

# BOARD OF GOVERNORS OF THE FEDERAL RESERVE SYSTEM

DIVISION OF MONETARY AFFAIRS

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**Date:** March 31, 2020  
**To:** Vice Chair Clarida  
**From:** Marcel Pribsch<sup>1</sup>  
**Subject:** Does Central Bank Policy Imply a Lower Bound on Longer-Term Yields?

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## Summary

- Long-term bond yields can be shown to be subject to the same lower bound as the short rate, provided that
  - future short rates are constrained by the lower bound with certainty (the lower bound is “hard”), and
  - there are no arbitrage opportunities.
- This memo argues that, in practice, these assumptions are best viewed as approximations; hence, longer-term yields are more likely subject to a “soft” lower bound whose level may, moreover, differ from the lower bound on the policy rate.
- Nevertheless, it is plausible that there is a limit to how low long-term Treasury yields in the U.S. can get while credibly communicating a lower bound on the federal funds rate and supporting smooth functioning of Treasury markets.

## Introduction

In a recent piece, [Gagnon and Jeanne \(2020\)](#) argue that the scope for bond yields to fall below zero is strictly limited by the market’s perception of how far below zero the central bank is willing to set its policy rate, even when the central bank is able to

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<sup>1</sup> Don Kim provided very helpful advice and comments.

influence expectations and term premiums through forward guidance and asset purchases.<sup>2</sup>

This memo reviews the authors' main argument and provides examples that illustrate the underlying assumptions, including the euro area's experience with negative rates. Overall, the authors' argument has merit, suggesting that the extent to which long-term yields can fall is limited. However, this should not be interpreted as implying that tools such as large-scale asset purchases are of little value when longer-term yields are low.

### **Lower Bound on Longer-Term Yields**

This section reviews the core property derived by Gagnon and Jeanne (2020). Let the time  $t$  short rate underlying the yield curve be denoted by  $r_t$ , and suppose  $r_t$  is subject to a lower bound  $\underline{r}$ . Then, the zero-coupon yield on a long-term bond maturing at time  $t + \tau$ ,  $y_t^\tau$ , must obey the same lower bound, that is,  $y_t^\tau \geq \underline{r}$ , as long as

1. there is zero probability that future short rates will fall below  $\underline{r}$ ,<sup>3</sup> and
2. there are no arbitrage opportunities.

The proof is by contradiction: If the property were ever violated (that is,  $y_t^\tau < \underline{r}$ ), it would be an arbitrage strategy to short-sell the long-term bond (thus effectively borrowing at rate  $y_t^\tau$  from time  $t$  to time  $t + \tau$ ) and roll over the proceeds in a series of short-term investments. Since, by assumption, a short-term investment is guaranteed to return at least  $\underline{r}$  period by period, it must also return at least  $\underline{r}$  on average over the life of the long-term bond, even if the short rate path (and thus the overall return on the rolling investment) is otherwise random.<sup>4</sup> The strategy will thus yield strictly more than

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<sup>2</sup> Gagnon, Joseph E., and Olivier Jeanne (2020), "Central Bank Policy Sets the Lower Bound on Bonds Yields," Working Paper 20-2, Peterson Institute for International Economics. Available at: <https://www.piie.com/publications/working-papers/central-bank-policy-sets-lower-bound-bond-yields>

<sup>3</sup> Probabilities in this memorandum are "real-world" or "physical" probabilities unless otherwise indicated.

<sup>4</sup> This is a general property of arithmetic averages and mathematical expectations, known as "monotonicity."

required to cover the short bond position at time  $t + \tau$ . With no net outlays before time  $t + \tau$ , this is therefore an arbitrage opportunity by definition.

The property can be proved more formally and in greater generality. It does not require a specific asset pricing model and it holds regardless of how risk averse investors are. However, Gagnon and Jeanne's (2020) conclusion that "if a central bank communicates a credible commitment to keeping its policy rate above a given level under all circumstances, then bond yields must be higher than that level" requires another implicit assumption, namely that

3. the policy rate is the relevant short rate for the yield curve under consideration.

The three assumptions together allow the authors to argue that the central bank in large part controls the lower bound on yields across the maturity spectrum.<sup>5</sup> The following sections will review the assumptions and discuss how plausible they are in practice.

### **Assumption 1: Hard Bound on Short Rate**

The first assumption underlying Gagnon and Jeanne's (2020) argument concerns the lower bound  $\underline{r}$  on the short rate. All future short rates must be subject to the lower bound with certainty. Such a hard bound, however, must be viewed as an approximation. For example, in the U.S., the policy rate (the federal funds target rate or range) has never been set below zero, suggesting a historical ELB at or slightly above zero.<sup>6</sup> However, as shown in Figure 1, the probability distribution for the federal funds rate at the end of 2020 implied by federal funds futures options currently assigns non-negligible probabilities to modestly negative future rates.<sup>7</sup>

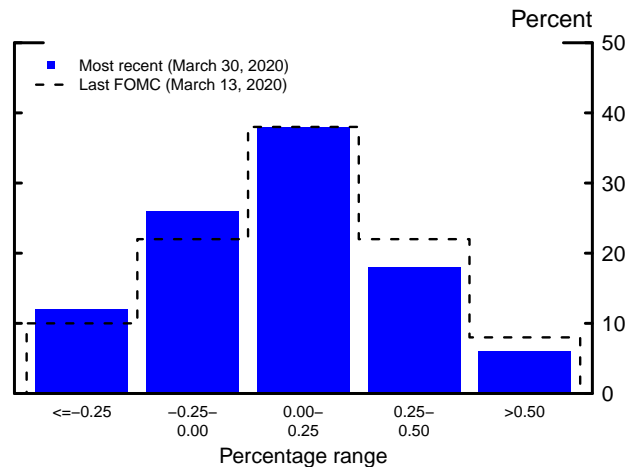
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<sup>5</sup> Using an analogous no-arbitrage argument, it follows that if a central bank issues perfectly credible forward guidance that the policy rate will remain at its ELB for at least  $\tau$  periods, then all long-term bonds with maturities less than or equal to  $\tau$  must have yields *equal to* the policy rate's ELB.

<sup>6</sup> Here we do not distinguish between policy rate and short rate, but the distinction is discussed in detail later as part of Assumption 3.

<sup>7</sup> The option-implied probabilities should be interpreted as "risk neutral," that is, subject to risk premiums. However, a strictly positive risk-neutral probability of some event requires a strictly positive real-world probability of that same event (although the two will likely differ in their exact magnitude).

Figure 1: Option-Implied Probability Distribution for the Federal Funds Rate at the End of 2020



Source: CME Group; Federal Reserve Board staff computations.

Similarly, respondents to the Desk's March surveys of primary dealers and market participants attached average probabilities of about 5 percent to negative federal funds rate outcomes at the ends of 2021 and 2022, respectively.<sup>8</sup> Indeed, while most respondents have viewed the ELB to be at zero or slightly above in recent surveys (including those predating the current crisis), individual responses have been as low as -1 percent.

The fact that, in reality, the lower bound is not hard is also demonstrated by the euro area's experience, in which investor perceptions of the level of the ELB on the policy rate have moved over time.<sup>9</sup> Figure 2 shows the time series of the spot EONIA rate, the one-year-forward EONIA rate, and the ECB's deposit facility rate (DFR).<sup>10</sup> The ECB first set the DFR to zero in 2012. For a period of about two years, both spot EONIA and one-year-forward EONIA remained above zero. Indeed, estimates place ELB

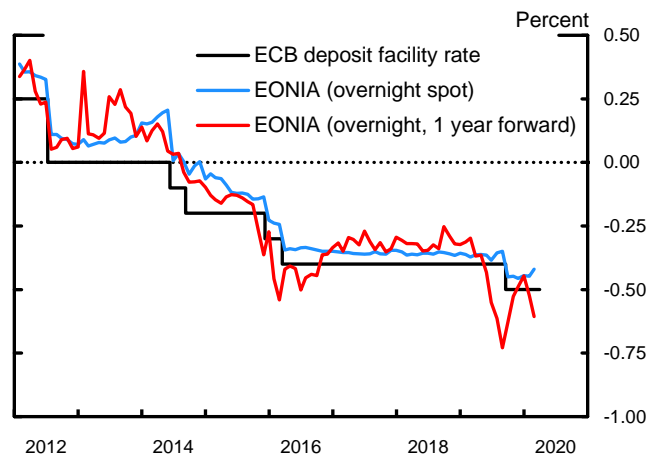
<sup>8</sup> The surveys' response options for federal funds rate outcomes at the end of 2020 were not sufficiently granular to identify negative rate probabilities.

<sup>9</sup> The euro area is chosen as an illustrative case study. Complementary empirical evidence comes from other countries with a history of negative policy rates, including Sweden, Denmark, and Switzerland.

<sup>10</sup> EONIA is a weighted average of all unsecured overnight euro lending in the euro area interbank market. The DFR is the lowest of the ECB's three policy rates and has historically served as a floor for spot EONIA, with particularly tight spreads in recent years amid excess liquidity.

perceptions at the time at or close to zero.<sup>11</sup> Over subsequent years, however, the ECB lowered its policy rate further to its current level of  $-0.5$  percent. As shown in Figure 2, spot EONIA has remained slightly above the contemporaneous deposit facility rate throughout, but the one-year forward rate has at times been notably lower, thus reflecting investor expectations of further reductions in the deposit facility rate consistent with a reassessment of the perceived ELB.<sup>12</sup> Figure 3 further illustrates the concept of the *perceived level of the lower bound*, with a graph of the forward rate curve for EONIA as of mid-2019. While the short end of the curve is anchored slightly above the then-current DFR of  $-0.4$  percent, the curve reflects expectations of declines well below that level over the next 1–2 years, to about  $-0.55$  percent, before sloping up. The lowest point on the forward rate curve provides an upper limit on the market’s perceived lower bound at the time.<sup>13</sup>

Figure 2: Euro Area Policy and Interbank Lending Rates



Note: EONIA series are end-of-month observations.

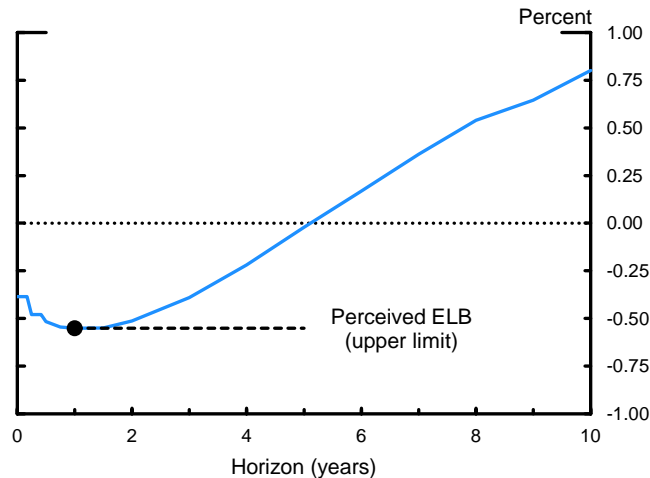
Source: J.P. Morgan Chase & Co., MorganMarkets,  
<https://www.hub.com/hub/>.

<sup>11</sup> See, for example, Lemke, W., and A. L. Vladu (2017), “Below the zero lower bound: a shadow-rate term structure model for the euro area,” ECB Working Paper Series No. 1991. Available at: <https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp1991.en.pdf>

<sup>12</sup> Forward rates generally reflect a term premium component in addition to expected future rates. Nevertheless, forward rates are subject to a lower bound under the same conditions as long-term yields. In particular, therefore, if the lower bound is hard, term premiums cannot become so negative as to push forward rates below the ELB. On the other hand, if the ELB is soft, the presence of term premiums may confound the identification of the level of the perceived lower bound.

<sup>13</sup> Investors may be placing some odds on EONIA rate realizations *below* the forward rate.

Figure 3: EONIA-OIS Forward Curve (Mid-2019)



Source: J.P. Morgan Chase & Co., MorganMarkets,  
<https://www.hub.com/hub/>.

Whether, in light of this representative evidence from the euro area, the assumption of a hard lower bound is acceptable may be an empirical question and depend on the application. The assumption is commonly used, for example, in so-called shadow rate models of the term structure, including Lemke and Vladu (2017) (see footnote 11 above). Such a model is able to capture well the shape of the forward rate curve in Figure 3, in which the forward rate rises monotonically at horizons beyond the point corresponding to the perceived lower bound.

## **Assumption 2: Absence of Arbitrage Opportunities**

The market for U.S. Treasury securities is considered the deepest and most liquid market for government securities in the world. This is reflected in generally high trading volumes and tight bid-ask spreads. The no-arbitrage assumption is thus likely a reasonable first approximation.<sup>14</sup> Gagnon and Jeanne (2020) point out that it might be possible for a central bank to overpower arbitrageurs by purchasing a dominating fraction of outstanding securities, or imposing regulatory holding requirements on banks, thus effectively limiting market forces that would otherwise ensure the absence of arbitrage

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<sup>14</sup> This has held true even at times of heightened market stress (including the recent coronavirus episode).

opportunities.<sup>15</sup> However, the FOMC has stressed that its asset purchases are, in part, intended to support smooth functioning of the Treasury market in order to help the transmission of monetary policy. Therefore, even if it might be possible to identify transient deviations from model-implied no-arbitrage restrictions (for example, on days of high volatility, or for high-demand securities such as on-the-run Treasury securities), systematic and persistent deviations from no-arbitrage (at least those that would be large enough to allow  $y_t^T < \underline{r}$  for longer-term yields) seem unlikely in U.S. Treasury markets.

### **Assumption 3: The Policy Rate is the Relevant Short Rate**

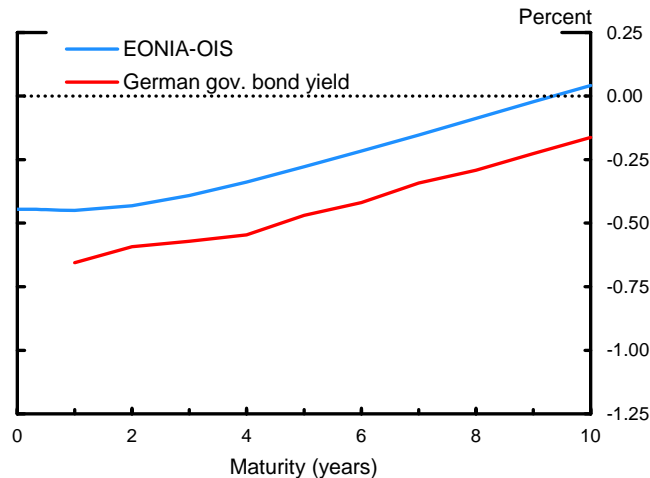
The third assumption is that the policy rate is the relevant discount rate for long-term bonds. Gagnon and Jeanne (2020) invoke this assumption when saying that “central bank policy sets the lower bound for bond yields.” In the U.S., the policy rate (federal funds rate) is the short rate underlying the OIS curve; the assumption therefore does not hold exactly if the bond yields we have in mind are Treasury yields. The federal funds rate can deviate somewhat from short-term Treasury yields as the two markets differ along a number of dimensions including access, credit risk, and liquidity-driven demands.

That said, as long as short-term yields corresponding to the yield curve of interest (such as T-bill yields in the case of the Treasury curve) move roughly in lockstep with the policy rate, we might still argue that the central bank in large part controls the lower bound for bond yields, although, in that case, the perceived lower bound for the policy rate could have a level difference from the perceived lower bound that is implicit in the government bond yield curve.

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<sup>15</sup> For example, if the central bank were to set a target for long-term yields below their no-arbitrage lower bound, it would only be able to defend this target by altogether depleting the market of securities available for borrowing and short-selling by arbitrageurs.

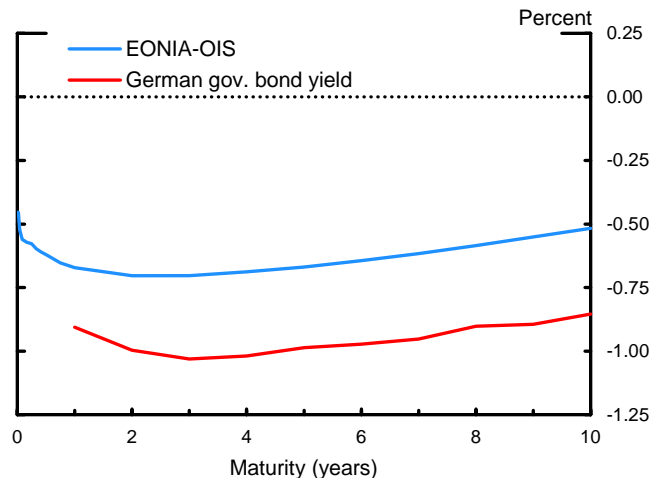
Figure 4: Euro Area Yield Curves (Year-End 2019)



Source: J.P. Morgan Chase & Co., MorganMarkets,  
<https://www.hub.com/hub/>.

The recent behavior of the EONIA-OIS yield curve and the German government yield curve provides a case in point: During the coronavirus-related market turmoil, Germany's 10-year government bond yield fell to a level significantly below the ECB's current deposit facility rate. However, even during normal times the German yield curve has had a notable spread to the euro area OIS curve, as seen in Figure 4. When, on March 9, 2020, the OIS curve became inverted for near-term maturities, the entire German yield curve fell below the current ECB policy rate (Figure 5). This can be plausibly accounted for by a combination of investor expectations of further declines in the policy rate (as implied by the initially downward-sloping OIS curve) and the usual spread between the two curves (likely amplified somewhat by safe-haven flows).

Figure 5: Euro Area Yield Curves (March 9, 2020)



Source: J.P. Morgan Chase & Co., MorganMarkets,  
<https://www.hub.com/hub/>.



## Discussion and Conclusion

As reviewed above, under appropriate technical assumptions, Gagnon and Jeanne's (2020) main theoretical argument—that a lower bound on the policy rate implies a lower bound on longer-term yields—is sound. The assumptions are unlikely to hold exactly in practice so that there is some leeway for, say, long-term government bond yields to fall below the level of the lower bound on the current policy rate (as perceived by investors). However, even if long-term Treasury yields in the U.S. are subject to only a soft lower bound, based on the authors' arguments policymakers face potential tension between communicating a credible lower bound on short-term rates and attempting to lower long-term rates through large-scale asset purchases. When longer-term yields approach their perceived lower bound, the likely effects of asset purchases on term premiums (and possibly on short rate expectations, via a signaling channel) will become nonlinear; put differently, this policy tool faces decreasing returns along the dimension of lowering the level of longer-term yields (unless asset purchases by the central bank reach the point of inducing significant scarcity effects). In light of this tension, [Bernanke \(2020\)](#) has recently noted that it might be helpful for policymakers to maintain “constructive ambiguity” about the future use of negative short-term rates.<sup>16</sup> He argues that such ambiguity might help ease the (perceived) ELB constraint on the policy rate, thereby creating somewhat more policy space at the long end of the yield curve. Lastly, it is worth emphasizing that lowering the level of longer-term yields is only one of several goals the FOMC has articulated for its large-scale asset purchases. Even with a low level of yields relative to their perceived ELB, asset purchases may continue to play important roles in supporting smooth market functioning and in alleviating upward pressure on yields. In particular, projected increases in Treasury issuance corresponding to coronavirus-related fiscal stimulus spending are likely to create substantial room for further purchases by the Federal Reserve without approaching any lower bound on Treasury yields.

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<sup>16</sup> Bernanke, B. (2020), “The new tools of monetary policy,” *Brookings* (blog), January 4, <https://www.brookings.edu/blog/ben-bernanke/2020/01/04/the-new-tools-of-monetary-policy/>