Banks, Liquidity Management and Monetary Policy*

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Preliminary

Abstract

We develop a new framework for studying the implementation of monetary policy through the banking sector. Banks are subject to a maturity mismatch problem leading to precautionary holdings of reserves. Through various instruments, monetary policy alters tradeoffs banks face between lending, holding reserves, holding deposits and paying dividends. This translates into the real economy via effects on real interests and lending. We study how these instruments interact with shocks to the volatility in the payments system, bank losses, the demand for loans and with capital requirements. We use a calibrated version of the model to answer, quantitatively, why have banks held onto a substantial increase in reserves while not increasing lending since 2008.

Keywords: Banks, Monetary Policy, Liquidity, Capital Requirements

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1 Introduction

The conduct of monetary policy around the world is changing. The last five years have witnessed banking systems experiencing unprecedented losses together with frozen interbank markets. Banks cut back on lending as these events materialized. In an attempt to preserve financial stability and reinvigorate lending, the central banks of the US and Europe have reduced policy rates to almost zero and continuously purchased private loans from banks. In response to these unprecedented policy interventions, banks seem to have mostly accumulated central bank reserves without renewing lending as intended by policy.[1] Why and what can central banks do about this remains an open question.

Unsurprisingly, the role of banks in the transmission of monetary policy has been at the centerpiece of policy debates. Unfortunately, there are not many modern macroeconomic models that enable the study of monetary policies implemented through the liquidity management of banks as occurs in practice. In this paper, we present a model that fills this gap. We use this model to answer a number of theoretical issues: How does the transmission of monetary policy depend on the decisions of commercial banks? What type of shocks can induce banks to hold more reserves and lend less? How will the strength of monetary be affected by these shocks? What is the connection between monetary policy and regulatory capital requirements?

We use the lessons derived from our model to quantitatively investigate why banks are not lending despite all aforementioned policy efforts. Our model is able to validate different hypotheses that are informally discussed in policy and academic circles. Through the lens of the model, we evaluate the plausibility of each of the following:

Hypothesis 1 - Bank Equity Losses: The lack of lending responds to optimal behavior by banks given the increase in bank leverage which followed from the large equity losses in 2008.

Hypothesis 2 - Interbank Uncertainty: Banks hold more reserves relative to their lending because their is substantial uncertainty about potential costs of accessing the interbank market.

Hypothesis 3 - Capital Requirements: The effective and/or expected path of capital requirements are leading banks to hold more reserves and lend less. Central banks are constrained by other regulatory constraints.

Hypothesis 4 - Weak Demand: Banks face a weaker effective demand for loans. The lack of demand is caused by the lack of borrowers that meet credit standards or borrowers do not want to borrow.

We calibrate our model and fit it with shocks we associate with each hypothesis. We use the properties of the model to uncover which shocks can explain why lending is weak while the volume of excess reserves has increased by several factors. Our model suggests that a combination of an early increase in the uncertainty about interbank payments followed by a substantial contraction in loan demand is the most empirically plausible combination of shocks that explains this facts.

[1] As is well known, the Bank of Japan had been facing similar issues since the early nineties.

The Mechanism. The building block of our model is a liquidity management problem. This problem constitutes finding the optimal mix between lending, deposit issuances and holdings of central bank reserves. Banks tradeoff the profit on a loan against potential liquidity risks. Liquidity risks are associated with financial losses that follow when deposits are withdrawn from one bank that lacks sufficient reserves to meet its payment-settlement needs. Bank lending reacts to monetary policy because policy instruments alter the tradeoffs underlying this problem.

Liquidity management is recognized as one of the fundamental problems in banking.[2] When a bank grants a loan, it must create or obtain a liability in the form of a credit line or a demand deposit. Granting a loan is profitable because a higher interest is charged on the loan than what is paid on deposits. However, the trade-off is that more lending relative to a given amount of reserves also increases liquidity risks: when deposits are transferred to another bank, the issuing bank must transfer some asset to settle the transaction. We assume, as occurs in practice, that loans cannot be sold immediately due to various frictions. Hence, central bank reserves are critical to clear settlements. This friction implies that with an increase in deposits that follows from additional lending comes additional liquidity risk. The lower the liquidity ratio of a bank –its deposits-to-reserve ratio– the more likely it isto be short of reserves. Banks short of reserves, incur financial losses as they must incur in expensive borrowing from other banks or the Central Bank's discount window.

We introduce this liquidity management problem into a tractable dynamic general equilibrium model with rational profit-maximizing heterogeneous banks and an inter-bank market. Bank liquidity management is captured through a portfolio problem with non-linear returns. We show how different instruments operate by altering the incentives banks face to grant loans. Short-run monetary policy effects result from the ability that central banks have to supply reserves or alter market rates. Long-run monetary-policy effects are also present because bank equity returns and the size of the financial sector evolves in response to these policies.

Implementing Monetary Policy. Monetary policy in our model is implemented through various tools. In particular, the Central Bank is equipped with discount rates, interests on reserves, open-market operations (conventional and unconventional), liquidity facilities and reserve requirements. All these instruments have a common effect: they tilt the balance towards more or less lending. Their macroeconomic effects result from changes in lending volumes and interest rates. However, as much as monetary policy can stimulate lending, its power may be altered by exogenous shocks associated with hypotheses 1 through 4. The richness in this set of tools allows us to provide a close description to the central banks's policies during the last five years once we test these hypotheses.

Testable Implications. The model delivers a rich set of descriptions for banking and monetary indicators. For individual banks, it explains the behavior of their reserve holdings, leverage

[2]See Saunders and Cornett (2010).

and dividends rates. For the banking industry as a whole, it provides descriptions for aggregate lending, interbank lending volumes and excess reserves. It also delivers predictions about interbank and non-interbank borrowing and lending rates. The model also provides a description for financial indicators for banks: return on loans, return on equity, dividend ratios and book and market values for equity. It also yields predictions about the size of the financial sector relative to the rest of the economy. At the macroeconomic level, it provides implications for the evolution of monetary aggregates, M0 and M1 so it delivers an endogenous money multiplier.

We use this richness of descriptions to explain the dynamic effects of aggregate outcomes to changes in different monetary policy instruments, financial regulation and to explain the pass-through of these policies when we alter the volatility of bank withdrawals, the demand for loans or shock the equity of banks.

Monetary Policy 2008-2013. The quantitative analysis section tries to shed light on which of the four hypotheses fits best the patterns we saw in the US data since the 2008-2009 financial crisis. We argue that a combination of hypotheses 1 and 4, that is, increased interbank uncertainty and a weak demand for loans, was the most likely shock to have been in place during the crisis.

Organization. The paper is organized as follows. The following section analyzes how our model fits with the literature. Section 2, presents our model of banking. Section 3 presents the calibration and empirical analysis. We study the steady state and policy functions in sections 4.We use this environment to study the effects of deterministic shock paths in section 5. Finally, in 6 we evaluate quantitatively the four different hypothesis to explain why financial intermediation remains depressed after the crisis.

1.1 Related Literature

There is a tradition in economics that dates at least to Bagehot (1873) which stresses the importance of analyzing monetary policy together with financial intermediaries. A classic attempt to study policy in a model with a full description of households, firms and banks is Gurley and Shaw (1964). With few exceptions, modeling banks was a practice abandoned by macroeconomics for many years. Until the Great Recession, questions about the macroeconomic effects of monetary policy and how this policy is implemented through banks were treated independently.[3]

In the aftermath of the crisis, there have been numerous calls for writing models with an explicit role for banks with the goal to improve our understanding of the implementations of monetary policy.[4] Some of the earliest steps have been taken by Gertler and Karadi (2009) and Curdia and Woodford (2009) followed by a burgeoning literature. The focus in most of this literature is on studying conventional and unconventional monetary policy that mitigate

[3]This simplification seemed natural. In the US, banking did not seem to matter for macroeconomic performance. For example, the banking industry was among the most stable industries in terms of solvency. More importantly, the pass-through from key policy rates and to lending terms seemed straight.

[4]See for example Woodford (2010) and Mishkin (2011).

financial shocks that affect bank leverage constraints and propagate by interrupting financial intermediation or increasing spreads. In contrast, the key friction in our model is a liquidity mismatch.Thus, monetary policy in our framework works differently by reducing illiquidity risks. This liquidity management is related to illiquidity problems that arise in classic models of banking.[5] Our contribution to bring the insights of the liquidity-managment literature into a full equilibrium dynamics and general equilibrium effects that can be used to analyze policy and aggregate shocks.

Brunnermeier and Sannikov (2012) introduce inside money and outside money into a dynamic macro model with financial intermediaries. Their focus is on the real effects of monetary policy through the redistributive effects of inflation. In contrast, in our setup outside money are central bank reserves which only serve a role as an instruments to hedge illiquidity risks. This maturity mismatch problem explains why monetary policy affects bank lending. Our paper is also related to Corbae and D'Erasmo (2013) who study a model of the banking industry's dynamics featuring rich heterogeneity.

Finally, our paper is also builds on insights from the literature that studies search theoretic foundations for money. A theoretical model where reserves emerge as an essential tool for credit creation is found in Cavalcanti and Andres Erosa (1999). Williamson (2012) studies an environment where assets of different maturity have different properties as mediums of payments. Rocheteau and Rodriguez-Lopez (2013) has a spillover from liquidity needs in a OTC market to the labor market where firms are issuing loans to hire workers. Like us, they use these frameworks to study the liquidity effects of different monetary policy tools. Along that dimension, our model is also related to Stein (2012) and Stein et al. (2013) who study environments where there is and exogenous demand for safe short-term liquid assets with implications for policy. A common feature in all of these papers is that the classic Modigliani-Miller theorem for open-market operations (see Wallace (1981)) is broken. Finally, we build on Afonso and Lagos (2012) who studies the intraday allocation of reserves and pricing of overnight loans using a dynamic equilibrium search-theoretic framework. Our market for reserves is a simplified version of theirs, but here we study the implications the management of reserves for the macroeconomy.

2 The Model

The description of our model begins with a partial equilibrium dynamic model of banks. The focus of the section is on explaining the supply of loans and the demand for liquidity as functions of

[5]Early classic papers that studied static liquidity (reserve) management problem of individual banks are Poole (1968) and Frost (1971). There are many textbooks for banking practitioners that deal with this subject. See for example, Saunders and Cornett (2010) for a managerial perspective and Duttweiler (2009) for an operations research perspective. Modern banking papers have focuses on bank rusn. See for example Diamond and Dybvig (1983), Allen and Gale (1998), Ennis and Keister (2009), or Holmstrm and Tirole (1998). Gertler and Kiyotaki (2013) is a recent paper that incorporates bank runs into DSGE models. We do not classic bank run problems in this paper but rather focus on liquidity-risk management.

prices, central bank policies and aggregate shocks. We then close the model introducing a demand for loans from the real sector.

2.1 Environment

Time is discrete, is indexed by t and there is an infinite horizon. Each period is divided into two stages: a lending stage (l) and a balancing stage (b). The economy is populated by a continuum of competitive banks whose identity is denoted by z. Banks face an exogenous demand for loans (for now) and a vector of shocks that we describe later. There is an exogenous deterministic monetary policy chosen by the monetary authority which we refer to as the FED. There are thre types of assets, deposits, loans and central bank reserves. Deposits and loans are denominated in real terms. Reserves are denominated in nominal terms. Deposits play the role of a numeraire. We discuss the properties of these assets below.

Banks. A bank's preferences over real dividend streams $\{DIV_t\}_{t\geq 0}$ are evaluated via an expected utility criterion:

$$\mathbb{E}_0 \sum_{t\geq 0} \beta^t U\left(DIV_t\right)$$

where $U\left(DIV\right) \equiv \frac{DIV^{1-\gamma}}{1-\gamma}$ and DIV_t is the banker's consumption at date t.[6] Banks hold a portfolio of loans, B_t, and central bank reserves, C_t, as part of their assets. Demand deposits, D_t, are their only form of liabilities. These holdings are the individual state variables of a bank.

Loans. Loans are granted during the lending stage. Loans constitute a promise to repay the bank $I_t\left(1-\delta\right)\delta^n$ in period $t+1+n$ for all $n \geq 0$, in units of the numeraire. Hence, loans promise a geometrically decaying stream of payments as in the Leland-Toft model (see Leland and Toft (1996)). We denote by B_t the loans that banks hold at time t. Given this structure for payments, the value of loans follow

$$B_{t+1} = \delta B_t + I_t$$

This law of motion is a tractable recursive representation for the value of all future coupon payments.[7] Banks grant new loans I_t taking the market price q_t as given.When giving out a loan, banks give out the borrower demand deposits which amount to $q_t^l I_t$. The bank's immediate profits are given by $\left(1-q_t^l\right) I_t$.

A first key feature of our model is that bank loans are illiquid during the balancing stage.[8] The lack of a liquid market for loans in the balancing stage can be rationalized by financial frictions. For

[6] The curvature in the objective function is important to generate smooth dividends, as empiricaly observed. One way to rationalize this is by modelling undiversified investors that invest their entire income in a bank shares. Other ways would be to introduce equity adjustment costs as commonly found in the corporate finance literature.

[7] To see this compute B_{t+1} as function of $I_t, I_{t-1}...$ and substract δB_t.

[8] We maintain the assumption that loans can be sold during the lending stage, which allows us to reduce the state space of the model. In particular, we will show that it will not be necessary to keep track of the composition but only the size of the balance sheet thanks to this assumption. We could break this assumption and obtain non-degenerate cross-sectional distribution of reserves and loan holdings.

example, one can argue that banks may specialize in certain customers or face private information or moral-hazard constraints that lead to this illiquidity.[9]

Demand Deposits. Deposits are callable upon demand. Deposits pay its holder a real interest of r^d. Behind the scenes, banks enable transactions between third parties. We have in mind that when granting loans, borrowers simultaneously receive deposits. These deposits are effectively mediums of payments (deposits) which enables them to purchase goods. In the aggregate, deposits are only created when banks provide loans as noted above.[10] This means that banks create loans (a liability for the borrower) and issue their own liabilities (an asset ultimately held by a third party). The borrower may use deposits to purchase goods. The holder of these deposits can, in turn, transfer the funds to others accounts, make payments and so on. Thus, bank deposits play a role as a medium of exchange. Deposits are reduced as borrowers make payments to banks as their loans mature.

A second key feature is that while banks choose how much deposits to create during the lending stage, they face a random deposit-withdrawal shock in the balancing stage. In particular, banks receive a withdrawal of $\omega_t D_t$ where $\omega_t \sim F_t(\cdot)$ with support in $(-\infty, 1]$. Here, F_t is an exogenous time-varying distribution for withdrawals. For simplicity F_t is common to all banks.[11] When ω_t is positive (negative), the bank loses (receives) deposits in that amount. The shock ω_t is capturing the complexity of transactions in the payments system which leads to randomness in payments. This stochastic process is natural process because, in practice, deposits are being constantly transferred to settle transactions outside the banking system. For simplicity, we assume that deposits do not leave the banking sector:

Assumption 1 (Deposit Conservation)**.** *Deposits remain within banks:* $\int_0^\infty \omega_t F_t(d\omega) = 0$, $\forall_t$.

This assumption implies that there are no withdrawals of reserves from the banking system.[12]

When a deposit it transferred from one bank to another, the receptor bank is absorbing a liability issued by another bank. Therefore, this transaction needs to be settled with a transfer of an asset. Thus, because banks are illiquid, when a deposit is transferred from one bank to another the receptor bank will request reserves to clear out the transaction. The illiquidity of loans induces a demand for reserves.

[9] For various formal arguments see Holmstrom and Tirole (1997) for a model of moral hazard where bankers are required to have a stake on the loans they make; Diamond (1984) and Williamson (1987) who introduce specialized monitoring technologies, and Bolton and Freixas (2009) who introduce a differentiated role for different bank liabilities following from asymmetric information.

[10] Individually, banks can raise deposits and then lend this funds. This is also a possibility in our model: A bank receiving an inflow of deposits will receive also reserves from other banks to settle the transaction. The banker can then sell reserves and purchase or make new loans. In the aggregate, loan creation is deposit creation.

[11] We could assume that F is a function of the bank's liquidity or leverage ratio. This would add complexity to the bank's decisions but would not break any aggregation result. This tractability is lost if F_t is a function of the bank's size.

[12] This assumption can be relaxed without problem to allow for a demand for currency or system wide bank-runs at an extreme.

Reserves. Reserves are special assets in that they are always liquid. During the lending stage, banks can acquire these reserves from the market in exchange for deposits. We denote by p_t be the price of reserves in terms of deposits. Because deposits are in real terms, p is the inverse of the price level.

We assume that banks are required to hold a minimum amount of reserves at the end of the period so that $p_t C_t \geq \rho D_t(1-\omega_t)$, where $\rho \in [0,1]$ is a reserve requirement chosen by the FED. Thus, if ω_t is large, reserves may be insufficient to settle the outflow of deposits. In turn, banks that receive a large inflow of deposits will hold reserves in excess of this requirement. The case of $\rho = 0$ requires banks not to finish with an overdraft in the account for reserves.[13]

To meet reserve requirements or allocate reserves in excess banks can lend and borrow from one another or from the FED. As part of its toolbox, the FED chooses two policy instruments: a lending rate, r_t^{DW}, and a borrowing rate, r_t^{ER} that satisfy $r_t^{DW} > r_t^{ER}$.[14] These rates are intraperiod and paid within the balancing stage periods. The lending rate specifies the rate paid in deposits for a bank that borrows reserves from the FED. The borrowing rate is the interest paid by the FED when banks hold excess reserves that they don't lend to other banks.

Interbank Market. Banks count the dollars of reserves and deficits and decide the fraction to borrow or lend to or from the FED and the fraction they take to the interbank market. We assume that the interbank market for reserves is a directed over-the-counter market.[15] In particular, banks decide to place orders at the borrowing side or the lending side. Orders are placed on a per-unit basis as in Atkeson, Eisfeldt and Weil (2013*).* Once orders are directed to their corresponding sides, orders are matched and banks use Nash bargaining to split the surplus. The bargaining problem for those matches is:

Problem 1 *The interbank bargaining problem solves:*

$$\max_{r^{FF}} \left(m_l r_t^{DW} - m_l r^{FF}\right)^{\xi} \left(m_b r^{FF} - m_b r_t^{ER}\right)^{1-\xi}.$$

In this problem, m_l is the marginal value of bank lending reserves and m_b the corresponding term for banks with excess reserves. The terms of this bargaining problem are the FED funds rate r^{FF}. Banks bargain only about the net rates of loans because the principle of the loan is returned overnight. The objective in the bargaining problem is the following. The surplus borrowing a unit of reserves is $r^{FF} - r_t^{DW}$ because this is the opportunity cost of not borrowing from the FED. The lender gains $r^{FF} - r_t^{ER}$ because if the unit of reserves is not lent to another bank, the lender can lend to the FED.

[13]For operating frameworks that allow reserve averaging over a mainteinance period, banks' choices in our model would correspond to averages over the period.

[14]This determines what in practice is know as the corridor system.

[15]The idea that of modeling the interbank market is inherited from work by Afonso and Lagos (2013), Ashcraft and Duffie (2013) and Duffie (2012).

The first order conditions of this problem are immediate:

$$\frac{\left(r^{FF}-r_t^{ER}\right)^{\xi}}{\left((1+r_t^{DW})-(1+r^{FF})\right)^{1-\xi}}=\frac{(1-\xi)}{\xi}.$$

This condition yields an implicit solution for the FED funds market: r^{FF}. Since $\frac{(1-\xi)}{\xi}$ is positive, it is clear that r^{FF} will fall within the FED's corridor of interest rates. Hence, for example, if $\xi=1/2$, then $r^{FF}=\frac{r_t^{ER}+r_t^{DW}}{2}$. In general, $r^{FF}\in\left[r_t^{DW},r_t^{ER}\right]$.

The probabilities that a lending orders meet borrowing orders depends on the relative masses on either side of the market. In particular, γ^- is the probability that a dollar in deficit finds a matching dollar as surplus. We denote M^+ the mass of lending orders and M^- the mass of borrowing orders. The probability that a borrowing order finds a lending order is given by $\gamma^-=\min\left(1,M^+/M^-\right)$. The probability that a lending order finds a borrowing order is $\gamma^+=\min\left(1,M^-/M^+\right)$. If an order does not find a match, it does not lose the opportunity to lend/borrow from/to the FED. No bank places orders beyond its needs.

Bank Equity and Payouts. Book value equity evolves according to the realization of bank profits that follow from lending to customers and borrowing from the interbank market. The market value of equity is defined as $E_t=q_tB_t+p_tC_t-D_t$ which will evolve depending on the movements in prices. Finally, profits are realized during the lending stage as well as the payment of dividends, DIV_t. Dividends are paid by issuing deposits to share holders.

2.2 Timing, Laws of Motion and Bank Problems

The model can be expressed recursively so we drop time subscripts to explain the problems of banks. If Z is a predetermined state variable at the beginning of the period, $\tilde{Z}$ is the value of this variable by the end of the lending stage and the beginning of the balancing stage. In turn, Z' is the value of that variable when the period ends at the balancing stage. The aggregate state includes monetary policy variables, F_t, and a level of (exogenous) loan demand to be specified below. This aggregate state is summarized in the vector X. We denote by V^l the value function of the bank during the lending stage and V^b the value during the balancing stage.

Lending Stage. Banks enter the period during the lending stage with currency, C, a portfolio of loans, B, and a stock of deposits, D, as their individual states. The problem of the bank consists of choosing dividend payments, DIV, loan issuances, I, and purchases of reserves, φ. The purchase of reserves, φ, occurs during the lending stage and, therefore, is different from reserves borrowed or lent in the interbank market that opens in the balancing stage. The evolution of deposits depends on these decisions. As noted earlier, when providing a loan, banks are simulaneously issuing deposits. When purchasing (selling) reserves, banks are issuing (are paid) deposits to finance the transaction. In addition, dividends to shareholders are paid with deposits. Finally, deposits evolve as loans issued earlier by the bank mature. This leads to the following intra-period law of motion for demand deposits:

$$\tilde{D} = D + qI + DIV + \varphi p - B(1-\delta).$$

Thus, a bank that begins with D as deposits at the beginning of the stage ends with $\tilde{D}$ at the end through the following sources. It credits qI the account of his borrower (or whomever he trades with him), after a loan of size I. It also pays dividends to shareholders in amount DIV. It issues $p\varphi$ liabilities to other banks if it borrows φ_t in cash. Finally, it earns $B(1-\delta)$ deposits from the payment of previously issued loans wich substract from its stock of liabilities.

The evolution of bank reserves is given by the sum of the previous stock plus cash purchases, $\tilde{C} = C_t + \varphi$. Loans evolve according to the fraction of the original stock that has not matured yet plus the newly issued loans, $\tilde{B} = \delta B + I$. Banks choose $\{I, DIV, \varphi\}$ subject to these laws of motion and a regulatory constraints. Capital requirement constraints impose an upper bound, κ, on the amount of leverage (marked-to-market) the bank can take so $\tilde{D} \leq \kappa \tilde{E}$. The problem of a bank during the lending stage is:[16]

Problem 2 *Banks solve the following problem during the lending stage:*

$$\begin{aligned}
V^l(C, B, D; X) &= \max_{I, DIV, \varphi} U(DIV) + \mathbb{E}V^b(\tilde{C}, \tilde{B}, \tilde{D}; \tilde{X}) \\
\frac{\tilde{D}}{(1+r^d)} &= D + qI' + DIV + p\varphi - B(1-\delta) \\
\tilde{C} &= C + \varphi \\
\tilde{B} &= \delta B + I \\
\tilde{D} &\leq \kappa(\tilde{B}q + p\tilde{C} - \tilde{D}); \tilde{B}, \tilde{C}, \tilde{D} \geq 0.
\end{aligned}$$

Balancing Stage. During the balancing state, the decision of the bank consists of placing orders at the interbank market or at the FED. Loans remain unchanged in their balance sheet but the withdrawal $\omega\tilde{D}$ causes as shift in deposits and reserves.

Let x be the deficit in reserves during the balancing stage. Note that for a given bank, this deficit is the result of a shock to the withdrawal of a given bank. A bank's reserve deficit is given by:

$$x = \rho \underbrace{\left(\tilde{D} - \omega\tilde{D}\right)}_{\substack{\text{End-of-Stage} \\ \text{Desposits}}} - \underbrace{\left(\tilde{C}p - \omega\tilde{D}\right)}_{\substack{\text{End-of-Stage} \\ \text{Reserves}}}.$$

Thus, the average return to a dollar in excess reserves is:

$$\chi_l = \gamma^+ r^{FF} + \left(1-\gamma^+\right) r_t^{ER}$$

[16]Technically, the leverage constraints bounds the problem of the banks and prevents a Ponzi-scheme. It is important to note that if the bank arrives to a node with negative equity, the problem is not well defined. However, in choosing its policies, it will make decisions such that it is guaranteed that it doesn't run out of equity. Implicitly, it is assumed that if it violates any constraint, the bank goes bankrupt.

which is the average of the FED funds rate and the FED's lending rate where the average is determined by γ^{+}, the probability that that reserves is allocated in the interbank market. The average interest on a dollar in deficit is, by analogy:

$$\chi_b = \gamma^{-} r^{FF} + \left(1 - \gamma^{-}\right) r_t^{DW}.$$

These averages determine the wedge between the value of having a unit of reserves in excess compared to a unit in deficit. [17] It should be clear that banks with deficits (surplusses) will place borrowing (lending) orders for the full amount of x before contacting the FED because $r^{FF} \in \left[r_t^{DW}, r_t^{ER}\right]$. Hence, we can write the value function for the bank during the balancing stage without reference to the choice of participating in the interbank market or the bargaining process:

Problem 3 *The value of the Bank's problem during the balancing stage is:*

$$\begin{aligned}
V^b(\tilde{C}, \tilde{B}, \tilde{D}; \tilde{X}) &= \beta \mathbb{E}\left[V^l(C', B', D'; X') | X\right] \\
D' &= \tilde{D}(1 - \omega) + \chi(x) \\
B' &= \tilde{B} \\
x &= \rho\left(\tilde{D} - \omega \tilde{D}\right) - \left(\tilde{C}p - \omega \tilde{D}\right) \\
C' &= \tilde{C} - \frac{\omega \tilde{D}}{p}
\end{aligned}$$

where χ represents a "penalty function" that depend on endogenous choices in the lending stage as well as policy tools $\left(r_t^{DW}, r_t^{ER}\right)$:

$$\chi(x) = \begin{cases} \chi_l x \ if \ \ x \leq 0 \\ \chi_b x \ \ if \ \ x > 0 \end{cases}$$

Taking advantage of the simple rule that characterizes the balancing stage, we can write the entire problem of a bank through a single Bellman equation that collapses all of its decisions.

[17] Afonso and Lagos (2012) provide a detailed model for the overnight interbank market.

Problem 4 *The bank's problem during the lending stage is:*

$$
\begin{aligned}
V^{l}(C,B,D) &= \max_{\left\{I,DIV,\tilde{C},\tilde{D}\right\}\in\mathbb{R}^{4}_{+}} U\left(DIV\right)\ldots \\
&\quad +\beta\mathbb{E}\left[V^{l}(\tilde{C}-\frac{\omega'\tilde{D}}{p},\tilde{B},\tilde{D}+\chi((\rho+\omega'\left(1-\rho\right))\tilde{D}-\tilde{C}p);X')|X\right] \\
\frac{\tilde{D}}{1+r^{d}} &= D+qI+DIV_{t}+p\varphi-B(1-\delta) \\
\tilde{B} &= \delta B+I \\
\tilde{C} &= \varphi+C \\
\tilde{D} &\leq \kappa(q\tilde{B}+\tilde{C}-\tilde{D}).
\end{aligned}
$$

We are ready to characterize the bank's problem.

2.3 Characterization of Problems

The recursive problem of banks can be characterized through a single state variable $E\equiv pC+B(1-\delta+\delta q)-D$. To see this, note that the budget constraint can be rearranged as

$$E=q\tilde{B}+\tilde{C}p+DIV-\tilde{D}$$

where the level of equity at the beginning of the period constitutes that can be used to purchase loans, reserves, repay deposits or pay dividends. Updating equity one period forward, we obtain the following single state value function:

Proposition 1 (Single-State Representation)

$$
\begin{aligned}
V(E) &= \max_{\tilde{C},B',\tilde{D},DIV\in\mathbb{R}^{4}_{+}} u(DIV)+\beta\mathbb{E}\left[V(E')|X\right] \\
E &= q\tilde{B}+p\tilde{C}+DIV-\tilde{D} \\
E' &= \left(q'\delta+1-\delta\right)\tilde{B}+p'\tilde{C}-\tilde{D}(1+r_{d})-\chi((\rho+\omega'\left(1-\rho\right))\tilde{D}-\tilde{C}p)(1+r_{d}) \\
\tilde{D} &\leq \kappa(\tilde{B}q+\tilde{C}p-\tilde{D})
\end{aligned}
$$

This problem resembles a standard consumption-savings where dividends, in this case, are financed with holdings of two assets, $\left(\tilde{B},\tilde{C}\right)$ and borrowing $\tilde{D}$ subject to leverage and liquidity constraints. The budget constraint is linear in E and the objective is homothetic in dividends. Thus, by theorems in Alvarez and Stokey (1998) we have that the solution to this problem exists and is unique and we have that policy functions are linear in equity. We have the following proposition.

Proposition 2 (Homogeneity—γ) *The value function $V(E;X)$ satisfies*

$$V(E;X) = v(X)\, E^{1-\gamma}$$

where $v(X)$ is the value of

$$\max_{\tilde{c},\tilde{b},\tilde{d},div\in\mathbb{R}^4_+} u(div) + \beta\mathbb{E}\left[v(X')\,|X\right]\mathbb{E}_{\omega'}\left(e'\right)^{1-\gamma}$$

subject to

$$\begin{aligned} 1 &= q\tilde{b} + p\tilde{c} + div - \frac{\tilde{d}}{1+r^d} \\ e' &= q'\delta + (1-\delta)\tilde{b} + p'\tilde{c} - \tilde{d} - \chi((\rho + \omega'(1-\rho))\,\tilde{d} - p\tilde{c}) \\ \tilde{d} &\leq \kappa(q\tilde{b} + \tilde{c}p - \tilde{d}) \end{aligned}$$

Moreover, all the policy functions of $V^l(E)$ satisfy $X = xE$. In the expression above, $\mathbb{E}_{\omega'}$ is the expectation under F.

The fact that the bank's problem is homogeneous in the bank's market-value of equity has several implications. First, it implies that two banks with different equity values will be scaled versions of a bank with one unit of equity. This also implies that the distribution of equity is not a state variable, but rather only the aggregate value of equity. Moreover, although there is no invariant distribution for bank equity as the variance of distribution may grow over time, the model does have a prediction about the growth rate of its cross-sectional dispersion.

An additional useful property of the banker's problem is that it satisfies a separation theorem whereby the policy function for dividends can be analyzed independently from the policy functions of deposit issuance, cash holdings and loans. Using the principle of optimality, we can break the Bellman equation in two parts.

Proposition 3 (Separation) *The value function $v(\cdot)$ solves:*

$$v(X) = \max_{div\in\mathbb{R}_+} U(div) + \beta\mathbb{E}\left[v(X')\,|X\right]\Omega(X)^{1-\gamma}\left(1-div\right)^{1-\gamma}$$

where $\Omega(X)$ is the value of the certainty-equivalent portfolio of the bank:

$$\max_{\dot{d},\dot{b},\dot{c}\in\mathbb{R}^3_+}\left\{\mathbb{E}_{w'}\left[\left(q'\delta + (1-\delta)\right)\dot{b} + p'\dot{c} - \dot{d}\left(1+r^d\right) - \chi((\rho+\omega'(1-\rho))\,\dot{d} - p\dot{c})\right]^{1-\gamma}\right\}^{\frac{1}{1-\gamma}}$$

This maximization problem consists of choosing portfolio shares among assets of different returns. We obtain this portfolio problem via change of variables that we can later invert to obtain the specific values of $\tilde{c}, \tilde{b}, \tilde{d}$. Define the following portfolio shares: $w_b \equiv q\dot{b}$, $w_c \equiv \dot{c}/p$ and

$w_d \equiv -\dot{d}$.

Return on Loans. Let R^B be there return on a loan with maturity δ. This return is given by the sum of the coupon payment plus resale price so that: $R^B \equiv \left(\delta q + (1-\delta)\right)/q$.

Return on Reserves and Deposits. The return on reserves and deposit components of the portfolios is determined jointly. These returns depend on the withdrawal shock ω. They can be separated into an independent return and a joint return component that follows from the penalty. The independent return on reserves is $R^C \equiv p'/p$, which captures the revaluation component given by the fact that deposits are denominated in real term while reserves are in nominal terms. Hence, R^C is the inverse of inflation. For most of the paper $p' = p$, so that reserves have a return equal to one. Deposits yield a return (cost) of R^D, given by $R^D \equiv (1 + r^d)$.

Portfolio Illiquidity Cost. Finally, the joint return component is captured by potential cost (or benefit) of running out of reserves. This illiquidity cost is given by,

$$R^{\chi}\left(w_d, w_c, \omega'\right) \equiv \chi\left(\left(\rho + (1-\rho)\,\omega'\right) w_d - w_c\right).$$

Return on Equity. Finally, the return on the bank's equity is given by $R^E\left(\omega'; w_b, w_d, w_c\right) \equiv R^B w_b + R^C w_c - R^D w_d - R^{\chi}\left(w_d, w_c, \omega'\right)$ which is given by the weighted sum of returns minus the illiquidity cost.

Using the returns on the banks assets, we obtain the following liquidity management portfolio problem:

Proposition 4 $\Omega\left(X\right)$ *solves the following liquidity-management portfolio problem:*

$$\Omega^*\left(X\right) = \max_{\{w_b, w_d, w_c\}} \left\{\mathbb{E}_{\omega'}\left[\left(R^B w_b + R^C w_c - R^D w_d - R^{\chi}\left(w_d, w_c\right)\right)\right]^{1-\gamma}\right\}^{\frac{1}{1-\gamma}}$$

subject to,

$$\begin{aligned} 1 &= w_b + w_c - w_d \\ w_d &\leq \kappa\left(w_b + w_c - w_d\right), w_d, w_c, w_b \geq 0 \end{aligned}$$

The objective of the problem is to maximize the certainty equivalent of $R^E\left(\omega'\right)$. However, this portfolio problem is not a standard portfolio problem as it features non-linear returns resulting from the interaction between joint determination of the return on reserves and deposits. The intuition behind the main mechanism in this paper can be transparently understood by analyzing the strategies from this problem, which we analyze in the following section. Once we solve the policy functions of this portfolio problem, we can reverse the solution for $\left\{\tilde{c}, \tilde{b}, \tilde{d}\right\}$ via following formulas: $\tilde{b} = (1 - div)\,w_b/q$, $\tilde{c} = (1 - div)\,w_c/p$ and $\tilde{d} = -\,(1 - div)\,w_d/(1 + r^d)$.

The separation between the dividend-payment problem and the portfolio problem for the bank implies that the value function is given by:

$$v^{l}(X)=\max_{div}U(div)+\beta\mathbb{E}\left[v^{l}(X')\,|X\right](\Omega^{*}(X)(1-div))^{1-\gamma}.$$

We can characterize dividends and the value of the bank, without further reference to the choice of deposits, loans or reservers. We have the following proposition.

Proposition 5 (Portfolio) *Given the solution to $\Omega^{*}(X)$, the dividend ratio and value of bank equity are given by:*

$$div(X)=\frac{1}{1+\left[\beta\mathbb{E}\left[v^{l}(X')\,|X\right]\Omega^{*}(X)\right]^{1/\gamma}}$$

and

$$v^{l}(X)^{\frac{1}{\gamma}}=1+(\beta\Omega^{*}(X))^{\frac{1}{\gamma}}\,\mathbb{E}\left[v^{l}(X')\,|X\right]^{\frac{1}{\gamma}}$$

for $\gamma>0$. For the risk-neutral case, $\gamma=0$,

$$div(X)=\begin{cases}0 \textit{ if } \beta\mathbb{E}\left[v^{l}(X')\,|X\right]\Omega^{*}(X)<1\\ 1 \textit{ if } \beta\mathbb{E}\left[v^{l}(X')\,|X\right]\Omega^{*}(X)>0\\ \in[0,1] \textit{ if } \beta\mathbb{E}\left[v^{l}(X')\,|X\right]\Omega^{*}(X)=1\end{cases}.$$

The policy functions by banks will determine a demand for loans and for central bank reserves. This concludes the partial equilibrium description of the bank's problem. We now describe the actions of the FED.

2.4 Central Bank Balance Sheet and Operations

The FED's balance sheet satisfies the following identity:

$$\underbrace{D_{t}^{FED}+B_{t}^{FED}}_{\text{Assets}}=\underbrace{M_{t}^{0}+E_{t}^{FED}}_{\text{Liabilities}}.$$

Here M_t^0 stands for the amount of reserves (or high power money) issued by the FED, D_t^{FED} are its holdings of commercial bank deposits, and B_t^{FED} are holdings of private loans. The variable E_t^{FED} corresponds to the FED's equity. The FED has a monopoly over the supply of reserves, M_t^0 and alters this quantity via open-market operations . The balance sheet constraint of the fed is given by

$$p_{t}\left(M_{t+1}^{0}-M_{t}^{0}\right)=D_{t+1}^{FED}-D_{t}^{FED}+q_{t}\left(B_{t+1}^{FED}-\delta B_{t}^{FED}\right)+T_{t}-r_{t}^{ER}\phi_{t}+r_{t}^{DW}\psi_{t}$$

where ϕ_t represents loans by the FED in the discount window and ψ_t represents deposits taken by the FED in the form of reserves. The term T_t correspond to transfers to the fiscal authority, which is the analogue of FED dividends.

Unconventional Open-Market Operations. Since there are no T-Bills so far, only unconventional monetary operations are available to the FED.[18] Thus, unconventional open-market is the purchase of loans without increasing the the deposits held by the FED.

Open-Market Liquidity Facilities. Liquidity facilities are swap of liabilities of the FED for deposits.

FED Profits. In equilibrium, the Fed can experience profits or losses (e.g. if they hold bonds, or treasuries they make a return at steady state), or via the charge on penalties to commercial banks.

FED Targets. We assume that the FED choses targets for the FED funds rate r^{FF} and for the value of reserves, $p_t = p$.

2.5 Market Clearing and Evolution of Bank Equity

Loan Demand. We consider a demand for loans of the form:

$$q_t = \Theta_t \left(I_t^D\right)^{\epsilon}, \epsilon > 0, \Theta_t > 0. \tag{1}$$

Since the yield on loans depend negatively on q_t this demand function is increasing on q_t. The appendix provides a derivation for this demand for loans. In the quantitative analysis, we will consider shocks to Θ_t to uncover whether demand or supply shocks were more relevant in the context of the US recent credit crunch. Market clearing for the loans market requires us to equate I_t^D to the supply of new loans by banks and the FED. Hence, we have $I_t^D = B_{t+1} - \delta B_t + B_{t+1}^{FED} - \delta^T B_t^{FED}$.

Money Market. Similarly, the market for reserves must clear. Since reserves are not leant outside the banking system equilibrium, requires

$$\int_0^1 \tilde{c}_t\left(z\right) E_t\left(z\right) dz = M_t^0 \longrightarrow \tilde{c}_t \bar{E}_t = M_t^0.$$

Given that the model does not have a role for coins and currency, banks reserves represent the whole of the monetary base. Deposits, instead correspond to the monetary creation by banks, $M_t^1 \equiv \int_0^1 \tilde{d}_t\left(z\right) E_t\left(z\right) dz$. Hence, similarly to Brunnermeier and Sannikov (2012), the model yields and endogenous money multiplier $\mu_t = \frac{M_t^1}{M_t^0}$.

Interbank Market. The relevant equilibrium conditions for the interbank market are given by the relative masses of banks in surpluss and deficti. Thus, we need to obtain the expressions for γ^+ and γ^-. The probability that a bank has a deficit is given by the value of ω that sets $x = 0$, which is the mass under the cutoff shock $\omega^* = \left(\tilde{C}/p - \rho\tilde{D}\right)/\left(1-\rho\right)$. Since we already showed that banks are identical replicas of each other, scaled up by their equity. Thus, for every value

[18] Incorporating Treasury Bills (T-Bills) into our model is quite straight forward. Appendix xxx, discusses two extensions. One in which deposits have the liquidity properties of reserves and the other when they don't.

of E, there's an identical distribution of banks short and long of reserves. This implies that the mass of reserves in deficit in is given by:

$$M^{-}=\mathbb{E}\left[x|x>0\right]\left(1-F\left(\frac{\tilde{C}/p-\rho\tilde{D}}{(1-\rho)}\right)\right)E$$

and the mass of surplus reserves is,

$$M^{+}=\mathbb{E}\left[x|x<0\right]F\left(\frac{\tilde{C}/p-\rho\tilde{D}}{(1-\rho)}\right)E,$$

a number that is endogenous to the choice of $\left(\tilde{C},\tilde{D}\right)$. The term $\mathbb{E}\left[x|x>0\right]=(1-\rho)\,\mathbb{E}\left[\omega|\omega>\omega^{*}\right]$ and $\mathbb{E}\left[x|x<0\right]=(1-\rho)\,\mathbb{E}\left[\omega|\omega<\omega^{*}\right]$.

Discount window loans must be equal to the deficit in reserve holdings in the balancing stage $\phi=\max(C/p-\rho D,0)$ and $\psi=\max(\rho D-C/p,0)$

Bank Equity Evolution. The distribution of bank equity evolves according to $E_{t+1}(z)=e_{t}E_{t}(z)$. The growth rate e_t uses the scale invariance property described in the previous section. The measure of equity holdings at each bank evolves is denoted by Γ_t. Since the model is scale invariant, we just need to keep track of the evolution of average equity, $\int_0^1 E_t(z)\,dz$, which by independence grows at rate $\mathbb{E}_{\omega}\left[\Psi_t\right]$.[19]

Using this expression and the results from the previous section, we can express the supply of new loans in period t through:

$$I_t^S=\tilde{b}_t\int_0^1 E_t(z)\,dz-(1-\delta)\,\tilde{b}_{t-1}\int_0^1 E_{t-1}(z)\,dz.$$

Notice that the supply of new loans, which determines q_t, is the difference between current demand for the stock of loans minus the fraction of previously existing loans that has not matured yet.

2.6 Equilibrium

The definition of equilibrium is as follows:

Definition. *A competitive equilibrium is a sequence of bank policy rules* $\left\{\tilde{c}_t,\tilde{b}_t,\tilde{d}_t,div_t\right\}_{t\geq0}$*, bank values* $\{v_t\}_{t\geq0}$*, government policies* $\left\{\rho_t,D_t^{FED},B_t^{FED},M_t^0,\kappa_t,r_t^{ER},r_t^{DW}\right\}_{t\geq0}$*, aggregate shocks* $\{\Theta_t,F_t\}_{t\geq0}$*, measures of equity distributions* $\{\Gamma_t\}_{t\geq0}$*, measures of reserve surplusses and deficits* $\{M^+,M^-\}_{t\geq0}$ *and prices* $\left\{q_t,p_t,r_t^{FedFunds}\right\}_{t\geq0}$*, such that: (1) Given price sequences* $\left\{q_t,p_t,r_t^{FedFunds}\right\}_{t\geq0}$ *and policies* $\left\{\rho_t,D_t^{FED},B_t^{FED},M_t^0,\kappa_t,r_t^{ER},r_t^{DW}\right\}_{t\geq0}$*, the policy functions* $\left\{\tilde{c}_t,\tilde{b}_t,\tilde{d}_t,div_t\right\}_{t\geq0}$ *are solutions to Problem 4. Moreover,* v_t *is the value in Proposition 3. (2) The Money Market Clears:* $\tilde{c}_t\bar{E}_t=M_t$,

[19] A limiting distribution for Γ_t is not well defined unless one adapts the process for equity growth.

(3) The loan Market Clears: $I_t^S = \Theta_t^{-1} q_t^{\frac{1}{\epsilon}}$, *(4) The measure* Γ_t *evolves consistently with* $\Psi_t(\omega)$, *the masses* $\{M^+, M^-\}_{t\geq 0}$ *are consistent with policy functions and the sequence of distributions* F_t.

Moreover, all the policy functions of Problem 4 satisfy $X = xE$.

2.7 Theoretical Analysis

To gain more intuition, it is convenient to analyze the portfolio problem of the bank derived in Proposition 4.

Excess Returns in $\Omega^*(X)$. Fix any given state X. It is useful to re-write the portfolio problem $\Omega(X)$ by substituting the weight on loans. We obtain:

$$\max_{\{w_d, w_c\}} \left(\mathbb{E}_{\omega'} \left[\left(\underbrace{R^B}_{\text{Return to Equity}} \underbrace{- \left(R^B - R^C\right) w_c}_{\text{Cash Opportunity Cost}} + \underbrace{\left(R^B - R^D\right)}_{\text{Arbitrage}} w_d - \underbrace{R^\chi(w_d, w_c, \omega')}_{\text{Liquidity Cost}} \right)^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}$$

subject to $w_d \leq \kappa$ The objective in $\Omega(X)$ clear. If banks hold all their equity on loans, they would obtain R^B per unit of equity. Issuing an additional deposit yields an arbitrage opportunity when the spread between return on loans and return on deposits is positive: $\left(R^B - R^D(\omega')\right)$. Reserve holdings have the classical opportunity cost of cash, $\left(R^B - R^C\right)$ but they yield the benefit of reducing the exposure to liquidity risk by reducing the expected liquidity cost $R^\chi(w_d, w_c, \omega')$.

Liquidity Premium. In the context of our model, reserves have a liquidity premium relative to loans. To see this, we can derive the first order conditions of the problem above. The conditions for w_c and w_d are respectively

$$\mathbb{E}_{\omega'}\left[m' \cdot \left\{\left(R^B - R^D\right) - R_d^\chi(w_d, w_c, \omega')\right\}\right] + \kappa\mu = 0 \tag{2}$$

and

$$\mathbb{E}_{\omega'}\left[m' \cdot \left\{\left(R^B - R^C\right) + R_c^\chi(w_d, w_c, \omega')\right\}\right] = 0. \tag{3}$$

where μ is the multiplier on the capital requirement constraint, and $m' = \beta\left(div' R^E(1-div)E\right)^{-\gamma} / (div)^{-\gamma}$ is the stochastic discount factor. We can use these expressions to derive a liquidity premium, i.e. a difference between the market return on loans and cash. Rearranging (3) we obtain:

$$\begin{aligned} \underbrace{R^B - R^C}_{\text{Cash Opportunity Cost}} &= -\frac{\mathbb{E}_{\omega'}\left[m' \cdot R_c^\chi(w_d, w_c, \omega')\right]}{\mathbb{E}_{\omega'}[m']} \\ &= \underbrace{\mathbb{E}_{\omega'}\left[R_c^\chi(w_d, w_c, \omega')\right]}_{\text{Direct Liquidity Effect}} - \underbrace{\frac{\mathbb{COV}_{\omega'}\left[m' \cdot R_c^\chi(w_d, w_c, \omega')\right]}{\mathbb{E}_{\omega'}[m']}}_{\text{Liquidity Risk Premium}}. \end{aligned}$$

This expression is close to standard asset-pricing equations and can be used to obtain measures of the increase in the perceived liquidity risk. The return on reserves has two terms, a direct return, R^C and an additional additive term that follows from the reduction in liquidity risk for the bank, $R_c^\chi(w_d, w_c, \omega')$. The expression above says that the excess return on loans, $R^B - R^C$, the opportunity cost of holding reserves, equals the additional benefit of holding reserves $R_c^\chi(w_d, w_c, \omega')$ which is adjusted by the risk-premium associated with the withdrawal shocks. A similar expression can be derived for the spread between loans and deposits:

$$\underbrace{R^B - R^D}_{\text{Arbitrage}} = \underbrace{\mathbb{E}_{\omega'}\left[R_d^\chi(w_d, w_c, \omega')\right]}_{\text{Direct Liquidity Effect}} - \underbrace{\frac{\mathbb{COV}_{\omega'}\left[m \cdot \left(R_d^\chi(w_d, w_c, \omega') - R^D(\omega')\right)\right]}{\mathbb{E}_{\omega'}[m']}}_{\text{Liquidity Risk Premium}}.$$

This expression states that the arbitrage on loans by borrowing with deposits is limited by the direct expected increase in liquidity costs $\mathbb{E}_{\omega'}\left[R_d^\chi(w_d, w_c, \omega')\right]$ adjusted by the liquidity risk premium on deposits.

Notice also, that when the capital requirement constraint is binding, there is a larger excess return between loans and deposits.

3 Calibration and Quantitative Analysis

3.1 Dispersion of Deposit Growth

Calibrating our model requires an empirical counterpart for the random-withdrawal process for deposits, F_t. We use information from individual commercial bank Call Reports collected by the Federal Deposit Insurance Corporation (FDIC) to describe the evolution of deposit withdrawals. The Data Call Reports present balance-sheet information for all commercial banks in the US. Publicly available data spans all the quarters from 1990 until 2011. The Appendix, provides more details on how we construct the data we report in this section and other aspects of the data that are relevant for the paper.

We take the stance of calibrating F_t using information from the volatility of Total Deposits. To justify this choice, we need first to discuss Figure 1. The bars in the figure contain pre-crisis sample (2000Q1-2007Q4) information. The solid line that shifts to the left reports Great Recession (post-crisis) (2008Q1-2010Q4) information. The units of observation in Figure 1 correspond to quarter-bank observations on Total Deposits. The histogram plots the empirical frequencies of cross-sectional deviations of the growth rates of each bank quarter from the average growth rate for a given quarter in the sample.

In our model, banks feature the same growth rates of equity as the average bank unless they experience a withdrawal shock. Thus, in the model, a bank showing an increase in deposit growth higher than the mean is a bank experiencing an inflow of deposits, and viceversa. Hence, the

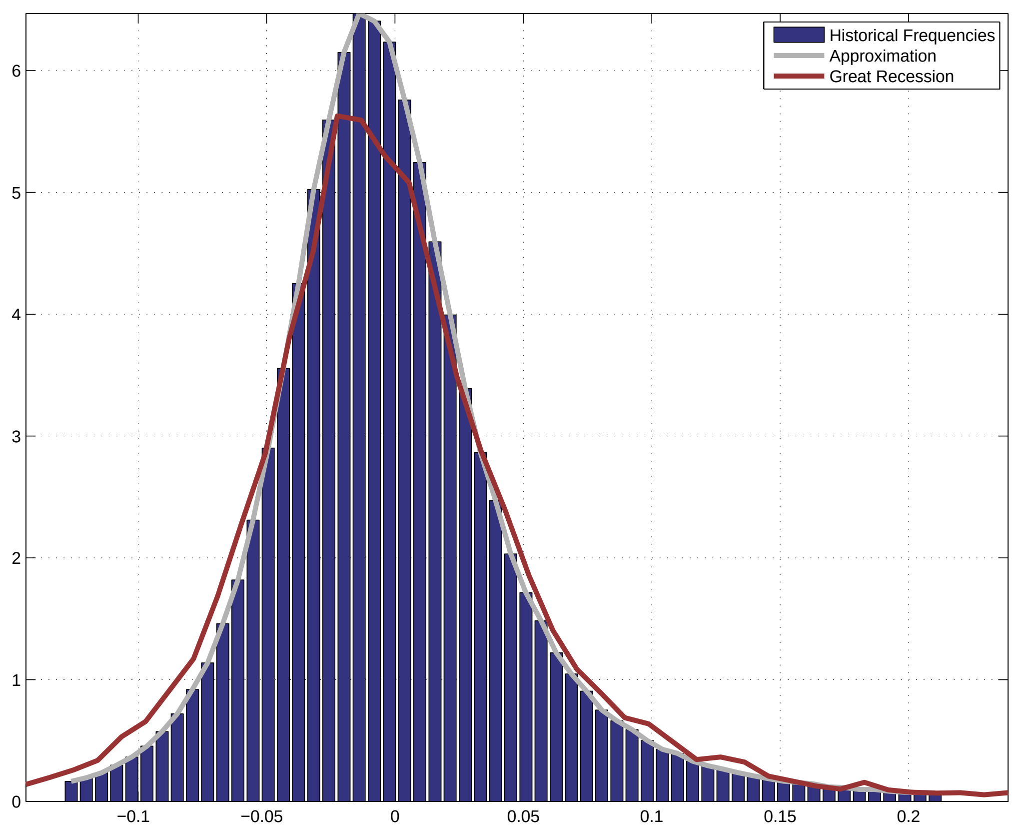

Figure 1: Cross-Sectional Distribution of Deviation from Cross-Sectional Average Growth Rates

deviations from average growth rates have a one-to-one map to the withdrawal shocks. Banks in our model have only one form of liability, demand deposits. In practice, commercial banks have other forms of liabilities that include bonds, long-term deposits (savings deposits) and other variable income securities. For this reason, we must be careful in our choice of the data counterpart of F_t. We choose total deposits as our counterpart.

In the Appendix, we show that Total Deposits in the data are substantially less volatility than Demand Deposits. Second, Total Deposits feature a trend which is consistent with the growth of bank liabilities whereas this is not the case for demand deposits. In practice, total deposits may include savings for short periods of time, or may also be removed from a bank at a cost.

Given the substantial variation in the volatility of total deposits observed in Figure 1 we believe this is a relevant measure to capture illiquidity risk. Thus, we use this empirical histogram of quarterly deviations of today deposits, to calibrate F_t, the process for withdrawal shocks, non-parametrically. Our model also predicts that the behavior of equity should be perfectly correlated with the behavior of deposits. We report this correlation in the Appendix. We find that the correlation is positive, as suggested by our model significantly lower from one which is reasonable given that equity captures variations in the prices of securities, credit risks and operating costs that we don't include in the model. The data appendix discusses this point as well as the validity of the growth independence assumption. A final thing to note is that the variation in deposit growth has moved to the left.

3.2 Banking Sector Data Moments

Figure 2 plots the evolution of some of key ratios for banks. These moments have a direct mapping to our model. We also use these moments to estimate changes in Θ_t in the data once we perform the quantitative investigation of our model. A noticeable fact to keep in mind is the increase in the liquidity ratios, together with sharp declines on return on assets, leverage and dividend ratios during the crisis.

3.3 Calibration

We are not finished with the calibration of the model. The values of all parameters are listed in Table 1. We need to assign values to eleven parameters $\left\{\kappa, \beta, \delta, \gamma, \epsilon, \rho, r^d, r^b, r^l\right\}$. Each period in the model represents a quarter. We set the capital requirement and the reserve requirement according to standard regulatory measures. In particular, we set $\kappa = 24$, which corresponds to the Tier 1 Capital Ratio, $\rho = 10$ percent. The discount factor is set to 0.99. The risk aversion is set to 4 and $r^d = 0$.

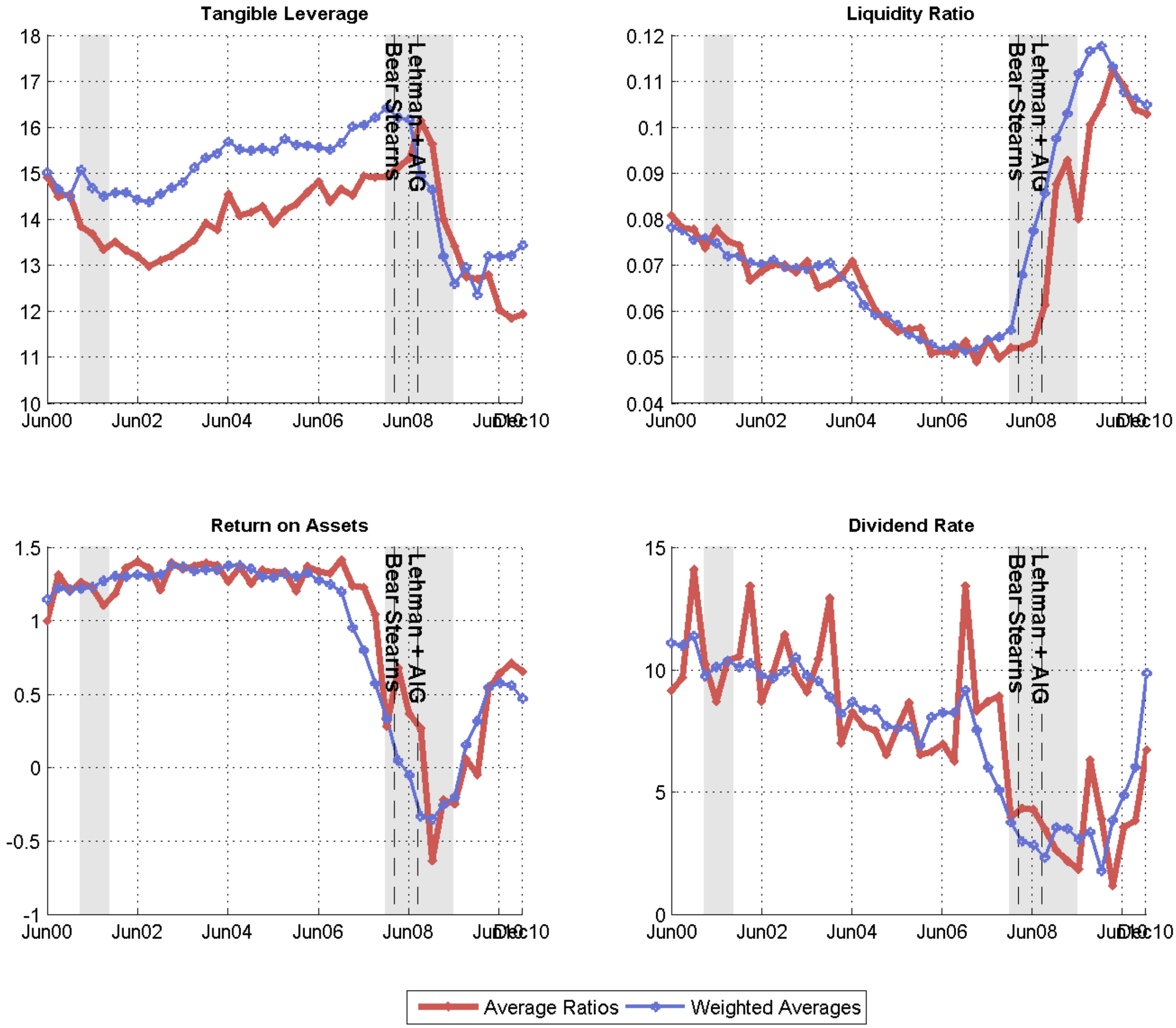

Figure 2: Evolution of Key Ratios for the banking sectors during the last decade.

Table 1: Parameter Values

	Value	Reference
Capital requirement	$\kappa = 20$	5% Tier-1 Capital
Discount factor	$\beta = 0.995$	Benchmark
Risk aversion	$\gamma = 4$	Benchmark
Loan Maturity	$\delta = 0.87$	3.5 year mat
Liquidity Requirement	$\rho = 0.10$	Res. Req.
Loan Demand Elasticity	$\epsilon = 0.01$	
Discount window rate	$r^b = 5$	
Withdrawal-shock volatility F_t	Non-Param	Data

We set r^b, the interest rate on discount window to 5 percent annually, which is the rate at the beginning 2007. The loan average maturity is set to 2 years, implying a value for $\delta = 0.87$. The value of the loan demand elasticity $\epsilon = 0.01$. Finally, for the withdrawal we use a non-parametric estimation as described above.

4 Policy Functions - Prices Given

We start with a partial equilibrium analysis of the model by showing banks policy functions at different prices. Figure 3 reports decisions for cash, loans, dividend, as well as liquidity and leverage ratios, the value of the asset portfolio, liquidity risk, expected returns and expected equity growth. These policies correspond to the solution to the Bellman equation (4) for different values of loan prices q.

A first observation that emerge from Figure 3 is that the supply of loans is decreasing in the loan price whereas dividends and cash ratios are increasing in the loan price. As loan prices decrease, loans become relatively more profitable leadings banks to keep a lower fraction of its assets in relatively low return assets, i.e., cash. Moreover, banks cut on dividend rate payments to allocate more funds to loan issuances and experience higher equity growth. The exposure to liquidity risk, measured as the expected penalty costs from falling short of cash holdings, is also decreasing in loan prices, reflecting the fact that banks' asset portfolio becomes relatively more liquid.

Another key observation is that there is a kink in banks' policies at the value of the loan price where the capital requirement ceases to bind. In particular, when the price of loans is sufficiently high, high profits from intermediation lead banks to reduce deposits to the point in which the capital requirement is not binding. In this region, a decrease in the loan price leads to a sharp increase in lending ratios. To the left of this point, decreases in the loan price lead to a less of a sharper increase in lending rates due to the fact that higher lending needs to be financed with either lower dividend payments or a reduction in cash holdings, which exposes the banks to more severe effects of withdrawal shocks. In other words, the capital requirement generates an asymmetric response to changes in the loan price depending on whether the capital requirement is binding or not.

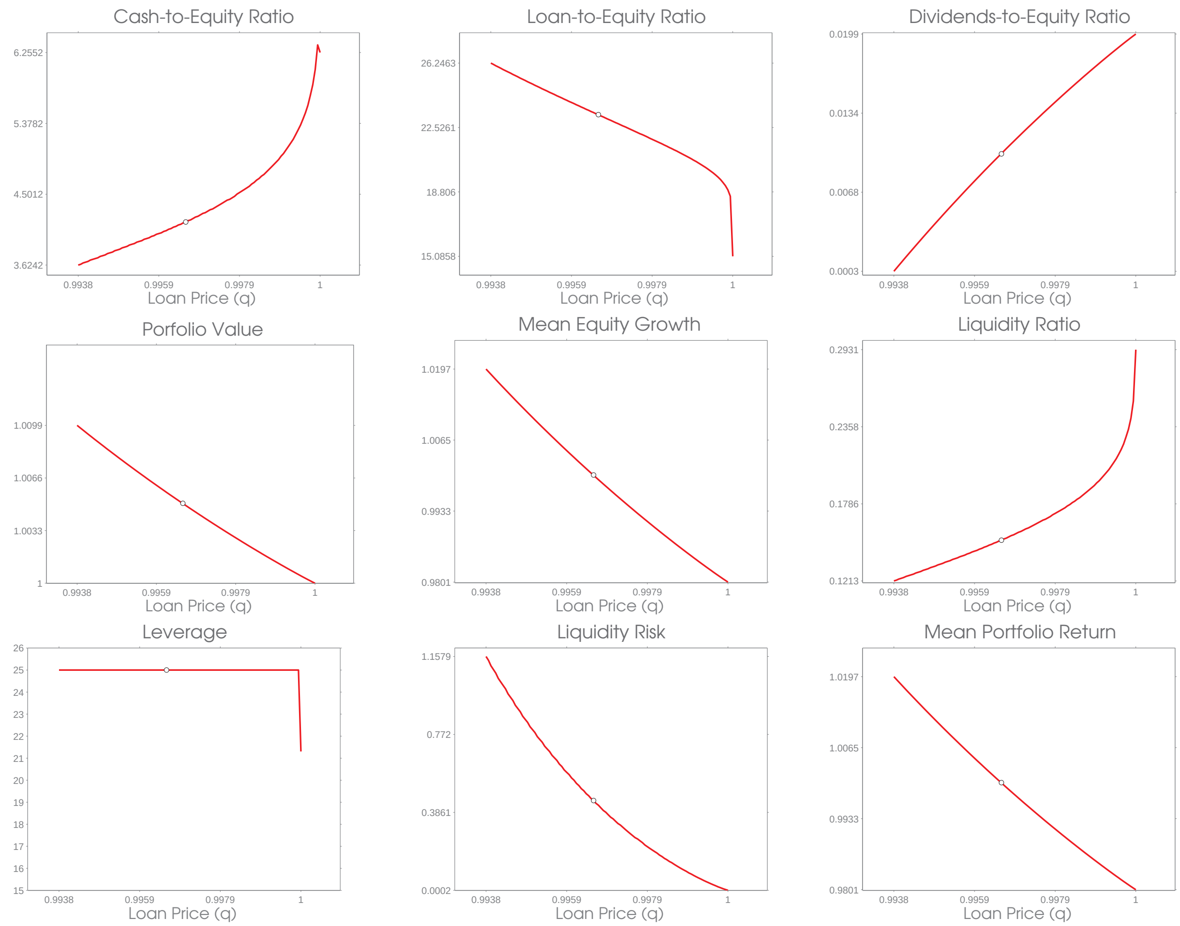

Figure 3: Policy function for different Loan Prices

5 Transitional Dynamics

This section studies how the economy responds to different shocks. For this purpose, we consider an economy that is at steady state at period $t = 0$ and experience an unanticipated shock. The shocks we consider are equity losses, credit demand shocks, a tightening of capital requirements, uncertainty shocks and rise in funding costs. For each shock, we analyze the transitional dynamics of banking and monetary indicators, as illustrated in Figures 4-??. The superior panel for each figure shows aggregates for equity, lending, cash and the return on loans. The inferior panel shows the ratios for cash-to-equity ratio, loan-to-equity ratio, dividends-to-equity ratio as well as portfolio value, banks' marginal value and liquidity risk.

5.1 Equity Losses

The first shock we consider is a sudden unexpected decline in bank equity. This shock can be interpreted as capturing an unexpected rise in non-performing loans or losses from other sources of risk.[20] To the extent that equity is the key state variable, the analysis of the transition dynamics to an equity loss would also be relevant to understand the dynamics of the economy in response to other shocks as well.

Figure 4 illustrates the response of the economy to equity losses of 4 percent. The economy experiences on impact a drop in loan prices, i.e. an increase in loan returns, which further reduces the value of equity. Moreover, aggregate lending and cash holdings also decline. Because of the marked-to-market capital requirement constraint, the initial shock causes a fire sale effect as the decline in equity, loan prices and lending mutually reinforce each other. After about 6 years, the economy converges back to the initial steady state.

Let us analyze the transitional dynamics in more detail. What explains the immediate increase in loan returns? Suppose loan returns did not change at all. In this case, banks would keep the same lending ratios and the volume of new loan issuances $E_1b_1 - E_0b_0(1-\delta)$ would fall. If loan return do not vary, however, the demand for loans would remain unchanged leading to an excess demand for loans. In equilibrium, this means that loan returns need to rise on impact. High return on loans also imply low dividend rates and equity growth. Hence, as the financial sector gradually recovers from the equity losses, the economy experiences a decrease in loan returns until it converges to the same initial steady state.

It is important to notice that the volume of bank lending follows a persistent decline. In fact, the volume of total lending falls on impact following the contraction of the balance sheets and continue to fall after the initial shock. The reason for this persistent decline is due to the long-maturity of loans that introduces a "stickiness" in the behavior of the stock of loans. In particular, the flow of lending can be decomposed into the difference between the new issuances and the repayment of old loans. As the return on loans increase on impact, there is a sharp decline in the demand for new issuances, which is larger than the repayments of previous loans. This explains the initial fall in the stock of loans. As new issuances remain below repayments, the stock of loans continue to decline. Once the stock of loans become low enough, new issuances become larger than loan repayments, and this leads to a monotonic increase in the stock of lending.

Equity losses have other important implications for banks' liquidity management. In particular, since loans return are high along the transition, banks invest a lower fraction of their equity in

[20] One way to incorporate this explicitly in the model would be to consider an aggregate shock to the default rate on loans.

cash and invest relatively more in loans. As a result, banks become more exposed to illiquidity risk.

5.2 Tightening of Capital Requirements

Next, we consider a sudden tightening of capital requirements, i.e., a reduction in κ.[21] This shock can be interpreted as a tightening in regulatory requirements, or alternatively as reflecting a rise in solvency concerns about the banking sector due to standard agency problems.

Figure 5 illustrates the effects of an decrease in κ of 30 percent, which is associated with an increase in the capital ratio of 2.5 percent, which is the extra capital buffer imposed by Basel III. We assume that the reduction in capital requirements are permanent and follow a gradual increase, modelled as a deterministic AR(1) process, such that the the adjustment in κ is almost complete after 5 years.

Before analyzing the transitional dynamics, let us first analyze the effects on the steady state of the economy. A central observation, apparent in Figure 5, is that the new steady state features a higher return on loans. What explains the increase in the return on loans is the fact that the contraction in capital requirements reduces the banks' loanable funds. As the supply of credit is reduced, this requires a higher return on loans to clear the loans market. Moreover, higher return on loans also imply lower dividend rates as well as lower cash holdings.

What does the transition to the new steady state look like? On impact, that there is a sharp increase in the return on loans that exceed the long-run increase, i.e., there is an overshooting in the increase in the return on loans. The overshooting result is a key prediction of our model. To understand the intuition behind this result, it is useful to consider an alternative transition where the return on loans adjusts immediately to the new steady state. On the supply side, a constant return on loans implies a constant loan-to-equity ratio and because equity is below the new steady state value, this implies that total supply of new lending is lower on impact compared to the new steady state value. On the demand side, a constant return on loans imply a constant demand for loans. Because the economy is in equilibrium at the new steady state, this implies that under the scenario of a constant return on loans, there would be on impact an excess demand for loans. As a result, the return on loans have to experience a sharp increase on impact and then decline towards the new steady state.

The overshooting in the return on loans has implications for the dynamics of banks' balance sheet positions as well. In particular, lending rates b_t are characterized by a drop on impact followed by a continued drop. This gradual decline in lending rates is consistent with the behavior of loan return. That is, the sharp increase on impact in the return on loans mitigate the contraction in lending rates caused by the tightening of capital requirements. Over time, as the return on loans is reduced, lending rates continue to fall. Total lending B_t follows a relatively similar pattern with

[21] Recall that the capital requirement constraint is binding in the initial steady state.

the key difference that equity growth along the transition path partially compensates the drop in lending rates. As a result, the stock of loans experience a sharpe decline although after a few periods, there is an increase in the stock of loans. In the new steady state, total lending remains below the initial steady state reflecting the tighter capital requirement constraint.

On the other hand, cash holdings both in absolute value and relative to assets and equity fall more in the short run than in the long run. The asymmetry between the response of reserve and lending rates is due to the general equilibrium effects of the tightening in capital requirements. While the tightening of capital requirements limits the expansion of banks' balance sheets, the increase in the return on loans generate a reallocation in banks portfolio towards loans. There is another effect at work beyond the price effects on the portfolio decisions. Because banks have lower leverage, this also reduces the liquidity risk premium. leading banks to invest relatively more in loans. Hence, banks have a lower liquidity risk premium and become relatively more exposed to illiquidity risk.

5.3 Bank-Run Probability

The next shock we consider is an increase in uncertainty in the process for withdrawals. In particular, we assume that there is a probability that banks would face a bank-run can lose the entire stock of deposits.[22] This shock arguably played a role in the US financial crises as confidence in the financial sector plummeted, and is consistent with the dispersion on deposit growth documented in section 3 . Figure 6 shows the effects of an increase in the bank-run probability from zero to 10 percent. We assume that this shock follows a deterministic AR(1) process so that the shock dies out after about 2 years.

The introduction of a probability of a bank-run generates a more precautionary behavior by banks as the risk of illiquidity becomes larger. Banks respond by reallocating their portfolio from loans to cash . The decline in the supply of loans generate an increase in loan return, i.e. a decline in loan prices and the value of equity, which tightens the capital requirement constraint and amplifies the initial drop in lending. As explained above, the long-maturity of loans imply that the drop in loans is persistent over time.

The reallocation of assets towards cash also generate a persistent decline in equity. Notice that once the bank-run shock dies out, the banking sector has a level of equity which is below the steady state value. As a result, the return on loans remains above steady state and lending (cash) rates are above (below) steady state.

Finally, the introduction of a bank-run probability generates a decline in dividend rates. This follows again from a precautionary behavior in response to a more uncertain value of banks.

[22] To keep the distribution somewhat symmetric, we also assume that with the same probability banks can receive a large inflow of deposits that would double the stock of deposits.

5.4 Credit Demand Shocks

We now study the effects of negative credit demand shock, i.e. a decline Θ_t. This shock can be though as capturing a decline in total factor productivity that reduces the demand of loans by firms. However, a credit demand shock could also have a financial origin, e.g., a decline in the value of firm's or household collateral that limit their ability to borrow. The explicit modelling of the demand shock is beyond the scope of this paper. Figure 8 illustrates the effects of a temporary decline in credit demand. The shock follows a deterministic AR(1) process that lasts for about 7 years.

The effects of credit demand shocks contrast sharply with the effect of the shocks considered above. In particular, banks respond to a lower demand of loans by paying higher dividends and investing in cash. A negative credit demand shock also generates a rise in loan prices as well as an increase in the value of equity, which also makes the capital requirement *less* tighter.

How does the economy converge to the new steady state? Because the demand shock generate low portfolio returns, banks increase dividend rates and experience a reduction in equity. Moreover, as equity is reduced along the transition, loan prices fall leading to increasing portfolio returns until the steady state. Finally, an important observation is that there is on impact a sharp increase in the reserve rate and in the liquidity ratio. Intuitively, banks respond to the decline in credit demand by reallocating their portfolio towards cash and away from loans.

6 Monetary Policy during 2008-2013

This section describes the patterns observed for monetary policy in the US during the last 5 years. Then, we introduce a particular sequence of shocks into our model and describe how far does our model go in explaining this facts depending on the shocks that we introduce.

7 Conclusions

The bulk of money-macro models has developed independently from the insights from literature on banking (Diamond and Dybvig (1983)). In doing so, the profession has lacked an explicit modeling of monetary policy through the financial system, and for good reasons. For a long time it did not seem to make much difference, monetary policy seemed to be carried with ease and with stable banks, equity growth, leverage, bank dividends and interest premia. Thus, in absence of crises, the activities of the financial sector can appear irrelevant for long stretches of time. The crisis, revealed that having a model that allows us to study banks in conjunction with banking may be a desired tool in our theoretical toolbox.

This paper has borrowed from the banking literature, to introduce a model where by maturity mismatch between assets and liabilities cause banks to demand reserves. A Central Bank can influence real activity by altering the trade-off in bank lending via various inputs. We continue to investigate the quantitative implications of the model and the interaction of different shocks with monetary policy.

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A Additional Figures

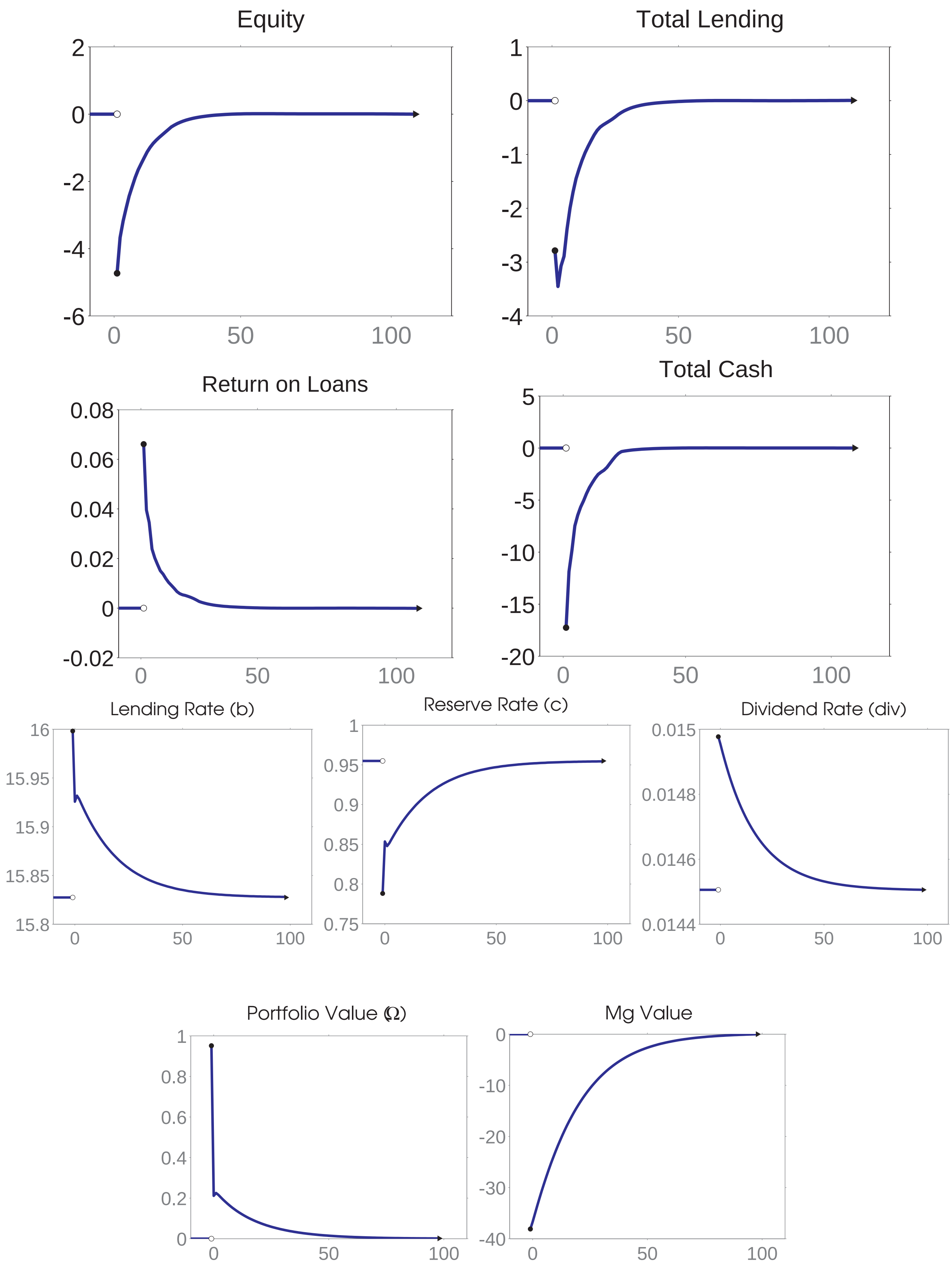

Figure 4: Impulse Response to Equity Losses

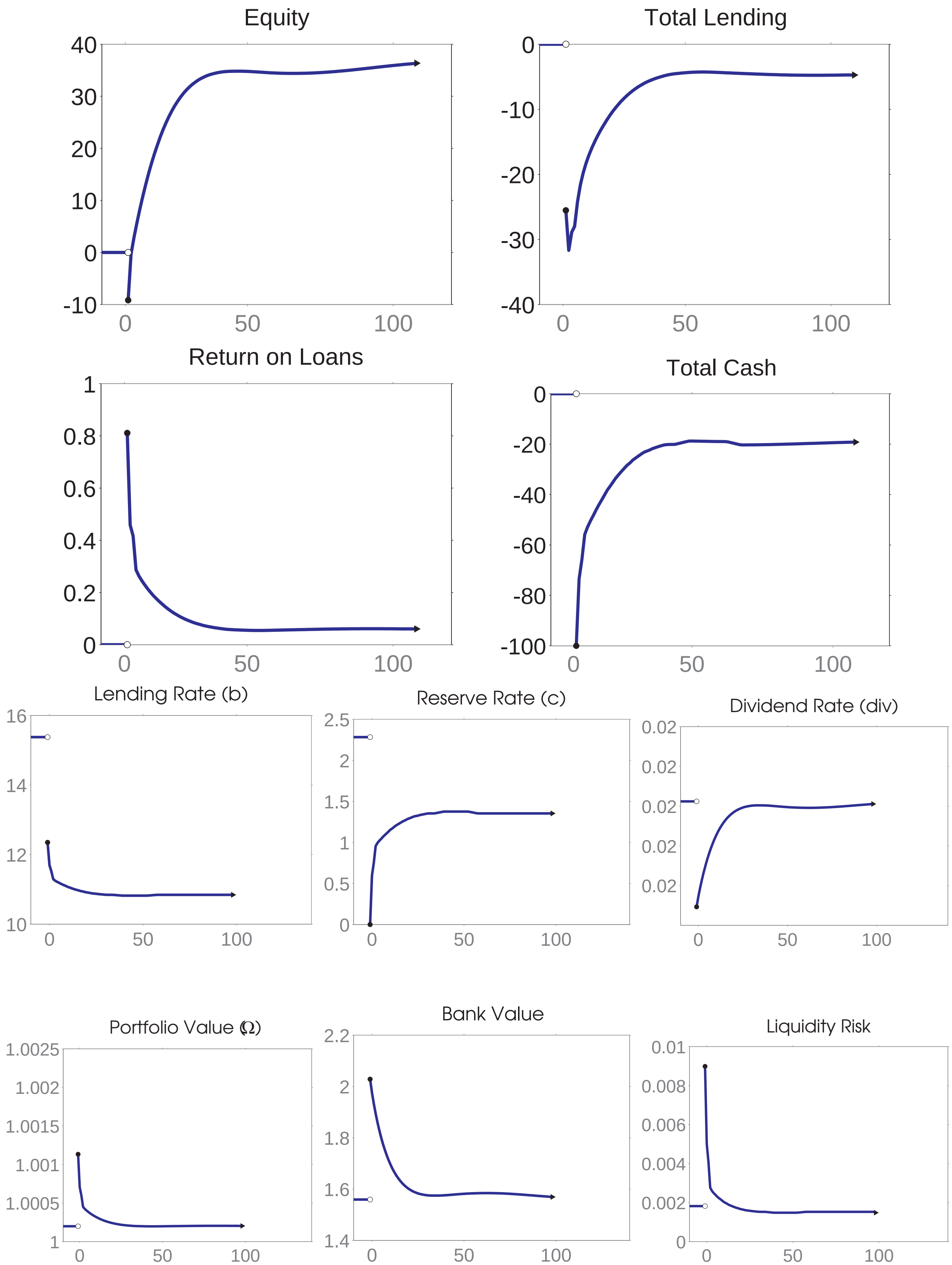

Figure 5: Impulse response to permanent gradual increase in capital requirements

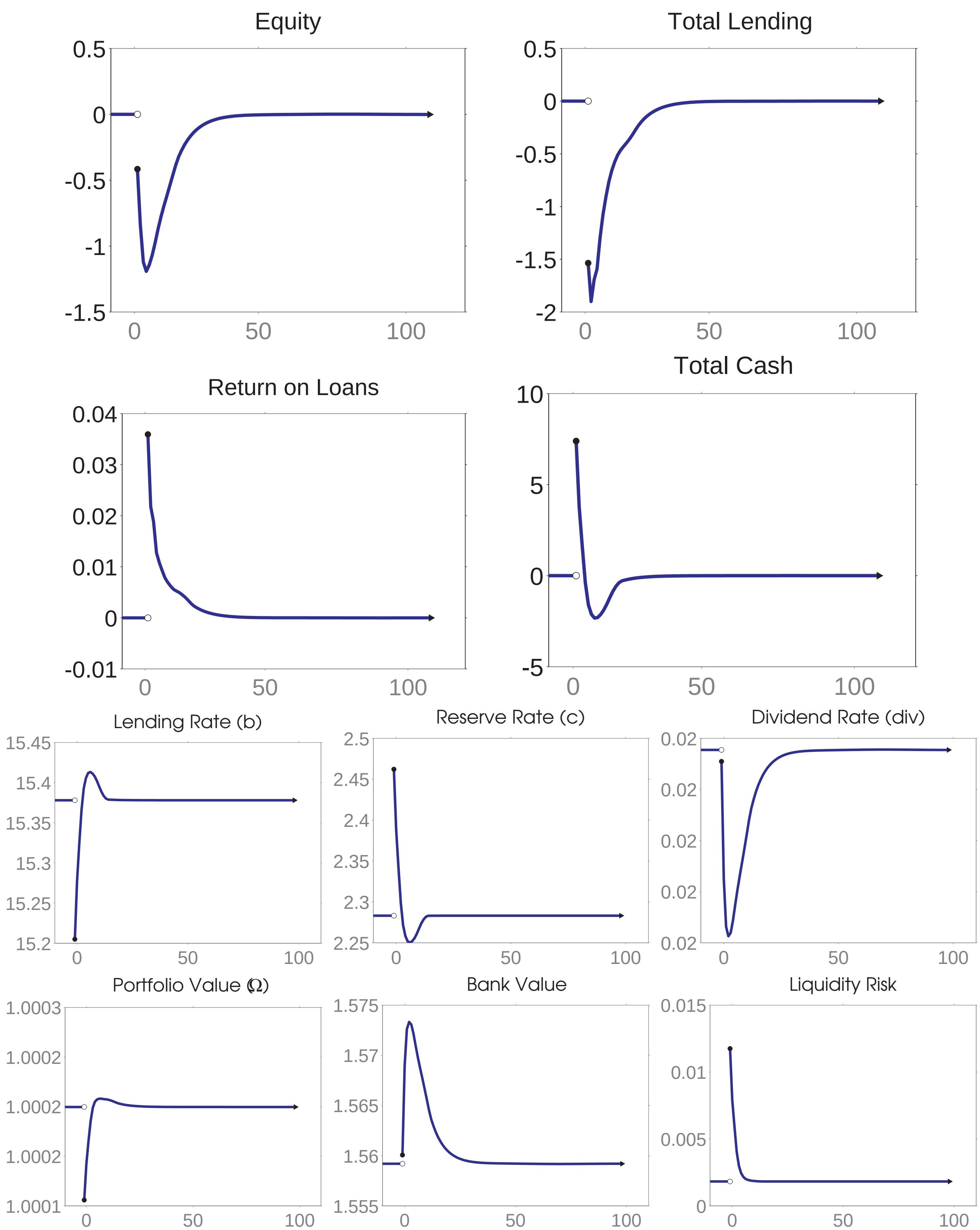

Figure 6: Impulse response to temporary increase in the probability of a bank-run

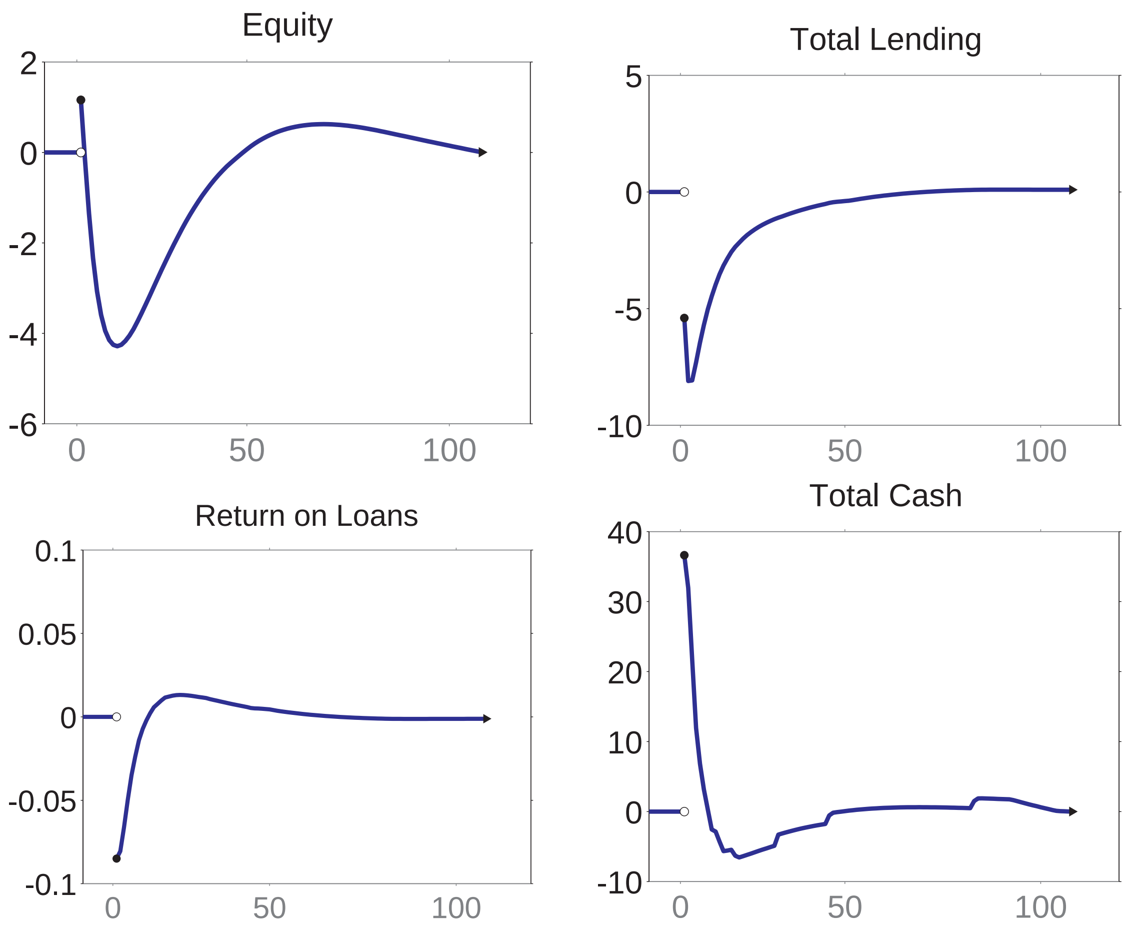

Figure 7: Temporary decrease in Loan Demand (Aggregates)

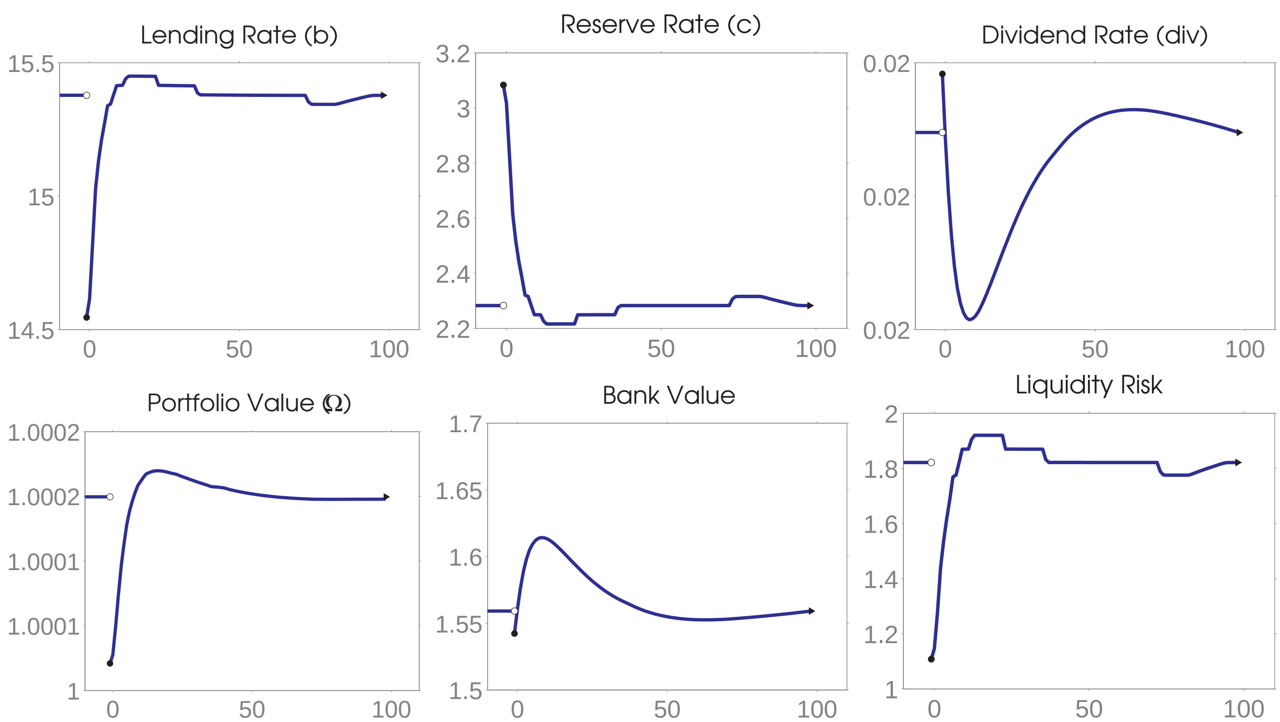

Figure 8: Impulse response to temporary decrease in loan demand

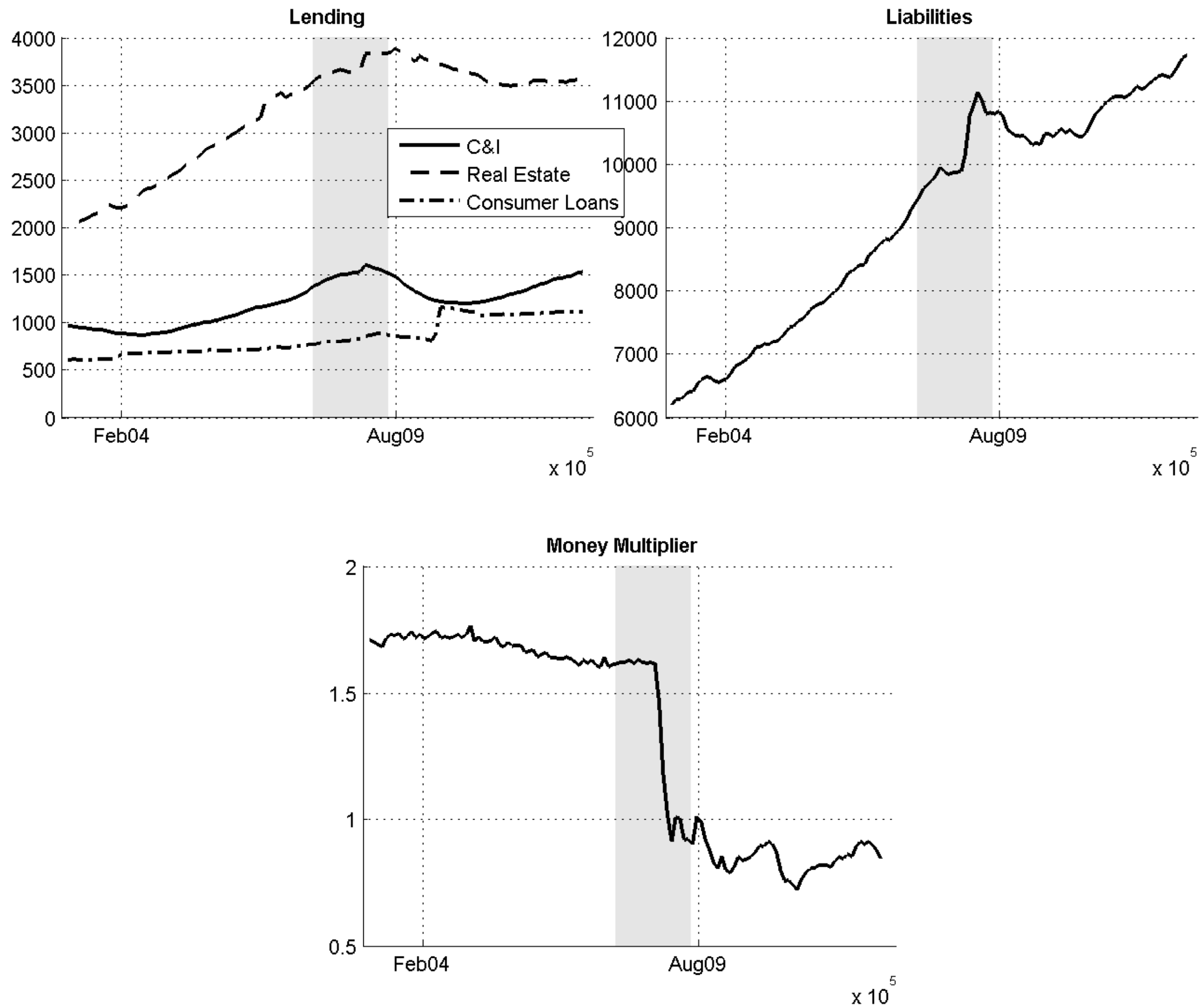

Figure 9: **Commercial Bank Assets: 2002-2012.** The figure shows several measures of commercial bank lending.

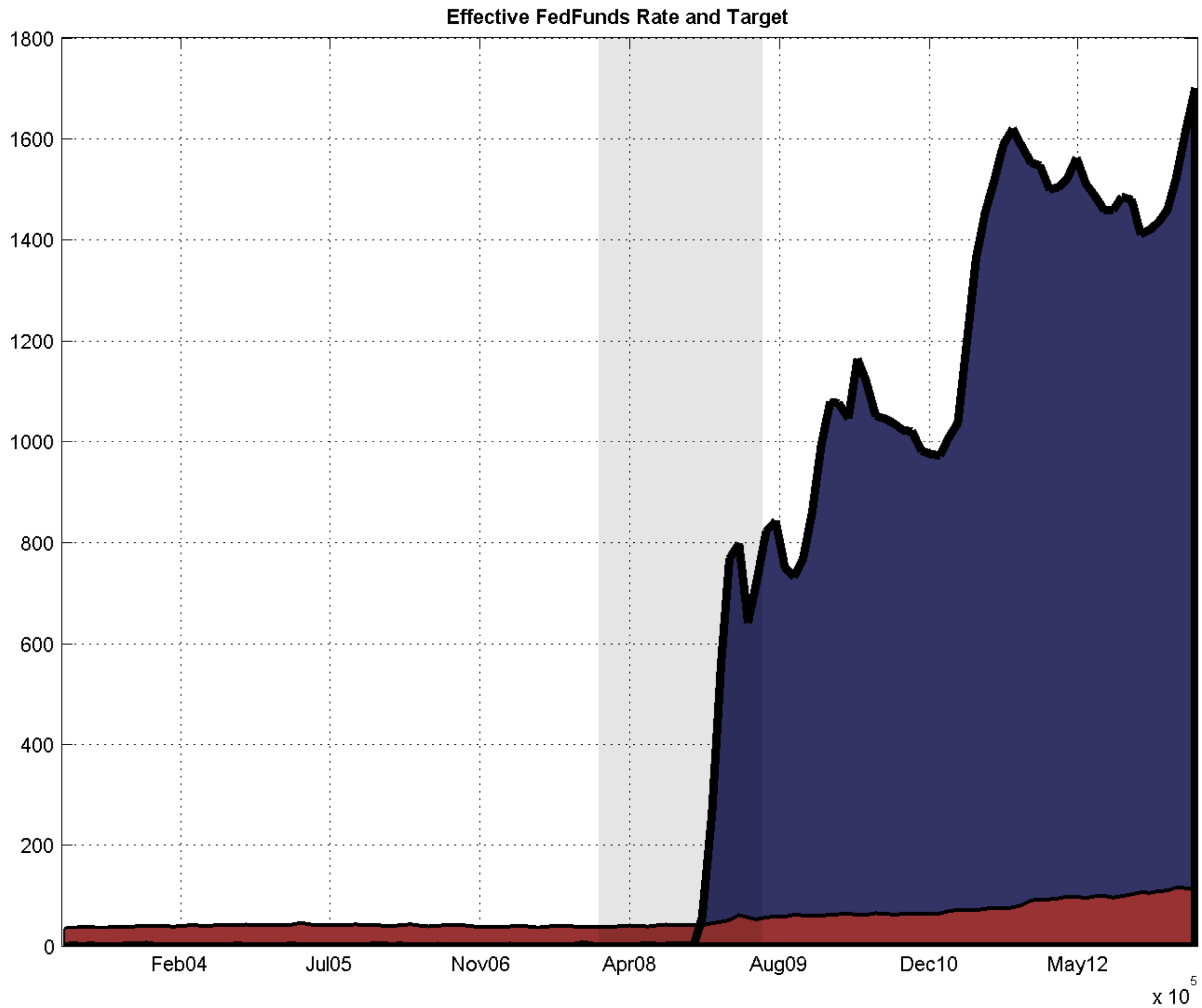

Figure 10: **Federal Reserve Assets: 2002-2012.** The figure shows the evolution of the Commercial Bank excess (blue) and required (red) reserve holding.

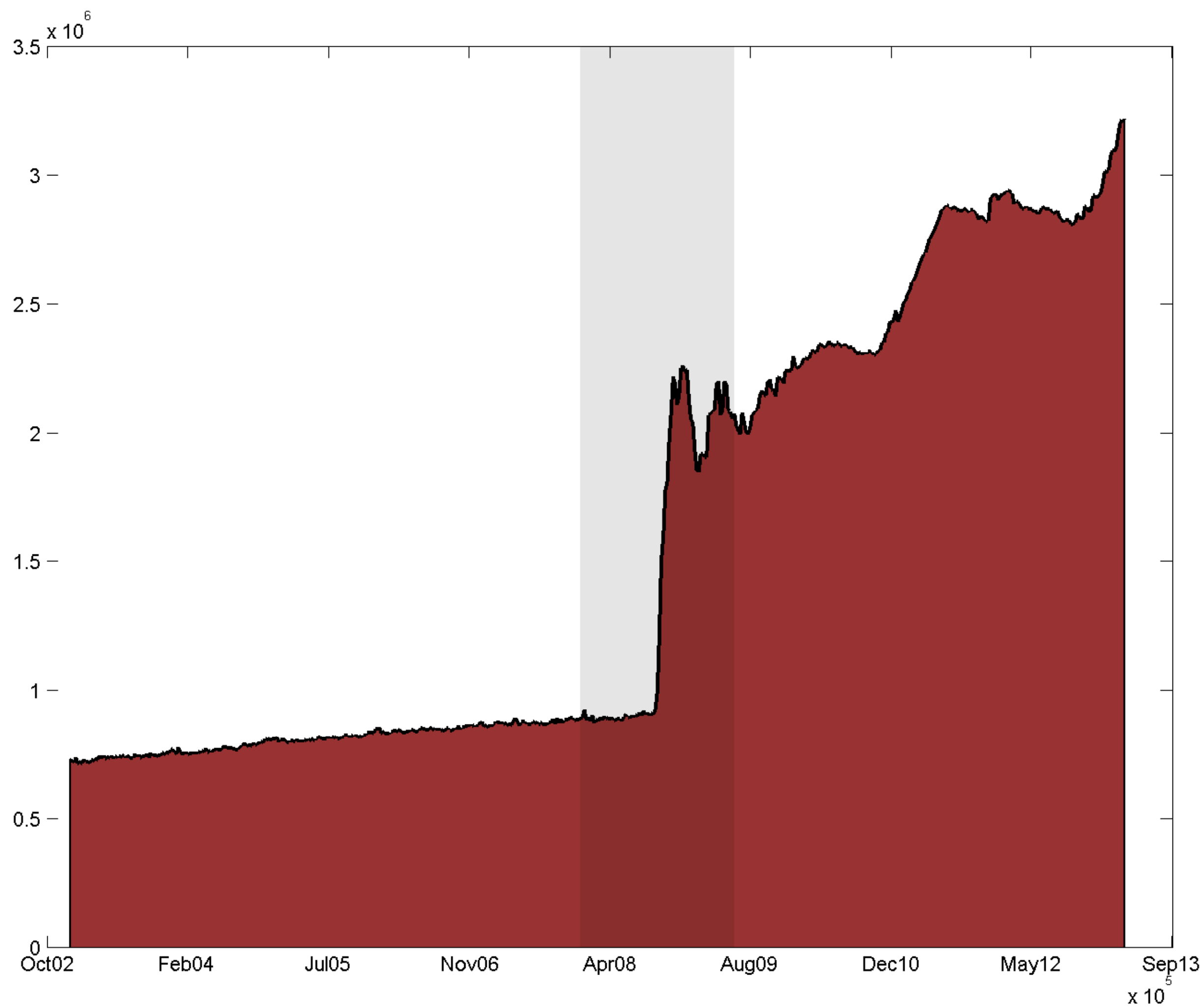

Figure 11: **Federal Reserve Assets: 2002-2012.** The figure shows the expansion in the FED's asset holdings. The magnitudes are in Millions of US$.

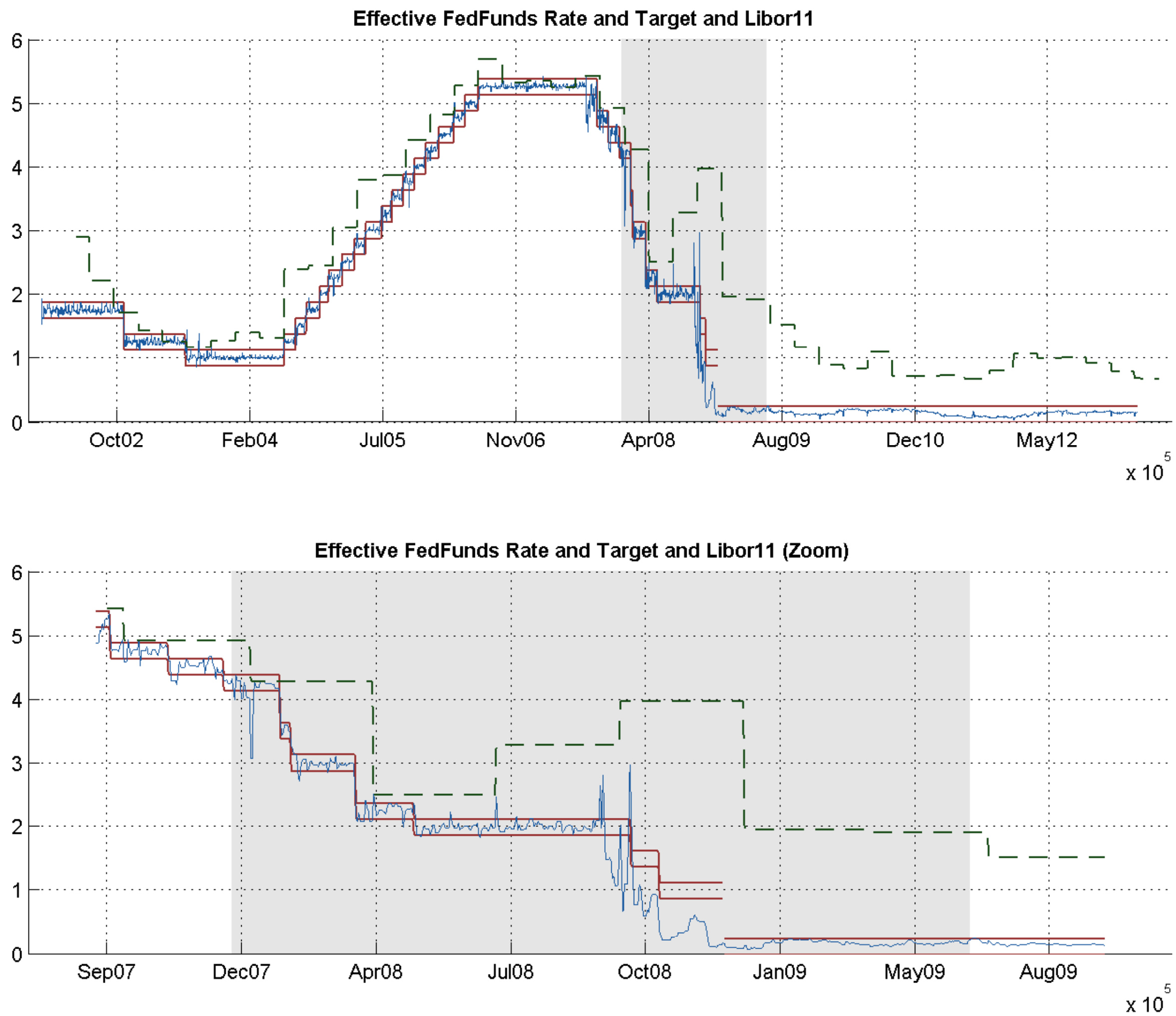

Figure 12: **Fed Funds Rate 2002-2012.** The figures plot the evolution of the FED Funds rate, the 11-month Libor rate and the overnight lending and borrowing rates.

B Proofs

B.1 Proof of Propositions 1, 2 and 3

This section provides a proof of the optimal policies described in Section 3.4. The proof of Proposition 1 is straight forward by noticing that once E is determined, the banker does not care how he came-up with those resources. The proof of Propositions 2 and 3 is presented jointly and the strategy is guess and verify. Let X be the aggregate state. We guess the following. $V(E;X) = v(X)E^{1-\gamma}$ where $v(X)$ is the slope of the value function, a function of the aggregate state that will be solved for implicitly. Policy functions are given by: $DIV(E;X) = div(X)E$, $\tilde{B}(E;X) = \tilde{b}(E;X)E$, $\tilde{D}(E;X) = \tilde{d}(E;X)E$ and $\tilde{C}(E;X) = \tilde{c}(E;X)E$.

B.1.1 Proof of Proposition 2

Given the conjecture for functional form of the value function, the value function satisfies:

$$
\begin{aligned}
V(E;X) \quad &= \quad \max_{DIV,\tilde{C},\tilde{B},\tilde{D}} U(DIV) + \beta\mathbb{E}\left[v(X')(E')^{1-\gamma})|X\right] \\
\text{Budget Constraint:} \quad & \quad E = q\tilde{B} + \tilde{C}p + DIV - \tilde{D} \\
\text{Evolution of Equity} \quad &: \quad E' = (q'\delta + (1-\delta))\tilde{B} + \tilde{C}p' - \tilde{D}(1+r_d) - \chi(\rho + \omega'(1-\rho))\tilde{D} - p\tilde{C}) \\
\text{Capital Requirement} \quad &: \quad \tilde{D} \leq \kappa(\tilde{B}q + \tilde{C}p - \tilde{D})
\end{aligned}
$$

where the form of the continuation value follows from our guess. We can express all of the constraints in the problem as linear constraints in the ratios of E. Dividing all of the constraints by E, we obtain:

$$
\begin{aligned}
1 \quad &= \quad div + q\tilde{b} + p\tilde{c} - \tilde{d} \\
E'/E \quad &= \quad (q'\delta + 1 - \delta)\tilde{b} + \tilde{c}p' - \tilde{d}(1+r_d) - \chi((\rho + \omega'(1-\rho))\tilde{D} - p\tilde{C}) \\
\tilde{d} \quad &\leq \quad \kappa(\tilde{B}q + \tilde{C}p - \tilde{D})
\end{aligned}
$$

where $div = DIV/E$,$\tilde{b} = \tilde{B}/E, \tilde{c} = \tilde{C}/E$ and $\tilde{d} = \tilde{D}/E$. Since, E is given at the time of the decisions of B,C,D and DIV, we can express the value function in terms of choice of these ratios. Substituting the evolution of E' into the objective function, we obtain:

$$\begin{aligned}
V(E;X) &= \max_{div,\tilde{c},\tilde{b},\tilde{d}} U(divE) + \beta\mathbb{E}\left[v(X')\left(R(\omega,X,X')E\right)^{1-\gamma})|X\right] \\
1 &= div + q\tilde{b} + p\tilde{c} - \tilde{d} \\
\tilde{d} &\leq \kappa(\tilde{B}q + \tilde{C}p - \tilde{D})
\end{aligned}$$

where we use the fact that E' can be written as:

$$E' = R(\omega,X,X')E$$

where $R(\omega,X,X')$ is the realized return to the bank's equity and defined by:

$$R(\omega,X,X') \equiv (q(X')\delta + (1-\delta))\tilde{b} + (1+r(X'))\tilde{c} - (1+r_d)\tilde{d} - \chi(\rho+\omega'(1-\rho))\tilde{D} - p\tilde{C}).$$

We can do this factorization for E because the evolution of equity on hand is linear in all the term where prices appear. Moreover, it is also linear in the penalty χ also. To see this, observe that $\chi\left((\rho+\omega'(1-\rho))\tilde{D} - p\tilde{C}\right) = \chi\left((\rho+\omega'(1-\rho))\tilde{d} - \tilde{c}E\right)$ by definition of $\left\{\tilde{d},\tilde{c}\right\}$. Since, $E \geq 0$ always, we have that

$$(\rho+\omega'(1-\rho))\tilde{D} - \tilde{C} \leq 0 \leftrightarrow \left((\rho+\omega'(1-\rho))\tilde{d} - \tilde{c}\right) \leq 0.$$

Thus, by definition of χ,

$$\begin{aligned}
\chi((\rho+\omega'(1-\rho))\tilde{D} - \tilde{C}) &= \begin{cases} E\underline{\chi}\left((\rho+\omega'(1-\rho))\tilde{d} - \tilde{c}\right) & \text{if } \left((\rho+\omega'(1-\rho))\tilde{d} - \tilde{c}\right) \leq 0 \\ E\overline{\chi}\left((\rho+\omega'(1-\rho))\tilde{d} - \tilde{c}\right) & \text{if } \left((\rho+\omega'(1-\rho))\tilde{d} - \tilde{c}\right) > 0 \end{cases}. \\
&= E\chi\left((\rho+\omega'(1-\rho))\tilde{d} - \tilde{c}\right).
\end{aligned}$$

Hence, the evolution of $R(\omega,X,X')$ is a function of the portfolio ratios b, c and d but not of the level of E. With this properties, one is allowed to factor out, $E^{1-\gamma}$ from the objective because it is a constant when decisions are made. Thus, the value function may be written as:

$$\begin{aligned}
V(E;X) &= E^{1-\gamma}\left[\max_{div,\tilde{c},\tilde{b},\tilde{d}} U(div) + \beta\mathbb{E}\left[v(X')R(\omega,X,X')^{1-\gamma})|X\right]\right] && (4) \\
1 &= div + q\tilde{b} + p\tilde{c} - \tilde{d} \\
\tilde{d} &\leq \kappa(\tilde{B}q + \tilde{C}p - \tilde{D}) && (5)
\end{aligned}$$

Then, let an arbitrary $\tilde{v}(X)$ be the solution to:

$$\begin{aligned}
\tilde{v}(X) &= \max_{div,\tilde{c},\tilde{b},\tilde{d}} U(div) + \beta \mathbb{E}\left[\tilde{v}(X')\, R(\omega, X, X')^{1-\gamma})|X\right] \\
1 &= div + q\tilde{b} + p\tilde{c} - \tilde{d} \\
\tilde{d} &\leq \kappa(\tilde{B}q + \tilde{C}p - \tilde{D})
\end{aligned}$$

We now shows that if $\tilde{v}(X)$ exists, $v(X) = \tilde{v}(X)$ verifies the guess to our Bellman equation. Substituting $v(X)$ for the particular choice of $\tilde{v}(X)$ in (4) allows us to write $V(E; X) = \tilde{v}(X)\, E^{1-\gamma}$. Note this is true because maximizing over $div, \tilde{c}, \tilde{b}, \tilde{d}$ yields a value of $\tilde{v}(X)$. Since, this also shows that $div, \tilde{c}, \tilde{b}, \tilde{d}$ and independent of E, and $DIV = divE$, $\tilde{B} = \tilde{b}E$, $\tilde{C} = \tilde{c}E$ and $\tilde{D} = \tilde{d}E$.

B.1.2 Proof of Proposition 3

We now solve for $\tilde{v}(X)$ and show that the value of equity on hand can be written as a consumption-savings problem and an independent portfolio problem. To do so, let $\dot{b}$, $\dot{c}$ and $\dot{d}$ be the fraction of loans, reserves and deposits invested by the bank after paying dividends. To simplify notation, we suppress the reference to the aggregate state X. These functions are mathematically defined as:

$$\dot{b} \equiv \frac{\tilde{b}}{(1-div)},\ \dot{c} \equiv \frac{\tilde{c}}{(1-div)} \text{ and } \dot{d} \equiv \frac{\tilde{d}}{(1-div)}.$$

and collecting terms we obtain:

$$div + (1-div)\left(q\dot{b} + p\dot{c} - \dot{d}\right) = 1.$$

Thus, the resource constraint for the bank can also be written as,

$$q\dot{b} + p\dot{c} - \dot{d} = 1.$$

Similarly, we can multiply the capital and liquidity constraints by $(1-div)$ and express the constraints in :

$$\dot{d} \leq \kappa(\dot{b} + \dot{c} - \dot{d})$$

To make further progress, we employ the principle of optimality and solve for $\left\{\dot{b}, \dot{c}, \dot{d}\right\}$, assuming we already know div. Let's assume an arbitrary div^o as the optimal choice in $v(X)$. Then $v(X)$ also satisfies:

$$v(X) = \max_{\dot{b},\dot{c},\dot{d}} U(div^o) + (1-div^o)^{1-\gamma}\, \beta \mathbb{E}\left[v(X')\, R(\omega, X, X')^{1-\gamma})|X\right]$$

subject to:

$$\begin{aligned} 1 &= q\dot{b} + \dot{c}p - \dot{d} \\ \dot{d} &\leq \kappa(\dot{b} + \dot{c} - \dot{d}) \end{aligned}$$

We note that $R(\omega, X, X') = (q(X')\delta + (1-\delta))\dot{b} + (1+r(X'))\dot{c} - (1+r^d)\dot{d} - \chi((\rho+\omega'(1-\rho))\dot{d} - \dot{c}))(1 - div^o)$. Since $R(\omega, X, X')$ only enterers in the continuation utility, then, $\left\{\dot{b}, \dot{c}, \dot{d}\right\}$ must solve:

$$\max_{\dot{b},\dot{c},\dot{d}} \mathbb{E}\left[v(X')(q(X')\delta + (1-\delta))\dot{b} + (1+r(X'))\dot{c} - \dot{d}(1+r^d) - \chi((\rho+\omega'(1-\rho))\dot{d} - \dot{c}))^{1-\gamma})|X\right].$$

subject to

$$\begin{aligned} 1 &= q\dot{b} + \dot{c}p - \dot{d} \\ \dot{d} &\leq \kappa(\dot{b} + \dot{c} - \dot{d}) \end{aligned}$$

if it is part of a solution, otherwise there is a better solution to $v(X)$.

When, X' is deterministic $v(X')$ is known at stage X. In such cases, we can factor out this problem out of the max. However, we must be careful with the sign of $v(X')$ since the max operator switches to a min operator when we change signs. We will show that when $\gamma > 1$, $v(X')$ is negative, so we need to minimize term inside the brackets. To prevent changing the max operator to a min operator, we use the certainty equivalent operator. Thus, $\left\{\dot{b}, \dot{c}, \dot{d}\right\}$ are solutions to:

$$\Omega(X) = \max_{\dot{b},\dot{c},\dot{d}} \mathbb{E}_{\omega}\left[(q(X')\delta + (1-\delta))\dot{b} + (1+r(X'))\dot{c} - \dot{d}(1+r^d) - \chi((\rho+\omega'(1-\rho))\dot{d} - \dot{c}))^{1-\gamma})\right]^{\frac{1}{1-\gamma}}.$$

subject to

$$\begin{aligned} 1 &= q\dot{b} + \dot{c}p - \dot{d} \\ \dot{d} &\leq \kappa(\dot{b} + \dot{c} - \dot{d}) \end{aligned}$$

When, $(1-\gamma) < 0$, the solution to $\Omega(X)$ will be equivalent to minimizing the objective. For $\gamma \to 1$, the objective becomes:

$$\Omega(X) = \exp\left\{\mathbb{E}_{\omega}\left[\log\left(R(\omega, X, X')\right)\right]\right\}.$$

Note that we can only do this separation when X is deterministic because otherwise we need to account for the correlation between $v(X')$ and $R(\omega, X, X')$. However, for now we assume the problem is deterministic. Since, the solution to $\Omega(X)$ is the same for any div, and not just the optimal, div^o. The objective can be written as

$$v(X) = \max_{div} U(div) + (1-div)^{1-\gamma}\beta\mathbb{E}\left[v(X')\,\Omega(X)^{1-\gamma})|X\right],$$

which is the formulation in Proposition 3. Proposition 5 shows an explicit solution for div and $v(X)$.

B.2 Equivalence in Problems

We now show that using our guess for policy functions we can recover the consumption savings problem above. In the original problem, the first order conditions for $\tilde{b}$ is:

$$\left(\tilde{b}\right): q(X)\,U^{'}(div(X)\,E)\,E = \beta\left(\mathbb{E}\left[v(X)\,u'(E')\,\frac{\partial E'}{\partial \tilde{b}}\right]\right) + \mu_\kappa(X)\,E^{1-\gamma}\kappa - \mu_\eta(X)\,E^{1-\gamma}\eta. \quad (6)$$

Now, we know from xxx that $\frac{\partial E'}{\partial \tilde{b}}$ is:

$$(q(X')\,\delta + (1-\delta))\,E.$$

Hence, the first order condition becomes:

$$q(X)\,U^{'}(div(X))\,E^{1-\gamma} = \beta\mathbb{E}\left[v(X)\,u'(R(\omega,X,X')\,E)\,(q(X')\,\delta+(1-\delta))\,E\right] + \mu_\kappa(X)\,E^{1-\gamma}\kappa - \mu_\eta(X)\,E^{1-\gamma}\eta,$$

and simplifying E this equation further more, we obtain:

$$q(X)\,U^{'}(div(X)) = \beta\mathbb{E}\left[v(X)\,R(\omega,X,X')^{-\gamma}(q(X')\,\delta+(1-\delta))\right] + \mu_\kappa(X)\,\kappa - \mu_\eta(X)\,\eta.$$

Similarly, the first order condition for $\tilde{c}(X)$ yields,

$$(\tilde{c}): (1+r(X))U^{'}(div(X)) = \beta\mathbb{E}\left[v(X)\,R(\omega,X,X')^{-\gamma}\,\frac{\partial R(\omega,X,X')}{\partial\tilde{c}}\right] + \mu_\kappa(X)\,\kappa - \mu_\eta(X)\,(\eta-1)\,, \quad (7)$$

and a similar expression can be found for $\tilde{d}(X)$:

$$\left(\tilde{d}\right): U^{'}(div(X)) = \beta\mathbb{E}\left[v(X)\,R(\omega,X,X')^{-\gamma}\,\frac{\partial R(\omega,X,X')}{\partial\tilde{d}}\right] + \mu_\kappa(X)\,(1-\kappa) + \mu_\eta(X)\,\eta. \quad (8)$$

Note that 6, 7 and 8 are independent of E. One can multiply equation 6 by $\dot{b}$, equation 7 by $\dot{c}$ and equation 8 by $\dot{d}$, and obtain:

$$U^{'}\left(div\right)=\left(1-div\right)^{1-\gamma}\beta\mathbb{E}\left[v\left(X\right)\Omega\left(X\right)^{1-\gamma}\right]$$

To see this, note that we are using the definitions

$$\mu_{\kappa}\left(X\right)\left[\kappa(\tilde{b}\left(E;X\right)+\tilde{c}\left(E;X\right)-\tilde{d}\left(E;X\right))-\tilde{d}\left(E;X\right)\right]=0$$

and

$$\mu_{\eta}\left(X\right)\left[\tilde{c}\left(E;X\right)-\eta(\tilde{b}\left(E;X\right)+\tilde{c}\left(E;X\right)-\tilde{d}\left(E;X\right))\right]=0$$

and that $q\dot{b}+p\dot{c}-\dot{d}=1$. Since, $U^{'}\left(div\right)=\left(1-div\right)^{-\gamma}\beta\mathbb{E}\left[v\left(X\right)\Omega\left(X\right)^{1-\gamma}\right]$ is the same first order condition for the for the problem in Proposition 5, we know that the optimal dividend choice satisfies the conditions in that problem also.

B.3 Proof of Proposition 5

Taking first order conditions we obtain:

$$div=\beta\mathbb{E}\left[v^{l}\left(X'\right)|X\right]\Omega^{*}\left(X\right)^{-(1-\gamma)/\gamma}\left(1-div\right)$$

and therefore one obtains:

$$div=\frac{1}{1+\left[\beta\mathbb{E}\left[v^{l}\left(X'\right)|X\right]\left(\Omega^{*}\left(X\right)\right)^{1-\gamma}\right]^{1/\gamma}}.$$

Substituting this expression for dividends, one obtains a functional equation for the value function:

$$
\begin{aligned}
v^{l}\left(X\right) &= \frac{1}{\left(1+\left[\beta\mathbb{E}\left[v^{l}\left(X'\right)|X\right]\left(\Omega^{*}\left(X\right)\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}\right)^{1-\gamma}} \\
&\quad +\beta\mathbb{E}\left[v^{l}\left(X'\right)|X\right]\left(\Omega^{*}\left(X\right)\right)^{1-\gamma}\left[\frac{\left[\beta\mathbb{E}\left[v^{l}\left(X'\right)|X\right]\left(\Omega^{*}\left(X\right)\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}}{\left(1+\left[\beta\mathbb{E}\left[v^{l}\left(X'\right)|X\right]\left(\Omega^{*}\left(X\right)\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}\right)}\right]^{1-\gamma} \\
&= \left(\frac{1+\left(\left[\beta\mathbb{E}\left[v^{l}\left(X'\right)|X\right]\left(\Omega^{*}\left(X\right)\right)^{1-\gamma}\right]^{\frac{\gamma+(1-\gamma)}{\gamma(1-\gamma)}}\right)^{1-\gamma}}{\left(1+\left[\beta\mathbb{E}\left[v^{l}\left(X'\right)|X\right]\left(\Omega^{*}\left(X\right)\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}\right)^{1-\gamma}}\right) \\
&= \left(\frac{1+\left[\beta\mathbb{E}\left[v^{l}\left(X'\right)|X\right]\left(\Omega^{*}\left(X\right)\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}}{\left(1+\left[\beta\mathbb{E}\left[v^{l}\left(X'\right)|X\right]\left(\Omega^{*}\left(X\right)\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}\right)^{1-\gamma}}\right) \\
&= \left(1+\left[\beta\mathbb{E}\left[v^{l}\left(X'\right)|X\right]\left(\Omega^{*}\left(X\right)\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}\right)^{\gamma}
\end{aligned}
$$

Therefore, we obtain the following map:

$$
v^{l}\left(X\right)^{\frac{1}{\gamma}} = 1+\left(\beta\Omega^{*}\left(X\right)\right)^{\frac{1}{\gamma}}\mathbb{E}\left[v^{l}\left(X'\right)|X\right]^{\frac{1}{\gamma}}.
$$

Performing a change of variables, and denoting $\upsilon\left(X\right)\equiv v^{l}\left(X\right)^{\frac{1}{\gamma}}$, we have the following functional equation:

$$
\upsilon\left(X\right) = 1+\left(\beta\left(\Omega^{*}\left(X\right)\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}\mathbb{E}\left[\upsilon\left(X'\right)^{\gamma}|X\right]^{\frac{1}{\gamma}}.
$$

We can treat the right hand side of this functional equation as an operator. This operator will be a contraction depending on the values of $\left(\beta\left(\Omega^{*}\left(X\right)\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}$. Theorems in Alvarez and Stokey guarantee that this operator satisfies the dynamic programming arguments.

In a non-stochastic steady state we obtain:

$$
v^{l}\left(X\right) = \left(\frac{1}{1-\left(\beta\left(\Omega^{*}\left(X\right)\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}}\right)^{\gamma}.
$$

C Evolution of Bank Equity Distribution

Because the economy displays equity growth, equity is unbounded and thus, the support of this measure is the positive real line. Let $\mathcal{B}$ be the Borel σ-algebra on the positive real line. Then, define as $Q_t(e, \boldsymbol{E})$ as the probability that an individual bank with current equity e transits to the set $\boldsymbol{E}$ next period. Formally $Q_t : \mathbb{R}_+ \times \mathcal{B} \to [0, 1]$, and

$$Q(e, \boldsymbol{E}) = \int_{-1}^{1} \mathbb{I}\left\{\Psi_t\left(\omega\right) e \in \boldsymbol{E}\right\} F\left(d\omega\right)$$

where $\mathbb{I}$ is the indicator function of the event in brackets. Then Q is a transition function and the associated T^* operator for the evolution of bank equity is given by:

$$\Gamma_{t+1}\left(\boldsymbol{E}\right) = \int_{0}^{1} Q(e, \boldsymbol{E})\Gamma_{t+1}\left(e\right) de.$$

The distribution of equity is fanning out and the operator is unbounded. Gibrat's law shows that for t large enough Γ_{t+1} is approximated well by a log-normal distribution. Moreover, by introducing more structure into the problem, we could easily obtain a Power law distribution for $\Gamma_{t+1}\left(\boldsymbol{E}\right)$. We will use this properties in the calibrated version of the model.

D Data Analysis

1990-2010 Sample Averages. The banking industry underwent a consolidation over the last two decades. Due to the substantial amount of mergers in the industry and measurement error, aggregating balance sheet information directly would be misleading. Hence, we isolates the effects of mergers on the increase in the volatility of different bank liabilities by eliminating observations with observations that lie more than four standard deviations or that exhibit negative entries.

The summary statistics for the quarterly growth rate of the aggregate time series is presented in Table 2.

Variable	**Mean**	**Std. Dev.**	**N**
TD	1.023	0.085	771410
DD	1.029	0.191	778467
TL	1.022	0.07	773629
TE	1.017	0.083	769077
RTE	1.017	0.097	766806
E	1.018	0.072	774407
RE	1.018	0.086	769338

Table 2: Summary statistics .

The data exhibits very similar patterns for the growth of total deposits and total liabilities. Demand deposits are 2.5 times more volatile than all the deposits. This may respond to a stronger seasonality in this variables. One of the reasons why we use total deposits as our data counterpart for deposits in our model is that it is substantially volatile, the standard deviation is 8.5% per quarter, and it is close to the volatility of total liabilities, 7.0%. Moreover, demand deposits may be exchanged for deposits of longer maturity, which explains why total deposits are less volatile (being a sub-account), but through the lens of our model, this would be as a change in one account for another within the same bank. This can be observed from in the Table 3.

Figure 13 presents the evolution of the growth rates of these time series substracting the growth rate of the GDP deflator. All the series show a strong seasonal component. The wider curve presents the Hodrick-Prescott filtered series. The trends reveal a decline in the growth rates towards the end of the sample, corresponding to the period of the Great Recession and onwards.

Variables	TD	DD	TL	TE	RTE	E	RE
TD	1.000						
DD	0.393	1.000					
TL	0.844	0.350	1.000				
TE	0.077	0.010	0.133	1.000			
RTE	0.046	0.002	0.096	0.858	1.000		
E	0.144	0.031	0.234	0.737	0.647	1.000	
RE	0.112	0.024	0.184	0.647	0.777	0.885	1.000

Table 3: Cross-Sectional correlation for Quarter-Bank observations

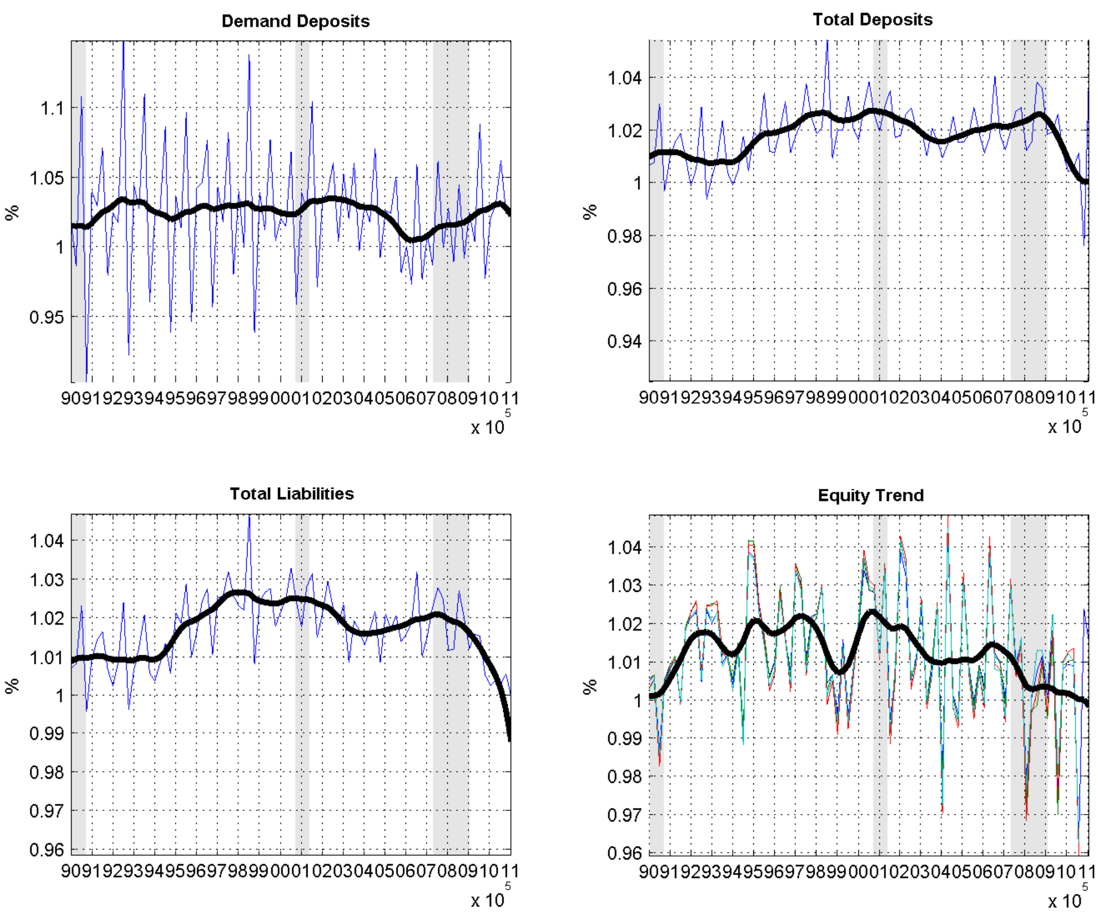

Figure 13: Cross-Sectional Average Growth Rates

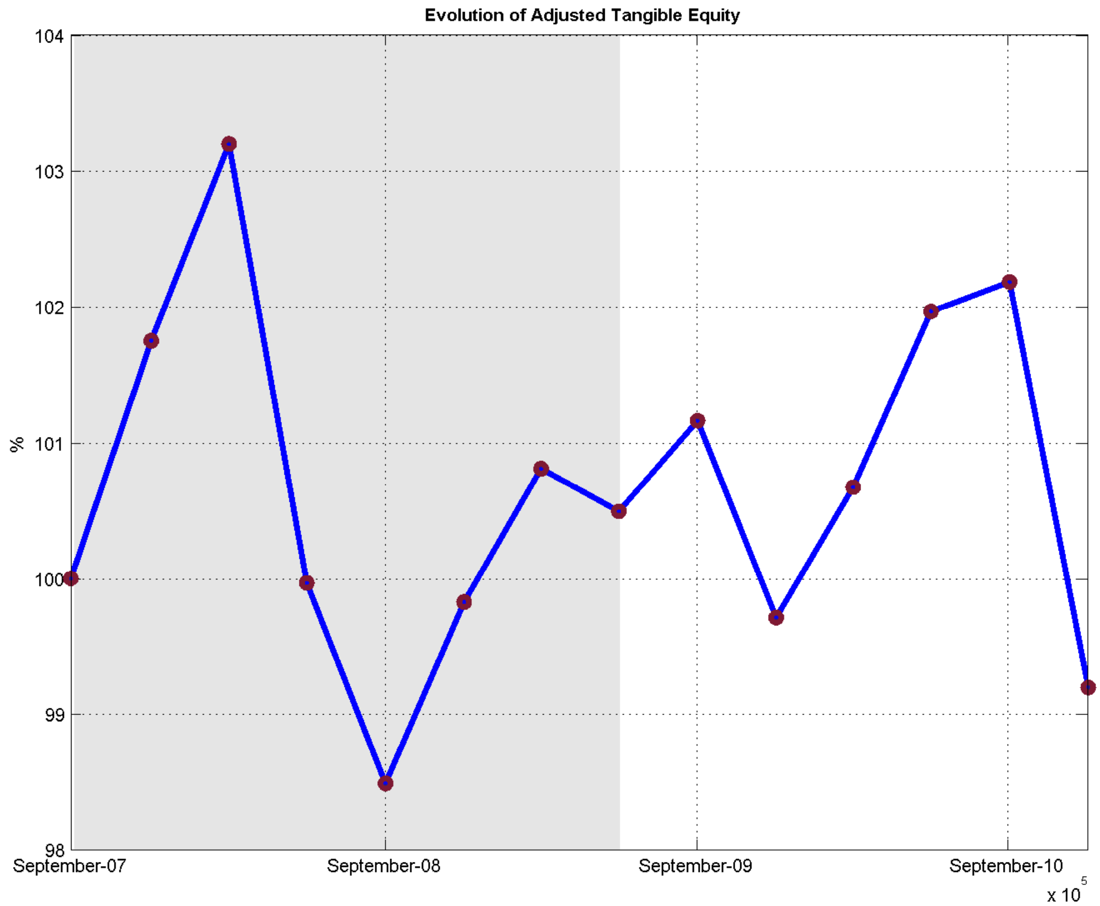

Figure 14: Evolution of RTE during the Great Recession.

For our quantitative analysis, we are particularly interested in the behavior of the series for bank equity. The figure shows a decline in filtered growth rates, but due to the strong volatility in the period, the filtered series does not show a decline in levels. One of the hypothesis that we consider in our quantitative analysis is the decline in equity during the Great Recession. A snapshot of the compounded growth rates reveals a mild decline in the book value of tangible equity even adjusting for Loan and Loss Allowances. Figure 14 shows the pattern. The evolution of the book value may be distorted by other factors such as TARP, and not materialized losses outside the book value. Hence, we will stretch the results and show use a benchmark of 5% for our book value equity losses.

Quarterly Cross-Sectional Deviations. Part of the variation in the bank-quarter statistics presented above have a common component, including seasonality, nominal changes in the time series and aggregate trends. To decompose the variation of these liabilities into their common component, we present the summary statistics in terms of deviations of these variables from their quarterly cross-sectional averages. Table 4 presents the results:

Variable	**Mean**	**Std. Dev.**	**N**
dev TD	0	0.084	771369
dev DD	0	0.183	777932
dev TL	0	0.069	773629
dev TE	0	0.081	769073
dev RTE	0	0.096	766803
dev E	0	0.071	774401
dev RE	0	0.085	769329

Table 4: Summary statistics

One thing one gathers from this table is that most of the variation is preserved even when one substracts the evolution of aggregate averages. This shows the substantial amount of idiosyncratic volatility among banks. Except for the behavior of demand deposits, the volatility of the liabilities of banks is almost exclusively idiosyncratic. This can be seen from Table 5 which shows the behavior of the correlation in cross-sectional deviations from quarterly means. These correlations are essentially the same as the correlations for historical growth rates implying that the idiosyncratic component is quite high.

The correlation in the data between deviation of tangible equity growth and the deviation in the growth rate of total deposits ranges from 5% to 15% depending on the definition of equity that we use. In the model, this correlation will be very high (though not 1) because deposit volatility is the only source of risk for banks. In practice, banks face other important sources of risks such as loan risk, duration risk and trading risk. This figure however implies that deposit withdrawal risks are non-negligible risks for banks. Figure 1 in the body of the paper, reports the empirical histograms for every quarter-bank growth observation and decomposes the data into two samples pre-crisis (1990Q1-2007Q4) and crisis (2008Q1-2010Q4).

Variables	dev TD	dev DD G	dev TL	dev TE	dev RTE	dev E	dev RE
dev T D	1.000						
dev DD	0.389	1.000					
dev TL	0.844	0.345	1.000				
dev TE	0.082	0.027	0.135	1.000			
dev RTE	0.050	0.016	0.097	0.854	1.000		
dev E	0.152	0.052	0.238	0.727	0.635	1.000	
dev RE	0.118	0.040	0.187	0.635	0.769	0.881	1.000

Table 5: Correlation for Cross-Sectional Deviations from Means

We use the empirical histogram of the quarterly deviations of TD to calibrate F_t, the process for withdrawal shocks in our model. In the quantitative analysis, we contrast the behavior of equity volatility that results as an outcome with the corresponding histogram and correlation in the date for this variable.

We also analyze the evolution of the volatility in the variables. This analysis provides the basis for our calibration of the increase in withdrawals shocks during the Great Recession, one of the hypothesis that we test. Figure 15 shows the time series for cross-sectional dispersion in growth rates in all of the series that we study. As the cross-sectional averages these series display a high seasonal component. The Hodrick-Prescott filter of the series reveals non-negligible increases in cross-sectional dispersion, in particular after 2009. The cross-sectional dispersion of all the measures of equity show a 60% increase at the peak of the Great Recession.

Tests for Growth Independence. Our models assumes that the withdrawal process is i.i.d over time and banks. This assumption implies that if we substract the common growth rates of all the balance sheet variables in our model, the residual should be serially uncorrelated. We test the independence of the deviations-from-means quarterly growth rates using an OLS estimation procedure. We run the deviations in quarterly growth rates from the cross-sectional averages against their lags. The evidence from OLS auto-regressions support the assumption that of time-independent growth. Tables 6 we report coefficients that are statistically identical to zero (with zeros lying within the two standard deviation bounds) for all of the variables of that we study.

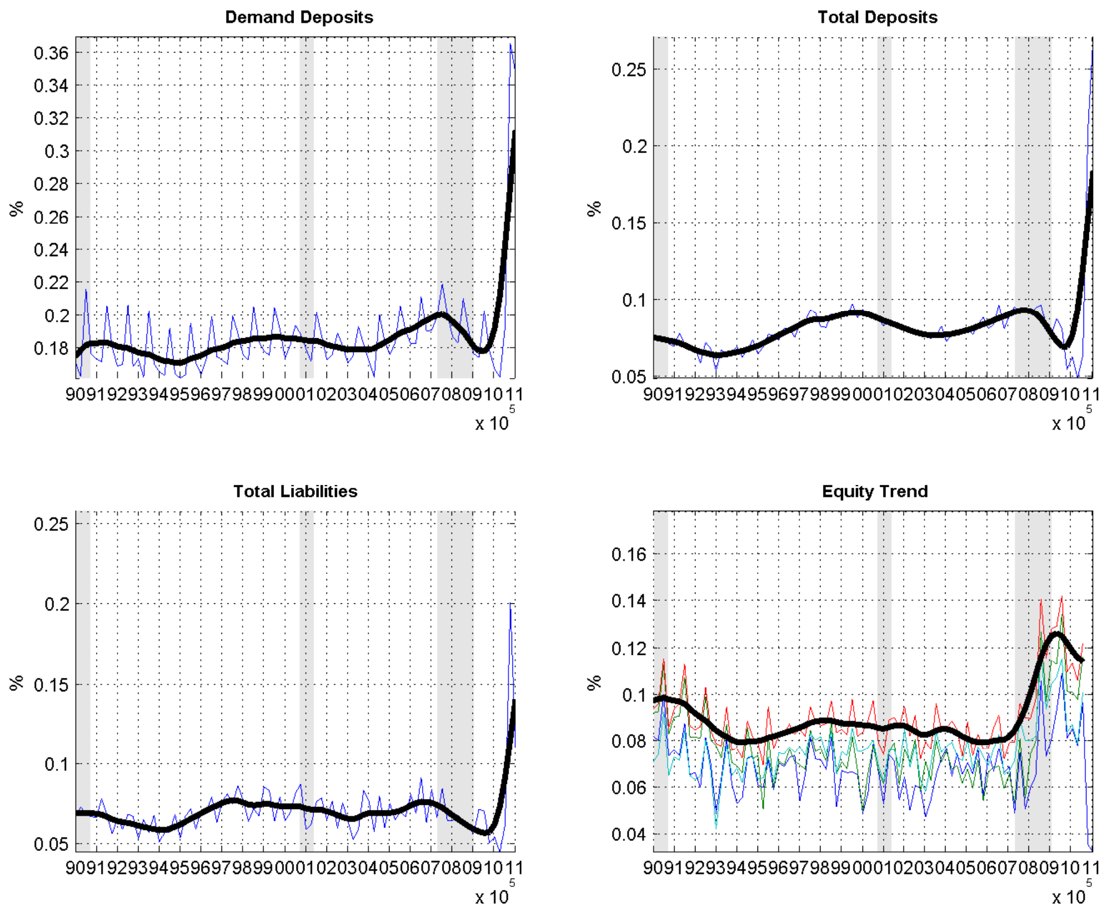

Figure 15: Quarterly Cross-Sectional Dispersions

Variable	**Coefficient**	**(Std. Err.)**
TD	0.001	(0.001)
DD	0.000	(0.001)
TL	-0.001	(0.001)
RTE	0.000	(0.001)
TE	-0.001	(0.001)
E	-0.001	(0.001)
RE	0.000	(0.001)

Table 6: Autocorrelation Estimates for First Lag of Growth Rates Cross-Sectional Deviations

E Algorithm for Computing Transitional Dynamics

F Derivation of Loan Demand

Section Pending.