The Zero Bound on Interest Rates and Optimal Monetary Policy *

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The consequences for the proper conduct of monetary policy of the existence of a lower bound of zero for overnight nominal interest rates has recently become a topic of lively interest. In Japan, the call rate (the overnight cash rate that is analogous to the federal funds rate in the U.S.) has been within 50 basis points of zero since October 1995, so that little room for further reductions in short-term nominal interest rates has existed since that time, and has been essentially equal to zero for most of the past four years. (See Figure 1 below.) At the same time, growth has remained anemic in Japan over this period, and prices have continued to fall, suggesting a need for monetary stimulus. Yet the usual remedy — lower short-term nominal interest rates — is plainly unavailable. Vigorous expansion of the monetary base (which, as shown in the figure, is now more than twice as large, relative to GDP, as in the early 1990s) has also seemed to do little to stimulate demand under these circumstances.

The fact that the federal funds rate has now been reduced to only 1.25 percent in the U.S., while signs of recovery remain exceedingly fragile, has led many to wonder if the U.S. could not also soon find itself in a situation where interest-rate policy would no longer be available as a tool for macroeconomic stabilization. A number of other nations face similar questions. The result is that a problem that was long treated as a mere theoretical curiosity after having been raised by Keynes (1936) — namely, the question of what can be done to stabilize the economy when interest rates have fallen to a level below which they cannot be driven by further monetary expansion, and whether monetary policy can be effective at all under such circumstances — now appears to be one of urgent practical importance, though one with which theorists have become unfamiliar.

The question of how policy should be conducted when the zero bound is reached — or when the possibility of reaching it can no longer be ignored — raises many fundamental issues for the theory of monetary policy. Some would argue that awareness of the possibility of hitting the zero bound calls for fundamental changes in the way that policy is conducted even when the bound has not yet been reached. For example, Krugman (2003) refers to deflation as a "black hole", from which an economy cannot expect to escape once it has

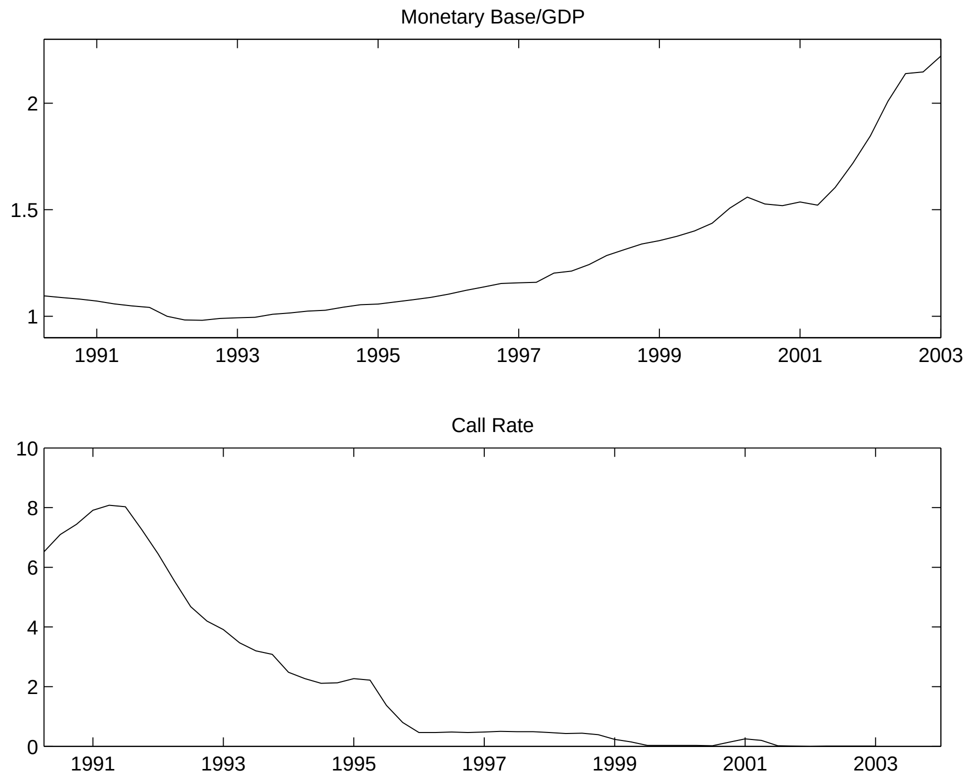

Figure 1: Evolution of the Japanese monetary base as a proportion of GDP [1992 = 1.0], and the associated path of the call rate on uncollateralized overnight loans.

been entered. A conclusion that is often drawn from this pessimistic view of the efficacy of monetary policy under circumstances of a liquidity trap is that it is vital to steer far clear of circumstances under which deflationary expectations could ever begin to develop — for example, by targeting a sufficiently high positive rate of inflation even under normal circumstances.

Others are more sanguine about the continuing effectiveness of monetary policy even when the zero bound is reached, but frequently defend their optimism on grounds that again imply that conventional understanding of the conduct of monetary policy is inadequate in important respects. For example, it is often argued that deflation need not be a "black hole" because monetary policy can affect aggregate spending and hence inflation through channels other than central-bank control of short-term nominal interest rates. Thus there has been much recent discussion — both among commentators on the problems of Japan, and among those addressing the nature of deflationary risks to the U.S. — of the advantages of vigorous expansion of the monetary base even when these are not associated with any further reduction in interest rates, of the desirability of attempts to shift longer-term interest rates through purchases of longer-maturity government securities by the central bank, and even of the possible desirability of central-bank purchases of other kinds of assets. Yet if these views are correct, they challenge much of the recent conventional wisdom regarding the conduct of monetary policy, both within central banks and among monetary economists, which has stressed a conception of the problem of monetary policy in terms of the appropriate adjustment of an operating target for overnight interest rates, and formulated prescriptions for monetary policy, such as the celebrated "Taylor rule" (Taylor, 1993), that are cast in these terms. Indeed, some have argued that the inability of such a policy to prevent the economy from falling into a deflationary spiral is a critical flaw of the Taylor rule as a guide to policy (Benhabib *et al.*, 2001).

Similarly, the concern that a liquidity trap can be a real possibility is sometimes presented as a serious objection to another currently popular monetary policy prescription, namely inflation targeting. The definition of a policy prescription in terms of an inflation target presumes that there is in fact an interest-rate choice that can allow one to hit one's target (or at least to be projected to hit it, on average). But, some would argue, if the zero interest-rate bound is reached under circumstances of deflation, it will not be possible to hit any higher inflation target, as further interest-rate decreases are not possible despite the fact that one is undershooting one's target. Is there, in such circumstances, any point in having an inflation target? This has frequently been offered as a reason for resistance to inflation targeting at the Bank of Japan. For example, Kunio Okina, director of the Institute for Monetary and Economic Studies at the BOJ, was quoted by Dow Jones News (8/11/1999) as arguing that "because short-term interest rates are already at zero, setting an inflation target of, say, 2 percent wouldn't carry much credibility."

Here we seek to shed light on these issues by considering the consequences of the zero lower bound on nominal interest rates for the optimal conduct of monetary policy, in the context of an explicit intertemporal equilibrium model of the monetary transmission mechanism. While our model remains an extremely simple one, we believe that it can help to clarify some of the basic issues just raised. We are able to consider the extent to which the zero bound represents a genuine constraint on attainable equilibrium paths for inflation and real activity, and to consider the extent to which open-market purchases of various kinds of assets by the central bank can mitigate that constraint. We are also able to show how the character of optimal monetary policy changes as a result of the existence of the zero bound, relative to the policy rules that would be judged optimal in the absence of such a bound, or in the case of real disturbances small enough for the bound never to matter under an optimal policy.

To preview our results, we find that the zero bound does represent an important constraint on what monetary stabilization policy can achieve, at least when certain kinds of real disturbances are encountered in an environment of low inflation. The possibility of expansion of the monetary base through central-bank purchases of a variety of types of assets does not do anything to expand the set of feasible equilibrium paths for inflation and real activity that are consistent with equilibrium under some policy. Hence the relevant tradeoffs can correctly be studied by simply considering what can be achieved by alternative anticipated state-contingent paths of the short-term nominal interest rate, taking into account the constraint that this quantity must be non-negative at all times.

Nonetheless, we argue that the extent to which this constraint restricts possible stabilization outcomes under sound policy is much more minimal than the deflation pessimists presume. Even though the set of feasible equilibrium outcomes corresponds to those that can be achieved through alternative interest-rate policies, monetary policy is far from powerless to mitigate the contractionary effects of the kind of disturbances that would make the zero bound a binding constraint. The key to dealing with this sort of situation in the least damaging way is to create the right kind of expectations regarding the way in which monetary policy will be used *subsequently*, at a time when the central bank again has room to maneuver. We use our intertemporal equilibrium model to characterize the kind of expectations regarding future policy that it would be desirable to create, and discuss a form of price-level targeting rule that — if credibly committed to by the central bank — should bring about the constrained-optimal equilibrium. We also discuss, more informally, ways in which other types of policy actions could help to increase the credibility of the central bank's announced commitment to this kind of future policy.

Our analysis will be recognized as a development of several key themes of Paul Krugman's (1998) treatment of the same topic in these pages a few years ago. Like Krugman, we give particular emphasis to the role of expectations regarding future policy in determining the severity of the distortions that result from hitting the zero bound. Our primary contribution, relative to Krugman's earlier treatment, will be the presentation of a more fully dynamic analysis. For example, our assumption of staggered pricing, rather than the simple hypothesis of prices that are fixed for one period as in the analysis of Krugman, allows for richer (and at least somewhat more realistic) dynamic responses to disturbances. In our model, unlike Krugman's, a real disturbance that lowers the natural rate of interest can cause output to remain below potential for years (as shown in Figure 2 below), rather than only for a single "period", even when the average frequency of price adjustments is more than once per year. These richer dynamics are also important for a realistic discussion of the kind of policy commitment that can help to reduce economic contraction during a "liquidity trap". In our model, a commitment to create subsequent inflation involves a commitment to *keep interest rates low* for a time in the future, whereas in Krugman's model, a commitment to a higher future price level does not involve any reduction in future nominal interest rates. We are also better able to discuss questions such as how the creation of inflationary expectations during the period that the zero bound is binding can be reconciled with maintaining the credibility of the central bank's commitment to long-run price stability.

1 Is "Quantitative Easing" a Separate Policy Instrument?

A first question that we wish to consider is whether expansion of the monetary base represents a policy instrument that should be effective in preventing deflation and associated output declines, even under circumstances where overnight interest rates have fallen to zero. According to the famous analysis of Keynes (1936), monetary policy ceases to be an effective instrument to head off economic contraction in a "liquidity trap," that can arise if interest rates reach a level so low that further expansion of the money supply cannot drive them lower. Others have argued that monetary expansion should increase nominal aggregate demand even under such circumstances, and the supposition that this is correct lies behind the explicit adoption in Japan since March 2001 of a policy of "quantitative easing" in addition to the "zero interest-rate policy" that continues to be maintained.[1]

Here we consider this question in the context of an explicit intertemporal equilibrium model, in which we model both the demand for money and the role of financial assets (including the monetary base) in private-sector budget constraints. The model that we use for this purpose is more detailed in several senses than the one used in subsequent sections to characterize optimal policy, in order to make it clear that we have not excluded a role for "quantitative easing" simply by *failing to model* the role of money in the economy. The model is discussed in more detail in Woodford (2003, chapter 4), where the consequences of various interest-rate rules and money-growth rules are considered under the assumption that disturbances are not large enough for the zero bound to bind.

Our model abstracts from endogenous variations in the capital stock, and assumes perfectly flexible wages (or some other mechanism for efficient labor contracting), but assumes monopolistic competition in goods markets, and sticky prices that are adjusted at random intervals in the way assumed by Calvo (1983), so that deflation has real effects. We assume a model in which the representative household seeks to maximize a utility function of the form

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[u(C_t, M_t/P_t; \xi_t) - \int_0^1 v(H_t(j); \xi_t) dj \right],$$

where C_t is a Dixit-Stiglitz aggregate of consumption of each of a continuum of differentiated

[1] See Kimura *et al.* (2002) for discussion of this policy, as well as an expression of doubts about its effectiveness.

goods,

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta}{\theta-1}} di \right]^{\frac{\theta-1}{\theta}},$$

with an elasticity of substitution equal to $\theta > 1$, M_t measures end-of-period household money balances,[2] P_t is the Dixit-Stiglitz price index,

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \tag{1.1}$$

and $H_t(j)$ is the quantity supplied of labor of type j. Real balances are included in the utility function, following Sidrauski (1967) and Brock (1974, 1975), as a proxy for the services that money balances provide in facilitating transactions.[3]

For each value of the disturbances ξ_t, $u(\cdot, \cdot; \xi_t)$ is concave function, increasing in the first argument, and increasing in the second for all levels of real balances up to a satiation level $\bar{m}(C_t; \xi_t)$. The existence of a satiation level is necessary in order for it to be *possible* for the zero interest-rate bound ever to be reached; we regard Japan's experience over the past several years as having settled the theoretical debate over whether such a level of real balances exists. Unlike many papers in the literature, we do not assume additive separability of the function u between the first two arguments; this (realistic) complication allows a further channel through which money can affect aggregate demand, namely an effect of real money balances on the current marginal utility of consumption. Similarly, for each value of ξ_t, $v(\cdot; \xi_t)$ is an increasing convex function. The vector of exogenous disturbances ξ_t may contain several elements, so that no assumption is made about correlation of the exogenous shifts in the functions u and v.

For simplicity we shall assume complete financial markets and no limits on borrowing against future income. As a consequence, a household faces an intertemporal budget con-

[2] We shall not introduce fractional-reserve banking into our model. Technically, M_t refers to the monetary base, and we represent households as obtaining liquidity services from holding this base, either directly or through intermediaries (not modelled).

[3] We use this approach to modelling the transactions demand for money because of its familiarity. As shown in Woodford (2003, appendix section A.16), a cash-in-advance model leads to equilibrium conditions of essentially the same general form, and the neutrality result that we present below would hold in essentially identical form were we to model the transactions demand for money after the fashion of Lucas and Stokey (1987).

straint of the form

$$E_t \sum_{T=t}^{\infty} Q_{t,T}[P_T C_T + \delta_T M_T] \leq W_t + E_t \sum_{T=t}^{\infty} Q_{t,T} \left[\int_0^1 \Pi_T(i) di + \int_0^1 w_T(j) H_T(j) dj - T_T \right]$$

looking forward from any period t. Here $Q_{t,T}$ is the stochastic discount factor by which the financial markets value random nominal income at date T in monetary units at date t, δ_t is the opportunity cost of holding money (equal to $i_t/(1+i_t)$, where i_t is the riskless nominal interest rate on one-period obligations purchased in period t, in the case that no interest is paid on the monetary base), W_t is the nominal value of the household's financial wealth (including money holdings) at the beginning of period t, $\Pi_t(i)$ represents the nominal profits (revenues in excess of the wage bill) in period t of the supplier of good i, $w_t(j)$ is the nominal wage earned by labor of type j in period t, and T_t represents the net nominal tax liabilities of each household in period t.

Optimizing household behavior then implies the following necessary conditions for a rational-expectations equilibrium. Optimal timing of household expenditure requires that aggregate demand Y_t for the composite good[4] satisfy an Euler equation of the form

$$u_c(Y_t, M_t/P_t; \xi_t) = \beta E_t \left[u_c(Y_{t+1}, M_{t+1}/P_{t+1}; \xi_{t+1})(1+i_t) \frac{P_t}{P_{t+1}} \right], \tag{1.2}$$

where i_t is the riskless nominal interest rate on one-period obligations purchased in period t.

Optimal substitution between real money balances and expenditure leads to a static first-order condition of the form

$$\frac{u_m(Y_t, M_t/P_t; \xi_t)}{u_c(Y_t, M_t/P_t; \xi_t)} = \frac{i_t}{1+i_t},$$

under the assumption that zero interest is paid on the monetary base, and that preferences are such that we can exclude the possibility of a corner solution with zero money balances. If both consumption and liquidity services are normal goods, this equilibrium condition can be solved uniquely for the level of real balances that satisfy it in the case of any positive

[4]For simplicity, we here abstract from government purchases of goods. Our equilibrium conditions directly extend to the case of exogenous government purchases, as shown in Woodford (2003, chap. 4).

nominal interest rate. The equilibrium relation can then equivalently be written as a pair of inequalities

$$\frac{M_t}{P_t} \geq L(Y_t, i_t; \xi_t), \tag{1.3}$$

$$i_t \geq 0, \tag{1.4}$$

together with the "complementary slackness" condition that at least one must hold with equality at any time. (Here we define $L(Y, 0; \xi) = \bar{m}(Y; \xi)$, the minimum level of real balances for which $u_m = 0$, so that the function L is continuous at $i = 0$.)

Household optimization similarly requires that the paths of aggregate real expenditure and the price index satisfy the bounds

$$\sum_{T=t}^{\infty} \beta^T E_t \left[u_c(Y_T, M_T/P_T; \xi_T)Y_T + u_m(Y_T, M_T/P_T; \xi_T)(M_T/P_T)\right] < \infty, \tag{1.5}$$

$$\lim_{T\to\infty} \beta^T E_t[u_c(Y_T, M_T/P_T; \xi_T)D_T/P_T] = 0 \tag{1.6}$$

looking forward from any period t, where D_t measures the total nominal value of government liabilities (monetary base plus government debt) at the end of period t. under the monetary-fiscal policy regime. (Condition (1.5) is required for the existence of a well-defined intertemporal budget constraint, under the assumption that there are no limitations on households' ability to borrow against future income, while the transversality condition (1.6) must hold if the household exhausts its intertemporal budget constraint.) Conditions (1.2) – (1.6) also suffice to imply that the representative household chooses optimal consumption and portfolio plans (including its planned holdings of money balances) given its income expectations and the prices (including financial asset prices) that it faces, while making choices that are consistent with financial market clearing.

Each differentiated good i is supplied by a single monopolistically competitive producer. There are assumed to be many goods in each of an infinite number of "industries"; the goods in each industry j are produced using a type of labor that is specific to that industry, and also change their prices at the same time. Each good is produced in accordance with a common production function

$$y_t(i) = A_t f(h_t(i)),$$

where A_t is an exogenous productivity factor common to all industries, and $h_t(i)$ is the industry-specific labor hired by firm i. The representative household supplies all types of labor as well as consuming all types of goods.[5]

The supplier of good i sets a price for that good at which it supplies demand each period, hiring the labor inputs necessary to meet any demand that may be realized. Given the allocation of demand across goods by of households in response to firm pricing decisions, on the one hand, and the terms on which optimizing households are willing to supply each type of labor on the other, we can show that the nominal profits (sales revenues in excess of labor costs) in period t of the supplier of good i are given by a function

$$\begin{aligned}\Pi(p_t(i), p_t^j, P_t; Y_t, M_t/P_t, \tilde{\xi}_t) \quad \equiv \quad & p_t(i)Y_t(p_t(i)/P_t)^{-\theta} \\ & -\frac{v_h(f^{-1}(Y_t(p_t^j/P_t)^{-\theta}/A_t);\xi_t)}{u_c(Y_t, M_t/P_t;\xi_t)} P_t f^{-1}(Y_t(p_t(i)/P_t)^{-\theta}/A_t),\end{aligned}$$

where p_t^j is the common price charged by the other firms in industry j.[6] (We introduce the notation $\tilde{\xi}_t$ for the complete vector of exogenous disturbances, including variations in technology as well as preferences.) If prices were fully flexible, $p_t(i)$ would be chosen each period to maximize this function.

Instead we suppose that prices remain fixed in monetary terms for a random period of time. Following Calvo (1983), we suppose that each industry has an equal probability of reconsidering its prices each period, and let $0 < \alpha < 1$ be the fraction of industries with prices that remain unchanged each period. In any industry that revises its prices in period t, the new price p_t^* will be the same. This price is implicitly defined by the first-order condition

$$E_t\left\{\sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi_1(p_t^*, p_t^*, P_T; Y_T, M_T/P_T, \tilde{\xi}_T)\right\} = 0. \tag{1.7}$$

We note furthermore that the stochastic discount factor used to price future profit streams

[5] We might alternatively assume specialization across households in the type of labor supplied; in the presence of perfect sharing of labor income risk across households, household decisions regarding consumption and labor supply would all be as assumed here.

[6] In equilibrium, all firms in an industry charge the same price at any time. But we must define profits for an individual supplier i in the case of contemplated deviations from the equilibrium price.

will be given by

$$Q_{t,T} = \beta^{T-t} \frac{u_c(C_T, M_T/P_T; \xi_T)}{u_c(C_t, M_t/P_t; \xi_t)}.$$

Finally, the definition (1.1) implies a law of motion for the aggregate price index of the form

$$P_t = \left[(1-\alpha)p_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{1.8}$$

Equations (1.7) – (1.8) jointly determine the evolution of prices given demand conditions, and represent the aggregate-supply block of our model.

It remains to specify the monetary and fiscal policies of the government. In order to address the question whether "quantitative easing" represents an additional tool of policy, we shall suppose that the central bank's operating target for the short-term nominal interest rate is determined by a feedback rule in the spirit of the Taylor rule (Taylor, 1993),

$$i_t = \phi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t), \tag{1.9}$$

where now $\tilde{\xi}_t$ may also include exogenous disturbances in addition to the ones listed above, to which the central bank happens to respond. We shall assume that the function ϕ is non-negative for all values of its arguments (otherwise the policy would not be feasible, given the zero lower bound), but that there are conditions under which the rule prescribes a zero interest-rate policy. Such a rule implies that the central bank supplies the quantity of base money that happens to be demanded at the interest rate given by this formula; hence (1.9) implies a path for the monetary base, in the case that the value of ϕ is positive. However, under those conditions in which the value of ϕ is zero, the policy commitment (1.9) implies only a lower bound on the monetary base that must be supplied. In these circumstances, we may ask whether it matters whether a greater or smaller quantity of base money is supplied.

We shall suppose that the central bank's policy in this regard is specified by a base-supply rule of the form

$$M_t = P_t L(Y_t, \phi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t); \xi_t)\psi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t), \tag{1.10}$$

where the multiplicative factor ψ satisfies

(i) $\psi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t) \geq 1$,

(ii) $\psi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t) = 1$ if $\phi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t) > 0$

for all values of its arguments. (Condition (ii) implies that $\psi = 1$ whenever $i_t > 0$.) Note that a base-supply rule of this form is consistent with both the interest-rate operating target specified in (1.9) and the equilibrium relations (1.3) – (1.4). The use of "quantitative easing" as a policy tool can then be represented by a choice of a function ψ that is greater than 1 under some circumstances.

We specify fiscal policy in terms of a rule that determines the evolution of total government liabilities D_t, as well as a rule that specifies the composition of non-monetary liabilities among different types of securities that might be issued by the government. We shall suppose that the evolution of total government liabilities is in accordance with a rule of the form

$$\frac{D_t}{P_t} = d\left(\frac{D_{t-1}}{P_{t-1}}, \frac{P_t}{P_{t-1}}, Y_t; \tilde{\xi}_t\right), \tag{1.11}$$

which specifies the acceptable level of real government liabilities as a function of the pre-existing level of real liabilities and various aspects of current macroeconomic conditions. This notation allows for such possibilities as an exogenously specified state-contingent target for real government liabilities as a proportion of GDP, or for the government budget deficit (inclusive of interest on the public debt) as a share of GDP, among others.

The part of total liabilities that consists of base money is specified by the base rule (1.10). We suppose, however, that the rest may be allocated among any of k different types of securities (of differing maturities, degrees of indexation, etc.). If ω_{jt} indicates the share of government debt (*i.e.,* non-monetary liabilities) at the end of period t that is of type j, and R_t is the vector of gross nominal returns on the securities of the k types between periods $t-1$ and t, then the flow government budget constraint takes the form

$$B_t = R_t'\omega_{t-1}B_{t-1} - T_t - (M_t - M_{t-1}),$$

where $B_t \equiv D_t - M_t$ is the total nominal value of end-of-period non-monetary liabilities, and T_t is the nominal value of the primary budget surplus (taxes net of transfers, if we abstract from government purchases). Net tax collections implied by a given rule (1.11) for aggregate public liabilities are then given by

$$T_t = R_t' \omega_{t-1}(D_{t-1} - M_{t-1}) + M_{t-1} - D_t,$$

which depends in general on the composition of the debt as well, indicated by the vector ω_{t-1}.

We suppose that debt management policy (*i.e.*, the determination of the composition of the government's non-monetary liabilities at each point in time) is specified by a function

$$\omega_t = \omega(P_t/P_{t-1}, Y_t; \tilde{\xi}_t), \tag{1.12}$$

specifying the shares as a function of aggregate conditions, where the vector-valued function ω has the property that its components sum to 1 for all possible values of its arguments. Note that we may allow for a possibility that a different debt management policy is pursued under the circumstances of a "liquidity trap"; for the arguments of the function ω include all of the arguments of ϕ, making it possible for ω to take a different value precisely under the circumstances in which the zero lower bound on nominal interest rates is binding. Together, the two relations (1.11) and (1.12) provide a complete specification of fiscal policy. A consideration of the way in which equilibrium may depend on the specifications of the functions ψ and ω allows us to consider both the consequences of "quantitative easing" and the related question of the extent to which it matters which kinds of assets the central bank may acquire from the public if it decides to expand the monetary base. (Purchases of different types of assets by the central bank correspond to different specifications of the function ω, as such actions change the composition of the securities portfolio left in the hands of the public.[7])

We may now define a rational-expectations equilibrium as a collection of stochastic processes $\{p_t^*, P_t, Y_t, i_t, M_t, D_t, \omega_t\}$, with each endogenous variable specified as a function of the history of exogenous disturbances to that date, that satisfy each of conditions (1.2) – (1.6)

[7] We might, of course, introduce separate notation for the composition of securities issued by the Treasury and those held by the central bank, but it should be evident that it is only the net supply of securities to the private sector — the securities issued by the Treasury that are *not* held by the central bank — that can matter for equilibrium determination.

of the aggregate-demand block of the model, conditions (1.7) – (1.8) of the aggregate-supply block, conditions (1.9) – (1.10) specifying monetary policy, and conditions (1.11) – (1.12) specifying fiscal policy each period. We then obtain the following irrelevance result for the specification of certain aspects of policy.

Proposition. The set of paths for the variables $\{p_t^*, P_t, Y_t, i_t, D_t\}$ that are consistent with the existence of a rational-expectations equilibrium are independent of the specification of the functions ψ in equation (1.10) and ω in equation (1.12).

The reason for this is fairly simple. The set of restrictions on the processes $\{p_t^*, P_t, Y_t, i_t, D_t\}$ implied by our model can be written in a form that does not involve the variables $\{M_t, \omega_t\}$, and hence that does not involve the functions ψ or ω.

To show this, let us first note that for all $m \geq \bar{m}(C;\xi)$,

$$u(C, m; \xi) = u(C, \bar{m}(C;\xi); \xi),$$

as additional money balances beyond the satiation level provide no further liquidity services. By differentiating this relation, we see further that $u_c(C, m; \xi)$ does not depend on the exact value of m either, as long as m exceeds the satiation level. It follows that in our equilibrium relations, we can replace the expression $u_c(Y_t, M_t/P_t; \xi_t)$ by

$$u_c(Y_t, L(Y_t, \phi(P_t/P_{t-1}, Y_t; \xi_t); \xi_t); \xi_t),$$

using the fact that (1.3) holds with equality at all levels of real balances at which u_c depends on the level of real balances. Hence we can write u_c as a function of variables other than M_t/P_t, without using the relation (1.10), and so in a way that is independent of the function ψ.

We can similarly replace the expression $u_m(Y_t, M_t/P_t; \xi_t)(M_t/P_t)$ that appears in (1.5) by

$$u_m(Y_t, L(Y_t, \phi(P_t/P_{t-1}, Y_t; \xi_t); \xi_t); \xi_t)L(Y_t, \phi(P_t/P_{t-1}, Y_t; \xi_t); \xi_t),$$

since M_t/P_t must equal $L(Y_t, \phi(P_t/P_{t-1}, Y_t; \xi_t); \xi_t)$ when real balances do not exceed the satiation level, while $u_m = 0$ when they do. Using these two substitutions, we can write each of the equilibrium relations (1.2), (1.5), (1.6), and (1.7) in a way that no longer makes reference to the money supply.

We then have a system of requirements that the variables $\{p_t^*, P_t, Y_t, i_t, D_t\}$ must satisfy each period, consisting of equations (1.2), (1.4) – (1.6), (1.7) – (1.8), (1.9), and (1.11). None of these equations involve the variables $\{M_t, \omega_t\}$, nor do they involve the functions ψ or ω. Furthermore, this is the complete set of restrictions on these variables that are required in order for them to be consistent with a rational-expectations equilibrium. For given any processes $\{p_t^*, P_t, Y_t, i_t, D_t\}$ that satisfy the equations just listed in each period, the implied path of the money supply is given by (1.10), which clearly has a solution; and this path for the money supply necessarily satisfies (1.3) and the complementary slackness condition, as a result of our assumptions about the form of the function ψ. Similarly, the implied composition of the public debt at each point in time is given by (1.12). We then have a set of processes that satisfies all of the requirements for a rational-expectations equilibrium, and the result is established.

This proposition implies that neither the extent to which quantitative easing is employed when the zero bound binds, nor the nature of the assets that the central bank may purchase through open-market operations, has any effect on whether a deflationary price-level path will represent a rational-expectations equilibrium. Hence the notion that expansions of the monetary base represent an additional tool of policy, independent of the specification of the rule for adjusting short-term nominal interest rates, is not supported by our general-equilibrium analysis of inflation and output determination.

It is, of course, important to note that our irrelevance proposition depends on an assumption that interest-rate policy is specified in a way that implies that these open-market operations have no consequences for interest-rate policy, either immediately (which is trivial, since it would not be possible for them to *lower* current interest rates, which is the only effect that would be desired) *or at any subsequent date* either. We have also specified fiscal policy in a way that implies that the contemplated open-market operations have no effect on the evolution of total government liabilities $\{D_t\}$ either — again, neither immediately nor at any later date. While we think that these definitions make sense, as a way of isolating the pure effects of open-market purchases of assets by the central bank from either interest-rate policy on the one hand and from fiscal policy on the other, it is important to note that someone who recommends monetary expansion by the central bank may intend for this to have consequences of one or both of these other sorts.

For example, when it is argued that surely nominal aggregate demand could be stimulated by a "helicopter drop of money", the thought experiment that is usually contemplated is not simply a change in the function ψ in our policy rule (1.10). First of all, it is typically supposed that the expansion of the money supply will be *permanent*. If this is the case, then the function ϕ that defines interest-rate policy is also being changed, in a way that will become relevant at some future date, when the money supply no longer exceeds the satiation level.[8] Second, the assumption that the money supply is increased through a "helicopter drop" rather than an open-market operation implies a change in *fiscal* policy as well. The operation increases the value of nominal government liabilities, and it is generally at least tacitly assumed that this is a permanent increase as well. Hence the experiment that is imagined is not one that our irrelevance proposition implies should have no effect on the equilibrium path of prices.

It is sometimes argued that central-bank purchases of longer-term bonds should surely be able to affect the economy, even if open-market purchases of short-term Treasury bills will not, in the case that longer-term bond yields remain well above zero.[9] The idea is that as long as any bond yields remain positive, it should be possible to drive them lower

[8] This explains the apparent difference between our result and the one obtained by Auerbach and Obstfeld (2003) in a similar model. These authors assume explicitly that an increase in the money supply while the zero bound binds carries with it the implication of a permanently higher money supply, and also that there exists a future date at which the zero bound ceases to bind, so that the higher money supply will imply a different interest-rate policy at that later date.

[9] Cecchetti (2003) is one of many examples of commentators who argue that this channel for the effectiveness of Fed policy would remain available even if the federal funds rate were to reach zero in the U.S.

through aggressive open-market purchases; and lowering long-term bond yields should stimulate spending. In the model that we have presented (with its stipulation regarding future monetary policy), this would not occur, no matter how large the open-market purchases, because of the indifference of private parties as to the composition of their portfolios, as long as all assets are correctly priced (*i.e.*, in a way consistent with the stochastic discount factor implied by the marginal rate of substitution of the representative household between consumption at different dates and in different states of the world). Of course, one could imagine a situation in which long-term Treasury securities would simply cease to be held by private parties, owing to the willingness of the central bank to purchase them at an above-market price. But this would do nothing to stimulate expenditure, in the absence of any change in private-sector expectations about the way future monetary policy would be conducted. Private expenditure would still depend on the perceived relative price at which current and future income could be traded off by the private sector — what one might call the "shadow long bond rate" — even if there were an official price of long-term government bonds at which no private parties were willing to hold them.

The key to lowering long-term interest rates, in a way that would actually provide an incentive for increased spending, would be by changing expectations regarding the likely future path of short rates. As a logical matter, this need not require any open-market purchases of long-term bonds at all.[10] On the other hand, such purchases could help to stimulate demand if they helped to change private-sector expectations regarding future (short-term) interest-rate policy. In our analysis above, there is no question of such an effect — not only is future policy specified by a rule (1.9) which does not allow the prior open-market purchases to have any effect, but this rule is treated as being fully understood by the private sector. In practice, the management of private-sector expectations is an art of considerable subtlety, and shifts in the portfolio of the central bank could be of some value in making credible to

[10] And in fact, most central banks now accept the principle that it is best to target only the shortest-term interest rates, typically only an overnight rate, allowing long rates to be determined by arbitrage considerations in financial markets, even if they often pay attention to long bond rates as an indicator of the degree to which their policy is affecting private-sector expectations in the desired way.

the private sector the central bank's own commitment to a particular kind of future policy, as we discuss further in section 6. "Signalling" effects of this kind are often argued to be an important reason for the effectiveness of interventions in foreign-exchange markets, and might well provide a justification for open-market policy when the zero bound binds.

We do not wish, then, to argue that asset purchases by the central bank are necessarily pointless under the circumstances of a binding zero lower bound on short-term nominal interest rates. However, we do think it important to observe that insofar as such actions can have any effect, it is not because of any necessary or mechanical consequence of the shift in the portfolio of assets in the hands of the private sector itself. Instead, any effect of such actions must be due to the way in which they change expectations regarding future interest-rate policy, or, perhaps, the future evolution of total nominal government liabilities. In sections 6 and 7 we discuss reasons why open-market purchases by the central bank might plausibly have consequences for expectations of these types. But since it is only through effects on expectations regarding future policy that these actions can matter, we shall focus our attention on the question of what kind of commitments regarding future policy are in fact to be desired. And this question can be addressed without explicit consideration of the role of open-market operations by the central bank of any kind. Hence we shall simplify our model — abstracting from monetary frictions and the structure of government liabilities altogether — and instead consider how it is desirable for interest-rate policy to be conducted, and what kind of commitments about this policy it is desirable to make in advance.

2 How Severe a Constraint is the Zero Bound?

We turn now to the question of the way in which the existence of the zero bound restricts the degree to which a central bank's stabilization objectives, with regard to both inflation and real activity, can be achieved, even under ideal policy. It follows from our discussion in the previous section that the zero bound does represent a genuine constraint. The differences among alternative policies that are relevant to the degree to which stabilization objectives are achieved having only to do with the implied evolution of short-term nominal interest rates, and the zero bound obviously constrains the ways in which this instrument can be used, though it remains to be seen how relevant this constraint may be.

Nonetheless, we shall see that it is not at all the case that there is nothing that a central bank can do to mitigate the severity of the destabilizing impact of the zero bound. The reason is that inflation and output do not depend solely upon the current level of short-term nominal interest rates, or even solely upon the history of such rates up until the current time (so that the current level of interest rates would be the only thing that could possibly changed in response to an unanticipated disturbance). The expected character of *future* interest-rate policy is also a critical determinant of the degree to which the central bank achieves its stabilization objectives, and this allows an important degree of scope for policy to be improved upon, even when there is little choice about the current level of short-term interest rates.

In fact, the management of expectations is the key to successful monetary policy at *all* times, and not just in those relatively unusual circumstances when the zero bound is reached. The effectiveness of monetary policy has little to do with the direct effect of changing the level of overnight interest rates, since the current cost of maintaining cash balances overnight is of fairly trivial significance for most business decisions. What actually matters is the private sector's anticipation of the *future* path of short rates, as this determines equilibrium long-term interest rates, as well as equilibrium exchange rates and other asset prices — all of which are quite relevant for many current spending decisions, hence for optimal pricing behavior as well. The way in which short rates are managed matters because of the signals that it gives about the way in which the private sector can expect them to be managed in the future. But there is no reason to suppose that expectations regarding future monetary policy, and hence expectations regarding the future evolution of nominal variables more generally, should change only insofar as the current level of overnight interest rates changes. A situation in which there is no decision to be made about the current level of overnight rates (as in Japan at present) is one which brings the question of what expectations regarding future policy one should wish to create more urgently to the fore, but this is in fact the correct way to think about sound monetary policy at all times.

Of course, there is no question to be faced about what future policy one should wish for people to expect if there is no possibility of committing oneself to a different sort of policy in the future than one would otherwise have pursued, as a result of the constraints that are currently faced (and that make desirable the change in expectations). This means that the private sector must be convinced that the central bank will not conduct policy in a way that is *purely forward-looking, i.e.,* taking account at each point in time only of the possible paths that the economy could follow from that date onward. For example, we will show that it is undesirable for the central bank to pursue a certain inflation target, once the zero bound is expected no longer to prevent it from being achieved, even in the case that the pursuit of this target *would* be optimal if the zero bound did not exist (or would never bind under an optimal policy). The reason is that an expectation that the central bank will pursue the fixed inflation target after the zero bound ceases to bind gives people no reason to hold the kind of expectations, while the bound is binding, that would mitigate the distortions created by it. A history-dependent inflation target[11] — if the central bank's commitment to it can be made credible — can instead yield a superior outcome.

But this too is an important feature of optimal policy rules more generally (see, *e.g.,* Woodford, 2003, chapter 7). Hence the analytical framework and institutional arrangements used to make monetary policy need not be changed in any fundamental way in order to deal with the special problems created by a "liquidity trap". As we explain in section 4, the optimal policy in the case of a binding zero bound can be implemented through a targeting procedure that represents a straightforward generalization of a policy that would be optimal even if the zero bound were expected never to bind.

[11] As we shall see, it is easier to explain the nature of the optimal commitment if it is described as a history-dependent price-level target.

2.1 Feasible Responses to Fluctuation in the Natural Rate of Interest

In order to characterize the way in which stabilization policy is constrained by the zero bound, we shall make use of a log-linear approximation to the structural equations of section 2, of a kind that is often employed in the literature on optimal monetary stabilization policy (see, *e.g.,* Clarida *et al.,* 1999; Woodford, 2003). Specifically, we shall log-linearize the structural equations of our model (except for the zero bound (1.4)) around the paths of inflation, output and interest rates associated with a zero-inflation steady state, in the absence of disturbances ($\xi_t = 0$). We choose to expand around these particular paths because the zero-inflation steady state represents optimal policy in the absence of disturbances. In the event of small enough disturbances, optimal policy will still involve paths in which inflation, output and interest rates are at all times close to those of the zero-inflation steady state. Hence an approximation to our equilibrium conditions that is accurate in the case of inflation, output and interest rates near those values will allow an accurate approximation to the optimal responses to disturbances in the case that the disturbances are small enough.

In the zero-inflation steady state, it is easily seen that the real rate of interest is equal to $\bar{r} \equiv \beta^{-1} - 1 > 0$, and this is also the steady-state nominal interest rate. Hence in the case of small enough disturbances, optimal policy will involve a nominal interest rate that is always positive, and the zero bound will not be a binding constraint. (Optimal policy in this case is characterized in the references cited in the previous paragraph.) However, we are interested in the case in which disturbances are at least occasionally large enough for the zero bound to bind, *i.e.,* for it to prevent attainment of the outcome that would be optimal in the absence of such a bound. A case in which it is possible to rigorously consider this problem using only a log-linear approximation to the structural equations is that in which we suppose that the lower bound on nominal interest is not much below $\bar{r}$. We can arrange for this gap to be as small as we may wish, without changing other crucial parameters of the model such as the assumed rate of time preference, by supposing that interest is paid on the monetary base at a rate $i^m \geq 0$ that cannot (for some institutional reason) be reduced. Then the lower bound on interest rates actually becomes

$$i_t \geq i^m \tag{2.1}$$

We shall characterize optimal policy subject to a constraint of the form (2.1), in the case that *both* a bound on the amplitude of disturbances $||\xi||$ and the size of the steady-state opportunity cost of holding money $\bar{\delta} \equiv (\bar{r} - i^m)/(1 + \bar{r}) > 0$ are small enough. Specifically, both our structural equations and our characterization of the optimal responses of inflation, output and interest rates to disturbances will be required to be exact only up to a residual of order $\mathcal{O}(||\xi, \bar{\delta}||^2)$. We shall then hope (without here seeking to verify this) that our characterization of optimal policy in the case of a small opportunity cost of holding money and small disturbances is not too inaccurate in the case of an opportunity cost of several percentage points (the case in which $i^m = 0$) and disturbances large enough to cause the natural rate of interest to vary by several percentage points (as will be required in order for the zero bound to bind).

As shown in Woodford (2003), the log-linear approximate equilibrium relations may be summarized by two equations each period, a forward-looking "IS relation"

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n), \tag{2.2}$$

and a forward-looking "AS relation" (or "New Keynesian Phillips curve")

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t. \tag{2.3}$$

Here $\pi_t \equiv \log(P_t/P_{t-1})$ is the inflation rate, x_t is a welfare-relevant output gap, and i_t is now the continuously compounded nominal interest rate (corresponding to $\log(1 + i_t)$ in the notation of section 2). The terms u_t and r_t^n are composite exogenous disturbance terms that shift the two equations; the former is commonly referred to as a "cost-push disturbance", while the latter indicates exogenous variation in the Wicksellian "natural rate of interest", *i.e.*, the equilibrium real rate of interest in the case that output is at all times equal to the natural rate of output. The coefficients σ and κ are both positive, while $0 < \beta < 1$ is again the utility discount factor of the representative household.

Equation (2.2) is a log-linear approximation to (1.2), while (2.3) is derived by log-linearizing (1.7) – (1.8) and then eliminating $\log(p^*t/P_t)$. We omit the log-linear version of the money-demand relation (1.3), since we are here interested solely in characterizing the possible equilibrium paths of inflation, output, and interest rates, and we may abstract from the question of what the required path for the monetary base may be that is associated with any such equilibrium in considering this. (It suffices that there exist a monetary base that will satisfy the money-demand relation in each case, and this will be true as long as the interest-rate bound is satisfied.) The other equilibrium requirements of section 2 can be ignored in the case that we are interested only in possible equilibria that remain forever near the zero-inflation steady state, as they are automatically satisfied in that case.

Equations (2.2) – (2.3) represent a pair of equations each period to determine inflation and the output gap, given the central bank's interest-rate policy. We shall seek to compare alternative possible paths for inflation, the output gap, and the nominal interest rate that satisfy these two log-linear equations together with the inequality (2.1). Note that our conclusions will be identical (up to a scale factor) in the event that we multiply the amplitude of the disturbances and the steady-state opportunity cost $\bar{\delta}$ by any common factor; alternatively, if we measure the amplitude of disturbances in units of $\bar{\delta}$, our results will be independent of the value of $\bar{\delta}$ (to the extent that our log-linear approximation remains valid). Hence we choose the normalization $\bar{\delta} = 1 - \beta$, corresponding to $i^m = 0$, to simplify the presentation of our results. In the case, the lower bound for the nominal interest rate is again given by (1.4).

2.2 Deflation under Forward-Looking Policy

We begin by considering the degree to which the zero bound impedes the achievement of the central bank's stabilization objectives in the case that the bank pursues a strict inflation target. We interpret this as a commitment to adjust the nominal interest rate so that

$$\pi_t = \pi^* \tag{2.4}$$

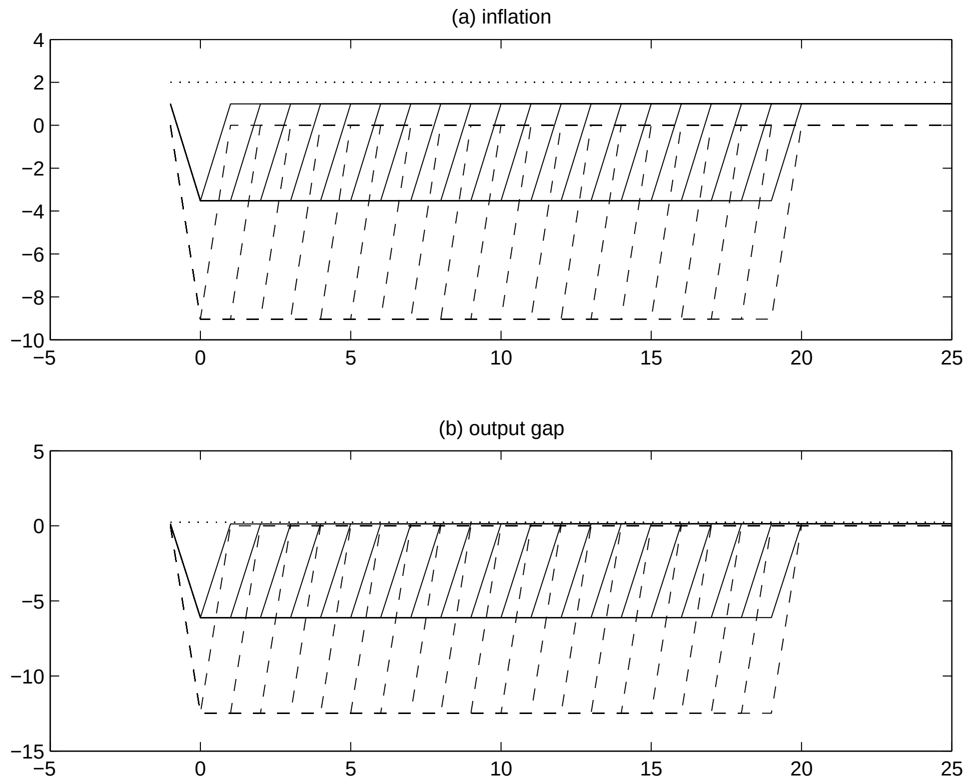

Figure 2: Dynamics of inflation and the output gap under strict inflation targeting, for three alternative inflation targets.

each period, insofar as it is possible to achieve this with some non-negative interest rate. It is easy to verify, by the IS and AS equation, that a necessary condition for this target to be satisfied is:

$$i_t = r_t^n + \pi^* \tag{2.5}$$

When inflation is on target, the real rate is equal to the natural real rate at all times and the output gap at its long run level. The zero bound, however, prevents (2.5) from holding if $r_t^n < -\pi^*$. Thus if the natural rate of interest is low, the zero bound frustrates the Central Bank's ability to implement an inflation target. Suppose the inflation target is zero so that $\pi^* = 0$. Then the zero bound is binding if the natural rate of interest is negative, and the Central Bank is unable to achieve its inflation target.

To illustrate this, let us consider the following experiment: Suppose the natural rate of interest is unexpectedly negative in period 0 and reverts back to the steady-state value $\bar{r} > 0$

with a fixed probability in every period. Figure 2 shows the state-contingent paths of the output gap and inflation in the case of three different possible inflation targets π^*. In the figure we assume in period 0 that the natural rate of interest becomes -2 percent per annum and then reverts back to the steady-state value of +4 percent per annum with a probability 0.1 each quarter. Thus the natural rate of interest is expected to be negative for 10 quarters on average at the time that the shock occurs.

The dashed lines in Figure 2 show the state-contingent evolution of the output gap and inflation if the central bank targets zero inflation.[12] The first dashed line shows the equilibrium if the natural rate of interest returns back to steady state in period 1, the next line if it returns in period 2, and so on. The inability of the central bank to set a negative nominal interest rate results in a 12 percent per output gap and 9 percent annual deflation. Since there is a 90 percent chance of the natural rate of interest to remain negative for the next quarter, this creates expectation of future deflation and negative output gap which creates even further deflation. Even if the central bank lowers the short-term nominal interest rate to zero the real rate of return is positive because the private sector expects deflation. The solid line in the figure shows the equilibrium if the central bank targets a one percent inflation target. In this case the private sector expect one percent inflation once out of the trap. This, however, is not enough to offset the minus two percent negative natural rate of interest, so that in equilibrium the private sector expect deflation instead of inflation. The result of this and a negative natural rate of interest is 3 percent annual deflation (when the natural rate of interest is negative) and an output gap of more than 5 percent.

Finally the dotted line shows the evolution of output and inflation if the central bank targets 2 percent inflation. In this case the central bank can satisfy equation (3.14) even

[12] In our numerical analysis, we interpret periods as quarters, and assume coefficient values of $\sigma = 0.5$, $\kappa = 0.02$, and $\beta = 0.99$. The assumed value of the discount factor implies a long-run real rate of interest of $\bar{r}$ equal to four percent per annum, as noted in the text. The assumed value of κ is consistent with the empirical estimate of Rotemberg and Woodford (1997). The assumed value of σ represents a relatively low degree of interest-sensitivity of aggregate expenditure. We prefer to bias our assumptions in the direction of only a modest effect of interest rates on the timing of expenditure, so as not to exaggerate the size of the output contraction that is predicted to result from an inability to lower interest rates when the zero bound binds. As Figure 2 shows, even for this value of σ, the output contraction that results from a slightly negative value of the natural rate of interest is quite substantial.

when the natural rate of interest in negative. When the natural rate of interest is minus two percent, the central bank lowers the nominal interest rate to zero. Since the inflation target is two percent, the real rate is minus two percent, which is enough to close the output gap and keep inflation on target. If the inflation target is high enough, therefore, the central bank is able to accommodate a negative natural rate of interest. This is the argument given by Summers (1991) for a positive inflation target. Krugman (1998) makes a similar argument, and suggests more concretely that Japan needs a positive inflation target of 4 percent to achieve negative real rates and curb deflation.

While we see that commitment to a higher inflation target will indeed guard against the need for a negative output gap in periods when the natural rate of interest falls, the price of this solution is the distortions created by the inflation, both when the natural rate of interest is negative and under more normal circumstances as well. Hence the optimal inflation target (from among the strict inflation targeting policies just considered) will be some value that is at least slightly positive, in order to mitigate the distortions created by the zero bound when the natural rate of interest is negative, but not so high as to keep the zero bound from ever binding (see Table 1). In the case of an intermediate inflation target, however (like the one percent target considered in the figure), there is *both* a substantial recession when the natural rate of interest becomes negative, and chronic inflation at all other times. Hence no such policy allows a complete solution of the problem posed by the zero bound in the case that the natural rate of interest is sometimes negative.

Nor can one do better through commitment to any policy rule that is *purely forward-looking* in the sense discussed by Woodford (2000). A purely forward-looking policy is one under which the central bank's action at any time depends only on an evaluation of the possible paths for the central bank's target variables (here, inflation and the output gap) that are possible from the current date forward — neglecting past conditions except insofar as they constrain the economy's possible evolution from here on. In the log-linear model presented above, the possible paths for inflation and the output gap from period t onward depend only on the expected evolution of the natural rate of interest from period t onward. If we assume a Markovian process for the natural rate, as in the numerical analysis above, then any purely forward-looking policy will result in an inflation rate, output gap, and nominal interest rate in period t that depend only on the natural rate in period t — in our numerical example, on whether the natural rate is still negative or has already returned to its long-run steady-state value. It is easily shown in the case of our 2-state example that the optimal state-contingent evolution for inflation and output from among those with this property will be one in which the zero bound binds if and only if the natural rate is in the low state; hence it will correspond to a strict inflation target of the kind just considered, for some π^* between zero and two percent.

But one can actually do considerably better, through commitment to a *history-dependent* policy, in which the central bank's actions will depend on past conditions even though these are irrelevant to the degree to which its stabilization goals could in principle be achieved from then on. We characterize the optimal form of history-dependent policy, and determine the degree to which it improves upon the stabilization of both output and inflation, in the next section.

2.3 The Optimal Policy Commitment

We now characterize optimal monetary policy. We assume that the government minimizes:

$$\min E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2) \right\} \tag{2.6}$$

This loss function can be derived by a second order Taylor expansion of the utility of the representative household. The optimal program can be found by a Lagrangian method, extending the methods used in Clarida *et al.* (1999) and Woodford (1999; 2003, chapter 7) to the case in which the zero bound can sometimes bind, as shown by Jung *et al.* (2001). Let us combine the zero bound and the IS equation to yield the inequality:

$$x_t \leq E_t x_{t+1} + \sigma(r_t^n + E_t \pi_{t+1})$$

The Lagrangian for this problem is then:

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2}[\pi_t^2 + \lambda x_t^2] + \phi_{1t}[x_t - x_{t+1} - \sigma\pi_{t+1} - \sigma r_t^n] + \phi_{2t}[\pi_t - \kappa x_t - \beta\pi_{t+1}] \right\}$$

The first order conditions for an optimal policy commitment are shown by Jung *et al.* to be:

$$\pi_t + \phi_{2t} - \phi_{2t-1} - \beta^{-1}\sigma\phi_{1t-1} = 0 \tag{2.7}$$

$$\lambda x_t + \phi_{1t} - \beta^{-1}\phi_{1t-1} - k\phi_{2t} = 0 \tag{2.8}$$

$$\phi_{1t} \geq 0, \quad i_t \geq 0, \quad \phi_{1t}i_t = 0 \tag{2.9}$$

One can not apply standard solution methods for rational expectation models to solve this system due to the complications of the nonlinear constraint (2.9). The numerical method that we use to solve these equations is described in the appendix.[13] Here we discuss the results that we obtain for the particular numerical experiment considered in the previous section.

What is apparent from the first order conditions (2.7)-(2.8) is that optimal policy is history dependent, so that the optimal choice of inflation, the output gap and the nominal interest rates depends on the past values of the endogenous variables. This can be seen by the appearance of lagged value of the Lagrange multipliers in the first order conditions. To get a sense of how this history dependence matters, it is useful to consider the numerical exercise from the last section: Suppose the natural rate of interest becomes negative in period 0 and then reverts back to steady state with a fixed probability in each period.

Figure 3 shows the optimal output gap, inflation and the price level from period 0 to period 25. One observes that the optimal policy involves committing to the creation of an output boom once the natural rate again becomes positive, and hence to the creation of future inflation. Such a commitment stimulates aggregate demand and reduces deflationary pressures while the economy remains in the "liquidity trap", through each of several channels. As Krugman (1998) points out, creating the expectation of future inflation can lower real interest rates, even when the nominal interest rate cannot be reduced. In the context of Krugman's model, it might seem that this requires that inflation be promised quite quickly

[13] Jung *et al.* (2001) discuss the solution of these equations only for the case in which the number of periods for which the natural rate of interest will be negative is known with certainty at the time that the disturbance occurs. Here we show how the system can be solved in the case of a stochastic process for the natural rate of a particular kind.

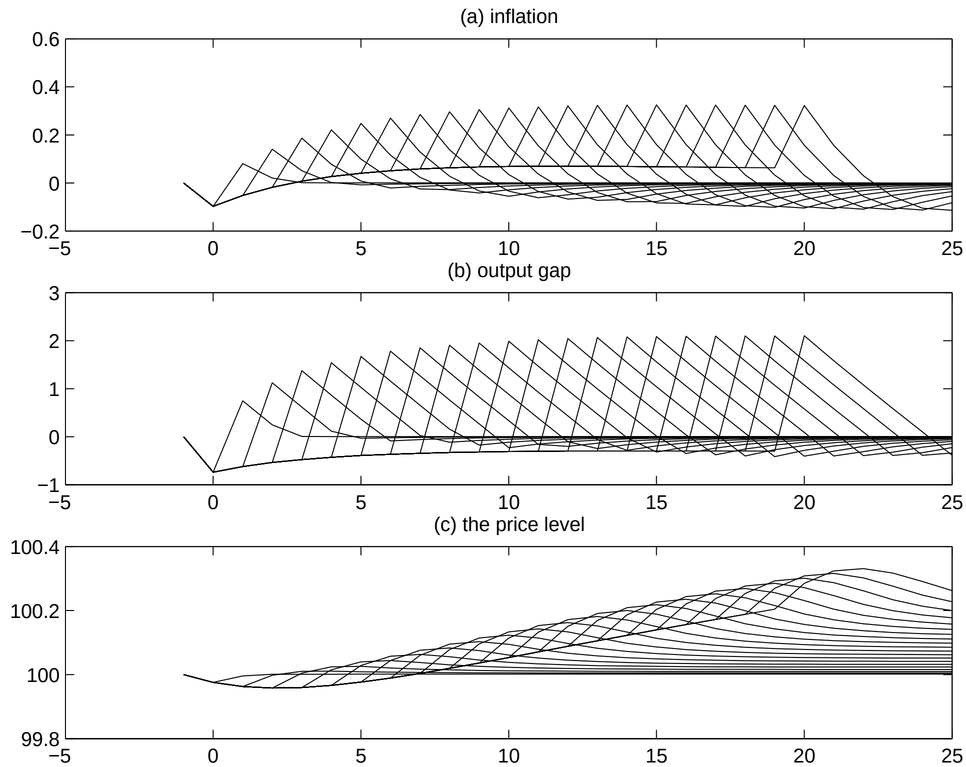

Figure 3: Dynamics of the output gap and inflation under an optimal policy commitment.

(by the following "period"). Our fully intertemporal model shows how even the expectation of later inflation — nominal interest rates are not expected to rise to offset it — can stimulate current demand, since in our model current spending decisions depend on real interest-rate expectations far in the future. For the same reason, the expectation that nominal interest rates will be kept low later, when the central bank might otherwise have raised them, will also stimulate spending while the zero bound still binds. And finally, the expectation of higher future income should stimulate current spending, in accordance with the permanent income hypothesis. In addition, prices are less likely to fall, even given the current level of real activity, insofar as future inflation is expected. This reduces the distortions created by deflation itself.

On the other hand, these gains from the change in expectations during the "trap" can be achieved (given rational expectations on the part of the private sector) only if the central bank is expected to actually pursue the inflationary policy after the natural rate returns to its

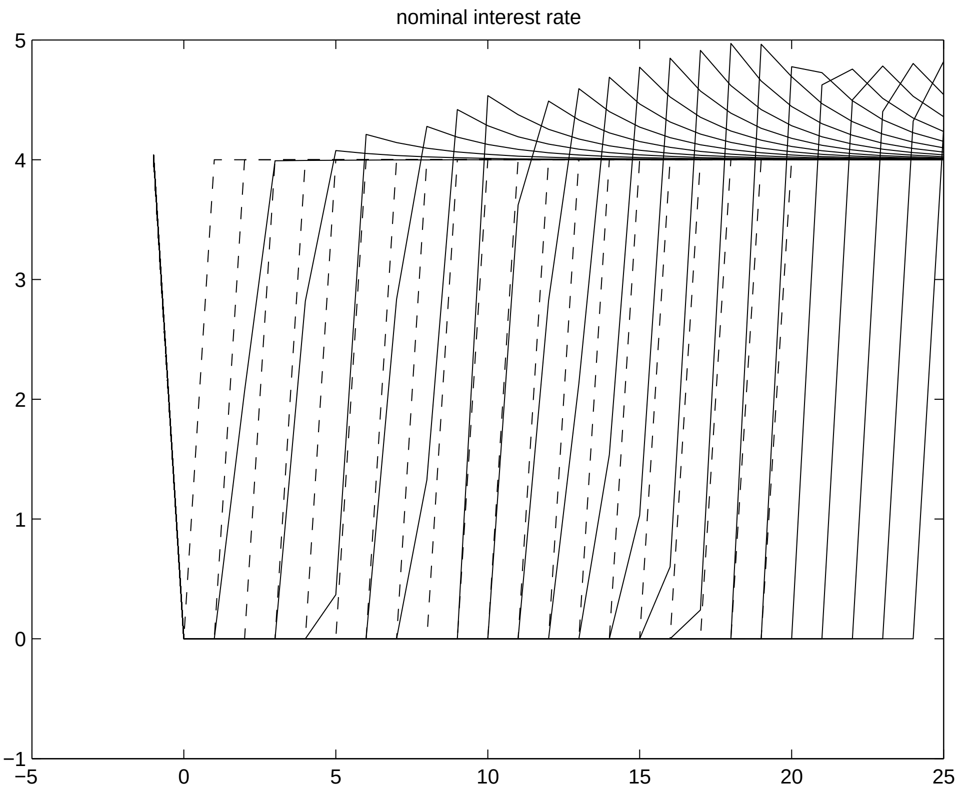

Figure 4: The associated state-contingent path of the short-term nominal interest rate, under the policy shown in Figure 3 [solid line], and under the zero inflation target shown in Figure 2 [dashed line].

normal level. This will in turn create distortions then, which limits the extent to which this tool is used under an optimal policy. Hence some contraction of output and some deflation occur during the period that the natural rate is negative, even under the optimal policy commitment. It is also worth noting that while the optimal policy involves commitment to a higher price level in the future, the price level will ultimately be stabilized. This is in sharp contrast to a constant positive inflation target that would implt an ever-increasing price level.

Figure 4 shows the corresponding state-contingent nominal interest rate under the optimal commitment, and contrasts it to the evolution of the nominal interest rate under a zero inflation target. To increase inflation expectations in the trap, the central bank commits to keeping the nominal interest rates at zero after the natural rate of interest becomes

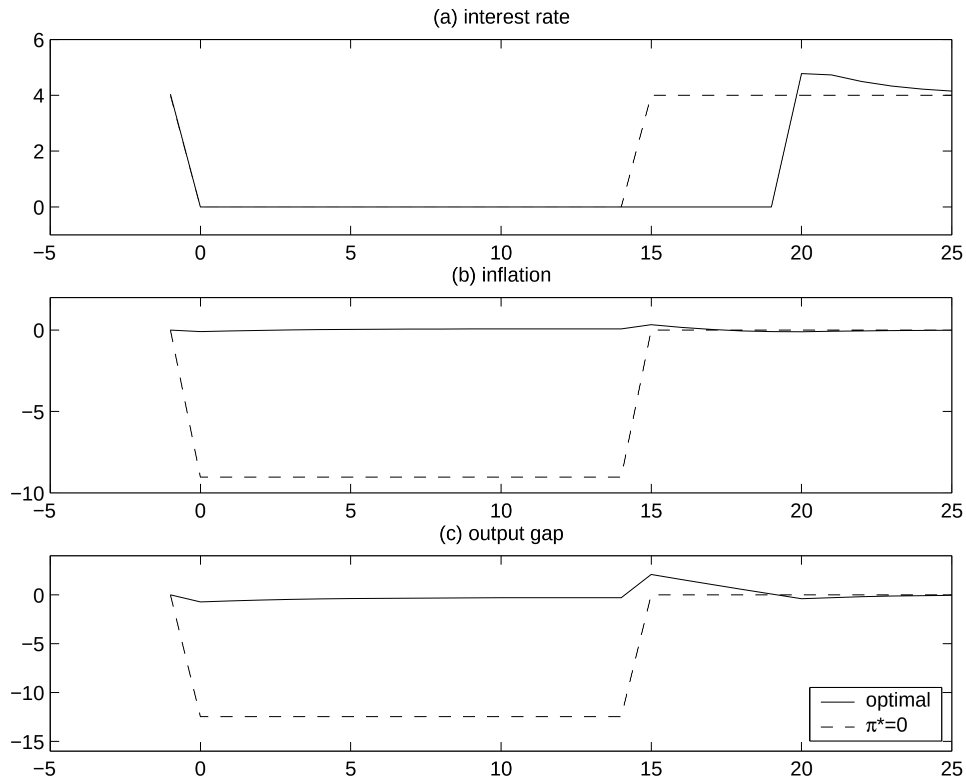

Figure 5: Comparison of the state-contingent paths under the two policies compared in Figure 4, in the case that the natural rate of interest is negative for 15 quarters.

positive again. In contrast, if the central bank targets zero inflation, it raises the nominal interest rate as soon as the natural rate of interest becomes positive again. The optimal commitment is an example of history-dependent policy, in which the central bank commits to raise the interest rates slowly at the time the natural rate becomes positive in order to affect expectations when the zero bound is binding.

The nature of the additional history-dependence of the optimal policy may perhaps be more easily seen if we consider the evolution of inflation, output and interest rates under a single possible realization of the random fundamentals. Figure 5 compares the equilibrium evolution of all three variables, both under the zero inflation target and under optimal policy, in the case that the natural rate of interest is negative for 15 quarters ($t = 0$ through 14), though it is not known until quarter 15 that the natural rate will return to its normal level in that quarter. Under the optimal policy, the nominal interest rate is kept at zero for five more quarters ($t = 15$ through 19), whereas it immediately returns to its long-run steady-state level in quarter 15 under the forward-looking policy. The consequence of the anticipation of policy of this kind is that both the contraction of real activity and the deflation that occur under the strict inflation target are largely avoided, as shown in the second and third panels of the figure.

3 Implementing Optimal Policy

How can the optimal policy be implemented? One may be tempted to believe that our suggested policy is not entirely realistic or operational. Figures 3 and 4, for example, indicate that the optimal policy involves a complicated state contingent plan for the nominal interest rate, that may be hard to communicate to the public. Furthermore, it may appear that it depends on a knowledge of a special statistical process for the natural rate of interest, that is in practice hard to estimate. Our discussion of the fixed inflation target suggest that the effectiveness of increasing inflation expectation to close the output gap depends on the difference between the announced inflation target and the natural rate of interest. It may, therefore, seem crucial to estimate the natural rate of interest to implement the optimal policy. Below we show the striking result that the optimal policy rule can be implemented without any estimate or knowledge of the statistical process for the natural rate of interest. This is an example of a robustly optimal direct policy rule of the kind discussed in Giannoni and Woodford (2002) for the case of a general class of linear-quadratic policy problems. An interesting feature of the present example is that we show how to construct an robustly optimal rule in the same spirit, in a case where not all of the relevant constraints are linear (owing to the fact that the zero bound binds at some times and not at others).

3.1 An Optimal Targeting Rule

To implement the rule proposed here the central bank need only observe the price level and the output gap. The rule suggested replicates exactly the history dependence discussed in last section. The rule is implemented as follows:

[i] In each an every period, there is a predetermined price-level target p_t^*. The Central Bank chooses interest rate i_t to achieve the target relation

$$\tilde{p}_t = p_t + \frac{\lambda_x}{\kappa} x_t = p_t^* \tag{3.10}$$

if possible; if this is not possible even by lowering the nominal interest rates to zero, then $i_t = 0$.

[ii] The target for next period is determines as

$$p_{t+1}^* = p_t^* + \beta^{-1}(1 + \kappa\sigma)\Delta_t - \beta^{-1}\Delta_{t-1} \tag{3.11}$$

where Δ_t is the period t target shortfall

$$\Delta_t \equiv p_t^* - p_t \tag{3.12}$$

It can be verified that this rule does indeed achieve the optimal commitment solution. If the price level target is not reached, due to the zero bound, the bank increases its target for the next period. This in turn, increases inflation expectations further in the trap which is exactly what is needed to reduce the real interest rate.

Figure 6 shows how the modified price-level target p_t^* would evolve over time, depending on the number of periods for which the natural rate of interest remains negative, in the same numerical experiment as in Figure 3. (Here the solid lines show the evolution of the actual modified price level $\tilde{p}_t$, while the dashed lines show the evolution of p_t^*.) One observes that the target price level is ratcheted up to steadily higher levels in the period in which the natural rate continues to be negative, as the actual price level continues to fall below the target by an increasing amount. Once the natural rate of interest becomes positive again, the degree to which the actual price level undershoots the target begins to shrink, although the target often continues to be undershot (as the zero bound continues to bind) for several more quarters. (The length of time for which this is true depends on how high the target price level has risen relative to the actual price level, which will be higher the longer the time for which the natural rate has been negative.) As the degree of undershoot begins to shrink,

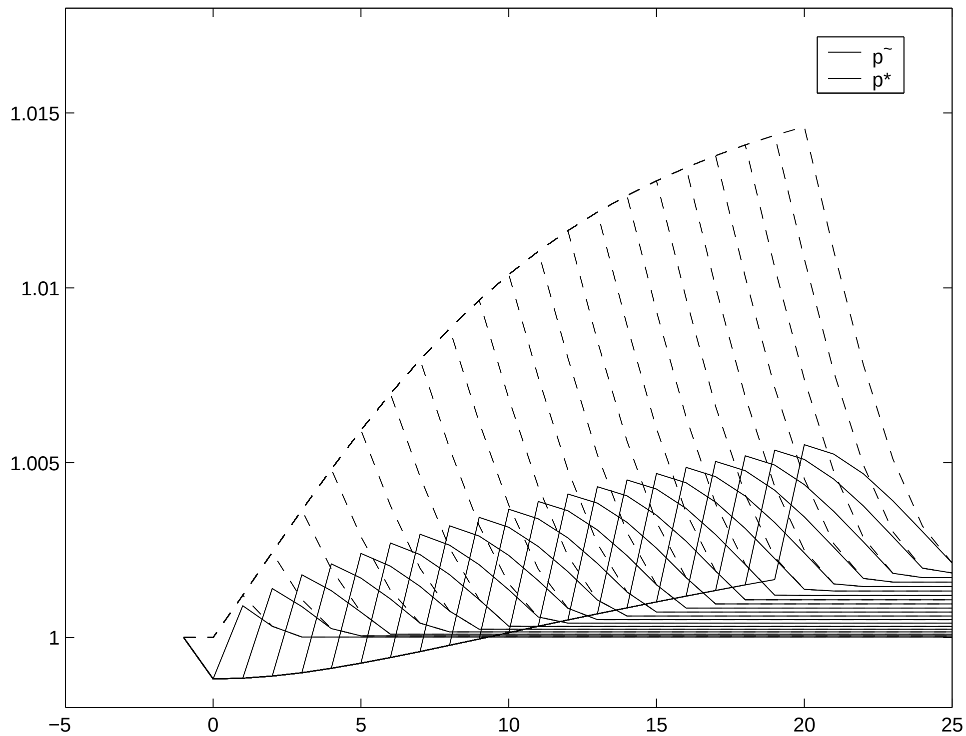

Figure 6: The associated state-contingent path of the short-term nominal interest rate, under the policy shown in Figure 3 [solid line], and under the zero inflation target shown in Figure 2 [dashed line].

the modified price-level target begins to fall again, as a result of the dynamics specified by (3.11). This hastens the date at which the target can actually be hit with a non-negative interest rate. Once the target ceases to be undershot any longer, it no longer changes, and the central bank targets and achieves a new constant value for the modified price level $\tilde{p}_t$, slightly higher than the target prior to the occurrence of the disturbance.

Note that this approach to implementing optimal policy gives an answer to the question whether there is any point in announcing an inflation target (or price-level target) if one knows that it is extremely unlikely that in the short run it can be achieved, owing to the fact that the zero bound is likely to continue to bind. The answer here is yes. The central bank wishes to make the private sector aware of its commitment to the time-varying price-level target described by (3.10) – (3.12), since eventually it *will* be able to hit the target, and the anticipation of that fact (*i.e.*, of the level that the price level will eventually reach, as a result of the policies that the bank will follow after the natural rate of interest again becomes positive) while the natural rate is still negative is important in mitigating the distortions caused by the zero bound. The fact that the target is not hit immediately should not create doubts about the meaningfulness of central-bank announcements regarding its target, if it is explained that the bank is committed to hitting the target *if* this is possible at a non-negative interest rate, so that at each point in time, either the target will be attained or a zero-interest-rate policy will be followed. The existence of the target is relevant even when it is not being attained, as it allows the private sector to judge how close the central bank is to a situation in which it would feel justified in abandoning the zero-interest-rate policy; hence the current gap between the actual and target price level should shape private-sector expectations of the time for which interest rates are likely to remain low.

Would the private sector have any reason to believe that the central bank was serious about the price-level target, if each period all that is observed is a zero nominal interest rate and yet another target shortfall? The best way of making a rule credible is for the central bank to conduct policy over time in a way that demonstrates its commitment. Ideally, the central bank's commitment to the price-level targeting framework would be demonstrated before the zero bound came to bind (at which time the central bank would have frequent opportunities to show that the target did determine its behavior). The rule proposed above is one that would be equally optimal under normal circumstances as in the case of the relatively unusual kind of disturbance that causes the natural rate of interest to be substantially negative.

To understand how the rule works out of the trap it is useful to note that when the nominal interest rates is positive then $\Delta_t = 0$ at all times. The central bank, therefore, should demonstrate a commitment to subsequently undo overshoots and undershoots of the price-level target. In this case, deflation that occurs when the economy finds itself in a liquidity trap should create expectations of future inflation, as mandated by optimal policy.

The additional term Δ_t implies that when the zero bound is binding, the central bank should raise its long run price level target even further, thus increasing inflation expectations even more.

It may be wondered why we discuss our proposal in terms of a modified price-level target, rather than an inflation target. In fact, we could equivalently describe the policy in terms of a time-varying target for the modified inflation rate $\tilde{\pi}_t \equiv \tilde{p}_t - \tilde{p}_{t-1}$. The reason that we prefer to describe the rule as a price-level targeting rule is that the essence of the rule is easily described in those terms. As we show below, a *fixed* target for the modified price level would actually represent quite a good approximation to optimal policy, whereas a fixed inflation target would not come close, as it would fail to allow for any of the history-dependence of policy that is necessary to mitigate the distortions resulting from the zero bound.

3.2 A Simpler Proposal

One may argue that an unappealing aspect of the rule suggested above is that it involves the term Δ_t, *i.e.*, the change in the price-level target, that is only non-zero when the zero bound is binding. Suppose that the central bank's commitment to a policy rule can only become credible over time through repeated demonstrations of its commitment to acting in accordance with it. In that case, the part of the rule that involves the adjustment of the target in response to target shortfalls when the zero bound binds might not come to be understood well by the private sector for a very long time, since the occasions on which the zero bound binds will presumably be relatively infrequent.

Fortunately, most of the benefits that can be achieved in principle through a credible commitment to the optimal targeting rule can be achieved through commitment to a much simpler rule, which would not involve any special provisos that are invoked only in the event of a liquidity trap. Let us consider the following simpler rule,

$$p_t + \frac{\lambda_x}{\kappa} x_t = p^*, \tag{3.13}$$

where now the price-level target is fixed at all times. The advantage of this rule, although not fully optimal when the zero bound is binding, is that it may be more easily communicated to the public. Note that the simple rule is fully optimal in the absence of the zero bound. In fact, even if the zero bound occasionally binds, this rule results in distortions only a bit more severe than those associated with the fully optimal policy.

Figure 7 and 8 compares the result for these two rules. The dotted line shows the equilibrium under the constant price level target rule in (3.13) whereas the solid line shows the fully optimal rule in (3.10)-(3.12). As can be seen by these figures the constant price-level targeting rule results in state-contingent responses of output and inflation that are very close to those under the optimal commitment, even if under this rule the price level falls farther during the period while the zero bound binds, and only asymptotically returns from below to the level that it had prior to the disturbance. Table 1 shows that most of the welfare gain achieved by the optimal policy, relative to what can be achieved by a purely forward-looking policy such as a strict inflation target, is already achieved by the simple rule. The table reports the value of expected discounted losses (2.6), conditional on the occurrence of the disturbance in period zero, under the three policies shown in Figure 2, the optimal policy characterized in Figure 3, and under the constant price-level targeting rule. Both of the latter two history-dependent policies are vastly superior to any of the strict inflation targets. While it is true that losses remain twice as large under the simple rule as under the optimal rule, we are referring to fairly small losses at this point.

As with the fully optimal rule, no estimate of the natural rate of interest is needed to implement the constant price level targeting rule. At first, it may seem puzzling, that a constant price level targeting rule does well since no account is taken of the size of the disturbance to the natural rate of interest. This is because a price level targeting commits the government to undo any deflation by subsequent inflation; a larger disturbance, that creates a larger initial deflation, automatically creates greater inflation expectations in response. Thus there is an "automatic stabilizer" build into the price level target, that is lacking under a strict inflation targeting regime.

A proper communication strategy for the central bank about is objectives and targets when outside the trap is of crucial importance for this policy rule to be successful. To see

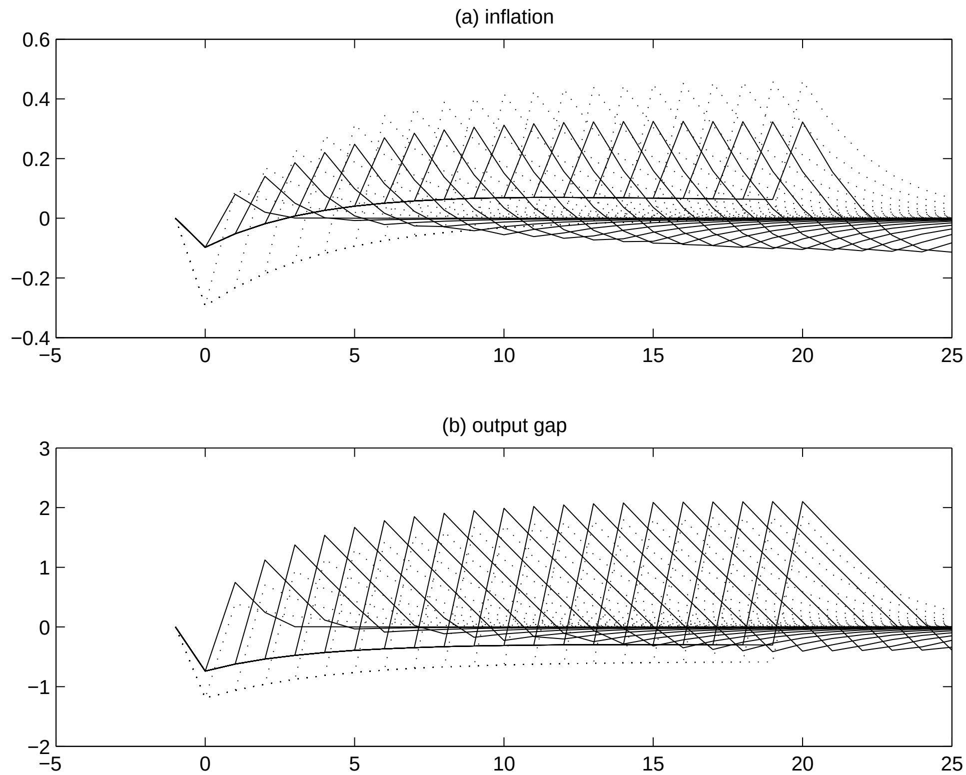

Figure 7: State-contingent paths of inflation and the output gap under the optimal targeting rule [solid lines] and under the simple rule [dotted lines].

this consider a rule that is equivalent to (3.13) when the zero bound is not binding. Taking the difference of (3.13) we obtain:

$$\pi_t + \frac{\lambda_x}{\kappa}(x_t - x_{t-1}) = 0 \tag{3.14}$$

Although this rule results in an identical equilibrium to the constant price level targeting rule when the zero bound is not binding, the result is dramatically different when the zero bound is binding. This is because this rule implies that the inflation rate is proportional to the negative of the growth rate of output. Thus it mandates deflation when there is growth in the output gap. This implies that the central bank will deflate once out of a liquidity trap since this is a period of output growth. This is exactly opposite to what is optimal as we have observed above. Thus the outcome under this rule is even worse than a strict zero inflation target, even if this rule replicates the price level targeting rule when out of

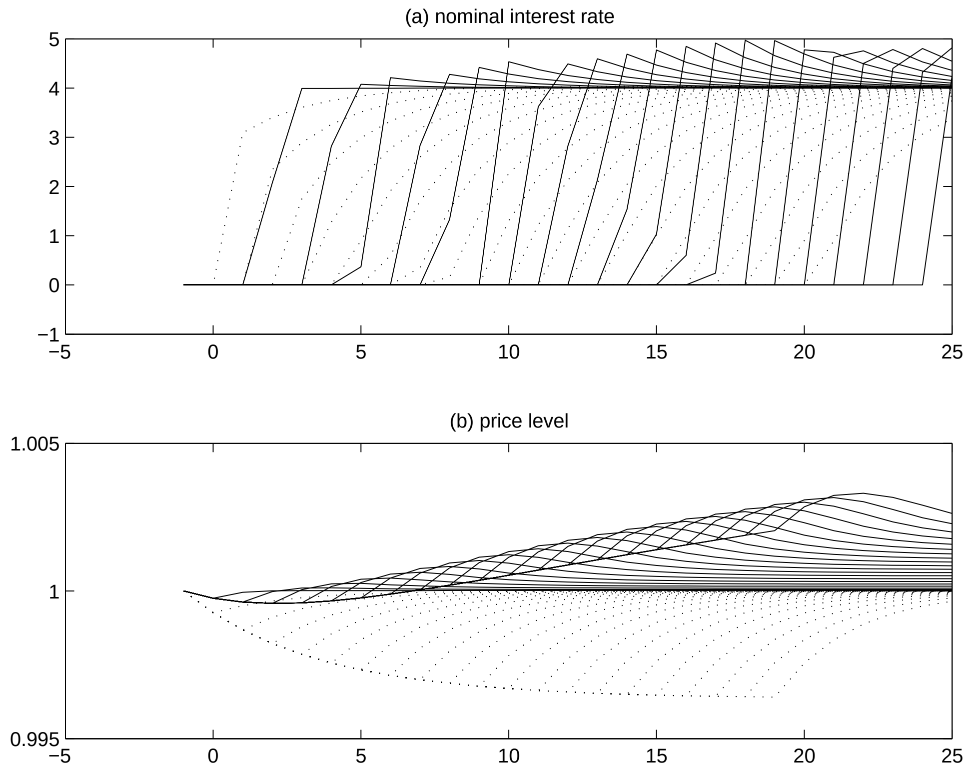

Figure 8: State-contingent paths of the nominal interest rate and the price level under the same two policies as in Figure 7.

the trap. What this underlines is that it is not enough to replicate the equilibrium behavior that correspond to (3.13) at normal times to induce the correct set of expectations when the zero bound is binding. It is crucial to communicate to the public that the government is committed to a long run price level target. This commitment is exactly what creates the desired inflation expectations when the zero bound is binding.

3.3 Should a Central Bank "Keep Powder in the Keg?"

Thus far we have only considered alternative policies that might be followed from the date at which the natural rate of interest unexpectedly falls to a negative value, causing the zero bound to bind. A question of considerable current interest in countries like the U.S., however, is how policy should be affected by the *anticipation* that the zero bound might well bind before long, even if this is not yet the case. Some commentators have argued that

Table 1: Relative losses under alternative policies [loss under zero inflation target = 100].

Strict Inflation Targets	
$\pi^* = 0$	100
$\pi^* = 1$	25.5
$\pi^* = 2$	43.3
Price-Level Targeting Rules	
constant target	0.098
optimal rule	0.049

in such circumstances the Fed should be cautious about lowering interest rates all the way to zero too soon, in order to "save its ammunition" for future emergencies. This suggests that the anticipation that the zero bound could bind in the future should lead to tighter policy than would otherwise be justified given current conditions. Others argue alternatively that policy should instead be more inflationary than one might otherwise prefer, in order to reduce the probability that a further negative shock can result in a situation where the zero bound binds.

Our above characterization of the optimal targeting rule can shed light on this debate. Recall that the rule (3.10) – (3.12) describes optimal policy *regardless*) of the assumed stochastic process for the natural rate of interest, and not only in the case of the particular two-state Markov process assumed in Figure 3. In particular, the same rule is optimal in the case that information is received indicating the likelihood of the natural rate of interest becoming negative before this actually occurs. How should the conduct of policy be affected by that news? Under the optimal targeting rule, the optimal target for $\tilde{p}_t$ is *unaffected* by such expectations, as long as a situation has not yet been reached in which the zero bound binds, since it is only target shortfalls that have already occurred that can justify a change in the target value p_t^*. Thus an increased assessment of the likelihood of a binding zero bound over the coming year or two would *not* be a reason for increasing the price-level target (or the implied target rate of inflation).

On the other hand, the evolution of inflation, output and interest rates will be affected

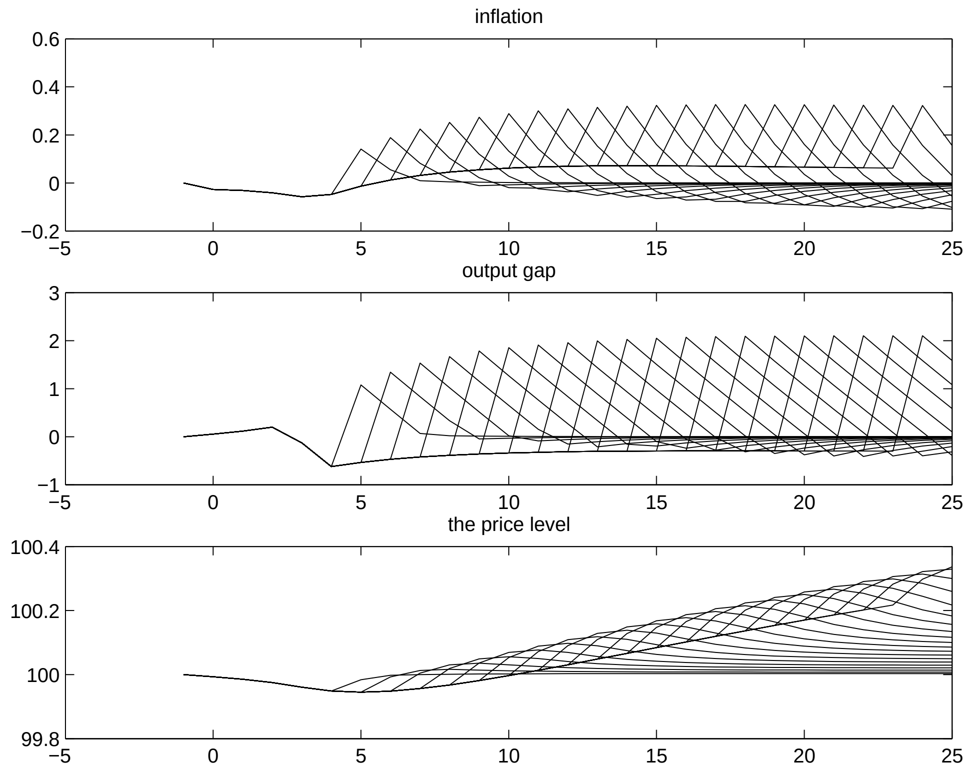

Figure 9: State-contingent paths of inflation and output under optimal policy, when the decline in the natural rate of interest can be anticipated four quarters in advance.

by this news, even in the absence of any immediate change in the central bank's price level-target owing to the effect on forward-looking private-sector spending and pricing decisions. The anticipation of a coming state in which the natural rate of interest will be negative, and actual interest rates will not be able to fall as much, owing to the zero bound, will reduce both desired real expenditure (at unchanged short-term interest rates) and desired price increases, as a result of the anticipated negative output gaps and price declines in the future. This change in the behavior of the private sector's outlook will require a change in the way that the central bank must conduct policy in order to hit its *unchanged* target for the modified price level, likely in the direction of a pre-emptive loosening of policy.

This is illustrated by the numerical experiment shown in Figure 9. Here we suppose that in quarter zero it is learned (by both the central bank and the private sector) that the natural rate of interest will fall to the level of -2 percent per annum only in period 4. It is known that the natural rate will remain at its normal level, +4 percent per annum until then; after the drop, it will return to the normal level with a probability of 0.1 each quarter, as in the case considered earlier. We now consider the character of optimal policy from period zero onward, given this information. Figure 9 again shows the optimal state-contingent paths of inflation and output in the case that the disturbance to the natural rate, when it arrives lasts for one quarter, two quarters, and so on.

We observe that under the optimal policy commitment, prices begin to decline mildly as soon as the news of the coming disturbance is received. The central bank is nonetheless able to avoid undershooting its target for $\tilde{p}_t$ at first, by stimulating an increase in real activity sufficient to justify the mild deflation. (Given the shift to pessimism on the part of the private sector, this is the policy dictated by the targeting rule, given that even a mild immediate increase in real activity is insufficient to prevent price declines, owing to the anticipated decline in real demand when the disturbance hits.) By quarter 3, this is no longer possible, and the central bank undershoots its target for $\tilde{p}_t$ (as both prices and output decline), even though the nominal interest rate is at zero. Thus optimal policy involves driving the nominal interest rate to zero *even before* the natural rate of interest has turned negative, when that development can already be anticipated for the near future. The fact that the zero bound binds even before the natural rate of interest becomes negative means that the price-level target is higher than it otherwise would have been at the time that the disturbance to the natural rate arrives. As a result, the deflation and output gaps during the period in which the natural rate is negative are less severe than in the case in which the disturbance is unanticipated. In this scenario, optimal policy is somewhat more inflationary after the disturbance occurs than in the case considered in Figure 3, for in this case the optimal policy commitment takes into account the contractionary effects in periods *before* the disturbance takes effect of anticipations that the disturbance will result in price-level and output declines. The fact that optimal policy after the disturbance occurs is different in this case, despite the fact that the disturbance has exactly the same effects as before from quarter 4 onward, is another illustration of the history-dependence of optimal policy.

4 Preventing a Self-Fulfilling Deflationary Trap

In our analysis thus far, we have assumed that the real disturbance results in a negative natural rate of interest only temporarily. We have therefore supposed that price-level stabilization will eventually be consistent with positive nominal interest rates, and accordingly that a time will foreseeably be reached at which it is possible for the central bank to create inflation by keeping short-term nominal rates at a low (but non-negative) level. Some may ask, however, if it is not possible for the zero bound to bind *forever* in equilibrium, not because of a permanently negative natural rate, but simply because deflation continues to be (correctly) expected indefinitely. If so, it might seem that the central bank's commitment to a non-decreasing price-level target would be irrelevant; the actual price level would fall further and further short of the target, but because of the binding zero bound, there would never be anything the central bank could do about this.

In the model presented in section 2, a self-fulfilling permanent deflation is indeed consistent with both the Euler equation (1.2) for aggregate expenditure, the money-demand relation (1.3) and the pricing relations (1.7) – (1.8). Suppose that from some date τ onward, all disturbances $\xi_t = 0$ with certainty, so that the natural rate of interest is expected to take the constant value $\bar{r} = \beta^{-1} - 1 > 0$, as in the scenarios considered in section 3. Then possible paths for inflation, output, and interest rates consistent with each of the relations just listed in all periods $t \geq \tau$ is given by

$$i_t = 0,$$

$$P_t/P_{t-1} = \beta < 1,$$

$$p_t^*/P_t = \tilde{p}^* \equiv \left(\frac{1 - \alpha\beta^{\theta-1}}{1-\alpha}\right)^{\frac{1}{1-\theta}} < 1,$$

$$Y_t = \tilde{Y}$$

for all $t \geq \tau$, where $\tilde{Y} < \bar{Y}$ is implicitly defined by the relation

$$\Pi_1(\tilde{p}^*, \tilde{p}^*, 1; \tilde{Y}, \bar{m}(\tilde{Y}; 0), 0) = 0.$$

Note that this deflationary path is consistent with monetary policy as long as real balances satisfy $M_t/P_t \geq \bar{m}(\tilde{Y};0)$ each period; faster growth of the money supply does nothing to prevent consistency of this path with the requirement that money supply equal money demand each period.

There remains, however, one further requirement for equilibrium in the model of section 2, the transversality condition (1.6), or equivalently the requirement that households exhaust their intertemporal budget constraints. Whether the deflationary path is consistent with this condition as well depends, properly speaking, on the specification of fiscal policy: it is a matter of whether the government budget results in contraction of the nominal value of total government liabilities D_t at a sufficient rate asymptotically. Under some assumptions about the character of fiscal policy, such as the "Ricardian" fiscal policy rule assumed by Benhabib *et al.*, the nominal value of government liabilities will necessarily contract along with the price level, so that (1.6) is also satisfied, and the processes described above will indeed represent a rational-expectations equilibrium. In such a case, then, a commitment to the price-level targeting rule proposed in the previous section will be equally consistent with more than one equilibrium: if people expect the optimal price-level process characterized earlier, then that will indeed be an equilibrium, but if they expect perpetual deflation, this will be an equilibrium as well.

We can, however, exclude this outcome through a suitable commitment with regard to the asymptotic evolution of total government liabilities. Essentially, there needs to be a commitment to policies that ensure that the nominal value of government liabilities cannot contract at the rate required for satisfaction of the transversality condition despite perpetual deflation. One example of a commitment that would suffice is a commitment to a balanced-budget policy of the kind analyzed by Schmitt-Grohé and Uribe (2000). These authors show that self-fulfilling deflations are not possible under commitment to a Taylor rule, together with the balanced-budget fiscal commitment. The key to their result is that the fiscal rule includes a commitment *not to allow budget surpluses* any more than budget deficits would be allowed; hence it is not possible for the nominal value of government liabilities to contract, even when the price level falls exponentially forever.

The credibility of this sort of fiscal commitment might be doubted, and so it is worth mentioning that another way of maintaining a floor on the asymptotic nominal value of total government liabilities is through a commitment *not to contract the monetary base,* together with a commitment of the government to maintain a non-negative asymptotic present value of the public debt. In particular, suppose that the central bank commits itself to follow a base-supply rule of the form

$$M_t = P_t^* \bar{m}(Y_t; \xi_t) \tag{4.1}$$

in each period when the zero bound binds (*i.e.*, when it is not possible to hit the price-level target with a positive nominal interest rate), where

$$P_t^* \equiv \exp\left\{p_t^* - \frac{\lambda}{\kappa} x_t\right\}$$

is the current price-level target implied by the adjusted price-level target p_t^*. When the zero bound does not bind, the monetary base is whatever level is demanded at the nominal interest rate required to hit the price-level target. This is a rule in the same spirit as (1.10), specifying a particular level of excess supply of base money in the case that the zero bound binds, but letting the monetary base be endogenously determined by the central bank's other targets at all other times. Equation (4.1) is a more complicated formula than is necessary to make our point, but it has the advantage of making the monetary base a continuous function of other aggregate state variables at the point where the zero bound just ceases to bind.

This particular form of commitment has the advantage that it may be considered less problematic for the central bank to commit itself to maintain a particular nominal value for its liabilities than for the Treasury to do so. It can also be justified as a commitment that is entirely consistent with the central bank's commitment to the price-level targeting rule; even when the target cannot be hit, the central bank supplies the quantity of money *that would be demanded if the price level were at the target level.* Doing so — refusing to contract the monetary base even under circumstances of deflation — is a way of signalling to the public that the bank is serious about its intention to see the price level restored to the target level.

If we then assume a fiscal commitment that guarantees that

$$\lim_{T\to\infty} E_t Q_{t,T} B_T = 0, \tag{4.2}$$

i.e., that the government will asymptotically be neither creditor nor debtor, the transversality condition (1.6) reduces to

$$\lim_{T\to\infty} \beta^T E_t[u_c(Y_T, M_T/P_T; \xi_T) M_T/P_T] = 0. \tag{4.3}$$

In the case of the base-supply rule (4.1), this condition is violated in the candidate equilibrium described above, since the price-level and output paths specified would imply that

$$\begin{aligned} \beta^T E_t[u_c(Y_T, M_T/P_T; \xi_T) M_T/P_T] &= \beta^\tau u_c(\tilde{Y}, \bar{m}(\tilde{Y}; 0); 0)\bar{m}(\tilde{Y}; 0) P_T^*/P_\tau \\ &\geq \beta^\tau u_c(\tilde{Y}, \bar{m}(\tilde{Y}; 0); 0)\bar{m}(\tilde{Y}; 0) P_\tau^*/P_\tau, \end{aligned}$$

where the last inequality makes use of the fact that under the price-level targeting rule, $\{p_t^*\}$ is a non-decreasing series. Note that the final expression on the right-hand side is independent of T, for all dates $T \geq \tau$. Hence the series is bounded away from zero, and condition (4.3) is violated.

Thus a commitment of this kind can exclude the possibility of a self-fulfilling deflation of the sort described above as a possible rational-expectations equilibrium. It follows that there is a possible role for "quantitative easing" — understood to mean supply of base money beyond the minimum quantity required for consistency with the zero nominal interest rate — as an element of an optimal policy commitment. A commitment to supply base money in proportion to the *target* price level, and not the actual current price level, in a period in which the zero bound prevents the central bank from hitting its price-level target, can be desirable both as a way of ruling out self-fulfilling deflations and as a way of signalling the central bank's continuing commitment to the price-level target, even though it is temporarily unable to hit it.

Note that this result does not contradict the irrelevance proposition of section 2, for we have here made a different assumption about the nature of the fiscal commitment than the one made in section 2. Condition (4.2) implies that the evolution of total nominal government liabilities will not be independent of the central bank's target for the monetary base. As a consequence, the neutrality proposition of section 2 no longer holds. The import of that proposition is that expansion of the monetary base when the economy is in a liquidity trap is necessarily pointless; rather, it is that any effect of such action must depend either on changing expectations regarding future interest-rate policy *or* on changing expectations regarding the future evolution of total nominal government liabilities. The present discussion has illustrated circumstances under which expansion of the monetary base — or at any rate, a commitment not to contract it — could serve both of these ends.

Nonetheless, the present discussion does *not* support the view that the central bank should be able to hit its price-level target at all times, simply by flooding the economy with as much base money as is required to prevent the price level from falling below the target at any time. Our analysis in section 3 still describes all of the possible paths for the price level consistent with rational-expectations equilibrium, and we have seen that even if the central bank were able to choose the expectations that the private sector should have (as long as it were willing to act in accordance with them), the zero bound would prevent it from being able to fully stabilize inflation and the output gap. Furthermore, the degree of base expansion during a "liquidity trap" called for by rule (4.1) is quite modest. The monetary base will be gradually raised, if the zero bound continues to bind, as the price-level target is ratcheted up to steadily higher levels. But our calibrated example above indicates that this would typically involve only quite a modest increase in the monetary base, even in the case of a "liquidity trap" that lasts for several years. There would be no obvious benefit to the kind of rapid expansion of the monetary base actually tried in Japan over the past two years. An expansion of the monetary base of this kind is evidently *not* justified by any intentions regarding the future price level, and hence regarding the size of the monetary base once Japan exits from the "trap." But an injection of base money that is expected to be removed again once the zero bound ceases to bind should have little effect on spending or pricing behavior, as shown in section 2.

5 Further Aspects of the Management of Expectations

In section 2, we argued that neither expansion of the monetary base as such nor purchases of particular types of assets through open-market purchases should have any effect on either inflation or real activity, *except* to the extent that such actions might result in changes in expectations regarding future interest-rate policy (or possibly expectations regarding the asymptotic behavior of total nominal government liabilities, and hence the question of whether the transversality condition should be satisfied). Because of this, we were able, in sections 3 and 4, to characterize the optimal policy commitment without any reference to the use of such instruments of policy; a consideration of the different possible joint paths of interest rates, inflation and output that would be consistent with rational-expectations equilibrium sufficed to allow us to determine the best possible equilibrium that one could hope to arrange, and to characterize it in terms of the interest-rate policy that one should wish for the private sector to expect.

However, this does not mean that other aspects of policy — beyond a mere announcement of the rule according to which the central bank wishes to be understood to be committed in setting future interest-rate policy — cannot matter. They may matter insofar as certain kinds of present actions may help to make it more credible that the central bank is indeed committed to the kind of future policy that the optimal equilibrium requires people to expect. We have given one example of this already, in the previous section. Adjustment of the supply of base money during the period in which the zero bound binds so as to keep the monetary base proportional to the *target* price level rather than the actual current price level can be helpful, even though it is irrelevant as far as interest-rate control is concerned, as a way of making visible to the private sector the central bank's belief about whether the price level ought properly to be (and hence, the quantity of base money that the economy ought to need). By making the existence of the price-level target more salient, such an action can help to create the expectations regarding future interest-rate policy that are necessary in order to mitigate the distortions created by the binding zero bound.

Similarly, actions that change the balance sheet of the central bank, or the structure of financial claims on the government in the hands of the public, may be relevant to shaping expectations regarding future policy in a desirable way, even if these actions have relatively little consequence for equilibrium determination otherwise. One way that such actions may help to render the central bank's commitment to an optimal policy more credible is by providing the bank with a *motive* to behave in the future in the way that it would currently wish that people would expect it to behave. Here we briefly discuss how policy actions that are possible while the economy remains in a "liquidity trap" may be helpful in this regard. Our perspective is not so much that the central bank is in need of a "commitment technology" because it will itself be unable to resist the temptation to break its commitments later in the absence of such a constraint — a capacity for discipline and principled behavior is generally a trait that is sought in central bankers — but that it may well be in need of a way of making its commitment *visible* to the private sector. Taking actions now that imply that the central bank will be disadvantaged later if it were to deviate from the policy to which it wishes to commit itself can serve this purpose.

To consider what kind of current actions provide useful incentives, it is helpful to analyze (Markov) equilibrium under the assumption that policy is conducted by a discretionary optimizer, unable to commit its future actions at all. Eggertsson (2003a, b) presents two examples of such exercises, that show how either immediate tax cuts or open-market purchases of certain kinds of assets could help to shift expectations regarding future monetary policy in the desired direction under circumstances of a "liquidity trap". Here we provide a brief summary of his results.

5.1 Deflation as a Credibility Problem

We first consider what a Markov equilibrium under discretionary optimization would be like, in the case that the only policy instrument is the choice each period of a short-term nominal interest rate, and the objective of the central bank is the minimization of the loss function (2.6). As shown in section 3, if credible commitment of future interest-rate policy is possible, this problem has a solution in which the zero bound does not result in too serious a distortion, though it does bind.

Under discretion, however, the outcome will be much inferior. Note that discretionary policy (under the assumption of Markov equilibrium in the dynamic policy game) is an example of a purely forward-looking policy. It then follows from our argument in section 3 that the equilibrium outcome will correspond to the kind of equilibrium discussed there in the case of a strict inflation target. More specifically, it is obvious that the equilibrium is the same as under a strict inflation target $\pi^* = 0$, since this is the inflation rate that will be chosen by the discretionary optimizer once the natural rate is again at its steady-state level. (From that point onward, a policy of zero inflation clearly minimizes the remaining terms in the discounted loss function.)

As shown in Figure 2, an expectation by the private sector that the central bank will behave in this fashion results in a deep and prolonged contraction of economic activity and a sustained deflation, in the case that the natural rate of interest remains negative for several quarters. We have also seen that these effects could largely be avoided, even in the absence of other policy instruments, if the central bank were able to credibly commmit itself to a history-dependent monetary policy in later periods. Thus, in the kind of situation considered here, there is a *deflationary bias* to discretionary monetary policy, although, at its root, the problem is again the one identified in the classic analysis of Kydland and Prescott (1977). We now wish instead to consider the extent to which the outcome could be improved, even in a Markov equilibrium with discretionary optimization, by changing the nature of the policy game.

5.2 Using Fiscal Policy to Create Inflation Expectations

One example of a current policy action, available even when the zero bound binds, that can help to shift expectations regarding future policy in a desirable way is for the government to cut taxes and issue additional nominal debt. Alternatively, the tax cut can be financed by money creation — for when the zero bound binds, there is no difference between expanding the monetary base and issuing additional short-term Treasury debt at a zero interest rate. This is essentially the kind of policy imagined when people speak of a "helicopter drop" of additional money on the economy; but it is the *fiscal* consequence of such an action with which we are here concerned.

Of course, if the objective of the central bank in setting monetary policy remains as assumed above, this will make no difference to the discretionary equilibrium — the optimal policy once the natural rate of interest becomes positive again will once more appear to be the immediate pursuit of a strict zero inflation target. However, if the central bank also cares about reducing the social costs of increased taxation — whether due to collection costs or other distortions — as it ought if it really takes social welfare into account, the result is different. As shown in Eggertsson (2003a), the tax cut will then increase inflation expectations, even if the government cannot commit to future policy.

The logic behind the result is quite simple. Suppose that in addition to announcing a target price level 10 percent higher than the current level, the government issues a quantity of nominal debt. In this case it has an incentive to bring about the promised increase in the price level, for leaving the price level where it is would increase the real value of government debt by 10 percent relative to what it will be if the government fulfills its commitment. Sooner or later the government would have to make up for this by raising taxes, which would not be preferred even by forward-looking policymakers, assuming that those in control of monetary policy care about tax distortions along with their concern for inflation and output-gap stabilization. Hence deficit spending represents a straightforward way of credibly increasing inflationary expectations. This is an example of a non-Keynesian effect of fiscal policy, since it increases output by changing expectations about future monetary policy.

What is the relevance of this analysis for Japan? Figure 10 shows that government debt has doubled in Japan over the last ten years, from roughly 64.5% in 1990 to over 140% in 2002, largely due to deficit spending. This is the highest level of gross government debt in the G7 countries, as illustrated in Table 2, showing data for 2002. These numbers suggest

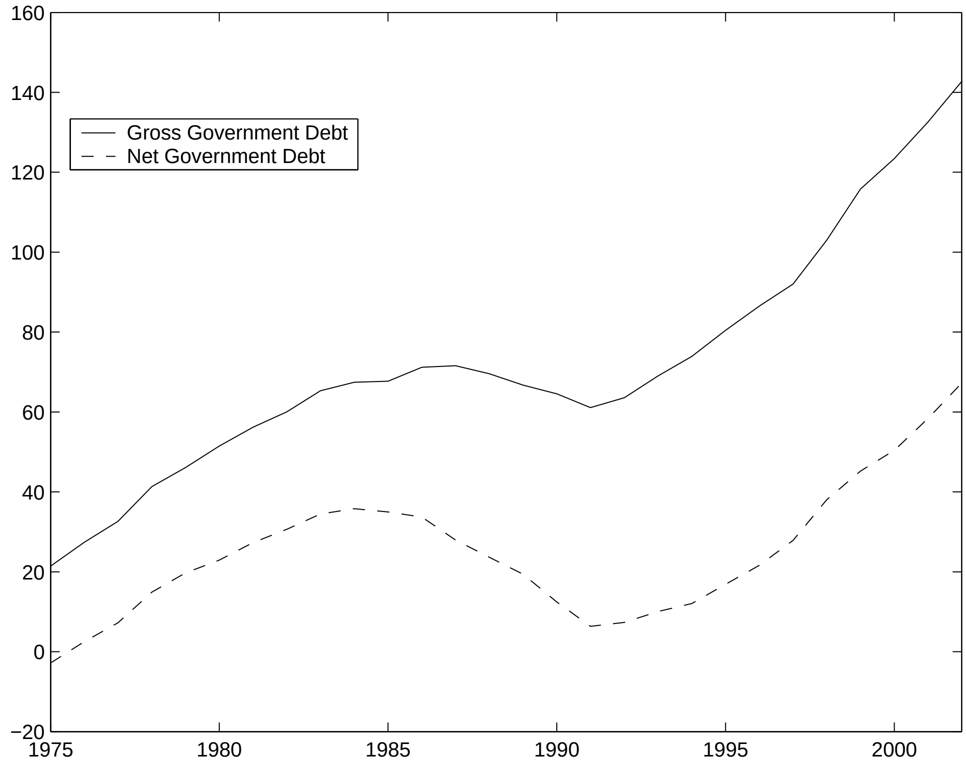

Figure 10: Net and gross Japanese government debt as a share of GDP.

that the Japanese government should already have a substantial incentive to inflate.

To study the inflation incentive further we need to look further at the structure of the Japanese public debt. Figure 11 shows the maturity structure of outstanding debt in Japan, *i.e.*, the nominal value of debt due to be paid in the period 2003-2023. It is simple to calculate the government gains from inflation from this data if we make some simple assumption about the evolution of the natural rate of interest. Figure 12 illustrates how much the real debt would be reduced under different inflation rates. The underlying assumption is that the natural real rate will remain negative for 5 years at -2% and then return to a positive rate. The figure shows the real value of the debt in 2023 if it is rolled over from 2003 onwards. We express this value as a fraction of the real value of the debt if there were to be zero inflation over that period, and compute this fraction for the cases of 3, 4, 5, 10 and 20 percent inflation per annum. As illustrated in the figure, there would be a substantial reduction in the real value of the debt under even relative modest rates of inflation; for example, it is reduced by

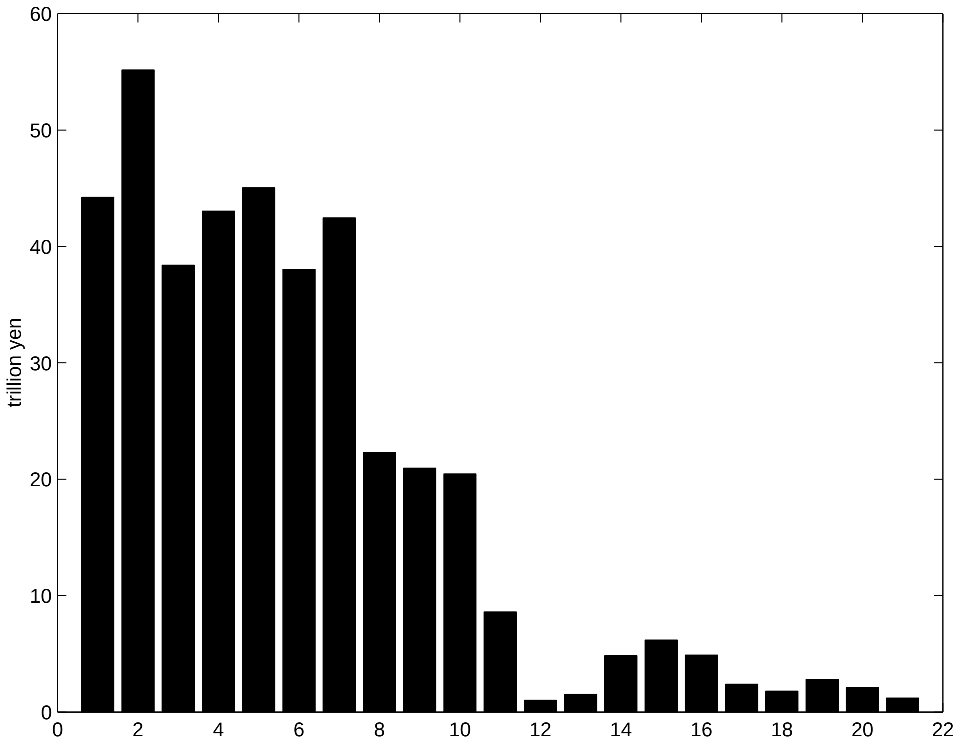

Figure 11: Maturity structure of outstanding Japanese government debt [in years].

more than a quarter in the case of the 4 percent inflation rate advocated by Krugman (1998). This suggests that the Japanese government should currently have fairly large incentives to adhere to an announced inflation target in this range.

There is, however, an important caveat. Although gross nominal debt over GDP is 140 percent in Japan today, this does not reflect the true inflation incentives of the government. The ratio of gross national debt to GDP *overestimates* the inflation incentives of the government, because a substantial portion of Treasury debt is held by other governmental institutions.[14] The ratio of *net* government debt to GDP is perhaps a more realistic measure.

[14]Government institutions such as Social Security, Postal Savings, Postal Life Insurance and the Trust Fund Bureau hold a large part of this nominal debt. If the part of the public debt that is held by these institutions is subtracted from the total value of gross government debt it turns out that the "net" government debt over output is only 51 percent. The important thing to notice is that most of the government institutions that hold the government nominal debt have *real liabilities*. For example, Social Security (that holds roughly 25% of the nominal debt held by the government itself) pays Japanese pensions and medical expenses. Those pensions are *indexed to the CPI*. If inflation increases, the real value of Social Security assets will decrease but the real value of most its liabilities remain unchanged. Thus the Ministry of Finance would eventually have to step in to make up for any loss in the value of Social Security assets if the government is to keep its

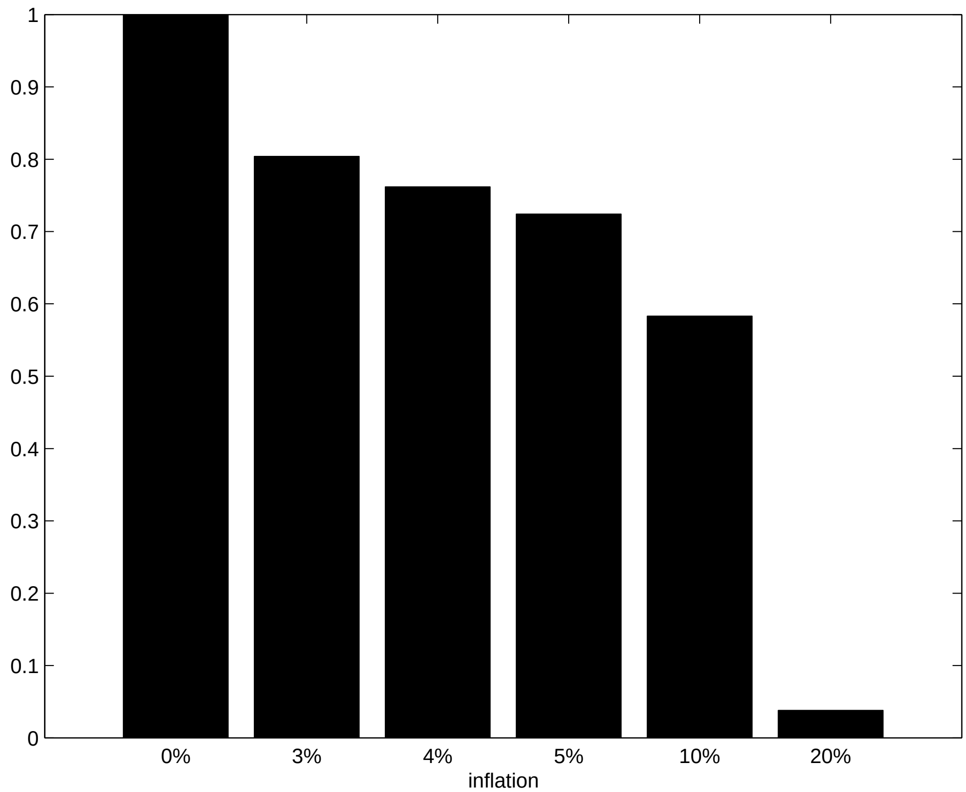

Figure 12: Reduction in the real value of Japanese government debt if rolled over to 2021.

Table 2 also contrasts net government debt in Japan to net debt in the other G7 countries. In fact, net government debt is not so large relative to that of the other G7 countries. Thus trying to evaluate the inflation incentives of the government in Japan from data on gross debt may be quite misleading. In order for an inflation target to be credible, further deficit spending might still be appropriate.

Another possible reason for the continued low expectations of inflation in Japan at present, despite the current size of the nominal public debt, has to do with the assumption that the central bank can be expected to care about reducing the burden of the public debt when determining future monetary policy. The Bank of Japan may not be believed by the public to have such an objective; the expressed resistance of the Bank to suggestions that it increase its purchases of Japanese government bonds, on the ground that this could

pension program unchanged. Therefore, the gains of reducing the real value of outstanding debt is partly offset by a decrease in the real value of the assets of government institutions such as Social Security.

Table 2: Gross and net government debt in the G7 countries in 2002 [as percentage of GDP].

	Gross Debt/GDP	Net Debt/GDP
Canada	81.2	41.1
France	66.7	39.4
Germany	62.4	47.2
Italy	109.6	97.3
Japan	142.7	67.2
UK	50.8	29.2
US	60.7	44.3
average	82.0	52.3

encourage a lack of fiscal discipline, certainly suggests that reducing the burden of government finance is not among its highest priorities. As Eggertsson (2003a) stresses, in order for fiscal policy to be effective as a means of increasing inflationary expectations, fiscal and monetary policy must be coordinated to maximize social welfare. The consequences of a narrow concern with inflation stabilization on the part of the central bank, together with in ability to credibly commit future monetary policy, can be dire, even from the point of view of the bank's own stabilization objectives.

5.3 Buying Real Assets or Foreign Exchange to Make Inflation Credible

Another instrument that may be used to change expectations regarding future monetary policy is open-market purchases of real assets or foreign exchange. An open-market purchase of real assets (say, real estate) with newly created yen can affect inflation expectations in much the same way as deficit spending, as discussed in Eggertsson (2003a), as it is again a way of increasing nominal government liabilities. The alternative approach has the advantage of not worsening the overall fiscal position of the government — a current concern in Japan, owing to the size of the existing gross debt — while still increasing the fiscal incentive for inflation. Once again, however, success of this method in changing expectations would depend on a perceived concern to reduce tax distortions when future monetary policy decisions are made.

An assumed central-bank concern to reduce tax distortions is not even needed to show that open-market operations in foreign exchange could be used to affect expectations, as shown by Eggertsson (2003b). Open-market purchases of foreign assets give the central bank an incentive to inflate in the future in order to obtain capital gains at the expense of foreigners. Under rational expectations, of course, no such gains are realized on average. Still, the purchase of foreign assets can work as a commitment device, because reneging on its inflation commitment would cause capital losses if the government holds foreign assets. Purchases of foreign assets are thus a way of committing the government to looser monetary policy in the future. This creates a reason for purchases of foreign exchange to cause a devaluation (which will also stimulate current demand), even without any assumption of a deviation from interest-rate parity, of the kind relied upon by several authors in recommending this kind of policy for Japan ([ADD REFERENCES]). Of course, the argument does depend on an assumption that future monetary policy would be made with a view to attaining these capital gains. But such a motive would follow from social welfare maximization by a discretionary central bank, even if taxes are not at all distorting. In addition, even a central bank with narrower objectives could have an incentive to fulfill its commitment to create inflation, if it acquires foreign assets for its own balance sheet, and so stands to suffer capital losses itself in the event of an unexpectedly tight monetary policy.

A Appendix: The Numerical Solution Method

Here we illustrate a solution method for the optimal commitment solution. This method can also be applied, following the same steps, to find the solution if the government commits to the constant price level target rule and the strict inflation target. We assume that the natural rate of interest becomes unexpectedly negative in period 0 and the reverts back to normal with probability α_t in every period t. There is a final date S in which the natural rate becomes positive with probability one (this date can be arbitrarily far into the future). The solution takes the form:

$$\begin{array}{rclccl} i_t & = & 0 & \forall & \text{if} & 0 \leq t < \tau + k \\ i_t & > & 0 & \forall & \text{if} & t \geq \tau + k \end{array}$$

It follows that:

$$\begin{array}{rclcl} E_t x_{t+1} - x_t + \sigma(E_t \pi_{t+1} + r_t^n) & = & 0 & \text{if} & t < \tau + k \\ \phi_{1t} & = & 0 & \text{if} & t \geq \tau \end{array}$$

Here τ is he stochastic date at which the natural rate of interest returns to steady state. We assume that τ can take any value between 1 and the terminal date S that can be arbitrarily far into the future. The number $\tau + k_\tau$ is the period in which the zero bound stops being binding in the contingency when the natural rate of interest becomes positive in period τ. Note that the value of k_τ can depend on the value of τ. We will first show the solution for the problem as if we knew the sequence $\{k_\tau\}_{\tau=1}^{S}$. We then describe a numerical method to find the sequence $\{k_\tau\}_{\tau=1}^{S}$.

A.0.1 Solution for $t \geq \tau + k_\tau$

The system can be written in the form:

$$\begin{bmatrix} E_t Z_{t+1} \\ P_t \end{bmatrix} = M \begin{bmatrix} Z_t \\ P_{t-1} \end{bmatrix} \tag{A.4}$$

If there are two eigenvalues of the matrix M outside the unit circle this system has a unique bounded solution of the form:

$$P_t = \Omega^0 P_{t-1} \tag{A.5}$$

$$Z_t = \Lambda^0 P_{t-1} \tag{A.6}$$

A.0.2 Solution for $\tau \leq t < \tau + k$

Again this is a perfect foresight solution but with the zero bound binding. The solution satisfies the equations:

$$\begin{gathered} \pi_t = \kappa x_t + \beta \pi_{t+1} \\ x_t = \sigma(r_t^n + \pi_{t+1}) + x_{t+1} \\ \pi_t + \phi_{2t} - \phi_{2t-1} - \beta^{-1}\sigma\phi_{1t-1} = 0 \\ \lambda_x x_t + \phi_{1t} - \beta^{-1}\phi_{1t-1} - \kappa\phi_{2t} = 0 \end{gathered} \tag{A.7}$$

The system can be written as:

$$\begin{bmatrix} P_t \\ Z_t \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} P_{t-1} \\ Z_{t+1} \end{bmatrix} + \begin{bmatrix} M \\ V \end{bmatrix}$$

This system has a solution of the form:

$$P_{\tau+j} = \Omega^{k_\tau - j} P_{t-1} + \Phi^{k_\tau - j} \tag{A.8}$$

$$Z_{\tau+j} = \Lambda^{k_\tau - j} P_{\tau,t-1} + \Theta^{k_\tau - j} \tag{A.9}$$

where $j = 0, 1, 2, ..., k$. Here $\Omega^{k_\tau - j}$ is the coefficient in the solution when there are $k_\tau - j$ periods until the zero bound stops being binding (i.e. when $j - k_\tau = 0$ the zero bound is not binding anymore and the solution is equivalent to (A.5)-(A.6)). We can find the numbers $\Lambda^j, \Omega^j, \Theta^j$ and Φ^j for $j = 2, 3, ..., k$ by solving the equations below using the initial conditions $\Phi^0 = \Theta^0 = 0$ for $j = 0$ and the initial conditions for Λ^j and Ω^j given in (A.5)-(A.6):

$$\Omega^j = [I - B\Lambda^{j-1}]^{-1} A$$

$$\Lambda^j = C + D\Lambda^{j-1}\Omega^j$$

$$\Phi^j = (I - B\Lambda^{j-1})^{-1}[B\Theta^{j-1} + M]$$

$$\Theta^j = D\Lambda^{j-1}\Phi^j + D\Theta^{j-1} + V$$

A.0.3 Solution for $t < \tau$

The solution satisfies the following equations:

$$\tilde{\pi}_t = \kappa\tilde{x}_t + \beta\{(1-\alpha_{t+1})\tilde{\pi}_{t+1} + \alpha_{t+1}(\Lambda_{11}^{k_{t+1}}\tilde{\phi}_{1t} + \Lambda_{12}^{k_{t+1}}\tilde{\phi}_{2t} + \Theta_1^{k_{t+1}})\}$$

$$\tilde{x}_t = \sigma\{r_t^{nL} + (1-\alpha_{t+1})\tilde{\pi}_{t+1} + \alpha_{t+1}(\Lambda_{11}^{k_{t+1}}\tilde{\phi}_{1t} + \Lambda_{12}^{k_{t+1}}\tilde{\phi}_{2t} + \Theta_1^{k_{t+1}})\} + \{(1-\alpha_{t+1})\tilde{x}_{t+1} + \alpha_{t+1}(\Lambda_{21}^{k_{t+1}}\tilde{\phi}_{1t} + \Lambda_{22}^{k_{t+1}}\tilde{\phi}_{2t} + \Theta_2^{k_{t+1}})\}$$

$$\tilde{\pi}_t + \tilde{\phi}_{2t} - \tilde{\phi}_{2t-1} - \beta^{-1}\sigma\tilde{\phi}_{1t-1} = 0$$

$$\lambda_x\tilde{x}_t + \tilde{\phi}_{1t} - \beta^{-1}\tilde{\phi}_{1t-1} - \kappa\tilde{\phi}_{2t} = 0$$

Here hat on the variables refers to the value of each variable contingent on that the natural rate of interest is negative. $\Lambda_{ij}^{k_{t+1}}$ is the ijth element of the matrix $\Lambda^{k_{t+1}}$. The value k_{t+1} depends on for how many additional periods the zero bound is binding (recall that here we are solving for the equilibrium assuming that we know the value of the sequence $\{k_\tau\}_{\tau=1}^{S}$). We can write the system as:

$$\begin{bmatrix} \tilde{P}_t \\ \tilde{Z}_t \end{bmatrix} = \begin{bmatrix} A_t & B_t \\ C_t & D_t \end{bmatrix} \begin{bmatrix} \tilde{P}_{t-1} \\ \tilde{Z}_{t+1} \end{bmatrix} + \begin{bmatrix} M_t \\ V_t \end{bmatrix}$$

We can solve this backwards from the date S in which the natural rate returns back to normal with probability one. We can then calculate the path for each variable to date 0. Note that.

$$B_{S-1} = D_{S-1} = 0$$

By recursive substitution we can find a solution of the form:

$$\tilde{P}_t = \Omega_t\tilde{P}_{t-1} + \Phi_t \quad \text{(A.10)}$$

$$\tilde{Z}_t = \Lambda_t\tilde{P}_{t-1} + \Theta_t \quad \text{(A.11)}$$

where the coefficients are time dependent. To find the numbers $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t consider the solution of the system in period $S-1$ when $B_{S-1} = D_{S-1} = 0$. We have:

$$\Omega_{S-1} = A_{S-1}$$

$$\Phi_{S-1} = M_{S-1}$$

$$\Lambda_{S-1} = C_{S-1}$$

$$\Theta_{S-1} = V_{S-1}$$

We can find of numbers $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t for period 0 to $S-2$ by solving the system below (using the initial conditions shown above for $S-1$):

$$\Omega_t = [I - B_t\Lambda_{t+1}]^{-1}A_t$$

$$\Lambda_t = C_t + D_t\Lambda_{t+1}\Omega_t$$

$$\Phi_t = (I - B_t\Lambda_{t+1})^{-1}[B_t\Theta_{t+1} + M_t]$$

$$\Theta_t = D_t\Lambda_{t+1}\Phi_t + D_t\Theta_{t+1} + V_t$$

Using the initial condition $\tilde{P}_{-1} = 0$ we can solve for each of the endogenous variables under the contingency that the trap last to period S by (A.10) and (A.11). We then use the solution from (A.5)-(A.9) to solve for each of the variables when the natural rate reverts back to steady state.

A.0.4 How to find $\{k_\tau\}_{t=0}^{\infty}$?

A simple way to look find the value for $\{k_\tau\}_{\tau=1}^{\infty}$ is to first assume that k_τ is the same for all τ and find the k so that the zero bound is never violated. Suppose that the system has converged at t=25 (i.e. the response of each of the variables is the same). Then we can move to 24 and see if $k_\tau = 4$ for $\tau = 1, 2, ...24$ is a solution that never violates the zero bound. If not move to 23 and try the same thing and so on. For preparing this paper we wrote a routine in MATLAB that applied this method to find the optimal solution and verified that the results satisfied all the necessary conditions.

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