Implementation of a Unique Monetary Policy with Heterogeneous Banking System

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Preliminary Version

Abstract: *The paper presents a model of endogenous credit allocation with heterogeneous lenders. We consider three classes of agents' -firms, individual investors and banks. Banks differ according to their level of capital and monitoring technology. In a setting of moral hazard with limited liability, we stress that firms' ability to obtain external funds is conjointly determined by their own wealth, bank capital, and monitoring technology. We show that small (medium) firms invest with the small (large) bank and pay a high (low) interest rate whereas large firms are financed by the financial market. Moreover, we stress that restrictive monetary policy leads to a contraction in aggregate investment and to a credit reallocation mechanism, between the two banks and the market, similar to a "flight to quality" effect. This restrictive policy has a strong effect not on bank-dependent firms but on small bank-dependent firms.*

JEL: E5, D8, G2

1. INTRODUCTION

The adoption of the euro on 1st January 1999 for the countries of the monetary union was associated with the conduct of a single monetary policy. However, assessments of the European banking industry describe it as a composite of banks that differ in size, capital, and liquidity, and suggest that European banking system remains highly heterogeneous[1]. Specialization effects in the monitoring of different kinds of borrowers may explain the persistence of size heterogeneity[2]. Indeed, lending to small, informationally opaque borrowers and lending to large, informationally transparent borrowers are two different activities that require the use of distinct monitoring technologies. In particularly, it appears that small banks are specialized in financing

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[1] See Ehrmann, Gambacorta, Martinez-Pagés, Sevestre and Worms (2001). They show that the share of small and large banks as their characteristics in terms of level of liquidity and capitalisation is highly different according to European countries.

[2] If small banks have a cost advantage in providing relationship loans to small businesses, consolidation will not lead necessarily to the disappearance of small banks; they will continue to play a vital role at the small end of the lending market (Strahan and Weston (1996)).

small businesses while large banks tend to be specialized in financing loans to medium or large businesses[3]. This results from the better ability of small banks to meet the credit needs of small businesses as their organizational structures allow them to be better delegated monitors than large banks[4]. For the US economy, Peek and Rosengren (1996) and Berger, Kashyap and Scalise (1995) show a strong link between banking institution size and the supply of credit for small businesses, since, compared to smaller financial institutions, large banks tend to devote proportionally less of their assets to small businesses. This relationship is also found for the main European countries in recent studies concerning the role of the banking system in the transmission of monetary policy[5].

These facts suggest that banking system heterogeneity strongly affects the ability of firms to obtain external funds. Indeed, the relative share of small and large banks in the banking system may determine the availability of total credit. Similarly, it is important to take this heterogeneity into account in order to evaluate precisely the indirect and asymmetric effect of monetary policy on firms’ investment level. In this paper we develop an equilibrium model of endogenous credit allocation with heterogeneous lenders in order to appraise the impact of monetary policy on firms’ financing opportunities. The major questions we address in this framework are the following: How is the optimal credit allocation in a heterogeneous banking system determined? In what way does this allocation determine the relative strength of monetary policy across intermediaries and impact on the level of firms' investment?

Our model is built on Holmstrom and Tirole (1997). It adds to this initial contribution concerned with the existence of heterogeneous lenders. Specifically, we assume that our financial intermediaries (banks) differ by their amount of capital and by their size. A first category of intermediaries (named "the small bank") is specialized in monitoring small firms whereas a second category (named "the large bank") is specialized in monitoring large businesses. Moreover, the small bank is also the less capitalized. Three main results emerge from our analysis.

First, we stress that firms' ability to obtain external funds is conjointly determined by their own wealth, bank capital, and monitoring technology. We model the optimal borrowing contract for a firm that can choose between financial market, the large bank, and the small bank. Firms are characterized by a different amount of internal funds and have access to an identical fixed size investment project. These projects require external funds in order to be undertaken. External investors are uninformed

[3] See Berger, Miller, Petersen, Rajan and Stein (2002), and Berger and Udell (2002) for a study concerning US banking industry.

[4] Because of legal lending limits and problems of diversification, small banks have equally fewer opportunities to make loans to large business than do large banks.

[5] See Loupias, Savignac and Sevestre (2002) for the case of France, or Gaiotti and Generale (2001) for Italy, and Ehrmann, Gambacorta, Martinez-Pagés, Sevestre and Worms (2001), Chatelain, Generale, Hernando, Von Kalckreuth and Vermulen (2001) for euro zone.

about the expected profitability of a specific project, which leads to a moral hazard problem between entrepreneurs and financiers as the former can divert project resources towards private uses. According to the financial intermediation literature[6], banks can alleviate this entrepreneurial moral hazard problem owing to a specific monitoring technology. However, because monitoring is costly for the banks, there is a second moral hazard between intermediaries and investors in the provision of these monitoring services. In order to reduce this problem, banks must invest a part of their own capital in the project it monitors. Thus, both monitoring technology and bank capital must be considered in the banks' ability to lend. In order to finance their investment, we show that firms with high wealth rationally choose the financial market, firms with middle wealth choose to be monitored by the large bank, and firms with low private wealth are monitored by the small bank. Finally, firms without sufficient wealth cannot invest. This result is explained in terms of relative efficiency of banks' monitoring compared to the relative level of banks' capitalization.

Second, we stress that in equilibrium, the cost of borrowing from the large bank (well-capitalized) is lower than the cost of borrowing from the small bank (low-capitalized) whereas the financial market interest rate is the lowest one. This spread in the interest rate between the two banks is compatible with a no-arbitrage condition, as it is linked with equality of banks' payoff.

Our final point is related to the impact of the monetary policy[7] on the level of firms' investment. Restrictive monetary policy leads to a contraction in aggregate investment and to a credit reallocation mechanism, between the two banks and the market, similar to a "flight to quality" effect. This restrictive policy has a strong effect not on bank-dependent firms but on small bank-dependent firms. Small firms are the most affected because they have to be financed by the small bank and cannot substitute other sources of financing. More generally, this result underlines that a same monetary policy must have a differential impact on aggregate investment according to the structure of the financial system. This point is important in order to evaluate the conduct of monetary policy in the EMU. It suggests that a unified monetary policy will have an asymmetric impact as long as European financial system remains diversified.

Few theoretical papers have tried to encompass elements of banking heterogeneity. Concerning bank expertise, Dell'Ariccia and Marquez (2001) have built a model where a lender with an informational advantage competes with a lender with worse information but with a cost advantage. They examine how credit allocation reacts to changes in the relative cost of funds for the two lenders. Boot and Thakor (2000) study the incidence of increased competition on a bank's choice between different modes of lending and specialization. They define a relationship loan as a loan that

[6] See Boot (2000) for a comprehensive survey.

[7] We interpret a restrictive monetary policy as an increase in the riskless interest rate (See Holmstrom and Tirole (1997)).

permits the bank to use its expertise to improve a borrower's project payoff. They show that this kind of loan is reduced with capital market competition. In a different view, other theoretical papers explore how bank capital plays an important role to mitigate information imperfections (Bernanke and Gertler (1987), or Stein (1998)). These models insist on the role of capital as an imperfect substitute to expertise or as a signal for depositors. Linked with these two approaches, Almazan (2002) has developed a model of banking competition with two banks that differ by their expertise and their amount of capital. In this framework, he focuses on the bank's optimal geographic position. Similarly, Ennis (2001b) studies a model that supports the view that the existence of small banks is justified even in an unregulated environment[8].

The paper is organized as follows. Section 2 presents the basic incentive model. Section 3 characterizes the optimal contracts. Section 4 describes the optimal choice of financing between entrepreneurs and lenders, and examines the impact of a restrictive monetary policy. Section 5 concludes and gives some macroeconomic implications. Mathematical proofs are contained in the appendix.

2. THE MODEL

We consider three classes of agents' -firms, individual investors and banks - and two periods. In the first period, firms need external funds in order to invest in a risky project. They have access to capital market and/or bank loan. During this period, financial contracts are signed between lenders and borrowers and investment decisions are made. In the second period, returns on investment are realized and firms have to pay for their external funds. We assume that all parties are risk neutral and protected by limited liability.

2.1. Firms' behaviour

There is a continuum of firms, all of which having access to the same technology. The only difference among firms relies on their private wealth (θ) that is supposed to be cash[9]. We assume that $\theta \in [0,1]$ which means that firms are uniformly distributed on [0,1] according to their wealth. Under a uniform law of distribution, $[0,\theta]$ also indicates the proportion of firms with asset less than θ.

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[8] See equally Winton (1995) for a model in which capital can substitute bank diversification and Winton (2000) for a financial institution's choice between diversification and specialization as the source of competitive advantage for intermediaries.

[9] As in Holmstrom and Tirole (1997), we keep this assumption for simplicity. However, private wealth could be any type of asset that can be used as collateral (see Chen (2001)).

In order to undertake a risky project a specific firm must invest an amount of capital equal to 1. We take for granted that firms lack capital. In period 1, a firm with a wealth θ needs at least $(1-\theta)$ in external fund to invest. In period 2, the investment generates a stochastic payoff equal to 0 (in the case of project failure) or $R>0$ (If project is successful). In case of success, investment's payoff also depends on the behaviour of the entrepreneur who runs the firm. If the entrepreneur undertakes a good action (state h) the probability of success of the project is equal to p_h, whereas a bad action (state l) leads to a success probability equal to p_l, where $p_h > p_l$. A moral hazard problem occurs because entrepreneurs enjoy a private benefice equal to $B>0$ if they act improperly (bad action) and nothing if they behave properly (good action)[10]. Define γ as the minimum rate of return on investors' capital (γ is exogenously given by monetary policy conditions and is defined as the riskless interest rate), we assume that only the good action leads to an economically viable investment project.

Assumption 1. Investment projects have positive-net-value during the second period only if entrepreneurs do not shrink

$$p_h R - \gamma > 0 > p_l R + B - \gamma \tag{1}$$

which requires that

$$R > \frac{B}{\Delta p}, \text{ with } \Delta p = p_h - p_l > 0 \tag{2}$$

2.2. Individual investors

Individual investors (or uninformed investors) are endowed with a large amount of capital named *uninformed* capital. These investors are not able to reduce hazard moral problem. In order to accept lending to entrepreneurs, uninformed investors must receive at least the rate of return γ on their capital. In case of project's failure, none of the parties is paid. In case of success, investment's payoff is shared between entrepreneurs and investors. Define R_e and R_f respectively as the firm and the investors' payoff if investment succeeds, we have

$$R = R_e + R_f$$

Incentive condition for investor to finance a risky project requires that

$$p_h R_f \geq I_f \gamma \tag{3}$$

[10] B may also be interpreted as an opportunity cost for entrepreneurs if they manage their firm properly.

This condition means that the expected payoff received by an investor who finances a risky project $\left(p_h R_f\right)$ is at least equal to the opportunity cost of the loaned funds $\left(I_f \gamma\right)$, I_f being the amount lent by an investor.

Moreover, according to (1), investors will finance a project only if entrepreneur prefers to be diligent. Indeed, on the opposite case, condition (3) is never fulfilled (the project is not economically viable). An entrepreneur behaves properly (undertakes the good action) only if the return linked with the good action is higher than the return linked with the bad action. The following condition has to be realized for an entrepreneur to be diligent

$$p_h R_e \geq p_l R_e + B \tag{4}$$

which requires that

$$R_e \geq \frac{B}{\Delta p} \tag{5}$$

Equation (5) is an incentive condition for entrepreneurs. It defines the minimum amount an entrepreneur must receive in order to choose the good action when it borrows uninformed capital.

2.3. Financial sector

We consider that the financial sector consists of two banks (indexed by $i \in \{0,1\}$) competing on a competitive market[11] and differing by their capital endowment and by their size. Specifically, we assume that the small bank is also the less capitalized one whereas the large bank is the most capitalized.

Assumption 2. Bank 0 (small bank) is endowed with an amount K_0 of capital while bank 1 (large bank) is endowed with an amount K_1, with $K_1 > K_0$.

For banks' capital to play a role in their lending ability, we assume, following Holmstrom and Tirole (1997) and Almazan (2002), that all projects are perfectly correlated. Without some degree of correlation, banks would not need to put any capital in a project (Diamond (1984), Williamson (1986) and Ennis (2001a))[12].

[11] In the following, we assume that financial and bank markets are competitive. Consequently, neither uninformed investors nor the two banks can impose their prices. The equilibrium price of bank capital is determined by the market (see also Boot and Thakor (2000) and Almazan (2002) for other models of banks competition).

[12] To simplify our analysis, we assume perfect correlation even if only a certain degree of correlation is necessary.

The function of bank is to monitor firms and consequently to alleviate the moral hazard problem. We suppose that bank's monitoring reduces firm's opportunity cost (or private benefice) from B to d (with $B > d > 0$). This means that the minimum amount an entrepreneur must receive in order to choose the good action decreases. Denote by R_{ei} entrepreneurs' minimum expected return in order to be diligent when investment project is monitored by bank i. Firms incentive condition requires that

$$p_h R_{ei} \geq p_l R_{ei} + d \text{ which leads to } R_{ei} \geq \frac{d}{\Delta p} \tag{6}$$

In order to monitor firms' projects, banks have access to a specific monitoring technology. In this paper, we assume that, according to their size, both banks do not possess the same monitoring technology[13]. Indeed, following Stein (2002) we suppose that efforts to coordinate lending in large institutions could lead to standardized credit policies based on easily observable, verifiable, and transmittable data. This kind of information could be named "hard information" as it is based on relatively objective ratio, such as collateral ratio or credit scoring[14]. According to this approach, it becomes possible to conclude that large banks are disadvantaged in the lending relationship for small opaque firms because this kind of lending often requires "soft information". Soft information is not easily observed and verified by others. Consequently, soft information is difficult to transmit through the communication channels of large organizations.

We model these two sorts of specialization by providing respectively the large bank (bank 1) and the small bank (bank 0) with a cost advantage in monitoring large business (small business). First, we assume, following the existing literature, that banks monitoring costs are decreasing functions of firms' wealth. Consequently, the lower the wealth of a borrower the higher the monitoring cost for a bank to reduce moral hazard problem. Second, because of their respective specialisation, small bank's monitoring costs are lower that large bank's monitoring costs for a given range of firms.

Assumption 3. Banks i (with $i \in \{0,1\}$) can reduce entrepreneur's private benefice from B to d $(B > d)$ suffering a cost $c_i(\theta) > 0, \forall\theta$ with $c_i(\theta)$ monotonous and continuous on $[0,1]$. $c_i(\theta)$ has the following properties:

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[13] Specifically, small banks have an informational advantage in evaluating and monitoring loan quality of opaque small firms (Nakamura (1994)). For an empirical investigation see Jayaratne and Wolken (1999) and McNulty, Akhigbe and Verbrugge (2001).

[14] For a distinction between hard and soft information see also Berger and Udell (2002) and Stein (2002). Tirole (1986) and Laffont and Rochet (1997) use the distinction between hard and soft information in order to expose the condition of collusion in small and large organisations.

1- $0 < c_0(0) < c_1(0) \to +\infty$ and $c_0(1) > c_1(1) = 0$, which means that the large bank (small bank) has a cost advantage in monitoring highly (weakly) capitalized firms.

2- $\frac{dc_0(\theta)}{d\theta} = c_0^{'}(\theta) < 0$, $\frac{dc_1(\theta)}{d\theta} = c_1^{'}(\theta) < 0$ and $\left|c_1^{'}(\theta)\right| > \left|c_0^{'}(\theta)\right|$, that is, the cost of monitoring is decreasing for the two banks along with firm's net wealth.

3- $\frac{d^2 c_0(\theta)}{d\theta^2} = c_0^{''}(\theta) > 0$ and $\frac{d^2 c_1(\theta)}{d\theta^2} = c_1^{''}(\theta) > 0$, which means that banks operate with decreasing return to scale in the monitoring technology.

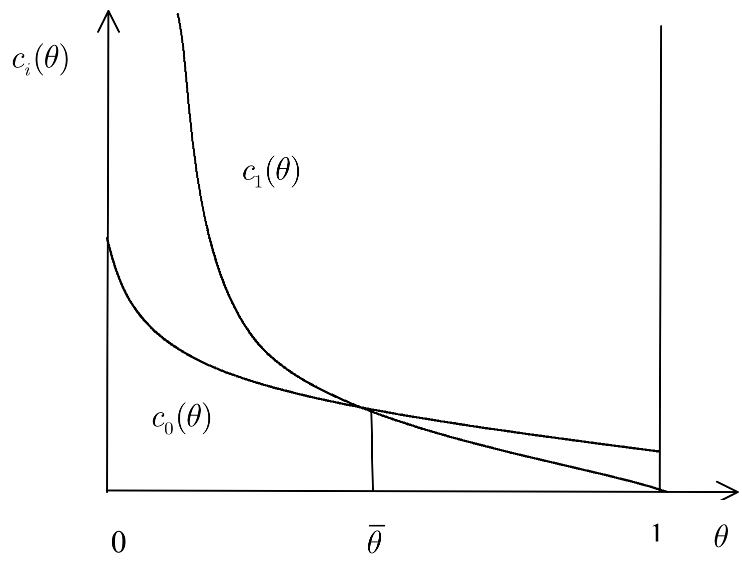
4- Cost functions are integrable on $[0,1]$ and

$C_0(\theta) = \int c_0(\theta)\, d\theta > 0$ with $C_0(\theta)$ an increasing function of θ.

$C_1(\theta) = \int c_1(\theta)\, d\theta > 0$ with $C_1(\theta)$ an increasing function of θ.

$C_0(\theta)$ and $C_1(\theta)$ represent the total cost of monitoring support respectively for the small bank and for the large bank.

Under these conditions there exist a critical wealth value $\left(\overline{\theta} \in [0,1]\right)$ for which the cost of the two banks are identical $\left(c_0\left(\overline{\theta}\right) = c_1\left(\overline{\theta}\right)\right)$. A possible graphical representation of the two cost functions is given below.



As monitoring is costly, banks will monitor a firm only if the payoff linked with monitoring is higher than the payoff obtained when there is no monitoring. Denoting R_{bi} as the expected payoff received by bank i in payment of monitoring activity, the following condition must be fulfilled for bank i

$$p_h R_{bi} - c_i(\theta) \geq p_l R_{bi} \text{ which leads to } R_{bi} \geq \frac{c_i(\theta)}{\Delta p} \tag{7}$$

A moral hazard problem arises since monitoring is costly for banks and not verifiable by uninformed investor. To make monitoring credible for the financial market, a bank must invest a part of its own capital in financing project it monitors. Bank capital becomes a key element in our framework because it appears as a signal of bank willingness to monitor.

We can now focus attention on the nature of the optimal contract between firms, banks and uninformed investors.

3. OPTIMAL CONTRACTS

Consider a firm who needs external funds and must contract with a lender. First, the optimal contract between the firm and the lender must specify how much each party should invest and how much it should be paid in case of success. Second, entrepreneurs compare the financial conditions offered by all lenders (financial market and the two banks) before choosing the best one. We will show that firms' choice between uninformed and informed capital is endogenously determined according to their own wealth.

3.1. Optimal contract with direct finance

Consider a firm θ, that wishes to finance its investment's project on the financial market. According to incentive constraints, an optimal contract is the solution of the following problem

$$\max_{M,M'} R_e(\theta) \tag{8}$$

Subject to constraints

$$I_f(\theta) + \theta \geq 1 \tag{9}$$

$$R_e(\theta) + R_f(\theta) \leq R \tag{10}$$

$$p_h R_f(\theta) \geq \gamma I_f \tag{11}$$

$$R_e(\theta) \geq \frac{B}{\Delta p} \tag{12}$$

$M \equiv \left[I_f(\theta) \right]$ represents the amount of uninformed capital invested in the project whereas $M' \equiv \left[R_f(\theta), R_e(\theta) \right]$ is the allocation of the payoff between uninformed investor and entrepreneur if the project succeeds.

We can interpret the constraints of program (8). Eq. (9) describes how the financing of investment is shared between both parties. Eq. (10) shows how the project's payoff must be shared between agents. Eq. (11) ensures that uninformed investors' expected outcome must be at least equal to the market value of the funds. If this equation is not enforced, uninformed investors are not encouraged to invest their funds in the risky project. Finally, eq. (12) guarantees that the entrepreneur behaves fairly since its minimum payoff must be at least equal to the opportunity cost of being diligent (cf. *supra*). The optimal contract defines the amount invested by uninformed investor and the payoff it receives in equilibrium. Using (8) and (11) we have

$$I_f^*(\theta) = 1 - \theta \qquad\qquad R_f^*(\theta) = (1-\theta)\frac{\gamma}{p_h}$$

Optimal contract requires that: i) firm invests its entire "cash" in its project whereas uninformed investors finance the difference, ii) uninformed investors payoff is just equal to the opportunity cost of the funds.

The equilibrium payoff for the firm is calculated by substituting the equilibrium value in (10)

$$R_e^*(\theta) = R - R_f^*(\theta) \equiv R - (1-\theta)\frac{\gamma}{p_h} \tag{13}$$

Equation (13) represents equilibrium firms' payoff when investment is financed by uninformed investor. Obviously, this payoff is an increasing function of firms' net wealth.

Substituting (12) in (13) and binding the resulting equation, we obtain in equilibrium the necessary and sufficient condition for a firm to have access to direct finance:

$$\left[R - \frac{B}{\Delta p} \right] \geq (1-\theta)\frac{\gamma}{p_h} \text{ and}$$

$$\theta_M = 1 - \frac{p_h}{\gamma}\left[R - \frac{B}{\Delta p} \right] \tag{14}$$

$\theta_M^* \leq 1$ is always true because $R \geq \dfrac{B}{\Delta p}$ (*assumption 1*). $\theta_M^* \geq 0$ requires that $p_h R - \gamma < p_h \dfrac{B}{\Delta p}$ which means that the total surplus of a project $\left(p_h R - \gamma\right)$ is less than the minimum amount a firm must be paid to behave diligently[15]. Consequently, $\theta_M \in [0;1]$.

LEMMA 1. Firms with a wealth $\theta \geq \theta_M$ can ensure to uninformed agents the minimum incentive payoff and should resort to finance only. Firms with a wealth $\theta < \theta_M$ don't invest.

3.2. Optimal contract with indirect finance

When a firm has an insufficient wealth, it cannot raise funds in financial market. Because of their specific monitoring technology banks can help these firms to obtain external funds. Indeed, we have shown that monitoring activity reduces firm's opportunity cost of being diligent by reducing private benefit from *B* to *d*. The total surplus available for uninformed investor increases and the incentive condition is fulfilled for firms with wealth lower than θ_M. In this context, we assume that investors deposit their money at the banks, which invest, along with its own funds, in the firms it monitors. Therefore, in the case of indirect finance, there are three parties to the financial contract: the firm, one of the two banks, and the uninformed investors. We assume that firms take as granted both the rate of return on bank capital $\left(\beta_i\right)$, and the cost of uninformed capital $\left(\gamma\right)$.

Consider a firm θ who wants to have its project financed by bank i. The optimal contract between the three parties (firm, bank and uninformed investors) is solution of the following optimization problem

$$\max_{H,H'} R_{ei}\left(\theta\right) \tag{15}$$

Subject to constraints

$$I_{bi}(\theta) + I_{fi}(\theta) + \theta \geq 1 \tag{16}$$

$$R_{ei}\left(\theta\right) + R_{bi}\left(\theta\right) + R_{fi}\left(\theta\right) \leq R \tag{17}$$

$$p_h R_{fi}\left(\theta\right) \geq \gamma I_{fi} \tag{18}$$

[15] See Holmstrom and Tirole (1997).

$$p_h R_{bi}(\theta) \geq \beta_i I_{bi} \tag{19}$$

$$R_{bi} \geq \frac{c_i(\theta)}{\Delta p} \tag{20}$$

$$R_{ei}(\theta) \geq \frac{d}{\Delta p} \tag{21}$$

Where $H \equiv \left[I_{bi}(\theta), I_{fi}(\theta) \right]$ represents the amount invested in the project by the bank (large or small) and by the uninformed investor. $H' \equiv \left[R_{bi}(\theta), R_{fi}(\theta), R_{ei}(\theta) \right]$ denotes the payoff, of bank, uninformed investors, and the firm, if the project is successful. Interpretation of program (15) is straightforward. Eq. (16) refers to the respective amounts invested by bank, uniformed investors and firm. Eq. (17) describes how the cash flow must be shared among all. Eq. (18) states the participation constraint for uninformed investors if they finance a part of the investment with the bank. As usual, it means that the expected cash flow received by investors financing risky projects must be at least equal to the opportunity cost of the lending funds. Identically, knowing that β_i is the rate of return on banks informed capital, eq. (19) stresses that bank's expected payoff must cover the cost of its informed capital. Eq. (20) determines the minimum amount the bank must be paid in order to monitor. Finally, eq. (21) still describes the minimum return a firm has to receive. Binding equations (16) to (20) and substituting (18), (19) (20) into (16) gives the optimal contract between a θ firm, the bank and the uninformed investors.

LEMMA 2. The optimal financial contract is defined by the amount of funds provided by the bank, by the uninformed investor, and by their corresponding shares in the project's payoff.

$$I_{bi}^*(\theta) = \frac{p_h c_i(\theta)}{\beta_i \Delta p}, \qquad R_{bi}^*(\theta) = \frac{c_i(\theta)}{\Delta p} \tag{22}$$

$$I_{fi}^*(\theta) = (1-\theta) - \frac{p_h c_i(\theta)}{\beta_i \Delta p}, \quad R_{fi}^*(\theta) = \left((1-\theta) - \frac{p_h c_i(\theta)}{\beta_i \Delta p} \right) \frac{\gamma}{p_h} \tag{23}$$

Three observations emerge from lemma 2:

i)- Optimal contract requires that firms invest all their wealth in the project whereas bank and uninformed investors put up the balance (respectively I_{bi} and I_{fi}).

ii)- Banks' payoff is just equal to the minimum amount required to enforce monitoring and uninformed investors just receive the opportunity cost of the funds.

iii) According to the rate of return on informed capital, firms' demand of informed capital just allows to pay banks at the minimum incentive payoff.

To find the equilibrium firm's payoff we must substitute (22) and (23) into (17) and binding the equations we obtain

$$R_{ei}^{*}\left(\theta\right)=R-R_{bi}^{*}-R_{fi}^{*}\equiv R-\frac{c_i\left(\theta\right)}{\Delta p}-\left(\left(1-\theta\right)-\frac{p_h c_i\left(\theta\right)}{\beta_i\Delta p}\right)\frac{\gamma}{p_h} \tag{24}$$

Eq. (24) gives firm's equilibrium payoff if it finances a share of its investment thanks to a bank and uninformed investors. It can be underlined that firm's payoff is an increasing function of its net wealth (see Appendix A-1).

Moreover, monitoring is costly for banks. Therefore, β_i must be high enough to make banks prefer monitoring rather than investing their capital on the open market where they would earn a rate of return equal to γ. The minimum acceptable rate of return on informed capital is determined, for both banks, by the following condition

$$p_h R_{bi}^{*}-c_i\left(\theta\right)=\gamma I_{bi}^{*} \text{ substituting equilibrium values}$$

$$p_h\frac{c_i\left(\theta\right)}{\Delta p}-c_i\left(\theta\right)=\gamma\frac{p_h c_i\left(\theta\right)}{\Delta p\beta_i} \text{ with } \Delta p=p_h-p_l>0 \text{ which means that}$$

$$\beta_i=\frac{p_h}{p_l}\gamma>\gamma$$

The cost of informed capital being higher than the cost of uninformed capital, firms submitted to monitoring exactly demand the minimum level of bank capital. This level is decreasing with firm's wealth, as the cost of monitoring is a decreasing function of firms' net wealth. Consequently, the relative share of uninformed capital in investment project, compared to informed capital, increases with firms' wealth as $I_{fi}^{*}(\theta)=\left(1-\theta\right)-I_{bi}^{*}(\theta)$. In equilibrium, uninformed investors provide this amount because there is a sufficient quantity of informed capital ensuring that the hazard moral problem is alleviated.

4. OPTIMAL CREDIT ALLOCATION AND MONETARY POLICY

Our aim is to determine the level of aggregate investment, the choice of financing for a specific firm, and the price of informed capital. In order to compute the equilibrium we must determine firm's optimal choice between the different lenders.

4.1. Endogenous choice between market, the small bank, and the large bank

Knowing the financial conditions offered by the market and the two banks, a firm θ must choose rationally the contract that allows it to achieve a maximal payoff. We take for granted that firms with net wealth higher than θ_M will finance their investment with uninformed capital only (*via* financial market). Indeed, whatever its wealth, the payoff a firm can achieve on financial market is always higher than the payoff associated to a mix between bank and market (see Appendix A-2).

We focus our attention on a firm with a wealth lower than θ_M. Such a firm must be monitored in order to obtain external finance. However, it can resort to large or small banks. A firm will choose to be monitored by the large bank as long as its equilibrium payoff is higher than the one it can achieve if the small bank monitors it. We must find the marginal firm $\left(\theta_c^*\right)$, which is indifferent between the two banks. For this indifferent firm it must be

$$R_{e1}^*\left(\theta_c^*\right) = R_{e0}^*\left(\theta_c^*\right)$$

$$R - \frac{c_0\left(\theta_c^*\right)}{\Delta p} - \left(\left(1-\theta_c^*\right) - \frac{p_h c_0\left(\theta_c^*\right)}{\beta_0 \Delta p}\right)\frac{\gamma}{p_h} = R - \frac{c_1\left(\theta_c^*\right)}{\Delta p} - \left(\left(1-\theta_c^*\right) - \frac{p_h c_1\left(\theta_c^*\right)}{\beta_1 \Delta p}\right)\frac{\gamma}{p_h}$$

or, put differently

$$g\left(\theta_c^*\right) \equiv \frac{c_0\left(\theta_c^*\right)}{\Delta p} - \frac{c_1\left(\theta_c^*\right)}{\Delta p} + \left(\frac{p_h c_1\left(\theta_c^*\right)}{\beta_1 \Delta p} - \frac{p_h c_0\left(\theta_c^*\right)}{\beta_0 \Delta p}\right)\frac{\gamma}{p_h} = 0 \qquad (25)$$

Positive relationships between firm's payoff and their own wealth implies that all firms with wealth $\left(\theta > \theta_c^*\right)$ finance their investment by the large bank[16]. Consequently, firms with wealth $\left(\theta < \theta_c^*\right)$ will resort to the small bank. When monitoring takes place, the incentive condition requires that the minimum payoff a firm must receive in order to behave diligently is equal to $\frac{d}{\Delta p}$. Thus, we must find the last firm $\left(\theta_m^*\right)$ for which this incentive condition is fulfilled. $\left(\theta_m^*\right)$ is determined by the realisation of the following condition

$$R_{e0}^*\left(\theta_m^*\right) \equiv R - \frac{c_0\left(\theta_m^*\right)}{\Delta p} - \left(\left(1-\theta_m^*\right) - \frac{p_h c_0\left(\theta_m^*\right)}{\beta_0 \Delta p}\right)\frac{\gamma}{p_h} = \frac{d}{\Delta p}$$

or expressed differently

[16] See proof of proposition 1 in appendix B.

$$f\left(\theta_m^*\right) \equiv R - \frac{c_0\left(\theta_m^*\right)}{\Delta p} - \left(\left(1-\theta_m^*\right) - \frac{p_h c_0\left(\theta_m^*\right)}{\beta_0 \Delta p}\right)\frac{\gamma}{p_h} - \frac{d}{\Delta p} = 0 \tag{26}$$

Finally, knowing the respective shares of the financial market, the large bank, and the small bank in the financing, we can compute the equilibrium price of informed capital. In this perspective, we must turn our attention to bank's behaviour. As monitoring is costly and the rate of return on informed capital is higher than the one on uninformed capital, banks are encouraged to finance risky projects. Idle capital is incompatible with profit maximizing behaviour in perfect competition. Thus, small bank and large bank devote all their capital in the financing of the risky project. The equilibrium rate of return for the two banks $\left(\beta_1^*, \beta_0^*\right)$ is given by the following two conditions

$$\int_{\theta_c}^{\theta_M} I_{b1}^*\left(\theta\right) d\theta = K_1 \tag{27}$$

$$\int_{\theta_m}^{\theta_c} I_{b0}^*\left(\theta\right) d\theta = K_0 \tag{28}$$

Formally, equilibrium is fully defined by the quadruplet (β_1^*, β_0^*, θ_c^*, θ_m^*), which is the solution of the system (25) to (28). Once $\left(\theta_c^*\right)$ and $\left(\theta_m^*\right)$ is obtained, equilibrium rates of return are given by

$$\beta_1^* = \frac{p_h}{K_1 \Delta p}\left[C_1\left(\theta_M^*\right) - C_1\left(\theta_c^*\right)\right] \tag{29}$$

$$\beta_0^* = \frac{p_h}{K_0 \Delta p}\left[C_0\left(\theta_c^*\right) - C_0\left(\theta_m^*\right)\right] \tag{30}$$

where $\left[C_1\left(\theta_m^*\right) - C_1\left(\theta_c^*\right)\right]$ and $\left[C_0\left(\theta_c^*\right) - C_0\left(\theta_m^*\right)\right]$ respectively accounts for the total cost of funds for the large and the small bank.

Figure 1 is a graphical representation of this equilibrium whereas propositions 1 and 2 expose its main characteristics.

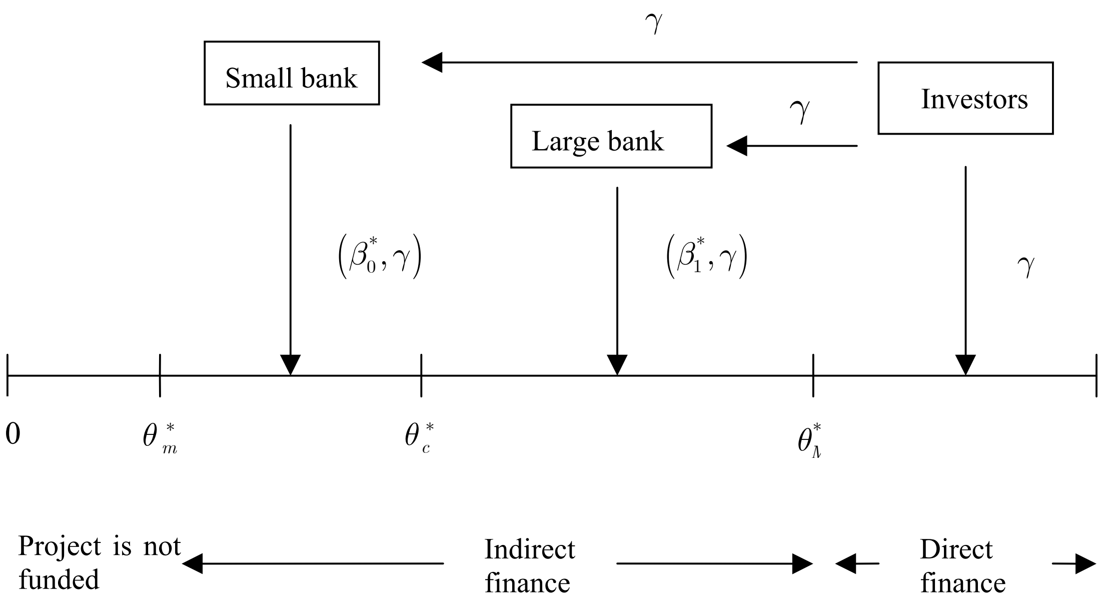


Figure –1–

PROPOSITION 1. *Firms with net wealth lower than* θ_m^* *are not financed at all, firms with net wealth between* θ_m^* *and* θ_c^* *(small firms) are financed by the small bank and uninformed investors; firms with net wealth between* θ_c^* *and* θ_M^* *(middle firms) are financed by the large bank and uninformed investors. Large firms with a wealth between* θ_M^* *and 1 resort to financial market only (uninformed investors). The total amount of investment undertaken in the economy is given by* $\left[1-\theta_m^*\right]$.

PROOF : See Appendix B.

PROPOSITION 2. *The cost of small bank capital is higher than the cost of large bank capital. As the rate of return on uninformed capital is the less expensive we have* $\gamma < \beta_1^* < \beta_0^*$.

PROOF : See Appendix C.

Firms with large wealth go to financial market (and use uninformed capital only) because they have little information problem. This means they do not need to be monitored by a bank as they can credibly provide the minimal payoff to uninformed investors. Thus, they obtain the larger payoff at the minimal cost.

For firms whose wealth is lower than θ_M^*, the trade-off between large bank and small bank is based on a basic arbitrage condition. A firm will address to large bank as long as

$$R_{e1}^* \geq R_{e0}^* \text{ or, put differently}$$

$$p_h\left[\frac{c_0(\theta)}{\Delta p}-\frac{c_1(\theta)}{\Delta p}\right] \geq \left[I_{b0}^* - I_{b1}^*\right]\gamma \tag{31}$$

The left part of equation (31) reflects the difference in the banking rent a firm must pay. It is an indicator of the large bank cost advantage compared to the small bank in monitoring a θ firm. The right part expresses the opportunity cost, in term of uninformed capital, if the firm uses the large bank rather than the small bank. A firm will choose the large bank whenever the cost advantage is higher than the opportunity cost. It comes from this equation that θ_c^* can be lower or higher than $\overline{\theta}$ (the value for which large and small banks monitoring costs are equal). Particularly, under our hypothesis, we have $\overline{\theta} > \theta_c^*$ (see proof of proposition 2). This allows us to understand the importance of monitoring and capital in our model. Monitoring cost advantage is not sufficient to explain a firm's choice between the two banks. We must also consider the two banks relative amount and prices of capital. This trade off between monitoring cost advantage and capital drives the arbitrage condition.

It results from proposition 2 that the small bank's capital equilibrium rate of return is higher than the large bank one. This result seems paradoxical in an equilibrium situation. Actually, one can notice that this difference is required in order to equal the total payoff received by the two banks (see appendix C). In equilibrium, the total payoff received by the two banks is equal, which implies that all arbitrages possibilities are exploited: there is no incentive to transfer bank capital from large bank to small bank. Moreover, even if $\beta_0^* > \beta_1^*$ small firms (with wealth lower than θ_c^*) prefer addressing to the small bank, because the cost of monitoring is lower. Consequently, for these firms, the payoff associated to small bank is higher than the one corresponding to large bank.

4.2. Monetary policy

The bank-lending channel operates if a monetary policy contraction forces banks to curtail lending and if borrowers cannot find alternative sources of financing. Our model aims at improving this analysis; showing how bank credit is reallocated when a monetary policy becomes restrictive in a context of heterogeneous banking system. Consider a change in γ we relate to a change in monetary policy conditions.

PROPOSITION 3. *An increase in the riskless rate implies a decrease in global level's investment as* $\frac{d\theta_m^*}{d\gamma} > 0$ *and* $\left[1-\theta_m^*\right]$ *is reduced. Moreover, there is a* *reallocation mechanism leading to a "flight to quality" effect as small and large bank tend to finance some wealthier borrowers* $\left(\frac{d\theta_m^*}{d\gamma} > 0\right)$, $\left(\frac{d\theta_c^*}{d\gamma} > 0\right)$, $\left(\frac{d\theta_M}{d\gamma} > 0\right)$.

Proposition 3 portrays the global effect of monetary policy on aggregate investment and on the level of firm's investment. Let's first have a look to the reallocation effect. As the opportunity cost of uninformed capital rises, firms with a wealth close to θ_M cannot ensure the minimum incentive payoff to uninformed investors. These firms cannot invest *via* direct finance any more. They must switch toward the large bank in order to fulfil incentive conditions and credibly convince uninformed investors to invest. At the same time, according to eq. (31), the rise in γ implies an increase in the opportunity cost of uninformed capital whereas monitoring costs remain fixed for a firm of given wealth. Thus, firms close to θ_c^* choose to switch toward the small bank in order to obtain a better payoff. As a result, the heterogeneous banking system allows medium firms to find alternative sources of financing. This compositional change reduces the impact of monetary policy on these borrowers. Finally, firms close to θ_m^* cannot ensure to uninformed investors that they will behave properly and are supplanted of the credit market. These borrowers are precisely the most opaque because they are small bank-dependent and are not able to signal to other lenders. Simultaneously, the relative optimal repartition of external funds between informed and uninformed capital is modified. As the banks payoff remains fixed, an increase in γ leads to a bank capital relatively less expensive than uninformed capital. To finance their investment, firms use more informed capital than before. As banks are capital constrained, the number of projects a bank can finance fall. These two effects lead to an increase of the global wealth to obtain external funds similar to a flight to quality mechanism[17].

Usual results on bank lending channel argue that the tightening of monetary conditions has a stronger effect on bank dependent firms[18]. Our approach completes this channel. Indeed, proposition 3 emphasizes the asymmetric role of monetary policy on small firms. If capital and monitoring technology are divided not equally among banks, monetary policy leads to reallocations effects and to a flight to quality. We show that monetary policy contraction has a strong effect not on bank-dependent firms but on small bank-dependent firms. Small firms are the most affected because they are exclusively financed by the small bank and cannot substitute for other sources of financing.

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[17] Our definition of this mechanism is somewhat different from those described in the literature. In the standard flight to quality literature, there is a decline in the share of credit of borrowers with high agency costs to borrowers with greater access to credit market (Bernanke, Gertler and Gilchrist (1996)).

[18] See Besanko and Kanatas (1993) and Repullo and Suarez (2000).

5. CONCLUDING REMARKS

Our model of credit allocation with heterogeneous lenders analyses how monetary policy influences the distribution of credit to small firms. We have characterized a credit market equilibrium in which firms with high wealth prefer market lending, those with middle wealth get large bank lending, and those with little wealth prefer small bank lending. The remainder of firms are unable to invest. In accordance with the empirical literature, we show that small bank interest rate is higher than large bank's one[19]. Moreover, this analysis sheds light on an asymmetric impact of monetary policy. The general contraction of firms' investment is explained by an optimal credit reallocation and a flight to quality mechanism. On the one hand, small borrowers will suffer disproportionately from a restrictive monetary policy because they are at the small-end of the credit market. On the other hand, medium firms are less constrained because they could find some alternative external funds. These points suggest that small bank specialization is important to allow some opaque borrowers to obtain finance. Compare to large intermediaries, these small banks must be higher capitalized in order to better absorb monetary shocks. Moreover, it appears that banking consolidation by reducing the quantity of small banks could affect the availability of small business credit[20]. More generally, our results underline that a same monetary policy must have a differential impact on aggregate investment according to the structure of the financial system. This point is important in order to evaluate the conduct of monetary policy in the EMU[21]. It suggests that a unified monetary policy will have an asymmetric impact as long as European financial system remains diversified.

This model is highly stylized in order to capture the essential mechanisms of credit allocation and some important points are missing. A first caveat concerns the exogenous level of bank capital. In particular, we do not model the liability side of bank's balance sheets. Banks may refinance on markets. It has been suggested that banks with more transparent portfolios may have cheaper access to funds and may therefore be favoured by regulators. In such case, this possibility could mitigate our results. A second caveat concerns the nature of firm's wealth. Indeed, we do not taking into account the use of real asset as collateral. Actually, a complete model should integrate the impact of the interest rate on real assets. We leave these extensions for future research.

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[19] See Hubbard, Kuttner and Palia (1999), and Berger and Udell (1996) for empirical investigation.

[20] See Sapienza (2002), Berger, Rosen and Udell (2001), and Berger, Saunders, Scalise and Udell (1998) among others.

[21] See Kashyap and Stein (1997) and Cechetti (1999).

APPENDIX

APPENDIX A-1: Firms' payoff is an increasing function of their wealth when it finances with bank capital.

$$R_{ei}^{*}(\theta)=R-\frac{c_i(\theta)}{\Delta p}-\left((1-\theta)-\frac{p_h c_i(\theta)}{\beta_i \Delta p}\right)\frac{\gamma}{p_h} \text{ and}$$

$$\frac{\partial R_{ei}^{*}}{\partial \theta}=-\frac{c_i^{'}(\theta)}{\Delta p}\left[1-\frac{\gamma}{\beta_i}\right]+\frac{\gamma}{p_h}>0 \qquad \text{A-1}$$

as $\beta_i > \gamma$ (eq.) and $c_i^{'}(\theta)<0$ (*assumption 3)* ■

APPENDIX A-2: Firms payoff achieves on capital market is always higher than the one achieves with bank capital.

$$R-(1-\theta)\frac{\gamma}{p_h}>R-\frac{c_i(\theta)}{\Delta p}-\left((1-\theta)-\frac{p_h c_i(\theta)}{\beta_i \Delta p}\right)\frac{\gamma}{p_h} \text{ or}$$

$$\frac{c_i(\theta)}{\Delta p}\left(1-\frac{\gamma}{\beta_i}\right)>0 \text{ as } c_i(\theta)>0 \text{ ■}$$

Appendix B: Proof of PROPOSITION 1.

In order to prove Proposition 1, we must impose restrictions on model's parameters. These restrictions consist in maintaining our exogenous values in plausible range.

B1: $p_h\left[R-\frac{c_0(0)}{\Delta p}\right]>\gamma>p_h\left[R-\frac{d}{\Delta p}\right]$. Firms with no wealth cannot be financed by the small bank.

B2: $R_e^{*}\left(\theta_c^{*}\right)<\frac{B}{\Delta p}$. Incentive constraint (12) is not realised by the marginal firm θ_c^{*}.

B3: $p_h\left[R-\frac{c_0\left(\theta_c^{*}\right)+d}{\Delta p}\right]>\left(1-\theta_c^{*}\right)\gamma$. Incentive constraints are realised for all parties if the marginal firm θ_c^{*} is monitored by the small bank.

The proof of proposition 1 consists in three parts.

1- Existence and uniqueness of θ_c^* .

Consider the following equation

$$g(\theta) \equiv R_{e1}^*(\theta) - R_{e0}^*(\theta) = \frac{c_0(\theta)}{\Delta p} - \frac{c_1(\theta)}{\Delta p} + \left(\frac{p_h c_1(\theta)}{\beta_1 \Delta p} - \frac{p_h c_0(\theta)}{\beta_0 \Delta p}\right)\frac{\gamma}{p_h} \qquad \text{A-2}$$

We search for θ_c^* such that $g(\theta_c^*) = 0$. In order to prove the existence of this value, let's first turn our attention on the main properties of firms' payoff.

R_{e1}^* and R_{e0}^* are continuous on $[0,1]$. Moreover, according to appendix A-1, we know that

$$\frac{\partial R_{e1}^*(\theta)}{\partial \theta} = -\frac{c_1^{'}(\theta)}{\Delta p}\left[1 - \frac{\gamma}{\beta_1}\right] + \frac{\gamma}{p_h} > 0 \text{ and } \frac{\partial R_{e0}^*(\theta)}{\partial \theta} = -\frac{c_0^{'}(\theta)}{\Delta p}\left[1 - \frac{\gamma}{\beta_0}\right] + \frac{\gamma}{p_h} > 0$$

Finally, part 3 of *assumption 3* ensures there is no inflexion point, as $\frac{\partial^2 R_{e1}^*(\theta)}{\partial \theta^2} = -\frac{c_1^{''}(\theta)}{\Delta p}\left[1 - \frac{\gamma}{\beta_1}\right] < 0$ and $\frac{\partial^2 R_{e0}^*(\theta)}{\partial \theta^2} = -\frac{c_0^{''}(\theta)}{\Delta p}\left[1 - \frac{\gamma}{\beta_0}\right] < 0$

Therefore, R_{e1}^* and R_{e0}^* are continuous increasing concave function of θ.

Let's analyse now the behaviour of firm's rent on the limit of the wealth interval $[0,1]$. For a firm with no wealth, we have

$$\lim_{\theta \to 0} R_{e0}^*(\theta) = R - \frac{c_0(0)}{\Delta p}\left[1 - \frac{\gamma}{\beta_0}\right] - \frac{\gamma}{p_h} > 0 \text{ under restriction B1.}$$

$$\lim_{\theta \to 0} R_{e1}^*(\theta) = R - \frac{c_1(0)}{\Delta p}\left[1 - \frac{\gamma}{\beta_1}\right] - \frac{\gamma}{p_h} \to -\infty \text{ as } c_1(0) \to +\infty \text{ and } \beta_1 > \gamma .$$

For a firm with a wealth equal to 1, we have

$$\lim_{\theta \to 1} R_{e0}^*(\theta) = R - \frac{c_0(1)}{\Delta p}\left[1 - \frac{\gamma}{\beta_0}\right] > 0 \text{ as } R \geq R_{b0} \geq \frac{c_0(\theta)}{\Delta p}$$

$$\lim_{\theta \to 1} R_{e1}^*(\theta) = R - \frac{c_1(1)}{\Delta p}\left[1 - \frac{\gamma}{\beta_1}\right] \to R \text{ as } c_1(1) = 0 .$$

We conclude that as $\lim_{\theta \to 0} R_{e0}^*(\theta) > \lim_{\theta \to 0} R_{e1}^*(\theta)$ and $\lim_{\theta \to 1} R_{e1}^*(\theta) > \lim_{\theta \to 1} R_{e0}^*(\theta)$ there is at least one value on $[0,1]$ (named θ_c^*) for which $R_{e1}^*(\theta_c^*) = R_{e0}^*(\theta_c^*)$ and $g(\theta_c^*) = 0$.

Moreover, as the two functions are increasing in θ and there is no inflexion point, we conclude that $\frac{\partial R_{e1}^{*}(\theta)}{\partial\theta} > \frac{\partial R_{e0}^{*}(\theta)}{\partial\theta}$. This leads to the conclusion that the equilibrium is unique as $\frac{\partial g(\theta)}{\partial\theta} = g^{'}(\theta) = \frac{c_0^{'}(\theta)}{\Delta p} - \frac{c_1^{'}(\theta)}{\Delta p} + \left(\frac{p_h c_1^{'}(\theta)}{\beta_1 \Delta p} - \frac{p_h c_0^{'}(\theta)}{\beta_0 \Delta p}\right)\frac{\gamma}{p_h} > 0$ A-3

Finally, according to $g^{'}(\theta) > 0$, all firms with wealth $\theta > \theta_c^{*}$ will choose to be monitored by the large bank, the other preferring the small bank.

2- Existence and uniqueness of θ_m^{*}.

We search for the last firm that are able to be financed by the small bank.

Consider the following equation

$$f(\theta) = R - \frac{c_0(\theta)}{\Delta p} - \left((1-\theta) - \frac{p_h c_0(\theta)}{\beta_0 \Delta p}\right)\frac{\gamma}{p_h} - \frac{d}{\Delta p} \qquad \text{A-4}$$

We search the value θ_m^{*} for which $f(\theta_m^{*}) = 0$. Let's study first the behaviour of function $f(.)$ on 0 and 1.

$\lim_{\theta\to 0} f(\theta) = R - \frac{c_0(0)}{\Delta p}\left[1 - \frac{\gamma}{\beta_0}\right] - \frac{\gamma}{p_h} - \frac{d}{\Delta p} < 0$ under restriction B1.

$\lim_{\theta\to 1} f(\theta) = R - \frac{c_0(1)}{\Delta p}\left[1 - \frac{\gamma}{\beta_0}\right] - \frac{d}{\Delta p} > 0$ under restriction B3.

Moreover, according to appendix A-1, we have

$\frac{\partial f(\theta)}{\partial\theta} = -\frac{c_0^{'}(0)}{\Delta p}\left[1 - \frac{\gamma}{\beta_0}\right] + \frac{\gamma}{p_h} > 0$.

Consequently, there is an unique value, θ_m^{*}, for which $f(\theta_m^{*}) = 0$. This value gives us the characteristic of the last firm that are able to be financed by the small bank.

3- Proof that $\theta_M^{*} > \theta_c^{*} > \theta_m^{*}$

a- According to condition B2 we have

$R_e^{*}(\theta_c^{*}) = R - (1-\theta_c^{*})\frac{\gamma}{p_h} < \frac{B}{\Delta p} = R_e^{*}(\theta_M^{*})$.

As firms' payoff is an increasing function of θ, it means that $\theta_M^* > \theta_c^*$.

b- According to condition B3 we have

$$R_{e0}^*\left(\theta_c^*\right) = R - \frac{c_0\left(\theta_c^*\right)}{\Delta p} - \left(\left(1-\theta_c^*\right) - \frac{p_h c_0\left(\theta_c^*\right)}{\beta_0 \Delta p}\right)\frac{\gamma}{p_h} > \frac{d}{\Delta p} = R_{e0}^*\left(\theta_m^*\right).$$

As firms' payoff is an increasing function of θ, it means that $\theta_c^* > \theta_m^*$.

Consequently, it result that $\theta_M^* > \theta_c^* > \theta_m^*$ ■

APPENDIX C: Proof of PROPOSITION 2.

a- The first part of the proof shows that, in equilibrium, the cost of capital of the small bank is higher that the cost of capital of the large bank. Equilibrium interest rate are given by (29) and (30). Consequently, in order to show that $\beta_0^* > \beta_1^*$ we have to prove that the following condition is realized

$$\frac{K_1}{K_0} > \frac{\left[C_1\left(\theta_M^*\right) - C_1\left(\theta_c^*\right)\right]}{\left[C_0\left(\theta_c^*\right) - C_0\left(\theta_m^*\right)\right]} \qquad \text{A-5}$$

Compute $g(\theta)$ for equilibrium interest rates gives

$$g(\theta) = \frac{c_0(\theta)}{\Delta p} - \frac{c_1(\theta)}{\Delta p} + \left(\frac{p_h c_1(\theta)}{\beta_1^* \Delta p} - \frac{p_h c_0(\theta)}{\beta_0^* \Delta p}\right)\frac{\gamma}{p_h} \text{ or using (29) and (30)}$$

$$g(\theta) = \frac{c_0(\theta)}{\Delta p} - \frac{c_1(\theta)}{\Delta p} + \left[\frac{K_1 c_1(\theta)}{\left[C_1\left(\theta_M^*\right) - C_1\left(\theta_c^*\right)\right]} - \frac{K_0 c_0(\theta)}{\left[C_0\left(\theta_c^*\right) - C_0\left(\theta_m^*\right)\right]}\right]\frac{\gamma}{p_h}$$

Taking the derivative of this function leads to

$$g'(\theta) = \frac{c_0'(\theta)}{\Delta p}\left[1 - \frac{\Delta p \gamma K_0}{p_h\left[C_0\left(\theta_c^*\right) - C_0\left(\theta_m^*\right)\right]}\right] - \frac{c_1'(\theta)}{\Delta p}\left[1 - \frac{\Delta p \gamma K_1}{p_h\left[C_1\left(\theta_M^*\right) - C_1\left(\theta_c^*\right)\right]}\right]$$

According to A-3 we know that $g'(\theta) > 0$ consequently we have

$$\frac{c_0^{'}(\theta)}{\Delta p}\left[1-\frac{\Delta p\gamma K_0}{p_h\left[C_0\left(\theta_c^*\right)-C_0\left(\theta_m^*\right)\right]}\right] > \frac{c_1^{'}(\theta)}{\Delta p}\left[1-\frac{\Delta p\gamma K_1}{p_h\left[C_1\left(\theta_M^*\right)-C_1\left(\theta_c^*\right)\right]}\right]$$ which leads to $\frac{\left[1-\frac{\Delta p\gamma K_0}{p_h\left[C_0\left(\theta_c^*\right)-C_0\left(\theta_m^*\right)\right]}\right]}{\left[1-\frac{\Delta p\gamma K_1}{p_h\left[C_1\left(\theta_M^*\right)-C_1\left(\theta_c^*\right)\right]}\right]} > \frac{c_1^{'}(\theta)}{c_0^{'}(\theta)} > 1$ as $\frac{c_1^{'}(\theta)}{c_0^{'}(\theta)} > 1$ (see *assumption 3*).

This condition is realised if and only if

$$\left[\frac{\Delta p\gamma K_1}{p_h\left[C_1\left(\theta_M^*\right)-C_1\left(\theta_c^*\right)\right]}\right] > \left[\frac{\Delta p\gamma K_0}{p_h\left[C_0\left(\theta_c^*\right)-C_0\left(\theta_m^*\right)\right]}\right]$$ which implies that

$$\frac{K_1}{K_0} > \frac{\left[C_1\left(\theta_M^*\right)-C_1\left(\theta_c^*\right)\right]}{\left[C_0\left(\theta_c^*\right)-C_0\left(\theta_m^*\right)\right]}$$

Thus, condition A-5 is fulfilled and $\beta_0^* > \beta_1^*$ ■

b- We have to show that the positive spread between small and large banks interest rates is compatible with a no-arbitrage condition. Indeed, in equilibrium, the two banks' total rents are identical. Define $p_h R_{e0}^* - c_0(\theta) - \gamma I_{b0}$ as the unitary rent achieves by bank 0 when it finances a θ's wealth firm and W_0 as the total rent, earn in equilibrium, by bank 0. We have

$$W_0 = \int_{\theta_m^*}^{\theta_c^*}\left[p_h R_{e0}^* - c_0(\theta) - \gamma I_{b0}\right] d\theta$$ or

$$W_0 = \int_{\theta_m^*}^{\theta_c^*}\left[p_h\frac{c_0(\theta)}{\Delta p} - c_0(\theta) - \gamma\frac{p_h c_0(\theta)}{\Delta p\beta_0^*}\right] d\theta \qquad \text{A-6}$$

Simplifying equation A-6 leads to

$$W_0 = \frac{1}{\Delta p}\left[p_l - \frac{\gamma p_h}{\beta_0^*}\right]\int_{\theta_m^*}^{\theta_c^*} c_0(\theta)\, d\theta = \frac{1}{\Delta p}\left[p_l - \frac{\gamma p_h}{\beta_0^*}\right]\left[C_0\left(\theta_c^*\right)-C_0\left(\theta_m^*\right)\right]$$

Using the fact that, in equilibrium, $\left[C_0\left(\theta_c^*\right)-C_0\left(\theta_m^*\right)\right]=\dfrac{\Delta p K_0\beta_0^*}{p_h}$ we obtain the total rent earn by the small bank

$$W_0=K_0\left[\frac{p_l}{p_h}\beta_0^*-\gamma\right] \qquad \text{A-7}$$

The same reasoning for the large bank leads to the following value of its total rent

$$W_1=K_1\left[\frac{p_l}{p_h}\beta_1^*-\gamma\right] \qquad \text{A-8}$$

According to the no-arbitrage condition, the value of the two rents must be equal in equilibrium, which leads to the following condition

$$W_0=K_0\left[\frac{p_l}{p_h}\beta_0^*-\gamma\right]=K_1\left[\frac{p_l}{p_h}\beta_1^*-\gamma\right]=W_1 \qquad \text{A-9}$$

A-9 is realized if

$$\frac{K_1}{K_0}=\frac{\left[{}^{p_l}\!/_{p_h}\,\beta_0^*-\gamma\right]}{\left[{}^{p_l}\!/_{p_h}\,\beta_1^*-\gamma\right]}>1$$

Which is true if and only if $\beta_0^*>\beta_1^*$. Consequently, difference in bank equilibrium interest rates is a necessary condition to the rent equality and the no-arbitrage condition ■

APPENDIX D : Proof of PROPOSITION 3.

Taking the total derivative of equations (14), (25), (26), (29) and (30) leads to

$$d\beta_o=\left[\frac{\beta_0^*c_0\left(\theta_c^*\right)}{C_0\left(\theta_c^*\right)-C_0\left(\theta_m^*\right)}\right]d\theta_c-\left[\frac{\beta_0^*c_0\left(\theta_m^*\right)}{C_0\left(\theta_c^*\right)-C_0\left(\theta_c^*\right)}\right]d\theta_m-\left[\frac{\Delta p\beta_0^{*2}}{p_h\left(C_0\left(\theta_c^*\right)-C_0\left(\theta_m^*\right)\right)}\right]dK_0$$

$$d\beta_1 = \left[\frac{\beta_1^* c_1\left(\theta_M^*\right)}{C_1\left(\theta_M^*\right) - C_1\left(\theta_c^*\right)}\right] d\theta_M - \left[\frac{\beta_1^* c_1\left(\theta_c^*\right)}{C_1\left(\theta_M^*\right) - C_1\left(\theta_c^*\right)}\right] d\theta_c - \left[\frac{\Delta p \beta_1^{*2}}{p_h\left(C_1\left(\theta_M^*\right) - C_1\left(\theta_c^*\right)\right)}\right] dK_1$$

$$d\theta_c = \left[\frac{c_1\left(\theta_c^*\right)\gamma}{g'\left(\theta_c^*\right)\Delta p \beta_1^{*2}}\right] d\beta_1 - \left[\frac{c_0\left(\theta_c^*\right)\gamma}{g'\left(\theta_c^*\right)\Delta p \beta_0^{*2}}\right] d\beta_0 - \left[\left(\frac{1}{g'\left(\theta_c^*\right)}\right)\left(\frac{c_1\left(\theta_c^*\right)}{\Delta p \beta_1^*} - \frac{c_0\left(\theta_c^*\right)}{\Delta p \beta_0^*}\right)\right] d\gamma$$

$$d\theta_m = \left[\frac{c_0\left(\theta_m^*\right)\gamma}{f'\left(\theta_m^*\right)\Delta p \beta_0^{*2}}\right] d\beta_0 - \left[\left(\frac{1}{f'\left(\theta_m^*\right)}\right)\left(\frac{c_0\left(\theta_m^*\right)}{\Delta p \beta_0^*} - \frac{\left(1-\theta_m^*\right)}{p_h}\right)\right] d\gamma$$

$$d\theta_M = \left[\left(R - \frac{B}{\Delta p}\right)\frac{p_h}{\gamma^2}\right] d\gamma$$

Taking the fact that $dK_0 = dK_1 = 0$, we can simplify this system

$$d\beta_0 = A_1 d\theta_c - A_2 d\theta_m \qquad \text{A-10}$$

$$d\beta_1 = B_1 d\theta_M - B_2 d\theta_c \qquad \text{A-11}$$

$$d\theta_c = \frac{1}{g'\left(\theta_c^*\right)}\left[C_1 d\beta_1 - C_2 d\beta_0 - C_3 d\gamma\right] \qquad \text{A-12}$$

$$d\theta_m = \frac{1}{f'\left(\theta_m^*\right)}\left[D_1 d\beta_0 - D_2 d\gamma\right] \qquad \text{A-13}$$

$$d\theta_M = E d\gamma \qquad \text{A-14}$$

with $\left(A_1, A_2\right) \equiv \left(\frac{\beta_0^* c_0\left(\theta_c^*\right)}{C_0\left(\theta_c^*\right) - C_0\left(\theta_m^*\right)}, \frac{\beta_0^* c_0\left(\theta_m^*\right)}{C_0\left(\theta_c^*\right) - C_0\left(\theta_m^*\right)}\right)$

$\left(B_1, B_2\right) \equiv \left(\frac{\beta_1^* c_1\left(\theta_M^*\right)}{C_1\left(\theta_M^*\right) - C_1\left(\theta_c^*\right)}, \frac{\beta_1^* c_1\left(\theta_c^*\right)}{C_1\left(\theta_M^*\right) - C_1\left(\theta_c^*\right)}\right)$

$\left(C_1, C_2, C_3\right) \equiv \left(\frac{c_1\left(\theta_c^*\right)\gamma}{\Delta p \beta_1^{*2}}, \frac{c_0\left(\theta_c^*\right)\gamma}{\Delta p \beta_0^{*2}}, \frac{c_1\left(\theta_c^*\right)}{\Delta p \beta_1^*} - \frac{c_0\left(\theta_c^*\right)}{\Delta p \beta_0^*}\right)$

$$\left(D_1, D_2\right) \equiv \left(\frac{c_0\left(\theta_m^*\right)\gamma}{\Delta p \beta_0^{*2}}, \frac{c_0\left(\theta_m^*\right)}{\Delta p \beta_0^*} - \frac{\left(1-\theta_m^*\right)}{p_h}\right)$$

$$E \equiv \left(R - \frac{B}{\Delta p}\right)\frac{p_h}{\gamma^2}$$

and $A_1, A_2, B_1, B_2, C_1, C_2, C_3, D_1, E > 0$ and $D_2 < 0$.

From A-14, it is straightforward that $\frac{d\theta_M}{d\gamma} > 0$.

Substituting A-13 in A-10, A-14 in A-11, and A-10 and A-11 in A-12, we obtain

$$d\theta_c\left[g'\left(\theta_c^*\right) + B_2 C_1 + \frac{A_1 C_2}{1 + \frac{A_2 D_1}{f'\left(\theta_m^*\right)}}\right] = d\gamma\left[B_1 C_1 E - C_3 - \frac{A_2 C_2 D_2}{f'\left(\theta_m^*\right) + A_2 D_1}\right] \qquad \text{A.15}$$

with $g'\left(\theta_c^*\right) + B_2 C_1 + \frac{A_1 C_2}{1 + \frac{A_2 D_1}{f'\left(\theta_m^*\right)}} > 0$, and $\frac{A_2 C_2 D_2}{f'\left(\theta_m^*\right) + A_2 D_1} < 0$

$\frac{d\theta_c^*}{d\gamma} > 0$ as long as $B_1 C_1 E - C_3 > 0$.

$$B_1 C_1 E - C_3 = \left[\frac{\beta_1^* c_1\left(\theta_M^*\right)}{C_1\left(\theta_M^*\right) - C_1\left(\theta_c^*\right)}\right]\left[\frac{c_1\left(\theta_c^*\right)\gamma}{\Delta p \beta_1^{*2}}\right]\left[\left(R - \frac{B}{\Delta p}\right)\frac{p_h}{\gamma^2}\right] + \frac{c_0\left(\theta_c^*\right)}{\Delta p \beta_0^*} - \frac{c_1\left(\theta_c^*\right)}{\Delta p \beta_1^*}$$

Simplifying this expression, we obtain

$$\left[\frac{\beta_1^* c_1\left(\theta_M^*\right)}{C_1\left(\theta_M^*\right) - C_1\left(\theta_c^*\right)}\right]\left[\frac{c_1\left(\theta_c^*\right)\gamma}{\Delta p \beta_1^{*2}}\right]\left[\left(R - \frac{B}{\Delta p}\right)\frac{p_h}{\gamma^2}\right] + \frac{c_0\left(\theta_c^*\right)}{\Delta p \beta_0^*} > \frac{c_1\left(\theta_c^*\right)}{\Delta p \beta_1^*}$$

Substituting (29) and (30) in this equation, $B_1 C_1 E - C_3 > 0$ if and only if

$$\frac{K_0}{K_1} > \frac{\left(C_0\left(\theta_c^*\right) - C_0\left(\theta_m^*\right)\right)c_1\left(\theta_c^*\right)}{\left(C_1\left(\theta_M^*\right) - C_1\left(\theta_c^*\right)\right)c_0\left(\theta_c^*\right)} - \frac{\left(p_h\left(R\Delta p - B\right)c_1\left(\theta_M^*\right)\right)}{\left(C_1\left(\theta_M^*\right) - C_1\left(\theta_c^*\right)\right)^2 \gamma \Delta p c_0\left(\theta_c^*\right)} \qquad \text{B.4}$$

We assume that this condition is fulfilled. Thus, $\frac{d\theta_c}{d\gamma} > 0$.

Finally, we have

$$d\theta_m \left[f'\left(\theta_m^*\right) + \frac{A_2 D_1}{1 + \dfrac{A_1 C_2}{g'\left(\theta_c^*\right) + B_2 C_1}} \right] = d\gamma \left[\frac{A_1 D_1 \left(B_1 C_1 E - C_3\right)}{g'\left(\theta_c^*\right) + B_2 C_1 + A_1 C_2} - D_2 \right]$$

with $f'\left(\theta_m^*\right) + \dfrac{A_2 D_1}{1 + \dfrac{A_1 C_2}{g'\left(\theta_c^*\right) + B_2 C_1}} > 0$, $B_1 C_1 E - C_3 > 0$ (B.4), and $D_2 < 0$

Consequently, $\dfrac{d\theta_m}{d\gamma} > 0$ ■

REFERENCES

Almazan, A. (2002). A Model of Competition in Banking: Bank Capital Vs Expertise, *Journal of Financial Intermediation*, **11**, 87-121.

Basset, B. and Brady, T. (2001). The Economic Performance of Small Banks, 1985-2000, *Federal Reserve Bulletin*, 719-728.

Berger, A., Kashyap, A. and Scalise, J. M. (1995). The Transformation of the Us Banking Industry: What a Long Strange Trip It's Been, *Brookings Papers on Economic Activity*, **2**, 55-218.

Berger, A., Miller, N. H., Petersen, M. A., Rajan, R. and Stein, J. C. (2002). Does Function Follow Organizational Form ? Evidence from the Lending Practices of Large and Small Banks, *NBER working paper*, **8752**,

Berger, A., Rosen, R. and Udell, G. F. (2001). The Effect of Banking Market Size Structure on Bank Competition: The Case of Small Business Lending, *Board of Governors of the Federal Reserve System, Working Paper*,

Berger, A., Saunders, A., Scalise, J. M. and Udell, G. F. (1998). The Effects of Banks Mergers and Acquisitions on Small Lending, *Journal of Financial Economics*, **50**, 187-229.

Berger, A. and Udell, G. F. (2002). Small Business Credit Availability and Relationship Lending: The Importance of Bank Organisational Structure, *Economic Journal*, **112**, 32-53.

Bernanke, B. S., Gertler, M. and Gilchrist, S. (1996). The Financial Accelerator and the Flight to Quality, *The Review of Economics and Statistics*, **78**, 1-15.

Besanko, D. and Kanatas, G. (1993). Credit Market Equilibrium with Bank Monitoring and Moral Hazard, *Review of Financial studies*, **6**, 213-232.

Boot, A. W. A. (2000). Relation Banking: What Do We Know?, *Journal of Financial Intermediation*, **9**, 7-25.

Boot, A. W. A. and Thakor, A. V. (2000). Can Relationship Banking Survive Competition ?, *Journal of Finance*, **55**, 679-713.

Cechetti, S. (1999). Legal Structure, Financial Structure, and the Monetary Transmission Mechanism, *Federal Reserve Bank of New York, Economic Policy Review*, 9-28.

Chatelain, J.B., A. Generale, I. Hernando, U. Von Kalckreuth and P. Vermulen (2001). Firm Investment and Monetary Transmission in the Euro Zone, *European Central Bank, Working Paper* **112**, 1-49.

Chen, N.-K. (2001). Bank Net Worth, Asset Prices and Economic Activity., *Journal of Monetary Economics*, **48**, 415-436.

Dell'Ariccia, G. and Marquez, R. (2001). Flight to Quality or to Captivity: Information and Credit Allocation, *International Monetary Fund, Working Paper*, 2-25.

Diamond, D. (1984). Financial Intermediation and Delegated Monitoring, *Review of Economic Studies*, **51**, 393-414.

Ehrman, M., L. Gambacorta, J. Martinez-Pagés, P. Sevestre and A. Worms (2001). Financial System and the Role of Monetary Policy Transmission in the Euro Area, *European Central Bank, Working Paper 105*, 1-60.

Ennis, H. M. (2001a). Loanable Funds, Monitoring and Banking, *European Finance Review*, **5**, 79-114.

Ennis, H. M. (2001b). On the Size Distribution of Banks, *Federal Reserve Bank of Richmond, Economic Quaterly*, **87**, 1-25.

Gaiotti, E. and A. Generale (2001). Does Monetary Policy Have Asymmetric Effects? A Look at the Investment Decisions of Italian Firms, *European Central Bank, Working Paper* **110**, 1-58.

Gertler, M. and Gilchrist, S. (1994). Monetary Policy, Business Cycles, and the Behaviour of Small Manufacturing Firms, *Quaterly Journal of Economics*, **109**, 309-40.

Holmstrom, B. and Tirole, J. (1997). Financial Intermediation, Loanable Funds, and the Real Sector, *Quaterly Journal of Economics*, **112**, 663-691.

Hubbard, R. G., Kuttner, K. N. and Palia, D. N. (1999). Are There Banks Effects in Borrowers Costs of Funds?, *Federal Reserve Bank of New-York*, **78**,

Jayaratne, J. and Wolken, J. D. (1999). How Important Are Small Banks to Small Business Lending?New Evidence from a Survey of Small Firms, *Journal of Banking and Finance*, **23**, 427-458.

Kashyap, A. and Stein, J. C. (1995). The Impact of Monetary Policy on Bank Balance Sheets, *Canergie-Rochester Conference Series on Public Policy*, **42**, 151-95.

Kashyap, A. and Stein, J. C. (1997). The Role of Banks in Monetary Policy: A Survey with Implications for the European Monetary Union, *Federal Reserve Bank of Chicago, Economic Perspectives*, 3-18.

Kashyap, A. and Stein, J. C. (2000). What Do a Million Observations on Banks Say About the Transmission of Monetary Policy, *American Economic Review*, **90**, 407-28.

Kishan, R. P. and Opiela, T. P. (2000). Bank Size, Bank Capital, and the Bank Lending Channel, *Journal of Money, Credit and Banking*, **32**, 121-141.

Laffont, J. J. and Rochet, J. C. (1997). Collusion in Organizations, *Scandinavian Journal of Economics*, **99**, 485-495.

Loupias, C., F. Savignac and P. Sevestre (2002). Is there a Bank Lending Channel in France? Evidence from Bank Panel Data, *Banque de France, Note d'Etude et de Recherche* **92**, 1-14.

McNulty, J. E., Akhigbe, A. O. and Verbrugge, J. A. (2001). Small Bank Loan Quality in a Deregulated Environment: The Information Advantage Hypothesis, *Journal of Economics and Business*, **53**, 325-339.

Nakamura, L. (1994). Small Borrowers and the Survival of the Small Banks: Is Mouse Bank Mighty or Mickey?, *Federal Reserve Bank of Philadelphia, Business Review*, 3-

15.

Repullo, S. and Suarez, J. (2000). Entrepreneurial Moral Hazard and Bank Monitoring: A Model of the Credit Channel, *European Economic Review*, **44**, 1931-1950.

Sapienza, P. (2002). The Effects of Banking Margers on Loans Contracts, *Journal of Finance*, **57**, 329-350.

Stein, J. C. (1998). An Adverse Selection Model of Bank Asset and Liability Management with Implications for the Transmission of Monetary Policy, *Rand Journal od Economics*, **29**, 466-486.

Stein, J. C. (2002). Information Production and Capital Allocation: Decentralized Vs. Hierarchical Firms, *Journal of Finance*, **forthcoming**,

Strahan, P. E. and Weston, J. (1996). Small Business Lending and Bank Consolidation: Is There Cause for Concern?, *Federal Reserve Bank of New York, Current Issues in Economics and Finance 2*, 1-6.

Tirole, J. (1986). Hierarchies and Bureaucracies: On the Role of Collusion in Organizations, *Journal of Law, Economics and Organization*, **2**, 181-214.

Williamson, S. D. (1986). Increasing Returns to Scale in Financial Intermediation and the Non-Neutrality of Government Policy, *Review of Economic Studies*, **53**, 863-875.

Winton, A. (1995). Delegated Monitoring and Bank Structure in a Finite Economy, *Journal of Financial Intermediation*, **4**, 158-187.

Winton, A. (2000). Don't Put All Eggs in One Basket? Diversification and Specialization in Lending, *mimeograph*.