Monetary Policy Rules in an Interdependent World

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Abstract

This paper analyzes the welfare effects of monetary policy rules, in a quantitative business cycle model of a two-country world. The model features staggered price setting, and shocks to productivity and to the uncovered interest rate parity (UIP) condition. UIP shocks have a sizable negative effect on welfare, when trade links are strong. An exchange rate peg may raise world welfare, if the peg eliminates the UIP shocks. The model explains the empirical finding that more open economies are more likely to adopt a peg.

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1. Introduction

What policy rule is best suited for maximizing welfare in open economies—especially: should central banks stabilize the exchange rate? Recent work has addressed this normative question, using general equilibrium models of open economies in which monetary policy affects real variables because of sticky prices—a literature often referred to as "New Open Economy Macroeconomics" (NOEM). Because of its rigorous microeconomic foundations, that approach is better suited for normative issues than the traditional Keynesian models. However, existing normative NOEM studies use highly stylized (often static) models (that permit to derive closed form solutions) which underpredict sharply the high volatility of exchange rates observed during the post-Bretton Woods period; this may cast doubts on the relevance of these models for assessing the welfare consequences of floating exchange rates.

A first step towards studying welfare effects of monetary policy using richer, more realistic quantitative (calibrated) models was made by Kollmann (2002a) who considered a small open economy with staggered price setting. The present paper extends that analysis by studying a two-country world. A two-country model allows to examine the effect of monetary policy on world welfare.

A key feature of the model here is that (besides the standard productivity shocks) there are shocks to the uncovered interest parity (UIP) condition; these "UIP shocks" can be interpreted as reflecting biased exchange rate forecasts by households. These disturbances enable the model to generate highly volatile nominal and real exchange rates. Other features that enhance the realism of the present model—and that distinguish the model here from those typically used in previous normative NOEM studies—are incomplete international risk sharing (due to the assumption that international financial transactions are restricted to trade in bonds) and physical capital.

Model variants with weak trade links between the two countries (1% imports/GDP ratio) and with strong trade links (20% trade share) are considered. These variants shed, inter alia, light on optimal monetary arrangements between the US and Europe (low trade), and on optimal arrangements among European economies (strong trade links).

Monetary policy is described by 'simple' rules under which a country's interest rate is set as a function of inflation, of GDP, and of the rate of depreciation of the nominal exchange rate. The parameters of both central banks' policy rules are set at the values that maximize world welfare (defined as the sum of the expected values of Home and Foreign household utility). An exchange rate peg is also considered, in which the policy parameters are set at the values that maximize world welfare, subject to the constraint that the exchange rate has to be kept constant.

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1 See Lane (2001), Sarno (2001) and Ganelli and Lane (2002) for surveys.
3 Several recent papers have studied quantitative NOEM business cycle models; however, these papers do not compute welfare (and thus do not determine welfare maximizing policy rules). See, for example, Batini et al. (2000,2001), Benigno (1999), Bergin (2001), Betts and Devereux (2001), Chari et al. (2000), Collard and Dellas (2002), Dedola and Leduc (2001), Duarte and Stockman (2001), Erceg and Levin (2001), Faia (2001), Ghironi and Rebucci (2001), Hairault et al. (2001), Kollmann (2001a,b), Laxton and Pesenti (2002), Lubik (2000), McCallum and Nelson (1999, 2000), Monacelli (1999), Schmitt-Grohé and Uribe (2001a), and Smets and Wouters (2000, 2001). With the exception of the models by Batini et al. and by McCallum and Nelson—who like the paper here assume interest parity shocks (see discussion below)—these models do not capture the strong exchange rate volatility observed in the post-Bretton Woods period. After the research here was completed, I received papers by Bergin and Tchakarov (2002) and by Tchakarov (2002) that likewise conduct welfare analyses of quantitative two-country NOEM models based on the same numerical technique as the paper here.
UIP shocks raise the volatility of consumption, of the real exchange rate, and of inflation, and they reduce world welfare. When the world economy is subjected to exogenous UIP shocks, then optimized policy entails exchange rate floating—the welfare gain from optimized policy compared to a peg corresponds to a permanent 0.46% [0.22%] consumption increase, in the variant of the baseline model with weak [strong] trade links.

However, the key issue for welfare is whether a peg affects the UIP shocks. Departures from interest rate parity were markedly smaller in the Bretton Woods [BW] era than in the post-BW period (e.g., Kollmann, 2002b). (Under the interpretation that UIP shocks reflect biased exchange rate forecasts, this finding can easily be rationalized—under a (credible) peg there is much less scope for irrational exchange rate forecasts than under a float.) In the model here, a peg is optimal if a peg eliminates the UIP shocks. The baseline model predicts that the welfare gain from an exchange rate peg that eliminates UIP shocks would be very slightly positive between the US and Europe—the equivalent of a permanent 0.004% consumption increase (compared to an optimized floating rate regime): within Europe, the predicted welfare gain from such a peg corresponds to a permanent 0.29% consumption increase. In the model, UIP shocks are more harmful in more open economies—the welfare gain from a peg that eliminates the UIP shocks is thus predicted to be higher the greater the degree of external openness. Empirically, the likelihood that a country pegs its exchange rate is positively linked to openness; see e.g. Edwards (1996). The model here can rationalize this finding.

The model is solved using Sims' (2000) algorithm that is based on second-order Taylor expansions of the equilibrium conditions. In contrast to the linear, certainty-equivalent approximations that are widely used in macroeconomics, this approach allows to capture the effect of risk on mean values of endogenous variables—that effect turns out to be crucial for welfare. Compared to other non-linear methods (see Judd, 1998, for an overview), a key advantage of the method used here is the much greater ease and speed with which it allows to solve models with a large number of state variables. This allows me to numerically determine the welfare maximizing monetary policy parameters, in the rich business cycle model considered here.

Section 2 of this paper describes the model. Section 3 presents the results and Section 4 concludes.

2. The model

I consider a world with two countries, referred to as "Home" and "Foreign". In each country there are firms, a representative household and a central bank (the structure of preferences and technologies follows Kollmann, 2002a, 2001a). Each country produces a continuum of tradable intermediate goods indexed by $s \in [0,1]$. In each country there are competitive firms that bundle domestic and imported intermediate goods into a non-tradable final good that is consumed and used for investment. There is monopolistic competition in intermediate goods markets. Intermediate goods producers use domestic capital and labor as inputs (capital and labor are immobile internationally). In each country, the household owns all domestic producers and the capital stock, which it rents to producers. It also supplies labor. The markets for rental capital and for labor are competitive.

Preferences and technologies are symmetric across the countries. An asterisk denotes Foreign variables. The following description focuses on the Home country.

2.1. Final good production

The Home final good is produced using the aggregate technology

$$Z_t = Q^d_t/\alpha^d f^d (Q^m_t/\alpha^m)^{\alpha^m},$$

(1)
with $\alpha^d, \alpha^m > 0$, $\alpha^d + \alpha^m = 1$. $Z_t$ is final good output at date $t$; $Q^d_t, Q^m_t$ are quantity indices of domestic and imported intermediate goods, respectively: $Q^i_t = \left\{ \int_0^t q^i_t(s)^{\nu/(\nu-1)} ds \right\}^{1/(\nu-1)}$ with $\nu > 1$, for $i=d,m$, where $q^d_t(s)$ and $q^m_t(s)$ are quantities of the domestic and imported type $s$ intermediate goods. Let $p^d_t(s)$ and $p^m_t(s)$ be the prices of these goods in Home currency.

Cost minimization in final good production implies:

$$q^i_t(s) = (p^i_t(s)/P^i_t)^{-\nu} Q^i_t, \quad Q^i_t = \alpha^i P_t Z_t/P^i_t$$

for $i=d,m$.

(2)

$P^d_t$ [$P^m_t$] is a price index for domestic [imported] intermediate goods that are sold in the Home market. Perfect competition implies that the price of the Home final good is $P_t$ (its marginal cost is $P_t = (P^d_t)^{\alpha^d} (P^m_t)^{\alpha^m}$).

2.2. Intermediate goods firms

The technology of the firm that produces intermediate good $s$ in the Home country is:

$$y_t(s) = \theta_t K_t(s)^{\psi} L_t(s)^{1-\psi}, \quad 0 < \psi < 1.$$  

(4)

$y_t(s)$ is the firm's output at date $t$; $\theta_t$ is an exogenous productivity parameter that is identical for all Home intermediate goods producers; $K_t(s)$ and $L_t(s)$ are the amounts of capital and labor used by the firm.

Let $R_t$ and $W_t$ be the rental rate of capital and the wage rate. Cost minimization implies:

$$L_t(s)/K_t(s) = \psi^{-1}(1-\psi)R_t/W_t.$$  

(5)

The firm's marginal cost is: $MC_t = \psi W_t^{-1}(1-\psi)\psi^{-\psi}(1-\psi)^{-1}$. The firm’s good is sold in the domestic market and exported:

$$y_t(s) = q^d_t(s) + q^m_t(s),$$

(6)

where $q^d_t(s)$ [$q^m_t(s)$] is domestic [export] demand. The firm faces the following export demand function: $q^m_t(s) = (p^m_t(s)/P^m_t)^{-\nu} Q^m_t$, where $p^m_t(s)$ is the firm's export price, in Foreign currency.

The firm’s profit, $\pi_t$, is:

$$\pi_t(p^d_t(s), p^m_t(s)) = (p^d_t(s) - MC_t)(p^d_t(s)/P^d_t)^{-\nu} Q^d_t + (e_t p^m_t(s) - MC_t)(p^m_t(s)/P^m_t)^{-\nu} Q^m_t,$$

where $e_t$ is the nominal exchange rate, expressed as the Home currency price of Foreign currency.

Motivated by the empirical failure of the Law of One Price, and in particular by widespread pricing-to-market behavior (e.g., Knetter, 1993), it is assumed that intermediate goods producers can price discriminate between the domestic market and the export market ($p^d_t(s) \neq e_t p^m_t(s)$ is possible), and that they set prices in the currencies of their customers.

There is staggered price setting, à la Calvo (1983): intermediate goods firms cannot change prices (in buyer currency) unless they receive a random "price-change signal." The probability of receiving this signal in any particular period is $1-d$, a constant. Thus, the mean price-change-interval is $1/(1-d)$. Following Yun (1996) and Erceg et al. (2000) it is assumed that when a firm does not receive a "price-change signal," its price is automatically increased at the steady state growth factor of the price level (in the buyer's country). (Throughout this paper, the term "steady state" refers to the deterministic steady state.) Firms are assumed to meet all demand at posted prices.
Consider a Home country intermediate good producer that, at time \( t \), sets a new price \( p^d_{tt} \). If no "price-change signal" is received between \( t \) and \( t + \tau \), the price is \( p^d_{tt} \Pi^\tau \) at \( t + \tau \), where \( \Pi \) is the steady state growth factor of the Home price level.

The firm sets

\[
\text{Arg Max}_{p} \sum_{t=0}^{\infty} d^t E_t \{ \rho_{tt} \Pi_{tt} \} \left( \Pi^\tau, p^s_{tt}(s) / p^s_{tt} \right),
\]

where \( \Pi \) is a pricing kernel for valuing date \( t + \tau \) pay-offs (expressed in units of the Home final good) that equals the Home household’s marginal rate of substitution between consumption at \( t \) and at \( t + \tau \) (see discussion below).

Let \( \Xi_{t,tt}^d = \rho_{tt} \Pi_{tt} (P_t / P_{tt}) Q_{tt}^d (P_{tt}^s)^y \). The solution of the maximization problem regarding \( p^d_{tt} \) is:

\[
p^d_{tt} = (v/(v-1)) \left[ \sum_{t=0}^{\infty} (d \Pi^\nu)^t E_t \Xi_{t,tt}^d MC_{t+1}^d \right] / \left[ \sum_{t=0}^{\infty} (d \Pi^\nu)^t E_t \Xi_{t,tt}^d \right].
\]

Analogously, a Home intermediate good producer that gets to choose a new export price at date \( t \) sets that price at:

\[
p^m_{tt} = (v/(v-1)) \left[ \sum_{t=0}^{\infty} (d \Pi^\nu)^t E_t \Xi_{t,tt}^m MC_{t+1}^m e_t^m \right] / \left[ \sum_{t=0}^{\infty} (d \Pi^\nu)^t E_t \Xi_{t,tt}^m \right],
\]

where \( \Xi_{t,tt}^m = \rho_{tt} (P_t / P_{tt})(e_t / e_t^m) Q_{tt}^m (P_{tt}^m)^y \), while \( \Pi^* \) is the steady state growth factor of the Foreign price level.

The price indices \( P_t^d \), \( P_t^m \) (see (3)) evolve according to:

\[
(P_t^d)^{1-\nu} = d(P_{tt}^d \Pi^\nu) + (1-d)(p^d_{tt})^{1-\nu}; \quad (P_t^m)^{1-\nu} = d(P_{tt}^m \Pi^\nu) + (1-d)(p^m_{tt})^{1-\nu}.
\]

### 2.3. The representative household

The preferences of the Home household are described by:

\[
E_t \sum_{t=0}^{\infty} \beta^t U(C_t, L_t).
\]

\( E_t \) denotes the mathematical expectation conditional upon complete information pertaining to period \( t \) and earlier. \( C_t \) and \( L_t \) are period \( t \) consumption and labor effort. 0 < \( \beta < 1 \) is the subjective discount factor. \( U \) is a utility function given by:

\[
U(C_t, L_t) = \ln(C_t) - L_t.
\]

As indicated earlier, the household owns all domestic producers and it accumulates physical capital. The law of motion of the capital stock is:

\[
K_{t+1} + \phi(K_{t+1}, K_t) = K_t (1-\delta) + I_t,
\]

where \( I_t \) is gross investment, 0 < \( \delta < 1 \) is the depreciation rate of capital, and \( \phi \) is an adjustment cost function: \( \phi(K_{t+1}, K_t) = \Phi \{K_{t+1} - K_t\}^2 / K_t \), \( \Phi > 0 \).

The Home household holds nominal one-period bonds denominated in Home currency and in Foreign currency. Its period \( t \) budget constraint is:

\[
A_{t+1} + e_t B_{t+1} + P_t (C_t + I_t + F_t) = A_t (1 + i_{t-1}) + e_t B_t (1 + i^*_t) + R_t K_t + \int_0^1 \pi_t (s) ds + W_t L_t.
\]

\( A_t \) and \( B_t \) are stocks of Home and Foreign currency bonds that mature in period \( t \), while \( i_{t-1} \) and \( i^*_t \) are the interest rates on these bonds. The household bears a real cost (in Home final good units) of holding/issuing bonds, denoted \( F_t \): \( F_t \) is a quadratic function of \( A_{t+1} \) and \( B_{t+1} \): \( F_t = \frac{1}{2} \phi^A (A_{t+1}/P_t)^2 + \frac{1}{2} \phi^B (e_t B_t / P_t)^2 \), with \( \phi^A, \phi^B \geq 0 \), \( \phi^A + \phi^B > 0 \). This cost ensures
the existence of a stationary equilibrium, which allows to solve the model using the Sims (2000) method.

The household chooses a strategy \( \{A_{t+1}, B_{t+1}, K_{t+1}, C_t, L_t, \gamma_{t+1} \} \) to maximize its expected lifetime utility \((7)\), subject to constraints \((9)\) and \((10)\) and to initial values \(A_0, B_0, K_0\). Ruling out Ponzi schemes, the following equations are first-order conditions of this decision problem:

\[
1 = \frac{1+i}{1+\phi^A \cdot (A_{t+1}/P_t)} E_i\{\rho_{t+1} (P_t/P_{t+1})\},
\]

\[
1 = \frac{1+i^*}{1+\phi^B \cdot (e_t B_{t+1}/P_t)} E_i\{\rho_{t+1} (P_t/P_{t+1}) (e_{t+1}/e_t)\},
\]

\[
1 = E_i\{\rho_{t+1} (R_{t+1}/P_{t+1} + 1-\delta - \phi_{2,t+1})/(1+\phi_{1,t})\},
\]

\[
W_t/P_t = C_t,
\]

where \(\rho_{t+1} = \beta C_t/C_{t+1}, \phi_{1,t} = \partial K_{t+1}/\partial K_{t+1}, \phi_{2,t} = \partial K_{t+2}/\partial K_{t+1}\). (11)-(13) are Euler conditions, and (14) says that the household equates its marginal rate of substitution between consumption and leisure to the real wage rate.

### 2.4. Uncovered interest parity

Taking a (log-)linear approximation of \((11)\) and \((12)\) (around \(A_{t+1} = B_{t+1} = 0\)) yields:

\[
E_i \ln(e_{t+1}/e_t) \equiv i_t - i^*_t - \phi^A (A_{t+1}/P_t) + \phi^B (B_{t+1} e_t/P_t).
\]

Because of bond-holding costs (and because of the second order terms that have been suppressed in this approximation), uncovered interest parity (UIP) (i.e. the condition \(E_i \ln(e_{t+1}/e_t) = i_t - i^*_t\) does not hold in the model here. However, departures from UIP that are caused by bond-holding costs (and by second order terms) turn out to be very small, in the present model. Given the well-documented strong and persistent empirical departures from UIP during the post-Bretton Woods era (e.g., Lewis, 1995), variants of the model are explored in which the Home Euler condition for Foreign currency bonds \((12)\) is disturbed by a stationary exogenous stochastic random variable, \(\phi_t\) ("UIP shock," henceforth):

\[
1 = \frac{1+i^*_t}{1+\phi^B \cdot (e_t B_{t+1}/P_t)} \phi_t E_i\{\rho_{t+1} (P_t/P_{t+1}) (e_{t+1}/e_t)\}.
\]

Up to a (log-)linear approximation (around \(A_{t+1} = B_{t+1} = 0, \phi = 1\)) \((11)\) and \((15)\) imply

\[
E_i \ln(e_{t+1}/e_t) \equiv i_t - i^*_t - \phi^A (A_{t+1}/P_t) + \phi^B (B_{t+1} e_t/P_t) - \ln(\phi_t).
\]

\(\phi_t\) can be interpreted as reflecting a bias in the households' date \(t\) forecast of the date \(t+1\) exchange rate, \(e_{t+1}\). It is assumed that Home and Foreign households make identical exchange rate forecasts—and, thus that these forecasts exhibit the same bias.

The counterparts to \((11)\), \((15)\) and \((16)\), for the Foreign household are:

\[\text{Footnotes:}
4\text{ When the cost } F_t \text{ is zero (i.e. when } \phi^F = \phi^B = 0\), the decision problem of the household is a version of the permanent income theory of consumption, and asset positions and consumption are non-stationary.}
5\text{ Assume that household beliefs at } t \text{ about } e_{t+1} \text{ are given by a probability density function, } f_t \text{, that differs from the true pdf, } f_t, \text{ by a factor } \psi^\phi : f_t(e_{t+1}, \Omega) = f_t(e_{t+1} | \phi_t, \Omega)/\phi_t, \text{ where } \Omega \text{ is any other random variable. The Home [Foreign] Euler equation for foreign currency bonds is then given by } (15) [\text{(18)}].}
\]
\[ 1 - \frac{1 + i^*_t}{1 + \phi^{A^*_t}(B^*_t/P_t^*)} E_t\{\rho^*_{t+1}(P_t^*/P_{t+1}^*)\}, \]  
\[ 1 - \frac{1 + i_t}{1 + \phi^{A^*_t}(B^*_t/P_t^*)} E_t\{\rho^*_{t+1}(P_t^*/P_{t+1}^*)(e_t/e_{t+1})\}, \]  
\[ E_t\ln(e_{t+1}/e_t) \equiv i_t - i^*_t - \phi^{A^*_t}(e_t/P_t^*) + \phi^{A^*_t}(B_t^*/P_t^*) - \ln(\phi). \]  
(The Foreign household bears the following bond-holding cost, in units of the Foreign final good: \[ F_t^s = \frac{1}{2} \phi^{A^*_t}(A_t^*/(e_t/P_t^*))^2 + \frac{1}{2} \phi^{A^*_t}(B_t^*/P_t^*)^2. \])

2.5. Market clearing conditions

Supply equals demand in intermediate goods markets because intermediate goods firms meet all demand at posted prices. In the Home country, market clearing for the final good, labor, and rental capital requires:

\[ Z_t = C_t + I_t + F_t, \quad L_t = \int_0^1 L_t(s)ds, \quad K_t = \int_0^1 K_t(s)ds, \]

where \( Z_t, L_t \) and \( K_t \) are supplies of the final good, labor, and rental capital, respectively, while \( \int_0^1 L_t(s)ds \) and \( \int_0^1 K_t(s)ds \) represent total demand for labor and capital (by intermediate goods producers). Market clearing for bonds requires:

\[ A_t + A^*_t = 0, \quad B_t + B^*_t = 0, \]  
where \( A_t^*, B_t^* \) are the Foreign household’s stocks of Home currency bonds and of Foreign currency bonds, respectively.

2.6. Monetary policy rules

Much recent research on monetary policy regimes has centered on rules under which the nominal interest rate is set as a function of inflation and of real GDP (e.g., Taylor, 1993a, 1999). In the present study, I also include the exchange rate \( e_t \) as an argument in the policy rule, as this allows to study whether monetary authorities should respond (directly) to that variable. The following rules for Home and Foreign monetary policy are considered:

\[ i_t = \pi^*_t + \Pi^*_t Y_t^* + \Gamma P_t - \Gamma_1, \quad Y_t^* = \frac{Y_t - Y}{Y}, \]

with \( \Pi^*_t = (\Pi^*_t - \Pi)/\Pi, \quad \Pi^*_t = (Y_t - Y)/Y, \) where \( \Pi^*_t = P_t^*/P_{t+1}^* \) is the growth factor of the price index of Home-produced domestic intermediate goods that are sold in the Home market (i.e. gross Home domestic PPI inflation), and \( Y_t^* \) is Home real GDP. \( i_t \) and \( Y_t^* \) are the steady state Home nominal interest rate and steady state Home GDP, respectively. Throughout the paper, steady state values are denoted by variables without time subscripts, and \( \hat{x}_t = (x_t - x)/x \) is the relative deviation of a variable \( x_t \) from its steady state value, \( x \). \( \Gamma, \Gamma^*, \Gamma_e, \Gamma_e^* \) are parameters.

The central banks make a commitment to set the parameters of their policy rules at time-invariant values that maximize world welfare, defined as the sum of the unconditional expected values of Home and Foreign household utility, \( E(U(C_t, L_t)) + E(U(C^*_t, L^*_t)) \). I also

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6 Home nominal GDP equals the aggregate revenue of Home intermediate goods producers:

\[ Y^* = \int_0^1 q_t^*(s)q_t^*(s) + p_t^*(s)q_t^*(s)ds. \]  
Evaluating the quantities \( q_t^*(s) \) and \( q_t^*(s) \) at the prices of some baseline period gives real GDP. Here, I normalize all baseline prices at unity. Thus \( Y^* = \int_0^1 q_t^*(s) + q_t^*(s)ds \) (see (6)).
consider an (optimized) exchange rate peg, in which the policy parameters are set at the values that maximize world welfare, subject to the constraint that the exchange rate has to be kept constant through time.

As discussed in Kollmann (2002a), a fully optimal policy rule would allow for a response of the interest rate to all current and lagged state variables; I focus on “simple” rules (such as (21a,b)) because: (i) simple rules capture well actual central bank behavior (Taylor, 1999); (ii) the use of simple rules facilitates policy commitment; (iii) computationally, it does not seem feasible to determine fully optimal rules for the complex model considered here.

2.7. Welfare measures

A second-order Taylor expansion of the Home utility function around the steady state gives:

\[ E(U(C_t, L_t)) = U(C, L) + E(\hat{C}_t) - LE(\hat{L}_t) - \frac{1}{2} Var(\hat{C}_t), \]

where \( Var(\hat{C}_t) \) is the variance of \( \hat{C}_t \). (For the parameter values used below, \( L=0.74 \).

In what follows, welfare is expressed as the permanent relative change in consumption (compared to the steady state), \( \xi \), that yields expected utility \( (U(C_t, L_t)) \):

\[ ((1 + \xi)C_t, L_t) = U(C, L) + E(\hat{C}_t) - LE(\hat{L}_t) - \frac{1}{2} Var(\hat{C}_t). \]

\( \xi \) can be decomposed into components, denoted \( \xi^m \) and \( \xi^v \), that reflect the means of consumption and hours worked, and the variance of consumption, respectively:

\[ ((1 + \xi^m)C_t, L_t) = U(C, L) - \frac{1}{2} Var(\hat{C}_t), \]

\[ \ln(1 + \xi^v) = E(\hat{C}_t) - LE(\hat{L}_t) - \frac{1}{2} Var(\hat{C}_t), \]

and thus \( (1 + \xi) = (1 + \xi^m)(1 + \xi^v) \).

2.8. The resource cost of price variability

Under staggered price setting, time-varying Home inflation lowers welfare as it induces inefficient dispersion of prices across Home intermediate goods producers that raises the aggregate inputs of labor \( (L_t) \) and capital \( (K_t) \) that are required to produce given quantities of the aggregate Home intermediate goods \( Q_t^d \) and \( Q_t^{m*} \). I now derive a measure of that resource cost of price dispersion (Smets and Wouters (2002) present a closely related measure). Note that (2), (4), (5) and (6) imply:

\[ \theta_i K_t^{v} L_t^{1-v} = \delta^d_i Q_t^d + \delta^{m*}_i Q_t^{m*}, \]

with \( \delta^d_i = (P_i^d / P_t^d)^{-\nu}, \delta^{m*}_i = \int_0^1 p_i(s)^{1-\nu} ds \) for \( i=d,m*, \) The left-hand side of (22) equals Home real GDP, \( Y_t \). \( \delta^d_i \) [\( \delta^{m*}_i \)] is an index of the cross-firm dispersion of the domestic prices [export prices] charged by Home intermediate goods producers at date; it can be shown that \( \delta^d_i, \delta^{m*}_i \geq 1 \). \( \delta^d_i = 1, \delta^{m*}_i = 1 \) holds when there is no cross-firm price dispersion--as is the case under price flexibility or when domestic and export price inflation are constant at \( \Pi_t^d = \Pi, \Pi_t^{m*} = \Pi \), where \( \Pi_t^d = P_t^d / P_t^{d-1} \) for \( i=d,m* \) (in steady state: \( \delta^d = \delta^{m*} = 1 \)). \( E \delta^d_i \) and \( E \delta^{m*}_i \) are increasing functions of the degree of price stickiness \( (d) \) and of the variances of \( \Pi_t^d \) and \( \Pi_t^{m*} \), respectively: \( E \delta^d_i \equiv 1 + 0.5 \nu (d/(1-d))^2 Var(\Pi_t^d), \) for \( i=d,m* \).

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7 See Kollmann’s (2002a, p.998) discussion of the computational difficulties pertaining to fully optimal rules (in a related model).

8 A second-order expansion shows that \( \hat{\delta}_i \equiv \frac{1}{\nu} Var(\ln p_i^d(s)) \) for \( i=d,m* \), where \( Var_i \) denotes the variance across firms \( 0 \leq s \leq 1 \).

9 This formula too is based on a second order expansions (see Rotemberg and Woodford (1997) and Erceg et al. (2001) for derivations).
A second-order expansion of (22) yields
\[ \hat{Y} = (1 - \alpha^m) \hat{Q}_t^d + \alpha^m \hat{Q}_t^m + \delta_i^d, \]
with \( \delta_i^d = (1 - \alpha^m) \delta_i^d + \alpha^m \delta_i^m \). (23)
\( \delta_i^d = (1 - \alpha^m) \delta_i^d + \alpha^m \delta_i^m \) is a measure of the total resource cost of price dispersion across Home intermediate goods firms.

A policy that perfectly stabilizes \( \Pi_i^d \) (at \( \Pi \)) minimizes \( \delta_i^d \) (at \( \delta^d = 1 \)), while a policy that perfectly stabilizes \( \Pi_i^m \) (at \( \Pi^m \)) minimizes \( \delta_i^m \) (at \( \delta_i^m = 1 \)). Under pricing-to-market (as assumed here), firms generally charge domestic prices that differ from their export prices, and control over the two policy instruments \( i \) and \( i^* \) does not permit to fully eliminate all price dispersion across Home firms and across Foreign firms (as this would require attaining these four targets: \( \delta_i^d = \delta_i^m = \delta_i^{d^*} = \delta_i^{m^*} = 1 \)).

UIP shocks induce sizable, socially inefficient changes in the relative price between domestic and imported goods. These relative price changes trigger substitution effects between imported and domestic intermediate goods; as final good production is a concave function of intermediate goods inputs, these substitution effects raise the resource cost of producing the final good. Taking a second-order expansion of (1) and (2) yields:
\[ E \hat{Z}_t = (1 - \alpha^m) E \hat{Q}_t^d + \alpha^m E \hat{Q}_t^m - \delta^m \]
where \( \delta^m = \frac{1}{2} \alpha^m (1 - \alpha^m) \text{Var}(P_t^m/P_t^d) \).

This expression and (23) imply that in symmetric equilibria (in which \( E \hat{Q}_t^m = E \hat{Q}_t^m \)):
\[ E \hat{Y}_t = E \hat{Z}_t + E \delta_i^d + \delta^m \]
\( \delta^m \) can thus be interpreted as the effect of variability of the relative price between aggregate imported and domestic intermediate goods, \( P_t^m/P_t^d \), on the (mean) final good resource cost.\(^{10}\) Note that the resource cost is higher, the higher the variance of the relative price \( P_t^m/P_t^d \), and the higher the trade share, \( \alpha^m \) (for \( \alpha^m < 0.5 \)).

Across the model variants considered in Tables 1 and 2 below, \( E \delta_i^d + \delta^m \) is highly negatively correlated with welfare (correlation: -0.97); \( E \delta_i^d \) is highly negatively correlated with the welfare difference between sticky-price and flex-prices equilibria (correlation: -0.89).

2.9. Solution method and parameters (non-policy)
The model is solved using Sims’ (2000) algorithm/computer code that is based on second-order Taylor expansions of the equilibrium conditions. I numerically maximize the central banks' objective function (world welfare) with respect to the policy parameters (attention is restricted to parameter values for which a unique stationary equilibrium exists).

Preference and technology parameters are assumed to be symmetric across countries.

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\(^{10}\) In an efficient world economy, the relative price \( P_t^m/P_t^d \) responds to productivity shocks. The measure \( \delta^m \) does not distinguish between these efficient responses and the inefficient responses induced by UIP shocks. However, in the model simulations, UIP shocks have a much stronger effect on \( P_t^m/P_t^d \) than productivity shocks--\( \delta^m \) reflects thus, mainly, the effect of UIP shocks.

The effects of the exchange rate regime depend on the countries' openness to trade (imports/GDP ratio). A variant of the model is considered in which the (steady state) imports/GDP ratio is set at $\alpha^* = 0.01$ (see (1)), "low-trade-variant" henceforth, as well as a variant with $\alpha^* = 0.2$ ("high-trade-variant").

The "low-trade-variant" is (for example) suitable for analyzing monetary arrangements between the US and the European Union (EU); I calibrate that variant to data for the US and an aggregate of three large EU economies: France, Germany and Italy, 'EU3' henceforth (the ratio of US imports from the EU3 divided by US GDP and the ratio of EU3 imports from the US divided by EU3 GDP both averaged about 0.01 during the post-Bretton Woods era).

The "high-trade" variant allows to analyze the optimal exchange regime among EU countries (the ratio of total trade among EU members, divided by aggregate EU GDP is roughly 0.2).

The remaining technology parameters as well as preference parameters are set at identical values across these two variants.

The steady state value of the UIP shock is set at $\varphi = 1$ (in steady state, exchange rate expectations are thus unbiased). This implies that steady state stocks of bonds are zero ($A = B = A^* = B^* = 0$), and that the steady state real interest rate $r = (1 + \varphi^*) (1 + \varphi^*)$ is given by: $\beta (1 + \varphi^*) = 1$; the subjective discount factor is set at $\beta = (1.01)^{-1}$ which implies $r = 0.01$, a real interest rate that corresponds roughly to the long-run historical average quarterly return on capital.

The steady state price-marginal cost markup factor for intermediate goods is set at $\nu = \frac{1}{1.2}$, consistent with the findings of Martins et al. (1996) for the US and for European countries. The technology parameter $\psi$ (see (4)) is set at $\psi = 0.24$, which entails a 60% steady state labor income/GDP ratio, consistent with US and European data. Aggregate data suggest a quarterly capital depreciation rate of about 2.5%; thus, $\delta = 0.025$ is used. The capital adjustment cost parameter $\Phi$ is set at $\Phi = 8$ in order to match the fact that the standard deviation of Hodrick-Prescott filtered log investment is three to four times larger than that of GDP in the US and in Europe.

Symmetry of bond-holding-cost parameters across countries requires: $\phi^A = \phi^B^*$, $\phi^B = \phi^A^*$. Given this assumption, (16), (19) and (20) imply that, up to a (log-) linear approximation, the stocks of Home and Foreign currency bonds held by a given country each account for half its net asset position: $A_{t+1}/P_t \equiv \frac{1}{2} NFA_{t+1}/P_t$, $e_{t+1}/P_t \equiv \frac{1}{2} NFA_{t+1}/P_t$, where $NFA_{t+1} = A_{t+1} + e_{t+1}$ is the Home net foreign asset position (expressed in Home currency). Substituting these expression into (16) shows that, up to a (log-)linear approximation, the cross-country interest rate differential is linked to $NFA_{t+1}$:

$$i_t - i_t^* \approx E_t \ln(e_{t+1} / e_t) + \frac{1}{2} (\phi^A - \phi^B) NFA_{t+1} / P_t + \ln(\varphi_t). \quad (24)$$

Panel regressions (for 21 OECD countries) presented by Lane and Milesi-Ferretti (2001) [LMF] show that cross-country interest rate differentials are negatively related to net foreign assets (normalized by exports). This suggests that $\phi^A < \phi^B$, i.e. that (for a given country) holding a given stock of own-currency bonds is less costly than holding a stock of foreign-currency bonds of equal value. The LMF estimates imply that $\frac{1}{2} (\phi^A - \phi^B) = -0.0019 / Q^{m*}$, where $Q^{m*}$ is steady state Home exports (see Appendix). The LMF study does not allow to separately identify $\phi^A$ and $\phi^B$. I set $\phi^A$ and $\phi^B$ at the lowest possible (non-negative) values that are consistent with the LMF estimate for $(\phi^A - \phi^B)$: $\phi^A = 0$, $\phi^B = 0.0038 / Q^{m*}$. Note that this specification has the plausible (in my view) implication that the cost of international
financial transactions is lower the higher the degree of goods market integration (as $\phi^b$ is inversely related to exports).

Estimates of Calvo-style price setting equations for the US and for European countries suggest that the average price-change interval is about 4 quarters (e.g., Lopez-Salido (2000)). Hence, $d$ is set at $d=0.75$. The steady state growth factors of the Home and Foreign price levels are set at $\Pi = \Pi^* = 1$ ($\Pi$ and $\Pi^*$ have no effect on real variables, because of indexing).

Home and Foreign productivity are assumed to follow this process:

$$\begin{bmatrix} \ln(\theta_t) \\ \ln(\theta'_t) \end{bmatrix} = \begin{bmatrix} 0.81 & 0.03 \\ 0.03 & 0.81 \end{bmatrix} \begin{bmatrix} \ln(\theta_{t-1}) \\ \ln(\theta'_{t-1}) \end{bmatrix} + \begin{bmatrix} \varepsilon^\theta_t \\ \varepsilon'^\theta_t \end{bmatrix},$$

(25)

where $\varepsilon^\theta_t$ and $\varepsilon'^\theta_t$ are white noises with standard deviation 0.0059; the correlation between $\varepsilon^\theta_t$ and $\varepsilon'^\theta_t$ is 0.18. (25) is a "symmetrized" version of a VAR model that Kollmann (2002b) fitted to quarterly US and EU3 total factor productivity (1973-1994). Similar autoregressive processes for productivity have also been used in International Real Business Cycle models, as these processes fit well the behavior of productivity in industrialized countries (see, e.g., Backus et al. (1995), Kollmann (1996)). (25) is thus assumed in the "low-trade" variant as well as in the "high-trade" variant of the model.

Kollmann (2002b) constructs quarterly estimates of departures from UIP between the US and the EU3, for the period 1973-94. 13 The standard deviation of the estimated $\ln(\phi)$ series is 3.18%, and its autocorrelations $\rho(\tau)$ of order $\tau=1,\ldots,16$ are (standard errors in parentheses):

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(\tau)$</td>
<td>0.53</td>
<td>0.31</td>
<td>0.39</td>
<td>0.34</td>
<td>0.28</td>
<td>0.23</td>
<td>0.16</td>
<td>0.19</td>
<td>0.28</td>
<td>0.08</td>
<td>0.08</td>
<td>0.10</td>
<td>0.10</td>
<td>0.15</td>
<td>0.11</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.09)</td>
<td>(0.05)</td>
<td>(0.09)</td>
<td>(0.12)</td>
<td>(0.10)</td>
<td>(0.11)</td>
<td>(0.13)</td>
<td>(0.16)</td>
<td>(0.17)</td>
<td>(0.13)</td>
<td>(0.15)</td>
<td>(0.13)</td>
<td>(0.14)</td>
<td>(0.13)</td>
<td></td>
</tr>
</tbody>
</table>

The first-order autocorrelation is 0.53; the autocorrelation function decays gradually towards zero. As discussed below, the welfare effect of UIP shocks is sensitive to the persistence of these shocks—it is thus important to ensure that the simulation model captures adequately the serial correlation of the historical UIP shocks. The following two-factor structure suits that purpose; it expresses $\ln(\phi_t)$ as the sum of a serially correlated random variable and of an i.i.d. random variable:

$$\ln(\phi_t) = a_t + \omega_t, \quad a_t = \lambda a_{t-1} + \eta_t, \quad 0 < \lambda < 1$$

(26)

12 This specification captures LMF’s empirical relation between interest rate differentials and net foreign assets, normalized by exports (see Appendix).

13 Let $\nu_{i,t} = i_t - \bar{i}_t - \ln(e_{i,t}/e^*)$. (16) and $\phi^b = 0$ (as assumed in the simulations) imply:

$$\ln(\phi) \equiv E_{\nu_{i,t}} + \phi^b (B^e_{i,t} - P_t) \cdot \ln(\phi) \equiv E_{\nu_{i,t}} \quad \text{holds when } \phi^b \text{ is small. Kollmann (2002b) constructs an estimated }$$

$\ln(\phi_{i,t}) \text{ series by regressing } \nu_{i,t} \text{ on } \{\nu_{i,t-1}, \nu^*_{i,1-t}, \bar{Y}_{i,1-t}, Y^*_{i,1-t}\} \text{, where } \bar{Y}_{i,1-t} \text{ is linearly detrended log GDP.}$

14 As discussed in Sect. 3.3.1. a (fitted) AR(1) process is not suitable for that purpose, as it fails to capture the gradual decay of the empirical autocorrelation function. An AR process of order 3 (or higher) is needed to capture the shape of that function. Model simulations based on fitted high-order AR processes yield results that are roughly similar to those obtained using (26). As (26) is a more parsimoneous specification, (26) is used, in what follows.
where \( \omega_t \) and \( \eta_t \) are independent white noises with standard deviations \( \sigma_\omega \) and \( \sigma_\eta \), respectively. (26) implies \( \rho(\tau) = \lambda^\tau \Gamma \), for \( \tau \geq 1 \), where \( \Gamma = (\sigma_\eta^2/(1-\lambda^2))/\left(\sigma_\omega^2 + \sigma_\eta^2/(1-\lambda^2)\right) \).

Using Non-Linear Least Squares to fit the equation \( \rho(\tau) = \lambda^\tau \Gamma \) to the autocorrelations reported above yields these estimates: \( \lambda = 0.88 \), \( \Gamma = 0.52 \). Under (26), \( \text{Var}(\ln(\phi)) = \sigma_\omega^2 + \sigma_\eta^2/(1-\lambda^2) \). Setting \( \text{Var}(\ln(\phi)) \) at its historical value \( (0.0318)^2 \), then pins down \( \sigma_\omega \) and \( \sigma_\eta \): \( \sigma_\omega = 0.0220 \), \( \sigma_\eta = 0.0109 \). The "low-trade" (US-EU3) variant of the model uses these parameter values.

During the post-Bretton Woods era, EU countries have used a system of fixed-but-adjustable exchange rates (EMS), followed in 1999 by a currency union (EMU), to achieve bilateral exchange rate volatility that has been markedly lower than US-EU3 exchange rate volatility. The analysis here only considers irrevocable floats and pegs. I assume that, under a float, UIP shocks in the "high-trade" (EU) variant of the model would have the same stochastic properties as the post-Bretton Woods US-EU3 UIP shocks (the above estimates of \( \lambda, \sigma_\omega, \sigma_\eta \) are thus also used in the "high-trade" variant).

### 3. Results

Tables 1-2 report results. Because of the symmetric structure of the two countries, model predictions are only shown for the Home country. (The optimized policy parameters and welfare are identical across countries.) In the Tables, \( \Delta e_t = e_t/e_{t-1} \) is the depreciation factor of the nominal exchange rate. \( \text{RER}_t = e_tP_t^*/P_t \) is the (final good based) real exchange rate. \( A_{t+1} = A_{t+1}/(P_tY) \) and \( B_{t+1} = e_tB_{t+1}/(P_tY) \) are the Home household's stocks of Home-currency bonds and of Foreign-currency bonds, respectively, expressed in Home final good units, and normalized by steady state (quarterly) GDP.

Predicted standard deviations and/or mean values of these (and other) variables are shown. All variables are quarterly. The statistics for the domestic interest rate (\( i_t \)), for bond holdings \( (A_{t+1}, B_{t+1}) \) and the resource cost of the variability of imported-to-domestic relative prices \( (\delta^{\text{rel}}) \) refer to differences of these variables from steady state values (\( i_t \) is a quarterly rate expressed in fractional units), while statistics for the remaining variables refer to relative deviations from steady state values. All statistics are expressed in percentage terms.

Results are presented for simulations in which the world economy is simultaneously subjected to (Home and Foreign) productivity shocks and to UIP shocks, as well as for simulations with just productivity shocks, and for simulations with just UIP shocks (see Cols. labeled "\( \theta, \theta', \phi' \)", "\( \theta, \theta' \)" and "\( \phi \)", respectively).

#### 3.1. Results for the "low-trade" world (\( a^n=0.01 \))

Table 1 reports results for the "low-trade" world. Cols. 1-3 pertain to the optimized regime in which the exchange rate is not constrained to be constant; henceforth that regime is referred to as the "float". Cols. 5-6 consider the optimized exchange rate peg. These variants assume sticky prices. A flex-prices version of the model is considered in Cols. 6-8.

##### 3.1.1. Floating exchange rate regime

In the "low-trade" world (with sticky prices), welfare and the optimized policy parameters under the float are: \( \xi = -0.006\%, \Gamma_\omega = 7.93, \Gamma_\tau = -0.12, \Gamma_\eta = 0.00 \), when there are simultaneous productivity shocks and UIP shocks (see Col. 1). Welfare is thus slightly lower in the
stochastic economy than in the deterministic steady state. Optimized policy has an aggressive stance against PPI inflation—notice the high positive value of $\Gamma_*$. As a result, the standard deviation of PPI inflation ($\Pi^d$) is close to zero (0.01%), and the resource cost of cross-firm dispersion of domestic prices charged by Home intermediate goods firms is very low ($E\hat{\delta}^d=0.01\%$). The optimized response parameters on output and the nominal exchange rate ($\Gamma_*, \Gamma_e$) are markedly smaller than $\Gamma_e$; note especially that $\Gamma_e$ is very close to zero. (Setting $\Gamma_*, \Gamma_e=0$ has virtually no effect on welfare and on other model predictions.)

If the two economies were closed (i.e. under autarky), optimal monetary policy would (essentially) stabilize PPI inflation; that policy would eliminate price dispersion across intermediate goods producers located in the same country—and it would imply that the behavior of real variables (essentially) replicates the behavior under flexible prices. This helps to understand why optimized policy in the "low-trade" world likewise has a strict stance against PPI inflation, and why in that world most predicted statistics (including welfare) are virtually identical across the sticky-prices version and the flex-prices version, as can be seen by comparing Cols. 1-3 and Cols. 6-8. (Under flexible prices, the monetary policy rule does not affect real variables; in the flex-prices variant, I set the policy parameters at the values obtained for the optimized float, under sticky-prices, with simultaneous productivity shocks and UIP shocks—i.e. at the values used in Col. 1.)

In the "low-trade" world (with sticky prices), optimized policy entails that the standard deviations of GDP, consumption and investment are 1.39%, 1.06% and 3.64%, respectively (with simultaneous two types of shock); nominal and real exchange rates are markedly more volatile than these variables (standard deviations of $\Delta e, \text{RER}$: 7.44%, 12.44%). The real exchange rate is furthermore predicted to have a sizable positive autocorrelation (0.82). The model captures thus the fact that, during the post-Bretton Woods era, nominal and real US-EU3 exchange rates have been highly volatile, although it underpredicts the persistence of post-Bretton Woods exchange rate fluctuations. (Standard deviations of the growth factor of the nominal exchange rate and of the linearly detrended log real exchange rate between US and EU3, 1973-1994: 4.89% and 12.89%, respectively; autocorrelation of linearly detrended log real exchange rate: 0.95.)

Cols. 2-3 of Table 1 (where model versions with just productivity shocks, and with just UIP shocks are considered) show that, in the "low-trade" world (with sticky prices), productivity shocks account for about 99% of the variances of output, consumption and investment (that are generated under simultaneous productivity and UIP shocks), while UIP shocks explain 99% of the variances of nominal and real exchange rates.

The sizable volatility of the nominal exchange rate (when there are UIP shocks) implies that exports price inflation ($\Pi^m_*$) fluctuates much more than domestic PPI inflation (standard deviation of $\Pi^m_*$: 1.45%). The cross-form dispersion of exports prices that results from this under sticky prices has a noticeable effect on the resource cost of the aggregate export good: $E\hat{\delta}^m_*=0.86\%$; however, due to the small trade share in the "low-trade" world, the effect on the aggregate resource of price dispersion is very small ($E\hat{\delta}_*=0.01\%$). This

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15 See Rotemberg and Woodford’s (1997) analysis of optimal monetary policy in closed economies with staggered price setting.
16 The same remark applies also to the flex-prices version of the "high-trade" model discussed below.
17 In Cols. 2 and 3, the policy parameters are set at the values that maximize world welfare under simultaneous productivity shocks and UIP shocks (i.e. at the values used in Col. 1). Reoptimizing the policy coefficients when there are just productivity shocks (or just UIP shocks) hardly affects welfare (and other statistics).
(further) helps to understand why welfare in the sticky-prices economy is so close to that in the flex-prices economy.

UIP shocks induce sizable fluctuations of the relative price between (aggregate) imported and domestic goods (standard deviation of \( P_t^m/P_t^d \): 7% [12.5%] under flexible [sticky] prices). Due to the small trade share, these fluctuations only have a very small effect on the final good resource cost (\( \sigma_{\text{adh}} \leq 0.01\% \)).

Under the float, mean consumption, hours, GDP and the mean stock of physical capital differ only very slightly from steady state values (e.g. \( E\hat{C} = 0.01\% \)). Interestingly, UIP shocks affect the currency composition of international asset positions: on average, the Home country holds a short [long] position in Home [Foreign] currency bonds: \( EA_{t+1} = -0.48\%, EB_{t+1} = 0.48\% \).

3.1.2. Exchange rate peg
A peg can be achieved by picking "large" values of the policy parameters \( \Gamma_e \) and/or \( \Gamma_e^* \). In the limit, as \( \Gamma_e \) and/or \( \Gamma_e^* \) tend to infinity, the exchange rate is constant:

\[ e_t = e_{t-1}, \quad (27) \]

and the following interest rate rule holds:

\[ (1-\lambda) \gamma + \lambda^* \gamma^* = 1 + (1-\lambda) \left( \Gamma_e \hat{P}_t^d + \hat{Y}_t + \lambda \left( \Gamma_e^* \hat{P}_t^d + \hat{Y}_t^* \right) \right), \quad (28) \]

where \( \lambda^* \) is the limiting value of the ratio \( \Gamma_e^*/(\Gamma_e^* + \Gamma_e^*) \) (see Appendix). The peg discussed here is a model variant in which equations (21a), (21b) are replaced by (27), (28), and in which \( \lambda, \gamma, \gamma^*, \gamma_e^*, \gamma_e^* \) are set at the values that maximize world welfare (due to symmetry, optimization yields \( \lambda^* = 0.5 \)).

When the "low-trade" world (with sticky prices) is simultaneously subjected to productivity shocks and to UIP shocks, then welfare is noticeably lower under the peg (\( \zeta = -0.460\% \)) than under the (optimized) float (see Table 1, Col. 4). The low welfare under the peg is almost entirely due to UIP shocks (welfare with just UIP shocks: \( \zeta = -0.458\% \)); UIP shocks are thus markedly more detrimental for welfare, under the peg (compared to float).

Under the peg, UIP shocks have a much stronger effect on (Home and Foreign) nominal interest rates than under the float--basically because under the peg the cross-country interest rate differential adjusts roughly one-to-one to UIP shocks. (Standard deviation of \( i_t \) under peg [float]: 1.54% [0.14%].) Under the peg, UIP shocks induce thus markedly higher standard deviations of domestic PPI inflation (and of consumption); as a result, the

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\[ ^{18} \text{As discussed in the Appendix, the real exchange rate volatility triggered by UIP shocks raises the Home household's expected marginal rate of substitution between units of foreign currency available at dates t and t+1, which increases Home demand for Foreign currency bonds at t (analogously, real exchange rate volatility raises Foreign demand for Home currency bonds); thus } EA_{t+1} < 0, EB_{t+1} > 0. \]

\[ ^{19} \text{The optimized } \Gamma_e, \Gamma_e^* \text{ parameters under the peg are very large: } \Gamma_e = 453066.99, \Gamma_e^* = 121.51. \text{ World welfare is a very "flat" function of these parameters. Imposing "moderate" upper bounds on the absolute values of } \Gamma_e, \Gamma_e^* \text{ (such as } |\Gamma_e|, |\Gamma_e^*| \leq 50 \text{ ) leaves the model predictions basically unaffected. The same remark also applies to the "high-trade" variant of the model discussed below (Table 2).} \]

\[ ^{20} \text{Under the peg } i_t - \bar{i} = \gamma (\Phi - \Phi^*) NFA/P_t + \ln(\Phi) \text{ holds, up to a (log-)linear approximation (see (24)), and the behavior of } i_t - \bar{i} \text{ closely mimics that of } \ln(\Phi). \]
(aggregate) resource cost of (inefficient) cross-firm price dispersion ($E\hat{\delta}_{t}^d=0.24\%$) is higher under the peg than under the float (there: $E\hat{\delta}_{t}^d=0.01\%$). That efficiency loss is accompanied by a fall in mean consumption ($E\hat{C}_{t}=-0.36\%$ under the peg, compared to $E\hat{C}_{t}=-0.01\%$, under float). The welfare loss brought about by the peg (when there are UIP shocks) mainly reflects this reduction in mean consumption (and thus that loss mainly reflects a reduction in the "mean-component" of the welfare measure: $\zeta''=-0.394\%$ under the peg, compared to $\zeta''=-0.001\%$ under the float). The welfare cost of consumption variability is much smaller ($\zeta''=-0.066\%$).

Choice of exchange rate regime when the peg eliminates UIP shocks

As discussed by Kollmann (2002a), a key question in modeling a peg is whether it affects the variance of the UIP shocks. Departures from interest parity were markedly smaller in the Bretton Woods [BW] era than in the post-BW era (see, e.g., Kollmann, 2002b). This finding can easily be rationalized if UIP shocks reflect irrational exchange rate forecasts: under a (credible) peg there is obviously less scope for biased exchange rate forecasts than under a float. Col. 5 in Table 1 considers a version of the "low-trade" model, in which the peg eliminates the UIP shocks (in that variant, productivity shocks are the only disturbance). That peg generates higher welfare ($\zeta=-0.002\%$) than the optimized float with UIP shocks (there $\zeta=-0.006\%$).

According to the model here, it would thus be desirable to peg the exchange rate between the US and Europe--if that peg fully eliminated the UIP shocks. But note that the welfare gain from such a peg is predicted to be very small (it corresponds to a permanent 0.004% rise in consumption).

3.2. Results for the "high-trade" world ($\alpha^m=0.20$)

Table 2 shows results for the "high-trade" world. With simultaneous productivity shocks and UIP shocks, welfare and the optimized policy parameters under the float (under sticky prices) are: $\zeta=-0.188\%$, $\Gamma_\delta=34.59$, $\Gamma_\tau=0.27$, $\Gamma_\delta=0.27$ (see Col. 1); as in the "low-trade" variant, optimized policy has a clear stance against PPI inflation, and UIP shocks induce wide fluctuations in nominal and real exchange rates, and in the relative price between (aggregate) imported and domestic intermediate goods (standard deviations of $\Pi^d$, $\Delta e$, $RER$, $P_i^m/P_i^d$: 0.07%, 5.62%, 8.98%, 6.73%).

Welfare is lower than in the "low-trade" world (under flexible prices, $\zeta=-0.144\%$)--and that both under sticky prices and under flexible prices. The lower welfare (when $\alpha^m=0.2$) is almost entirely caused by UIP shocks (with just UIP shocks: $\zeta=-0.188\%$ [$\zeta=-0.146\%$] under sticky [flexible] prices). UIP shocks are thus more detrimental for welfare in the "high-trade" variant than in the "low-trade" variant.22

To understand this finding, recall (from Sect. 2.8) that $\delta_{dm}$, the resource cost of the (sizable) fluctuations in $P_i^m/P_i^d$ (largely triggered by UIP shocks), is an increasing function of $\alpha^m$. It appears that $\delta_{dm}$ is significantly greater in the "high-trade" world: there, $\delta_{dm}=0.04\%$ [$\delta_{dm}=0.10\%$] with sticky [flexible] prices, under UIP shocks (by contrast, $\delta_{dm}\leq0.01\%$ in

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21 The policy coefficients in Col. 5 have been re-optimized (with just productivity shocks), and differ thus from those used in Col. 4.  
22 In the "low-trade" world, with just UIP shocks $\zeta=-0.009\%$ [$\zeta=-0.010\%$] under sticky [flexible] prices.
the "low-trade" world). Note that $\delta_{dm}$ is higher under flexible prices, which is due to the fact that $P_t^d/P_t^d$ is more volatile under flexible prices.

An additional effect that operates under sticky prices is that the aggregate resource cost of the price dispersion across firms located in the same country (that arises under sticky prices) is markedly higher in the "high-trade" version ($\delta^h = 0.13\%$, compared to $\delta^l = 0.01\%$ in the "low-trade" version); this mainly reflects the fact that the (inefficient) cross-firm dispersion of exports prices (under sticky prices), has a greater effect on the aggregate resource cost of the intermediate goods sector, in the "high-trade" world. (Of course, $\delta^h = 0$, under flexible prices.)

The overall resource cost of (relative) price volatility ($\delta^f + \delta_{dm}$) is higher under sticky prices, which helps to understand why welfare is lower under sticky prices. Note that optimized policy in the "high-trade" world comes somewhat less close to replicating the flex-prices equilibrium (in welfare terms) than in the "low-trade" world. The welfare reduction induced by UIP shocks mainly reflects a reduction in the "mean-component" of the welfare measure, $\zeta^m$ (mean hours worked (as well as the mean capital stock and mean GDP) rise by about 0.3% relative to steady state; mean consumption changes less).

In the "high-trade" world with sticky prices, the exchange rate peg again (as in the "low-trade" world), markedly reduces welfare ($\zeta = -0.408\%$) when there are UIP shocks—see Col. 4. (That welfare loss again mainly reflects a reduction in the "mean-component" of the welfare measure.)

However, under the plausible assumption that a peg eliminates the UIP shocks (see discussion in Sect. 3.1.2.), welfare under the peg is $\zeta = 0.002\%$ (see Col. 5)—which represents a noticeable welfare improvement, compared to the optimized float with UIP shocks (recall that there $\zeta = -0.188\%$). Thus, the welfare gain from adopting a peg that eliminates UIP shocks is noticeably greater in the "high-trade" world than in the "low-trade" world.

The intuition for this is simple: as UIP shocks are more harmful the higher the degree of openness, the benefit from eliminating these shocks (by adopting a peg) are greater, the higher is openness. Empirically, the likelihood that a country pegs its exchange rate is positively linked to openness (e.g., Edwards (1996)). The model here can rationalize this fact.

### 3.3. Sensitivity analysis
#### 3.3.1. Alternative policy rules; maximizing conditional welfare
Experiments with interest rate rules that respond to additional state variables (beyond those included in (27a,b)) only generated small welfare gains (results available upon request). 23

The analysis here assumes that central banks maximize unconditional world welfare. Rotemberg and Woodford (1999, p.70) justify using unconditional welfare as a policy objective by pointing out that this objective is "not subject to any problem of time consistency". 24 However, as discussed by, i.a., Levin (2002) and Kim et al. (2002), this policy objective is not optimal if households discount future period utility ($\beta < 1$). 25

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23 These experiments included rules under which the interest rate is a function of: exports inflation, imports inflation, CPI inflation and employment; as well as rules under which each countries' interest rate is a function of domestic and foreign variables.

24 This objective function is widely assumed in the literature: see, e.g., Rotemberg and Woodford (1997), Benigno (1999), Clarida et al. (2001) and Smets and Wouters (2002).

25 Levin (2002) points out that the logic for this is the same as that of the suboptimality of the Golden-Rule of capital accumulation relative to the Modified-Golden-Rule.
I therefore considered a version of the model in which monetary authorities maximize the sum of the conditional expectation of Home and Foreign life-time utility, in the 'initial' period $t=0$: 

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) + \sum_{t=0}^{\infty} \beta^t U(C'_t, L'_t) \right\}.$$ 

I assume that the economy is in its (deterministic) steady state at $t=0$, and that the date 0 innovations to exogenous variables $(\varepsilon^p_t, \varepsilon^e_t, \omega_t, \eta_t)$ equal zero.

It appears that the policy implications of this alternative objective function are the same as those of the baseline objective: optimized policy still has a strict stance against PPI inflation, and it remains true that an exchange rate peg that eliminates UIP shocks yields higher welfare than the optimized float. Also, implied (unconditional) moments of macro variables are very similar to those predicted in the baseline model--the only difference is that conditional welfare is lower than unconditional welfare.

For example, in the "high-trade" variant with simultaneous productivity shocks and UIP shocks, maximization of conditional welfare yields these results: the standard deviations of domestic PPI inflation and of the real exchange rate are 0.06% and 9.12%, respectively, and unconditional welfare is $\zeta = -0.189\%$ (conditional welfare is: $\zeta = -0.060\%$); under the float (with just productivity shocks), unconditional (conditional) welfare is $\zeta = -0.002\%$ ($\zeta = -0.001\%$).

### 3.3.1. Persistence of UIP shocks

The welfare cost of UIP shocks, and the welfare gain from a peg (that eliminates the UIP shocks), are both positively linked to the persistence of these shocks. The empirical evidence in Section 2.9 suggests that UIP shocks are highly persistent. Persistent shocks are needed to capture the high empirical autocorrelation of real exchange rates. This is shown in the Table below where variants of the model with two alternative specifications for UIP shocks are considered:

1. **An estimated AR(1) process (Cols. 1-2).** Previous structural models with UIP shocks have mostly assumed that these shocks follow AR(1) processes. Fitting an AR(1) process to the historical US-EU3 UIP series described in Sect. 2.9 yields an autoregressive parameter of 0.53 (standard deviation of the regression residuals: 2.69%). Note that the autocorrelation function of the estimated AR(1) process decays faster than the empirical autocorrelation function of UIP shocks reported in Sect. 2.9. When these AR(1) parameters are used, the predicted standard deviation and autocorrelation of real exchange rate (about 5% and 0.5) are smaller than in the baseline model; the welfare cost of UIP shocks is noticeably smaller than in the baseline model, and the welfare gain from adopting a peg (that eliminates the UIP shocks) accordingly is likewise noticeably smaller—note that now the peg lowers welfare (very slightly) in the "low-trade" world (the welfare gain from the peg remains positive in the "high-trade" world; there $\zeta = -0.002\%$ (see Col. 5 in Table 1), compared to $\zeta = -0.014\%$ under float).

2. **A two-factor UIP process (see (26))** is considered whose parameters are selected in such a manner that the "low-trade" variant of the model (under float) replicates exactly the historical standard deviation (12.89%) and first-order autocorrelation (0.95) of the (linearly detrended and logged) post-Bretton Woods US-EU3 real exchange rate, as well as the historical standard deviation of the US-EU3 UIP shock (3.18%); see Cols. 3-4 in Table

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27 The autocorrelations of order 2, 3, and 4 implied by the fitted AR(1) process are 0.28, 0.14 and 0.08, while the corresponding historical autocorrelations are 0.31, 0.29 and 0.34.
below. In this variant, the welfare cost of UIP shocks (and the welfare gains of a peg that eliminates the UIP shocks) is roughly 5 to 7 times higher than in the baseline model (that gain represents a permanent 0.728% consumption increase when \( \alpha^m = 0.20 \)).

### Model predictions under optimized float—alternative assumptions about UIP process

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Notes: See Table 1.

### 3.3.2. Currency denomination of international bonds

The baseline model assumes that bonds denominated in both countries' currencies are traded internationally. In the real world, the largest industrialized economies borrow and lend internationally in terms of their domestic currency--other economies typically tend to use a foreign currency (typically the dollar or the Euro) for their international financial transactions (see Hartmann (1998)).

The following Table considers a variant of the model in which only bonds denominated in the currency of one of the countries ("Foreign") can be traded internationally.\(^{29}\)

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28 The parameters that achieve this are: \( \lambda = 0.998 \), \( \sigma_\delta = 0.11\% \), \( \sigma_\omega = 2.66\% \). These parameter values are used in the "low-trade" variant, as well as in the "high-trade" variant in the Table below.

29 In this variant, the Euler equation (18) is dropped from the model and, up to a (log-)linear approximation \( i_t - i_t^* \equiv E_t \ln(e_{t+1}/e_t) - \phi^g NFA_{t+1}/P_t + \ln(\phi) \) holds (instead of (24)), where \( NFA_{t+1} = B_{t+1} \). Based on Lane...
The restriction to Foreign currency bonds has little effect on world welfare, $\zeta + \zeta^*$ (compared to the baseline model). It remains true that a peg (that eliminates UIP shocks) raises world welfare (compared to the optimized float), and that the gain in world welfare generated by the peg is positively linked to the degree of openness.

However, the restriction to Foreign currency bonds affects the distribution of financial wealth and of welfare across countries. Under the float, Home holds (on average) a positive stock of external claims denominated in Foreign currency ($E_{B,s}\beta > 0$, as in the baseline model); when just Foreign currency bonds are traded internationally, Home is thus (on average) a net creditor vis-à-vis Foreign (under float)--and as a result Home enjoys higher welfare than Foreign. (In the symmetric baseline model, by contrast, Home's mean net asset position is zero, as its external assets denominated in Foreign currency are counterbalanced by external liabilities denominated in Home currency—and Home and Foreign enjoy equal welfare.)

The restriction to Foreign currency bonds has virtually no effect on the cross-country distribution of wealth and of welfare, under the peg (without UIP shocks), as productivity shocks have little effect on asset positions—both countries thus enjoy (basically) the same welfare under the peg; both in "low-trade" world and in the "high-trade" world, the peg raises Foreign welfare (by 0.014% and 0.305%, respectively); the peg very slightly lowers Home welfare in the "low-trade" world (by 0.003%), but it raises Home welfare in the "high-trade" world (by 0.077%).

### 3.3.3. Cross-country correlation of productivity

The literature on 'optimal currency areas' argues that two countries benefit more from a peg (i) the closer these countries are integrated in goods markets and (ii) the higher the cross-country correlation of productivity shocks (see Obstfeld and Rogoff (1996)). The simulations discussed above confirm point (i). Regarding point (ii), it can be noted that, in the model here, productivity shocks have a smaller effect on welfare than (persistent) UIP shocks. The ability of a peg to raise welfare hinges on its ability to eliminate the UIP shocks. In the baseline model, the adoption of a peg (that eliminates the UIP shocks) raises welfare, even in the

and Milesi-Ferretti's (2001) empirical findings, I set the coefficient of net foreign assets (NFA) in that equation at $-0.0019 / Q^{**}$ (see Sect. 2.9); thus: $\phi^* = 0.0019 / Q^{**}$, in this variant. (As in the baseline variant, $\phi^* = 0$.)
extreme case where productivity shocks are perfectly negatively correlated across countries. For example in a version of the "high-trade" world in which productivity shocks are perfectly negatively correlated across countries ($\text{Corr}(\varepsilon^p_t, \varepsilon^w_t) = -1$), welfare is $\zeta = -0.197\%$ under the optimized float, compared to $\zeta = -0.010\%$ under a peg without UIP shocks.

4. Conclusions

This paper has analyzed welfare effects of monetary policy rules, in a quantitative business cycle model of a two-country world. The model assumes staggered price setting, and shocks to productivity and to the uncovered interest rate parity (UIP) condition. UIP shocks have a sizable negative effect on welfare, when trade links are strong. An exchange rate peg raises world welfare, if the peg eliminates (or sufficiently reduces) the UIP shocks. The model explains the empirical finding that more open economies are more likely to adopt a peg.
APPENDIX

1. Estimation of $\frac{1}{2}(\phi^A - \phi^B)$ (see (24))

(24) implies that $\tilde{r}_r - \tilde{r}^* \equiv \frac{1}{2}(\phi^A - \phi^B)NFA/P_t + E[\ln(RER_{t+1}/RER_t) + \ln(\phi^r)]$, where $\tilde{r}_r = \tilde{r}_t - E[\ln(P_{t+1}/P_t)]$ and $\tilde{r}^* = \tilde{r}_t^* - E[\ln(P_{t+1}^*/P_t^*)]$ are Home and Foreign real interest rates, and $RER = e/P_t^*$ is the real exchange rate. Lane and Milesi-Ferretti (2001) fit this equation to a panel of 21 OECD economies, using annualized % interest rates and net foreign assets (NFA) normalized by annual exports. Based on instrumental variables (allowing for country fixed-effects), estimates of about -3 are obtained for the coefficient of the normalized NFA (Table 7, Cols. 5-8). In terms of the relation between quarterly fractional interest rate differentials and NFA normalized by quarterly exports, this implies a coefficient $\frac{1}{2}(\phi^A - \phi^B)Q^{\text{nr}} = -3/1600 = -0.0019$ (the value used in the simulations).

2. Explaining average asset stocks of external assets/liabilities

Following Kollmann (2002a, p.1012), note that (15) implies: $1 + \phi^B E(e_tB_{t+1}/P_t) = E(l^*_t\tilde{\xi}_{t+1})$, with $l^*_t = l + l^*_t$ and $\tilde{\xi}_{t+1} = \beta(C_{t+1}/C_t)(RER_{t+1}/RER_t)(\Pi^*_t)^{-1}\phi_t^r$; $l^*_t$ is the Home household's marginal rate of substitution between units of foreign currency available at $t$ and $t+1$. Second order approximations give: $\phi^B E(e_tB_{t+1}/P_t) \equiv E\tilde{\xi}_{t+1} + \Gamma_1^r$, with $\Gamma_1^r = E(l^*_t + \text{Cov}(\tilde{\xi}_{t+1}, \Gamma_t^r))$; $E\tilde{\xi}_{t+1} = \frac{1}{2}[\text{Var}(\tilde{\xi}_{t+1}) + \text{Var}(\Pi^*_t)] + \Gamma_2^r$, with $\Gamma_2^r = \text{Cov}(\phi_t^r, \Pi^*_t) - E(\Pi^*_t)$. $\Gamma_1^r$ and $\Gamma_2^r$ are small, as the variances of final good inflation and of nominal interest rates are small (the mean values of these variables are likewise small: $E(\Pi^*_t) = 0.00\%$, $E(\xi_t) = 0.00\%$; not shown in Tables). Note that $E(B_{t+1}^*/\Pi_t^*)$ is increasing in $E\tilde{\xi}_{t+1}$, and that the latter is increasing in Var$(\tilde{\xi}_{t+1})$.

When there are UIP shocks, real exchange rates and $\tilde{\xi}_{t+1}$ are highly volatile—and Var$(\tilde{\xi}_{t+1})$ dominates the terms $\Gamma_1^r$ and $\Gamma_2^r$, which implies $E\tilde{\xi}_{t+1} > 0$, $E(B_{t+1}^*/\Pi_t^*) > 0$ (and thus $EB_{t+1}^*/\Pi_t^* > 0$). The same logic explains why $E(A_{t+1}^*/(e_t/P_t)) < 0$ (and thus $EA_{t+1}^*/e_t < 0$).

3. Exchange rate peg

Substituting (21a) and (21b) into (24) yields:

$$\ln(e_t/e_{t-1}) = (1/(\Gamma^r + \Gamma_{t+1}^r)) \left[ E_t^r \ln(e_{t+1}/e_t) - (\Gamma^r \Pi^r_{t+1} + \Pi_{t+1}^*) - (\Gamma_{t+1}^r \hat{Y}_t - \Gamma_{t+1}^r \hat{y}_{t+1}) + \Psi_t \right],$$

where $\Psi_t = \text{Cov}(\phi_t^r, \Pi^*_t)^{-1}(\phi^A - \phi^B)NFA/P_t + (2nd \ and \ higher \ order \ terms)$. An exchange rate peg ($e_t = e_{t-1}$) obtains asymptotically when these four conditions are met: $|\Gamma^r + \Gamma_{t+1}^r| \to \infty$, $\Gamma^r / (\Gamma^r + \Gamma_{t+1}^r) \to 0$, $\Gamma_{t+1}^r / (\Gamma^r + \Gamma_{t+1}^r) \to 0$, $\Gamma_{t+1}^r / (\Gamma^r + \Gamma_{t+1}^r) \to 0$, $\Gamma_{t+1}^r / (\Gamma^r + \Gamma_{t+1}^r) \to 0$. Multiplying (21a) by $\Gamma_{t+1}^r / (\Gamma^r + \Gamma_{t+1}^r)$, and multiplying (21b) by $\Gamma_{t+1}^r / (\Gamma^r + \Gamma_{t+1}^r)$, and then summing the resulting equations gives:

$$(1-\Gamma^r / (\Gamma^r + \Gamma_{t+1}^r))(\pi_i + (\Gamma^r / (\Gamma^r + \Gamma_{t+1}^r)))(\Gamma_{t+1}^r \Pi^r_{t+1} + \Gamma_{t+1}^r \hat{Y}_t) + (\Gamma_{t+1}^r / (\Gamma^r + \Gamma_{t+1}^r)) (\Gamma_{t+1}^r \Pi_{t+1}^* + \Gamma_{t+1}^r \hat{y}_{t+1})$$

which yields (28) (when $|\Gamma^r + \Gamma_{t+1}^r| \to \infty$).
REFERENCES


Gaspar, J., Judd, K., 1997. Solving Large-Scale Rational Expectations Models, Macroeconomic Dynamics 1, 45-75.


Table 1. World with low trade shares ($\alpha'' = 0.01$)

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<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
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<td>$\Gamma_1$</td>
<td>7.93</td>
<td>7.93</td>
<td>7.93</td>
<td>4.5e5</td>
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<td>$\Gamma_2$</td>
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<td>$\Gamma_3$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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cost of price dispersion (across Home firms) in domestic/export market (\( \delta^i = \alpha^i \delta^i + (1-\alpha^i) \delta^e \)); \( \zeta, \zeta^-, \zeta^+ \): measures of Home welfare. **Standard deviations and means of** \( i, A, B \) **refer to differences from steady state values.** **statistics for the remaining variables refer to relative deviations from steady state values.** All statistics have been multiplied by 100, i.e. expressed in percentage terms.

Cols. labeled "\( \theta, \theta', \varphi \)" report model simulations with simultaneous (Home and Foreign) productivity shocks and UIP shocks; Cols. "\( \theta, \theta' \) ["\( \varphi \)] assume just (Home and Foreign) productivity shocks [just UIP shocks].
Table 2. World with high trade shares \( (\alpha^e = 0.2) \)

<table>
<thead>
<tr>
<th></th>
<th>Float</th>
<th>Peg</th>
<th>Flexible prices</th>
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<tbody>
<tr>
<td></td>
<td>( \theta, \varphi )</td>
<td>( \theta, \theta^* )</td>
<td>( \theta, \theta^* )</td>
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<tr>
<td>Standard deviations</td>
<td></td>
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<tr>
<td>( Y )</td>
<td>1.67</td>
<td>1.24</td>
<td>1.11</td>
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<tr>
<td>( C )</td>
<td>2.08</td>
<td>0.96</td>
<td>1.84</td>
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<tr>
<td>( I )</td>
<td>7.16</td>
<td>3.35</td>
<td>6.33</td>
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<tr>
<td>( Q^d )</td>
<td>1.96</td>
<td>1.34</td>
<td>1.43</td>
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<tr>
<td>( Q^{m*} )</td>
<td>7.68</td>
<td>1.21</td>
<td>7.58</td>
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<tr>
<td>( \Pi^d )</td>
<td>0.07</td>
<td>0.04</td>
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<tr>
<td>( \Pi^{m*} )</td>
<td>1.32</td>
<td>0.15</td>
<td>1.31</td>
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<tr>
<td>( \delta )</td>
<td>0.67</td>
<td>0.14</td>
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<tr>
<td>( \Delta e )</td>
<td>5.62</td>
<td>0.63</td>
<td>5.59</td>
</tr>
<tr>
<td>( RER )</td>
<td>8.98</td>
<td>1.01</td>
<td>8.92</td>
</tr>
<tr>
<td>( P/P^d )</td>
<td>6.73</td>
<td>0.73</td>
<td>6.69</td>
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<tr>
<td>( A )</td>
<td>14.48</td>
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<tr>
<td>( B )</td>
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<tr>
<td>Means (in %)</td>
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<tr>
<td>( Y )</td>
<td>0.25</td>
<td>0.01</td>
<td>0.24</td>
</tr>
<tr>
<td>( C )</td>
<td>0.01</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>( L )</td>
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<tr>
<td>( K )</td>
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<tr>
<td>( A )</td>
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<tr>
<td>( B )</td>
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<tr>
<td>( \delta^d )</td>
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<tr>
<td>( \delta^m )</td>
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<td>0.00</td>
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<tr>
<td>( \delta^{md} )</td>
<td>0.63</td>
<td>0.01</td>
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<td>First-order autocorrelations</td>
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<tr>
<td>( RER )</td>
<td>0.82</td>
<td>0.82</td>
<td>0.82</td>
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<tr>
<td>Welfare (% equivalent permanent variation in consumption)</td>
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<tr>
<td>( \zeta )</td>
<td>-0.188</td>
<td>-0.000</td>
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<tr>
<td>( \zeta^{m} )</td>
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<td>0.004</td>
<td>-0.171</td>
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<td>( \zeta^{v} )</td>
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<td>( \Gamma_l )</td>
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<tr>
<td>( \Gamma_i )</td>
<td>0.56</td>
<td>0.56</td>
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Notes: See Table 1.