

FRBNY DSGE Model: Research Directors Draft

June 2011

Overview

The FRBNY model is a fairly standard DSGE framework with nominal rigidities and financial frictions. Shocks originating in the financial sector were the main driver of the Great Recession, and their lingering effects impact the evolution of the model's forecast. Price and wage rigidities play a key role in the transmission of these shocks.

The FRBNY model projects 2.7% real growth for the year and a more moderate growth of about 2.0% in 2012-2013, with inflation remaining subdued for the entire forecast horizon. The current federal funds rate is constrained at the lower bound until 2012Q2; after that, a gradual tightening process takes place.

In this document we review the main features of the model; we also discuss how to use the model to interpret the Great Recession and the 2012-2013 forecast.

General Features of the Model

The FRBNY DSGE model is a medium-scale, one-sector, dynamic stochastic general equilibrium model. It builds on the neoclassical growth model by adding nominal wage and price rigidities, variable capital utilization, costs of adjusting investment, and habit formation in consumption. The model follows the work of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), but also includes credit frictions, as in the financial accelerator model developed by Bernanke, Gertler, and Gilchrist (1999). The actual implementation of the credit frictions closely follows Christiano, Motto, and Rostagno (2009).

In this section, we briefly describe the microfoundations of the model, including the optimization problem of the economic agents and the nature of the exogenous processes. The innovations to these processes, which we refer to as "shocks," are the drivers of macroeconomic fluctuations. The model identifies these shocks by matching the model dynamics with six quarterly data series: real GDP growth, core PCE inflation, the labor share, aggregate hours worked, the effective federal funds rate (FFR), and the spread between Baa corporate bonds and 10-year Treasury yields. Model parameters are estimated from 1984Q1 to the present using Bayesian methods. Details on the structure of the model, data sources,

and results of the estimation procedure can be found in the accompanying “FRBNY DSGE Model Documentation” note.

The economic units in the model are households, firms, banks, entrepreneurs, and the government. (Figure 1 describes the interactions among the various agents, the frictions and the shocks that affect the dynamics of this economy.)

Households supply labor services to firms. The utility they derive from leisure is subject to a random disturbance, which we call “labor supply” shocks (this shock is sometimes also referred to as a “leisure” shock). Labor supply shocks capture exogenous movements in labor supply due to such factors as demographics and labor market imperfections. The labor market is also subject to frictions because of nominal wage rigidities. These frictions play an important role in the extent to which various shocks affect hours worked. Households also have to choose the amount to consume and save. Their savings take the form of deposits to banks and purchases of government bills. Household preferences take into account habit persistence, a characteristic that affects their consumption smoothing decisions.

Monopolistically competitive firms produce intermediate goods, which a competitive firm aggregates into the single final good that is used for both consumption and investment. The production function of intermediate producers is subject to “total factor productivity” (TFP) shocks. Intermediate goods markets are subject to price rigidities. Together with wage rigidities, this friction is quite important in allowing demand shocks to be a source of business cycle fluctuations, as countercyclical mark-ups induce firms to produce less when demand is low. Inflation evolves in the model according to a standard, forward-looking New Keynesian Phillips curve, which determines inflation as a function of marginal costs, expected future inflation, and “mark-up” shocks. Mark-up shocks capture exogenous changes in the degree of competitiveness in the intermediate goods market. In practice, these shocks capture unmodeled inflation pressures, such as those arising from fluctuations in commodity prices.

Financial intermediation involves two actors, *banks* and *entrepreneurs*, whose interaction captures imperfections in financial markets. These actors should not be interpreted in a literal sense, but rather as a device for modeling credit frictions. Banks take deposits from

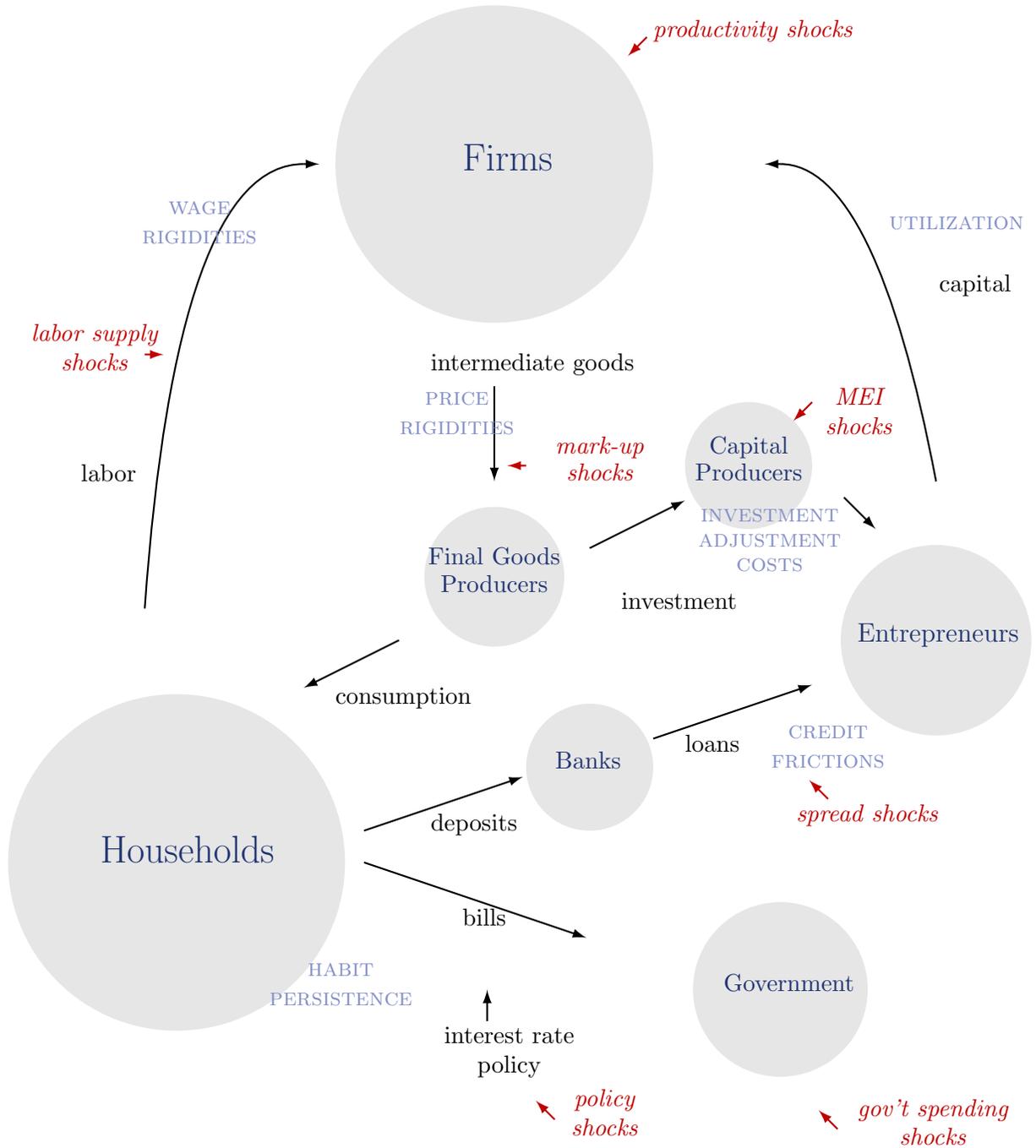
households and lend them to entrepreneurs. Entrepreneurs use their own wealth and the loans from banks to acquire capital. They then choose the utilization level of capital and rent the capital to intermediate good producers. Entrepreneurs are subject to idiosyncratic disturbances in their ability to manage the capital. Consequently, entrepreneurs' revenue may not be enough to repay their loans, in which case they default. Banks protect against default risk by pooling loans to all entrepreneurs and charging a spread over the deposit rate. Such spreads vary endogenously as a function of the entrepreneurs' leverage, but also exogenously depending on the entrepreneurs' riskiness. Specifically, mean-preserving changes in the volatility of entrepreneurs' idiosyncratic shocks lead to variations in the spread (since banks only focus on the downside risk). We refer to these exogenous movements as "spread" shocks. Spread shocks capture financial intermediation disturbances that affect entrepreneurs' borrowing costs. Faced with higher borrowing costs, entrepreneurs reduce their demand for capital, and investment drops. With lower aggregate demand, there is a contraction in hours worked and real wages. Wage rigidities imply that hours worked fall even more (because nominal wages do not fall enough). Price rigidities mitigate price contraction, further depressing aggregate demand.

Capital producers transform general output into capital goods, which they sell to the entrepreneurs. Their production function is subject to investment adjustment costs: producing capital goods is more costly in periods of rapid investment growth. It is also subject to exogenous changes in the "marginal efficiency of investment" (MEI). These MEI shocks capture exogenous movements in the productivity of new investments in generating new capital. A positive MEI shock implies that fewer resources are needed to build new capital, leading to higher real activity and inflation, with an effect that persists over time. Such MEI shocks reflect both changes in the relative price of investment versus that of consumption goods (although the literature has shown the effect of these relative price changes to be small), and most importantly financial market imperfections that are not reflected in movements of the spread.

Finally, the *government* sector comprises a monetary authority that sets short-term interest rates according to a Taylor-type rule and a fiscal authority that sets public spending and collects lump-sum taxes to balance the budget. Exogenous changes in government spending are called "government" shocks (more generally, these shocks capture exogenous movements

in aggregate demand). All exogenous processes are assumed to follow independent AR(1) processes with different degrees of persistence, except for i.i.d. “policy” shocks, which are exogenous disturbances to the monetary policy rule.

Figure 1: Model Structure



The Model's Interpretation of the Great Recession

The Great Recession was characterized by a severe financial crisis that impaired the flow of credit, depressing aggregate demand and employment. The presence of credit intermediation frictions enables the FRBNY model to capture the majority of these events, attributing the economic downturn primarily to one shock, the spread shock, which reflects higher perceived riskiness of borrowers and which causes disruptions in financial intermediation. The model identifies this shock by matching the behavior of the Baa corporate bond rate over 10-year Treasuries, and the spread's comovement with output growth, inflation, and the other observables. Figure 4 plots the standardized innovations (that is, the innovations measured in standard deviation units) of the shocks in the model, from 2007 on. The figure shows that realizations of the spread shock are indeed positive throughout the Great Recession, with large spikes in late 2007 and particularly in 2008Q4, in the aftermath of the Lehman episode.

Recall that spread shocks work through the model by increasing the expected cost of capital and reducing entrepreneurs' borrowing, hence decrease their capital accumulation and their ability to channel resources to the productive sector. Figure 5 shows the impulse responses of the variables used in the estimation to a one-standard-deviation shock. A shock of this size increases the spread by roughly 35 basis points (bottom right panel). This leads to a reduction in investment and consequently to a prolonged reduction in output growth (top left panel) and hours worked (top right panel). The drop in economic activity due to the spread shock leads to a prolonged decline in real marginal costs - which in this model map one-to-one into the labor share (middle left panel)- and, via the New Keynesian Phillips curve, in inflation (middle right panel). Finally, policymakers endogenously respond to the change in the inflation and real activity outlook by cutting the federal funds rate (bottom left panel).

Figure 2 quantifies the importance of each shock for output growth, core PCE inflation, and the federal funds rate (FFR) from 2007 on. Specifically, in each of the three panels the solid line (black for realized data, red for mean forecast) shows the variable in deviation from its steady state (for output, the numbers are per capita, as the model takes population growth as exogenous; for both output and inflation, the numbers are quarter-to-quarter annualized). The bars represent the contribution of each shock to the deviation of the variable

from steady state, that is, the counterfactual values of output growth, inflation, and the federal funds rate (in deviations from the mean) obtained by setting all other shocks to zero. By construction, for each observation the bars sum to the value on the solid line. The figure shows that spread shocks (dark purple) indeed capture most of the drop in output growth and inflation during the recession.

Other shocks also contributed to the Great Recession. Figure 2 shows that TFP shocks (dark red bars) played an important role in the decline in output, particularly in 2008. Figure 4 shows that TFP shocks indeed were largely negative during this period. However, productivity shocks cannot fully account for the Great Recession because a drop in productivity leads to an increase in inflation, rather than the decline that was observed. This is evident from Figure 6, which shows the impulse responses to a one-standard-deviation positive TFP shock. Conversely, one can see that a one-standard deviation negative TFP shock leads to a substantial drop in output but also to an increase in inflation. Moreover, because of nominal rigidities, the impact response of hours worked to the negative shock in productivity is very small, if not positive.

“Labor supply” shocks are also not a major driver of the Great Recession, as they cannot replicate the observed comovement between inflation and output during this episode. Figure 7 shows that positive labor supply shocks (exogenous inward shifts in labor supply, possibly due to unmodeled labor market imperfections) lead to a decline in output and hours, but to an increase in inflation. This is because marginal costs for firms rise following a contraction in labor supply.

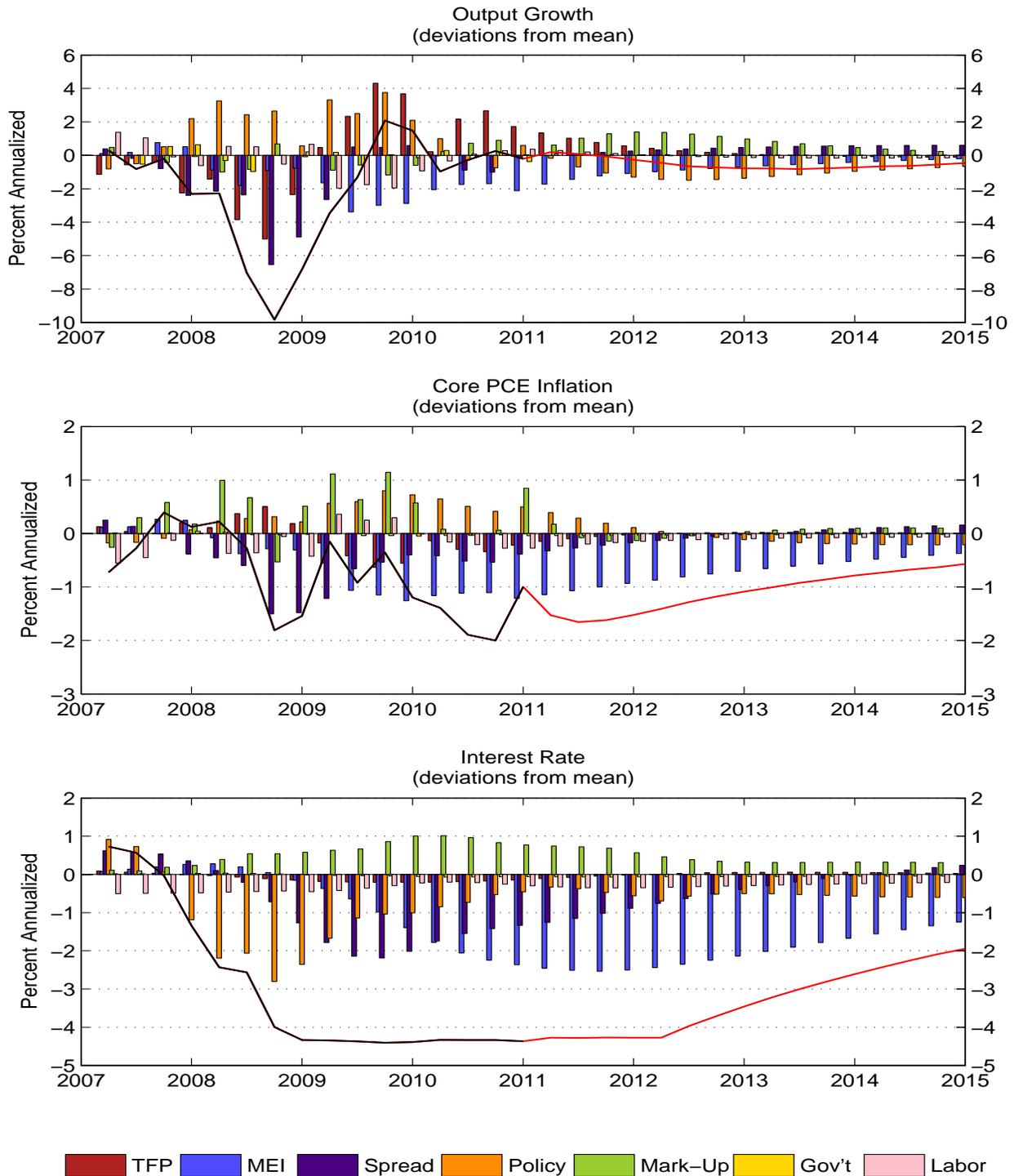
While the model attributes a smaller role to MEI shocks at the onset of the Great Recession, these shocks did have a negative impact on output growth in 2008Q3 and an even stronger and more persistent negative impact in the second half of 2009. MEI shocks also contributed importantly to reductions in the inflation rate from 2009Q2 on.

In summary, the model attributes the sharp decline in output growth that occurred in 2008 primarily to an increase in the cost of capital, together with negative productivity shocks that shifted the production function of intermediate goods. Later on, toward the official end of the recession and through the initial phase of the recovery, while the finan-

cial system was stabilizing, MEI shocks constituted the principal drag on economic activity. While spread shocks had a strong negative effect on inflation in late 2008 and early 2009, the model attributes to MEI shocks the main role in keeping inflation low from 2009Q2 on. The delay in the inflation decline appears to be due to the effect of positive shocks to price setters' mark-ups at the onset of the recession.

Finally, monetary policy shocks played an important countervailing role during the recession (see the orange bars in Figure 2). The contraction of inflation and output growth was not enough to explain the reduction in the federal funds rate observed in the recession. Hence, the model identifies a series of negative monetary policy shocks as the primary drivers of the rate's sharp decline by the end of 2008. The large drop in interest rates boosted output growth and, albeit with a lag, had a substantial effect on inflation.

Figure 2: Shock Decomposition



The solid lines (black for realized data, red for mean forecast) show each variable in deviation from its steady state. The bars represent the shock contributions; specifically, the bars for each shock represent the counterfactual values for the observables (in deviations from the mean) obtained by setting all other shocks to zero.

Forecasts

The model forecasts are mainly driven by the negative 'headwinds' from the financial crisis. As credit spreads normalize at the end of 2009, the effects of spread shocks, the main driver of the recession, subside and headwinds are captured primarily by the shocks to the efficiency of investment. Financial headwinds keep output below potential and inflation subdued throughout the forecast horizon, while the interest rate increase only gradually after a liftoff in 2012Q2.

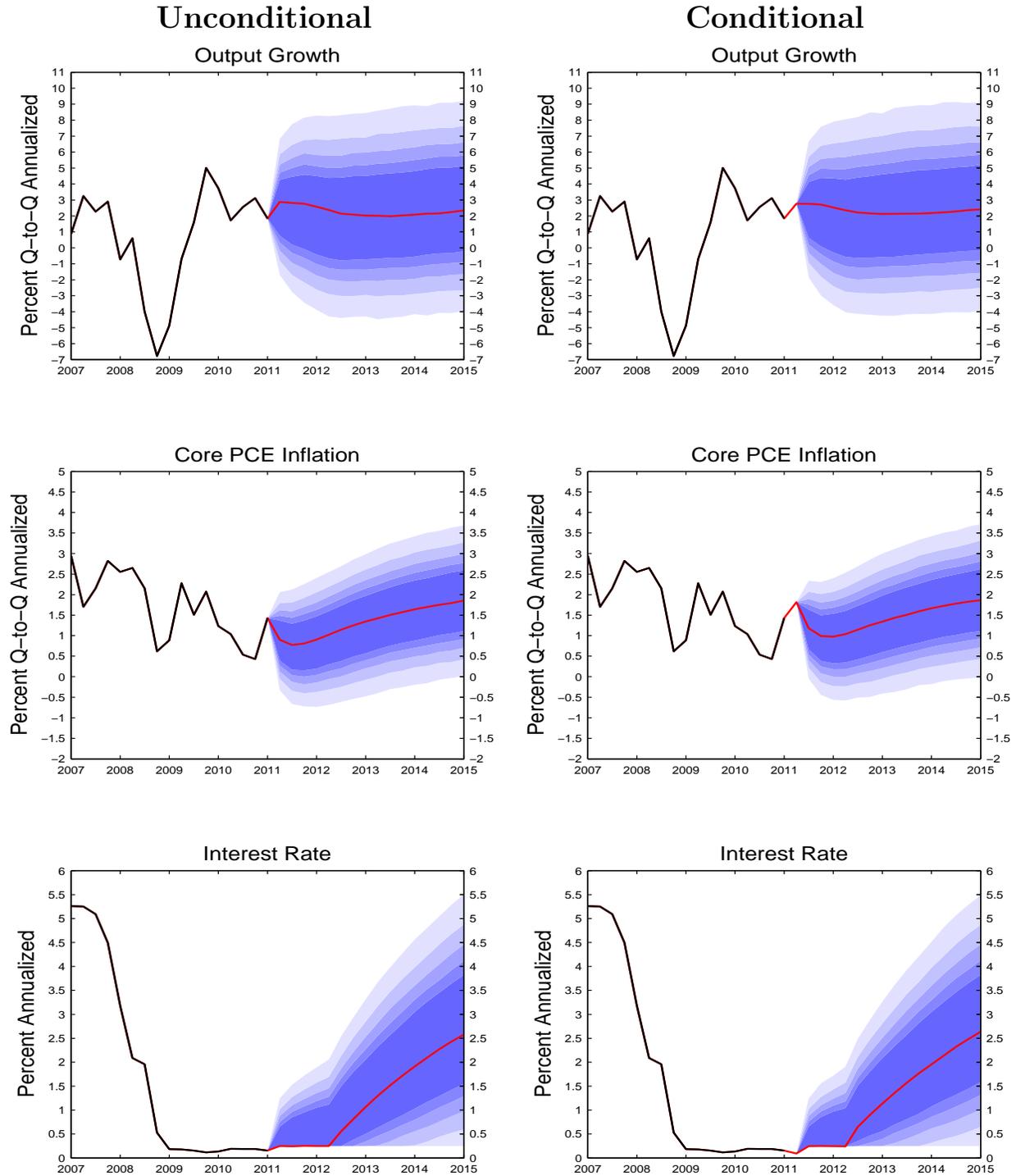
We detail the forecast of three main variables over a three-year horizon: real GDP growth, core PCE inflation and the federal funds rate. The federal funds rate is constrained to be near zero through 2012Q2, and follows the estimated policy rule afterwards. We impose the lower bound on the nominal interest rate by adding anticipated monetary policy shocks to the central bank's reaction function, following Laseen and Svensson (2009).

	Unconditional Forecast					
	2011 (Q4/Q4)		2012 (Q4/Q4)		2013 (Q4/Q4)	
	June	May	June	May	June	May
Core PCE Inflation	0.8	0.9	1.2	1.2	1.5	1.5
	(0.2,1.4)	(0.2,1.5)	(0.3,1.9)	(0.3,1.9)	(0.6,2.4)	(0.6,2.4)
Real GDP Growth	2.7	2.6	2.1	2.1	2.0	2.0
	(-0.2,4.8)	(-0.3,4.7)	(-1.3,5.0)	(-1.4,4.9)	(-1.2,5.4)	(-1.2,5.3)

	Conditional Forecast*					
	2011 (Q4/Q4)		2012 (Q4/Q4)		2013 (Q4/Q4)	
	June	May	June	May	June	May
Core PCE Inflation	1.2	1.3	1.2	1.3	1.5	1.6
	(0.7,1.7)	(0.8,1.7)	(0.3,1.9)	(0.5,2.0)	(0.6,2.4)	(0.7,2.4)
Real GDP Growth	2.7	3.3	2.2	2.1	2.1	2.0
	(0.5,4.2)	(1.1,4.9)	(-1.1,5.0)	(-1.3,4.9)	(-1.0,5.5)	(-1.2,5.3)

*The conditional forecasts treat the FRBNY staff forecast for output, inflation, and hours worked for 2011Q2 as data. Numbers in parentheses indicate 68 percent probability intervals.

Figure 3: Forecasts



Black lines indicate data, red lines indicate mean forecasts, and shaded areas mark the uncertainty associated with our forecast as 50, 60, 70, 80, and 90 percent probability intervals.

The table below presents Q4/Q4 forecasts for real GDP growth and inflation for 2011-2013, with 68 percent probability intervals. We include two sets of forecasts. The *unconditional* forecasts use data up to 2011Q1, the quarter for which we have the most recent GDP release. In the *conditional* forecasts, we also include the 2011Q2 FRBNY staff projections for GDP growth, PCE inflation, and hours worked as additional data points (the staff projections for 2011Q2 are 2.8% for output growth, 1.8% for core PCE inflation, and 2.1% growth for hours worked). Treating the staff forecasts as data allows us to incorporate into the DSGE forecasts information about the current quarter that is not yet available in the data. In addition to providing the current forecasts (June), for comparison we report our earlier forecasts made in May using data from the advance GDP release for 2011Q1.

Figure 3 presents quarterly forecasts, both unconditional (left panels) and conditional (right panels). In the graphs, the black line represents data, the red line indicates the mean forecast, and the shaded areas mark the uncertainty associated with our forecast as 50, 60, 70, 80 and 90 percent probability intervals. Output growth and inflation are expressed in terms of percent annualized rates, quarter to quarter. (Note that, unlike in figure 2, where the decomposition in the top graph refers to per capita output growth, the forecast is for aggregate output growth.) The interest rate is the annualized quarterly average. The bands reflect both parameter uncertainty and shock uncertainty.

Our forecasts project real GDP growth to be 2.8% in 2011Q2 and about 2.7% for the second half of 2011, while inflation is forecast to be near 1.0% in 2011Q2 and to remain well below 2% throughout the forecast horizon. There is significant uncertainty around the forecasts: the 25th percentile of the output growth forecast distribution is around 0% through the forecast horizon, while the 75th percentile is about 4.5%. There is less uncertainty about inflation: in the short run, the 50% bands range from 0.2% to 1.6%, and from 0.9% to 2.4% throughout the forecast horizon. The likelihood of negative readings of core PCE inflation in 2011Q2 is about 15%. Notice that the staff forecast suggests higher real GDP growth and inflation for 2011Q2 relative to the unconditional forecast, but other than that the conditional and unconditional forecasts are similar.

Interpreting the Forecasts

To understand the forecasts, we return to the shock decomposition shown in Figure 2 which describes the extent to which each of the disturbances contributes to keeping the variables from their long-run equilibrium. As of 2011Q1, the figure shows that both the inflation and interest rate variables are still below their steady-state values, while output *growth* is currently roughly at steady state (recall that the figure shows *per capita* output growth in deviations from steady state).

The major shocks responsible for the Great Recession also play a prominent role in the recovery and in the forecasts, given the persistence of their effects. Recall that Figure 5 shows that spread shocks have highly persistent effects on output growth, fading after about 8 quarters, and especially on inflation, lasting longer than 3 years. These shocks also have persistent effects on aggregate hours worked and the interest rate. MEI shocks have similarly persistent effects on output and inflation. The estimated effects of TFP shocks are not as persistent, lasting about a year and a half (see Figure 6).

Looking first at inflation, we see that the model forecasts a subdued path, despite the pick-up observed in the first quarter. Core inflation is forecast to decline in the near term before resuming a moderate increase; nonetheless, it remains below steady state for the entire forecast horizon. The main drag on the inflation forecasts comes from MEI shocks. Recall that the model identifies a string of very negative realizations of MEI shocks from late 2008 through late 2009 (see Figure 4). We can interpret negative MEI shocks as capturing headwinds from the financial crisis, which persistently depress both real activity and inflation. These shocks thus imposed a significant constraint on output's recovery, and their lingering effects will continue to restrain growth and core inflation throughout 2011 and 2012. Past spread shocks will also contribute to holding inflation below its steady-state value in the next few quarters, although to a lesser extent.

While MEI, spread, and TFP shocks affect inflation indirectly in the model, by influencing marginal costs, mark-up shocks affect inflation directly. Mark-up shocks have large positive effects on inflation on impact, but they are relatively short lived, with effects disappearing about 2 to 3 quarters after the shock (see Figure 10). As a result, the large positive mark-up shock behind the up-tick in inflation in the recent quarter has almost no effect on

our inflation forecasts. The recent sharp increase in commodity prices, which the model captures via the mark-up shock, is therefore forecast not to have a significant effect on the core PCE deflator, assuming that we will not continue to observe large increases in these prices. In the near term the main shock that is forecast to push inflation upward is the monetary policy shock, reflecting the very low level of short-term interest rates. Importantly, we impose the condition that expectations for the nominal interest rate are at the lower bound (25 basis points) through 2012Q2 by adding anticipated monetary policy shocks to the central bank's reaction function. The contribution of policy shocks therefore incorporates both contemporaneous and anticipated policy shocks.

Turning to output growth, the model attributes to TFP shocks, as well as to continued accommodative policy shocks, the turnaround of real growth in the second half of 2009 (see Figure 2). Figure 4 (upper left panel) shows that a string of positive productivity disturbances occurred in 2009 and again at the end of 2010. While the effects of TFP shocks tend to dissipate after about a year and a half, they have the largest impact on output growth (a one-standard-deviation shock raises growth by more than 1.5% above steady state on impact). Hence, productivity was not only the primary driver of the recovery in output growth in the second half of 2009, but the model also suggests that past TFP shocks working through the transmission mechanism are the main positive force behind output growth in 2011. In addition, forecasts of output growth appear sustained by past price mark-up shocks in the near term. Price mark-up shocks have been largely positive since 2009 and their effect on output growth, although negative on impact, turns slightly positive after about a year, as the level of output tends to return to its steady-state path. The model attributes a positive role to these shocks in 2012 and 2013.

The positive contribution of technology and mark-up shocks to output growth will offset the lingering drag of MEI shocks. As observed above, MEI shocks were largely negative from late 2008 through late 2009; given these shocks' persistent effects, our forecasts indicate that they will continue to restrain growth throughout 2011 and 2012. In addition, policy shocks are responsible for some negative pressure on output growth from the end of 2011 through the forecast horizon. While this result may seem counter-intuitive, it is the consequence of the fact that the impact of expansionary monetary policy on the *level* of output, while still positive, is fading, implying that the effect on the *growth rate* is currently negative. This is

partly because the stimulative effect of the “extended period” language is front-loaded, and hence had most impact in 2009.

The decomposition of the expected federal funds rate path shows that spread and MEI shocks are again behind the deviation of future interest rates from steady state over the forecast horizon. This is not surprising, since movements in the federal funds rate are largely driven by movements in inflation. Hence, the same shocks that drive inflation affect the federal funds rate.

Figure 4: Shock Histories

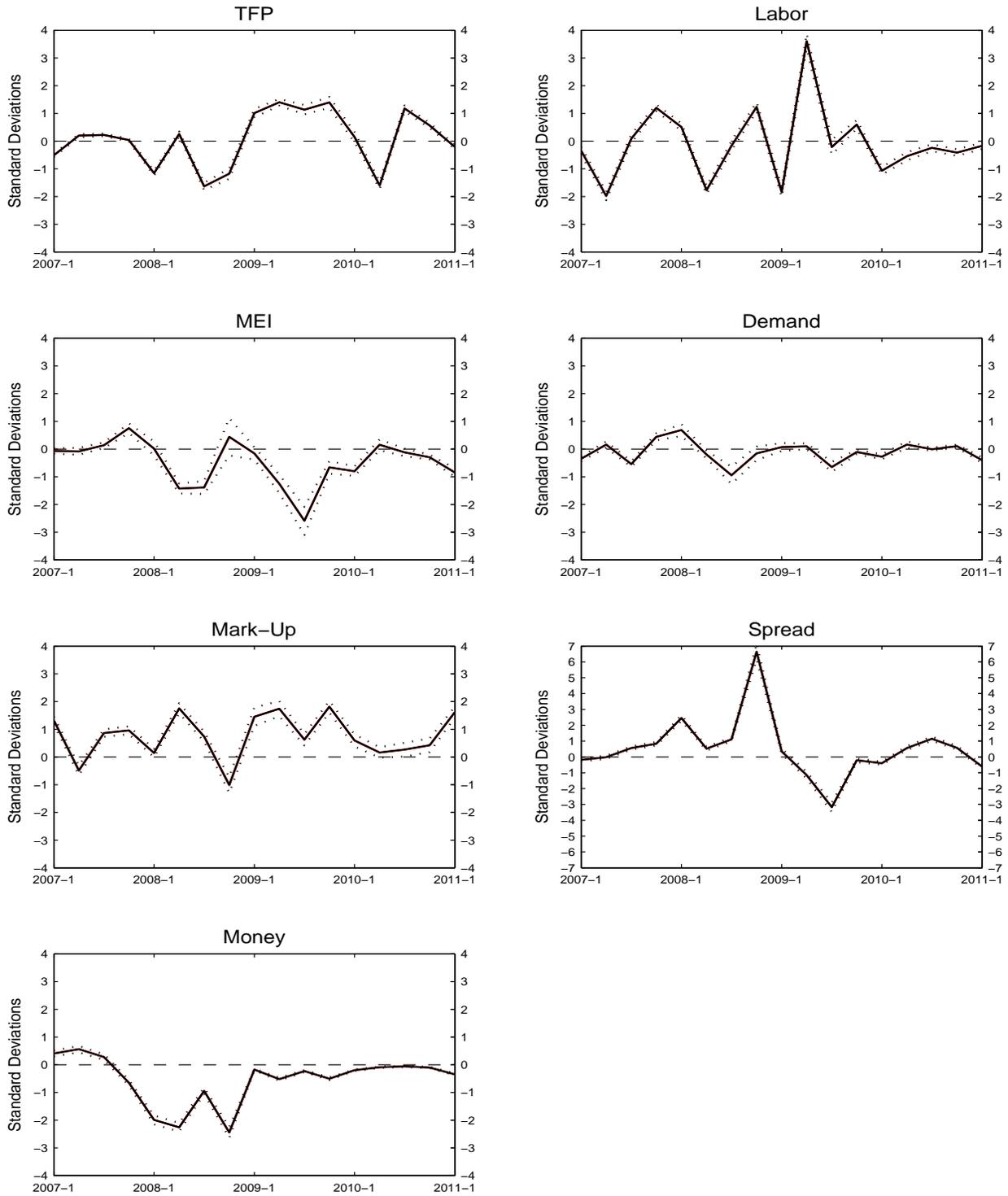


Figure 5: Responses to a Spread Shock

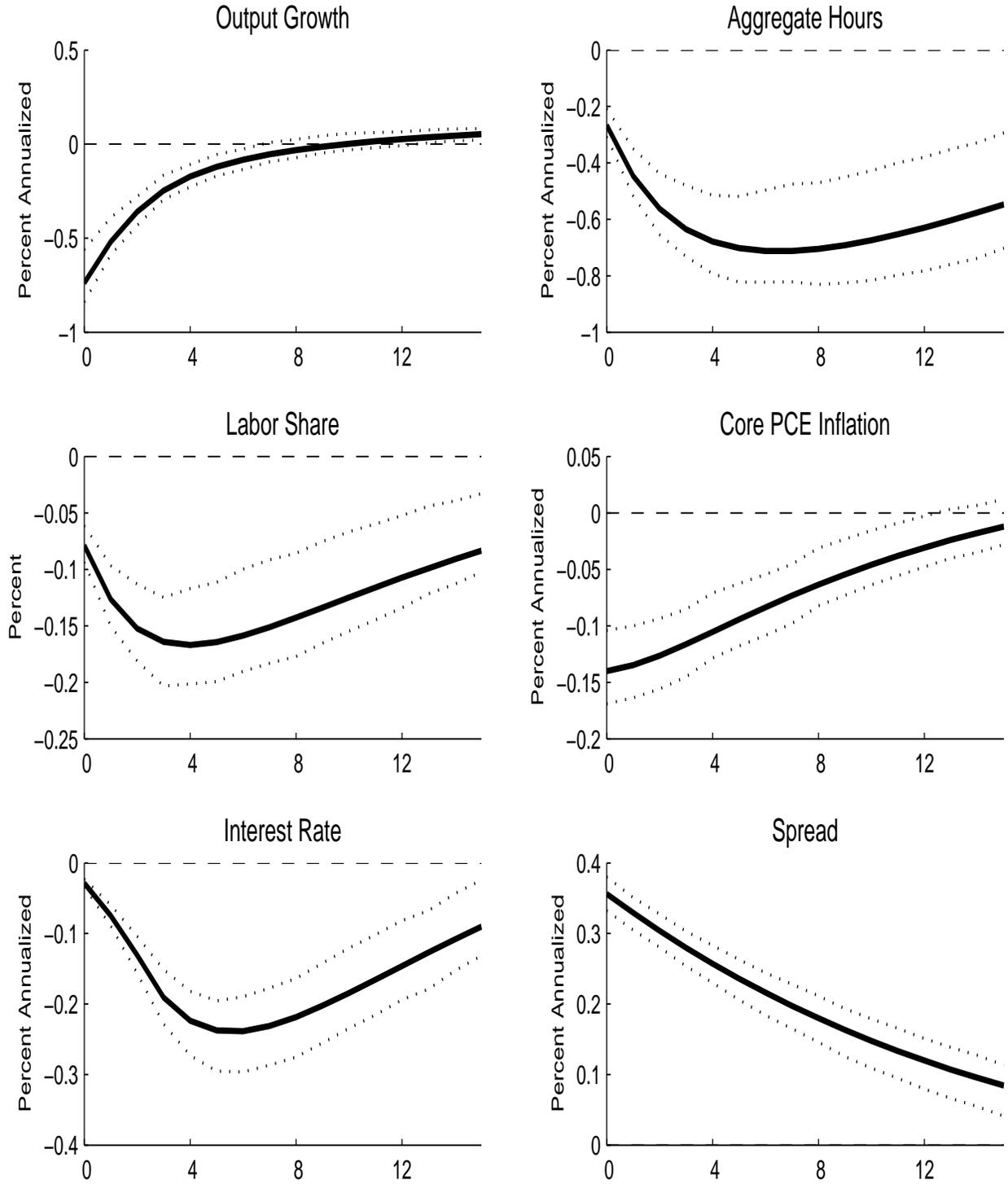


Figure 6: Responses to a TFP Shock

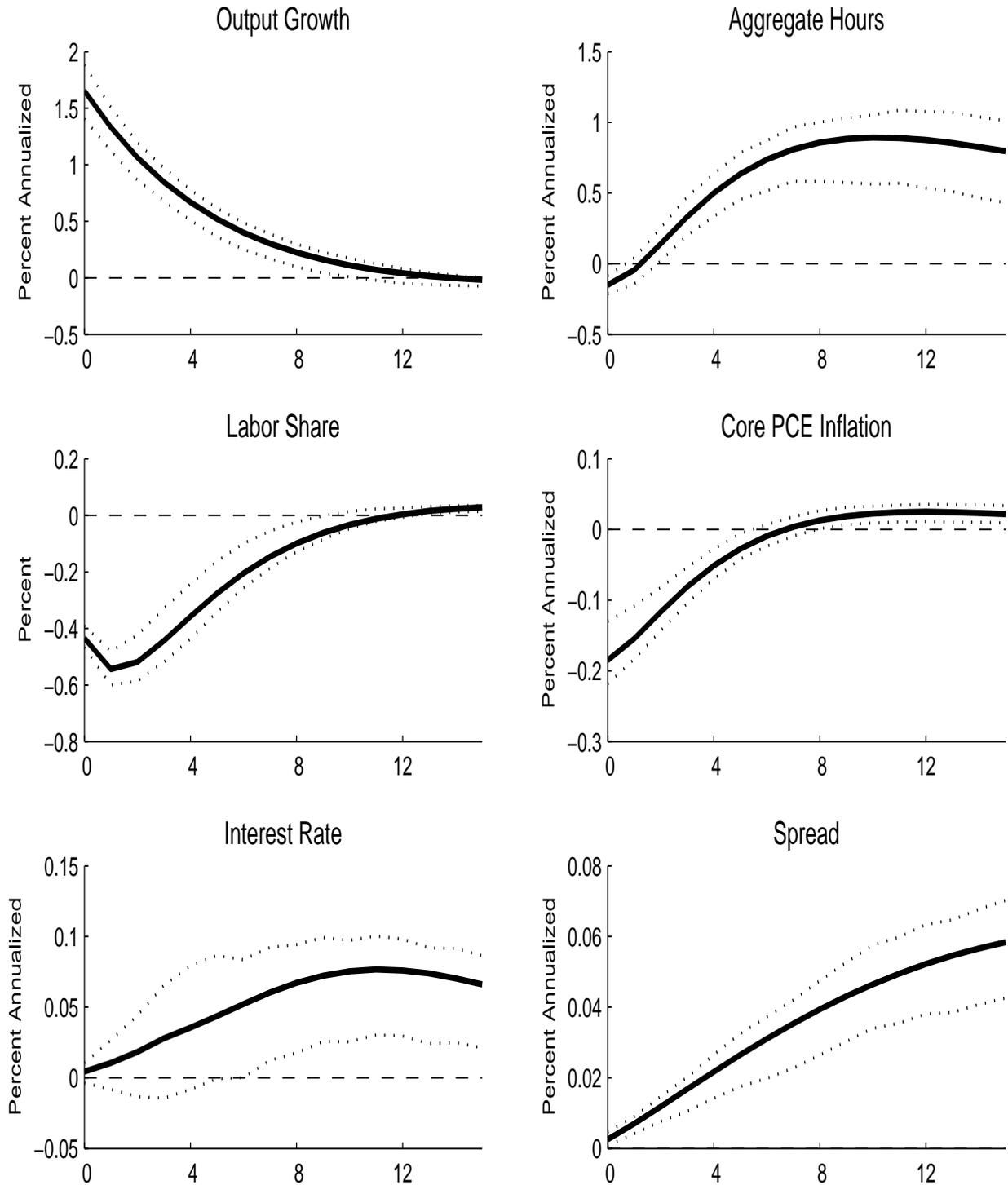


Figure 7: Responses to a Labor Supply Shock

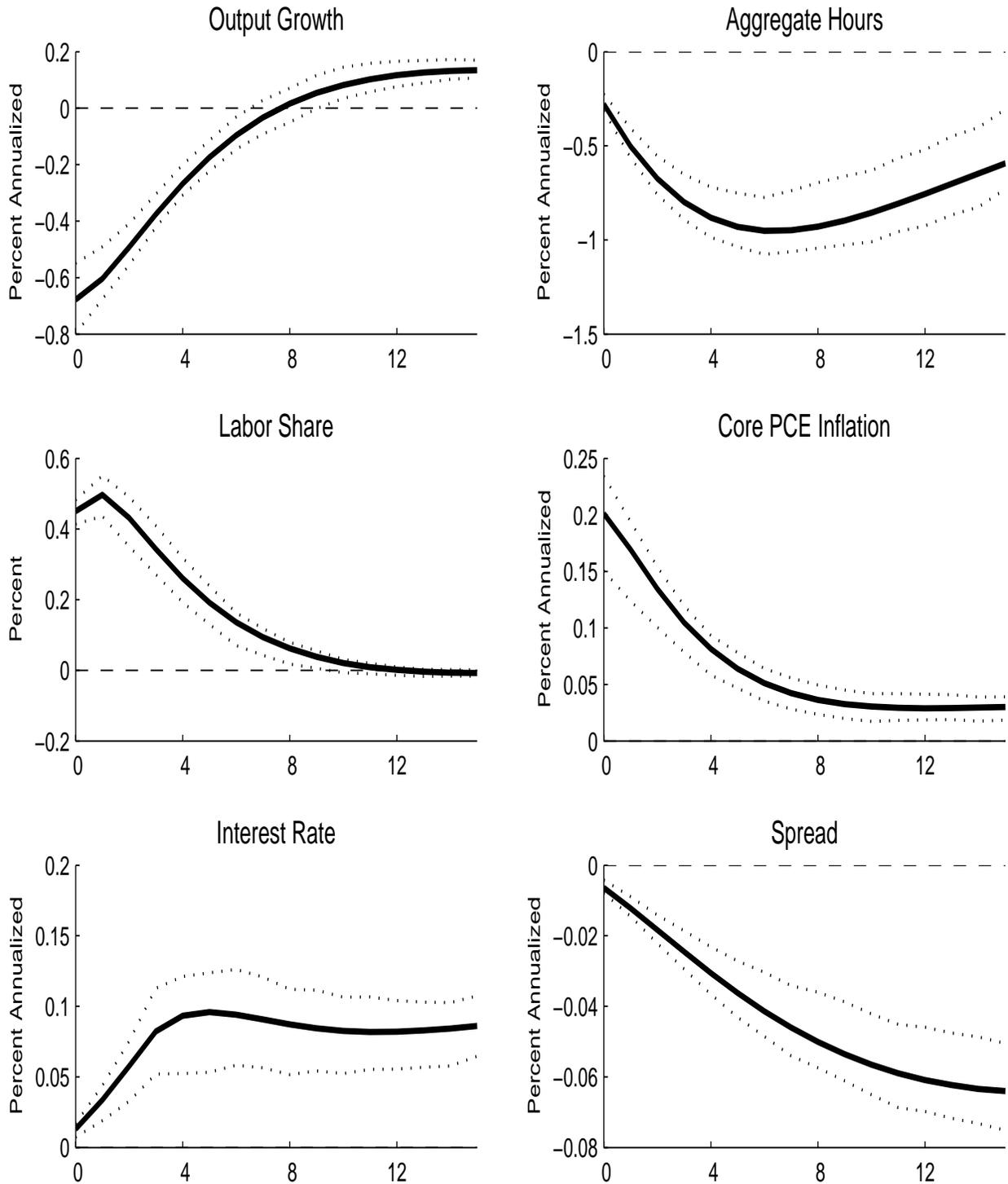


Figure 8: Responses to a Monetary Policy Shock

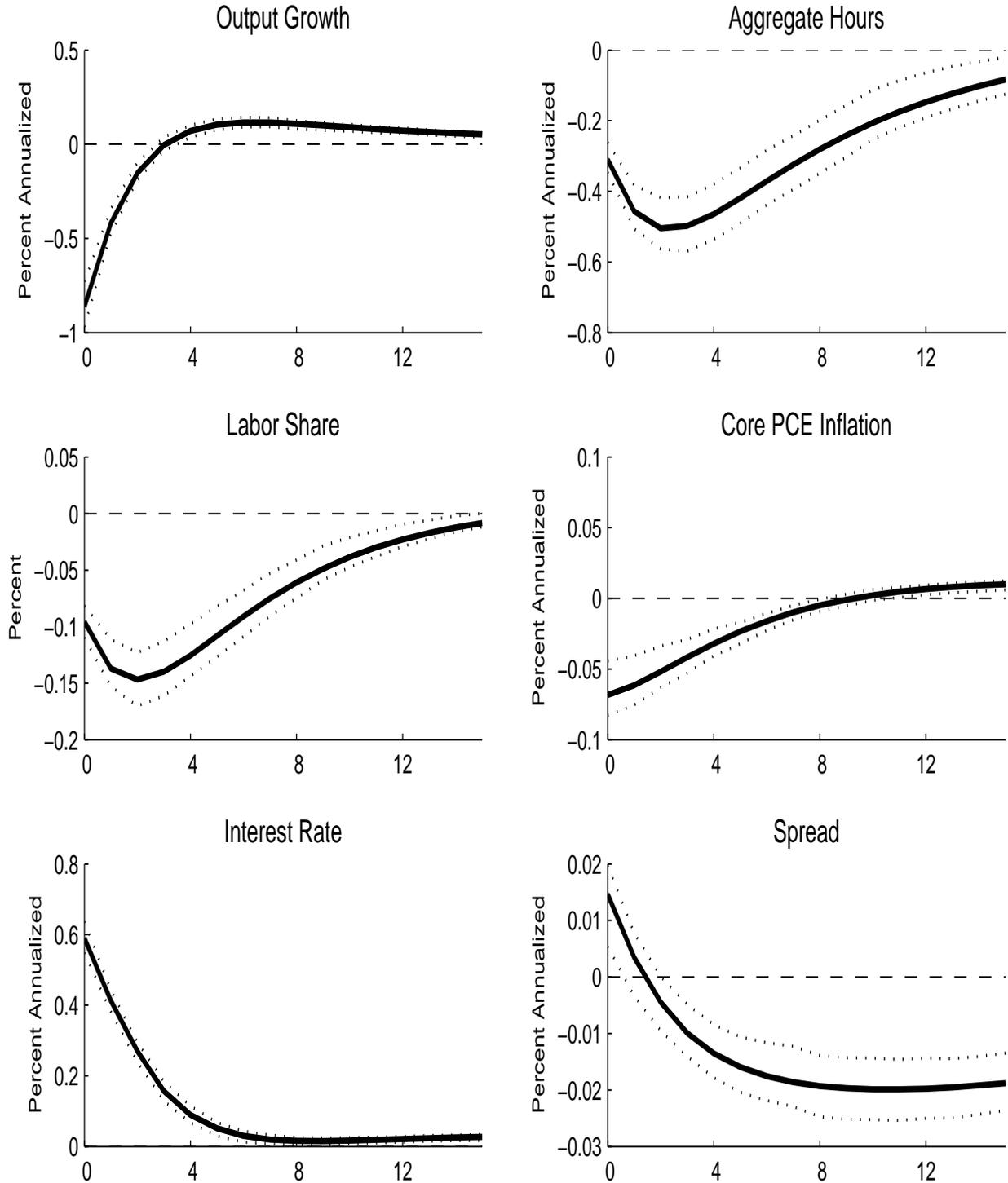


Figure 9: Responses to an MEI Shock

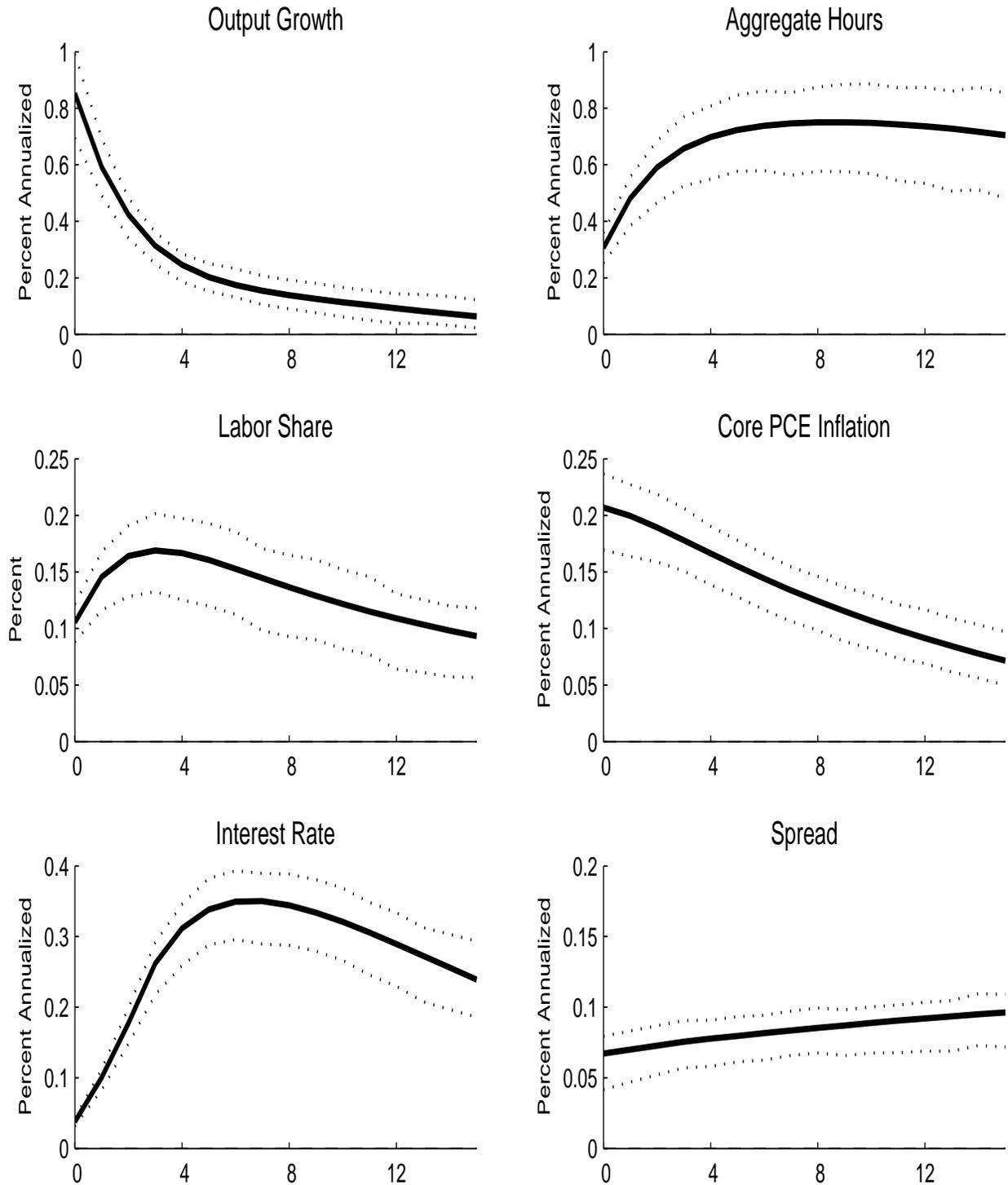


Figure 10: Responses to a Mark-up Shock

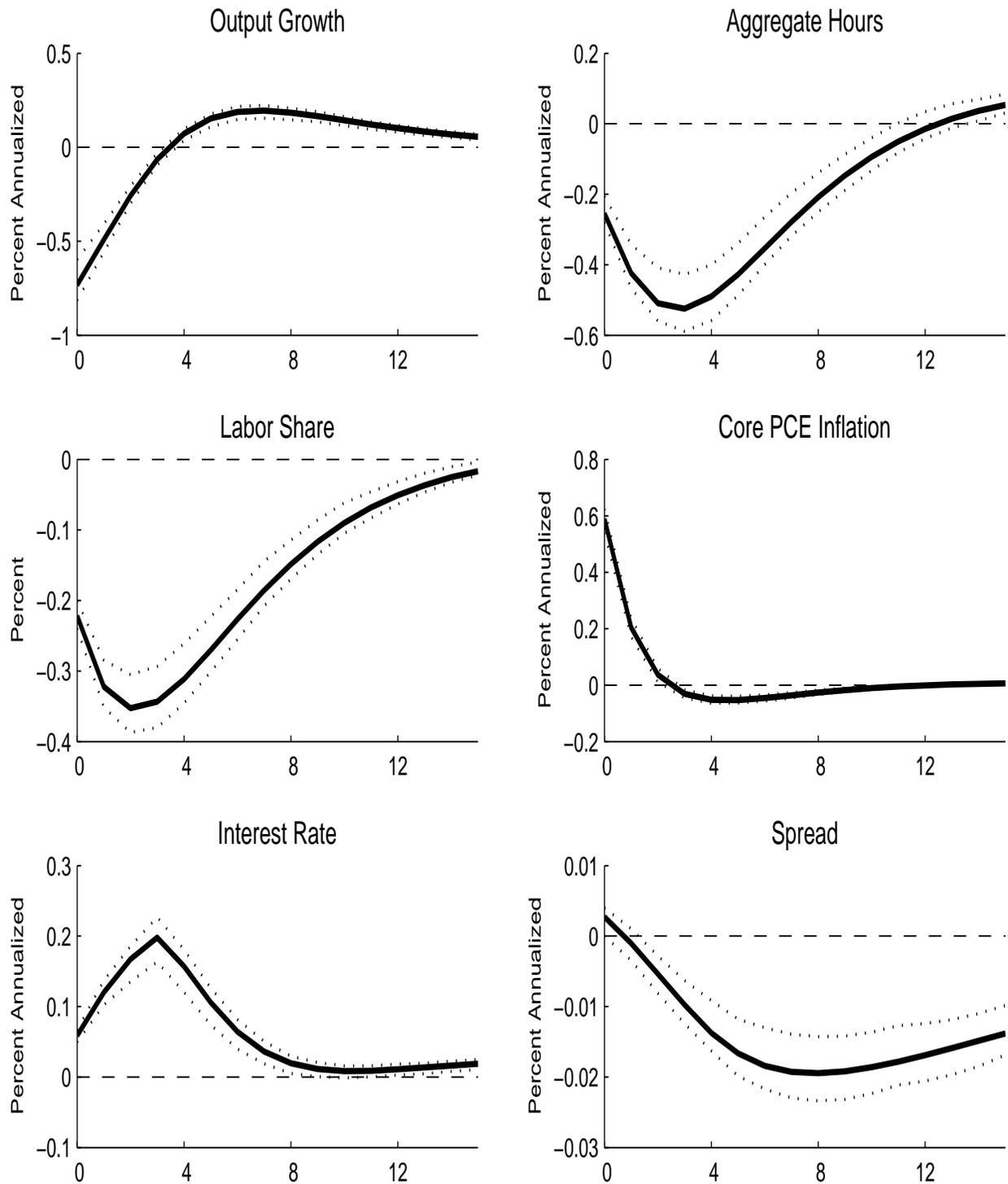
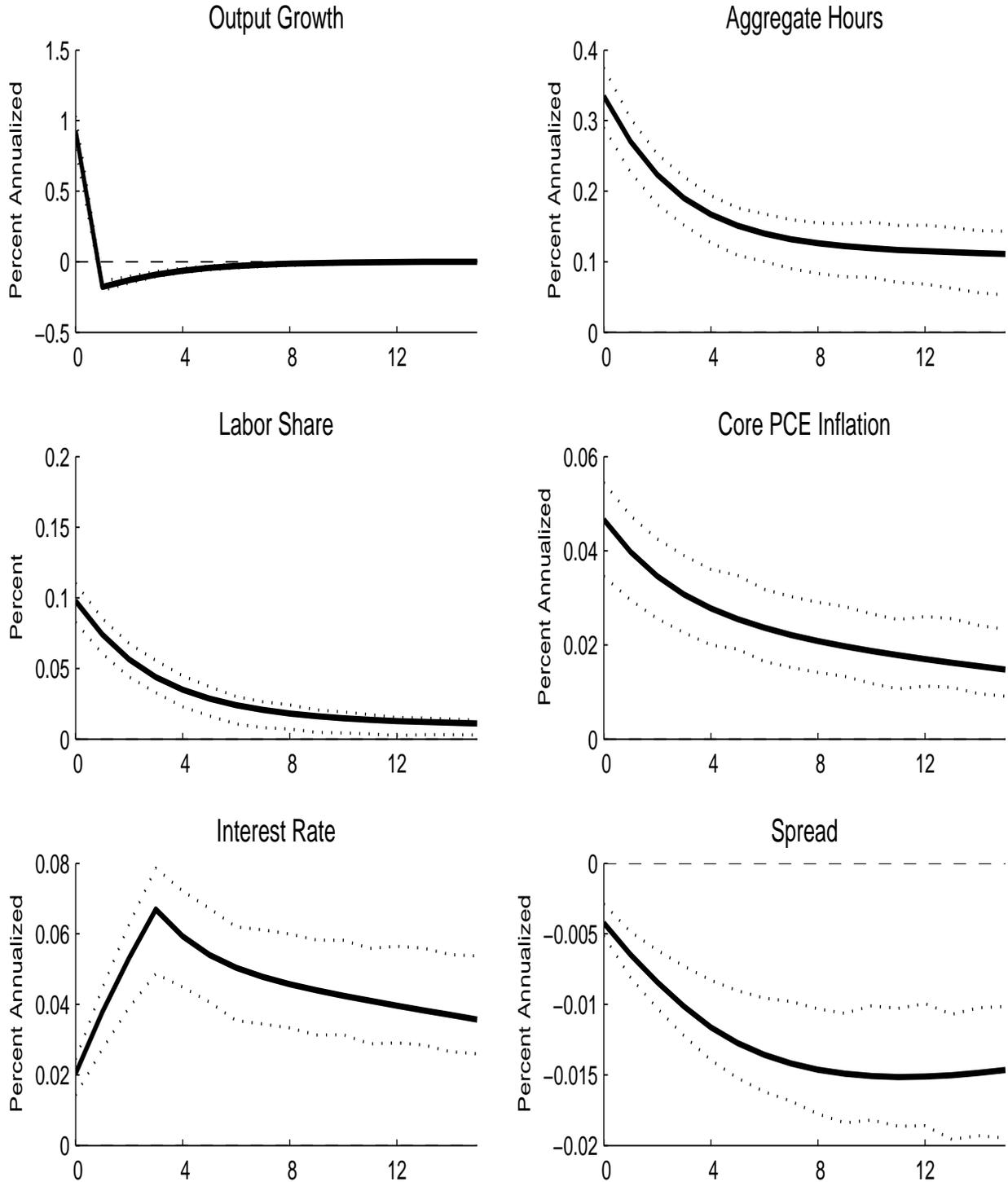


Figure 11: Responses to a Government Spending Shock



References

- [1] **Bernanke, Ben, Mark Gertler and Simon Gilchrist**, “The Financial Accelerator in a Quantitative Business Cycle Framework,” in J.B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*, vol. 1C, Amsterdam: North-Holland, 1999.
- [2] **Calvo, Guillermo**, “Staggered Prices in a Utility-Maximizing Framework,” *Journal of Monetary Economics*, 1983, *12*, 383–398.
- [3] **Christiano, Lawrence, Martin Eichenbaum, and Charles Evans**, “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy,” *Journal of Political Economy*, 2005, *113*, 1–45.
- [4] **Christiano, Lawrence, Roberto Motto, and Massimo Rostagno**, “Financial Factors in Economic Fluctuations,” Unpublished, 2009.
- [5] **Laseen, Stefan and Lars E. O. Svensson**, “Anticipated Alternative Instrument-Rate Paths in Policy Simulations,” NBER Working Paper No. w14902, 2009.
- [6] **Smets, Frank and Raphael Wouters**, “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” *American Economic Review*, 2007, *97* (3), 586 – 606.

FRBNY DSGE Model Documentation

1 The DSGE Model

1.1 General Structure

The FRBNY DSGE model is a medium scale, one-sector dynamic stochastic general equilibrium model. It builds on the neo-classical growth model by adding nominal wage and price rigidities, variable capital utilization, costs of adjusting investment, habit formation in consumption, and credit frictions. The core of the model is based on the work of Smets and Wouters (2007) and Christiano, Eichenbaum, and Evans (2005), but the model also includes credit frictions as in the *financial accelerator* model developed by Bernanke, Gertler and Gilchrist (1999). The actual implementation of the credit frictions follows closely Christiano, Motto and Rostagno (2009). In this section we describe the problem faced by each of the agents, in order to emphasize the micro-foundations of the model and the role of the exogenous processes. The innovations to these processes, which we refer to as ‘shocks’, are the drivers of macroeconomic fluctuations.

The model economy is populated by eight classes of agents, whose behaviour we describe subsequently: 1) a continuum of households, who consume and supply differentiated labor; 2) competitive labor aggregators that combine labor supplied by individual households; 3) competitive final good-producing firms that aggregate the intermediate goods into a final product; 4) a continuum of monopolistically competitive intermediate good producing firms; 5) competitive capital producers that convert final goods into capital; 6) a continuum of entrepreneurs who purchase capital using both internal and borrowed funds and rent it to intermediate good producing firms; 7) a representative bank collecting deposits from the households and lending funds to the entrepreneurs; and finally 8) a government, composed of a monetary authority that sets short-term interest rates and a fiscal authority that sets public spending and collects taxes.

Households and labor aggregators. There is a continuum of households indexed by $j \in [0, 1]$. Households have identical preferences, which are separable in consumption, leisure, and real money balances. The household objective function is

$$E_t \sum_{s=0}^{\infty} \beta^s \left[\log(C_{t+s}(j) - hC_{t+s-1}(j)) - \frac{\varphi_{t+s}}{1 + \nu_l} L_{t+s}(j)^{1+\nu_l} + \frac{\chi}{1 - \nu_m} \left(\frac{M_{t+s}(j)}{Z_{t+s} P_{t+s}} \right)^{1-\nu_m} \right], \quad (1)$$

FRBNY DSGE Model Documentation

where $C_t(j)$ is consumption, $L_t(j)$ is labor supply (total available hours are normalized to one), and $M_t(j)$ is money holdings. Households' preferences display habit persistence in consumption, captured by the parameter h . Real money balances enter the utility function deflated by the (stochastic) trend growth of the economy Z_t , so to make real money demand stationary. Since money balances enter separably, and we do not use money balances as an observable in estimating the model, the “demand for money” is irrelevant for the dynamics of the model, and we will subsequently ignore it.

Households' preferences are subject to a stochastic preference shifter φ_t affecting the marginal utility of leisure, which we assume follows an AR(1) process. We refer to innovations to this process as *labor supply shocks*.

Household j chooses $\{C_t(j), L_t(j), M_t(j), B_t(j), D_t(j)\}_{t=0}^{\infty}$ to maximize the expected utility (1) subject to the following budget constraint, written in nominal terms

$$\begin{aligned}
 P_{t+s}C_{t+s}(j) + B_{t+s}(j) + D_{t+s}(j) + M_{t+s}(j) &\leq R_{t+s}B_{t+s-1}(j) + R_{t+s}^d D_{t+s-1}(j) \\
 + M_{t+s-1}(j) + \Pi_{t+s} + W_{t+s}(j)L_{t+s}(j) + \mathcal{T}_{t+s}, & \tag{2}
 \end{aligned}$$

where $B_t(j)$ is holdings of government bonds and $D_t(j)$ is holdings of deposits in the banking sector. R_t is the gross nominal interest rate on government bonds, R_t^d is the gross nominal interest rate on bank deposits, Π_t is the per-capita profit the household receives from owning the firms (we assume that households pool their firm shares so that they all receive the same profits), $W_t(j)$ is the nominal wage, \mathcal{T}_t are transfers (or taxes, if negative) from the government, and T_t are net per-capita lump-sum transfers from the entrepreneurs (discussed later). We also assume that households have assets to the full menu of state-contingent securities. To simplify notation we do not explicitly add the state contingent securities to the household's budget constraint.

Because households have assets to the full menu of state-contingent securities, all households will make the same choice of consumption and money demand. However, we assume wage rigidity a la Calvo, which implies that chosen leisure and wage will differ across households.

Labor used by the intermediate goods producers, L_t , is a composite of labor services supplied by the households. We assume that there exist competitive labor aggregators (which we also refer to as “employment agencies”) who buy labor from households and combine it

into an aggregate L_t , which they sell to the intermediate goods producers

$$L_t = \left[\int_0^1 L_t(j)^{\frac{1}{1+\lambda_w}} dj \right]^{1+\lambda_w}, \quad (3)$$

where $\lambda_w \in (0, \infty)$ affects the elasticity of substitution between differentiated labor services.

From the FOC of the employment agencies' problem one obtains a labor demand schedule for labor services of household j :

$$L_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\frac{1+\lambda_w}{\lambda_w}} L_t, \quad (4)$$

where $W_t(j)$ is the wage associated with labor services $L_t(j)$ and W_t is the aggregate wage, defined as

$$W_t = \left[\int_0^1 W_t(j)^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}. \quad (5)$$

Every household has market power in choosing its nominal wage subject to the demand constraint (4). However, nominal wage rigidity à la Calvo implies that it can only readjust wages with probability $1 - \zeta_w$ in each period. The households that cannot optimally reset their wages increase $W_t(j)$ at the steady state rate increase in aggregate wages (equal to steady state inflation π^* times the growth rate of the economy e^γ). The problem of the households that can set an optimal wage is to choose a wage $\widetilde{W}_t(j)$ to maximize:

$$E_t \sum_{s=0}^{\infty} (\zeta_w \beta)^s b_{t+s} \left[-\frac{\varphi_{t+s}}{1 + \nu_l} L_{t+s}(j)^{1+\nu_l} \right], \quad (6)$$

subject to the budget constraint (2), the labor demand equation (4), and the evolution of its chosen wage

$$W_{t+s}(j) = (\pi^* e^\gamma)^s \widetilde{W}_t(j) \quad (7)$$

for $s = 1, \dots, \infty$.

Final good producers. The competitive final good producing firms combine intermediate goods $Y_t(i)$ using the technology

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{f,t}}} di \right]^{1+\lambda_{f,t}}. \quad (8)$$

Profit maximization implies that the demand for intermediate goods is

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_{f,t}}{\lambda_{f,t}}} Y_t. \quad (9)$$

Combining condition (9) with the zero profit condition, the price of the composite good is obtained as

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda_{f,t}}} di \right]^{-\lambda_{f,t}}. \quad (10)$$

We indicated by $\lambda_{f,t}$ the desired mark-up that intermediate goods producers would like to charge over marginal costs. This mark-up is time varying: we assume that it follows an AR(1) process and refer to innovations to this process as *mark-up shocks*. As monopolists, optimizing firms would charge a higher markup if facing a more rigid demand (a higher $\lambda_{f,t}$), leading to higher prices.

Intermediate good producers. A continuum of firms indexed by i produce differentiated intermediate goods by combining capital and labor via a common Cobb-Douglas production function with capital elasticity α

$$Y_t(i) = K_t(i)^\alpha (Z_t L_t(i))^{1-\alpha}, \quad (11)$$

where Z_t is an exogenous technological progress, assumed non-stationary. The growth rate of productivity $z_t = \ln(Z_t/Z_{t-1})$ is modeled as an AR(1) process. We refer to the innovations to this process \tilde{z}_t as *productivity shocks*, with $\tilde{z}_t \sim N(0, \sigma_z^2)$, *i.i.d.*

The intermediate goods producers hire labor and rent capital in competitive markets and face an identical nominal wage, W_t , and rental rate for capital, R_t^k . The profit function for each firm i is therefore

$$P_t(i)Y_t(i) - W_t L_t(i) - R_t^k K_t(i). \quad (12)$$

Following Calvo(1983), we assume that in every period a fraction $(1 - \zeta_p)$ of the intermediate goods producers optimize their prices and the remainder ζ_p adjusts prices to steady state inflation π^* . The firms that are able to optimize choose prices $\tilde{P}_t(i)$ to maximize the expected discounted sum of future profits:

$$\Xi_t^p(\tilde{P}_t(i) - MC_t)Y_t(i) + \mathbb{E}_t \sum_{s=1}^{\infty} \zeta_p^s \beta^s \Xi_{t+s}^p(\tilde{P}_t(i)\pi^{*s} - MC_{t+s})Y_{t+s}(i)$$

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subject to

$$Y_{t+s}(i) = \frac{\tilde{P}_t(i)\pi^{*s}}{P_{t+s}}^{-\left(1+\frac{1}{\lambda_{f,t+s}}\right)} Y_{t+s}.$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$, MC_t is firms' nominal marginal cost, and $\beta^s \Xi_{t+s}^p$ is a discount factor (Ξ_t^p is the Lagrange multiplier associated with the households' nominal budget constraint).

Capital producers. Capital producers are competitive firms which transform general output – which is bought from final goods producers – into new capital via the technology:

$$x = x + \mu_t \left(1 - S \left(\frac{I_t}{I_{t-1}} \right) \right) I_t. \quad (13)$$

where x is the initial capital purchased from entrepreneurs in period t , and x is the new stock of capital, which they sell back to entrepreneurs at the end of the same period. $I_t(j)$ is investment spending, and $S(\cdot)$ is the cost of adjusting investment, with $S(\cdot) > 0$, $S'(\cdot) > 0$. The exogenous process μ_t affects the efficiency by which a foregone unit of consumption contributes to capital accumulation. We assume that this process follows an AR(1) process, and label its innovation μ_t^i marginal efficiency of investment (MEI) shocks, with $\mu_t^i \sim N(0, \sigma_{\mu}^2)$, *i.i.d.* Capital producers choose investment as to maximize their profits, expressed in terms of consumption goods,

$$\Pi_t^k = \frac{Q_t^k}{P_t} x - \frac{Q_t^k}{P_t} x - I_t, \quad (14)$$

where Q_t^k is the price of capital.

Entrepreneurs. There is a continuum of entrepreneurs indexed by e . Each entrepreneur buys installed capital $\bar{K}_{t-1}(e)$ from the capital producers at the end of period $t-1$ using her own net worth $N_{t-1}(e)$ and a loan $B_{t-1}^d(e)$ from the banking sector:

$$Q_{t-1}^k \bar{K}_{t-1}(e) = B_{t-1}^d(e) + N_{t-1}(e) \quad (15)$$

where net worth is expressed in nominal terms. In the next period she rents capital to firms, earning a rental rate R_t^k per unit of effective capital. In period t an i.i.d. (across entrepreneurs and over time) random variable $\omega_t(e)$ increases or shrinks entrepreneurs' capital. We assume that $\log \omega_t(e) \sim N(m_{\omega,t-1}, \sigma_{\omega,t-1}^2)$, where $m_{\omega,t-1}$ is such that $\mathbb{E}\omega(e)_t = 1$, and $\sigma_{\omega,t}^2$ is an exogenous process which we will discuss later. We denote by $F_{t-1}(\omega)$ the

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cumulative distribution function of ω at time t , which needs to be known at time $t - 1$. In addition, after observing the shock the entrepreneur chooses a level of capital utilization $u_t(e)$ by paying a cost in terms of general output equal to $a(u_t(e))$ per-unit-of-capital. At the end of period t the entrepreneur sells undepreciated capital to the capital producers. Entrepreneurs' revenues in period t are therefore:

$$\omega_t(e)\tilde{R}_t^k(e)Q_{t-1}^k\bar{K}_{t-1}(e) \quad (16)$$

where

$$\tilde{R}_t^k(e) = \frac{R_t^k u_t(e) + (1 - \delta)Q_t^k - P_t a(u_t(e))}{Q_{t-1}^k} \quad (17)$$

is the gross nominal return to capital for entrepreneurs. From the profit function it is clear that the choice of the utilization rate is independent from the amount of capital purchased or the ω_t shock. Consequently in what follows we drop the index e from the return \tilde{R}_t^k .

The debt contract undertaken by the entrepreneur in period $t - 1$ consists of the triplet $(B_{t-1}^d(e), R_t^d(e), \bar{\omega}_t(e))$ where $R_t^d(e)$ represents the contractual interest rate, and $\bar{\omega}_t(e)$ the threshold level of $\omega_t(e)$ below which the entrepreneur cannot pay back, which is therefore defined by the equation:

$$\bar{\omega}_t(e)\tilde{R}_t^k Q_{t-1}^k \bar{K}_{t-1}(e) = R_t^d(e)B_{t-1}^d(e). \quad (18)$$

The Banks. The representative bank collects deposits from households and lends to entrepreneurs. For $\omega_t(e) < \bar{\omega}_t(e)$ the bank monitors the entrepreneurs and extracts a fraction $(1 - \mu^e)$ of their revenues $\tilde{R}_t^k Q_{t-1}^k \bar{K}_{t-1}(e)$, where μ^e represents exogenous bankruptcy costs. The bank's zero profit condition implies that:

$$[1 - F_{t-1}(\bar{\omega}_t(e))] R_t^d(e)B_{t-1}^d(e) + (1 - \mu^e) \int_0^{\bar{\omega}_t(e)} \omega dF_{t-1}(\omega) \tilde{R}_t^k Q_{t-1}^k \bar{K}_{t-1}(e) = R_{t-1} B_{t-1}^d(e) \quad (19)$$

where R_{t-1} is the rate paid by the bank to the depositors.

Entrepreneurs' expected profits (before the realization of the shock ω_t) can be written as:

$$\int_{\bar{\omega}(e)_t}^{\infty} \left[\omega(e)_t \tilde{R}_t^k(e) Q_{t-1}^k \bar{K}_{t-1}(e) - R_t^d(e) B_{t-1}^d(e) \right] dF_{t-1}(\omega_t(e)) \quad (20)$$

The contract that maximizes expected net worth for the entrepreneurs maximizes the

FRBNY DSGE Model Documentation

entrepreneurs' expected profits subject to the the bank's zero profit condition.

Aggregate entrepreneurs' equity evolves according to:

$$V_t = \int_{\bar{\omega}_t}^{\infty} \omega_t \tilde{R}_t^k Q_{t-1}^k \bar{K}_{t-1}(e) dF_{t-1}(\omega_t) - [1 - F_{t-1}(\bar{\omega}_t)] R_t^d(e) B_{t-1}^d(e) \quad (21)$$

Each period a fraction $1 - \gamma^e$ of entrepreneurs exits the economy and fraction γ^e survives to continue operating. A fraction Θ of the total net worth owned by exiting entrepreneurs is consumed upon exit and the remaining fraction of their network is transferred as a lump sum to the households. Each period new entrepreneurs enter and receive a net worth transfer W_t^e . Because W_t^e is small, this exit and entry process ensures that entrepreneurs do not accumulate enough net worth to escape the financial frictions. Aggregate entrepreneurs' net worth evolves accordingly as:

$$N_t = \gamma^e V_t + W_t^e, \quad (22)$$

and net transfers from entrepreneurs to households are equal to

$$\mathcal{T}_t = (1 - \Theta)(1 - \gamma^e)V_t - W_t^e. \quad (23)$$

We assume that the volatility of the idiosyncratic random productivity, $\sigma_{\omega,t}^2$, changes exogenously over time according to an AR(1) process. A (mean-preserving) increase in volatility implies that a larger fraction of entrepreneurs will default, and hence will increase the cost of capital (relative to the risk-free rate) given entrepreneurs' leverage. We therefore refer to innovations to this process as *spread shocks*.

The government. The central bank follows a standard feedback rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_R} \left[\left(\prod_{j=0}^3 \frac{\pi_{t-j}}{\pi^*} \right)^{\psi_\pi} \left(\log \frac{Y_t}{Y_{t-4}} \right)^{\psi_Y} \right]^{1-\rho_R} e_t^R, \quad (24)$$

where the interest rate responds to deviations of inflation from target and deviation of output growth from its steady state. In (24) R is the steady state (gross) nominal interest rate, $\prod_{j=0}^3 \pi_{t-j}$ is the 4-quarter rate of gross inflation and $\log \frac{Y_t}{Y_{t-4}}$ is the 4-quarter growth rate of output. e_t^R is a *monetary policy shock*, where $e_t^R \sim N(0, \sigma_r^2)$, *i.i.d.*, and π^* is the inflation target.

Fiscal policy is fully Ricardian. The government balances the budget in every period:

$$G_t + T_t = 0. \quad (25)$$

Public spending is determined exogenously as a time-varying fraction of aggregate output

$$G_t = (1 - 1/g_t)Y_t, \quad (26)$$

where government spending g_t follows an AR(1) process, and we refer to the innovations to this process, g_t , as *demand shocks*, with $g_t \sim N(0, \sigma_g^2)$, *i.i.d.*.

Market clearing. By combining the government's and the households' budget constraints with the zero profit condition of the final goods producers and of the employment agencies, we obtain the aggregate resource constraint

$$C_t + I_t + a(u_t)\bar{K}_{t-1} = \frac{1}{g_t}Y_t. \quad (27)$$

The optimization conditions of the model result in dynamic relationships among macroeconomic variables. Together with market clearing conditions, they completely characterize the equilibrium behavior of the model economy.

Methodology. The model has a source of non-stationarity in the process for technology Z_t , which has a unit root. Hence consumption, investment, capital, real wages and output inherit stochastic growth. To solve the model we first rewrite its equilibrium conditions in terms of stationary variables, and then solve for the non-stochastic steady state of the transformed model. Finally we take a log-linear approximation of the transformed model around its steady state. This approximation gives us a set of log-linear equations that we solve to obtain the model's state-space representation. We use the state-space representation in our estimation procedure.

The model's log-linear equations are as follows:

1. $\hat{m}c_t = (1 - \alpha)\hat{w}_t + \alpha\hat{r}_t^k$
2. $\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \frac{(1-\zeta_p\beta)(1-\zeta_p)}{\zeta_p}\hat{m}c_t + \frac{1}{\zeta_p}\tilde{\lambda}_{f,t}$
3. $\hat{k}_t = -(1 - \frac{i^*}{k^*})\hat{z}_t + (1 - \frac{i^*}{k^*})\hat{k}_{t-1} + \frac{i^*}{k^*}\hat{\mu}_t + \frac{i^*}{k^*}\hat{i}_t$
4. $\hat{k}_t = \hat{u}_t - \hat{z}_t + \hat{k}_{t-1}$

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5. $(e^\gamma - h\beta)(e^\gamma - h)\hat{\xi}_t = e^\gamma(e^\gamma - h)\hat{b}_t - (e^{2\gamma} + \beta h^2)\hat{c}_t + h e^\gamma \hat{c}_{t-1} - h e^\gamma \hat{z}_t - \beta h(e^\gamma - h)\mathbb{E}_t[\hat{b}_{t+1}] + \beta h e^\gamma \mathbb{E}_t[\hat{c}_{t+1}] + \beta h e^\gamma \mathbb{E}_t[\hat{z}_{t+1}]$
6. $v_m \hat{m}_t = \hat{\chi}_t + \hat{b}_t - \frac{1}{R^* - 1} \hat{R}_t - \hat{\xi}_t$
7. $\hat{\xi}_t = \hat{R}_t + \mathbb{E}_t[\hat{\xi}_{t+1}] - \mathbb{E}_t[\hat{z}_{t+1}] - \mathbb{E}_t[\hat{\pi}_{t+1}]$
8. $\hat{i}_t = \frac{1}{1+\beta} \mathbb{E}_t[\hat{i}_{t-1} - \hat{z}_t] + \frac{\beta}{1+\beta} \mathbb{E}_t[\hat{i}_{t+1} + \hat{z}_{t+1}] + \frac{1}{(1+\beta)S} \frac{1}{e^{2\gamma}} \hat{q}_t^k + \frac{1}{(1+\beta)S} \frac{1}{e^{2\gamma}} \hat{\mu}_t$
9. $r^{k*} \hat{r}_t^k = a \hat{u}_t$
10. $(1 + \nu_l \frac{1+\lambda_w}{\lambda_w}) \hat{w}_t + (1 + \zeta_w \beta \nu_l (\frac{1+\lambda_w}{\lambda_w})) \hat{w}_t = \zeta_w \beta (1 + \nu_l \frac{1+\lambda_w}{\lambda_w}) \mathbb{E}_t[\hat{w}_{t+1} + \hat{w}_{t+1}] + (1 - \zeta_w \beta) (e^{2\gamma} + h^2 \beta) \frac{e^{-\gamma}}{e^\gamma - h} \hat{b}_t + \hat{\varphi}_t + (1 - \zeta_w \beta) (\nu_l \hat{L}_t - \hat{\xi}_t) + \zeta_w \beta (1 + \nu_l \frac{1+\lambda_w}{\lambda_w}) \mathbb{E}_t[\hat{\pi}_{t+1} + \hat{z}_{t+1}]$
11. $\hat{w}_t = \hat{w}_{t-1} - \hat{\pi}_t + \frac{1 - \zeta_w}{\zeta_w} \hat{w}_t$
12. $\hat{k}_t = \hat{w}_t - \hat{r}_t^k + \hat{L}_t$
13. $\hat{y}_t = \hat{g}_t + \frac{c^*}{c^* + i^*} \hat{c}_t + \frac{i^*}{c^* + i^*} \hat{i}_t + \frac{r^{k*} k^*}{c^* + i^*} \hat{u}_t$
14. $\hat{y}_t = \alpha \frac{y^* + \Phi}{y^*} \hat{k}_t + (1 - \alpha) \frac{y^* + \Phi}{y^*} \hat{L}_t$
15. $\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) (\psi_\pi (\hat{\pi}_t^{4q} - \pi^*) + \psi_y \hat{y}_t^{4q}) + \frac{R}{t}$
16. $\mathbb{E}_t \left[\hat{R}_{t+1}^k - \hat{R}_t \right] = \zeta_{sp,b} \left(\hat{q}_t^k + \hat{k}_t - \hat{n}_t \right) + \tilde{\sigma}_{\omega,t}$
17. $\hat{n}_t = \zeta_{n,\tilde{R}^k} \left(\hat{R}_t^k - \pi_t \right) - \zeta_{n,R} \left(\hat{R}_{t-1} - \pi_t \right) + \zeta_{n,qK} \left(\hat{q}_{t-1}^k + \hat{k}_{t-1} \right) + \zeta_{n,n} \hat{n}_{t-1} - \gamma^e \frac{v_*}{n_*} \hat{z}_t - \frac{\zeta_{n,\sigma_\omega}}{\zeta_{sp,\sigma_\omega}} \tilde{\sigma}_{\omega,t-1}^2$
18. $\hat{R}_t^k - \pi_t = \frac{r_*^k}{r_*^k + (1 - \delta)} \hat{r}_t^k + \frac{(1 - \delta)}{r_*^k + (1 - \delta)} \hat{q}_t^k - \hat{q}_{t-1}^k$

Equation 1 represents the evolution of marginal costs (or labor share). Equation 2 is a Phillips curve, where we used the following re-parameterization of the cost-push shock: $\tilde{\lambda}_{f,t} = [(1 - \zeta_p \beta)(1 - \zeta_p) \lambda_f / (1 + \lambda_f)] \lambda_{f,t}$ where λ_f is the steady state value of the markup shock. Equations 3 and 4 are respectively the evolution of the capital stock and the evolution of effective capital. The marginal utility of consumption $\hat{\xi}_t$ is defined in equation 5, and equation 7 is the Euler equation. Equation 6 is optimal money demand and equation 8 is optimal investment. Equation 9 is capital utilization, equation 10 is the wage equation, and

equation 11 is the aggregate wage equation. Equation 12 defines the capital-labor ratio and equation 13 is the resource constraint. The production function is equation 14 and equation 15 represents the policy rule. Equation 16 defines the expected excess return on capital as a function of the leverage of the firms (the ratio of the value of capital to net worth) and exogenous shocks. Equation 17 shows the evolution firms' net worth and equation 18 links the realized return on capital to the capital rental rate and the evolution of the price of capital.

State space representation and shock processes. We use the method in Sims (2002) to solve the system of log-linear approximate equilibrium conditions and obtain the transition equation:

$$s_t = \mathcal{T}(\theta)s_{t-1} + \mathcal{R}(\theta) \epsilon_t. \quad (28)$$

We collect all the DSGE model parameters in a vector θ and stack the structural shocks in a vector ϵ_t . The state-space representation for our vector of observables y_t , which we describe in the next section, is composed of the transition equation (28), which summarizes the evolution of the states s_t , and of a system of measurement equations:

$$y_t = \mathcal{D}(\theta) + \mathcal{Z}(\theta)s_t, \quad (29)$$

mapping the states into the observables. The vector of ϵ_t is composed of seven exogenous shocks: a productivity shock z_t , a labor supply shock φ_t , a marginal efficiency of investment (MEI) shock μ_t , a government policy shock g_t , a price mark-up shock $\lambda_{f,t}$, a spread shock $\sigma_{\omega,t}$, and a monetary policy shock $R_{t,t}$.

1.2 The Data

The variables we use for estimation are as follows. $\Delta \ln Y_t$ is annualized real GDP per capita growth, where the real gross domestic product is computed as the ratio of nominal GDP (SAAR) to the chain-type price index (2005=100) from the BEA. $\ln L_t$ is (log) labor hours, measured as per capita hours in non-farm payroll. $\ln LS_t$ is the (log) labor share, computed as the ratio of compensation of employees (\$bil) to nominal GDP (\$bil), from the BEA. π_t^{Core} is the annualized rate of change of the core PCE deflator (PCE excluding food and energy, but including purchased meals and beverages), seasonally adjusted, 2005=100. FFR_t is the effective federal funds rate, percent annualized, computed

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from daily data. SP_t is the spread between the Baa rate and the rate on 10 year Treasuries. Finally, per capita variables are obtained by dividing through the civilian non-institutionalized population over 16. Haver mnemonics for the data are as follows: Real GDP (GDP@USECON/JGDP@USECON); Labor Hours (LHTNAGRA@USECON); Labor share (YCOMP@USECON/GDP@USECON); Core PCE deflator (JCXFE@USNA); FFR (FFED@DAILY); Civilian non-institutionalized population over 16 (LN16N@USECON); Baa (FBAA@USECON); 10yT (FCM10@USECON). The measurement equations are specified as follows:

$$\begin{array}{llll}
 \text{Real output growth (\%, annualized)} & 400(\ln Y_t - \ln Y_{t-1}) & = & 400(\hat{y}_t - \hat{y}_{t-1} + z_t) \\
 \text{Hours (\%)} & 100 \ln L_t & = & 100(L_t + \ln L^{adj}) \\
 \text{Labor Share (\%)} & 100 \ln LS_t & = & 100(\hat{L}_t + \hat{w}_t - \hat{y}_t + \ln LS_*) \\
 \text{Inflation (\%,annualized)} & \pi_t^{Core} & = & 400(\hat{\pi}_t + \ln \pi_*) \\
 \text{Interest Rate (\%,annualized)} & FFR_t & = & 400(\hat{R}_t + \ln R_*), \\
 \text{Spread (\%,annualized)} & SP_t & = & 400(\mathbb{E}_t \left[\hat{R}_{t+1}^k - \hat{R}_t \right] + SP_*),
 \end{array}$$

where the parameter L^{adj} captures the units of measured hours.

We use Bayesian methods to characterize the posterior distribution of the structural parameters. The posterior distribution combines the likelihood function with prior information.

In what follows, we report some information on the empirical properties of the estimated model. The variance decomposition graphs illustrate the relative contribution of the exogenous shocks to the overall variability of output, inflation and the nominal interest rate. The table summarizes prior and posterior for the model parameters. We also report the impulse response functions of the observable variables to the various shocks.

1.3 Anticipated Policy Shocks

This section describes the introduction of anticipated policy shocks in the model. We modify the policy rule (24) so to incorporate anticipated policy shocks. In log-linear form the new

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policy rule writes:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)(\psi_\pi(\hat{\pi}_t^{4q} - \pi^*) + \psi_y \hat{y}_t^{4q}) + \overset{R}{r}_t + \sum_{k=1}^K \overset{R}{r}_{k,t-k}, \quad (30)$$

where $\overset{R}{r}_t$ is the usual contemporaneous policy shock, and $\overset{R}{r}_{k,t-k}$ is a policy shock that is known to agents at time $t - k$, but affects the policy rule k periods later, that is, at time t . We assume that $\overset{R}{r}_{k,t-k} \sim N(0, \sigma_{k,r}^2)$, *i.i.d.*

In order to solve the model we need to express the anticipated shocks in recursive form. For this purpose, we augment the state vector s_t with K additional states $\nu_t^R, \dots, \nu_{t-K}^R$ whose law of motion is as follows:

$$\begin{aligned} \nu_{1,t}^R &= \nu_{2,t-1}^R + \overset{R}{r}_{1,t} \\ \nu_{2,t}^R &= \nu_{3,t-1}^R + \overset{R}{r}_{2,t} \\ &\vdots \\ \nu_{K,t}^R &= \overset{R}{r}_{K,t} \end{aligned}$$

and rewrite the policy rule (31) as

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)(\psi_\pi(\hat{\pi}_t^{4q} - \pi^*) + \psi_y \hat{y}_t^{4q}) + \overset{R}{r}_t + \nu_{1,t-1}^R, \quad (31)$$

It is easy to verify that $\nu_{1,t-1}^R = \sum_{k=1}^K \overset{R}{r}_{k,t-k}$, that is, $\nu_{1,t-1}^R$ is a “bin” that collects all anticipated shocks that affect the policy rule in period t . The model’s solution can then again be expressed in terms of the transition equation (28).

In order to estimate the importance of anticipated shocks and their effect on the variables of interest, we augment the measurement equation (29) with the expectations for the policy rate:

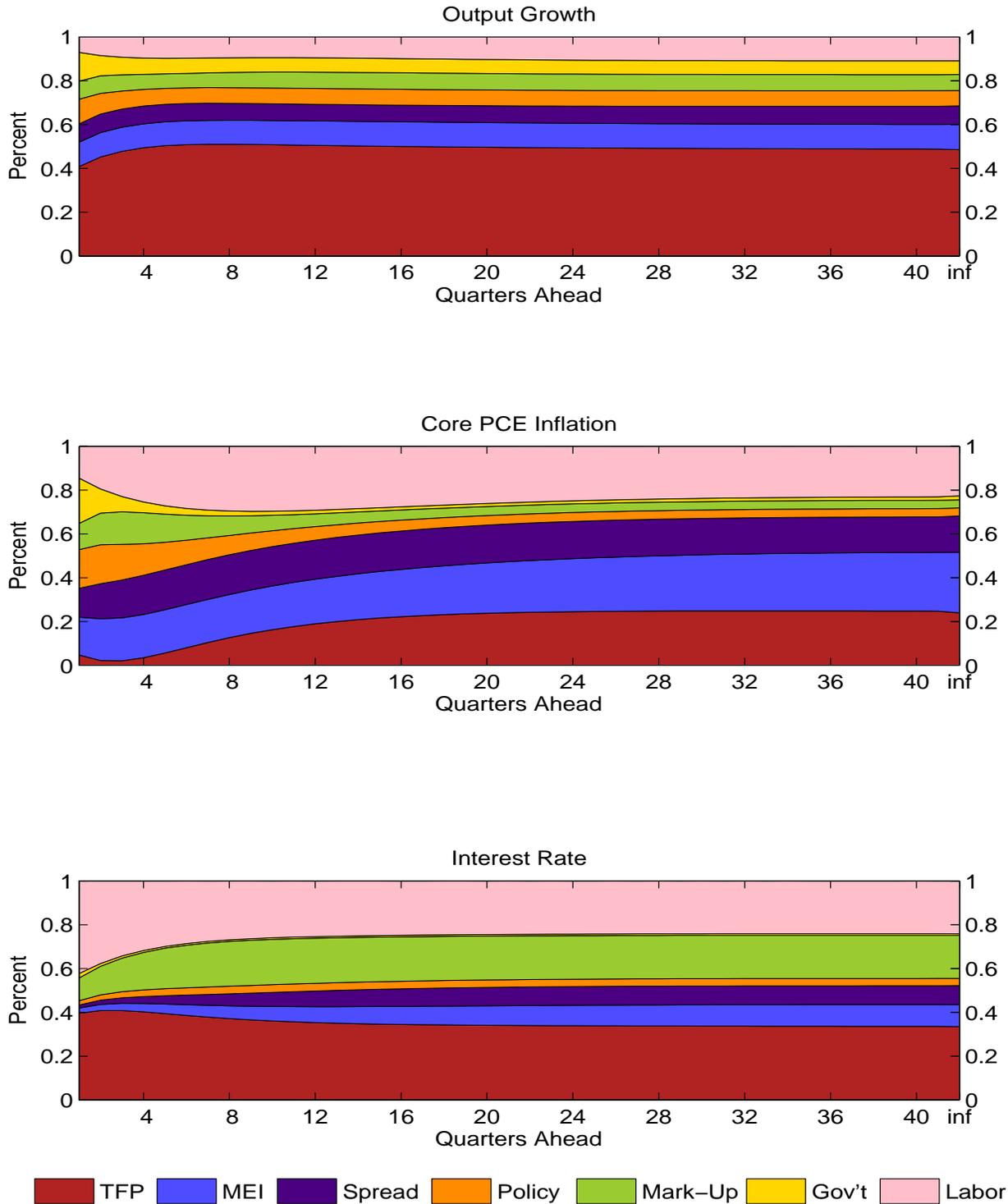
$$\begin{aligned} FFR_{t,t+1}^e &= 400(\mathcal{Z}(\theta)_{R,\cdot} \mathcal{T}(\theta)^1 s_t + R^*), \\ &\vdots \\ FFR_{t,t+K}^e &= 400(\mathcal{Z}(\theta)_{R,\cdot} \mathcal{T}(\theta)^K s_t + R^*), \end{aligned} \quad (32)$$

where $FFR_{t,t+k}^e$ are the market’s expectations for the FFR k quarters ahead, and $\mathcal{Z}(\theta)_{R,\cdot}$ is the row of $\mathcal{Z}(\theta)$ corresponding to the interest rate. Since we use the anticipated policy shocks since 2008Q3 onward (one period before the implementation of the zero lower bound) we do not have estimates for the standard deviations $\sigma_{k,r}$ of the anticipated shocks. In the implementation, we assume that these shocks have the same standard deviation as the

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contemporaneous shock: $\sigma_{k,r} = \sigma_r$.

2 Variance Decomposition

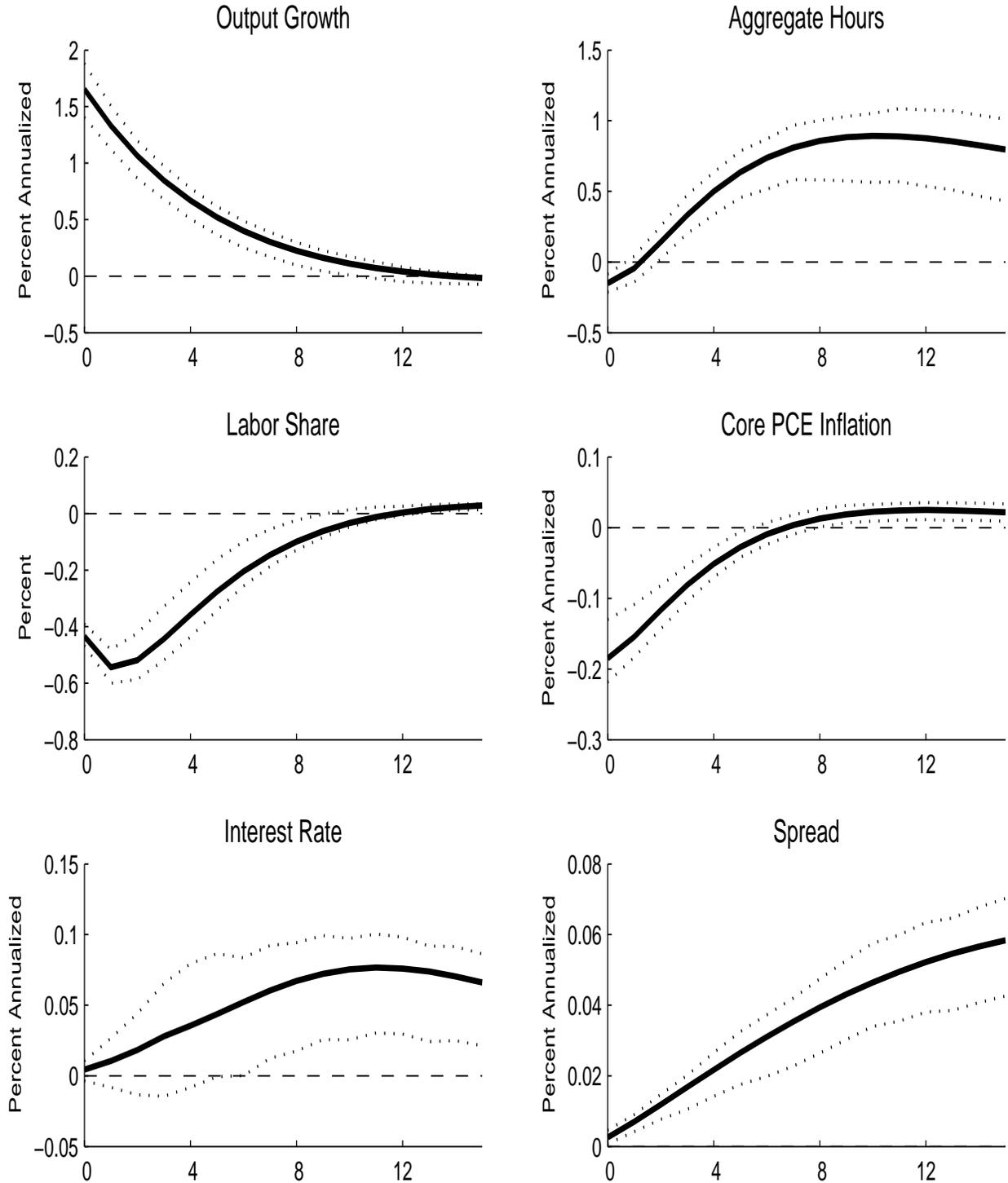


3 Prior and Posterior

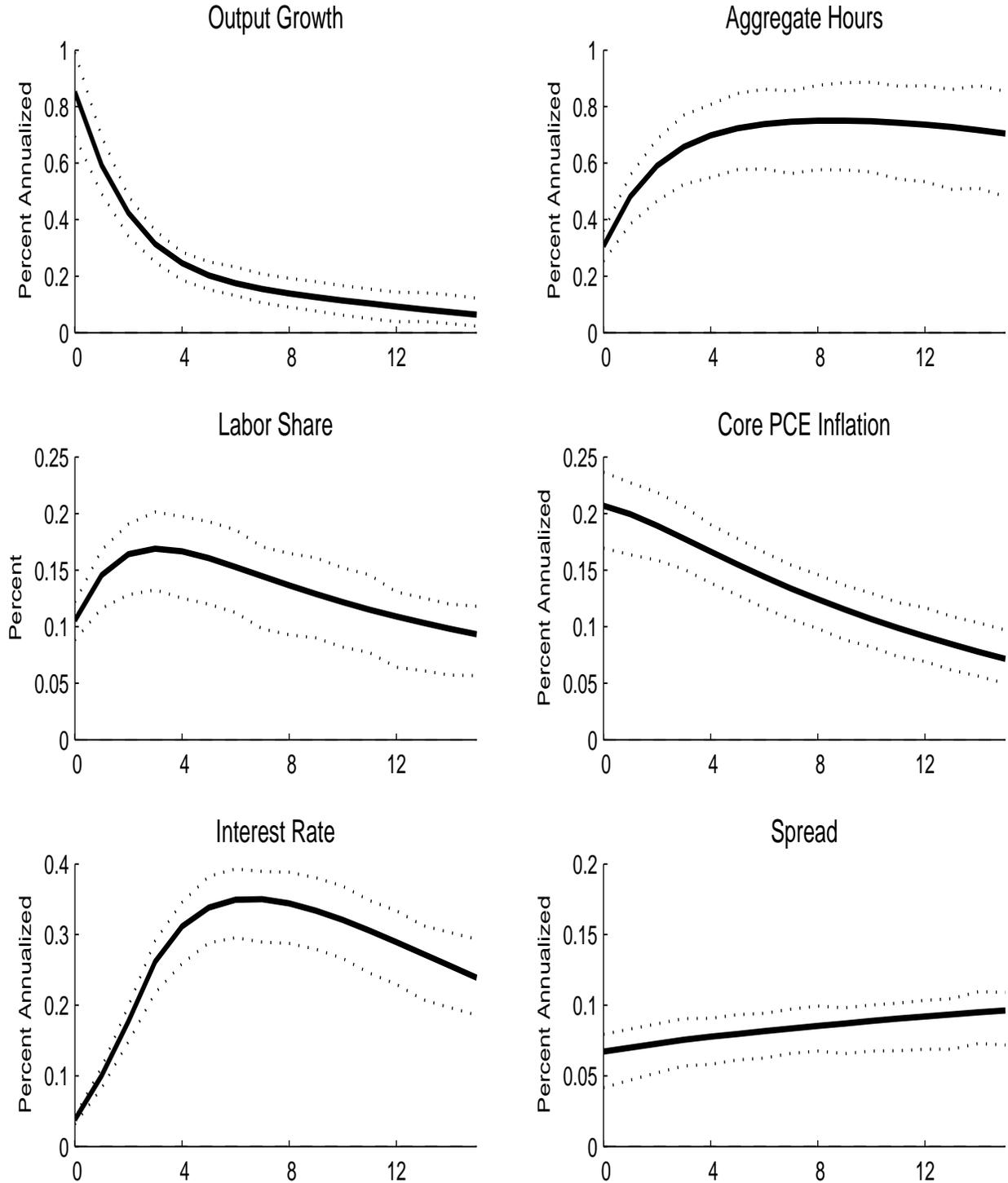
Parameters	Prior Mean	Prior Stdd	Post Mean	90% Lower Band	90% Upper Band
Policy Parameters					
ψ_1	2.000	0.250	2.014	1.771	2.241
ψ_2	0.200	0.100	0.267	0.167	0.371
ρ_r	0.500	0.200	0.761	0.718	0.805
σ_r	0.200	4.000	0.153	0.134	0.171
Nominal Rigidities Parameters					
ζ_p	0.750	0.100	0.878	0.847	0.908
ζ_w	0.750	0.100	0.892	0.849	0.933
Other “Endogenous Propagation and Steady State” Parameters					
α	0.330	0.020	0.350	0.345	0.355
a''	0.200	0.100	0.271	0.103	0.433
h	0.700	0.050	0.686	0.619	0.754
S''	4.000	1.500	3.020	2.057	3.946
ν_l	2.000	0.750	1.328	0.416	2.215
r^*	1.500	1.000	0.330	0.040	0.595
π^*	2.000	0.250	2.399	2.131	2.684
γ	2.750	0.500	1.793	1.389	2.216
g^*	0.300	0.100	0.196	0.090	0.297
ρs and σs					
ρ_z	0.400	0.250	0.489	0.365	0.617
ρ_ϕ	0.750	0.150	0.337	0.198	0.473
ρ_{λ_f}	0.750	0.150	0.480	0.367	0.592
ρ_μ	0.750	0.150	0.987	0.973	1.000
ρ_g	0.750	0.150	0.921	0.822	0.999
σ_z	0.300	4.000	0.787	0.694	0.882
σ_ϕ	3.000	4.000	26.664	7.979	46.859
σ_{λ_f}	0.200	4.000	0.088	0.072	0.103
σ_μ	0.750	4.000	0.376	0.276	0.472
σ_g	0.500	4.000	0.250	0.194	0.301

Note: The following parameters are fixed: $\delta = 0.025$, $\nu_m = 2$, $\lambda_w = 0.3$, $\chi = 0.1$, $\lambda_f = 0.15$. L^{adj} has a prior mean of 253.500, with standard deviation at 5.

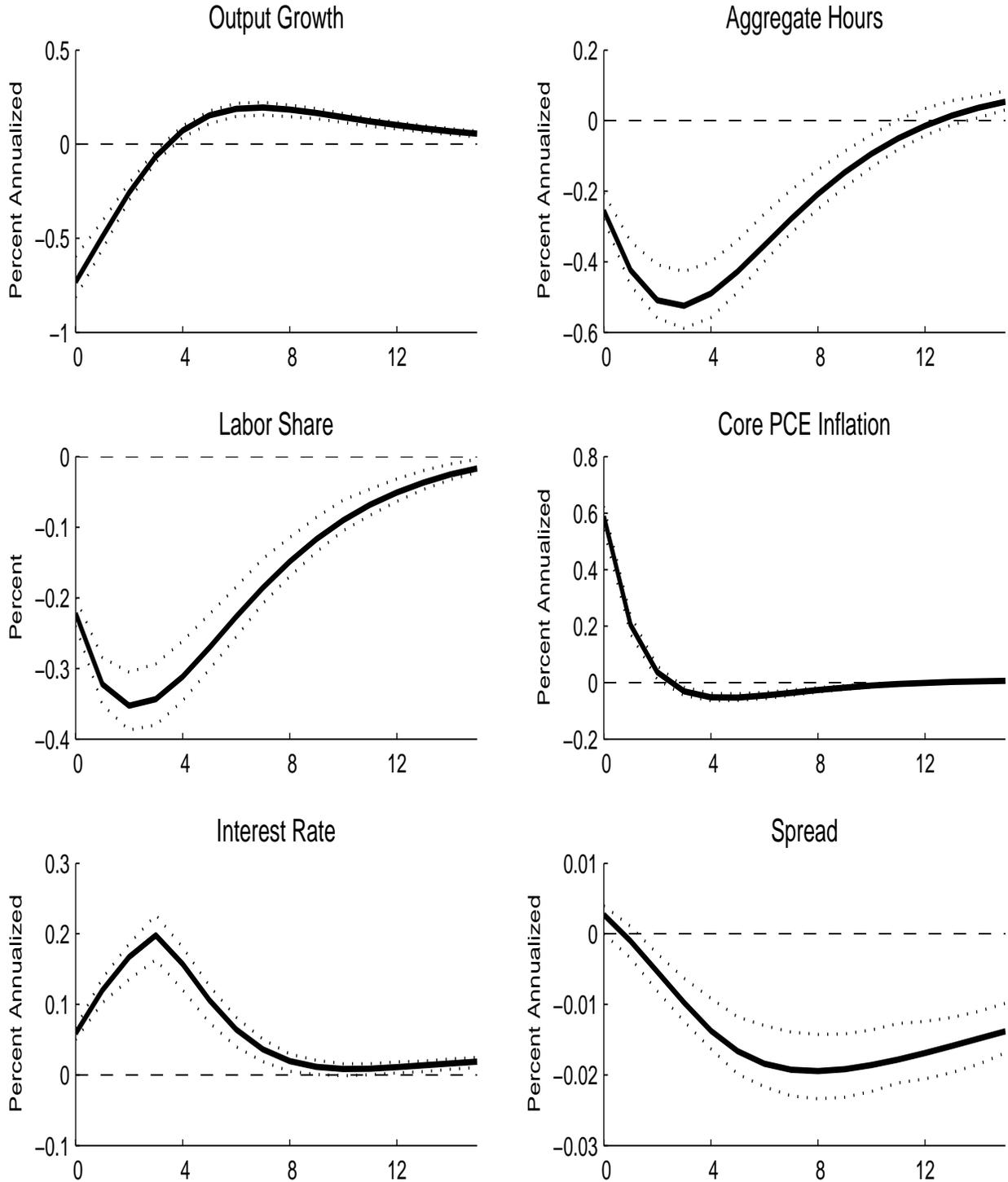
4 Responses to a Productivity Shock



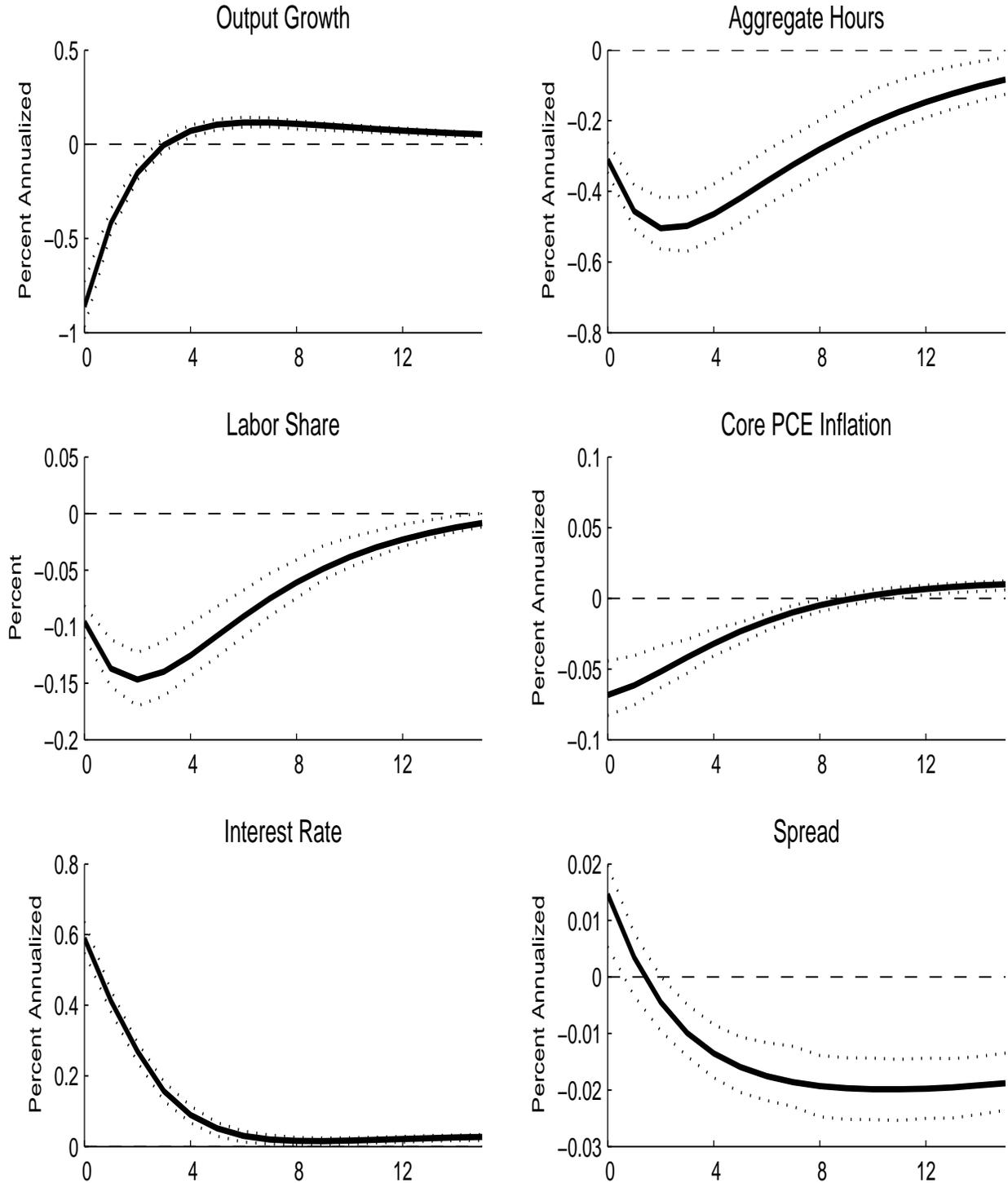
5 Responses to an MEI Shock



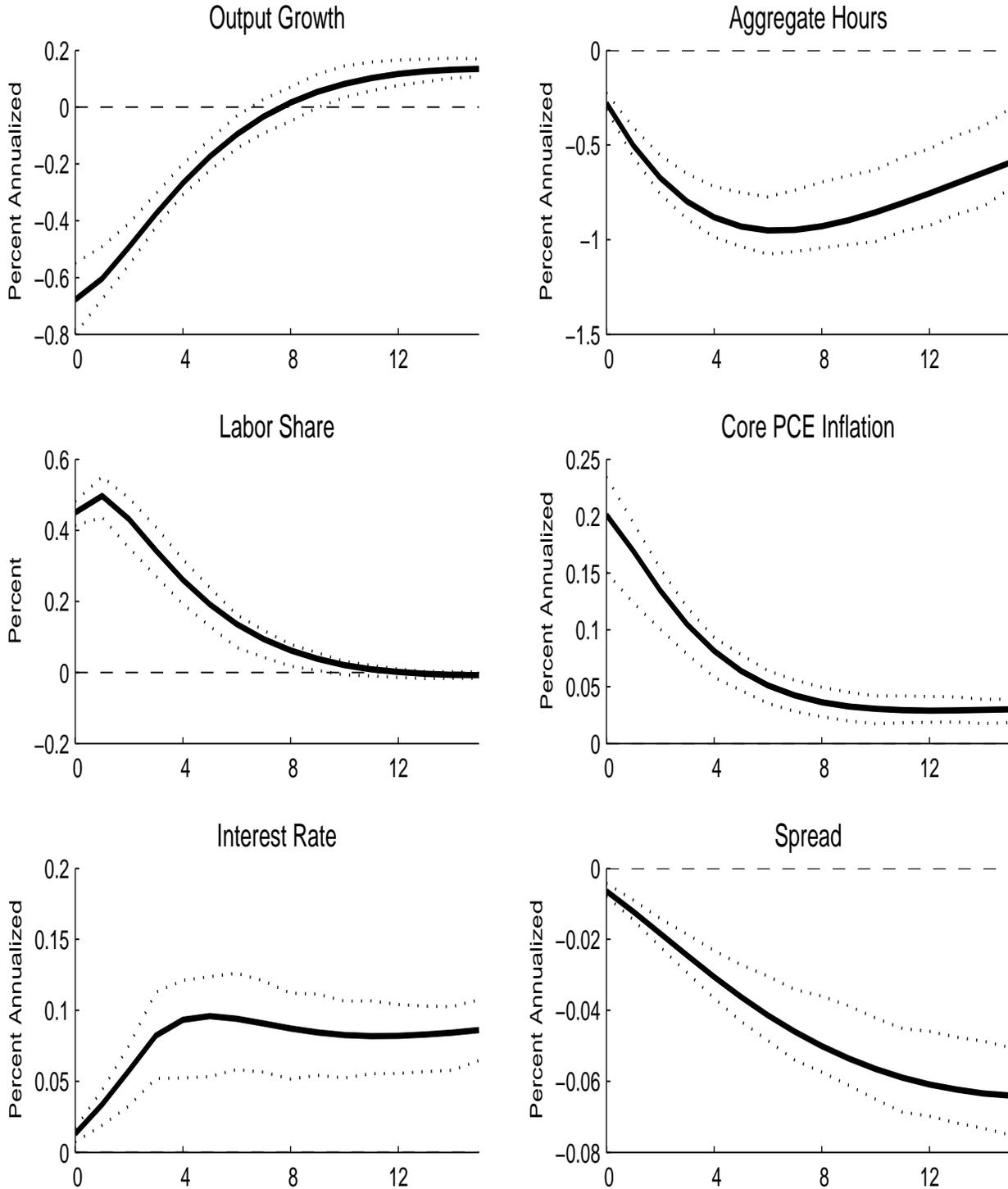
6 Responses to a Mark-up Shock



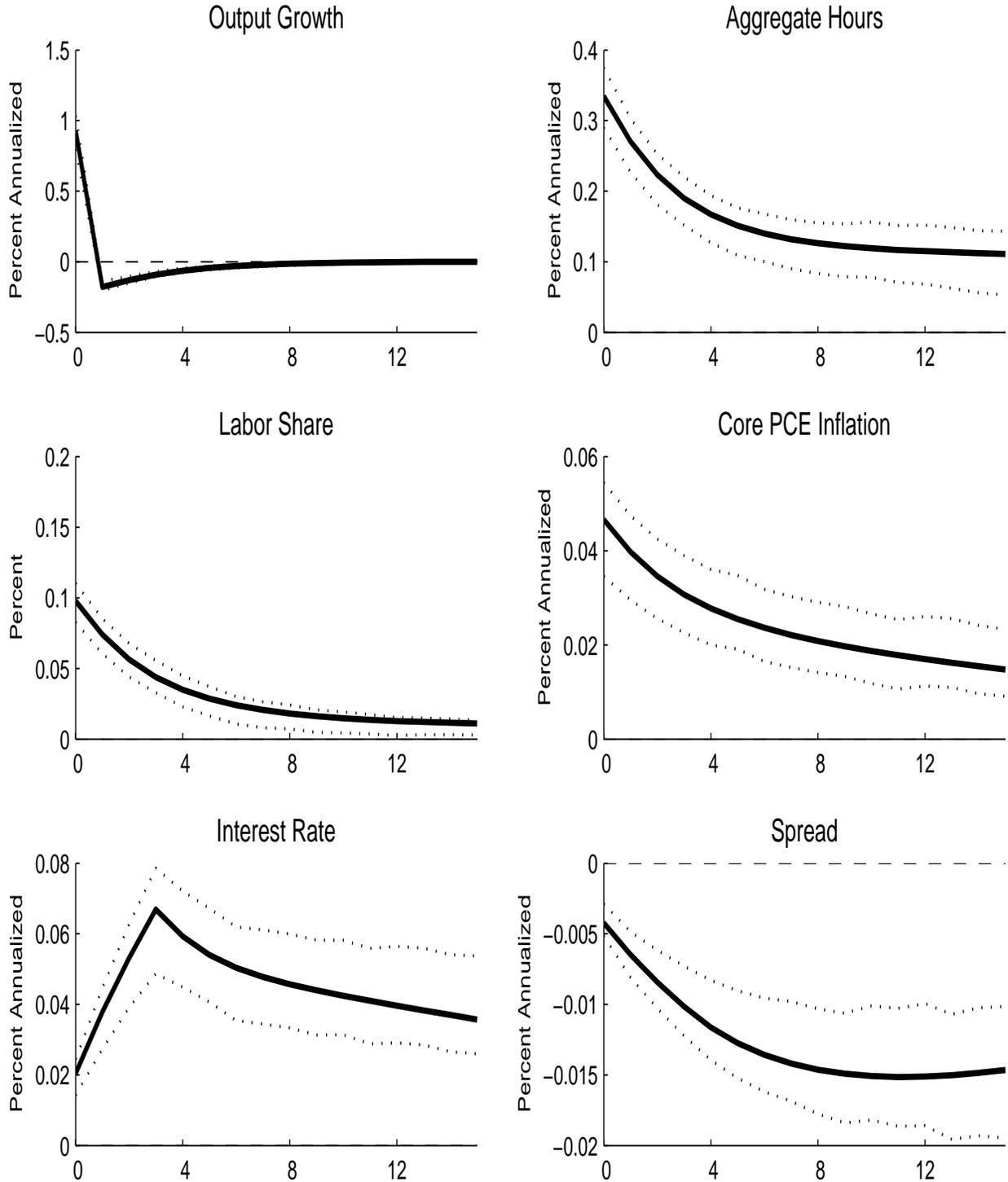
7 Responses to a Monetary Policy Shock



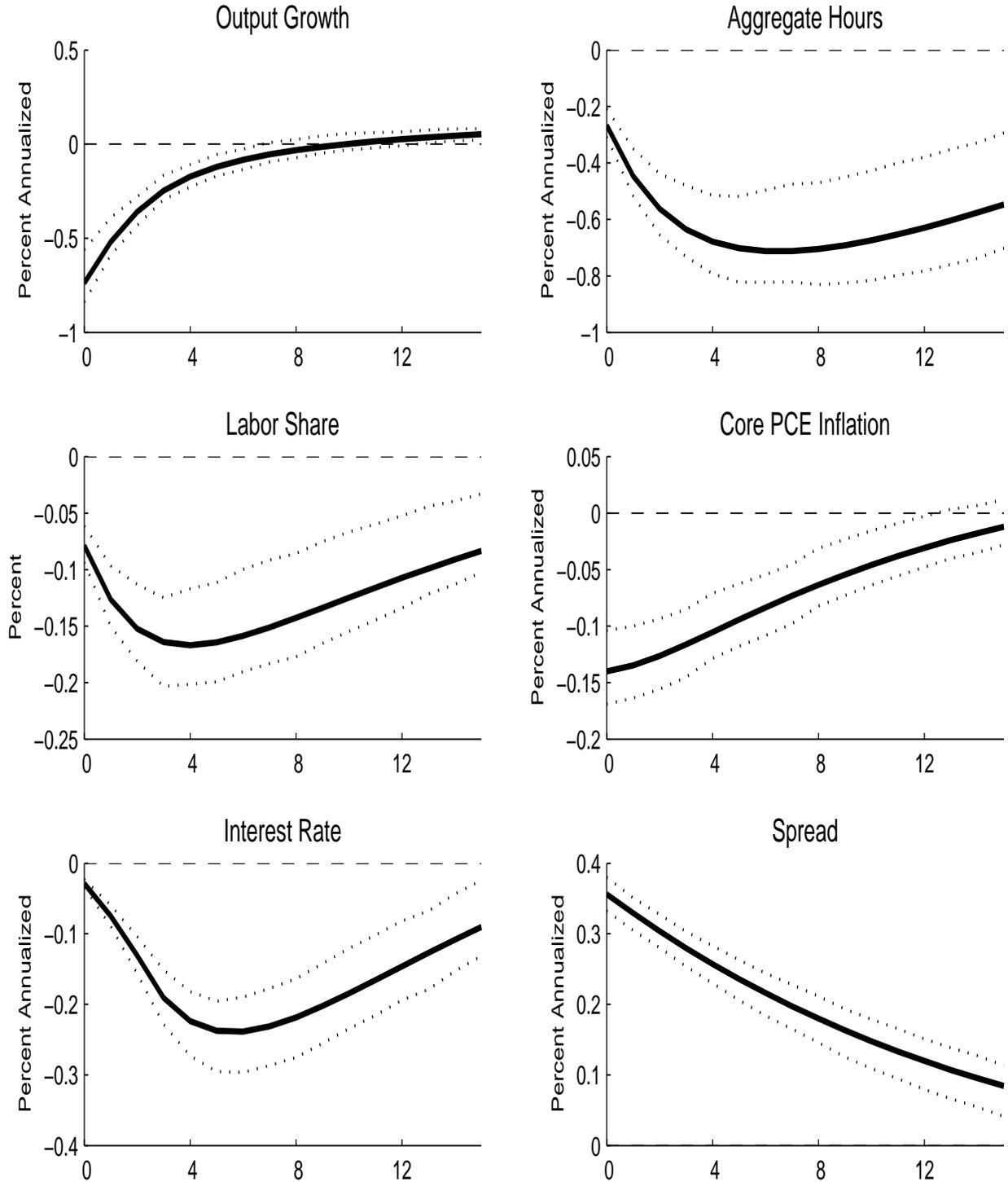
8 Responses to a Labor Supply Shock



9 Responses to a Demand Shock



10 Responses to a Spread Shock



References

- [1] **Bernanke, Ben, Mark Gertler and Simon Gilchrist**, “The Financial Accelerator in a Quantitative Business Cycle Framework,” in J.B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*, vol. 1C, Amsterdam: North-Holland, 1999.
- [2] **Calvo, Guillermo**, “Staggered Prices in a Utility-Maximizing Framework,” *Journal of Monetary Economics*, 1983, *12*, 383–398.
- [3] **Christiano, Lawrence, Martin Eichenbaum, and Charles Evans**, “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy,” *Journal of Political Economy*, 2005, *113*, 1–45.
- [4] **Christiano, Lawrence, Roberto Motto, and Massimo Rostagno**, “Financial Factors in Economic Fluctuations,” Unpublished, 2009.
- [5] **Del Negro, Marco, Frank Schorfheide, Frank Smets, and Raphael Wouters**, “On the Fit of New Keynesian Models,” *Journal of Business and Economic Statistics*, 2007, *25* (2), 123 – 162.
- [6] **Gurkaynak, Refet S., Brian Sack, and Eric Swanson**, “The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models,” *American Economic Review*, 2005, *95* (1), 425–36.
- [7] **Laseen, Stefan and Lars E. O. Svensson**, “Anticipated Alternative Instrument-Rate Paths in Policy Simulations,” NBER Working Paper No. w14902, 2009.
- [8] **Smets, Frank and Raphael Wouters**, “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” *American Economic Review*, 2007, *97* (3), 586 – 606.
- [9] **Sims, Christopher A.**, “Solving Rational Expectations Models,” *Computational Economics*, 2002, *20*, 1–20.