

# **Interest on Reserves: An Analytical Framework<sup>1</sup>**

**April 2008**

---

<sup>1</sup> Huberto Ennis (FRB Richmond) and Todd Keister (FRB New York) authored this paper as part of the work undertaken by the Interest on Reserves (IOR) workgroup.

This paper presents a simple, stylized framework for understanding the monetary policy implementation process. An explicit mathematical model of the demand for reserves by commercial banks is developed and presented using graphical methods. The objectives of this analysis are to illustrate the basic economics behind each of the proposed policy regimes and to highlight the similarities and differences each has with the current U.S. regime. The framework is also useful for analyzing the extent to which the different proposals are consistent with overall policy objectives.

We begin by presenting the basic framework in the context of the current U.S. policy regime. In the first section we describe the framework in its simplest form, which abstracts from reserve maintenance periods, daylight credit, and other important issues. In the second section we show how the framework can be extended to address these and other issues. In Section 3 we show how each of the policy proposals under consideration can be illustrated and analyzed in the context of the framework.

## 1. THE BASIC FRAMEWORK

The central component of the framework is a simple model of the demand for overnight reserves. In this section we describe the model in detail and discuss some of the issues abstracted from in the analysis. We then present a graphical illustration of the current U.S. policy regime; this case will serve as a base line for the analysis in the following two sections.

### 1.1 A SIMPLE MODEL OF RESERVE DEMAND

The model presented here focuses on *nonborrowed reserve balances*, that is, funds held by commercial banks on deposit at the Federal Reserve that have not been borrowed from the Federal Reserve. Banks hold these reserves primarily to satisfy reserve requirements although, as discussed below, other factors such as the desire to make interbank payments also play a role. Banks face uncertainty about the flows into and out of their reserve accounts and, therefore, are typically not able to exactly satisfy their reserve requirement. Instead, they must balance the possibility of holding excess reserve balances – and the associated opportunity cost – against the possibility of being penalized for a reserve deficiency. A bank's demand for reserves results from optimally balancing these two concerns. Banks are assumed to be risk neutral and to maximize expected profits.

The basic elements of the model are as follows:

*Interbank Market.* Each day, commercial banks can borrow and lend reserves in an interbank market. The Central Bank conducts open market operations and can thereby change the total supply of reserves. We model this market as being perfectly competitive, which is a reasonable assumption for most of the day. Toward the end of the day, this market closes and banks are unable to make further trades.

*Payment Shocks.* After the interbank market has closed, each bank experiences a payment shock  $P$  that affects its end-of-day reserve balance. The value of  $P$  can be either positive, indicating a net outflow of funds, or negative, indicating a net inflow. The assumption that the interbank market closes before the payments shocks are resolved is a simplified way of capturing the imperfections in the interbank market that become more severe near the end of the day. The important feature of the model is simply that banks are unable to perfectly target their end-of-day reserve balance. Uncertainty about the end-of-day balance creates a “smooth” demand for reserves.

We assume that the payment shock  $P$  is uniformly distributed on the interval  $[\underline{P}, \overline{P}]$ . The lower bound of this interval,  $\underline{P}$ , will typically be a negative number, meaning that late-day payment inflows are possible. We study the effects of other distributional assumptions later in this section. The value of this shock is not yet known when the interbank market is open; hence, a bank’s demand for reserves in this market is affected by the distribution of the shock and not the realization.

*Reserve Requirements.* To keep the presentation simple, we start with a model where reserve requirements must be met at the end of each day. Let  $K$  denote the level of reserves a typical bank is required to hold. We discuss multi-day maintenance periods in Section 2.2 below. One can also think of the case we study in this section as applying to the last day of a multi-day maintenance period. In this case, the reserve requirement  $K$  should be interpreted as the quantity of reserves the bank needs to hold on the last day of the maintenance period in order to satisfy the overall requirement, given the reserve holdings on previous days in the period.

If a bank finds itself holding fewer than  $K$  reserves at the end of the day, after the payment shock  $P$ , we assume the bank must borrow reserves to cover this deficiency at some “penalty” rate  $r_p$ . In the current U.S. system,  $r_p$  can be thought of as the rate charged on discount window loans, adjusted to take into account any stigma effects of borrowing from the discount window. Alternatively, a bank may instead pay the reserve-deficiency penalty, either by choice or because it does not have sufficient collateral posted at the discount window. For our framework, the important feature is simply that the bank is forced to make up any reserve deficiency at a penalty rate of interest. This rate could come from the discount window, a deficiency charge, or even from borrowing at a high rate in the Fed Funds market at the end of the day. Whatever the source of this penalty may be, we use  $r_p$  to denote the rate a bank must pay to cover the deficiency and satisfy its requirement  $K$ .

*Daylight credit.* In this section we assume that daylight credit is freely available to banks at no cost, so that the cost to the bank of making payments during the day is independent of its overnight reserve position. We study the case where daylight credit is costly and show how this changes the analysis in Section 2.3 below.

*Discussion.* The simple model used here abstracts from many features of reality. For example, we do not include vault cash in the analysis and, therefore, the required reserves in this framework should be interpreted as the requirement net of vault cash holdings. To

the extent that vault cash holdings are independent of the overnight rate, at least over short time horizons, including them in the model would have no effect. We also abstract from contractual clearing balances that, once set, act much like required reserves.

Other assumptions are perhaps less innocuous. The framework implicitly assumes, for example, that the only cost a bank faces when holding reserves is the opportunity cost of not lending the funds out. In reality, holding a much larger quantity of reserves might require a bank to raise more deposits and subject it to higher capital requirements. For most of the options under consideration, however, such effects would likely be small.

*Literature.* The model here uses the basic approach to reserve management introduced in Poole (1968). More recent contributions to this literature include Furfine (2000), Clouse and Dow (2002), Bartolini, Bertola and Prati (2002), and Whitesell (2006a). The model closely follows that in Ennis and Weinberg (2007).

## 1.2 THE CURRENT POLICY REGIME

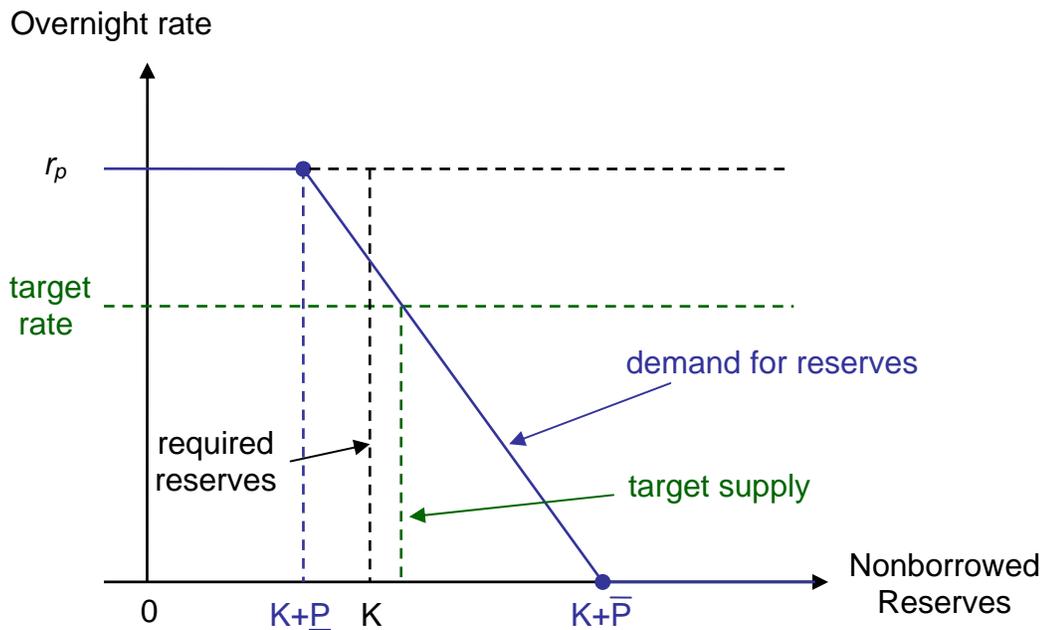
In the current U.S. policy regime, the FOMC selects a target for the overnight interest rate and instructs the Open Market Desk to adjust the quantity of reserve balances in order to achieve that target. Banks can borrow reserves from the discount window at a rate that is 100 basis points above the target Federal Funds rate. The penalty rate in our analysis  $r_p$  should thus be thought of as being at least 100 basis points above the target rate. Banks earn no interest on their reserve holdings.

This subsection presents a graphical illustration of the model demand for reserves under such a policy regime. Figure 1 depicts an individual bank's demand for nonborrowed reserves. To draw this curve, we ask: Given a particular value for the interest rate, what quantity of nonborrowed reserves would the bank demand to hold if that rate prevailed in the market? In most circumstances, a bank would be unwilling to hold any nonborrowed balances if the overnight rate were higher than the penalty rate  $r_p$ . If the market rate were higher than  $r_p$ , banks would choose to meet their requirements entirely through borrowing from the discount window. In fact, they would like to borrow even more than their requirement and lend at the higher market rate, but this fact is not important for the analysis. The important point is that there should be essentially no demand for nonborrowed reserves for any interest rate larger than  $r_p$ .

For interest rates below  $r_p$ , however, banks will choose to hold some nonborrowed reserves. This demand is "precautionary" in the sense that banks choose their reserve holdings to balance the possibility of falling short of the requirement against the possibility of having extra reserves that earn no interest. A bank will always choose to hold at least  $K + \underline{P}$  reserves in this case, because  $\underline{P}$  represents the smallest possible late-day outflow of funds from the bank's reserve account. (Note that the diagram is drawn under the assumption that  $\underline{P}$  is negative, so that  $-\underline{P}$  is the largest possible late-day inflow of funds.) If the bank held fewer than  $K + \underline{P}$  reserves, it would be certain to need to borrow at the penalty rate overnight, which would not be an optimal choice when the market rate is lower than  $r_p$ .

If the overnight rate were very low – close to zero – the opportunity cost of holding reserves would be very small. In this case, each bank would hold enough precautionary reserves so that it is virtually certain that unforeseen movements on its balance sheet will not decrease its reserves below the required level. In other words, the bank will hold  $K + \bar{P}$  reserves in this case. If the overnight interest rate were exactly zero there would be no opportunity cost of holding reserves. The demand curve is, therefore, flat along the horizontal axis after  $K + \bar{P}$ .

Figure 1: Basic Analysis of the Current Regime



In between these two extremes, the demand for reserve balances will vary inversely with the market interest rate; this portion of the demand curve is represented by the downward-sloping line segment in Figure 1. The curve is downward-sloping for two reasons. First, the market interest rate represents the opportunity cost of holding reserves overnight. When this rate is lower, finding itself with excess balances is less costly for the bank and, hence, the bank is more willing to hold precautionary balances. Second, when the market rate is lower, the relative cost of having to pay the penalty rate on a reserve deficiency is larger, which also tends to increase the bank’s precautionary demand for reserves.<sup>2</sup>

We assume for the moment that all banks are identical, so that the aggregate demand for reserves looks exactly like each individual bank’s demand. Figure 1 thus also represents

<sup>2</sup> The linearity of the downward-sloping part of the demand curve results from the assumption that the payment shock is uniformly distributed. We study the effects of other distributional assumptions below.

the *total* demand for non-borrowed reserves in the banking system, measured in per-bank terms. We discuss some of the issues that arise when banks are heterogeneous in Section 2.4 below. The equilibrium interest rate in the interbank market is determined by the height of this aggregate demand curve at the level of reserve balances supplied by the Central Bank. As shown in the diagram, there is a unique level of reserve supply that will lead the market to clear at a given target rate; this level of reserves is labeled the *target supply*.

Note that there are really two policies available to affect the (equilibrium) market rate: changing the supply of reserves and changing the discount rate. Suppose, for example, that the Central Bank wishes to decrease the market interest rate. It could either increase the supply of reserves (say, through open market operations), leading to a movement down the demand curve, or it could decrease the discount rate, which would tend to rotate the demand curve downward while leaving the supply of reserves unchanged. Both policies would – all else being equal – cause the market interest rate to fall.

In the current policy regime, the Federal Reserve sets the discount rate a fixed distance (100 basis points) above the target rate during normal times. In this case, changing the target rate involves an automatic change in the discount rate and, thus, an automatic shift in the demand curve. As a result, whether or not increasing the market interest rate requires a significant change in the supply of reserves depends on the elasticity of demand and other details. In general, no major change in the supply of reserves is to be expected. This fact helps explain some of the difficulties involved in empirically testing for the “liquidity effect” of open market operations (see Carpenter and Demiralp, 2006, and the references therein).

During periods of turmoil in financial markets, the stigma associated with borrowing from the discount window may increase.<sup>3</sup> In terms of Figure 1, an increase in stigma would be captured by an increase the penalty rate  $r_p$  and, thus, would cause the demand curve to rotate upward. Such an increase in demand would tend to increase the overnight interest rate, leading the Desk to increase the supply of reserves in order to bring the rate back down to the target level. In this way, the figure shows how periods of turmoil or increased uncertainty in the financial system will tend to be associated with higher levels of reserve holdings, as well as with a steeper demand curve for reserves.

### 1.3 UNCERTAINTY AND THE SHAPE OF THE DEMAND CURVE

The fact that the downward-sloping part of the demand curve in Figure 1 is a straight line derives from the assumption that the late-day payment shock is uniformly distributed. While this assumption is perhaps not the most realistic, it is useful for understanding the overall shape of the demand curve, as demonstrated above. To draw the curve in Figure 1, we only had to determine two points: where the curve diverges from the rate  $r_p$  and

---

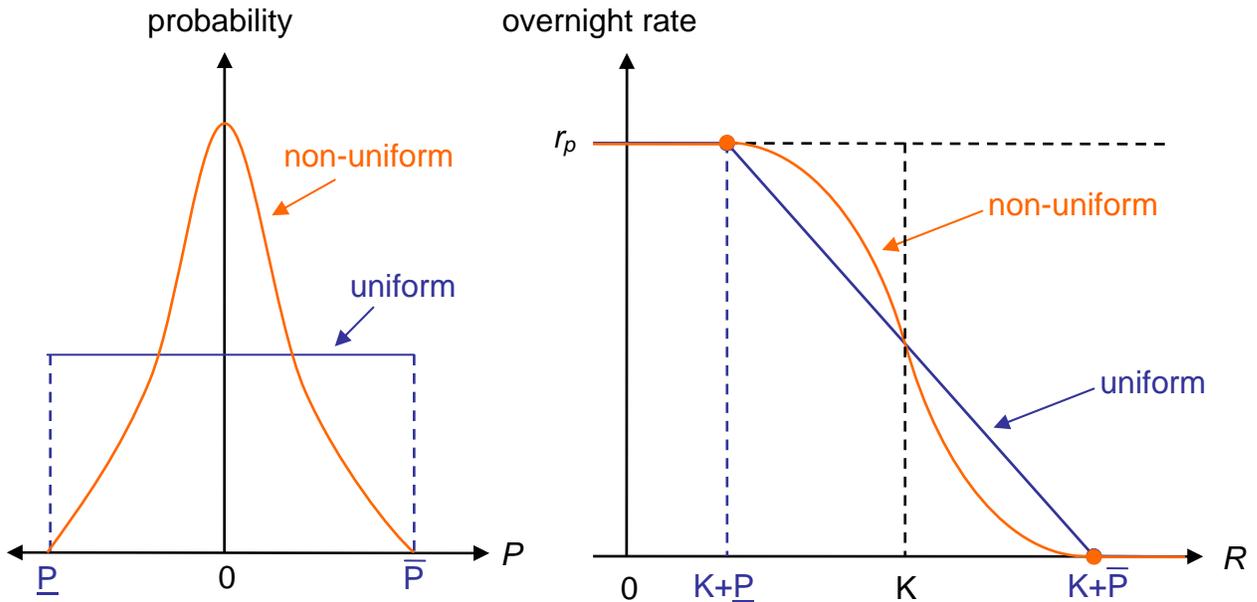
<sup>3</sup> Suppose, for example, that market participants believe recent movements in asset prices may make one or more banks insolvent, but do not know which banks were holding the relevant assets. A bank may then worry that if it borrows at the discount window, knowledge of the borrowing will leak out to market participants and lead them to infer that the bank is in poor financial condition.

where it hits the horizontal axis. The demand curve is then constructed by connecting these points with a straight line.

Under different distributional assumptions, the demand curve may have more “curvature”, but the overall shape will remain exactly as in the figure above. In particular, for any distribution of the payment shock with support  $[\underline{P}, \bar{P}]$ , the demand curve will be flat at  $r_p$  until the point  $K + \underline{P}$  and will be flat on the horizontal axis after the point  $K + \bar{P}$ . Between these two points, the demand curve will always be downward sloping. Different distributions merely change the shape of this downward-sloping part of the curve.

Suppose, for example, that the distribution of the late-day payment shock is hump-shaped, like the orange curve in the left panel of Figure 2. In this case, moderate values of  $P$  are more likely to occur than extreme values near either  $\underline{P}$  or  $\bar{P}$ . How would the demand curve change if the bank faced this type of uncertainty instead of the uniform distribution studied above?

Figure 2: Curvature of the Demand Function



The right panel in the figure presents the corresponding change in reserve demand. To understand the shape of this new demand curve, consider first a level of the overnight rate slightly below the penalty rate  $r_p$ . What quantity of reserves should the bank hold if this rate prevailed in the market? We argued above that the bank should hold at least  $K + \underline{P}$ , since if it held less than this amount it would be certain to face a deficiency after the payment shock is realized and be forced to borrow at the penalty rate. In addition, the bank will hold a small amount of “precautionary” reserves above the level  $K + \underline{P}$ . In doing

so, the bank exposes itself to the possibility of being left with excess reserves at the end of the day, which would occur if the realization of  $P$  is near  $\underline{P}$ . However, holding more reserves also decreases the amount the bank would have to borrow at the penalty rate for other values of the payment shock. A reserve balance slightly larger than  $K + \underline{P}$  optimally balances these two concerns.

Under the orange (non-uniform) distribution depicted in Figure 2, the probability of a payment shock near  $\underline{P}$  is very small. Compared to the uniform case, therefore, the bank is less concerned about a large payment inflow that would leave it holding excess reserves at the end of the day. As a result, the bank is willing to hold a larger quantity of reserves, which is why the orange demand curve in the right-hand panel lies above the blue line for values of the overnight rate near  $r_p$ .

Now suppose the market interest rate were close to zero. In this case, the bank will choose to hold almost  $K + \bar{P}$  reserves in order to prevent against the possibility of a large payment outflow that will force it to borrow at the penalty rate. How much less than  $K + \bar{P}$  the bank chooses to hold will depend on the likelihood it will be forced to borrow after the payment shock is realized. Under the non-uniform distribution in the figure, the probability of a large payment outflow (close to  $\bar{P}$ ) is very small. As a result, the bank will hold fewer reserves than it would in the uniform case, where large payment shocks are more likely. This corresponds to the fact that the orange demand curve in the right-hand panel lies below the blue line for values of the interest rate near zero.<sup>4</sup>

For the issues studied in this report, the specific assumptions about the distribution of the payment shock are largely unimportant. In what follows we use the assumption of a uniformly-distributed payment shock solely because it makes the graphs cleaner and easier to read. The assumption that the payment is bounded also makes it easy to see how changes in parameters affect the position of the demand curve. Suppose, for example, the entire distribution of the payment shock is shifted by some constant, so that  $P$  is distributed on  $[\underline{P} + c, \bar{P} + c]$ . This might happen, for example, if the bank knows that it must make a late day payment of size  $c$  in addition to the usual uncertainty. In this case, the entire demand curve shifts to the right. The quantity of reserves demanded at any given interest rate increases by exactly  $c$ .

Another interesting exercise is to suppose that the bank faces more uncertainty about its end-of-day payment flows, which can be modeled by increasing the support of the payment shock. To take a specific example, suppose the support changes to  $[\underline{P} - c, \bar{P} + c]$ , which corresponds to a mean-preserving spread in the case of the uniform distribution. It is not difficult to see that this change will necessarily make the slope of the downward-sloping part of the demand curve flatter. The effects of other changes, including changes to the policy regime, are also fairly straightforward to incorporate, as we show in the sections that follow.

---

<sup>4</sup> If the shock instead had an unbounded distribution, such as the normal distribution used by Whitesell (2006a) and others, the demand curve would again have this same shape, but would asymptote to the rate  $r_p$  and to the horizontal axis without ever intersecting them.

## 2. EXTENSIONS AND ISSUES

In this section, we discuss several extensions of the basic framework that enable it to address issues that are important to consider in designing a system of monetary policy implementation. We continue to focus on the current U.S. policy regime. We first look at factors affecting interest rate volatility, which leads naturally to the study of reserve maintenance periods. We then discuss the effect of charging fees for daylight credit and, finally, the differences between large and small banks in the context of the framework.

### 2.1 INTEREST RATE VOLATILITY

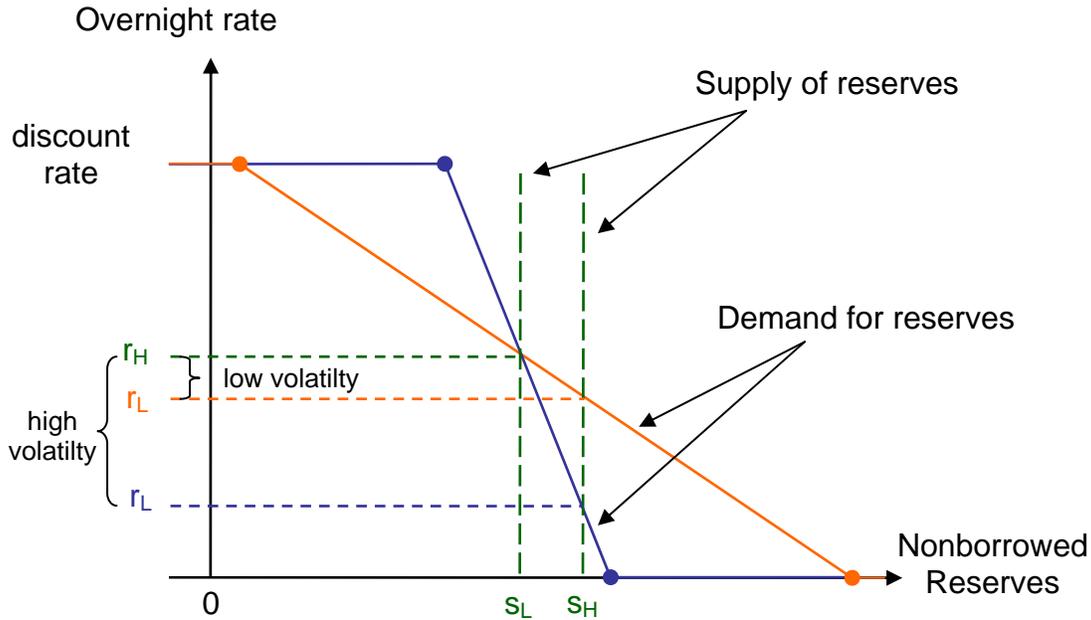
In this subsection we discuss interest rate volatility in the basic framework and how it relates to the slope of the demand curve in Figure 1. The Central Bank performs daily (or periodic) open market operations that aim to set the supply of reserves as close as possible to the target level. This process requires the Open Market Desk to accurately forecast both reserve demand and changes in the existing supply of reserves due to autonomous factors such as payments into and out of the Treasury's account. Forecasting errors will lead the actual supply to deviate from the target and, hence, will cause the market rate to differ from target, even if reserve demand is perfectly anticipated.

Figure 3 illustrates the fact that a flatter demand curve is associated with less volatility in the interest rate, given a particular level of uncertainty associated with autonomous factors. Suppose that uncertainty about the magnitude of the change in autonomous factors implies that, after a given open market operation, the total supply of reserves will be equal to either  $s_L$  or  $s_H$  in the figure. With the steeper (blue) demand curve, this uncertainty about the supply of reserves leads to a relatively wide range of uncertainty about the market rate. With the flatter (orange) demand curve, in contrast, the variation in the market rate is smaller. This simple result demonstrates that the slope of the demand curve, and those policies that affect the slope, are important determinants of the observed degree of volatility of the interest rate around the target.

As discussed in Section 1.2 above, the slope of the demand curve in the basic framework depends on the distribution of the late-day payment shock  $P$ . The orange line in Figure 3 represents demand under a wider support, that is, a situation where banks face greater uncertainty about the late day shock. This larger amount of uncertainty will lead to a flatter demand curve and, hence, lower volatility in the market interest rate due to forecast errors or unanticipated changes in the supply of reserves.

Central banks generally aim to minimize volatility in their target interest rate. For this reason, a variety of real-world arrangements have been designed in an attempt to decrease the slope of the demand curve, at least in the "relevant" region. Perhaps the most significant of these arrangements is reserve maintenance periods, which we discuss in the next subsection.

Figure 3: Interest Rate Volatility



## 2.2 RESERVE MAINTENANCE PERIODS

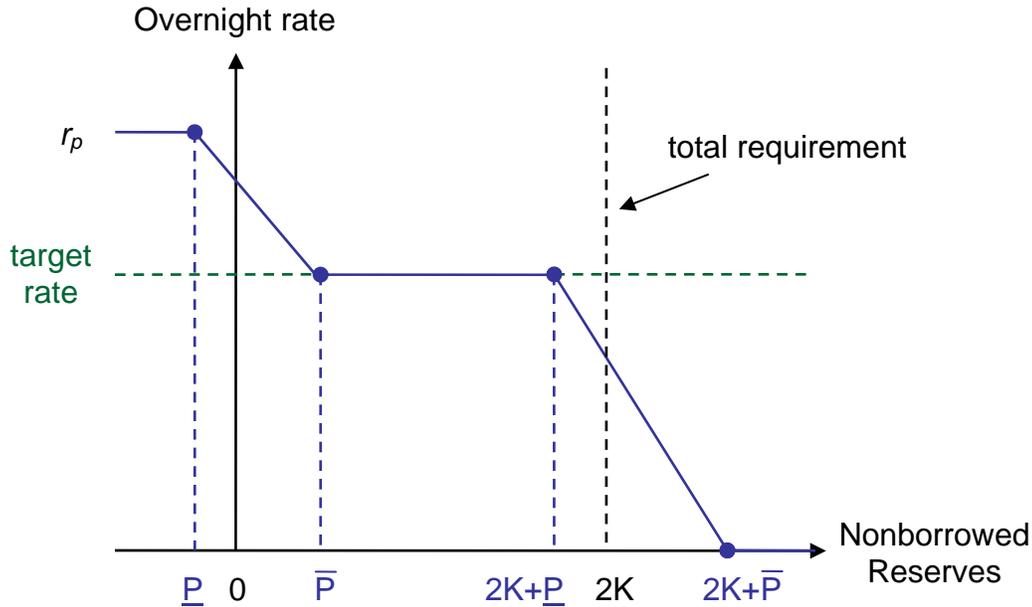
The basic framework can be adapted to study individual days in a reserve maintenance period. For expositional purposes we study a two-day maintenance period, but the insights generated here are helpful for understanding the effects of longer maintenance periods as well. The model for the second day of the period is exactly as above. In this subsection, we derive the demand for reserve balances on the first day of the period.

Let  $K$  denote the average daily requirement, so that the total requirement for the two-day maintenance period is  $2K$ . A bank's demand for reserves on the first day of the maintenance period depends crucially on its belief about what the market interest rate will be on the second day. The key insight is to realize that banks will try to hold reserves to satisfy more of the requirement on the day (within the maintenance period) in which they expect interest rates to be lowest.

Suppose a bank expects the market interest rate on the second day to equal the target rate. Figure 4 depicts the demand for reserves on the first day under this assumption. To understand the shape of this demand curve, suppose the interest rate in the market on the first day is very high, close to the discount rate. Then the bank will want to satisfy as much of its reserve requirement as possible on the second day, when it expects the rate to be substantially lower. However, if the bank's reserve balance after the payment shock is negative, it will be forced to go borrow at the penalty rate  $r_p$  to avoid having an overnight overdraft. As long as the market rate is below the penalty rate, therefore, the bank will choose a reserve position of at least  $\underline{P}$ . If it chose a position smaller than  $\underline{P}$ , it would be

certain to need to borrow funds at the end of the day, which cannot be an optimal choice as long as the market rate is below the penalty rate.<sup>5</sup>

Figure 4: First day of a Two-day maintenance period



For interest rates below  $r_p$ , but still larger than the target rate, the bank will choose to hold some “precautionary” reserves to decrease the probability that it will need to borrow at the penalty rate. This precautionary motive generates the first downward-sloping part of the demand curve in the figure. As long as the day-one interest rate is above the target rate, however, the bank will not hold more than  $\bar{P}$  in reserves on the first day. By holding  $\bar{P}$ , the bank is assured that it will have a positive reserve balance after the late-day payment shock. If the bank were holding more than  $\bar{P}$  on the first day, it could lend those reserves out at the (relatively high) market rate and meet its requirement by borrowing reserves on the second day, when the interest rate is expected to be at (lower) the target, yielding a positive profit. Hence, the first downward-sloping part of the demand curve must end at  $\bar{P}$ .

Now suppose the first-day interest rate is exactly equal to the target rate. In this case, the bank expects the rate to be the same on both days and is, therefore, indifferent between holding reserves on either day for the purpose of meeting requirements. In choosing its first-day reserve position, the bank will consider the following issues. First, it will

<sup>5</sup> Note that Figure 4, like the previous figures, is drawn under the assumption that  $P$  is negative. Choosing a negative reserve position should be interpreted as lending more reserves than the bank begins the day with and thus incurring a daylight overdraft when the funds are sent.

choose to hold at least enough reserves to ensure that it will not need to borrow from the discount window at the end of the first day. In other words, reserve holdings will be at least as large as the largest possible payment  $\bar{P}$ . The bank is willing to hold more reserves than  $\bar{P}$  for the purpose of satisfying some of its requirement. However, it wants to avoid the possibility of over-satisfying the requirement on the first day (that is, becoming “locked-in”), since it must hold a non-negative quantity of reserves on the second day. This implies that the bank will not be willing to hold more than the total requirement ( $2K$ ) plus the smallest possible payment shock ( $\underline{P}$ ) on the first day. (Note that if  $\underline{P}$  is negative, as drawn in the figure, the amount  $2K + \underline{P}$  is less than the total requirement  $2K$ .) The demand curve is flat between these two points (that is,  $\bar{P}$  and  $2K + \underline{P}$ ), indicating that the bank is indifferent between the various levels of reserves in this interval.

Finally, suppose the market interest rate on the first day is smaller than the target rate. Then the bank wants to satisfy most of the requirement the first day, since it expects the market rate to be *higher* on the second day. In this case, the bank will hold at least  $2K + \underline{P}$  reserves on the first day. If it held any less than this amount, it would be certain to have some requirement remaining on the second day, which would not be an optimal choice given that the rate will be higher on the second day. As the interest rate moves farther below the target rate, the bank will hold more reserves for the usual precautionary reasons. In this case, the bank is balancing the possibility of being locked-in after the first day against the possibility of needing to meet some of its requirement on the more-expensive second day. The larger the difference between the rates on the two days is, the larger the quantity the bank will choose to hold on the first day. This tradeoff generates the second downward-sloping part of the demand curve.

The flat portion of the demand curve in Figure 4 can help reduce interest rate volatility on days prior to the settlement day. As long as movements in autonomous factors are small enough that the supply of reserves stays in the flat area of the demand curve, interest rate fluctuations will be minimal. However, it should be noted this demand curve is flat at whatever interest rate is *expected* to obtain on the settlement day. Here we have assumed that rate is equal to the Central Bank’s target. If market participants expect a deviation from the target on the second day, the demand curve on the first day will reflect that deviation (see Bartolini, Bertola and Prati (2002) for an analysis of such effects).<sup>6</sup>

On the settlement day, the flat portion of the demand curve disappears and the curve reverts to that in Figure 1. This feature of the model indicates that the market interest rate is likely to be more volatile on settlement days, which matches observed data. In practice, however, clearing bands and carryover provisions are typically used in an attempt to limit this volatility. These provisions have the effect of creating a small range where the demand curve is flat, or nearly flat, even on a settlement day.

---

<sup>6</sup> Suppose, for example, that banks expect the settlement-day interest rate to be higher than the target. Then the flat portion of the first-day demand curve would lie at this higher rate. In such a case, implementing the target rate on the first day becomes more difficult for the Central Bank, since it would need to set reserve supply on a downward-sloping part of the demand curve instead of on the flat portion. This fact would tend to make the first-day interest rate more volatile.

It should be noted that Figure 4 is drawn under the assumption that the reserve requirement is relatively large. Specifically,  $2K > \bar{P} - \underline{P}$  is assumed to hold, which ensures that  $\bar{P}$  is less than  $2K - \underline{P}$ . In other words, the total reserve requirement for the period is assumed to be larger than the single-day uncertainty about the bank's reserve position. If this inequality were reserved, the flat portion of the demand curve would not exist. In this case, the two downward-sloping parts of the curve would overlap and the analysis becomes more complicated. In general, reserve maintenance periods are most useful as a policy tool when the underlying reserve requirements are sufficiently large relative to end-of-day balance uncertainty.

### 2.3 EFFECT OF FEES FOR DAYLIGHT CREDIT

All of the figures above were drawn under the assumption that the interest rate charged on daylight credit is zero, which is a close approximation of current U.S. policy. When the daylight credit rate is very small relative to the overnight interest rate, a bank's choice of (overnight) reserve position will be independent of its anticipated pattern of payment flows during the day. In this case, reserve demand can be studied without specifying the pattern of payments during the day and the corresponding usage of daylight credit.

If the interest rate on daylight credit is significant, however, a bank has an extra incentive to hold reserves overnight, as these reserves help it avoid incurring daylight overdrafts the following day.<sup>7</sup> In other words, costly daylight credit will tend to increase the precautionary demand for reserves. In such a situation, daylight credit policy and the pattern of payments during the day will affect the demand for reserves and the process of monetary policy implementation.

We can explicitly include daylight credit in the basic framework by adding daytime payments to the model. Suppose, for example, that each bank makes one payment and receives one payment during the "early" part of the day (this is in addition to the late-day payment shock  $P$  discussed above). To keep things as simple as possible, suppose that these two payment flows are of exactly the same size (call this size  $P_D$ ) and that this size is non-stochastic. However, the order in which these payments occur is random; some banks will receive the incoming payment before making the outgoing one, while others will make the outgoing payment before receiving the incoming one. Banks in the latter category will incur a daylight overdraft.

In terms of the model, let  $r_e$  denote the interest rate on daylight credit and  $R$  the level of reserves chosen by the bank in the interbank market. Let  $\delta$  denote the time period between the two payment flows and  $\pi$  the probability that a bank sends the outgoing payment before receiving the incoming one. Then the bank's expected cost of daylight

---

<sup>7</sup> We assume that interbank trades settle at the end of the day, so that a bank's choice of reserve position for the current day affects the size of its daylight overdrafts on the following day. However, the specific assumptions made about the timing of settlement are unimportant. As long as interbank trades are 24-hour loans and the expected pattern of payments is roughly the same on subsequent days, the timing of settlement has no effect on the analysis.

credit is  $\pi r_e \delta (P_D - R)$ . This expression shows the relationship between a bank's overnight reserve balance and its daylight credit charges. In particular, it shows that an additional dollar of reserve holdings will decrease the bank's expected cost of daylight credit by  $\pi r_e \delta$ .

We think of  $P_D$  as being large relative to  $K$  and  $P$ , reflecting the large demand for daylight credit observed in the data. Therefore, the term  $(P_D - R)$  will typically be positive and large in magnitude. If all payments made by one bank are received by another, the parameter  $\pi$  will be one-half. If, on the other hand, we think of some daytime payments as going to institutions that are net users of reserves during the day (CHIPS, etc.), the value of  $\pi$ , and thus the fraction of banks who make a payment before receiving one, would be larger than one-half.

The analysis here takes the size and timing of payments as given. Several papers have studied the interesting question of how banks respond to incentives (and to the actions of other banks) in choosing the timing of their outgoing payments and, hence, their daylight credit usage.<sup>8</sup> We abstract from such concerns here in order to keep the analysis tractable. A more complex model might also have multiple rounds of payments, so that a bank's reserve position would evolve throughout the day, perhaps randomly. While such an approach would be necessary for addressing certain questions, the simple approach presented here is a useful starting point and sufficient for illustrating the important issues.

Figure 5 shows how a bank's demand for reserves changes when daylight credit is costly. It is still true that there will be no demand for nonborrowed reserves whenever the market rate is above the penalty rate  $r_p$ . The interest rate measured on the vertical axis is (as in all of our figures) the rate for a 24-hour loan. If the market rate were above the penalty rate, a bank would prefer to lend out all of its reserves at the (high) market rate and satisfy its requirements by borrowing from the discount window. By arranging these loans to settle at approximately the same time on both days, this plan would have no effect on the bank's daylight credit usage and hence would generate a pure arbitrage profit.

It is also still true that whenever the market rate is below the penalty rate the bank will choose to hold at least  $K + \underline{P}$  reserves, since otherwise it would be certain to need to borrow after the payment shock is realized in order to meet its requirement. As the figure shows, the downward-sloping part of the demand curve is flatter when daylight credit is costly. For any market interest rate below the discount rate, the bank will choose to hold a higher quantity of reserves because these reserves now have the added benefit of reducing daylight credit fees.

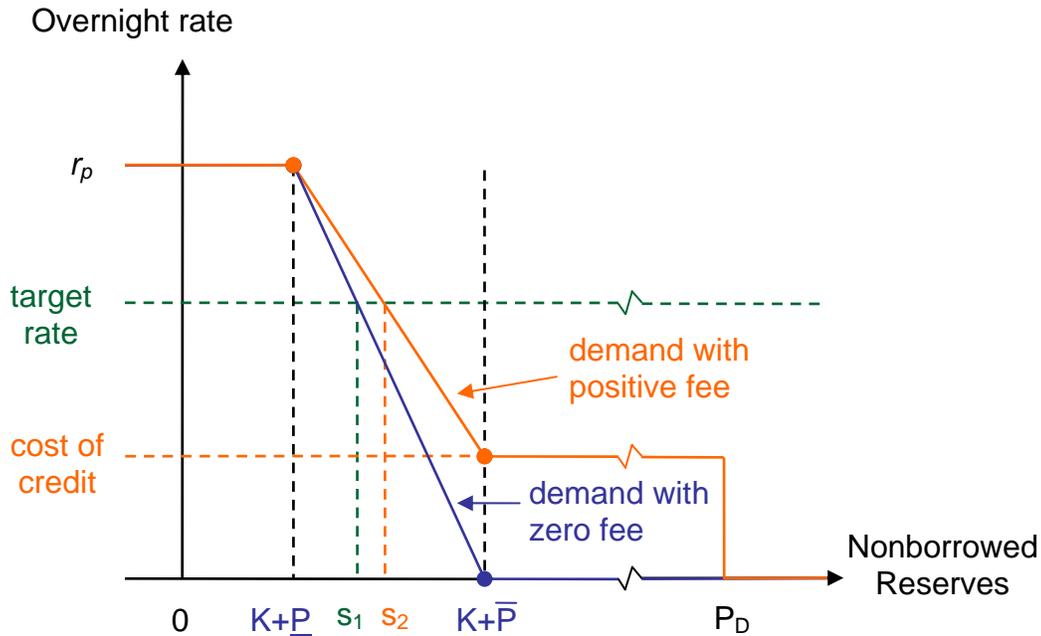
Rather than decreasing all the way to the horizontal axis as in Figure 1, the demand curve now becomes flat at the bank's expected marginal cost of intraday funds,  $\pi r_e \delta$ . As long as  $R$  is smaller than  $P_D$ , the bank would not be willing to lend out funds at an interest rate

---

<sup>8</sup> See, for example, McAndrews and Rajan (2000) and Bech and Garratt (2003).

below  $\pi r_e \delta$  because lending out these funds would increase the expected daylight credit fees the bank would have to pay by more than the interest earned on the loan. For values of  $R$  larger than  $P_D$ , the bank is holding sufficient reserves to cover all of its intraday payments and the demand curve drops to the horizontal axis. The “break” in the horizontal axis indicates that we view  $P_D$  as being much larger than the other quantities in the graph.

Figure 5: Effect of Costly Daylight Credit



As the figure shows, when daylight credit is costly the level of reserves required to implement the given target rate is higher ( $s_2$  rather than  $s_1$  in the diagram). In other words, costly daylight credit tends to increase the equilibrium level of precautionary reserve balances. However, given the small costs currently associated with daylight credit in most countries, this effect will typically be fairly small in practice.

The analysis above assumes a particular form of daylight credit usage; if an overdraft occurs, the size of the overdraft is constant over time. Alternative assumptions about the process of daytime payments would lead to slight changes in the figure. Suppose, for example, that a bank’s daylight overdraft is pyramid-shaped over the course of the day, building slowly to a peak level and then declining. In this case, the marginal cost of intraday funds would be decreasing in  $R$ , rather than being constant as in the case studied above. As a result, the portion of the demand curve beyond  $K + \bar{P}$  would be slightly

downward-sloping instead of flat. However, the qualitative properties of the figure would be largely unchanged.

## 2.4 HETEROGENEITY

The analysis above was based on the assumption that all banks are identical, so that the aggregate demand curve for reserve balances is identical to each individual bank's demand curve. Once we derived this individual demand curve, therefore, we could directly examine how changes in reserve supply affect the market interest rate. We now ask how the analysis changes when there is explicit heterogeneity among banks. In particular, we focus on the potential differences between large and small banks. We ask how such heterogeneity affects the aggregate demand for reserves and the process of monetary policy implementation in the context of our basic analytical framework. We also investigate how banks may be affected differently by the process of monetary policy implementation, and the Central Bank's choice of framework, depending on their size.

Banks can differ in several dimensions in the framework presented here. Perhaps the most natural way of capturing differences in bank size is by allowing for heterogeneity in the magnitude of the reserve requirement. Large banks will tend to have a large deposit base and, hence, be subject to larger requirements. Other ways in which banks may differ are the variance of the late-day payment shock they face and the penalty rate they pay when they need to borrow to make up a deficiency at the end of the day or of a reserve maintenance period. We address each of these potential sources of heterogeneity below.

*Size of Requirements.* Suppose that each bank  $i$  has a different reserve requirement  $K_i$ . Large banks will have higher values of  $K_i$ , and small banks will have lower values. This type of heterogeneity turns out to have no effect on the analysis above. In particular, the individual demand curves can be aggregated into the demand curve that would be generated by a "representative bank" whose requirement is exactly equal to the average of the individual bank requirements. All of the analysis above then applies equally to the representative bank and to each individual bank.

To see why this is true, consider a bank's "precautionary" reserve balance, which we define to be the difference between its chosen balance  $R_i$  and the requirement  $K_i$ . The difference  $(R_i - K_i)$  depends on the properties of the payment shock, but not on the size of the requirement  $K_i$ . In other words, suppose we compare two banks that face the same distribution of the late-day payment shock. Suppose one of the banks has a larger requirement  $K_i$  and will, therefore, hold a higher reserve balance  $R_i$ . However, both banks are facing essentially the same decision problem: choosing a quantity of precautionary reserves to optimally balance the risk of a reserve deficiency against the risk of being stuck holding excess reserves. Both banks will, therefore, choose to hold exactly the same quantity of precautionary reserves  $(R_i - K_i)$ . As a result, this particular type of heterogeneity has no effect on bank behavior and the process of monetary policy implementation in this framework, and all banks are affected equally by the Central Bank's choice of framework.

Adding heterogeneity in reserve requirements does generate an interesting implication for the distribution of excess reserve holdings across banks. Suppose the banking system is composed of a relatively small number of large banks and a much larger number of small banks, as is the case in the U.S. Then, under the assumption that most of these banks face comparable late-day payment uncertainty, the framework suggests that large and small banks should hold comparable quantities of precautionary reserves. After the payment shocks are realized, of course, some banks will end up holding excess reserves and others will end up needing to borrow. But it will necessarily be the case that the vast majority of excess reserves in the banking system will, on any given day, be held by small banks, simply because there are so many more of them. Even if large banks hold the majority of *total* reserve balances because of their larger requirements, the framework predicts that most of the *excess* reserve balances will be held by small banks. This prediction is broadly in line with the data for the U.S.

*Size of the Payment Shock.* Another way in which banks potentially differ from each other is the distribution of the late-day payment shock they face. Banks with larger and more complex operations, for example, might be expected to face a larger amount of uncertainty about their end-of-day reserve position. Of course, such banks tend to have sophisticated reserve management systems in place. As a result, the end-of-day uncertainty they face will tend to be much smaller in relative terms, compared to the size of their operations or their requirements. However, the important variable in the framework studied here is the *absolute* size of the uncertainty, not the relative size. Whether large banks or small banks face more absolute uncertainty is not clear. We now investigate the effects of introducing heterogeneity in the absolute size of uncertainty faced by banks into the model.

This type of heterogeneity can be introduced by allowing banks to have different bounds for the payment shock.<sup>9</sup> Suppose that the payment shock for bank  $i$  has support  $[\underline{P}_i, \overline{P}_i]$ , with the spread between these bounds being wider for larger banks. It can be shown that this type of heterogeneity also has no effect on the analysis presented above. In particular, when the distribution of the payments shock is uniform, the aggregate demand for reserves is exactly as presented in Figure 1 with the bounds  $\underline{P}$  and  $\overline{P}$  set to the average values of  $\underline{P}_i$  and  $\overline{P}_i$ , respectively. The slope of the demand curve is then determined by the “average” level of uncertainty that banks face. The process of monetary policy implementation is, therefore, completely unaffected by this type of heterogeneity.

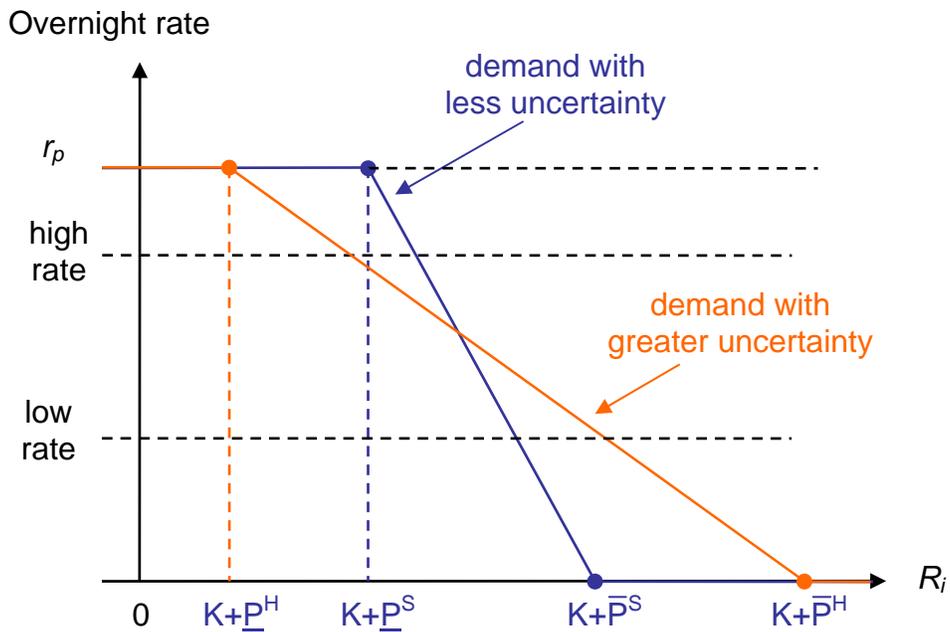
Having this type of heterogeneity does allow the model to address the question of how precautionary reserve balances are distributed across banks and how this distribution responds to changes in the market interest rate. Consider two banks, one that faces more

---

<sup>9</sup> When the distribution of the payment shock is uniform, changing the bounds of the support is equivalent to changing the variance of the distribution. With more general distributions, this need not be the case. One could imagine, for instance, that larger banks face a larger support of the shock but, due to their investment in reserve management capabilities, face a smaller variance. For most of the analysis, the variance of the distribution plays a more significant role than the support.

late-day payment uncertainty and one that faces less. The demand curves for each of these banks are plotted in Figure 6. The curves are scaled as if the two banks had the same level of requirements, but that is not important. The focus in each case is on the quantity of precautionary reserves held (that is, the difference between the reserve balance and the requirement). The figure shows that when the interest rate is high, the bank facing less uncertainty will hold more precautionary reserves, while the bank facing more uncertainty will hold more when the interest rate is low. Another way of stating this result is to say that the bank facing more uncertainty will adjust its reserve holdings more aggressively in response to changes in the market interest rate. A period of interest rate volatility would, therefore, be accompanied by large swings in the reserve position of such banks, and much smaller swings in the reserve position of banks that face less uncertainty. If we interpret smaller banks as facing less (absolute) uncertainty, this result would imply that the reserve demand of smaller banks is less sensitive to changes in the interest rate. Notice that this result obtains even though there are no costs of reserve management in the model.

Figure 6: The Size of the Payment Shock



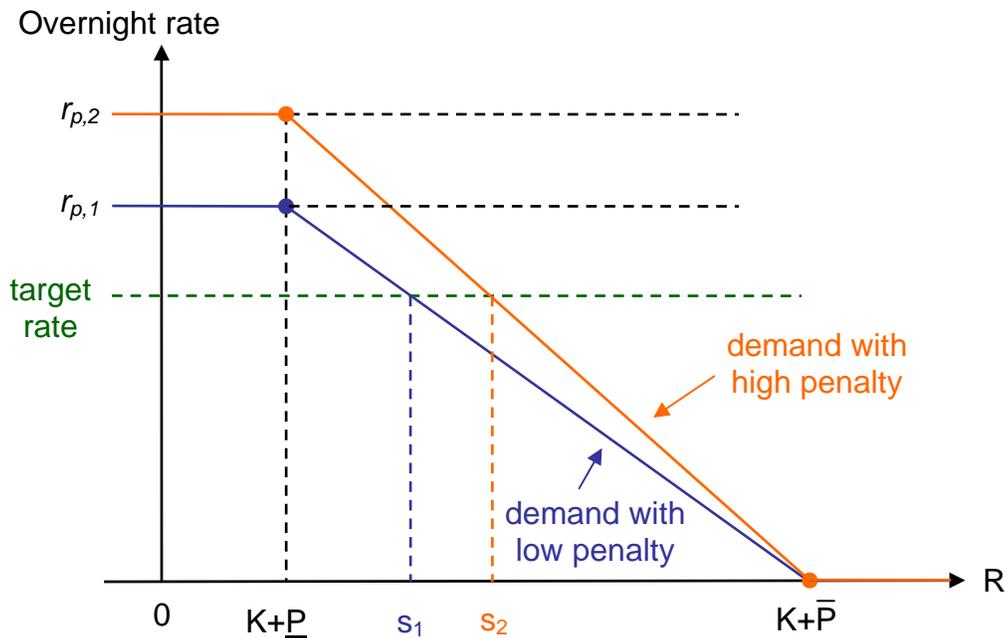
*Stigma and the Penalty Rate.* A third way in which banks might differ from each other is in the penalty rate they face if they need to borrow to avoid a reserve deficiency. To be eligible to borrow at the discount window, for example, a bank must establish an agreement with its reserve bank and post collateral. This fixed cost may lead some smaller banks to forgo accessing the discount window and instead pay the (higher) reserve deficiency fee when necessary. Smaller banks may also have difficulty borrowing in the Fed Funds market late in the day, leading them to pay higher rates or to face the deficiency fee when hit with a large payment outflow (see Ashcraft, McAndrews

and Skeie, 2007). In addition, there may be other, internal reasons why different banks would assign different non-pecuniary costs to borrowing from the discount window.

We can add this type of heterogeneity to the model by assuming that bank  $i$  faces an institution-specific penalty rate  $r_{p,i}$  when its reserve balance falls below its requirement. In this case, aggregating the individual bank demand curves into an aggregate demand for reserves is slightly more complex than before, but the outcome is qualitatively similar. The aggregate demand curve will still resemble that in Figure 1, and the slope of this curve will be based on a representative penalty rate  $r_p$ .<sup>10</sup> The process of monetary policy implementation is, therefore, largely unaffected by this type of heterogeneity.

Figure 7 shows how the behavior of individual banks will differ. Smaller banks, which we think of as facing a higher penalty, will hold more precautionary reserves for any given interest rate in order to decrease the probability that they will need to borrow. In the figure, the smaller bank will hold a quantity  $s_2$  while the larger bank holds only  $s_1$ , even though both face exactly the same uncertainty about their end-of-day balance. As a result, the distribution of excess reserves will tend to be skewed ever more heavily toward smaller banks. Notice also that the demand curve of smaller banks will have a steeper slope, meaning that their reserve demand is less sensitive to changes in the interest rate.

Figure 7: Heterogeneity in Deficiency Penalties



<sup>10</sup> This representative rate is not a simple average of the individual rates  $r_{p,i}$  because, in this case, the aggregation process is nonlinear.

### 3. ALTERNATIVE SYSTEMS OF MONETARY POLICY IMPLEMENTATION

This section shows how the central ideas of the policy proposals discussed in the body of the document can be illustrated in the context of the framework developed in this appendix. Rather than discussing the specific proposals, we focus on those features that lie within the scope of the basic framework. Some important distinctions between the various proposals, such as that between reserve requirements and contractual balances, lie completely outside this scope. Nevertheless, the framework can be used to illustrate the ways in which each proposal would likely affect the demand for reserves, as well as the procedure by which the target interest rate would be implemented under each proposal.

In the subsections that follow, we show how the interest rate paid on reserves, both required and excess, affects the shape of the aggregate demand curve for reserves. We also describe how the different proposals aim to limit interest rate volatility. While the figures below are each different from the basic model presented in Figure 1, they all share the feature that the demand for reserves is generally downward sloping. As we discussed in Sections 1 and 2, this demand curve can be quite steep, and this steepness makes implementing a target interest rate difficult because the Central Bank cannot precisely control the supply of reserve balances. Each of the policy proposals contains elements that aim to decrease the slope of the demand curve and thereby stabilize the market interest rate near the target. Some of the proposals rely on reserve maintenance period, as presented in Section 2.2 above, but others do not.

#### 3.1 PAYING INTEREST ON REQUIRED RESERVE BALANCES

One possible policy would be to keep the basic structure of monetary policy implementation unchanged, but to pay interest on the reserve balances held by banks that are used to meet requirements. An important issue that must be decided in this case is how to determine which part of a bank's reserve holdings is used to meet requirements (and hence will earn interest) and which part is considered to be excess. This matters because the FSRRA authorizes the payment of interest on reserve balances held at the Fed, but not on vault cash. One approach is to count vault cash toward requirements first. Under this rule, a bank will only earn interest on the difference between its reserve requirement and its vault cash holdings. Banks that are currently "unbound," in the sense of meeting their entire requirement with vault cash, would earn no interest under this policy.<sup>11</sup>

If we view the level of each bank's requirement as being fixed, this policy change has no effect whatsoever on the analysis presented in the previous two sections. The policy would increase banks' revenue, but it would have no impact on the decision problem a

---

<sup>11</sup> An alternative approach would be to allow each bank to choose which of its reserves to count toward the requirement. Under this rule, a bank would elect to apply its reserve balances (rather than vault cash) toward the requirement first in order to maximize the interest it receives. This approach could have the effect of removing the opportunity cost of holding additional reserve balances for some institutions, which would lead to substantial changes in the analysis.

bank faces. Each bank would continue to choose its precautionary reserve balance ( $R_i - K_i$ ) based on the factors described above. The demand curves in the various figures would be completely unaffected.

In reality, of course, the levels of reserve requirements are not fixed and can be influenced by banks' behavior. For example, banks might engage in less reserve avoidance activity under this policy, resulting in a higher level of required reserves; this effect may or may not be small. In addition, this policy would introduce an incentive for banks to economize on vault cash holdings. The effects of this incentive might be worth studying. However, as described in Section 2.4, changes in the level of requirements do not affect the decision problem a bank faces when choosing its level of precautionary reserves. As a result, the process of monetary policy implementation would proceed almost exactly as described above.

### 3.2 A CONVENTIONAL SYMMETRIC CORRIDOR

The key features of a corridor system are standing Central Bank facilities that lend to and accept deposits from commercial banks at fixed interest rates. Figure 8 depicts the demand curve for nonborrowed reserves under such a system. As in Figure 1, this curve represents the aggregate demand for reserve balances on the last day of a reserve maintenance period. The curve looks very similar to that in Figure 1. In particular, there is no demand for nonborrowed reserve balances if the market interest rate is higher than the rate at the lending facility.<sup>12</sup> For lower values of the market rate, banks will, on average, choose to hold the required level of reserves plus some precautionary balances. This precautionary demand is decreasing in the interest rate for exactly the same reasons as before. The big change from Figure 1 is that the demand curve now becomes flat at the deposit rate, rather than at the horizontal axis. If the market rate were below the deposit rate, each bank's demand for reserves would effectively be infinite, as they would try to borrow at the market rate and hold the reserves at the deposit facility overnight, making a pure profit.

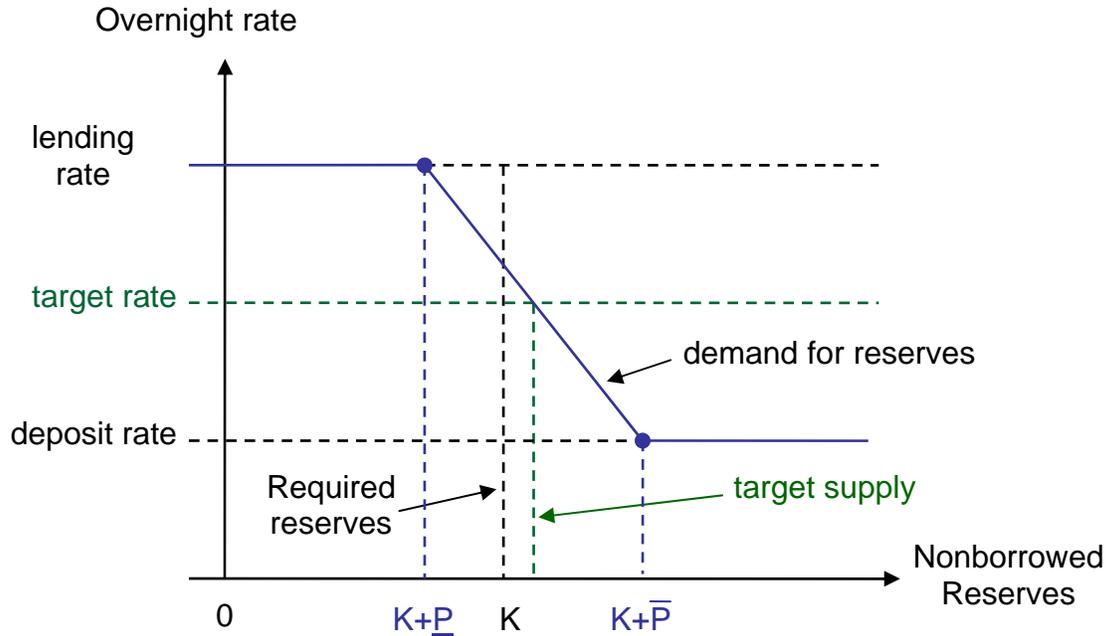
Notice that the width of the downward-sloping part of the demand curve is exactly the same as in Figure 1; it is again determined by the support of the late-day payment shock. This fact has an immediate implication for the slope of the demand curve: the presence of a deposit facility will make the demand curve less steep than before (assuming the lending facility rate is unchanged). The deposit facility makes holding excess reserves less costly and, therefore, leads each bank to demand more precautionary reserves. The lower the market interest rate is, the more important this effect becomes (precisely because the bank is choosing to hold more reserves). As a result, the demand curve becomes flatter and a bank's reserve position will be more responsive to changes in the market interest rate.

---

<sup>12</sup> This statement assumes that the lending facility is designed in such a way that banks attach no stigma or other non-pecuniary costs to borrowing from it. It is not clear to what extent converting the discount window in the U.S. to a lending facility in a corridor system would change the stigma levels banks perceive.

However, the slope of the demand curve remains steep enough that a Central Bank will most likely want to use some additional tool(s) to limit interest rate volatility. In a corridor system with required reserves, reserve maintenance periods can serve this purpose. As analyzed in Section 2.2, reserve maintenance periods tend to create a flat portion of the demand curve near the target rate on non-settlement days. On a settlement day, however, the demand curve would revert to that in Figure 8, unless some other tools (such as carryover provisions or clearing bands) are employed.

Figure 8: A Conventional Corridor



At the aggregate level, the equilibrium interest rate is determined exactly as before, by the height of the demand curve at the level of reserve balances supplied by the Central Bank. Notice that the introduction of a deposit facility creates a *floor* below which market interest rate will not fall. In Figure 1, if reserve supply turned out to be unexpectedly large, the market interest rate could fall all the way to zero.<sup>13</sup> In Figure 8, this cannot happen; even with a large deviation of reserve supply from the target, the market interest rate can only fall down to the deposit rate.

In practice, corridor systems are often *symmetric*, in that the lending and deposit rates are set an equal distance from the target rate (say,  $x$  basis points above and below the target rate, respectively). In such a case, changing the Central Bank's target rate effectively amounts to changing the levels of both the lending and deposit rates, which shifts the

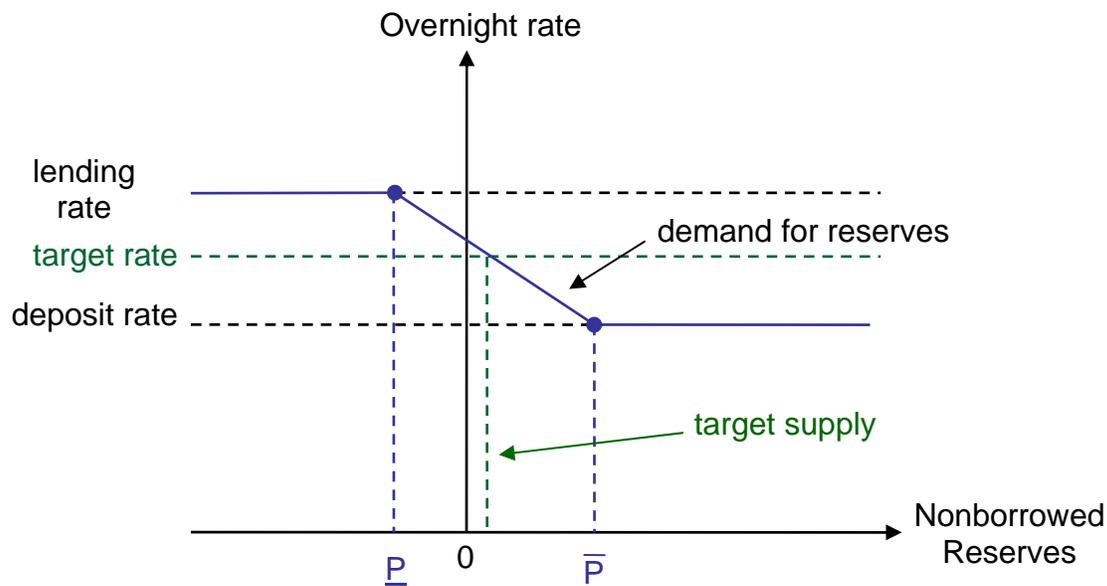
<sup>13</sup> While the effective Federal Funds rate rarely deviates from the target by a very large amount, in recent months there have been many occasions on which the market rate has approached 1 percent or lower during certain portions of the trading day. The presence of a deposit facility would likely put a floor on the interest rate for *all* trades, not just for the effective rate.

demand curve along with them. The supply of reserves may not need to change much in order to maintain the new target rate. In fact, in the simple model studied here, the target level of reserve supply would not change at all when the policy rate changes.

### 3.3 A NARROW CORRIDOR WITH NO RESERVE REQUIREMENT

Rather than introducing reserve maintenance periods, another way to limit interest rate volatility in a corridor system is to move both the lending and deposit rates closer to the target (see Figure 9). As discussed above, narrowing the corridor has two effects on the level of interest rate volatility. First, the lending and deposit rates create a ceiling and a floor, respectively, for the market interest rate. Moving these two rates closer together, therefore, will limit the potential for large deviations from the interest rate target. If, for example, the lending and deposit rates are both set 25 basis points away from the target, deviations from the target rate of more than 25 basis points should occur rarely, if at all.

Figure 9: A Narrow Corridor



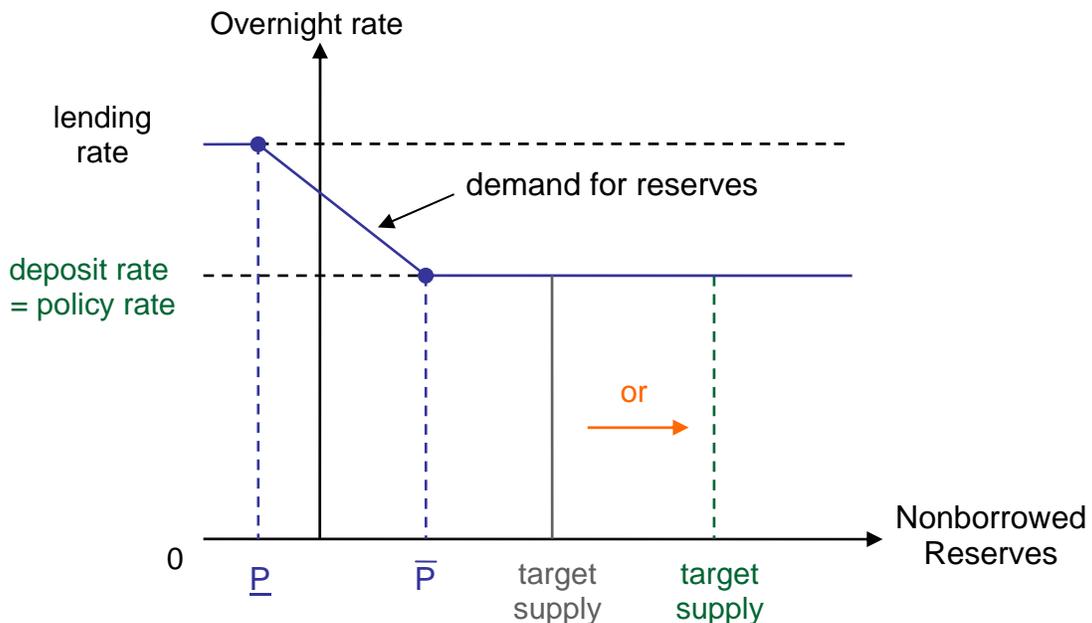
In addition, narrowing the corridor makes the demand curve flatter. The width of the downward-sloping part of the demand curve is always determined by the support of the payment shock; it is equal to  $\bar{P} - P$ . When the corridor is narrower, the demand curve falls by a smaller amount in this range and, hence, has a lower slope. Small deviations of reserve supply from the target level would thus lead to smaller variations in the interest rate compared to a wider corridor system as depicted in Figure 8. However, such deviations might still be large when compared to a non-settlement day in a system with reserve maintenance periods. If so, the narrow corridor approach could lead to more small deviations, but fewer large deviations from the target.

Countries that operate a narrow corridor system with no reserve maintenance period typically do not have any required reserves. For this reason, Figure 9 is drawn under the assumption that requirements are set to zero. One consequence of this approach is that it becomes more likely that unanticipated movements in autonomous factors will leave the total level of reserves in the system negative. When this happens, some banks must borrow from the discount window, even if interbank markets are operating well and all banks are behaving prudently. Having a narrow corridor implies that the penalty paid by banks in this situation is not unduly large.

### 3.4 THE “FLOOR” SYSTEM: A CORRIDOR WITH HIGH RESERVE BALANCES

Starting from the corridor system in Figure 9, notice that there is another way in which a Central Bank could limit the amount of interest-rate volatility that is caused by unexpected deviations in reserve supply. Suppose the Central Bank were to (i) set the deposit rate *equal* to the target interest rate, instead of below it, and (ii) choose reserve supply so that it intersects the flat part of the demand curve generated by the deposit rate, as illustrated in Figure 10, rather than intersecting the downward-sloping part as in the previous figures. Then small, or even large, deviations in reserve supply would have almost no effect on the market interest rate because the demand curve is perfectly flat in this region. This approach is called a “floor system” because it relies on the Central Bank supplying enough reserves to drive the market interest rate down to the “floor” created by the deposit facility.

Figure 10: A Floor System



The fact that these supply and demand curves cross at the target rate does not imply that trades in the interbank market would occur at exactly this rate, of course. A bank would require a small premium, reflecting transactions costs and credit risk, in order to be willing to lend out funds rather than simply holding them as (interest-bearing) reserves. As a result, the measured interest rate in the interbank market would generally be slightly above the deposit-facility rate. The deposit rate in this system should be referred to as the *policy rate* rather than the *target rate* in order to make this distinction clear.

Compared to all of the systems discussed above, including those with reserve maintenance periods, the floor system should be the most effective at maintaining the market interest rate near the target. Recall from Section 2.2 that the systems with reserve maintenance periods rely on an expectational effect: the flat portion of the demand curve on a non-settlement day lies at whatever interest rate market participants expect to prevail on later days. Under a floor system, in contrast, the flat part of the demand curve always lies at the policy rate chosen by the Central Bank. In addition, the flat portion is very wide and is the same every day, rather than changing each day in a reserve maintenance period.

Another unique feature of the floor system is that the quantity of reserves can, to a large degree, be chosen independently of the interest rate target. This fact is shown clearly in Figure 10, where the same policy rate is consistent with both a moderate level of reserve supply and a much larger one. This feature allows the Central Bank to use the supply of reserves to achieve other objectives, such as changing the amount of broad liquidity in the economy or affecting the level of daylight overdrafts in the payments system.

### 3.5 DAILY REQUIREMENTS WITH WIDE CLEARING BANDS

Clearing bands provide another way in which a Central Bank can generate a flat region in the demand curve for reserves without introducing a reserve maintenance period. Suppose the Central Bank establishes reserve requirements and operates a symmetric corridor, as in Figure 8. There is no reserve maintenance period, so the reserve requirements apply to each day.<sup>14</sup> Assume that interest is paid on required reserve balances at the target rate. The distinguishing feature of a clearing-band system is that a bank need not meet this requirement exactly. Instead, it faces no penalty as long as its final balance falls somewhere within a “clearing band” around the target.

For concreteness, suppose a bank has a reserve requirement  $K$ , but with a clearing band given by the interval  $[K_L, K_H]$ . These bounds might, for example, be set  $x\%$  below and above  $K$ , respectively. In general, the level of the requirements  $K$  is not important in this type of system; only the upper and lower bounds of the clearing band matter. The bank is required to hold at least  $K_L$  reserves each day; if it falls short of this amount it must borrow the difference at the penalty rate established by the lending facility. The bank is paid the target interest rate on all of these balances as well as any balances it holds up to

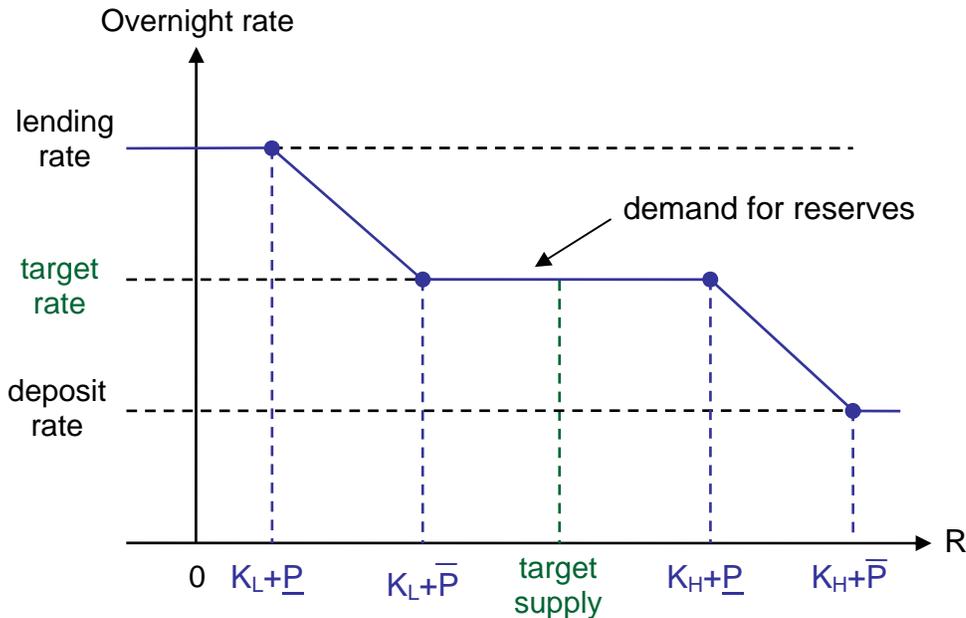
---

<sup>14</sup> These requirements could either be based on the deposits banks hold, as in the current U.S. system, or could be contractual in nature.

$K_H$ . Any reserve holdings above  $K_H$  are considered to be “excess” and earn the lower rate established at the deposit facility.

Figure 11 presents the demand for reserves under this system. We assume the clearing band is wide enough so that  $K_L + \bar{P}$  is less than  $K_H + \underline{P}$ . In other words, we assume the width of the clearing band is greater than the amount of uncertainty the bank faces about its late-day payments. While narrower bands could also potentially be effective tools for monetary policy implementation, the benefits of the clearing-band approach are most clear when the band is sufficiently wide. As shown in the figure, the demand curve for reserves is flat, at the target interest rate, in between  $K_L + \bar{P}$  and  $K_H + \underline{P}$ . To see why this happens, suppose the market rate were equal to the target and ask what quantity of reserves a bank would choose to hold. The bank would want to (1) hold enough reserves to ensure that a late-day payment shock will not force it to borrow from the discount window and (2) hold few enough reserves to ensure that a late-day payment receipt will not push its reserve balance above the upper bound of the band. The bank will be indifferent between any level of reserves that ensures these two things, which generates the flat portion on the demand curve in the figure.<sup>15</sup>

Figure 11: Wide Clearing Bands



<sup>15</sup> See Whitesell (2006b) for a similar system based on contractual balances, but where the lower bound  $K_L$  is set to zero.

## REFERENCES

Ashcraft, Adam, James McAndrews and David Skeie (2007) "Precautionary reserves and the interbank market," mimeo., Federal Reserve Bank of New York.

Bartolini, Leonardo, Giuseppe Bertola and Alessandro Prati (2002) "Day-to-day monetary policy and the volatility of the federal funds interest rate," *Journal of Money, Credit and Banking* 34, 137–159.

Bech, Morten L. and Rod Garratt (2003) "The intraday liquidity management game" *Journal of Economic Theory* 109, 198–219.

Carpenter, Seth and Selva Demiralp (2006) "The liquidity effect in the federal funds market: evidence from daily open market operations," *Journal of Money, Credit, and Banking* 38, 901-920.

Clouse, James A. and James P. Dow Jr. (1999) "Fixed costs and the behavior of the federal funds rate," *Journal of Banking & Finance* 23, 1015-1029.

Clouse, James A. and James P. Dow Jr. (2002) "A computational model of banks' optimal reserve management policy," *Journal of Economic Dynamics & Control* 26, 1787-1814.

Ennis, Huberto M. and John A. Weinberg (2007) "Interest on reserves and daylight credit," Federal Reserve Bank of Richmond *Economic Quarterly* 93 (Spring) 111-142.

Furfine, Craig H. (2000) "Interbank payments and the daily federal funds rate," *Journal of Monetary Economics* 46, 535-553.

McAndrews, James and Samira Rajan (2000) "The timing and funding of Fedwire funds transfers" Federal Reserve Bank of New York Richmond *Economic Policy Review* 6 (July) 17-32.

Poole, William (1968) "Commercial bank reserve management in a stochastic model: implications for monetary policy," *Journal of Finance* 23, 769-791.

Whitesell, William (2006a) "Interest rate corridors and reserves," *Journal of Monetary Economics* 53, 1177-1195.

Whitesell, William (2006b) "Monetary policy implementation without averaging or rate corridors," Finance and Economics Discussion Series Paper 2006-22, Federal Reserve Board.