Projections from EDO: Understanding the Current Outlook and the Great Recession in a DSGE Model

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1 The Outlook for 2011 to 2013

The EDO model projects economic growth a touch above trend and low inflation, with monetary policy gradually lifting the federal funds rate over the next several years. In particular, the current sizable shortfall of production relative to long-run sustainable levels abates as the aversion to risk-taking apparent in the elevated level of risk premia (and, implicitly, restrictions on credit availability) seen by EDO falls back to historically typical levels. Inflation remains low as wage pressures are weak relative to labor productivity, reflecting the declines in household wealth over the past several years, low level of hours worked anticipated over the next few years, and the rapid increases in productivity seen in 2009.

Conditional on the projected decline in risk premia, EDO projects that real GDP will advance at a pace modestly above trend going forward—about 3 percent, on average, over 2011-2013, as shown in figure 1. This improvement brings real activity closer to EDO’s estimate of its long-run trend (as shown in figure 2). Moreover, the above-trend pace of growth is accompanied by inflation just above 1 percent per year.

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substantially below the target of 2 percent, reflecting the labor market slack apparent in the output gap. Given these developments, the federal funds rate is projected to remain near zero until late in 2011 and only rises gradually thereafter.\footnote{The EDO model has been shown to forecast as well as, or better than, alternatives in a number of papers (e.g., Edge, Kiley, and Laforte (2010) and Wieland and Wolters (2010); however, forecasting is very challenging, and models generally perform similar to, but not better than, simple time series alternatives, or consensus forecasts.}

Figure 1: Recent History and Forecasts
The decomposition of the projections for these variables shown in figure 1 highlights the important role that the adverse shocks to financial conditions in 2008 and early 2009 play in shaping the recession in that period and the projected recovery. Specifically, the figures decompose the movements in real GDP, the federal funds rate, and core inflation into the contributions from financial (risk premium) shocks, monetary policy shocks, productivity movements, and other disturbances (largely markup, or Phillips-curve, shocks); the first two are traditional “demand” shocks, and the latter two are traditional “supply” shocks. As shown in the federal funds rate chart, the need to accommodate the adverse impact of the tightening in financial conditions (the red bars) is the most largest factor holding the federal funds rate at a low level through the projection; indeed, monetary policy “shocks” were largely positive – that
is, contractionary – in 2010, reflecting the binding zero-lower bound constraint. The recovery in real GDP projected for 2011-13 is essentially entirely the result of the projected step-up in demand that should accompany lower risk premia, again illustrated by the contribution of the red bars in the GDP chart.

Given that the outlook is largely driven by an unwinding of the factors that caused the Great Recession, the next section walks through the EDO model’s accounting for developments during that period.

2 The Great Recession and the Prospects for Recovery

Real GDP in the second quarter of 2009 was more than 4 percent below its level a year earlier, the sharpest four-quarter decline since the Great Depression. Over the same four quarters, payroll employment fell more than 7 percent and hours worked in the nonfarm business sector fell 8 percent.

What happened, and what is the outlook for 2011-2013?

The basic elements of the story are now well known (although the reasons why these developments occurred are not fully understood). Specifically, a financial crisis created sizable frictions all along the chain that connects savers and borrowers, and these financial frictions led to a collapse in spending on intermediated purchases, primarily investment-type expenditures; indeed, as emphasized elsewhere (e.g., Hall (2010)), these financial frictions became apparent in a widening of the spread between the costs of financial funds facing households and businesses and the rates on low-risk assets (such as Treasury securities or federal funds).

For example, the left column of figure 3 reports three measures of financial conditions facing households and firms: the spread between the average rate on personal loans from banks and the federal funds rate since 1985; the spread between the earnings yield on the S&P500 and the federal funds rate; and the spread between the interest rate on a one-year adjustable-rate mortgage (ARM) and the yield on a one-year U.S. Treasury note. Both the personal loan and earnings/price-based spreads have cyclical patterns and jumped during 2008 as the financial crisis worsened. The ARM spread increased sharply, as falling rates on low risk government obligations were not accompanied by substantially lower mortgage rates for households, prob-
ably reflecting both higher risk associated with loans backing home purchases and impaired financial positions at mortgage lenders. Looking at earlier periods, other notable developments are also apparent – for example, with the earnings/price spread reaching lows in the late 1990s as equity prices soared, an easing in financial conditions not apparent in a measure like the federal funds rate.

Figure 3: Financial Frictions

The right-hand column of figure 3 shows the movements in the indicators of financial frictions just discussed along with several of the risk-premium measures from EDO. these risk premia summarize EDO’s estimates of the role of financial frictions and are derived from the model’s structure and the data on activity, inflation, and
risk-free rates that inform EDO’s projections; as a result, the correspondence between observable measures of frictions and EDO’s estimates provide a cross-check on the model’s interpretation of the Great Recession. As shown in the top panel, the swings in the spread between the rate on personal loans and the federal funds rate correlate well with EDO’s aggregate risk premium. The middle panel reveals that the combined aggregate risk premium and premium on business investment corresponds well with the earnings-equity price/funds rate spread; and the model’s estimate of the risk premium attached to residential investment moves with the ARM spread.

The role played by a widening in risk premiums during the Great Recession was previously highlighted in figure 1. Specifically, the red bars show that the downturn was driven by the rise in risk premiums; particularly significant was the aggregate risk premium, shown in the upper right panel of figure 3. As shown in figure 4, an increase in the aggregate risk premium (that is, a shift in demand away from real assets and towards the nominal risk-free asset) causes the price level to fall, but, due to short-run nominal rigidities, insufficiently to completely offset the shift. The resulting fall in the demand for physical stocks lowers wages, hours and output throughout the economy, with a particularly sharp fall in the capital-producing sector. With the lower path for income, non-durable consumption growth falls below trend for several quarters.

Turning to price developments, much of the high-frequency movements in inflation since 2008 have reflected short-term noise, according to EDO, rather than the effects of economic slack. In particular, the model would have expected only modest downward pressure in the very short run on marginal costs following a transient risk premium shock. As a result, other factors, most notably “markup” shocks, account for a large portion of the weak readings on inflation seen in late 2008 (movements which were subsequently reversed, consistent with the “noise” interpretation of such shocks). Nonetheless, the low realizations of marginal cost associated with the strong performance of productivity relative to wages, on net, since mid-2009, combined with the sizable decline in wealth attributed to elevated risk premia for an extended period, exert sizable influences on the outlook for prices, as can be seen in the blue and red bars.

\[2\] These responses are to a one standard deviation shock, and the responses are expressed at a quarterly rate.
Our discussion of the exogenous disturbances that led to the recession may leave some questioning the economic mechanisms through which risk premia affect demand in EDO. To recap, EDO identifies the lead up to and the early stages of the recession as associated with increasingly tight terms on financing residential investment, i.e., as driven by an increase in the risk premium on residential investment. However, as the economic weakness broadened to include overall consumer spending and business investment, the primary driver of the weakness centered on an increase in the economywide risk premium.

Within the models interpretation of events, these shifts in fundamentals brought about the weakness in economic activity. The increase in the risk premium associated with residential investment directly depressed residential spending and real
estate prices by raising the cost of capital for such spending, but the overall macroeco-
nomic impact would have been fairly limited, according to EDO, if economywide 
risk premiums had not risen as well. In this regard, future work may wish to inves-
tigate the mechanisms that could link the weakness in housing to the more general 
macroeconomic fallout that followed the decline in house prices, perhaps through a 
more sophisticated modeling of financial intermediation.

That said, the sharp increase in the economywide risk premium estimated for 
the second half of 2008 through the middle of 2009 depressed consumer spending, 
residential investment, and business investment through a range of channels. First, 
this increase directly raised the cost of capital for residential and business investment 
and consumer durable outlays. In addition, higher risk premiums lowered household 
wealth (including equity claims on firms and the value of residential real estate), de-
pressing consumption of nondurables and services, and also led households to postpone 
consumption. These declines in spending were further exacerbated by the weakening 
in labor income.

All else equal, EDO would have expected a fairly strong recovery to have com-
menced after the first half of 2009, as risk premiums were projected to fall and mon-
etary policy would have been expected to provide continuous support to the recovery 
in normal times. However, three conditions contributed to a more moderate recovery. 
First, the zero lower bound limited the degree to which monetary policy could sup-
port the recovery. Second, risk premiums are estimated to have fallen more slowly 
than expected, restraining the recovery in demand. Finally, the persistently slow re-
covery led to a modest downward adjustment in the models estimate of the economys 
productive potential.

3 An Overview of Key Model Features

Figure 5 provides a graphical overview of the model. While similar to most related 
models, EDO has a more detailed description of production and expenditure than 
most other models.\(^3\)

Specifically, the model possesses two final good sectors in order to capture key 
long-run growth facts and to differentiate between the cyclical properties of different 

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\(^3\)Chung, Kiley, and Laforte (2011) provide much more detail regarding the model specification, 
estimated parameters, and model properties.
categories of durable expenditure (e.g., housing, consumer durables, and nonresidential investment). For example, technological progress has been faster in the production of business capital and consumer durables (such as computers and electronics).

The disaggregation of production (aggregate supply) leads naturally to some disaggregation of expenditures (aggregate demand). We move beyond the typical model with just two categories of (private domestic) demand (consumption and investment) and distinguish between four categories of private demand: consumer non-durable goods and non-housing services, consumer durable goods, residential investment, and non-residential investment. The boxes surrounding the producers in the figure illustrate how we structure the sources of each demand category. Consumer non-durable goods and services are sold directly to households; consumer durable goods, residential capital goods, and non-residential capital goods are intermediated through capital-goods intermediaries (owned by the households), who then rent these capital stocks to households. Consumer non-durable goods and services and residential capital goods are purchased (by households and residential capital goods owners, respectively) from the first of economy’s two final goods producing sectors, while consumer durable goods and non-residential capital goods are purchased (by consumer durable and residential capital goods owners, respectively) from the second sector.
In addition to consuming the non-durable goods and services that they purchase, households supply labor to the intermediate goods-producing firms in both sectors of the economy.

This remainder of this section provides an overview of the key properties of the model. In particular, the model has five key features:

- Production of goods and services occurs in two sectors, with differential rates of technological progress across sectors.

- A disaggregated specification of household preferences and firm production processes that leads to separate modeling of nondurables and services consumption, durables consumption, residential investment, and business investment.

- Risk premia associated with different investment decisions play a central role in the model. These include A) an aggregate risk-premium, or natural rate of interest, shock driving a wedge between the short-term policy rate and the interest rate facing private decisionmakers (as in Smets and Wouters (2007)) and B) fluctuations in the discount factor/risk premia facing the intermediaries financing household (residential and consumer durable) and business investment.

- A new-Keynesian structure for price and wage dynamics.

- A monetary policy that reacts to inflation and a measure of resource utilization.

3.1 Two-sector production structure

It is well known (e.g., Edge, Kiley, and Laforte (2010)) that real outlays for business investment and consumer durables have substantially outpaced those on other goods and services, while the prices of these goods (relative to others) has fallen. For example, real outlays on consumer durables have far outpaced those on other consumption, while prices for consumer durables have been flat and those for other consumption have risen substantially; as a result, the ratio of nominal outlays in the two categories has been much more stable, although consumer durable outlays plummeted in the Great Recession. Many models fail to account for this fact.

EDO accounts for this development by assuming that business investment and consumer durables are produced in one sector and other goods and services in another sector. Specifically, production by firm \( j \) in each sector \( s \) (where \( s \) equals \( kb \) for the
sector producing business investment and consumer durables sector and \( cbi \) for the sector producing other goods and services) is governed by a Cobb-Douglas production function with sector-specific technologies:

\[
X_t(j) = (Z_t^m Z_t^s L_t^s(j))^{1-\alpha} (K_t^{u,nr,s}(j))^{\alpha}, \text{ for } s = cbi, kb.
\]  

(1)

In 1, \( Z^m \) represents (labor-augmenting) aggregate technology, while \( Z^s \) represents (labor-augmenting) sector-specific technology; we assume that sector-specific technological change affects the business investment and consumer durables sector only; \( L^s \) is labor input and \( K^{u,nr,s} \) is capital input (that is, utilized non-residential business capital (and hence the \( nr \) and \( u \) terms in the superscript). Growth in this sector-specific technology accounts for the long-run trends, while high-frequency fluctuations allow the possibility that investment-specific technological change is an important source of business cycle fluctuations.

3.2 The structure of demand

EDO differentiates between several categories of expenditure. Specifically, business investment spending determines non-residential capital used in production, and households value consumer nondurables goods and services, consumer durable goods, and residential capital (e.g., housing). Differentiation across these categories is important, as fluctuations in these categories of expenditure can differ notably, with the cycles in housing and business investment, for example, occurring at different points over the last three decades.

Valuations of these goods and services, in terms of household utility, is given by the following utility function:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \varsigma^{cnn} \ln(E_{t}^{cnn}(i) - h E_{t-1}^{cnn}(i)) + \varsigma^{cd} \ln(K_{t}^{cd}(i)) \right. \\
+ \left. \varsigma^{r} \ln(K_{t}^{r}(i)) - \varsigma^{1}(L_{t}^{cbi}(i) + L_{t}^{kb}(i))^{1+\nu} \right] 
\]  

(2)

where \( E^{cnn} \) represents expenditures on consumption of nondurable goods and services, \( K^{cd} \) and \( K^{r} \) represent the stocks of consumer durables and residential capital (housing), \( L^{cbi} + L^{kb} \) represents the sum of labor supplied to each productive sector (with hours worked causing disutility), and the remaining terms represent parame-
ters (such as the discount factor, relative value in utility of each service flow, and the elasticity of labor supply).

By modeling preferences over these disaggregated categories of expenditure, EDO attempts to account for the disparate forces driving consumption of nondurables and durables, residential investment, and business investment – thereby speaking to issues such as the surge in business investment in the second half of the 1990s or the housing cycle the early 2000s recession and the most recent downturn. Many other models do not distinguish between developments across these categories of spending.

### 3.3 Risk premia, financial shocks, and economic fluctuations

The structure of the EDO model implies that households value durable stocks according to their expected returns, including any expected service flows, and according to their risk characteristics, with a premium on assets which have high expected returns in adverse states of the world. However, the behaviour of models such as EDO is conventionally characterized under the assumption that this second component is negligible. In the absence of risk adjustment, the model would then imply that households adjust their portfolios until expected returns on all assets are equal.

Empirically, however, this risk adjustment may not be negligible and, moreover, there may be a variety of factors, not explicitly modelled in EDO, which limit the ability of households to arbitrage away expected return differentials across different assets. To account for this possibility, EDO features several exogenous shocks to the rates of return required by the household to hold the assets in question. Following such a shock – an increase in the premium on a given asset, for example – households will wish to alter their portfolio composition to favor the affected asset, leading to changes in the prices of all assets and, ultimately, to changes in the expected path of production underlying these claims.

The “sector-specific” risk shocks affect the composition of spending more than the path of GDP itself. This occurs because a shock to these premia leads to sizable substitution across residential, consumer durable, and business investment; for example, an increase in the risk premia on business investment leads households to shift away from business investment and towards residential investment and consumer durables. Consequently, it is intuitive that a large fraction of the non-cyclical, or idiosyncratic, component of investment flows to physical stocks will be accounted for by movements

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12 of 54
in the associated premia.

Shocks to the required rate of return on the nominal risk-free asset play an especially large role in EDO. Following an increase in the premium, in the absence of nominal rigidities, the households’ desire for higher real holdings of the risk-free asset would be satisfied entirely by a fall in prices, i.e., the premium is a shock to the natural rate of interest. Given nominal rigidities, however, the desire for higher risk-free savings must be offset, in part, through a fall in real income, a decline which is distributed across all spending components. Because this response is capable of generating comovement across spending categories, the model naturally exploits such shocks to explain the business cycle. Reflecting this role, we denote this shock as the “aggregate risk-premium”.

### 3.4 New-Keynesian Price and Wage Phillips Curves

As in most of the related literature, nominal prices and wages are both “sticky” in EDO. This friction implies that nominal disturbances – that is, changes in monetary policy – have effects on real economic activity. In addition, the presence of both price and wage rigidities implies that stabilization of inflation is not, in general, the best possible policy objective (although a primary role for price stability in policy objectives remains).

Given the widespread use of the New-Keynesian Phillips curve, it is perhaps easiest to consider the form of the price and wage Phillips curves in EDO at the estimated parameters. The price Phillips curve (governing price adjustment in both productive sectors) has the form:

\[
\pi_{t}^{p,s} = 0.22\pi_{t-1}^{p,s} + 0.76E_t\pi_{t+1}^{p,s} + 0.017mc_t + \theta_t^{s} \tag{3}
\]

where \(mc\) is marginal cost and \(\theta\) is a markup shock. As the parameters indicate, inflation is primarily forward-looking in EDO.

The wage (\(w\)) Phillips curve for each sector has the form:

\[
\Delta w_t^s = 0.01\Delta w_{t-1}^s + 0.95E_t\Delta w_{t+1}^s + 0.012\left(mrs_{t}^{c,l} - w_t^s\right) + \theta_t^w + adj.\ costs. \tag{4}
\]
where $mrs$ represents the marginal rate of substitution between consumption and leisure. Wages are primarily forward looking and relatively insensitive to the gap between households’ valuation of time spent working and the wage.

### 3.5 The Monetary Policy Rule

The estimated monetary policy rule has standard features – the policy interest rate responds inertially to inflation and a deviation of output from a trend level:

$$r_t = 0.76r_{t-1} + (1 - 0.76) \left(1.50 \Delta P_t^{PCE} + 1.20 (y_t - trend)\right) + \delta_t^{Rshock}. \quad (5)$$

$$\delta_t^{Rshock} = \rho^{Rshock} \delta_{t-1}^{Rshock} + \epsilon_t^R \quad (6)$$

The long-run responses to the output gap and inflation are very similar to those in the literature. The measure of trend output is based on a production-function concept – that is, trend output is the level of output consistent with labor input and the utilization of capital at long-run levels, given the current level of productive capital; this output concept is a Divisia aggregate of production in the two sectors discussed earlier.

### 3.6 Summary of Model Specification

To summarize, fluctuations in all economic variables are driven by eleven structural shocks. It is most convenient to summarize these shocks into four broad categories:

- **Permanent technology shocks**: This category consists of shocks to aggregate and investment-specific (or fast-growing sector) technology.

- **Financial, or intertemporal, shocks**: This category consists of shocks to risk premia. In EDO, variation in risk premia – both the premium households’ receive relative to the federal funds rate on nominal bond holdings and the additional variation in discount rates applied to the investment decisions of capital intermediaries – are purely exogenous. Nonetheless, the specification captures aspects of related models with more explicit financial sectors (e.g., Bernanke, Gertler, and Gilchrist (1999)).

- **Monetary policy shocks**.
• Other shocks: This category is dominated by shocks to price and wage markups, or Phillips curve shock; it also includes the shock to autonomous demand, which is quantitatively not important in EDO.

4 Estimation: Data and Properties

4.1 Data

The empirical implementation of the model takes a log-linear approximation to the first-order conditions and constraints that describe the economy’s equilibrium, casts this resulting system in its state-space representation for the set of (in our case 12) observable variables, uses the Kalman filter to evaluate the likelihood of the observed variables, and forms the posterior distribution of the parameters of interest by combining the likelihood function with a joint density characterizing some prior beliefs. Since we do not have a closed-form solution of the posterior, we rely on Markov-Chain Monte Carlo (MCMC) methods.

Because of the detailed modeling of demand, EDO can consider more data on expenditure than other related models to inform its parameter estimates and projections. The model is estimated using 12 data series over the sample period from 1984:Q4 to 2011:Q1. The series are:

1. The growth rate of real gross domestic product (ΔGDP);
2. The growth rate of real consumption expenditure on non-durables and services (ΔC);
3. The growth rate of real consumption expenditure on durables (ΔCD);
4. The growth rate of real residential investment expenditure (ΔRes);
5. The growth rate of real business investment expenditure (ΔI);
6. Consumer price inflation, as measured by the growth rate of the Personal Consumption Expenditure (PCE) price index (ΔP_{C,total});
7. Consumer price inflation, as measured by the growth rate of the PCE price index excluding food and energy prices (ΔP_{C,core});
8. Inflation for consumer durable goods, as measured by the growth rate of the PCE price index for durable goods (ΔP_{cd});
9. Hours, which equals hours of all persons in the non-farm business sector from the Bureau of Labor Statistics ($H$);\(^4\)

10. The growth rate of real wages, as given by compensation per hour in the non-farm business sector from the Bureau of Labor Statistics divided by the GDP price index ($\Delta RW$);

11. The federal funds rate ($R$).


Our implementation adds measurement error processes to the likelihood implied by the model for all of the observed series used in estimation except the short-term nominal interest rate series.

Figure 6 presents the observed data (in blue) and the observable data net of the model’s estimated measurement error (in black), along 95 percent confidence intervals. For series other than overall PCE price inflation, measurement error is a moderate portion of movements in the series. The larger role for measurement error in accounting for the path of PCE price inflation reflects the absence of separate sectors for food and energy in the model.

### 4.2 Estimates of shocks and exogenous fundamentals

Figures 7 and 8 report modal estimates of the model’s structural shocks and the persistent exogenous fundamentals (i.e., risk premia and autonomous demand). These series have recognizable patterns for those familiar with U.S. economic fluctuations. For example, the risk premia jump at the end of the sample, reflecting the financial crisis and the model’s identification of risk premia, both economy-wide and for housing, as key drivers.

Of course, these stories from a glance at the exogenous drivers yield applications for alternative versions of the EDO model and future model enhancements. For example, the exogenous risk premia can easily be made to have an endogenous component following the approach of Bernanke, Gertler, and Gilchrist (1999) (and indeed we have considered models of that type). At this point we view incorporation of such

\(^4\)We remove a low-frequency trend from hours via the Hodrick-Prescott filter with a smoothing parameter of 128000; our model is not designed to capture low frequency trends in population growth or labor force participation.
mechanisms in our baseline approach as premature, pending ongoing research on financial frictions, banking, and intermediation in dynamic general equilibrium models. Nonetheless, the EDO model captured the key financial disturbances during the last several years in its current specification, and examining the endogenous factors that explain these developments will be a topic of further study.

### 4.3 Variance Decompositions and impulse responses

We provide detailed variance decompositions and impulse response in Chung, Kiley, and Laforte (2011), and only highlight the key results here.
Volatility in hours per capita is accounted for primarily by the economy-wide risk premium and business investment risk premium shocks at horizons between one and sixteen quarters. The large role for risk premia shocks in the forecast error decomposition at business cycle horizons illustrates the importance of this type of “demand” shock for volatility in the labor market. This result is notable, as hours per capita is the series most like a “gap” variable in the model – that is, house per capita shows persistent cyclical fluctuations about its trend value.

Volatility in aggregate GDP growth is accounted for primarily by the technology shocks in each sector, although the economy-wide risk premium shock contributes non-negligibly to the unconditional variance of GDP growth.
Volatility in core inflation is accounted for primarily by the markup shocks in the short run and technology shocks in the long run.

Volatility in the federal funds rate is accounted for primarily by the economywide risk premium.

Volatility in expenditures on consumer non-durables and non-housing services is, in the near horizon, accounted for predominantly by economy-wide and non-residential investment specific risk-premia shocks.

Volatilities in expenditures on consumer durables, residential investment, and non-residential investment are, in the near horizon, accounted for predominantly by their own sector specific risk-premium shocks.
With regard to impulse responses, we previously highlight the responses to the most important shock, the aggregate risk premium, in figure 4. As we noted, this shock looks like a traditional demand shock, with an increase in the risk premium lowering real GDP, hours worked, and inflation; monetary policy offsets these negative effects somewhat by becoming more accommodative. As for responses to other disturbances, the impulse responses to a monetary policy innovation captures the conventional wisdom regarding the effects of such shocks. In particular, both household and business expenditures on durables (consumer durables, residential investment, and nonresidential investment) respond strongly (and with a hump-shape) to a contractionary policy shock, with more muted responses by nondurables and services consumption; each measure of inflation responds gradually, albeit more quickly than in some analyses based on vector autoregressions (VARs).  

Shocks to sectoral risk premia principally depress spending in the associated category of expenditure (e.g., an increase in the residential risk premium lowers residential investment), with offsetting positive effects on other spending (which is “crowded in”).

Following an economy-wide technology shock, output rises gradually to its long-run level; hours respond relatively little to the shock (in comparison to, for example, output), reflecting both the influence of stick prices and wages and the offsetting income and substitution effects of such a shock on households willingness to supply labor.

References


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5This difference between VAR-based and DSGE-model based impulse responses has been highlighted elsewhere – for example, in the survey of Boivin, Kiley, and Mishkin (2010).
tween Federal Reserve staff forecasts, simple reduced-form models, and a DSGE model. Journal of Applied Econometrics.


Estimated Dynamic Optimization-based (EDO) Model

This appendix provides documentation for the EDO model, that is, the Estimated Dynamic Optimization-based model developed at the Federal Reserve Board; for further details, see Chung, Kiley, and Laforte (2010).

1 Overview

The EDO model builds on the Smets and Wouters (2007) model. Households have preferences over nondurable consumption services, durable consumption services, housing services, and leisure and feature internal habit in each service flow. Production occurs in two sectors that experience different (stochastic) rates of technological progress, thereby allowing the model to match the faster rate of growth (in real, or constant dollar, terms) for some expenditure components (like nonresidential investment); growth is balanced in nominal, rather than real, terms. Expenditures on nondurable consumption, durable consumption, residential investment, nonresidential investment, and the remainder of demand are each modeled, with the last category exogenous.

Individuals’ wages and firms’ prices are sticky in the sense of Rotemberg, with indexation to a weighted average of long-run inflation and lagged inflation. This structure, for the two productive sectors of the economy, yield four Phillips curves: two for wage inflation and two for price inflation. The deviation of marginal cost from its steady-state value plays its usual role in the price Phillips curve. In the wage Phillips curve, the deviation of the wage from the marginal rate of substitution between consumption and leisure is the driving fundamental.

A simple monetary policy reaction function governs monetary policy choices. The Federal Funds rate responds to it’s value in the previous quarter, the current and lagged value of the output gap (defined as the deviation of output from its long-run

Beveridge-Nelson (stochastic) trend), and the deviation of inflation from the assumed objective of 2 percent (at an annual rate).

The exogenous shocks/processes in the model include the monetary policy shock, the growth rates of economywide and investment-specific technologies, financial shocks which include an economywide risk premium and risk premia that affect the intermediaries for consumer durable, residential investment, and nonresidential investment, and other shocks including autonomous aggregate demand and price and wage markup shocks.

The model is estimated (using Bayesian methods) over the sample period 1984Q4 to 2008Q4. The data used in estimation include the following: Real GDP; Real consumption of nondurables and services excluding housing; Real consumption of durables; Real residential investment; Real business investment; Aggregate hours worked in the nonfarm business sector (per capita); PCE price inflation; core PCE price inflation; Percent change in PCE durables price index; Compensation per hour divided by GDP price index; and federal funds rate. Each expenditure series is measured in per capita terms, using the (smoothed) civilian noninstitutional population over the age of 16. We remove a very smooth trend from hours per capita prior to estimation.

2 The Structure of the Model

The model possesses two final good sectors in order to capture key long-run growth facts and to differentiate between the cyclical properties of different categories of durable expenditure (e.g., housing, consumer durables, and nonresidential investment). For example, technological progress has been faster in the production of business capital and consumer durables (such as computers and electronics). Edge, Kiley, and Laforte (2008 and 2010) discuss this motivation in greater detail. The first sector is the slow-growing sector—called “CBI” because most of these goods are used
for consumption (C) and because they are produced by the business and institutions (BI) sector—and the second is the fast-growing sector—called “KB” because these goods are used for capital (K) accumulation and are produced by the business (B) sector. The goods are produced in two stages by intermediate- and then final-goods producing firms. As in most new-Keynesian models, the introduction of intermediate and final goods producers facilitates the specification of nominal rigidities.

The disaggregation of production (aggregate supply) leads naturally to some disaggregation of expenditures (aggregate demand). We move beyond the typical model with just two categories of (private domestic) demand (consumption and investment) and distinguish between four categories of private demand: consumer non-durable goods and non-housing services, consumer durable goods, residential investment, and non-residential investment. Consumer non-durable goods and services are sold directly to households; consumer durable goods, residential capital goods, and non-residential capital goods are intermediated through capital-goods intermediaries (owned by the households), who then rent these capital stocks to households. Consumer non-durable goods and services and residential capital goods are purchased (by households and residential capital goods owners, respectively) from the first of economy’s two final goods producing sectors, while consumer durable goods and non-residential capital goods are purchased (by consumer durable and residential capital goods owners, respectively) from the second sector. In addition to consuming the non-durable goods and services that they purchase, households supply labor to the intermediate goods-producing firms in both sectors of the economy.

This remainder of this section provides an overview of the decisions made by each of the agents in our economy. Given some of the broad similarities between our model and others, our presentation is selective.

**The Final Goods Producers’ Problem.** The economy produces two final goods and services: slow-growing “consumption” goods and services, \( X_{cbi}^t \), and fast-growing “capital” goods, \( X_{kb}^t \). These final goods are produced by aggregating (accord-
ing to a Dixit-Stiglitz technology) an infinite number of sector-specific differentiated intermediate inputs, \( X_t^s(j) \) for \( s = cbi, kb \), distributed over the unit interval. The representative firm in each of the consumption and capital goods producing sectors chooses the optimal level of each intermediate input, taking as given the prices for each of the differentiated intermediate inputs, \( P_t^s(j) \), to solve the cost-minimization problem:

\[
\min_{\{X_t^s(j)\}} \int_0^1 P_t^s(j)X_t^s(j)dj \quad \text{subject to} \quad \left( \int_0^1 (X_t^s(j))^{s_t^s} \, dj \right)^{s_t^s} \geq X_t^s, \quad \text{for } s = cbi, kb.
\]

(1)

The term \( \Theta_t^s \) is the stochastic elasticity of substitution between the differentiated intermediate goods inputs used in the production of the consumption or capital goods sectors. Letting \( \theta_t^s \equiv \ln \Theta_t^s - \ln \Theta_t^s * \) denote the log-deviation of \( \Theta_t^s \) from its steady-state value of \( \Theta_t^s * \), we assume that

\[
\theta_t^s = \epsilon_t^{\theta_t^s}, \quad \text{for } s = cbi, kb,
\]

(2)

where \( \epsilon_t^{\theta_t^s} \) is a shock process. A stochastic elasticity of substitution introduces transitory markup shocks into the pricing decisions of intermediate-goods producers.

**The Intermediate Goods Producers’ Problem.** The intermediate goods entering each final goods technology are produced by aggregating (according to a Dixit-Stiglitz technology) an infinite number of differentiated labor inputs, \( L_t^s(j) \) for \( s = cbi, kb \), distributed over the unit interval and combining this aggregate labor input (via a Cobb-Douglas production function) with utilized non-residential capital, \( K_t^{nr,s} \). Each intermediate-good producing firm effectively solves three problems: two factor-input cost-minimization problems (over differentiated labor inputs and the aggregate labor and capital) and one price-setting profit-maximization problem.

In its first cost-minimization problem, an intermediate goods producing firm chooses the optimal level of each type of differential labor input, taking as given
the wages for each of the differentiated types of labor, $W_i^s(i)$, to solve:

$$\min_{\{L_t^s(i,j)\}} W_i^s(i) L_t^s(i,j) \text{subject to } 0 \leq \frac{\partial L_t^s(i,j)}{\partial i} \geq 1, \text{ for } s = cbi, kb. \quad (3)$$

The term $\Theta^l_t$ is the stochastic elasticity of substitution between the differentiated labor inputs. Letting $\Theta^l_t \equiv \ln \Theta^l_t - \ln \Theta^l_s$ denote the log-deviation of $\Theta^l_t$ from its steady-state value of $\Theta^l_s$, we assume that

$$\Theta^l_t = \epsilon^l_t. \quad (4)$$

where $\epsilon^l_t$ is a shock process. A stochastic elasticity of substitution introduces transitory wage markup shocks into the wage decisions of households.

In its second cost-minimization problem, an intermediate-goods producing firm chooses the optimal levels of aggregated labor input and utilized capital, taking as given the wage, $W_t^s$, for aggregated labor, $L_t^s$ (which is generated by the cost function derived the previous problem), and the rental rate, $R_t^{nr,s}$, on utilized capital, $K_t^{u,nr,s}$, to solve:

$$\min_{\{L_t^s(j), K_t^{u,nr,s}(j)\}} W_t^s L_t^s(j) + R_t^{nr,s} K_t^{u,nr,s}(j) \text{subject to } (Z_t^m Z_t^{iu} L_t^s(j))^{1-\alpha} (K_t^{u,nr,s}(j))^{\alpha} \geq X_t(j), \text{ for } s = cbi, kb, \text{ with } Z_t^{cbi} \equiv 1. \quad (5)$$

The parameter $\alpha$ is the elasticity of output with respect to capital, while the $Z_t$ variables denote the level of productivity. The level of productivity has two components. The first, $Z_t^m$, is common to both sectors and thus represents the level of economy-wide technology. The second, $Z_t^s$, is sector specific; we normalize $Z_t^{cbi}$ to one, while $Z_t^{kb}$ is not restricted.

The exogenous productivity terms contain a unit root, that is, they exhibit permanent movements in their levels. We assume that the stochastic processes $Z_t^m$ and $Z_t^{kb}$ evolve according to

$$\ln Z_t^n - \ln Z_{t-1}^n = \ln \Gamma_t^z \equiv \ln (\Gamma_t^z \cdot \exp[\epsilon^z_t]) = \ln \Gamma_t^z + \epsilon^z_t, \text{ for } s = kb, m \quad (6)$$
where $\Gamma^{z,n}_s$ and $\epsilon^{z,n}_t$ are the steady-state and stochastic components of $\Gamma^{z,n}_t$. The stochastic component $\epsilon^{z,n}_t$ is an i.i.d shock process.

The unit-root in technology in both sectors yields a non-trivial Beveridge-Nelson permanent/transitory decomposition. The presence of capital-specific technological progress allows the model to generate differential trend growth rates in the economy’s two production sectors. In line with historical experience, we assume a more rapid rate of technological progress in capital goods production by calibrating $\Gamma^{z,kb}_t > 1$, where (as is the case for all model variables) an asterisk on a variable denotes its steady-state value.

In its price-setting (or profit-maximization) problem, an intermediate goods producing firm chooses its optimal nominal price and the quantity it will supply consistent with that price. In doing so it takes as given the marginal cost, $MC^s_t(j)$, of producing a unit of output, $X_t^s(j)$, the aggregate price level for its sector, $P_t^s$, and households’ valuation of a unit of nominal profits income in each period, which is given by $\Lambda_t^{cnn}/P_t^{cbi}$ where $\Lambda_t^{cnn}$ denotes the marginal utility of non-durables and non-housing services consumption. Specifically, firms solve:

$$
\max_{\{P_t^s(j),X_t^s(j)\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t^{cnn} \left\{ P_t^s(j)X_t^s(j) - MC^s_t(j)X_t^s(j) \right\}
- \frac{100 \cdot \chi^p}{2} \left( \frac{P_t^s(j)}{P_{t-1}^s(j)} - \eta^p \Pi_{t-1}^{p,s} - (1-\eta^p)\Pi_t^{p,s} \right)^2 P_t^s X_t^s
$$

subject to $X_t^s(j) = (P_t^s(j)/P_t^{\tau})^{-\Theta^s} X_\tau^s$ for $\tau = 0, 1, \ldots, \infty$ and $s = cb_i, kb$.

The profit function reflects price-setting adjustment costs (the size which depend on the parameter $\chi^p$ and the lagged and steady-state inflation rate). The constraint against which the firm maximizes its profits is the demand curve it faces for its differentiated good, which derives from the final goods producing firm’s cost-minimimization problem. This type of price-setting decision delivers a new-Keynesian Phillips curve. Because adjustment costs potentially depend upon lagged inflation, the Phillips curve can take the “hybrid” form in which inflation is linked to its own lead and lag as well
as marginal cost.

**The Capital Owners’ Problem.** We now shift from producers’ decisions to spending decisions. There exists a unit mass of non-residential capital owners (individually denoted by k, with k distributed over the unit interval) who choose investment in non-residential capital, \( E^{nr}_t \), the stock of non-residential capital, \( K^{nr}_t \) (which is linked to the investment decision via the capital accumulation identity), and the amount and utilization of non-residential capital in each production sector, \( K^{nr,cbi}_t \), \( U^{cbi}_t \), \( K^{nr,kb}_t \), and \( U^{kb}_t \). (Recall, that the firm’s choice variables in equation 5 is utilized capital \( K^{a,nr,s}_t = U^s_t K^{nr,s}_t \).) The mathematical representation of this decision is described by the following maximization problem (in which capital owners take as given the rental rate on non-residential capital, \( R^{nr}_t \), the price of non-residential capital goods, \( P^{kb}_t \), and households’ valuation of nominal capital income in each period, \( \Lambda^{cn}_t / P^{cbi}_t \), and the exogenous risk premium specific to non-residential investment, \( A^{nr}_t \)):

\[
\max_{(E^{nr}_t(k),K^{nr+1}_t(k),K^{nr,cbi}_t(k),K^{nr,kb}_t(k)U^{cbi}_t(k),U^{kb}_t(k))_{t=0}^\infty} \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda^{cn}_t}{A^{nr}_t P^{cbi}_t} \left\{ R^{nr}_t U^{cbi}_t(k) K^{nr,cbi}_t(k) + R^{nr}_t U^{kb}_t(k) K^{nr,kb}_t(k) - P^{kb}_t E^{nr}_t(k) \right. \\
- \kappa \left( \frac{U^{cbi}_t(k)^{1+\psi} - 1}{1 + \psi} \right) Q^{nr}_t K^{nr,cbi}_t(k) - \kappa \left( \frac{U^{kb}_t(k)^{1+\psi} - 1}{1 + \psi} \right) Q^{nr}_t K^{nr,kb}_t(k) \left\} \\
\text{subject to} \quad K^{nr+1}_t(k) = (1-\delta^{nr}) K^{nr}_t(k) + E^{nr}_t(k) - \frac{100 \cdot \chi^{nr}}{2} \frac{E^{nr}_t(k) - E^{nr}_{t-1}(k)}{K^{nr}_t} K^{nr}_t(k) + K^{nr,kb}_t(k) = K^{nr}_t(k) \quad \text{for } \tau = 0,1,\ldots,\infty. \quad (8)
\]

The parameter \( \delta^{nr} \) in the capital-accumulation constraint denotes the depreciation rate for non-residential capital, while the parameter \( \chi^{nr} \) governs how quickly investment adjustment costs increase when \( (E^{nr}_t(k) - E^{nr}_{t-1}(k)) \) rises above zero; note that these adjustment costs include a term for the stochastic growth rate of the trend in the level of the output in sector KB, \( \Gamma^{x,kb}_t \) equal to \( \Gamma^{z,m}_t \Gamma^{z,kb}_t \). The variable \( A^{nr}_t \) is
a stochastic element reflecting a risk premium on non-residential investment. Letting $a_{t}^{nr} \equiv \ln A_{t}^{nr}$ denote the log-deviation of $A_{t}^{nr}$ from its steady-state value of unity, we assume that:

$$a_{t}^{nr} = \rho^{nr} a_{t-1}^{nr} + \epsilon_{t}^{nr}.$$  \hspace{1cm} (9)

Higher rates of utilization incur a cost (reflected in the last two terms in the capital owner’s profit function). We assume that utilization is unity in the steady-state, implying $\kappa = R^{nr}_{s} / Q^{nr}_{s}$.

The time-variation in utilization, along with the imperfect competition in product and labor markets, implies that direct measurement of total factor productivity may not provide an accurate estimate of technology; as a result, the EDO model can deliver smoother estimates of technology that might be implied by a real-business-cycle model.

The problems solved by the consumer durables and residential capital owners are slightly simpler than the non-residential capital owner’s problems. Since utilization rates are not variable for these types of capital, their owners make only investment and capital accumulation decisions. Taking as given the rental rate on consumer durables capital, $R_{t}^{cd}$, the price of consumer-durable goods, $P_{t}^{kb}$, and households’ valuation of nominal capital income, $\Lambda^{cnn}_{t} / P_{cbi}^{t}$, and the exogenous risk premia specific to consumer durables investment, $A_{t}^{cd}$, the capital owner chooses investment in consumer durables, $I_{t}^{cd}$, and its implied capital stock, $K_{t}^{cd}$, to solve:

$$\max_{\{E_{t}^{cd}(k), K_{t+1}^{cd}(k)\}_{t=0}^{\infty}} \mathbb{E}_{0}^{t} \sum_{t=0}^{\infty} \beta^{t} \frac{\Lambda^{cnn}_{t}}{A_{t}^{cd} P_{cbi}^{t}} \left\{ R_{t}^{cd} K_{t}^{cd}(k) - P_{t}^{kb} E_{t}^{cd}(k) \right\}$$

subject to

$$K_{\tau+1}^{cd}(k) = (1 - \delta^{cd}) K_{\tau}^{cd}(k) + E_{\tau}^{cd}(k) - \frac{100 \cdot \chi^{cd}}{2} \left( \frac{E_{\tau}^{cd}(k) - E_{\tau-1}^{cd}(k) \Gamma^{x,kb}_{\tau}}{K_{\tau}^{cd}} \right)^{2} K_{\tau}^{cd}$$

for $\tau = 0, 1, \ldots, \infty$.  \hspace{1cm} (10)
The residential capital owner’s decision is analogous:

\[
\max_{\{E_t^r(k), K_{t+1}^r(k)\}_{t=0}^{\infty}} \beta^t \frac{\Lambda_t^{cnm}}{A_t^r P_t^{ebl}} \left\{ R_t^r K_t^r(k) - P_t^{ebl} E_t^r(k) \right\}
\]

subject to

\[
K_{t+1}^r(k) = (1 - \delta^r) K_t^r(k) + E_t^r(k) - \frac{100 \cdot \chi^r}{2} \left( \frac{E_t^r(k) - E_{t-1}^r(k) \Gamma_{\tau}^{x, ebl}}{K_{\tau}^{cd}} \right)^2 K_{\tau}^{cd}
\]

for \( \tau = 0, 1, \ldots, \infty \).

The notation for the consumer durables and residential capital stock problems parallels that of non-residential capital. In particular, the asset-specific risk premia shocks, \( A_t^{cd} \) and \( A_t^r \), follow an autoregressive process similar to that given in equation (9).

The Households’ Problem. The final group of private agents in the model are households who make both expenditure and labor-supply decisions. Households derive utility from four sources: their purchases of the consumer non-durable goods and non-housing services, the flow of services from their rental of consumer-durable capital, the flow of services from their rental of residential capital, and their leisure time, which is equal to what remains of their time endowment after labor is supplied to the market. Preferences are separable over all arguments of the utility function. The utility that households derive from the three components of goods and services consumption is influenced by the habit stock for each of these consumption components, a feature that has been shown to be important for consumption dynamics in similar models.

A household’s habit stock for its consumption of non-durable goods and non-housing services is equal to a factor \( h \) multiplied by its consumption last period \( E_{t-1}^{cnm} \). Its habit stock for the other components of consumption is defined similarly.

Each household chooses its purchases of consumer non-durable goods and services, \( E_t^{cnm} \), the quantities of residential and consumer durable capital it wishes to rent, \( K_t^r \) and \( K_t^{cd} \), its holdings of bonds, \( B_t \), its wage for each sector, \( W_t^{ebl} \) and \( W_t^{kb} \), and the supply of labor consistent with each wage, \( L_t^{ebl} \) and \( L_t^{kb} \). This decision is made subject to the household’s budget constraint, which reflects the costs of adjusting wages and the mix of labor supplied to each sector, as well as the demand curve the household
faces for its differentiated labor. Specifically, the $i$th household solves:

$$
\begin{align*}
\max_{\{E^c_{t+i}, K^c_{t+i}, K^l_{t+i}, \{W^s_{t+i}(i), L^s_{t+i}(i)\}_{s=cbi, kb}, B_{t+1}(i)\}}_{t=0}^\infty &
= \mathcal{E}_t \sum_{t=0}^\infty \beta \left\{ \varsigma^c \ln \left( E^c_{t+i}(i) - h E^c_{t+1}(i) \right) + \varsigma^d \ln \left( K^c_{t+i}(i) - h K^c_{t+1}(i) \right) \right. \\
&\quad \left. + \varsigma^r \ln \left( K^l_{t+i}(i) - h K^l_{t+1}(i) \right) - \varsigma^l \left( \frac{L^c_{t+i}(i) + L^kb_{t+i}(i)}{1 + \nu} \right) \right\}
\end{align*}
$$

subject to:

$$
\begin{align*}
\frac{B_{t+1}(i)}{R_t \Omega_t} &= B_t(i) + \sum_{s=cbi, kb} W^s_t(i)L^s_t(i) + \text{Capital and Profits Income}_t(i) - P^cbi(i)E^c_{t+i}(i) \\
- R^c_t K^c_{t+i}(i) - R^r_t K^r_{t+i}(i) &= \sum_{s=cbi, kb} \frac{100 \cdot \chi^w}{2} \left( \frac{W^s_{t+i}(j)}{W^s_{t+1}(j)} - \eta^w \Pi^w_{t-1} - (1 - \eta^w) \Pi^w_s \right) W^s_t L^s_t \\
- 100 \cdot \chi^l &\left( \frac{L^c_{t+i}(i)}{L^c_t(i)} + \frac{L^kb_{t+i}(i)}{L^kb_t(i)} \right) \left( \frac{L^c_{t+i}(i)}{L^c_t(i)} + \frac{L^kb_{t+i}(i)}{L^kb_t(i)} \right) \frac{L^kb_t(i)}{L^cb_t(i)} \\
L^cb_t(i) &= (W^cb_t(i)/W^cb_t(i))^{-\Theta^c_l} L^cb_t(i), \quad \text{and} \quad L^cb_t(i) = (W^cb_t(i)/W^cb_t(i))^{-\Theta^c_l} L^cb_t(i), \\
\end{align*}
$$

for $\tau = 0, 1, \ldots, \infty$. \hspace{1cm} (12)

In the utility function the parameter $\beta$ is the household’s discount factor, $\nu$ denotes its inverse labor supply elasticity, while $\varsigma^c$, $\varsigma^d$, $\varsigma^r$, and $\varsigma^l$ are scale parameter that tie down the ratios between the household’s consumption components.

The stationary, unit-mean, stochastic variable $\Omega_t$ represents an aggregate risk-premium shock that drives a wedge between the policy short-term interest rate and the return to bonds received by a household. Letting $\omega_t = \ln \Omega_t - \ln \Omega_s$ denote the log-deviation of $\Omega_t$ from its steady-state value of $\Omega_s$, we assume that

$$
\omega_t = \rho^\omega \omega_{t-1} + \epsilon^\omega_t. \hspace{1cm} (13)
$$

The variable $\epsilon^\omega_t$ is a shock process, and $\rho^\omega$ represents the persistence of $\Omega_t$.

The household’s budget constraint reflects wage setting adjustment costs, which depend on the parameter $\chi^w$ and the lagged and steady-state wage inflation rate, and the costs in changing the mix of labor supplied to each sector, which depend on the
parameter $\chi^l$. The costs incurred by households when the mix of labor input across sectors changes may be important for sectoral comovements.

**Gross Domestic Product.** The demand and production aspects of the model are closed through the exogenous process for demand other than private domestic demand and the GDP identity. $\bar{X}_t^{HG}$ represents exogenous demand (i.e., GDP other than private domestic demand, the aggregate of $E_t^{cnm}$, $E_t^d$, $E_t^r$, and $E_t^{nr}$). Exogenous demand is assumed to follow the process:

$$\ln \bar{X}_t^{HG} - \ln \bar{X}_*^{HG} = \rho^{HG} \left( \ln \bar{X}_t^{HG} - \ln \bar{X}_*^{HG} \right) + \epsilon_t^{HG}.$$  

We assume that the exogenous demand impinges on each sector symmetrically, and specifically that the percent deviation of exogenous demand proportionally affects demand for each sector’s ($s = cbi, kb$) output via the share of exogenous demand in total demand, $\omega_{HG}$. (In this formulation, $\bar{X}_t^{HG}$ represents the level of expenditure relative to the stochastic long-run trend, i.e., the model assumes balanced growth, so exogenous demand for each sector fluctuates around its long-run trend; for example, the long-run trend for sector KB is given by $Z_t^m Z_t^{kb}$).

The rate of change of Gross Domestic Product (real GDP) equals the Divisia (share-weighted) aggregate of production in the two sectors (and of final spending across each expenditures category), as given by the identity:

$$H_t^{gdp} = \left( \frac{X_t^{cbi}}{X_{t-1}^{cbi}} \right)^{p_{s}^{cbi} X_t^{cbi}} \left( \frac{X_t^{kb}}{X_{t-1}^{kb}} \right)^{p_{s}^{kb} X_t^{kb}} \frac{1}{p_{s}^{cbi} X_t^{cbi} + p_{s}^{kb} X_t^{kb}}.$$  

(14)

**Monetary Authority.** We now turn to the last important agent in our model, the monetary authority. It sets monetary policy in accordance with a Taylor-type interest-rate feedback rule. Policymakers smoothly adjust the actual interest rate $R_t$ to its target level $\bar{R}_t$

$$R_t = (R_{t-1})^{\phi^r} \left( \bar{R}_t \right)^{1-\phi^r} \exp \left[ \epsilon_t^r \right],$$  

(15)

where the parameter $\phi^r$ reflects the degree of interest rate smoothing, while $\epsilon_t^r$ represents a monetary policy shock. The central bank’s target nominal interest rate, $\bar{R}_t$
depends the deviation of output from its stochastic trend ($\tilde{X}^{bn}$, the output gap as defined by Beveridge and Nelson (1981))

$$\tilde{X}_t^{bn} = \mathcal{E}_t \left[ \sum_{\tau=-\infty}^{t} H^{gdp}_\tau - \sum_{\tau=-\infty}^{\infty} H^{gdp}_\tau \right].$$  \hspace{2cm} (16)

In equation 16, the deterministic, or steady-state, levels of growth are suppressed. Consumer price inflation and the change in the output gap also enter the target. The target equation is:

$$\tilde{R}_t = \left( \tilde{X}_t^{bn} \right)^{\phi_y} \left( \tilde{X}_t^{bn} / \tilde{X}_{t-1}^{bn} \right)^{\phi_{\Delta y}} \left( \frac{\Pi_t^e}{\Pi_t^f} \right)^{\phi^r} R_s.$$  \hspace{2cm} (17)

In equation (17), $R_s$ denotes the economy’s steady-state nominal interest rate and $\phi_y$, $\phi_{\Delta y}$, and $\phi^r$ denote the weights in the feedback rule. Consumer price inflation, $\Pi_t$, is the weighted average of inflation in the nominal prices of the goods produced in each sector, $\Pi_t^{p,cbi}$ and $\Pi_t^{p,kb}$:

$$\Pi_t^e = (\Pi_t^{p,cbi})^{1-w_{cd}} (\Pi_t^{p,kb})^{w_{cd}}.$$  \hspace{2cm} (18)

The parameter $w_{cd}$ is the share of the durable goods in nominal consumption expenditures.

**Structural Shocks.** The rich specification of structural shocks (to aggregate and investment-specific productivity, aggregate and sector-specific risk premiums, and mark-ups) and adjustment costs allows our model to be brought to the data with some chance of finding empirical validation.

Within EDO, fluctuations in all economic variables are driven by eleven structural shocks. It is most convenient to summarize these shocks into four broad categories:

- Permanent technology shocks: This category consists of shocks to aggregate and investment-specific (or fast-growing sector) technology.
- Financial, or intertemporal, shocks: This category consists of shocks to risk premia. In EDO, variation in risk premia – both the premium households’
receive relative to the federal funds rate on nominal bond holdings and the additional variation in discount rates applied to the investment decisions of capital intermediaries – are purely exogenous. Nonetheless, the specification captures important aspects of related models with more explicit financial sectors (e.g., Bernanke, Gertler, and Gilchrist (1999)), as we discuss in our presentation of the model’s properties below.

- Markup shocks: This category includes the price and wage markup shocks.
- Other demand shocks: This category includes the shock to autonomous demand and a monetary policy shock.

**Market Clearing.** There are a number of market clearing conditions that must be satisfied in our model. Market clearing in the slow-growing “consumption” goods and fast-growing “capital” goods sectors, given price- and wage-adjustment costs and variable utilization costs, implies that

\[
X_t^{cbi} = \frac{1}{E_t^{cm}}(i)di + \frac{1}{E_t^p(t)}dk + \tilde{X}_t^{HG} * Z_t^m * (Z_t^{kb})^\alpha
+ \frac{100}{2} \chi^p \left( \Pi_t^{p,cbi} - \eta^p \Pi_{t-1}^{p,cbi} - (1 - \eta^p) \Pi_t^{p,cbi} \right)^2 P_t^{cbi} X_t^{cbi}
+ \frac{100}{2} \chi^w \left( \Pi_t^{w,cbi} - \eta^w \Pi_{t-1}^{w,cbi} - (1 - \eta^w) \Pi_t^{w,cbi} \right)^2 W_t^{cbi} L_t^{cbi} - \kappa \left( \frac{U_t^{cbi}(k)^{1+\psi} - 1}{1 + \psi} \right) P_t^{cbi} K_t^{nr,cbi}
\]

and

\[
X_t^{kb} = \frac{1}{E_t^{cd}}(k)dk + \frac{1}{E_t^{nr}(k)dk} + \tilde{X}_t^{HG} * Z_t^m * Z_t^{kb}
+ \frac{100}{2} \chi^p \left( \Pi_t^{p,kb} - \eta^p \Pi_{t-1}^{p,kb} - (1 - \eta^p) \Pi_t^{p,kb} \right)^2 P_t^{kb} X_t^{kb}
+ \frac{100}{2} \chi^w \left( \Pi_t^{w,kb} - \eta^w \Pi_{t-1}^{w,kb} - (1 - \eta^w) \Pi_t^{w,kb} \right)^2 W_t^{kb} L_t^{kb} - \kappa \left( \frac{U_t^{kb}(k)^{1+\psi} - 1}{1 + \psi} \right) P_t^{kb} K_t^{nr,kb}
\]

The market clearing conditions for the labor and non-residential capital supplied and
demanded in sector $s$ are given by
\[ L^s_t(i) = \frac{1}{0} L^s_t(i, j) dj \quad \text{and} \quad U(k)^s_t K^rnr,s_t(k) dk = \frac{1}{0} K^{rsnr,s_t}_t(j) dj \quad \forall \ i \in [0, 1] \quad \text{and for} \ s = cbi, kb. \] (21)

The market clearing conditions for consumer durables and residential capital are
\[ \frac{1}{0} K^{cd}_t(k) dk = \frac{1}{0} K^{cd}_t(i) di \quad \text{and} \quad \frac{1}{0} K^{r}_t(k) dk = \frac{1}{0} K^{r}_t(i) di. \] (22)

The identities for inflation include:
\[ W^s_t(i) = \Pi^{w,s}_t(i) W^s_{t-1}(i) \quad \text{and} \quad W^s_t = \Pi^{w,s}_t W^s_{t-1} \quad \forall \ i \in [0, 1] \quad \text{and for} \ s = cbi, kb, \quad \text{and} \quad (23) \]
\[ P^s_t(j) = \Pi^{p,s}_t(j) P^s_{t-1}(j) \quad \text{and} \quad P^s_t = \Pi^{p,s}_t P^s_{t-1} \quad \forall \ j \in [0, 1] \quad \text{and for} \ s = cbi, kb. \] (24)

### 3 Solution and Estimation

We estimate the log-linearized, symmetric and stationary version of the model described above. The log-linearization of our model equations is performed symbolically by the software that we use to parse the model into its estimable form. The steady-state solution to the symmetric and stationary version of the model is an input into the model’s estimation. The empirical implementation of the model takes a log-linear approximation to the first-order conditions and constraints that describe the economy’s equilibrium, casts this resulting system in its state-space representation for the set of (in our case 11) observable variables, uses the Kalman filter to evaluate the likelihood of the observed variables, and forms the posterior distribution of the parameters of interest by combining the likelihood function with a joint density characterizing some prior beliefs. Since we do not have a closed-form solution of the posterior, we rely on Markov-Chain Monte Carlo (MCMC) methods.

**Data.** The model is estimated using data over the sample period from 1984:Q4 to 2008:Q4. There are 11 data series: real gross domestic product; real consumption expenditure on non-durables and services excluding housing services; real consumption expenditure on durables; real residential investment expenditure; real business
investment expenditure; the Personal Consumption Expenditure (PCE) price index; the PCE price index excluding food and energy prices; the PCE price index for durable goods; real compensation per hour in the non-farm business sector (that is, nominal compensation deflated by the GDP price index); detrended hours of work in the non-farm business sector; and the federal funds rate. Our implementation adds measurement error processes to the likelihood implied by the model for all of the observed series used in estimation except the nominal interest rate series.

**Calibrated Parameters.** The calibrated structural parameters of the model are presented in Table 1. Some important determinants of steady-state behavior were calibrated to yields growth rates of GDP and associated price indexes that corresponded to “conventional” wisdom in policy circles, even though slight deviations from such values would have been preferred (in a “statistically significant” way) to our calibrated values. In other cases, parameters were calibrated based on how informative the data were likely to be on the parameter and/or identification and overparameterization issues.

The standard deviations of the measurement errors for observable variables are reported in Table 2. These standard deviations were calibrated to ensure a moderate contribution of such errors to the overall variability of the data (according to our model) while also preserving desirable forecast properties.

**Estimated Parameters.** The first three columns of Table 3 and 4 outline our assumptions about the prior distributions of the estimated parameters, the remaining columns describe the parameters’ posterior distributions.

First, consider the estimated parameters related to household and business spending decisions. The habit-persistence parameter is moderate, near 0.6. Investment.

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2We remove a low-frequency trend from hours via the Hodrick-Prescott filter with a smoothing parameter of 64000, because our model is not designed to capture low frequency trends in population growth or labor force participation.

3See Kiley (2010a) for a discussion of issues related to identification of the habit parameter using frequentist techniques.
adjustment costs are large for residential investment but small for business investment. This finding highlights once advantage of our disaggregated approach. In addition, this result is importantly driven by the inclusion of inventory investment in business investment; this is a very cyclically important component of GDP and was an important element in early investigations of dynamic general equilibrium models (e.g., Kydland and Prescott (1982)), but is typically ignored in similar DSGE models.

The estimated value of the inverse of the labor supply elasticity implies quite elastic labor supply. We also find a role for the sectoral adjustment costs to labor: In our multisector setup, shocks to productivity or preferences in one sector of the economy result in strong shifts of labor towards that sector, which conflicts with the high degree of sectoral co-movement in the data.

Finally, adjustment costs to prices and wages are both estimated to be important. Our estimate of the price adjustment cost is equivalent to a Calvo pricing setting where a bit more than half of the firms cannot update their prices each period. The estimated quadratic costs in wages imply a slightly larger frequency of adjustments for the suppliers of labor. We also find only a modest role for lagged inflation in our adjustment cost specification (around 1/4), equivalent to modest indexation to lagged inflation in other sticky-price specifications. This differs from some other estimates, perhaps because of the focus on a more recent post-1983 sample (similar to results in Kiley (2007) and Laforte (2007)).

4 Variance Decompositions

We have computed forecast error variance decompositions at various (quarterly) horizons at the posterior mode of the parameter estimates for key variables and shocks.

Volatility in aggregate GDP growth is accounted for primarily by the technology shocks in each sector, although the economy-wide risk premium shock contributes non-negligibly to the unconditional variance of GDP growth.
Volatility in hours per capita is accounted for primarily by the economy-wide risk premium and business investment risk premium shocks at horizons between one and sixteen quarters. Technology shocks in each sector contribute appreciably to the unconditional variance. The large role for risk premia shocks in the forecast error decomposition at business cycle horizons illustrates the importance of this type of “demand” shock for volatility in the labor market. This result is notable, as hours per capita is the series most like a “gap” variable in the model – that is, house per capita shows persistent cyclical fluctuations about its trend value.

Volatility in core inflation is accounted for primarily by the markup shocks in the short run and technology shocks in the long run.

Volatility in the federal funds rate is accounted for primarily by the economy-wide risk premium.

Volatility in expenditures on consumer non-durables and non-housing services is, in the near horizon, accounted for predominantly by economy-wide and non-residential investment specific risk-premia shocks. In the far horizon, volatility is accounted for primarily by capital-specific and economy-wide technology shocks.

Volatilities in expenditures on consumer durables, residential investment, and non-residential investment are, in the near horizon, accounted for predominantly by their own sector specific risk-premium shocks. At farther horizons, their volatilities are accounted for by capital-specific technology shocks.

5 Impulse Responses

We now turn to the impulse responses of some of the key observable variables to the exogenous shocks that drive fluctuations in the model. In each case we consider unit shocks; the reader is referred to the reported estimates of the standard deviation of the shocks for information that will scale these responses to units consistent with a standard deviation shock. Expenditure variables are reported as percent deviations
from initial values (in natural log points); inflation variables and the federal funds rate are reported at quarterly (not annual) rates.

The impulse responses to a monetary policy innovation (shown in figure 4) captures the conventional wisdom regarding the effects of such shocks. In particular, both household and business expenditures on durables (consumer durables, residential investment, and nonresidential investment) respond strongly (and with a hump-shape) to a contractionary policy shock, with more muted responses by nondurables and services consumption; each measure of inflation responds gradually, albeit more quickly than in some analyses based on vector autoregressions (VARs). (This difference between VAR-based and DSGE-model based impulse responses has been highlighted elsewhere – for example, in the survey of Boivin, Kiley, and Mishkin (2010)).

Figures 1 to 6 present the impulse responses of key variables to the model’s four risk premia shocks ($\Omega_t, A_t^{nr}, A_t^{cd}$, and $A_t^r$), the autonomous spending shock ($X_{HG}$), price and wage mark-up shocks ($\Theta_t^{cbi}, \Theta_t^{cbi}$, and $\Theta_t^l$), and technology shocks ($\Gamma_t^{zm}$ and $\Gamma_t^{z,kb}$).

The aggregate risk premium shock (figure 1) depresses spending across the board, lowering hours appreciably; inflation and the federal funds rate fall in response. (As in the model of Smets and Wouters (2007), the aggregate risk premium drives down the flexible-price nominal interest rate one-for-one, and hence the downward move in the nominal funds rate facilitates moving the economy toward its flexible price outcome).

Shocks to sectoral risk premia (figures 9, 10 and 11) principally depress spending in the associated category of expenditure, with offsetting positive effects on other spending (which is “crowded in”).

The impulse responses to a capital-specific technology shock (shown in figure 5) are a touch more gradual, as the embodied component of this type of technological progress implies a need for nonresidential capital accumulation. (In addition, the long-run responses of nonresidential investment and consumer durables are much larger
than those of other spending, reflecting the biased nature of this technology shock).

Following an economy-wide technology shock (figure 6), output rises gradually to its long-run level; hours respond relatively little to the shock (in comparison to, for example, output, reflecting both the influence of stick prices and wages and the offsetting income and substitution effects of such a shock on households willingness to supply labor.

References


Table 1: Calibrated Parameters

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<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</tr>
<tr>
<td>(\alpha)</td>
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<td>(\Pi^*_z)</td>
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Table 2: Measurement Errors on Observable Variables

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<td>(GDP)</td>
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<td>1.5</td>
<td>1.5</td>
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<tr>
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Table 3: Prior and Posterior Distributions of the Behavioral and Policy Parameters

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<th>Parameter</th>
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<th>Posterior Distribution</th>
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<td>(\nu)</td>
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Table 4: Prior and Posterior Distributions of the Parameters corresponding to the Exogenous Processes

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<tr>
<td>$\rho^{cd}$</td>
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<tr>
<td>$\rho^{HG}$</td>
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<tr>
<td>$\rho^f$</td>
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<td>$\sigma_{\theta,l}$</td>
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Figure 1: Impulse Responses: Risk-premium
Figure 2: Impulse Responses: Exog. Demand
Figure 3: Impulse Responses: Wage Markup
Figure 4: Impulse Responses: Funds Rate
Figure 5: Impulse Responses: Capital Goods Technology
Figure 6: Impulse Responses: Overall TFP
Figure 7: Impulse Responses: Non-Invest. Price Markup
Figure 8: Impulse Responses: Invest. Price Markup
Figure 9: Impulse Responses: Housing Risk-Premium
Figure 10: Impulse Responses: Durables Risk-Premium
Figure 11: Impulse Responses: Capital Risk-Premium