{\frac{1}{\nu}}\psi_b^{-\frac{1}{\nu}} + (1-\pi_b)\psi_s^{-\frac{1}{\nu}}\right]^{\nu}}. \tag{2.25}$$

Given our calibration of s_c and s_c^{bs}, we can write:

$$s_s = \frac{s_c}{\pi_b s_c^{bs} + (1-\pi_b)}, \tag{2.26}$$

and $s_b = s_c^{bs}s_c$.

The resources constraint implies

$$1 - s_c - s_g = \mathbf{1}_{(\bar{\Xi}>0)}\frac{\bar{\omega}}{\eta}\rho_b, \tag{2.27}$$

where $\mathbf{1}_{(\bar{\Xi}>0)}$ is an indicator function that takes the value one if $\bar{\Xi} > 0$ and zero otherwise.

Zero inflation implies that

$$\bar{\Delta} = 1. \tag{2.28}$$

The debt equation is

$$\left[1 + \pi_b \bar{\omega} - \delta\left(1 + \bar{\omega}\right)\right] \rho_b = \pi_b \left(1 - \pi_b\right) \frac{B\left(\bar{\lambda}^b, \bar{\lambda}^s, \bar{Y}, \bar{\Delta}; 0\right)}{\bar{Y}} - \pi_b \rho_b^g \left[1 - \delta\left(1 + \bar{r}^d\right)\right],$$

with

$$\frac{B\left(\bar{\lambda}^b, \bar{\lambda}^s, 1, 1; 0\right)}{\bar{Y}} = s_b - s_s - \frac{\bar{\Omega}^{\frac{1}{\nu}} \psi_b^{-\frac{1}{\nu}} - \psi_s^{-\frac{1}{\nu}}}{\pi_b \bar{\Omega}^{\frac{1}{\nu}} \psi_b^{-\frac{1}{\nu}} + \left(1 - \pi_b\right) \psi_s^{-\frac{1}{\nu}}} \frac{1 - \bar{\tau}}{\mu^p \left(1 + \omega_y\right)},$$

implying that

$$\rho_b = \frac{\pi_b \left(1 - \pi_b\right) \left(s_b - s_s - \frac{\bar{\Omega}^{\frac{1}{\nu}} \psi_b^{-\frac{1}{\nu}} - \psi_s^{-\frac{1}{\nu}}}{\pi_b \bar{\Omega}^{\frac{1}{\nu}} \psi_b^{-\frac{1}{\nu}} + (1-\pi_b) \psi_s^{-\frac{1}{\nu}}} \frac{1-\bar{\tau}}{\mu^p (1+\omega_y)}\right) - \pi_b \rho_b^g \left[1 - \delta\left(1 + \bar{r}^d\right)\right]}{1 + \pi_b \bar{\omega} - \delta\left(1 + \bar{\omega}\right)\left(1 + \bar{r}^d\right)}, \tag{2.29}$$

which is then used to solve for real debt according to $\bar{b} = \rho_b \bar{Y}$. Given $\bar{b}$ and s_c, the real resources equation (2.27) determines $s_g = 1 - s_c - \mathbf{1}_{(\bar{\Xi}>0)} \frac{\bar{\omega}}{\bar{\eta}} \rho_b$.

Furthermore, we set $\bar{\sigma}$, hence

$$\sigma_s = \frac{\bar{\sigma}}{\pi_b s_b \sigma_{bs} + \left(1 - \pi_b\right) s_s}, \tag{2.30}$$

$$\sigma_b = \sigma_{bs} \sigma_s. \tag{2.31}$$

Finally,

$$\bar{C}^b = s_b \left(\bar{\lambda}^b\right)^{\sigma_b}, \tag{2.32}$$

$$\bar{C}^s = s_b \left(\bar{\lambda}^s\right)^{\sigma_s}, \tag{2.33}$$

$$\bar{K} = \frac{\lambda\left(\bar{\lambda}^b, \bar{\lambda}^s\right) \mu^p \left(1 + \omega_y\right) \psi \mu^w \tilde{\lambda}\left(\bar{\lambda}^b, \bar{\lambda}^s\right)^{-1} \frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}}}{1 - \alpha\beta}, \tag{2.34}$$

$$\bar{F} = \frac{\lambda\left(\bar{\lambda}^b, \bar{\lambda}^s\right)\left(1 - \bar{\tau}\right)}{1 - \alpha\beta}. \tag{2.35}$$

We further set ψ_b / ψ_s such that the labor supply is the same in steady state, which implies that

$$\frac{\bar{\lambda}^b}{\psi_b} = \frac{\bar{\lambda}^s}{\psi_s} \Leftrightarrow \frac{\bar{\lambda}^b}{\bar{\lambda}^s} = \frac{\psi_b}{\psi_s} \Rightarrow \frac{\psi_b}{\psi_s} = \bar{\Omega}, \tag{2.36}$$

hence

$$\psi_s = \left[\pi_b \bar{\Omega}^{-\frac{1}{\nu}} + (1-\pi_b)\right]^{\nu}, \tag{2.37}$$

$$\psi_b = \bar{\Omega}\psi_s, \tag{2.38}$$

$$\bar{\lambda}^s = \frac{\mu^p (1+\omega_y)\, \mu^w \frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}}}{(1-\bar{\tau})\left[\pi_b \bar{\Omega}^{\frac{1}{\nu}} \psi_b^{-\frac{1}{\nu}} + (1-\pi_b)\, \psi_s^{-\frac{1}{\nu}}\right]^{\nu}}, \tag{2.39}$$

$$\bar{\lambda}^b = \bar{\Omega}\bar{\lambda}^s, \tag{2.40}$$

$$\Lambda\left(\bar{\lambda}^b, \bar{\lambda}^s\right) = \pi_b \bar{\lambda}^b + (1-\pi_b)\, \bar{\lambda}^s. \tag{2.41}$$

2.3 Log-linear equations

In this section we present all the log-linear relations of the model, in which we linearize around the zero inflation steady state.

Full system

The full system of log-linear equation is given by:

$$\hat{\lambda}_t^b = \hat{\imath}_t^d + \hat{\omega}_t - E_t \pi_{t+1} + \chi_b E_t \hat{\lambda}_{t+1}^b + (1-\chi_b)\, E_t \hat{\lambda}_{t+1}^s, \tag{2.42}$$

$$\hat{\lambda}_t^s = \hat{\imath}_t^d - E_t \pi_{t+1} + (1-\chi_s)\, E_t \hat{\lambda}_{t+1}^b + \chi_s E_t \hat{\lambda}_{t+1}^s, \tag{2.43}$$

$$\begin{aligned} \hat{K}_t \;=\; & (1-\alpha\beta)\left[\hat{\Lambda}_t - \widehat{\bar{\lambda}}_t + \hat{\mu}_t^w - \nu \bar{h}_t + (1+\omega_y)\left(\hat{Y}_t - z_t\right)\right] \\ & + \alpha\beta E_t\left[\theta\,(1+\omega_y)\,\pi_{t+1} + \hat{K}_{t+1}\right], \end{aligned} \tag{2.44}$$

$$\hat{F}_t = (1-\alpha\beta)\left[\hat{\Lambda}_t - \hat{\tau}_t + \hat{Y}_t\right] + \alpha\beta E_t\left[(\theta-1)\,\pi_{t+1} + \hat{F}_{t+1}\right], \tag{2.45}$$

$$\begin{aligned} (1+\pi_b\bar{\omega})\,\hat{b}_t \;=\; & \pi_b\,(1-\pi_b)\,\rho_b^{-1}\hat{B}_t - \pi_b\,(1+\bar{\omega})\,\hat{\omega}_t \\ & + \delta\left(1+\bar{r}^d\right)\left[(1+\bar{\omega}) + \pi_b \rho_b^g/\rho_b\right]\left(\hat{\imath}_{t-1}^d - \pi_t\right) \\ & + \delta\left(1+\bar{r}^d\right)(1+\bar{\omega})\left(\hat{b}_{t-1} + \hat{\omega}_{t-1}\right) \\ & - \pi_b \rho_b^{-1}\left[\hat{b}_t^g - \delta\left(1+\bar{r}^d\right)\hat{b}_{t-1}^g\right], \end{aligned} \tag{2.46}$$

$$\hat{Y}_t = \pi_b s_b\left(\bar{c}_t^b - \sigma_b \hat{\lambda}_t^b\right) + (1-\pi_b)\, s_s\left(\bar{c}_t^s - \sigma_s \hat{\lambda}_t^s\right) + \widehat{\bar{\Xi}}_t + \eta s_\Xi \hat{b}_t + \hat{G}_t, \tag{2.47}$$

$$\hat{\Delta}_t = \alpha \hat{\Delta}_{t-1}, \tag{2.48}$$

$$\pi_t = \frac{1-\alpha}{\alpha}\frac{1}{1+\omega_y\theta}\left(\hat{K}_t - \hat{F}_t\right), \tag{2.49}$$

$$\hat{\omega}_t = \frac{(1+\varkappa)\,\bar{\chi}\bar{b}^{\varkappa}}{1+\bar{\omega}}\left(\frac{\zeta_\chi}{\bar{\chi}}\widehat{\tilde{\chi}}_t + \varkappa\hat{b}_t\right) + \frac{\eta\bar{\Xi}\bar{b}^{\eta-1}}{1+\bar{\omega}}\left(\frac{\zeta_\Xi}{\bar{\Xi}}\widehat{\tilde{\Xi}}_t + (\eta-1)\,\hat{b}_t\right). \tag{2.50}$$

Auxiliary equations:

$$\begin{aligned}\hat{B}_t \;=\;& s_b\left(\bar{c}_t^b - \sigma_b\hat{\lambda}_t^b\right) - s_s\left(\bar{c}_t^s - \sigma_s\hat{\lambda}_t^s\right) \qquad (2.51)\\ &-\psi\overline{\tilde{\lambda}}^{-1}\bar{\mu}^w\bar{H}^{-\nu}\left(\frac{\bar{Y}}{\bar{Z}}\right)^{1+\omega_y}\bar{Y}^{-1}\frac{1}{\nu}\left[\left(\frac{\psi}{\psi_b}\frac{\bar{\lambda}^b}{\overline{\tilde{\lambda}}}\right)^{\frac{1}{\nu}}\left(\hat{\lambda}_t^b - \widehat{\tilde{\lambda}}_t\right) - \left(\frac{\psi}{\psi_s}\frac{\bar{\lambda}^s}{\overline{\tilde{\lambda}}}\right)^{\frac{1}{\nu}}\left(\hat{\lambda}_t^s - \widehat{\tilde{\lambda}}_t\right)\right]\\ &-\psi\overline{\tilde{\lambda}}^{-1}\bar{\mu}^w\bar{H}^{-\nu}\left(\frac{\bar{Y}}{\bar{Z}}\right)^{1+\omega_y}\bar{Y}^{-1}\left[\left(\frac{\psi}{\psi_b}\frac{\bar{\lambda}^b}{\overline{\tilde{\lambda}}}\right)^{\frac{1}{\nu}} - \left(\frac{\psi}{\psi_s}\frac{\bar{\lambda}^s}{\overline{\tilde{\lambda}}}\right)^{\frac{1}{\nu}}\right]\times\\ &\times\left[\hat{\mu}_t^w - \nu\bar{h}_t - \widehat{\tilde{\lambda}}_t + (1+\omega_y)\left(\hat{Y}_t - z_t\right) + \hat{\Delta}_t\right], \qquad (2.52)\end{aligned}$$

$$\widehat{\tilde{\lambda}}_t = \pi_b\left(\frac{\psi}{\psi_b}\frac{\bar{\lambda}^b}{\overline{\tilde{\lambda}}}\right)^{\frac{1}{\nu}}\hat{\lambda}_t^b + (1-\pi_b)\left(\frac{\psi}{\psi_s}\frac{\bar{\lambda}^s}{\overline{\tilde{\lambda}}}\right)^{\frac{1}{\nu}}\hat{\lambda}_t^s, \tag{2.53}$$

$$\hat{\Lambda}_t = \pi_b\frac{\bar{\lambda}^b}{\bar{\lambda}}\hat{\lambda}_t^b + (1-\pi_b)\frac{\bar{\lambda}^s}{\bar{\lambda}}\hat{\lambda}_t^s, \tag{2.54}$$

$$\widehat{\tilde{\Lambda}}_t = \pi_b\frac{\psi_b^{-\frac{1}{\nu}}\left(\bar{\lambda}^b\right)^{\frac{1+\nu}{\nu}}}{\psi^{-\frac{1}{\nu}}\overline{\tilde{\Lambda}}^{\frac{1+\nu}{\nu}}}\hat{\lambda}_t^b + (1-\pi_b)\frac{\psi_s^{-\frac{1}{\nu}}\left(\bar{\lambda}^s\right)^{\frac{1+\nu}{\nu}}}{\psi^{-\frac{1}{\nu}}\overline{\tilde{\Lambda}}^{\frac{1+\nu}{\nu}}}\hat{\lambda}_t^s, \tag{2.55}$$

$$\hat{c}_t^b = \bar{c}_t^b - \sigma_b\hat{\lambda}_t^b, \tag{2.56}$$

$$\hat{c}_t^s = \bar{c}_t^s - \sigma_s\hat{\lambda}_t^s. \tag{2.57}$$

The exogenous variables all follow an AR(1) process as follows:

$$\xi_t = \rho_\xi\xi_{t-1} + \varepsilon_t \tag{2.58}$$

In the above equations we consider the following definitions

$$\hat{\imath}_t^\tau \equiv \ln\left(\left(1+i_t^\tau\right)/\left(1+\bar{\imath}_t^\tau\right)\right), \tag{2.59}$$

$$\hat{\omega}_t \equiv \ln\left(\left(1+\omega_t\right)/\left(1+\bar{\omega}\right)\right), \tag{2.60}$$

$$\pi_t \equiv \ln\Pi_t, \tag{2.61}$$

$$\bar{\lambda}_t^\tau \equiv \ln\left(\lambda_t^\tau/\bar{\lambda}^\tau\right), \tag{2.62}$$

$$\hat{Y}_t \equiv \ln\left(Y_t/\bar{Y}\right), \tag{2.63}$$

$$\hat{F}_t \equiv \ln\left(F_t/\bar{F}\right), \tag{2.64}$$

$$\hat{K}_t \equiv \ln\left(K_t/\bar{K}\right), \tag{2.65}$$

$$\hat{b}_t \equiv \ln\left(b_t/\bar{b}\right), \tag{2.66}$$

$$\bar{h}_t \equiv \ln\left(\bar{H}_t/\bar{H}\right), \tag{2.67}$$

$$z_t \equiv \ln\left(Z_t/\bar{Z}\right), \tag{2.68}$$

$$\hat{\tau}_t \equiv -\log\left(\left(1-\tau_t\right)/\left(1-\bar{\tau}\right)\right), \tag{2.69}$$

$$\hat{b}_t^g \equiv \left(b_t^g - \bar{b}\right)/\bar{Y}, \tag{2.70}$$

$$\bar{c}_t^\tau \equiv \ln\left(\bar{C}_t^\tau/\bar{C}^\tau\right), \tag{2.71}$$

$$\hat{\mu}_t^w \equiv \ln\left(\mu_t^w/\bar{\mu}^w\right), \tag{2.72}$$

$$\hat{G}_t \equiv \left(G_t - \bar{G}\right)/\bar{Y}, \tag{2.73}$$

$$\widehat{\tilde{\Xi}}_t \equiv \frac{\bar{b}^\eta}{\bar{Y}}\left(\tilde{\Xi}_t - \overline{\tilde{\Xi}}\right), \tag{2.74}$$

$$\widehat{\tilde{\chi}}_t \equiv \left(1+\varkappa\right)\bar{b}^{\varkappa}\left(\tilde{\chi}_t - \overline{\tilde{\chi}}\right), \tag{2.75}$$

and

$$\chi_\tau \equiv \beta\left(1+\bar{r}^\tau\right)\left[\delta + \left(1-\delta\right)\pi_\tau\right]. \tag{2.76}$$

Alternative (simplified) log-linear system

We can write the required equations as

$$\hat{\imath}_t^{avg} = \hat{\imath}_t^d + \pi_b\hat{\omega}_t, \tag{2.77}$$

$$\hat{\Omega}_t = \hat{\omega}_t + \hat{\delta}E_t\hat{\Omega}_{t+1}, \tag{2.78}$$

$$\begin{aligned}\hat{Y}_t \;=\;& E_t\hat{Y}_{t+1} - \bar{\sigma}\left(\hat{\imath}_t^{avg} - E_t\pi_{t+1}\right) - E_t\Delta g_{t+1} - E_t\Delta\hat{\Xi}_{t+1} \\ & -\bar{\sigma}s_\Omega\hat{\Omega}_t + \bar{\sigma}\left(s_\Omega + \psi_\Omega\right)E_t\hat{\Omega}_{t+1},\end{aligned} \tag{2.79}$$

$$\pi_t = \beta E_t\pi_{t+1} + u_t + \kappa\left(\hat{Y}_t - \hat{Y}_t^n\right) - \xi\bar{\sigma}^{-1}\hat{\Xi}_t + \xi\left(s_\Omega + \pi_b - \gamma_b\right)\hat{\Omega}_t, \tag{2.80}$$

$$\hat{\omega}_t = \omega_b\hat{b}_t + \omega_\chi\widehat{\tilde{\chi}}_t + \omega_\Xi\widehat{\tilde{\Xi}}_t, \tag{2.81}$$

$$\begin{aligned}\hat{b}_t \;=\;& \varrho_r\left(\hat{\imath}_{t-1}^d - \pi_t\right) + \varrho_Y\hat{Y}_t + \varrho_\Omega\hat{\Omega}_t + \varrho_\omega\hat{\omega}_t + \varrho_b\left(\hat{b}_{t-1} + \hat{\omega}_{t-1}\right) \\ & +\varrho_\xi\left[\pi_b\left(1-\pi_b\right)s_c\bar{c}_t + B_\lambda\bar{\sigma}^{-1}\left(g_t + \hat{\Xi}_t\right) - \tilde{B}_u\left[\hat{\mu}_t^w - \nu\bar{h}_t - \left(1+\omega_y\right)z_t\right]\right] \\ & -\pi_b\varrho_\xi\left[\hat{b}_t^g - \delta\left(1+\bar{r}^d\right)\hat{b}_{t-1}^g\right],\end{aligned} \tag{2.82}$$

with

$$\hat{Y}_t^n \equiv \left(\omega_y + \bar{\sigma}^{-1}\right)^{-1}\left[\bar{\sigma}^{-1}g_t + \nu\bar{h}_t + \left(1+\omega_y\right)z_t\right], \tag{2.83}$$

$$s_c\bar{c}_t = \pi_b s_b\bar{c}_t^b + \left(1-\pi_b\right)s_s\bar{c}_t^s, \tag{2.84}$$

$$g_t = s_c \bar{c}_t + \hat{G}_t, \tag{2.85}$$

$$u_t \equiv \xi \left(\hat{\mu}_t^w + \hat{\tau}_t \right), \tag{2.86}$$

$$\Delta g_t \equiv g_t - g_{t-1}, \tag{2.87}$$

$$\Delta \widehat{\bar{\Xi}}_t \equiv \widehat{\bar{\Xi}}_t - \widehat{\bar{\Xi}}_{t-1}, \tag{2.88}$$

$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t. \tag{2.89}$$

and

$$\hat{\delta} \equiv \chi_b + \chi_s - 1, \tag{2.90}$$

$$\psi_\Omega \equiv \pi_b \left(1 - \chi_b\right) - \left(1 - \pi_b\right)\left(1 - \chi_s\right), \tag{2.91}$$

$$s_\Omega \equiv \pi_b \left(1 - \pi_b\right) \frac{s_b \sigma_b - s_s \sigma_s}{\bar{\sigma}}, \tag{2.92}$$

$$\xi \equiv \frac{1-\alpha}{\alpha} \frac{1-\alpha\beta}{1+\omega_y \theta}, \tag{2.93}$$

$$\gamma_b \equiv \pi_b \left(\frac{\psi}{\psi_b} \frac{\bar{\lambda}^b}{\bar{\tilde{\lambda}}} \right)^{\frac{1}{\nu}}, \tag{2.94}$$

$$\kappa \equiv \xi \left(\omega_y + \bar{\sigma}^{-1} \right), \tag{2.95}$$

$$\omega_b \equiv \frac{\varkappa \bar{\chi}' \left(\bar{b} \right) + \eta \left(\eta - 1 \right) \frac{s_\Xi}{\rho_b}}{1 + \bar{\omega}}, \tag{2.96}$$

$$\omega_\chi \equiv \frac{1}{1+\bar{\omega}}, \tag{2.97}$$

$$\omega_\Xi \equiv \frac{1}{1+\bar{\omega}} \frac{\eta}{\rho_b}, \tag{2.98}$$

$$\tilde{B}_\Omega \equiv \psi \bar{\tilde{\lambda}}^{-1} \bar{\mu}^w \bar{H}^{-\nu} \left(\frac{\bar{Y}}{\bar{Z}} \right)^{1+\omega_y} \bar{Y}^{-1} \frac{1}{\nu} \gamma_b \left(1 - \gamma_b\right), \tag{2.99}$$

$$\tilde{B}_u \equiv \psi \bar{\tilde{\lambda}}^{-1} \bar{\mu}^w \bar{H}^{-\nu} \left(\frac{\bar{Y}}{\bar{Z}} \right)^{1+\omega_y} \bar{Y}^{-1} \left(\gamma_b - \pi_b \right), \tag{2.100}$$

$$B_\Omega \equiv s_\Omega \pi_b - s_b \sigma_b \pi_b \left(1 - \pi_b\right) - \tilde{B}_\Omega + \tilde{B}_u \left(\gamma_b - \pi_b \right), \tag{2.101}$$

$$B_\lambda \equiv \tilde{B}_u - s_\Omega, \tag{2.102}$$

$$\varrho_r \equiv \frac{\delta \left(1 + \bar{r}^d\right)}{1 + \pi_b \bar{\omega}} \left[\left(1 + \bar{\omega}\right) + \pi_b \frac{\rho_b^g}{\rho_b} \right], \tag{2.103}$$

$$\varrho_Y \equiv -\varrho_\xi \left(\tilde{B}_u \left(1 + \omega_y\right) + B_\lambda \bar{\sigma}^{-1} \right), \tag{2.104}$$

$$\varrho_\Omega \equiv \varrho_\xi \left(B_\Omega - B_\lambda s_\Omega \right), \tag{2.105}$$

$$\varrho_\omega \equiv -\frac{\pi_b\left(1+\bar{\omega}\right)}{1+\pi_b\bar{\omega}}, \tag{2.106}$$

$$\varrho_\xi \equiv \frac{1}{\rho_b\left(1+\pi_b\bar{\omega}\right)}, \tag{2.107}$$

$$\varrho_b \equiv \frac{\delta\left(1+\bar{r}^d\right)\left(1+\bar{\omega}\right)}{1+\pi_b\bar{\omega}}. \tag{2.108}$$

2.4 Calibration

The paper discusses the strategy for the calibration. Here we present the exact values for all the parameters.

Notice that unless otherwise mentioned, all exogenous disturbances follow an AR(1) process with autocorrelation coefficient equal to ρ_ξ, which in the baseline calibration is set to 0.9. The only expection to this is the autocorrelation coefficient of the monetary policy shock, in the case of a Taylor rule, in which case we consider an autocorrelation coefficient of 0.6.

Exogenous, takes resources

The spread in the FF model is exogenous and consumes resources, i. e. $\varkappa = 0$, $\bar{\chi} = 0$, $\bar{\Xi} = \bar{\omega}$ and $\eta = 1$. The full list of parameters is:

ϕ^{-1}	0.75	$1+\bar{\omega}$	$(1.02)^{1/4}$	s_Ξ	0.0159	$\bar{\sigma}^{-1}$	0.16
α	0.66	δ	0.975	ρ_b^g	0	σ	8.9286
ω_y	0.473	π_b	0.5	s_c	0.7	σ_b	13.8019
ν	0.1048	ρ_b	3.2	s_b	0.7821	σ_s	2.7604
$(\theta-1)^{-1}$	0.15	η	1	s_s	0.6179	σ_b/σ_s	5
μ^p	1.15	$\varkappa$	0	s_b/s_s	1.2657	$\bar{\lambda}_b/\bar{\lambda}_s$	1.2175
$\bar{r}^d$	0.01	$\bar{\mu}^w$	1	ψ_b	1.1492	$\bar{Z}$	1
β	0.9874	$\bar{\tau}$	0.2	ψ_s	0.9439	$\bar{H}$	1
$\bar{Y}$	1	s_g	0.2841	ψ_b/ψ_s	1.2175	ρ_ξ	0.9

Endogenous, takes resources

The spread in the FF model is endogenous and takes resources, i.e. $\eta > 1$ (such that the spread elasticity to debt is 1/4), $\varkappa = 0$ and $\bar{\chi} = 0$. The full list of parameters is:

ϕ^{-1}	0.75	$1+\bar{\omega}$	$(1.02)^{1/4}$	s_Ξ	0.0003	$\bar{\sigma}^{-1}$	0.16
α	0.66	δ	0.975	ρ_b^g	0	σ	8.9286
ω_y	0.473	π_b	0.5	s_c	0.7	σ_b	13.8019
ν	0.1048	ρ_b	3.2	s_b	0.7821	σ_s	2.7604
$(\theta-1)^{-1}$	0.15	η	51.623	s_s	0.6179	σ_b/σ_s	5
μ^p	1.15	$\varkappa$	0	s_b/s_s	1.2657	$\bar{\lambda}_b/\bar{\lambda}_s$	1.2175
$\bar{r}^d$	0.01	$\bar{\mu}^w$	1	ψ_b	1.1492	$\bar{Z}$	1
β	0.9874	$\bar{\tau}$	0.2	ψ_s	0.9439	$\bar{H}$	1
$\bar{Y}$	1	s_g	0.2997	ψ_b/ψ_s	1.2175	ρ_ξ	0.9

3 NoFF model: heterogeneous agents and no credit frictions

This section describes the model with heterogeneous agents but no credit frictions. This model is referred in the paper as the "NoFF" model. In this version of the model there is no spread nor any costs of financial intermediation, so that the deposit and borrowing rates are equal at all times. Because there is no spread at any time then it must be true that the marginal utilities of consumption of the two types are equal in equilibrium, hence we know that $\lambda_t^b = \lambda_t^s = \lambda_t$.

3.1 All equations

The objective:

$$\tilde{U}_t = \pi_b \frac{\lambda_t^{1-\sigma_b} \bar{C}_t^b}{1-\sigma_b^{-1}} + (1-\pi_b) \frac{\lambda_t^{1-\sigma_s} \bar{C}_t^s}{1-\sigma_s^{-1}} - \frac{1}{1+\nu} \bar{H}_t^{-\nu} \left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} \Delta_t \tag{3.1}$$

The equations describing the economy are summarized below:

$$0 = \left(1+i_t^d\right) \beta E_t \left[\frac{\lambda_{t+1}}{\Pi_{t+1}}\right] - \lambda_t \tag{3.2}$$

$$0 = \mu^p (1+\omega_y) \psi \mu_t^w \bar{H}_t^{-\nu} \left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} + \alpha\beta E_t \left[\Pi_{t+1}^{\theta(1+\omega_y)} K_{t+1}\right] - K_t \tag{3.3}$$

$$0 = \lambda_t (1-\tau_t) Y_t + \alpha\beta E_t \left[\Pi_{t+1}^{\theta-1} F_{t+1}\right] - F_t \tag{3.4}$$

$$0 = \pi_b (1-\pi_b) \left[\bar{C}_t^b (\lambda_t)^{-\sigma_b} - \bar{C}_t^s (\lambda_t)^{-\sigma_s}\right] - \pi_b b_t^g + \delta \left[b_{t-1} + \pi_b b_{t-1}^g\right] \frac{1+i_{t-1}^d}{\Pi_t} - b_t \tag{3.5}$$

$$0 = \pi_b \bar{C}_t^b \lambda_t^{-\sigma_b} + (1-\pi_b) \bar{C}_t^s \lambda_t^{-\sigma_s} + G_t - Y_t \tag{3.6}$$

$$0 = \alpha \Delta_{t-1} \Pi_t^{\theta(1+\omega_y)} + (1-\alpha) \left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta(1+\omega_y)}{\theta-1}} - \Delta_t \tag{3.7}$$

$$0 = \frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha} - \left(\frac{F_t}{K_t}\right)^{\frac{\theta-1}{1+\omega_y\theta}} \tag{3.8}$$

Auxiliary:

$$c_t^b = \bar{C}_t^b \lambda_t^{-\sigma_b} \tag{3.9}$$

$$c_t^s = \bar{C}_t^s \lambda_t^{-\sigma_s} \tag{3.10}$$

3.2 Zero inflation steady state

We consider the solution to steady state in which we simply assume zero inflation. For simplification of the analysis consider the following definitions

$$s_c \equiv \pi_b s_b + (1-\pi_b)\, s_s,$$

$$s_b \equiv \bar{c}^b/\bar{Y},$$

$$s_s \equiv \bar{c}^s/\bar{Y},$$

$$s_c^{bs} \equiv s_b/s_s,$$

$$\bar{\sigma} \equiv \pi_b s_b \sigma_b + (1-\pi_b)\, s_s \sigma_s,$$

$$\sigma_{bs} \equiv \sigma_b/\sigma_s,$$

$$\rho_b \equiv \bar{b}/\bar{Y},$$

$$\rho_b^g \equiv \bar{b}^g/\bar{Y},$$

$$s_g \equiv \bar{G}/\bar{Y},$$

Without any loss of generality we calibrate the following values:

$$\bar{Y} = 1,$$

$$\psi = 1.$$

The values of s_c, s_b/s_s and σ_b/σ_s are set according to the calibration described in the paper.

For the interest rate we have the standard $1+\bar{r}^d = \beta^{-1}$. We use this steady-state relation to calibrate β, given assumed values for δ, π_b and $\bar{r}^d$. The nominal deposit rate is equal to the real rate:

$$1+\bar{\imath}^d = 1+\bar{r}^d. \tag{3.11}$$

Using $\bar{F} = \bar{K}$,

$$\lambda = \frac{\mu^p\,(1+\omega_y)\,\psi\mu_t^w}{(1-\bar{\tau})}\frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}}. \tag{3.12}$$

Given our calibration of s_c and s_c^{bs}, we can write:

$$s_s = \frac{s_c}{\pi_b s_c^{bs} + (1-\pi_b)}, \tag{3.13}$$

and $s_b = s_c^{bs} s_c$.

The resources constraint implies

$$s_g = 1 - s_c. \tag{3.14}$$

Zero inflation implies that

$$\bar{\Delta} = 1. \tag{3.15}$$

The debt equation is

$$(1-\delta)\,\rho_b = \pi_b\,(1-\pi_b)\,\frac{s_b - s_s}{\bar{Y}} - \pi_b \rho_b^g \left[1 - \delta\left(1+\bar{r}^d\right)\right],$$

implying that

$$\rho_b = \frac{\pi_b\,(1-\pi_b)\,(s_b - s_s) - \pi_b \rho_b^g \left[1 - \delta\left(1+\bar{r}^d\right)\right]}{1-\delta\left(1+\bar{r}^d\right)}, \tag{3.16}$$

which is then used to solve for real debt according to $\bar{b} = \rho_b \bar{Y}$.

Furthermore, we set $\bar{\sigma}$, hence

$$\sigma_s = \frac{\bar{\sigma}}{\pi_b s_b \sigma_{bs} + (1-\pi_b)\,s_s}, \tag{3.17}$$

$$\sigma_b = \sigma_{bs}\sigma_s. \tag{3.18}$$

Finally,

$$\bar{C}^b = s_b \bar{\lambda}^{\sigma_b}, \tag{3.19}$$

$$\bar{C}^s = s_b \bar{\lambda}^{\sigma_s}, \tag{3.20}$$

$$\bar{K} = \frac{\mu^p\,(1+\omega_y)\,\psi \mu^w \frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}}}{1-\alpha\beta}, \tag{3.21}$$

$$\bar{F} = \frac{\bar{\lambda}\,(1-\bar{\tau})}{1-\alpha\beta}. \tag{3.22}$$

3.3 Log-linear equations

In this section we present all the log-linear relations of the model, in which we linearize around the zero inflation steady state.

Full system

The full system of log-linear equation is given by:

$$\hat{\lambda}_t = \hat{\imath}_t^d - E_t \pi_{t+1} + E_t \hat{\lambda}_{t+1}, \tag{3.23}$$

$$\hat{K}_t = (1-\alpha\beta)\left[\hat{\mu}_t^w - \nu \bar{h}_t + (1+\omega_y)\left(\hat{Y}_t - z_t\right)\right] + \alpha\beta E_t\left[\theta\,(1+\omega_y)\,\pi_{t+1} + \hat{K}_{t+1}\right], \tag{3.24}$$

$$\hat{F}_t = (1-\alpha\beta)\left[\hat{\lambda}_t - \hat{\tau}_t + \hat{Y}_t\right] + \alpha\beta E_t\left[(\theta-1)\,\pi_{t+1} + \hat{F}_{t+1}\right], \tag{3.25}$$

$$\begin{aligned}\hat{b}_t \;=\;& \pi_b\,(1-\pi_b)\,\rho_b^{-1}\left[s_b \bar{c}_t^b - s_s \bar{c}_t^s - (s_b\sigma_b - s_s\sigma_s)\,\hat{\lambda}_t\right] \\ &+\delta\left(1+\bar{r}^d\right)\pi_b \rho_b^g/\rho_b\left(\hat{\imath}_{t-1}^d - \pi_t\right) - \pi_b \rho_b^{-1}\left[\hat{b}_t^g - \delta\left(1+\bar{r}^d\right)\hat{b}_{t-1}^g\right] \\ &+\delta\left(1+\bar{r}^d\right)\hat{b}_{t-1},\end{aligned} \tag{3.26}$$

$$\hat{Y}_t = \pi_b s_b \bar{c}_t^b + (1-\pi_b)\, s_s \bar{c}_t^s - \left[\pi_b s_b \sigma_b + (1-\pi_b)\, s_s \sigma_s\right] \hat{\lambda}_t + \hat{G}_t, \tag{3.27}$$

$$\hat{\Delta}_t = \alpha \hat{\Delta}_{t-1}, \tag{3.28}$$

$$\pi_t = \frac{1-\alpha}{\alpha} \frac{1}{1+\omega_y \theta} \left(\hat{K}_t - \hat{F}_t\right). \tag{3.29}$$

Auxiliary equations:

$$\hat{c}_t^b = \bar{c}_t^b - \sigma_b \hat{\lambda}_t, \tag{3.30}$$

$$\hat{c}_t^s = \bar{c}_t^s - \sigma_s \hat{\lambda}_t. \tag{3.31}$$

Alternative (simplified) log-linear system

We can write the required equations as

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \bar{\sigma}\left(\hat{\imath}_t^d - E_t \pi_{t+1}\right) - E_t \Delta g_{t+1}, \tag{3.32}$$

$$\pi_t = \beta E_t \pi_{t+1} + u_t + \kappa \left(\hat{Y}_t - \hat{Y}_t^n\right), \tag{3.33}$$

$$\begin{aligned} \hat{b}_t \;=\; & \pi_b (1-\pi_b)\, \rho_b^{-1} \left[s_b \bar{c}_t^b - s_s \bar{c}_t^s + \frac{s_b \sigma_b - s_s \sigma_s}{\bar{\sigma}} \left(\hat{Y}_t - g_t\right) \right] \\ & + \delta \left(1+\bar{r}^d\right) \pi_b \rho_b^g / \rho_b \left(\hat{\imath}_{t-1}^d - \pi_t\right) - \pi_b \rho_b^{-1} \left[\hat{b}_t^g - \delta \left(1+\bar{r}^d\right) \hat{b}_{t-1}^g\right] \\ & + \delta \left(1+\bar{r}^d\right) \hat{b}_{t-1}, \end{aligned} \tag{3.34}$$

with

$$\hat{Y}_t^n \equiv \left(\omega_y + \bar{\sigma}^{-1}\right)^{-1} \left[\bar{\sigma}^{-1} g_t + \nu \bar{h}_t + (1+\omega_y)\, z_t\right], \tag{3.35}$$

$$s_c \bar{c}_t = \pi_b s_b \bar{c}_t^b + (1-\pi_b)\, s_s \bar{c}_t^s, \tag{3.36}$$

$$g_t = s_c \bar{c}_t + \hat{G}_t, \tag{3.37}$$

$$u_t \equiv \xi \left(\hat{\mu}_t^w + \hat{\tau}_t\right), \tag{3.38}$$

$$\Delta g_t \equiv g_t - g_{t-1}, \tag{3.39}$$

$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t. \tag{3.40}$$

and

$$s_\Omega \equiv \pi_b (1-\pi_b) \frac{s_b \sigma_b - s_s \sigma_s}{\bar{\sigma}}, \tag{3.41}$$

$$\xi \equiv \frac{1-\alpha}{\alpha} \frac{1-\alpha\beta}{1+\omega_y \theta}, \tag{3.42}$$

$$\kappa \equiv \xi \left(\omega_y + \bar{\sigma}^{-1}\right), \tag{3.43}$$

3.4 Calibration

The paper discusses the strategy for the calibration. Here we present the exact values for all the parameters.

Notice that unless otherwise mentioned, all exogenous disturbances follow an AR(1) process with autocorrelation coefficient equal to ρ_ξ, which in the baseline calibration is set to 0.9. The only expection to this is the autocorrelation coefficient of the monetary policy shock, in the case of a Taylor rule, in which case we consider an autocorrelation coefficient of 0.6.

The full parameter values are the following ones:

ϕ^{-1}	0.75	$1+\bar{\omega}$	1	s_Ξ	0	$\bar{\sigma}^{-1}$	0.16
α	0.66	δ	0.975	ρ_b^g	0	σ	8.9286
ω_y	0.473	π_b	0.5	s_c	0.7	σ_b	13.8019
ν	0.1048	ρ_b	3.2	s_b	0.7821	σ_s	2.7604
$(\theta-1)^{-1}$	0.15	η	–	s_s	0.6179	σ_b/σ_s	5
μ^p	1.15	$\varkappa$	–	s_b/s_s	1.2657	$\bar{\lambda}_b/\bar{\lambda}_s$	1
$\bar{r}^d$	0.01	$\bar{\mu}^w$	1	ψ_b	1	$\bar{Z}$	1
β	0.99	$\bar{\tau}$	0.2	ψ_s	1	$\bar{H}$	1
$\bar{Y}$	1	s_g	0.3	ψ_b/ψ_s	1	ρ_ξ	0.9

4 RepHH model: representative agent and no credit frictions

This section describes the model with a representative agent and no credit frictions. This model is referred in the paper as the "RepHH" model. In this version of the model there is no spread nor any costs of financial intermediation, so that the deposit and borrowing rates are equal at all times. Furthermore we consider a representative household. In a representative household environment there is no intrahousehold debt, hence b_t is not defined.

4.1 All equations

The objective:

$$\tilde{U}_t = \frac{\lambda_t^{1-\sigma}\bar{C}_t}{1-\sigma^{-1}} - \frac{1}{1+\nu}\bar{H}_t^{-\nu}\left(\frac{Y_t}{Z_t}\right)^{1+\omega_y}\Delta_t \tag{4.1}$$

The equations describing the economy are summarized below:

$$0 = \left(1+i_t^d\right)\beta E_t\left[\frac{\lambda_{t+1}}{\Pi_{t+1}}\right] - \lambda_t \tag{4.2}$$

$$0 = \mu^p\left(1+\omega_y\right)\psi\mu_t^w\bar{H}_t^{-\nu}\left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} + \alpha\beta E_t\left[\Pi_{t+1}^{\theta(1+\omega_y)}K_{t+1}\right] - K_t \tag{4.3}$$

$$0 = \lambda_t (1 - \tau_t) Y_t + \alpha\beta E_t \left[\Pi_{t+1}^{\theta-1} F_{t+1}\right] - F_t \tag{4.4}$$

$$0 = \bar{C}_t \lambda_t^{-\sigma} + G_t - Y_t \tag{4.5}$$

$$0 = \alpha\Delta_{t-1}\Pi_t^{\theta(1+\omega_y)} + (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta(1+\omega_y)}{\theta-1}} - \Delta_t \tag{4.6}$$

$$0 = \frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha} - \left(\frac{F_t}{K_t}\right)^{\frac{\theta-1}{1+\omega_y\theta}} \tag{4.7}$$

Auxiliary:

$$c_t = \bar{C}_t \lambda_t^{-\sigma} \tag{4.8}$$

4.2 Zero inflation steady state

We consider the solution to steady state in which we simply assume zero inflation. For simplification of the analysis consider the following definitions

$$s_c \equiv \bar{c}/\bar{Y},$$

$$\bar{\sigma} \equiv s_c \sigma,$$

$$\rho_b^g \equiv \bar{b}^g/\bar{Y},$$

$$s_g \equiv \bar{G}/\bar{Y},$$

Without any loss of generality we calibrate the following:

$$\bar{Y} = 1.$$

The values of s_c is set according to the calibration described in the paper.

For the interest rate we have the standard $1+\bar{r}^d = \beta^{-1}$. We use this steady-state relation to calibrate β, given assumed values for $\bar{r}^d$. The nominal deposit rate is equal to the real rate:

$$1 + \bar{\imath}^d = 1 + \bar{r}^d. \tag{4.9}$$

Using $\bar{F} = \bar{K}$,

$$\lambda = \frac{\mu^p (1+\omega_y)\psi\mu_t^w}{(1-\bar{\tau})}\frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}}. \tag{4.10}$$

The resources constraint implies

$$s_g = 1 - s_c. \tag{4.11}$$

Zero inflation implies that

$$\bar{\Delta} = 1. \tag{4.12}$$

Finally,

$$\bar{C} = s_c \bar{\lambda}^{\sigma}, \tag{4.13}$$

$$\bar{K} = \frac{\mu^p \left(1+\omega_y\right) \psi \mu^w \frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}}}{1-\alpha\beta}, \tag{4.14}$$

$$\bar{F} = \frac{\bar{\lambda}\left(1-\bar{\tau}\right)}{1-\alpha\beta}. \tag{4.15}$$

4.3 Log-linear equations

In this section we present all the log-linear relations of the model, in which we linearize around the zero inflation steady state.

Full system

The full system of log-linear equation is given by:

$$\hat{\lambda}_t = \hat{\imath}_t^d - E_t \pi_{t+1} + E_t \hat{\lambda}_{t+1}, \tag{4.16}$$

$$\hat{K}_t = (1-\alpha\beta)\left[\hat{\mu}_t^w - \nu \bar{h}_t + (1+\omega_y)\left(\hat{Y}_t - z_t\right)\right] + \alpha\beta E_t\left[\theta\left(1+\omega_y\right)\pi_{t+1} + \hat{K}_{t+1}\right], \tag{4.17}$$

$$\hat{F}_t = (1-\alpha\beta)\left[\hat{\lambda}_t - \hat{\tau}_t + \hat{Y}_t\right] + \alpha\beta E_t\left[(\theta-1)\pi_{t+1} + \hat{F}_{t+1}\right], \tag{4.18}$$

$$\hat{Y}_t = s_c \bar{c}_t - s_c \sigma \hat{\lambda}_t + \hat{G}_t, \tag{4.19}$$

$$\hat{\Delta}_t = \alpha \hat{\Delta}_{t-1}, \tag{4.20}$$

$$\pi_t = \frac{1-\alpha}{\alpha}\frac{1}{1+\omega_y\theta}\left(\hat{K}_t - \hat{F}_t\right). \tag{4.21}$$

Auxiliary equations:

$$\hat{c}_t = \bar{c}_t - \sigma \hat{\lambda}_t, \tag{4.22}$$

Alternative (simplified) log-linear system

We can write the required equations as

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \bar{\sigma}\left(\hat{\imath}_t^d - E_t \pi_{t+1}\right) - E_t \Delta g_{t+1}, \tag{4.23}$$

$$\pi_t = \beta E_t \pi_{t+1} + u_t + \kappa\left(\hat{Y}_t - \hat{Y}_t^n\right), \tag{4.24}$$

with

$$\hat{Y}_t^n \equiv \left(\omega_y + \bar{\sigma}^{-1}\right)^{-1}\left[\bar{\sigma}^{-1} g_t + \nu \bar{h}_t + \left(1+\omega_y\right) z_t\right], \tag{4.25}$$

$$g_t = s_c \bar{c}_t + \hat{G}_t, \tag{4.26}$$

$$u_t \equiv \xi\left(\hat{\mu}_t^w + \hat{\tau}_t\right), \tag{4.27}$$

$$\Delta g_t \equiv g_t - g_{t-1}, \tag{4.28}$$

$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t. \tag{4.29}$$

and

$$\xi \equiv \frac{1-\alpha}{\alpha}\frac{1-\alpha\beta}{1+\omega_y\theta}, \tag{4.30}$$

$$\kappa \equiv \xi\left(\omega_y + \bar{\sigma}^{-1}\right), \tag{4.31}$$

4.4 Calibration

The paper discusses the strategy for the calibration. Here we present the exact values for all the parameters.

Notice that unless otherwise mentioned, all exogenous disturbances follow an AR(1) process with autocorrelation coefficient equal to ρ_ξ, which in the baseline calibration is set to 0.9. The only expection to this is the autocorrelation coefficient of the monetary policy shock, in the case of a Taylor rule, in which case we consider an autocorrelation coefficient of 0.6.

The full parameter values are the following ones:

ϕ^{-1}	0.75	$1+\bar{\omega}$	–	s_Ξ	0	$\bar{\sigma}^{-1}$	0.16
α	0.66	δ	–	ρ_b^g	0	σ	8.9286
ω_y	0.473	π_b	–	s_c	0.7	σ_b	–
ν	0.1048	ρ_b	–	s_b	–	σ_s	–
$(\theta-1)^{-1}$	0.15	η	–	s_s	–	σ_b/σ_s	–
μ^p	1.15	$\varkappa$	–	s_b/s_s	–	$\bar{\lambda}_b/\bar{\lambda}_s$	–
$\bar{r}^d$	0.01	$\bar{\mu}^w$	1	ψ_b	–	$\bar{Z}$	1
β	0.99	$\bar{\tau}$	0.2	ψ_s	–	$\bar{H}$	1
$\bar{Y}$	1	s_g	0.3	ψ_b/ψ_s	–	ρ_ξ	0.9

References

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