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To: Research Directors

From: Matthew M. Luecke

Subject: Supporting Documents for DSGE Models Update

The attached documents support the update on the projections of the DSGE models.

System DSGE Memo -- Research Directors Drafts

EDO (Board), New York Fed, Prism (Philadelphia Fed), Chicago Fed

September 2020

The Current Outlook in EDO: September 2020 FOMC Meeting Class II FOMC – Restricted (FR)

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1 The EDO Forecast from 2020 to 2023

Reflecting the huge movements in economic activity in the data for 2020:Q2 and the staff's nowcast for 2020:Q3 – and informed by the staff's assessment of the likely effects of social distancing over the next several quarters –, the EDO model forecast calls for GDP to fall 2.9 percent this year and then to rebound 5.4 percent in 2021. Inflation is subdued, hovering around 1.5 percent through the end of 2023. The federal funds rate remains at the effective lower bound (ELB) until the end of 2022, reflecting both an accommodative monetary policy stance and the sluggish pace of economic activity following the rebound.[1] The EDO model forecast conditions on the data and nowcast for 2020:Q2 and 2020:Q3 and must therefore attempt to extrapolate from the unprecedented turmoil in those quarters. Because the disruption associated with the pandemic lies far outside the model's estimation sample and structure, we guide the model using the staff's July Tealbook projection for social-distancing effects on consumption, investment, and employment through the end of 2021. In particular, in the model, we represent this sequence of effects by anticipated shocks to technology and household preferences for consumption and investment, recognized by private agents in 2020:Q2.

With the federal funds rate at the effective lower bound in 2020:Q2, we also assume that the public in that quarter expects the federal funds rate to remain at the ELB until the middle of 2022, in line with survey evidence suggesting expectations of an ELB episode of several years. The expectation of an extended spell at the ELB arises, in large part, from the arrival of news about the future stance of monetary policy, which the model views as unusually accommodative. When calculating the distribution of outcomes over the forecast horizon, we assume that monetary policy keeps the federal funds rate at the ELB until mid-2022 without reference to particular exit conditions.

*The author is affiliated with the Division of Research and Statistics of the Federal Reserve Board. Sections 2 and 3 contain background material on the EDO model, as in previous rounds. These sections were co-written with Jean-Philippe Laforte.

[1] The Alternative Models exhibit in the Risks and Uncertainty section of the Tealbook reports an alternative forecast using the EDO model, but conditioning on the staff's forecast for 2020 as a whole.

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Uncertainty about the path of the pandemic and its attendant macroeconomic effects is a central element of the model projection in these circumstances. In particular, motivated by the substantial probability that secondary epidemics may trigger renewed bouts of intense social distancing, we assume that a second wave may begin in 2020Q4 and 2021Q1 with a 25 percent probability each quarter; the course of the second wave follows that of the first, but with a scale uniformly distributed between 25 and 75 percent the size of the first wave. These assumptions contribute to a large adverse bias away from the modal forecast in 2020:Q4 and 2021:Q1, apparent in the low growth rates for both of those years.

2 An Overview of Key Model Features

Figure 1 provides a graphical overview of the model. While similar to most related models, EDO has a more detailed description of production and expenditure than most other models.[2]

Specifically, the model possesses two final good sectors in order to capture key long-run growth facts and to differentiate between the cyclical properties of different categories of durable expenditure (for example, housing, consumer durables, and nonresidential investment). For example, technological progress has been faster in the production of business capital and consumer durables (such as computers and electronics).

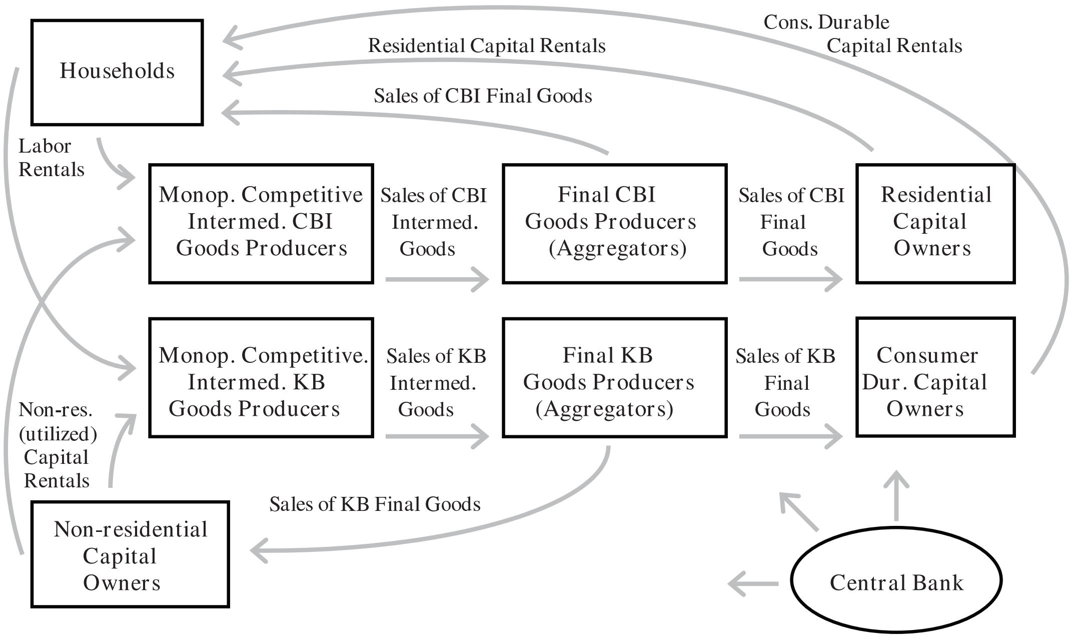
The disaggregation of production (aggregate supply) leads naturally to some disaggregation of expenditures (aggregate demand). We move beyond the typical model with just two categories of (private domestic) demand (consumption and investment) and distinguish between four categories of private demand: consumer nondurable goods and nonhousing services, consumer durable goods, residential investment, and nonresidential investment. The boxes surrounding the producers in the figure illustrate how we structure the sources of each demand category. Consumer nondurable goods and services are sold directly to households; consumer durable goods, residential capital goods, and nonresidential capital goods are intermediated through capital-goods intermediaries (owned by the households), who then rent these capital stocks to households. Consumer nondurable goods and services and residential capital goods are purchased (by households and residential capital goods owners, respectively) from the first of economy's two final goods-producing sectors, while consumer durable goods and nonresidential capital goods are purchased (by consumer durable and residential capital goods owners, respectively) from the second sector. In addition to consuming the nondurable goods and services that they purchase, households supply labor to the intermediate goods-producing firms in both sectors of the economy.

The remainder of this section provides an overview of the main properties of the model. In particular, the model has five key features:

- A New-Keynesian structure for price and wage dynamics. Unemployment measures the difference between the amount workers are willing to be employed and firms' employment demand. As a result, unemployment is an indicator of wage and, hence, price pressures as in Gali (2011).

[2] Chung, Kiley, and Laforte (2010) provide much more detail regarding the model specification, estimated parameters, and model properties.

Figure 1: Model Overview



- Production of goods and services occurs in two sectors, with differential rates of technological progress across sectors. In particular, productivity growth in the investment and consumer durable goods sector exceeds that in the production of other goods and services, helping the model match facts regarding long-run growth and relative price movements.

- A disaggregated specification of household preferences and firm production processes that leads to separate modeling of nondurables and services consumption, durables consumption, residential investment, and business investment.

- Risk premiums associated with different investment decisions play a central role in the model. These include, first, an aggregate risk premium, or natural rate of interest, shock driving a wedge between the short-term policy rate and the interest rate faced by private decisionmakers (as in Smets and Wouters (2007)) and, second, fluctuations in the discount factor/risk premiums faced by the intermediaries financing household (residential and consumer durable) and business investment.

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2.1 Two-sector production structure

It is well known (for example, Edge, Kiley, and Laforte (2008)) that real outlays for business investment and consumer durables have substantially outpaced those on other goods and services, while the prices of these goods (relative to others) has fallen. For example, real outlays on consumer durables have far outpaced those on other consumption while prices for consumer durables have been flat and those for other consumption have risen substantially; as a result, the ratio of nominal outlays in the two categories has been much more stable, although consumer durable outlays plummeted in the Great Recession. Many models fail to account for this fact.

EDO accounts for this development by assuming that business investment and consumer durables are produced in one sector and other goods and services in another sector. Specifically, production by firm j in each sector s (where s equals kb for the sector producing business investment and consumer durables and cbi for the sector producing other goods and services) is governed by a Cobb-Douglas production function with sector-specific technologies:

$$X_t^s(j) \ = \left(Z_t^m Z_t^s L_t^s(j)\right)^{1-\alpha} \left(K_t^{u,nr,s}(j)\right)^{\alpha}, \text{ for } s = cbi, kb. \tag{1}$$

In equation (1), Z^m represents (labor-augmenting) aggregate technology, while Z^s represents (labor-augmenting) sector-specific technology; we assume that sector-specific technological change affects the business investment and consumer durables sector only. L^s is labor input and $K^{u,nr,s}$ is capital input (that is, utilized *nonresidential business* capital (and hence the nr and u terms in the superscript). Growth in this sector-specific technology accounts for the long-run trends, while high-frequency fluctuations allow for the possibility that investment-specific technological change is a source of business cycle fluctuations, as in Fisher (2006).

2.2 The structure of demand

EDO differentiates between several categories of expenditure. Specifically, business investment spending determines nonresidential capital used in production, and households value consumer nondurables goods and services, consumer durable goods, and residential capital (for example, housing). Differentiation across these categories is important, as fluctuations in these categories of expenditure can differ notably, with the cycles in housing and business investment, for example, occurring at different points over the last three decades.

Valuations of these goods and services, in terms of household utility, is given by the following utility function:

$$\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \varsigma^{cnn} \ln(E_t^{cnn}(i) - h E_{t-1}^{cnn}(i)) + \varsigma^{cd} \ln(K_t^{cd}(i)) + \varsigma^{r} \ln(K_t^{r}(i)) - \Lambda_t^{Lpref} \Theta_t^H \sum_{s=cbi,kb} \int_0^1 \varsigma^{l,s} L_t^s(i)^{\frac{1+\sigma_N}{1+\frac{\sigma_N}{1+\sigma_h}}} di \Big\}, \tag{2}$$

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where E^{cnn} represents expenditures on consumption of nondurable goods and services, K^{cd} and K^{r} represent the stocks of consumer durables and residential capital (housing), Λ_t^{Lpref} represents a labor supply shock, Θ_t is an endogenous preference shifter whose role is to reconcile the existence of a long-run balance growth path with a small short-term wealth effect[3], L^{cbi} and L^{kb} represent the labor supplied to each productive sector (with hours worked causing disutility), and the remaining terms represent parameters (such as the discount factor, relative value in utility of each service flow, and the elasticity of labor supply). Gali, Smets, and Wouters (2011) state that the introduction of the endogenous preference shifter is key in order to match the joint behavior of the labor force, consumption, and wages over the business cycle.

By modeling preferences over these disaggregated categories of expenditure, EDO attempts to account for the disparate forces driving consumption of nondurables and durables, residential investment, and business investment —thereby speaking to issues such as the surge in business investment in the second half of the 1990s or the housing cycle in the early 2000s recession and the most recent downturn. Many other models do not distinguish between developments across these categories of spending.

2.3 Risk premiums, financial shocks, and economic fluctuations

The structure of the EDO model implies that households value durable stocks according to their expected returns, including any expected service flows, and according to their risk characteristics, with a premium on assets that have high expected returns in adverse states of the world. However, the behavior of models such as EDO is conventionally characterized under the assumption that this second component is negligible. In the absence of risk adjustment, the model would then imply that households adjust their portfolios until expected returns on all assets are equal.

Empirically, however, this risk adjustment may not be negligible and, moreover, there may be a variety of factors, not explicitly modeled in EDO, that limit the ability of households to arbitrage away expected return differentials across different assets. To account for this possibility, EDO features several exogenous shocks to the rates of return required by the household to hold the assets in question. Following such a shock —an increase in the premium on a given asset, for example —households will wish to alter their portfolio composition to favor the affected asset, leading to changes in the prices of all assets and, ultimately, to changes in the expected path of production underlying these claims.

The "sector specific" risk shocks affect the composition of spending more than the path of GDP itself. This occurs because a shock to these premiums leads to sizable substitution across residential, consumer durable, and business investment; for example, an increase in the risk premiums on residential investment leads households to shift away from residential investment and toward other types of productive investment. Consequently, it is intuitive that a large fraction of the non-cyclical, or idiosyncratic, component of investment flows to physical stocks will be accounted for by movements in the associated premiums.

[3]The endogenous preference shifter is defined as $\Theta_t^H = Z_t \Lambda_t^{cnn}$, where $Z_t = \frac{Z_{t-1}^{1-\nu}}{\Lambda_t^{cnn}}$ and Λ_t^{cnn} is the shadow price of nondurable consumption. The importance of the short-term wealth effect is determined by the parameter $\nu \in (0, 1]$.

Shocks to the required rate of return on the nominal risk-free asset play an especially large role in EDO. Following an increase in the premium, in the absence of nominal rigidities, the households' desire for higher real holdings of the risk-free asset would be satisfied entirely by a fall in prices, that is, the premium is a shock to the natural rate of interest. Given nominal rigidities, however, the desire for higher risk-free savings must be offset, in part, through a fall in real income, a decline which is distributed across all spending components. Because this response is capable of generating co-movement across spending categories, the model naturally exploits such shocks to explain the business cycle. Reflecting this role, we denote this shock as the "aggregate risk-premium."

Movements in financial markets and economic activity in recent years have made clear the role that frictions in financial markets play in economic fluctuations. This role was apparent much earlier, motivating a large body of research (for example, Bernanke, Gertler, and Gilchrist (1999)). While the range of frameworks used to incorporate such frictions has varied across researchers studying different questions, a common theme is that imperfections in financial markets —for example, related to imperfect information on the outlook for investment projects or earnings of borrowers —drives a wedge between the cost of riskless funds and the cost of funds facing households and firms. Much of the literature on financial frictions has worked to develop frameworks in which risk premiums fluctuate for endogenous reasons (for example, because of movements in the net worth of borrowers). Because the risk-premium shocks induces a wedge between the short-term nominal risk-free rate and the rate of return on the affected risky rates, these shocks may thus also be interpreted as a reflection of financial frictions not explicitly modeled in EDO. The sector-specific risk premiums in EDO enter the model in much the same way as does the exogenous component of risk premiums in models with some endogenous mechanism (such as the financial accelerator framework used Boivin, Kiley, and Mishkin (2010)), and the exogenous component is quantitatively the most significant one in that research.[4]

2.4 Labor market dynamics in the EDO model

This version of the EDO model assumes that labor input consists of both employment and hours per worker. Workers differ in the disutility they associate with employment. Moreover, the labor market is characterized by monopolistic competition. As a result, unemployment arises in equilibrium – some workers are willing to be employed at the prevailing wage rate, but cannot find employment because firms are unwilling to hire additional workers at the prevailing wage.

As emphasized by Gali (2011), this framework for unemployment is simple and implies that the unemployment rate reflects wage pressures: When the unemployment rate is unusually high, the prevailing wage rate exceeds the marginal rate of substitution between leisure and consumption, implying that workers would prefer to work more.

The new preference specification and the incorporation of labor force participation in the information set impose discipline in the overall labor market dynamics of the EDO model. The estimated short-run wealth effect on labor supply is relatively attenuated with respect to previous versions of

[4]Specifically, the risk premiums enter EDO to a first-order (log)linear approximation in the same way as in the cited research if the parameter on net worth in the equation determining the borrowers cost of funds is set to zero; in practice, this parameter is often fairly small in financial accelerator models.

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the EDO model. Therefore, the dynamics of both labor force participation and employment are more aligned with the empirical evidence.

In addition, in our environment, nominal wage adjustment is sticky, and this slow adjustment of wages implies that the economy can experience sizable swings in unemployment with only slow wage adjustment. Our specific implementation of the wage adjustment process yields a relatively standard New Keynesian wage Phillips curve. The presence of both price and wage rigidities implies that stabilization of inflation is not, in general, the best possible policy objective (although a primary role for price stability in policy objectives remains).

While the specific model on the labor market is suitable for discussion of the links between employment and wage/price inflation, it leaves out many features of labor market dynamics. Most notably, it does not consider separations, hires, and vacancies, and is hence not amenable to analysis of issues related to the Beveridge curve.

The decline in employment during the Great Recession primarily reflected, according to the EDO model, the weak demand that arose from elevated risk premiums that depressed spending, as illustrated by the light blue and red bars in figure **??**. The role played by these demand factors in explaining the cyclical movements in employment is only determinant during the 1980s and during the Great Recession. As apparent in figure **??**, the most relevant drivers of employment in the remaining of the sample are labor supply (preference) and markup shocks as shown by the blue bars. Specifically, favorable supply developments in the labor market are estimated to have placed upward pressure on employment until 2010; these developments have reversed, and some of the currently low level for employment growth is, according to EDO, attributable to adverse labor market supply developments. As discussed previously, these developments are simply exogenous within EDO and are not informed by data on a range of labor market developments (such as gross worker flows and vacancies).

2.5 New Keynesian price and wage Phillips curves

As in most of the related literature, nominal prices and wages are both "sticky" in EDO. This friction implies that nominal disturbances —that is, changes in monetary policy —have effects on real economic activity. In addition, the presence of both price and wage rigidities implies that stabilization of inflation is not, in general, the best possible policy objective (although a primary role for price stability in policy objectives remains).

Given the widespread use of the New Keynesian Phillips curve, it is perhaps easiest to consider the form of the price and wage Phillips curves in EDO at the estimated parameters. The price Phillips curve (governing price adjustment in both productive sectors) has the form

$$\pi_t^{p,s} = 0.22\pi_{t-1}^{p,s} + 0.76E_t\pi_{t+1}^{p,s} + .017mc_t^s + \theta_t^s \tag{3}$$

where mc is marginal cost and θ is a markup shock. As the parameters indicate, inflation is primarily forward looking in EDO.

The wage (w) Phillips curve for each sector has the form

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$$\triangle w_t^s = 0.01\triangle w_{t-1}^s + 0.95E_t\triangle w_{t+1}^s + .012\left(mrs_t^{c,l} - w_t^s\right) + \theta_t^w + adj.\,costs. \tag{4}$$

where mrs represents the marginal rate of substitution between consumption and leisure. Wages are primarily forward looking and relatively insensitive to the gap between households' valuation of time spent working and the wage.

The top right panel of figure **??** presents the decomposition of inflation fluctuations into the exogenous disturbances that enter the EDO model. As can be seen, aggregate demand fluctuations, including aggregate risk premiums and monetary policy surprises, contribute little to the fluctuations in inflation according to the model. This is not surprising: In modern DSGE models, transitory demand disturbances do not lead to an unmooring of inflation (so long as monetary policy responds systematically to inflation and remains committed to price stability). In the short run, inflation fluctuations primarily reflect transitory price and wage shocks, or markup shocks in the language of EDO. Technological developments can also exert persistent pressure on costs, most notably during and following the strong productivity performance of the second half of the 1990s, which is estimated to have lowered marginal costs and inflation through the early 2000s. More recently, disappointing labor productivity readings over the course of 2011 have led the model to infer sizable negative technology shocks in both sectors, contributing noticeably to inflationary pressure over that period (as illustrated by the blue bars in figure **??**).

2.6 Monetary authority and a long-term interest rate

We now turn to the last agent in our model, the monetary authority. It sets monetary policy in accordance with an Taylor-type interest rate feedback rule. Policymakers smoothly adjust the actual interest rate R_t to its target level $\bar{R}_t$

$$R_t = (R_{t-1})^{\rho^r}\left(\bar{R}_t\right)^{1-\rho^r}\exp\left[\epsilon_t^r\right], \tag{5}$$

where the parameter ρ^r reflects the degree of interest rate smoothing, while ϵ_t^r represents a monetary policy shock. The central bank's target nominal interest rate, $\bar{R}_t$ depends on the deviation of output from the level consistent with current technologies and "normal" (steady-state) utilization of capital and labor ($\tilde{X}^{pf}$, the "production function" output gap). Also, the change in the output gap and consumer price inflation enter the target. The target equation is

$$\bar{R}_t = \left(\tilde{X}_t^{pf}\right)^{r^y}\left(d\tilde{X}_t^{pf}\right)^{r^{dy}}\left(\frac{\Pi_t^c}{\Pi_*^c}\right)^{r^\pi} R_*. \tag{6}$$

In equation (6), R_* denotes the economy's steady-state nominal interest rate, $d\tilde{X}_t^{pf}$ denotes the change in the output gap and r^y, r^{dy} and r^π denote the weights in the feedback rule. Consumer price inflation, Π_t^c, is the weighted average of inflation in the nominal prices of the goods produced

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in each sector, $\Pi_t^{p,cbi}$ and $\Pi_t^{p,kb}$:

$$\Pi_t^c = (\Pi_t^{p,cbi})^{1-w_{cd}} (\Pi_t^{p,kb})^{w_{cd}}. \tag{7}$$

The parameter w^{cd} is the share of the durable goods in nominal consumption expenditures.

The model also includes a long-term interest rate (RL_t), which is governed by the expectations hypothesis subject to an exogenous term premium shock:

$$RL_t = \mathcal{E}_t \left[\Pi_{\tau=0}^{N} R_\tau \right] \cdot \Upsilon_t. \tag{8}$$

where Υ is the exogenous term premium, governed by

$$Ln\left(\Upsilon_t\right) = \left(1 - \rho^{\Upsilon}\right) Ln\left(\Upsilon_*\right) + \rho^{\Upsilon} Ln\left(\Upsilon_{t-1}\right) + \epsilon_t^{\Upsilon}. \tag{9}$$

In this version of EDO, the long-term interest rate plays no allocative role; nonetheless, the term structure contains information on economic developments useful for forecasting (for example, Edge, Kiley, and Laforte (2010)), and hence RL is included in the model and its estimation.

2.7 Summary of model specification

Our brief presentation of the model highlights several points. First, although our model considers production and expenditure decisions in a bit more detail, it shares many similar features with other DSGE models in the literature, such as imperfect competition, nominal price and wage rigidities, and real frictions like adjustment costs and habit-persistence. The rich specification of structural shocks (to aggregate and investment-specific productivity, aggregate and sector-specific risk premiums, and markups) and adjustment costs allows our model to be brought to the data with some chance of finding empirical validation.

Within EDO, fluctuations in all economic variables are driven by 13 structural shocks. It is most convenient to summarize these shocks into five broad categories:

- Permanent technology shocks: This category consists of shocks to aggregate and investment-specific (or fast-growing sector) technology.

- A labor supply shock: This shock affects the willingness to supply labor. As was apparent in our earlier description of labor market dynamics and in the presentation of the structural drivers below, this shock captures the dynamics of the labor force participation rate in the sample and those of employment. While EDO labels such movements labor supply shocks, an alternative interpretation would describe these as movements in the labor force and employment that reflect structural features not otherwise captured by the model.

- Financial, or intertemporal, shocks: This category consists of shocks to risk premiums. In EDO, variation in risk premiums —both the premium households receive relative to the federal funds rate on nominal bond holdings and the additional variation in discount rates applied to the investment decisions of capital intermediaries —are purely exogenous. Nonetheless,

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the specification captures aspects of related models with more explicit financial sectors (for example, Bernanke, Gertler, and Gilchrist (1999)), as we discuss in our presentation of the model's properties below.

- Markup shocks: This category includes the price and wage markup shocks.
- Other demand shocks: This category includes the shock to autonomous demand and a monetary policy shock.

3 Estimation: Data and Properties

3.1 Data

The empirical implementation of the model takes a log-linear approximation to the first-order conditions and constraints that describe the economy's equilibrium, casts this resulting system in its state-space representation for the set of (in our case, 13) observable variables, uses the Kalman filter to evaluate the likelihood of the observed variables, and forms the posterior distribution of the parameters of interest by combining the likelihood function with a joint density characterizing some prior beliefs. Since we do not have a closed-form solution of the posterior, we rely on Markov-Chain Monte Carlo (MCMC) methods.

The model is estimated using 13 data series over the sample period from 1984:Q4 to 2015:Q3. The series are the following:

1. The growth rate of real gross domestic product (ΔGDP);
2. The growth rate of real consumption expenditure on nondurables and services (ΔC);
3. The growth rate of real consumption expenditure on durables (ΔCD);
4. The growth rate of real residential investment expenditure (ΔRes);
5. The growth rate of real business investment expenditure (ΔI);
6. Consumer price inflation, as measured by the growth rate of the Personal Consumption Expenditure (PCE) price index ($\Delta P_{C,total}$);
7. Consumer price inflation, as measured by the growth rate of the PCE price index excluding food and energy prices ($\Delta P_{C,core}$);
8. Inflation for consumer durable goods, as measured by the growth rate of the PCE price index for durable goods (ΔP_{cd});
9. Hours, which equals hours of all persons in the nonfarm business sector from the Bureau of Labor Statistics (H);
10. Civilian employment-population ratio, defined as civilian employment from the Current Population Survey (household survey) divided by the noninstitutional population, age 16 and over (N);
11. Labor force participation rate;
12. The growth rate of real wages, as given by compensation per hour in the non-farm business sector from the Bureau of Labor Statistics divided by the GDP price index (ΔRW); and
13. The federal funds rate (R).

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Our implementation adds measurement error processes to the likelihood implied by the model for all of the observed series used in estimation except the short-term nominal interest rate series.

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New York Fed DSGE Model: Research Directors Draft

September 4, 2020

Introduction

This document describes the New York Fed DSGE model, which we use both for internal forecasting and for creating our contributions to the System DSGE memo distributed quarterly to the FOMC. The document is structured as follows. First, we provide a description and interpretation of the forecast for the current forecast horizon. Next, we describe the structure of the DSGE model followed by the impulse response functions to various shocks.

Model Forecast

The New York Fed model forecasts are obtained using data released through 2020Q2, augmented for 2020Q3 with the August Philadelphia Fed Survey of Professional Forecasters (SPF) median forecasts for real GDP growth and core PCE inflation (adjusted for the difference between the Blue Chip and SPF GDP deflator inflation forecasts, since the former incorporates the information in the August CPI release), the August consensus Financial Blue Chip forecasts for the GDP deflator, and the yields on 10-year Treasuries and Baa corporate bonds based on 2020Q3 averages up to August 27.[1] Moreover, the forecast is conditional on federal funds rate expectations derived from OIS data through 2021Q4.

As mentioned in the June memo, the model was changed in order to address the implications of the COVID-19 shock. In particular, the model was augmented with a number of both demand and supply shocks that are purely transitory and hit the economy in 2020Q1, Q2, and Q3, in order to capture the partly temporary nature of the COVID-19 shock. The demand shocks are so-called discount rate shocks that affect intertemporal consumption decisions, while the supply shocks are both productivity shocks and labor supply shifters. The standard deviations of these transitory shocks are drawn from a relatively uninformative

[1] The conditional number for GDP growth in 2020Q3 is interpreted as a noisy estimate of actual 2020Q3 GDP growth, as in Del Negro and Schorfheide (2013), section 5.3. We set the standard deviation of the noise to 2.0 (annualized).

prior distribution, allowing for uncertainty in the interpretation of the shutdown as a supply- or demand-driven phenomenon.

The degree to which the COVID-19 shock will have persistent effects on growth and inflation is very uncertain, because little is known about either the channels of transmission of the shock, or the likelihood of recurrence (i.e., future waves of contagion). This uncertainty is captured in the NY Fed DSGE forecasts using a combination of three scenarios, which are referred to as the "Temporary Shutdown", "Shutdown with Business Cycle Dynamics", and "Second Wave" scenarios. The "Temporary Shutdown" scenario explains the decline in economic activity in 2020Q1 and Q2 using predominantly the transitory shocks mentioned above, and intentionally limiting the role of standard shocks in these two quarters. This yields a relatively rapid recovery, with 2020 Q4/Q4 GDP growth of -3.8 percent and further rebound in economic activity in 2021 and 2022. In the "Shutdown with Business Cycle Dynamics" the usual set of shocks that populate the model (which have much more persistent dynamics than the COVID-19 shocks) play a larger role. This yields more persistent effects, with 2020 Q4/Q4 GDP growth in the neighborhood of -5 percent. It is worth noting that the forecast differences between these two scenarios are less stark than they were in June, at least for 2020, mostly because the second scenario projects higher output growth than it did back then. This indicates that the data so far point toward a robust recovery. In the medium run however, the two scenarios remain markedly different, with the second scenario predicting much more modest growth in 2021 and 2022.

Finally, the "Second Wave" scenario builds upon the "Temporary Shutdown" scenario by additionally assuming a renewed weakness in demand in 2020Q4, reflecting a resurgence of the pandemic in that quarter. We implement this scenario by imposing that the current quarter expectation for real GDP growth in Q4 coincides, up to some measurement error, with the 10th percentile of the cross-sectional distribution of SPF point forecasts (-0.36 percent, annualized). This scenario yields 2020 Q4/Q4 GDP growth in the neighborhood of -5.5 percent, not very distant from that in the second scenario. Differently from the second scenario, the "Second Wave" scenario features a stronger rebound of the economy in 2021 and 2022, as the effects of the second wave shock are transitory.

Note that the "Second Wave" scenario replaces the "Persistent Demand Shortfall" scenario featured in the June forecast, which turned out to be counterfactual (at least assuming the median SPF projections are broadly correct) in that this demand shortfall did not quite materialize in the current quarter. In all three scenarios the model allows for both the

COVID-19 and the standard business cycle shocks to be active in Q3, although both sets of shocks play a relatively small role in this quarter as the models projections were largely in line with the SPF forecasts.

The three scenarios are combined using weights (80, 10, and 10 percent, respectively) that are loosely informed by the SPF average probability distribution for 2020 year-over-year real GDP growth. In the combined forecast real GDP growth is expected to be -4.1 percent in 2020 on a Q4/Q4 basis, compared with a -6.2 percent projection in June. In 2021 and 2022, GDP growth is projected to recover to 5.9 and 4.4 percent respectively, much faster than predicted in June (2.1 and 0.8 percent, respectively). Core inflation is projected to be 0.8 percent in 2020, below the June forecast of 1.5 percent, and is expected to remain subdued throughout the forecast horizon, at 0.7 and 1.0 percent in 2021 and 2022, respectively. The small slope of the Phillips curve in the DSGE model implies that the drop in activity has a modest (relative to the size of the contraction) but prolonged effect on inflation.

The projections for all variables are surrounded by a large degree of uncertainty (although this has fallen somewhat relative to the June forecast for growth in 2020). For instance, the 68 percent probability interval ranges from -5.7 to -3 percent for 2020 GDP growth, and from 1.9 to 8.1 percent for 2021 GDP growth.

While a priori the COVID-19 shock can be interpreted as a combination of both supply and demand shocks, the model mostly leans on the latter in order to explain the data. As a consequence, the real natural rate falls temporarily by a large amount, reflecting the transitory nature of the shocks, although it recovers relatively rapidly. The real natural rate is -3.7 percent in 2020, and rises to -1.5 and -0.5 percent in 2021 and 2022, respectively. The output gap is estimated to be persistently negative, rising gradually from -5.6 percent in 2020Q4 to -1.4 percent in 2023Q4.

The model description part provides some detail on the modeling of the transitory COVID-19 shocks. The memo also contains at the end information about each of the three scenarios.

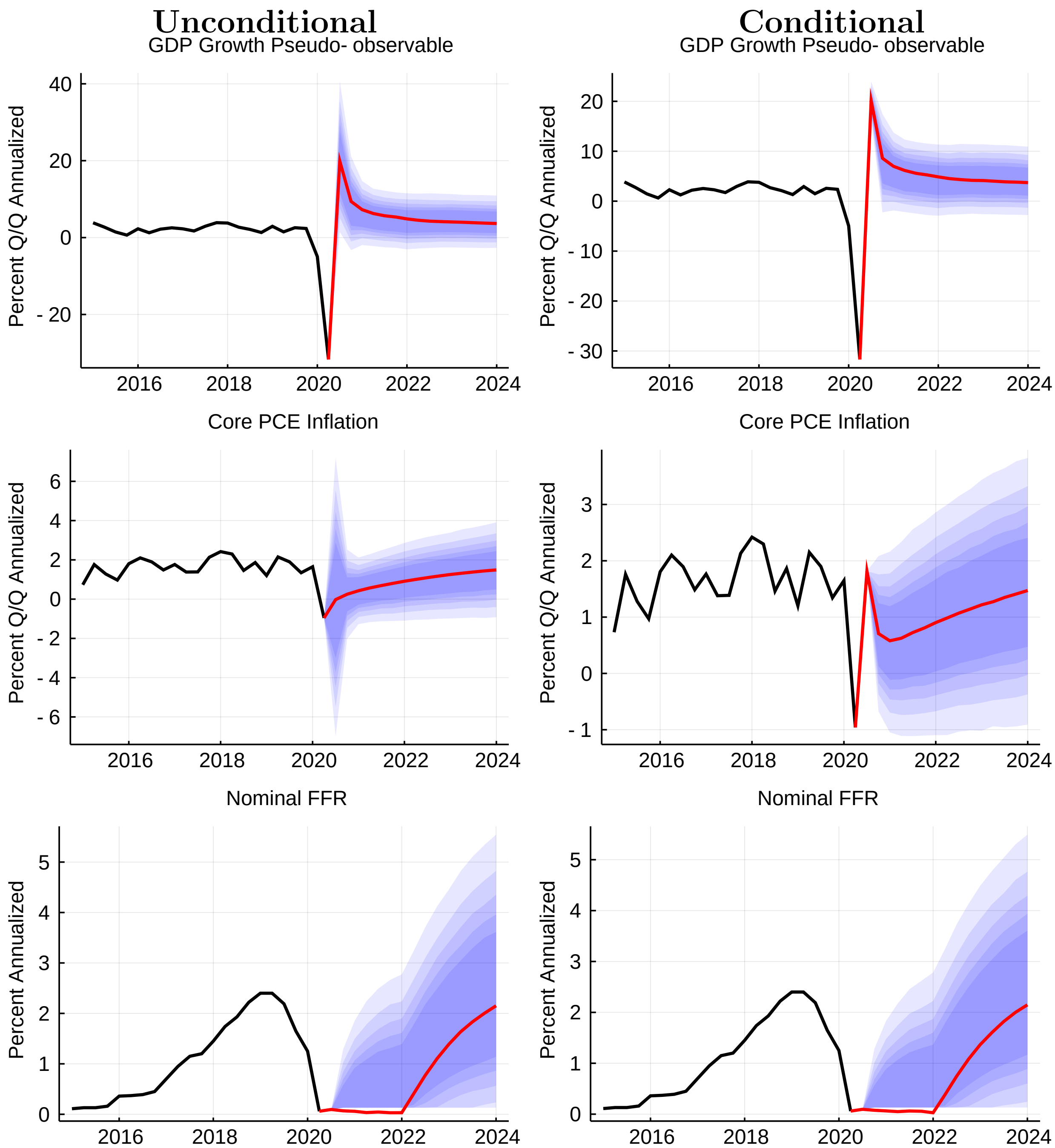
Table 1: Forecasts

Unconditional Forecast	2020		2021		2022		2023	
	Sep.	Jun.	Sep.	Jun.	Sep.	Jun.	Sep.	Jun.
Real GDP Growth (Q4/Q4)	−4.1 (−6.1,−2.5)	−3.3 (−5.7,−1.7)	6.1 (1.9,8.3)	1.6 (−2.1,3.6)	4.5 (1.2,7.2)	0.7 (−2.1,3.3)	3.9 (1.2,7.0)	1.2 (−1.4,4.1)
Core PCE Inflation (Q4/Q4)	0.2 (−1.1,1.5)	1.2 (0.7,1.6)	0.6 (−0.3,1.5)	1.0 (0.1,1.9)	1.0 (−0.1,2.1)	1.1 (−0.0,2.2)	1.3 (0.1,2.5)	1.3 (0.0,2.5)
Federal Funds Rate (Q4)	0.1 (0.1,0.8)	0.0 (0.0,1.1)	0.0 (0.1,1.7)	0.1 (0.0,1.8)	1.1 (0.2,3.0)	1.1 (0.1,3.1)	2.0 (0.6,4.1)	1.7 (0.4,3.8)
Real Natural Rate (Q4)	−3.9 (−7.4,−0.3)	−2.4 (−4.2,−0.6)	−1.5 (−3.0,0.0)	−0.1 (−1.6,1.3)	−0.5 (−2.1,1.2)	0.3 (−1.3,1.8)	0.2 (−1.5,1.9)	0.5 (−1.1,2.2)
Output Gap (Q4)	−5.6 (−7.5,−4.1)	−3.2 (−5.6,−1.7)	−3.3 (−7.2,−1.7)	−2.9 (−7.0,−1.1)	−2.2 (−7.2,0.1)	−3.3 (−8.2,−1.0)	−1.3 (−6.4,1.6)	−3.4 (−8.6,−0.4)

Conditional Forecast	2020		2021		2022		2023	
	Sep.	Jun.	Sep.	Jun.	Sep.	Jun.	Sep.	Jun.
Real GDP Growth (Q4/Q4)	−4.1 (−5.7,−3.0)	−6.2 (−9.4,−4.0)	5.9 (1.9,8.1)	2.1 (−1.5,4.2)	4.4 (1.2,7.1)	0.8 (−2.1,3.4)	3.9 (1.2,7.0)	1.3 (−1.2,4.3)
Core PCE Inflation (Q4/Q4)	0.8 (0.6,1.0)	1.5 (1.1,1.9)	0.7 (−0.2,1.6)	1.1 (0.2,2.0)	1.0 (−0.1,2.1)	1.1 (0.0,2.2)	1.3 (0.1,2.6)	1.3 (−0.0,2.5)
Federal Funds Rate (Q4)	0.1 (0.1,0.8)	0.0 (0.0,1.2)	0.0 (0.1,1.7)	0.1 (0.0,1.8)	1.1 (0.2,3.0)	1.1 (0.1,3.1)	2.0 (0.6,4.1)	1.8 (0.4,3.8)
Real Natural Rate (Q4)	−3.7 (−6.7,−0.7)	−3.3 (−4.9,−1.8)	−1.5 (−3.0,0.0)	−0.3 (−1.8,1.3)	−0.5 (−2.1,1.2)	0.2 (−1.4,1.9)	0.2 (−1.5,1.9)	0.5 (−1.1,2.2)
Output Gap (Q4)	−5.6 (−7.5,−4.1)	−5.7 (−9.3,−3.3)	−3.4 (−7.2,−1.7)	−4.8 (−9.8,−2.2)	−2.3 (−7.2,0.0)	−4.9 (−10.4,−2.0)	−1.4 (−6.4,1.6)	−4.8 (−10.2,−1.6)

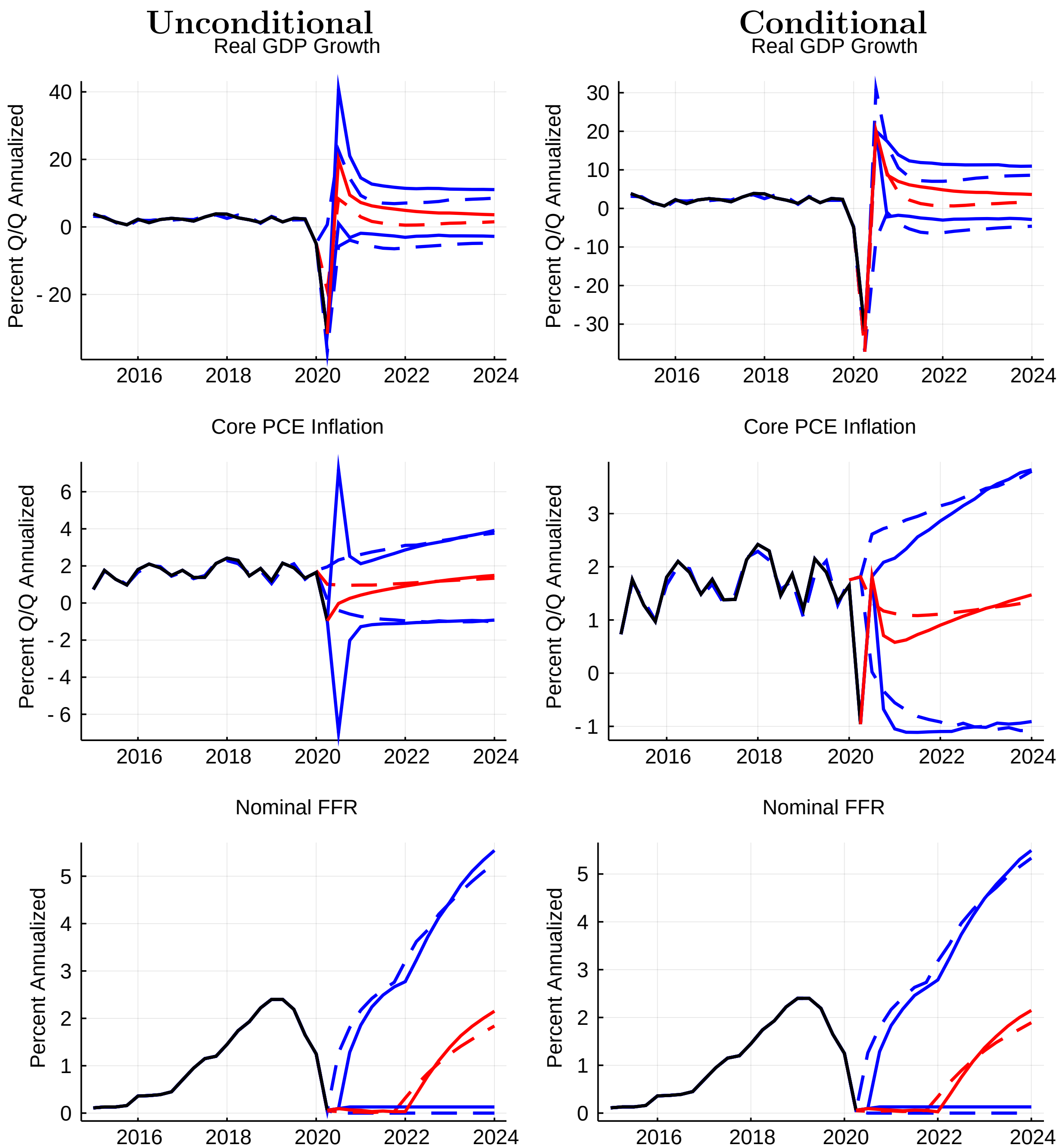
The unconditional forecasts use data up to the quarter for which we have the most recent GDP release, as well as the federal funds rate, 10-year Treasury yield, and spreads data for the following ("current") quarter. In the conditional forecasts, we further include the August Philadelphia Fed Survey of Professional Forecasters (SPF) median forecasts for real GDP growth and core PCE inflation (adjusted for the difference between the Blue Chip and SPF GDP deflator inflation forecasts, since the former incorporates the information in the August CPI release) and the August consensus Financial Blue Chip forecasts for the GDP deflator as additional data points for the current quarter. The conditional number for GDP growth in 2020Q3 is interpreted as a noisy estimate of actual 2020Q3 GDP growth, as in Del Negro and Schorfheide (2013), section 5.3. Numbers in parentheses indicate 68 percent probability intervals.

Figure 1: Forecasts



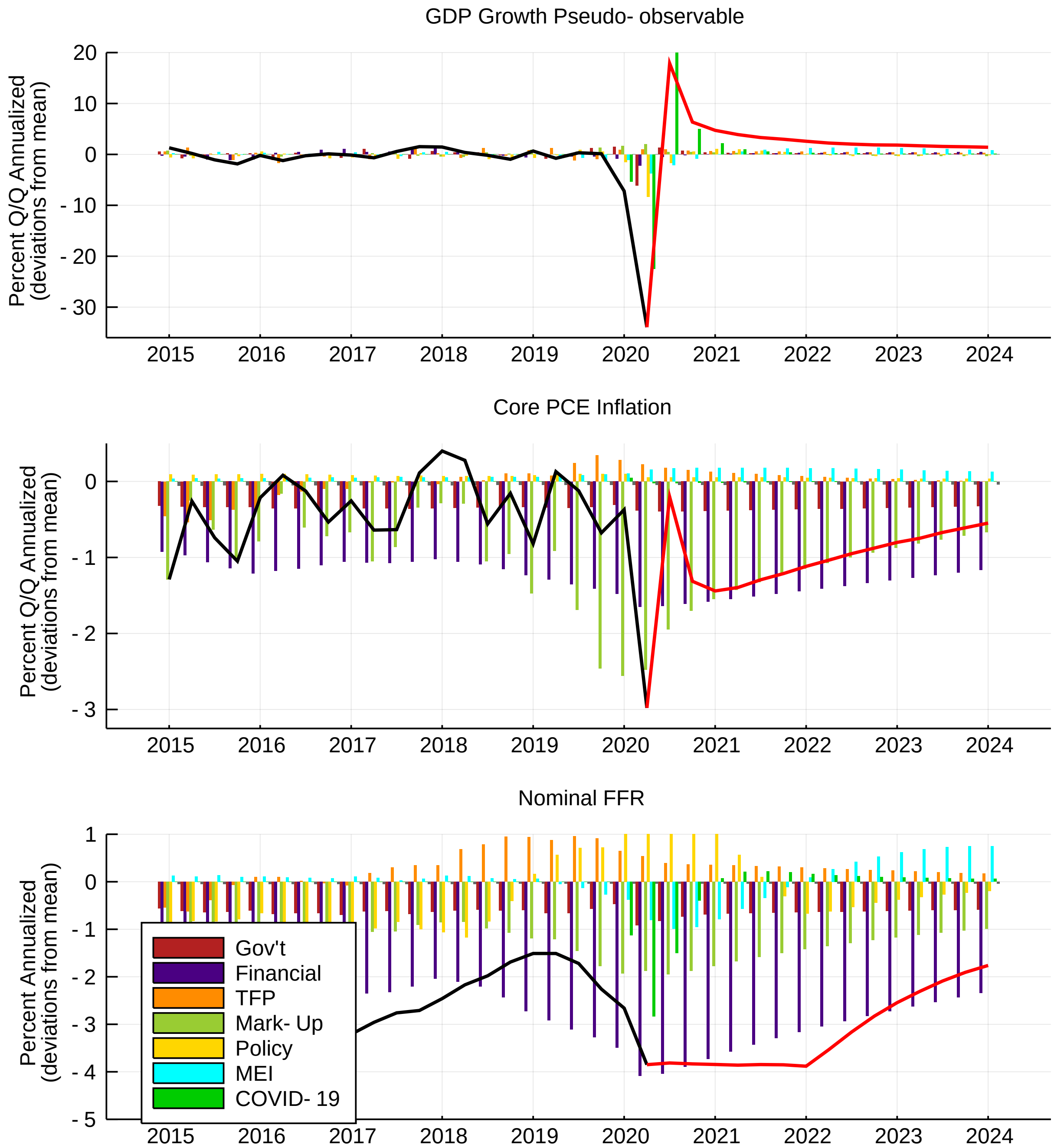
Quarterly forecasts, both unconditional (left panels) and conditional (right panels). The black line represents data, the red line indicates the mean forecast, and the shaded areas mark the 50, 60, 70, 80 and 90 percent probability intervals for the forecasts, reflecting both parameter and shock uncertainty.

Figure 2: Change in Forecasts



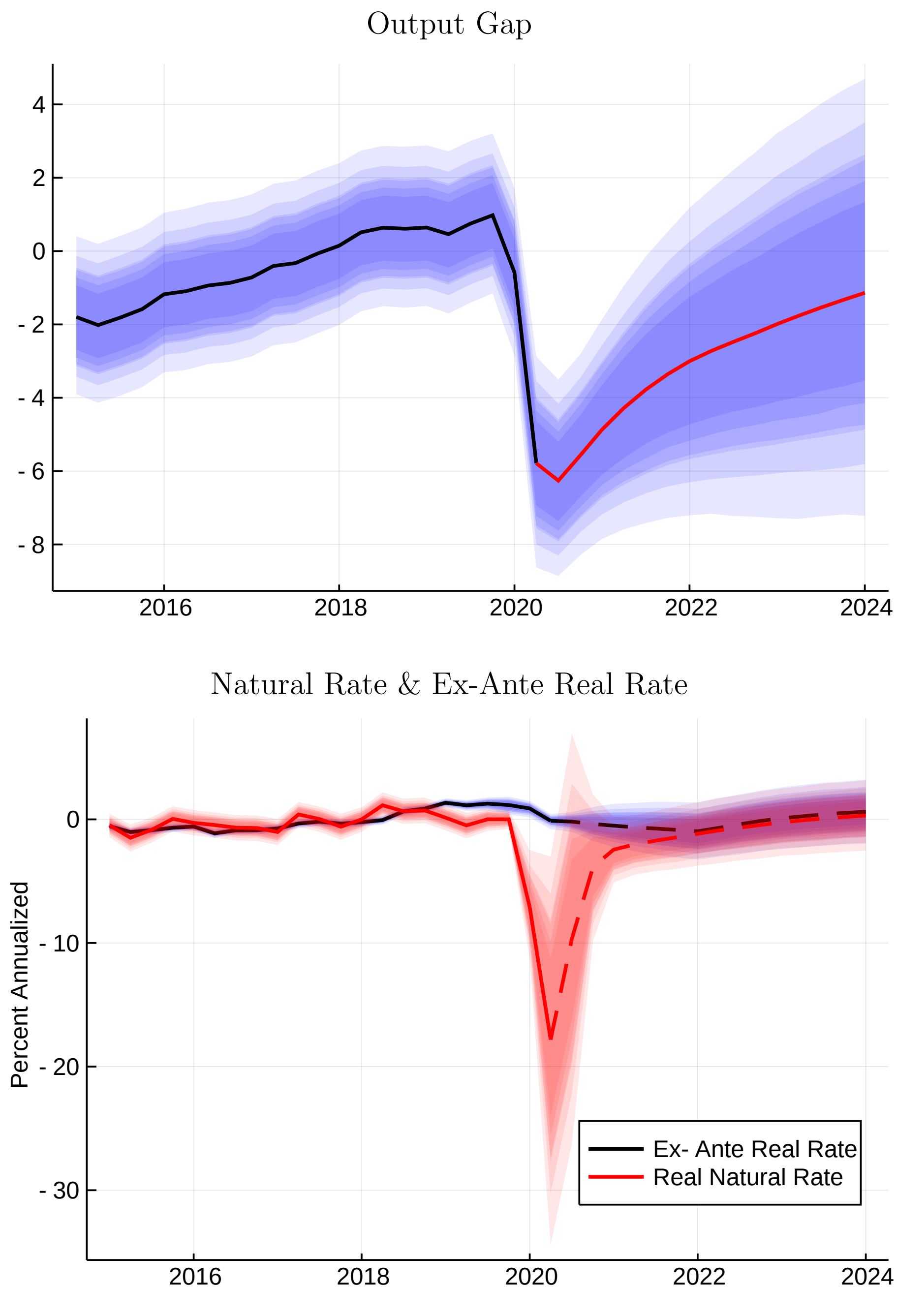
Comparison of current and previous quarterly forecasts. Solid (dashed) red and blue lines represent the mean and the 90 percent probability intervals, respectively, of the current (previous) forecast.

Figure 3: Shock Decomposition



Shock decomposition of the conditional forecast. The solid lines (black for realized data, red for mean forecast) show each variable in deviation from its steady state. The bars represent the shock contributions; specifically, the bars for each shock represent the counterfactual values for the observables (in deviations from the mean) obtained by setting all other shocks to zero.

Figure 4: Output Gap and Natural Interest Rate



Historical estimates and forecasts of the output gap (upper panel) and the real natural rate of interest and the ex-ante real interest rate (lower panel). In the upper panel, the black line represents the mean historical estimate, the red line the mean forecast. In the lower panel, the solid lines represent historical estimates and the dashed lines represent forecasts of the natural rate (red) and ex-ante rate (black). In both panels, the shaded areas mark the 50, 60, 70, 80, and 90 percent probability intervals for the historical estimates and forecasts, reflecting both parameter and shock uncertainty.

The Model

The following section contains a description of the New York Fed DSGE model and plots of impulse response functions.

General structure

The New York Fed DSGE model is a medium scale, one-sector dynamic stochastic general equilibrium model which is based on the New Keynesian model with financial frictions used in Del Negro et al. (2015). The core of the model is based on the work of Smets and Wouters (2007) (henceforth SW) and Christiano et al. (2005): It builds on the neo-classical growth model by adding nominal wage and price rigidities, variable capital utilization, costs of adjusting investment, habit formation in consumption. The model also includes credit frictions as in the *financial accelerator* model developed by Bernanke et al. (1999b) where the actual implementation of credit frictions follows closely Christiano et al. (2014), and accounts for forward guidance in monetary policy by including anticipated policy shocks as in Laseen and Svensson (2011).

The current version of the model has several features that improve upon the version presented in the New York Fed Staff Report no. 647. It features both a deterministic and a stochastic trend in productivity and allows for exogenous movements in risk premia; the inflation target is time-varying, following Del Negro and Schorfheide (2012); households preferences are non-separable in consumption and leisure; the Dixit-Stiglitz aggregator of intermediate goods has been replaced by the more flexible Kimball aggregator; we include indexation in the price and wage adjustment processes.

Here is a brief overview. The model economy is populated by eight classes of agents: 1) a continuum of households, who consume and supply differentiated labor; 2) competitive labor aggregators that combine labor supplied by individual households; 3) competitive final good-producing firms that aggregate the intermediate goods into a final product; 4) a continuum of monopolistically competitive intermediate good producing firms; 5) competitive capital producers that convert final goods into capital; 6) a continuum of entrepreneurs who purchase capital using both internal and borrowed funds and rent it to intermediate good producing firms; 7) a representative bank collecting deposits from the households and lending funds to the entrepreneurs; and finally 8) a government, composed of a monetary authority that sets short-term interest rates and a fiscal authority that sets public spending and collects taxes.

Growth in the economy is driven by technological progress. We specify a process for technology Z_t^* which includes both a deterministic and a stochastic trend, and a stationary component:

$$Z_t^* = e^{\frac{1}{1-\alpha}\tilde{z}_t} Z_t^p e^{\gamma t}, \tag{1}$$

where γ is the steady state growth rate of the economy, Z_t^p is a stochastic trend and $\tilde{z}_t$ is the stationary component.

The *production function* is

$$Y_t(i) = \max\{e^{\tilde{z}_t} K_t(i)^\alpha \left(L_t(i) e^{\gamma t} Z_t^p\right)^{1-\alpha} - \Phi Z_t^*, 0\}, \tag{2}$$

where ΦZ_t^* is a fixed cost.

Trending variables are divided by Z_t^* to express the model's equilibrium conditions in terms of the stationary variables. In what follows we present a summary of the log-linearized equilibrium conditions, where all variables are expressed in log deviations from their non-stochastic steady state.

Log-linear equilibrium conditions

The stationary component of productivity $\tilde{z}_t$ evolves as:

$$\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \sigma_z \varepsilon_{z,t}. \tag{3}$$

Since Z_t^p is a non stationary process, we define its growth rate as $z_t^p = log(Z_t^p / Z_{t-1}^p)$ and assume that it follows an AR(1) process:

$$z_t^p = \rho_{z^p} z_{t-1}^p + \sigma_{z^p} \epsilon_{z^p,t},\ \epsilon_{z^p,t} \sim N(0,1). \tag{4}$$

It follows that

$$z_t \equiv log(Z_t^* / Z_{t-1}^*) - \gamma = \frac{1}{1-\alpha}(\rho_z - 1)\tilde{z}_{t-1} + \frac{1}{1-\alpha}\sigma_z \epsilon_{z,t} + z_t^p, \tag{5}$$

where γ is the steady-state growth rate of the economy. Steady-state values are denoted by $*$-subscripts, and steady-state formulas are provided in the technical appendix of Del Negro and Schorfheide (2012), which is available online.

The *optimal allocation of consumption* satisfies the following consumption Euler equation:

$$c_t = -\frac{(1-he^{-\gamma})}{\sigma_c(1+he^{-\gamma})}\left(R_t - I\!\!E_t[\pi_{t+1}] + b_t\right) + \frac{he^{-\gamma}}{(1+he^{-\gamma})}\left(c_{t-1} - z_t\right) + \frac{1}{(1+he^{-\gamma})} I\!\!E_t\left[c_{t+1} + z_{t+1}\right] + \frac{(\sigma_c - 1)}{\sigma_c(1+he^{-\gamma})}\frac{w_* L_*}{c_*}\left(L_t - I\!\!E_t[L_{t+1}]\right), \quad (6)$$

where c_t is consumption, L_t is labor supply, R_t is the nominal interest rate, and π_t is inflation. The exogenous process b_t drives a wedge between the intertemporal marginal utility of consumption and the riskless real return $R_t - I\!\!E_t[\pi_{t+1}]$, and is meant to capture risk-premium shocks.[2] This shock follows an AR(1) process with parameters ρ_b and σ_b. The parameters σ_c and h capture the degree of relative risk aversion and the degree of habit persistence in the utility function, respectively.

The *optimal investment decision* satisfies the following relationship between the level of investment i_t, measured in terms of consumption goods, and the value of capital in terms of consumption q_t^k:

$$i_t = \frac{q_t^k}{S'' e^{2\gamma}(1+\bar{\beta})} + \frac{1}{1+\bar{\beta}}\left(i_{t-1} - z_t\right) + \frac{\bar{\beta}}{1+\bar{\beta}} I\!\!E_t\left[i_{t+1} + z_{t+1}\right] + \mu_t. \quad (7)$$

This relationship shows that investment is affected by investment adjustment costs (S'' is the second derivative of the adjustment cost function) and by an exogenous process μ_t, which we call "marginal efficiency of investment", that alters the rate of transformation between consumption and installed capital (see Greenwood et al. (1998)). The shock μ_t follows an AR(1) process with parameters ρ_μ and σ_μ. The parameter $\bar{\beta}$ depends on the intertemporal discount rate in the household utility function, β, on the degree of relative risk aversion σ_c, and on the steady-state growth rate γ: $\bar{\beta} = \beta e^{(1-\sigma_c)\gamma}$.

The *capital stock*, $\bar{k}_t$, which we refer to as "installed capital", evolves as

$$\bar{k}_t = \left(1 - \frac{i_*}{\bar{k}_*}\right)\left(\bar{k}_{t-1} - z_t\right) + \frac{i_*}{\bar{k}_*} i_t + \frac{i_*}{\bar{k}_*} S'' e^{2\gamma}(1+\bar{\beta})\mu_t, \quad (8)$$

where $i_*/\bar{k}_*$ is the steady state investment to capital ratio.

Capital is subject to variable capacity utilization u_t; *effective capital* rented out to firms,

[2] In the code, the b_t shock is normalized to be in the same units as consumption, i.e., we estimate the shock $\tilde{b}_t = -\frac{(1-he^{-\gamma})}{\sigma_c(1+he^{-\gamma})} b_t$.

k_t, is related to $\bar{k}_t$ by:

$$k_t \;=\; u_t - z_t + \bar{k}_{t-1}. \tag{9}$$

The optimality condition determining the *rate of capital utilization* is given by

$$\frac{1-\psi}{\psi} r_t^k = u_t, \tag{10}$$

where r_t^k is the rental rate of capital and ψ captures the utilization costs in terms of foregone consumption.

Real marginal costs for firms are given by

$$mc_t \;=\; w_t + \alpha L_t - \alpha k_t, \tag{11}$$

where w_t is the real wage and α is the income share of capital (after paying mark-ups and fixed costs) in the production function.

From the optimality conditions of goods producers it follows that all firms have the same *capital-labor ratio*:

$$k_t \;=\; w_t - r_t^k + L_t. \tag{12}$$

We include financial frictions in the model, building on the work of Bernanke et al. (1999a), Christiano et al. (2003), De Graeve (2008), and Christiano et al. (2014). We assume that banks collect deposits from households and lend to entrepreneurs who use these funds as well as their own wealth to acquire physical capital, which is rented to intermediate goods producers. Entrepreneurs are subject to idiosyncratic disturbances that affect their ability to manage capital. Their revenue may thus turn out to be too low to pay back the loans received by the banks. The banks therefore protect themselves against default risk by pooling all loans and charging a spread over the deposit rate. This spread may vary as a function of entrepreneurs' leverage and riskiness.

The *realized return on capital* is given by:

$$\tilde{R}_t^k - \pi_t = \frac{r_*^k}{r_*^k + (1-\delta)} r_t^k + \frac{(1-\delta)}{r_*^k + (1-\delta)} q_t^k - q_{t-1}^k, \tag{13}$$

where $\tilde{R}_t^k$ is the gross nominal return on capital for entrepreneurs, r_*^k is the steady state value of the rental rate of capital r_t^k, and δ is the depreciation rate.

The *excess return on capital* (the spread between the expected return on capital and the riskless rate) can be expressed as a function of the entrepreneurs' leverage (i.e. the ratio of the value of capital to nominal net worth) and exogenous fluctuations in the volatility of entrepreneurs' idiosyncratic productivity:

$$E_t\left[\tilde{R}^k_{t+1} - R_t\right] = b_t + \zeta_{sp,b}\left(q^k_t + \bar{k}_t - n_t\right) + \tilde{\sigma}_{\omega,t}, \tag{14}$$

where n_t is entrepreneurs' net worth, $\zeta_{sp,b}$ is the elasticity of the credit spread to the entrepreneurs' leverage $(q^k_t + \bar{k}_t - n_t)$, and $\tilde{\sigma}_{\omega,t}$ captures mean-preserving changes in the cross-sectional dispersion of ability across entrepreneurs (see Christiano et al. (2014)). $\tilde{\sigma}_{\omega,t}$ follows an AR(1) process with parameters ρ_{σ_ω} and σ_{σ_ω}.

Entrepreneurs' net worth n_t evolves according to:

$$\begin{aligned} n_t &= \zeta_{n,\tilde{R}^k}\left(\tilde{R}^k_t - \pi_t\right) - \zeta_{n,R}\left(R_{t-1} - \pi_t + b_{t-1}\right) + \zeta_{n,qK}\left(q^k_{t-1} + \bar{k}_{t-1}\right) + \zeta_{n,n}n_{t-1} \\ &\quad -\gamma_*\tfrac{v_*}{n_*}z_t - \tfrac{\zeta_{n,\sigma_\omega}}{\zeta_{sp,\sigma_\omega}}\tilde{\sigma}_{\omega,t-1}, \end{aligned} \tag{15}$$

where the ζ's denote elasticities, that depend among others on the entrepreneurs' steady-state default probability $F(\bar{\omega})$, where γ_* is the fraction of entrepreneurs that survive and continue operating for another period, and where v_* is the entrepreneurs' real equity divided by Z^*_t, in steady state.

The *production function* is

$$y_t = \Phi_p\left(\alpha k_t + (1-\alpha)L_t\right), \tag{16}$$

where $\Phi_p = \frac{y_* + \Phi}{y_*}$, and the *resource constraint* is:

$$y_t = g_* g_t + \frac{c_*}{y_*}c_t + \frac{i_*}{y_*}i_t + \frac{r^k_* k_*}{y_*}u_t. \tag{17}$$

where $g_t = log(\frac{G_t}{Z^*_t y_* g_*})$ and $g_* = 1 - \frac{c_* + i_*}{y_*}$.

Government spending g_t is assumed to follow the exogenous process:

$$g_t = \rho_g g_{t-1} + \sigma_g \varepsilon_{g,t} + \eta_{gz}\sigma_z\varepsilon_{z,t}.$$

The *price and wage Phillips curves* are, respectively:

$$\pi_t = \kappa\ mc_t + \frac{\iota_p}{1+\iota_p\bar{\beta}}\pi_{t-1} + \frac{\bar{\beta}}{1+\iota_p\bar{\beta}}\mathbb{E}_t[\pi_{t+1}] + \lambda_{f,t}, \tag{18}$$

and

$$w_t = \frac{(1-\zeta_w\bar{\beta})(1-\zeta_w)}{(1+\bar{\beta})\zeta_w((\lambda_w-1)\epsilon_w+1)}\left(w_t^h - w_t\right) - \frac{1+\iota_w\bar{\beta}}{1+\bar{\beta}}\pi_t + \frac{1}{1+\bar{\beta}}\left(w_{t-1} - z_t + \iota_w\pi_{t-1}\right) + \frac{\bar{\beta}}{1+\bar{\beta}}\mathbb{E}_t\left[w_{t+1} + z_{t+1} + \pi_{t+1}\right] + \lambda_{w,t}, \tag{19}$$

where $\kappa = \frac{(1-\zeta_p\bar{\beta})(1-\zeta_p)}{(1+\iota_p\bar{\beta})\zeta_p((\Phi_p-1)\epsilon_p+1)}$, the parameters ζ_p, ι_p, and ϵ_p are the Calvo parameter, the degree of indexation, and the curvature parameter in the Kimball aggregator for prices, and ζ_w, ι_w, and ϵ_w are the corresponding parameters for wages. w_t^h measures the household's marginal rate of substitution between consumption and labor, and is given by:

$$w_t^h = \frac{1}{1-he^{-\gamma}}\left(c_t - he^{-\gamma}c_{t-1} + he^{-\gamma}z_t\right) + \nu_l L_t, \tag{20}$$

where ν_l characterizes the curvature of the disutility of labor (and would equal the inverse of the Frisch elasticity in the absence of wage rigidities). The mark-ups $\lambda_{f,t}$ and $\lambda_{w,t}$ follow exogenous ARMA(1,1) processes:

$$\lambda_{f,t} = \rho_{\lambda_f}\lambda_{f,t-1} + \sigma_{\lambda_f}\varepsilon_{\lambda_f,t} - \eta_{\lambda_f}\sigma_{\lambda_f}\varepsilon_{\lambda_f,t-1},$$

and

$$\lambda_{w,t} = \rho_{\lambda_w}\lambda_{w,t-1} + \sigma_{\lambda_w}\varepsilon_{\lambda_w,t} - \eta_{\lambda_w}\sigma_{\lambda_w}\varepsilon_{\lambda_w,t-1},$$

respectively.

Finally, the monetary authority follows a generalized *policy feedback rule*:

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\Big(\psi_1(\pi_t - \pi_t^*) + \psi_2(y_t - y_t^f)\Big) + \psi_3\Big((y_t - y_t^f) - (y_{t-1} - y_{t-1}^f)\Big) + r_t^m. \tag{21}$$

where y_t^f is the flexible price/wage output, obtained from solving the version of the model without nominal rigidities and markup shocks (that is, Equations (6) through (20) with

$\zeta_p = \zeta_w = 0$, and $\lambda_{f,t} = \lambda_{w,t} = 0$), and the residual r_t^m follows an AR(1) process with parameters ρ_{r^m} and σ_{r^m}.

In this version of the model we have replaced a constant inflation target with a time-varying inflation target π_t^*, to capture the rise and fall of inflation and interest rates in the estimation sample. Although time-varying target rates have been frequently used for the specification of monetary policy rules in DSGE model (e.g., Erceg and Levin (2003) and Smets and Wouters (2003), among others), we follow the approach of Aruoba and Schorfheide (2008) and Del Negro and Eusepi (2011) and include data on long-run inflation expectations as an observable for the estimation of the model. At each point in time, long-run inflation expectations essentially determine the level of the target inflation rate. To the extent that long-run inflation expectations at the forecast origin contain information about the central bank's objective function, e.g. the desire to stabilize inflation at 2%, this information is automatically included in the forecast.

The time-varying *inflation target* evolves according to:

$$\pi_t^* = \rho_{\pi^*}\pi_{t-1}^* + \sigma_{\pi^*}\epsilon_{\pi^*,t}, \tag{22}$$

where $0 < \rho_{\pi^*} < 1$ and $\epsilon_{\pi^*,t}$ is an iid shock. We model π_t^* as a stationary process, although our prior for ρ_{π^*} will force this process to be highly persistent. The assumption that the changes in the target inflation rate are exogenous is, to some extent, a short-cut. For instance, the learning models of Sargent (1999) or Primiceri (2006) imply that the rise in the target inflation rate in the 1970's and the subsequent drop is due to policy makers learning about the output-inflation trade-off and trying to set inflation optimally. We are abstracting from such a mechanism in our specification.

Anticipated policy shocks

This section describes the introduction of anticipated policy shocks in the model, which follows Laseen and Svensson (2011). We modify the exogenous component of the policy rule (21) as follows:

$$r_t^m = \rho_{r^m} r_{t-1}^m + \epsilon_t^R + \sum_{k=1}^{K} \epsilon_{k,t-k}^R, \tag{23}$$

where ϵ_t^R is the usual contemporaneous policy shock, and $\epsilon_{k,t-k}^R$ is a policy shock that is known to agents at time $t-k$, but affects the policy rule k periods later, that is, at time t.

We assume that $\epsilon^R_{k,t-k} \sim N(0, \sigma^2_{k,r}),\ i.i.d.$

In order to solve the model we need to express the anticipated shocks in recursive form. For this purpose, we augment the state vector s_t (described below) with K additional states ν^R_t,...,ν^R_{t-K} whose law of motion is as follows:

$$\begin{aligned} \nu^R_{1,t} &= \nu^R_{2,t-1} + \epsilon^R_{1,t} \\ \nu^R_{2,t} &= \nu^R_{3,t-1} + \epsilon^R_{2,t} \\ &\vdots \\ \nu^R_{K,t} &= \epsilon^R_{K,t} \end{aligned}$$

and rewrite the exogenous component of the policy rule (23) as[3]

$$r^m_t = \rho_{r^m} r^m_{t-1} + \epsilon^R_t + \nu^R_{1,t-1}.$$

Adding COVID-19 Shocks

Some of the model modifications needed to capture the COVID-19 shock (at least within the narrow-minded framework on this one sector DSGE model) simply amount to adding i.i.d. shocks. These shocks are i.i.d. because the COVID-19 related economic disruptions (e.g., lockdown of productive capacity, impossibility to consume some goods/services) are temporary. Note however that even purely temporary shocks may have longer lasting effects on the economy via the model's dynamics. Moreover, some of the shocks hit the economy for more than one period. So while the impulse responses of these shocks reflect the fact that they have no exogenous persistence, the sequence of shocks affecting the economy is not i.i.d. at all. In fact, we assume that some of these shocks are anticipated. For instance, in 2020Q1 agents expect that a set of disturbances twice the size of those affecting the economy in the current quarter will also hit it in the following quarter. In 2020Q2 a new set of disturbances will hit the economy on top of the shocks that were anticipated in the previous quarter.

We introduce two shocks: a so-called "discount factor" shock $\tilde{\beta}_t$ and a "labor supply" shock $\hat{\varphi}_t$. The first one enters as a stochastic addition to the discount rate β, and the second

[3] It is easy to verify that $\nu^R_{1,t-1} = \sum_{k=1}^K \epsilon^R_{k,t-k}$, that is, $\nu^R_{1,t-1}$ is a "bin" that collects all anticipated shocks that affect the policy rule in period t.

as a labor (dis)utility shifter. These shocks modify the Euler equation and the intratemporal condition as follows:

$$\hat{c}_t = -\frac{(1-he^{-z_*^*})}{\sigma_c(1+he^{-z_*^*})}\left(\hat{R}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]\right) + \frac{he^{-z_*^*}}{(1+he^{-z_*^*})}\left(\hat{c}_{t-1} - \hat{z}_t^*\right) + \hat{b}_t + \hat{\beta}_t$$
$$+ \frac{1}{(1+he^{-z_*^*})}\mathbb{E}_t\left[\hat{c}_{t+1} + \hat{z}_{t+1}^*\right] + \frac{(\sigma_c-1)}{\sigma_c(1+he^{-z_*^*})}\frac{w_*L_*}{c_*}\left(\hat{L}_t - \mathbb{E}_t[\hat{L}_{t+1}]\right)$$
$$+ \frac{(\sigma_c-1)}{\sigma_c(1+he^{-z_*^*})}\frac{w_*L_*}{c_*}\left(\hat{\varphi}_t - \mathbb{E}_t[\hat{\varphi}_{t+1}]\right), \quad (24)$$

and

$$\frac{1}{1-he^{-z_*^*}}\left(\hat{c}_t - he^{-z_*^*}\hat{c}_{t-1} + he^{-z_*^*}\hat{z}_t^*\right) + \nu_l\hat{L}_t + \nu_l\hat{\varphi}_t = \hat{w}_t^h. \quad (25)$$

Note that φ_t enters the wage Phillips curve in the same way as a wage mark-up shock via $\hat{w}_t^h$. However, differently from $\hat{\lambda}_{w,t}$ it also enters the Euler equation.

In addition, we add an additional *stationary* i.i.d. productivity disturbance $\check{z}_t$. As a consequence total productivity growth becomes:

$$\hat{z}_t^* = \frac{1}{1-\alpha}(\tilde{z}_t - \tilde{z}_{t-1}) + z_t^p + \frac{1}{1-\alpha}(\check{z}_t - \check{z}_{t-1}). \quad (26)$$

All the shocks are i.i.d. (that is, $\rho_\beta = \rho_\varphi = \rho_{\check{z}} = 0$). As mentioned, some of the scenarios feature anticipated shocks:

$$\check{z}_t = \rho_{\check{z}}\check{z}_{t-1} + \sigma_{\check{z}}\epsilon_{\check{z},t} + \sum_{k=1}^{K}\sigma_{\check{z},k}\varepsilon_{k,t-k}^{\check{z}}$$
$$\hat{\beta}_t = \rho_\beta\hat{\beta}_{t-1} + \sigma_\beta\varepsilon_{\beta,t} + \sum_{k=1}^{K}\sigma_{\beta,k}\varepsilon_{k,t-k}^{\beta}$$
$$\hat{\varphi}_t = \rho_\varphi\hat{\varphi}_{t-1} + \sigma_\varphi\varepsilon_{\varphi,t} + \sum_{k=1}^{K}\sigma_{\varphi,k}\varepsilon_{k,t-k}^{\varphi}.$$

We use $K = 1$ (only one anticipated shock) and set the anticipated shock to be a proportion ϕ of the current shock, e.g. $\sigma_{\check{z},1}\varepsilon_{1,t}^{\check{z}} = \phi\sigma_{\check{z}}\epsilon_{\check{z},t}$.

Parameters

The following tables describe the parameters used in the New York Fed DSGE model. Table 2 gives the prior distributions for each parameter. Table 3 gives the posterior mean, 5th percentile, and 95th percentile for each parameter.

Table 2: Priors

	Dist	Mean	Std Dev		Dist	Mean	Std Dev
Policy Parameters							
ψ_1	Normal	1.50	0.25	ρ_{r^m}	Beta	0.50	0.20
ψ_2	Normal	0.12	0.05	σ_{r^m}	InvG	0.10	2.00
ψ_3	Normal	0.12	0.05	σ_{ant1}	InvG	0.20	4.00
ρ_R	Beta	0.75	0.10				
Nominal Rigidities Parameters							
ζ_p	Beta	0.50	0.10	ζ_w	Beta	0.50	0.10
ι_p	Beta	0.50	0.15	ι_w	Beta	0.50	0.15
ϵ_p	-	10.00	fixed	ϵ_w	-	10.00	fixed
Other Endogenous Propagation and Steady State Parameters							
100γ	Normal	0.40	0.10	S''	Normal	4.00	1.50
α	Normal	0.30	0.05	ψ	Beta	0.50	0.15
$100(\beta^{-1}-1)$	Gamma	0.25	0.10	π_*	-	0.50	fixed
σ_c	Normal	1.50	0.37	γ_{gdpdef}	Normal	1.00	2.00
h	Beta	0.70	0.10	δ_{gdpdef}	Normal	0.00	2.00
ν_l	Normal	2.00	0.75	$\bar{L}$	Normal	-45.00	5.00
δ	-	0.03	fixed	λ_w	-	1.50	fixed
Φ_p	Normal	1.25	0.12	g_*	-	0.18	fixed
Financial Frictions Parameters							
$F(\bar{\omega})$	-	0.03	fixed	$\zeta_{sp,b}$	Beta	0.05	0.00
SP_*	Gamma	2.00	0.10	γ_*	-	0.99	fixed
Exogenous Process Parameters							
ρ_g	Beta	0.50	0.20	σ_g	InvG	0.10	2.00
ρ_b	Beta	0.50	0.20	σ_b	InvG	0.10	2.00
ρ_μ	Beta	0.50	0.20	σ_μ	InvG	0.10	2.00
$\rho_z til$	Beta	0.50	0.20	$\sigma_{\tilde{z}}$	InvG	0.10	2.00
ρ_{σ_ω}	Beta	0.75	0.15	σ_{σ_ω}	InvG	0.05	4.00
ρ_{π_*}	-	0.99	fixed	σ_{π_*}	InvG	0.03	6.00
ρ_{z^p}	Beta	0.50	0.20	σ_{z^p}	InvG	0.10	2.00
ρ_{λ_f}	Beta	0.50	0.20	σ_{λ_f}	InvG	0.10	2.00
ρ_{λ_w}	Beta	0.50	0.20	σ_{λ_w}	InvG	0.10	2.00
η_{λ_f}	Beta	0.50	0.20	η_{gz}	Beta	0.50	0.20
η_{λ_w}	Beta	0.50	0.20				
Measurement Error Parameters							

Note: For Inverse Gamma prior mean and SD, τ and ν reported.

σ_{ant1} through σ_{ant12} all have the same distribution.

	Dist	Mean	Std Dev		Dist	Mean	Std Dev
$\mathcal{C}_{me}$	-	1.00	fixed	ϱ_{gdp}	Normal	0.00	0.40
ρ_{gdp}	Normal	0.00	0.20	σ_{gdp}	InvG	0.10	2.00
ρ_{gdi}	Normal	0.00	0.20	σ_{gdi}	InvG	0.10	2.00
ρ_{10y}	Beta	0.50	0.20	σ_{10y}	InvG	0.75	2.00
ρ_{tfp}	Beta	0.50	0.20	σ_{tfp}	InvG	0.10	2.00
ρ_{gdpdef}	Beta	0.50	0.20	σ_{gdpdef}	InvG	0.10	2.00
ρ_{pce}	Beta	0.50	0.20	σ_{pce}	InvG	0.10	2.00

Note: For Inverse Gamma prior mean and SD, τ and ν reported.

Table 3: Posteriors

	Mean	(p5, p95)		Mean	(p5, p95)
Policy Parameters					
ψ_1	1.51	(1.32, 1.72)	σ_{ant1}	0.08	(0.08, 0.09)
ψ_2	0.05	(0.04, 0.07)	σ_{ant2}	0.08	(0.07, 0.08)
ψ_3	0.26	(0.22, 0.29)	σ_{ant3}	0.08	(0.08, 0.09)
ρ_R	0.68	(0.62, 0.74)	σ_{ant4}	0.08	(0.07, 0.08)
ρ_{r^m}	0.27	(0.17, 0.38)	σ_{ant5}	0.08	(0.08, 0.08)
σ_{r^m}	0.23	(0.22, 0.24)	σ_{ant6}	0.10	(0.10, 0.11)
Nominal Rigidities Parameters					
ζ_p	0.92	(0.90, 0.94)	ζ_w	0.94	(0.93, 0.95)
ι_p	0.25	(0.14, 0.39)	ι_w	0.54	(0.33, 0.76)
ϵ_p	10.00	fixed	ϵ_w	10.00	fixed
Other Endogenous Propagation and Steady State Parameters					
100γ	0.38	(0.32, 0.44)	S''	3.50	(2.52, 4.43)
α	0.18	(0.17, 0.19)	ψ	0.49	(0.33, 0.64)
$100(\beta^{-1}-1)$	0.13	(0.07, 0.19)	π_*	0.50	fixed
σ_c	0.90	(0.72, 1.08)	γ_{gdpdef}	1.04	(1.00, 1.08)
h	0.49	(0.41, 0.57)	δ_{gdpdef}	-0.00	(-0.04, 0.03)
ν_l	2.25	(1.54, 2.97)	$\bar{L}$	-48.50	(-50.47, -46.53)
δ	0.02	fixed	λ_w	1.50	fixed
Φ_p	1.09	(1.05, 1.12)	g_*	0.18	fixed
Financial Frictions Parameters					
$F(\bar{\omega})$	0.03	fixed	$\zeta_{sp,b}$	0.05	(0.05, 0.06)
SP_*	1.82	(1.71, 1.93)	γ_*	0.99	fixed
Exogenous Process Parameters					
ρ_g	0.99	(0.99, 0.99)	σ_g	2.16	(1.99, 2.30)
ρ_b	0.96	(0.96, 0.96)	σ_b	0.03	(0.03, 0.03)
ρ_μ	0.77	(0.72, 0.83)	σ_μ	0.44	(0.40, 0.48)
$\rho_z til$	0.97	(0.96, 0.97)	$\sigma_{\tilde{z}}$	0.59	(0.56, 0.64)
ρ_{σ_ω}	0.99	(0.99, 1.00)	σ_{σ_ω}	0.03	(0.03, 0.04)
ρ_{π_*}	0.99	fixed	σ_{π_*}	0.03	(0.03, 0.04)
ρ_{z^p}	0.93	(0.91, 0.95)	σ_{z^p}	0.11	(0.09, 0.13)
ρ_{λ_f}	0.84	(0.77, 0.92)	σ_{λ_f}	0.07	(0.06, 0.07)
ρ_{λ_w}	0.40	(0.18, 0.62)	σ_{λ_w}	0.39	(0.36, 0.42)
η_{λ_f}	0.72	(0.61, 0.84)	η_{gz}	0.38	(0.11, 0.64)
η_{λ_w}	0.45	(0.26, 0.64)			
Measurement Error Parameters					
$\mathcal{C}_{me}$	1.00	fixed	ϱ_{gdp}	-0.08	(-0.71, 0.52)

	Mean	(p5, p95)		Mean	(p5, p95)
ρ_{gdp}	-0.01	(-0.20, 0.19)	σ_{gdp}	0.24	(0.21, 0.27)
ρ_{gdi}	0.94	(0.92, 0.96)	σ_{gdi}	0.30	(0.29, 0.32)
ρ_{10y}	0.97	(0.96, 0.97)	σ_{10y}	0.12	(0.12, 0.12)
ρ_{tfp}	0.20	(0.14, 0.28)	σ_{tfp}	0.75	(0.71, 0.79)
ρ_{gdpdef}	0.37	(0.27, 0.49)	σ_{gdpdef}	0.17	(0.16, 0.17)
ρ_{pce}	0.23	(0.07, 0.38)	σ_{pce}	0.11	(0.10, 0.12)

Impulse Responses

The following figures depict impulse response functions to various shocks. Figure 5 depicts the response of the economy to a discount factor shock, Figure 6 to a spread shock, Figure 7 to a shock to the marginal efficiency of investment (MEI), Figure 8 to a TFP shock, Figure 9 to a price markup shock, Figure 11 to a monetary policy shock, Figure 12 to an iid Euler equation shock, Figure 13 to an iid labor supply shock, and Figure 14 to an iid TFP shock. Figures 12, 13, and 14 are the impulse responses of the COVID-19 shocks.

Figure 5: Responses to a Discount Factor Shock b_t

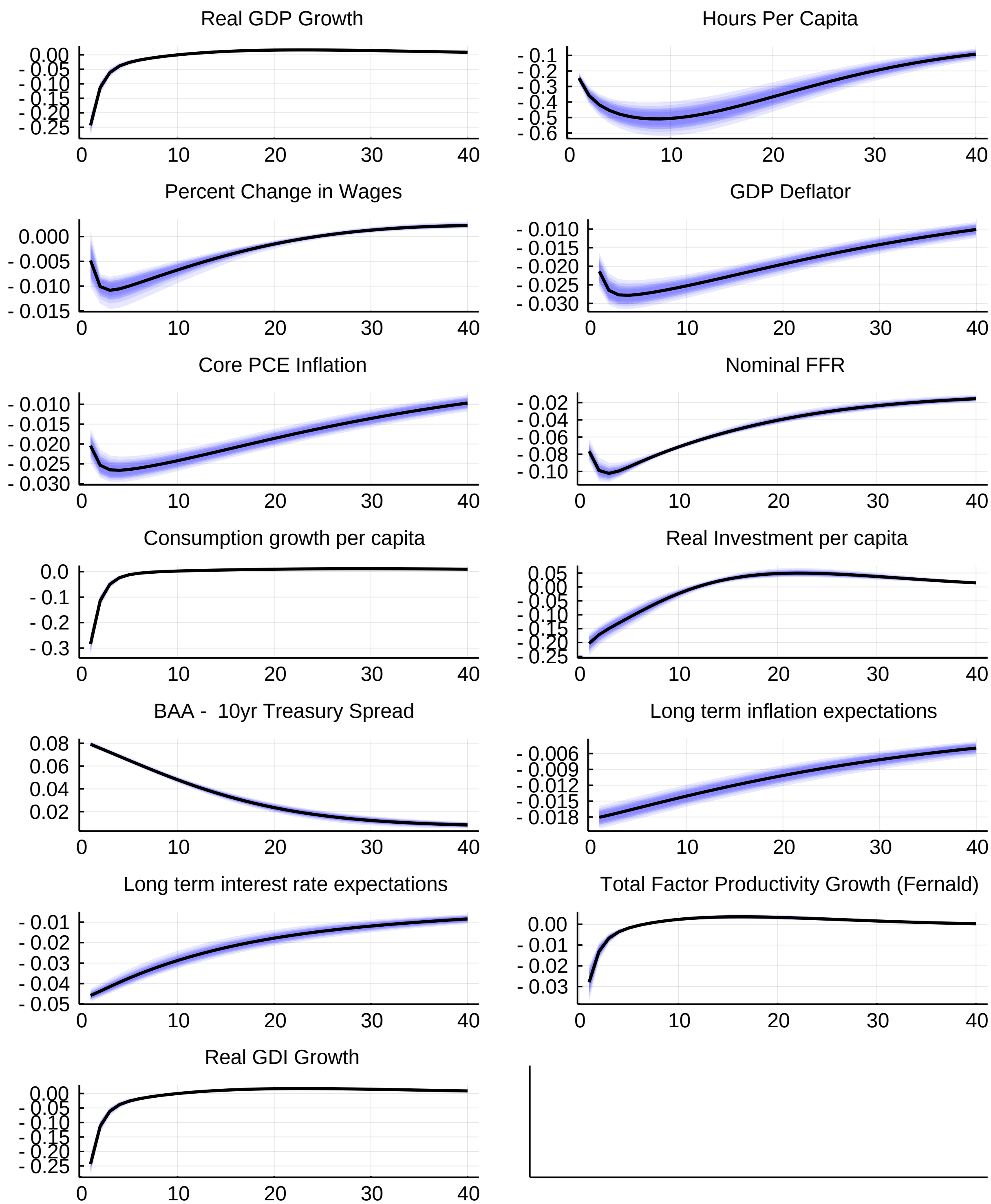


Figure 6: Responses to a Spread Shock $\tilde{\sigma}_{\omega,t}$

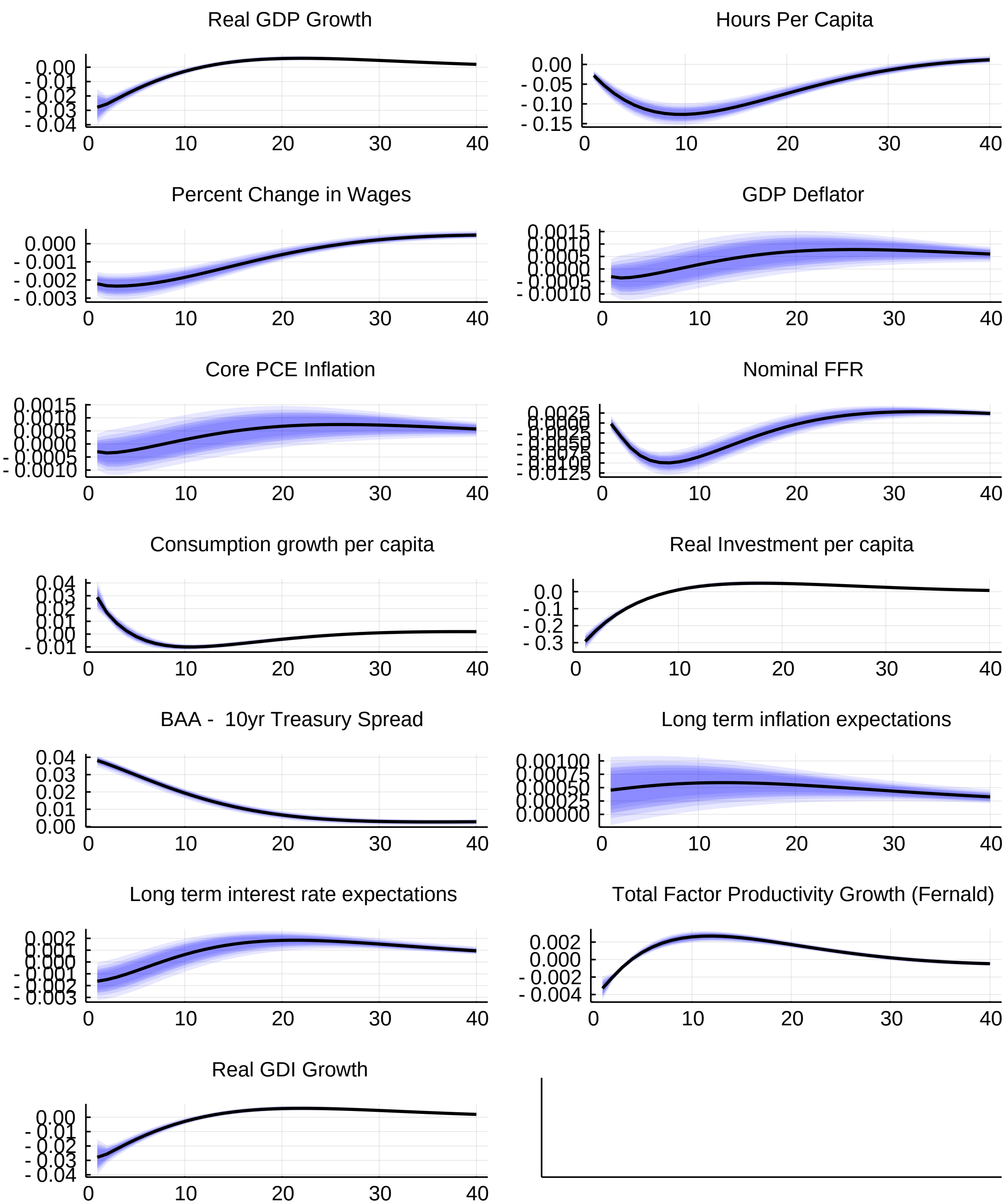


Figure 7: Responses to an MEI Shock μ_t

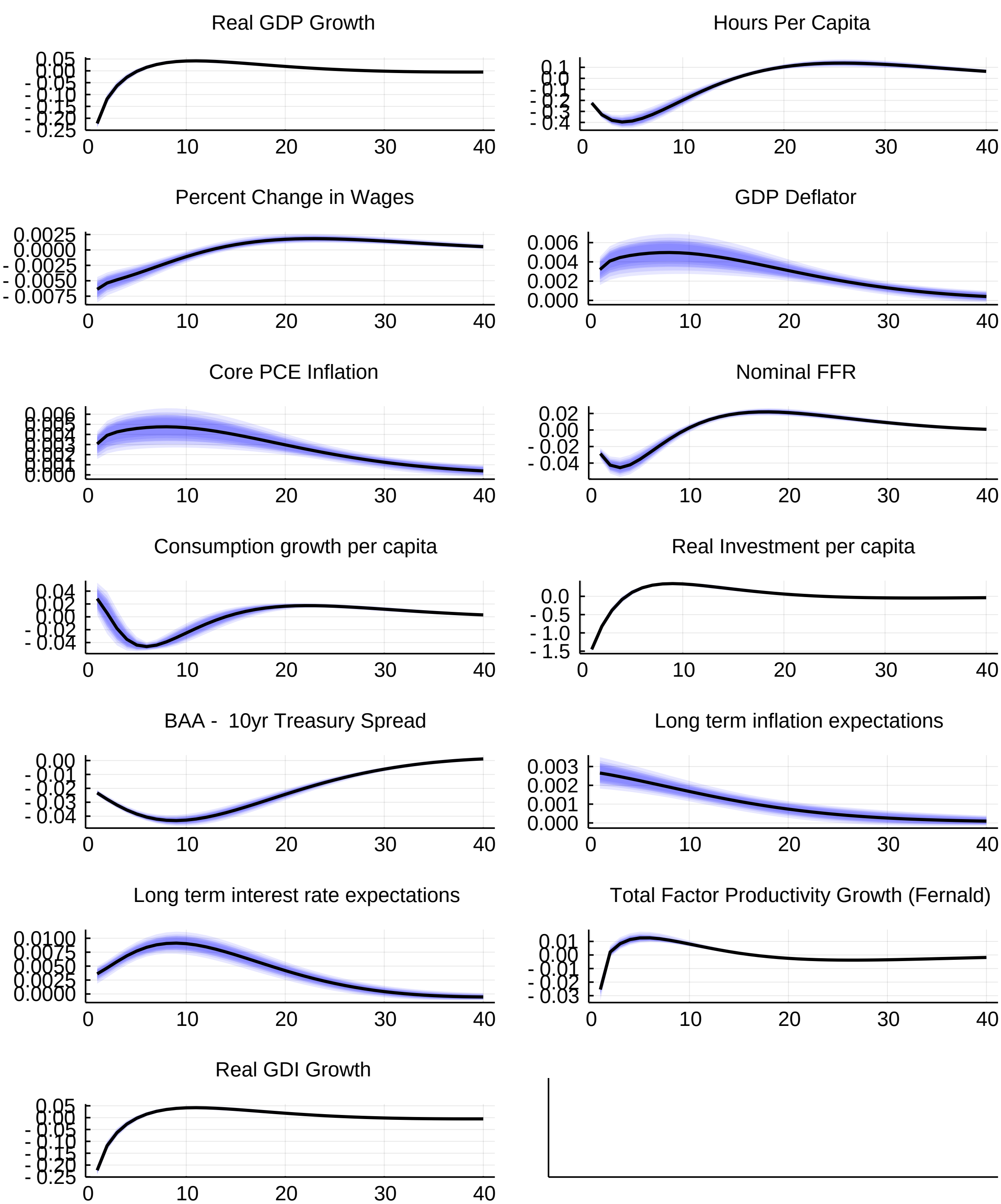


Figure 8: Responses to a TFP Shock $\tilde{z}_t$

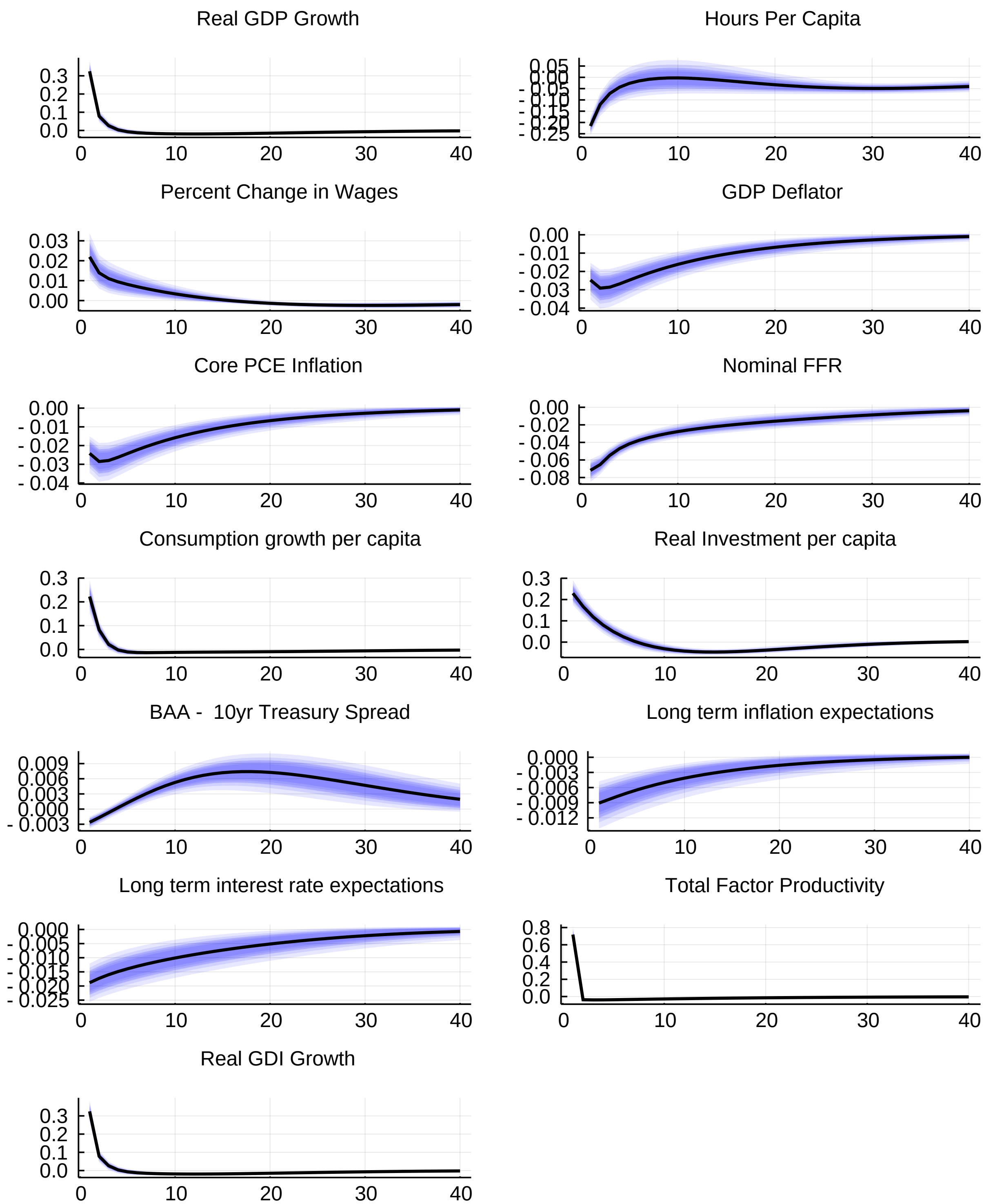


Figure 9: Responses to a Price Markup Shock $\lambda_{f,t}$

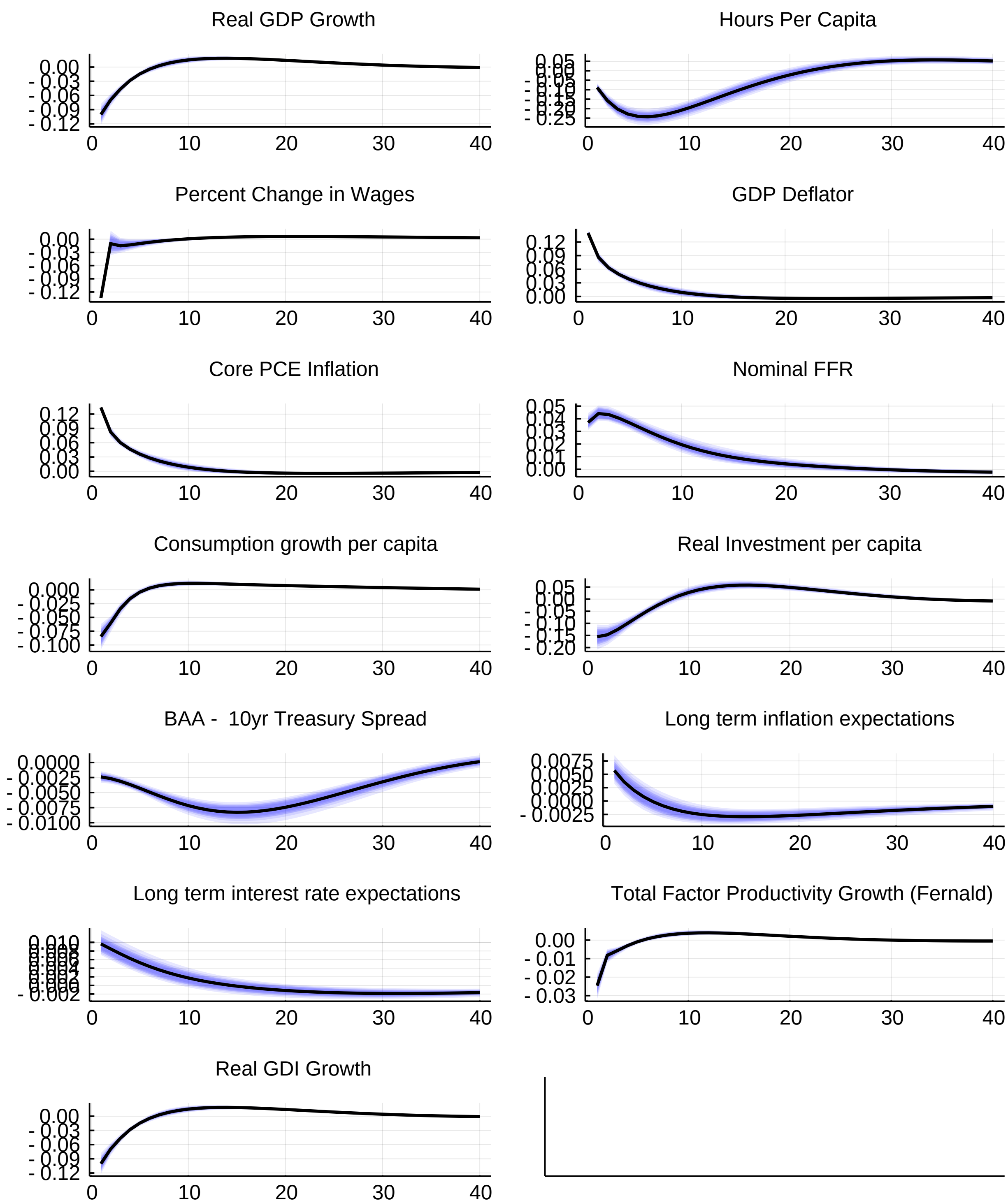


Figure 10: Responses to a Wage Markup Shock $\lambda_{w,t}$

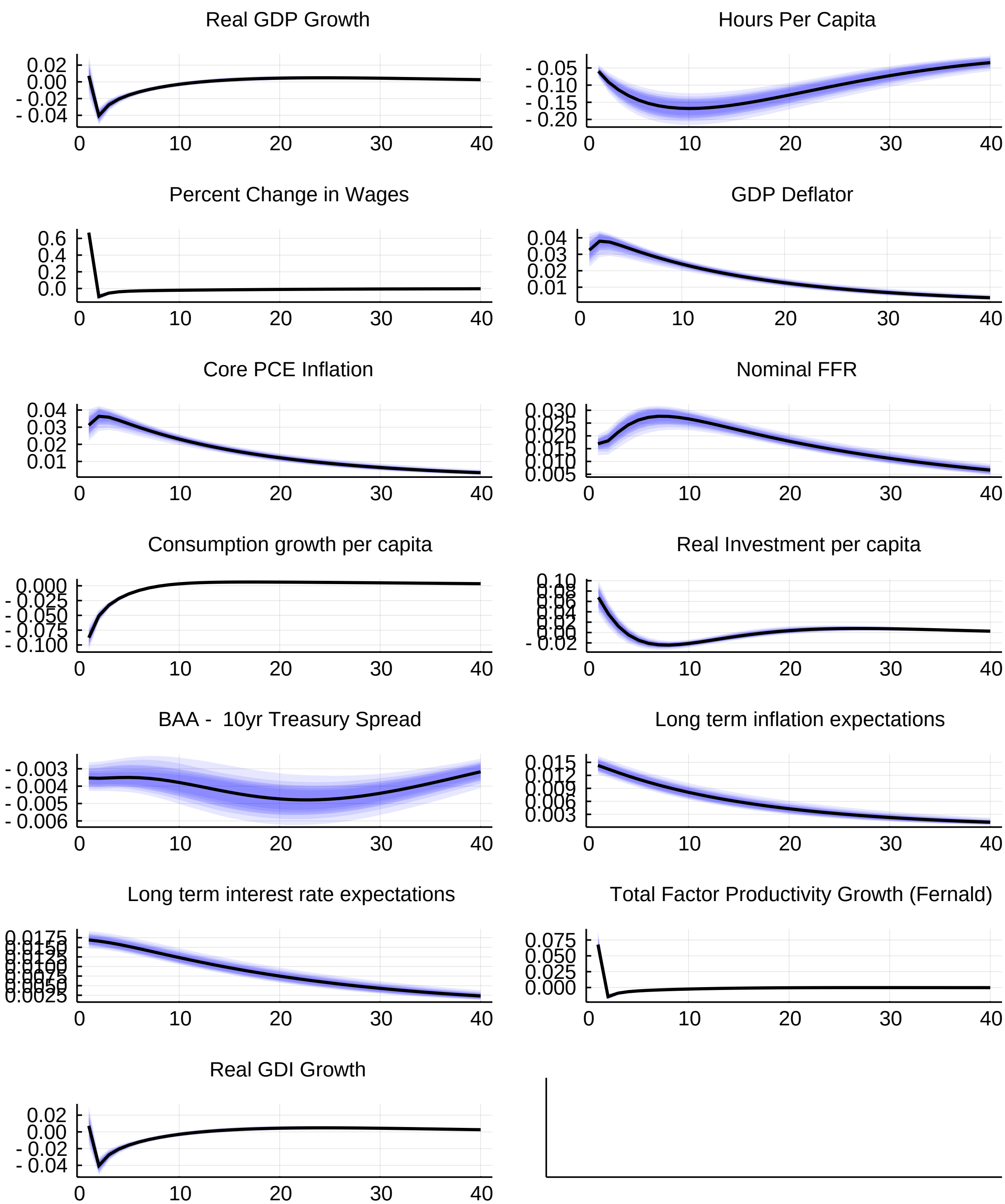


Figure 11: Responses to a Monetary Policy Shock r_t^m

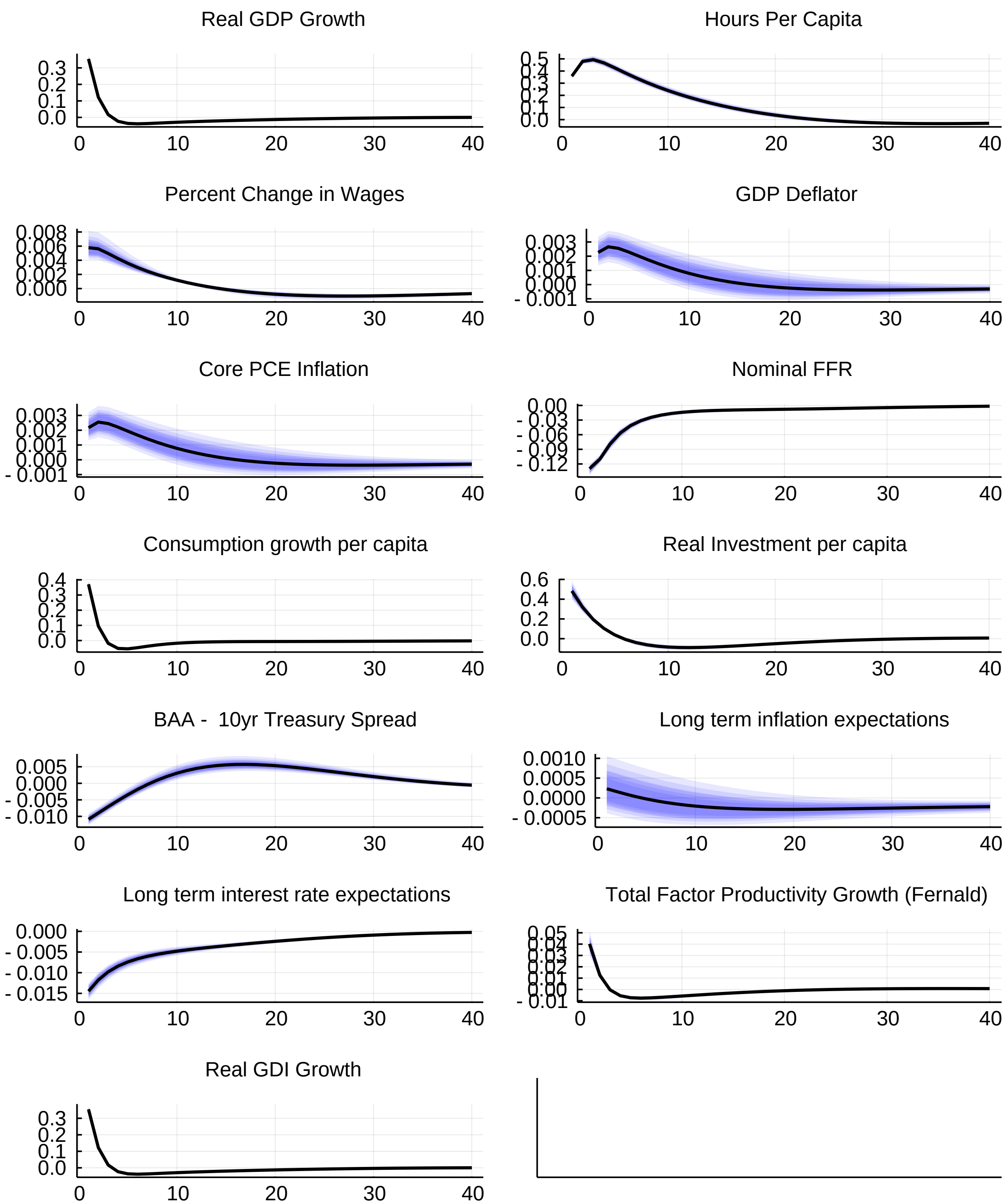


Figure 12: Responses to an iid Euler Equation Shock $b^{iid}_{c,t}$

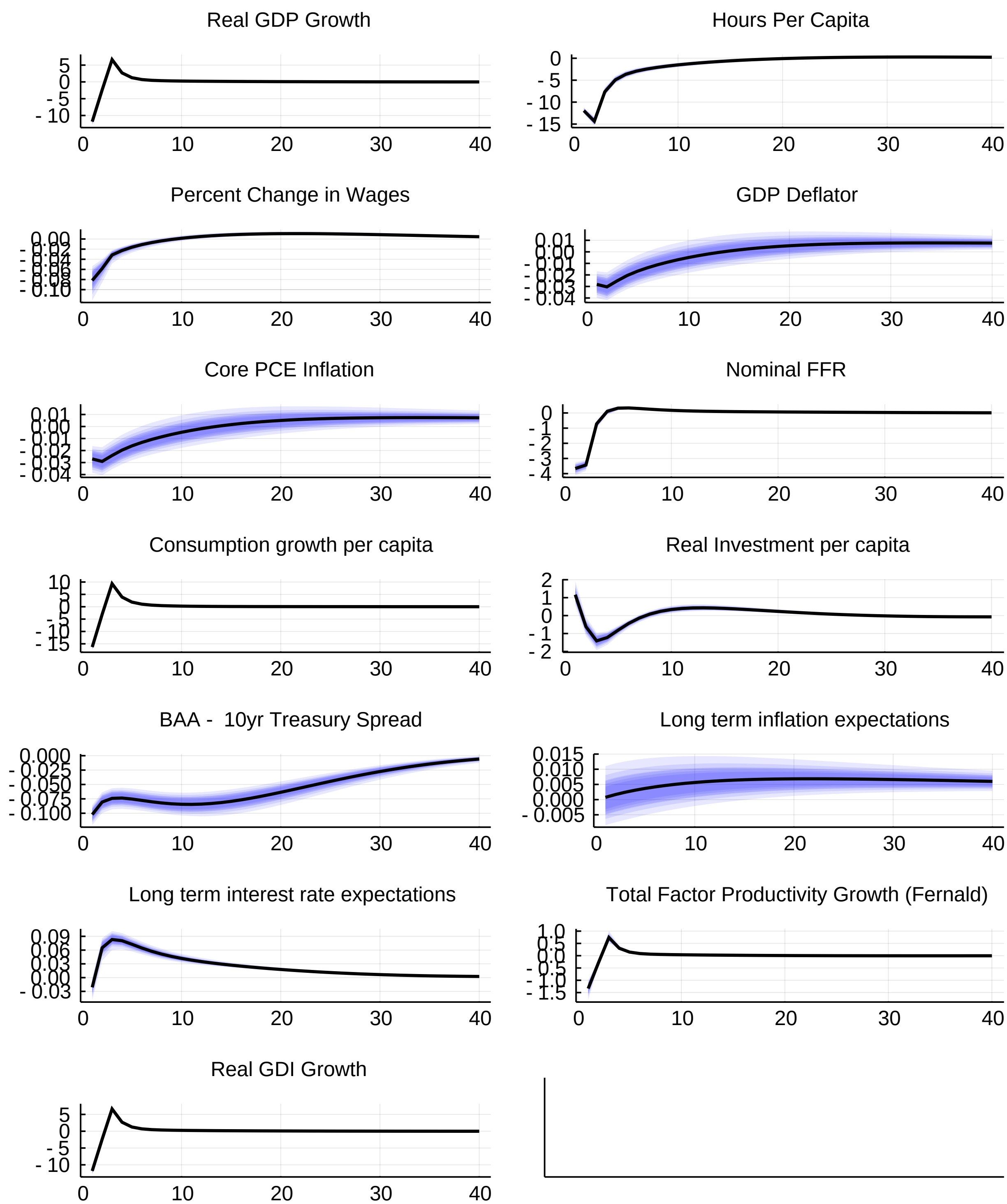


Figure 13: Responses to an iid Labor Supply Shock φ_t^{iid}

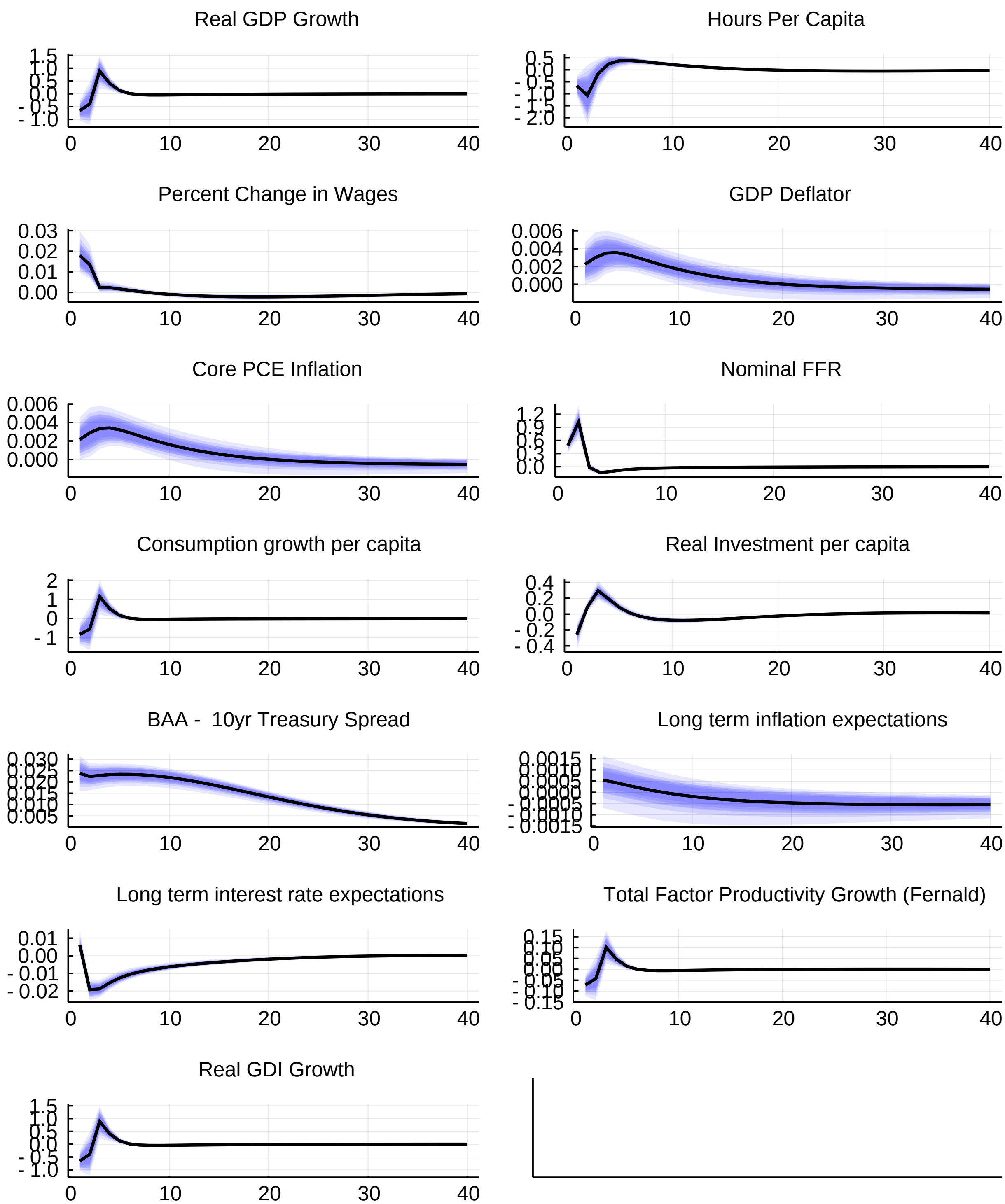
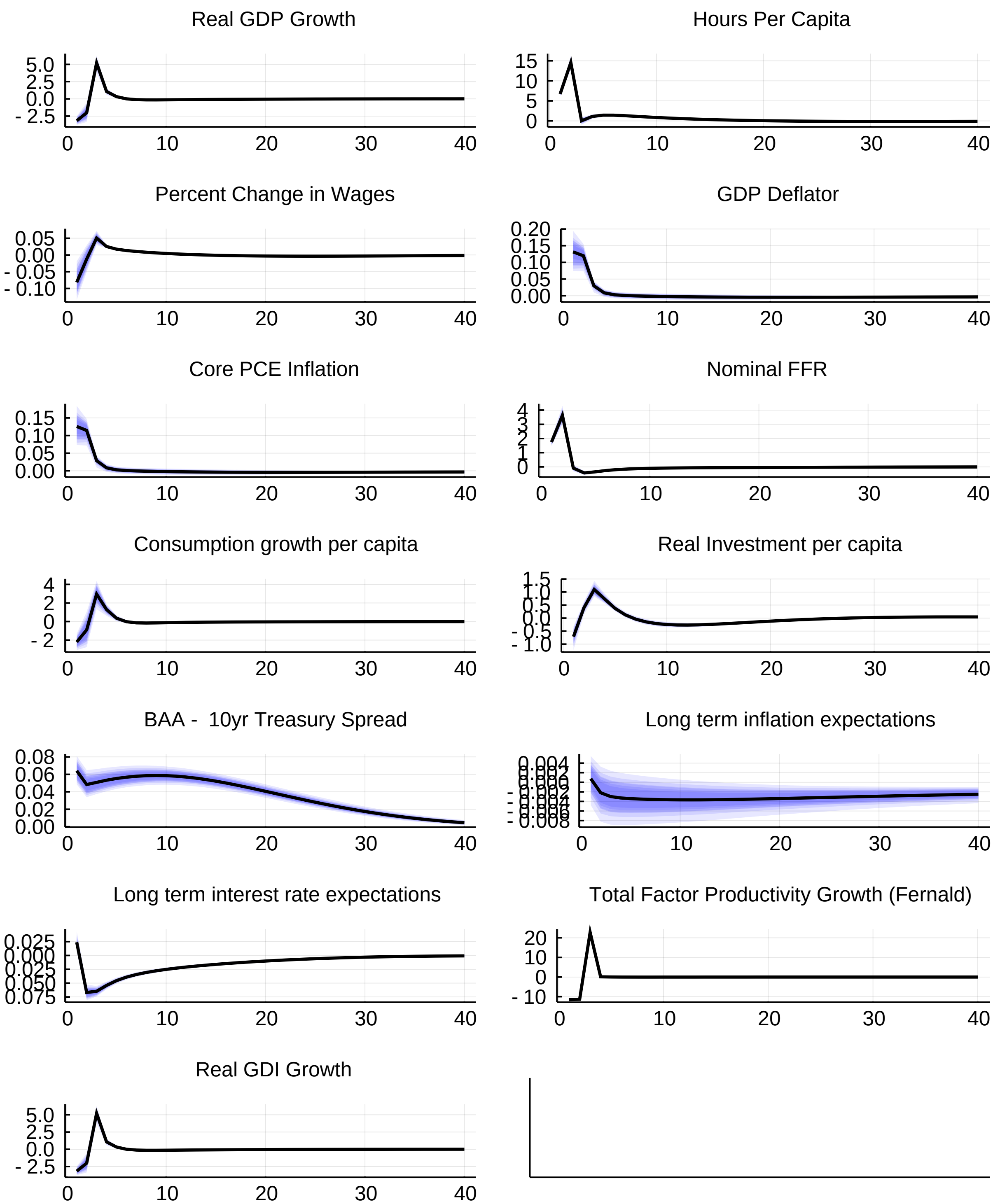


Figure 14: Responses to an iid TFP Shock z_t^{iid}



Description of the Scenarios

Scenario 1: Temporary Shutdown

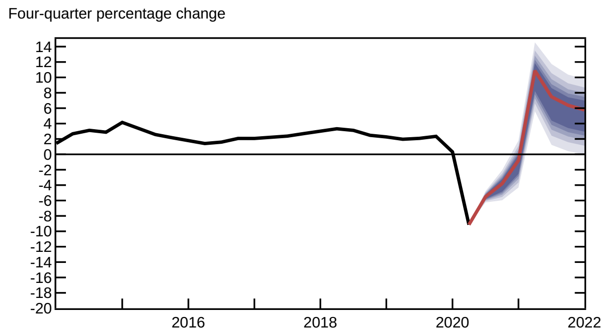
The "Temporary Shutdown" scenario explains the decline in economic activity in 2020Q1 and Q2 using predominantly the transitory demand and supply shocks mentioned in the model section (the "discount factor" shock $\tilde{\beta}_t$, the "labor supply" shock $\hat{\varphi}_t$, and the productivity disturbance $\check{z}_t$, which we will refer to as the "COVID-19" shocks), and intentionally limiting the role of standard shocks in these two quarters. This scenario also uses the transitory shocks in conjunction with the standard shocks (now at "normal" values) to help explain an economic rebound in 2020Q3.

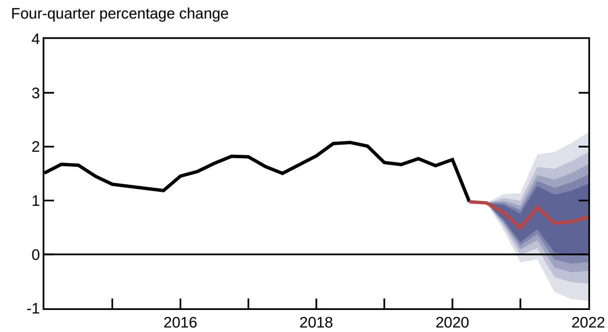
Specifically, we assume that the COVID-19 shocks hit the economy in 2020Q1-Q3 *only*. In 2020Q1 agents expect that a set of disturbances twice the size of those affecting the economy in the current quarter will also hit it in the following quarter. In 2020Q2 a new set of disturbances will hit the economy on top of the shocks that were anticipated in the previous quarter. The standard deviations of these transitory shocks are drawn from a relatively uninformative prior distribution, allowing for uncertainty in the interpretation of the shutdown as a supply- or demand-driven phenomenon. Finally, in 2020Q3 unanticipated COVID-19 shocks again hit the economy.

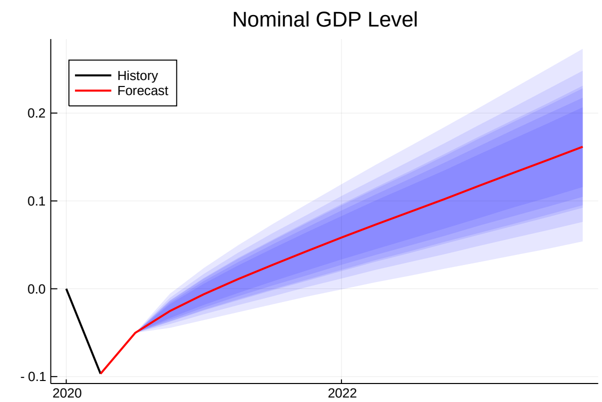
In 2020Q1 and Q2 the standard deviations of the usual set of shocks are set to 1/4 their estimated value, and the standard deviation of persistent growth rate productivity shocks is set to zero. In 2020Q3 and onward, the standard deviations of the usual set of shocks are set to their estimated values.

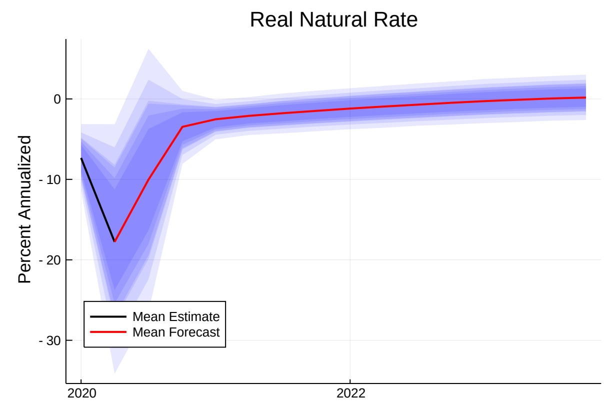
Finally, to allow for additional uncertainty in 2020Q3, the conditional number for GDP growth in 2020Q3 is interpreted as a noisy estimate of actual 2020Q3 GDP growth, as in Del Negro and Schorfheide (2013), section 5.3. We set the standard deviation of the noise to 2.0 (annualized), which is twice the estimated standard deviation of the GDP measurement error we normally use.

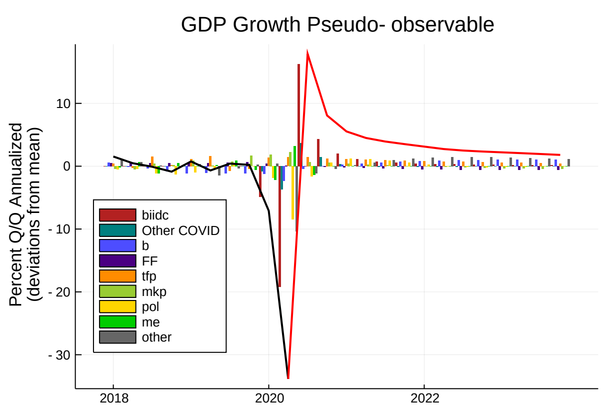
	2020		2021		2022		2023	
	Sep	Jun	Sep	Jun	Sep	Jun	Sep	Jun
GDP growth	-3.8	-4.6	6.4	2.1	4.7	0.3	4.0	1.0
(Q4/Q4)	(-5.2,-2.8)	(-6.4,-3.6)	(2.5,8.3)	(-1.5,4.1)	(1.6,7.3)	(-2.4,2.9)	(1.5,7.2)	(-1.5,3.9)
Core PCE inflation	0.8	1.6	0.6	1.2	1.0	1.2	1.3	1.3
(Q4/Q4)	(0.6,1.0)	(1.2,1.9)	(-0.3,1.5)	(0.3,2.1)	(-0.2,2.1)	(0.1,2.3)	(0.0,2.5)	(0.1,2.5)
Federal funds rate	0.1	0.0	0.0	0.0	1.1	1.1	1.9	1.8
(Q4)	(0.1,0.8)	(0.0,1.1)	(0.1,1.7)	(0.0,1.8)	(0.2,3.0)	(0.1,3.1)	(0.6,4.0)	(0.4,3.8)
Real natural rate	-3.5	-3.6	-1.5	0.1	-0.5	0.4	0.2	0.6
of interest (Q4)	(-6.1,-0.8)	(-5.0,-2.2)	(-3.0,0.0)	(-1.4,1.5)	(-2.0,1.1)	(-1.2,2.0)	(-1.5,1.8)	(-1.0,2.3)
Output gap	-5.4	-4.0	-3.1	-3.2	-1.8	-3.8	-0.8	-3.9
(Q4)	(-7.3,-4.1)	(-6.1,-2.9)	(-6.9,-1.6)	(-7.2,-1.6)	(-6.8,0.2)	(-8.6,-1.5)	(-5.9,1.9)	(-9.0,-1.1)

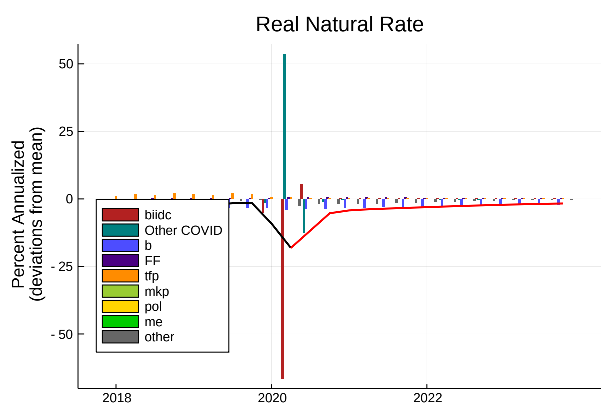

Four-quarter percentage change
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
2016
2018
2020
2022


Four-quarter percentage change
4
3
2
1
0
-1
2016
2018
2020
2022


Nominal GDP Level
History
Forecast
0.2
0.1
0.0
- 0.1
2020
2022


Real Natural Rate
Percent Annualized
0
- 10
- 20
- 30
Mean Estimate
Mean Forecast
2020
2022

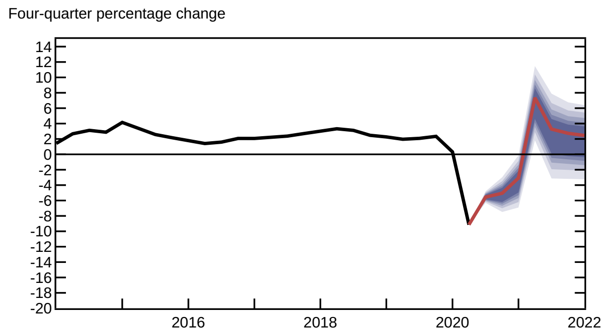

GDP Growth Pseudo- observable
Percent Q/Q Annualized
(deviations from mean)
10
0
- 10
- 20
- 30
biidc
Other COVID
b
FF
tfp
mkp
pol
me
other
2018
2020
2022

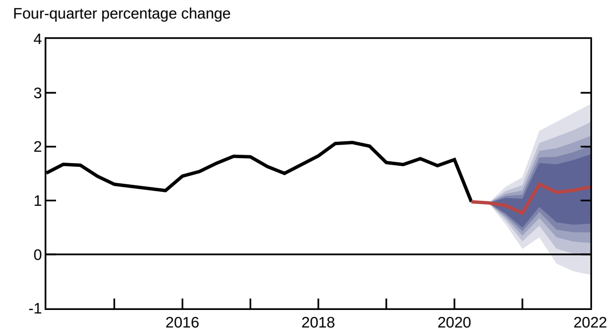

Real Natural Rate
Percent Annualized
(deviations from mean)
50
25
0
- 25
- 50
biidc
Other COVID
b
FF
tfp
mkp
pol
me
other
2018
2020
2022

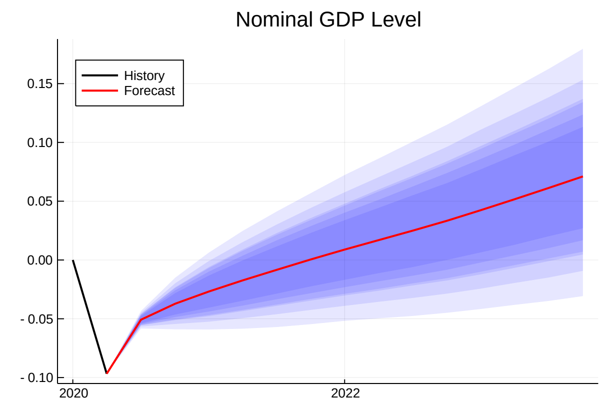
Scenario 2: Shutdown with Business Cycle Dynamics

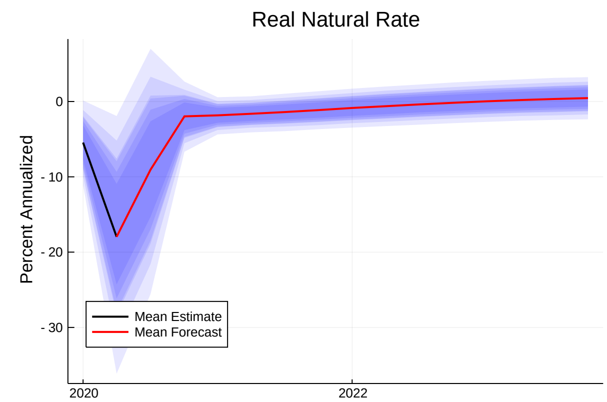
In the "Shutdown with Business Cycle Dynamics" we allow for the usual set of shocks that populate the model to play a larger role, yielding more persistent effects. Specifically, this scenario is constructed exactly like the "Temporary Shutdown" scenario, except that we do not constrain the standard deviation of the usual set of shocks that populate the model, including persistent productivity growth shocks.

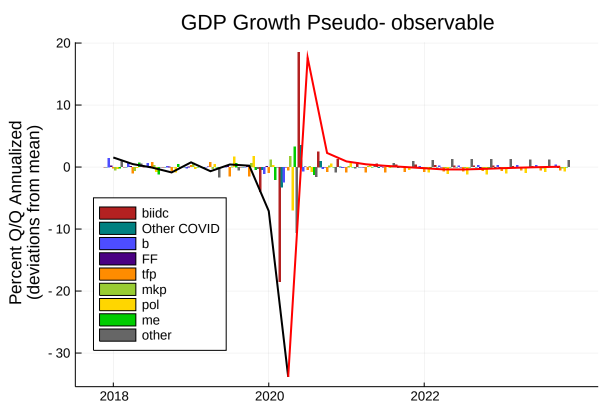
	2020		2021		2022		2023	
	Sep	Jun	Sep	Jun	Sep	Jun	Sep	Jun
GDP growth	-5.1	-6.4	2.7	2.4	1.9	1.3	2.1	1.7
(Q4/Q4)	(-6.6,-3.9)	(-8.7,-4.9)	(-1.1,4.8)	(-1.2,4.5)	(-1.1,4.5)	(-1.6,3.9)	(-0.3,5.1)	(-0.9,4.6)
Core PCE inflation	0.9	1.4	1.2	1.0	1.4	1.1	1.5	1.2
(Q4/Q4)	(0.7,1.1)	(1.1,1.8)	(0.3,2.0)	(0.0,1.9)	(0.3,2.5)	(-0.1,2.1)	(0.3,2.8)	(-0.1,2.4)
Federal funds rate	0.1	0.0	0.0	0.1	1.3	1.1	2.2	1.8
(Q4)	(0.1,0.8)	(0.0,1.1)	(0.1,1.7)	(0.0,1.8)	(0.3,3.3)	(0.1,3.1)	(0.7,4.3)	(0.4,3.9)
Real natural rate	-2.0	-3.6	-1.1	-0.5	-0.2	0.1	0.4	0.4
of interest (Q4)	(-4.7,0.7)	(-5.0,-2.1)	(-2.6,0.4)	(-2.0,1.0)	(-1.8,1.5)	(-1.5,1.7)	(-1.2,2.2)	(-1.3,2.1)
Output gap	-5.5	-6.4	-4.5	-5.3	-4.4	-5.1	-4.0	-4.7
(Q4)	(-7.5,-4.0)	(-8.9,-4.7)	(-8.5,-2.8)	(-9.6,-3.4)	(-9.3,-2.1)	(-10.1,-2.6)	(-9.1,-1.2)	(-9.9,-1.7)

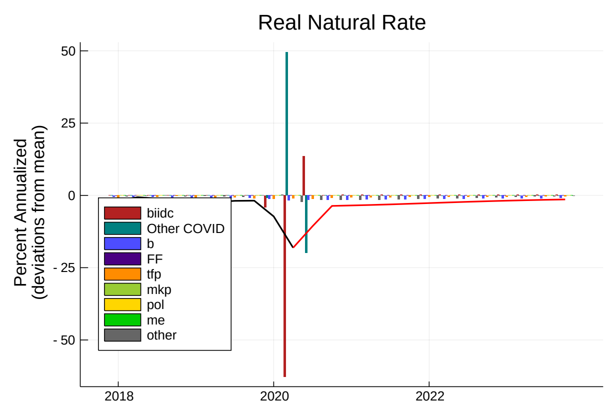

Four-quarter percentage change
2016
2018
2020
2022


Four-quarter percentage change
2016
2018
2020
2022


Nominal GDP Level
History
Forecast
2020
2022


Real Natural Rate
Percent Annualized
Mean Estimate
Mean Forecast
2020
2022


GDP Growth Pseudo- observable
Percent Q/Q Annualized
(deviations from mean)
biidc
Other COVID
b
FF
tfp
mkp
pol
me
other
2018
2020
2022

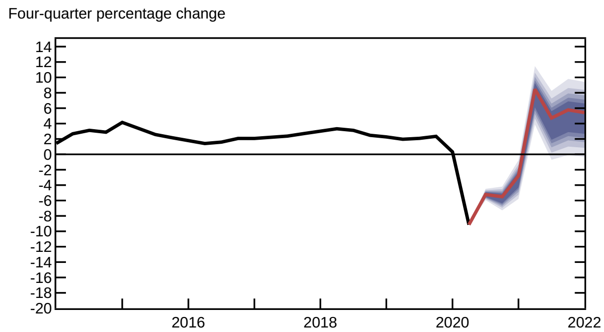

Real Natural Rate
Percent Annualized
(deviations from mean)
biidc
Other COVID
b
FF
tfp
mkp
pol
me
other
2018
2020
2022

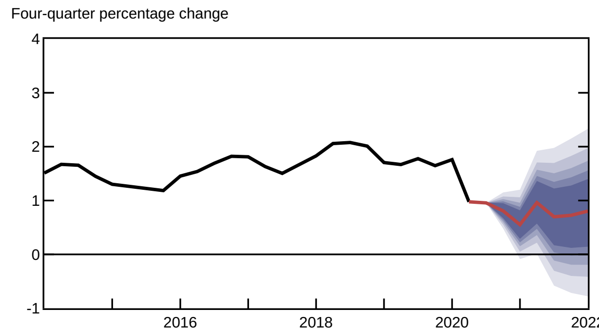
Scenario 3: Second Wave

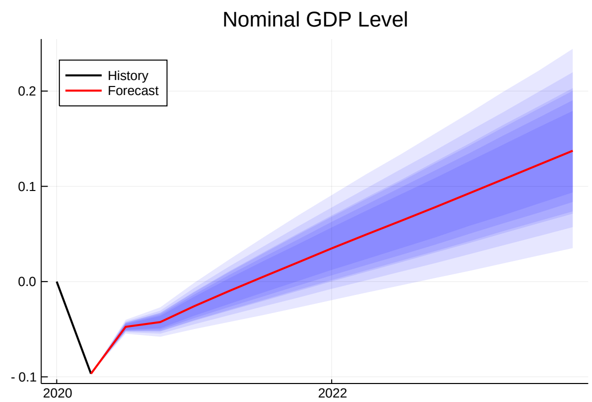
The "Second Wave" scenario builds upon the "Temporary Shutdown" scenario by additionally assuming a renewed weakness in demand in 2020Q4, reflecting a resurgence of the pandemic in that quarter. We implement this scenario by imposing that the current quarter expectation for real GDP growth in Q4 coincides with the 10th percentile of the cross-sectional distribution of SPF point forecasts (-0.36 percent, annualized). This scenario yields 2020 Q4/Q4 GDP growth in the neighborhood of -5.5 percent, not very distant from that in the second scenario. Differently from the second scenario, the "Second Wave" scenario features a stronger rebound of the economy in 2021 and 2022, as the effects of the second wave shock are transitory. To allow for additional uncertainty about 2020Q4, the number for expected GDP growth in 2020Q4 is interpreted as a noisy estimate of expected 2020Q4 GDP growth, as in Del Negro and Schorfheide (2013), section 5.3. We set the standard deviation of the noise to 2.0 (annualized), which is twice the estimated standard deviation of the GDP measurement error we normally use.

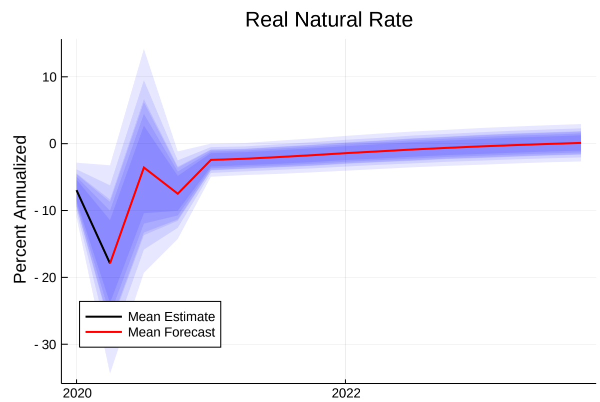
Note that the "Second Wave" scenario replaces the "Persistent Demand Shortfall" scenario featured in the June forecast, which turned out to be counterfactual (at least assuming the median SPF projections are broadly correct) in that this demand shortfall did not quite materialize in the current quarter. In all three scenarios the model allows for both the COVID-19 and the standard business cycle shocks to be active in Q3, although both sets of shocks play a relatively small role in this quarter as the models projections were largely in line with the SPF forecasts.

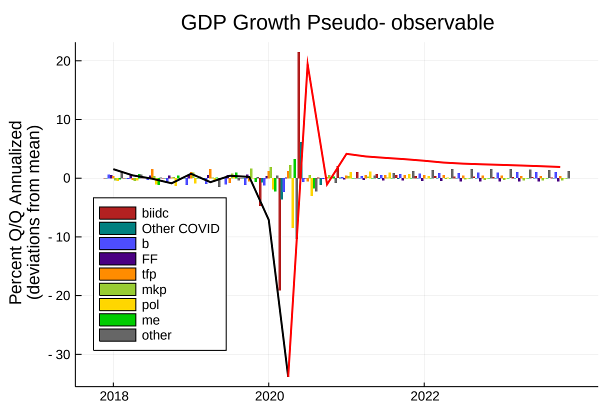
	2020		2021		2022		2023	
	Sep	Jun	Sep	Jun	Sep	Jun	Sep	Jun
GDP growth	-5.5	-12.2	5.8	1.5	4.7	1.7	4.2	2.2
(Q4/Q4)	(-6.7,-4.8)	(-14.6,-10.5)	(1.9,7.8)	(-2.1,3.7)	(1.5,7.3)	(-1.2,4.3)	(1.5,7.1)	(-0.4,5.1)
Core PCE inflation	0.8	1.4	0.7	0.8	1.1	0.9	1.3	1.0
(Q4/Q4)	(0.6,1.0)	(1.0,1.8)	(-0.2,1.6)	(-0.1,1.8)	(-0.1,2.1)	(-0.2,2.0)	(0.1,2.6)	(-0.2,2.3)
Federal funds rate	0.1	0.0	0.0	0.0	1.1	1.1	2.0	1.8
(Q4)	(0.1,0.8)	(0.0,1.1)	(0.1,1.8)	(0.0,1.8)	(0.2,3.1)	(0.1,3.0)	(0.7,4.1)	(0.4,3.9)
Real natural rate	-7.5	-2.0	-1.7	-1.2	-0.6	-0.3	0.1	0.2
of interest (Q4)	(-11.3,-3.5)	(-3.7,-0.4)	(-3.2,-0.2)	(-2.7,0.3)	(-2.3,1.0)	(-1.9,1.2)	(-1.6,1.8)	(-1.5,1.8)
Output gap	-6.6	-11.6	-4.3	-10.3	-3.0	-9.3	-1.9	-8.0
(Q4)	(-8.5,-5.2)	(-14.3,-9.7)	(-8.1,-2.7)	(-14.7,-8.2)	(-7.7,-0.9)	(-14.4,-6.6)	(-6.8,0.8)	(-13.3,-4.9)

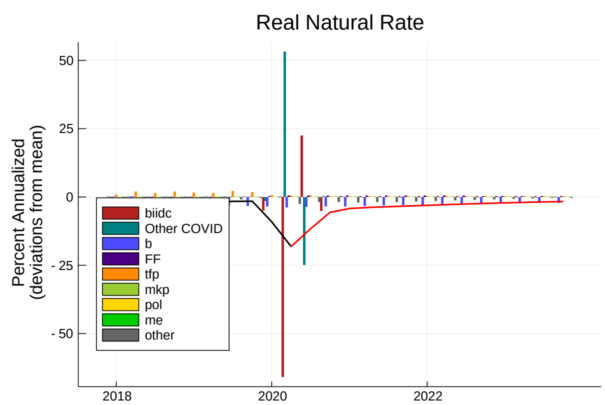

Four-quarter percentage change


Four-quarter percentage change


Nominal GDP Level
History
Forecast


Real Natural Rate
Percent Annualized
Mean Estimate
Mean Forecast


GDP Growth Pseudo- observable
Percent Q/Q Annualized
(deviations from mean)
biidc
Other COVID
b
FF
tfp
mkp
pol
me
other


Real Natural Rate
Percent Annualized
(deviations from mean)
biidc
Other COVID
b
FF
tfp
mkp
pol
me
other

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Philadelphia Forecast Detailed Summary

Philadelphia Empirical Macro Group*

2020-09-04

Overview of the Model

The second generation forecasting model (PRISM-II) is a medium-scale NKDSGE model that is inspired by Gertler, Sala, and Trigari (2008). It adds to the core Smets-Wouters style model a meaningful role for unemployment that arises from labor market search frictions. The model and features of its estimation are described in detail in the Technical Appendix at the end of this document. The model incorporates the following seven shocks: TFP, matching efficiency, household discount factor, investment specific technology, price markup, monetary policy, and government spending.

In light of the unusual macro dynamics caused by the COVID-19 pandemic, we made a modification to the model by introducing an additional short-lived and self-correcting shock that allows us to replicate the large volatility of the economy in 2020. We call this shock the COVID-19 shock and it is generated simply as a combination of four underlying structural shocks (three existing shocks plus one additional shock). Each of the three existing shocks follows a MA (1) process with a negative coefficient instead of the conventional persistent AR (1) process while the additional shock follows an i.i.d process. We calibrate these underlying shock processes, such that the macroeconomic behavior in 2020 is dominated by the combination of these shocks.

The COVID-19 Shock

The COVID-19 shock is a combination of (i) the discount factor shock, (ii) the investment-specific technology shock, (iii) the matching efficiency shock, and (iv) the vacancy posting cost shock. The last one is new for the current purpose. As mentioned above, each of the first three shocks follow an MA (1) process with a negative coefficient, while the shock to the vacancy posting cost is assumed to be i.i.d. We chose these four shocks and calibrated the stochastic processes, such

*Jonas Arias, Shigeru Fujita, Thorsten Drautzburg, and Keith Sill.

that, by combining these four shocks, the model generates the sharp contraction of real GDP in the second quarter of 2020 and the subsequent partial recovery in the following quarter. We also make certain that the observed patterns of the unemployment rate and inflation for the first half of this year and their forecasts for the second half are mostly driven by the COVID-19 shock and in line with the staff's view about the short-run evolution of the economy. The negative MA (1) coefficients allow us to replicate the partial correction of large negative shocks. We also assume that the COVID-19 shock operates only through the third quarter of 2020. Forecasts thereafter are driven by the model's internal dynamics that arise from the existing AR (1) shocks and the COVID-19 shocks. Details on the construction of the COVID-19 shock as well as the impulse response functions are provided below in the Appendix A.

Conditioning

We also pin down the forecasts for key model variables ex ante to bring the forecasts in line with our staff view on the near-term evolution of the economy. Specifically, (1) we fix the federal funds rate at the ELB until 2023Q4 and the two-year Treasury rate accordingly and (2) we also utilize the IHS nowcasts (third quarter values) for real GDP growth, core inflation, the unemployment rate, and the growth rate of government expenditures and net exports.

Forecasts

The model projects that Q4/Q4 real GDP growth for this year will be -4.1 percent. Growth is expected to rebound at a 23 percent annual rate in the third quarter and then decelerate to 5.6 percent in the fourth quarter. The model expects above-trend growth over the next three years (2.7 percent in 2021, 3.4 percent in 2022, and 3.3 percent in 2023) (Figure 1). Under this projection, the economy reaches its pre-pandemic level of real GDP in the first half of 2022.

Core PCE inflation is expected to be well below the FOMC target throughout the forecast horizon. Inflation runs at a 0.6 percent pace in 2020 and falls further to 0.3 percent in 2021, before it rises to 0.9 percent in 2022 and 1.5 percent in 2023 (Figure 2).

As noted above, the federal funds rate is pegged at the ELB over the forecast horizon (Figure 3), while fiscal policy is determined by fixing government expenditures and net exports at values provided by IHS for the third quarter. Specifically, these two components together grow at an annualized rate of 14 percent in the third quarter. The model then generates a dynamic path conditioned on this value.

The output gap, measured as the log deviation of output from its flexible price counterfactual level, is estimated to have narrowed to -0.4 percent in the current quarter from its recent bottom (-1.7 percent) in the second quarter. It is, however, expected to widen again through the third quarter of 2021 before it begins to close. At the end of 2023, the gap is expected to be substantially smaller, albeit remaining negative at -0.3 percent (Figure 4). The real natural rate of interest – measured as the real interest rate that would obtain in a counterfactual flexible price economy – took an extraordinary dive in the second quarter in response to the COVID-19 shock, according to the model. The natural rate rebounds in the current quarter to -1.5 percent, but remains negative until the first quarter of 2021. It is expected to increase to 1.5 percent at the end of 2023 (Figure 5).

The unemployment rate is endogenous in the Philadelphia model. After a large spike in the second quarter, the unemployment rate is expected to fall to 9.5 percent in the current quarter and 8.1 percent in the final quarter of this year. The pace of the declines thereafter is expected be gradual (Figure 6). The model forecasts the unemployment rate to be 5.2 percent at the end of 2023, about one percentage point above the CBO estimate of the natural rate of unemployment.

To generate uncertainty bands around our baseline forecast in Figures 1-6, we draw i.i.d normally distributed shocks for all shock processes except for monetary policy and the COVID-19 shocks, and we take parameter draws from the posterior distributions for parameter uncertainty. However, note that these uncertainty bands are based on historical data and thus may not adequately capture the heightened level of uncertainty in the current environment. Thus, for the numbers reported in the forecast template, we inflate the uncertainty bands for all the variables except the fed funds rate by the factor that the standard errors of density forecasts in the SPF have risen between 2020Q1 and 2020Q3.

Shock Decomposition

The key shocks driving historical and forecast output growth are shown in Figure 7. The large swing in output growth this year is driven by the COVID-19 shock as intended. Outside the COVID-19 shocks, the model assigns negative contributions to Q3 growth from the government expenditure shock, the investment-specific technology shock, and the markup shock. Going forward, the contribution of the investment-specific technology shock turns positive throughout the forecast horizon, while the markup shock remains to be a drag to the economy. Under the baseline scenario, the impact of the COVID-19 pandemic largely disappears by the end of next year.

Consumption dynamics closely follow that of output and are driven mostly by the same factors (Figure 10). The model attributes only half of the drop in investment in 2020Q2 to the short-lived COVID-19 shock, but it explains most of the drop in investment in 2020Q1 and accounts for more than the estimated increase in investment growth in 2020Q3. The remaining dynamics of investment are largely driven by the evolution of investment-specific technology shocks. But the undoing of negative contributions from markup shocks and accommodative monetary policy also contribute noticeably to the above-trend investment growth from 2021 to 2023 (Figure 11).

For inflation, the COVID-19 shocks pull inflation down by an average of -0.7 percentage point in 2021 and -0.2 percentage point even in 2023. Apart from the pandemic-related shocks, declining contributions from markup shocks and TFP shocks partly account for the weakening inflation in the near term. In addition, monetary policy, constrained by the ELB, keeps inflation down as well (Figure 8).

As mentioned above, the forecast is implemented with the federal funds rate pegged at the ELB through the end of 2023. Under this constrained path, negative contributions from COVID-19 shocks, investment-specific technology shocks, and government spending shocks are offset by positive contributions from markup and monetary policy shocks. Absent the ELB constraint, the model's policy rule calls for a substantially negative federal funds rate over the forecast horizon (Figure 9)

The sharp increase in the unemployment rate in the second quarter and the subsequent drop in the third quarter are driven mostly by COVID-19 shocks. However, monetary policy shocks, matching efficiency shocks, government spending shocks, and investment-specific technology shock all make significant contributions in keeping the unemployment rate higher than the natural rate in the medium run. The labor market recovery is expected to be swift through the end of this year, as the COIVD-19 shocks dissipate but the recovery thereafter is expected to be gradual as the traditional persistent business cycle shocks become more dominant.

The natural rate of interest drops sharply in the first half of 2020 in response to the short-lived but large COVID-19 shocks. As these shocks dissipate, the natural rate rises to -1.5 percent in the current quarter. The dynamics of the natural rate of interest over the medium term are dominated by the investment-specific technology shocks (Figure 13).

The significant negative output gap in the short run is accounted for by the COVID-19 shocks. (In the absence of the COVID-19 shocks, the output gap is roughly zero). Outside the COVID-19 shocks, monetary policy shocks and investment-specific technology shocks push down the output gap in the short run, offsetting the positive effects of markup shocks (Figure 14).


Real GDP Growth
30
20
10
0
-10
-20
-30
-40
-50
2014
2015
2016
2017
2018
2019
2020
2021
2022
2023

FIGURE 1

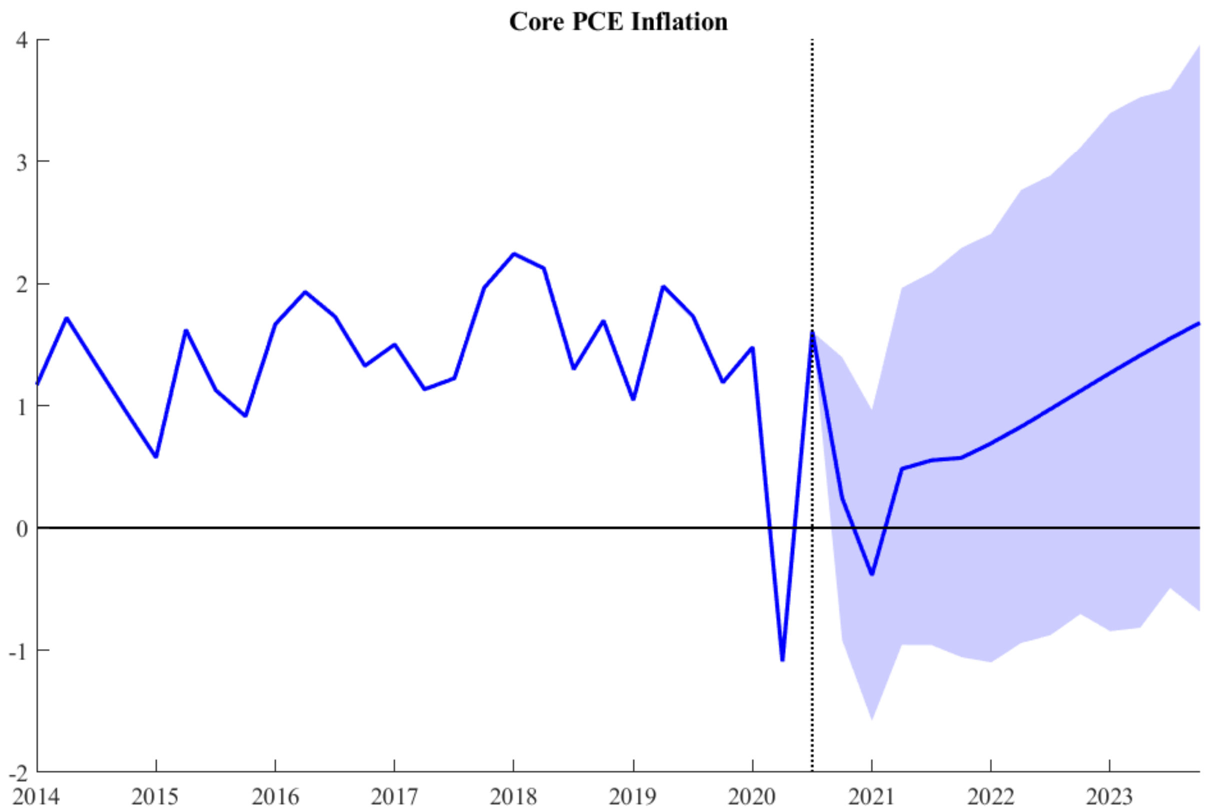

Core PCE Inflation
4
3
2
1
0
-1
-2
2014
2015
2016
2017
2018
2019
2020
2021
2022
2023

FIGURE 2

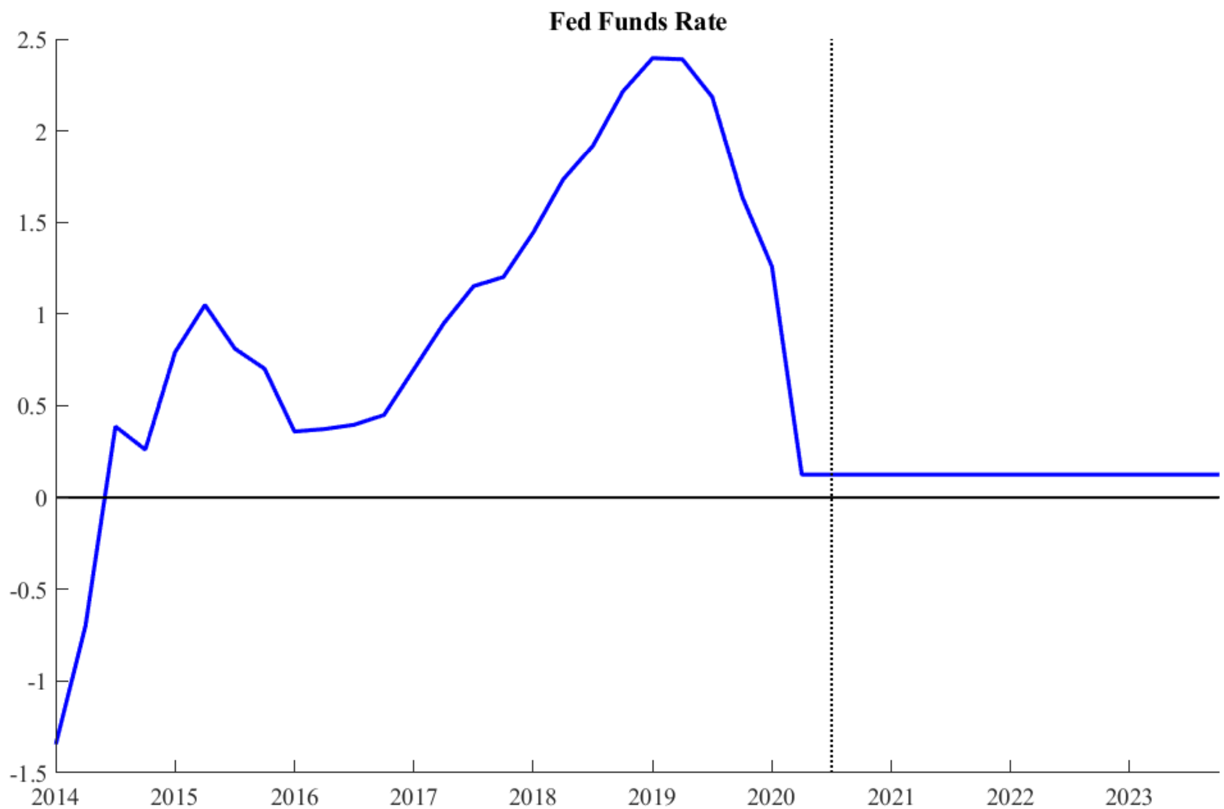

Fed Funds Rate
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
2014
2015
2016
2017
2018
2019
2020
2021
2022
2023

FIGURE 3

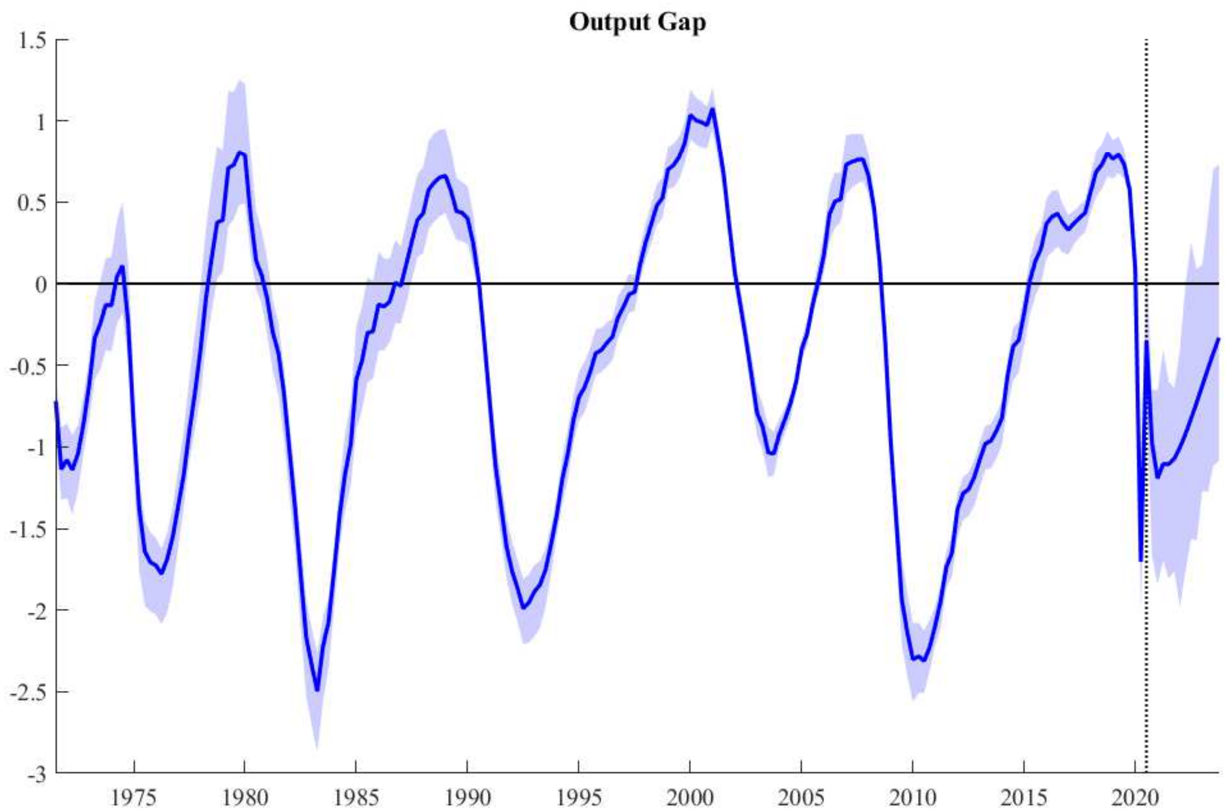

Output Gap
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020

FIGURE 4

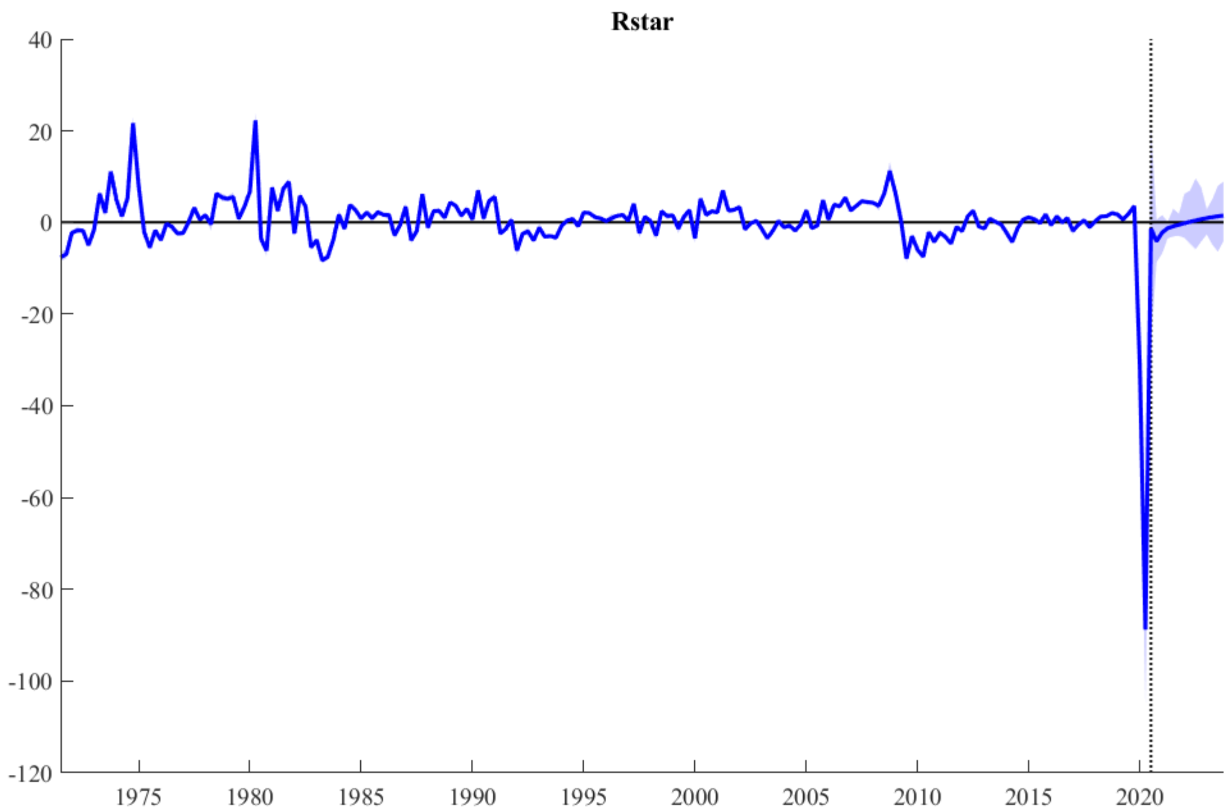
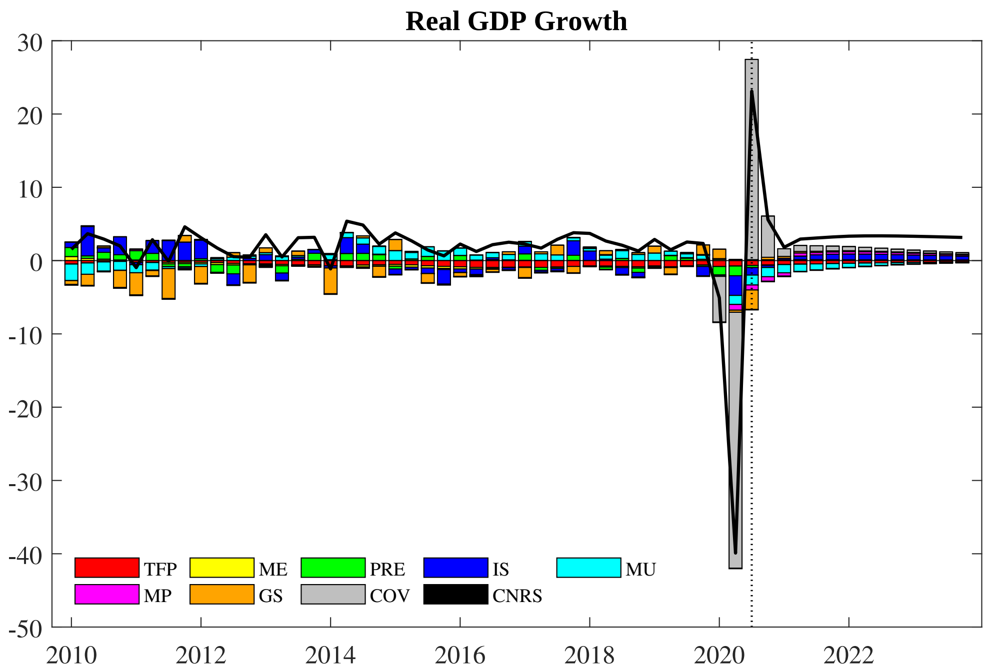

Rstar
40
20
0
-20
-40
-60
-80
-100
-120
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020

FIGURE 5


Unemployment Rate
14
12
10
8
6
4
2
0
2014
2015
2016
2017
2018
2019
2020
2021
2022
2023

FIGURE 6


Real GDP Growth
30
20
10
0
-10
-20
-30
-40
-50
2010
2012
2014
2016
2018
2020
2022
TFP
ME
PRE
IS
MU
MP
GS
COV
CNRS

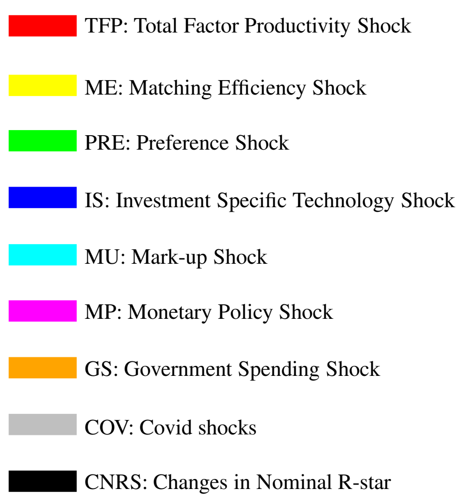
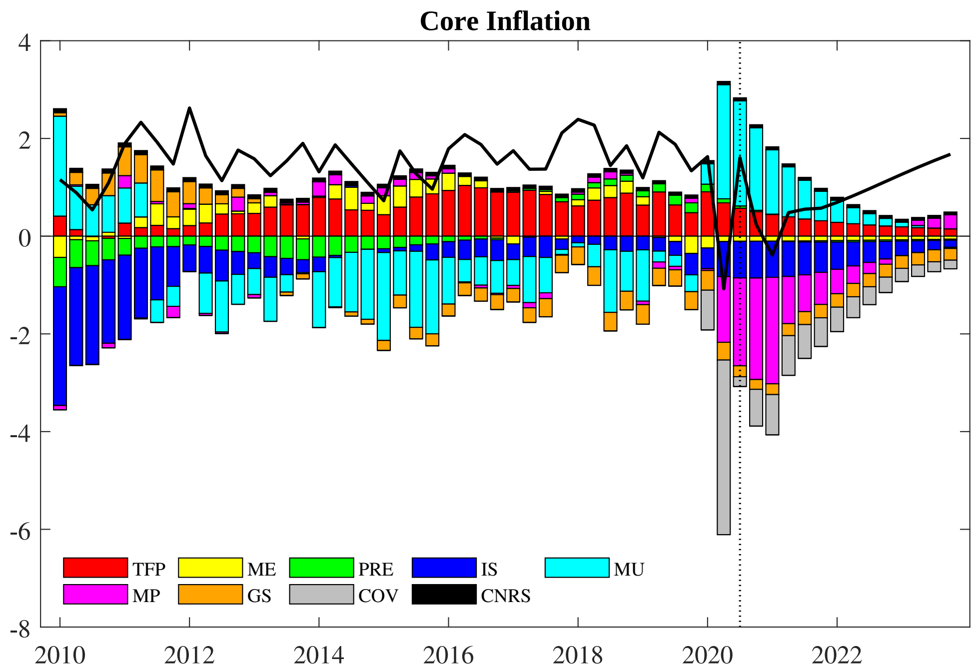

TFP: Total Factor Productivity Shock
ME: Matching Efficiency Shock
PRE: Preference Shock
IS: Investment Specific Technology Shock
MU: Mark-up Shock
MP: Monetary Policy Shock
GS: Government Spending Shock
COV: Covid shocks
CNRS: Changes in Nominal R-star

FIGURE 7



TFP: Total Factor Productivity Shock

ME: Matching Efficiency Shock

PRE: Preference Shock

IS: Investment Specific Technology Shock

MU: Mark-up Shock

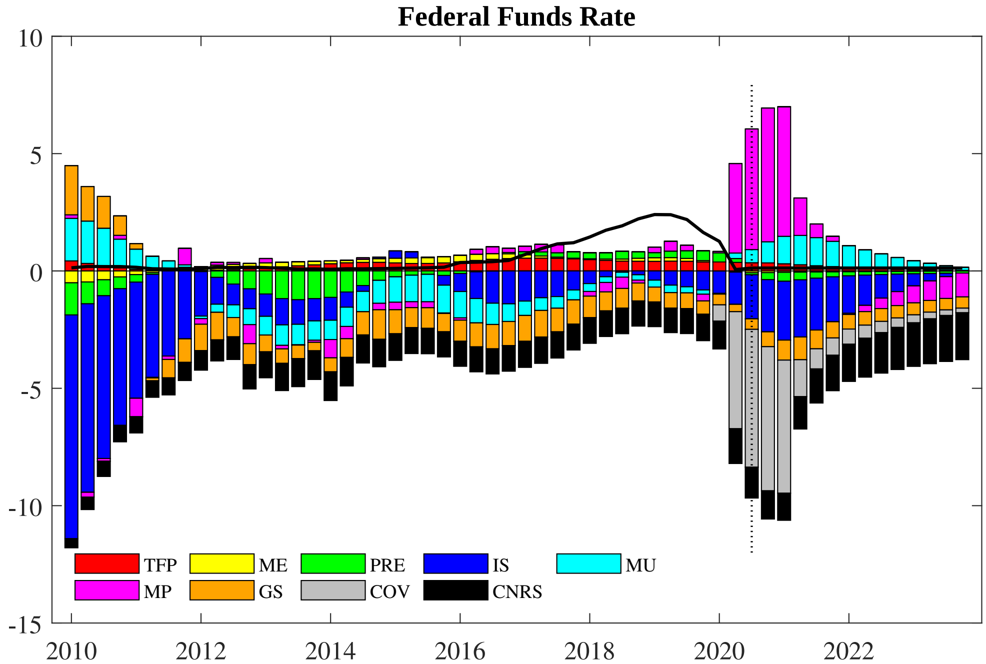
MP: Monetary Policy Shock

GS: Government Spending Shock

COV: Covid shocks

CNRS: Changes in Nominal R-star

FIGURE 8



TFP: Total Factor Productivity Shock

ME: Matching Efficiency Shock

PRE: Preference Shock

IS: Investment Specific Technology Shock

MU: Mark-up Shock

MP: Monetary Policy Shock

GS: Government Spending Shock

COV: Covid shocks

CNRS: Changes in Nominal R-star

FIGURE 9

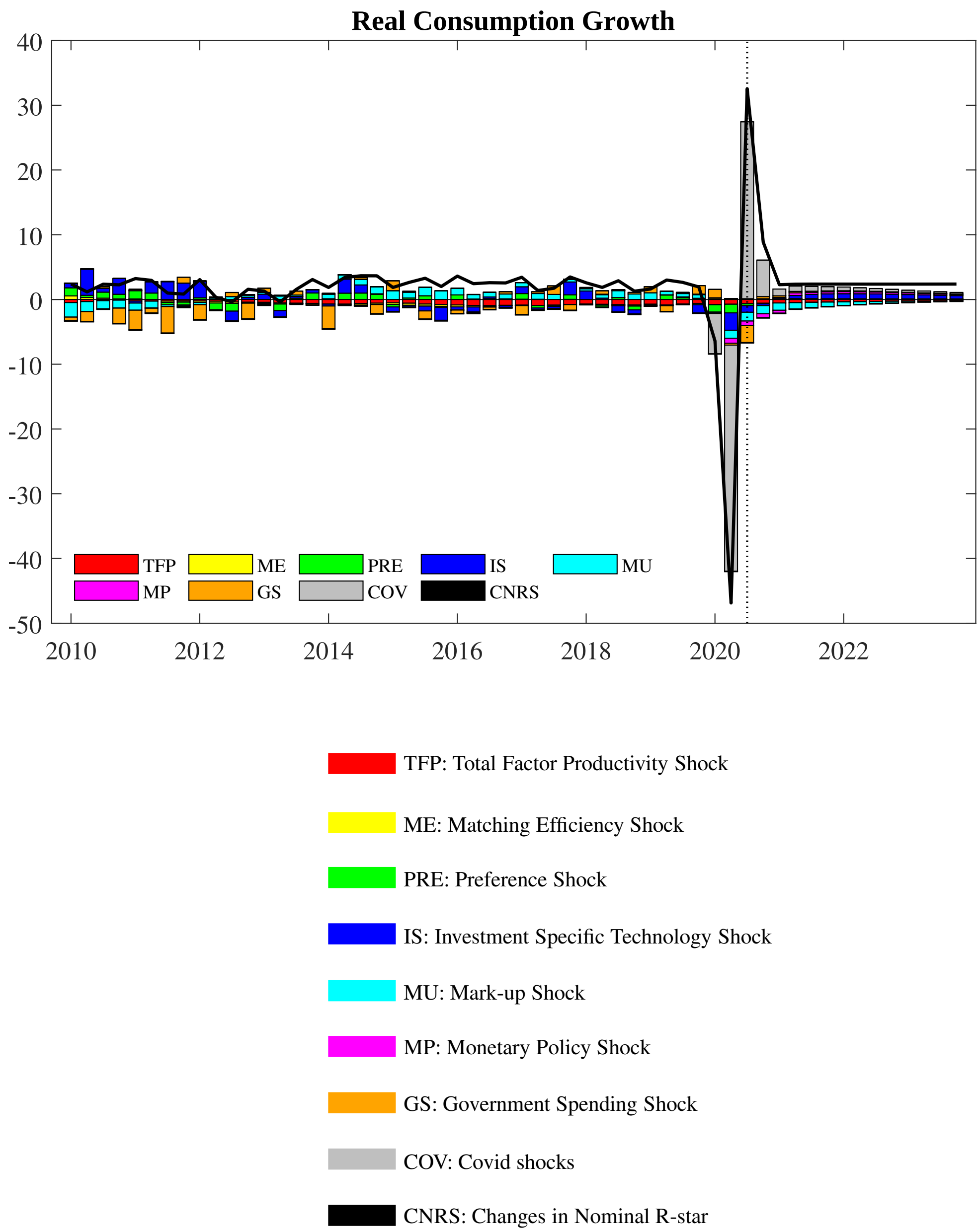

Real Consumption Growth
40
30
20
10
0
-10
-20
-30
-40
-50
2010
2012
2014
2016
2018
2020
2022
TFP
ME
PRE
IS
MU
MP
GS
COV
CNRS
TFP: Total Factor Productivity Shock
ME: Matching Efficiency Shock
PRE: Preference Shock
IS: Investment Specific Technology Shock
MU: Mark-up Shock
MP: Monetary Policy Shock
GS: Government Spending Shock
COV: Covid shocks
CNRS: Changes in Nominal R-star

FIGURE 10

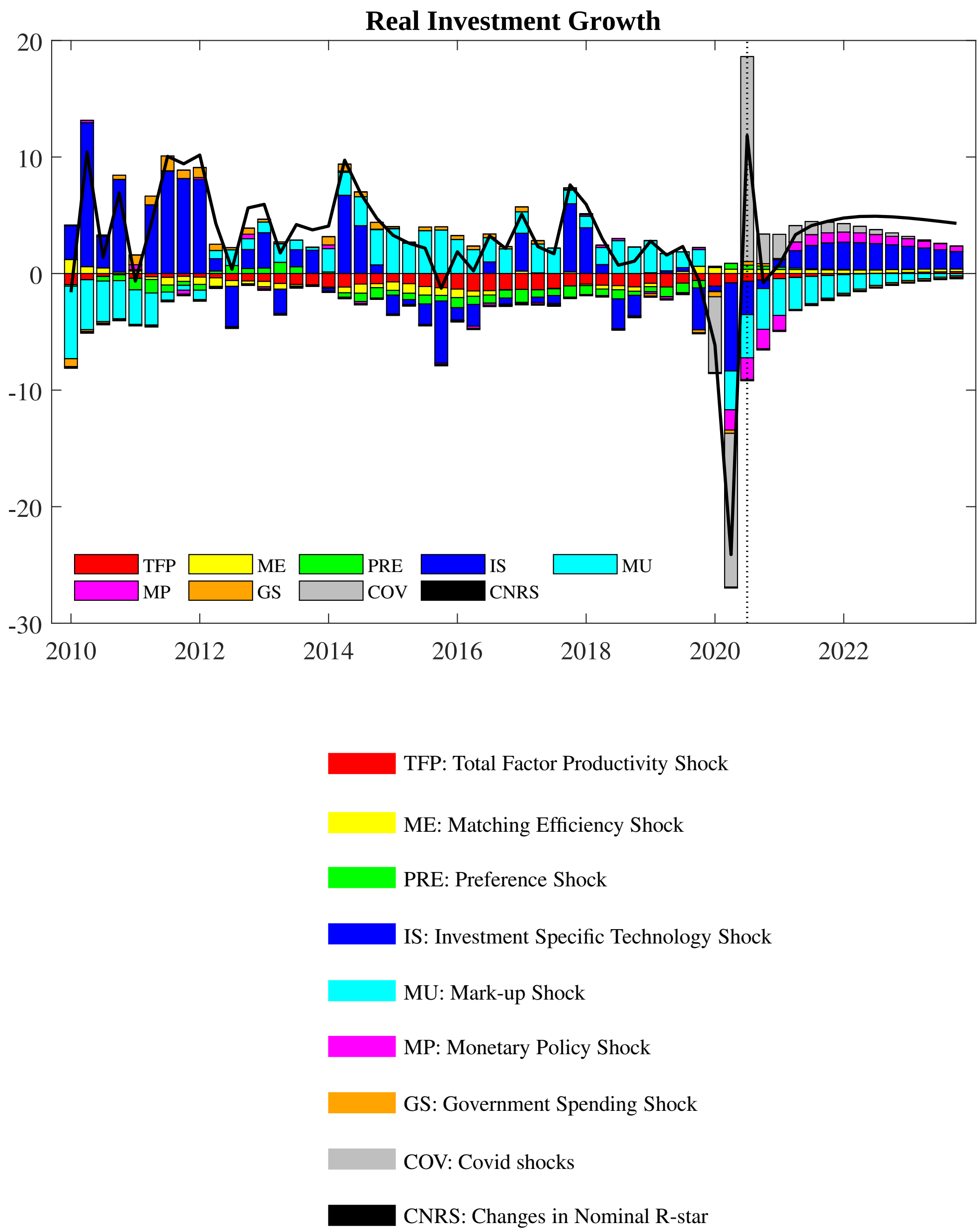

Real Investment Growth
20
10
0
-10
-20
-30
2010
2012
2014
2016
2018
2020
2022
TFP
ME
PRE
IS
MU
MP
GS
COV
CNRS
TFP: Total Factor Productivity Shock
ME: Matching Efficiency Shock
PRE: Preference Shock
IS: Investment Specific Technology Shock
MU: Mark-up Shock
MP: Monetary Policy Shock
GS: Government Spending Shock
COV: Covid shocks
CNRS: Changes in Nominal R-star

FIGURE 11

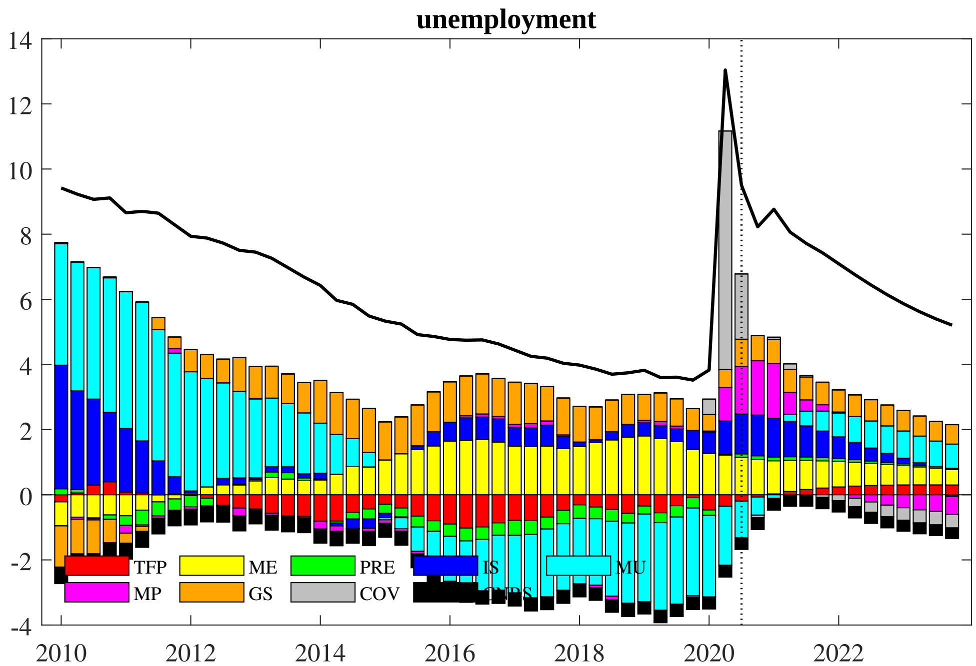


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FIGURE 12

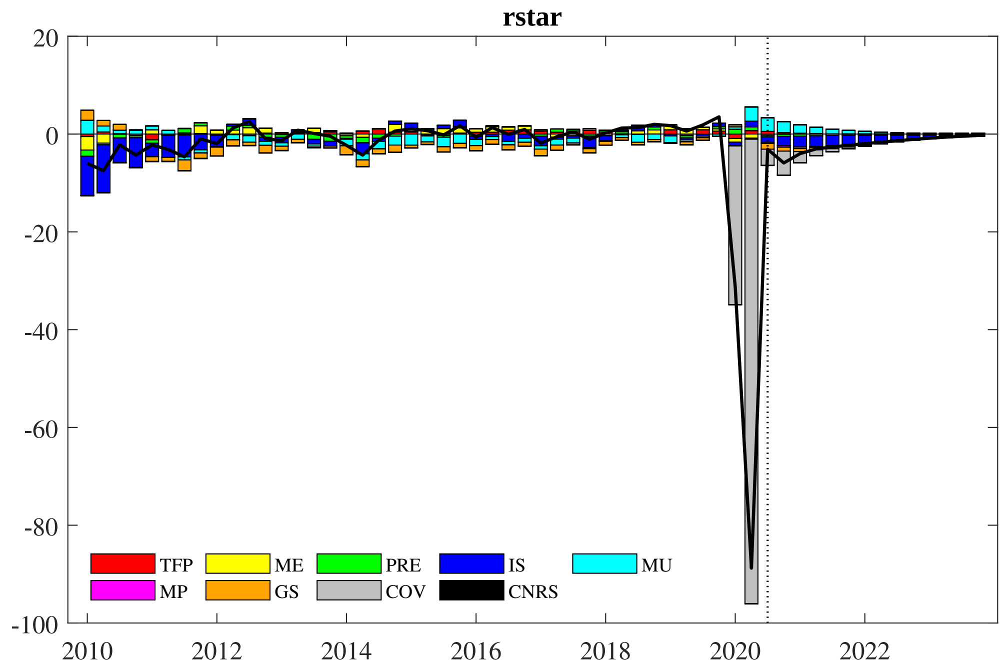

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FIGURE 13

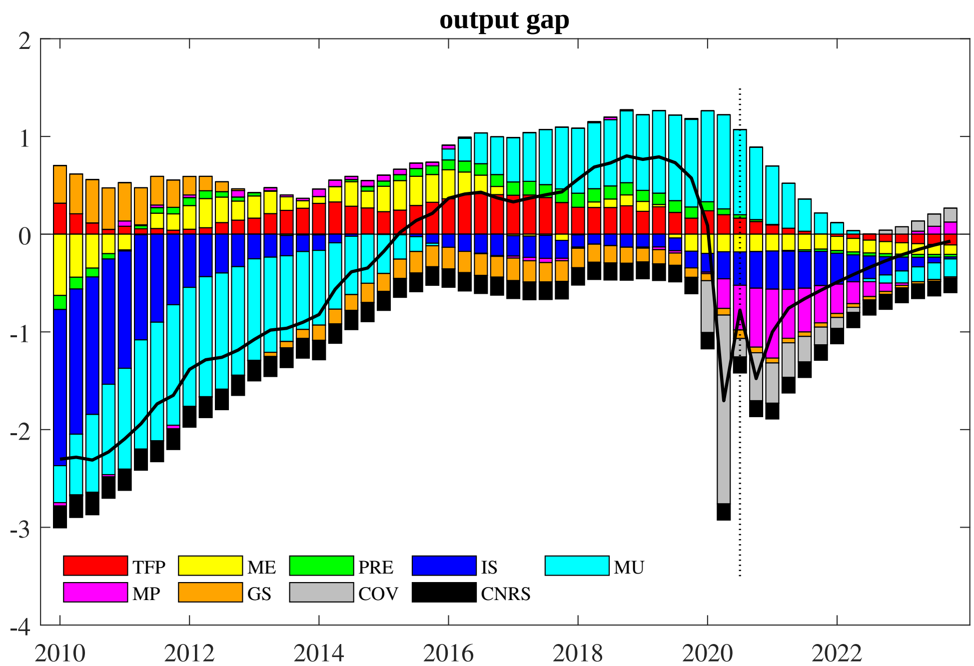

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FIGURE 14

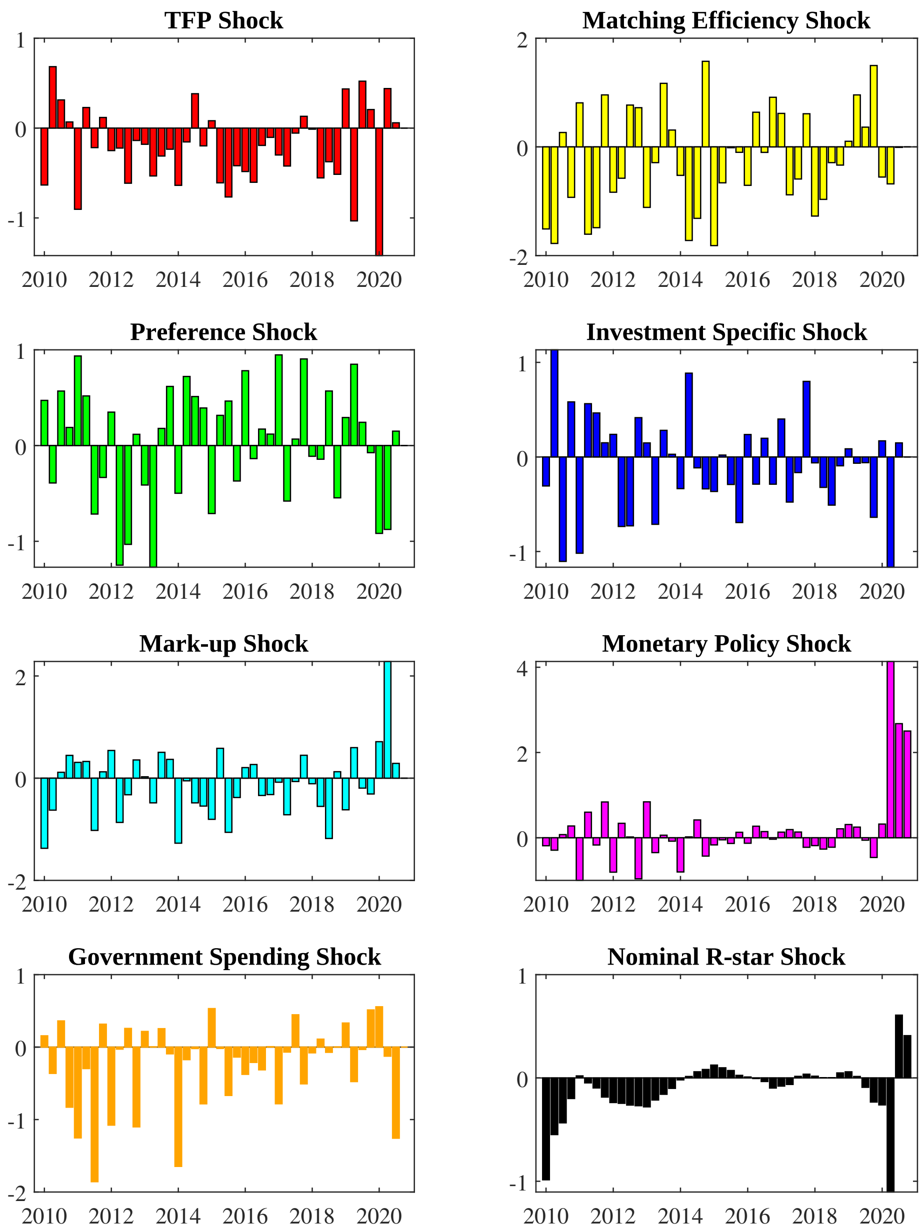

TFP Shock
Matching Efficiency Shock
Preference Shock
Investment Specific Shock
Mark-up Shock
Monetary Policy Shock
Government Spending Shock
Nominal R-star Shock
2010
2012
2014
2016
2018
2020

FIGURE 15

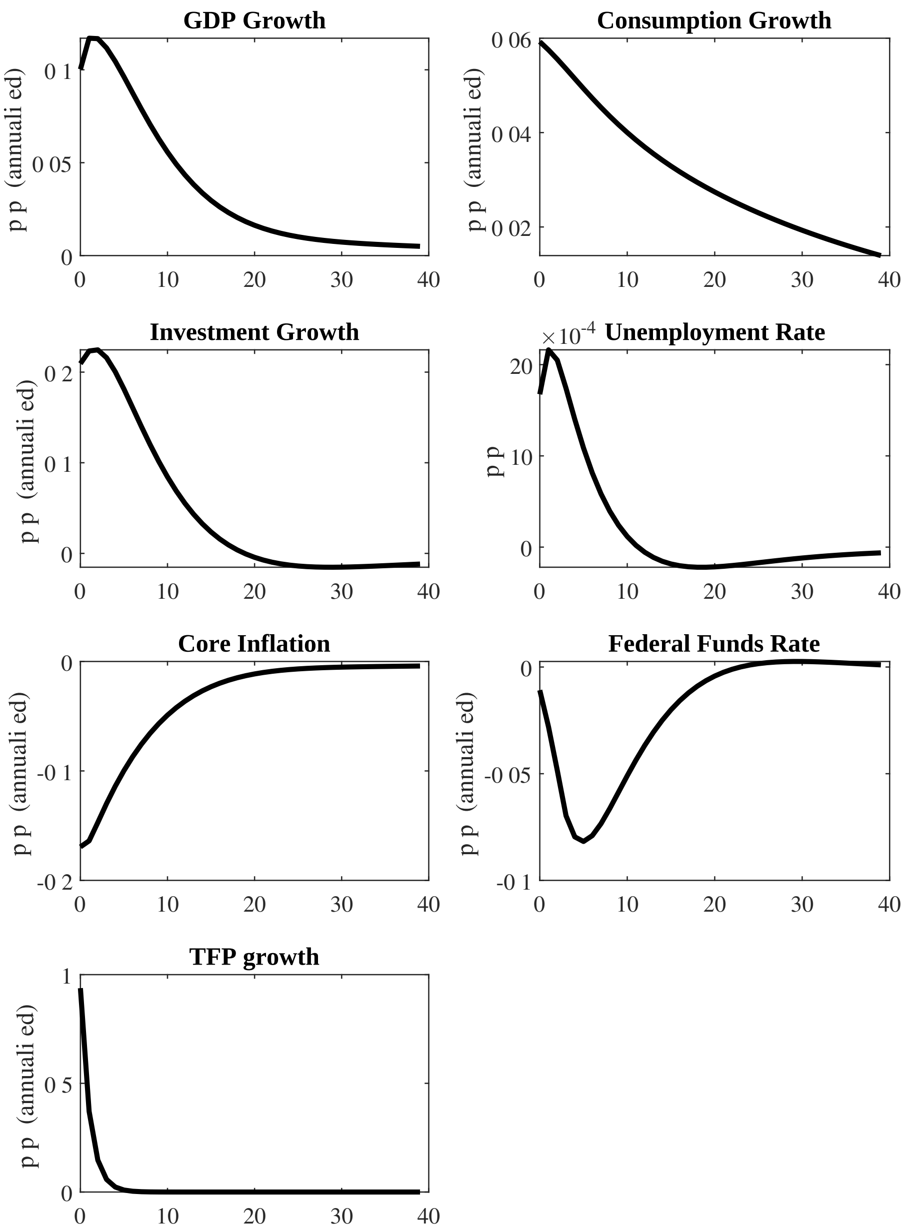


FIGURE 13A: Selected Impulse Responses to a Total Factor Productivity (TFP) Shock

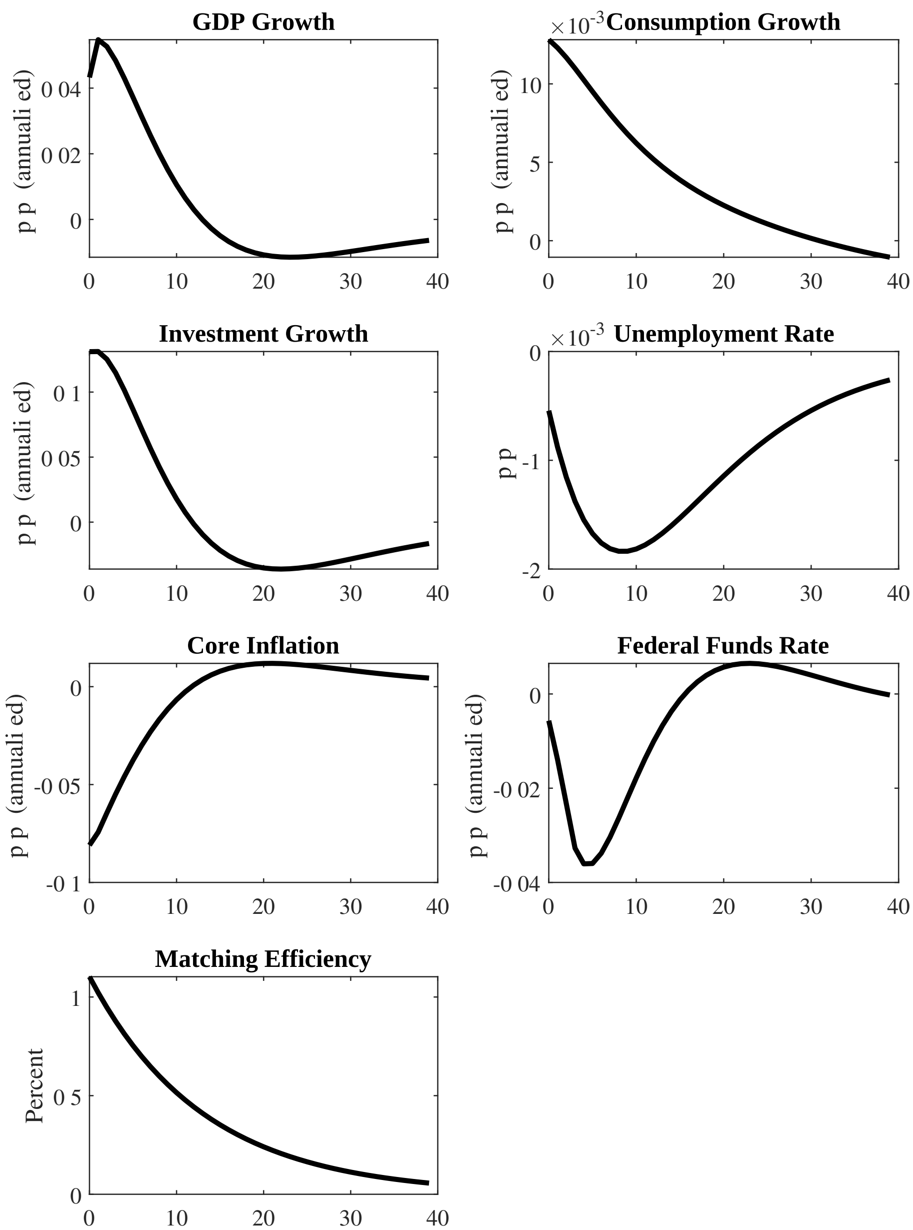


FIGURE 13B: Selected Impulse Responses to a Matching Efficiency Shock

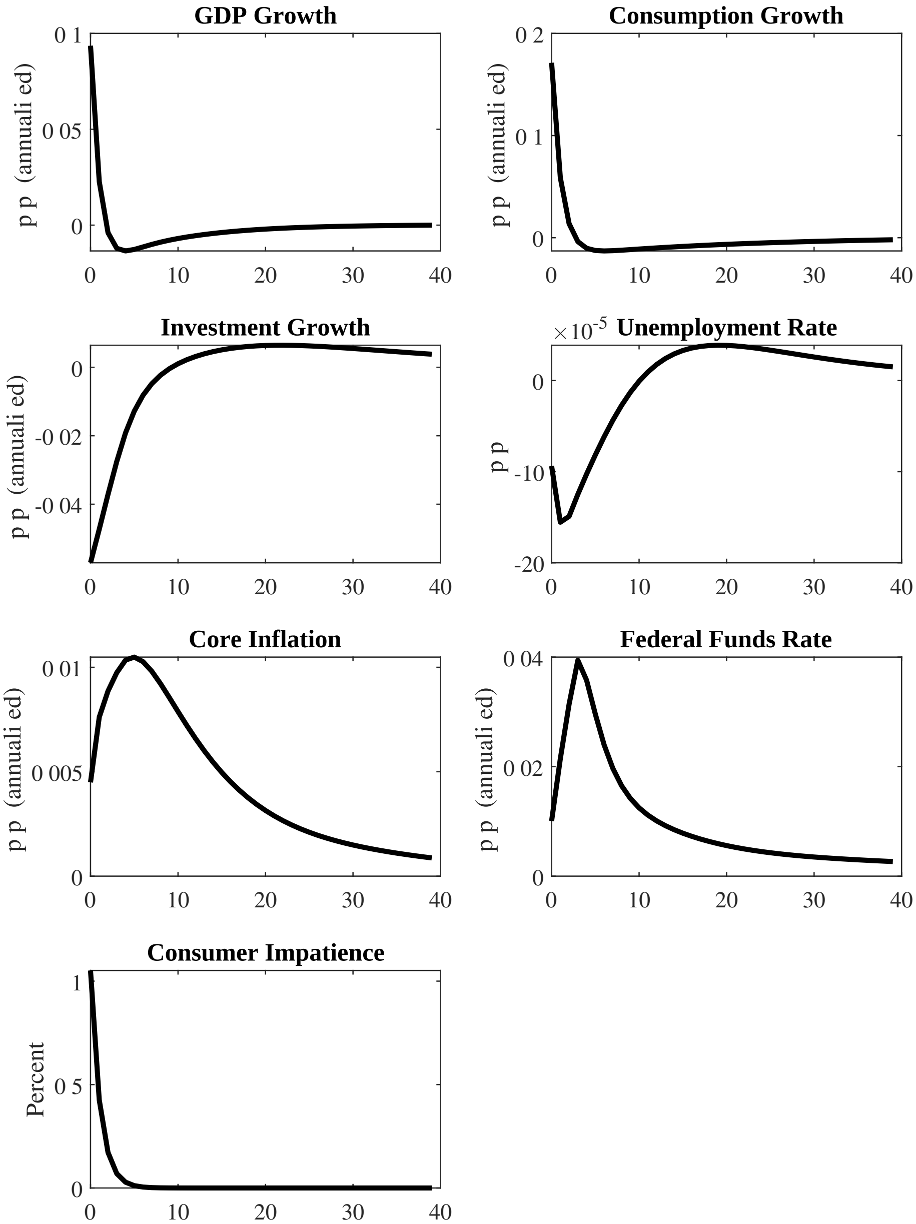


FIGURE 13C: Selected Impulse Responses to a Preference Shock

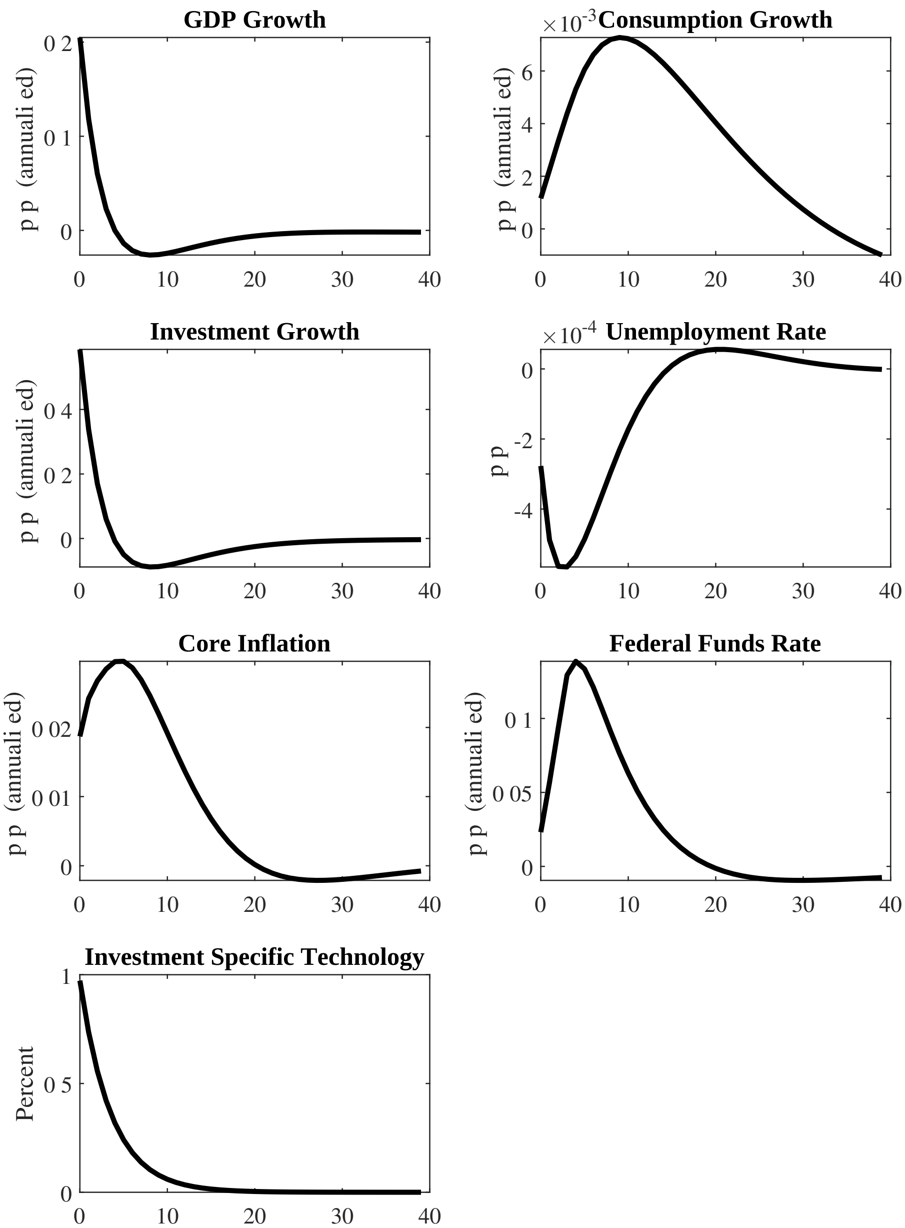


FIGURE 13D: Selected Impulse Responses to an Investment Specific Technology Shock

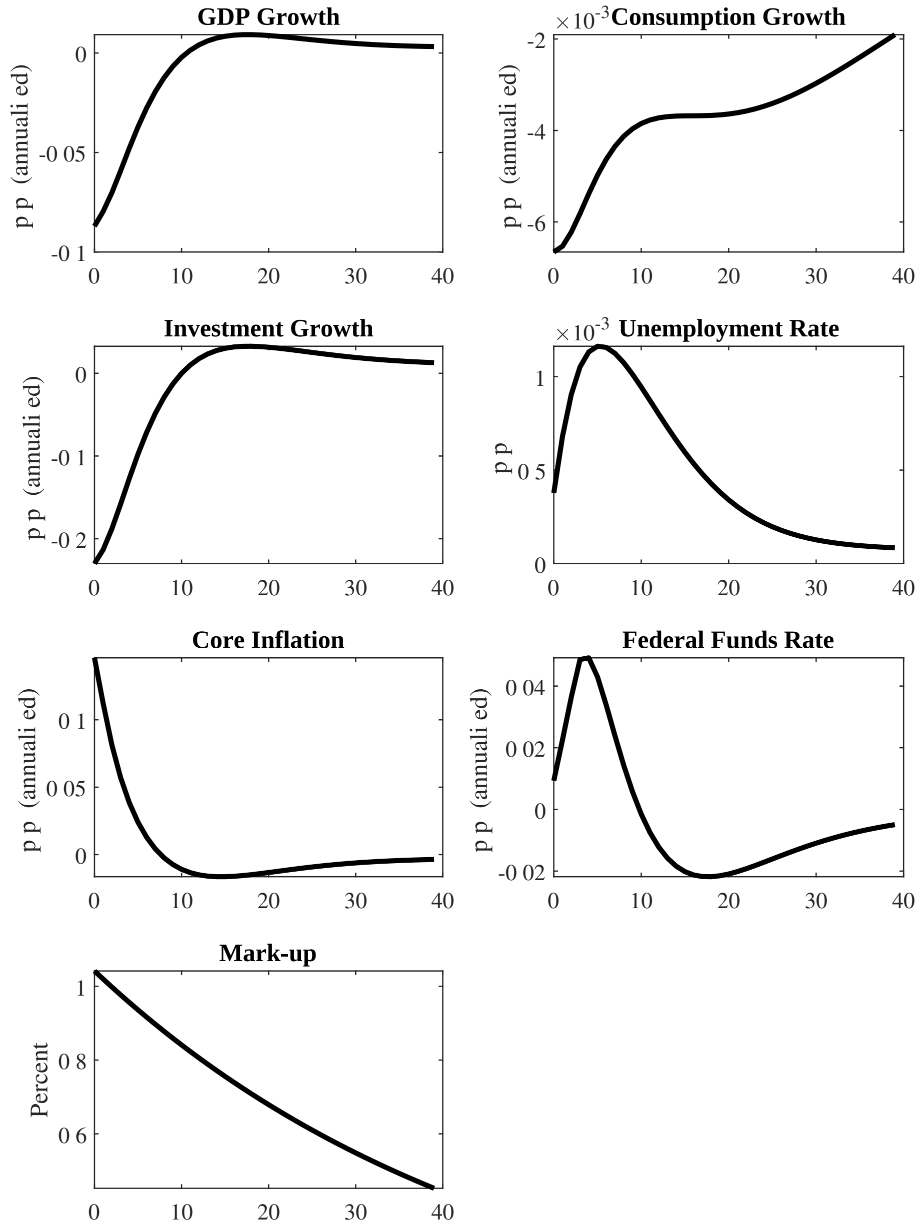
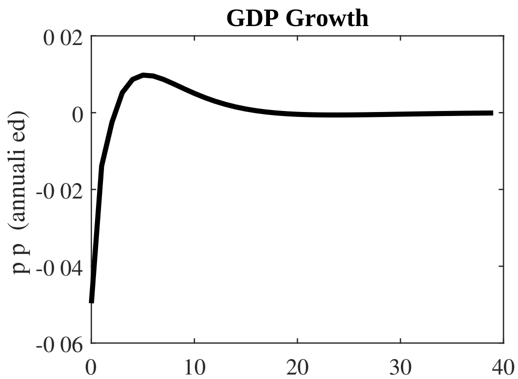
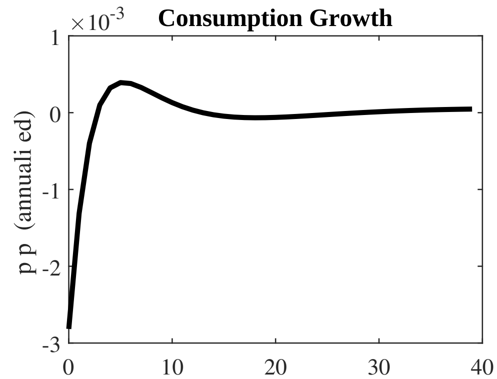
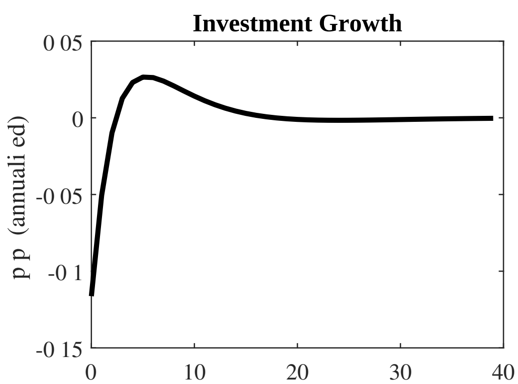
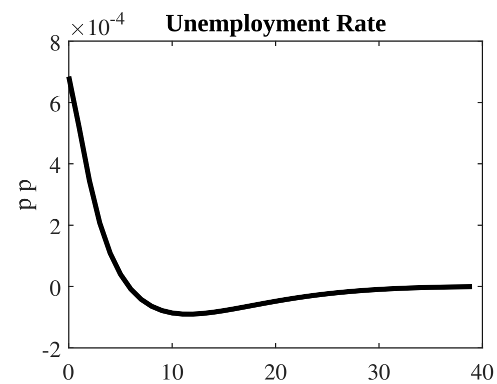


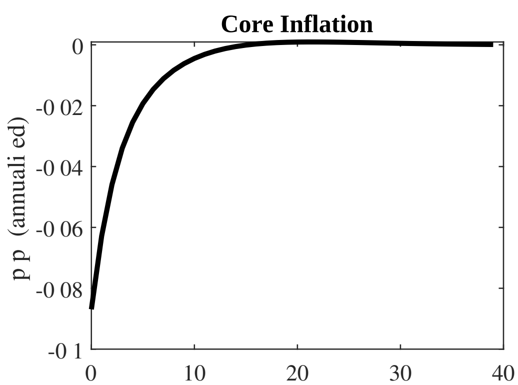
FIGURE 13E: Selected Impulse Responses to a Mark-up Shock











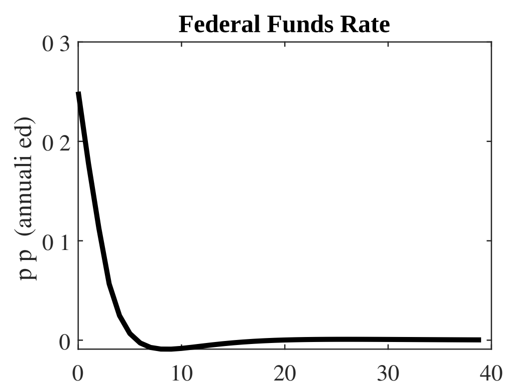


FIGURE 13F: Selected Impulse Responses to a Monetary Policy Shock

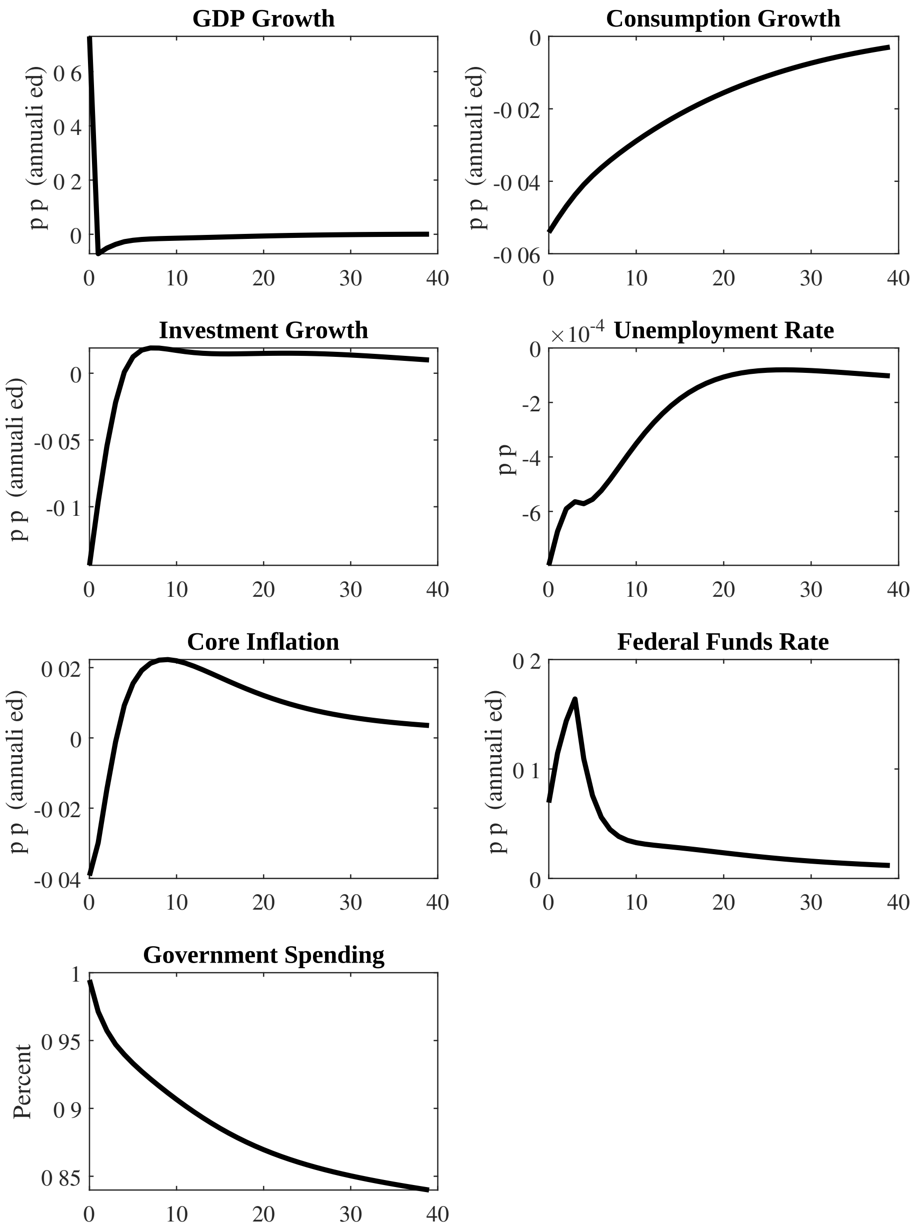


FIGURE 13G: Selected Impulse Responses to a Government Spending Shock

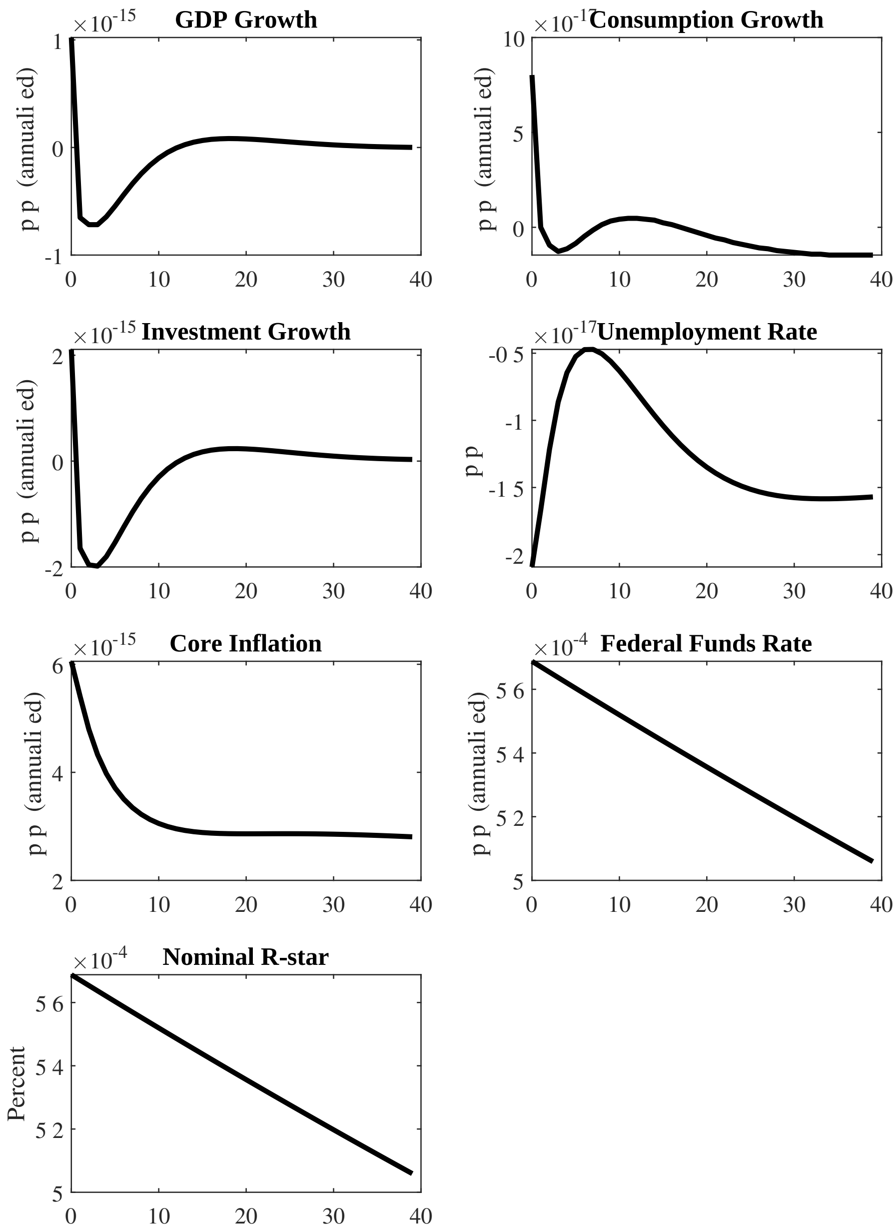


FIGURE 13H: Selected Impulse Responses to a Nominal R-star Shock

Appendix A: Construction of the COVID-19 Shock

September 4, 2020

Model Modification

The PRISM-II model features the seven structural shocks, all of which follow persistent AR (1) processes. The estimated AR (1) processes are not necessarily adequate to describe the macroeconomic dynamics caused by the COVID-19 pandemic. To address this issue, we introduce the copies of the three existing shocks and assume that they follow MA (1) processes:

$$\ln x_t = \epsilon_t^x + \theta\epsilon_{t-1}^x, \theta \leq 0.$$

Note that the MA (1) process together with $\theta \leq 0$ yields "short-lived and self-correcting dynamics." This process appears to be a reasonable characterization of the macroeconomic dynamics in 2020. Those three shocks are the discount factor shock, the investment-specific technology shock, and the matching efficiency shock. The MA (1) coefficients for the three shocks are set to $-3/4, -1/3$ and $-1/2$, respectively. In addition to these three shocks, we also introduce a shock to the cost of vacancy creation. This shock is assumed to be i.i.d, (or $\theta = 0$).

The COVID-19 shock is simply a combination of these four shocks. Combining these four shocks with their negative MA (1) coefficients allows us to produce the patterns of key observables (most notably, GDP growth, the unemployment rate, inflation, and the natural rate of interest) that are consistent with our staff short-term judgmental forecasts as well as their realized values in the first half of this year. We also assume that, in the model, the COVID-19 shocks exist only in the first quarter through the third quarter of this year. The model freely infers their magnitude given a prior of high volatility. The inferred shocks are plotted in Figure A.1.

Remarks on the Labor Market Shocks

Specifically, we introduce a shock to the vacancy posting cost as a "perceived" hiring cost by the firm. A higher cost of vacancy creation reduces vacancy posting and, endogenously, lowers the job finding probability as firms choose to post fewer vacancies. However, vacancy posting is a form of investment that takes up resources in the model. Thus, a stark increase in vacancy posting could, counterfactually, cause an increase in output. To get around this implication, we assume that the increased cost of vacancy creation is only perceived to be high without affecting output. The higher perceived cost of hiring and the negative self-correcting matching efficiency shock are largely responsible for the dynamics of the unemployment rate in 2020, although the other two components also play important roles.

In reality, the large spike and the subsequent quick but partial recovery in the unemployment rate are driven by the dynamics of the job separation rate. Our model, however, assumes the constant separation rate and thus is unable to replicate this underlying mechanism. We explored the idea of introducing the exogenously time-varying separation rate into the model. This specification, however, introduces several counterfactual implications on other variables, most notably, on job vacancies. Our current solution is to use the hiring cost shock and the matching efficiency shock to generate plausible dynamics of the unemployment rate.

Impulse Response Functions

Figures A.2 through A.5 plot the impulse response functions to the underlying four shocks. They are expressed as responses to contractionary shocks in the sense that they all result in increases in the unemployment rate at least initially. While the discount factor shock drives consumption lower and investment higher, the investment-specific technology shock lowers both consumption and investment. These two shocks together lower both demand components. The two labor market shocks result in declines in these demand components as well, but are inflationary. They are inflationary because these shocks make hiring more costly and thus raise marginal costs. Consequently, inflation rises, and in response, the funds rate increases. These effects are offset by the combination of the first two shocks (particularly the discount factor shock) that lower inflation and thus the funds rate.

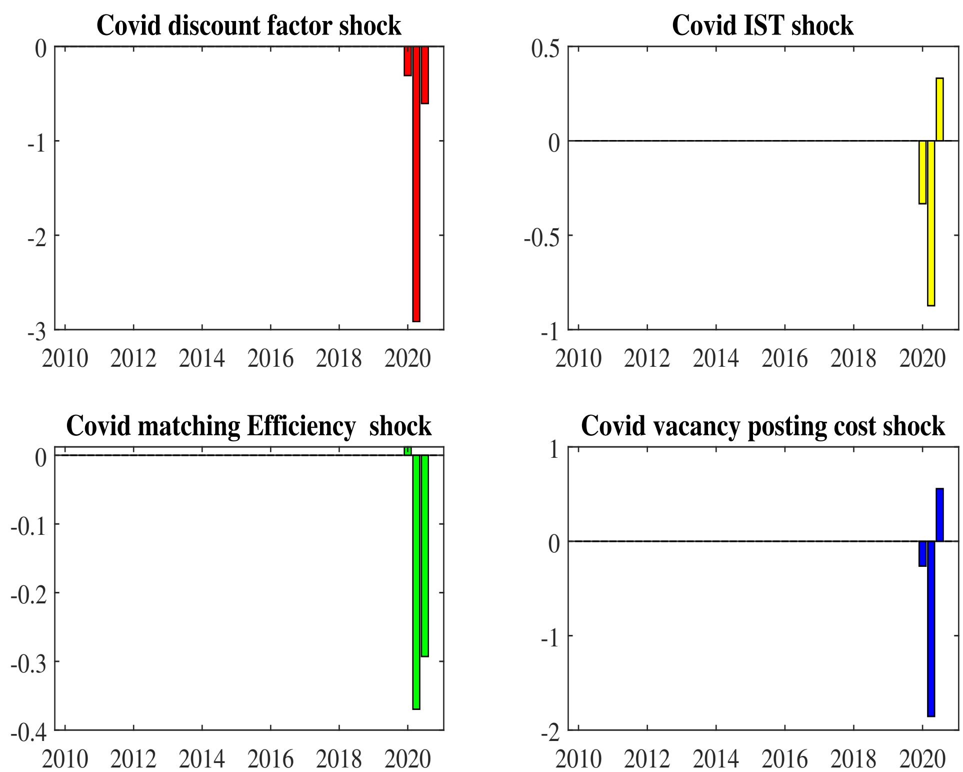


Figure A.1: Paths of Components of the COVID-19 Shock

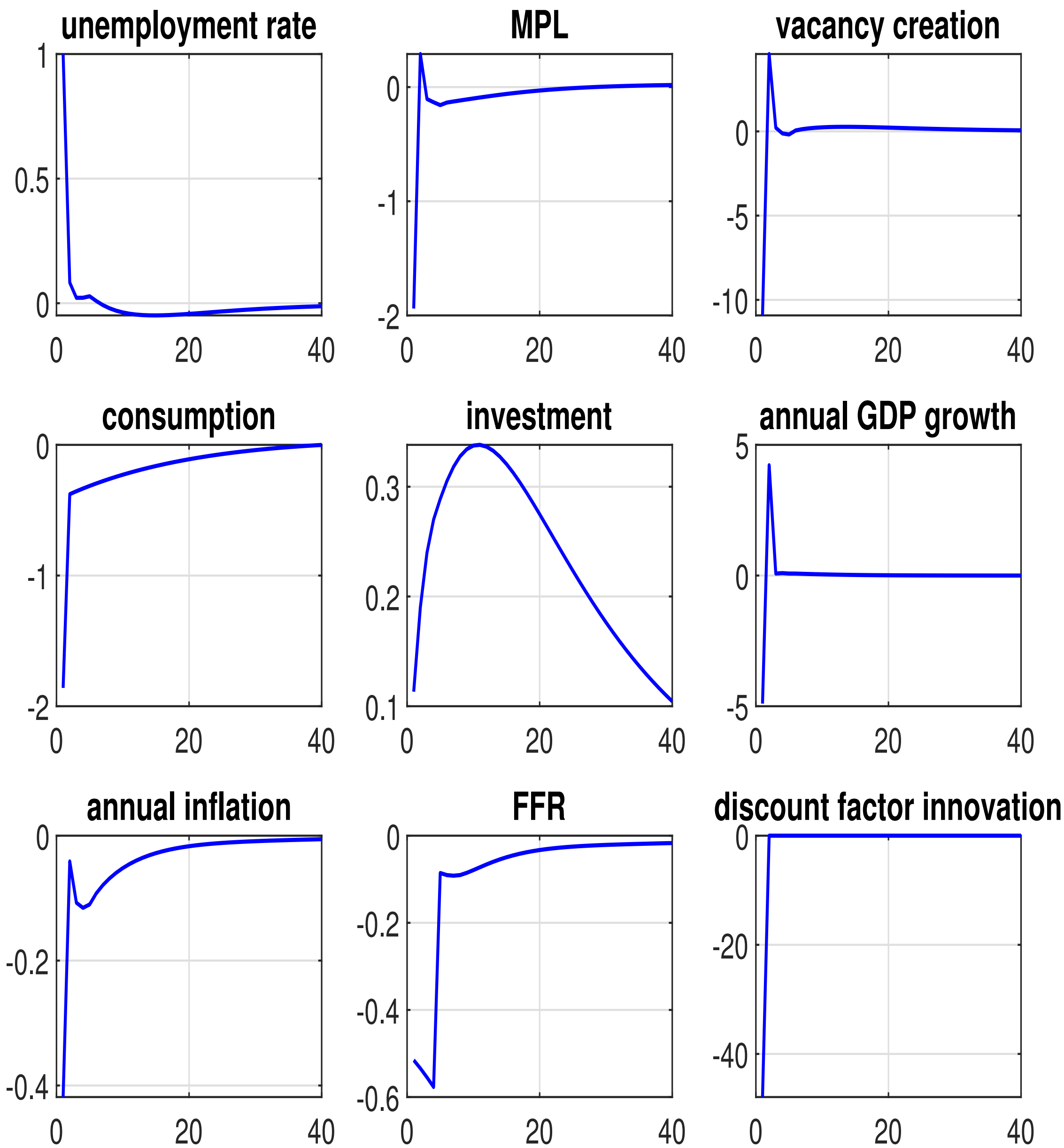


Figure A.2: Responses to the COVID discount factor shock

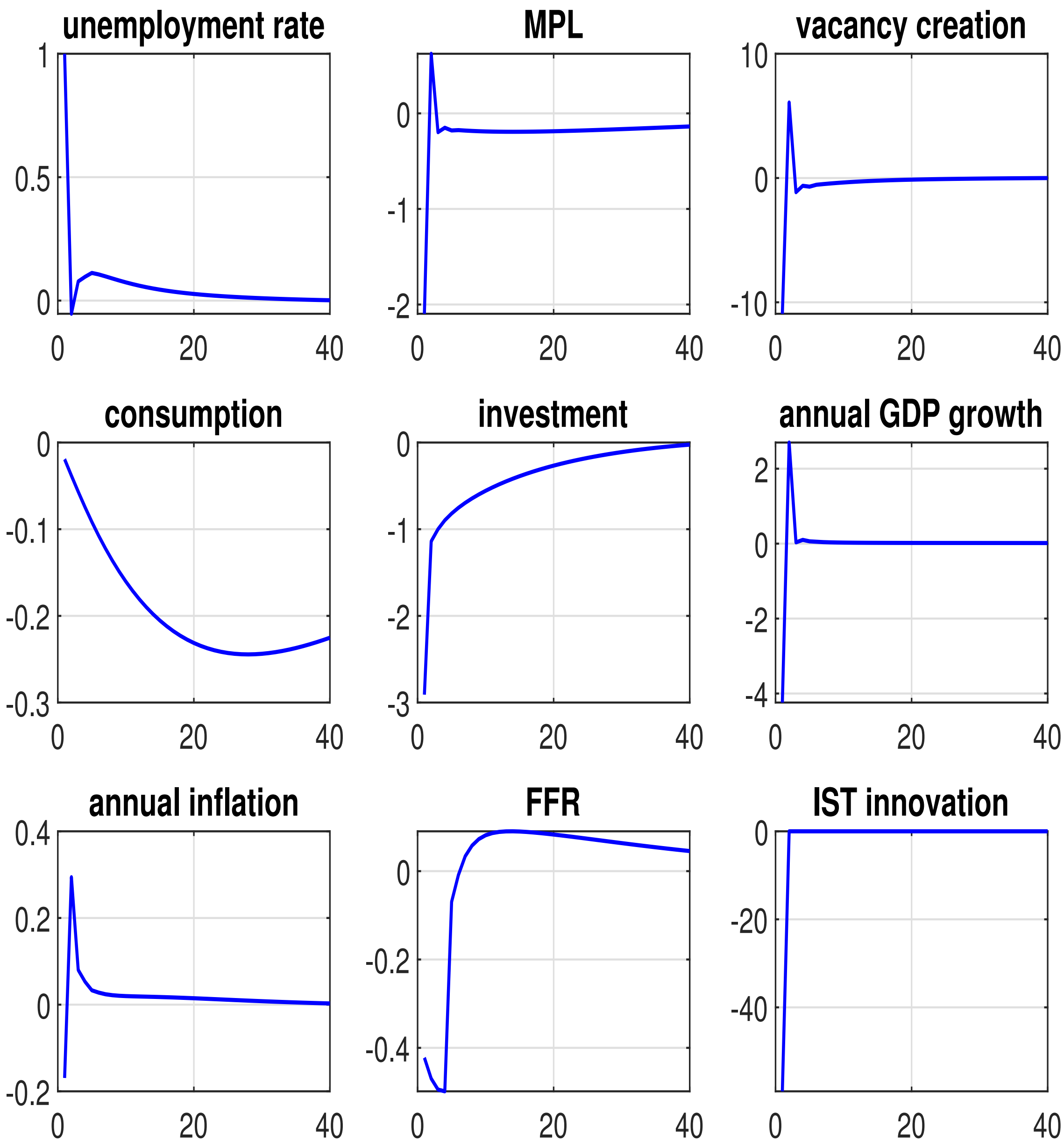


Figure A.3: Responses to the COVID investment-specific technology shock

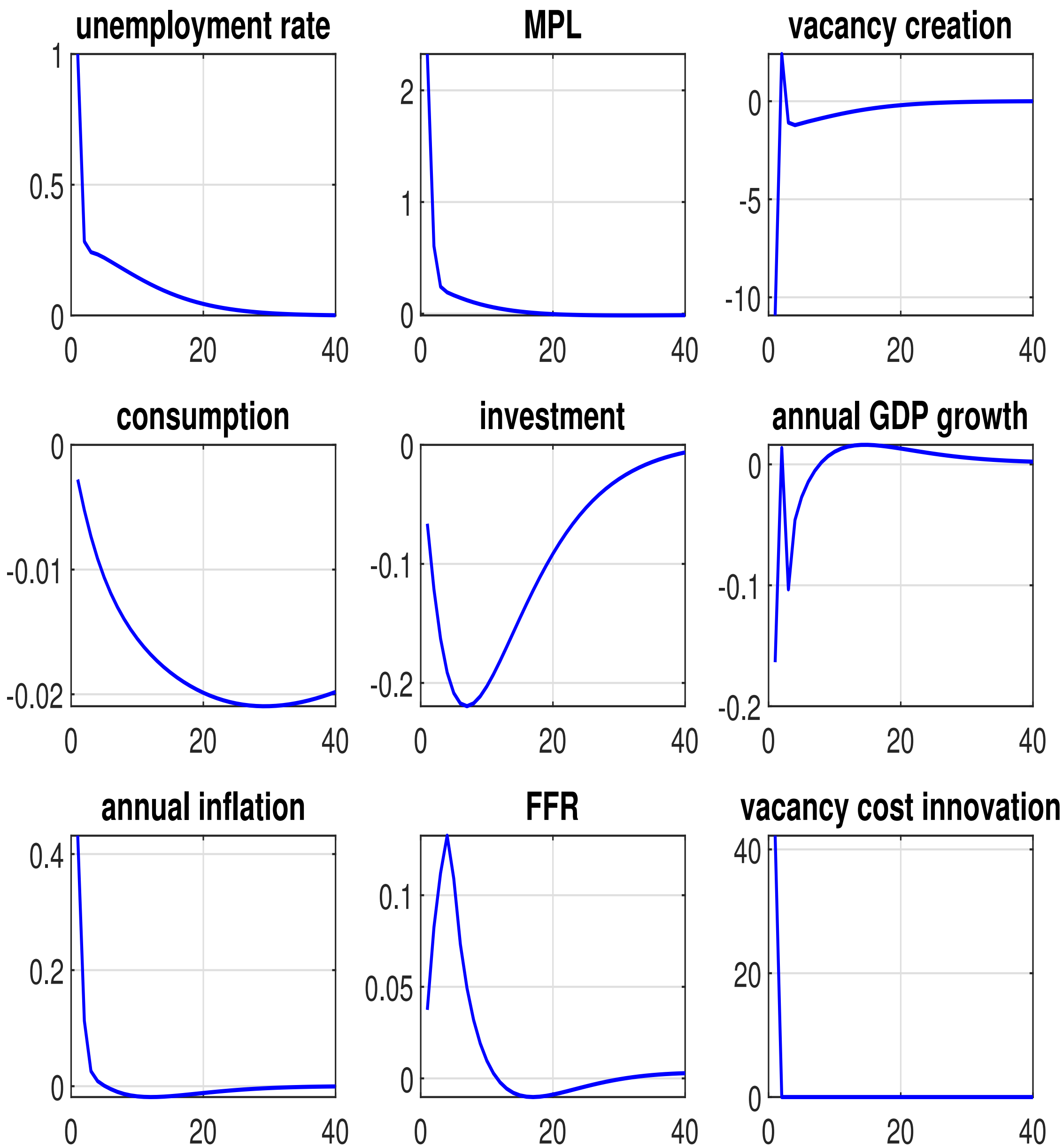


Figure A.4: Responses to the COVID-19 vacancy posting cost shock

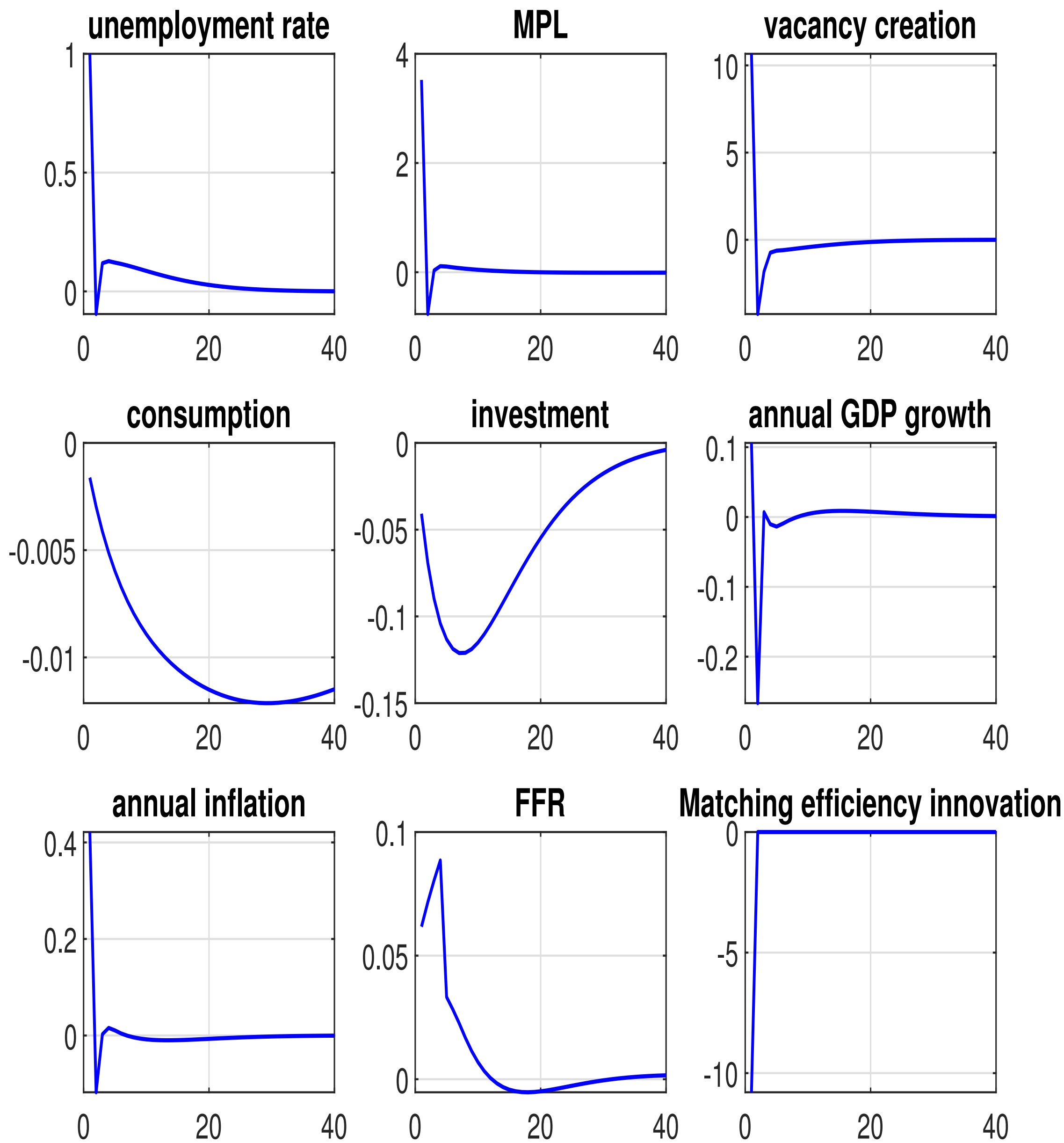


Figure A.5: Responses to the COVID-19 matching efficiency shock

Technical Appendix: PRISM-II Documentation

Jonas Arias Thorsten Drautzburg Shigeru Fujita Keith Sill

March 1, 2019

1 Introduction

This document describes the second-generation DSGE model (PRISM-II) that is developed and maintained by the Real Time Data Research Center (RTDRC) and by the Research Department of the Federal Reserve Bank of Philadelphia. PRISM-II is a medium-scale DSGE model—inspired by Gertler et al. (2008)—that features the various nominal and real frictions that were present in the first-generation PRISM, but that in addition explicitly incorporates a role for unemployment arising from labor market search frictions. This document lays out the model and explains the estimation procedure.

2 Model

The economy consists of an intermediate goods sector, a representative household, a retail sector, and a government.

2.1 Intermediate Goods Sector

The production technology of each of the firms in the intermediate goods sector is assumed to take the Cobb-Douglas form:

$$Y_t = K_t^{\alpha} Z_t^{1-\alpha} (h_t n_t)^{1-\alpha}, \tag{1}$$

where Y_t is the intermediate good, K_t is the current-period effective units of physical capital, Z_t is total factor productivity (TFP), n_t is employment, and h_t represents hours of work per worker. The TFP series obeys:

$$\ln Z_t - \ln Z_{t-1} = (1-\rho_z)\ln \gamma_z + \rho_z(\ln Z_{t-1} - \ln Z_{t-2}) + \varepsilon_{z,t},$$

where $\ln \gamma_z$ is the unconditional mean of the stochastic process $z_t = \ln Z_t - \ln Z_{t-1}$. The objective of each firm is to maximize the present discounted value of the stream of profits, $\Pi(.)$, written as:

$$\Pi(n_{t-1}, W_t; Z_t) = \max_{n_t, h_t, v_t, K_t} p_t^w Y_t - W_t h_t n_t - \frac{c_t^v v_t^{1+\epsilon^v}}{1+\epsilon^v} - r_t^k K_t + \mathbb{E}_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \Pi(n_t, W_{t+1}; Z_{t+1}),$$

where p_t^w is the price of the intermediate good, W_t is real wage per hour, $\frac{c_t^v v_t^{1+\epsilon^v}}{1+\epsilon^v}$ represents hiring costs as a function of the number of job openings v_t; c_t^v is a scale parameter of the hiring cost function and equals $c^v Z_t$, ϵ^v is its elasticity parameter, r_t^k is the rental rate of capital, β is the discount factor, and Λ_t is marginal utility of the representative household's consumption. The real wage W_t is a state variable due to the dependence on its past, as discussed below. This optimization problem is subject to the following law of motion for employment:

$$n_t = n_{t-1} - sn_{t-1} + v_t q(\theta_t), \tag{2}$$

where s is a constant separation rate and $q(\theta_t)$ is the job filling rate. The first-order conditions (FOCs) to the problem are:

$$r_t^k = \alpha \frac{p_t^w Y_t}{K_t}, \tag{3}$$

$$\frac{c_t^v}{q_t} v_t^{\epsilon^v} = (1-\alpha)\frac{p_t^w Y_t}{n_t} - W_t h_t + \mathbb{E}_t \beta \frac{\Lambda_{t+1}}{\Lambda_t}(1-s)\frac{c_{t+1}^v}{q_{t+1}} v_{t+1}^{\epsilon^v}, \tag{4}$$

$$W_t = (1-\alpha)\frac{p_t^w Y_t}{h_t n_t}. \tag{5}$$

Equation (3) is the FOC for the demand of capital, Equation (4) is the job creation (labor demand) condition, and Equation (5) characterizes the firm's demand for hours from each worker.

2.2 Labor Flows and Stocks

The search friction is represented by the following aggregate matching function:

$$m_t \tilde{u}_t^{\phi} v_t^{1-\phi},$$

where m_t denotes the time-varying matching efficiency and $\tilde{u}_t$ is the number of job seekers in the current period, which is written as:

$$\tilde{u}_t = 1 - n_{t-1} + sn_{t-1}. \tag{6}$$

Equation (6) assumes that workers who lost their job at the beginning of period t enter the matching market in the same period. We separately define the unemployment rate u_t as:

$$u_t = 1 - n_t. \tag{7}$$

Given the above matching function, the job filling rate $q(\theta_t)$ is written as:

$$q(\theta_t) = \frac{m_t \tilde{u}_t^{\phi} v_t^{1-\phi}}{v_t} = m_t \left(\frac{v_t}{\tilde{u}_t}\right)^{-\phi} = m\theta_t^{-\phi}. \tag{8}$$

Note that θ_t is the ratio between the number of job openings and the number of job seekers, and hence it represents the labor market tightness. Similarly, the job finding rate is written as:

$$f(\theta_t) = \frac{m_t \tilde{u}_t^{\phi} v_t^{1-\phi}}{\tilde{u}_t} = m_t \theta_t^{1-\phi}. \tag{9}$$

From the household's point of view, the stock of employment evolves according to:

$$n_t = (1-s)n_{t-1} + [1-(1-s)n_{t-1}]f(\theta_t). \tag{10}$$

The matching efficiency series obeys:

$$\ln m_t = (1-\rho_m)\ln \bar{m} + \rho_m \ln m_{t-1} + \varepsilon_{t,m}. \tag{11}$$

Time-varying matching efficiency is useful to explicitly allow for unemployment fluctuations that cannot be accounted for by other shocks. Furlanetto and Groshenny (2016) also introduce the matching efficiency shock to the model similar to ours and argue that it plays an important role in explaining labor market fluctuations.

2.3 Household

It is assumed that members of the representative household pool their incomes from all sources, thus allowing each member to be insured against unemployment risk. The household value function is written as follows:

$$\begin{aligned} V(C_{t-1}, K^p_{t-1}, H_{t-1}, I_{t-1}, \chi_t, \zeta_t) &= \max_{C_t, K^p_t, h_t, H_t, I_t, \nu_t} \chi_t \left[\ln(C_t - lC_{t-1}) - \bar{h}\frac{h_t^{1+\upsilon}}{1+\upsilon} n_t \right] \\ &+ \beta \mathbb{E}_t V(C_t, K^p_t, H_t, I_t, \chi_{t+1}, \zeta_{t+1}). \end{aligned} \tag{12}$$

This optimization problem is subject to the following constraints:

$$C_t + I_t + \frac{H_t}{r_t P_t} = W_t h_t n_t + (1-n_t)B_t + r^k_t \nu_t K^p_{t-1} + D_t + T_t - \mathcal{A}(\nu_t)K^p_{t-1} + \frac{H_{t-1}}{P_t} \tag{13}$$

$$K^p_t = (1-\delta)K^p_{t-1} + \zeta_t \left[1 - \mathcal{S}\Big(\frac{I_t}{I_{t-1}}\Big)\right] I_t, \tag{14}$$

$$K_t = \nu_t K^p_{t-1}, \tag{15}$$

$$\ln \chi_t = \rho_\chi \ln \chi_{t-1} + \varepsilon_{t,\chi}, \tag{16}$$

$$\ln \zeta_t = \rho_\zeta \ln \zeta_{t-1} + \varepsilon_{t,\zeta}, \tag{17}$$

where C_t is consumption, K^p_t is physical capital, H_t is nominal bond holdings, I_t is gross investment, ν_t is the utilization rate of the capital stock, χ_t is the intertemporal preference shock, ζ_t is the investment specific technology shock, l is a habit parameter, $\bar{h}$ is a scale parameter for the disutility of hours worked, and $1/\upsilon$ is the Frisch (intensive-margin) elasticity of labor supply, B_t is a flow value of unemployment (UI benefits), r^k_t is the rental rate of capital, P_t is the price level of the final good, r_t is the gross nominal interest rate, D_t is dividends paid by the retail sector, T_t is the lump sum transfers from the government, $\mathcal{A}(.)$ represents the cost of capital utilization, $\mathcal{S}(.)$ is the adjustment cost function for investment. It is assumed that $B_t = bZ_t$. We choose $\mathcal{A}$ such that the utilization rate ν_t is normalized to one along the balanced growth path and has no resource costs, i.e., we set $\mathcal{A}(1) = 0$, $\mathcal{A}(1) = \bar{r}_k$. We denote the elasticity by $\xi_A \equiv \mathcal{A}'(1)/\mathcal{A}''(1)$. Note also that $\mathcal{S}(\gamma_z) = \mathcal{S}'(\gamma_z) = 0$ and that $\mathcal{S}''(\gamma_z) = \xi_S$.

The first-order conditions of this problem are:

$$\Lambda_t = \chi_t \frac{1}{C_t - lC_{t-1}} - \beta \mathbb{E}_t \chi_{t+1} \frac{l}{C_{t+1} - lC_t}, \tag{18}$$

$$\Lambda_t = r_t \beta \mathbb{E}_t \Big(\frac{\Lambda_{t+1} P_t}{P_{t+1}}\Big), \tag{19}$$

$$\bar{h}\chi_t h_t^\upsilon = \Lambda_t W_t \tag{20}$$

$$\mathcal{A}'(\nu_t) = r^k_t, \tag{21}$$

$$\omega_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[(1-\delta)\omega_{t+1} + r^k_{t+1}\nu_{t+1} - \mathcal{A}(\mu_{t+1})\right], \tag{22}$$

$$\omega_t \zeta_t \left[1 - S\Big(\frac{I_t}{I_{t-1}}\Big)\right] = \omega_t \zeta_t \frac{I_t}{I_{t-1}} \mathcal{S}'\Big(\frac{I_t}{I_{t-1}}\Big) + 1 - \beta \mathbb{E}_t \omega_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \zeta_{t+1} \mathcal{S}'\Big(\frac{I_{t+1}}{I_t}\Big)\Big(\frac{I_{t+1}}{I_t}\Big)^2, \tag{23}$$

where ω_t represents Tobin's Q.

2.4 Wages

To determine the wage, first define earnings $\overline{W}_t$ as:

$$\overline{W}_t = h_t W_t. \tag{24}$$

We assume that the worker and the firm bargain over $\overline{W}_t$. To derive the expression for $\overline{W}_t$, we write the values of employment (N_t), unemployment (U_t), and a filled job (J_t) as follows:

$$\begin{aligned}
N_t &= \overline{W}_t - \frac{\bar{h}\chi_t}{\Lambda_t}\frac{h_t^{1+\upsilon}}{1+\upsilon} + \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\left[(1-s+sf(\theta_{t+1}))N_{t+1} + s(1-f(\theta_{t+1}))U_{t+1}\right],\\
U_t &= B_t + \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\left[f(\theta_{t+1})N_{t+1} + (1-f(\theta_{t+1}))U_{t+1}\right],\\
J_t &= (1-\alpha)\frac{p_t^w Y_t}{n_t} - \overline{W}_t + (1-s)\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}J_{t+1}.
\end{aligned}$$

The interpretation is straightforward. If employed this period, the worker receives $\overline{W}_t$, and in the following period, she obtains the value N_{t+1} if either she did not lose the job with probability $1-s$, or finds a job within the same period after separation, which occurs with probability $sf(\theta_{t+1})$. In the third equation, the first two terms correspond to the firm's flow profits and the next term captures the future value after imposing the free entry condition.

Following Hall (2005), we allow for equilibrium wage (earnings) rigidity of the following form:

$$\overline{W}_t = \rho^w z_t \overline{W}_{t-1} + (1-\rho^w)\overline{W}_t^f, \tag{25}$$

where $\overline{W}_t^f$ is (hypothetical) period-by-period flexible Nash bargained wage (i.e., "reference" wage); ρ^w measures the degree of its rigidity. We can obtain the flexible Nash bargained wage payment $\overline{W}_t^f$ by using the surplus sharing rule:

$$\eta J_t = (1-\eta)(N_t - U_t),$$

where η is the bargaining power of the worker. Using the three value functions above in this equation, one can get:

$$\overline{W}_t^f = \eta(1-\alpha)\frac{p_t^w Y_t}{n_t} + (1-\eta)\left[\frac{\bar{h}\chi_t}{\Lambda_t}\frac{h_t^{1+\upsilon}}{1+\upsilon} + B_t\right] + \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\left[\eta(1-s)c_t^{\upsilon}\theta_{t+1}v_{t+1}^{\epsilon^{\upsilon}}\right]. \tag{26}$$

Note that Equation (25) implies the following indexation of nominal earnings:

$$P_t\overline{W}_t = \rho^w \pi_t z_t P_{t-1}\overline{W}_{t-1} + (1-\rho^w)P_t\overline{W}_t^f. \tag{27}$$

2.5 Hours Per Worker

From Equations (5) and (20), we have the following equilibrium condition for hours per worker.

$$(1-\alpha)\frac{p_t^w Y_t}{n_t} = \frac{\bar{h}\chi_t h_t^{1+\upsilon}}{\Lambda_t}. \tag{28}$$

As described in the previous section, earnings $\overline{W}_t$ are determined through bargaining, while Equation (28) determines hours per worker. The implied hourly wage rate is then determined by Equation (24).

2.6 Retail Sector

There is a continuum of monopolistically competitive retailers indexed by j on the unit interval. Retailers buy the intermediate goods at price p_t^w, differentiate them with a technology that transforms them into consumption goods, and then sell them to the household. Each retailer faces the following demand function:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\epsilon_t} Y_t, \tag{29}$$

where ϵ_t is the elasticity of substitution, which is related to the markup μ_t as follows:

$$\epsilon_t = \frac{1+\mu_t}{\mu_t}. \tag{30}$$

The variable μ_t evolves according to:

$$\ln \mu_t = (1-\rho_\mu)\ln \bar{\mu} + \rho_\mu \ln \mu_{t-1} + \varepsilon_{t,\mu}. \tag{31}$$

The firm sets its price subject to a quadratic price adjustment cost, maximizing the following expression:

$$\Pi_{jt}^R(P_{jt-1}) = \max_{P_{jt}} \frac{P_{jt}}{P_t} Y_{jt} - p_t^w Y_{jt} - \frac{\tau}{2}\left(\frac{P_{jt}}{\pi_{t-1}^{\psi}(\pi^*)^{1-\psi} P_{jt-1}} - 1\right)^2 Y_t + \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \Pi_{jt}^R(P_{jt}),$$

where $\pi_t = \frac{P_t}{P_{t-1}}$; π^* is central bank's target inflation rate; ψ is a degree of backwardness. The first-order condition under the symmetric equilibrium is:

$$1-\epsilon_t-\tau\pi_t\left(\frac{\pi_t}{\pi_{t-1}^{\psi}(\pi^*)^{1-\psi}}-1\right)\frac{1}{\pi_{t-1}^{\psi}(\pi^*)^{1-\psi}}+p_t^w\epsilon_t+\mathbb{E}_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}\tau\left(\frac{\pi_{t+1}}{\pi_t^{\psi}(\pi^*)^{1-\psi}}-1\right)\frac{Y_{t+1}}{Y_t}\frac{\pi_{t+1}}{\pi_t^{\psi}(\pi^*)^{1-\psi}}=0. \tag{32}$$

2.7 Government

The central bank sets the nominal interest rate as follows:

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho_r}\left[\left(\prod_{j=0}^{3}\frac{\pi_{t-j}}{\pi^*}\right)^{r_\pi}\left(\frac{Y_t}{Y_{t-4}}\frac{1}{\gamma_z^4}\right)^{r_{gy}}\right]^{1-\rho_r}\kappa_t, \tag{33}$$

where r is the steady-state nominal interest rate, ρ_r is the degree of monetary policy inertia embedded in the monetary policy equation, r_π is the response of the nominal interest rate to deviations of inflation from the inflation target (π^*), r_{gy} is the response of the nominal interest rate to deviations of output growth from the growth rate of the economy at the steady-state (γ_z), and κ_t is an exogenous monetary policy shock. The monetary policy shock is assumed to follow:

$$\ln \kappa_t = \rho_\kappa \ln \kappa_{t-1} + \varepsilon_{t,\kappa}. \tag{34}$$

The government expenditures G_t obeys:

$$G_t = \left(1-\frac{1}{x_t}\right)Y_t, \tag{35}$$

where x_t varies according to:

$$\ln x_t = (1-\rho_x)\ln \bar{x} + \rho_x \ln x_{t-1} + \varepsilon_{t,x}. \tag{36}$$

2.8 Resource Constraint

The following resource constraint closes the model.

$$Y_t = C_t + I_t + \frac{c_t^v {v_t}^{1+\epsilon^v}}{1+\epsilon^v} + A(\nu_t)K_{t-1}^p + \frac{\tau}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\psi}(\pi^*)^{1-\psi}} - 1\right)^2 Y_t. \tag{37}$$

3 Detrended Model

The model is rendered stationary by detrending the level equations above by TFP, Z_t. The lower case letters represent stationary variables.

3.1 Intermediate Goods Sector

- Production function:

$$y_t = k_t^{\alpha}(h_t n_t)^{1-\alpha}. \tag{38}$$

- TFP:

$$\ln z_t = (1-\rho_z)\ln\gamma_z + \rho_z \ln z_{t-1} + \varepsilon_{z,t}, \tag{39}$$

  where

$$z_t = \frac{Z_t}{Z_{t-1}}.$$

- Demand for capital:

$$r_t^k = \alpha p_t^w \frac{y_t}{k_t}. \tag{40}$$

- Job creation condition:

$$\frac{c^v}{q_t}v_t^{\epsilon^v} = (1-\alpha)\frac{p_t^w y_t}{n_t} - \bar{w}_t + (1-s)\mathbb{E}_t\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{c^v}{q_{t+1}}v_{t+1}^{\epsilon^v}, \tag{41}$$

  where $\lambda_t = \Lambda_t Z_t$.

3.2 Labor Market Flows

The equations here are mostly the same as in the previous section, but are listed below for completeness.

- Job filling rate:

$$q_t = m_t\theta_t^{-\phi}. \tag{42}$$

- Job finding rate:

$$f_t = m_t\theta_t^{1-\phi}. \tag{43}$$

- Employment evolution:

$$n_t = (1-s)n_{t-1} + [1-(1-s)n_{t-1}]f(\theta_t). \tag{44}$$

- The number of job seekers:

$$\tilde{u}_t = 1 - n_{t-1} + s n_{t-1}. \tag{45}$$

- The unemployment rate:

$$u_t = 1 - n_t. \tag{46}$$

- Matching efficiency:

$$\ln m_t = (1-\rho_m)\ln \bar{m} + \rho_m \ln m_{t-1} + \varepsilon_{t,m}. \tag{47}$$

3.3 Household

- Effective capital services:

$$k_t = \frac{\nu_t}{z_t} k^p_{t-1}. \tag{48}$$

- Evolution of physical capital:

$$k^p_t = (1-\delta)\frac{1}{z_t}k^p_{t-1} + \zeta_t\left[1 - S\left(z_t \frac{i_t}{i_{t-1}}\right)\right] i_t. \tag{49}$$

- Capital utilization:

$$\mathcal{A}'(\nu_t) = r^k_t. \tag{50}$$

- Tobin's Q:

$$\omega_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t z_{t+1}}\left[(1-\delta)\omega_{t+1} + r^k_{t+1}\nu_{t+1} - \mathcal{A}(\nu_{t+1})\right]. \tag{51}$$

- Investment:

$$\omega_t\zeta_t\left[1-\mathcal{S}\left(z_t\frac{i_t}{i_{t-1}}\right)\right] = \omega_t\zeta_t z_z \frac{i_t}{i_{t-1}}\mathcal{S}'\left(z_t\frac{i_t}{i_{t-1}}\right) + 1 - \beta\mathbb{E}_t\omega_{t+1}\frac{\lambda_{t+1}}{\lambda_t z_{t+1}}\zeta_{t+1}\mathcal{S}'\left(z_{t+1}\frac{i_{t+1}}{i_t}\right)\left(z_{t+1}\frac{i_{t+1}}{i_t}\right)^2. \tag{52}$$

- Consumption:

$$\lambda_t = \frac{\chi_t z_t}{c_t - lc_{t-1}} - \beta h \mathbb{E}_t \frac{\chi_{t+1}}{c_{t+1}z_{t+1} - lc_t}. \tag{53}$$

- Euler equation:

$$1 = r_t \beta \mathbb{E}_t\left(\frac{1}{z_{t+1}}\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\pi_{t+1}}\right). \tag{54}$$

- Preference shock:

$$\ln \chi_t = \rho_\chi \ln \chi_{t-1} + \epsilon_{t,\chi}.. \tag{55}$$

- Investment specific technology shock:

$$\ln \zeta_t = \rho_\zeta \ln \zeta_{t-1} + \epsilon_{t,\zeta}. \tag{56}$$

3.4 Wages

- Earnings

$$\overline{w}_t = h_t w_t. \tag{57}$$

- Nash bargained earnings:

$$\overline{w}_t^f = \eta(1-\alpha)p_t^w \frac{y_t}{n_t} + (1-\eta)\left[b + \frac{\bar{h}\chi_t}{\lambda_t}\frac{h_t^{1+\upsilon}}{1+\upsilon}\right] + \beta(1-s)\eta\mathbb{E}_t\frac{\lambda_{t+1}}{\lambda_t}c^v\theta_{t+1}v_{t+1}^{\epsilon^v}. \tag{58}$$

- Actual earnings:

$$\overline{w}_t = \rho^w \overline{w}_{t-1} + (1-\rho^w)\overline{w}_t^f. \tag{59}$$

3.5 Hours Per Worker

- Hours per worker

$$(1-\alpha)\frac{p_t^w y_t}{n_t} = \frac{\bar{h}\chi_t h_t^{1+\upsilon}}{\lambda_t}. \tag{60}$$

3.6 Retail Sector

- Inflation:

$$1 - \epsilon_t - \tau\pi_t\left(\frac{\pi_t}{\pi_{t-1}^{\psi}(\pi^*)^{1-\psi}} - 1\right)\frac{1}{\pi_{t-1}^{\psi}(\pi^*)^{1-\psi}} + p_t^w\epsilon_t + \mathbb{E}_t\beta\frac{\lambda_{t+1}}{\lambda_t}\tau\left(\frac{\pi_{t+1}}{\pi_{t-1}^{\psi}(\pi^*)^{1-\psi}} - 1\right)\frac{y_{t+1}}{y_t}\frac{\pi_{t+1}}{\pi_{t-1}^{\psi}(\pi^*)^{1-\psi}} = 0. \tag{61}$$

- Elasticity of substitution:

$$\epsilon_t = \frac{1+\mu_t}{\mu_t}. \tag{62}$$

- Markup:

$$\ln\mu_t = (1-\rho_\mu)\ln\bar{\mu} + \rho_\mu\ln\mu_{t-1} + \varepsilon_{t,\mu}. \tag{63}$$

3.7 Government and Resource Constraint

- Monetary policy:

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho_r}\left[\left(\prod_{j=0}^{3}\frac{\pi_{t-j}}{\pi^*}\right)^{r_\pi}\left(\frac{y_t}{y_{t-4}}\prod_{j=0}^{3}\frac{z_{t-j}}{\gamma_z}\right)^{r_{gy}}\right]^{1-\rho_r}\kappa_t. \tag{64}$$

- Monetary policy shock:

$$\ln\kappa_t = \rho_\kappa\ln\kappa_{t-1} + \varepsilon_{t,\kappa}. \tag{65}$$

- Government expenditures:

$$g_t = \left(1 - \frac{1}{x_t}\right)y_t. \tag{66}$$

- The government expenditure shock:

$$\ln x_t = (1-\rho_x)\ln x + \rho_x \ln x_{t-1} + \varepsilon_{t,x}. \tag{67}$$

- The resource constraint:

$$y_t = c_t + i_t + \frac{c^v v_t^{1+\epsilon^v}}{1+\epsilon^v} + \mathcal{A}(\nu_t)\frac{k^p_{t-1}}{z_t} + \frac{\tau}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\psi}(\pi^*)^{1-\psi}} - 1\right)^2 y_t. \tag{68}$$

4 Empirical Application

We estimate the log-linearized version of the model described using standard Bayesian method implemented in Adjemian et al. (2011). For the current model, the sample period starts in 1971Q3. We re-estimate the model every quarter as we receive more data. The sample period for the results below ends at the third quarter of 2018.

4.1 Calibrated Parameters

Some parameters are calibrated prior to the estimation either directly or through steady-state restrictions. Table 1 summarizes these parameters. The capital share parameter α and the depreciation rate of the physical capital δ are set to 0.33 and 0.025, respectively, both of which are standard in macro. The value of the discount factor β is selected to be 0.9996. This pins down the nominal interest rate, given inflation expectations and growth along the balanced growth path. The economy is assumed to grow 0.4 percent per quarter along the balanced growth path, and thus $\gamma_z = 1.004$.

The quarterly employment separation probability s is set to 0.195. In the model, those workers that separate at the beginning of the period may find a job within the same period, which occurs with probability f_t. The steady-state value of f_t is targeted to 0.75 and thus the probability that an employed worker at the beginning of the period ends up in the unemployment pool at the end of the period is 0.0488. Note also that $s = 0.195$ and $f = 0.75$ imply the unemployment rate equals 6.1 percent at the steady state. The scale parameter of the matching function is set to 0.75 because the steady-state value of labor market tightness θ is normalized to 1, which implies $\bar{m} = f$. The elasticity of the matching function with respect to $\tilde{u}_t$ is set to 0.5. The hiring cost function is assumed to be quadratic (thus $\epsilon^v = 1$) as in Gertler et al. (2008). The level of unemployment benefits b is set to 0.2145. This value is computed by imposing the restriction that the worker's flow outside value including the value of not-working, measured in terms of the consumption good amounts to 71 percent of the steady-state earnings level (see the expression in the square bracket in (58)). This value has often been used in the literature (e.g., Hall and Milgrom (2008)). The inverse of the elasticity of intensive-margin labor supply is fixed at 2. The labor-supply elasticity of 0.5 is in line with the evidence in micro-econometric studies.

The steady-state price markup ($\bar{\mu}$) is set to 0.2. The steady-state level of the exogenous government expenditure process $\bar{x}$ is set to 1.25, which implies the share of government expenditures in output being 19.3 percent. We fix the target inflation rate at 2 percent so that $\pi^* = 1.02^{\frac{1}{4}}$.

There are two parameters c^v and $\bar{h}$ that are endogenously determined after the estimation is completed; we discuss these parameters here because they are not directly estimated. The scale parameter of the hiring cost function c^v is selected so that the job creation condition holds, given all the parameters and the targeted steady-state job filling rate at 0.75. Similarly, the scale parameter of the labor supply function is chosen such that hours of work equal 1/3 at the steady state.

Parameter	Description	Value
α	Capital share	0.33
β	Discount factor	0.9996
δ	Depreciation rate	0.025
$\bar{m}$	Scale parameter of matching function	0.75
ϕ	Elasticity of matching function	0.5
s	Separation probability	0.195
b	UI benefits	0.2145
ϵ^v	Curvature of hiring cost	1
ν	Inverse of elasticity of labor supply	2
γ_z	Steady state TFP growth	1.004
$\bar{x}$	Steady-state level of government expenditures	1.25
$\bar{\mu}$	Steady-state level of markup	0.2
π^*	Target inflation rate	$1.02^{\frac{1}{4}}$

Table 1: Calibrated Parameters

4.2 Data

We use the following macroeconomic series to estimate the remaining parameters. Real output in the model corresponds to NIPA real GDP. We compute real GDP by dividing the nominal GDP series by the chained-price GDP deflator. It is converted into per capita real GDP by dividing it by population 16 years or older. Consumption in the model corresponds to total personal consumption expenditures less durable-goods consumption in the data. Investment is defined as gross private domestic investment plus durable-goods consumption. We take nominal consumption and investment series and divide both series by the chained-price GDP deflator and 16+ population to obtain real per-capita series. We use a geometrically smoothed version of the population series to deal with small discontinuities. We compute quarter-to-quarter growth rates as log difference of real per capita variables and multiply the growth rates by 100 to convert them into percentages.

For labor market variables, we use the unemployment rate, the vacancy rate, and real earnings per worker in the estimation. Specifically, the logged quarterly series of the unemployment rate, taken from the Current Population Survey, is with the CBO estimate of the natural rate of unemployment. This series is linked with log-deviations of u_t from its steady-state level. We detrend the unemployment rate because it exhibits low frequency movements due to factors, such as demographic changes, that our model does not explicitly model. For the vacancy rate, we use the total number of job openings from the JOLTS (Job Opening and Labor Turnover Survey). Since this series is available only from December 2000 onwards, we splice it with the Conference Board's help-wanted index series and extend the vacancy series backwards. We multiply the level of the latter series by a constant factor. The multiplicative factor is computed such that the average levels of the two series match up over the overlapping sample period (between December 2000 and December 2014). The total number of job openings is normalized by the labor force. Its quarterly average series is logged and HP filtered with the smoothing parameter set at 10^5. Similar to the unemployment rate, the vacancy rate series exhibits a low frequency trend that our model is not designed to capture. We remove this slow moving trend via the HP filter. The detrended series is equated with log deviations of v_t from its steady-state level. We compute quarter-over-quarter growth rates of real earnings per worker, using the data available through the Productivity and Cost Program of the BLS. We first obtain the real hourly earnings index, the aggregate hours index, and the aggregate employment index. Quarter-over-quarter log differences in these three indexes allow us to compute quarter-over-quarter log differences in real earnings per worker. We assume that this series is measured with some i.i.d. error and estimate the standard deviation (σ_{mew}) of the measurement error.

The effective Fed funds rate is used as the measure of the monetary policy rate. In quarters when the funds rate was constrained by the effective lower bound (ELB), we treat the funds rate as missing. Further, assuming that the expectation hypothesis of the term structure holds, we include the two-year treasury rate as a noisy measure of the expected funds rate over the next two years. We calibrate the noise to lie within a few basis points of the value implied by the expectation hypothesis, after taking out the average term premium. This measure of expected interest rates over the next two years ensures that the estimation is informed by variations in monetary policy expectations over the next two years even during the ELB period when the observations for the funds rate are missing.

Lastly, we use core-PCE inflation as the observable measure of inflation. We detrend the inflation rate by a measure of long-term PCE inflation expectations. Although trend inflation is constant at 2 percent in the model, trend inflation is likely to be time varying over longer sample periods and we capture this trend via long-term PCE expectations. For the period after 2007Q1, we use long-term PCE inflation expectations available through the SPF (Survey of Professional Forecasters). For the period between 1991Q4 and 2006Q4, we use CPI inflation expectations available also in the SPF. For the overlapping sample period, CPI inflation expectations are 20 basis points higher than PCE inflation expectations. We splice the two series after subtracting 20 basis points from the CPI inflation expectations for 1991Q4 to 2006Q4. Prior to 1994Q4, we use other sources to compute the long-term CPI inflation expectations. From 1979Q4 to 1991Q3, we use inflation expectations available from the Livingston and Blue Chip surveys (all available from the Federal Reserve Bank of Philadelphia). Whenever available, we use the Livingston survey and otherwise use the Blue Chip survey. If neither is available, we linearly interpolate between the combined surveys. Before 1979Q3, we use the historical break-even rates for inflation expectations computed by the Federal Reserve Bank of New York. In our estimation, the detrended core-PCE inflation rate is linked to the deviation of the inflation rate from its steady-state value (2 percent) in the model.

5 Estimated Parameters

The estimation results are presented in Tables 2 and 3. Our choice of prior distributions is standard. Posterior means are also roughly in line with the existing literature. The model introduces real wage rigidity, and the parameter ρ^w is indeed estimated to be fairly high at 0.88. The estimation results for exogenous processes are also roughly in line with the existing literature. The estimated parameter values for the matching efficiency process are similar to those estimated by Furlanetto and Groshenny (2016), although their model is different from ours and they use different observables to estimate the shock process. Another notable result is that in our estimation, the markup shock is estimated to be highly persistent and quite volatile. We find that this shock contributes significantly to overall variations of the model.

		Prior			Posterior
Parameter	Density	Mean	Std	Mean	90% Intv.
τ	Gamma	50.00	10.00	80.07	[65.18 , 97.59]
ψ	Beta	0.50	0.20	0.06	[0.01 , 0.12]
ℓ	Beta	0.50	0.20	0.95	[0.94 , 0.97]
ρ_w	Beta	0.50	0.10	0.88	[0.83 , 0.94]
κ	Gamma	2.00	2.00	12.41	[6.53 , 19.29]
η	Beta	0.50	0.20	0.73	[0.61 , 0.85]
r_π	Normal	1.50	0.25	2.62	[2.35 , 2.88]
r_{gy}	Normal	0.40	0.30	0.53	[0.44 , 0.62]
r_ρ	Beta	0.50	0.20	0.85	[0.83 , 0.87]

Table 2: Estimated Structural Parameters

		Prior			Posterior
Parameter	Distribution	Mean	Std	Mean	90% Intv.
ρ_m	Beta	0.50	0.20	0.93	[0.89 , 0.97]
ρ_χ	Beta	0.50	0.20	0.38	[0.28 , 0.48]
ρ_ζ	Beta	0.50	0.20	0.81	[0.77 , 0.85]
ρ_μ	Beta	0.50	0.20	0.98	[0.96 , 1.00]
ρ_x	Beta	0.50	0.20	0.99	[0.99 , 1.00]
ρ_z	Beta	0.50	0.20	0.44	[0.34 , 0.54]
σ_z	Inverse Gamma	0.01	2.00	0.0054	[0.0047 , 0.0061]
σ_m	Inverse Gamma	0.01	2.00	0.0220	[0.0202 , 0.0239]
σ_χ	Inverse Gamma	0.01	2.00	0.0880	[0.0607 , 0.1148]
σ_ζ	Inverse Gamma	0.01	2.00	0.1112	[0.0684 , 0.1535]
σ_κ	Inverse Gamma	0.01	2.00	0.0027	[0.0025 , 0.0030]
σ_x	Inverse Gamma	0.01	2.00	0.0060	[0.0055 , 0.0065]
σ_μ	Inverse Gamma	0.01	2.00	0.0615	[0.0512 , 0.0716]
σ_{mew}	Inverse Gamma	0.01	2.00	0.0092	[0.0084 , 0.0100]

Table 3: Estimated Exogenous Parameters

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Detailed Chicago Forecast Overview

September 2020

The Chicago Fed DSGE model is an estimated New Keynesian model which contains many features familiar to other DSGE analyses of monetary policy and business cycles. External habit in preferences, *i*-dot costs of adjusting investment, price and wage stickiness, and partial indexation of unadjusted prices and wages using recently observed price and wage inflation. The salient features which distinguish our analysis from many otherwise similar undertakings are: forward guidance shocks, investment-specific technological change and a mixed calibration-Bayesian estimation approach, a *synthetic* shock seizing the effects of the COVID-19 induced recession.[1]

The Chicago Fed DSGE model is used both for internal forecasts and for creating our contribution to the System DSGE memo distributed quarterly to the FOMC. The document is structured as follows. We first provide a brief summary of the forecast. In the following section, we characterize more in detail the forecasts, e.g. the conditioning assumptions and the forces that drive our projections. At the end of this document, we offer a technical guide that describes the bells and whistles of our modelling and empirical strategy.

Forecast Summary

The Chicago Fed DSGE model projects that real GDP will be at -2.3 percent in 2020. This number partly embeds our judgmental assumptions about 2020Q3 where we expect a rebound. The recovery extends to next year generating a GDP growth forecast of 4.1 percent for 2021. While monetary policy is constrained by the ELB in 2020, the expectation that the federal funds rate remains at the ELB until the end of the forecasting horizon more than offsets the contractionary effects of the ELB, leading monetary policy to positively contribute to our forecasts for GDP growth. Core PCE inflation is expected to be below the FOMC's target for the entire forecasting horizon. In particular, inflation averages from 2020 through 2023 0.8 percent, 1.8 percent, 1.5 percent and 1.6 percent respectively. We forecast the (real) natural rate of interest at the end of the year for 2020 through 2023 to equal -16.7 percent, -1.2 percent, -1.2 percent, and 0.5 percent. The model sees a widening of the output gap in the current year, with actual output being 2.5 percent below natural output by the end of 2020. The gap remains negative throughout the forecast horizon.

[1] These and other features of the model are described in the appendix at the end of this document.

Current Forecast and Shock Identification

The Chicago Fed DSGE model forecast is constructed using data through 2020Q2 supplemented by a number of assumptions based on market expectations, survey data and judgments for the third quarter of 2020. The assumption for GDP growth for 2020Q3 is 26.6 percent at an annualized rate based on Macro Advisers (MA). The forecast also incorporates assumptions for the main components of GDP growth, consumption and investment, based on MA forecasts and internal calculations. The federal fund rate is at the effective lower bound (ELB) and expected to stay there until the first semester of 2023, in line with the Survey of Market Participants. The conditioning assumptions also include 2020Q3 expected inflation, both one-quarter ahead and over the next 10 years, taken from the first quarter Survey of Professional Forecasters (SPF). Unlike previous forecast rounds, our information set has been augmented with the SPF expectations about GDP growth and inflation for the next four quarter; these expectations are crucial to identify parameters that govern the propagation of the pandemic as we discuss next.

The Chicago Fed model does not explicitly feature a pandemic outbreak and its propagation. We model COVID-19 as a synthetic disturbance whose realizations could (i) affect contemporaneously different margins and wedges of the economy (i.e. supply, demand or intertemporal decisions...) and (ii) embed news about its near term propagations.The COVID-19 shock has thus a hybrid nature and a different propagation from usual structural business cycles shocks. The identification of the parameters seizing its size and persistence is achieved through the use of expectations and narrative restrictions, i.e. by assuming that the COVID-19 shocks explains most of the variation in 2020Q2 both in the actual and expected macroeconomic aggregates. More precisely, the shock causing the COVID-19 recession is assumed to be dormant throughout our full sample, i.e. from 1993Q1 to 2020Q1. In the second quarter of 2020, the COVID-19 shock hits the US economy while the usual business cycles shocks are muted (this is achieved by reducing considerably the standard deviation of the structural business cycles shocks); its effects going forward unfold based on the SPF expectations about the likely course of the economy. Finally, we assume that this shock has a liquidity preference (demand) and a permanent neutral (supply) component. The standard deviation of the COVID-19 shock, its anticipation structure and the relative loadings on demand and supply shocks are chosen to maximize the likelihood function over the second quarters of 2020.

The defining features of this synthetic COVID-19 shock and its anticipation structure are pinned down by recent data, including the SPF one-, two-, three-, and four-quarters-ahead forecasts of GDP growth and inflation. The standard deviation of the COVID-19 shock, its anticipation structure, are chosen in quarters 2020Q2 and 2020Q3 so as to maximize the likelihood function in each of these quarters. The relative loadings on demand and supply shocks, which define the COVID-19 shock, are analogously estimated, but for these parameters only 2020Q2 data are used. As the model's projections were largely informed by the SPF forecasts, which were only marginally revised in 2020Q3, the COVID-19 shock turns out to play a relatively small role in this quarter.

Motivated by the positive probability of the virus resurgence in the fall of this year, the point forecasts presented in this memo are constructed combining two scenarios: a neutral tendency where no more COVID-19 shocks are expected in the future ("baseline scenario") and a more pessimistic view where a second wave (half of the size of the first one) will hit the US economy in 2020Q4 ("second wave scenario"). We assume that the probability of the former (latter) is 75 (25) percent. In both scenarios, monetary policy shocks are wangled in order to respect the Effective Lower Bound (ELB) until the first quarter of 2023.

Figures 1-5 report the current point (mean) forecasts for output growth, core PCE inflation, the federal funds rate the real natural rate and the output gap as well as the 68% probability coverage bands.[2] Figures 6-8 report the shocks decomposition of the forecast of output growth, core PCE inflation and the federal funds rate. The black vertical line indicates the last observation which in our case is 2020Q3; the black line with dots denotes the observed data and its forecast and the black dashed line the steady states; the red line denotes the forecast conditioning on 2015Q1 information. The difference between the black and the red line is entirely due to the shocks that materialized between 2015Q1 and 2020Q3. The colored bars decompose this difference in terms of structural shocks. All variables are expressed in quarterly values.

The estimated COVID-19 shock in 2020Q2 is extremely large and explains the deep trough in 2020Q2 as well as the strong rebound in 2020Q3. The recovery in the second half of the year however is not sufficient to bring output growth in positive territory yielding a Q4/Q4 GDP growth of -2.3 percent in 2020. It is important to highlight that the second wave scenario build in our combined forecast contributes to the negative growth number in 2020 and adds momentum in 2021. The economy rebounds in 2021 where we forecast GDP growth at 4.0 percent. While most of the economic recovery starting in the third quarter of 2020 is COVID-19 induced (i.e. the result of progressively relaxing the social distancing measures), part of it is also explained by a positive technology shock. Moreover, even if constrained by the ELB in 2020, monetary policy remains supportive of growth. In fact, the expectations that the federal funds rate will remain at the ELB until the first half of 2023 more than offset the contractionary effects of the ELB, leading monetary policy to positively contribute to the model's forecasts for GDP growth. However, the removal of this large monetary accommodation acts as a drag for the real economy in 2022 and 2023. As a result, the model forecasts GDP growth at 1.5 percent in 2022 and 1.4 percent in 2023. While monetary policy is constrained by the ELB in 2020, the expectation that

[2]The probability coverage bands are constructed simulating the model out of sample by drawing from the theoretical distribution of the shocks 50,000 times.

the federal funds rate remains at the ELB until the end of the forecasting horizon more than offsets the contractionary effects of the ELB, leading monetary policy to positively contribute to our forecasts for GDP growth (see Figure 6 and Figure 7). However, the removal of this large monetary accommodation acts as a severe drag for the real economy in 2022. As a result, the model forecasts GDP growth at 0.4 percent in 2022.

The forecast for Q4/Q4 core PCE inflation is substantially below target in 2020, i.e. at 0.8 percent. The temporary weakness in inflation comes from both negative markup shocks and discount factor shocks (increase in the desire to save); these deflationary forces are counterbalanced by the supply side of the COVID-19 shock that exerts a positive upward pressure on inflation. As a result, inflation is expected to rise modestly in 2021 approaching 1.8 percent. In the medium term however, inflation is forecast to remain at subdued levels, settling at 1.5 percent in 2022 and 2023. This is mostly due to the effect of the estimated positive technology shock in 2020Q3, which push inflation down in the medium term.

Fluctuations in the natural rate are huge and entirely driven by the COVID-19 shock. Since the magnitude of the COVID-19 shock is estimated to be extremely large by any historical standards, the estimated drop of the natural rate 2020 is very pronounced, much larger than any historical records. In particular, the model forecasts that the (real) natural rate of interest at the end of the year for 2020 through 2023 will equal -16.7 percent, -1.2 percent, -1.2 percent, and -0.5 percent respectively. The model sees that the output gap will not close throughout the entire forecasting horizon; we forecast end-of-year output gaps for 2020 through 2023, at -2.5 percent, -1.2 percent, -1.4 percent and -1.6 percent respectively.

The uncertainty surrounding these forecasts is very large.

Figure 1: Model Forecasts with 68% bands

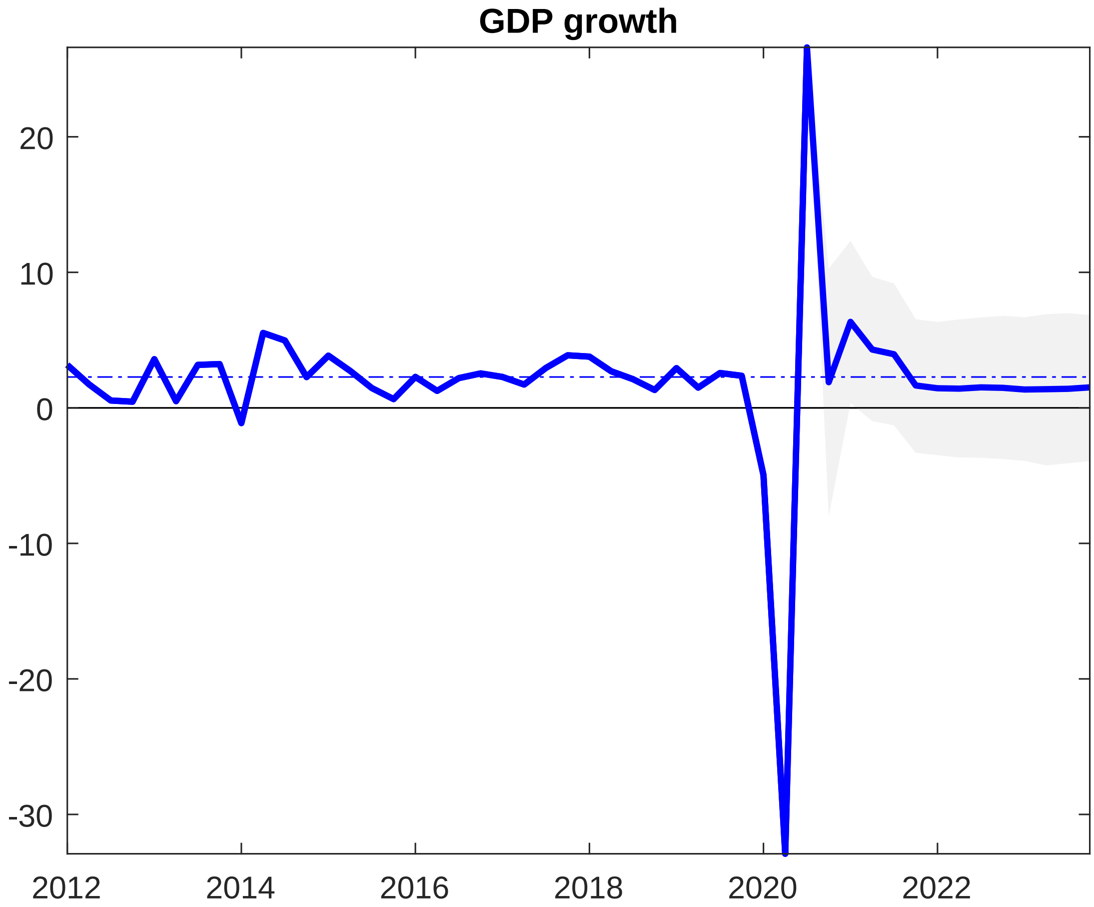


Figure 2: Model Forecasts with 68% bands

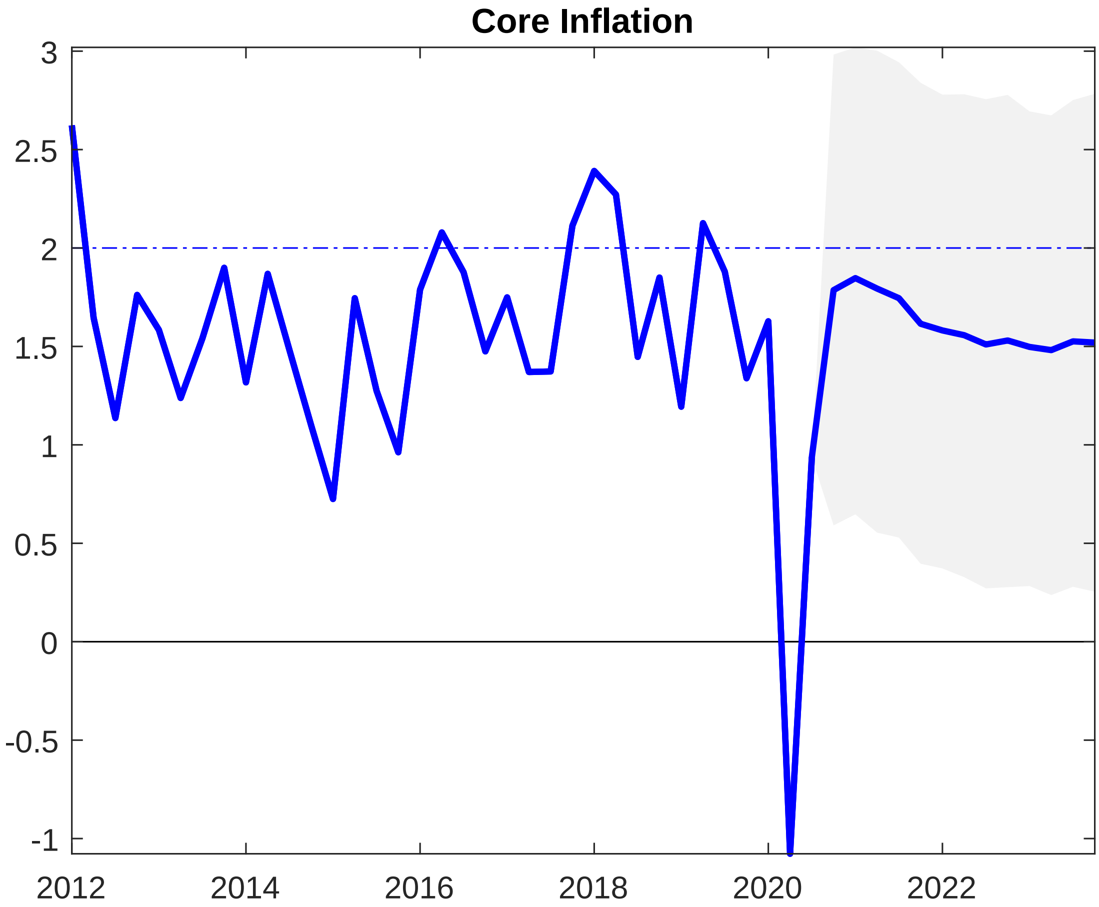


Figure 3: Model Forecasts with 68% bands

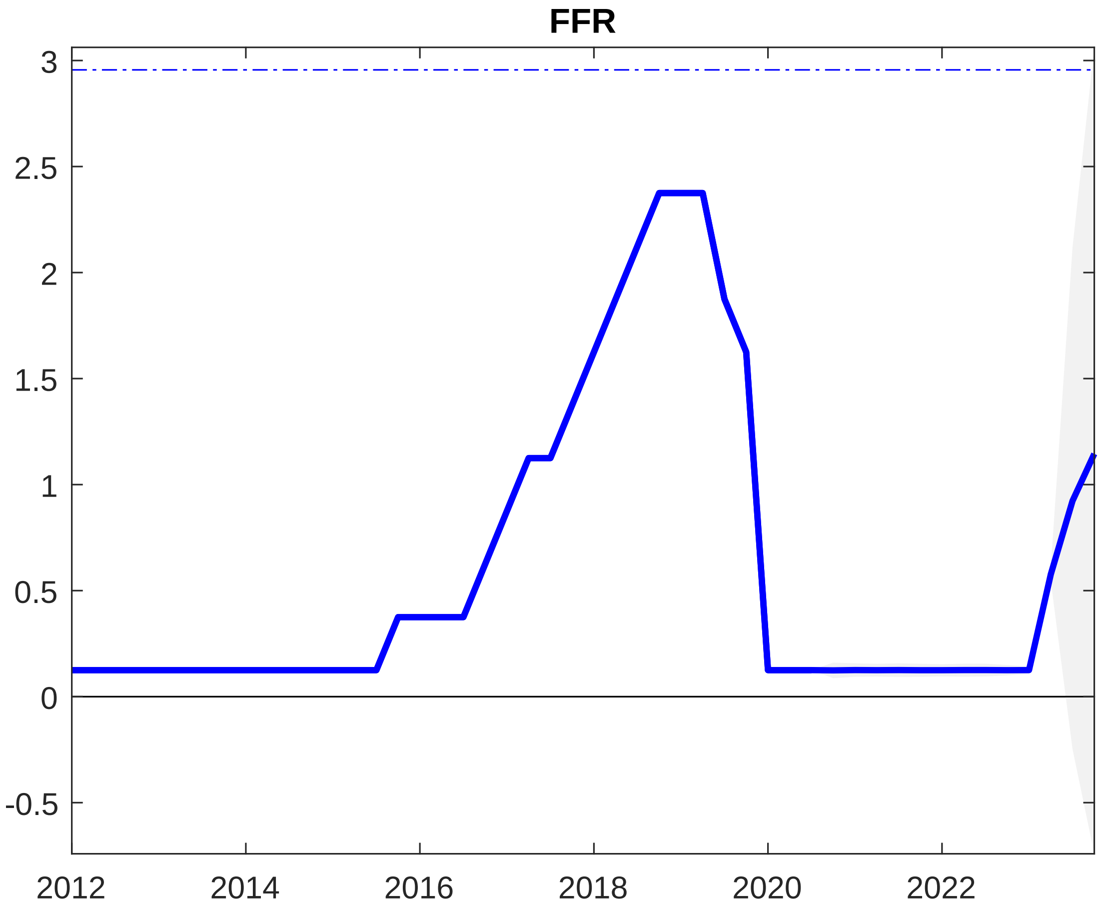


Figure 4: Model Forecasts with 68% bands

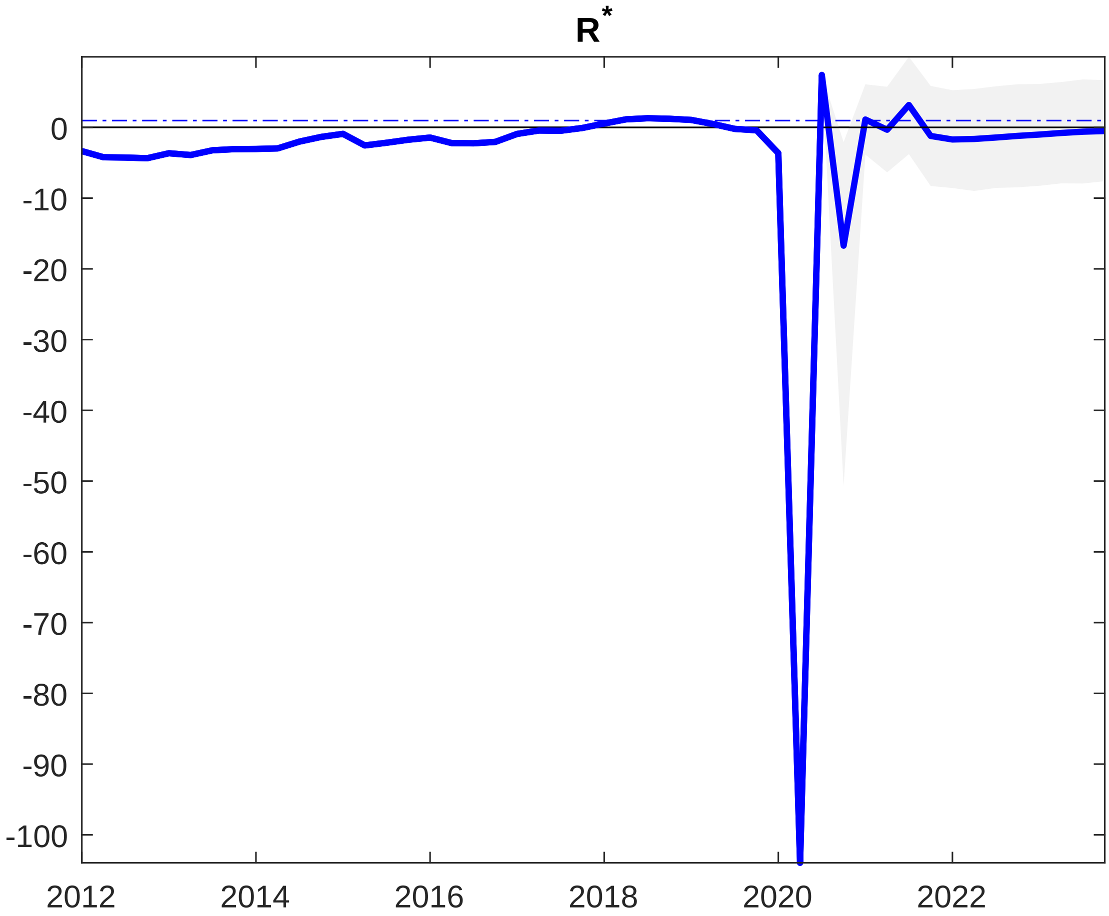


Figure 5: Model Forecasts with 68% bands

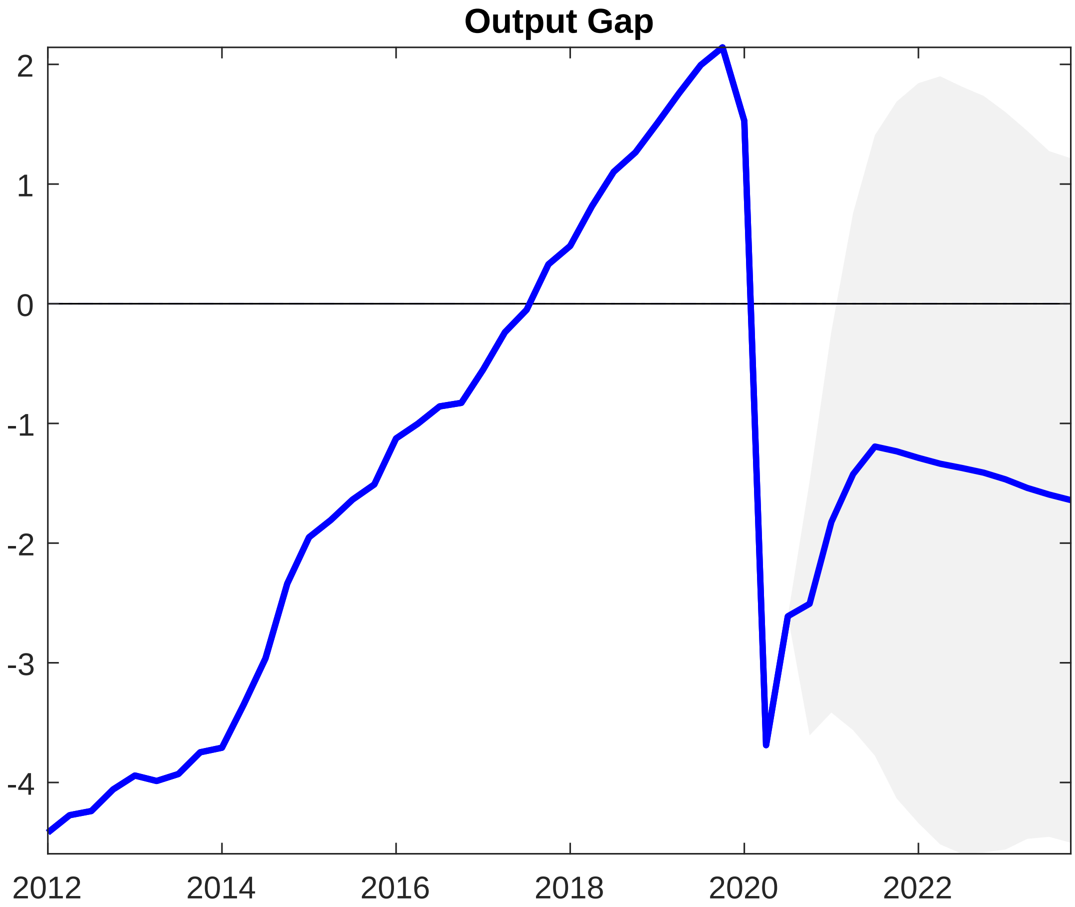
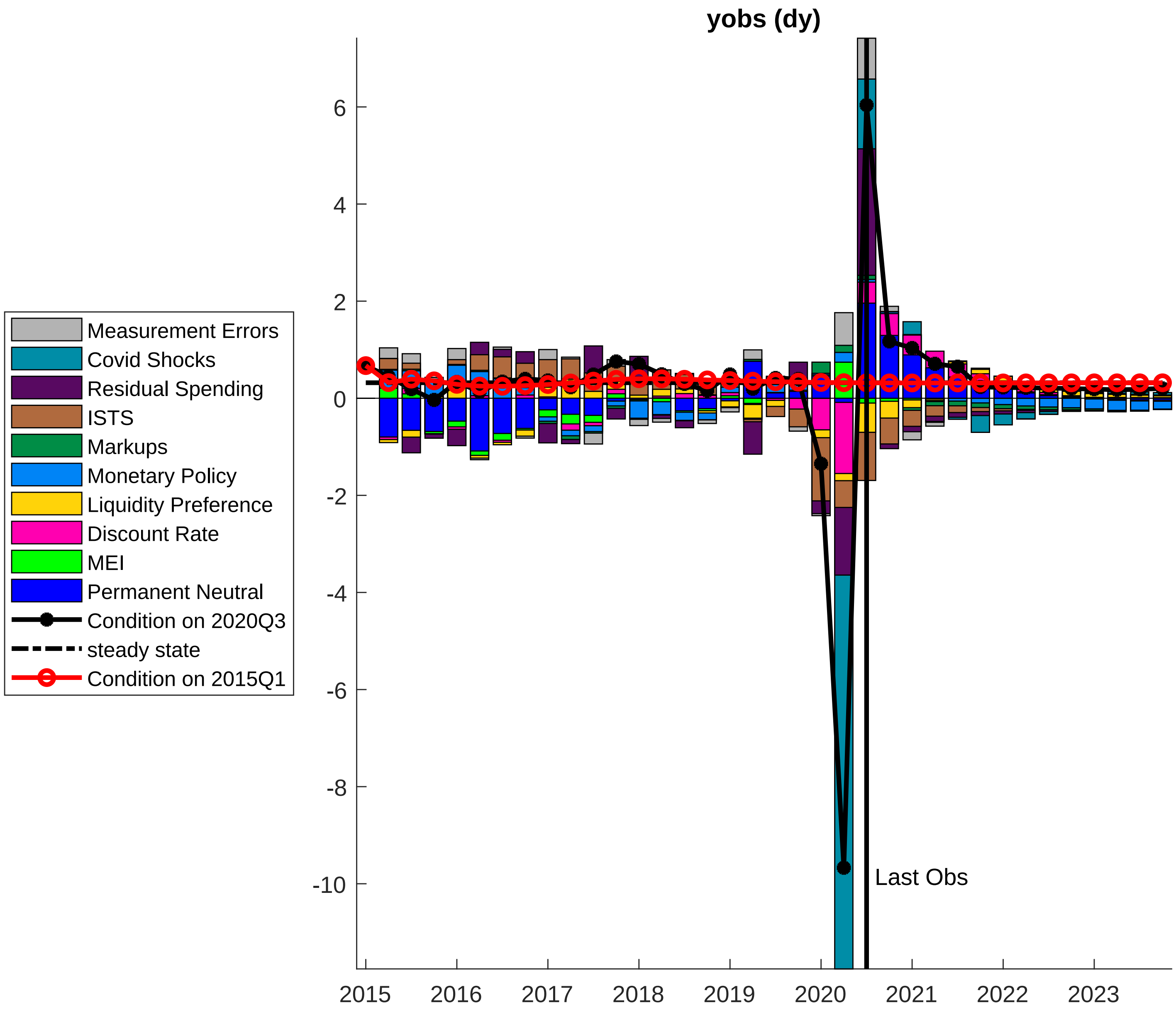


Figure 6: Shock Decomposition of the forecast: The black vertical line indicates the last observation, the black line with dots denotes the observed data and its forecast and the black dashed line the steady states; the red line denotes the forecast launching from 2015Q1. The colored bars decompose the difference between the black and the red line in terms of structural shocks. All variables are expressed in quarterly values.



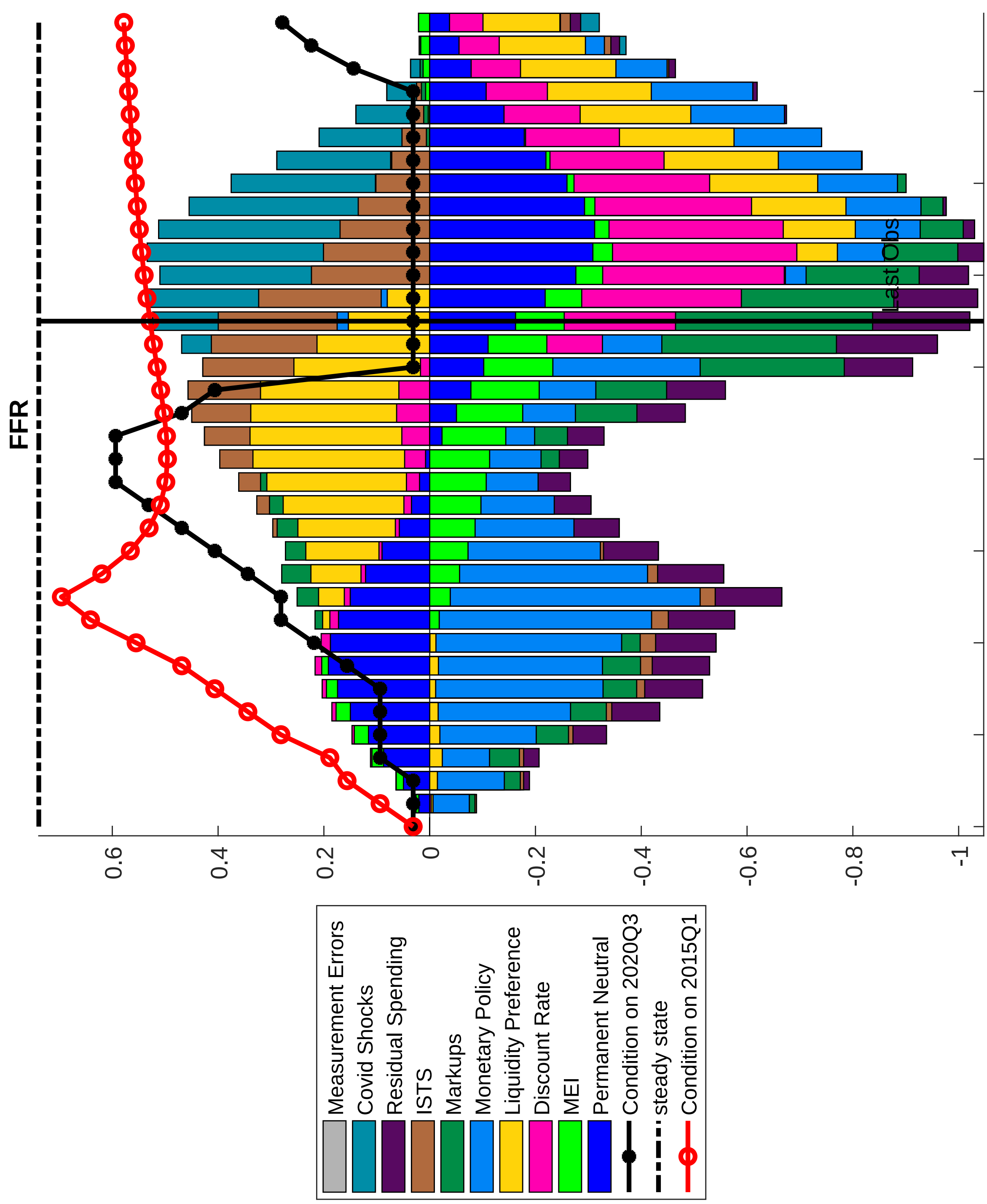
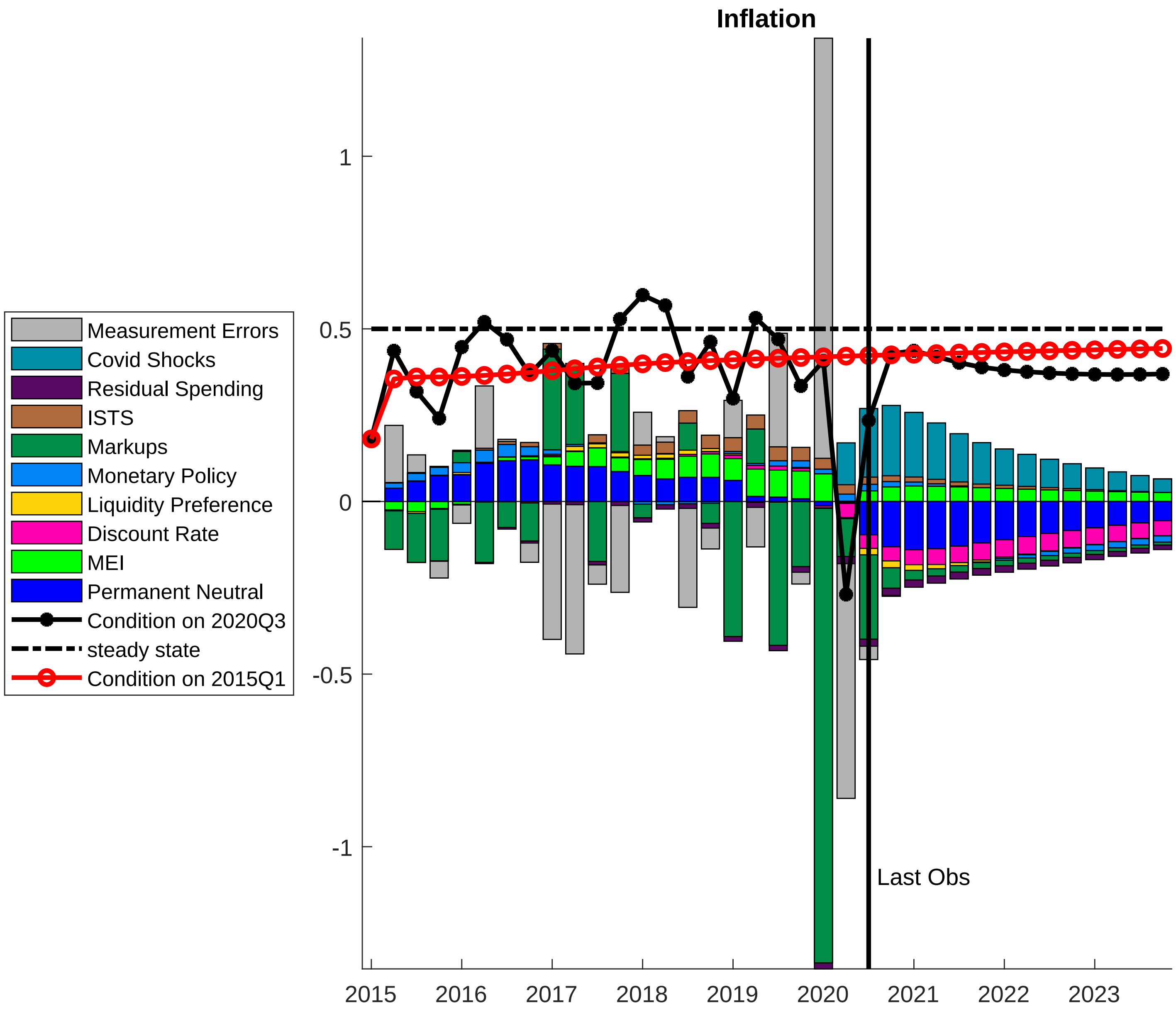


Figure 7: Shock Decomposition of the forecast: The black vertical line indicates the last observation, the black line with dots denotes the observed data and its forecast and the black dashed line the steady states; the red line denotes the forecast launching from 2015Q1. The colored bars decompose the difference between the black and the red line in terms of structural shocks. All variables are expressed in quarterly values.

Figure 8: Shock Decomposition of the forecast: The black vertical line indicates the last observation, the black line with dots denotes the observed data and its forecast and the black dashed line the steady states; the red line denotes the forecast launching from 2015Q1. The colored bars decompose the difference between the black and the red line in terms of structural shocks. All variables are expressed in quarterly values.



Research Directors' Guide to the Chicago Fed DSGE Model*

Filippo Ferroni   Jonas D. M. Fisher   Leonardo Melosi

Version 2020.09, September 4, 2020

This guide describes the construction and estimation of the Chicago Fed's DSGE model, which we use both for internal forecasting and for creating our contributions to the System DSGE memo distributed quarterly to the FOMC. The model has been in use and under ongoing development since 2010. Originally, it was largely based on Justiniano, Primiceri, and Tambalotti (2010). We published results based on simulations from the estimated model in Campbell, Evans, Fisher, and Justiniano (2012) and in Campbell, Fisher, Justiniano, and Melosi (2016).

The model contains many features familiar from other DSGE analyses of monetary policy and bussiness cycles. External habit in preferences, i-dot costs of adjusting investment, price and wage stickiness based on Calvo's (1983) adjustment probabilities, and partial indexation of unadjusted prices and wages using recently observed price and wage inflation. The features which distinguish our analysis from many otherwise similar undertakings are

- **Forward Guidance Shocks:** An interest-rate rule which depends on recent (and expected future) inflation and output and is subject to stochastic disturbances governs our model economy's monetary policy rate. Standard analysis prior to the great recession restricted the stochastic disturbances to be unforecastable. Our model deviates from this historical standard by including forward guidance shocks, as in Laséen and Svensson (2011). A j-quarter ahead forward guidance shock revealed to the public at time t influences the interest-rate rule's stochastic intercept only at time $t + j$. Each period, the model's monetary authority reveals a vector of these shocks with one element for each quarter from the present until the end of the forward guidance horizon. The

*This is a living document under continual revision. Jeffrey Campbell and the late Alejandro Justiniano made fundamental contributions to this project. We thank May Tysinger for her assistance. The views expressed herein are the authors'. They do not necessarily represent those of the Federal Reserve Bank of Chicago, the Federal Reserve System, or its Board of Governors.

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vector's elements may be correlated with each other, so the monetary authority could routinely reveal persistent shifts in the interest-rate rule's stochastic intercept. However, the forward guidance shocks are serially uncorrelated *over time*, as is required for them to match the definition of "news."

- **Investment-Specific Technological Change:** As in the Real Business Cycle models from which modern DSGE models decend (King, Plosser, and Rebelo, 1988a), stochastic trend productivity growth both short-run and long-run fluctuations. Our model features two such stochastic trends, one to Hicks-neutral productivity (King, Plosser, and Rebelo, 1988b) and one to the technology for converting consumption goods into investment goods (as in Fisher (2006)). This investment-specific technological change allows our model to reproduce the dynamics of the relative price of investment goods to consumption goods, which is a necessary input into the formula we use to create Fisher-ideal chain-weighted index of real GDP.

- **A Mixed Calibration-Bayesian Estimation Empirical Strategy:** Bayesian estimation of structural business cycle models attempts to match all features of the data's probability distribution using the model's parameters. Since no structural model embodies Platonic "truth," this exercise inevitably requires trading off between the model's ability to replicate first moments with its fidelty to the business cycles in second moments. Since the criteria for this tradeoff are not always clear, we adopt an alternative "first-moments-first" strategy. This selects the values of model parameters which govern the model's steady-state growth path, such as the growth rates of Hicks-neutral and investment-specific technology, to match estimates of selected first moments. These parameter choices are then fixed for Bayesian estimation, which chooses values for model parameters which only influence second moments, such as technology innovation variances. (Since we employ a log linear solution of our model and all shocks to its primitives have Gaussian distributions, our analysis has no non-trivial implications for third and higher moments of the data.)

- **Pandemic shocks:** We construct a synthetic shock approximating the expected macroeconomic effects of the COVID-19 pandemic. The synthetic shock is a combination of unexpected and anticipated surprises; to economize on the parameters to estimate we assume a correlated (factor) structure across the various horizons of the surprises. Moreover, we assume that the synthetic shock has *hybrid* nature meaning that it could affect contemporaneously different margins of the economy, e.g. demand and supply sides. We estimate the parameters that capture the magnitude and transmission of the COVID-19 shock using plugs for the second quarter of 2020 and SPF expectations about the likely evolution of GDP and inflation over the next four quarters. Akin to empirical literature that uses narrative restrictions or event studies, we identify the COVID-19 shock by assuming that it is the dominant source of fluctuations in the second and third quarter of 2020; to this aim we reduce the standard deviation of the usual business cycles shocks by a factor of 4.

The guide proceeds as follows. The next section presents the model economy's primitives, while Section 2 presents the agents' first-order conditions. Section 3 gives the formulas used to remove nominal and technological trends from model variables and thereby induce model stationarity, and Sections 4 and 5 discuss the stationary economy's steady state and the log linearization of its equilibrium necessary conditions around it. Section 6 discusses measurement issues which arise when comparing model-generated data with data measured by the BEA and BLS. Section 7 describes our mixed Calibration-Bayesian Estimation empirical strategy and presents the resulting parameter values we use for model simulations and forecasting.

1 The Model's Primitives

Eight kinds of agents populate the model economy:

- Households,
- Investment producers,
- Competitive final goods producers,
- Monopolistically-competitive differentiated goods producers,
- Labor Packers,
- Monopolistically-competitive guilds,

- a Fiscal Authority and
- a Monetary Authority.

These agents interact with each other in markets for

- final goods used for consumption
- investment goods used to augment the stock of productive capital
- differentiated intermediate goods
- capital services
- raw labor
- differentiated labor
- composite labor
- government bonds
- privately-issued bonds, and
- state-contingent claims.

The households have preferences over streams of an aggregate consumption good, leisure, and the real value of the fiscal authority's bonds in their portfolios. Our specification for preferences displays balanced growth. They also feature *external* habit in consumption; which creates a channel for the endogenous propagation of shocks. Our bonds-in-the-utility-function preferences follow those of Fisher (2015), and they allow us to incorporate a persistent spread between the monetary policy rate and the return on productive capital. The aggregate consumption good has a single alternative use, as the only input into the linear production function operated by investment producers. These firms sell their output to the households. In turn, households produce capital services from their capital stocks, which they then sell to differentiated goods producers. Producers of final goods operate a constant-returns-to-scale technology with a constant elasticity of substitution between its inputs, which are differentiated goods produced by the monopolistically-competitive firms. These firms operate technologies with affine cost curves (a constant fixed cost and linear marginal cost), which employs capital services and composite labor as inputs. The labor packers produce composite labor using a constant-returns-to-scale technology with a constant elasticity of substitution between its inputs, the differentiated labor sold by guilds. Each of these produces differentiated labor from the raw labor provided by the households with a linear technology, and they sell their outputs to the labor packers. There is a nominal unit of account, called the "dollar." The fiscal authority issues one-period nominally risk-free bonds, provides public goods through government spending, and assesses lump-sum taxes on households. The monetary authority sets the interest rate on the fiscal authority's one-period bond according to an interest-rate rule.

All non-financial trade is denominated in dollars, and all private agents take prices as given with two exceptions: the monopolistically-competitive differentiated-goods producers and guilds. These choose output prices to maximize the current value of expected future profits taking as given their demand curves and all relevant input prices. Financial markets are complete, but all securities excepting equities in differentiated-goods producers are in zero net supply. These producers' profits and losses are rebated to the households (who own the firms' equities) lump-sum period-by-period, as are the profits and losses of the guilds. Given both a process for government spending and taxes and a rule for the monetary authority's interest rate choice, a competitive equilibrium consists of allocations and prices that are consistent with households' utility maximization, firms' profit maximization, guilds' profit maximization, and market clearing.

The economy is subject to stochastic disturbances in technology, preferences, and government policy. Without nominal rigidities, the economy's real allocations in competitive equilibrium can be separated from inflation and other dollar-denominated variables. Specifically, monetary policy only influences inflation. To connect real and nominal variables in the model and thereby consider the impact of monetary policy on the business cycle, we introduce Calvo-style wage and price setting. That is, nature endows both differentiated goods producers and guilds with *stochastic* opportunities to incorporate all available information into their nominal price choices. Those producers and guilds without such a opportunity must set their prices according to simple indexing formulas. These two pricing frictions create two forward-looking Phillips curves, one for prices and another for wages, which form the core of the new Keynesian approach to monetary policy analysis.

The model economy is stochastic and features complete markets in state-contingent claims. To place these features on a sound footing, we base all shocks on a general Markovian stochastic process s_t. Denote the history of this vector from an initial period 0 through τ with $s^\tau \equiv (s_0, s_1, \ldots, s_\tau)$. All model shocks are implicit functions of s_t, and all endogenous variables are implicit functions of s^t. We refer to all such implicit functions as "state-contingent sequences." We use braces to denote such a sequence. For example, $\{X_t\}$ represents the state-contingent sequence for a generic variable X_t.

1.1 Households

Our model's households are the ultimate owners of all assets in positive net supply (the capital stock, differentiated goods producers, and guilds). They provide labor and divide their current after-tax income (from wages and assets) between current consumption, investment in productive capital, and purchases of financial assets, both those issued by the government and those issued by other households. The individual household divides its current resources between consumption and the available vehicles for intertemporal substitution (capital and financial assets) to maximize a discounted sum of current and expected future felicity.

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \varepsilon^b_{t+\tau} \left(U_{t+\tau} + \varepsilon^s_{t+\tau} L \left(\frac{B_{t+\tau}}{P_{t+\tau} R_{t+\tau}} \right) \right) \right]$$

with

$$U_t = \frac{1}{1-\gamma_c} \left((C_t - \varrho \bar{C}_{t-1})(1 - H_t^{1+\gamma_h}) \right)^{(1-\gamma_c)} \tag{1}$$

The function $L(\cdot)$ is strictly increasing, concave, and differentiable everywhere on $[0, \infty)$. In particular, $L'(0)$ exists and is finite. Without loss of generality, we set $L'(0)$ to one. The argument of $L(\cdot)$ equals the real value of government bonds in the household's portfolio: their period $t+1$ redemption value B_t divided by their nominal yield R_t expressed in units of the consumption good with the nominal price index P_t. The time-varying coefficient multiplying this felicity from bond holdings, ε^s_t, is the liquidity preference shock introduced by Fisher (2015). A separate shock influences the household's discounting of future utility to the present, ε^b_t. Specifically, the household discounts a certain utility in $t+\tau$ back to t with $\beta^\tau \mathbb{E}_t \left[\varepsilon^b_{t+\tau} / \varepsilon^b_t \right]$. In logarithms, these two preference shocks follow independent autoregressive processes.

$$\ln \varepsilon_t^b = (1-\rho_b)\ln \varepsilon_*^b + \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b, \eta_t^b \sim \mathbb{N}(0,\sigma_b^2) \quad (2)$$
$$\ln \varepsilon_t^s = (1-\rho_s)\ln \varepsilon_*^s + \rho_s \ln \varepsilon_{t-1}^s + \eta_t^s, \eta_t^s \sim \mathbb{N}(0,\sigma_s^2). \quad (3)$$

A household's wealth at the beginning of period t consists of its nominal government bond holdings, B_t, its net holdings of privately-issued financial assets, and its capital stock K_{t-1}. The household chooses a rate of capital utilization u_t, and the capital services resulting from this choice equal $u_t K_{t-1}$. The cost of increasing utilization is higher depreciation. An increasing, convex and differentiable function $\delta(U)$ gives the capital depreciation rate. We specify this as

$$\delta(u) = \delta_0 + \delta_1(u-u_\star) + \frac{\delta_2}{2}(u-u_\star)^2.$$

A household can augment its capital stock with investment, I_t. Investment requires paying adjustment costs of the "i-dot" form introduced by Christiano, Eichenbaum, and Evans (2005). Also, an *investment demand shock* alters the efficiency of investment in augmenting the capital stock. Altogether, if the household's investment in the previous period was I_{t-1}, and it purchases I_t units of the investment good today, then the stock of capital available in the *next* period is

$$K_t = (1-\delta(u_t))K_{t-1} + \varepsilon_t^i\left(1 - S\left(\frac{A_{t-1}^K I_t}{A_t^K I_{t-1}}\right)\right) I_t. \quad (4)$$

In (4), A_t^K equals the productivity level of capital goods production, described in more detail below, and ε_t^i is the investment demand shock. In logarithms, this follows a first-order autoregression with a normally-distributed innovation.

$$\ln \varepsilon_t^i = (1-\rho_i)\ln \varepsilon_*^i + \rho_i \ln \varepsilon_{t-1}^i + \eta_t^i, \eta_t^i \sim \mathbb{N}(0,\sigma_i^2) \quad (5)$$

1.2 Production

The producers of investment goods use a linear technology to transform the final good into investment goods. The technological rate of exchange from the final good to the investment good in period t is A_t^I. We denote $\Delta \ln A_t^I$ with ω_t, which we call the *investment-specific technology shock* and which follows first-order autogregression with normally distributed innovations.

$$\omega_t = (1 - \rho_\omega)\omega_\star + \rho_\omega \omega_{t-1} + \eta_t^\omega, \eta_t^\omega \sim \mathbb{N}(0, \sigma_\omega^2) \tag{6}$$

Investment goods producers are perfectly competitive.

Final good producers also operate a constant-returns-to-scale technology; which takes as inputs the products of the differentiated goods producers. To specify this, let Y_{it} denote the quantity of good i purchased by the representative final good producer in period t, for $i \in [0, 1]$. The representative final good producer's output then equals

$$Y_t \equiv \left(\int_0^1 Y_{it}^{\frac{1}{1+\lambda_t^p}} \, di \right)^{1+\lambda_t^p}.$$

With this technology, the elasticity of substitution between any two differentiated products equals $1 + 1/\lambda_t^p$ in period t. Although this is constant across products within a time period, it varies stochastically over time according to an ARMA$(1, 1)$ in logarithms.

$$\ln \lambda_t^p = (1 - \rho_p) \ln \lambda_\star^p + \rho_p \ln \lambda_{t-1}^p - \theta_p \eta_{t-1}^p + \eta_t^p, \eta_t^p \sim \mathbb{N}(0, \sigma_p^2) \tag{7}$$

Given nominal prices for the intermediate goods P_{it}, it is a standard exercise to show that the final goods producers' marginal cost equals

$$P_t = \left(\int_0^1 P_{it}^{-\frac{1}{\lambda_t^p}} \, di \right)^{-\lambda_t^p} \tag{8}$$

Just like investment goods firms, the final goods' producers are perfectly competitive. Therefore, profit maximization and positive final goods output together require the competitive output price to equal P_t. Therefore, we can define inflation of the nominal final good price as $\pi_t \equiv \ln(P_t/P_{t-1})$.

The intermediate goods producers each use the technology

$$Y_{it} = (K_{it}^e)^\alpha \left(A_t^Y H_{it}^d\right)^{1-\alpha} - A_t \Phi \tag{9}$$

Here, K_{it}^e and H_{it}^d are the capital services and labor services used by firm i, and A_t^Y is the level of neutral technology. Its growth rate, $\nu_t \equiv \ln(A_t^Y / A_{t-1}^Y)$, follows a first-order autogregression.

$$\nu_t = (1 - \rho_\nu)\,\nu_* + \rho_v \nu_{t-1} + \eta_t^\nu, \eta_t^\nu \sim \mathbb{N}(0, \sigma_\nu^2), \tag{10}$$

The final term in (9) represents the fixed costs of production. These grow with

$$A_t \equiv A_t^Y \left(A_t^I\right)^{\frac{\alpha}{1-\alpha}}. \tag{11}$$

We demonstrate below that A_t is the stochastic trend in equilibrium output and consumption, measured in units of the final good. We denote its growth rate with

$$z_t = \nu_t + \frac{\alpha}{1-\alpha}\omega_t \tag{12}$$

Similarly, define

$$A_t^K \equiv A_t A_t^I \tag{13}$$

In the specification of the capital accumulation technology, we labelled A_t^K the "productivity level of capital goods production." We demonstrate below that this is indeed the case with the definition in (13).

Each intermediate goods producer chooses prices subject to a Calvo (1983) pricing scheme. With probability $\zeta_p \in [0,1]$, producer i has the opportunity to set P_{it} without constraints. With the complementary probability, P_{it} is set with the indexing rule

$$P_{it} = P_{it-1} \pi_{t-1}^{\iota_p} \pi_\star^{1-\iota_p}. \tag{14}$$

In (14), $\pi_\star$ is the gross rate of price growth along the steady-state growth path, and $\iota_p \in [0,1]$.[1]

[1] To model firms' price-setting opportunities as functions of s_t, define a random variable u_t^p which is independent over time and uniformly distributed on $[0,1]$. Then, firm i gets a price-setting opportunity if either $u_t^p \geq \zeta_p$ and $i \in [u_t^p - \zeta_p, u_t^p]$ or if $u_t^p < \zeta_p$ and $i \in [0, u_t^p] \cup [1 + u_t^p - \zeta_p, 1]$.

1.3 Labor Markets

Households' hours worked pass through two intermediaries, guilds and labor packers, in their transformation into labor services used by the intermediate goods producers. The guilds take the households' homogeneous hours as their only input and produce differentiated labor services. These are then sold to the labor packers, who assemble the guilds' services into composite labor services.

The labor packers operate a constant-returns-to-scale technology with a constant elasticity of substitution between the guilds' differentiated labor services. For its specification, let H_{it} denote the hours of differenziated labor purchased from guild i at time t by the representative labor packer. Then that packer's production of composite labor services, H_t^s are given by

$$H_t^s = \left(\int_0^1 (H_{it})^{\frac{1}{1+\lambda_t^w}} di \right)^{1+\lambda_t^w}.$$

As with the final good producer's technology, an ARMA$(1,1)$ in logarithms governs the constant elasticity of substitution between any two guilds' labor services.

$$\ln \lambda_t^w = (1-\rho_w) \ln \lambda_\star^w + \rho_w \ln \lambda_{t-1}^w - \theta_w \eta_{t-1}^w + \eta_t^w, \eta_t^w \sim \mathbb{N}(0, \sigma_w^2) \tag{15}$$

Just as with the final goods producers, we can easily show that the labor packers' marginal cost equals

$$W_t = \left(\int_0^1 (W_{it})^{-\frac{1}{\lambda_t^w}} di \right)^{-\lambda_t^w}. \tag{16}$$

Here, W_{it} is the nominal price charged by guild i per hour of differentiated labor. Since labor packers are perfectly competitive, their profit maximization and positive output together require that the price of composite labor services equals their marginal cost.

Each guild produces it's differentiated labor service using a linear technology with the household's hours worked as its only input. A Calvo (1983) pricing scheme similar to that of the differentiated goods producers constrains their nominal prices. Guild i has an unconstrained opportunity to choose its nominal price with probability $\zeta_w \in [0,1]$. With the complementary probability, W_{it} is set with an indexing rule based on π_{t-1} and last period's trend growth rate, z_{t-1}.

$$W_{it} = W_{it-1} \left(\pi_{t-1} e^{z_{t-1}}\right)^{\iota_w} \left(\pi_\star e^{z_\star}\right)^{1-\iota_w}. \tag{17}$$

In (17), $z_\star \equiv \nu_\star + \frac{\alpha}{1-\alpha}\omega_\star$ is the unconditional mean of z_t and $\iota_w \in [0,1]$.

1.4 Fiscal and Monetary Policy

The model economy hosts two policy authorities, each of which follows exogenously-specified rules that receive stochastic disturbances. The fiscal authority issues bonds, B_t, collects lump-sum taxes T_t, and buys "wasteful" public goods G_t. Its period-by-period budget constraint is

$$G_t + B_{t-1} = T_t + \frac{B_t}{R_t}. \tag{18}$$

The left-hand side gives the government's uses of funds, public goods spending and the retirement of existing debt. The left-hand side gives the sources of funds, taxes and the proceeds of new debt issuance at the interest rate R_t. We assume that the fiscal authority keeps its budget balanced period-by-period, so $B_t = 0$. Furthermore, the fiscal authority sets public goods expenditure equal to a stochastic share of output, expressed in consumption units.

$$G_t = (1 - 1/g_t)Y_t, \tag{19}$$

with

$$\ln g_t = (1-\rho_g)\ln s^g_\star + \rho_g \ln g_{t-1} + \eta^g_t, \eta^g_t \sim \mathbb{N}(0, \sigma^2_g). \tag{20}$$

The monetary authority sets the nominal interest rate on government bonds, R_t. For this, it employs a Taylor rule with interest-rate smoothing and forward guidance shocks.

$$\ln R_t = \rho_R \ln R_{t-1} + (1-\rho_R)\ln R^n_t + \sum_{j=0}^{M} \xi^j_{t-j}. \tag{21}$$

The monetary policy disturbances in (21) are $\xi^0_t, \xi^1_{t-1}, \ldots, \xi^M_{t-M}$. The public learns the value of ξ^j_{t-j} in period $t-j$. The conventional unforecastable shock to current monetary policy is ξ_t^0, while for $j \geq 1$, these disturbances are *forward guidance shocks.* We gather all monetary shocks revealed at time t into the vector ε_t^R. This is normally distributed and *i.i.d.* across time. However, *its elements may be correlated with each other.* That is,

$$\varepsilon_t^R \equiv \left(\xi_t^0, \xi_t^1, \ldots, \xi_t^M\right) \sim \mathbb{N}(0, \Sigma_\varepsilon). \tag{22}$$

The off-diagonal elements of Σ^1 are not necessarily zero, so forward-guidance shocks need not randomly impact expected future monetary policy at two adjacent dates independently. Current economic circumstances influence R_t through the notional interest rate, R_t^n.

$$\ln R_t^n = \ln r_\star + \ln \pi_t^\star + \frac{\phi_1}{4}\mathbb{E}_t \sum_{j=-2}^{1} (\ln \pi_{t+j} - \ln \pi_t^\star) + \frac{\phi_2}{4}\mathbb{E}_t \sum_{j=-2}^{1} (\ln Y_{t+j} - \ln y^\star - \ln A_{t+j}). \tag{23}$$

The constant $r_\star$ equals the real interest rate along a steady-state growth path, and $\pi_t^\star$ is the central bank's intermediate target for inflation. We call this the *inflation-drift shock.* it follows a first-order autoregression with a normally-distributed innovation. Its unconditional mean equals $\pi_\star$, the inflation rate on a steady-state growth path.

$$\ln \pi_t^\star = (1 - \rho_\pi)\pi_\star + \rho_\pi \ln \pi_{t-1}^\star + \eta_t^\pi, \eta_t^\pi \sim \mathbb{N}(0, \sigma_\pi^2) \tag{24}$$

Allowing $\pi_t^\star$ to change over time enables our model to capture the persistent decline in inflation from the early 1990s through the early 2000s engineered by the Greenspan FOMC.

1.5 Other Financial Markets and Equilibrium Definition

All households participate in the market for nominal risk-free government debt. Additionally, they can buy and sell two classes of privately issued assets without restriction. The first is one-period nominal risk-free *private* debt. We denote the value of household's net holdings of such debt at the beginning of period t with B_{t-1}^P and the interest rate on such debt issued in period t maturing in $t+1$ with R_{t+1}^P. The second asset class consists of a complete set of *real* state-contingent claims. As of the end of period t, the household's ownership of securities that pay off one unit of the aggregate consumption good in period τ if history s^{τ} occurs is $Q_t(s^{\tau})$, and the nominal price of such a security in the same period is $J_t(s^{\tau})$.

We define an equilibrium for our economy in the usual way: Households maximize their utility given all prices, taxes, and dividends from both producers and guilds; final goods producers and labor packers maximize profits taking their input and output prices as given; differentiated goods producers and guilds maximize the market value of their dividend streams taking as given all input and financial-market prices; differentiated goods producers and guilds produce to satisfy demand at their posted prices; and otherwise all product, labor, and financial markets clear.

2 First Order Conditions

In this section we present the first-order conditions associated with the optimization problems that the agents in our model solve.

2.1 Households

Given initial financial asset holdings holdings, a stock of productive capital, investment in the previous period (which influences investment adjustment costs), and the external habit stock; households' choices of consumption, capital investment, capital utilization, hours worked, and financial investments maximize utility subject to the constraints of the capital accumulation and utilization technology and a sequence of one-period budget constraints. To specify these budget constraints, denote the nominal wage-per-hour paid by labor guilds to households with W_t^h, the nominal rental rate for capital services with R_t^k, the nominal price of investment goods with P_t^I, and the dividends paid by labor guilds added to those paid by differentiated good producers with D_t. With this notation, writing the period t budget constraint with uses of funds on the left and sources of funds on the right yields

$$C_t + \frac{P_t^I I_t}{P_t} + \frac{B_t}{R_t P_t} + \frac{B_t^P}{R_t^P P_t} + \frac{T_t}{P_t} \leq \frac{B_{t-1}}{P_t} + \frac{B_{t-1}^P}{P_t} + \frac{W_t^h H_t}{P_t} + \frac{R_t^k u_t K_{t-1}}{P_t} + \frac{D_t}{P_t} \tag{25}$$

Denote the Lagrange multiplier on (25) with $\beta^t \Lambda_t^1$, and that on the capital accumulation constraint in (4) with $\beta^t \Lambda_t^2$. With these definitions, the first-order conditions for a household's utility maximization problem are

$$
\begin{aligned}
\Lambda_t^1 &= \varepsilon_t^b \left((C_t - \varrho \bar{C}_{t-1})(1-\varepsilon_t^h H_t^{1+\gamma_h})\right)^{-\gamma_c} (1-\varepsilon_t^h H_t^{1+\gamma_h}) \\
\Lambda_t^1 \frac{W_t^h}{P_t} &= (1+\gamma_h)\varepsilon_t^b \left((C_t - \varrho \bar{C}_{t-1})(1-\varepsilon_t^h H_t^{1+\gamma_h})\right)^{-\gamma_c} (C_t - \varrho \bar{C}_{t-1})\varepsilon_t^h H_t^{\gamma_h} \\
\frac{\Lambda_t^1}{R_t P_t} - \varepsilon_{t+q}^b L'(\frac{B_t}{R_t P_t})\frac{\varepsilon_t^s}{R_t P_t} &= \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}^1}{P_{t+1}}\right] \\
\frac{\Lambda_t^1}{R_t^P P_t} &= \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}^1}{P_{t+1}}\right] \\
\Lambda_t^2 &= \beta \mathbb{E}\left[\Lambda_{t+1}^1 \frac{R_{t+1}^k u_{t+1}}{P_{t+1}} + \Lambda_{t+1}^2 (1-\delta(u_{t+1}))\right] \\
\frac{\Lambda_t^1 R_t^k}{P_t} &= \Lambda_t^2 \delta'(u_t) \\
\Lambda_t^1 &= \varepsilon_t^i \Lambda_t^2 \left((1 - S_t(\cdot)) - S_t'(\cdot)\frac{i_t}{i_{t-1}}\right) \\
&\quad + \beta \mathbb{E}_t \left[\varepsilon_{t+1}^i e^{(1-\gamma_C)z_{t+1}} \lambda_{t+1}^2 S_{t+1}'(\cdot)\frac{i_{t+1}^2}{i_t^2}\right]
\end{aligned}
$$

In equilibrium, $\bar{C}_t = C_t$ always.

2.2 Goods Sector

2.2.1 Final Goods Producers

The nominal marginal cost of final goods producers equals the right-hand side of (8). A producer of final goods maximizes profit by shutting down if P_t is less than this marginal cost and can make an arbitrarily large profit if P_t exceeds it. When (8) holds, an individual final goods producer's output is indeterminate.

Final goods producers' demand for intermediate goods takes the familiar constant-elasticity form. If we use Y_t to denote total final goods output, then the amount of differentiated good i demanded by final goods producers is

$$Y_{it} = Y_t \left(\frac{P_{it}}{P_t}\right)^{-\frac{1+\lambda_t^p}{\lambda_t^p}}.$$

Given the choice of a reset price, we wish to calculate the overall price level.

All intermediate goods producers with a price-setting opportunity choose $\tilde{P}_t$. The remaining producers use the price-indexing rule in (14). The aggregate price level is given by

$$P_t = \left[(1-\zeta_p)\tilde{P}_t^{\frac{1}{\lambda_{p,t-1}}} + \zeta_p \left((\pi_{t-1})^{\iota_p} (\pi_*)^{1-\iota_p} P_{t-1} \right)^{\frac{1}{\lambda_{p,t-1}}} \right]^{\lambda_{p,t-1}}$$

where $\tilde{P}_t$ is the optimal reset price

2.2.2 Intermediate Goods Producers

Intermediate goods producers' cost minimization reads as follows:

$$\max_{H_{t,i},K^e_{i,t}} W_t H^d_{t,i} + R^k_t K^e_{i,t}$$
$$\text{s.t. } Y_{t,i} = \varepsilon^a_t (K^e_{t,i})^\alpha \left(A^y_t H^d_{t,i} \right)^{1-\alpha} - A_t \Phi$$

We get the following optimal capital-labor ratio.

$$\frac{\alpha}{1-\alpha} \frac{W_t}{R^k_t} = \frac{(K^e_{it})^s}{H^d_{t,i}}$$

Notice how for each firm, the idiosyncratic capital to labor ratio is not a function of any firm-specific component. Hence, each firm has the same capital to labor ratio. In equilibrium,

$$K^e_t = u_t K_{t-1}$$

To find the marginal cost, we differentiate the variable part of production with respect to output, and substitute in the capital-labor ratio.

$$MC_{t,i} = (\varepsilon^a_t)^{-1} (A^y_t)^{-(1-\alpha)} W_t^{1-\alpha} R_t^{k\alpha} \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)}$$

Again, notice that each firm as the same marginal cost.

Given cost minimization, a differentiated goods producer with an opportunity to adjust its nominal price does so to maximize the present-discounted value of profits earned until the next opportunity to adjust prices arrives. Formally,

$$\max_{\tilde{P}_{t,i}} E_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\beta^s \Lambda_{t+s}^1 P_t}{\Lambda_t^1 P_{t+s}} \left[\tilde{P}_{t,i} X_{t,s}^y - MC_{t+s}\right] Y_{t+s,i}$$

$$\text{s.t. } Y_t(i) = \left(X_{t,s}^y \frac{\tilde{P}_{t,i}}{P_t} \right)^{\frac{\lambda_{p,t}}{1-\lambda_{p,t}}} Y_t$$

$$\text{where } X_{t,s}^y = \left\{ \begin{array}{ll} 1 & : s = 0 \\ \prod_{l=1}^{s} \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p} & : s = 1, \ldots, \infty \end{array} \right\}$$

This problem leads to the following price-setting equation for firms that are allowed to reoptimize their price:

$$0 = E_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\beta^s \Lambda_{t+s}^1 P_t}{\Lambda_t^1 P_{t+s}} Y_{it+s} \left[\lambda_{p,t+s} MC_{t+s} - X_{t,s} \tilde{P}_{it} \right]$$

It can be shown that the producers that are allowed to reoptimize choose the same price. So henceforth, $\tilde{P}_{it} = \tilde{P}_t$.

2.2.3 Investment Goods Producers

Characterizing the profit-maximizing choices of investment goods and final goods producers is straightforward. If $P_t^I > P_t / A_t^I$, then each investment goods producer can make infinite profit by choosing an arbitrarily large output. On the other hand, if $P_t^I < P_t / A_t^I$, then investment goods producers maximize profits with zero production. Finally, their profit-maximizing production is indeterminate when

$$P_t^I = P_t / A_t^I. \tag{26}$$

The relative price of investment to consumption is equal to $(A_t^I)^{-1}$. Making this substitution into the household f.o.c and noting that $P_t Y_t^I$ is an intermediate input that will not be reflected in the aggregate resource constraint, it suffices to substitute the relative price $(A_t^I)^{-1}$ in the constraint for the household.

2.3 Labor Sector

2.3.1 Labor Packers

The labor packers choose the the labor inputs supplied by guilds, pack them into a composite labor service to be sold to the intermediate goods producers. Formally, labor packers' problem reads as follows:

$$\max_{H_t^s, H_{it}} W_t H_t^s - \int_0^1 W_{it} H_{it} di$$
$$s.t. \left[\int_0^1 H_{it}^{\frac{1}{1+\lambda_{w,t}}} di \right]^{1+\lambda_{w,t}} = H_t^s$$

We obtain the following labor demand equation for guild i:

$$H_{it} = \left(\frac{W_{it}}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} H_t \tag{27}$$

As for the goods sector, we can show that aggregate wage is given by the following equation:

$$W_t = \left[(1-\zeta_w) \tilde{W}_t^{-\frac{1}{\lambda_{w,t}}} + \zeta_w \left(\left(e^{z_{t-1}} \pi_{t-1} \right)^{\iota_w} \left(\pi_* e^{z_*} \right)^{1-\iota_w} W_{t-1} \right)^{-\frac{1}{\lambda_{w,t}}} \right]^{-\lambda_{w,t}}$$

where $\tilde{W}$ is the optimal reset wage for guilds.

2.3.2 Guilds

Each guild with an opportunity to set its nominal price does so to maximize the current value of the stream of dividends returned to the household. Formally, their problem reads

$$\max_{\tilde{W}_{it}} E_t \sum_{s=0}^{\infty} \zeta_w^s \left(\frac{\beta^s \Lambda_{t+s}^1 P_t}{\Lambda_t^1 P_{t+s}} \right) \left[X_{t+s}^l \tilde{W}_{it} - W_{t+s}^h \right] H_{it+s}$$
$$\text{s.t. } H_{it+s} = \left(\frac{X_{t,s}^l \tilde{W}_{it}}{W_{t+s}} \right)^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} H_{t+s}$$
$$\text{where } X_{t,s}^l = \left\{ \begin{array}{ll} 1 & : s = 0 \\ \prod_{j=1}^s \left(\pi_{t+j-1} \frac{A_{t+j-1}}{A_{t+j-2}} \right)^{1-\iota_w} (\pi e^{\gamma})^{\iota_w} & : s = 1, \ldots, \infty \end{array} \right\}$$

$\tilde{W}_t$ is the optimal reset wage. This optimal wage is chosen by the guilds who are allowed, with probability ζ_w, to change their prices in a given period. Also, we index the nominal wage inflation rate with ι_w.

This maximization problem gives a wage-setting equation that reads as follows:

$$0 = E_t \sum_{s=0}^{\infty} \zeta_w^s \frac{\beta^s \Lambda_{t+s}^1 P_t}{\Lambda_t^1 P_{t+s}} H_{it+s} \frac{1}{\lambda_{w,t+s}} \left((1 + \lambda_{w,t+s}) W_{t+s}^h - X_{t,s}^l \tilde{W}_{it} \right)$$

It can be shown that the guilds that are allowed to reoptimize choose the same wage. So henceforth, $\tilde{W}_{it} = \tilde{W}_t$.

3 Detrending

To remove nominal and real trends, we deflate nominal variables by their matching price deflators, and we detrend any resulting real variables influenced permanently by technological change. All scaled versions of variables are the lower-case counterparts.

$$c_t = \frac{C_t}{A_t} \qquad i_t = \frac{I_t}{A_t A_t^I}$$

$$k_t = \frac{K_t}{A_t A_t^I} \qquad k_t^e = \frac{K_t^e}{A_t A_t^I}$$

$$w_t = \frac{W_t}{A_t P_t} \qquad \tilde{w}_t = \frac{\tilde{W}_t}{A_t P_t}$$

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t} \qquad \pi_t = \frac{P_t}{P_{t-1}}$$

$$y_t = \frac{Y_t}{A_t} \qquad mc_t = \frac{MC_t}{P_t}$$

$$r_t^k = \frac{R_t^k A_t^I}{P_t} \qquad w_t^h = \frac{W_t^h}{A_t P_t}$$

$$\lambda_t^1 = \Lambda_t^1 A_t^{\gamma_C} \qquad \lambda_t^2 = \Lambda_t^2 A_t^{\gamma_C} A_t^I$$

$$\varepsilon_t^s = A_t^{\gamma_C} \varepsilon_t^s$$

3.1 Detrended Equations

The detrended equations describing our model are listed in the following sections.

Households' FOC

$$\lambda_t^1 = \varepsilon_t^b \left[\left(c_t - \varrho \frac{c_{t-1}}{e^{z_t}} \right) \left(1 - \varepsilon_t^h h_t^{1+\gamma_h} \right) \right]^{-\gamma_c} \left(1 - \varepsilon_t^h h_t^{1+\gamma_h} \right)$$

$$\lambda_t^1 w_t^h = (1+\gamma_h) \varepsilon_t^b \left[\left(c_t - \varrho \frac{c_{t-1}}{e^{z_t}} \right) \left(1 - \varepsilon_t^h h_t^{(1+\sigma_h)} \right) \right]^{-\gamma_c} \left(c_t - \varrho \frac{c_{t-1}}{e^{z_t}} \right) \varepsilon_t^h h_t^{\gamma_h}$$

$$\frac{\lambda_t^1}{R_t^P} = \beta E_t \left[\frac{\lambda_{t+1}^1 e^{-\gamma_C z_{t+1}}}{\pi_{t+1}} \right]$$

$$\frac{\lambda_t^1}{R_t} - L'(0) \frac{\varepsilon_t^b \varepsilon_t^s}{R_t} = \beta E_t \frac{\lambda_{t+1}^1}{\pi_{t+1}} e^{-z_{t+1} \gamma_C}$$

$$\lambda_t^1 = \varepsilon_t^i \lambda_t^2 \left((1 - S_t(\cdot)) - S_t'(\cdot) \frac{i_t}{i_{t-1}} \right) + \beta E_t \left[\varepsilon_{t+1}^i e^{(1-\gamma_C) z_{t+1}} \lambda_{t+1}^2 S_{t+1}'(\cdot) \frac{i_{t+1}^2}{i_t^2} \right]$$

$$\lambda_t^2 = \beta E_t \left[e^{-\gamma_C z_{t+1} - \omega_{t+1}} \left(\lambda_{t+1}^1 r_{t+1}^k u_{t+1} + \lambda_{t+1}^2 (1 - \delta(u_{t+1})) \right) \right]$$

$$\lambda_t^1 r_t^k = \lambda_t^2 \delta'(u_t)$$

$$k_t = (1 - \delta(u_t)) k_{t-1} e^{-z_t - \omega_t} + \varepsilon_t^i (1 - S(\cdot)) i_t$$

$$k_t^e = u_t k_{t-1} e^{-z_t - \omega_t}$$

Final Goods Price Index

$$1 = \left[(1 - \zeta_p) \tilde{p}_t^{\frac{1}{1-\lambda_{p,t}}} + \zeta_p \left(\pi_{t-1}^{\iota_p} \pi^{*(1-\iota_p)} \pi_t^{-1} \right)^{\frac{1}{1-\lambda_{p,t}}} \right]^{1-\lambda_{p,t}}$$

Intermediate Goods Firms: Capital-Labor Ratio

$$\frac{k_t^e}{h_t^d} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k}$$

Intermediate Goods Firms: Real Marginal Costs

$$mc_t = \frac{w_t^{1-\alpha} (r_t^k)^\alpha}{\varepsilon_t^a \alpha^\alpha (1-\alpha)^{1-\alpha}}$$

Intermediate Goods Firms: Price-Setting Equation

$$0 = E_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \lambda_{t+s}^1 \frac{\tilde{y}_{t,t+s}}{\lambda_{p,t+s} - 1} \left(\frac{A_{t+s}}{A_t} \right)^{1-\gamma_C} \left[\lambda_{p,t+s} mc_{t+s} - \tilde{X}_{t,s}^p \tilde{p}_t \right]$$

where

$$\tilde{X}_{t,s}^p = \left\{ \begin{array}{ll} 1 & : s = 0 \\ \frac{\prod_{j=1}^s \pi_{t+j-1}^{1-\iota_p} \pi_*^{\iota_p}}{\prod_{j=1}^s \pi_{t+j}} & : s = 1, \ldots, \infty \end{array} \right\}$$

$\tilde{y}_{t,t+s}$ denotes the time $t+j$ output sold by the producers that have optimized at time t the last time they have reoptimized. Since it can be shown that optimizing producers all choose the same price, then we do not have to carry the i-subscript.

Labor Packers: Aggregate Wage Index

$$w_t = \left[(1 - \zeta_w) \tilde{w}_t^{-\frac{1}{\lambda_{w,t}}} + \zeta_w \left(e^{\iota_w z_{t-1} - z_t} e^{(1-\iota_w) z_*} \pi_{t-1}^{\iota} \pi_t^{-1} \pi_*^{1-\iota_w} w_{t-1} \right)^{-\frac{1}{\lambda_{w,t}}} \right]^{-\lambda_{w,t}}$$

Guilds: Wage-Setting Equation

$$0 = E_t \sum_{s=0}^{\infty} \zeta_w^s \beta \lambda_{t+s}^1 \left(\frac{A_{t+s}}{A_t} \right)^{1-\gamma_C} \frac{\tilde{h}_{t,t+s}}{\lambda_{w,t+s}} \left((1 + \lambda_{w,t+s}) w_{t+s}^h - \tilde{X}_{t,s}^l \tilde{w}_t \right)$$

where

$$\tilde{X}_{t,s}^l = \left\{ \begin{array}{ll} 1 & : s = 0 \\ \frac{\prod_{j=1}^s \left(\pi_{t+j-1} e^{z_{t+j-1}} \right)^{1-\iota_w} (\pi\gamma)^{\iota_w}}{\prod_{j=1}^s \pi_{t+j} e^{z_{t+j}}} & : s = 1, \ldots, \infty \end{array} \right\}$$

$\tilde{h}_{t,t+s}$ denotes the time $t+j$ labor supplied by the guild that have optimized at time t the last time they have reoptimized. Since it can be shown that optimizing guilds all choose the same wage, then we do not have to carry the i-subscript.

Monetary Authority

$$R_t = R_{t-1}^{\rho_R}\left[r_*\pi_t^*\left(\prod_{j=-2}^{1}\frac{\pi_{t+j}}{\pi_t^*}\right)^{\frac{\psi_1}{4}}\left(\prod_{j=-2}^{1}\frac{y_{t+j}}{y^*}\right)^{\frac{\psi_2}{4}}\right]^{1-\rho_R}\prod_{j=0}^{M}\xi_{t-j,j}$$

The Aggregate Resource Constraint

$$\frac{y_t}{g_t} = c_t + i_t$$

Production Function

$$y_t = \varepsilon_t^a \left(k_t^e\right)^\alpha \left(h_t^d\right)^{1-\alpha} - \Phi$$

Labor Market Clearing Condition

$$h_t = h_t^d$$

4 Steady State

We normalize most shocks and the utilization rate:

$$u_\star = 1 \qquad \varepsilon^i = 1$$
$$\varepsilon^a = 1 \qquad \varepsilon^b = 1$$

Next, we set the following restriction on adjustment costs:

$$S(\cdot_*) \equiv 0$$
$$S'(\cdot_*) \equiv 0$$

4.1 Prices and Interest Rates

Given β, z_*, γ_C, and π_*, we can solve for the steady-state nominal interest rate on private bonds R^P_* by using the FOC on private bonds:

$$R^P_* = \frac{\pi_*}{(\beta e^{-\gamma_C z_*})} \tag{28}$$

From the definition of $\delta(u)$, we have

$$\begin{aligned} \delta(1) &= \delta_0 \\ \delta'(1) &= \delta_1. \end{aligned}$$

Next, given ω_*, δ_0, and the above, we can solve for the real return on capital r^k_* using the FOC on capital:

$$r^k_* = \frac{e^{\gamma_C z_* + \omega_*}}{\beta} - (1 - \delta_0) \tag{29}$$

4.2 Ratios

Moving to the production side, we can use the aggregate price equation to solve for $\tilde{p}_*$:

$$\tilde{p}_* = 1$$

Using this result and given $\lambda_{p,*}$, we can use the price Phillips curve to solve for mc_*:

$$mc_* = \frac{1}{1 + \lambda_{p,*}} \tag{30}$$

Given values for α and ε^a_*, we can use the marginal cost equation to solve for w_*:

$$w_* = \left(mc_* \alpha^\alpha (1-\alpha)^{1-\alpha} (r^k_*)^{-\alpha}\right)^{\frac{1}{1-\alpha}} \tag{31}$$

The definition of effective capital gives us a value for k_*^e in terms of k_*:

$$k_*^e = k_* e^{-z_* - \omega_*}$$

Calculating y_* using the labor share of output $1 - \alpha$:

$$y_* = \frac{w_* h_*}{1 - \alpha}$$

Using capital shares based off our value of α, we can calculate the output to capital ratio as follows:

$$\frac{y_*}{k_*^e} = \frac{r_*^k}{\alpha}$$
$$\frac{y_*}{k_*} = e^{-z_* - \omega_*} \frac{r_*^k}{\alpha}$$

Using the capital accumulation equation, we can get a value for $\frac{i_*}{k_*}$

$$\frac{i_*}{k_*} = 1 - (1 - \delta_0) e^{-z_* - \omega_*}$$

Using the resource constraint, we can get $\frac{c_*}{k_*}$:

$$\frac{c_*}{k_*} = \frac{y_*}{k_* s_*^g} - \frac{i_*}{k_*}$$

These ratios will give us the remaining steady-state levels and ratios:

$$k_* = y_* \left(\frac{y_*}{k_*}\right)^{-1} \qquad i_* = \frac{i_*}{k_*} k_*$$
$$c_* = \frac{c_*}{k_*} k_* \qquad g_* = g_y y_*$$

4.3 Liquidity Premium

Using the aggregate wage equation, we can get the following for $\tilde{w}_*$:

$$\tilde{w}_* = w_*$$

Combining this result with the wage Phillips curve, we get the following:

$$w^h_* = \frac{w_*}{1 + \lambda_{w,*}}$$

We can use the FOC for consumption and the labor supply to pin down ε^h and λ^1_*

$$\begin{aligned} \varepsilon^b \left[c_* \left(1 - \frac{\varrho}{e^z} \right) \right]^{-\gamma_c} \left(1 - \varepsilon^h h_*^{(1+\gamma_h)} \right) - \lambda^1_* = 0 \\ -(1+\gamma_h)\varepsilon^b c_*^{(1-\gamma_c)} \left(1 - \frac{\varrho}{\varepsilon^z} \right)^{(1-\gamma_c)} \left(1 - \varepsilon^h h_*^{(1+\gamma_h)} \right)^{-\gamma_c} \varepsilon^h h_*^{\gamma_h} + \lambda^1_* w^h_* = 0 \end{aligned}$$

Finally, the government bond rate is calculated from

$$\begin{aligned} \lambda^1_* - \varepsilon^b_* \varepsilon^s_* &= \beta R_* \frac{\lambda^1_*}{\pi_*} e^{-\gamma_C z} \\ \underbrace{\frac{\pi_*}{\beta e^{-\gamma_C z}}}_{R^P_*} - \varepsilon^b_* \varepsilon^s_* \frac{\pi_*}{\beta e^{-\gamma_C z} \lambda^1_*} &= R_* \end{aligned}$$

Noting that $R^P_* = \frac{\pi_*}{\beta e^{-\gamma_C z}}$ we can write

$$\frac{R^P_* - R_*}{R^P_*} = \frac{\varepsilon^b_* \varepsilon^s_*}{\lambda^1_*}.$$

This is the liquidity premium in steady state.

5 Log Linearization

Hatted variables refer to log deviations from steady-state ($\hat{x} = \ln\left(\frac{x_t}{x_*}\right)$):

$$\ln \varepsilon^j_t = \rho_j \ln \varepsilon^j_{t-1} + \eta^j_t$$

In the cases of z_t, ω_t, and ν_t, we have that $\hat{x} = x_t - x_*$ as these variables are already in logs.

Households' First Order Conditions

$$\hat{\varepsilon}_t^b - \hat{\lambda}_t^1 - \gamma_c \frac{1}{1 - \frac{\varrho}{e^z}} \hat{c}_t + \gamma_c \frac{\frac{\varrho}{e^z}}{1 - \frac{\varrho}{e^z}} (\hat{c}_{t-1} - \hat{z}_t) \tag{32}$$

$$\hat{\lambda}_t^1 + \hat{w}_t^h - \hat{\varepsilon}_t^b - \hat{\varepsilon}_t^h - \frac{1 - \gamma_c}{1 - \frac{\varrho}{e^z}} \hat{c}_t + (1 - \gamma_c) \frac{\frac{\varrho}{e^z}}{1 - \frac{\varrho}{e^z}} (\hat{c}_{t-1} - \hat{z}_t) \tag{33}$$

$$- \left(\gamma_h + \gamma_c (1 + \gamma_h) \frac{\varepsilon^h h_*^{1+\gamma_h}}{(1 - \varepsilon^h h_*^{1+\gamma_h})^2} \right) \hat{h}_t = 0$$

$$\hat{\lambda}_t^1 = \frac{R_*^P - R_*}{R_*^P} (\hat{\varepsilon}_t^s + \hat{\varepsilon}_t^b) + \frac{R_*}{R_*^P} (\hat{R}_t + E_t[(\hat{\lambda}_{t+1}^1 - \hat{\pi}_{t+1} - \gamma_C \hat{z}_{t+1}]) \tag{34}$$

$$\hat{\lambda}_t^1 = E_t \left[\hat{\lambda}_{t+1}^1 - \gamma_C \hat{z}_{t+1} + \hat{R}_t - \hat{\pi}_{t+1} \right] \tag{35}$$

$$\hat{\lambda}_t^1 = \left(\ln \varepsilon_t^i + \hat{\lambda}_t^2 \right) - S'' (\hat{\imath}_t - \hat{\imath}_{t-1}) + \beta e^{(1-\gamma_C)\gamma} S'' E_t (\hat{\imath}_{t+1} - \hat{\imath}_t) \tag{36}$$

$$\lambda_*^2 \hat{\lambda}_t^2 = \beta e^{-\gamma_C z_* - \omega_*} \left[\lambda_*^1 u_* r_*^k E_t \left(-\gamma_C \hat{z}_{t+1} - \hat{\omega}_{t+1} + \hat{\lambda}_{t+1}^1 + \hat{r}_{t+1}^k + \hat{u}_{t+1} \right) \right] + \tag{37}$$

$$+ \beta e^{-\gamma_C z_* - \omega_*} \left[(1 - \delta_0) \lambda_*^2 E_t \left(-\gamma_C \hat{z}_{t+1} - \hat{\omega}_{t+1} + \hat{\lambda}_{t+1}^2 \right) - \lambda_*^2 \delta_1 u_* E_t \hat{u}_{t+1} \right]$$

$$\hat{\lambda}_t^1 = \hat{\lambda}_t^2 + \frac{\delta_2}{\delta_1} u_* \hat{u}_t - \hat{r}_t^k \tag{38}$$

$$\hat{k}_t = \left(1 - \frac{\varepsilon_*^i i_*}{k_*} \right) \left(\hat{k}_{t-1} - \hat{z}_t - \hat{\omega}_t \right) + \frac{\varepsilon_*^i i_*}{k_*} \left(\hat{\varepsilon}_t^i + \hat{\imath}_t \right) - \delta_1 u_* e^{-z_* - \omega_*} \hat{u}_t \tag{39}$$

$$\hat{k}_t^e = \hat{u}_t + \hat{k}_{t-1} - \hat{z}_t - \hat{\omega}_t \tag{40}$$

Capital-Labor Ratio

$$\hat{k}_t^e = \hat{w}_t - \hat{r}_t^k + \hat{h}_t^d \tag{41}$$

Real Marginal Costs

$$\widehat{mc}_t = (1 - \alpha) \hat{w}_t + \alpha \hat{r}_t^k - \hat{\varepsilon}_t^a \tag{42}$$

The New Keynesian Phillips Curve for Inflation

$$\hat{\pi}_t = \frac{(1 - \beta \zeta_p e^{(1-\gamma_C) z_*})(1 - \zeta_p)}{(1 + \beta \iota_p e^{(1-\gamma_C) z_*}) \zeta_p} \left[\frac{\lambda_{p,*}}{1 + \lambda_{p,*}} \hat{\lambda}_{p,t} + \widehat{mc}_t \right] + \tag{43}$$

$$+ \frac{\iota_p}{1 + \beta \iota_p e^{(1-\gamma_C) z_*}} \hat{\pi}_{t-1} + \frac{\beta e^{(1-\gamma_C) z_*}}{1 + \beta \iota_p e^{(1-\gamma_C) z_*}} E_t \hat{\pi}_{t+1}$$

Wage Mark-Up

$$\hat{\mu}_t^w = \hat{w}_t - \hat{w}_t^h \tag{44}$$

The New Keynesian Phillips Curve for Wages

$$\hat{w}_t = \frac{1}{1+\beta e^{(1-\gamma_C)z_*}}\hat{w}_{t-1} + \frac{\beta e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}\hat{w}_{t+1} + \frac{\beta e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}(E_t\hat{\pi}_{t+1} + E_t\hat{z}_{t+1}) + \tag{45}$$

$$\frac{\iota_w}{1+\beta e^{(1-\gamma_C)z_*}}(\hat{\pi}_{t-1} + \hat{z}_{t-1}) - \frac{1+\iota_w\beta e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}(\hat{\pi}_t + \hat{z}_t) +$$
$$\frac{1-\beta\zeta_w e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}\frac{1-\zeta_w}{\zeta_w}\left[\frac{\lambda_{w,*}}{1+\lambda_{w,*}}\hat{\lambda}_{w,t} - \hat{\mu}_t^w\right]$$

The Aggregate Resource Constraint

$$\frac{y_*}{g_*}(\hat{y}_t - \hat{g}_t) = \frac{c_*}{c_*+i_*}\hat{c}_t + \frac{i_*}{c_*+i_*}\hat{\imath}_t \tag{46}$$

The Production Function

$$\hat{y}_t = \frac{1}{mc_*}\left(\ln\varepsilon_t^a + \alpha\hat{k}_t^e + (1-\alpha)\,\hat{h}_t^d\right) \tag{47}$$

Labor Market Clearing Condition

$$\hat{h}_t = \hat{h}_t^d \tag{48}$$

Monetary Authority's Reaction Function

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left[(1-\psi_1)\,\hat{\pi}_t^* + \frac{\psi_1}{4}\left(\sum_{j=-2}^{1}\hat{\pi}_{t+j}\right) + \frac{\psi_2}{4}\left(\sum_{j=-2}^{1}\hat{y}_{t+j}\right)\right] + \sum_{j=0}^{M}\hat{\xi}_{t-j,j} \tag{49}$$

6 Measurement

6.1 National Income Accounts

The model economy's basic structure, with the representative household consuming a single good and accumulating capital using a different good, differs in some important ways from the accounting conventions of the Bureau of Economic Analysis (BEA) underlying the National Income and Product Accounts (NIPA). In particular

1. The BEA treats household purchases of long-lived goods inconsistently. It classifies purchases of residential structures as investment and treats the service flow from their stock as part of Personal Consumption Expenditures (PCE) on services. The BEA classifies households purchases of all other durable goods as consumption expenditures. No service flow from the stock of household durables enters measures of current consumption. In the model, all long-lived investments add to the productive capital stock.

2. The BEA treats all government purchases as government consumption. However, government at all levels makes purchases of investment goods on behalf of the populace. In the model, these should be treated as additions to the single stock of productive capital.

3. The BEA sums PCE and private expenditures on productive capital (Business Fixed Investment and Residential Investment), with government spending, inventory investment, and net exports to create Gross Domestic Product. The model features only the first three of these.

To bridge these differences, we create four *model consistent* NIPA measures from the BEA NIPA data.

1. Model-consistent GDP. Since the model's capital stock includes both the stock of household durable goods and the stock of government-purchased capital, a model-consistent GDP series should include the value of both stocks' service flows. To construct these, we followed a five-step procedure.

   (a) We begin by estimating a constant (by assumption) service-flow rate by dividing the nominal value of housing services from NIPA Table 2.4.5 by the beginning-of-year value of the residential housing stock from the

BEA's Fixed Asset Table 1.1. We use annual data and average from 1947 through 2014. The resulting estimate is 0.096. That is, the annual value of housing services equals approximately 10 percent of the housing stock's value each year.

(b) In the second step, we estimate estimate constant (by assumption) depreciation rates for residential structures, durable goods, and government capital. We constructed these by first dividing observations of value lost to depreciation over a calendar year by the end-of-year stocks. Both variables were taken from the BEA's Fixed Asset Tables. (Table 1.1 for the stocks and Table 1.3 for the deprecation values.) We then averaged these ratios from 1947 through 2014. The resulting estimates are 0.021, 0.194, and 0.044 for the three durable stocks.

(c) In the third step, we calculated the average rates of real price depreciation for the three stocks. For this, we began with the nominal values and implicit deflators for PCE Nondurable Goods and PCE Services from NIPA Table 1.2. We used these series and the Fisher-ideal formula to produce a chain-weighted implicit deflator for PCE Nondurable Goods and Services. Then, we calculated the price for each of the three durable good's stocks in consumption units as the ratio of the implicit deflator taken from Fixed Asset Table 1.2 to this deflator. Finally, we calculated average growth rates for these series from 1947 through 2014. The resulting estimates equal 0.0029, −0.0223, and 0.0146 for residential housing, household durable goods, and government-purchased capital.

(d) The fourth combines the previous steps' calculations to estimate constant (by assumption) service-flow rates for household durable goods and government-purchased capital. To implement this, we assumed that all *three* stocks yield the same financial return along a steady-state growth path. These returns sum the per-unit service flow with the appropriately depreciated value of the initial investment. This delivers two equations in two unknowns, the two unknown service-flow rates. The resulting estimates are 0.29 and 0.12 for household durable goods and government-purchased capital.

(e) The fifth and final step uses the annual service-flow rates to calculate real and nominal service flows from the real and nominal stocks of durable goods and government-purchased capital reported in Fixed Asset Table 1.1. This delivers an annual series. Since the stocks are measured as of the end of the calendar year, we interpret these as the service flow values in the *next* year's first quarter. We create quarterly data by linearly interpolating between these values.

With these real and nominal service flow series in hand, we create nominal model-consistent GDP by summing the BEA's definition of nominal GDP with the nominal values of the two service flows. We create the analogous series for model-consistent real GDP by applying the Fisher ideal formula to the nominal values and price indices for these three components.

2. Model-consistent Investment. The nominal version of this series sums nominal Business Fixed Investment, Residential Investment, PCE Durable Goods, and government investment expenditures. The first three of these come from NIPA Table 1.1.5, while government investment expenditures sums Federal Defense, Federal Nondefense, and State and Local expenditures from NIPA Table 1.5.5. We construct the analogous series for real Model-consistent Investment by combining these series with their real chain-weighted counterparts found in NIPA Tables 1.1.3 and 1.5.3 using the Fisher ideal formula. By construction, this produces an implicit deflator for Model-consistent investment as well.

3. Model-consistent Consumption. The nominal version of this series sums nominal PCE Nondurable Goods, PCE Services, and the series for nominal services from the durable goods stock. The first two of these come from NIPA Table 1.1.5. We construct the analogous series for real Model-consistent consumption by combining these series with their real chain-weighted counterparts using the Fisher ideal formula. The two real PCE series come from NIPA Table 1.1.3. Again, this produces an implicit deflator for Model-consistent consumption as a by-product.

4. Model-consistent Government Purchases. Conceptually, the model's measure of Government Purchases includes all expenditures not otherwise classified as Investment or Consumption: Inventory Investment, Net Exports, and actual Government Purchases. We construct the nominal version of this series simply

by subtracting nominal Model-consistent Investment and Consumption from nominal Model-consistent GDP. We calculate the analogous real series using "chain subtraction." This applies the Fisher ideal formula to Model-consistent GDP and the *negatives* of Model-consistent Consumption and Investment.

Our empirical analysis requires us to compare model-consistent series measured from the NIPA data with their counterparts from the model's solution. To do this, we begin by solving the log-linearized system above, and then we feed the model specific paths for all exogenous shocks starting from a particular initial condition. for a given such simulation, the growth rates of Model-consistent Consumption and Investment equal

$$\begin{aligned} \Delta \ln C_t^{obs} &= z_* + \Delta \hat{c}_t + z_t \text{ and} \\ \Delta \ln I_t^{obs} &= z_* + \omega_* + \Delta \hat{i}_t + z_t + \omega_t \end{aligned}$$

The measurement of GDP growth in the model is substantially more complicated, because the variables Y_t and y_t denote model output *in consumption units.* In contrast, we mimic the BEA by using a chain-weighted Fisher ideal index to measure model-consistent GDP. Therefore, we construct an analogus chain-weighted GDP index from model data. Since such an ideal index is invariant to the units with which nominal prices are measured, we can normalize the price of consumption to equal one and employ the prices of investment goods and government purchases relative to current consumption. Our model identifies the first of these relative prices as with investment-specific technology. However, the model characterizes only government purchases *in consumption units*, because private agents do not care about their division into "real" purchases and their relative price. For this reason, we use a simple autoregression to characterize the evolution of the price of government services in consumption units. Denote this price in quarter t with P_t^g. We construct this for the US economy by dividing the Fisher-ideal price index for model-consistent government purchases by that for model-consistent consumption. Then, our model for its evolution is

$$\pi_t^{g,obs} = \ln(P_t^g / P_{t-1}^t) = (1-\beta_{2,1}-\beta_{2,2})\pi_g^* + \beta_{2,1}\ln(P_{t-1}^g / P_{t-2}^g) + \beta_{2,2}\ln(P_{t-2}^g / P_{t-3}^g) + u_t^g. \quad (50)$$

Here, $u_t^g \sim \mathbb{N}(0, \sigma_g^2)$. Given an arbitrary normalization of P_t^g to one for some time period, simulations from (50) can be used to construct simulated values of P_t^g for all other time periods. With these and a simulation from the model of all other variables in hand, we can calculate the simulation's values for Fisher ideal GDP growth using

$$\frac{Q_t}{Q_{t-1}} \equiv \sqrt{\dot{Q}_t^P \dot{Q}_t^L}, \tag{51}$$

where the Paasche and Laspeyres indices of quantity growth are

$$\dot{Q}_t^P \equiv \frac{C_t + P_t^I I_t + P_t^G (G_t / P_t^G)}{C_{t-1} + P_t^I I_{t-1} + P_t^G (G_{t-1} / P_{t-1}^g)} \text{ and} \tag{52}$$

$$\dot{Q}_t^L \equiv \frac{C_t + P_{t-1}^I I_t + P_{t-1}^G (G_t / P_t^G)}{C_{t-1} + P_{t-1}^I I_{t-1} + P_{t-1}^G (G_{t-1} / P_{t-1}^G)}. \tag{53}$$

In both (52) and (53), P_t^I is the relative price of investment to consumption. In equilibrium, this always equals A_t^I.

The above gives a complete recipe for *simulating* the growth of model-consistent real GDP growth. However, we also embody its insights into our estimation with a log-linear approximation. For this, we start by removing stochastic trends from all variables in (52) and (53), and we proceed by taking a log-linear approximation of the resulting expression. Details are available from the authors upon request.

6.2 Hours Worked Measurement

Empirical work using DSGE models like our own typically measure labor input with hours worked per capita, constructed directly from BLS measures of hours worked and the civilian non-institutional population over age 16. However, this measure corresponds poorly with business cycle models because it contains underlying low frequency variation. This fact led us to construct a new measure of hours for the model using labor market trends produced for the FRB/US model and for the Chicago Fed's in-house labor market analysis.

We begin with a multiplicative decomposition of hours worked per capita into hours per worker, the employment rate of those in the labor force, and the labor-force participation rate. The BLS provides CPS-based measures of the last two rates for the US as a whole. However, its measure of hours per worker comes from the Establishment Survey and covers only the private business sector. If we use hours per worker in the business sector to approximate hours per worker in the economy as a whole, then we can measure hours per capita as

$$\frac{H_t}{P_t} = \frac{H_t^E}{E_t^E}\frac{E_t^C}{L_t^C}\frac{L_t^C}{P_t^C}.$$

Here, H_t and P_t equal total hours worked and the total population, H_t^E/E_t^E equals hours per worker measured with the Establishment survey, E_t^C/L_t^C equals one minus the CPS based unemployment rate, and L_t^C/P_t^C equals the CPS based labor-force participation rate. Our measure of *model-relevant* hours worked deflates each component on the right-hand side by an exogenously measured trend. The trend for the unemployment rate comes from the Chicago Fed's Microeconomics team, while those for hours per worker and labor-force participation come from the FRB/US model files.

6.3 Inflation

Our empirical analysis compares model predictions of price inflation, wage inflation, inflation in the price of investment goods relative to consumption goods, and inflation expectations with their observed values from the U.S. economy. We describe our implementations of these comparisons sequentially below.

6.3.1 Price Inflation

Our model directly characterizes the inflation rate for Model-consistent Consumption. In principle, this is close to the FOMC's preferred inflation rate, that for the implicit deflator of PCE. However, in practice the match between the two inflation rates is poor. In the data, short-run movements in food and energy prices substantially influences the short-run evolution of PCE inflation. Our model lacks such a volatile sector, so if we ask it to match observed short-run inflation dynamics, it will attribute those to transitory shocks to intermediate goods' producers' desired markups driven by λ_t^p.

To avoid this outcome, we adopt a different strategy for matching model and data inflation rates, which follows that of Justiniano, Primiceri, and Tambalotti (2013). This relates three observable inflation rates – core CPI inflation, core PCE inflation, and market-based PCE inflation – to Model-consistent consumption inflation using auxiliary observation equations. For core PCE inflation, this equation is

$$\pi_t^{1,obs} = \pi_* + \pi_*^1 + \beta^{\pi,1}\hat{\pi}_t + \gamma^{\pi,j}\pi_t^{d,obs} + u_t^{\pi,1}, \tag{54}$$

In (54) as elsewhere, π_* equals the long-run inflation rate. The constant π_*^1 is an adjustment to this long-run inflation rate which accounts for possible long-run differences between realized inflation and the FOMC's goal of $\pi_\star$ (for PCE inflation π_*^1 is set to zero). The right-hand side's inflation rates, $\hat{\pi}_t$ and $\pi_t^{d,obs}$ equal Model-consistent consumption inflation and PCE Durables inflation. We refer to the coefficients multiplying them, $\beta^{\pi,1}$ and $\gamma^{\pi,1}$, as the *inflation loadings*. We include PCE Durables inflation on the right-hand side of (54) because the principle adjustment required to transform Model-consistent inflation into core PCE inflation is the replacement of the price index for durable goods services with that for durable goods purchases. The disturbance term $u_t^{\pi,1}$ follows a zero-mean first-order autoregressive process.

The other two observed inflation measures, market-based PCE inflation and core CPI inflation, have identically specified observation equations. We use 2 and 3 in superscripts to denote these equations parameters and error terms, and we use the same expressions as subscripts to denote the parameters governing the evolution of their error terms. We assume that the error terms $u_t^{\pi,1}$, $u_t^{\pi,2}$, and $u_t^{\pi,3}$ are independent of each other at all leads and lags.

To produce forecasts of inflation with these these three observation equations, we must forecast their right-hand side variables. The model itself gives forecasts of $\hat{\pi}_t$. The forecasts of durable goods inflation come from a second-order autoregression.

$$\pi_t^{d,obs} = (1 - \beta_{1,1} - \beta_{1,2})\pi_*^d + \beta_{1,1}\pi_{t-1}^{d,obs} + \beta_{1,2}\pi_{t-2}^{d,obs} + u_t^d \tag{55}$$

Its innovation is normally distributed and serially uncorrelated.

6.3.2 Wage Inflation

Although observed wage inflation does not feature the same short-run variability as does price inflation, it does include the influences of persistent demographic labor-market trends which we removed ex ante from our measure of hours worked. Therefore, we follow the same general strategy of relating observed measures of wage inflation to the model's predicted wage inflation with a error-augmented observation equation. For this, we employ two measures of compensation per hour, Earnings per Hour and Total Compensation per Hour. In parallel with our notation for inflation measures, we use 1 and 2 to denote these two wage measures of wage inflation. The observation equation for Earnings per Hour is

$$\Delta \ln w_t^{1,obs} = z_* + w_*^j + \beta^{w,1}\left(\hat{w}_t - \hat{w}_{t-1} + \hat{z}_t\right) + u_t^{w,1}, \tag{56}$$

where "Δ" is the first difference operator. Just as with the price inflation measurement errors, $u_t^{w,1}$ follows a zero-mean first-order autoregressive process. The observation equation for Total Compensation per Hour is analogous to (56).

6.3.3 Relative Price Inflation

To empirically ground investment-specific technological change in the model, we use an error-augmented observation equation to relate the relative price of investment to consumption, both model-consistent measures constructed from NIPA and Fixed Asset tables as described above, with the model's growth rate of the rate of technological transformation between these two goods, ω_t.

$$\pi_t^{i,obs} = \omega_* + \hat{\omega}_t + u_t^{c/i};$$

Here, $\pi_t^{i,obs}$ denotes the price of *c*onsumption relative to *i*nvestment. The measurement error $u_t^{c/i}$ follows a i.i.d. zero-mean normally-distributed innovation.

6.3.4 Inflation Expectations

We also discipline our model's inferences about the state of the economy by comparing expectations of one-year and 10-year inflation from the Survey of Professional Forecasters with the analogous expectations from our model. Just as with all of the other inflation measures, we allow these two sets of expectations to differ from each other by including serially correlated measurement errors. The observation equations are

$$\pi_t^{l,j,obs} = \pi_* + \pi_*^{l,j} + \frac{\beta^{l,j}}{l}\sum_{i=1}^{l} E_t\hat{\pi}_{t+i} + u_t^{l,j,\pi},\ j = 1,2,\ l = 1,40;$$

The two measurement errors follow mutually-independent first-order autoregressive processes.

6.4 Interest Rates and Monetary Policy Shocks

Since our model features forward guidance shocks, it has non-trivial implications for the current policy rate as well as for expected future policy rates. To discipline the parameters governing their realizations, the elements of Σ_ε, using data, we compare the model's monetary policy shocks to high-frequency interest-rate innovations informed by event studies, such as that of Gürkaynak, Sack, and Swanson (2005). Those authors applied a factor structure to innovations in implied expected interest rates from futures prices around FOMC policy announcement dates. Specifically, they show that the vector of M implied interest rate changes following an FOMC policy announcement, Δr_t, can be written as

$$\Delta r_t = \Lambda f_t + \eta_t$$

Where f is a 2×1 vector of factors, Λ is a $H \times 2$ matrix of factor loadings, and η is an $H \times 1$ vector of mutually independent shocks. Denoting the 2×2 diagonal variance covariance matrix of f with Σ_f and the $H \times H$ diagonal variance-covariance matrix of η with Ψ, we can express the observed variance-covariance matrix of Δr as $\Lambda \Sigma_f \Lambda' + \Psi$.

Our model has implications for this same variance covariance matrix. For this, use the model's solution to express the changes in current and future expected interest rates following monetary policy shocks as $\Delta r = \Gamma_1 \varepsilon^R$. Here, ε_t^R is the vector which collects the current monetary policy shock with $M-1$ forward guidance shocks, and Γ_1 is an $H \times H$ matrix. In general, Γ_1 does *not* simply equal the identity matrix, because current and future inflation and output gaps respond to the monetary policy shocks and thereby influence future monetary policy "indirectly" through the interest rate rule.

We assume that a factor structure determines the cross-correlations among monetary policy shocks. Specifically, we assume

$$\varepsilon_{R,t}^j = \alpha_j f_t^\alpha + \beta_j f_t^\beta + \eta_t^j,$$

where the factors f_t^α and f_t^β and factor loadings α_i and β_i are scalars, η_t^j is a measurement error. The factors and shocks have zero means and are independent and normally distributed. In matrix notation, we have

$$\varepsilon_t^R = \boldsymbol{\alpha} f_t^\alpha + \boldsymbol{\beta} f_t^\beta + \eta_t,$$

where $\boldsymbol{\alpha} = [\alpha_0, \ldots, \alpha_H]'$, $\boldsymbol{\beta} = [\beta_0, \ldots, \beta_H]'$. Let $\Sigma_\eta = E(\eta_t \eta_t')$ denote the variance-covariance matrix of the idiosyncratic shocks, and σ_α^2 (σ_β^2) denote the variance of f_t^α (f_t^β). Therefore we have that

$$\Lambda \Sigma_f \Lambda' + \Psi = \Gamma_1 (\boldsymbol{\alpha}\boldsymbol{\alpha}' \sigma_\alpha^2 + \boldsymbol{\beta}\boldsymbol{\beta}' \sigma_\beta^2) \Gamma_1' + \Gamma_1 \Sigma_\eta \Gamma_1'$$

6.5 Measurement Equations Synthesis

To summarize the measurement equations are as follows:

$$
\begin{aligned}
\Delta \ln Q_t^{obs} &= f\left(\hat{c}_t, \hat{c}_{t-1}, \hat{i}_t, \hat{i}_{t-1}, \hat{g}_t, \hat{\omega}_t, \hat{\pi}_t^{g,obs}\right); \\
\Delta \ln C_t^{obs} &= z_* + \Delta \hat{c}_t + \hat{z}_t; \\
\Delta \ln I_t^{obs} &= z_* + \omega_* + \Delta \hat{\imath}_t + \hat{z}_t + \hat{\omega}_t; \\
\log H_t^{obs} &= \hat{H}_t; \\
\pi_t^{i,obs} &= \omega_* + \hat{\omega}_t + u_t^i; \\
R_t^{obs} &= R_* + \hat{R}_t; \\
R_t^{j,obs} &= R_* + E_t \hat{R}_{t+j}, \ j = 1, 2, \ldots, H; \\
\pi_t^{l,j,obs} &= \pi_* + \pi_*^{l,j} + \frac{\beta^{l,j}}{l} \sum_{i=1}^{l} E_t \hat{\pi}_{t+i} + u_t^{l,j,\pi}, \ j = 1, 2, \ l = 1, 40; \\
\pi_t^{j,obs} &= \pi_* + \pi_*^j + \beta^{\pi,j} \hat{\pi}_t + \gamma^{\pi,j} \pi_t^{d,obs} + u_t^{j,p}, \text{ with } \beta^{\pi,1} = 1, j = 1, 2, 3; \\
\Delta \ln w_t^{j,obs} &= z_* + w_*^j + \beta^{w,j} \left(\hat{w}_t - \hat{w}_{t-1} + \hat{z}_t\right) + u_t^{j,w}, \text{ with } \beta^{w,1} = 1, j = 1, 2; \\
\pi_t^{d,obs} &= (1 - \beta_{1,1} - \beta_{1,2}) \pi_*^d + \beta_{1,1} \pi_{t-1}^{d,obs} + \beta_{1,2} \pi_{t-2}^{d,obs} + u_t^d; \\
\pi_t^{g,obs} &= (1 - \beta_{2,1} - \beta_{2,2}) \pi_*^g + \beta_{2,1} \pi_{t-1}^{g,obs} + \beta_{2,2} \pi_{t-2}^{g,obs} + u_t^g.
\end{aligned}
$$

The left hand side variables represent data (Q denotes chain-weighted GDP). The function f in the first equation represents the linear approximation to the chain-weighted GDP formula. As previously discussed, two variables are included to complete the mapping from model to data but are not endogenous to the model.

Specifically, the consumption price of government consumption plus net exports, $\pi_t^{g,obs}$, helps map model GDP to our model-consistent measure of chain-weighted GDP, and inflation in the consumption price of consumer durable goods, $\pi_t^{d,obs}$, is used to complete the mapping from model inflation to measured inflation.

The measurement equations indicate we use 21 time series to estimate the model in the first sample. In addition to the real quantities and federal funds rate that are standard in the literature our estimation includes multiple measures of wage and consumer price inflation, two measures each of average inflation expected over the next ten years and over one quarter, and $H = 4$ quarters of interest rate futures. Our second sample estimation is restricted to estimating the parameters of the stochastic process for forward guidance news with $H = 10$ plus the processes driving $\pi_t^{g,obs}$ and $\pi_t^{d,obs}$. This estimation uses the measurement equations involving the current federal funds rate and 10 quarters of expected future policy rates plus the last two equations. We take into account the change in steady state but keep the remaining structural parameters at their first sample values. Because our estimation forces data on real activity, wages and prices to coexist with the interest rate futures data, we expect the estimation to mitigate the forward guidance puzzle. Finally, it is worth stressing that our estimation respects the ELB in the second sample. This is because we measure expected future rates in the model, the $E_t\hat{R}_{t+j}$, using the corresponding empirical futures rates, $R_t^{j,obs}$, and we use futures rates extending out 10 quarters.

6.6 Data Synopsis

Dates:

- Our dates are quarterly and formatted as YYYY as quarter 1, YYYY.25 as quarter 2, YYYY.5 as quarter 3, and YYYY.75 as quarter 4 for the year YYYY.

Model-Consistent Output: gdp_pcLD100

- The DSGE model output is the chained sum of conventional GDP with government capital services and durable goods services. This series is detrended by population growth.

Model-Consistent Consumption: cons_pcLD100

- DSGE consumption is defined as the chained sum of conventional PCE nondurable goods with PCE services and durable goods services. This series is de-trended by population growth.

Model-Consistent Investment: inv_pcLD100

- Model-consistent Investment is the chained sum of durable goods purchases, fixed investment, and government investment. This series is de-trended by population growth.

Model-Consistent Residual Output Inflation: gnx_CONSINF

- The residual output is the chained difference of model consumption and investment from model GDP. Residual output reflects government spending and net exports.

Relative Price of Consumption to Investment: RPCtoI_LD100

- The relative price is constructed by dividing the consumption price series and investment price series.

Deflators for Consumer Durables: JCD_LD100

- We take the log difference[2] of the PCE Durable Goods Chain Price Index for the deflators for consumer durables.

Inflation Expectations: inf_10YQ_PCE, ASAF1CPX, inf_10YQ_CPI, ASAF1CX

- Our inflation expectations series are quarterly inflation expectations data from the Survey of Professional Forecasters at the Philadelphia Fed. They report inflation expectations at various horizons for both PCE and CPI measures. We use measures of 1Q ahead and 40Q ahead CPI and core PCE inflation expectations in the model. The 40Q ahead series are the ten-year ahead expectations, not the annual average over the next ten years. The SPF did not report expectations for core PCE prior to 2007, so we do not have many observations for the first sample of our data. However, we continue to include these few observations in order to initialize the kalman filter for second sample estimation. We have the full data for CPI expectations.

[2] All log differenced series are multipled by 100.

Real Wages: lepriva_CORE, ls_CORE

- We have two different measures of wages in the model - average hourly earnings and employment compensation. We take the average hourly earnings and divide by the chain price index of core PCE, then take the log difference.
- We repeat the same steps to calculate employment compensation but use the employment cost index for the compensation of civilian workers.

Price Inflation: JCXFE_LD100, JCMXFE_LD100, PCUSLFE_LD100

- We use three different measures of price inflation: Core PCE, Market-Based Core PCE, and Core CPI.

Hours: hours_L

- We construct our hours series with the methodology as described in *Forward Guidance and Macroeconomic Outcomes Since the Financial Crisis* (Campbell et al., 2016).

Effective Federal Funds Rate: ffed_q

- For the first sample (1993q1-2008q3), we use the federal funds target rate observed as the average over the last month of the quarter.
- For the second sample (2008q4-2018q4), we use the federal funds target rate observed at the end of the quarter.
- We divide the series by 4 to convert to quarterly rates.

Expected Federal Funds Rate (FFR): 1-10QAhead

- From 1993Q1 to 2005Q4, our 4-quarter ahead path comes from Eurodollar futures. Eurodollar futures have expiration dates that lie about two weeks before the end of each quarter. Eurodollar rate is closely tied to expectations for the Federal Funds rates over the same period, so the Eurodollar futures rate corresponds with the Fed Funds rate at the middle of the last month of each quarter.

- Beginning with 2006Q1, our 4-quarter ahead, and later, 10-quarter ahead path comes from the Overnight Index Swaps (OIS). The OIS data are converted into a point estimate of the Fed Funds for a particular date using a Svensson term structure model. The dates of the OIS data reflect the middle of the quarter values, and we interpolate to obtain the end of quarter values.
- From 2014Q1, we began to use the expected Fed Funds from the Survey of Market Participant (SMP). The SMP correspond to the survey participants' expected Fed Funds at the end of the quarter.
- The path for the current forecasting quarter is the most recently released SMP path adjusted with the difference between the SMP date OIS and the forecasting date OIS.
- All expected FFR series are in quarterly rates.

7 Calibration and Bayesian Estimation

As we discussed, we follow a two-stage approach to the estimation of our model's parameters. In a calibration stage, we set the values of selected parameters so that the model has empirically-sensible implications for long-run averages from the U.S. economy. In this stage, we also enforce several normalizations and a judgemental restriction on one of the measurement error variances. In the second stage, we estimate the model's remaining parameters using standard Bayesian methods.

7.1 Calibration

Our calibration strategy is the same as in Campbell, Fisher, Justiniano, and Melosi (2016) except that we address the well-known evidence of secular declines in economic growth and rates of return on nominally risk free assets. We address these developments by imposing a change in steady state in 2008q4 (the choice of this date is motivated in the next subsection). Steady state GDP growth is governed by the mean growth rates of the neutral and investment-specific technologies, ν_* and ω_*. We adjust ω_* down to account for the slower decline in the relative price of investment since 2008q4. Given this change we then lower ν_* so that steady state GDP growth is reduced to 2%. To match a lower real risk-free rate of 1% we increase the steady state marginal utility of government bonds using ε^s_*.[3] These adjustments leave the other calibrated parameters unchanged but do change the steady state values of the endogenous variables and therefore the point at which the economy is log-linearized.[4]

We observe the long-run average of the following aggregates: nominal federal funds rate, labor share, government spending share, investment spending share, the capital-output ratio, real per-capita GDP growth (g_y), inflation in price of government, net exports and inventory investment relative to non-durables and services consumption, and the growth rate of the consumption-investment relative price.

- The labor share can be used to calibrate the parameter α.
- The government spending share determines s^g_*.
- The government price growth rate pins down π^g_*.
- The growth rate of the consumption-investment relative price pins down ω_*.
- The investment share pins down i_*/y_*.
- The capital output ratio pins down k_*/y_*.
- Calculate the consumption-output share

$$\frac{c_*}{y_*} = \left(1 - \frac{i_*}{y_*} - \frac{g_*}{y_*}\right). \tag{57}$$

- The growth rate of real chain-weighted GDP is used to pin down the growth rate of the common trend z_*. First

$$g_y = e^{z_*}\sqrt{\frac{\frac{c_*}{y_*} + e^{\omega}\frac{i_*}{y_*} + \left(\pi^g_*\right)^{-1}\frac{g_*}{y_*}}{\frac{c_*}{y_*} + e^{-\omega}\frac{i_*}{y_*} + \pi^g_*\frac{g_*}{y_*}}}$$

[3]The targets for steady state GDP growth and risk-free rate reflect a variety of evidence including the Fed's Summary of Economic Projections.

[4]Our re-calibration changes the return on private assets by a little. This small change is consistent with **?** who show that rates of return on private capital have stayed roughly constant in the face of declines in risk free rates.

All the variables in this equation are known except for z_*. So we can solve for z_*:

$$z_* = g_y - \frac{1}{2}\ln\left(\frac{\frac{c_*}{y_*} + e^{\omega}\frac{i_*}{y_*} + \left(\pi_*^g\right)^{-1}\frac{g_*}{y_*}}{\frac{c_*}{y_*} + e^{-\omega}\frac{i_*}{y_*} + \pi_*^g\frac{g_*}{y_*}}\right) \tag{58}$$

- The growth rate of the labor-augmenting technology ν_* can be easily obtained by exploiting the following equation:

$$z_* = \upsilon_* + \frac{\alpha}{1-\alpha}\omega_*. \tag{59}$$

- We are now in a position to identify the depreciation rate δ_0 using the steady-state equation pinning down the investment capital ratio:

$$\begin{aligned}\frac{i_*}{k_*} &= 1 - (1-\delta_0)e^{-z_*-\omega_*} \\ &\Rightarrow \delta_0 = 1 + \left(\frac{i_*}{k_*} - 1\right)e^{z_*+\omega_*}\end{aligned}$$

where the investment capital ratio is obtained combining the investment share and the capital output ratio:

$$\frac{i_*}{k_*} = \frac{i_*/y_*}{k_*/y_*}. \tag{60}$$

- From the steady-state equilibrium we have that

$$\frac{y_*}{k_*} = e^{-z_*-\omega_*}\frac{\delta_1}{\alpha}. \tag{61}$$

Therefore

$$\delta_1 = \alpha\left(\frac{k_*}{y_*}\right)^{-1} e^{z_*+\omega_*} \tag{62}$$

where the capital output ratio is given above.

- In steady state, the real rate of return on private bonds is derived from the first order condition for private bonds:

$$r_*^p \equiv \frac{R_*^P}{\pi_*} = \frac{e^{\gamma_c z_*}}{\beta}. \tag{63}$$

In steady state the real rental rate of capital is derived from the first order condition for capital:

$$r_*^k = \left[\frac{e^{\gamma_c z_*}}{\beta}\right] e^{\omega_*} - (1 - \delta_0) \tag{64}$$

Combining these last two equations yields

$$r_*^k = r_*^p e^{\omega_*} - (1 - \delta_0)$$

and hence

$$r_*^p = \left[r_*^k + 1 - \delta_0\right] e^{-\omega_*}.$$

Note that $r_*^k = \delta_1$ from the first order condition for capacity utilization. It follows that

$$r_*^p \quad = \quad (1 - \delta_0 + \delta_1)\, e^{-\omega_*}$$

- The liquidity premium in steady state (i.e., $\frac{R_*/\pi_*}{r_*^p}$) can be computed now by assuming a *nominal* average federal funds rate, R_*, and an annualized average inflation rate.

- Using equation (64) and the fact that $r_*^k = \delta_1$, we can calibrate the discount factor β :

$$\beta = (1 - \delta_0 + \delta_1)^{-1}\, e^{\omega_*} e^{\gamma_c z_*}$$

where γ_c is a parameter of the utility function to be estimated.

7.2 Bayesian Estimation

Our Bayesian estimation uses the same split-sample strategy as in Campbell, Fisher, Justiniano, and Melosi (2016) except that we incorporate the change in steady state described above and one other change noted below. As in Campbell, Fisher, Justiniano, and Melosi (2016) our sample begins in 1993q1. This date is based on the availability and reliability of the overnight interest rate futures data. The sample period ends in 2016q4 but we impose a sample break in 2008q4. Our choice of this latter date is motivated by three main considerations. First, there is the evidence that points to lower interest rates and economic growth later in the sample. Second, it seems clear that the horizon over which forward guidance was communicated by the Fed lengthened substantially during the ELB period. Finally, the downward trends in inflation and inflation expectations from the early 1990s appear to come to an end in the mid-2000s. Splitting the sample in 2008q4 and assuming some parameters change at that date is our way of striking a balance between parsimony and addressing the multiple structural changes that seem to occur around the same time.

We estimate the full suite of non-calibrated structural parameters in the first sample under the assumption that forward guidance extends for $H = 4$ quarters. Starting in 2008q4 we assume the model environment changes in three ways. First we assume the change in the steady state described above. Second, forward guidance lengthens to $H = 10$ quarters Third, the time-varying inflation target from the first sample becomes a constant equal to the steady state rate of inflation, 2% at an annual rate. All three changes are assumed to be unanticipated and permanent.

We employ standard prior distributions, but those governing monetary policy shocks deserve further elaboration. Our estimation requires the variance-covariance matrix of monetary policy shocks to be consistent with the factor-structure of interest rate innovations used by Gürkaynak, Sack, and Swanson (2005), as described above. Therefore, we parameterize Σ_ε in terms of factors STD (σ_α and σ_β), factor loadings (α and β) and STD of the idiosyncratic errors ($\sigma_{\eta,j}$). We then center our priors for these parameters at their estimates from event-studies. However, we do not require our estimates to equal their prior values. Our Bayesian estimation procedure employs quarterly data on expected future interest rates, the posterior likelihood function includes them as free parameters. It is well known that factors STD and loadings are not separately identified, so we impose two scale normalizations and one rotation normalization on α and β. The rotation normalization requires that the first factor, which we label "Factor A", is the only factor influence the current policy rate. That is, the second factor, "Factor B" influences only future policy rates. Gürkaynak, Sack, and Swanson (2005) call Factors A and B the "target" and "path" factors.

7.3 Posterior Estimates

We report the results of our two-stage two-sample estimation in a series of tables. Table 1 reports our most notable calibration targets. The long-run policy rate equals 1.1 percent on a quarterly basis. We target a two percent growth rate of per capita GDP. Given an average population growth rate of one percent per year, this implies that our potential GDP growth rate equals three percent. The other empirical moments we target are a nominal investment to output ratio of 26 percent and nominal government purchases to output ratio of 15 percent. Finally, we target a capital to output ratio of approximately 10 on a quarterly basis.

Table 2 lists the parameters which we calibrate along with their given values. The table includes many more parameters than there are targets in Table 1. This is because Table 1 omitted calibration targets which map one-to-one with particular parameter values. For example, we calibrate the steady-state capital depreciation rate (δ_0) using standard methods applied to data from the Fixed Asset tables. It is also because Table 2 lists several parameters which are normalized prior to estimation. Most notable among these are the three factor loadings listed at the table's bottom. Tables 3 and 6 report prior distributions and posterior modes for the model's remaining parameters, for the first and second samples respectively. Table 7 reports various measures of model fit for the first and second samples. In particular, the log marginal likelihood, the log posterior kernel and the one-step ahead prediction error for key variables. The one-step ahead prediction error is normalized so that it is bounded from above. More formally, we compute

$$F = 1 - \frac{(y - y^f)'(y - y^f)}{(y - \bar{y})'(y - \bar{y})}$$

where y is the time series of the observable over the estimation sample, y^f is the (in-sample) one-step-ahead forecast at each date in the sample, and $\bar{y}$ is the model's steady state value of the observable. If we forecast next quarter's growth coincides with the value next period then the prediction error is zero and $F = 1$. If we forecast next quarter's growth to be the steady state at each date then $F = 0$. Any positive value between zero and one indicate a decent forecasting performance. By this metric the model does quite well forecasting real variables such as GDP, consumption, investment and hours in the first sample. In the second sample, the forecasts for real variables deteriorate significantly.

8 Pandemic scenario analysis

We propose an event-study research design to identify the propagation of synthetic Covid-19 shocks in Chicago Fed DSGE models. The initial outbreak is represented as the onset of a new shock process where the shock is defined as a linear combination of the model's other structural shocks. Realizations of the pandemic shock come with news about its propagation. We identify pandemic shocks and their propagation with revisions to private sector forecasts of GDP and inflation. The event-study assumptions comprise priors on the pandemic shock's contribution to aggregate dynamics. Alternative scenarios for the path of the pandemic, such as a second wave, can be modeled as possible future realizations of the pandemic shock and its propagation. We discuss the use of this framework next.

8.1 Methodology and Assumptions

We start by introducing some modeling assumptions about the new COVID-19 shock. These assumptions introduce enough structure to be able to separately parameterize the nature of COVID shock and its expected persistence.

Definition 1 *The propagation of COVID–19 is modeled as a combination of anticipated iid shocks* ψ_t^j *that are governed by the factor model*

$$\psi_t^j = \lambda(j) f_t, \quad j \in \{0, ..., n\} \tag{65}$$

where $\lambda = \{\lambda(j)\}_{j=0}^n$ *denotes the loadings for the* n *anticipated shocks.* $f_t \overset{iid}{\sim} \mathcal{N}(0,1)$ *is the COVID shock that is assumed to be* $f_t = 0$ *in the pre-COVID period that is set* *to zero in the periods preceding the onset of the COVID-19 pandemic. Notationally,* ψ_t^j *denotes shocks that are known at time* t *and will hit the economy in period* $t+j$*.*

Definition 2 *We introduce a (new) set of current and anticipated shocks in our DSGE model* $\{\varepsilon_t(i)\}_{i=0}^m$*, with anticipation horizons* $j \in \{0,...,n\}$ *and* $i \in \mathcal{S}^C$*, where* $\mathcal{S}^C$ *denotes the subset of DSGE shocks chosen to approximate the propagation of COVID-19.*

While the shocks in the set $\mathcal{S}^C$ are shocks that have never realized before the start of the COVID-19 pandemic, their nature is identical to that of the original set of shocks in our DSGE models (e.g., liquidity preference shocks or temporary technology shocks). Furthermore, the set $\mathcal{S}^C$ includes anticipated version of those shocks.

Definition 3 *We assume that the current and anticipated iid shocks* ψ_t^j*, capturing the propagation of COVID-19, are linear combinations of the current and anticipated shocks in our DSGE model* $\{\varepsilon_t(i)\}_{i=0}^m$*. Formally,*

$$\varepsilon_t^j(i) = \phi_i \psi_t^j, \quad j \in \{0,...,n\} \tag{66}$$

where $\varepsilon_t^j(i)$ *denotes the* $i-th$ *shock in the set* $\mathcal{S}^C$ *with anticipation horizon* $j \in \{0,...,n\}$ *and* ϕ_i *is a scalar that controls the weight of the* $i-th$ *shock in affecting the shocks* ψ_t^j*, which approximate the propagation of COVID-19.*

Note that these weights $\{\phi_i\}_{i=0}^m$ are shock-specific and do not depend on the anticipation horizon of the shocks. We make this assumption in order to economize on the number of parameters needed to be estimated. This assumption implies that the composition of the DSGE shocks, which is used to approximate the propagation of the COVID-19, does not vary across anticipation horizons. This seems a very natural assumption to make.

To sum up, the loadings λ identify the *persistence* of the COVID-19 shock. The vector $\phi = \{\phi_i\}_{i=1}^m$ identifies the *nature* of the COVID shocks - defined as a particular combination of shocks in the subset $\mathcal{S}^C$. The exogenous iid variable f_t should be interpreted as the forecasters' *revision* of their expectations regarding the magnitude of the COVID-19 shock.

8.2 Estimation and Identification

This structure leaves us with $m + n$ new parameters to be estimated. We estimate these parameters using our standard set of plugs plus a set of observed expectations data about future GDP and inflation (we might add also consumption, investment, wages etc). There is virtually no limit to how many observations we can use here (we can use multiple indicators to combine multiple expectations about economic activity or inflation). An idea is to use the forecasts Spence presented the last time around. We call this data set X. Apart from the larger dimensionality, this estimation step is pretty much an extension of what we already do for the system forecasts.

One way to go is to denote the parameters that we need to estimate as $\Xi = [\lambda,\phi]$ and use the Bayes theorem to obtain a distribution of these parameters conditional on the expectation data X.[5] In symbols

$$p(\Xi|X, \Theta, s_{t-1}; \mathcal{M}) \propto \mathcal{L}(X|\Xi, \Theta, s_{t-1}; \mathcal{M})\, p(\Xi), \quad (67)$$

where Θ denotes the parameters of the DSGE model, s_{t-1} denotes our model's state vector estimated one quarter earlier (initial conditions), and $\mathcal{M}$ denotes our DSGE model. The density $p(\Xi)$ is a prior on the new parameters capturing the propagation of COVID-19 in our DSGE model. The density $\mathcal{L}(X|\Xi, \Theta; \mathcal{M})$ denotes the likelihood function associated with the data set X.

To start, we can assume that $p(\Xi)$ is a diffuse prior and so we only need to maximize the likelihood $\mathcal{L}(X|\Xi, \Theta; \mathcal{M})$. If the thing works, we can be more formal and approximate the posterior $p(\Xi|X, \Theta; \mathcal{M})$ using the Metropolis Hastings.

The intuition here is to find the combination of DSGE shocks that can best explain the propagation of COVID-19 expected by a large set of leading forecasters and our judgement reflecting our best knowledge about the effects of the pandemic on macro variables.

To sum up, at this stage we use the data X and our model $(s_{t-1}, \mathcal{M})$ to pin down both the nature of the COVID shock, ϕ, and its expected persistence, λ, as well as forecasters' *revisions* of their expectations regarding the magnitude of the

[5] In a later stage, it would be nice to choose the $\mathcal{S}^C$ so as to maximize the model's fit of the expectation data/judgement used to estimate Ξ.

COVID-19 shock, f_t. In every quarters for which macro forecasts are made available (i.e., the sample size of X), we can use the smoother to evaluate forecasters' revision of their expectations regarding the magnitude of the COVID-19 shock.

The full list of assumptions of our estimation exercises is reported below

- We estimate the parameters of the COVID-19 shock using 2020Q2 and 2020Q3 data.
- We consider SPF inflation expectations on CPI and PCE from 1 to 4 quarters ahead: we calibrate the measurement parameters of the inflation expectations for horizon 2 to 4 using at the estimated values of the measurement parameters of inflation expectations for the next quarter.
- We consider SPF GDP growth expectations: We calibrate the parameters of the measurement equations that bridge the model-consistent GDP expectations to the BEA-consistent GDP expectations by setting $c_{SPF} = \alpha + 1$, $\lambda_{SPF} = 4\beta$, $\mu_{SPF} = \varphi_{SPF} = 0$. α and β are set at the OLS estimator of the regression of BEA GDP growth on model consistent growth using the sample 1993Q1:2019Q4. These values are $\alpha = -0.138$ and $\beta = 1.060$. We calibrate the STD of the measurement equation of the GDP growth expectation is set to the standard deviation of the estimated error.
- Our identification strategy relies on the assumption that in 2020Q2 and 2020Q3 the dominant shock explaining the movements in the observed variables is the COVID-19 shock. To implement this assumption we reduced the standard deviations of the structural business cycles shocks by a factor of 4.
- The COVID-19 shock loads on Permanent Neutral and Liquidity Preference only.

Table 1: First Sample Calibration Targets

Description	Expression	Value
Fixed Interest Rate (quarterly, gross)	R^*	1.011
Per-Capita Steady-State Output Growth Rate (quarterly)	Y_{t+1}/Y_t	1.005
Investment to Output Ratio	I_t/Y_t	0.260
Capital to Output Ratio	K_t/Y_t	10.763
Fraction of Final Good Output Spent on Public Goods	G_t/Y_t	0.153
Growth Rate of Relative Price of Consumption to Investment	P_C/P_I	0.371

Table 2: First Sample Calibrated Parameters

Parameter	**Symbol**	**Value**
Discount Factor	β	0.986
Steady-State Measured TFP Growth (quarterly)	z_*	0.489
Investment-Specific Technology Growth Rate	ω_*	0.371
Elasticity of Output w.r.t Capital Services	α	0.401
Steady-State Wage Markup	λ_*^w	1.500
Steady-State Price Markup	λ_*^p	1.500
Steady-State Scale of the Economy	H_*	1.000
Steady-State Inflation Rate (quarterly)	π_*	0.500
Steady-State Depreciation Rate	δ_0	0.016
Steady-State Marginal Depreciation Cost	δ_1	0.039
Core PCE, 1Q Ahead and 10Y Ahead Expected PCE		
Constant	$\pi_*^1, \pi_*^{l,1}$	0.000
Loading 1	$\beta^{\pi,1}, \beta^{l,1}$	1.000
Core CPI, 1Q Ahead and 10Y Ahead Expected CPI		
Constant	$\pi_*^2, \pi_*^{l,2}$	0.122
10Y Ahead Expected CPI and PCE		
Standard Deviation of $u_t^{40,j,\pi}$		0.010
PCE Durable Goods Inflation		
1st Lag Coefficient	$\beta_{1,1}$	0.418
2nd Lag Coefficient	$\beta_{1,2}$	0.379
Inflation in Relative Price of Government, Inventories and Net Exports to Consumption		
1st Lag Coefficient	$\beta_{2,1}$	0.311
2nd Lag Coefficient	$\beta_{2,2}$	0.006
Compensation		
Constant	w_*^1	-0.202
Loading	$\beta^{w,1}$	1.000
Earnings Constant	w_*^2	-0.237
Loading 0 Factor A	α_0	0.981
Loading 0 Factor B	β_0	0.000
Loading 4 Factor B	β_4	0.951

Table 3: First Sample Estimated Parameters

Parameter	Symbol	Density	Prior Mean	Prior Std.Dev	Posterior Mode
Depreciation Curve	$\frac{\delta_2}{\delta_1}$	G	1.0000	0.150	0.474
Active Price Indexation Rate	ι_p	B	0.5000	0.150	0.409
Active Wage Indexation Rate	ι_w	B	0.5000	0.150	0.077
External Habit Weight	λ	B	0.7500	0.025	0.780
Labor Supply Elasticity	γ_H	N	0.6000	0.050	0.589
Price Stickiness Probability	ζ_p	B	0.8000	0.050	0.831
Wage Stickiness Probability	ζ_w	B	0.7500	0.050	0.914
Adjustment Cost of Investment	φ	G	3.0000	0.750	5.354
Elasticity of Intertemporal Substitution	γ_c	N	1.5000	0.375	1.319
Interest Rate Response to Inflation	ψ_1	G	1.7000	0.150	1.791
Interest Rate Response to Output	ψ_2	G	0.2500	0.100	0.398
Interest Rate Smoothing Coefficient	ρ_R	B	0.8000	0.100	0.801
Autoregressive Coefficients of Shocks					
Discount Factor	ρ_b	B	0.5000	0.250	0.813
Inflation Drift	ρ_π	B	0.9900	0.010	0.998
Exogenous Spending	ρ_g	B	0.6000	0.100	0.887
Investment-Demand	ρ_i	B	0.5000	0.100	0.791
Liquidity Preference	ρ_s	B	0.6000	0.200	0.887
Price Markup	ρ_{λ_p}	B	0.6000	0.200	0.136
Wage Markup	ρ_{λ_w}	B	0.5000	0.150	0.469
Neutral Technology	ρ_ν	B	0.3000	0.150	0.492
Investment Specific Technology	ρ_ω	B	0.3500	0.100	0.303
Moving Average Coefficients of Shocks					
Price Markup	θ_{λ_p}	B	0.4000	0.200	0.307
Wage Markup	θ_{λ_w}	B	0.4000	0.200	0.391
Standard Deviations of Innovations					
Discount Factor	σ_b	U	0.5000	2.000	1.768
Inflation Drift	σ_π	I	0.0150	0.0075	0.077
Exogenous Spending	σ_g	U	1.0000	2.000	4.139

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

First Sample Estimated Parameters (Continued)

Parameter	**Symbol**	**Density**	**Prior Mean**	**Prior Std.Dev**	**Posterior Mode**
Investment-Demand	σ_i	I	0.2000	0.200	0.549
Liquidity Preference	σ_s	U	0.5000	2.000	0.341
Price Markup	σ_{λ_p}	I	0.1000	1.000	0.101
Wage Markup	σ_{λ_w}	I	0.1000	1.000	0.035
Neutral Technology	σ_ν	U	0.5000	0.250	0.530
Investment Specific Technology	σ_ω	I	0.2000	0.100	0.259
Relative Price of Cons to Inv	$\sigma_{\frac{c}{i}}$	I	0.0500	2.000	0.675
Monetary Policy					
Unanticipated	σ_{η_0}	N	0.0050	0.0025	0.012
1Q Ahead	σ_{η_1}	N	0.0050	0.0025	0.012
2Q Ahead	σ_{η_2}	N	0.0050	0.0025	0.008
3Q Ahead	σ_{η_3}	N	0.0050	0.0025	0.009
4Q Ahead	σ_{η_4}	N	0.0050	0.0025	0.012
Compensation					
Standard Deviation of $u_t^{1,w}$		I	0.0500	0.100	0.194
AR(1) Coefficient of $u_t^{1,w}$		B	0.4000	0.100	0.458
Earnings					
Loading 1	$\beta^{w,2}$	N	0.8000	0.100	0.904
Standard Deviation of $u_t^{2,w}$		I	0.0500	0.100	0.143
AR(1) Coefficient of $u_t^{2,w}$		B	0.4000	0.100	0.674
Core PCE					
Loading 2	$\gamma^{\pi,1}$	N	0.0000	1.000	0.045
Standard Deviation of $u_t^{1,p}$		I	0.0500	0.100	0.046
AR(1) Coefficient of $u_t^{1,p}$		B	0.2000	0.100	0.108
Core CPI					
Loading 1	$\beta^{\pi,2}$	N	1.0000	0.100	0.808
Loading 2	$\gamma^{\pi,2}$	N	0.0000	1.000	0.087
Standard Deviation of $u_t^{2,p}$		I	0.1000	0.100	0.077
AR(1) Coefficient of $u_t^{2,p}$		B	0.4000	0.200	0.586
Market-Based Core PCE					

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

First Sample Estimated Parameters (Continued)

Parameter	Symbol	Density	Prior Mean	Std.Dev	Posterior Mode
Constant	π_*^3	N	-0.1000	0.100	-0.037
Loading 1	$\beta^{\pi,3}$	N	1.0000	0.100	1.121
Loading 2	$\gamma^{\pi,3}$	N	0.0000	1.000	0.015
Standard Deviation of $u_t^{3,p}$		I	0.0500	0.100	0.035
AR(1) Coefficient of $u_t^{3,p}$		B	0.2000	0.100	0.144
1Q Ahead Expected PCE					
Standard Deviation of $u_t^{1,1,\pi}$		I	0.0500	0.100	0.026
AR(1) Coefficient of $u_t^{1,1,\pi}$		B	0.2000	0.100	0.196
1Q Ahead Expected CPI					
Loading	$\beta^{1,2}$	N	1.0000	0.100	0.980
Standard Deviation of $u_t^{1,2,\pi}$		I	0.0500	0.100	0.062
AR(1) Coefficient of $u_t^{1,2,\pi}$		B	0.2000	0.100	0.198
10Y Ahead Expected PCE					
AR(1) Coefficient of $u_t^{40,1,\pi}$		B	0.2000	0.100	0.271
10Y Ahead Expected CPI					
Loading	$\beta^{40,2}$	N	1.0000	0.100	1.021
AR(1) Coefficient of $u_t^{40,2,\pi}$		B	0.2000	0.100	0.213
PCE Durable Goods Inflation					
Constant	π_*^d	N	-0.3500	0.100	-0.360
Standard Deviation of u_t^d		I	0.2000	2.000	0.286

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

First Sample Estimated Parameters (Continued)

Parameter	**Symbol**	**Density**	**Prior Mean**	**Prior Std.Dev**	**Posterior Mode**
Inflation in Relative Price of Government, Inventories and Net Exports to Consumption					
Constant	π_*^g	N	0.1980	1.000	-0.666
Standard Deviation of u_t^g		I	0.5000	2.000	1.861
Factor A					
Loading 1	α_1	N	0.6839	0.200	1.305
Loading 2	α_2	N	0.5224	0.200	0.877
Loading 3	α_3	N	0.4314	0.200	0.306
Loading 4	α_4	N	0.3243	0.200	-0.012
Standard Deviation	σ_α	N	0.1000	0.0750	0.040
Factor B					
Loading 1	β_1	N	0.3310	0.200	0.656
Loading 2	β_2	N	0.6525	0.200	1.104
Loading 3	β_3	N	0.8059	0.200	1.162
Standard Deviation	σ_β	N	0.1000	0.0750	0.078

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

Table 4: Second Sample Calibration Targets (Different from First Sample)

Description	Expression	Value
Fixed Interest Rate (quarterly, gross)	R^*	1.007
Per-Capita Steady-State Output Growth Rate (quarterly)	Y_{t+1}/Y_t	1.003
Growth Rate of Relative Price of Consumption to Investment	P_C/P_I	0.171

Table 5: Second Sample Calibrated Parameters (Different from First Sample)

Parameter	Symbol	Value
Steady-State Measured TFP Growth (quarterly)	z_*	0.415
Investment-Specific Technology Growth Rate	ω_*	0.171
Steady-State Marginal Depreciation Cost	δ_1	0.038
Core CPI, 1Q Ahead and 10Y Ahead Expected CPI		
Constant	$\pi_*^2, \pi_*^{l,2}$	0.060
10Y Ahead Expected CPI and PCE		
Standard Deviation of $u_t^{40,j,\pi}$		0.020
PCE Durable Goods Inflation		
1st Lag Coefficient	$\beta_{1,1}$	0.000
2nd Lag Coefficient	$\beta_{1,2}$	0.000
Inflation in Relative Price of Government, Inventories and Net Exports to Consumption		
1st Lag Coefficient	$\beta_{2,1}$	0.320
2nd Lag Coefficient	$\beta_{2,2}$	-0.240
Compensation Loading	$\beta^{w,1}$	1.000
Loading 5 Factor A	α_5	0.932
Loading 8 Factor B	β_8	0.210
Loading 10 Factor B	β_{10}	0.000

Table 6: Second Sample Estimated Parameters

Parameter	Symbol	Prior Mean	Prior Std.Dev	Posterior Mode
Compensation				
Constant	w_*^1	-0.2023	0.100	-0.129
Standard Deviation of $u_t^{1,w}$		0.1941	0.100	0.267
AR(1) Coefficient of $u_t^{1,w}$		0.4579	0.100	0.388
Earnings				
Constant	w_*^2	-0.2370	0.100	-0.131
Loading 1	$\beta^{w,2}$	0.9039	0.100	0.721
Standard Deviation of $u_t^{2,w}$		0.1434	0.100	0.255
AR(1) Coefficient of $u_t^{2,w}$		0.6741	0.100	0.600
Core PCE				
Loading 2	$\gamma^{\pi,1}$	0.0449	0.100	0.211
Standard Deviation of $u_t^{1,p}$		0.0457	0.100	0.247
AR(1) Coefficient of $u_t^{1,p}$		0.1081	0.150	0.180
Core CPI				
Loading 1	$\beta^{\pi,2}$	0.8083	0.150	0.192
Loading 2	$\gamma^{\pi,2}$	0.0868	0.100	0.252
Standard Deviation of $u_t^{2,p}$		0.0770	0.100	0.096
AR(1) Coefficient of $u_t^{2,p}$		0.5856	0.150	0.625
Market PCE				
Constant	π_*^3	-0.0367	0.100	-0.120
Loading 1	$\beta^{\pi,3}$	1.1213	0.150	0.292
Loading 2	$\gamma^{\pi,3}$	0.0153	0.100	0.245
Standard Deviation of $u_t^{3,p}$		0.0349	0.100	0.096
AR(1) Coefficient of $u_t^{3,p}$		0.1436	0.150	0.196
1Q Ahead Expected PCE				
Standard Deviation of $u_t^{1,1,\pi}$		0.0259	0.020	0.070
AR(1) Coefficient of $u_t^{1,1,\pi}$		0.1960	0.050	0.256
1Q Ahead Expected CPI				
Loading	$\beta^{1,2}$	0.9803	0.080	0.993

Second Sample Estimated Parameters (Continued)

Parameter	Symbol	Prior Mean	Prior Std.Dev	Posterior Mode
Standard Deviation of $u_t^{1,2,\pi}$		0.0622	0.020	0.101
AR(1) Coefficient of $u_t^{1,2,\pi}$		0.1982	0.050	0.220
10Y Ahead Expected PCE				
AR(1) Coefficient of $u_t^{40,1,\pi}$		0.2711	0.050	0.310
10Y Ahead Expected CPI				
Loading	$\beta^{40,2}$	1.0207	0.100	1.062
AR(1) Coefficient of $u_t^{40,2,\pi}$		0.2133	0.050	0.212
PCE Durable Goods Inflation				
Constant	π_*^d	-0.4500	0.200	-0.451
Standard Deviation of u_t^d		0.5000	0.150	0.316
Inflation in Relative Price of Government, Inventories and Net Exports to Consumption				
Constant	π_*^g	0.8900	0.400	0.067
Standard Deviation of u_t^g		0.8143	0.150	1.267
Factor A				
Loading 0	α_0	0.0180	0.250	0.135
Loading 1	α_1	0.0574	0.250	0.120
Loading 2	α_2	0.1941	0.250	0.284
Loading 3	α_3	0.3996	0.250	0.460
Loading 4	α_4	0.6520	0.250	0.760
Loading 6	α_6	1.2266	0.250	1.127
Loading 7	α_7	1.5237	0.250	1.465
Loading 8	α_8	1.8139	0.250	1.697
Loading 9	α_9	2.0914	0.250	1.919
Loading 10	α_{10}	2.3523	0.250	2.742
Standard Deviation	σ_α	0.0442	0.100	0.055
Factor B				
Loading 0	β_0	-0.0181	0.300	0.029
Loading 1	β_1	0.2211	0.300	0.033
Loading 2	β_2	0.3679	0.300	0.070

Second Sample Estimated Parameters (Continued)

Parameter	**Symbol**	**Prior** **Mean**	**Std.Dev**	**Posterior** **Mode**
Loading 3	β_3	0.4424	0.300	0.103
Loading 4	β_4	0.4612	0.300	0.126
Loading 5	β_5	0.4370	0.300	0.137
Loading 6	β_6	0.3817	0.300	0.162
Loading 7	β_7	0.3032	0.300	0.179
Loading 9	β_9	0.1074	0.300	0.212
Standard Deviation	σ_β	0.0334	0.100	0.439
Standard Deviations of Monetary Policy Innovations				
Unanticipated	σ_{η_0}	0.0061	0.005	0.011
1Q Ahead	σ_{η_1}	0.0021	0.005	0.010
2Q Ahead	σ_{η_2}	0.0004	0.005	0.009
3Q Ahead	σ_{η_3}	0.0019	0.005	0.010
4Q Ahead	σ_{η_4}	0.0001	0.005	0.010
5Q Ahead	σ_{η_5}	0.0025	0.005	0.000
6Q Ahead	σ_{η_6}	0.0019	0.005	0.010
7Q Ahead	σ_{η_7}	0.0011	0.005	0.010
8Q Ahead	σ_{η_8}	0.0001	0.005	0.000
9Q Ahead	σ_{η_9}	0.0014	0.005	0.003
10Q Ahead	$\sigma_{\eta_{10}}$	0.0028	0.005	0.009

Table 7: Measures of fit

	I sample	II sample
Log Posterior Kernel	536.0	588.1
Marginal Log Likelihood	536.4	558.0
yobs (dy)	0.1	-0.7
cobs (dc)	0.5	0.3
iobs (di)	0.1	-0.9
Hours	0.9	0.9
FFR	1.0	1.0
PCE Inflation	0.2	0.5

Figure 1: DiscountFactor

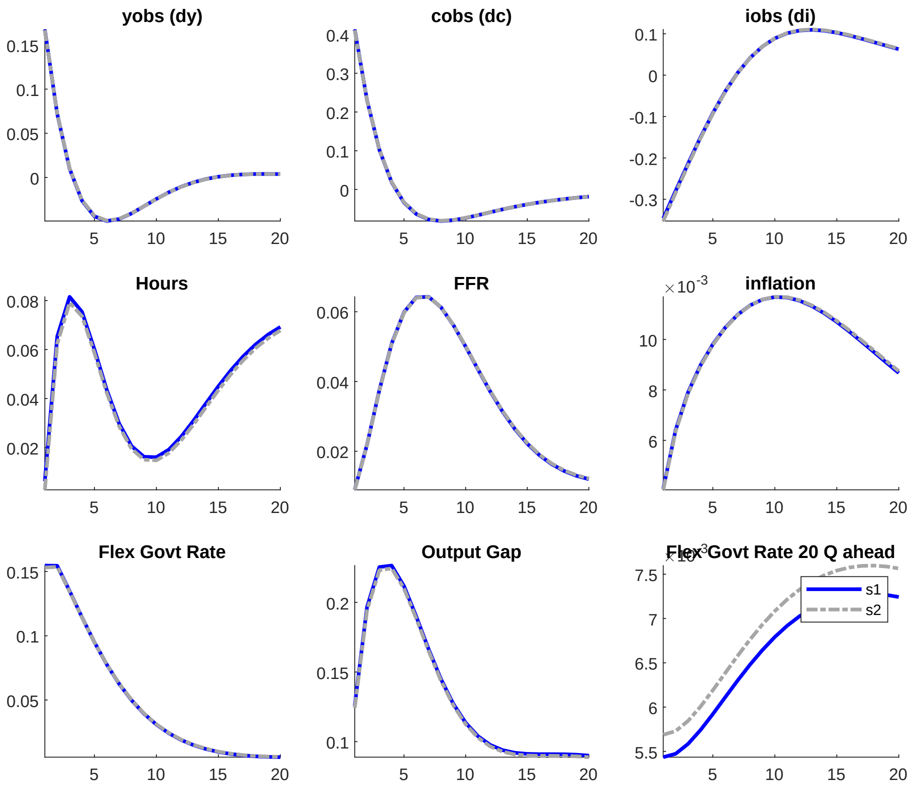


Figure 2: InflationDrift

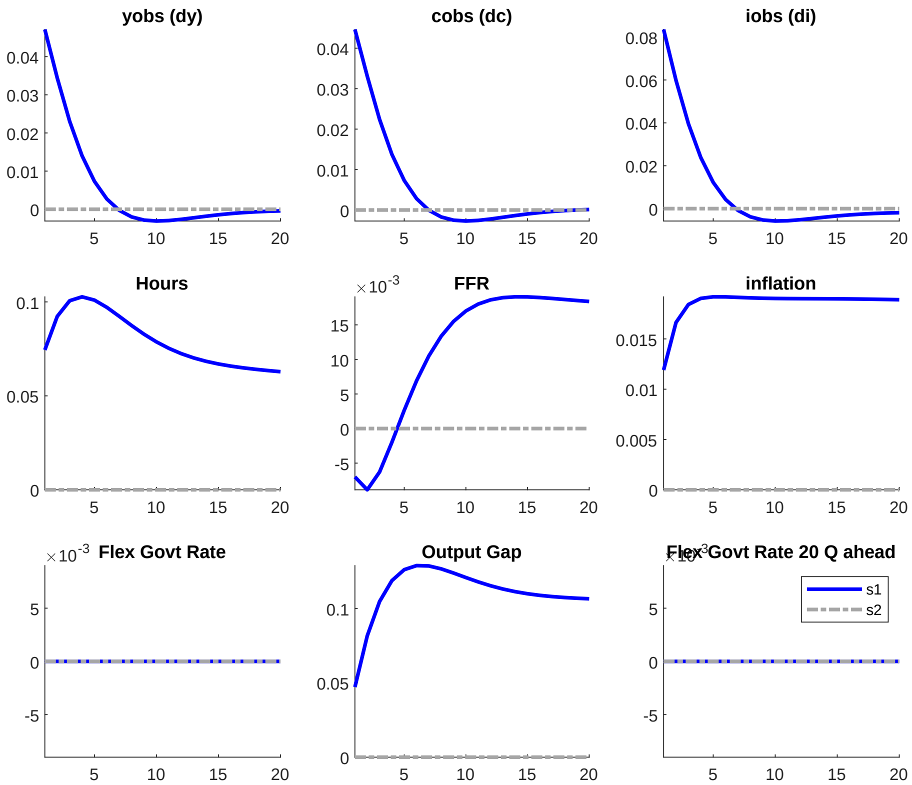


Figure 3: FactorA

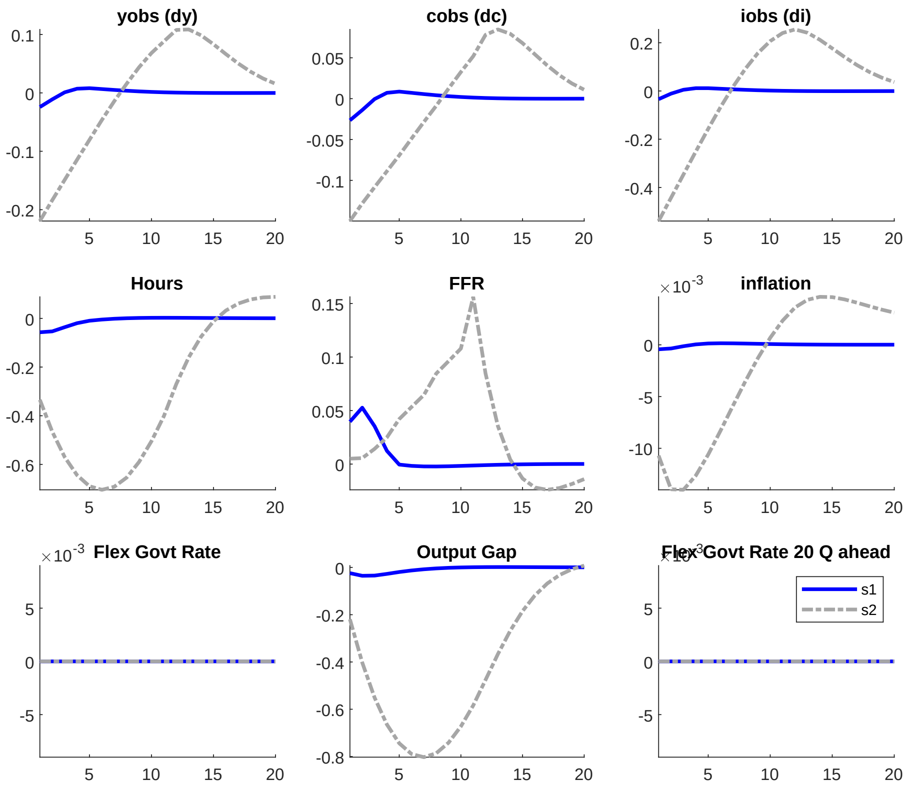


Figure 4: FactorB

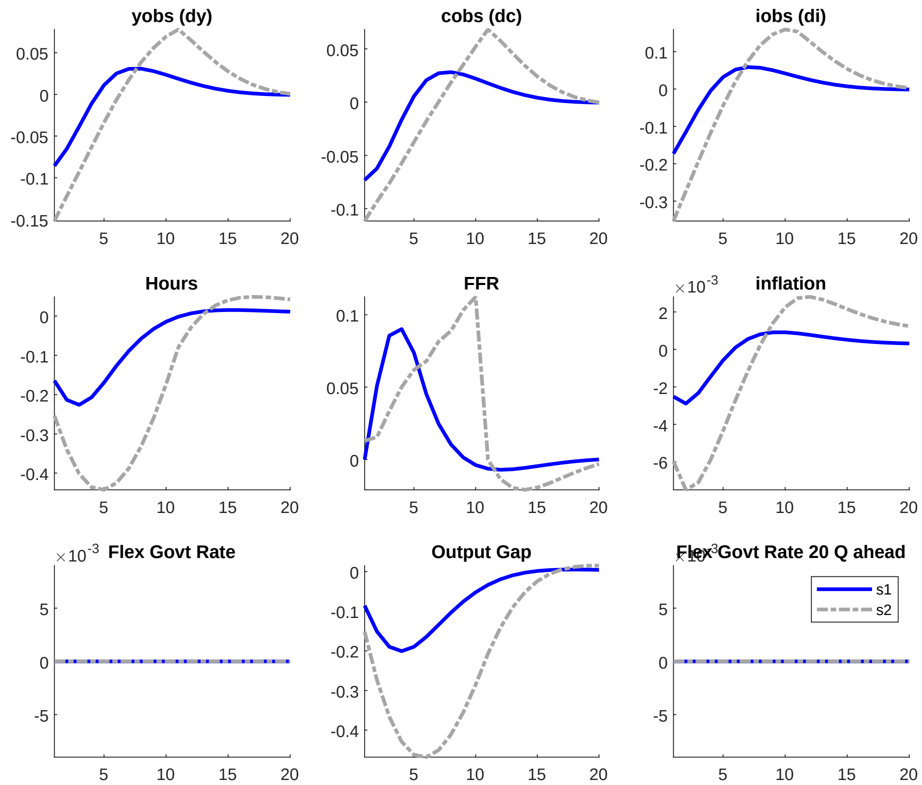


Figure 5: InvestmentShock

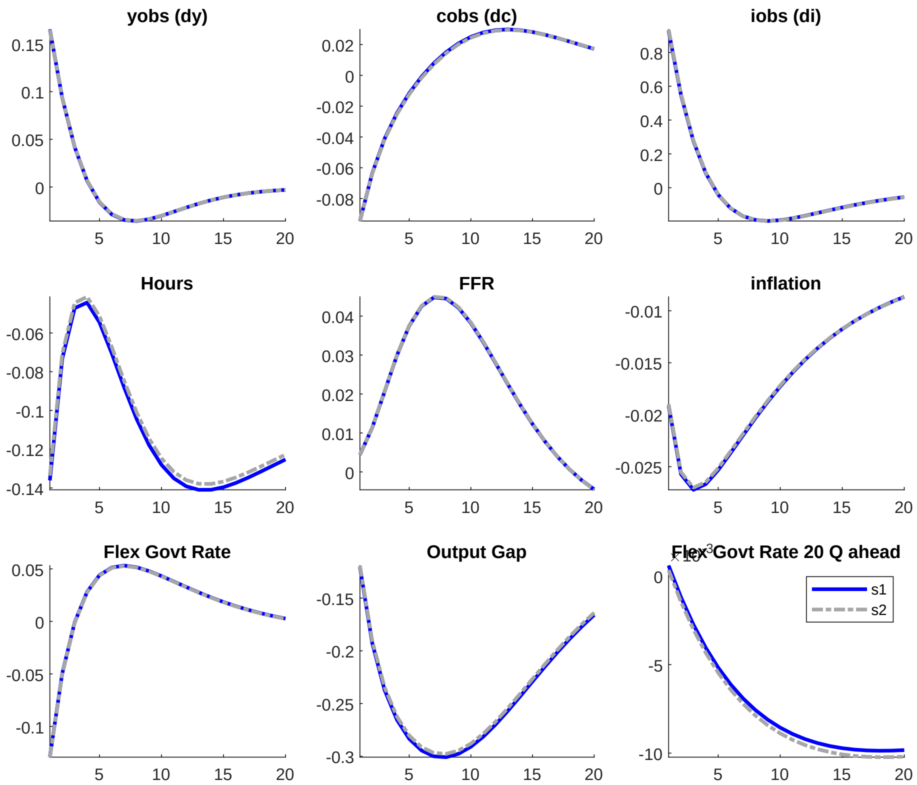


Figure 6: PermanentNeutral

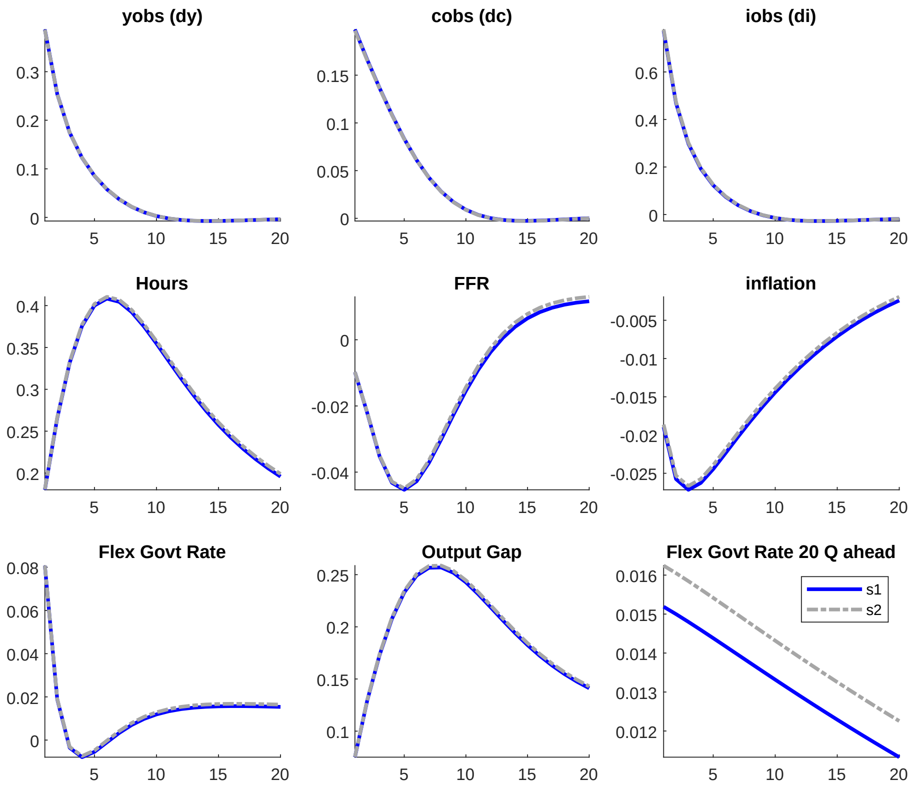


Figure 7: PriceMarkup

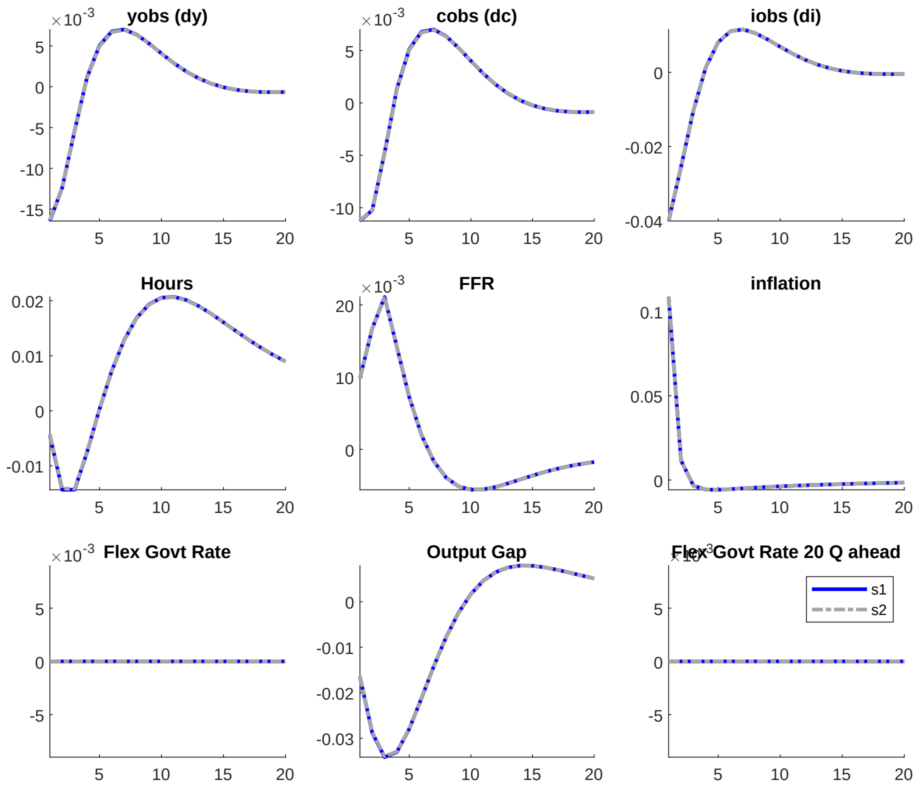


Figure 8: WageMarkup

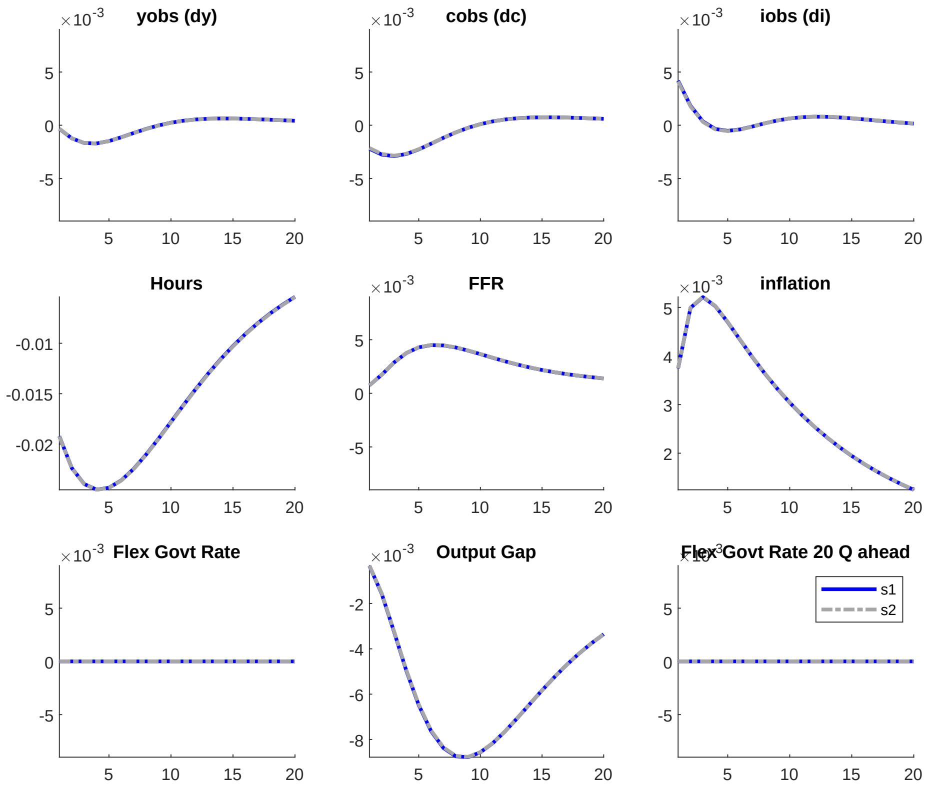


Figure 9: LiquidityPreference

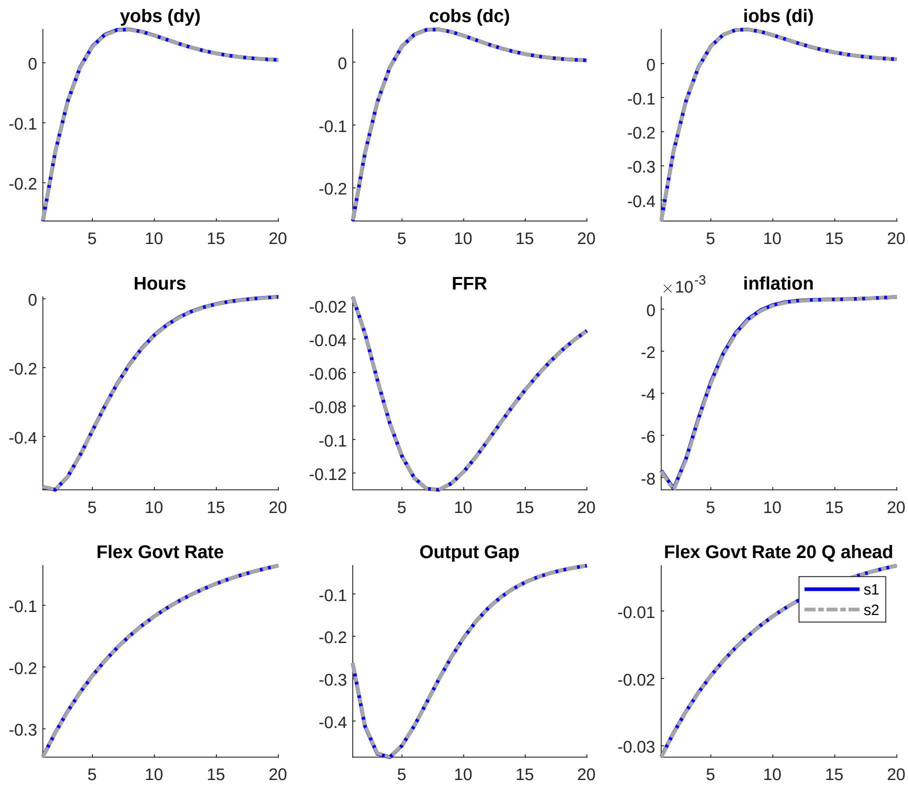


Figure 10: ISTS

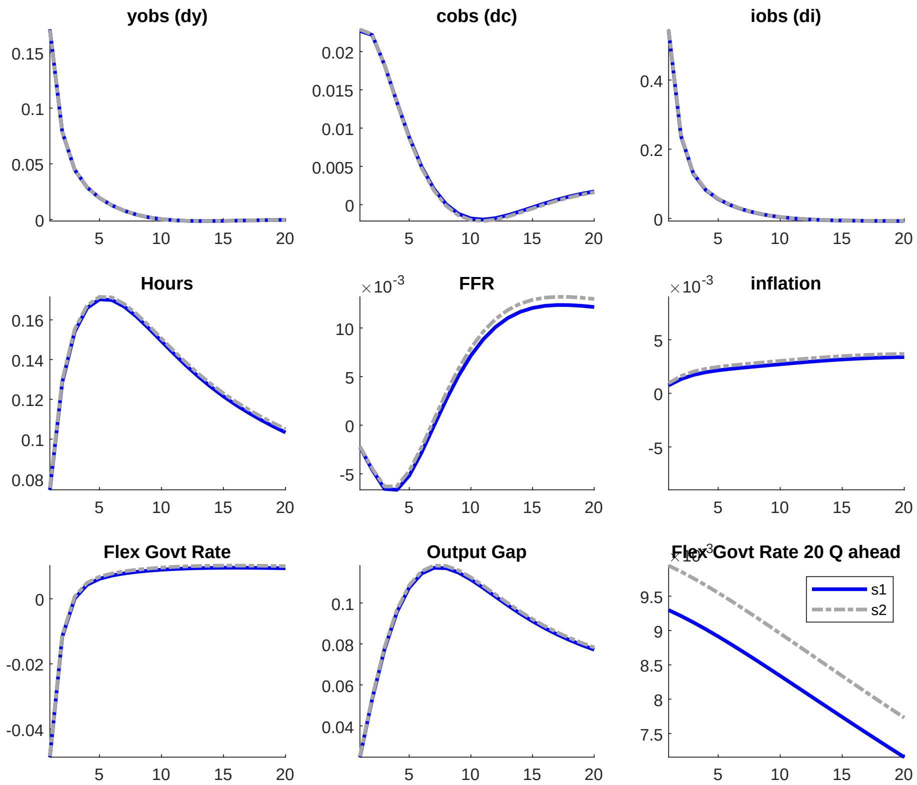
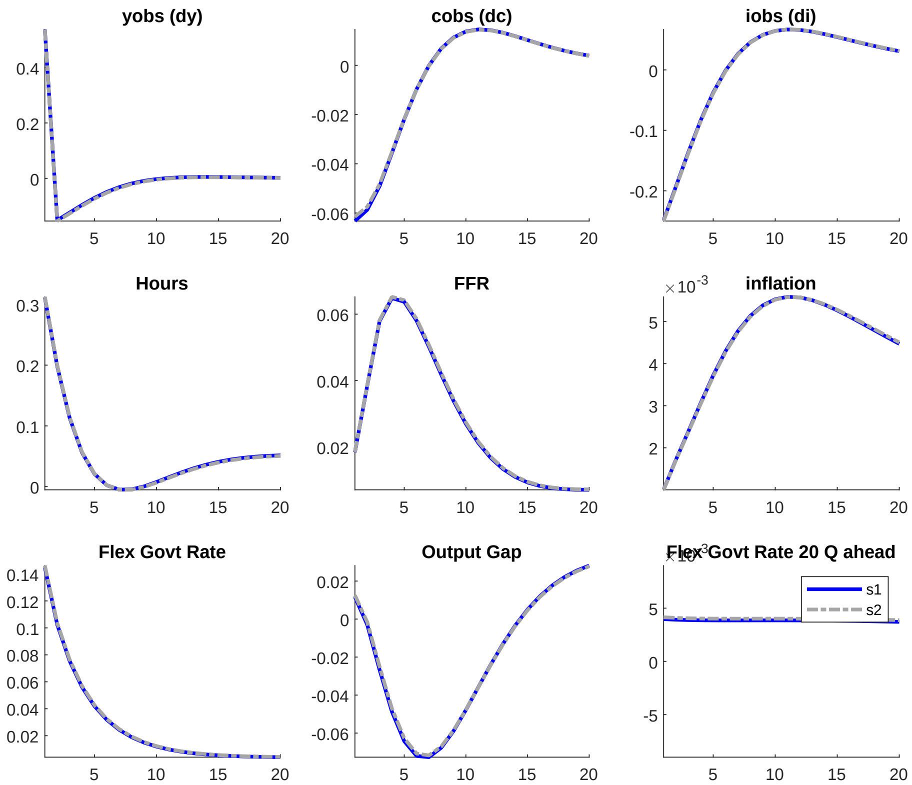


Figure 11: GovernmentSpending



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