Real Exchange Rate Persistence and Systemic Monetary Policy Behaviour*

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Abstract

In this paper we estimate forward-looking monetary policy rules for Germany over the 1979-1998 period and for the United Kingdom for the periods 1979-1990 and 1992-1998. The estimation results indicate that there are substantial differences in systemic monetary policy both between Germany and the United Kingdom as well as for the United Kingdom solely over this period. The implications of these estimated policy rules for real exchange rate behaviour are analysed within an open economy dynamic stochastic general equilibrium model. The analysis shows that real exchange rate persistence itself is due to the persistence of real shocks and interest rate smoothing behaviour of central banks. However, the observed cross-country asymmetry in systemic monetary policy behaviour elevates real exchange rate persistence to realistic levels whereas changes in this asymmetric policy behaviour changes the character of persistence. The results of our analysis sheds light on the role of systemic monetary policy in the widely observed persistence of real exchange rates in the post-Bretton Woods era.

Keywords: Dynamic stochastic general equilibrium models, GMM, monetary policy feedback rules, real exchange rate persistence.

JEL classification: E52, F31, F33, F41.

*Part of the research was done while the second author was a PhD intern at the Bank of England. This paper has benefited from helpful comments by Gianluca Benigno, Charles Engel and seminar participants at the Bank of England Exchange Rate Research Workshop. Research assistance by James Cookson is gratefully acknowledged. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Bank of England or members of the Monetary Policy Committee.

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1 Introduction

The very persistent nature of real exchange rates across industrialised countries under the post-Bretton Woods float has been a topic of extensive research in the international economics literature. Mark (1990), Papell (1997) and O’Connell (1998), amongst others, apply time series-based and panel data-based unit root tests on bilateral real exchanges rates amongst the major OECD countries, and these studies generally are unable to reject a unit root in these real exchange rates. As purchasing power parity [PPP] implies that real exchange rates should be mean reverting around a constant mean, one can interpret the cited evidence from the literature that long-run PPP does not hold.

One reason of this apparent failure of PPP could be that there are real forces that permanently shift the real exchange rate. Balassa (1964) and Samuelson (1964) argue that in fast growing economies productivity growth in the traded goods sector is higher than in the non-traded goods sector and consequently the relative price of non-traded/traded goods for such an economy would exhibit an acceleration in its growth rate. If the economy at home grows faster than the economy abroad, the corresponding bilateral real exchange rate will exhibit a sustained appreciation and *vice versa*. Nonetheless, the international relative price of traded goods should be mean reverting around a constant level according to the law of one price [LOOP]. However, the empirical evidence on LOOP is not wide spread. Engel (1999) finds for the major United States [US] dollar bilaterals that even in the long-run real exchange rate variability is determined by LOOP deviations. Groen and Lombardelli (2002) report evidence that within a monthly 1976-2002 panel of the United Kingdom [UK] based real exchanges of Canada, the Euro-area (or Germany), Japan and the US there is a long-run relationship between the real exchange rate and the relative price ratio of non-tradable/tradable goods. However, Groen and Lombardelli (2002) also find that the observed deviations between real exchange rates and the corresponding relative price ratio of non-tradable/tradable goods are very persistent, indicating that LOOP deviations are long lasting.

Given the aforementioned evidence from the literature, there must be short-to-medium run factors operating that induce substantial deviations from LOOP and PPP, and identifying a candidate factor is the focus of this paper. One obvious candidate for these short-to-medium run factors is nominal price stickiness. Chari *et al.* (2002) build an open economy dynamic stochastic general equilibrium [DSGE] model with price stickiness, local currency pricing, and a rich, disaggregated real sector with capital in order to generate artificial real exchange rate series. Their artificial real exchange rate series indeed are shown to be persistent albeit not enough to match the empirically observed degree of real exchange rate persistency. Therefore factors beyond price stickiness and local currency pricing are needed to explain the observed deviations from LOOP and PPP.

One possibility is to enrich the degree of price stickiness such that the degree of price stickiness varies across countries and across domestically-produced and imported goods. Benigno (2002) shows that such an asymmetric degree of price stickiness in combination with a high degree of persistency in the bilateral interest rate differential is capable of elevating the persistence of the artificial real exchange series from his model to realistic...
levels of real exchange rate persistence. Another approach is to include real rigidities in the model in the form of international transaction costs between spatially separated markets otherwise known as shipping costs, see e.g. Dumas (1992). In this approach the prediction is that the real exchange rate will be non-mean reverting within a certain range, in which arbitrage trade profits do not outstrip the shipping costs, and mean reverting outside this range. Utilising non-linear autoregressive time series models Obstfeld and Taylor (1997) and Taylor et al. (2001), in the context of LOOP and PPP respectively, claim to have found empirical evidence for this prediction of the shipping costs view on real exchange rate persistence.

Rather than focusing on asymmetric price stickiness or real rigidities, we take in this paper a different approach in that we focus on how real exchange rate persistence is affected by cross-country differences in how national central banks interact with their respective economies. We use a two-country DSGE model of the ‘new open economy’ strand in which we have no cross-country and cross-sector differences in price stickiness, and a real sector which is identical across countries. However, the monetary policy rules, i.e. the feedback mechanisms of the central bank relative to the rest of the economy, are asymmetric across countries. After calibrating the real side of our model and estimating the persistence and variance of the real shocks as well as each of the monetary policy rules, we analyse how the real exchange rate behaves in the face of a common real shock, a relative real shock and policy shocks. One implication of the usage of asymmetric monetary policy rules is that the real exchange rate will react to a common real shock, which would not have been the case under symmetric monetary policy rules. Next to that, we also analyse what the implications are for both the real exchange rate to each of the aforementioned shocks and the general half-life of the real exchange rate when in one country the central bank changes its feedback behaviour. Our analysis is very much in the spirit of Clarida et al. (2000), who calibrate a closed-economy DSGE model for the US and analyse the consequences for inflation and output dynamics when they feed into this model monetary policy rules that are estimated under different US monetary policy regimes.

The emphasis of the analysis in this paper is on the Germany/UK real exchange rate over the 1979-1998 sample. During the period this period German systemic monetary policy behaviour has been broadly stable. In the UK, on the other hand, the feedback mechanism of monetary policy has undergone some significant changes over the 1979-1998 sample, as exemplified by Nelson (2003). We identify three phases in the systemic conduct of UK monetary policy. The first is the period 1979-1990 in which the UK unsuccessfully tried several means to stabilise inflation. During 1990-1992 the UK fixed its exchange rate to the Deutsche Mark [DM] with the ERM. In the period 1992-1998, finally, inflation targeting was introduced and the Bank of England forcefully brought inflation down to low and stable levels. From the analysis in Mussa (1986) of real exchange rate behaviour under the Bretton Woods fixed exchange rate regime and the post-Bretton Woods float, we know that real exchange rates are much more persistent and volatile under flexible exchange rates than under fixed exchange rates. The low real exchange rate persistence under fixed
exchange rates is not surprising, as in this case domestic monetary policy is implicitly used to get PPP to hold. Given the aforementioned uncertainty in the literature to what causes the high real exchange rate persistence under flexible exchange rates, however, we focus in our on the first and third UK monetary policy regimes during which the exchange was more or less flexible. Thus, we estimate a German monetary policy rule over 1979-1998 and UK monetary policy rules over the periods 1979-1990 and 1992-1998. After feeding these estimated policy rules into our calibrated open economy DSGE model and ceteris paribus the preference parameters and the real shock processes, we show that real exchange rate persistence is high under both UK policy regimes although the character of this persistence is markedly different over the two regimes.

The remainder of this paper is organised as follows. In Section 2 we define an interest rate-based forward looking monetary policy rule and estimate it for Germany and the UK. The structure of the open economy DSGE model, which we utilise in our analysis of real exchange rate persistence, is set out in Section 3. After estimating the real shock processes, calibrating the real parameters and feeding in the different estimated monetary policy rules, we use in Section 4 impulse response functions and Monte Carlo simulations to analyse the impact of systemic monetary policy behaviour on real exchange rate persistence. We end with concluding remarks in Section 5.

2 Empirical Monetary Policy Behaviour

Most central banks view the nominal short-term interest rate as their main operating instrument for which they specify a target level in order to reach their medium-to-long term policy objectives. Central banks typically use their base or discount rate and open market operations to move the short-term interest rate towards the perceived target level. Hence, in modeling the systemic behaviour of central banks and its effect on real exchange rates, we specify the central bank reaction function in terms of short-term interest rates and estimate it.

In Section 2.1 we show how to specify the central bank reaction function in terms of the short-term interest rate and how to estimate it. Section 2.2 reports the estimation results for Germany and for the UK, and for the latter under different identified monetary policy regimes.

2.1 Central Bank Policy Reaction Functions: Specification and Estimation Issues

Most central banks aim to influence short-term interest rates towards a certain target level in order to achieve their medium-to-long term objectives. These target levels are formulated in terms of expected future inflation and the expected future level of the output gap (see e.g. Clarida et al. (2000)), i.e.

\[ i_t^* = \bar{i}^* + \phi^* E_t(\pi_{t+k1}) + \psi^* E_t(\dot{Y}_{t+k2}), \]

(1)
where $\pi_{t+k1}$ is the year-on-year inflation rate $k1$ periods ahead, $\dot{Y}_{t+k2}$ is output gap $k2$ periods ahead and $E_t$ is the conditional expectation based on the available information set in period $t$. From e.g. Clarida et al. (1998a) we know that when the inflation coefficient $0 < \phi^* < 1$ the central bank reaction function will destabilise the economy and generate persistent self-fulfilling outbursts in the inflation rate, as the central bank fully accommodates every rise in inflation expectations. Thus, when $\phi^* > 1$ monetary policy itself will not be a source of macroeconomic instability. Note, however, that when $\phi^*$ is slightly bigger than 1, i.e. just outside the destabilising range, the economy can still be relatively instable as the central bank is close to fully accommodating inflationary pressures, see Clarida et al. (2000, pp. 174-177).

In general, central banks tend not to influence short-term interest rates so strongly that it equals the target level in each period, partly because central banks want to avoid destabilising effects of sudden changes in interest rates and partly because central banks do not have perfect control over interest rates. We therefore assume that short-term interest rates evolve as a weighted average of its lag and the target level (1):

$$i_t = \gamma \bar{i}_{t-1} + (1 - \gamma) i_t^* + \varepsilon_t,$$

with $0 < \gamma < 1$. After substituting in (1) we can write (2) as

$$i_t = (1 - \gamma) \bar{i}^* + \gamma \bar{i}_{t-1} + (1 - \gamma) \phi^* E_t(\pi_{t+k}) + (1 - \gamma) \psi^* E_t(\dot{Y}_{t+k2}) + \nu_t,$$

with

$$\nu_t = \varepsilon_t + (1 - \gamma) \phi^* (\pi_{t+k} - E_t(\pi_{t+k})) + (1 - \gamma) \psi^* (\dot{Y}_{t+k2} - E_t(\dot{Y}_{t+k2})).$$

The variable $\varepsilon_t$ in (2) and (3) is a stationary, zero-mean disturbance which we interpret as an exogenous monetary policy shock. Following Clarida et al. (1998b) and Christiano et al. (2000) we can interpret $\varepsilon_t$ in several ways. One interpretation is that $\varepsilon_t$ reflect exogenous shocks to the preferences of policy makers.\(^1\) Second, it may reflect the inability of policy makers to keep the interest on target, for example when money demand shocks occur and policy makers not only rely on the interest rate to achieve their policy objectives. Next, it could also reflect decision by policy makers to deliberately temporarily deviate from target levels. Finally, variable $\varepsilon_t$ also can reflect technical factors such as measurement error.

The composite error term $\nu_t$ is a linear term of forecast errors which should by definition be uncorrelated with any of variables in the current information set $I_t$. When we have a vector $Z_t$ of instruments variables such that $Z_t \subset I_t$, we can write based on (3) a set of orthogonality conditions,

$$\left( i_t - (1 - \gamma) \bar{i}^* - \gamma \bar{i}_{t-1} - (1 - \gamma) \phi^* E_t(\pi_{t+k}) - (1 - \gamma) \psi^* E_t(\dot{Y}_{t+k2}) \right) Z_t = 0.$$

We now can use the Generalized Method of Moments [GMM] approach, see Hansen (1982), to estimate the parameters in (3) through the orthogonality conditions (4). The weighting

\(^1\)For example, in present-day UK this could occur due to changes in the composition of MPC members.
matrix for our GMM estimators are based on the Newey and West (1987) disturbance covariance matrix estimator which is asymptotically robust to heteroskedasticity and autocorrelation in the disturbances.

### 2.2 Estimation Results

The focus of this paper is on the Germany/UK real exchange rate relationship. We do not consider the Euro-area policy rule, and thus the Euro/UK relationship, as the sample is too short. We use monthly data, covering the period July 1979 - December 1998. Only the period prior to the launch of EMU will be considered. Simply splicing the Bundesbank policy rule to that of the ECB would be inappropriate as Faust et al. (2001) have shown that these are most likely to be different.

The inflation rate \( \pi_t \) is the log difference between the CPI level in the same month over two consecutive years in percentages. As our measure of the output gap \( \dot{Y}_t \) we use the percent deviation of log industrial production from its Hodrick and Prescott (1997) filtered value. Our interest rate \( i_t \) is the 1-month maturity Euro-market interest rate, as these are not disturbed by institutional factors and opens up the possibility of a cross-country comparison.

Estimating (3) with GMM through orthogonality conditions (4) means that we have to take a stand on the contents of instrument vector \( Z_t \). We consider \( Z_t \) to consist of the lags of the variables in (3), i.e. \( Z_t = (i_{t-1}, \ldots, i_{t-l_1}, \pi_{t-1}, \ldots, \pi_{t-l_2}, \dot{Y}_t, \ldots, \dot{Y}_{t-l_3})' \). Thus, selecting the optimal \( Z_t \) instrument vector boils down to selecting optimal lag orders \( l_1, l_2 \) and \( l_3 \). Normally one would use the Hansen (1982) \( J \)-test to the optimal lag orders \( l_1, l_2 \) and \( l_3 \), and therefore optimal number of instrument variables. The finite sample properties of this test, however, are known to be very poor and often results in overrejection of the selected number of orthogonality conditions. As an alternative selection procedure of the appropriate number of instruments we apply the GMM-BIC criterion of Andrews (1999). This GMM-BIC criterion is the GMM analogue of the well known Bayesian-Schwarz Information Criterion, and it yields consistent estimates of the optimal number orthogonality conditions and has better finite sample properties than the \( J \)-test. Andrews (1999) also proposes a GMM analogue of the Akaike Information Criterion, but this GMM-AIC criterion is asymptotically inconsistent and has a tendency to select too few orthogonality conditions.

The first row in Table 1 contains the GMM estimates of (3). The estimation results show that the German Bundesbank overall pursued a stabilising strategy relative to inflation, and also took into account news about economic prospects in general. Estimation results for the UK over the 1979-1990 regime are reported in the second row of Table 1. These parameter estimates makes it clear that UK monetary authorities have been fairly

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2The Euro-market interest rates are supplied by Datastream, whereas all the other series are from the International Financial Statistics CD-rom of the IMF.

3Andrews (1999) also proposes a GMM analogue of the Akaike Information Criterion, but this GMM-AIC criterion is asymptotically inconsistent and has a tendency to select too few orthogonality conditions.
accommodating towards inflationary pressures during this periods as the inflation coefficient $\phi^*$ is slightly smaller than 1. For the 1992-1998 inflation targeting regime, on the other hand, we have to add an extra lag to (3) for the UK,

$$i_t = (1 - \gamma_1 - \gamma_2)i^* + \gamma_1i_{t-1} + \gamma_2i_{t-2} + (1 - \gamma_1 - \gamma_2)\phi^*E_t(\pi_{t+k}) + (1 - \gamma_1 - \gamma_2)\psi^*E_t(\dot{Y}_{t+k2}) + \upsilon_t. \quad (5)$$

The corresponding results can be found in the third row of Table 1 and they show that UK monetary authorities have put much more emphasis on inflation stabilisation resulting in a high inflation coefficient $\phi^*$ and an insignificant output gap coefficient $\psi^*$.

### 3 The Structure of the Model

We use a two-country DSGE model, with countries denoted as Home and Foreign, based on Benigno (2002). Infinitely lived households are drawn from unit interval $[0, 1]$ with a fraction of $n$ living in Home and $1-n$ living in Foreign and indexed by $z$. Each household owns a firm monopolistically supplying a consumer good $z$ as in Blanchard and Kiyotaki (1987). A continuum of consumption goods are traded internationally and we abstract from non-traded goods. This is because we believe that real exchange rate fluctuation is coming mostly from violation of the law of one price. Engel (1993) finds that relative price of different goods within a country is generally less volatile than the relative price of same kind of goods across countries. Engel (1999) also shows that the almost none of the real exchange rate movement is attributable to the relative-relative price of non-traded goods.\[^{4}\]

\[^{4}\]This is the reason why we abstract from non-traded goods.

A firm produces a consumer good using labor and set its price in advance. We assume Calvo (1983)-type price stickiness\[^{5}\] with local currency pricing. There are some criticisms against local currency pricing because of the counterfactual movement of the terms of trade. However, by introducing distributors like Devereux et al. (1997), Devereux and Engel (2002), we can fix the problem to some degree since the terms of trade at the port level moves as observed in data\[^{6}\]. In our model, we do not introduce distributors for simplicity because there is no significant difference under the complete nominal asset market assumption\[^{7}\].

Monetary policy is described by a variant of the familiar Taylor rule. We allow each monetary authority to adopt its own variant of the Taylor rule. Our main focus is on the effect of two monetary authorities adopting different monetary policies. We also investigate a shift in monetary policy. This becomes possible when we allow two countries to have different policies otherwise we have to assume the shifts are simultaneous.

Before going to detail, let us define some notation. $E_t$ is expected value conditional on time $t$ information. Let $C_t$ be the index of consumption basket at time $t$, $M_t$ be nominal

\[^{5}\]See Taylor (1999) for detailed discussion on Calvo Pricing.

\[^{6}\]See Matsumoto (2002) for detailed discussion.

\[^{7}\]See NBER working paper version of Devereux et al. (1997) for further discussion.
money balance, $P_t$ be the unit price index corresponding to $C_t$, and $L_t$ be labor. Generally subscripts are time script but the subscript for consumption goods $C$ and price $P$ indicates the country of goods produced if subscript is $H$ or $F$. Superscript $i \in H, F$ denotes the amount or value facing Home and Foreign consumer, while superscript $j \in W, R$ denotes world aggregate and Home relative to Foreign. So, for example, $C^H_{F,t}$ indicates consumption amount of Foreign goods by Home consumer at time $t$, while $C_t(z)$ is a goods produced by z household at time $t$. $\hat{X}_t$ indicates the log deviation from the steady state, while $\bar{X}$ indicates value at the steady state.

### 3.1 Households

A household $z$ owns a firm producing goods $z$ and receives profit $PR_t(z)$ from it. Because of this, there is idiosyncratic shock among households, but we assume domestically complete asset market so that there is no heterogeneity among agents in a country. Households also receive transfer from the government. A household receives utility from consumption, real balance, and leisure. Let $U$, $N$, and $V$ denote sub-utility functions for each argument, which are twice differentiable and concave. A representative household in country $i$ solves the following maximization problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ U(C^i_t) + N \left( \frac{M^i_t}{P^i_t} \right) - V(L^i_t) \right],$$

subject to its budget constraint,

$$A^i_t + M^i_t + P^i_t C^i_t = A^i_{t-1} R^i_{t-1} + M^i_{t-1} + W^i_t L^i_t + PR^i_t + TR^i_t,$$

where $A_t$ is the sum of the nominal value of financial assets excluding the money balance, $R_{t-1}$ is the weighted average gross nominal return on the financial assets carried from last period, $W_t$ is nominal wage, $PR_t$ denotes nominal firm profit and $TR_t$ denotes the nominal government transfer from government. The government supplies money to meet household demand, and transfers the value of seignorage per capita to each individual: $TR^i_t = \Delta M^i_t \equiv M^i_t - M^i_{t-1}$.

The consumption index is defined by a CES function,

$$C^i \equiv \left[ n^{1/\varsigma} C^i_H^{(\varsigma-1)/\varsigma} + (1 - n)^{1/\varsigma} C^i_F^{(\varsigma-1)/\varsigma} \right]^{\varsigma/(\varsigma-1)} \quad \varsigma \geq 1$$

where $C^i_H$ and $C^i_F$ are sub index defined by

$$C^i_H \equiv \left[ \frac{1}{n} \int_0^n C^i(z)^{(\sigma-1)/\sigma} dz \right]^{\sigma/(\sigma-1)} \quad C^i_F \equiv \left[ \frac{1}{1-n} \int_n^1 C^i(z)^{(\sigma-1)/\sigma} dz \right]^{\sigma/(\sigma-1)}.$$
where, \( \sigma > 1 \) is an elasticity of substitution among within-country goods, and \( C_i^i(z) \) is a per capita consumption of goods \( z \) in country \( i \).

Given the prices of goods and total consumption \( C \), the optimal consumption for the representative household in country \( i \) of a good \( z \) is

\[
C_i^i(z) = \left( \frac{P_i^i(z)}{P_{i'}^i(z)} \right)^{\sigma} C_i^{i'}, \quad C_i^{i'} = \left( \frac{P_i^{i'}}{P_i^i} \right)^{-\sigma} C_i^i,
\]

(10)

where \( C_i^{i'} \) denotes per capita demand in country \( i \) for goods produced in country \( i' \), and \( i' = H \) iff \( z \leq n \) and \( i' = F \) otherwise.

The price index corresponding to the consumption basket is

\[
P_i^i = \left[ n P_H^i \right]^{1-\sigma} + (1 - n) P_F^i \right]^{1/(1-\sigma)},
\]

(11)

where \( P_H^i \) and \( P_F^i \) are price sub-indices of Home and Foreign goods in country \( i \),

\[
P_H^i = \left[ \frac{1}{n} \int_0^n P_i^i(z)^{1-\sigma} dz \right]^{1/(1-\sigma)}, \quad P_F^i = \left[ \frac{1}{1-n} \int_n^1 P_i^i(z)^{1-\sigma} dz \right]^{1/(1-\sigma)}.
\]

(12)

The first order conditions with respect to labor and money balance imply,

\[
N_M \left( \frac{M_i^i}{P_t^i} \right) = U_C(C_i^i) - \beta E_t \left[ U_C(C_{i+1}^i) \frac{P_i^i}{P_{i+1}^i} \right],
\]

(13)

\[
V_L(L_i^i) = U_C(C_i^i) \frac{W_i}{P_t^i}.
\]

(14)

The first order condition for nominal bond holdings gives us a usual Euler equation,

\[
U_C(C_i^i) = (1 + i_t) \beta E_t \left[ U_C(C_{i+1}^i) \frac{P_i^i}{P_{i+1}^i} \right],
\]

(15)

where \( i_t \) is the nominal interest rate.

Combining this with equation (13), we can get the “LM equation”,

\[
N_M \left( \frac{M_i^i}{P_t^i} \right) = \frac{i_t}{1+i_t} U_C(C_i^i).
\]

(16)

In our model, the LM equation does not play a significant role. However, this can be interpreted as a bridge from money supply rule to interest rate rule\(^{10}\).

In addition to complete markets within a country such that every individual in country \( i \) has same consumption level, we allow nominal contingent claims to be internationally traded as in Chari et al. (2002). With a complete set of nominal contingent claims traded, we have

\[
Q_t \equiv \frac{S_i^t P_F^t}{P_H^t} = \kappa \frac{U_C(C_F^t)}{U_C(C_H^t)}, \quad \kappa = \frac{S_0^t P_F^t U_C(C_H^0)}{P_H^0 U_C(C_H^0)}
\]

(17)

\(^{10}\)See Woodford (2002, Chapter 2) for more discussion
where the real exchange rate proportional to the ratio of the marginal utility of consumption between two countries, $S_t$ is a nominal exchange rate and $\kappa$ is a constant that depicts initial condition, which we assume to be unity. We assume nominal complete markets for simplicity, although this assumption generates the “consumption-real exchange rate anomaly” as pointed out in Chari et al. (2002). Devereux and Engel (2002) study exchange rate movements under incomplete markets. By adding noise traders, they show that the correlation between consumption and the real exchange rate can be low. Since our main focus is on the real exchange rate persistence, we do not try to solve this problem here.

3.2 Firms and Production Technology

A firm in country $i$ uses linear technology,

$$Y^i = A^iL^i,$$

where $A^i$ is the productivity of labor specific to country $i$. The labor force is assumed to be homogeneous within a country. Since there is no specific factor, the labor market is assumed to be perfectly competitive. Thus, the wages are set in order to satisfy the labor supply condition in equation (14).

With probability $1 - \alpha$ in each period, a firm receives a signal to change its price. The firm set the price for its good in the Home market and the Foreign market in the currency of consumers. These prices will be fixed until the firm receives another signal. Let $\Xi^i_{t,s} = \beta^s \frac{U_C(C^i_{t+s})}{U_C(C^i_t)} \frac{P^i_t}{P^i_{t+s}}$ be the stochastic discount factor of a firm in country $i$. A Home firm, if it receives a signal, will set the prices of the its good in the Home and Foreign markets at $t$ in order to maximize the expected value of discounted profits:

$$E_t \sum_{s=t}^{\infty} \alpha^s \Xi^H_{t,s} \left[ P^H_t(z)nC^H_{t+s}(z) + S_{t+s}P^F_t(z)(1-n)C^F_{t+s}(z) \right] - \frac{W^H_{t+s}}{A^H_{t+s}} \left( nC^H_{t+s}(z) + (1-n)C^F_{t+s}(z) \right).$$

(18)

Since a firm has measure 0, it cannot affect aggregate price level and therefore regards the aggregate price level as given. The first order conditions yield the optimal pricing policy for a Home firm, $z \in [0,n]$:

$$P^H_t(z) = \frac{\sigma}{\sigma - 1} \frac{E_t \sum_{s=t}^{\infty} \alpha^s \Xi^H_{t,s} \frac{W^H_{t+s}}{A^H_{t+s}} C^H_{t+s}(z)}{E_t \sum_{s=t}^{\infty} \alpha^s \Xi^H_{t,s} C^H_{t+s}(z)},$$

(19)

$$P^F_t(z) = \frac{\sigma}{\sigma - 1} \frac{E_t \sum_{s=t}^{\infty} \alpha^s \Xi^H_{t,s} \frac{W^H_{t+s}}{A^H_{t+s}} C^F_{t+s}(z)}{E_t \sum_{s=t}^{\infty} \alpha^s \Xi^H_{t,s} S_{t+s} C^F_{t+s}(z)}.$$  

(20)
Analogous conditions hold for a Foreign firm, \( z \in (n, 1) \):  

\[
P_t^H(z) = \frac{\sigma}{\sigma - 1} E_t \sum_{s=t}^{\infty} \alpha^s \Xi^F_{i,t,s} \frac{W^F_{t+s}}{F^H_{t+s}} C^H_{t+s}(z),
\]

(21)  

\[
P_t^F(z) = \frac{\sigma}{\sigma - 1} E_t \sum_{s=t}^{\infty} \alpha^s \Xi^F_{i,t,s} \frac{W^F_{t+s}}{F^F_{t+s}} C^F_{t+s}(z).
\]

(22)  

Since there are assumed to be an infinite number of firms, the law of large number holds for the aggregate price level. A fraction \( \alpha \) of firms cannot change price, thus the prices are the same as in the previous period. A fraction \( 1 - \alpha \) of firms set new price, \( P_i^t(z) \), which should be same among themselves. Thus, equations (12) can be rewritten,

\[
P_{i',t}^i = \left( \alpha P_{i,t-1}^{i'} (1-\sigma) + (1 - \alpha) P_t^i(z)^{(1-\sigma)} \right)^{1/(1-\sigma)}.
\]

(23)  

### 3.3 Linear Dynamic System

We now log-linearize the system in order to derive a linear system of equations. First, we define \( \rho \equiv -\frac{\nu_{CC}(\bar{C})C}{\nabla C} \), the relative risk averse parameter, and \( \eta \equiv \frac{\nu_{LL}(\bar{L})L}{\nabla L} \). We omit time subscript if the equation is intratemporal.

#### Households

Log linearizing the Euler equation (15), we can get,

\[
E_t \hat{C}_{i,t+1} - \hat{C}_i^i = \frac{1}{\rho} (\hat{r}_i^i - E_t \pi_{i,t+1}^i),
\]

(24)  

\[
E_t \hat{C}_{w,t+1} - \hat{C}_w^w = \frac{1}{\rho} (\hat{r}_w^w - E_t \pi_{w,t+1}^w).
\]

(25)  

We use the world Euler equation so that the system consists of ‘world’ and ‘relative’ terms.

We can also get log approximation of the real exchange rate from equation (17),

\[
\hat{Q} = \rho (\hat{C}^H - \hat{C}^F).
\]

(26)  

As we have mentioned this is real exchange implies relative consumption.

Combining equations (24) and (26), we can get uncovered interest parity (UIP),

\[
E_t (\Delta S_{t+1}) = \hat{i}_{t}^H - \hat{i}_{t}^F.
\]

(27)  

UIP holds in our model because of our complete market assumption and rational expectation. Devereux and Engel (2002) introduce noise-traders as in Jeanne and Rose (2002) under incomplete market, and show that can be the source of exchange rate volatility.
Output Gap

Potential output is defined as the level of output under fully flexible prices. The output gap is defined as the log difference between actual output under sticky prices and potential output. Below we define output gap and discuss its economic implications. A detailed derivation of some of the variables can be found in Appendix A.

By log-linearizing demand function (10) in country level, the output under sticky prices can be written as:

\[
\hat{Y}^H = n \left\{ -\varsigma (\hat{P}^H_H - \hat{P}^H_H) + \hat{C}^H \right\} + (1 - n) \left\{ -\varsigma (\hat{P}^F_F - \hat{P}^F_F) + \hat{C}^F \right\} \\
= \hat{C}^W + (1 - n)\varsigma \hat{T}^W, \tag{28}
\]

\[
\hat{Y}^F = n \left\{ -\varsigma (\hat{P}^H_F - \hat{P}^H_H) + \hat{C}^H \right\} + (1 - n) \left\{ -\varsigma (\hat{P}^F_F - \hat{P}^F_F) + \hat{C}^F \right\} \\
= \hat{C}^W - n\varsigma \hat{T}^W, \tag{29}
\]

where

\[
\hat{T}^W \equiv n \ln \left( \frac{P^H_F}{P^H_H} \right) + (1 - n) \ln \left( \frac{P^F_F}{P^F_H} \right) \tag{30}
\]

is a weighted average of the relative price of Foreign to Home goods of each market.

Let \( \tilde{X} \) be the log deviation from steady state under flexible price in which per capita consumption is the same across the countries. While the real exchange rate is constant, the relative price\(^{11}\) fluctuates to reflect relative productivity. Log-linearizing flexible price model around the steady state, we can get,

\[
\tilde{C}_t = \frac{\eta + 1}{\eta + \rho} \left[ n \hat{A}^H + (1 - n)\hat{A}^F \right], \tag{31}
\]

\[
\tilde{T}_t = \frac{\eta + 1}{\eta \varsigma + 1} \left[ \hat{A}^H - \hat{A}^F \right]. \tag{32}
\]

As we can see easily, these summarize the world productivity and the relative productivity. We assume AR1 processes for \( \tilde{C}, \) and \( \tilde{T}. \)\(^{12}\)

\[
\tilde{C}_t = \varrho_C \tilde{C}_{t-1} + \nu_t^C, \quad |\varrho_C| < 1, \tag{33}
\]

\[
\tilde{T}_t = \varrho_T \tilde{T}_{t-1} + \nu_t^T, \quad |\varrho_T| < 1, \tag{34}
\]

where, \( \nu \)'s are mean zero and bounded variance i.i.d. processes.

We can also describe the potential output with these two terms,

\[
\tilde{Y}^H = \tilde{C} + (1 - n)\varsigma \tilde{T}, \quad \tilde{Y}^F = \tilde{C} - n\varsigma \tilde{T}. \tag{35}
\]

Therefore, output gaps, denoted as \( \check{Y}, \) can be written as:

\[
\check{Y}^H = (\hat{C}_t^W - \tilde{C}_t) + (1 - n)\varsigma (\hat{T}_t^W - \tilde{T}_t), \quad \check{Y}^F = (\hat{C}_t^W - \tilde{C}_t) - n\varsigma (\hat{T}_t^W - \tilde{T}_t). \tag{36}
\]

\(^{11}\)Relative price of each market is equal since flexible price guarantees the law of one price.

\(^{12}\)Notice that per capita consumption level and terms of trade under flexible price are same for Home, Foreign and world.
There are two sources for our output gap measure. The world output gap is common to both the Home and Foreign country. The other is relative productivity. Engel and West (2002) adopt relative productivity shocks as a factor driving the relative output gap.

**Phillips Curve**

By log-linearizing the price equations, we can get the forward looking Phillips curve or supply curve. Let, \( \pi_t = \ln(P_t/P_{t-1}) \).

\[
\begin{align*}
\pi_t^H &= \frac{\eta + \rho}{1 + \sigma \eta} \zeta (\hat{C}_t^W - \hat{C}_t^H) + \zeta (1 - n) \hat{Q}_t + \beta E_t \pi_{t+1}^H, \\
\pi_t^F &= \frac{\eta + \rho}{1 + \sigma \eta} \zeta (\hat{C}_t^W - \hat{C}_t^F) - \zeta n \hat{Q}_t + \beta E_t \pi_{t+1}^F,
\end{align*}
\]

where \( \zeta = \frac{1 - \alpha \beta}{1 - \alpha} \). Inflation of both countries are function of inflation expectation, world output gap and real exchange rate. It is useful to write these Phillips curves in a following way to see the effect of each term separately;

\[
\begin{align*}
\pi_t^W &= \frac{\eta + \rho}{1 + \sigma \eta} \zeta (\hat{C}_t^W - \hat{C}_t^H) + \beta E_t \pi_{t+1}^W, \\
\pi_t^R &= \zeta \hat{Q}_t + \beta E_t \pi_{t+1}^R.
\end{align*}
\]

In order to complete linear dynamic system, we should note that

\[
\begin{align*}
\hat{Q}_t &= \hat{Q}_{t-1} + \Delta S_t - \pi_t^R, \\
\hat{T}_t^W &= \hat{T}_{t-1} + n(\pi_{t,H}^H - \pi_{t,F}^H) + (1 - n)(\pi_{t,F}^F - \pi_{t,H}^F) \\
&= \hat{T}_{t-1} + \frac{1 + \sigma \eta}{1 + \sigma \eta} \zeta (\hat{T}_{t-1}^W - \hat{T}_t) + \beta E_t (\hat{T}_{t+1}^W - \hat{T}_t).
\end{align*}
\]

Under certain conditions we can rewrite the second equation as

\[
\hat{T}_t^W = \lambda_1 \hat{T}_{t-1}^W + \lambda_1 \frac{1 + \sigma \eta}{1 + \sigma \eta} \zeta \frac{1}{1 - \beta \lambda_1} \hat{T}_t,
\]

where \( \lambda_1 \in (0, 1) \).

### 3.4 Monetary Policy Rules

We adopt a variation of a Taylor rule as our monetary policy rules:

\[
\begin{align*}
\hat{i}_t^H &= \xi \hat{i}_{t-1}^F + \gamma_t^H \hat{i}_{t-1}^H + \phi_t E_t (\pi_{t+1}^H) + \psi_t \hat{Y}_t^H + \varepsilon_t^H, \\
\hat{i}_t^F &= \gamma_t^F \hat{i}_{t-1}^F + \phi_t E_t (\pi_{t+1}^F) + \psi_t \hat{Y}_t^F + \varepsilon_t^F,
\end{align*}
\]

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where $\pi$ is the inflation rate, $\dot{Y}$ is the output gap, and $\phi$ and $\psi$ are related to $\phi^*$ and $\psi^*$ from (3) or (5) through $\phi = (1 - \gamma)\phi^*$ and $\psi = (1 - \gamma)\psi^*$. We assume for the monetary policy rule shock

$$\varepsilon_t^i \sim N(0, \sigma_{\varepsilon_t^i}^2), \quad \text{for } i = H, F.$$  \hspace{1cm} (46)

While most of the theoretical work assume identical parameters\footnote{See, for example, Engel and West (2002).}, we allow for asymmetric values of the parameters in our monetary policy rules to show the potential source of real exchange rate persistence.

Now substituting expected inflation and output gap according to equations (36, 37-40), we get,

$$\hat{i}_t^H = \xi^F_t \hat{i}_t^F + \gamma_H \hat{i}_{t-1}^H + \phi_H E_t(\pi_{t+1}^H) + \psi_H \dot{Y}_t^H + \varepsilon_t^H$$

$$= \gamma_H \hat{i}_{t-1}^H + \frac{\phi_H}{\beta} \{ \pi_t^W + (1 - n)\pi_t^R \} - \{ \frac{\phi_H}{\beta} \zeta (1 - n) \} \hat{Q}_t$$

$$+ \left( \psi_H - \frac{\phi_H}{\beta} (1 + \sigma_n^2) \right) (\hat{C}_t^W - \hat{C}_t) + \psi_H (1 - n) \zeta (\hat{T}_t^W - \hat{T}_t) + \varepsilon_t^H.$$ \hspace{1cm} (44)

Here, we emphasize some important aspects of forward looking Taylor rule. Unlike the feedback rule from inflation, world output gap have some impact on the interest rate through the impact on inflation expectation even if the coefficient on the output gap in Taylor rule, $\psi$ is zero.

### 3.5 Some Analytical Result

We can write the system of linear equations (44-46,25,27,33,34,39-41,43) in a following way\footnote{We use the programs from Woodford’s web site: http://www.princeton.edu/~woodford/Tools/},

$$AE_t \left( \begin{array}{c} y_{t+1} \\ x_t \end{array} \right) = B \left( \begin{array}{c} y_t \\ x_{t-1} \end{array} \right) + C \nu_t,$$ \hspace{1cm} (47)

where, $A$ is a $12 \times 12$ matrix, $B$ is a $12 \times 12$ matrix, $C$ is a $12 \times 4$ matrix, $y = (\hat{C}^W, \pi^W, \pi^R, \Delta \hat{S})'$, $x = (\hat{i}_t^H, \hat{i}_t^F, \hat{Q}, \hat{T}^W, \hat{C}, \hat{T}, \varepsilon^H, \varepsilon^F)'$, and $\nu = (\nu^T, \nu^C, \nu^H, \nu^F)$. We can reduce this linear system to the following,

$$E_t \left( \begin{array}{c} y_{t+1} \\ x_t \end{array} \right) = D \left( \begin{array}{c} y_t \\ x_{t-1} \end{array} \right) + F \nu_t,$$ \hspace{1cm} (48)

where $D = A^{-1}B$ and $F = A^{-1}C$. As shown in Blanchard and Kahn (1980), assuming $D$ has 8 eigenvalue inside the unit circle and 4 outside, we have a unique bounded rational expectation solution given $x_0$. Since our focus is not the determinacy or bubble, we just assume that parameters of our interest satisfy the condition hereafter.
Terms of Trade Indepency

As shown in Benigno (2002), $\hat{T}_t^W$ only depends on the relative productivity shock as in (43),

$$\hat{T}_t^W = \lambda_1 \hat{T}_{t-1}^W + \lambda_1 \frac{\zeta(1+\varsigma \eta)}{1+\sigma \eta} \frac{1}{1-\beta \lambda_1 \rho} \hat{T}_t,$$

where $\lambda_1$ is the smaller\textsuperscript{15} root of $x^2 - \left[1 + \frac{1}{\beta} + \frac{1}{\beta} \frac{\zeta(1+\varsigma \eta)}{1+\sigma \eta}\right] x + \frac{1}{\beta} = 0$. This is because our assumption of common $\alpha$ for all goods. Benigno (2002) considers the case where price stickiness is different across and within countries. In addition to the fact that terms of trade $\hat{T}_t^W$ is exogenous as well as $\hat{C}$, $\hat{T}$, $\varepsilon^H$, $\varepsilon^F$ for the dynamic system, it is important to notice that even if the productivity shock is i.i.d., the terms of trade follow AR1 process, since $\lambda_1$ cannot be zero.

Symmetric Monetary Policy

When monetary policies of Home and Foreign are identical, we ignore some of the effect from the source which is common to both Home and Foreign. We show that the world productivity shock is such a variable.

\textbf{Definition 3.1} Monetary policies are \textbf{symmetric} if $\gamma_H = \gamma_F \equiv \gamma$, $\phi_H = \phi_F \equiv \phi$, $\psi_H = \psi_F \equiv \psi$.

\textbf{Proposition 3.1} If monetary policies are symmetric, the real exchange rate is independent of world consumption $\hat{C}^W$ and world productivity $\hat{C}$.

\textbf{Proof.} We can easily see,

$$\hat{i}_t^R = \gamma \hat{i}_{t-1}^R + \phi E_t(\pi_{t+1}^R) + \psi_\varsigma (\hat{T}_t^W - \hat{T}_t) + \varepsilon^R. \hspace{1cm} (50)$$

Since we can close our linear dynamic system including real exchange rate $\hat{Q}$ only with $\pi^R$, $i^R$, $\Delta S$, and exogenous variables $\hat{T}^W$, $\hat{T}$, $\varepsilon^H$, $\varepsilon^F$. By focusing on symmetric Taylor rule, we might miss some important source of persistence.

\textbf{Non-Smoothing}

While our focus is not building a model which generates persistence without persistent exogeneous shocks, it is important to know what kind of source can generate it. Is that possible to have real exchange rate persistence only from monetary shock? Answer is no, unless monetary authorities adopt smoothing.

\textsuperscript{15}It is easy to see that $\lambda_1 \in (0, 1)$. 

\hspace{1cm} 15
Definition 3.2  Monetary policies are **non-smoothing** if $\xi = 0$, $\gamma_H = \gamma_F = 0$, $\mu_H = \mu_F = 0$.

If monetary policies are non-smoothing, monetary authority try to immediately adjust misalignment of economy. They try to adjust three key economic variables – the output gap, expected inflation and the real exchange rate – which are different from flexible price level.

Proposition 3.2  If monetary policies of both countries are non-smoothing, then the only source of real exchange rate persistence comes from exogenous variables $T^W, \hat{T}, \hat{C}, \varepsilon^i$, given we have a unique rational expectation solution.

Proof.  By examining equations, we can reduce our system further and the system can be described as

$$E_t \left( \begin{array}{c} y_{t+1} \\ \hat{Q}_t \end{array} \right) = G \left( \begin{array}{c} y_t \\ \hat{Q}_{t-1} \end{array} \right) + H \varepsilon_t,$$

where $y_t = (\Delta S_t, \hat{C}^W_t, \pi^W_t, \pi^R_t)'$, is a $5 \times 1$ vector, $H$ is a $5 \times 5$ matrix, $\varepsilon_t = (\hat{T}^W_t, \tilde{C}_t, \tilde{T}_t, \varepsilon^H_t, \varepsilon^F_t)'$ and

$$G = \begin{pmatrix} -\frac{\zeta}{\beta} \Phi_1 & \Psi_1 - \frac{k_C}{\beta} \Phi_2 & \frac{1}{\beta} \Phi_2 & \frac{\zeta + 1}{\beta} \Phi_1 & -\frac{\zeta}{\beta} \Phi_1 \\ -\frac{\zeta}{\beta} \Phi_3 & \Psi_2 - \frac{k_C}{\beta} \Phi_4 & \frac{1}{\beta} \Phi_4 & \frac{\zeta + 1}{\beta} \Phi_3 & -\frac{\zeta}{\beta} \Phi_3 \\ 0 & -k_C & \frac{1}{\beta} & 0 & 0 \\ -\frac{\zeta}{\beta} & 0 & 0 & \frac{\zeta + 1}{\beta} & -\frac{\zeta}{\beta} \\ 1 & 0 & 0 & -1 & 1 \end{pmatrix},$$

where $k_C = \frac{\eta + \rho}{1 + \sigma \eta} \zeta$, $\Phi_1 = (1-n)\phi_H + n\phi_F$, $\Phi_2 = \phi_H - \phi_F$, $\Phi_3 = \frac{(1-n)n}{\rho} \Phi_2$, $\Phi_4 = n\phi_H + (1-n)\phi_F - 1$, $\Psi_1 = \psi_H - \psi_F$ and $\Psi_2 = n\psi_H + (1-n)\psi_F - 1$. Now it is easy to see $G$ has its rank at most 4. On the other hand, the other 4 eigenvalues are by assumption outside of unit circle. Since $\hat{Q}$ is the only one predetermined variable in this dynamic system (51), following Blanchard and Kahn (1980), real exchange can be written as

$$\hat{Q}_t = a_Q \hat{Q}_{t-1} + b_Q \varepsilon_t,$$

where $a_Q$ is a scalar, $b_Q$ is a $5 \times 1$ vector. But $a_Q$ is an eigenvalue of $G$ inside unit circle, which should be zero because of its rank. Since $a_Q = 0$, the source of real exchange rate persistence comes only from the exogenous variables. ■

In our model, we can have real exchange rate persistence if we have monetary policy which react to $\hat{T}^W$ which is typically the case with $\psi > 0$. This is because $\hat{T}^W$ follows at least AR1 process even if we only have non-persistent productivity shocks $\hat{C}, \hat{T}$ and

$$\text{16}.$$
monetary policy shocks $\varepsilon^H, \varepsilon^F$. Once we assume persistent relative productivity shock and $\psi > 0$, then we have real exchange rate persistency. On the other hand, $T^W$ is not particular persistent if we have assume $\hat{T}$ is i.i.d. If we assume persistency in $\hat{C}$, then non-symmetric monetary policies are necessary to transmit the persistence.

While we emphasize asymmetry that can transmit the persistence from an ignored source, asymmetry per se is not sufficient for the real exchange rate persistence. As proposition 3.2 tells, as far as monetary policy is non-smoothing then we need persistence from exogenous source including $\hat{T}^W$. That is if monetary authority adjust interest rate to the optimal level at once then the monetary shock does not have persistent effect on the real exchange rate.

4 The Interaction between Real Exchange Rates and Systemic Monetary Policy

In this section we analyse the implications of the estimated German and UK monetary policy rules from Section 2 for the real exchange within the structure of the DSGE model from Section 3. We focus in particular on how the different types of cross-country asymmetry in monetary policy rules affect the transmission of nominal and real shocks through the real exchange and how these differences affect the degree of real exchange rate persistence.

Section 4.1 describes how we calibrate our DSGE model, how identify the real shock processes on our data, and how we want to analyse the aforementioned phenomena. The results of our analysis are reported in Section 4.2.

4.1 Parameterization and Stylised Facts

In order to be able to assess the effect of systemic monetary policy on real exchange rate persistence we have to parameterize the DSGE model from Section 3. The Home country size $n$ is computed using the 2000 PPP-based US dollar values of real GDP for Germany and the UK. Based on these values the German economy is roughly two times as large as the UK economy and thus we set $n = \frac{1}{3}$. Following the calibration in Benigno (2002) and Chari et al. (2002) we assume the following values for our preference parameters:

- The discount rate $\beta$ is set equal to $\beta = 0.996$, which implies an annual discount rate of 0.953 in our monthly setting.

- The relative risk averse parameter (otherwise known as the intertemporal rate of substitution) $\rho$ is set equal to $\rho = 6$.

- The labour supply parameter $\eta$ is set equal to $\eta = 2$.

- We assume a value of 1.5 for the elasticity of substitution between Home goods and Foreign goods, i.e. $\varsigma = 1.5$. 

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• Finally, the elasticity between the variety of goods from each country $\sigma$ is assumed to be 10, i.e. $\sigma = 10$.

• We estimate (33) by estimating an AR(1) model on a quadratically detrended weighted average of German and UK HP-filter trend values of log industrial production, which is valid through (35), which yields $\varrho_c = 0.96$ and $\text{STD}(\nu_c) = 0.012$.\footnote{We detrend the HP-filter trends as we do not allow for population growth and capital formation in our DSGE model.}

• We estimate (34) by estimating an AR(2) model on the difference the UK and German HP-filter trend values of log industrial production, which is valid through (35), which yields $\varrho_c = 0.95$ (equals the sum of the two autoregressive parameters) and $\text{STD}(\nu_T) = 0.0179)$. We then rescale the process by $1/\varsigma$, see (35).

One crucial feature is the value of the Calvo price stickiness parameter $\alpha$. As a baseline case we set this parameter equal to a value $\alpha = 0.94$, which implies an average price rigidity of a little less than 1.5 years. How does our DSGE model compare to the data when use the aforementioned values of the preference parameters, $\alpha = 0.94$, and the parameter values of the estimated monetary policy feedback rules for Germany and the UK from Section 2? In the upper panel of Table 2 we report some stylised facts on the Germany/UK real exchange rate relationship over the three identified phases in UK monetary policy: pre-ERM, ERM and inflation targeting. The real exchange rate is a high autocorrelation in the first and third period, whereas under the ERM the real exchange rate is not very persistent and less volatile which is not surprising given the results in Mussa (1986). As we are interested solely in the effects of systemic monetary policy under floating rates, we focus on the effects of the first and third regimes in our DSGE model. In the middle panel of Table 2 we show the main result of our calibration exercise under $\alpha = 0.94$. We observe a very high correlation between $\Delta S_t$ and $\Delta Q_t$, and this is because we have a volatile nominal exchange rate relative to the inflation differential $\pi^R$. This result is in compliance with, for example, Engel and West (2002). However, the observed ranking of first order real exchange rate autocorrelation does not match the data. In order to check whether this ranking is due to an improper calibration of the preference parameters in our DSGE model, we show Figure 1 the implied first order autocorrelation and standard deviation of the real exchange rate from our the two regimes in our DSGE model (using $\alpha = 0.94$) over a wide range of different preference parameter values. From this figure it becomes apparent that the ranking of real exchange rate persistence and volatility over the two regimes is not affected by variations in the preference parameters.

Given the observed switch in systemic monetary policy behaviour, see Table 1, it is doubtful that the Calvo price stickiness parameter remains constant over time. In our model we do not allow for endogenous price rigidity, and as such we simply have to pick different Calvo price stickiness parameters over the different monetary regimes. Under high and variable inflation firms have the incentive to change their prices more frequently, whereas under low and stable rates of inflation it becomes less necessary for firms to...
change prices. As it is realistic to assume that over the two UK regimes that we have identified that the degree of price stickiness changes. In order to investigate this shift in the degree of price stickiness over the regimes, we recalculate the implied stylised facts from our DSGE model with $\alpha = 0.92$ under regime 1 and with $\alpha = 0.96$ under regime 3. A Calvo parameter $\alpha = 0.92$ implies an expected price rigidity of about 1 year for 1979.03-1990:10, whereas $\alpha = 0.96$ implies an expected price rigidity of about 2 years for 1992:11-1998:12. When we use these values of $\alpha$ over the two regime we can see from the lower panel of Table 2 that in terms of the first order autocorrelation of the real exchange rate the implied ranking from the DSGE model matches the observed ranking in the data.

4.2 Characterising Real Exchange Rate Persistence under Different Regimes

Based on the model-implied real exchange rate stylised facts from Section 4.1, we will analyse within our DSGE model the implications of different types of systemic monetary policy on real exchange rate persistence with $\alpha = 0.92$ for the pre-ERM regime and $\alpha = 0.96$ for the inflation targeting regime plus the corresponding estimated monetary policy rules from Section 2.2. We will compute the implied response of real exchange rate under each regime to each of the 4 shocks in our DSGE model: a common real shock, a relative real shock, a Home monetary shock and Foreign monetary shock. Next, we utilise Monte Carlo experiments to back out the implied half life of the real exchange rate under each regime when the system is hit at random by the four aforementioned shocks

In order to investigate the effect from each shock, we use the impulse response functions in our model, and these are reported in Figure 2 for the 1979-1990 pre-ERM regime or regime 1 and in Figure 3 for the 1992-1998 inflation targeting regime or regime 3. The implied DM/UK real exchange rate reaction to either Home or Foreign monetary shocks does not seem to differ much between regime 1 and regime 3. While the Home and Foreign monetary shocks themselves are not persistent they still generate persistent effects on the real exchange rate, mainly due to interest smoothing and local currency pricing. Nonetheless, the real exchange rate persistence caused by these shocks in Figures 2 and 3 is not large.

The effect of a common real shock, however, differs significantly among the two regimes, see the left hand graph in the upper rows of Figures 2 and 3. In regime 1, the DM/UK real exchange rate depreciates then come back to its original level within 2 years, after which it slightly appreciates and returns to its steady state value. Compared to regime 3, however, the effect of the common real shock is generally small. In regime 3, the common shock results in a real appreciation of sterling and it only returns back to its initial level very slowly, i.e. the real exchange rate returns to its steady state value after approximately 6.5 years. From the right hand side graphs in the upper rows of Figures 2 and 3 it becomes apparent that the magnitude of the relative real shock is relatively higher in the pre-ERM regime 1 than under the inflation targeting regime 3. The effect of the relative real shock is slightly more persistent under the UK inflation targeting regime than
under regime 1, where under the latter the seizes to exercise an effect after approximately 3 years as supposed to approximately 5 years under the former. Note, however, that in contrast to the common real shock the direction of change of the real exchange rate as a consequence of a relative real shock is not different across the monetary policy regimes. One also must be aware of the fact that the observation that real shocks in our DSGE model have a more persistent effect on the real exchange rate is due to the persistent effect of these shocks on the steady state values of consumption and the terms-of-trade, i.e. $\bar{C}$ and $\bar{T}$. The difference in systemic monetary policy behaviour matters, as it affects the dynamics after the occurrence of the real shocks.

Figures 4 to 7 report how inflation and interest rates at home and abroad react to the four above described shocks. In particular in the case of the common real shock UK monetary authorities seem to react in a lot more accommodating way than Germany under the pre-ERM regime 1 than under the inflation targeting regime 3. Under regime 3 German and UK monetary authorities react in a fairly similar direction and as a result the persistency of the common real shock is fully transmitted to the real exchange rate.

A commonly used measure of the degree of real exchange rate persistence generated by DSGE models is the first order autocorrelation of the real exchange rate, see Table 2. This is, however, a very crude measure of persistence as it ignores higher order dynamics and potential non-stationarity. An alternative measure used in the empirical exchange rate literature is the augmented Dickey and Fuller (1979) [ADF] unit root test, which is a t-test of $H_0: \rho - 1 = 0$ in

$$\Delta q_t = \delta_0 + (\rho - 1)q_{t-1} + \sum_{j=1}^{p-1} \delta_j \Delta q_{t-j} + \epsilon_t,$$

where $q_t$ is the log real exchange rate and $\rho$ measures the sum of the autoregressive parameters of the AR($p$) model of the level of $q_t$. In the upper panel of Table 3 we report the ADF unit root test result for each of the two subperiods, i.e. the 1979.07-1990.10 pre-ERM period and the 1992.11-1998.12 inflation targeting period, both within (54) where the lag order $p - 1$ is selected through the Akaike Information Criterion [AIC] as well as the Bayesian-Schwarz Information Criterion [BIC]. These empirical unit root test results indicate that for the first pre-ERM regime we have a pretty robust rejection of the unit root in the real exchange rate, whereas for the inflation targeting regime 3 we have the opposite result. The estimated $\rho$ from (54) can also be used to construct a measure of the half life of a shocks to the real exchange rate through

$$HL = \frac{\ln(0.5)}{\ln(\rho)},$$

and these half life measures are also reported in the upper panel of Table 3. For regime 1 we have a robust finding of a half life of 1 year, whereas under regime 3 half life estimates are much more variable and range from 1.33 years up to about 4.5 years. Hence, it most likely has been the case that real persistence has gone up to a certain extent with the
introduction of inflation targeting by the end of 1992 relative to systemic monetary policy behaviour in the pre-ERM period.

Can we reproduce this different pattern in real exchange rate persistence across policy regimes within our DSGE setting? In order to be able to answer this, we do some Monte Carlo analysis with our DSGE model both under the first regime based on $\alpha = 0.92$ and the corresponding policy rule estimates from Section 2.2 as well as under regime 3 with $\alpha = 0.96$ and corresponding policy rule estimates. We draw the real shocks from normal distributions which are parameterized through the calibration of Section 4.1, and are assumed to be identical over the two regimes. The Home and Foreign monetary shocks are drawn from zero-mean normal distributions with a standard deviation set equal to the estimated standard error of the policy rule regressions from Table 2.2.

The first type of Monte Carlo experiment we do is estimating the empirical power of the ADF unit root test on the finite sample generated from our DSGE model for each of the 2 regimes. As the DSGE model imposes long-run PPP, and therefore generates a stationary real exchange rate series, the empirical power ratio at a certain significance level is simply the number of rejections of the null hypothesis of a unit root across the number of Monte Carlo iterations. The results of this Monte Carlo-based power analysis at the 5% significance level based on either using AIC or BIC in (54), utilising 10,000 simulations, can be found in the lower panel of Table 3.\textsuperscript{17} The first row in the lower panel of this table reports the empirical power for the empirical sample size under each regime. At the empirical sample size the ADF unit root test has certainly more power to reject the false null of a unit root in the real exchange rate under the pre-ERM regime than under the inflation targeting regime. However, one can partly explain the larger power under the first pre-ERM regime based on the longer empirical sample size. We therefore repeated the exercise under an identical finite sample of 100 observations for each regime. From the second row for each regime in the lower panel of Table 3 it becomes clear that the results remain qualitatively unchanged.

Related to the first Monte Carlo experiment we also conduct an analysis of the distribution of the half life shocks to the real exchange rate based on Monte Carlo simulations. The set-up is comparable to the previous Monte Carlo simulation but we now use (54) to compute the half life through (55) based on 1,000 observations per iteration. In the lower panel of Table 3 we report the mean of the half life estimates across the 10,000 Monte Carlo samples. The results indicate that even in large samples the half life is bigger under the inflation targeting regime than under the pre-ERM regime.

## 5 Conclusions

The persistence of real exchange rates since the break down of the Bretton Woods system of fixed exchange rates has been a long standing issue in exchange rate research. Although nominal price stickiness plays a major role in real exchange rate persistence, studies like

\textsuperscript{17}In each iteration we generate $T+100$ observations on the artificial real exchange rate and then discard the first 100 in order to deal with initial value bias.
Chari et al. (2002) have shown that this is not enough to explain the full extent of the observed degree of persistence. In this paper we try to investigate the role of cross-country differences in systemic monetary policy behaviour in generating real exchange rate persistence. Our approach is similar to the analysis in Clarida et al. (2000) for the US, in that we use a dynamic stochastic general equilibrium model and combine it with estimated monetary policy rules to investigate how real exchange rate persistence is affected by relative differences in the feedback mechanism of monetary policy at home and abroad.

We focus on the Germany/UK real exchange rate relationship over the period 1979-1998 and in particular on subsamples when the corresponding nominal exchange rate was more or less flexible, i.e. the 1979-1990 pre-ERM period and the 1992-1998 inflation targeting era. As a first step we estimate interest rate-based forward looking monetary policy rules of the Clarida et al. (1998b, 2000) type for Germany over the whole sample and for the UK over the aforementioned subsamples. The estimation results indicate that for Germany and the UK under inflation targeting monetary policy had a stabilising effect on inflation whereas under the pre-ERM regime it almost fully accommodated inflationary pressures. Next we build a two-country DSGE model with local currency pricing, Calvo-type price stickiness, persistent real shock processes and asymmetric monetary policy rules. In analysing the model we show that real exchange rate persistence depends on the degree of interest rate smoothing by central banks at home and abroad as well as the persistent real shocks. Finally, we calibrate the preference parameters and the real shocks as realistically as possible. Given these calibrations we feed the estimated policy rules across different regimes into the DSGE model and analyse how the general persistence of the real exchange rate is affected by this. The result of our analysis shows that, like in the data, our DSGE model has the result that the degree of real exchange rate persistence is higher under the inflation targeting regime than under the pre-ERM regime. This is mainly due to the fact that common real shocks are transmitted differently across the two regimes. Under the inflation targeting regime UK monetary authorities react almost identical to the German authorities and as a consequence the nominal exchange rate will not counter-move the effect of the common real shock.

In future work we want to analyse other real exchange rate relationships, in particular the Germany/US relationship, in order to investigate the robustness of our results. We also want to make our DSGE model slightly more realistic in that we want to allow in the simulations for cross-country differences in nominal price stickiness, where we now assume that this identical across countries. Another extension is to assume incomplete asset markets and to add a noise component to the UIP relationship.
Appendix

A Flexible Price Model

Under flexible price, the nominal complete market assumption becomes usual complete market assumption, which guarantees $C^H = C^F$ and $P^H = S^F (\tilde{Q} = 0)$ for all time. However, as productivity shocks hit, relative price will change. Labor supply equation (14) can be linearised as,

$$\eta \tilde{L}^i + \rho \tilde{C} = \tilde{W}^i - \tilde{P}^i. \quad (A.1)$$

By constructing the world variable and the relative variable, we get

$$\eta L^W + \rho \tilde{C} = \tilde{W}^W - \tilde{P}^H + (1-n)\tilde{S} \quad (A.2)$$

$$\eta L^R = \tilde{W}^R - \tilde{S} \quad (A.3)$$

The law of one price holds and prices reflect the marginal cost and exchange rate in case of import goods.

$$\tilde{P}_H^H = \tilde{W}^H - \tilde{A}^H \quad (A.4)$$

$$\tilde{P}_F^H = \tilde{W}^F - \tilde{A}^F + \tilde{S} \quad (A.5)$$

Since $\tilde{P}^H = n\tilde{P}_H^H + (1-n)\tilde{P}_F^H$,

$$\tilde{P}^H = \tilde{W}^W + (1-n)\tilde{S} - n\tilde{A}^H - (1-n)\tilde{A}^F \quad (A.6)$$

Combining (A.2) and (A.6),

$$\eta \tilde{L}^W + \rho \tilde{C} = n\tilde{A}^H + (1-n)\tilde{A}^F \quad (A.7)$$

Market clearing conditions for Home goods and Foreign goods will give us,

$$\tilde{A}^H + \tilde{L}^H = -\varsigma (\tilde{P}_H^i - \tilde{P}^i) + \tilde{C} \quad (A.8)$$

$$\tilde{A}^F + \tilde{L}^F = -\varsigma (\tilde{P}_F^i - \tilde{P}^i) + \tilde{C} \quad (A.9)$$

where, $i$ could be either $H$ or $F$. They will give us

$$n\tilde{A}^H + (1-n)\tilde{A}^F + \tilde{L}^W = \tilde{C}. \quad (A.10)$$

Using this and (A.7), we can get equation (31).

$$\tilde{C} = \frac{\eta + 1}{\eta + \rho} [n\tilde{A}^H + (1-n)\tilde{A}^F].$$

Now using (A.4) and (A.5),

$$\tilde{T} = \tilde{P}_F^H - \tilde{P}_H^H = -\tilde{W}^R + \tilde{A}^H - \tilde{A}^F + \tilde{S} \quad (A.11)$$
Combining with (A.3), we get
\[ \hat{T} = \hat{A}^H - \hat{A}^F - \eta \hat{L}^R \] (A.12)

Equations (A.8) and (A.9) give us,
\[ \hat{A}^H - \hat{A}^F + \hat{L}^R = \varsigma \hat{T} \] (A.13)

Thus, we can get equation (32).
\[ \hat{T} = \frac{\eta + 1}{\eta \varsigma + 1} [\hat{A}^H - \hat{A}^F] \]

B Phillips Curve

We take the first order Taylor expansion around the steady state value. The followings are key results. Following Benigno (2002), we can turn price equations (19)-(22) into Philips Curves.

\[ \pi_{H,t}^H = k_C(\hat{C}_t^W - \hat{C}_t) + k_{H,T}^H(\hat{T}_t^H - \hat{T}_t) + k_{H,F}^H(\hat{T}_t^F - \hat{T}_t) + k_Q^H \hat{Q}_t + \beta E_t \pi_{H,t+1} \] (B.1)
\[ \pi_{F,t}^H = k_C(\hat{C}_t^W - \hat{C}_t) + k_{H,T}^H(\hat{T}_t^H - \hat{T}_t) + k_{H,F}^H(\hat{T}_t^F - \hat{T}_t) + k_Q^H \hat{Q}_t + \beta E_t \pi_{H,t+1} \] (B.2)
\[ \pi_{H,t}^F = k_C(\hat{C}_t^W - \hat{C}_t) + k_{H,T}^F(\hat{T}_t^H - \hat{T}_t) + k_{F,T}^F(\hat{T}_t^F - \hat{T}_t) + k_Q^F \hat{Q}_t + \beta E_t \pi_{H,t+1} \] (B.3)
\[ \pi_{F,t}^F = k_C(\hat{C}_t^W - \hat{C}_t) + k_{F,T}^H(\hat{T}_t^H - \hat{T}_t) + k_{F,T}^F(\hat{T}_t^F - \hat{T}_t) + k_Q^F \hat{Q}_t + \beta E_t \pi_{F,t+1} \] (B.4)

where, \( \hat{T}^H = \ln \left( \frac{P_{H,F}^H}{P_{H,H}^H} \right) \), the terms of trade in Home, and \( \hat{T}^F = \ln \left( \frac{P_{F,F}^H}{P_{F,H}^H} \right) \), the inverse of the terms of trade in Foreign. Also,

\[ k_C = \frac{\eta + \rho}{1 + \sigma \eta} \zeta \]
\[ k_{H,T}^H = \frac{\zeta(1-n)}{1 + \sigma \eta} [1 + \varsigma \eta \sigma + \sigma \eta (1-n)] \]
\[ k_{H,F}^H = -\frac{\zeta n}{1 + \sigma \eta} [1 + \varsigma \eta \sigma + \sigma \eta (1-n)] \]
\[ k_{H,T}^F = \frac{\zeta(1-n)^2}{1 + \sigma \eta} \eta [\varsigma - \sigma] \]
\[ k_{H,F}^F = -\frac{\zeta n(1-n)}{1 + \sigma \eta} \eta [\varsigma - \sigma] \]
\[ k_{F,T}^H = \frac{\zeta(1-n)}{1 + \sigma \eta} \eta [\varsigma - \sigma] \]
\[ k_{F,T}^F = \frac{\zeta(1-n)}{1 + \sigma \eta} [1 + \varsigma \eta (1-n) + \sigma \eta \sigma] \]
\[ k_Q^H = (1-n) \zeta \]
\[ k_Q^F = -n \zeta \]

Using them we can get Phillips Curves for both countries, world and relative.
References


Table 1: Estimated monetary policy rules, 1979.07-1998.12.\textsuperscript{a}

<table>
<thead>
<tr>
<th></th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$\phi^*$</th>
<th>$\psi^*$</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>0.94***</td>
<td>–</td>
<td>1.48***</td>
<td>0.45***</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td></td>
<td>(0.14)</td>
<td>(0.12)</td>
<td></td>
</tr>
<tr>
<td>UK, 1979.03-1990.10</td>
<td>0.91***</td>
<td>–</td>
<td>0.99***</td>
<td>0.82**</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td></td>
<td>(0.19)</td>
<td>(0.32)</td>
<td></td>
</tr>
<tr>
<td>UK, 1992:11-1998:12</td>
<td>1.29***</td>
<td>−0.36***</td>
<td>2.08***</td>
<td>0.62</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.06)</td>
<td>(0.61)</td>
<td>(0.44)</td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{a} The table contains GMM estimates of (3) or in case of the UK for 1992.11-1998.12 period (5). The values in parentheses are Newey and West (1987)-based standard errors with a truncation lag of 12. The set of instrument variables are selected based on minimizing the Andrews (1999) GMM-BIC criterion. For Germany we have $k_1 = 12$, $k_2 = 0$, $l_1 = 12$, $l_2 = 11$ and $l_3 = 11$, for 1979.07-1990.10 UK we have $k_1 = 12$, $k_2 = 3$, $l_1 = 7$, $l_2 = 7$ and $l_3 = 5$, and for 1992.11-1998.12 UK we have $k_1 = 6$, $k_2 = 3$, $l_1 = 8$, $l_2 = 6$ and $l_3 = 3$ (see (3) or (5)).
Table 2: Real exchange rate facts under different UK monetary policy regimes.\(^a\)

<table>
<thead>
<tr>
<th>Period</th>
<th>Corr (Q_t, Q_{t-1})</th>
<th>STD (Q)</th>
<th>STD (\Delta Q)</th>
<th>STD (\Delta S)</th>
<th>Corr (\Delta S_t, \Delta Q_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979:07-1990:10</td>
<td>0.962</td>
<td>6.09%</td>
<td>2.33%</td>
<td>2.22%</td>
<td>0.960</td>
</tr>
<tr>
<td>1990:11-1992:10</td>
<td>0.503</td>
<td>2.92%</td>
<td>1.67%</td>
<td>2.23%</td>
<td>0.972</td>
</tr>
<tr>
<td>1992:11-1998:12</td>
<td>0.973</td>
<td>8.17%</td>
<td>1.90%</td>
<td>1.93%</td>
<td>0.956</td>
</tr>
</tbody>
</table>

**Empirical real exchange rate facts.**

**Real exchange rate facts from DSGE model: \(\alpha = 0.94\)**

<table>
<thead>
<tr>
<th>Period</th>
<th>Corr (Q_t, Q_{t-1})</th>
<th>STD (Q)</th>
<th>STD (\Delta Q)</th>
<th>STD (\Delta S)</th>
<th>Corr (\Delta S_t, \Delta Q_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979:07-1990:10</td>
<td>0.935</td>
<td>4.87%</td>
<td>1.75%</td>
<td>1.82%</td>
<td>0.992</td>
</tr>
<tr>
<td>1992:11-1998:12</td>
<td>0.919</td>
<td>1.25%</td>
<td>0.50%</td>
<td>0.52%</td>
<td>0.991</td>
</tr>
</tbody>
</table>

**Real exchange rate facts from DSGE model: different \(\alpha\)’s**

<table>
<thead>
<tr>
<th>Period</th>
<th>Corr (Q_t, Q_{t-1})</th>
<th>STD (Q)</th>
<th>STD (\Delta Q)</th>
<th>STD (\Delta S)</th>
<th>Corr (\Delta S_t, \Delta Q_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979:07-1990:10 ((\alpha = 0.92))</td>
<td>0.914</td>
<td>3.75%</td>
<td>1.55%</td>
<td>1.64%</td>
<td>0.986</td>
</tr>
<tr>
<td>1992:11-1998:12 ((\alpha = 0.96))</td>
<td>0.946</td>
<td>1.95%</td>
<td>0.64%</td>
<td>0.65%</td>
<td>0.995</td>
</tr>
</tbody>
</table>

\(^a\) The values are based on monthly data.
Table 3: Persistence of DM/UK Real Exchange Rate under Different UK Monetary Policy Regimes.\textsuperscript{a}

<table>
<thead>
<tr>
<th></th>
<th>ADF-AIC</th>
<th>Lags</th>
<th>HL-AIC</th>
<th>ADF-BIC</th>
<th>Lags</th>
<th>HL-BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Historical persistence</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1979.07-1990.10</td>
<td>−2.94**</td>
<td>1</td>
<td>12</td>
<td>−2.94**</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>1992.11-1998.12</td>
<td>−1.92</td>
<td>8</td>
<td>16</td>
<td>−0.63</td>
<td>1</td>
<td>53</td>
</tr>
</tbody>
</table>

Persistence implied by DSGE model: Simulated power ADF and mean half life

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>pre-ERM regime</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pre-ERM regime</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T = 136$</td>
<td>36.2%</td>
<td>7.6</td>
<td>42.8%</td>
<td>7.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T = 100$</td>
<td>26.5%</td>
<td></td>
<td>30.4%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

|                          |          |       |        |          |       |        |
| inflation targeting regime |          |       |        |          |       |        |
| inflation targeting regime |          |       |        |          |       |        |
| $T = 74$                 | 18.2%    | 12.2  | 17.5%  | 12.1     |
| $T = 100$                | 19.0%    |       | 18.9%  |          |

\textsuperscript{a} The column denoted with ‘ADF-AIC’ report in the upper panel the empirical value of the ADF unit root t-test based AIC lag order selection, where * (**) [***] denotes a rejection of the unit root null hypothesis based on MacKinnon (1991) critical values, whereas in the lower panel the values in this column report the empirical power ratios based on Monte Carlo simulations of our DSGE model. The column denoted with ‘HL-AIC’ reports in the upper panel the empirical half life estimate through (55) via (54) using AIC lag order selection, whereas in the lower panel we report the Monte Carlo mean of the half life from our DSGE model. Similar statistics can be found in columns ‘ADF-BIC’ and ‘HL-BIC’ based on BIC lag order selection.
Figure 1: Model Sensitivity to the Real Parameters

Persistence of the Real Exchange Rate

Variance of the Real Exchange Rate

Persistence of the Real Exchange Rate

Variance of the Real Exchange Rate

Persistence of the Real Exchange Rate

Variance of the Real Exchange Rate

Persistence of the Real Exchange Rate

Variance of the Real Exchange Rate

Persistence of the Real Exchange Rate

Variance of the Real Exchange Rate

Persistence of the Real Exchange Rate

Variance of the Real Exchange Rate
Figure 2:

IRF of Q, Regime 1, 1 STD shock from $\nu^C$

IRF of Q, Regime 1, 1 STD shock from $\nu^F$

IRF of Q, Regime 1, 1 STD shock from $\nu^H$

Figure 3:

IRF of Q, Regime 3, 1 STD shock from $\nu^C$

IRF of Q, Regime 3, 1 STD shock from $\nu^F$

IRF of Q, Regime 3, 1 STD shock from $\nu^H$
Figure 4: IRF of Inflation, regime 1

Figure 5: IRF of Interest Rate, regime 1
Figure 6: IRF of Inflation, regime 3

Figure 7: IRF of Interest Rate, regime 3