# BOARD OF GOVERNORS OF THE FEDERAL RESERVE SYSTEM

DIVISION OF RESEARCH AND STATISTICS

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**Date:** December 14, 2000

**To:** Members of the Board

**From:** Thomas Laubach and John Williams

**Subject:** Estimates of a Time-varying Equilibrium Real Federal Funds Rate

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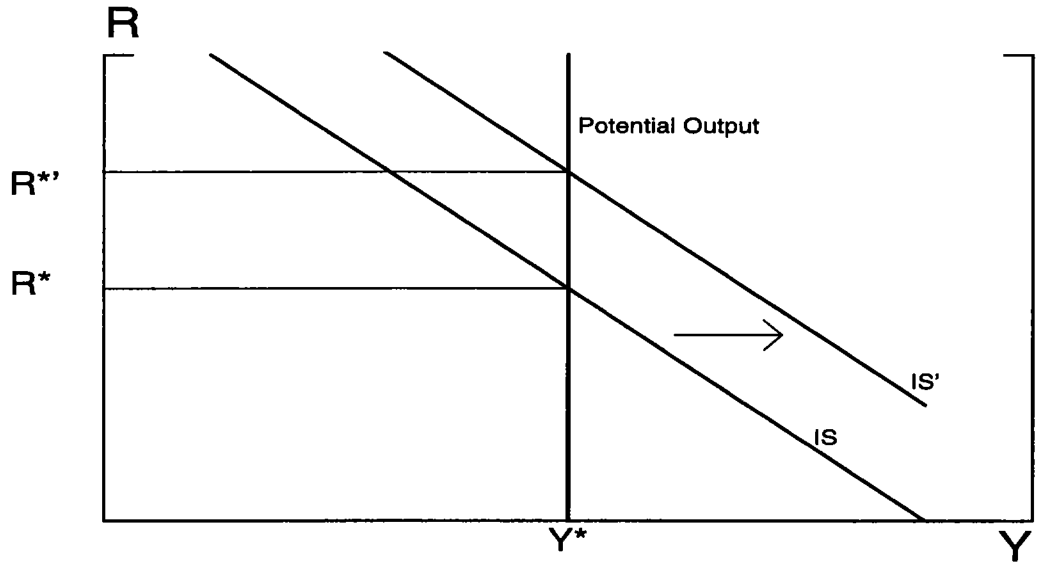
This memo is a first report on a broader project to study alternative definitions and estimates of the equilibrium real rate ($R^*$) and to evaluate their usefulness in the conduct of monetary policy. The approach pursued here imposes relatively little theoretical structure on the estimation problem; a complementary but more structural approach using the FRB/US model is currently underway.[1] In this study, we define the equilibrium real rate to be the real funds rate at which the output gap would gradually return to zero, barring further disturbances to aggregate demand and supply. Much of the literature on monetary policy rules has assumed that the equilibrium real rate is constant; for example, the Taylor rule includes a fixed value of 2 percent. Economic theory, however, suggests that highly persistent shifts in aggregate demand or supply, such as the fiscal expansion of the 1980s and the productivity acceleration of the late 1990s, might cause the equilibrium real rate to vary over time. Because the equilibrium real rate is not directly observed, we apply statistical methods to infer changes in the equilibrium real rate from co-movements in the output gap and actual real interest rates. The empirical evidence indicates that the equilibrium real rate has varied significantly over the past four decades; the current estimate is 4-1/4 percent.

## 1. Motivation and Specification of a Time-varying Equilibrium Real Rate

The basic idea underlying the equilibrium real rate can be illustrated by a standard textbook IS curve. Figure 1 shows a downward-sloping IS curve (labeled IS) reflecting the negative relationship between output ($Y$) and the real interest rate ($R$). For

[1] See Antulio Bomfim's January 28, 1998 memo to Governor Meyer – "Equilibrium Real Fed Funds Rate in the FRB/US Model" – for a preliminary report on the FRB/US-based approach to estimating $R^*$.

Figure 1: Determination of the Equilibrium Real Rate




simplicity, we assume that potential output (denoted $Y^*$) is invariant to the real interest rate and can therefore be represented by a vertical line. The output gap – the percent deviation between output and potential – equals zero where the IS curve and potential output intersect, and the real interest rate at that point is the equilibrium real rate, denoted by $R^*$. Transitory disturbances to demand or supply shift the IS or potential output curves for a time, but then the curves return to their initial positions, leaving the equilibrium real rate unchanged.

Permanent shifts in demand or supply, however, can affect the equilibrium real rate. For example, a permanent increase in government spending shifts the IS curve to the right – represented by the curve labeled IS′ in the figure – driving up the equilibrium real rate to $R^{*\prime}$. This is the standard "crowding out" effect. Note that some economic developments, such as an increase in structural productivity growth, would likely shift both the IS and potential output curves, with the net effect on the equilibrium real rate positive, zero, or negative, depending on the relative magnitude of the demand and supply effects.

For the purposes of estimation, we implement the basic characterization of temporary and permanent aggregate disturbances described above by specifying a simple reduced-form dynamic equation for the output gap,

$$X_t = a_1 X_{t-1} + a_2 X_{t-2} + b\tilde{R}_{t-1} + P_t + U_t \tag{1}$$

where:

- $X_t$ denotes the staff's estimate of the output gap (the percent deviation of actual from potential GDP),
- $\tilde{R}_t$ is the two-quarter average of the real federal funds rate (nominal funds rate less the annualized quarterly growth rate of the core PCE price index),
- $P_t$ is the unobserved permanent component of a shift in the output gap relation, and
- $U_t$ is the transitory component.

We use lags of the output gap to capture short-run output dynamics (with $a_1+a_2 < 1$). Equation (1) can be rewritten in terms of the equilibrium real rate

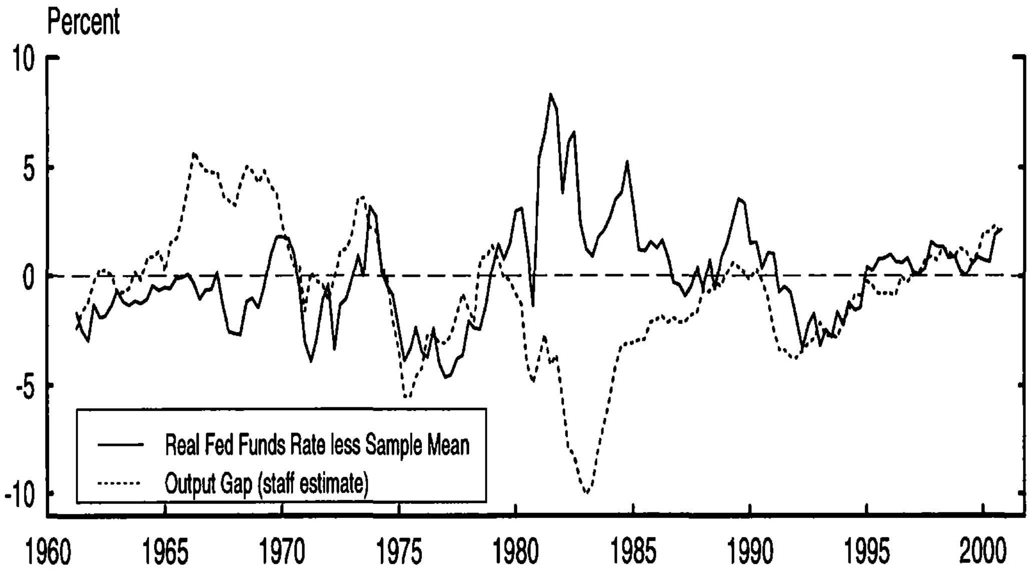
$$X_t = a_1 X_{t-1} + a_2 X_{t-2} + b(\tilde{R}_{t-1} - R^*_{t-1}) + U_t, \tag{2}$$

where $R^*_{t-1} = -\frac{P_t}{b}$, implying that $R^*$ cannot be directly observed, but must be inferred from the data.[2]

One key consideration in characterizing the equilibrium real rate is the parsing of disturbances into temporary versus permanent components. In equation (2), a transitory disturbance affects the output gap for many quarters. Indeed, if the sum of the coefficients on the lagged output gaps is near unity, such "temporary" disturbances have very long-lived effects, and could well be considered permanent for the purposes of monetary policy decisions. In fact, as discussed below, our estimate of $a_1 + a_2$

[2]Arguably, a number of financial variables – including stock market wealth, real bond rates, and the exchange value of the dollar – should appear in such an equation. We focus on the real federal funds rate for reasons of parsimony and because it is the FOMC's primary lever for affecting all these financial variables. An alternative specification might be to use the real long bond rate as a summary statistic of financial market conditions. However, it is difficult to measure the longer-term inflation expectations needed to construct ex ante real bond rates, in contrast to the real funds rate, for which shorter-run inflation expectations can be reasonably proxied by current inflation. (In principle, survey data on long-term inflation expectations could be used to estimate real bond rates, but such information is available only since the late 1970s.)

Figure 2: The Real Funds Rate and the Output Gap




is fairly low – about 0.9 – implying that half of any transitory shock has dissipated after two years, and more than two-thirds after three years. This rate of decay is rapid enough to provide a reasonable cutoff between transitory and highly persistent components, thereby allowing us to identify movements in $R^*$.

To provide some empirical evidence regarding potential time variation in the equilibrium real rate, figure 2 plots the real federal funds rate less its sample mean (the solid line) and the staff's estimate of the output gap (the dotted line). The mean level of the real funds rate over the past 40 years is 3 percent. Some of the developments shown in the chart suggest that the equilibrium real rate has not in fact been constant over time. For example, the actual real funds rate has been consistently above its long-run average since 1995, yet over that period the output gap has steadily drifted higher, suggesting the equilibrium real rate has been above its long-run average of late. Similarly, the real funds rate was around zero during the mid-1970s, implying a sizable monetary stimulus assuming a constant equilibrium real rate of around 3 percent, but the output gap closed only slowly during that period. Of course, these "ocular regressions" are only suggestive, so we now turn to formal econometric estimation.

## 2. Estimation

In order to proceed with the estimation, we need to specify processes for the two unobserved terms, $R^*$ and $U$. In the first estimation exercise, we assume that $R^*$ follows a random walk with innovation $V$, that $U$ is serially uncorrelated, and that $U$ and $V$ have no contemporaneous correlation. Specifically, the law of motion for $R^*$ is given by

$$R_t^* = R_{t-1}^* + V_t. \tag{3}$$

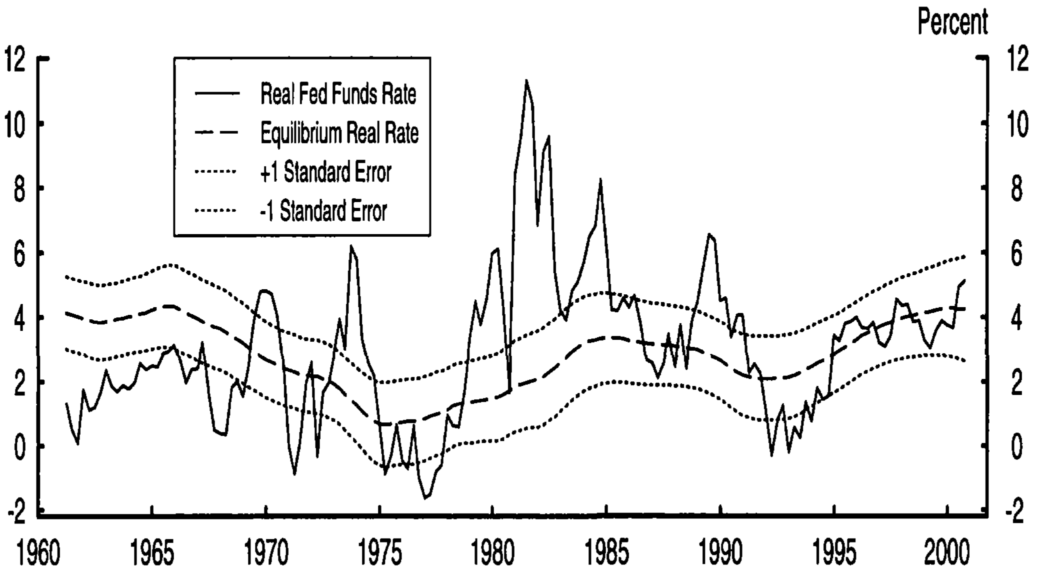
Equations (2) and (3), combined with the assumptions regarding the covariance of $U$ and $V$, define the model to be estimated.

We estimate by maximum likelihood the free parameters of the model – the coefficients $a_1, a_2$, and $b$, and the standard deviations of $U$ and $V$. Specifically, the estimated values of $R_t^*$ are computed using the Kalman filter, a standard approach for signal extraction problems such as this.[3] Based on a given "signal-to-noise ratio" (the ratio of the variance of $V$ to that of $U$), the Kalman filter parses the "surprises" in the movement of the output gap – that is, the actual output gap less the prediction generated by equation (2) given an estimate of $R_{t-1}^*$ – into transitory and permanent components. If the signal-to-noise ratio is estimated to be very low, then, by implication, nearly all output disturbances are transitory. As a result, the updating rule would correspondingly attribute output gap surprises mostly to transitory disturbances, implying that the estimate of $R^*$ would be nearly constant over time. In contrast, if the ratio is estimated to be relatively high, a significant share of output gap surprises would be attributed to permanent shocks, and the estimate of $R^*$ would vary appreciably over time.[4] Under certain assumptions, the Kalman filter approach yields the best "one-sided filter" estimate of $R^*$ – that is to say, the best estimate

[3] This approach has been applied to the problem of estimating a time-varying NAIRU based on a Phillips curve; see, for example, King, Stock, and Watson, "Temporal Instability of the Unemployment-Inflation Relationship," *Economic Perspectives*, Federal Reserve Bank of Chicago, May–June 1995.

[4] In estimation, we assume that the signal-to-noise ratio – and hence the parsing of output gap surprises to transitory and permanent components – is constant over time. The only exception is that during the early part of the sample the parsing of information to transitory and permanent components varies slightly, reflecting uncertainty about the initial value of $R^*$.

Figure 3: Estimated Equilibrium Real Funds Rate




that uses only past and present data on the output gap and real interest rates to compute the current value of the equilibrium real rate.

Although the one-sided Kalman filter is the best means of estimating the *current* $R^*$, we can improve our estimates of *past* values of $R^*$ using a "two-sided filter" that incorporates information from subsequent periods to revise the entire estimated historical path of $R^*$. The two-sided filter differs from the one-sided filter because subsequent observations of the output gap can be informative regarding the temporary versus permanent nature of past disturbances. Such revisions tend to become smaller for estimates of $R^*$ further back in the past. The two-sided filter estimates are commonly referred to as "smoothed" estimates; for the remainder of this memo, references to estimates of $R^*$ refer to smoothed estimates, based on observations through 2000q3. Of course, the one-sided and two-sided filter estimates are identical for the current value of $R^*$. An important implication of this approach is that $R^*$ is never known with complete certainty, even in hindsight and with complete knowledge of all model parameters. In practice, of course, the model parameters are not known with certainty either, but are themselves estimated, and these estimates are surrounded by uncertainty as in ordinary regressions. The standard errors that we report for our equilibrium real rate estimates take account of both these sources of uncertainty.

Table 1: Estimation Results

| | Baseline | | Growth and Fiscal Effects | |
|---|---|---|---|---|
| Parameter | Est. | SE | Est. | SE |
| $a_1 + a_2$ | .91 | .034 | .91 | .031 |
| $b$ | -.13 | .026 | -.13 | .022 |
| SD($U$) | .74 | .033 | .74 | .032 |
| SD($V$) | .44 | .090 | .29 | .086 |
| $c$ | 0 | | 1.4 | |
| $d$ | 0 | | -.4 | |
| Memo: 2000q2 values for $R_t^*$ | 4.24 | 1.6 | 3.98 | 1.3 |

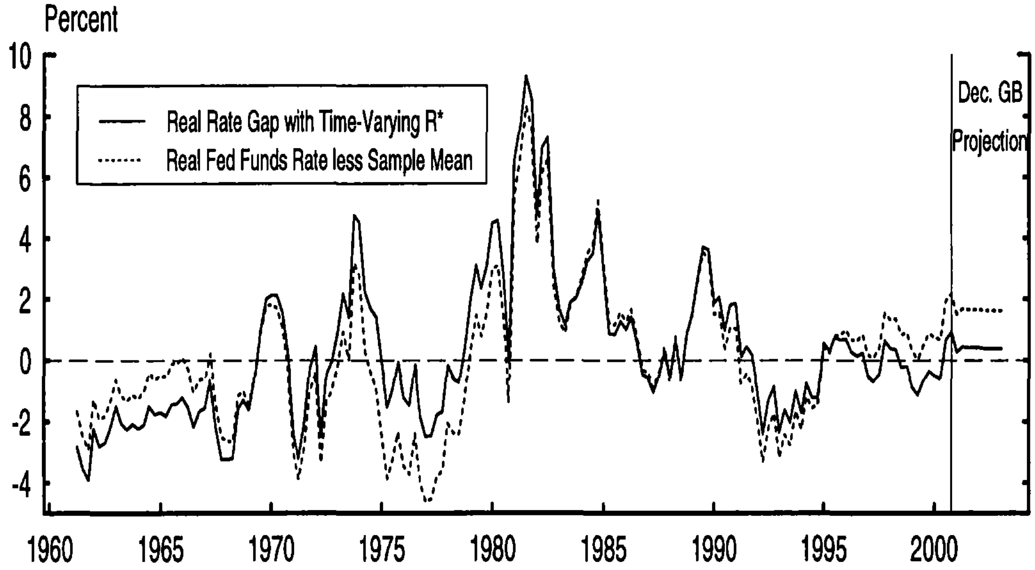
Estimation results are reported in the column of table 1 labeled "Baseline" and are shown in figure 3.[5] The estimation period is 1961q1–2000q3. The estimated sum of the coefficients $a_1$ and $a_2$ is .91, indicating that only half of the effect of a transitory shock to the output gap remains after 8 quarters, and only one-fifth remains after 16 quarters. The coefficient on the lagged two-quarter average of the real funds rate less $R_{t-1}^*$ is -.13. All of the model parameters are statistically significant and estimated with precision. The standard error of the regression, using the estimated equilibrium real rate, is 3/4 of a percentage point.

As shown in the table, the standard deviation of the innovation to the equilibrium real rate – SD(V) – is significantly different from zero, indicating that the hypothesis of a constant equilibrium real rate can be rejected. Still, highly persistent shocks are estimated to account for only about 7 percent of all shocks to the output gap equation. Despite the precise estimates of the model coefficients, the uncertainty around the current estimate of the equilibrium real rate is sizeable, with a standard error of about 1-1/2 percentage points.

The estimated equilibrium real rate series is shown as the dashed line in figure 3; the dotted lines are the estimates plus or minus one standard error. For comparison,

[5]We have estimated this model using core CPI to construct the real funds rate. The resulting core CPI-based estimates of the equilibrium real rate are broadly similar to those based on core PCE price inflation, with the differences largely accounted for by changes in the gap between core CPI and core PCE price inflation. The "real rate gaps" – the differences between the real rate and the estimated equilibrium rate – implied by the two price measures are therefore nearly identical.

Figure 4: Estimated Real Rate Gap




the solid line plots the actual real federal funds rate. The estimates of the equilibrium real rate fluctuate by more than 3-1/2 percentage points over the sample, falling from a peak of 4.3 percent in 1965 to 0.7 percent by the end of 1974. The sample mean of the estimated equilibrium real rate, 2-3/4 percent, is almost identical to a simple OLS estimate of a constant $R^*$. The most recent value, for the second quarter of 2000, is 4-1/4 percent.[6] According to our estimate, the equilibrium real rate has increased by a bit more than 2 percentage points since the early 1990s.

Figure 4 compares the estimate of the real rate gap – the difference between the real funds rate and our estimate of the equilibrium real rate (the solid line) – to the real funds rate less its sample mean (the dotted line). We have extended the sample through the end of 2002, incorporating the December Greenbook projections for the real federal funds rate. The stance of monetary policy appears appreciably different during three periods – the mid-1960s, the mid-1970s, and the late 1990s – according to whether the equilibrium real rate is assumed to be constant or to vary over time. For example, while the average real funds rate of about 3-3/4 percent over the past

[6]Estimates of the current level of the equilibrium real rate are sensitive to the assumed path of potential output. We are using the staff's estimated potential output series for all the results reported in this memo. To explore this issue further, we are in the process of estimating a model in which both potential output and $R^*$ are unobserved series.

five years exceeds the long-run average of 3 percent, it is slightly below the average equilibrium real rate of 4 percent estimated for that period.

## 3. Comparison to the December Greenbook Forecast

The result that the current estimate of the equilibrium real rate is well above its average historical value has important implications for the outlook. Table 2 compares the projected values of the output gap in the December Greenbook with a projection derived from our simple output gap equation that incorporates a time-varying equilibrium real rate. For the simple output gap model, we hold $R^*$ fixed at its current estimated value of 4-1/4 percent, and project the real federal funds rate to follow the Greenbook path through 2002. (Over this period, the real funds rate averages about 4-1/2 percent.)

Table 2: Output Gap Projections

| | 2000q4 | 2001q4 | 2002q4 |
|---|---|---|---|
| December Greenbook | 1.3 | -.0 | -.4 |
| Time-varying $R^*$ model | 1.4 | .6 | .1 |

The projection of the output gap derived from the simple model is a bit stronger than that of the December Greenbook, suggesting that, in the context of the model, the value of the equilibrium real rate implicit in the Greenbook projection is a bit below our current 4-1/4 percent estimate. To gauge the magnitude of this difference, we have extended the estimation period through the end of 2002, treating the December Greenbook projection as actual data. The result of this exercise is to reduce the estimate of the prevailing equilibrium real rate by 1/4 percentage point, to 4 percent.

## 4. The Role of Trend Growth and Fiscal Policy

Evidence for an acceleration of trend productivity growth in the U.S. economy during the 1990s is an important reason for constructing estimates of a time-varying equilibrium real rate. The neoclassical growth model provides a framework in which the steady-state real interest rate varies positively with the growth rate. In versions of the model with utility-maximizing households, the factor of proportionality of real rate changes to changes in growth is determined by households' degree of risk aversion. Even without assuming utility maximization, the basic result holds that an acceleration in trend growth raises the equilibrium real rate, where the sensitivity is usually held to be one-for-one or higher.[7]

The effects of fiscal policy on the equilibrium real rate are more controversial. In some theoretical models, changes in fiscal policy are completely offset by private sector spending decisions, leaving the equilibrium real rate unaffected. Other models, however, suggest that fiscal policy could affect the equilibrium real rate. If a sufficiently large share of households is liquidity constrained, for example, a change in fiscal policy could have a large and persistent effect on spending by households and therefore on the equilibrium real rate.[8]

To explore the role of these two factors in explaining time variation in the equilibrium real rate, we modify the law of motion (3) for $R^*$ as follows

$$R_t^* = c\, g_t + d\, f_t + Z_t, \quad (4)$$

$$Z_t = Z_{t-1} + V_t. \quad (5)$$

Equations (4) and (5), combined with the output gap equation (2), constitute the model to be estimated. In this formulation, the equilibrium real rate is determined by the four-quarter moving average of the staff's estimate of potential GDP growth, $g_t$, the four-quarter moving average of the staff's estimate of the ratio of the full-employment federal budget surplus to potential GDP, $f_t$, and a highly persistent component, $Z_t$, that captures other factors affecting $R^*$. $Z$ is modeled as an unobserved random walk.

In the column of table 1 labeled "Growth and Fiscal Effects," we report results estimated using this modified specification. We do not estimate coefficients $c$ and $d$, but instead chose values based on results from the FRB/US model.[9] The estimates

[7]See "The Analytics of Growth and the Real Rate of Interest," Memo to Governors Gramlich and Meyer by Flint Brayton, May 3, 2000.

[8]A recent synthesis of the different views on the effects of fiscal policy is provided by Mankiw, "The Savers-Spenders Theory of Fiscal Policy," *American Economic Review*, May 2000.

[9]See "Recent U.S. Macroeconomic Performance and Its Implications for the Future: The FRB/US View," FOMC memo by Flint Brayton and David Reifschneider, June 19, 2000. Estimation of the

of coefficients $a_1, a_2$, and $b$ are nearly identical to those obtained from estimating the "baseline" model reported above. The precision of the parameter estimates improves and the standard deviation of the random walk component of $R^*$ declines by about 1/3 relative to the baseline model, which helps to reduce the standard error of the current estimate of $R^*$ by about 1/4 percentage point.

The estimated values of $R^*$ based on equations (4) and (5) are shown by the dashed line in the top panel of figure 5. These estimates are broadly similar to those shown in figure 3. In particular, the current estimate of $R^*$ (for 2000q2) is 4 percent, just a little under that of the baseline estimate. The main difference from the previous estimates is that trend growth and fiscal policy now explain a large fraction of the movements in $R^*$, and accordingly the residual component is only two thirds as volatile compared to that in the baseline estimation.

The bottom panel of figure 5 shows the contributions of the three components to the overall movements in $R^*$. The solid line shows the contribution from trend growth, $c\ g_t$, the dashed line shows the contribution from fiscal policy, $d\ f_t$, and the dotted line the unobserved random walk component, $Z_t$. Changes in trend growth explain a significant portion of the decline in the equilibrium real rate from the mid-1960s to the mid-1970s and the later increase in the 1990s. However, the increase in $R^*$ in the mid-1980s can be mostly attributed to fiscal policy; the subsequent return of fiscal discipline has exerted significant downward pressure on $R^*$.

As noted, inclusion of terms accounting for trend growth and fiscal policy considerably reduces the magnitude of estimated movements in $R^*$ owing to the unobserved component and improves the precision with which the equilibrium real rate is estimated. We are currently investigating other possible observable determinants of the equilibrium real rate, such as interactions of inflation with the tax code, and equity and bond risk premia, that may further help explain time variation in $R^*$.

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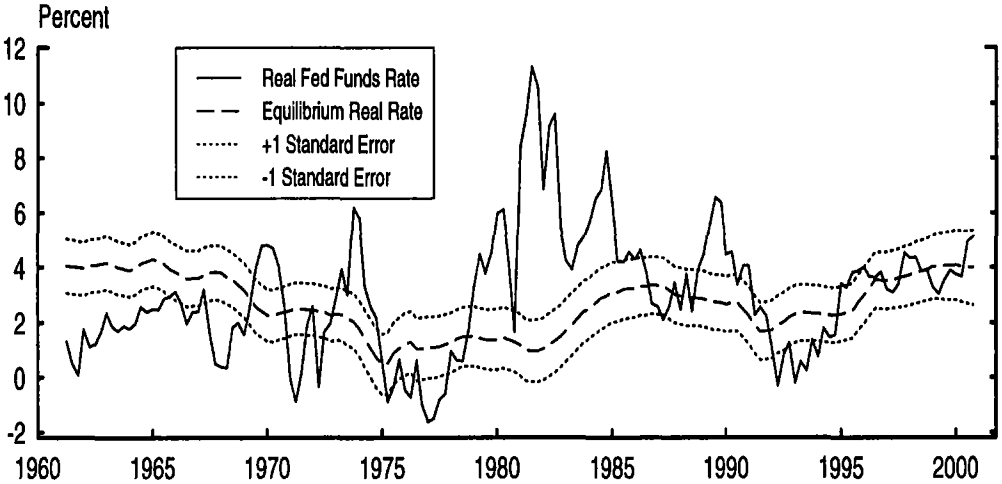
coefficients $c$ and $d$ leads to values about 4-1/2 times as large as those shown in Table 1, implying implausibly large movements in the equilibrium real rate. The values used here, consistent with the FRB/US model, are not rejected by a likelihood ratio test at the 10% level. The hypothesis that $c = d = 0$ that is implicit in equation (2), however, is rejected at the 2% level.

Figure 5: Equilibrium Real Funds Rate with Structural Determinants




Contributions to the Estimated Equilibrium Real Rate