Ramsey Monetary Policy and International Relative Prices

Ester Faia*
Universitat Pompeu Fabra

Tommaso Monacelli†
IGIER, Università Bocconi

March 2003 (PRELIMINARY)

Abstract

In an economy with sticky prices and monopolistic competition we analyze how openness affects the welfare maximizing monetary policy. Openness, by driving a wedge between consumer and producer prices, entails per se an additional distortion that we label international relative price distortion. Throughout the analysis we remain consistent to a public finance approach by an explicit consideration of all the distortions that are relevant to the Ramsey planner. We then analyze the impact of such distortions on the optimal monetary policy setting both in the long-run and in the short run. We employ perturbation methods (as in Sims(2001) and Schmitt-Grohe and Uribe (2002)) to characterize the solution to the system of (non-linear) equations that describe the optimal Ramsey allocation.

Preliminary results show that the rate of inflation associated to the efficient steady-state lies always below the one of the corresponding closed economy. Furthermore there exist multiple values of the degree of openness such that, unlike in a closed economy, a zero-inflation steady-state is also efficient. In the short-run, the optimal response to productivity shocks entails deviations from a (producer) price stability policy that depend on the elasticity of substitution betwen domestic and foreign goods.

Keywords. Monetary and Fiscal Policy Regimes, Markov Switching, Equilibrium Determinacy, Inflation.

JEL Classification Number: C32, E60, E62.

*Correspondence to Universitat Pompeu Fabra, Ramon Trias Fargas 25, Barcelona, Spain. E-mail: ester.faia@econ.upf.es, homepage: http://www.econ.upf.es/~faia/.

†Correspondence to IGIER Universita' Bocconi, Via Salasco 3/5, 20136 Milan, Italy. Email: tommaso.monacelli@uni-bocconi.it.

1 Introduction

There has been a growing interest recently for the analysis of the foundations of optimal monetary policy. The use of dynamic stochastic general equilibrium models with nominal rigidities has naturally opened the avenue to a serious investigation of the welfare effects of alternative policies. Recent work by Woodford (2002) has shown how to conduct welfare analysis by resorting to second order approximations of the household's utility while maintaining the tractability of the log-linear approximation of the economy's equilibrium conditions. In this case neglecting the (local) non-linearities can be justified by resorting to specific assumptions on the magnitude of the inefficiences that characterize the (non-stochastic) steady-state relative to the driving stochastic forces in the model. Conducting welfare analysis in DSGE models of the open economy has proved to be a much more daunting task (Kim and Kim (2002), Henderson and Kim(2002)). The New Open Economy Macroeconomics literature may be thought of as divided into two (complementary) streams. The first, exemplified by the work of Obstfeld and Rogoff (1995) and Corsetti and Pesenti (2001), takes the form of elegant but highly stylized models in which the analysis of optimal monetary policy is typically simplified by the assumption that prices (or wages) are predetermined one-period. This is obviously a restrictive assumption. For it typically gives rise to a Lucas-type aggregate supply curve in which the forward-looking nature of inflation is neglected, and along with it the channel through which the anticipation of future policy conduct comes to play a role. This is just an example of how restrictive this class of models is with respect to the introduction of additional dynamics.

A second class of models in the NOEM literature tries to incorporate the full program of the New Neoclassical Synthesis in the analysis of optimal policy. Examples are, among many others, Benigno and Benigno (2002), Gali and Monacelli (2002), Smets and Wouters (2002), Monacelli (2002). However, in these models, obtaining second-order accurate approximations of welfare along the lines of Woodford (2002) is feasible only under very specific knife-edge conditions that allow to drop the first order terms in the obtained expression for households' welfare. One such condition typically imposes log preferences and unitary international elasticity of substitution. In this case the welfare analysis neglects an additional factor that distorsts the incentives of the monetary authority (besides market power and sticky prices): the possibility of influencing the terms of trade in a way beneficial to domestic residents. This possibility is a consequence of the imperfect substitutability between domestic and foreign goods (combined with sticky prices) and genuinely pertains to the openness dimension of the economy. The ability to affect the terms of trade may create at the margin an incentive to deviate from the flexible price allocation (and thus from strict producer price stability) in response to a shock. In that case the monetary authority has to weigh the gains from that terms of trade (or consumer's purchasing power) effect against the losses from non-zero output gap and inflation.

In this paper we take a different, more general, route to the analysis of optimal monetary

policy in an open economy. We employ a full-fledged *Ramsey approach*. Recently there has been a resurgence of interest for such an approach in the analysis of optimal policy in sticky-price models (see e.g., Scmitt-Grohe and Uribe (2002), Siu (2002)). Yet to our knowledge this method has found no application to the study of optimal monetary policy in the NOEM literature.

In our work we remain consistent to a public finance setup by an explicit consideration of all the distortions that are relevant to the solution of the Ramsey planner problem. The crucial element to be emphasized is that this approach allows to work with a steady state that is not necessarily efficient. This is a fundamental difference with respect to a recurrent strategy in the recent optimal monetary policy literature (see RotembergWoodford (2002)) of specifying lump-sum subsidies that are meant to neutralize the existing market power distortion.

We analyze the impact of all the relevant distortions on the optimal monetary policy setting both in the *long-run* and in the *short-run*. The optimal design of monetary policy under commitment is based on a multistage approach along the lines highlighted in Lucas and Stockey (1983). First one needs to determine the conditions that characterize the competitive economy for a given policy, i.e., optimizing conditions for households and firms, resource constraints and policy constraints. Then such conditions are manipulated to get rid off prices and policy instruments. They are summarized by feasibility constraints and implementability constraints on real quantities. Finally the monetary authority maximizes the sum of discounted expected utility of agents subject to the real allocations characteriaing the (imperfectly) competitive economy.

1.1 Setup and Solution Method

Below we describe the setup of the model and of the relevant Ramsey problem for a world economy composed of two countries: a small open economy and a foreign (rest-of-the world) economy (which can be approximated by an almost closed economy). After describing the set of (non-linear) first order conditions we employ perturbation methods recently developed by Sims (2001), Anderson and Levin (2002), Schmitt-Grohe and Uribe (2002) to solve for the optimal allocation.[1] Recently there have been other applications of second order approximation/perturbation techniques to open economy models of the NOEM type. See for instance Kollmann (2002, 2003), Bergin and Tchakarov (2003). What distinguishes our approach is that we search for second order solutions to the already characterized Ramsey planner problem. The a priori description of the Ramsey planner problem entails at least two advantages. First it adds a signifcant amount of discipline and transparency to the description of the relevant problem. Second it often allows to characterize a wider array of analytical results before employing the full-blown solution of the model.

[1]Specifically we employ and adapt the Matlab codes developed by Schmitt-Grohe and Uribe (2002) available at http://www.econ.upenn.edu/~uribe/closing.htm

1.1.1 Preliminary results

Our project (which is still in a preliminary stage) has already allowed to reach few results. First, we have shown that the rate of inflation associated to the efficient steady-state lies always below the one of the corresponding closed economy. Second, there exist multiple values of the degree of openness such that, unlike in a closed economy, a zero-inflation steady-state is also efficient. Third, in the short-run, the optimal response to productivity shocks entails deviations from a (producer) price stability policy that depend on the elasticity of substitution betwen domestic and foreign goods.

2 The Model

The world economy is divided into two regions, a small open economy and the rest of the world where agents consume and produce only domestically. Each economy is populated by infinite-lived agents, whose total measure is normalized to unity.

2.0.2 The Households

Agents consume different varieties of domestic and foreign goods which are imperfectly substitutable. Optimal allocation of expenditure within each variety goods yields:

$$C_H(i) = \left(\frac{P_H(i)}{P_H}\right)^{-\varepsilon} C_H;\ C_F(i) = \left(\frac{P_F(i)}{P_F}\right)^{-\varepsilon} C_F; \tag{1}$$

where $C_H \equiv \int_0^1 [C_H(i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$ and $C_F \equiv \int_0^1 [C_F(i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$.

Optimal allocation of expenditure between domestic and foreign goods yields:

$$C_H = (1-\alpha)\left(\frac{P_H}{P}\right)^{-\eta} C;\ C_F = \alpha\left(\frac{P_F}{P}\right)^{-\eta} C \tag{2}$$

where $P \equiv [(1-\alpha)P_H^{1-\eta} + \alpha P_F^{1-\eta}]^{\frac{1}{1-\eta}}$ is the CPI index, $C \equiv [(1-\alpha)^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}}]^{\frac{\eta-1}{\eta}}$ is the composite aggregate consumption index, α is the balanced-trade steady state share of imported goods and η is the elasticity of substitution between domestic and foreign goods.

Let $s^t = \{s_0, ...s_t\}$ denote the history of events up to date t, where s_t denotes the event realization at date t. The date 0 probability of observing history s^t is given by $\rho(s^t)$. The initial state s^0 is given so that $\rho(s^0) = 1$.

Agents maximize the following expected discounted sum of utilities:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \rho(s^t) U\left(C(s^t), N(s^t)\right) \tag{3}$$

that we assume separable in consumption and labor, so that $U_{cn} = U_{nc} = 0$. At the beginning of time t the households receive a nominal labor income of $W(s^t)N(s^t)$. To insure their consumption pattern against random shocks at time t they decide to spend $\sum_{s^{t+1}} \nu(s^{t+1}|s^t)B(s^{t+1})$ in nominal state contingent securities, where $\nu(s^{t+1}|s^t)$ is the price kernel. Each state contingent asset $B(s^{t+1})$ pays one unit of domestic currency at time $t+1$ and in state s^{t+1}. Hence the sequence of budget constraints assumes the following form:

$$\begin{aligned} & \int_0^1 \{P_H(i,s^t)C_H(i,s^t) + P_F(i,s^t)C_F(i,s^t)\}di + \sum_{s_{t+1}} \nu(s^{t+1}|s^t)B(s^{t+1}) \\ \leq \quad & W(s^t)N(s^t) + T(s^t) + B(s^t) + \varkappa(s^t) \end{aligned} \tag{4}$$

where $T(s^t)$ are government transfers of domestic currency and $\Phi(s^t)$ are the profits of the domestic monopolistic firms, whose shares are owned by the domestic residents. By considering the optimal expenditure conditions (1) and (2) the sequence of budget constraints can be written:

$$P(s^t)C(s^t) + \sum_{s_{t+1}} \nu(s^{t+1}|s^t)B(s^{t+1}) \leq W(s^t)N(s^t) + T(s^t) + B(s^t) + \Phi(s^t) \tag{5}$$

Households choose the set of processes $\{C(s^t), N(s^t)\}_{t=0}^{\infty}$ and bonds $\{B(s^{t+1})\}_{t=0}^{\infty}$, taking as given the set of processes $\{P(s^t),\ W(s^t),\ v(s^{t+1}|s^t)\}_{t=0}^{\infty}$ and the initial wealth $B(s_0)$ so as to maximize (3) subject to (5).

The following optimality conditions must hold:

$$U_c(s^t)\frac{W(s^t)}{P(s^t)} = -U_n(s^t) \tag{6}$$

$$\beta\rho(s^{t+1}|s^t)\frac{P(s^t)}{P(s^{t+1})}\frac{U_c(s^{t+1})}{U_c(s^t)} = \nu(s^{t+1}|s^t) \tag{7}$$

$$\frac{1}{R^n(s^t)} = \sum_{s_{t+1}} \nu(s^{t+1}|s^t) \tag{8}$$

$$\lim_{j\to\infty} \sum_{s_{t+j}} \nu(s^{t+j}|s^t)B(s^{t+j}) = 0 \tag{9}$$

Equation (6) defines the optimal choice of labor supply. Equation (7) defines the price of the Arrow-Debreu security, with a conditional expected return given by (8). Optimality requires that the first order conditions and the No-Ponzi game conditions, as from equation (9), are simultaneously satisfied. The nominal interest rate $R^n(s^t) = R(s^t)\frac{P(s^{t+1})}{P(s^t)}$

2.1 Law of One Price, the Real Exchange Rate and Risk Sharing

We assume throughout that the *law of one price* holds, implying that $P_F(i) = \mathcal{E}\ P_F^*(i)$ for all $i \in [0,1]$, where $\mathcal{E}$ is the nominal exchange rate, i.e., the price of foreign currency in terms of home currency, and $P_F^*(i)$ is the price of foreign good i denominated in foreign currency.

It is important to clarify that, as in Gali and Monacelli (2002), we treat the rest of the world as an approximately closed economy, with goods produced in the small open economy representing only a negligible fraction of the world's consumption basket. This implies that the foreign price index P^* coincides with P_F^*, i.e., the foreign currency price of foreign goods.

Let's denote by B^* foreign households 'holdings of the state contingent bond denominated in domestic currency. The budget constraint of the foreign representative household will read:

$$P^*(s^t)C^*(s^t) + \sum_{s_{t+1}} \nu(s^{t+1}|s^t)\frac{B^*(s^{t+1})}{\mathcal{E}(s^t)} \tag{10}$$

$$\leq \quad W^*(s^t)N^*(s^t) + T^*(s^t) + \frac{B^*(s^t)}{\mathcal{E}(s^t)} + \Phi^*(s^t) \tag{11}$$

The efficiency condition for bonds' holdings is

$$\beta\rho(s^{t+1}|s^t)\frac{P(s^t)\mathcal{E}(s^t)}{P(s^{t+1})\mathcal{E}(s^{t+1})}\frac{U_c^*(s^{t+1})}{U_c^*(s^t)} = \nu(s^{t+1}|s^t) \tag{12}$$

After equating (12) with (7) one can exploit the completeness feature of asset markets and iterate obtaining a consumption risk sharing condition

$$Q(s^t) = \kappa\frac{U_c^*(s^t)}{U_c(s^t)} \tag{13}$$

where $Q(s^t) \equiv \frac{\mathcal{E}(s^t)P^*(s^t)}{P(s^t)}$ is the real exchange rate and $\kappa \equiv \frac{\mathcal{E}(s^0)P^*(s^0)U_c(s^0)}{P(s^0)U_c^*(s^0)}$. Therefore the marginal utility of consumption will differ across countries due to movements of the real exchange rate. In our framework, despite the law of one price holding at each point in time, deviations from PPP are possible due to the asymmetry in the aggregate consumption baskets between the domestic and the foreign economy.

2.2 The Domestic Production Sector

Each monopolistic firm i produces a homogenous good according to:

$$Y(i, s^t) = A\ N(i, s^t) \tag{14}$$

The cost minimizing choice of labor input implies:

$$\frac{W(s^t)}{P_H(s^t)} = mc(s^t)A(s^t) \tag{15}$$

where mc denotes the real marginal cost.

Changing output prices is subject to some costs. We follow Rotemberg (1982) and model the cost of adjusting prices for each firm i equal to:

$$PAC(i, s^t) \equiv \frac{\theta}{2}(\frac{P(i, s^t)}{P(s^t)} - 1)^2 \tag{16}$$

where the parameter θ measures the degree of price stickiness. The higher the θ the more sluggish is the adjustment of nominal prices. If $\theta = 0$ prices are flexible.

The cost of price adjustment renders the domestic producer's pricing problem dynamic. Each producer chooses the price $P_H(i)$ of variety i to maximize its total market value:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \lambda(s^t) \ \frac{D(i, s^t)}{P_H(s^t)} \tag{17}$$

subject to the constraint

$$Y(i, s^t) \leq \left(\frac{P_H(i, s^t)}{P_H(s^t)}\right)^{-\varepsilon} C^W(s^t) \tag{18}$$

where $C^W \equiv C_H + C_H^*$ is the world total demand for domestic goods, $\beta^t \lambda(s^t)$ measures the marginal utility value to the representative producer of additional profits expressed in domestic currency, and where

$$\frac{D(i, s^t)}{P_H(s^t)} \equiv \frac{P_H(i, s^t)Y(i, s^t)}{P_H(s^t)} - \frac{W(s^t)}{P_H(s^t)}N(s^t) - \frac{\theta}{2}\left(\frac{P_H(i, s^t)}{P_H(i, s^{t-1})} - 1\right)^2$$

The first order condition of the above problem reads

$$\begin{aligned} 0 \ = \ & \lambda(s^t)\left(\frac{P_H(i, s^t)}{P_H(s^t)}\right)^{-\epsilon} \frac{C^W(s^t)}{P_H(s^t)}\left((1-\varepsilon) + \varepsilon\frac{W(s^t)}{A(s^t)P_H(s^t)}\left(\frac{P_H(i, s^t)}{P_H(s^t)}\right)^{-1}\right) \\ & -\lambda(s^t)\theta\left(\frac{P_H(i, s^t)}{P_H(i, s^{t-1})} - 1\right)\frac{1}{P_H(i, s^{t-1})} \ + \ \beta\lambda(s^{t+1})\theta\left(\frac{P_H(i, s^{t+1})}{P_H(i, s^t)} - 1\right)\frac{P_H(i, s^{t+1})}{P_H(i, s^t)^2} \end{aligned} \tag{19}$$

Let's define $\widetilde{p}_H \equiv \frac{P_H(i)}{P_H}$ as the relative price of domestic variety i and $\pi_H(s^t) \equiv \frac{P_H(s^t)}{P_H(s^{t-1})}$ as the gross domestic inflation rate. It is useful to see that the above condition can be rewritten as

$$
\begin{aligned}
0 &= \lambda(s^t)C^W(s^t)\widetilde{p}_H(s^t)^{-\varepsilon}\left((1-\varepsilon)+\varepsilon\frac{W(s^t)}{A(s^t)P_H(s^t)}\right)- \\
&\lambda(s^t)\theta\left(\pi_H(s^t)\frac{\widetilde{p}_H(s^t)}{\widetilde{p}_H(s^{t-1})}-1\right)\frac{\pi_H(s^t)}{\widetilde{p}_H(s^{t-1})} \\
&+\beta\lambda(s^{t+1})\theta\left(\pi_H(s^{t+1})\frac{\widetilde{p}_H(s^{t+1})}{\widetilde{p}_H(s^t)}-1\right)\pi_H(s^{t+1})\frac{\widetilde{p}_H(s^{t+1})}{\widetilde{p}_H(s^t)^2}
\end{aligned} \tag{20}
$$

2.3 The Monetary Authority

The monetary regime consists in the announcement of a state contingent plans for the nominal interest rate $\{R(s^t)\}_{t=0}^{\infty}$. The fiscal authority makes transfer to agents, $T(s^t)$, whose discounted value sum to infinity is equal to zero.

3 Equilibrium

3.1 Equilibrium in the Rest of The World

We first show the set of equations for the symmetric equilibrium of the foreign economy. Optimality conditions for consumers read:

$$C^*(i,s^t) = (\frac{P^*(i,s^t)}{P^*(s^t)})^{-\eta}C^*(s^t) \tag{21}$$

$$U_c(C^*(s^t))\frac{W^*(s^t)}{P^*(s^t)} = -U_n^*((s^t)) \tag{22}$$

$$\beta\rho(s^{t+1}|s^t)\frac{P^*(s^t)}{P^*(s^{t+1})}\frac{U_c(C^*(s^{t+1}))}{U_c(C^*(s^t))}\frac{1}{e(s^{t+1})} = \frac{\nu(s^{t+1}|s^t)}{e(s^t)} \tag{23}$$

$$R^{n*}(s^t)^{-1} = \sum_{s_{t+1}}\nu(s^{t+1}|s^t) \tag{24}$$

The resource constraint implies that total consumption plus aggregate costs of changing prices must equate total output:

$$Y^*(s^t) = C^*(s^t) + \frac{\theta}{2}\left(\pi^*(s^t)-1\right)^2 \tag{25}$$

Efficiency conditions for pricing and labor demand are standard:

$$
\begin{aligned}
&\lambda^*(s^t)[(1-\varepsilon)(\frac{P_i^*(s^t)}{P^*(s^t)})^{-\epsilon}\frac{Y^*(s^t)}{P^*(s^t)} + \frac{1}{A^*(s^t)}\frac{W^*(s^t)}{P^*(s^t)}\varepsilon(\frac{P_i^*(s^t)}{P^*(s^t)})^{-\epsilon-1}\frac{Y^*(s^t)}{P^*(s^t)} \\
&-\theta(\frac{P_i^*(s^t)}{P_i^*(s^{t-1})}-1)\frac{1}{P^*(s_i^{t-1})}] - \beta\lambda^*(s^{t+1})\theta(\frac{P_i^*(s^{t+1})}{P_i^*(s^t)}-1)\frac{P_i^*(s^{t+1})}{P_i^*(s^t)^2} = 0
\end{aligned} \tag{26}
$$

$$\frac{W^*(s^t)}{P^*(s^t)} = mc^*(s^t)A^*(s^t) \tag{27}$$

An optimal consumption plan implies the following Euler equation

$$U_c^*(s^t) = \beta \sum_{s_{t+1}} \rho(s^{t+1}|s^t)R^*(s^t)U_c^*(s^{t+1}) \tag{28}$$

By combining (26) and (27) and assuming a symmetric equilibrium we obtain:

$$\begin{aligned} U_c^*(s^t)\pi^*(s^t)((\pi^*(s^t)-1) &= \beta U_c^*(s^{t+1})\pi^*(s^{t+1})(\pi^*(s^{t+1})-1) - \\ & \frac{\varepsilon N^*(s^t)U_c^*(s^t)A^*(s^t)}{\theta}\left(\frac{U_{n^*}(s^t)}{U_c^*(s^t)A^*(s^t)} + \frac{\varepsilon-1}{\varepsilon}\right) \end{aligned} \tag{29}$$

At this point we can define the competitive symmetric equilibrium for the rest of the world as the one that occurs when all firms set the same prices.

Definition 1 *Given the initial value of wealth $B(s^0)$, the eovlution of the aggregate shock $\{A^*(s^t)\}_{t=0}^{\infty}$ and the state contingent policy $\{R^*(s^t)\}_{t=0}^{\infty}$, a symmetric competitive equilibirum for the rest of the world is given by the allocation $\{C^*(s^t),\ Y^*(s^t), N^*(s^t),\ mc^*(s^t), \pi^*(s^t)\}_{t=0}^{\infty}$ and a price system $\{P^*(s^t)$,$W^*(s^t)$,$\nu(s^{t+1} \mid s^t)\ \}_{t=0}^{\infty}$ that satisfies equations (22), (28), (29), (25), (27).*

3.2 Equilibrium in the Small Economy

We focus our attention on a *symmetric* equilibrium where all producers charge the same price, adopt the same technology and therefore choose the same demand for labor. This implies that

$$\widetilde{p}_H = 1 \tag{30}$$

$$N(i) = N \text{ all } i \tag{31}$$

Hence in equilibrium equation 20 will simplify to

$$\lambda(s^t)\pi_H(s^t)(\pi(s^t)-1) = \beta\lambda(s^{t+1})\pi_H(s^{t+1})(\pi_H(s^{t+1})-1) + \frac{\lambda(s^t)\varepsilon A(s^t)N(s^t)}{\theta}\left(mc(s^t) - \frac{\varepsilon-1}{\varepsilon}\right) \tag{32}$$

The total net supply of bonds must satisfy

$$B(s^t) + B^*(s^t) = 0 \tag{33}$$

Market clearing for the domestic variety i must satisfy

$$
\begin{aligned}
Y(i,s^t) &= C_H(i,s^t) + C_H^*(i,s^t) + PAC(i,s^t) \qquad (34)\\
&= \left(\frac{P_H(i,s^t)}{P_H(s^t)}\right)^{-\varepsilon}\left[\left(\frac{P_H(s^t)}{P(s^t)}\right)^{-\eta}(1-\alpha)C(s^t) + \left(\frac{P_H(s^t)}{\mathcal{E}(s^t)P^*(s^t)}\right)^{-\eta}\alpha^* Y^*(s^t)\right] + PAC(i,s^t)
\end{aligned}
$$

for all $i \in [0,1]$ and t.

In a symmetric equilibrium $\widetilde{p}_H(i,s^t) = 1$ and $PAC(i) = PAC = \frac{\theta}{2}\left(\pi_H(s^t) - 1\right)^2 \ \forall\ i$. This implies that plugging (34) into the definition of aggregate output $Y \equiv \left[\int_0^1 Y(i)^{1-\frac{1}{\varepsilon}}\, di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ we can express the resource constraint as

$$
Y(s^t) = \left[\left(\frac{P_H(s^t)}{P_t(s^t)}\right)^{-\eta}(1-\alpha)\ C(s_t) + \left(\frac{P_H(s^t)}{\mathcal{E}(s^t)P^*(s^t)}\right)^{-\eta}\alpha^* Y^*(s^t)\right] + \frac{\theta}{2}\left(\pi_H(s^t) - 1\right)^2 \qquad (35)
$$

We have now the elements to define *a symmetric competitive equilibrium.*

Definition 2 *Given the initial financial wealth $B(s^0)$, the evolution of the aggregate shock $\{A(s^t)$, $A^*(s^t)\}_{t=0}^{\infty}$, the set of allocations describing a symmetric equilibrium in the world economy $\{C^*(s^t)$, $Y^*(s^t)$, $R^{n*}(s^t)$, $\pi^*(s^t)$, $mc^*(s^t)$, $N^*(s^t)\}_{t=0}^{\infty}$ and the state contingent policy $\{R(s^t)\}_{t=0}^{\infty}$ a symmetric competitive equilibrium for the small open economy is a set of allocations $\{C(s^t)$, $Y(s^t)$, $N(s^t)$, $C_H(s^t)$, $C_F(s^t)$, $C_H^*(s^t)$, $mc(s^t)$, $Q(s^t)\}_{t=0}^{\infty}$, a price system $\{P_H(s^t)$, $P_F(s^t)$, $P(s^t)$, $W(s^t)$, $\nu(s^{t+1}|s^t)$, $\mathcal{E}(s^t)\}_{t=0}^{\infty}$ that solves the following equations:*

$$
C_H(i) = \left(\frac{P_H(i)}{P_H}\right)^{-\varepsilon} C_H;\ C_F(i) = \left(\frac{P_F(i)}{P_F}\right)^{-\varepsilon} C_F; \qquad (36)
$$

$$
C_H = (1-\alpha)\left(\frac{P_H}{P}\right)^{-\eta} C;\ C_F = \alpha\left(\frac{P_F}{P}\right)^{-\eta} C \qquad (37)
$$

$$
U_c(s^t)\frac{W(s^t)}{P(s^t)} = -U_n(s^t) \qquad (38)
$$

$$
\beta\rho(s^{t+1}|s^t)\frac{P(s^t)}{P(s^{t+1})}\frac{U_c(s^{t+1})}{U_c(s^t)} = \nu(s^{t+1}|s^t) \qquad (39)
$$

$$
R^n(s^t)^{-1} = \sum_{s_{t+1}} \nu(s^{t+1}|s^t) \qquad (40)
$$

$$
\lambda(s^t)\pi_H(s^t)(\pi_H(s^t)-1) = \beta\lambda(s^{t+1})\pi_H(s^{t+1})(\pi_H(s^{t+1})-1) + \frac{\lambda(s^t)\varepsilon A(s^t)N(s^t)}{\theta}\left(mc(s^t) - \frac{\varepsilon-1}{\varepsilon}\right) \qquad (41)
$$

$$\frac{W(s^t)}{P_H(s^t)} = mc(s^t)A(s^t) \tag{42}$$

$$Y(s^t) = \left[\left(\frac{P_H(s^t)}{P_t(s^t)}\right)^{-\eta}(1-\alpha)\ C(s_t) + \left(\frac{P_H(s^t)}{\mathcal{E}(s^t)P^*(s^t)}\right)^{-\eta}\alpha^* Y^*(s^t)\right] + \frac{\theta}{2}\left(\pi_H(s^t)-1\right)^2 \tag{43}$$

$$\lim_{j\to\infty}\sum_{s_{t+j}}\nu(s^{t+j}|s^t)B(s^{t+j}) = 0 \tag{44}$$

3.3 The Primal Form

As mentioned before the optimal policy is determined by a monetary authority that maximizes the discounted sum of utilities of all agents in the economy given the constraints of the competitive economy. Our next task is to select the relations that represent the relevant constraints for the central planner. This amounts to describing the competitive equilibrium in terms of a minimal set of relations involving only real allocations as in the primal approach described Lucas and Stokey (1983). Let's first proceed by defining a series of relationships linking real quantities with the relevant relative prices in our framework.

3.3.1 Primal Form of the Small Open economy.

The terms of trade is the relative price of imported goods:

$$\mathcal{T}(s^t) \equiv \frac{P_F(s^t)}{P_H(s^t)} = \frac{\mathcal{E}(s^t)P^*(s^t)}{P_H(s^t)} \tag{45}$$

Notice that we can write

$$\Phi(\mathcal{T}(s^t)) \equiv \frac{P(s^t)}{P_H(s^t)} = [(1-\alpha)+\alpha\mathcal{T}(s^t)^{1-\eta}]^{\frac{1}{1-\eta}} \tag{46}$$

Notice also that the terms of trade and the real exchange rate can be related as follows:

$$\begin{aligned}\mathcal{T}(s^t) &= Q(s^t)\Phi(\mathcal{T}(s^t)) \\ &= \left(\frac{(1-\alpha)Q(s^t)^{1-\eta}}{1-\alpha Q(s^t)^{1-\eta})}\right)^{\frac{1}{1-\eta}}\end{aligned} \tag{47}$$

By combining (47) with (13) we can finally relate the terms of trade to the ratio of marginal utilities of consumption as follows

$$\mathcal{T}(s^t) = \Omega\left(C(s^t),\ C^*(s^t)\right) \equiv [\frac{(1-\alpha)\left(\kappa\frac{U_c^*(s^t)}{U_c(s^t)}\right)^{1-\eta}}{1-\alpha\left(\kappa\frac{U_c^*(s^t)}{U_c(s^t)}\right)^{1-\eta}}]^{\frac{1}{1-\eta}} \tag{48}$$

The CPI level can be linked to the domestic price level and aggregate consumption as follows

$$\begin{aligned} P(s^t) &= P_H(s^t)\ g(\mathcal{T}(s^t)) \\ &= P_H(s^t)\ \Phi(C(s^t), C^*(s^t)) \end{aligned} \tag{49}$$

where $\Phi(C(s^t), C^*(s^t)) \equiv g(\Omega(\bullet))$.

Let's define gross CPI inflation as $\pi(s^t) \equiv \frac{P(s^t)}{P(s^{t-1})}$. This can be linked to domestic producer inflation and aggregate relative consumption as follows:

$$\pi(s^t) = \pi_H(s^t)\frac{\Phi(C(s^t), C^*(s^t))}{\Phi(C(s^{t-1}), C^*(s^{t-1}))} \tag{50}$$

The condition on optimal bond investment can then be rearranged accordingly. By taking conditional expectations of (7) we obtain

$$U_c(s^t) = \beta \sum_{s_{t+1}} \rho(s^{t+1}|s^t) R(s^t) U_c(s^{t+1}) \tag{51}$$

where

$$R(s^t) = \frac{R^n(s^t)P(s^t)}{P(s^{t+1})} \tag{52}$$

is the CPI-based gross real interest rate.

Next we need to rearrange the optimality condition for the production sector. This requires first to express the real marginal cost and the real wage in terms of aggregate real quantities. Hence by combining (38) and (42) we can write

$$mc(s^t) = -\frac{U_n(s^t)}{U_c(s^t)A(s^t)}\Phi(C(s^t), C^*(s^t)) \tag{53}$$

This implies that the aggregate condition for optimal pricing (41) can be rewritten as

$$\begin{aligned} U_c(s^t)\pi_H(s^t)((\pi_H(s^t)-1) &= \beta U_c(s^{t+1})\pi_H(s^{t+1})(\pi_H(s^{t+1})-1) + \\ &\quad \frac{U_c(s^t)\varepsilon A(s^t)N(s^t)}{\theta}\left(\frac{-U_n(s^t)}{U_c(s^t)A(s^t)}\Phi(C(s^t), C^*(s^t)) - \frac{\varepsilon-1}{\varepsilon}\right) \end{aligned} \tag{54}$$

Finally let's turn to the resource constraint (43). This can first be rewritten as

$$Y(s^t) = \mathcal{T}(s^t)^\eta\ \left[(1-\alpha)\ C(s^t)Q(s^t)^{-\eta} + \alpha^* Y^*(s^t)\right] + \frac{\theta}{2}\left(\pi_H(s^t)-1\right)^2 \tag{55}$$

By using (13) and (48) we can rewrite:

$$Y(s^t) = \Omega^{\eta}(C(s^t), C^*(s^t)) \ \left[(1-\alpha)\ C(s^t)(\frac{U_c^*(s^t)}{U_c(s^t)})^{-\eta} + \alpha^*\ Y^*(s^t)\right] + \frac{\theta}{2}\left(\pi_H(s^t) - 1\right)^2 \tag{56}$$

This last equation will represent the *feasibility constraint* in the Ramsey problem.

The following proposition establishes a mapping between the primal form expressed above and the set of allocations describing the competitive equilibrium as from Definition 1.

Proposition 1. **[Part A]** *Any competitive equilibrium allocation* $\{C(s^t),\ Y(s^t),\ N(s^t),\ C_H(s^t), C_F(s^t),\ mc(s^t),\ Q(s^t), \pi_H(s^t)\}_{t=0}^{\infty}$ *satisfying equations (36) through (44) also satisfies equations (54) and (56).***[Part B]** *Using allocations* $\{C(s^t),\ N(s^t),\ \pi_H(s^t)\}_{t=0}^{\infty}$ *that satisfy equations (54) and (56) it is possible to construct all the remaining real allocations, nominal variables and policy instruments.*

Proof. Appendix A.

3.3.2 Primal Form for the Rest of the World.

Using a similar argument one can prove the following proposition of the rest of the world.

Proposition 2. **[Part A]** *Any competitive equilibrium allocation* $\{C^*(s^t),\ Y^*(s^t),\ N^*(s^t),\ mc^*(s^t),\ \pi^*(s^t)\}_{t=0}^{\infty}$ *satisfying equations (22), (28), (29), (25), (27) also satisfies equations:*

$$U_c^*(s^t)\pi^*(s^t)((\pi^*(s^t)-1) + \frac{U_c^*(s^t)\varepsilon A^*(s^t)N^*(s^t)}{\theta}\left(\frac{U_n^*(s^t)}{U_c^*(s^t)A^*(s^t)} + \frac{\varepsilon - 1}{\varepsilon}\right) -$$
$$-\lambda_p^*(s^t)\beta U_c^*(s^{t+1})\pi^*(s^{t+1})(\pi^*(s^{t+1})-1) = 0 \tag{57}$$

$$A^*(s^t)N^*(s^t) - C^*(s^t) - \frac{\theta}{2}\left(\pi^*(s^t) - 1\right)^2 \tag{58}$$

[Part B] *Using allocations* $\{C^*(s^t),\ N^*(s^t),\ \pi^*(s^t)\}_{t=0}^{\infty}$ *that satisfy equations (57) and (58) it is possible to construct all the remaining real allocations, nominal variables and policy instruments.*

4 Ramsey Monetary Policy

Having defined the constraints that the monetary authority faces in maximizing welfare we can now turn to define the general set-up for the optimal monetary policy design under commitment. Next we will study the problem in the long run and in the short run.

4.1 The Small Open Economy

In the following we formally define the Ramsey problem for the small open economy.

Definition 1 *Let* $\lambda_p(s^t)$ *and* $\lambda_f(s^t)$ *represent the Lagrange multipliers on the constraints (54) and (56). For given optimal allocations* $\{A(s^t),\ C^*(s^t), A^*(s^t)\}_{t=0}^{\infty}$*, plans for the control variables*

$\{C(s^t),\ \pi_H(s^t),\ N(s^t)\}_{t=0}^{\infty}$ *and for the costate variables* $\{\lambda_p(s^t), \lambda_f(s^t)\}_{t=0}^{\infty}$ *represent a first best constrained allocation if they solve the following maximization problem:*

Choose $\{C(s^t),\ \pi_H(s^t),\ N(s^t)\}_{t=0}^{\infty}$ and $\{\lambda_p(s^t), \lambda_f(s^t)\}_{t=0}^{\infty}$ to

$$Max \sum_{t=0}^{\infty} \sum_{s^t} \rho(s^t)\beta^t \{[U(C(s^t), N(s^t))] +$$

$$\lambda_p(s^t)[U_c(s^t)\pi(s^t)((\pi(s^t)-1) - \beta U_c(s^{t+1})\pi(s^{t+1})(\pi(s^{t+1})-1) +$$
$$+\frac{U_c(s^t)\varepsilon A(s^t)N(s^t)}{\theta}\left(\frac{U_n(s^t)\ \Phi(C(s^t), C^*(s^t))}{U_c(s^t)A(s^t)} + \frac{\varepsilon - 1}{\varepsilon}\right)]$$

$$+\lambda_f(s^t)[A(s^t)N(s^t) - \Omega^{\eta}(C(s^t), C^*(s^t))\ \left[(1-\alpha)\ C(s^t)(\frac{U_c^*(s^t)}{U_c(s^t)})^{-\eta} + \alpha^* Y^*(s^t)\right] + \frac{\theta}{2}\left(\pi_H(s^t) - 1\right)^2]\}$$

Appendix A shows first order conditions for this plan.

4.2 Foreign Economy

Let us define a stationary Ramsey problem for the foreign economy.

Definition 2 *Let* $\lambda_p^*(s^t)$ *and* $\lambda_f^*(s^t)$ *represent the Lagrange multipliers on the constraints (29) and (25). For given allocations* $\{A^*(s^t)\}_{t=0}^{\infty}$*, plans for the control variables* $\{C^*(s^t),\ \pi^*(s^t),\ N^*(s^t)\}_{t=0}^{\infty}$ *and for the costate variables* $\{\lambda_p^*(s^t), \lambda_f^*(s^t)\}_{t=0}^{\infty}$ *represent a first best constrained allocation if they solve the following maximization problem:*

Choose $\{C^*(s^t),\ \pi^*(s^t),\ N^*(s^t)\}_{t=0}^{\infty}$ and $\{\lambda_p^*(s^t), \lambda_f^*(s^t)\}_{t=0}^{\infty}$ to

$$Max \sum_{t=0}^{\infty} \sum_{s^t} \rho(s^t)\beta^t \{[U(C^*(s^t), N^*(s^t))]$$

$$\lambda_p^*(s^t)\left\{U_c^*(s^t)\pi^*(s^t)((\pi^*(s^t)-1) + \frac{U_c^*(s^t)\varepsilon A^*(s^t)N^*(s^t)}{\theta}\left(\frac{U_n^*(s^t)}{U_c^*(s^t)A^*(s^t)} + \frac{\varepsilon-1}{\varepsilon}\right)\right\}$$
$$-\lambda_p^*(s^t)\beta U_c^*(s^{t+1})\pi^*(s^{t+1})(\pi^*(s^{t+1})-1)$$
$$+\lambda_f^*(s^t)\left\{A^*(s^t)N^*(s^t) - C^*(s^t) - \frac{\theta}{2}\left(\pi^*(s^t)-1\right)^2\right\}$$

Appendix (**??**) shows first order conditions for this plan.

4.3 Optimal Monetary Policy in the Long-Run

It is instructive to begin the analysis of the optimal monetary policy problem by illustrating how the different distortions should be managed in order to achieve the efficient steady state.

4.4 Efficient Steady-State in the Foreign Economy

The Foreign economy is a prototypical sticky-price economy with imperfect competition. As such it is characterized by two distortions that drive a wedge between the first-best efficient allocation and the constrained Pareto optimum allocation: the rigidity of prices (which generates an output adjustment cost) and the presence of market power (which renders the long-run level of output inefficiently low).

Our discussion at this stage follows King and Wolman (1998). However while they introduce the nominal rigidity in the form of price staggering a la Calvo (which produces a distortion in terms of relative price misalignment) we postulate quadratic adjustment (output) costs a la Rotemberg (1982). Hence in our framework, unlike in the Calvo setup, all firms get the opportunity of adjusting the price at each point in time, but they have to do so by optimally balancing marginal revenues and marginal costs.

A characterization of the efficient steady state requires an appropriate weighing of the two distortions. The cost-of-adjusting-price distortion, summarized by the quadratic term $\frac{\theta}{2}(\pi^* - 1)^2$ in the resource constraint, is obviously minimized at zero net inflation (i.e., $\pi^* = 1$). In order to pin down the rate of inflation that minimizes the markup distortion let's consider the steady-version of the Phillips curve (29), which can be interpreted (in a symmetric equilibrium) as the optimal pricing condition for a representative imperfectly competitive firm. Recall that in our economy the marginal cost is the same across firms and hence the markup is the inverse of the real marginal cost. By considering this equation (29) can be written:

$$\pi^*(\pi^* - 1)(1 - \beta) = \frac{\varepsilon N^* A^*}{\theta}\left(\frac{1}{\mu^*} - \frac{\varepsilon - 1}{\varepsilon}\right) \tag{59}$$

where μ^* is the steady-state markup. Rearranging the above expression one can derive the following relationship between markup and inflation:

$$\mu^*(\pi^*, N^*) = \frac{\varepsilon N^*}{\theta\pi^*(\pi^* - 1)(1 - \beta) + (\varepsilon - 1)N^*} \tag{60}$$

Hence one notice that the particular case of constant steady-state markup and equal to $\frac{\varepsilon}{\varepsilon - 1}$ obtains either when $\theta = 0$ (pure flexible prices) or when $\pi^* = 1$, i.e. when net inflation is zero.

The discussion so far points to the critical tension between the price-stickiness distortion and the markup distortion that characterizes the steady state of the Foreign (closed) economy. While the presence of a price adjustment cost (which is convex in the gross rate of inflation) calls for setting a long-run inflation rate of zero, the presence of market power requires setting a positive rate of inflation.[2]

Therefore we have the following proposition.

[2] See King and Wolman (1998).

Proposition 3. *In a closed economy with price adjustment costs and imperfect competition the efficient steady state must be characterized by a positive rate of inflation,i.e.,* $\pi^* > 1$.

Proof. See Appendix

The policy that will implement such efficient rate of inflation will result from maximizing utility subject to two constraints: the steady-state version of (25), which is the relevant feasibility constraint, and equation (59), which is the relevant implementability constraint. Appendix describes this problem formally.

To derive an intuition one can set $\mu^*(\pi^*, N^*) = 1$ from (60) and derive the rate of inflation that is consistent with efficiency as

$$\overline{\pi}^*(\overline{\pi}^* - 1) = \frac{\overline{N}^*}{\theta(1-\beta)} \tag{61}$$

where a bar denotes a variable evaluated in the efficient steady state. Hence we see that unless the price adjustment cost is infinite (i.e., $\theta \to \infty$) it must be that $\overline{\pi}^* > 1$. Intuitively while the price rigidity distortion drives down the desired rate of inflation ($\overline{\pi}^*$ is decreasing in θ) the markup distortion pushes the tension in the opposite direction. Hence the rate of inflation that maximizes households' welfare must be positive but lie below the one that would minimize the markup distortion.[3]

An equivalent, yet useful, way to look at the tension between market power and price stickiness is to recall that we can write the inverse of the real marginal cost function, i.e., the *steady-state markup function*, as

$$\mu^*(A^*, C^*, N^*) = \left(\frac{-U_n^*(N^*)}{U_c^*(C^*)}\right)^{-1} \tag{62}$$

Efficiency requires that

$$\mu^*(A^*, C^*, N^*) = 1 \tag{63}$$

In a steady state with zero inflation, which implies a constant markup $\mu^* = \frac{\varepsilon}{\varepsilon - 1} > 1$, the above condition is clearly not satisfied.

[3]This proposition calls for a further clarification. Notice that the μ is a concave function of π^*, with $\frac{\partial \mu}{\pi^*} > (<) 0$ for $\pi^* < (>) \frac{1}{2}$. Hence technically the markup could be minimized also for extremely high (yet unrealistic) rates of deflation. The form of the relationship bewteen steady-state markup and inflation is specific to the kind of (convex) price adjustment cost a la Rotemberg that we have introduced. King and Wolman (1998) conduct the same analysis but in a context with price adjustment costs a la Calvo. In that case the relationship between steady-state markup and inflation is convex, with the markup rising for sufficiently high levels of inflation. This is the result of inflation affecting the relative price of adjusting vs. non adjusting firms, with higher inflation inducing a higher markup for the adjusting firm but eroding the one of the non-adjusting firms. Recall that with a Rotemberg-type cost of changing prices all firms get to revise the price at each point in time. However, and unlike in Calvo's model, this happens only partially, or in other words in such a way that the marginal cost is equated to the marginal revenue.

4.5 Efficient Steady-State in the Small Open Economy

What distinguishes the analysis of the long-run optimal policy in the small economy is the presence of an additional distortion that is able to drive a wedge in the marginal rate of substitution between consumption and leisure. This distortion derives from *consumer* prices being different from *producer* prices in an open economy, a wedge that, as such, creates the possibility of affecting the marginal rate of substitution via the *international relative prices* (terms of trade).

To understand this it is instructive to derive the markup function from the competitive equilibrium of the small open economy. By combining the steady-state version of (38) and (42) we can write

$$\frac{-U_n(C)}{U_n(N)} = \frac{1}{\mu}\frac{P_H}{P} = \frac{1}{\mu(\pi_H, N)}\left[\Phi(\mathcal{T})\right]^{-1}$$

where, similarly to (60), $\mu(\pi_H, N)$ derives from the steady-state version of (41) as

$$\mu(\pi_H, N) = \frac{\varepsilon N}{\theta\pi_H(\pi_H - 1)(1-\beta) + (\varepsilon - 1)N} \tag{64}$$

Hence long-run efficiency in the small open economy requires

$$\mu(\pi_H, N)\Phi(\mathcal{T}) = 1 \tag{65}$$

To understand how the terms of trade distortion (summarized by the term $\Phi(\mathcal{T})$ which is increasing in $\mathcal{T}$) interacts with the markup distortion, let's assume for the sake of argument that prices are fully flexible, so that the markup is always equal to a constant value of $\frac{\varepsilon}{\varepsilon-1}$. The only distortions left interacting are therefore the ones summarized by the markup and the international relative prices. By making use of (46) we can rewrite a relationship between the desired terms of trade and the markup which reads

$$\mathcal{T} = \left(\frac{\alpha}{\mu^{\eta-1} - (1-\alpha)}\right)^{\frac{1}{\eta-1}} \tag{66}$$

Hence we see that, in the efficient steady state, a higher markup calls for more depreciated terms of trade (see *Figure 1)*. The intuition is simple. The presence of imperfectly competitive output markets makes desirable to expand output towards the perfectly competitive efficient level. While this is always welfare improving in an closed economy with flexible prices, in an open economy a rise in output relative to the rest of the world requires (in equilibrium) also a *depreciation* of the terms of trade, which hurts the purchasing power of domestic consumers. Equation (66) shows that, at the margin, it is optimal to have the terms of trade *depreciate less,* or equivalently let output expand less relatively to the case of a (imperfectly competitive) closed economy.

On the other hand such relative appreciation of the terms of trade has consequences on the inflation rate associated to the efficient steady state of the small open economy. Consider the expression for the real marginal cost (53). Given that Φ is increasing in $\mathcal{T}$, and for given consumption and labor, a relative appreciation of the terms of trade is associated to lower steady-state real marginal cost and inflation. Intuitively, then, the steady state of the small open economy should be associated with lower inflation relatively to the one of the corresponding closed economy characterized by the same distortions.

To formalize this argument we can define the efficient steady state of the small open economy as the triplet

$$\{\overline{\pi}_H, \overline{C}, \overline{N}\} = \arg\max\{U(\overline{C}, \overline{N})\}$$

subject to an optimal pricing-implementability condition

$$\overline{\pi}_H(\overline{\pi}_H - 1)(1-\beta) = \frac{\varepsilon \overline{N}}{\theta}\left(\frac{-U_n(\overline{N})\Phi(\overline{C}, C^*)}{U_c(\overline{C})} - \frac{\varepsilon - 1}{\varepsilon}\right) \tag{67}$$

and to a feasibility constraint

$$N = \Phi(\overline{C}, C^*)^{\eta}\left(\frac{\kappa U_c^*(C^*)}{U_c^*(C^*)}\right)^{\eta}\left[(1-\alpha)\ C(s^t)\left(\frac{\kappa U_c^*(C^*)}{U_c^*(C^*)}\right)^{-\eta} + \alpha^* C^*\right] + \frac{\theta}{2}\left(\pi_H - 1\right)^2 \tag{68}$$

where it is understood that C^* is taken as exogenous.

Appendix (.) shows the first order conditions of this problem. *Figures 2 and 3* display selected variables in the efficient steady state as a function of a key parameter, the degree of openness α. Hence we see that for $\alpha = 0$ the rate of (producer) inflation that maximizes welfare coincides exactly with the one of the corresponding closed economy with sticky prices and monopolistic competition. As α turns positive, i.e., the economy becomes open, the desired steady-state inflation rate decreases below the one of the corresponding closed economy.

Therefore we have the following proposition:

Proposition 4. *In a small open economy with price adjustment costs and monopolistic competition the (producer) inflation rate in the efficient steady-state lies below the one of the corresponding closed economy.*

Figure 2 shows that the efficient inflation rate reaches a minimum for an intermediate value of $\alpha = \frac{1}{2}$. At this degree of openness the optimal rate of inflation is negative. The intuition lies in the specific structure of our world economy. For $\alpha \to 0$ the domestic economy is closed to international trade, while for $\alpha \to 1$ it mimics exactly the dynamic of the rest of the world. In the latter case no variation in international relative prices is necessary in equilibrium to achieve a reallocation of consumption across countries.

The interesting aspect that the openness dimension brings about is that while in a closed economy with sticky prices and monopolistic competition a zero-inflation steady state is never optimal it can indeed be optimal when the economy is open. This is the result of the international relative price distortion pushing the inflation rate in the efficient steady state below the one implied by the corresponding closed economy.

To summarize this result we have the following proposition.

Proposition 5. *The (producer) inflation rate in the efficient steady state of the small open economy is concave in the degree of openness. Hence there exist multiple values of α such that a zero-inflation steady-state is also the efficient one.*

5 Ramsey Policy and Short-Run Dynamics

We move now to characterize numerically the dynamic pattern of Ramsey allocations in response to a productivity shock. To derive an accurate description of the dynamics under the optimal allocation we solve the non-linear system of the first order conditions using *perturbation methods* as described in Sims (2000), Kollman (2002) and Schmitt-Grohe and Uribe (2002). In practice we compute dynamics by solving second-order logarithmic approximation to the Ramsey policy function around a non-stochastic Ramsey steady state. We define as *constrained social optimum* the solution to the system of first order conditions of the Ramsey plan evaluated at the non-stochastic steady state .

Indeed notice that since the monetary authority is maximizing welfare given the distortions in the economy the solution to the planning problem will be Pareto superior to the solution of the competitive economy but Pareto inferior to the one of the efficient allocation. Such a second best allocation is usually defined as *constrained social optimum.*

To show that the long run solution to the constrained social optimum differs from the one of the efficient allocation one can verify that for both economies, the rest of the world and the small open economy, the system of first order conditions of the Ramsey plan evaluated at the non-stochastic steady state is satisfied only at a zero inflation equilibrium. On the other side the efficient solution characterized before is associated to a positive long run level of inflation.

5.1 Calibration

The time unit is meant to be quarters. We assign values for the key parameters and for long run values of variables for the small open economy using as reference some papers in the open economy literature. Preference parameters are assigned as in the Gali' and Monacelli (2002). The elasticity of consumption in the utility function is 1, the inverse elasticity of labor supply is 3. The elasticity of substitution between domestic and foreign goods is 1.5. The degree of price stickiness is taken from Schmitt-Grohe and Uribe (2002). The value of the parameters in the adjustment cost function

is 17.5. The elasticity between varieties produced by the monopolistic sector is 6. The degree of openness identified by the steady ate ration of exports over GDP is set to 0.4. The same values for preferences and technology are assigned to the rest of the world.

5.2 Responses to a Productivity Shock in the Rest of the World under the Ramsey Plan

The dynamic of the policy functions is derived by second order approximations around the constrained pareto optimum. We proceed as follows. We first simulate the Ramsey allocation - i.e. optimal policy - for the rest of the wrold and then simulate the Ramsey allocation for the small open economy taking as given the optimal allocation for the rest of the world.

Figure 4 shows impulse responses of Foreign variables to a positive productivity shock in the Rest of the World. We As in King and Wolman (1998), and as expected, movements in inflation are negligible. Indeed the rest of the world behaves as a closed economy. That implies that we do expect inflation under the Ramsey allocation for the rest of the world to behave as inflation would behave in a closed economy.

(more to be added)

5.3 Responses to a Productivity Shock in the Small Open Economy under the Ramsey Plan (to be added)

6 Conclusions (to be added)

A Proof of Proposition 1

The proof of part A just follows from the substitutions and the rearrangements done in section 3 in order to reduce the number of constraints. Notice that the foreign variables are taken as given and their dynamics will be given by the solution of the competitive equilibrium of the foreign country for a given policy.

As for part B, given the sequence $\{C(s^t), N(s^t), \pi_H(s^t)\}$ that satisfy constraints (54) and (56), it is possible to construct all of the remaining equilibrium (real) allocation, price and policy variables for given $\{C^*(s^t), Y^*(s^t), \pi^*(s^t)\}$.

For a given productivity process $A(s^t)$ and allocation for labor $N(s^t)$ satisfying the Ramsey problem we can obtain the optimal allocation for output the real marginal cost and the real wage:

$$A(s^t)N(s^t) = Y(s^t) \tag{69}$$

$$mc(s^t) = A(s^t)U_n(N(s^t)) \tag{70}$$

$$\frac{U_n(N(s^t))}{U_c(C(s^t))} = \frac{W(s^t)}{P(s^t)} \tag{71}$$

We can then use equations (13) and (47) to obtain the optimal pattern of the real exchange rate and the terms of trade, and in turn solve for the relative prices:

$$\left(\frac{P(s^t)}{P_H(s^t)}\right)^{1-\eta} = \left((1-\alpha) + \alpha\ \mathcal{T}(s^t)^{\ 1-\eta}\right) \tag{72}$$

$$\left(\frac{P(s^t)}{P_F(s^t)}\right)^{1-\eta} = \left(\frac{(1-\alpha)}{\mathcal{T}(s^t)^{\ 1-\eta}} + \alpha\right) \tag{73}$$

Optimal demands for domestic and foreign goods can be obtained from:

$$C_H(s^t) = (1-\alpha)(\frac{P_H(s^t)}{P(s^t)})^{-\eta}(C(s^t)); C_F(s^t) = \alpha(\frac{P_F(s^t)}{P(s^t)})^{-\eta}(C(s^t)) \tag{74}$$

The optimal program for CPI inflation $\pi(s^t)$ follows from equation (50). It is then possible to recover the CPI based real interest rate from equation (52).

In turn, the optimal nominal interest rate set by the monetary authority obtains from:

$$R^n(s^t) = \frac{R(s^t)}{\Pi(s^t)}$$

with $\Pi(s^t) = \frac{P(s^t)}{P(s^{t-1})}$.

B Efficient Steady State

B.1 Efficient steady-state in the Foreign economy

Let's define λ_p^* and λ_f^* as the Lagrange multipliers associated with the steady-state implementability and feasibility constraint respectively Hence we can set up the Lagrangian

$$\begin{aligned}\mathcal{L}^* &= U(C^*, N^*) + \lambda_p^* \left\{ \pi^*(\pi^* - 1)(1-\beta) + \frac{\varepsilon N^*}{\theta} \left(\frac{-U_n^*(N^*)}{U_c^*(C^*)} - \frac{\varepsilon - 1}{\varepsilon} \right) \right\} \\ &\quad + \lambda_f^* \left\{ N^* - C^* - \frac{\theta}{2} (\pi^* - 1)^2 \right\}\end{aligned}$$

First order necessary conditions for optimality read as follows:

- C :

$$U_c^* + \lambda_p^* \frac{\varepsilon N^* U_n^*}{\theta} \frac{U_{cc}^*}{(U_c^*)^2} - \lambda_f^* = 0 \tag{75}$$

- N :

$$U_n^* - \lambda_p^* \frac{\varepsilon}{\theta U_c^*} (U_n^* + N^* U_{nn}^*) - \lambda_p^* \frac{\varepsilon - 1}{\theta} + \lambda_f^* = 0 \tag{76}$$

- π^* :

$$(1-\beta)(2\pi^* - 1)\lambda_p^* - \lambda_{f,t}^* \theta (\pi_t^* - 1) = 0 \tag{77}$$

- λ_p^* :

$$\pi^*(\pi^* - 1)(1-\beta) + \frac{\varepsilon N^*}{\theta} \left(\frac{-U_n^*}{U_c^*} - \frac{\varepsilon - 1}{\varepsilon} \right) = 0 \tag{78}$$

- λ_f^* :

$$N - C^* - \frac{\theta}{2} (\pi^* - 1)^2 = 0 \tag{79}$$

Solving the above first order conditions gives the inflation rate that minimizes all distortions in the economy.

B.2 Efficient Steady State in the Small Open Economy

Let λ_p and λ_f be the Lagrange multipliers associated with the steady-state implementability and feasibility constraint respectively. Hence we can set up the Lagrangian

$$
\begin{aligned}
\mathcal{L} \;=\; & U(C,N) + \lambda_p \left\{ \pi(\pi-1)(1-\beta) + \frac{\varepsilon N}{\theta} \left(\frac{-U_n(N^*)\Phi(C,C^*)}{U_c(C^*)} - \frac{\varepsilon-1}{\varepsilon} \right) \right\} \\
& + \lambda_f \left\{ N - \Phi(\overline{C},C^*)^{\eta} \left(\frac{\kappa U_c^*(C^*)}{U_c^*(C^*)} \right)^{\eta} \left[(1-\alpha)\; C(s^t) \left(\frac{\kappa U_c^*(C^*)}{U_c^*(C^*)} \right)^{-\eta} + \alpha^* C^* \right] - \frac{\theta}{2} \left(\pi_H - 1\right)^2 \right\}
\end{aligned}
$$

First order necessary conditions for optimality read as follows:

$C:$

$$
\begin{aligned}
& U_c + \lambda_p \frac{\varepsilon N U_n}{\theta} \left(\Phi_c U_c^{-1} - \Phi U_c^{-2} U_{cc} \right) - \lambda_f (1-\alpha) \left(\eta \Phi^{\eta-1} \Phi_c C + \Phi^{\eta} \right) - \\
& -\lambda_f \alpha^* C^* \kappa^{\eta} U_c^{*\eta} (\eta \Phi^{\eta-1} \Phi_c U_c^{-\eta} - \eta U_c^{-\eta-1} U_{cc} \Phi^{\eta}) = 0
\end{aligned} \tag{81}
$$

$N:$

$$
U_n - \lambda_p \frac{\varepsilon A \Phi}{\theta U_c} (U_n + N U_{nn}) - \lambda_p \frac{\varepsilon - 1}{\theta} + \lambda_f = 0 \tag{82}
$$

$\pi_{H:}$

$$
\lambda_p (1-\beta)(2\pi_H - 1) - \lambda_f \theta (\pi_H - 1) = 0 \tag{83}
$$

$\lambda_p:$

$$
\pi_H (\pi_H - 1)(1-\beta) + \frac{\varepsilon N U_N \Phi}{\theta U_c} + \frac{AN(\varepsilon - 1)}{\theta} = 0 \tag{84}
$$

$\lambda_f:$

$$
AN - \kappa^{\eta}(1-\alpha) C \Phi^{\eta} - \alpha^* C^* \Phi^{\eta} \kappa^{\eta} U_c^{*\eta} U_c^{-\eta} - \frac{\theta}{2} \left(\pi_H - 1\right)^2 = 0 \tag{85}
$$

C The Ramsey Problem

Notice that the first constraint of the Ramsey problem bears future expectations on inflation. This renders the maximization problem inherently non-stationary since the set of policy equations is constrained by future expectations. Time inconsistency arises according to the definition of Kydland and Prescott (1980). If the state space is expanded to include some pseudo co-state variables the solution to the new problem is stationary in the new state space. We transform the problem using the recursive approach methodology as defined in Marcet and Marimon (2000).[4] The recursive approach converts the non stationary problem in a recursive saddle point problem.

[4] Other important applications of this approach are Kehoe and Perri (2000), Kahn, King and Wolman (2000).

C.1 First Order Conditions of the Ramsey Problem in the Small Open Economy

In the following, and for the sake of notation, we abstract from indexing variables with the state s^t. Let us define $\Omega_t \equiv \Omega(C(s^t), C^*(s^t))$, $\Phi_t \equiv \Phi(C(s^t), C^*(s^t))$, $\Omega_{c,t} \equiv \frac{\partial \Omega(C(s^t),C^*(s^t))}{\partial C(s^t)}$, $\Phi_{c,t} \equiv \frac{\partial \Phi(C(s^t),C^*(s^t))}{\partial C(s^t)}$, $U_{c,t} \equiv \frac{\partial U(C(s^t),N(s^t))}{\partial C(s^t)}, U_{n,t} \equiv \frac{\partial U(C(s^t),N(s^t))}{\partial N(s^t)}$, $U_{cc,t} \equiv \frac{\partial U_{c,t}}{\partial C(s^t)}$, $U_{nn,t} \equiv \frac{\partial U_{n,t}}{\partial N(s^t)}$, $\delta_t^* \equiv \left(\kappa U_c^*(s^t)\right)^{1-\eta}$. In particular Ω_t is given by (48) while the expression for Φ_t is given by

$$\Phi_t \equiv \left((1-\alpha) + \frac{\alpha(1-\alpha)\left(\kappa \frac{U_c^*(s^t)}{U_c(s^t)}\right)^{1-\eta}}{1-\alpha\left(\kappa \frac{U_c^*(s^t)}{U_c(s^t)}\right)^{1-\eta}} \right)^{\frac{1}{1-\eta}}$$

It follows that

$$\Phi_{c,t} = -\frac{(1-\alpha)^{\frac{1}{1-\eta}} \alpha U_{c,t}^{\eta-2} U_{cc,t}}{(1-\alpha\kappa_t^* U_{c,t}^{\eta-1})^{\frac{2-\eta}{1-\eta}}}$$

First order conditions for the Ramsey problem in the small open economy read as follow:

- (C_t) :

$$\begin{aligned} U_{c,t} + U_{cc,t}\ \pi_{H,t}(\pi_{H,t}-1)\ (\lambda_{p,t} - \lambda_{p,t-1}) + \lambda_{p,t}\frac{\epsilon N_t\ U_{n,t}\Phi_{c,t}}{\theta} \\ + \frac{(\varepsilon-1)\lambda_{p,t}A_tN_t}{\theta} - \lambda_{f,t}\ \kappa_t^*(1-\alpha)\Omega_t{}^{\eta}U_{c,t}^{\eta}\left(1+\eta C_t\ \left(\Omega_t^{-1}\Omega_{c,t} + U_{c,t}^{-1}U_{cc,t}\right)\right) \\ -\lambda_{f,t}\alpha\theta Y_t^*\eta\Omega_t^{\eta-1}\Omega_{c,t} = 0 \end{aligned} \tag{86}$$

- (N_t) :

$$U_{n,t} + \frac{\lambda_{p,t}U_{c,t}\varepsilon A_t}{\theta}\left(\frac{\Phi_t(U_{n,t}+N_tU_{nn,t})}{U_{c,t}A_t} + \frac{\varepsilon-1}{\varepsilon}\right) + \lambda_{f,t}A_t = 0 \tag{87}$$

- $(\pi_{H,t})$:

$$U_{c,t}\ (2\pi_{H,t}-1)\ (\lambda_{p,t} - \lambda_{p,t-1}) - \lambda_{f,t}\theta(\pi_{H,t}-1) = 0 \tag{88}$$

- $(\lambda_{p,t})$:

$$\begin{aligned} U_{c,t}\ \pi_{H,t}(\pi_{H,t}-1) = \beta U_{c,t}\ \pi_{H,t+1}(\pi_{H,t+1}-1) \\ + \frac{U_{c,t}\varepsilon A_tN_t}{\theta}\left(\frac{U_{n,t}\Phi_t}{U_{c,t}A_t} + \frac{\varepsilon-1}{\varepsilon}\right) \end{aligned} \tag{89}$$

- $(\lambda_{f,t})$:

$$A_tN_t = \Omega_t^{\eta}\left[C_t\ (1-\alpha)\ \kappa_t^*U_{c,t}^{\eta} + \alpha^*\ Y_t^*\right] + \frac{\theta}{2}\left(\pi_{H,t}-1\right)^2 = 0 \tag{90}$$

C.2 First Order Conditions of the Ramsey Problem in the Foreign Economy

Let's define $U^*_{c,t} \equiv \frac{\partial U(C^*(s^t),N^*(s^t))}{\partial C^*(s^t)}, U^*_{n,t} \equiv \frac{\partial U(C^*(s^t),N^*(s^t))}{\partial N^*(s^t)}$, $U^*_{cc,t} \equiv \frac{\partial U^*_{c,t}}{\partial C^*(s^t)}$, $U^*_{nn,t} \equiv \frac{\partial U^*_{n,t}}{\partial N^*(s^t)}$. The first order conditions of the Ramsey problem in the Foreign economy are as follows:

- (C^*_t) :

$$U^*_{c,t} + U^*_{cc,t}\ \pi^*_t(\pi^*_t - 1)[\lambda^*_{p,t} - \lambda^*_{p,t-1}] + \lambda^*_{p,t}\left(\frac{\varepsilon - 1}{\theta}\right) A^*_t N^*_t U^*_{cc,t} - \lambda^*_{f,t} = 0$$

- (N^*_t) :

$$U^*_{n,t} + \frac{\lambda^*_{p,t}\varepsilon}{\theta}[U^*_{n,t} + \frac{\varepsilon - 1}{\varepsilon} A^*_t U^*_{c,t} + U^*_{nn,t} N^*_t] + \lambda^*_{f,t} A^*_t = 0 \tag{91}$$

- (π^*_t) :

$$U^*_{c,t}(2\pi^*_t - 1)[\lambda^*_{p,t} - \lambda^*_{p,t-1}] - \lambda^*_{f,t}\theta(\pi^*_t - 1) = 0 \tag{92}$$

- $(\lambda^*_{p,t})$:

$$U^*_{c,t}\ \pi^*_t(\pi^*_t - 1) = \beta U^*_{c,t+1}\pi^*_{t+1}(\pi^*_{t+1} - 1) - \frac{\varepsilon U^*_{c,t} A^*_t N^*_t}{\theta}\left(\frac{U^*_{n,t}}{U^*_{c,t} A^*_t} + \frac{\varepsilon - 1}{\varepsilon}\right) \tag{93}$$

- $(\lambda^*_{f,t})$:

$$A^*_t N^*_t = C^*_t + \frac{\theta}{2}\left(\pi^*_t - 1\right)^2 \tag{94}$$

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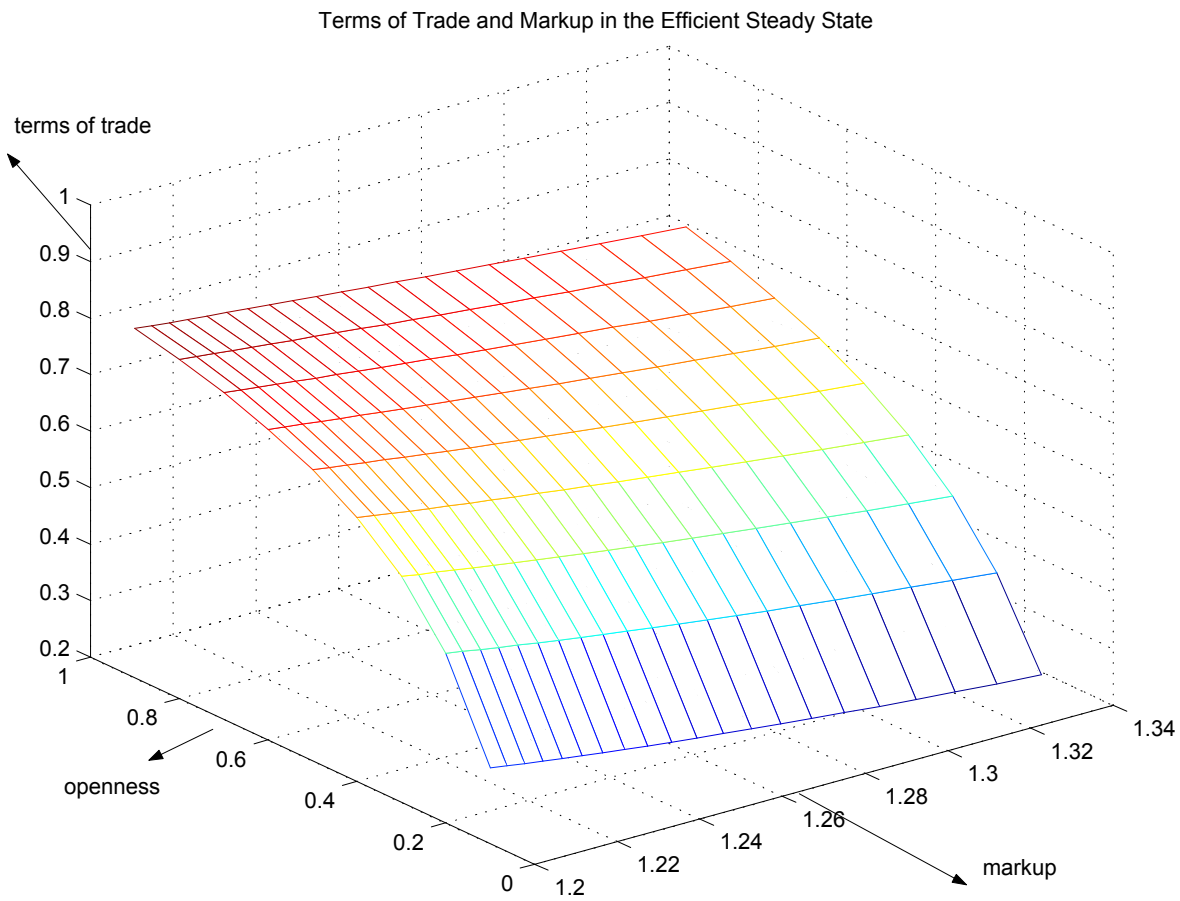

Figure 1: **Terms of Trade and Markup in the Efficient Steady-State**

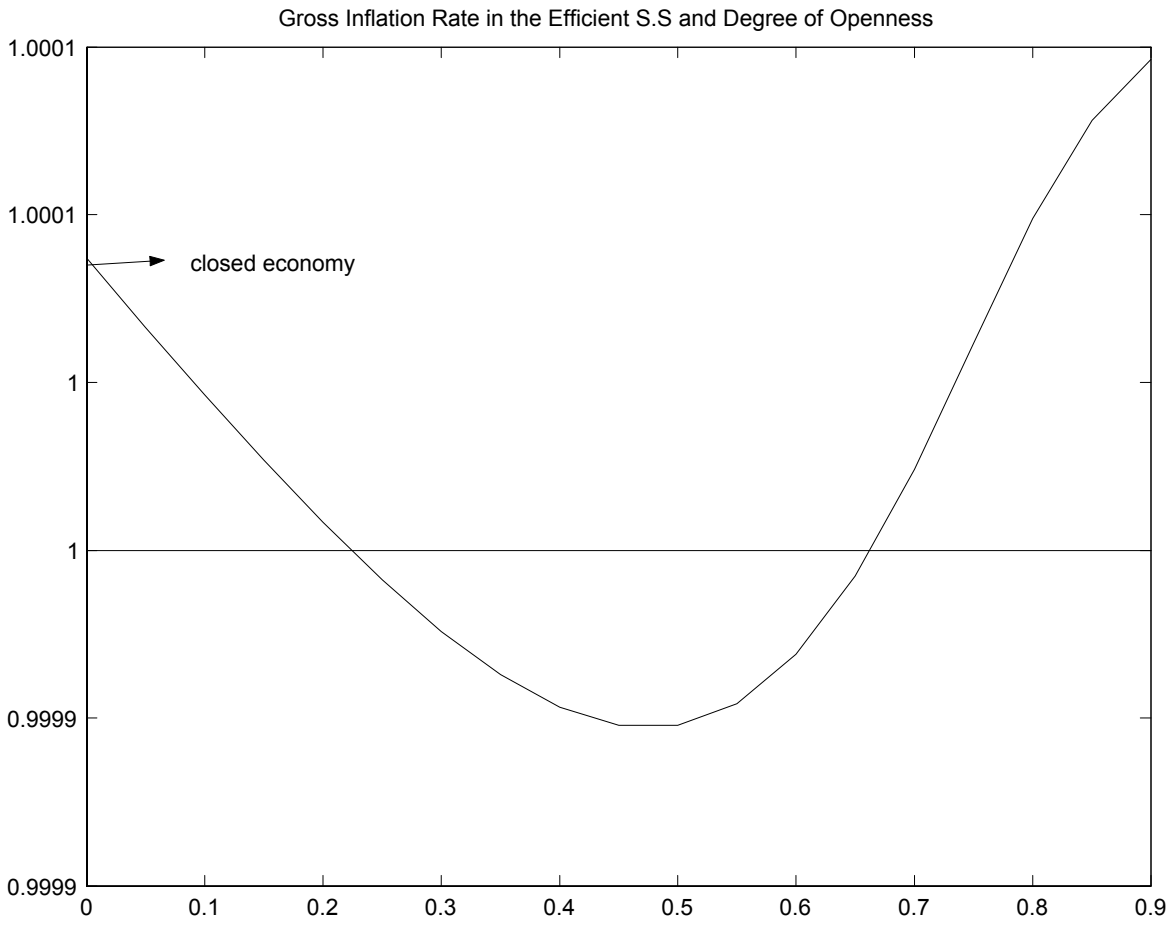

Figure 2: **Openness and Gross Inflation Rate in the Efficient Steady-State of the Small Open Economy**

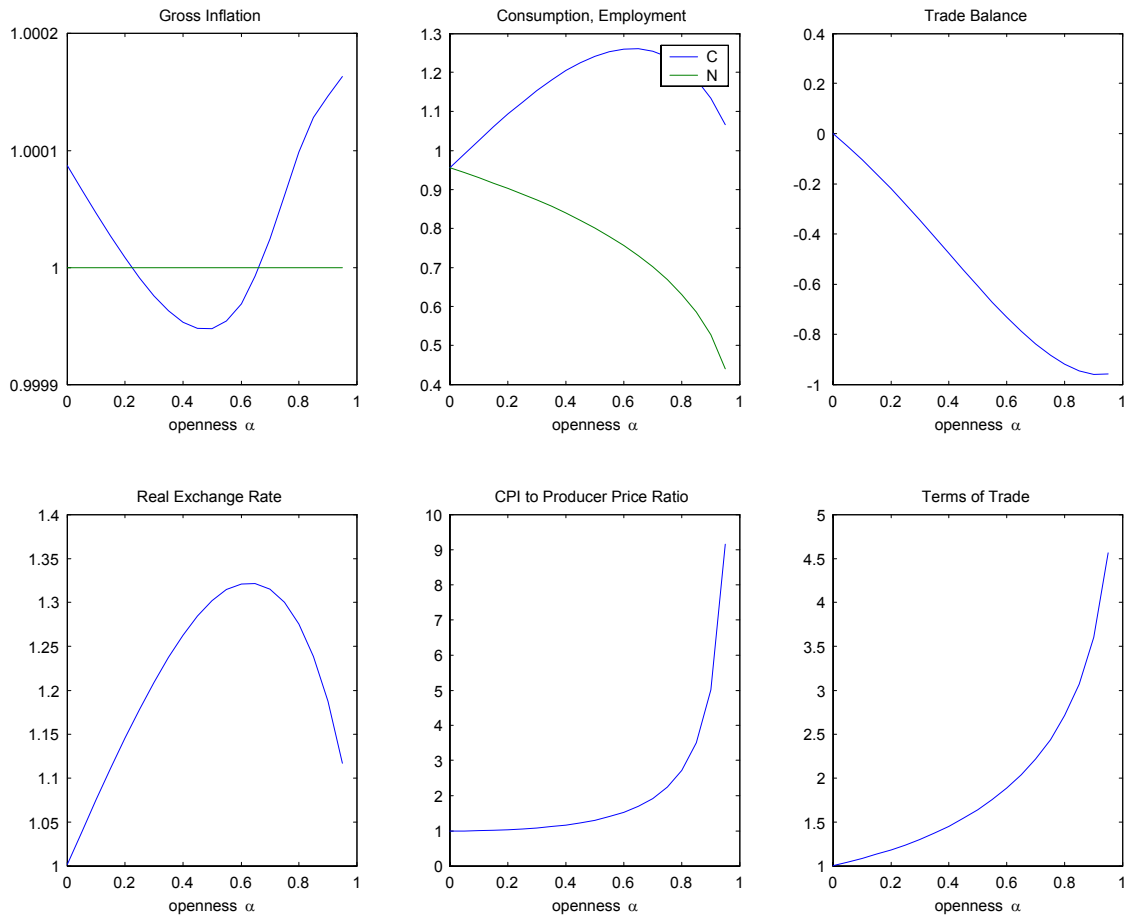

Figure 3: **Effect of Varying Opennness on the Efficient Steady-State of the Small Open Economy.**

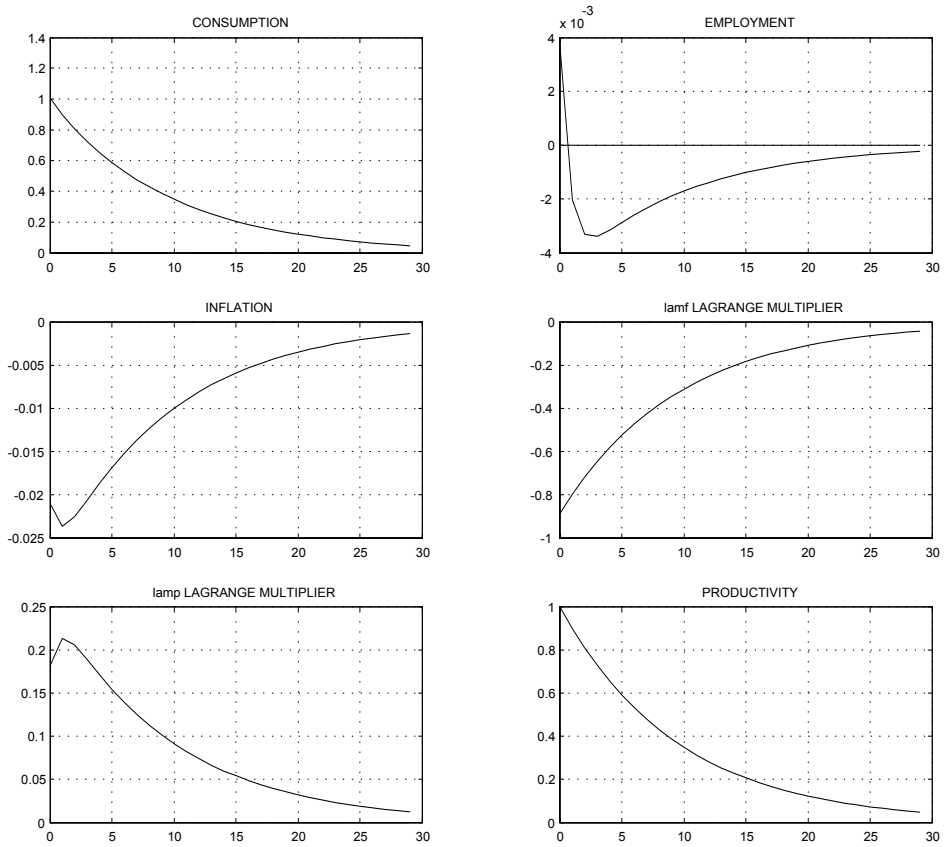

Figure 4: **Impulse Responses to a 1% Positive Productivity Shock under the Ramsey Policy (Rest of the World).**