Information variables for monetary policy in a small structural model of the Euro area.

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Abstract

This paper estimates a small New-Keynesian model with imperfect information and optimal discretionary policy using data for the Euro area. The estimated model is used to: (1) compare the values of key parameters concerning the strucutre of the economy and monetary policy targets with those commonly used in calibrations. (2) assess the imperfect information problem and the usefulness of monetary aggregates and unit labor costs as information variables for monetary policy.

JEL Classification Numbers: E5

Key Words: monetary policy, Kalman filter, inflation, output gap.

1. Introduction

This paper estimates a small New-Keynesian model with imperfect information and optimal discretionary policy using data for the Euro area. We think that our exercise adds two useful elements to existing analyses.

First, while most available studies rest upon calibration, we use maximum likelihood to estimate the structural parameters. As several quantitative predictions of the model hinge on the value of some key parameters, e.g. the degree of forward looking behavior in the output and inflation equations, it is important to let the data "speak" about these magnitudes and to quantify the uncertainty which surrounds them.

Second, most existing models neglect a role for imperfect information. This is particularly troubling with the new keynesian model because one of its key variables, "potential output" (i.e. the flexible price level of output), is not directly observable. This adds on top of the fact that information about several other variables of interest (e.g. GDP or inflation in the current period) is usually available only with lags and subject to statistical revisions. Quantifying the relevance of the imperfect information problem is an empirical challenge. We deal with this issue in a consistent way by modeling the optimal processing of information by the monetary authority, providing the policy maker with various indicators of the variables of interest and letting the estimation determine the noisiness of each indicator. In particular, monetary aggregates and a measure of unit labor costs are considered as indicators of output, inflation and potential output. The estimates determine the noisiness of each indicator and, eventually, allow us to evaluate their information content and assess whether this information is welfare improving. These issues are distinct. In an economy with forward-looking agents the additional information provided by the indicators does not necessarily increase the policy maker's welfare. This happens because better information about current state variables may cause some forward-looking variables to be more responsive to new information, increasing their volatility.

Previous analyses, most notably Ehrmann and Smets (2001), assess the effects of imperfect information using a calibrated model. We address this empirical question by estimating the structural parameters of our model, which include the measurement errors that are present in the data, using the Kalman filter and data for the Euro area. Moreover, the joint presence of an optimization and a filtering problem within the theoretical model is an important difference with respect to Ireland (2001), where a small structural model for the US is estimated assuming perfect information and an exogenous policy rule.

Our methodological strategy is the following. We specify a dynamic stochastic monetary policy model of the "new keynesian" variety developed by e.g. Woodford (1999) and Clarida, Gali and Gertler (1999). The model is then extended to incorporate imperfect information, following a method proposed by Svensson and Woodford (2002). The solution of this theoretical model maps the structural parameters into a vector autoregression. We estimate these parameters using the Kalman filter following a methodology proposed by Sargent (1989) and Ireland (2001).

Our estimates suggest that inflation has a strong forward looking component, while this is not true for output for which the backward looking component is more important. The estimates for the weights in the monetary authority's objective function show that the weight attached to inflation is almost double than the weight attached to the output-gap target and about four times higher than the one attached to interest-rate-smoothing target. Moreover, the results reveal that monetary aggregates contain little information about the state variables of interest for the conduct of stabilization policy. The unit labor cost indicator, instead, contains information that helps reducing the forecast error in the output gap, a key non-observable variable in the new-keynesian model. This, in turn, increases the policy maker's welfare because, by allowing for a better identification of the potential output shocks, leads to a smaller variability of inflation and output.

2. The model

We model policy by assuming that the central banks aims at minimizing the intertemporal loss function

$$\Lambda_t = E[\sum_{\tau=0}^{\infty} \beta^{\tau} L_{t+\tau} \mid I_t] \tag{2.1}$$

where $\beta \in (0,1)$ is the intertemporal discount factor and period losses are given by

$$L_t \equiv \frac{1}{2}\left[(\pi_t)^2 + \lambda_y (y_t - \bar{y}_t)^2 + \lambda_i (i_t - i_{t-1})^2\right].$$

where π_t, y_t , $\bar{y}_t$ and i_t denote, respectively, inflation, output, potential output and the nominal interest rate.

Our benchmark model, taken from Ehrmann and Smets (2001), consists of the following structural equations:

$$\begin{aligned} y_t &= \delta y_{t-1} + (1-\delta)\, y_{t+1|t} - \theta\left(i_t - \pi_{t+1|t}\right) + u_{p,t} && (2.2)\\ \pi_t &= \alpha \pi_{t-1} + (1-\alpha)\, \pi_{t+1|t} + \kappa\,(y_t - \overline{y}_t) + u_{c,t} && (2.3)\\ \overline{y}_t &= \rho \overline{y}_{t-1} + u_{\overline{y},t} && (2.4)\\ m_t &= \gamma_1 m_{t-1} + \gamma_2 m_{t+1|t} + \gamma_y y_t - \gamma_i i_t + u_{m,t} && (2.5) \end{aligned}$$

where m_t is real money. There are four structural iid innovations in the model with covariance matrix Σ_u^2: a preference shock $u_{p,t}$, a cost-push shock $u_{c,t}$, a potential output shock $u_{\overline{y},t}$ and a money demand shock $u_{m,t}$.

Information about the variables in the economy is obtained from the following vector of measurables:

$$\begin{aligned} y_t^o &= y_{t-1} + v_{y,t} && (2.6a)\\ \pi_t^o &= \pi_t + v_{\pi,t} && (2.6b)\\ m_t^o &= m_t + v_{m,t} && (2.6c)\\ x_t^o &= y_{t-1} - \overline{y}_{t-1} + v_{x,t} && (2.6d) \end{aligned}$$

where y_t^o is the indicator output variable, given by a noisy observation on the previous period output level. This assumption models the fact that information on output y_t in a given quarter is not contemporaneously available and that, moreover, output observations are subject to revisions, which justifies the existence of noisy measurement. The indicators π_t^o and m_t^o posit that inflation and real money balances are observed contemporaneously, possibly with noise. Although no direct role for money exist in this model, as it does not affect any of the payoff relevant variables or their transmission mechanism, the monetary indicator may contain useful information on current output through the money demand equation (2.5), which may help reducing the imperfect information problem. Similarly, the last indicator, x_t^o, is a noisy measure of the previous period output gap, given by a lagged measure of real unit labor cost. Rotemberg and Woodford (1997) show, among others, that such costs are proportional to the output gap. Measurement errors contained in the vector v are assumed to be iid with covariance matrix Σ_v^2.

2.1. The Economy under a Discretionary Equilibrium

We focus on the discretionary (i.e. Markov perfect) equilibrium, whereby both the strategy of both the policy maker and the agents are constrained to be functions of the predetermined (natural) state variables alone (i.e. history-dependent strategies are ruled out).[1]

To solve the above model it is convenient to rewrite the system in the state-space form following a method by Svensson and Woodford (2000), defining the vector $X_t' \equiv \begin{bmatrix} y_{t-1} & \pi_{t-1} & m_{t-1} & \overline{y}_t & u_{p,t} & u_{c,t} & u_{m,t} & i_{t-1} & \overline{y}_{t-1} \end{bmatrix}$ of predetermined state variables and the vector $x_t' \equiv \begin{bmatrix} y_t & \pi_t & m_t \end{bmatrix}$ of non-predetermined (forward looking) variables (see Appendix A).

Information is described by the set $J_t \equiv \{Z_\tau, \Omega;\ \tau = t, t-1, ..., 0\}$ i.e. all agents in the model are supposed to know the model parameters

$$\Omega \equiv [\alpha, \beta, \delta, \gamma_1, \gamma_2, \gamma_y, \gamma_i, \lambda_y, \lambda_i, \kappa, \theta, \rho, \Sigma_u^2, \Sigma_v^2]$$

and the history of the four observable variables (2.6), stacked in the vector $Z_t' \equiv [y_t^o, \pi_t^o, m_t^o, x_t^o]$, up to and including period t.

We use the algorithms of Gerali and Lippi (2003) to solve for the optimal Markov perfect policy ($i_t = FX_{t|t}$) and to compute the equilibrium representation of the model, i.e. the law of motion of the state variables (X_t), forward-looking (x_t) variables and the optimal prediction for X_t computed by the Kalman filter:

[1]Alternatively, the model could be solved for the optimal Ramsey policy, under the assumption that the central bank can commit.

$$
\begin{aligned}
X_{t+1} &= HX_t + JX_{t|t} + C_u u_{t+1} && (2.7a)\\
x_t &= GX_{t|t} + G^1(X_t - X_{t|t}) && (2.7b)\\
X_{t|t} &= X_{t|t-1} + K[L(X_t - X_{t|t-1}) + v_t] && (2.7c)
\end{aligned}
$$

where the matrices F, H, J, C_u, G, G^1, L and K depend on the primitive parameters in Ω (see Svensson and Woodford, 2000).

The linear quadratic structure of this problem and the certainty equivalence principle imply that the optimal interest rate rule in this model, $i_t = FX_{t|t}$, is a linear function of the estimate of the states which does not depend on the uncertainty in the system. Of course uncertainty affects the way in which an innovation in the observables is mapped into an updated estimate of the state variables, which occurs through the Kalman gain matrix: K.

3. Bringing the model to the data

The evolution of the whole economic system (2.7) can be expressed in a compact notation using the following vector autoregression representation:

$$
Q_{t+1} = \hat{A}Q_t + \hat{G}w_{1,t+1} \tag{3.8}
$$

where

$$
Q_{t+1} \equiv \begin{bmatrix} X_{t+1} \\ X_{t+1|t} \end{bmatrix} \qquad \hat{A} \equiv \begin{bmatrix} H + JKL & J(I - KL) \\ (H+J)KL & (H+J)(I-KL) \end{bmatrix}
$$

$$
w_{1,t+1} \equiv \begin{bmatrix} u_{t+1} \\ v_t \end{bmatrix} \qquad \hat{G} \equiv \begin{bmatrix} C_u & JK \\ 0 & (H+J)K \end{bmatrix}
$$

The endogenous variables are linked to the states Q_t by:

$$
\begin{bmatrix} i_t \\ Z_t \\ x_{t|t} \end{bmatrix} = \begin{bmatrix} FKL & F(I-KL) \\ L + MKL & M(I-KL) \\ GKL & G(I-KL) \end{bmatrix} \begin{bmatrix} X_t \\ X_{t|t-1} \end{bmatrix} + \begin{bmatrix} FK \\ MK + I \\ GK \end{bmatrix} \begin{bmatrix} v_t \end{bmatrix} \tag{3.9}
$$

The data used in the estimation are given by the 3-month interest rate, taken to be a noisy measure of the monetary policy control variable and the four observables of the theoretical model, which are taken as noisy measures of the true (lagged) output, inflation, money and the (lagged) output gap (hence $d_t' = [Z_t' \; i_t]$). From the first and second row in (3.9):

$$\begin{aligned} d_t &= \hat{L} Q_t + w_{2,t} \\ w_{2,t} &\equiv \hat{M} v_t + e_t \end{aligned} \tag{3.10}$$

where the matrix $\hat{L}$ and $\hat{M}$ are

$$\hat{L} \equiv \begin{bmatrix} L + MKL & M(I - KL) \\ FKL & F(I - KL) \end{bmatrix} \text{ and } \hat{M} \equiv \begin{bmatrix} FK \\ MK + I \end{bmatrix}$$

and the vector $e_t \equiv [\begin{array}{ccccc} 0 & 0 & 0 & 0 & e_{i,t} \end{array}]'$ is a vector of measurement errors in the data. Since we already have measurement errors in the theoretical model (the vector v), the measurement errors in e associated to the Z variables are assumed to be identically zero to avoid redundancy. Instead, the introduction of a measurement error for the interest rate is needed to avoid a stochastic singularity problem, as the theoretical model predicts that the interest rate is a linear function of the state variables. By introducing the measurement error $e_{i,t}$ we create a wedge between the optimal rate predicted by the model and the actual rate recorded in the data which makes estimation possible. The standard deviation of the measurement error $e_{i,t}$ can be interpreted as a measure of the distance between actual policy and the optimal one.

Equations (3.8)-(3.10) represent, respectively, a state space system to which a Kalman filter can be applied to estimate the structural model parameters, Ω. The basic insight rests on the fact that the solution of the theoretical model maps the structural parameters Ω into the matrices $\hat{A}$, $\hat{G}, \hat{L}$, $\hat{M}$ and Σ_u^2 and Σ_v^2 which fully characterize the system dynamics (3.8) and (3.10). Given this system, the Kalman filter provides a convenient method to compute the likelihood function associated to a vector of observations on d_t. The estimation problem thus consists in finding the vector of parameters Ω that maximizes the likelihood function. The idea, originally due to Sargent (1989), McGrattan (1994) and Ireland (2001), is illustrated in more detail in Appendix (B).

3.1. Estimation results

The data used in the estimation of the model described by equations (3.8) and (3.10) are the euro area counterparts of the variables in the vector Z_t and i_t: output, which is measured by real GDP, the inflation rate, measured by the quarterly changes in the GDP deflator, real money, measured by the stock of M3 divided by the GDP deflator, the (lagged) output gap indicator, measured by (lagged) real unit labor costs and the nominal short-term interest rate. The data runs from 1981:1 to 2002:3. Stationarity of the time series is achieved by means of the Hodrick-Prescott filter with the only exception of the inflation rate for which we used deviations from an annual rate of 2 per cent and the nominal interest rate for which we used deviations from an annual rate of 4.1 per cent.[2] A figure of the detrended data is reported below.

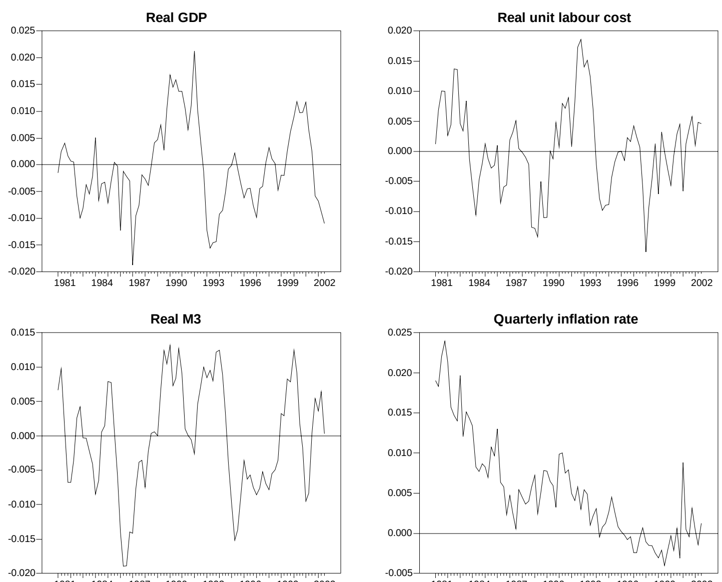

Figure 1: Detrended data

The likelihood function is constructed using the Kalman filter and is maximized with respect to the free parameters of the model. The discount factor in the loss function, β, is set to 0.9949 implying a steady state real interest rate of two per cent. The value was calibrated as the average three-month real interest

[2] The steady state interest rate is computed dividing the steady state inflation rate by the discount factor.

Table 1. Parameter estimates		
	Estimates	Standard errors
λ_y	0.2457	0.0091
λ_i	0.4952	0.0059
δ	0.7293	0.0969
θ	0.1045	0.0034
α	0.5244	0.0028
κ	0.0032	0.0001
γ_1	0.7965	0.0196
γ_2	0.0527	0.0001
γ_y	0.0616	0.0009
γ_i	0.0094	0.0005
$\rho_{\overline{y}}$	0.7663	0.0041
$\sigma_{u,p}$	0.0056	0.0001
$\sigma_{u,c}$	0.0013	0.0001
$\sigma_{u,\overline{y}}$	0.0098	0.0004
$\sigma_{u,m}$	0.0038	0.0002
$\sigma_{\tau,y}$	0.0196	0.0002
$\sigma_{\tau,\pi}$	0.0012	0.0000
$\sigma_{\tau,m}$	0.0006	0.0001
$\sigma_{\tau,x}$	0.0086	0.0002
$\sigma_{e,i}$	0.0106	0.0000

rate between 1998 and 2002 a period in which annual inflation rate was fluctuating around its steady state. The estimated parameters are reported in Table 1.[3] All the estimated parameters are statistically significant at conventional 5 per cent confidence level.

According to the estimated weights in the loss function, the monetary authority is more concerned with fluctuations in the inflation rate and the interest rate than the output gap. These values differ substantially from the ones used in the literature: for example in the benchmark calibration in Ehrmann and Smets the weights are set to 1 and 0.1 for, respectively, the output gap and the changes in the interest rate. Our estimates indicate a much smaller weight for the output gap (0.25) and a greater weight for the interest rate term (0.5).

[3] The likelihood function is maximized using the algorithm `csminwel.m` written by C. Sims. This routine is robust to discontinuities in the objective function although being a gradient-based method.

The estimates suggest that output exhibits a large degree of backwardness (high δ) and a low sensitivity to changes in the expected real interest rate (small θ). The first result contrasts with the estimates in Andres at al. (2001) who suggest a larger degree of forwardness in the output equation. With respect to the interest rate elasticity our value is smaller than the estimate in Andres at al. (2001) and Smets and Wouters (2002).

With respect to the parameters of the New Phillips curve equation we find a large degree of forwardness in inflation ($\alpha = 0.52$), as in Galí et al. (2000), Andres at al. (2001) and Smets and Wouters (2002). The estimated value of the elasticity of inflation to the output gap, κ, suggests a rather flat supply curve and is close to the value in Smets and Wouters (2002).

The estimated money demand equation suggests a large degree of backwardness (large γ_1) and a small interest rate elasticity. The long-run elasticity to output and the interest rate are equal to, respectively, 0.38 and -0.06.

The estimates of the standard deviation of the structural shocks are small, with innovations in potential output being the most volatile. The latter result is in line with the empirical findings of Ireland (2001) for the United States and of Smets and Wouters (2002) for the Euro area. The measurement errors in the observables are also small. The variable which is measured with the highest precision is real money (the standard deviation of the measurement error is 0.06 per cent) while the variable which is measured with the largest errors is output (2.0 per cent). The standard deviation of the measurement error in the interest rate, σ_{ei}, is equal to 1 per cent. The implications of these findings are discussed in Section 4.

As is the case for previous studies, the model forecasting performance within sample is rather modest. About half of the cyclical variability in inflation and real balances is captured, but much less is achieved for output (5 per cent) and the interest rate (13 per cent).[4] However the model performance with respect to an unconstrained VAR is reasonable: the ratio between the likelihood of our structural VAR and the likelihood of the corresponding unconstrained VAR is 0.83. This seems to suggest that a great portion of the cyclical volatility of these variables is not easy to fit.

[4]The ratio between the standard deviation of the forecast errors in a given variable and the standard deviation of that same variable is usually quite high, equal to 0.55 for inflation, 0.95 for output, 0.54 for real money, 0.78 for the real CLUP and 0.87 for the interest rate.

Table 2. The optimal policy function

	Coefficient	Standard error
$y_{t-1\|t}$	0.49	0.06
$\pi_{t-1\|t}$	0.77	0.01
$m_{t-1\|t}$	-	-
$\overline{y}_{t\|t}$	-0.27	0.00
$u_{p,t\|t}$	0.67	0.03
$u_{c,t\|t}$	1.39	0.16
$u_{m,t\|t}$	-	-
i_{t-1}	0.61	0.01
$\overline{y}_{t-1\|t}$	-	-

3.2. Analysis of the model

The estimated model is characterized by the optimal monetary policy rule $i_t = FX_{t|t}$, the coefficients of which are reported in the Table 2. The standard error are computed by means of Monte Carlo methods. The optimal rule reacts strongly to the cost-push shock which has important effects on inflation (also see Figure 5 below). The weight on lagged inflation is also large. The coefficient on potential output is negative and significant: an increase in potential output forces the central bank to accommodate the shock to stabilize inflation and the output gap. The coefficients on lagged real money and the money demand shock are zero: these two variables have no direct effect on the target variables. Therefore it is optimal for the central bank not to react to them.

Figure 2 below reports the time series for the interest rate that is implied by the optimal rule, together with 95 per cent confidence bands (dashed lines) and the realized 3-month interest rate (solid line) over the estimation period. It shows that the optimal rate implied by the theoretical model tracks the actual interest rate on average. The latter, however, appears to be less volatile: the interest rate was significantly higher than the optimal one in the 1993-99 period and below it in the 2001-02 period.

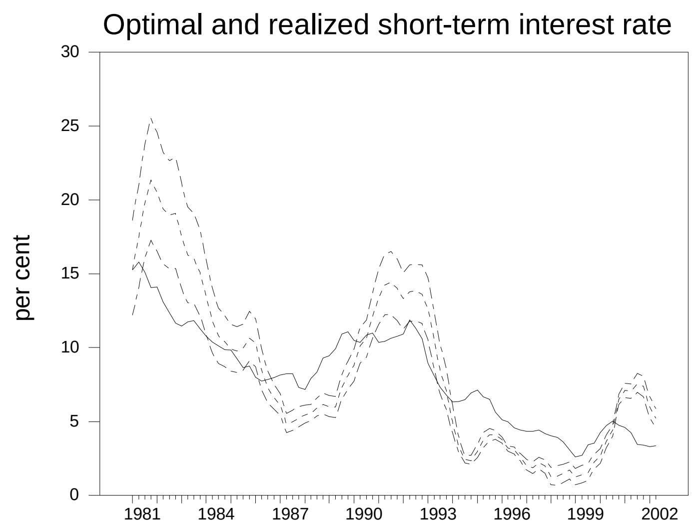

Figure 2

In order to have a more intuitive interpretation of the optimal monetary policy rule reported in Table 2, we estimated a Taylor-type of rule with data simulated using the model under the estimated coefficients reported in Table 1. This rule constrains the control variable (the interest rate) to be a linear function of the contemporaneous estimate of the output gap and inflation: $i_t = \phi_x x_{t|t} + \phi_\pi \pi_{t|t}$. The ordinary least square estimation explains about 80 per cent of the variability of the optimal rule. The estimated coefficients are 0.6 on the output gap and 1.6 on inflation. These values are remarkably close to those originally proposed by Taylor for the U.S. (0.5 and 1.5 for, respectively, the output gap and inflation).

The qualitative behavior of the estimated model can be described by means of impulse responses to the different shocks. An innovation in potential output (Figure 3) can be interpreted as a positive productivity shock which determines a decrease in real marginal costs, and hence inflation, and an increase in output. The central bank reduces the interest rate in order to increase output and stabilize the output gap. The initial decrease in the output gap reduces inflation. Real money increases as a consequence of the reduction in the interest rate and the increase in output.

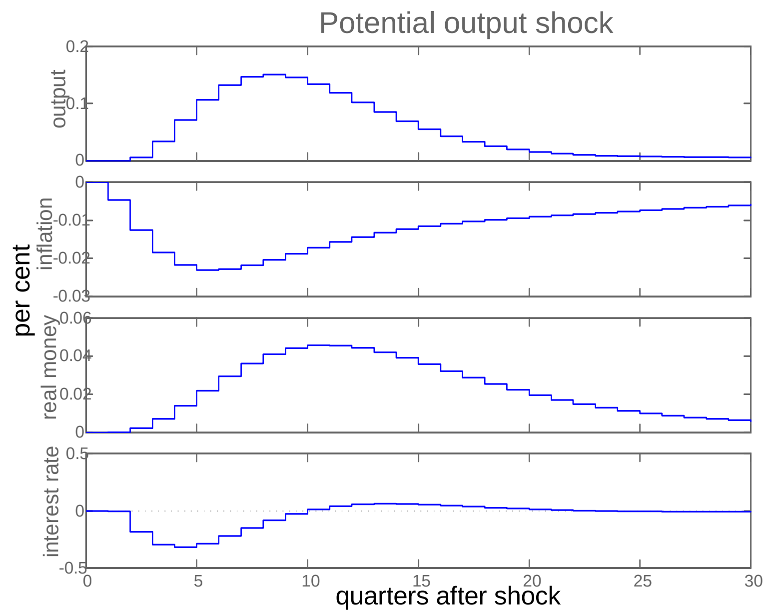

Figure 3

A positive demand shock (Figure 4), which in standard sticky price models is interpreted as a preference shock, increases output and the output gap and, through the Phillips curve, inflation. The central bank increases the interest rate to stabilize the target variables output gap and inflation. Real money increases reflecting mainly the increase in output which is partially compensated by the increase in the interest rate.

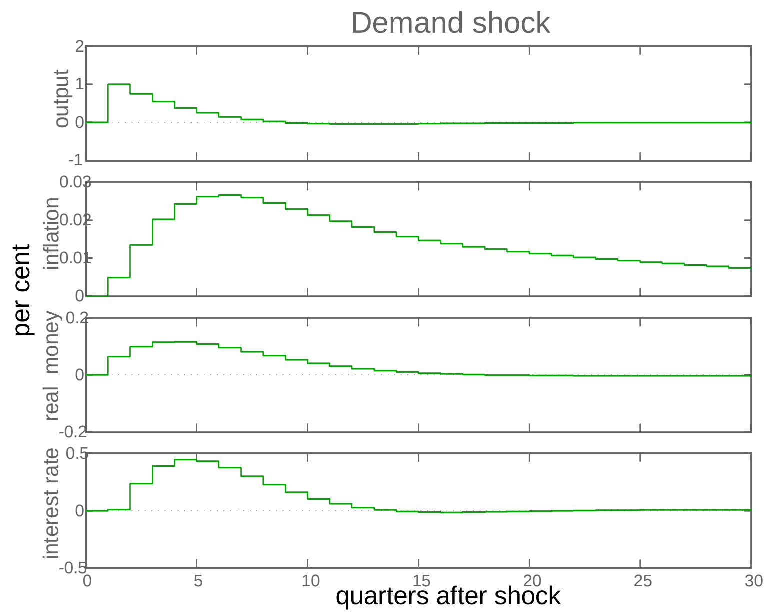

Figure 4

A positive cost-push shock (Figure 5) increases inflation on impact. The reaction of the monetary authority is to increase strongly the interest rate which reduces output and the output gap. As a result real money decreases.

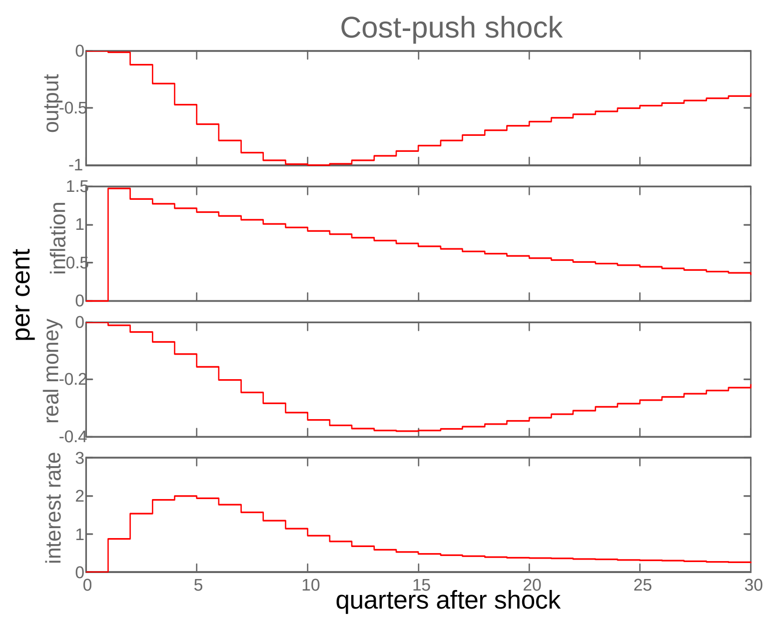

Figure 5

It is important to underline that in response to all the shocks, the central bank reacts gradually because changes in the interest rate are costly.

Table 3. Information and forecast error about fundamental shocks

	With Measurement Error			Without Meas. Error
Indicator:	*ULC and Money*	*ULC*	*Money*	
	{1}	{2}	{3}	{4}
$var\left[u_{p,t} - u_{p,t\|t}\right]$	0.316	0.319	0.316	0.316
$var\left[u_{c,t} - u_{c,t\|t}\right]$	0.008	0.008	0.008	0.000
$var\left[u_{\overline{y},t} - u_{\overline{y},t\|t}\right]$	1.381	1.385	2.313	0.957
$var\left[u_{m,t} - u_{m,t\|t}\right]$	0.007	0.141	0.007	0.001

4. The Role of Information

This section discusses the estimates of the measurement errors that are present in the model (Σ_v^2) and then proceeds to assess the role of these errors in affecting macroeconomic performance and the policy makers welfare. The estimates reported above provide quantitative information on the extent of the imperfect information problem in the model. Table 1 shows that the estimated measurement errors pertaining to output, inflation, money and the output gap are significant. The two largest errors pertain to output and the output gap, with standard deviation of respectively around 1.9 and 0.9 percent. This finding is not surprising given that these two variables are the ones for which no contemporaneous information is available. Much smaller measurement errors are computed for inflation and the monetary indicator. This is consistent with the empirical observation that information about these variables is available at higher frequency and that they are subject to much smaller statistical revisions.

The Kalman filter provides a convenient way to assess the consequences of these measurement errors for the information that the policy maker is able to extract about the fundamental shocks that hit the economy and, consequently, the true value of the state variables at each point in time. Column {1} of Table 3 reports the (unconditional) variance of the contemporaneous forecast errors about the fundamental shocks that the agents in the model face when information is processed optimally (using the Kalman filter) and both the monetary and the unit labor cost indicators are used. It appears that the largest forecast errors pertain to the innovations in potential output. This is partly due to the relatively large size of the innovations hitting this variable (see Table 1) and partly to the relative noisiness of the unit labor cost indicator that is used to form a forecast of this variable.

The other columns in this table analyze how the forecast errors change as we vary the information available to agents. When the monetary indicator is taken out of the vector of observables Z_t, the forecast errors concerning the money demand innovation obviously increase (they almost double, see column {2} but the forecast errors about the innovations in output (the preference shock) and potential output increase only by a tiny amount. This finding suggests that the M3 monetary aggregate contains relatively little information about the current and potential output, while it contains information on the innovation in the demand for real balances. This result is in stark contrast with the experiment reported in column {3}, in which the unit labor cost indicator is dropped from the information set of the policy maker. It appears that the forecast errors about potential output are almost doubled, while the forecast errors in the other variables are essentially unchanged.

Column {4} reports, as a benchmark of comparison, the variance of the forecast errors that are produced by the model if there is no measurement error on the vector of observables (i.e. when $\Sigma_v^2 = 0$). It shows that even with perfect measurement an incomplete information problem persists about actual and potential output given the assumption that information on this variables is available only with a lag. This benchmark shows that when the monetary indicator is used the forecast errors on output are as small as they would be if there was no measurement error on the lagged output indicator. Forecast errors about potential output instead remain above this benchmark even when the unit labor cost indicator is used (columns {1} and {2}).

4.1. Effects of information on outcomes and welfare

The forecast errors discussed above influence the unconditional variances of the main variables in the model. Table 4 reports the variance of the three goal variables (output gap, inflation and interest rate changes) together with the unconditional value of expected losses. The four columns of Table 4 report the values obtained under four alternative information assumptions. As before, the spirit of the exercise is to use the estimated model to analyze how economic performance (volatilities, welfare) changes in each of these scenarios.

Table 4. Targets volatility and the value of losses

	With Measurement Error		
Indicators:	*ULC and Money*	*ULC*	*Money*
	{1}	{2}	{3}
$var\left[y_t - \overline{y}_t\right]$	3.016	3.010	3.346
$var\left[\pi_t\right]$	0.413	0.413	0.412
$var\left[i_t - i_{t-1}\right]$	0.175	0.173	0.081
Λ_t	237.2	236.6	244.3

The results for the benchmark case in which both the monetary and the unit labor cost indicator used appear in column {1}. Let us compare the volatility of the goal variables for this case with the ones which are obtained when no monetary indicator is available and only the unit labor cost indicator is used. As Table 3 showed, this variation in the information set causes forecast errors about innovations in current and potential output to increase by a tiny amount. This (small) worsening in the information about the fundamental shocks causes monetary policy to be less active (smaller variability of interest rate changes) and the output gap volatility to be smaller. No effect is detected on the volatility of inflation. Smaller variances in two of the three goal variables lead to a moderate decrease in the losses enjoyed by the policy maker. Hence, less information about output innovations turns out to be good for welfare as it results in smaller volatility of target variables.

Quantitatively more noticeable consequences emerge when the output gap indicator is removed form the agent's information set (column 3). In this case, the greater noise surrounding the potential output indicator leads to a significant reduction in monetary policy activism (as indicated by the smaller volatility of interest rate changes) and to a significantly greater output gap volatility. Due to the certainty equivalence feature of our problem, policy effects stemming from imperfect information arise entirely from the way uncertainty influences the estimates of the states (i.e. through the matrix K in the updating equation (2.7c)), since the vector F of the optimal control rule ($i_t = FX_{t|t}$) does *not* depend on the uncertainty. As shown in the bottom line of the table, these changes increase the losses of the policy maker in comparison to the case in which both indicators are available. This finding indicates that the unit labor cost indicator is useful as it allows the policy maker to implement a welfare superior stabilization policy.

A. Appendix: State-space formulation of the ESM model

The model can be represented in state-space formulation:

$$\begin{bmatrix} X_{t+1} \\ x_{t+1|t} \end{bmatrix} = A^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + A^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + Bi_t + \begin{bmatrix} C_u \\ 0 \end{bmatrix} u_{t+1}$$

where the vector $X_t' \equiv \begin{bmatrix} y_{t-1} & \pi_{t-1} & m_{t-1} & \overline{y}_t & u_{p,t} & u_{c,t} & u_{m,t} & i_{t-1} & \overline{y}_{t-1} \end{bmatrix}$ and $x_t' \equiv \begin{bmatrix} y_t & \pi_t & m_t \end{bmatrix}$ denote, respectively, predetermined and non-predetermined (forward looking) variables at time t and i_t is the instrument controlled by the central bank.

The observables are stacked in the vector $Z_t \equiv \begin{bmatrix} y_t^o & \pi_t^o & m_t^o & x_t^o \end{bmatrix}$ according to:

$$Z_t = D^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + D^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + v_t$$

and target variables are collected in the vector $Y_t \equiv \begin{bmatrix} y_t - \overline{y}_t & \pi_t & i_t - i_{t-1} \end{bmatrix}$:

$$Y_t = C^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + C^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + C_i i_t$$

Mapping the model of Section 2 into this formulation yields the following matrices:

$$A_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{\delta}{1-\delta} & \frac{\alpha\theta}{\xi} & 0 & -\frac{\theta\kappa}{\xi} & -\frac{1}{1-\delta} & \frac{\theta}{\xi} & 0 & 0 & 0 & \frac{1-\alpha+\theta\kappa}{\xi} & -\frac{\theta}{\xi} & 0 \\
0 & -\frac{\alpha}{1-\alpha} & 0 & \frac{\kappa}{1-\alpha} & 0 & -\frac{1}{1-\alpha} & 0 & 0 & 0 & -\frac{\kappa}{1-\alpha} & \frac{1}{1-\alpha} & 0 \\
0 & 0 & \frac{\gamma_2}{\gamma_1} & 0 & 0 & 0 & -\frac{1}{\gamma_1} & 0 & 0 & -\frac{\gamma_y}{\gamma_1} & 0 & \frac{1}{\gamma_1}
\end{bmatrix},$$

where $\xi \equiv (1-\alpha)(1-\delta)$

$$A_2 = [0]\,, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ \frac{\theta}{1-\delta} \\ 0 \\ \frac{\gamma_i}{\gamma_1} \end{bmatrix} \qquad C_u = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \qquad D_2 = [0]$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad C_2 = [0] \qquad C_i = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

B. Appendix: Computing the likelihood function

[PRELIMINARY] In this section we describe how to compute the likelihood function for the model described in Section 2. The model, in its state-space representation, is defined by the following equations:

$$d_t = \hat{L}Q_t + w_{2,t} \qquad w_{2,t} \equiv \hat{M}v_t$$

$$Q_{t+1} = \hat{A}Q_t + \hat{G}w_{1,t+1}$$

$$w_{1,t+1} \equiv \begin{bmatrix} u_{t+1} \\ v_t \end{bmatrix}$$

where the first equation describe the law of motion of the unobserved states Q_{t+1} and the second equation is the observation equation linking the observed variables d_t to the states.

The vector of structural shocks u_t and the measurement errors v_t are assumed to be independent i.i.d processes with covariance matrices Σ_u^2 and Σ_v^2. The Kalman filter consists in a system of recursive equations that allows to forecast the unobserved state vector using the information contained in the observed variables.

The recursive system for computing the Kalman filter is given by

$$\begin{aligned} Q_{t+1|t} &= AQ_{t|t-1} + K_t\left(d_t - d_{t|t-1}\right) \\ K_t &= \left(A\Sigma^2_{t|t-1}C' + GV_3\right)\left(C\Sigma^2_{t|t-1}C' + V_2\right)^{-1} \\ \Sigma^2_{t+1|t} &= \left(A\Sigma^2_{t|t-1}A' + GV_1G'\right) - K_t\left(A\Sigma^2_{t|t-1}C' + GV_3\right)' \end{aligned}$$

where the matrix K_t is defined as the Kalman gain and $\Sigma^2_{t+1|t}$ is the covariance matrix of the forecast of next period state vector Q_{t+1} as of time t. The matrices V_1, V_2 and V_3 are given by:

$$V_1 = E\left(w_{1,t+1}w'_{1,t+1}\right) = \begin{bmatrix} \Sigma_u^2 & 0 \\ 0 & \Sigma_v^2 \end{bmatrix} \tag{B.1}$$

$$V_2 = E\left(w_{2,t}w'_{2,t}\right) = \hat{M}E\left[v_t v'_t\right]\hat{M}' = \hat{M}\Sigma_v^2\hat{M}' \tag{B.2}$$

$$V_3 = E\left(w_{1,t+1}w'_{2,t}\right) = \hat{M}E\begin{bmatrix} u_{t+1} \\ v_t \end{bmatrix} v'_t\hat{M}' = \begin{bmatrix} 0 \\ \Sigma_v^2\hat{M}' \end{bmatrix} \tag{B.3}$$

The prediction errors of the observed variables d_t, which are used to compute the likelihood function, are given by

$$a_t = d_t - dy_{t|t-1} = d_t - \hat{L}Q_{t|t-1} \tag{B.4}$$

and their covariance matrix by

$$E\left(a_t a'_t\right) = C\Sigma^2_{t|t-1}C' + V_2 = \Omega_t \tag{B.5}$$

Finally, the likelihood function is given by:

$$\log L = -\frac{nT}{2} ln\,(2\pi) - \frac{1}{2}\sum_{t=1}^{T} ln|\Omega_t| - \frac{1}{2}\sum_{t=1}^{T} a_t'\Omega_t^{-1}a_t \tag{B.6}$$

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