Bank Leverage Cycles*

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Abstract

We propose a general equilibrium framework with financial intermediaries subject to endogenous leverage constraints, and assess its ability to explain the observed fluctuations in intermediary leverage and real economic activity. In the model, intermediaries ('banks') borrow in the form of short-term risky debt. The presence of risk-shifting moral hazard gives rise to a leverage constraint, and creates a link between the volatility in bank asset returns and leverage. Unlike standard TFP shocks, volatility shocks produce empirically plausible fluctuations in bank leverage. The model is able to replicate the fall in leverage, assets and GDP during the 2007-9 financial crisis.

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The 2007-9 financial crisis witnessed a severe disruption of financial intermediation in many industrialized economies. This has led to a surge in both empirical and theoretical research aimed at understanding the causes and consequences of the financial crisis, evaluating the policy measures put in place to tackle its effects, and proposing further policy actions and new regulatory frameworks.

A particularly influential strand of the literature has focused on the role played by the deleveraging of the financial intermediation sector in the propagation of the financial turmoil. Before the crisis, a significant share of financial intermediaries funded their asset purchases primarily by means of collateralized debt with very short maturity, such as sale and repurchase (repo) agreements or asset backed commercial paper (ABCP).[1] As argued by Brunnermeier (2009), Gorton and Metrick (2010, 2012), Krishnamurthy, Nagel and Orlov (2012) and others, the initial losses suffered by some of the assets that served as collateral in repo or ABCP transactions, together with the uncertainty surrounding individual exposures to such assets, led the holders of that short-term debt (mostly institutional investors, such as money market funds) to largely stop rolling over their lending. This funding freeze forced the financial intermediaries to deleverage, with the resulting contraction in their balance sheets and ultimately in the credit flow to the real economy.

In fact, the observed deleveraging of financial intermediaries during the 2007-9 financial crisis is not an isolated episode. As documented by Adrian and Shin (2010, 2011), since the 1960s the leverage ratio (i.e. the ratio of total assets to equity capital) of important segments of the financial intermediation sector has exhibited a markedly procyclical pattern, in the sense

[1] This was especially true for the so-called 'shadow banking' sector, which comprises those financial intermediaries (investment banks, hedge funds, finance companies, off-balance-sheet investment vehicles, etc.) that have no access to central bank liquidity or public sector credit guarantees, and that are not subject to regulatory capital requirements. See Pozsar et al. (2012) for an in-depth analysis of 'shadow banking' in the United States.

that expansions (contractions) in balance sheet size have gone hand in hand with increases (decreases) in leverage.[2] Overall, this evidence points to the importance of leverage fluctuations for the cyclical behavior of financial intermediation and real economic activity.

The aim of this paper is to propose a general equilibrium framework with leverage-constrained financial intermediaries that is able to explain the observed fluctuations in intermediary leverage and its comovement with real economic activity. Intermediary leverage is modeled along the lines of Adrian and Shin's (2013) static, partial equilibrium framework. Our main theoretical contribution is to incorporate their model of endogenous leverage into a fully dynamic, quantitative general equilibrium framework in which banks have an instrumental role in the channeling of funds from savers to borrowers.

In the model, banks borrow in the form of short-term risky debt. The source of risk in bank debt is the following. Firms are segmented across 'islands' and are hit by island-specific shocks. Some firms are more exposed to island-specific risk than others. Those more exposed also have lower island-specific returns on average, such that financing them is inefficient. Firms can only obtain funds from banks on the same island, as only the latter have the technology to assess their risk exposure. Banks are thus instrumental in channeling funds to firms, but they are also exposed to island-specific risk. A fraction of them declare bankruptcy and default on their debt in each period.

As in Adrian and Shin (2013), banks are affected by risk-shifting moral hazard.[3] Due to limited liability, banks enjoy the upside risk in their assets over and above the face value of their debt, leaving their creditors to bear the downside risk. This provides banks with an incentive to finance firms that are more exposed to island-specific risk despite having lower expected returns, i.e. to engage in inefficient lending practices. Such an incentive increases with the bank's debt burden relative to the size of its balance sheet. In order to induce banks to invest efficiently, their creditors restrict

[2]This procyclicality has been particularly strong in the case of security brokers and dealers, a category that used to include investment banks.

[3]The risk-shifting theory was developed originally by Jensen and Meckling (1976). See Acharya and Viswanathan (2011) for another recent application within the finance literature.

bank indebtedness to a certain ratio of bank net worth, i.e. they impose a leverage constraint.

We then calibrate our model to the US economy and analyze its ability to replicate the fluctuations in leverage and its comovement with economic activity. In the data, intermediary leverage is characterized by relatively large fluctuations: it is several times more volatile than GDP, and has contributed more than equity to fluctuations in assets. Leverage is strongly procyclical with respect to assets (as originally documented by Adrian and Shin, 2010, 2011), and also mildly procyclical with respect to GDP. In the model, we consider two exogenous driving forces: total factor productivity (TFP), and time-varying volatility of island-specific shocks. While TFP shocks are fairly standard in business cycle modelling, a recent literature argues that exogenous changes in cross-sectional volatility are key in order to understand aggregate fluctuations.[4] Moreover, such changes in volatility can be interpreted as changes in 'uncertainty', which as argued before are considered to have played an important role in the recent financial crisis.

Our results show that TFP shocks by themselves are unable to replicate the volatility of leverage in the data, as well as its procyclicality with respect to GDP. Intuitively, TFP shocks barely affect banks' risk-taking incentives. On the contrary, shocks to cross-sectional volatility are able to produce fluctuations in leverage of a realistic size, as well as a positive comovement between leverage, assets and GDP. The mechanism, which we refer to as the 'volatility-leverage channel', is as follows. Consider e.g. an increase in island-specific volatility. Higher uncertainty regarding asset returns, coupled with limited liability, makes it more attractive for banks to engage in inefficiently risky lending practices. In order to prevent them from doing so, institutional investors impose a tighter constraint on banks' leverage. For given net worth, this deleveraging forces banks to contract their balance sheets, thus producing a fall in funding to firms. This leads to a fall in capital investment by firms, and in aggregate output. The consequence is

[4]See e.g. Bloom (2009), Bloom et al. (2012), Christiano, Motto and Rostagno (2003, 2014), Arellano, Bai and Kehoe (2012), Curdia (2007), Gilchrist, Sim and Zakrajsek (2010), Kiley and Sim (2011), and Pinter, Theodoridis and Yates (2013). Bloom (2009) refers to such disturbances as 'uncertainty shocks', whereas Christiano et al. (2013) label them 'risk shocks'. Our specification of cross-sectional volatility shocks is most closely related to Christiano, Motto and Rostagno (2003, 2014).

a positive comovement between leverage, assets and GDP.

We also study the model's ability to shed light on the 2008-9 recession. Our results show that the fall in TFP observed during that period explains part of the fall in GDP, but it fails to replicate its severity and duration. Moreover, the TFP shock is completely unable to explain the observed deleveraging in the banking sector, as well as the reduction in its total assets. Adding an increase in volatility starting at the beginning of 2008 allows the model to match well the large and protracted fall not only of GDP, but also of bank leverage and assets during the Great Recession.

Finally, we use our model as a laboratory for studying how the steady-state level of cross-sectional volatility affects both the mean level and the volatility of economic activity. We find that lower cross-sectional volatility raises the mean level of banks' leverage, through a channel very similar to the one described above. This produces an increase in the mean levels of intermediary assets, capital investment and GDP. Perhaps more surprisingly, lower cross-sectional volatility raises the volatility of GDP. Intuitively, lower perceived risk allows banks to increase their leverage, which generates larger responses in bank funding and output in the face of aggregate shocks. This result, which may be thought of as a 'risk diversification paradox', is reminiscent of Minsky's (1992) 'financial instability hypothesis,' according to which a lower perception of uncertainty leads to riskier investment practices, thus creating the conditions for the emergence of a financial crisis.

Literature review. Our paper contributes to the literature on the macroeconomic effects of financial frictions. A recent literature has provided theoretical explanations for the 'leverage cycles' discussed above, with contributions by Adrian and Shin (2013), Ashcraft, Garleanu and Pedersen (2011), Brunnermeier and Pedersen (2009), Dang, Gorton and Holmström (2011), Geanakoplos (2010), and Gorton and Ordoñez (2011), among others.[5] Most of these models consider some type of link between changes in 'uncertainty', typically defined as changes in the volatility of shocks, and the emergence of these leverage cycles. While these models provide important insights on the equilibrium behavior of leverage, they

[5]Some of these authors focus on the behavior of 'margins' or 'haircuts' in short-term collateralized debt contracts, which are closely related to the concept of 'leverage'.

are primarily aimed at illustrating theoretical mechanisms and are thus mainly qualitative. In particular, most of these papers consider two- or three-period economies, or two-period-lived agents in OLG setups; they also assume a partial equilibrium structure. We build on this literature by analyzing endogenous leverage cycles in a fully dynamic, general equilibrium model that can be compared to aggregate data, and that allows us to study the interaction between the real and financial sides of the economy.

Our paper is also related to a growing literature on the role of financial intermediaries in dynamic, stochastic, general equilibrium (DSGE) models. Early contributions to the macro-finance literature, such as Bernanke, Gertler and Gilchrist (1999), Kiyotaki and Moore (1997) and Carlstrom and Fuerst (1997), emphasized the importance of financial frictions for the macroeconomy, but largely obviated the role played by financial intermediaries. Since the recent financial crisis, a number of papers study how frictions arising in the financial intermediation sector affect credit flows to the real economy. Christiano, Motto and Rostagno (2010) model banks that face asymmetric information and agency problems in their lending activities and incur a cost when creating liquid liabilities such as deposits, but do not consider leverage constraints on financial intermediaries.

More closely related is the work of Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). These authors consider banks that borrow in the form of riskless debt, and that are leverage constrained due to a moral hazard problem stemming from bankers' ability to divert a fraction of deposits for personal use.[6] They focus their discussion on how adverse shocks may disrupt credit supply through their effect on bank equity capital, and how unconventional monetary policy interventions can mitigate the effects of such shocks on economic activity. By contrast, we propose a model where intermediaries face leverage constraints that limit their incentives to fund inefficiently risky activities, as in Adrian and Shin (2013), and we then assess its ability to explain the observed fluctuations in intermediary leverage and economic activity. The central feature of our model is the presence of risky debt contracts and the associated risk-shifting incentives, which gives rise to the volatility-leverage channel discussed above.

[6] Gertler and Kiyotaki (2010) also consider idiosyncratic bank liquidity shocks (as in Kiyotaki and Moore, 2008), which give rise to an interbank market for funding.

Brunnermeier and Sannikov (2011), He and Krishnamurthy (2012) and Boissay, Collard and Smets (2013) propose models with financially constrained intermediaries and characterize their dynamics in a fully nonlinear manner. Such nonlinearities generate rich and interesting dynamics.[7] Our framework is more standard in that it can be solved using standard perturbation methods, which facilitates simulation as well as likelihood-based estimation.

Our paper also contributes to the literature on the macroeconomic effects of second moment (volatility) shocks in quantitative general equilibrium models.[8] In our model, first moment (TFP) and second moment shocks map differently on real and financial variables: while both shocks produce sizable fluctuations in GDP, investment, etc., only volatility shocks produce large and procyclical fluctuations in bank leverage. In this sense, our analysis points to the usefulness of financial data for disentangling the effects of first and second moment shocks.

Finally, we extend the empirical analysis of leverage fluctuations by Adrian and Shin (2010), who first documented the positive correlation between leverage and assets for security broker/dealers. Building on their work, our paper provides a systematic analysis of business cycle statistics (including measures of volatility, in addition to correlations) for the main components of intermediary balance sheets (assets, equity, and leverage) and for a wider range of financial subsectors. Moreover, we adopt a more macro perspective by including real GDP in the analysis, which allows us to gauge the comovements between balance sheet variables and aggregate economic activity. This gives rise to new empirical findings, such as the large volatility of intermediary leverage (relative both to equity -the other determinant of balance sheet size- and GDP), and its procyclicality with respect to GDP.

[7]Brunnermeier and Sannikov (2011) construct a model with financial frictions where, due to nonlinear amplification effects, the economy is prone to instability and occasionally enters volatile crisis episodes. He and Krishnamurthy (2012) propose a framework with intermediaries subject to occasionally binding capital constraints, and use it to explain the nonlinear behavior of risk premia during crises and to evaluate different policy interventions. Boissay, Collard and Smets (2013) consider a model of heterogeneous financial intermediaries where moral hazard and asymmetric information may generate sudden interbank market freezes.

[8]See footnote 5 and the references therein. See also Williamson (1987) for an early theoretical analysis in a general equilibrium model with financial frictions.

The paper proceeds as follows. Section 1 presents empirical evidence on the cyclical behavior of financial intermediaries' balance-sheet aggregates and GDP in the US. Section 2 lays out a general equilibrium model with leverage constrained intermediaries and volatility shocks. Section 3 analyzes some of the theoretical properties of the model, including the volatility-leverage channel and a comparison with Gertler and Karadi (2011). Section 4 calibrates and simulates the model, assessing its ability to replicate the data. Section 5 concludes.

1 Bank leverage cycles in the US economy

In this section, we perform a systematic analysis of the cyclical fluctuations in the main balance sheet components of US financial intermediaries, with a special attention to the leverage ratio, and their comovement with real economic activity. Our analysis comprises the main subsectors in what Greenlaw et al. (2008) have termed the 'leveraged sector', including depository intermediaries such as US-chartered commercial banks and savings institutions, as well as non-depository intermediaries such as security brokers and dealers and finance companies.

The balance sheet size of financial intermediaries is the product of two components: equity capital and leverage ratio. We may thus write $A_t = \phi_t N_t$, where A_t denotes total assets, ϕ_t represents the leverage ratio, and N_t is equity capital. In logs, we have

$$\log(A_t) = \log(\phi_t) + \log(N_t). \tag{1}$$

Table 1 displays a number of statistics regarding the cyclical fluctuations in intermediary leverage, equity capital, total assets and GDP in the United States, for the period 1963:Q1-2011:Q3. Our leverage, equity and assets series are constructed using data from the US Flow of Funds.[9] We con-

[9]Leverage is total assets divided by equity capital (both in dollars). 'Assets' and 'Equity' are total assets and equity capital, both deflated by the GDP deflator. All series are from the US Flow of Funds, except real GDP and the GDP deflator which are from the Bureau of Economic Analysis. See Data Appendix for further details. Leverage, assets, equity and GDP have been logged and detrended with a band-pass filter that preserves cycles of 6 to 32 quarters and with lag length of 12 quarters (Baxter and King, 1999). Notice that the linear identity in (1) is preserved by the bandpass

sider four leveraged financial subsectors: US-chartered commercial banks, savings institutions, security brokers and dealers, and finance companies. US-chartered commercial banks and savings institutions are both depository intermediaries, whereas security broker/dealers and finance companies are non-depository ones.

Table 1: Business cycle statistics, 1963:Q1-2011:Q3

	Commercial banks	Savings institutions	Security broker/dealers	Finance companies
Standard deviations (%)				
Assets	1.88	4.00	8.93	3.85
Leverage	2.84	8.16	10.21	4.71
Equity	2.55	8.13	8.08	3.64
GDP: 1.48				
Correlations				
Leverage - Assets	0.48 *** (0.0000)	0.25 *** (0.0008)	0.65 *** (0.0000)	0.65 *** (0.0000)
Leverage - Equity	−0.76 *** (0.0000)	−0.88 *** (0.0000)	−0.54 *** (0.0000)	−0.60 *** (0.0000)
Leverage - GDP	0.18 ** (0.0188)	0.12 (0.1055)	0.26 *** (0.0006)	0.32 *** (0.0000)
Assets - GDP	0.63 *** (0.0000)	0.71 *** (0.0000)	0.42 *** (0.0000)	0.52 *** (0.0000)

Note: Data are from the US Flow of Funds and Bureau of Economic Analsyis. See Data Appendix for details. All series are logged and detrended with a bandpass filter (cycles of 6 to 32 quarters, lag length of 12 quarters). P-values of the test of no correlation against the alternative of non-zero correlation are reported in parenthesis. Asterisks denote statistical significance of non-zero correlation at the 1% (***) and 5% (**) confidence level.

The table reveals two main stylized facts regarding the cyclical fluctuations in the leverage ratio of financial intermediaries. First, leverage is volatile. Notice first that leverage fluctuates more than equity capital, which is the other determinant of balance sheet size. Moreover, leverage is several times more volatile than GDP. For broker/dealers and finance companies, the standard deviation of leverage is about 7 and 3 times larger than that of GDP, respectively. The leverage of savings institutions displays a high volatility too, due mainly to the savings and loans crisis episode in the 1980s. For commercial banks, the leverage ratio fluctuates comparatively less, although its standard deviation is still about twice that of GDP.

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filter.

Second, leverage is strongly procyclical with respect to total assets, as originally documented by Adrian and Shin (2010, 2011).[10] It is also mildly procyclical with respect to GDP. The comovement with assets is particularly strong for security broker/dealers and finance companies (with a correlation of 0.65), but is also significant for depository institutions. As explained by Adrian and Shin (2010), such a strong comovement reveals an active management of leverage as a means of expanding and contracting the size of balance sheets. The correlation of the different leverage ratios with GDP ranges from 0.12 to 0.36, and while they are comparatively small, they are all statistically significant (with the exception of savings institutions). We also note that leverage and equity comove *negatively* over the business cycle. This negative correlation is large for the four subsectors, ranging from -0.88 for savings institutions to -0.54 for security broker/dealers.

As a graphical illustration, Figure 1 shows the cyclical components of total assets and leverage for the two largest leveraged financial subsectors in the United States: US-chartered commercial banks, and security brokers and dealers.[11] The 2008-9 recession witnessed a sharp decline in the leverage ratio of security broker/dealers, and an incipient decline in that of commercial banks. A similar deleveraging process was observed during the mid-70s recession. However, other recessions such as the 1981-82 one did not have any noticeable effect on the leverage of these two subsectors. This explains their relatively low cyclicality with respect to GDP. Notice also that the strong correlation of commercial banks' assets and leverage at the beginning of the sample has weakened somewhat over time, while such comovement seems to have been more stable for security broker/dealers. Figure 2 displays the cyclical comovement between leverage and equity capital, again for commercial banks and security broker/dealers. The negative correlation between both variables is evident in the case of commercial

[10] Our treatment of the data differs somewhat from that of Adrian and Shin (2010). They focus on the comovement between the *growth rates* of leverage and *nominal* total assets. Here, we focus on the behavior of *real* total assets, due both to our interest in the comovement of financial variables with real GDP and for consistency with our subsequent theoretical model. Also, we use a standard band-pass filter so as to extract the cyclical component of assets and leverage.

[11] The sample period used in figures 1 and 2 runs through 2011:Q4. However, the lag length of the bandpass filter (12 quarters) implies that the last filtered observation corresponds to 2008:Q4. Shaded areas represent NBER-dated recessions.

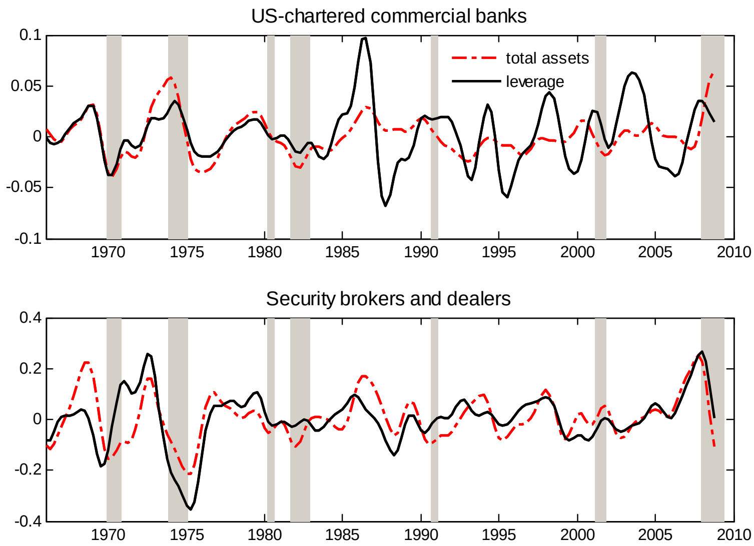

Figure 1: Cyclical components of intermediary leverage and total assets

Source: US Flow of Funds. See Data Appendix for details. Leverage and Total assets have been logged and detrended with a bandpass filter that preserves cycles of 6 to 32 quarters (lag length of 12 quarters). Shaded areas represent NBER-dated recessions.

banks, and less clear for broker/dealers. For instance, the recent recession featured an increase in equity of commercial banks, unlike that of broker/dealers. For the latter subsector, the negative correlation in Table 1 is the product of earlier historical episodes, such as the two recessions in the 1970s.

Our analysis of financial intermediaries is performed on a sectoral level. It would be interesting to consolidate the balance sheets of the different financial subsectors so as to study the cyclical properties of the leveraged financial system as a whole. Unfortunately, the Flow of Funds data does not allow this possibility, because asset and liability positions *between* the different subsectors are not netted out. As a result, simply adding assets and equity would lead to a double-counting of such cross positions. Nevertheless, it is important to emphasize that the stylized facts discussed above are robust across financial subsectors.

The above empirical findings are also robust in other dimensions. First, we have repeated the analysis using a Hodrick–Prescott filter instead of a bandpass one. Second, we have replaced 'total assets' by 'total financial

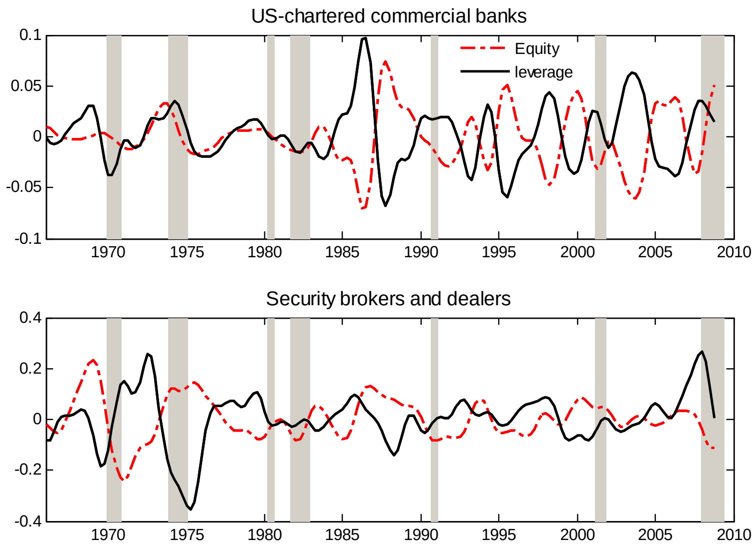

Figure 2: Cyclical components of intermediary leverage and equity

Source: US Flow of Funds. See Data Appendix for details. Leverage and Total assets have been logged and detrended with a bandpass filter that preserves cycles of 6 to 32 quarters (lag length of 12 quarters). Shaded areas represent NBER-dated recessions.

assets', which are also available in the Flow of Funds. In both cases quantitative results change very little.[12] Finally, we have restricted the sample period by starting it in 1984, instead of in 1963. Our motivation for doing so is the fact the US financial system has experienced substantial structural transformations during the postwar period, which raises the question as to how robust the business cycle statistics in Table 1 are to considering different subsamples. Results are shown in Table 2. Our stylized facts continue to hold. The only exception is that the correlation of commercial banks' leverage and GDP is no longer statistically significant.

Notice that the equity capital series from the Flow of Funds are of book equity, i.e. the difference between the value of intermediaries' portfolio of claims and their liabilities. An alternative measure of equity is the market capitalization, i.e. the market value of intermediaries' traded shares. As argued by Adrian, Colla and Shin (2012), book equity is the appropriate notion of equity if one is interested in the supply of bank credit, as we are here, whereas market capitalization would have been more appropriate

[12]Results are available upon request.

Table 2: Business cycle statistics, 1984:Q1-2011:Q3

	Commercial banks	Savings institutions	Security broker/dealers	Finance companies
Standard deviations (%)				
Assets	1.30	4.59	7.57	3.05
Leverage	3.12	8.61	7.62	5.34
Equity	3.12	8.35	5.27	4.58
GDP: 1.03				
Correlations				
Leverage - Assets	0.21 *	0.32 ***	0.76 ***	0.52 ***
	(0.0518)	(0.0023)	(0.0000)	(0.0000)
Leverage - Equity	−0.91 ***	−0.85 ***	−0.35 ***	−0.82 ***
	(0.0000)	(0.0000)	(0.0008)	(0.0000)
Leverage - GDP	−0.06	0.34 ***	0.22 **	0.24 **
	(0.5942)	(0.0014)	(0.0444)	(0.0252)
Assets - GDP	0.46 ***	0.73 ***	0.47 ***	0.41 ***
	(0.0000)	(0.0000)	(0.0000)	(0.0001)

Note: Data are from the US Flow of Funds and Bureau of Economic Analsyis. See Data Appendix for details. All series are logged and detrended with a bandpass filter (cycles of 6 to 32 quarters, lag length of 12 quarters). P-values of the test of no correlation against the alternative of non-zero correlation are reported in parenthesis. Asterisks denote statistical significance of non-zero correlation at the 1% (***), 5% (**) and 10% (*) confidence level.

if one were interested in new share issuance or mergers and acquisitions decisions.

A related issue is that, even if one focuses on the book value of equity, it matters how assets are valued. As argued e.g. by He, Khang and Krishnamurthy (2010), the fact that part of the assets on commercial banks' balance sheets are not subject to fair value accounting implies that official balance sheet data from the Flow of Funds or FDIC may overstate the true procyclicality of book leverage in that subsector. However, it is difficult to assess quantitatively how relevant this criticism is, given the lack of time series for the commercial banking sector that apply fair value accounting systematically. Also, this issue does not affect market-based intermediaries, such as security broker/dealers or finance companies, which constitute a significant share of the leveraged financial intermediary sector.[13] Therefore, it is reasonable to argue that, for the leveraged intermediary sector as a whole, leverage follows a procyclical pattern. In any case, we stress that

[13] As explained by Adrian and Shin (2013), book equity is fully marked-to-market for financial intermediaries that hold primarily marketable securities.

our measure of equity in the theoretical model will be consistent with the way equity is constructed in the data.

To summarize, our empirical analysis suggests the existence of a 'bank leverage cycle' in the postwar US economy, characterized by two main features. First, the leverage ratio of the different subsectors display large fluctuations, contributing more than equity capital to cyclical movements in total assets. Second, leverage is strongly procyclical with respect to total assets and mildly procyclical with respect to GDP; it is also negatively correlated with equity capital. In what follows, we present a general equilibrium model aimed at explaining these empirical patterns.

2 The model

The model economy is composed of five types of agents: households, final good producers ('firms' for short), capital producers, institutional investors, and banks. On the financial side, the model structure is as follows. Households lend to institutional investors in the form of deposits and equity. Institutional investors use the latter funds to lend to banks in the form of short-term debt. Banks combine this external funding and their own accumulated net worth so as to provide funding for firms. We assume no frictions in the relationship between banks and firms, such that the Modigliani-Miller theorem applies to firm financing. For simplicity and without loss of generality, we follow Gertler and Karadi (2011) in assuming that firms issue perfectly state-contingent debt only, which can be interpreted as equity. Banks and firms are segmented across a continuum of islands. Firms are hit by island-specific shocks to effective capital.[14] Banks are thus exposed to island-specific risk; each period, a fraction of them declare bankruptcy and default on their debt. Bank debt is not guaranteed and is therefore risky. As in Adrian and Shin (2013), moral hazard (of the risk-shifting type) creates a friction in the flow of funds from institutional investors to banks. Institutional investors operate economy-wide and diversify perfectly across islands; in fact, their only role in our model is to

[14] In introducing idiosyncratic shocks to effective capital, we follow Bernanke, Gertler and Gilchrist (1999). Christiano, Motto and Rostagno (2003, 2014) allow for exogenous variation over time in the dispersion of such idiosyncratic shocks.

insulate households from island-specific risk, which allows us to make use of the representative household construct.

The real side of the model is fairly standard. At the end of each period, after production has taken place, firms use borrowed funds to purchase physical capital from capital producers. At the beginning of the following period, firms combine their stock of capital and households' supply of labor to produce a final good. The latter is purchased by households for consumption purposes, and by capital producers. After production, firms sell their depreciated capital stock to capital producers, who use the latter and final goods to produce new capital. The markets for labor, physical capital and the final good are all economy-wide.

We now analyze the behavior of each type of agent. All variables are expressed in real terms, with the final good acting as the numeraire.

2.1 Households

The representative household's utility is $E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t) - v(L_t)\right]$, where C_t is consumption and L_t is labor supply. The budget constraint is

$$C_t + N_t^{II} + D_t = W_t L_t + R_t^N N_{t-1}^{II} + R_{t-1}^D D_{t-1} + \Pi_t^b,$$

where D_t and N_t^{II} are deposits and equity holdings respectively at institutional investors, R_{t-1}^D is the riskless gross deposit rate, R_t^N is the gross return on institutional investor equity, W_t is the wage, and Π_t^b are lump-sum net dividend payments from the household's ownership of banks. As we will see later on, Π_t^b incorporates any equity injections by households into banks. The first order conditions are

$$1 = E_t\left[\Lambda_{t,t+1} R_t^D\right], \quad 1 = E_t\left[\Lambda_{t,t+1} R_{t+1}^N\right], \quad W_t = \frac{v'(L_t)}{u'(C_t)},$$

where $\Lambda_{t,t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}$ is the stochastic discount factor.

2.2 Firms

The final good is produced by perfectly competitive firms. As in Kiyotaki and Moore (2008), we assume that firms are segmented across a continuum of 'islands', indexed by $j \in [0, 1]$.

In each island there are two types of firms, which we will refer to as 'standard' and 'substandard' firms. Each firm's type follows an iid process over time, with probability one half each; the type for time t is drawn at the end of $t-1$, after production has taken place. Both types differ in the following dimension. At the start of each period, firms receive an island-specific shock to effective capital. Let $\omega_t^j \geq 0$ and $\tilde{\omega}_t^j \geq 0$ denote the shock received by standard and substandard firms, respectively, in island j. Letting K_t^j denote the beginning-of-period capital stock of standard firms in island j, the shock ω_t^j changes their amount of effective capital to $\omega_t^j K_t^j$, and analogously for substandard firms. Both ω_t^j and $\tilde{\omega}_t^j$ are iid over time and across islands, and independent of each other. Let $F(\omega; \sigma_{t-1}) \equiv F_{t-1}(\omega)$ and $\tilde{F}(\tilde{\omega}; \sigma_{t-1}) \equiv \tilde{F}_{t-1}(\tilde{\omega})$ denote the cumulative distribution functions of time-t island-specific shocks received by standard and substandard firms, respectively. The term σ_{t-1} is an exogenous process that controls the time-t dispersion of both distributions; notice that the dispersion indicator is known one period in advance. The mean of both distributions is assumed to be time-invariant, and that of the standard technology is normalized to one: $\int \omega dF_t(\omega) \equiv E(\omega) = 1$, $\int \tilde{\omega} d\tilde{F}_t(\tilde{\omega}) \equiv E(\tilde{\omega})$.

The role of the two-types assumption will be made explicit in section 2.4, when we discuss the banking sector. As of now, it suffices to note that firms which draw the substandard type in a given period do not operate in equilibrium. Therefore, in what follows our notation refers to standard-type firms only, unless otherwise indicated.

At the beginning of period t, effective capital is combined with labor to produce units of final good, Y_t^j, according to a Cobb-Douglas technology,

$$Y_t^j = Z_t(\omega^j K_t^j)^\alpha (L_t^j)^{1-\alpha}, \tag{2}$$

where Z_t is an exogenous aggregate total factor productivity (TFP) process. The firm maximizes operating profits, $Y_t^j - W_t L_t^j$, subject to (2). The first order condition with respect to labor implies that the effective capital-labor ratio is equalized across islands,

$$\frac{\omega^j K_t^j}{L_t^j} = \left(\frac{W_t}{(1-\alpha) Z_t}\right)^{1/\alpha}, \tag{3}$$

for all j. The firm's profits are given by $Y_t^j - W_t L_t^j = \alpha Z_t (\omega^j K_t^j)^\alpha (L_t^j)^{1-\alpha} = R_t^k \omega^j K_t^j$, where $R_t^k \equiv \alpha Z_t \left[\frac{(1-\alpha) Z_t}{W_t}\right]^{(1-\alpha)/\alpha}$ is the return on effective capital, which is equalized too across islands. After production, the firm sells the depreciated effective capital $(1-\delta)\,\omega^j K_t^j$ to capital producers at price one. The total cash flow from the firm's investment project equals the sum of operating profits and proceeds from the sale of depreciated capital,

$$R_t^k \omega^j K_t^j + (1-\delta)\,\omega^j K_t^j = \left[R_t^k + (1-\delta)\right] \omega^j K_t^j. \tag{4}$$

Previously, at the end of period $t-1$, the firm bought K_t^j units of new capital at price one for production in t.[15] In order to finance this purchase, the firm issued a number A_{t-1}^j of claims on the period-t cash flow, equal to the number of capital units acquired. The firm's balance sheet constraint at the end of period $t-1$ is thus $K_t^j = A_{t-1}^j$. Since the capital purchase is financed entirely by state-contingent debt, the cash flow in (4) is paid off entirely to the lending banks.

2.3 Capital producers

There is a representative, perfectly competitive capital producer. In each period t, after production of final goods has taken place, the capital producer purchases the stock of depreciated capital $(1-\delta)\,K_t$ from firms. Used capital can be transformed into new capital on a one-to-one basis. Capital producers also purchase final goods in the amount I_t, which are used to produce new capital goods also on a one-to-one basis. Therefore, the total supply of new capital goods is given by $K_{t+1} = I_t + (1-\delta)\,K_t$. Capital goods are sold to firms at the end of the period for production in the following period. In equilibrium, the price of capital goods equals one and capital producers make zero profits.

[15] The assumption that firms purchase (or repurchase) their entire capital stock each period is standard in the macro-finance literature (see e.g. Bernanke, Gertler and Gilchrist, 1999; Gertler and Karadi, 2011; Christiano, Motto and Rostagno, 2014). As explained by Bernanke, Gertler and Gilchrist (1999), this modeling device ensures, realistically, that leverage restrictions or other financial constraints apply to the constrained borrowers (in this case, the banks) as a whole, not just to the marginal investment.

2.4 Banks

In each island there exists a representative bank. Only the bank on island j has the technology to obtain perfect information about firms on that island (including their type: standard or substandard), monitor them, and enforce their contractual obligations.[16] This effectively precludes firms from obtaining funding from other sources, including households or institutional investors. As indicated before, banks finance firms in the form of perfectly state-contingent debt. After production in period t, island j's firm pays the bank the entire cash flow from the investment project, $\left[R_t^k + (1-\delta)\right] \omega^j A_{t-1}^j$. The gross return on the bank's assets is thus the product of an aggregate component, $R_t^k + (1-\delta) \equiv R_t^A$, and an island-specific component, ω^j.

Regarding the liabilities side of its balance sheet, the bank borrows from institutional investors by means of one-period risky debt contracts.[17] Under the latter contract, at the end of period $t-1$ the bank borrows funds from the institutional investor in the amount B_{t-1}^j, and agrees to pay back a non-state-contingent amount $\bar{B}_{t-1}^j$ at the beginning of time t. At that point, the proceeds from the bank's assets, $R_t^A \omega^j A_{t-1}^j$, exceed the face value of its debt, $\bar{B}_{t-1}^j$, if and only if the island-specific shock ω^j exceeds a threshold level $\bar{\omega}_t^j$ given by

$$\bar{\omega}_t^j \equiv \frac{\bar{B}_{t-1}^j}{R_t^A A_{t-1}^j}. \tag{5}$$

The default threshold thus equals the face value of debt normalized by the bank's assets times their aggregate return. If $\omega^j \geq \bar{\omega}_t^j$ the bank honors its debt. If $\omega^j < \bar{\omega}_t^j$, the bank defaults, at which point the institutional investor seizes the bank's assets and cashes the resulting proceeds, $R_t^A \omega^j A_{t-1}^j$. The defaulting bank is then closed down. Notice that the default threshold $\bar{\omega}_t^j$ depends on R_t^A and is thus contingent on the aggregate state.

For non-defaulting banks, we assume that a random fraction $1-\theta$ of them close down for exogenous reasons each period, at which point the

[16] The costs of these activities for the bank are assumed to be negligible.

[17] Following Adrian and Shin (2013), we restrict our attention to standard debt contracts. The analysis of more general financial contracts is beyond the scope of this paper. Instead, our focus is on the limits on bank leverage resulting from the constraint placed by the investors, thereby limiting the size of the balance sheet for any given level of bank equity.

net worth accumulated in each bank is reverted to the household.[18] The remaining fraction θ of banks continue operating. For the latter, the flow of dividends distributed to the household is given by

$$\Pi_t^j = R_t^A \omega^j A_{t-1}^j - \bar{B}_{t-1}^j - N_t^j, \tag{6}$$

where N_t^j is net worth after dividends have been paid. We assume that continuing banks cannot issue new equity.[19] This implies the existence of a non-negativity constraint on dividends, $\Pi_t^j \geq 0$, or equivalently

$$N_t^j \leq R_t^A \omega^j A_{t-1}^j - \bar{B}_{t-1}^j. \tag{7}$$

Once the bank has decided how much net worth to hold, it purchases claims on firm cash flows, A_t^j, subject to its balance sheet constraint, $A_t^j = N_t^j + B_t^j$.

When borrowing from the institutional investor, the bank faces two constraints. First, a *participation constraint* requires that the expected payoff to the institutional investor from lending to the bank exceeds the expected payoff from lending at the riskless rate R_t^D. The latter is given by $E_t \Lambda_{t,t+1} R_t^D B_t^j = B_t^j = A_t^j - N_t^j$, where we have used the household's Euler equation and the bank's balance sheet constraint. Therefore, the participation constraint takes the form

$$E_t \Lambda_{t,t+1} \left\{ R_{t+1}^A A_t^j \int^{\bar{\omega}_{t+1}^j} \omega dF_t(\omega) + \bar{B}_t^j \left[1 - F_t\left(\bar{\omega}_{t+1}^j\right)\right] \right\} \geq A_t^j - N_t^j. \tag{8}$$

Second, we introduce a risk-shifting moral hazard problem on the part of the bank, in the spirit of Adrian and Shin (2013). We assume that, once the bank has received the funding, it may choose to invest in either of the two firm types (standard and substandard) within its island. As explained in section 2.2, both types differ in the distribution of island-specific shocks,

[18] The assumption of exogenous exit for borrowers (in our case, banks) is standard in the macro finance literature; see Bernanke, Gertler and Gilchrist (1999), and more recently Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). As explained by Bernanke, Gertler and Gilchrist (1999), this device is intended to preclude the possibility that the borrower will ultimately accumulate enough wealth to be fully self-financing. Indeed, as we show below, in equilibrium banks have no incentive to pay dividends.

[19] Such a constraint may be justified theoretically by the existence of agency or informational frictions in equity financing (see e.g. Myers and Majluf, 1984).

F_t and $\tilde{F}_t$. We make the following assumptions.

Assumption 1 *$\int \omega d\tilde{F}_t(\omega) < \int \omega dF_t(\omega)$.*

Assumption 2 *There exists a ω_t^* such that $F_t(\omega_t^*) = \tilde{F}_t(\omega_t^*)$ and $(F_t(\omega) - \tilde{F}_t(\omega))(\omega - \omega_t^*) > 0$ for all $\omega > 0$.*

Assumption 1 states that the substandard technology has lower mean return and is thus inefficient. Assumption 2 states that the distribution function F_t cuts $\tilde{F}_t$ precisely once from below, at a point denoted by ω_t^*.[20] As we will see, these assumptions imply that the bank may be tempted to invest in the substandard technology, despite its lower expected return. In order to induce the bank to invest in the standard firm type, the institutional investor imposes an *incentive compatibility* (IC) constraint. Let $V_{t+1}(\omega, A_t^j, \bar{B}_t^j)$ denote the value function at time $t+1$ of a non-defaulting, continuing bank, to be defined below. The IC constraint requires that the expected payoff of financing the standard firm exceeds that of financing the substandard one,

$$E_t \Lambda_{t,t+1} \int_{\bar{\omega}_{t+1}^j} \left\{ \theta V_{t+1}\left(\omega, A_t^j, \bar{B}_t^j\right) + (1-\theta)\left[R_{t+1}^A A_t^j \omega - \bar{B}_t^j\right] \right\} dF_t(\omega) \quad (9)$$
$$\geq E_t \Lambda_{t,t+1} \int_{\bar{\omega}_{t+1}^j} \left\{ \theta V_{t+1}\left(\omega, A_t^j, \bar{B}_t^j\right) + (1-\theta)\left[R_{t+1}^A A_t^j \omega - \bar{B}_t^j\right] \right\} d\tilde{F}_t(\omega).$$

To understand the bank's incentives to finance one firm type or another, notice that its expected net payoff (in a particular aggregate state at time $t+1$) from investing in standard firms can be expressed as $R_{t+1}^A A_t^j \int_{\bar{\omega}_{t+1}^j} (\omega - \bar{\omega}_{t+1}^j) dF_t(\omega)$. The integral represents the value of a *call option* on island-specific returns with strike price equal to the default threshold, $\bar{\omega}_{t+1}^j$, which in turn equals the normalized face value of debt, $\bar{B}_t^j / R_{t+1}^A A_t^j$.[21] Intuitively, limited liability implies that the bank enjoys the upside risk in asset returns over and above the face value of its debt, but does not bear the downside risk, which is transferred to the institutional investor. Furthermore, the

[20] It can be showed that both assumptions imply second-order stochastic dominance (SOSD) of F_t over $\tilde{F}_t$. The proof is available upon request.

[21] For a pioneering analysis of the payoff structure of defaultable debt claims, equity stakes, and their relationship to option derivatives, see Merton (1974).

value of the call option on island-specific risk may be expressed as

$$\int_{\bar{\omega}^j_{t+1}} (\omega - \bar{\omega}^j_{t+1}) dF_t(\omega) = E(\omega) + \pi_t(\bar{\omega}^j_{t+1}) - \bar{\omega}^j_{t+1},$$

where

$$\pi_t(\bar{\omega}^j_{t+1}) \equiv \pi(\bar{\omega}^j_{t+1}; \sigma_t) \equiv \int^{\bar{\omega}^j_{t+1}} (\bar{\omega}^j_{t+1} - \omega) dF_t(\omega) \tag{10}$$

is the value of the *put option* on island-specific returns with strike price $\bar{\omega}^j_{t+1}$.[22] Therefore, given the normalized face value of its debt, $\bar{\omega}^j_{t+1}$, the bank's expected net payoff increases with the mean island-specific return, $E(\omega)$, but also with the put option value $\pi_t(\bar{\omega}^j_{t+1})$. The same reasoning holds for the substandard technology; for the latter, the put option value $\tilde{\pi}_t(\bar{\omega}^j_{t+1})$ is defined analogously to (10), with $\tilde{F}_t$ replacing F_t. Under Assumptions 1 and 2, we obtain the following results.[23]

Lemma 1 *(i) The put option value is higher for the substandard investment strategy:* $\tilde{\pi}_t(\bar{\omega}^j_{t+1}) > \pi_t(\bar{\omega}^j_{t+1})$ *for all* $\bar{\omega}^j_{t+1} > 0$. *(ii) The difference in put option values* $\Delta\pi_t(\bar{\omega}^j_{t+1}) \equiv \tilde{\pi}_t(\bar{\omega}^j_{t+1}) - \pi_t(\bar{\omega}^j_{t+1})$ *cuts* $E(\omega) - E(\tilde{\omega})$ *precisely once from below.*

Part (i) of Lemma 1 implies that, when choosing between investment strategies, the bank trades off the *higher mean return* of investing in standard firms against the *lower put option value*. Part (ii) implies that the incentive to invest in substandard firms, as captured by the gain in put option value $\Delta\pi_t(\bar{\omega}^j_{t+1})$, *increases* initially with the normalized debt burden $\bar{\omega}^j_{t+1}$ and eventually surpasses the loss in mean return $E(\omega) - E(\tilde{\omega})$. This static reasoning contains the essence of the mechanism through which the bank may be tempted to finance substandard technologies, even though the actual IC constraint (eq. 9) is intrinsically dynamic due to the presence of the continuation value V_{t+1}.

We are ready to spell out the bank's maximization problem. Let $\bar{V}_t(N^j_t)$ denote the bank's value function after paying out dividends and at the time of borrowing from the institutional investor. We then have the following

[22] The relationship between the values of a European call option and a European put option is usually referred to as the 'put-call parity'.

[23] The proof is in Appendix B.

Bellman equations:

$$V_t\left(\omega^j, A_{t-1}^j, \bar{B}_{t-1}^j\right) = \max_{N_t^j} \left\{\Pi_t^j + \bar{V}_t\left(N_t^j\right)\right\},$$

subject to (6) and (7); and

$$\bar{V}_t\left(N_t^j\right) = \max_{A_t^j, \bar{B}_t^j} E_t \Lambda_{t,t+1} \int_{\bar{\omega}_{t+1}^j} \left[\theta V_{t+1}\left(\omega, A_t^j, \bar{B}_t^j\right) + (1-\theta)\left(R_{t+1}^A A_t^j \omega - \bar{B}_t^j\right)\right] dF_t(\omega),$$

subject to (5), (8) and (9). Let $\bar{b}_t^j \equiv \bar{B}_t^j / A_t^j$ denote the face value of debt normalized by the bank's assets. This allows us to express the default threshold as $\bar{\omega}_t^j = \bar{b}_{t-1}^j / R_t^A$. Appendix C proves the following result.

Proposition 1 (Solution to the bank's problem) *Assume the model parameters satisfy* $0 < \beta R^A - 1 < (1-\theta)\beta R^A \int_{\bar{\omega}^j} (\omega - \bar{\omega}^j)\, dF(\omega)$, *where* R^A *and* $\bar{\omega}^j$ *are the steady-state values of* R_t^A *and* $\bar{\omega}_t^j$, *respectively. Then the equilibrium dynamics of bank* j *in a neighborhood of the deterministic steady state are characterized by the following features:*

1. *The bank optimally retains all earnings,*

$$N_t^j = \left(\omega^j - \frac{\bar{b}_{t-1}}{R_t^A}\right) R_t^A A_{t-1}^j, \tag{11}$$

where $\bar{b}_{t-1}$ *is equalized across islands, such that* $\bar{\omega}_t^j = \bar{\omega}_t = \bar{b}_{t-1}/R_t^A < \omega_t^*$ *for all* j.

2. *The IC constraint holds with equality. In equilibrium, the latter can be expressed as*

$$1 - E(\tilde{\omega}) = E_t \left\{ \frac{\Lambda_{t,t+1} R_{t+1}^A (\theta \lambda_{t+1} + 1 - \theta)}{E_t \Lambda_{t,t+1} R_{t+1}^A (\theta \lambda_{t+1} + 1 - \theta)} \left[\tilde{\pi}\left(\frac{\bar{b}_t}{R_{t+1}^A}; \sigma_t\right) - \pi\left(\frac{\bar{b}_t}{R_{t+1}^A}; \sigma_t\right)\right]\right\}, \tag{12}$$

where λ_{t+1} *is the Lagrange multiplier associated to the participation constraint, which is equalized across islands.*

3. *The participation constraint holds with equality,*

$$A_t^j = \frac{1}{1 - E_t \Lambda_{t,t+1} R_{t+1}^A \left[\bar{\omega}_{t+1} - \pi\left(\bar{\omega}_{t+1}; \sigma_t\right)\right]} N_t^j \equiv \phi_t N_t^j. \tag{13}$$

According to (12), the (normalized) debt repayment $\bar{b}_t$ is set such that the gain in mean return from investing in the standard firm segment exactly compensates the bank for the loss in put option value. According to (13), the bank's demand for assets equals its net worth times a *leverage ratio* ϕ_t which is equalized across islands. Notice that leverage decreases with the left tail risk of the bank's portfolio, as captured by the put option value $\pi\left(\bar{\omega}_{t+1}; \sigma_t\right)$. Intuitively, since all the downside risk in the bank's assets is born by the institutional investor, a higher perception of such risk leads the latter to impose a tighter leverage constraint.

Once $\bar{b}_t$ and ϕ_t have been determined, it is straightforward to obtain the actual loan size, $B_t^j = \left(\phi_t - 1\right) N_t^j$, and its face value, $\bar{B}_t^j = \bar{b}_t A_t^j = \bar{b}_t \phi_t N_t^j$. Finally, the gross interest rate equals $\bar{B}_t^j / B_t^j = \bar{b}_t \phi_t / \left(\phi_t - 1\right)$.

2.5 Institutional investors

A representative, perfectly competitive institutional investor collects funds from households in the form of deposits and equity, and lends these funds to banks through short-term risky debt. Its balance sheet is thus $N_t^{II} + D_t = B_t$, where $B_t = \int_0^1 B_t^j dj$. There is no friction in the relationship between households and institutional investors. We assume that equity is sufficiently high to absorb aggregate risk and thus make deposits effectively safe.[24] The institutional investor operates economy-wide and hence perfectly diversifies its portfolio across islands.

The institutional investor's return from financing the island-j bank is $\min\left\{R_t^A \omega^j A_{t-1}^j, \bar{B}_{t-1}^j\right\} = R_t^A A_{t-1}^j \min\left\{\omega^j, \frac{\bar{b}_{t-1}}{R_t^A}\right\} = R_t^A \phi_{t-1} N_{t-1}^j \min\left\{\omega^j, \bar{\omega}_t\right\}$. Aggregating across islands and subtracting gross interest payments on deposits, we obtain the return on the institutional investor's equity,

$$
\begin{aligned}
R_t^N N_{t-1}^{II} &= R_t^A \phi_{t-1} \int_0^1 N_{t-1}^j \min\left\{\omega^j, \bar{\omega}_t\right\} dj - R_{t-1}^D D_{t-1} \\
&= R_t^A \phi_{t-1} N_{t-1} \left\{\left[1 - F_{t-1}\left(\bar{\omega}_t\right)\right] \bar{\omega}_t + \int^{\bar{\omega}_t} \omega dF_{t-1}\left(\omega\right)\right\} - R_{t-1}^D D_{t-1},
\end{aligned}
$$

where in the second equality we have used the fact ω^j is distributed inde-

[24] The relative shares of equity and deposits are undetermined in equilibrium. A way to guarantee that deposits are risk-free is to introduce a lump-sum tax on households that covers the potential losses in the deposits.

pendently from N_{t-1}^j, and where $N_{t-1} \equiv \int_0^1 N_{t-1}^j dj$ is aggregate bank net worth. The institutional investor distributes all earnings to the household in every period.

2.6 Aggregation and market clearing

Aggregate net worth of banks at the *end* of period t, N_t, is the sum of the net worth of both continuing and new banks: $N_t = N_t^{cont} + N_t^{new}$. From (11) and $A_{t-1}^j = \phi_{t-1} N_{t-1}^j$, we have that $N_t^j = R_t^A (\omega^j - \bar{\omega}_t) \phi_{t-1} N_{t-1}^j$. Aggregating across islands, we obtain the total net worth of continuing banks, $N_t^{cont} = \theta R_t^A \phi_{t-1} N_{t-1} \int_{\bar{\omega}_t} (\omega - \bar{\omega}_t) dF_{t-1}(\omega)$, where we have used the fact that ω^j is distributed independently from N_{t-1}^j. Banks that default or exit the market exogenously are replaced by an equal number of new banks, $1 - \theta [1 - F_{t-1}(\bar{\omega}_t)]$. We assume each new bank is endowed by households with a fraction τ of total assets at the beginning of the period, $A_{t-1} \equiv \int_0^1 A_{t-1}^j dj$.[25] Therefore, $N_t^{new} = \{1 - \theta [1 - F_{t-1}(\bar{\omega}_t)]\} \tau A_{t-1}$. We thus have

$$N_t = \theta R_t^A \phi_{t-1} N_{t-1} \int_{\bar{\omega}_t} (\omega - \bar{\omega}_t) dF_{t-1}(\omega) + \{1 - \theta [1 - F_{t-1}(\bar{\omega}_t)]\} \tau A_{t-1}. \tag{14}$$

New banks leverage their starting net worth with the same ratio as continuing banks. We thus have $A_t = \phi_t (N_t^{cont} + N_t^{new}) = \phi_t N_t$.

Aggregate net dividends to households from banks are given by $\Pi_t = (1-\theta) R_t^A \phi_{t-1} N_{t-1} \int_{\bar{\omega}_t} (\omega - \bar{\omega}_t) dF_{t-1}(\omega) - N_t^{new}$. Market clearing for capital requires that total demand by firms equals total supply by capital producers, $\int_0^1 K_t^j dj = K_t$. Total issuance of state-contingent claims by firms must equal total demand by banks, $K_{t+1} = A_t$. Using (3) to solve for firm j's labor demand L_t^j, aggregating across islands and imposing labor market clearing, we have

$$\int_0^1 L_t^j dj = \left(\frac{(1-\alpha) Z_t}{W_t} \right)^{1/\alpha} \int_0^1 \omega^j K_t^j dj = \left(\frac{(1-\alpha) Z_t}{W_t} \right)^{1/\alpha} K_t = L_t, \tag{15}$$

where we have used the facts that ω^j and K_t^j are distributed indepen-

[25] Our specification for equity injections into newly created banks follows Gertler and Karadi (2011).

dently and that ω^j has unit mean. Equations (3) and (15) then imply that $\omega^j K_t^j / L_t^j = K_t / L_t$. Using the latter and (2), aggregate supply of the final good equals $Y_t \equiv \int_0^1 Y_t^j dj = Z_t \left(\frac{L_t}{K_t}\right)^{1-\alpha} \int_0^1 \omega^j K_t^j dj = Z_t K_t^\alpha L_t^{1-\alpha}$. Finally, total supply of the final good must equal consumption demand by households and investment demand by capital producers, $Y_t = C_t + I_t$.

3 Model properties

Having laid out our model in the previous section, it is now worthwhile to explore some of its theoretical properties and transmission mechanisms.

3.1 Comparison to RBC model

Appendix D summarizes the equilibrium conditions in our model, and compares it with a standard RBC model. As we show there, the RBC model shares all its equilibrium conditions with our model, except for the investment Euler equation, given by

$$1 = E_t \left\{ \Lambda_{t,t+1} R_{t+1}^A \right\}, \tag{16}$$

where $R_{t+1}^A = (1-\delta) + \alpha Y_{t+1} / K_{t+1}$. The equation that determines leverage in our framework (13) can be rewritten in an analogous form,

$$1 = E_t \left\{ \Lambda_{t,t+1} R_{t+1}^A \left[\bar{\omega}_{t+1} - \pi \left(\bar{\omega}_{t+1}; \sigma_t \right) \right] \frac{\phi_t}{\phi_t - 1} \right\}, \tag{17}$$

where $\phi_t = K_{t+1}/N_t$. A comparison of equations (17) and (16) reveals that the term $\left[\bar{\omega}_{t+1} - \pi \left(\bar{\omega}_{t+1}; \sigma_t \right) \right] \frac{\phi_t}{\phi_t - 1} \equiv \Theta_{t+1}(\sigma_t)$ in equation (17) is a sufficient statistic for measuring the difference in equilibrium dynamics between both models. Using $\phi_t = A_t / N_t$ and the definition of the put option value $\pi_t(\bar{\omega}_{t+1})$ in equation (10), we can write

$$\begin{aligned} R_{t+1}^A \Theta_{t+1}(\sigma_t) &= R_{t+1}^A \left\{ \int^{\bar{\omega}_{t+1}} \omega dF_t(\omega) + \bar{\omega}_{t+1} \left[1 - F_t(\bar{\omega}_{t+1}) \right] \right\} \frac{A_t}{A_t - N_t} \\ &= \frac{R_{t+1}^A A_t \int^{\bar{\omega}_{t+1}} \omega dF_t(\omega) + \bar{B}_t \left[1 - F_t(\bar{\omega}_{t+1}) \right]}{B_t}, \end{aligned} \tag{18}$$

where in the second equality we have used $\bar{\omega}_{t+1} = \bar{B}_t/(R^A_{t+1}A_t)$ and $A_t - N_t = B_t$. Expression (18) is just the return on aggregate risky debt, B_t. Therefore, $\Theta_{t+1}(\sigma_t)$ captures the fraction of the total return on capital, R^A_{t+1}, that is received by the household (through the institutional investor). In the absence of financial frictions, households receive the entire return on capital and $\Theta_{t+1}(\sigma_t)$ is simply 1.[26] With financial frictions, we generally have $\Theta_{t+1}(\sigma_t) \neq 1$, which drives a wedge between investment decisions in our model and in the RBC model.

3.2 The volatility-leverage channel

A recent financially oriented literature argues that the volatility of returns on the assets held by borrowers is a key determinant of their leverage. For example, Brunnermeier and Pedersen (2009) analyze how an increase in the volatility of asset prices leads investors to demand higher margins, thus forcing borrowers to deleverage. Similarly, Fostel and Geanakoplos (2008) and Geanakoplos (2010) consider shocks that not only decrease the expected asset returns but also their volatility; these shocks, which the authors refer to as 'scary bad news', lead to tighter margins as lenders protect themselves against increased uncertainty. From a more macro perspective, recent work following the lead of Bloom (2009) suggests that exogenous changes in volatility are an important driving force behind business cycle fluctuations (see e.g. Arellano et al., 2012; Bloom et al., 2012; Christiano et al., 2014; Gilchrist et al., 2010; Kiley and Sim, 2011). Much of this literature is inspired by the 2007-9 financial crisis, in which higher 'uncertainty' or 'risk' perception (concepts that are closely related to that of volatility) about the solvency of financial intermediaries is widely believed to have played an important role.

In the standard RBC model, the absence of the financial wedge $\Theta_{t+1}(\sigma_t)$ implies that shocks to cross-sectional volatility, σ_t, have no effect whatsoever in the latter model. In our framework, on the contrary, the presence of financial frictions opens a link between cross-sectional volatility in asset returns, bank leverage, and investment dynamics.

[26] Assuming no financial frictions is equivalent to assuming that households themselves buy the capital and then rent it to firms, as is typically done in the RBC literature.

We now make two further assumptions on the distributions of island-specific shocks.

Assumption 3 *An increase in island-specific volatility increases the put option value of the standard technology:* $\frac{\partial \pi(\bar{\omega};\sigma)}{\partial \sigma} > 0$.

Assumption 4 *An increase in island-specific volatility increases the gain in put option value from the substandard technology:* $\frac{\partial \Delta\pi(\bar{\omega};\sigma)}{\partial \sigma} = \frac{\partial \tilde{\pi}(\bar{\omega};\sigma)}{\partial \sigma} - \frac{\partial \pi(\bar{\omega};\sigma)}{\partial \sigma} > 0$.

Both assumptions are relatively weak. They require that, following an increase in the dispersion of island-specific shocks, downside risk (as measured by the put option values π and $\tilde{\pi}$) goes up for both firm types, and that it does so by more for the substandard firms.

Under our distributional assumptions, an increase in the standard deviation of island-specific shocks, σ_t, induces a reduction in the leverage ratio of banks, via a mechanism sketched in Figure 3. The upper subplot represents the steady-state counterpart of the IC constraint (eq. 12). The upward-sloping line is the gain in left tail risk from investing in the substandard firm segment, $\Delta\pi(\bar{\omega};\sigma) = \tilde{\pi}(\bar{\omega};\sigma) - \pi(\bar{\omega};\sigma)$, which is an increasing function of the default threshold or (normalized) debt repayment, $\bar{\omega} = \bar{b}/R^A$.[27] The horizontal line is the loss in mean return, $E(\omega) - E(\tilde{\omega}) = 1 - \int \omega d\tilde{F}(\omega,\sigma)$. The IC constraint requires that the gain in left tail risk from investing in the substandard technology does not exceed the loss in mean return; since the constraint is binding in equilibrium, $\bar{\omega}$ is determined by the intersection of both lines. Consider now an increase in cross-sectional volatility, σ. Under Assumption 4, *ceteris paribus* the $\Delta\pi(\bar{\omega};\sigma)$ schedule rotates upwards and $\bar{\omega}$ goes down. Intuitively, since higher volatility makes it more attractive for the bank to invest inefficiently, the institutional investor reduces the (normalized) face value of debt so as to discourage the bank from doing so.

The lower subplot of Figure 3 represents the steady-state counterpart of the participation constraint, $\phi = \left\{1 - \beta R^A [\bar{\omega} - \pi(\bar{\omega};\sigma)]\right\}^{-1} \equiv PC(\bar{\omega};\sigma)$,

[27] Remember that $\Delta\pi'(\bar{\omega};\sigma) = \tilde{F}(\bar{\omega};\sigma) - F(\bar{\omega};\sigma) > 0$, where the inequality follows from Assumption 2 and the fact that in equilibrium $\bar{\omega} < \omega^*$.

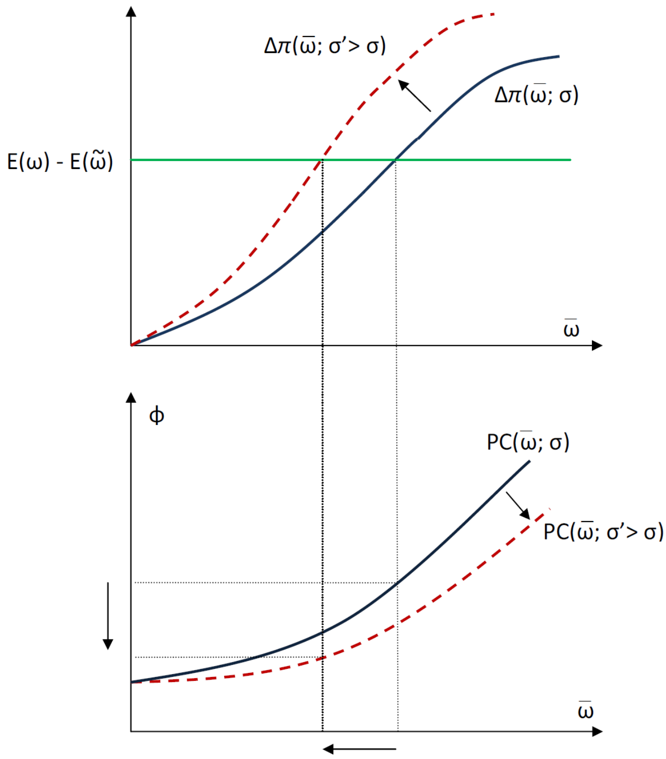

Figure 3: The volatility-leverage channel

in $(\bar{\omega}, \phi)$ space. The latter is an upward-sloping relationship.[28] *Ceteris paribus*, the increase in σ has a double effect on leverage. First, the participation constraint schedule rotates down as a result of Assumption 3, which reduces leverage for a given $\bar{\omega}$. Intuitively, higher volatility of island-specific shocks increases the downside risk $\pi(\bar{\omega};\sigma)$ of the bank's assets, which reduces the investor's expected payoff; in order to induce the investor to lend, the bank reduces its demand for funds as a fraction of its net worth. Second, the reduction in $\bar{\omega}$ through the IC constraint produces a leftward movement *along* the leverage schedule, thus further reducing leverage. Both effects are mutually reinforcing.

[28]The investor's expected payoff from lending to the bank equals the bank's assets times $\beta R^A [\bar{\omega} - \pi(\bar{\omega})]$. Since $\pi'(\bar{\omega}) = F(\bar{\omega}) < 1$, the latter payoff *increases* with $\bar{\omega}$. This allows the bank to borrow more as a fraction of its net worth (i.e. to increase its leverage) while still persuading the investor to lend the funds.

3.3 Comparison to Gertler and Karadi (2011)

In an influential paper, Gertler and Karadi (2011; GK) introduce leverage-constrained financial intermediaries (banks) in a standard DSGE framework. In their model, banks are leverage-constrained due to a moral hazard problem different from our risk-shifting specification. In this section, we compare both frameworks, focusing on the differences in modelling choices and the resulting implications for the transmission mechanisms.

GK abstract from cross-sectional uncertainty in firms' project returns and hence in banks' asset returns. An immediate implication is that bank leverage is independent of volatility shocks in their framework. In our model, the presence of cross-sectional uncertainty, coupled with limited liability and risk-shifting moral hazard on the part of banks, gives rise to a volatility-leverage channel, by which volatility shocks affect real economic activity via the leverage ratio of financial intermediaries.

Even if one abstracts from cross-sectional uncertainty in the payoff of the investment projects that are financed in equilibrium (those of standard-type firms), our framework continues to feature a volatility-leverage channel. To see this, consider the special case in which the standard technology has no island-specific uncertainty: $\omega_t^j = 1$ for all j and t, while the substandard technology retains its uncertainty.[29] In this case, bank debt is riskless and the participation constraint collapses to $E_t \Lambda_{t,t+1} \bar{B}_t^j \geq B_t^j = A_t^j - N_t^j$. Under parametric conditions analogous to those of Proposition 1, the participation constraint continues to bind in this simplified model version. Therefore, the interest rate on bank debt equals the risk-free rate, $\bar{B}_t^j / B_t^j = 1/E_t \Lambda_{t,t+1} = R_t^D$, and the face value of debt equals $\bar{B}_t^j = R_t^D (A_t^j - N_t^j)$, as in GK. The default threshold if the bank chooses to fund substandard projects can be expressed as $\bar{\omega}_{t+1}^j = \bar{B}_t^j / (R_{t+1}^A A_t^j) = \frac{R_t^D}{R_{t+1}^A}(1 - N_t^j / A_t^j)$.

To further facilitate the comparison between both models, we may assume that, conditional on funding the substandard firms, banks are forced to pay off the resulting earnings as dividends and close down, i.e. $\theta = 0$ *if*

[29] In order for our risk-shifting moral hazard problem to be well defined, there must be some island-specific uncertainty in the substandard technology.

the bank chooses the $\tilde{F}_t$ lottery. Our IC constraint then becomes

$$E_t \Lambda_{t,t+1} \left\{ \theta V_{t+1} \left(1, A_t^j, R_t^D (A_t^j - N_t^j) \right) + (1-\theta) \left[R_{t+1}^A A_t^j - R_t^D (A_t^j - N_t^j) \right] \right\}$$
$$\geq E_t \Lambda_{t,t+1} \int_{\frac{R_t^D}{R_{t+1}^A}(1-N_t^j/A_t^j)} \left[R_{t+1}^A A_t^j \omega - R_t^D (A_t^j - N_t^j) \right] d\tilde{F}(\omega; \sigma_t) = \Psi_t \left(N_t^j, A_t^j; \sigma_t \right) A_t^j, \tag{19}$$

with

$$\Psi_t \left(N_t^j, A_t^j; \sigma_t \right) \equiv E_t \Lambda_{t,t+1} R_{t+1}^A \int_{\frac{R_t^D}{R_{t+1}^A}(1-N_t^j/A_t^j)} \left[\omega - \frac{R_t^D}{R_{t+1}^A} \left(1 - \frac{N_t^j}{A_t^j} \right) \right] d\tilde{F}(\omega; \sigma_t).$$

The moral hazard problem in GK motivates an incentive compatibility constraint analogous to (19), with the value of 'behaving well' on the left-hand side of the inequality and the value of incurring in moral hazard on the right-hand side. The difference lies in the latter value. In GK, the moral hazard payoff can be expressed as ΨA_t^j, where Ψ is a parameter representing the fraction of assets that the bank manager can divert. In our framework, the moral hazard payoff $\Psi_t(N_t^j, A_t^j; \sigma_t) A_t^j$ is the value of funding substandard activities. Unlike in GK, the factor Ψ_t depends on the choice variable A_t^j, and is therefore not taken as given by the bank. Moreover, Ψ_t depends endogenously on the aggregate state of the economy, *including* volatility shocks σ_t. Once the simplified bank problem is solved, the IC constraint holds with equality and implicitly determines equilibrium bank leverage $A_t^j / N_t^j = \phi_t$.[30]

Therefore, the volatility-leverage channel *per se* does not depend on the presence of cross-sectional uncertainty in the projects that are financed in equilibrium. Allowing for the latter, however, introduces equilibrium default risk in bank debt, which in turn introduces a risk premium in the required return on bank debt.

[30] The solution of the bank's optimization problem in the simplified model version is available upon request.

4 Quantitative Analysis

4.1 Calibration and steady state

We calibrate our model to the US economy for the period 1984:Q1-2011:Q3. The parameters are shown in Table 3. We may divide the parameters between those that are standard in the real business cycle literature, and those that are particular to this model. From now onwards, we let variables without time subscripts denote steady-state values.

Table 3: Model parameters

Parameter	Value	Description	Source/Target
Standard parameters			
β	0.99	discount factor	$R^4 = 1.04$
α	0.36	capital share	$WL/Y = 0.64$
δ	0.025	depreciation rate	$I/K = 0.025$
φ	1	inverse labor supply elasticity	macro literature
$\bar{Z}$	0.5080	steady-state TFP	$Y = 1$
ρ_z	0.9297	autocorrelation TFP	FRBSF-CSIP TFP
σ_z	0.0067	standard deviation TFP	FRBSF-CSIP TFP
Non-standard parameters			
σ	0.0373	steady-state island-spec. volatility	average leverage (ϕ=18.3)
η	1.2691	variance substandard technology	$\left(\bar{R}/R\right)^4 - 1 = 0.25\%$
ψ	0.001	mean substandard technology	average volatility of equity
τ	0.0207	equity injections new banks	$I/Y = 0.2$
θ	0.75	continuation prob. banks	annual dividends ($\tau > 0$),
ρ_σ	0.9457	autocorr. island-specific volatility	NBER-CES manuf. TFP
σ_σ	0.0465	std. dev. island-specific volatility	NBER-CES manuf. TFP

We set the RBC parameters to standard values. In particular, we set $\beta = 0.99 = 1/R$, $\alpha = 0.36 = 1 - WL/Y$, $\delta = 0.025 = I/K$, which are broadly consistent with long-run averages for the real interest rate, the labor share, and the investment to capital ratio. We target a capital-output ratio of $K/Y = 8$, with is consistent with a ratio of investment over GDP of 20 percent, roughly in line with the historical evidence. We then have $R^A = \alpha\left(Y/K\right) + 1 - \delta = 1.02$. Our functional forms for preferences are standard: $u\left(C\right) = \log(C)$, $v(L) = L^{1+\varphi}/\left(1+\varphi\right)$. We set $\varphi = 1$, in line with other macroeconomic studies (e.g. Comin and Gertler, 2006). We assume an AR(1) process for the natural log of TFP, $\log\left(Z_t/\bar{Z}\right) =$

$\rho_z \log\left(Z_{t-1}/\bar{Z}\right) + \varepsilon_t^z$, where $\varepsilon_t^z \overset{iid}{\sim} N(0, \sigma_z)$. Our empirical counterpart for $\log\left(Z_t/\bar{Z}\right)$ is the Federal Reserve Bank of San Francisco-CSIP quarterly log TFP series, after being linearly detrended. We then choose ρ_z and σ_z so as to match their empirical counterparts. $\bar{Z}$ is chosen such that steady-state output is normalized to one.

Regarding the non-standard parameters in our model, our calibration strategy is as follows. Ideally, one would consolidate the balance sheets of the different financial subsectors so as to calibrate the model to the leveraged financial sector as a whole. As explained in section 1, this consolidation is however not feasible, due to the existence of cross-positions among financial subsectors and the need to avoid double-counting. For this reason, we choose the average across subsectors of the mean leverage ratios in the sample period as target for the steady-state leverage ratio: $\phi = 18.3$. If we were to interpret bank debt in the model as repo, then the haircut would be $\phi^{-1} = 5.46\%$; the latter is roughly in line with average haircuts for repos backed by corporate debt and private-label ABS, as documented by Krishnamurthy et al. (2012). The same authors show that the spread between repo rates for the same collateral categories and the Fed funds rate was close to zero in the pre-crisis period. Based on this, we target a spread in bank debt of 25 annualized basis points. The gross interest rate then equals $\bar{R} = R\,(1.0025)^{1/4}$. The face value of bank debt (normalized by assets) is then $\bar{b} = \bar{R}\,(\phi - 1)\,/\phi = 0.9555$. This implies a default threshold of $\bar{\omega} = \bar{b}/R^A = 0.9368$.

Island-specific shocks are assumed to be lognormally distributed, both for the standard and the substandard firm segment,

$$\log \omega \overset{iid}{\sim} N\left(\frac{-\sigma_t^2}{2}, \sigma_t\right), \quad \log \tilde{\omega} \overset{iid}{\sim} N\left(\frac{-\eta\sigma_t^2 - \psi}{2}, \sqrt{\eta}\sigma_t\right),$$

for $\psi > 0$ and $\eta > 1$. Therefore, $F(\omega; \sigma_t) = \Phi\left(\frac{\log(\omega)+\sigma_t^2/2}{\sigma_t}\right)$, where $\Phi(\cdot)$ is the standard normal cdf; and analogously for $\tilde{F}$. These functional forms are consistent with our assumptions in the model section. First, we have $E(\tilde{\omega}) = e^{-\psi/2} < 1 = E(\omega)$, i.e. both means are time-invariant (such that an increase in σ_t constitutes a mean-preserving spread) and the standard technology has a higher expected payoff. Also, the fact that $\eta > 1$ (i.e. the substandard technology has a higher variance) guarantees that F_t cuts $\tilde{F}_t$ once from below.[31] We have the following expressions for the values of the unit put options on island-specific risk,[32]

$$\pi\left(\bar{\omega}_t;\sigma_{t-1}\right)=\bar{\omega}_t\Phi\left(\frac{\log\left(\bar{\omega}_t\right)+\sigma_{t-1}^2/2}{\sigma_{t-1}}\right)-\Phi\left(\frac{\log\left(\bar{\omega}_t\right)-\sigma_{t-1}^2/2}{\sigma_{t-1}}\right), \quad (20)$$

$$\tilde{\pi}\left(\bar{\omega}_t;\sigma_{t-1}\right)=\bar{\omega}_t\Phi\left(\frac{\log\left(\bar{\omega}_t\right)+\frac{\psi+\eta\sigma_{t-1}^2}{2}}{\sqrt{\eta}\sigma_{t-1}}\right)-e^{-\psi/2}\Phi\left(\frac{\log\left(\bar{\omega}_t\right)+\frac{\psi-\eta\sigma_{t-1}^2}{2}}{\sqrt{\eta}\sigma_{t-1}}\right). \quad (21)$$

The standard deviation of island-specific shocks to standard firms is assumed to follow an AR(1) process in logs, $\log\left(\sigma_t/\sigma\right)=\rho_\sigma\log\left(\sigma_{t-1}/\sigma\right)+\varepsilon_t^\sigma$, where $\varepsilon_t^\sigma\overset{iid}{\sim}N(0,\sigma_\sigma)$. In order to calibrate σ, we notice that the participation constraint (eq. 13) in the steady state implies $\pi\left(\bar{\omega};\sigma\right)=\bar{\omega}-\left(1-1/\phi\right)/\beta R^A=0.0006$. Using the steady-state counterpart of (20), we can then solve for $\sigma=0.0373$. In order to calibrate the parameters governing the dynamics of island-specific volatility $(\rho_\sigma,\sigma_\sigma)$, we use the TFP series for all 4-digit SIC manufacturing industries constructed by the NBER and the US Census Bureau's Center for Economic Studies (CES).[33] We construct a time series for σ_t by calculating the cross-sectional standard deviation of the industry-level TFP series (in log deviations from a linear trend) at each point in time. Fitting an autoregressive process to the log of the resulting series, we obtain $\rho_\sigma=0.9457$ and $\sigma_\sigma=0.0465$.[34]

Regarding the parameters of the substandard technology, ψ and η, we make use of the IC constraint in the steady state, $1-e^{-\psi/2}=\tilde{\pi}\left(\bar{\omega};\sigma\right)-\pi\left(\bar{\omega};\sigma\right)$,where $\tilde{\pi}\left(\bar{\omega};\sigma\right)$ is given by expression (21) in the steady state. We thus have one equation for two unknowns, ψ and η. We set ψ to 0.001 so as to replicate the midpoint of the range of standard deviations of equity in Table 2, and then use the IC constraint to solve for $\eta=1.2691$. The

[31]It can be showed that $F_t\left(\omega\right)=\tilde{F}_t\left(\omega\right)$ if and only if $\omega=\exp\left(\frac{\psi/\sqrt{\eta}+\left(\sqrt{\eta}-1\right)\sigma_t^2}{2\left(1-1/\sqrt{\eta}\right)}\right)\equiv\omega_t^*>0$. It can also be showed that $F_t'\left(\omega_t^*\right)/\tilde{F}_t'\left(\omega_t^*\right)=\sqrt{\eta}>1$. Since $F_t\left(\omega_t^*\right)=\tilde{F}_t\left(\omega_t^*\right)$ and $F_t'\left(\omega_t^*\right)>\tilde{F}_t'\left(\omega_t^*\right)$, it follows that F_t crosses $\tilde{F}_t$ once from below at $\omega=\omega_t^*$.

[32]The proof is available upon request.

[33]See Bloom (2009) for a study that uses the NBER-CES manufacturing industry database to construct a measure of time-varying industry-specific volatility.

[34]See data appendix for details.

standard deviation of (log)shocks to standard firms is thus $\sqrt{\eta} = 1.1$ times that of their substandard counterparts.

Finally, the exogenous bank continuation rate θ and the bank equity injection parameter τ are calibrated as follows. In the steady state, the law of motion of bank net worth (eq. 14) becomes

$$\frac{1}{\phi} = \theta R^A \int_{\bar{\omega}} (\omega - \bar{\omega})\, dF(\omega;\sigma) + \{1 - \theta [1 - F(\bar{\omega};\sigma)]\}\,\tau, \tag{22}$$

where we have normalized by A. Equation (22) implies that τ is a decreasing function of θ, given the other parameters and steady state values. In the choice of θ, we are restricted by the requirement that $\tau \geq 0$, which holds for $\theta \leq 0.84$. We notice that in equilibrium $1/(1-\theta)$ represents the average frequency of dividend payments by banks. We set θ to 0.75, such that banks pay dividends once a year on average. We then use (22) to solve for $\tau = 0.0207$.

4.2 The effects of TFP shocks

We follow the lead of the traditional RBC literature by exploring how well TFP shocks can explain the unconditional patterns found in the data. Table 4 displays the second-order moments of interests.[35] For comparison, we also show the range of moments across subsectors for the 1984-2011 sample (see Table 2), as the model is calibrated with data from this period.

As shown by the third column of Table 4, conditional on TFP shocks the model replicates fairly well the standard deviation of GDP. However, the model fails dramatically at reproducing the volatility of intermediary leverage and equity. It also fails to produce any meaningful procyclicality in the leverage ratio, or to capture the high negative correlation between leverage and equity.[36]

[35] Model moments are computed using a second-order accurate solution of the model. Results are very similar if we use instead a first-order model approximation.

[36] The relatively high correlation between leverage and assets conditional on TFP (0.49) is actually produced by the bandpass filtering of the simulated data. Indeed, the *unfiltered* simulated series yield a conditional leverage-asset correlation of 0.03. The leverage-GDP and leverage-equity conditional correlations based on unfiltered series (-0.14 and -0.12, respectively) are very close to those in Table 4, so the latter do reflect the model's inherent propagation mechanisms.

Table 4: Business cycle statistics: data and model

	Data	Model		
	1984-2011	Both shocks	TFP	Volatility
Standard deviations (%)				
GDP	1.03	1.08	1.00	0.38
Assets	1.30 : 7.57	0.58	0.36	0.45
Leverage	3.12 : 8.61	6.09	0.21	6.09
Equity	3.12 : 8.35	5.83	0.32	5.82
Correlations				
Leverage - Assets	0.21 : 0.76	0.49	0.49	0.63
Leverage - Equity	−0.91 : −0.35	−0.996	−0.12	−0.998
Leverage - GDP	−0.06 : 0.34	0.31	−0.10	0.87
Assets - GDP	0.41 : 0.73	0.41	0.41	0.59

Note: Model statistics are obtained by simulating the model for 55,000 periods and discarding the first 5,000 observations to eliminate the effect of initial conditions. The model is solved using a second-order perturbation method in levels. Both data and model-simulated series have been logged and detrended with a band-pass filter that preserves cycles of 6 to 32 quarters (lag length = 12 quarters).

To understand these results, Figure 4 displays the impulse responses to a one-standard-deviation fall in TFP (black dashed line).[37] On impact, the fall in TFP produces a fall in the return on banks' assets, which in turn reduces their equity. However, the leverage ratio barely reacts. Intuitively, TFP shocks barely affect banks' incentives to invest in the substandard as opposed to the standard firm segment, and have thus little effect on the leverage constraint imposed by investors. Since bank leverage remains stable, bank assets basically reproduce the response of bank net worth; that is, the effects of TFP shocks on bank credit operate mainly through the bank equity channel.

4.3 The effects of volatility shocks

In section 3, we analyzed the volatility-leverage channel from a theoretical, partial equilibrium perspective. Here we assess such a mechanism from a quantitative, general equilibrium point of view. With this purpose, we simulate the model conditional on shocks to cross-sectional volatility. The

[37] To compute the impulse responses, the model is solved by means of a first-order perturbation method in levels. Responses are shown in percentage deviations from steady state, unless otherwise indicated.

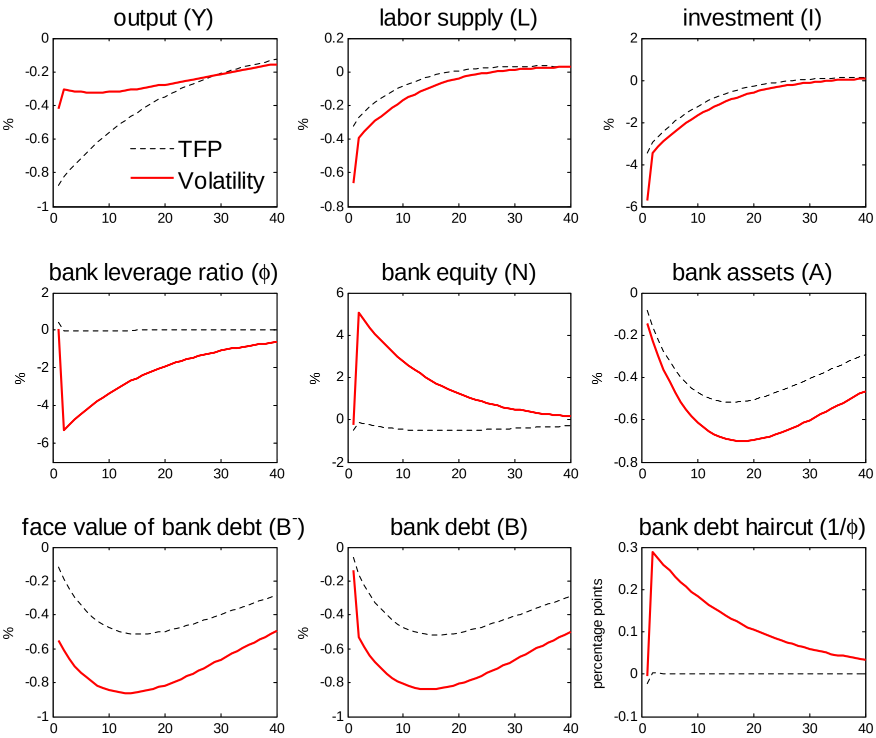

Figure 4: Impulse responses: TFP and volatility shock

results are shown in the fourth column of Table 4. Volatility shocks generate large fluctuations in the leverage ratio and equity of banks, comparable to those in the data. The fluctuations in output are relatively modest. In terms of correlations, volatility shocks produce a strong positive comovement between leverage and assets. It also generates a strong procyclicality in leverage with respect to GDP, which actually exceeds the upper bound in the empirical range.[38]

To understand these results, the solid line in Figure 4 displays the responses to a one-standard-deviation increase in cross-sectional volatility. The shock produces a drastic reduction in the leverage ratio of banks, which is mirrored by a symmetric increase in the haircuts imposed on bank debt. This results in a large fall in the amount of bank debt financed by investors, B_t. The associated reduction in debt repayments ($\bar{B}_t$) implies that bank net worth actually increases in subsequent periods. However, the drop in leverage dominates the increase in net worth, as evidenced by the fall in bank assets. This produces a contraction in the capital stock, investment and aggregate output.[39]

Finally, the second column in Table 4 shows the combined effects of both TFP and volatility shocks in the model. The existence of two uncorrelated sources of fluctuations reduces the correlation of leverage and GDP to a level within the empirical range. Regarding the standard deviations, the model underpredicts the volatility of bank assets, while capturing fairly well the size of fluctuations in bank leverage and equity.

4.4 The 2008-09 Recession in retrospect

So far we have studied the model's ability to replicate business cycle statistics in the postwar period. In this section we change our focus and ask the following question: how well can the model explain the evolution of financial intermediaries' balance sheets and GDP during the Great Recession of 2008-09?

[38] The conditional leverage-assets, leverage-equity, and leverage-GDP correlations based on unfiltered simulated series (0.76, -0.98, and 0.88, respectively) are very similar to those in Table 4.

[39] Aggregate output falls by less than in the case of TFP shocks, due to a smaller reduction in private consumption (not shown).

One implementation issue that arises in this exercise is the following. Our empirical proxy for the volatility process of island-specific shocks, the NBER-CES industry database, runs through 2005 only. We thus follow an alternative approach in order to infer the sequence of volatility shocks up until 2011. In particular, we use our model in order to filter out the sequence of both TFP and volatility shocks. We use as observables (i) the same TFP series that we used to calibrate the TFP process (Z_t), and (ii) a series of bank equity capital (N_t), both in log-deviations from a linear trend.[40] As regards the bank equity series, ideally one would like to have a consolidated equity capital series for the leveraged financial sector as a whole. However, as explained before, it is not possible to consolidate the balance sheets of the different leveraged subsectors, because the latter include cross positions that cannot be netted out. Thus, for the purpose of this exercise we choose the US-chartered commercial banking sector (by far the largest in the US in terms of balance sheet size) as an empirical counterpart for the bank balance sheet variables in the model (equity, leverage, and assets).[41] Since we are now using a different series to infer the volatility shocks, we estimate the parameters of the volatility process (ρ_σ, σ_σ) using Bayesian estimation; we also estimate the parameter θ, which is an important determinant of bank equity dynamics. For all three parameters we adopt priors centered around their calibrated values in Table 3. The sample period is again 1984:Q1-2011:Q3.

Table 5 reports the estimation results. The posterior means of the volatility process parameters are very close to their prior means. The posterior estimate of θ is sensibly lower than its prior mean. The reason is that in the model there is a positive relationship between θ and the standard deviation of bank equity.[42] Since the model is estimated with data on commercial banks' equity, and the latter has relatively low volatility (see Table 2), this leads to a relatively low posterior estimate for θ. It would be

[40] Since TFP is exogenous in the model, using only the TFP law of motion (as opposed to the full model) would yield exactly the same sequence of TFP shock innovations, ε_t^z.

[41] Accordingly, we also set the steady-state leverage ratio equal to its average value for US-chartered commecial banks (ϕ =10.3). The structural parameters consistent with this and the other targets, as well as the steady state values of other endogenous variables, are modified accordingly, following the procedure described in the calibration section.

[42] Results are available upon request.

Table 5: Results from Bayesian estimation

Param.	Prior dist.	Prior mean	Prior s.e.	Post. mean	Post. conf. interval
ρ_σ	Beta	0.9457	0.025	0.9275	0.8952 - 0.9613
σ_σ	Inv. gamma	0.0465	0.10	0.0451	0.0110 - 0.0829
θ	Beta	0.75	0.10	0.4024	0.3361 - 0.4703

Note: The Table reports results from Bayesian estimation. The observable variables are TFP (Z_t in model notation) and bank equity (N_t), both in log deviations from a linear trend. Bank equity refers to equity capital of US-chartered commercial banks, deflated by the GDP deflator. The sample period is 1984:Q1-2011:Q3. Statistics for the posterior distribution are obtained using a Metropolis-Hastings algorithm with two blocks of 100,000 iterations each.

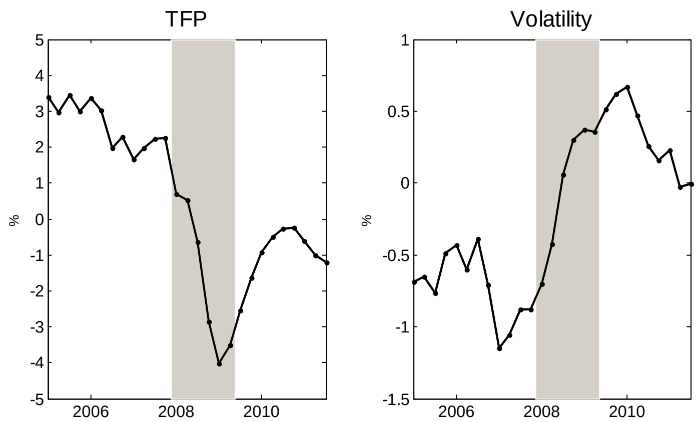

Figure 5: Smoothed series for the exogenous processes

natural to take this value as a lower bound.

Once the model has been estimated, we can use the Kalman smoothing algorithm to obtain the model-implied dynamics for real GDP (Y_t), bank leverage (ϕ_t), and bank assets (A_t), and then compare the latter with the actual historical series.[43] This provides an out-of-sample test for the model's ability to explain the observed dynamics in output and balance sheet aggregates.

The left panel of Figure 5 shows the historical series for TFP for the period 2005-2011, i.e. the years around the Great Recession. The first column of Figure 6 compares for the same period the historical GDP and

[43] The model-generated variables are in log deviations from the steady state, whereas the empirical series are in log deviations from a linear trend.

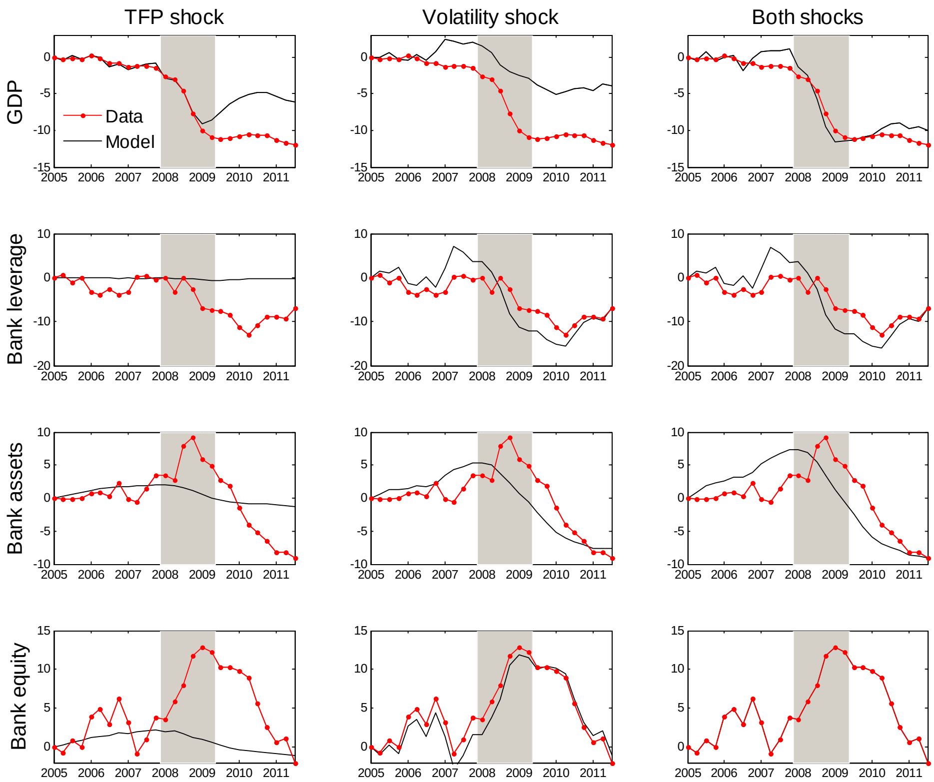

Figure 6: Historical and model-generated series for GDP and bank balance sheet variables

Note: data series are from US Flow of Funds and Bureau of Economic Analysis. See data appendix for details. All series are expressed as log changes relative to their 2005:Q1 values.

balance-sheet series with those generated by the model conditional on TFP shocks. All variables are expressed as log change relative to their 2005:Q1 value. The fall in TFP that started in 2008:Q1 explains part of the output fall during the recession, but it fails to replicate its depth and especially its duration. Moreover, the TFP shock is completely unable to explain the observed fall in leverage and total assets of the commercial banking sector during and after the recession.

The right panel of Figure 5 shows the smoothed series for the volatility process during 2005-2011. The increase in the latter during the 2008-9 recession is consistent with different measures of cross-sectional dispersion based on firm-level data (see e.g. Bloom et al. 2012, Christiano et al. 2014, and the references therein). The second column of Figure 6 performs the comparison between data and model-generated series conditional on volatility shocks. The sharp increase in volatility that starts at the beginning of 2008 explains part of the reduction in GDP during the Great Recession, as well as its prolonged duration. Furthermore, the rise in volatility explains well the contraction in bank leverage and assets that took place both during and after the recession. Once both shocks are combined (third column of Figure 6), we find that the model basically replicates the sharp and protracted fall in GDP, whereas it matches fairly well the actual dynamics in bank leverage and assets around the Great Recession period.

Summing up, the model is able to replicate the dynamics of both financial variables (banking assets and leverage) and real economic activity during the 2007-9 financial crisis through a combination of a fall in TFP and an increase in volatility. The latter is key to explain the sharp deleveraging and the contraction in banks' balance sheets, as well as the protracted duration of the recession.

4.5 The risk diversification paradox

The exercises presented above indicate that the model is able to roughly replicate the data in a number of dimensions. In particular, it can explain the observed intermediary leverage cycles as the result of exogenous changes in cross-sectional volatility. In this section, we use the model to analyze how different levels of *average* cross-sectional volatility may affect the macroeconomy. We may indeed consider a scenario in which financial innovation allows banks to better diversify their risks. In terms of the model, this amounts to a reduction in the steady-state volatility of island-specific shocks, σ. The question then is: what is the effect of this financial innovation both on the mean level *and* the volatility of output?

To answer this question, we study the behavior of the model as we lower σ from its baseline value in Table 3. For the purpose of this exercise, we simulate the model with both TFP and volatility shocks. Figure 7 displays the results.[44] The left panel displays the mean values of leverage (ϕ) and

[44]Model moments for each σ are computed by simulating 55,000 periods, discarding

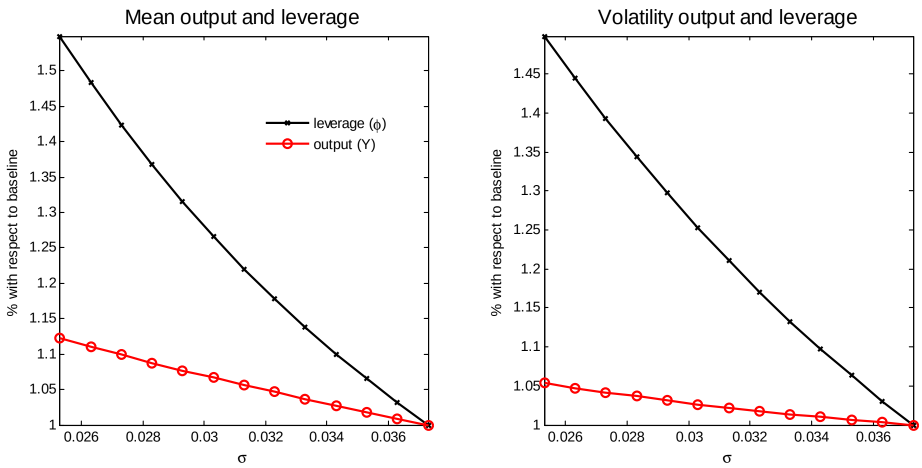

Figure 7: The effect of changes in steady-state cross-sectional volatility

output (Y), whereas the right panel displays their standard deviations. In this case the simulated series have not been filtered, as we need to preserve the means and we do not compare model results with filtered data.

As shown in the figure, a reduction in cross-sectional uncertainty allows banks to increase their leverage on average, through a mechanism very similar to the one explained before. For a given net worth, higher leverage allows banks to expand the size of their balance-sheets. This in turn leads to an increase in the stock of capital, and hence in the average level of output. Therefore, financial innovations that improve risk diversification induce an economic expansion on average via an increase in capital accumulation. This result is not controversial and has been confirmed by historical evidence, as discussed in Kindleberger (1986).

The effects on the volatilities are more striking. A reduction in cross-island volatility generates an *increase* in the volatility of output. For lack of a better name, we have named this effect 'the risk diversification' paradox, even though such a paradox is only apparent. A reduction in cross-island volatility increases the mean leverage of the banking sector, which in turn increases the size of fluctuations in leverage. The consequence is that a reduction in cross-island volatility leads to larger fluctuations in total intermediated assets. This in turn results in larger fluctuations in the capital

the first 5,000. The model is solved with a second-order perturbation method. Figure 7 shows moments normalized by their baseline value.

stock, and hence in aggregate output.[45]

The conclusion is that risk diversification has both a positive level effect on economic activity, and a negative effect through an increase in aggregate volatility, where the latter is due to higher mean leverage. The optimal size of risk diversification will depend on the degree risk aversion of the households, a point that we leave for further research.

5 Conclusions

We have presented a general equilibrium model with financial intermediation aimed at explaining the main features of the 'bank leverage cycle' in the US economy, characterized by large fluctuations in the leverage ratio of financial intermediaries and by a positive comovement between leverage, assets and GDP. Our main theoretical contribution is to introduce Adrian and Shin's (2013) model of intermediary leverage determination, based on a risk-shifting moral hazard problem, into a dynamic general equilibrium setting. Our results indicate that, unlike standard TFP shocks, volatility shocks generate volatile and procyclical bank leverage, thanks to a volatility-leverage channel in which bank default, limited liability and moral hazard play an important role. The model also replicates well the observed contractions in leverage, assets and GDP during the 2008-9 recession.

Consistently with most of the macro-finance literature (Bernanke, Gertler and Gilchrist, 1999; Kiyotaki and Moore, 1997; Gertler and Kiyotaki, 2010; Christiano, Motto and Rostagno, 2014, etc.), our model only considers short-term debt on the liabilities side of banks' balance sheets. In reality, financial intermediaries are also funded by long-term debt and other more stable sources of funding such as deposits. These debt contracts will be in general more "sticky" across the business cycle, thus reducing the procyclicality of leverage. This may also explain why leverage is more procyclical for intermediaries that rely heavily on short-term funding, such as security broker/dealers. The inclusion of long-term bank debt is therefore a model extension that is worth undertaking in further work.

Our analysis attributes a causal role to variations in cross-sectional

[45] The increase in unconditional output volatility holds also conditionally on TFP and volatility shocks. Results are available upon request.

volatility as a source of fluctuations in intermediary leverage and aggregate economic activity. It is in principle possible that such changes in cross-sectional volatility represent an endogenous response to alternative shocks; see the discussion on this issue in Christiano, Motto and Rostagno (2014) and the references therein. In this regard, Baker and Bloom (2011) provide some support for causal nature of volatility shocks.

Finally, this study has adopted a positive focus. We believe that understanding the effects of unconventional monetary policy interventions in this kind of framework may constitute an important topic for future research.

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Appendix for online publication

A. Data appendix

Data on equity capital and total assets of the four leveraged financial subsectors we consider (US-chartered commercial banks, savings institutions, security brokers and dealers, and finance companies) are from the Z.1 files of the US Flow of Funds.[46] The series corresponding to savings institutions are the sum of OTS and FDIC reporters. Data on *levels* in the Z.1 files (denoted by 'FL' in the series identifier) suffer from discontinuities that are caused by changes in the definition of the series. The Flow of Funds accounts correct for such changes by constructing *discontinuities* series (denoted by 'FD').[47] In particular, for each series the *flow* (denoted by 'FU') is equal to the change in level outstanding less any discontinuity. That is: $\mathrm{FU}_t = \mathrm{FL}_t$ - FL_{t-1} - FD_t. Therefore, the flow data are free from such discontinuities. In order to construct discontinuity-free level series, we take the value of the level in the first period of the sample and then accumulate the flows onwards.

For each subsector, the leverage ratio is the ratio between total assets and equity capital, both in dollars. In the tables and figures, 'assets' refer to real total assets, which are total assets (in dollars) divided by the GDP Implicit Price Deflator. The latter and Real GDP are both from the Bureau of Economic Analysis. Both series are readily available at the Federal Reserve Bank of St. Louis FRED database.[48]

In order to obtain an empirical proxy for aggregate log TFP, we use the quarterly change in the Business sector log TFP series (labelled 'dtfp') constructed by the Center for the Study of Income and Productivity (CSIP) at the Federal Reserve Bank of San Francisco.[49] We then accumulate the log changes to obtain the log level series.

Finally, in order to construct a proxy for island-specific volatility, we use the annual TFP series for all 4-digit SIC manufacturing industries constructed by the National Bureau of Economic Research (NBER) and the

[46] Website: http://www.federalreserve.gov/datadownload/Choose.aspx?rel=Z1

[47] For instance, changes to regulatory report forms and/or accounting rules typically trigger 'FD' entries for the affected series.

[48] Website: http://research.stlouisfed.org/fred2/

[49] Website: http://www.frbsf.org/csip/tfp.php

US Census Bureau's Center for Economic Studies (CES).[50] The data run through 2005, so our sample period in this case is 1984-2005. We discard those industries that exit the sample in the mid-nineties due to the change in industry classification from SIC to NAICS. We then log and linearly detrend each industry TFP series. Our proxy for the time series of (annual) island-specific volatility is the cross-sectional standard deviation of all industry TFP series in each year. We may denote the latter by σ_{τ}^{a}, where τ is the year subscript. Assuming that the underlying quarterly process is $\log \sigma_t = (1 - \rho_{\sigma}) \log \sigma + \rho_{\sigma} \log \sigma_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim iid\,(0, \sigma_{\sigma})$, and that each annual observation corresponds to the last quarter in the year, then the annual process satisfies $corr(\log \sigma_{\tau}^{a}, \log \sigma_{\tau-1}^{a}) = \rho_{\sigma}^{4}$, and $var\,(\log \sigma_{\tau}^{a}) = \frac{1+\rho_{\sigma}^{2}+\rho_{\sigma}^{4}+\rho_{\sigma}^{6}}{1-\rho_{\sigma}^{8}} \sigma_{\sigma}^{2}$. The sample autocorrelation and variance of $\log \sigma_{\tau}^{a}$ are 0.7997 and 0.0205, respectively, which imply $\rho_{\sigma} = 0.9457$ and $\sigma_{\sigma} = 0.0465$.

B. Proof of Lemma 1

In the text, we have defined $\pi_t(x) \equiv \int^{x} (x - \omega) dF_t\,(\omega)$. Using integration by parts, it is possible to show that

$$\pi_t(x) = \int^{x} F_t\,(\omega)\, d\omega.$$

We then have

$$\Delta\pi_t(x) \equiv \tilde{\pi}_t(x) - \pi_t(x) = \int^{x} \left(\tilde{F}_t\,(\omega) - F_t\,(\omega) \right) d\omega.$$

Notice first that $\Delta\pi_t(0) = 0$. We also have $\Delta\pi_t'(x) = \tilde{F}_t\,(x) - F_t\,(x)$. Thus, from Assumption 2, $\Delta\pi_t(x)$ is strictly increasing for $x \in (0, \omega_t^*)$, reaches a maximum at $x = \omega_t^*$, and then strictly decreases for $x > \omega_t^*$. Using integration by parts, it is also possible to show that

$$\begin{aligned} \lim_{x\to\infty} \Delta\pi_t(x) &= \int (1 - F_t\,(\omega))\, d\omega - \int \left(1 - \tilde{F}_t\,(\omega)\right) d\omega \\ &= \int \omega dF_t\,(\omega) - \int \omega d\tilde{F}_t\,(\omega) > 0, \end{aligned}$$

[50] Website: http://www.nber.org/data/nbprod2005.html

where the inequality follows from Assumption 1 in the main text. Therefore, for $x > \omega_t^*$ the function $\Delta\pi_t(x)$ decreases asymptotically towards $E(\omega) - E(\tilde{\omega})$. It follows that $\Delta\pi_t(x) > 0$ for all $x > 0$. It also follows that $\Delta\pi_t(x)$ cuts $E(\omega) - E(\tilde{\omega})$ precisely once and from below.

C. The bank's problem

We start by defining the ratio $\bar{b}_{t-1}^j \equiv \bar{B}_{t-1}^j / A_{t-1}^j$ and using the latter to substitute for $\bar{B}_{t-1}^j = \bar{b}_{t-1}^j A_{t-1}^j$. Given the choice of investment size A_t^j, the bank then chooses the ratio $\bar{b}_t^j$. With this transformation, and abusing somewhat the notation V_t and $\bar{V}_t$ in the main text, the bank's maximization problem can be expressed as

$$V_t\left(\omega, A_{t-1}^j, \bar{b}_{t-1}^j\right) = \max_{N_t^j} \left\{ \begin{array}{c} \left(\omega - \bar{b}_{t-1}^j / R_t^A\right) R_t^A A_{t-1}^j - N_t^j + \bar{V}_t\left(N_t^j\right) \\ +\mu_t^j \left[\left(\omega - \bar{b}_{t-1}^j / R_t^A\right) R_t^A A_{t-1}^j - N_t^j\right] \end{array} \right\}, \tag{23}$$

$$\bar{V}_t\left(N_t^j\right) = \max_{A_t^j, \bar{b}_t^j} E_t \Lambda_{t,t+1} \int_{\bar{b}_t^j / R_{t+1}^A} \left[\theta V_{t+1}\left(\omega, A_t^j, \bar{b}_t^j\right) + (1-\theta)\left(\omega - \bar{b}_t^j / R_{t+1}^A\right) R_{t+1}^A A_t^j\right] dF_t(\omega)$$

subject to the participation constraint,

$$E_t \Lambda_{t,t+1} R_{t+1}^A A_t^j \left\{ \int^{\bar{b}_t^j / R_{t+1}^A} \omega dF_t(\omega) + \frac{\bar{b}_t^j}{R_{t+1}^A} \left[1 - F_t\left(\frac{\bar{b}_t^j}{R_{t+1}^A}\right)\right] \right\} \geq A_t^j - N_t^j,$$

and the IC constraint

$$\begin{aligned} & E_t \Lambda_{t,t+1} \int_{\bar{b}_t^j / R_{t+1}^A} \left\{ \theta V_{t+1}\left(\omega, A_t^j, \bar{b}_t^j\right) + (1-\theta) R_{t+1}^A A_t^j \left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A}\right) \right\} dF_t(\omega) \\ \geq \; & E_t \Lambda_{t,t+1} \int_{\bar{b}_t^j / R_{t+1}^A} \left\{ \theta V_{t+1}\left(\omega, A_t^j, \bar{b}_t^j\right) + (1-\theta) R_{t+1}^A A_t^j \left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A}\right) \right\} d\tilde{F}_t(\omega). \end{aligned}$$

The first order condition with respect to N_t^j is given by

$$\mu_t^j = \bar{V}_t'\left(N_t^j\right) - 1.$$

We can now guess that $\bar{V}_t'(N_t^j) > 1$. Then $\mu_t^j > 0$ and the non-negativity constraint on dividends is binding, such that a continuing bank optimally decides to retain all earnings,

$$N_t^j = \left(\omega - \frac{\bar{b}_{t-1}^j}{R_t^A}\right) R_t^A A_{t-1}^j.$$

From (23), we then have $V_t(\omega, A_{t-1}^j, \bar{b}_{t-1}^j) = \bar{V}_t((\omega - \bar{b}_{t-1}^j/R_t^A)R_t^A A_{t-1}^j)$. Using the latter, we can express the Bellman equation for $\bar{V}_t(N_t^j)$ as

$$\begin{aligned}
\bar{V}_t(N_t^j) = & \max_{A_t^j, \bar{b}_t^j} \{ E_t \Lambda_{t,t+1} \int_{\bar{b}_t^j/R_{t+1}^A} \left[\theta \bar{V}_{t+1}\left(\left(\omega\text{-}\frac{\bar{b}_t^j}{R_{t+1}^A}\right) R_{t+1}^A A_t^j\right) + (1\text{-}\theta)\left(\omega\text{-}\frac{\bar{b}_t^j}{R_{t+1}^A}\right) R_{t+1}^A A_t^j\right] dF_t(\omega) \\
& + \lambda_t^j \left\{ E_t \Lambda_{t,t+1} R_{t+1}^A A_t^j \left[\int^{\bar{b}_t^j/R_{t+1}^A} \omega dF_t(\omega) + \frac{\bar{b}_t^j}{R_{t+1}^A}\left(1 - F_t\left(\frac{\bar{b}_t^j}{R_{t+1}^A}\right)\right)\right] - \left(A_t^j - N_t^j\right)\right\} \\
& + \xi_t^j E_t \Lambda_{t,t+1} \int_{\bar{b}_t^j/R_{t+1}^A} \left[\theta \bar{V}_{t+1}\left(\left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A}\right) R_{t+1}^A A_t^j\right) + (1-\theta) R_{t+1}^A A_t^j \left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A}\right)\right] dF_t(\omega) \\
& - \xi_t^j E_t \Lambda_{t,t+1} \int_{\bar{b}_t^j/R_{t+1}^A} \left[\theta \bar{V}_{t+1}\left(\left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A}\right) R_{t+1}^A A_t^j\right) + (1-\theta) R_{t+1}^A A_t^j \left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A}\right)\right] d\tilde{F}_t(\omega) \},
\end{aligned}$$

where λ_t^j and ξ_t^j are the Lagrange multipliers associated to the participation and IC constraints, respectively. The first order conditions with respect to A_t^j and $\bar{b}_t^j$ are given by

$$\begin{aligned}
0 = \; & E_t \Lambda_{t,t+1} R_{t+1}^A \int_{\bar{\omega}_{t+1}^j} \left[\theta \bar{V}_{t+1}'\left(N_{t+1}^j\right) + 1 - \theta\right] \left(\omega - \bar{\omega}_{t+1}^j\right) dF_t(\omega) \\
& + \lambda_t^j \left\{ E_t \Lambda_{t,t+1} R_{t+1}^A \left[\int^{\bar{\omega}_{t+1}^j} \omega dF_t(\omega) + \bar{\omega}_{t+1}^j \left[1 - F_t\left(\bar{\omega}_{t+1}^j\right)\right]\right] - 1 \right\} \\
& + \xi_t^j E_t \Lambda_{t,t+1} R_{t+1}^A \int_{\bar{\omega}_{t+1}^j} \left\{\theta \bar{V}_{t+1}'\left(N_{t+1}^j\right) + 1 - \theta\right\} \left(\omega - \bar{\omega}_{t+1}^j\right) dF_t(\omega) \\
& - \xi_t^j E_t \Lambda_{t,t+1} R_{t+1}^A \int_{\bar{\omega}_{t+1}^j} \left\{\theta \bar{V}_{t+1}'\left(N_{t+1}^j\right) + 1 - \theta\right\} \left(\omega - \bar{\omega}_{t+1}^j\right) d\tilde{F}_t(\omega),
\end{aligned}$$

$$
\begin{aligned}
0 \;=\; & -E_t\Lambda_{t,t+1}\int_{\bar{\omega}^j_{t+1}}\left[\theta\bar{V}'_{t+1}\left(N^j_{t+1}\right)+(1-\theta)\right]dF_t\left(\omega\right)-E_t\Lambda_{t,t+1}\theta\frac{\bar{V}_{t+1}\left(0\right)}{R^A_{t+1}A^j_t}f_t\left(\bar{\omega}^j_{t+1}\right)\\
& +\lambda^j_tE_t\Lambda_{t,t+1}\left[1-F_t\left(\bar{\omega}^j_{t+1}\right)\right]\\
& -\xi^j_tE_t\Lambda_{t,t+1}\int_{\bar{\omega}^j_{t+1}}\left\{\theta\bar{V}'_{t+1}\left(N^j_{t+1}\right)+(1-\theta)\right\}dF_t\left(\omega\right)-\xi^j_tE_t\Lambda_{t,t+1}\theta\frac{\bar{V}_{t+1}\left(0\right)}{R^A_{t+1}A^j_t}f_t\left(\bar{\omega}^j_{t+1}\right)\\
& +\xi^j_tE_t\Lambda_{t,t+1}\int_{\bar{\omega}^j_{t+1}}\left\{\theta\bar{V}'_{t+1}\left(N^j_{t+1}\right)+(1-\theta)\right\}d\tilde{F}_t\left(\omega\right)+\xi^j_tE_t\Lambda_{t,t+1}\theta\frac{\bar{V}_{t+1}\left(0\right)}{R^A_{t+1}A^j_t}\tilde{f}_t\left(\bar{\omega}^j_{t+1}\right),
\end{aligned}
$$

respectively, where we have used $\bar{b}^j_t/R^A_{t+1}=\bar{\omega}^j_{t+1}$. We also have the envelope condition

$$\bar{V}'_t\left(N^j_t\right)=\lambda^j_t.$$

At this point, we guess that in equilibrium $\bar{V}_t(N^j_t)=\lambda^j_tN^j_t$, and that the multipliers λ^j_t and ξ^j_t are equalized across islands: $\lambda^j_t=\lambda_t$ and $\xi^j_t=\xi_t$ for all j. Using this, the IC constraint simplifies to

$$E_t\Lambda_{t,t+1}R^A_{t+1}\left\{\theta\lambda_{t+1}+(1-\theta)\right\}\left[\int_{\bar{\omega}^j_{t+1}}\left(\omega-\bar{\omega}^j_{t+1}\right)dF_t\left(\omega\right)-\int_{\bar{\omega}^j_{t+1}}\left(\omega-\bar{\omega}^j_{t+1}\right)d\tilde{F}_t\left(\omega\right)\right]\geq 0. \tag{24}$$

The first order conditions then become

$$
\begin{aligned}
0 \;=\; & E_t\Lambda_{t,t+1}R^A_{t+1}\left[\theta\lambda_{t+1}+1-\theta\right]\int_{\bar{\omega}^j_{t+1}}\left(\omega-\bar{\omega}^j_{t+1}\right)dF_t\left(\omega\right) \qquad (25)\\
& +\lambda_t\left\{E_t\Lambda_{t,t+1}R^A_{t+1}\left[\int^{\bar{\omega}^j_{t+1}}\omega dF_t\left(\omega\right)+\bar{\omega}^j_{t+1}\left[1-F_t\left(\bar{\omega}^j_{t+1}\right)\right]\right]-1\right\},
\end{aligned}
$$

$$
\begin{aligned}
0 \;=\; & \lambda_tE_t\Lambda_{t,t+1}\left[1-F_t\left(\bar{\omega}^j_{t+1}\right)\right]-E_t\Lambda_{t,t+1}\left[\theta\lambda_{t+1}+1-\theta\right]\left[1-F_t\left(\bar{\omega}^j_{t+1}\right)\right]\\
& +\xi_tE_t\Lambda_{t,t+1}\left\{\theta\lambda_{t+1}+1-\theta\right\}\left[F_t\left(\bar{\omega}^j_{t+1}\right)-\tilde{F}_t\left(\bar{\omega}^j_{t+1}\right)\right], \qquad (26)
\end{aligned}
$$

where in (25) we have used the fact that ξ^j_t times the left-hand side of (24) must be zero as required by the Kuhn-Tucker conditions, and in (26) we have used the fact that, according to our guess, $\bar{V}_{t+1}\left(0\right)=0$. Solving for the Lagrange multipliers, we obtain

$$\lambda_t=\frac{E_t\Lambda_{t,t+1}R^A_{t+1}\left[\theta\lambda_{t+1}+1-\theta\right]\int_{\bar{\omega}^j_{t+1}}\left(\omega-\bar{\omega}^j_{t+1}\right)dF_t\left(\omega\right)}{1-E_t\Lambda_{t,t+1}R^A_{t+1}\left[\int^{\bar{\omega}^j_{t+1}}\omega dF_t\left(\omega\right)+\bar{\omega}^j_{t+1}\left[1-F_t\left(\bar{\omega}^j_{t+1}\right)\right]\right]}, \tag{27}$$

$$\xi_t = \frac{\lambda_t E_t \Lambda_{t,t+1}\left[1 - F_t\left(\bar{\omega}^j_{t+1}\right)\right] - E_t \Lambda_{t,t+1}\left[\theta\lambda_{t+1} + 1 - \theta\right]\left[1 - F_t\left(\bar{\omega}^j_{t+1}\right)\right]}{E_t \Lambda_{t,t+1}\left\{\theta\lambda_{t+1} + 1 - \theta\right\}\left[\tilde{F}_t\left(\bar{\omega}^j_{t+1}\right) - F_t\left(\bar{\omega}^j_{t+1}\right)\right]}. \tag{28}$$

In the steady state, the Lagrange multipliers are

$$\lambda = \frac{\beta R^A (1-\theta)\int_{\bar{\omega}^j}\left(\omega - \bar{\omega}^j\right) dF(\omega)}{1 - \beta R^A + (1-\theta)\beta R^A \int_{\bar{\omega}^j}\left(\omega - \bar{\omega}^j\right) dF(\omega)},$$

$$\xi = \frac{(\lambda - 1)(1-\theta)}{\theta\lambda + 1 - \theta}\frac{\left[1 - F\left(\bar{\omega}^j\right)\right]}{\tilde{F}\left(\bar{\omega}^j\right) - F\left(\bar{\omega}^j\right)},$$

where we have used $\int\left(\omega - \bar{\omega}^j\right) dF(\omega) = 1 - \bar{\omega}^j$. Provided the parameter values are such that

$$0 < \beta R^A - 1 < (1-\theta)\beta R^A \int_{\bar{\omega}^j}\left(\omega - \bar{\omega}^j\right) dF(\omega),$$

then $\lambda > 1$. Also, notice that for $\bar{\omega}^j \geq \omega^*$ we have $\Delta\pi\left(\bar{\omega}^j\right) > E(\omega) - E(\tilde{\omega})$ and thus the IC constraint is violated in the steady state.[51] Therefore, in equilibrium it must be the case that $\bar{\omega}^j < \omega^*$. But from Assumption 2 in the main text this implies $\tilde{F}\left(\bar{\omega}^j\right) > F\left(\bar{\omega}^j\right)$ which, together with $\lambda > 1$, implies in turn $\xi > 0$. That is, both the participation and IC constraints hold in the steady state.[52] Provided aggregate shocks are sufficiently small, we will also have $\lambda_t > 1$, $\xi_t > 0$ and $\bar{\omega}^j_t < \omega^*_t$ along the cycle. But if $\lambda_t > 1$, then our guess that $\bar{V}'_t(N^j_t) > 1$ is verified. Also, given that $\bar{\omega}^j_{t+1} = \bar{b}^j_t / R_{t+1}$, the ratio $\bar{b}^j_t$ is then pinned down by the IC constraint (equation 24) holding with equality. Since we have guessed that the multiplier λ_t is equalized across islands, so are $\bar{b}^j_t = \bar{b}_t$ and $\bar{\omega}^j_{t+1} = \bar{\omega}_{t+1} = \bar{b}_t / R_{t+1}$. But if $\bar{\omega}_{t+1}$ is equalized, then from (27) and (28) our guess that λ_t and ξ_t are symmetric across islands is verified too.

The participation constraint (holding with equality) is given by

$$E_t \Lambda_{t,t+1} R^A_{t+1} A^j_t \left\{\int^{\bar{\omega}_{t+1}} \omega dF_t(\omega) + \bar{\omega}_{t+1}\left[1 - F_t\left(\bar{\omega}_{t+1}\right)\right]\right\} = A^j_t - N^j_t.$$

[51] As shown in Appendix B, $\Delta\pi(x)$ increases initially, reaches a maximum at $x = \omega^*$ and then decreases asymptotically towards $E(\omega) - E(\tilde{\omega})$. This implies that $\Delta\pi(x) > E(\omega) - E(\tilde{\omega})$ for $x \geq \omega^*$.

[52] Our calibration in Table 3 implies $\lambda = 2.5528$ and $\xi = 7.6371$.

Using the latter to solve for A_t^j, we obtain

$$A_t^j = \frac{1}{1 - E_t \Lambda_{t,t+1} R_{t+1}^A \left\{ \bar{\omega}_{t+1} - \pi_{t+1} \left(\bar{\omega}_{t+1} \right) \right\}} N_t^j \equiv \phi_t N_t^j,$$

where we have also used the definition of the put option value, $\pi_t \left(\bar{\omega}_{t+1} \right) = \int^{\bar{\omega}_{t+1}} \left(\bar{\omega}_{t+1} - \omega \right) dF_t \left(\omega \right)$. Therefore, the leverage ratio $A_t^j / N_t^j = \phi_t$ is equalized across firms too. Finally, using $\bar{V}_{t+1}(N_{t+1}^j) = \lambda_{t+1} N_{t+1}^j$, $N_{t+1}^j = \left(\omega - \bar{\omega}_{t+1} \right) R_{t+1}^A A_t^j$ and $A_t^j = \phi_t N_t^j$, the value function $\bar{V}_t \left(N_t^j \right)$ can be expressed as

$$\bar{V}_t \left(N_t^j \right) = \phi_t N_t^j E_t \Lambda_{t,t+1} R_{t+1}^A \left[\theta \lambda_{t+1} + 1 - \theta \right] \int_{\bar{\omega}_{t+1}} \left(\omega - \bar{\omega}_{t+1} \right) dF_t \left(\omega \right),$$

which is consistent with our guess that $\bar{V}_t(N_t^j) = \lambda_t N_t^j$ only if

$$\begin{aligned} \lambda_t &= \phi_t E_t \Lambda_{t,t+1} R_{t+1}^A \left[\theta \lambda_{t+1} + 1 - \theta \right] \int_{\bar{\omega}_{t+1}} \left(\omega - \bar{\omega}_{t+1} \right) dF_t \left(\omega \right) \\ &= \frac{E_t \Lambda_{t,t+1} R_{t+1}^A \left[\theta \lambda_{t+1} + 1 - \theta \right] \left\{ 1 - \bar{\omega}_{t+1} + \pi_t \left(\bar{\omega}_{t+1} \right) \right\}}{1 - E_t \Lambda_{t,t+1} R_{t+1}^A \left\{ \bar{\omega}_{t+1} - \pi_t \left(\bar{\omega}_{t+1} \right) \right\}}. \end{aligned}$$

But the latter corresponds exactly with (27) without j subscripts, once we use the definition of $\pi_t \left(\bar{\omega}_{t+1} \right)$. Our guess is therefore verified.

D. Model summary and comparison to standard RBC model

Our model can be reduced to the following 11-equation system,

$$\frac{v'(L_t)}{u'(C_t)} = \left(1 - \alpha \right) \frac{Y_t}{L_t}, \tag{S1}$$

$$Y_t = Z_t L_t^{1-\alpha} K_t^{\alpha}, \tag{S2}$$

$$K_{t+1} = I_t + \left(1 - \delta \right) K_t, \tag{S3}$$

$$Y_t = C_t + I_t \tag{S4}$$

$$R_t^A = \left(1 - \delta \right) + \alpha \frac{Y_t}{K_t}, \tag{S5}$$

$$1=\beta E_t\left\{\frac{u'(C_{t+1})}{u'(C_t)}R^A_{t+1}\left[\bar{\omega}_{t+1}-\pi\left(\bar{\omega}_{t+1};\sigma_t\right)\right]\frac{\phi_t}{\phi_t-1}\right\}, \tag{S6}$$

$$1-\int\omega d\tilde{F}_t\left(\omega\right)=E_t\left\{\frac{u'(C_{t+1})R^A_{t+1}\left(\theta\lambda_{t+1}+1-\theta\right)}{E_t u'(C_{t+1})R^A_{t+1}\left(\theta\lambda_{t+1}+1-\theta\right)}\left[\tilde{\pi}_t\left(\frac{\bar{b}_t}{R^A_{t+1}}\right)-\pi_t\left(\frac{\bar{b}_t}{R^A_{t+1}}\right)\right]\right\}, \tag{S7}$$

$$K_{t+1}=\phi_t N_t, \tag{S8}$$

$$\bar{\omega}_t=\bar{b}_{t-1}/R^A_t, \tag{S9}$$

$$N_t=\theta R^A_t\left[1-\bar{\omega}_t+\pi_{t-1}\left(\bar{\omega}_t\right)\right]K_t+\left\{1-\theta\left[1-F_{t-1}\left(\bar{\omega}_t\right)\right]\right\}\tau K_t, \tag{S10}$$

$$\lambda_t=\frac{E_t\beta u'(C_{t+1})R^A_{t+1}\left[\theta\lambda_{t+1}+1-\theta\right]\left\{1-\bar{\omega}_{t+1}+\pi_t\left(\bar{\omega}_{t+1}\right)\right\}}{u'(C_t)-E_t\beta u'(C_{t+1})R^A_{t+1}\left\{\bar{\omega}_{t+1}-\pi_t\left(\bar{\omega}_{t+1}\right)\right\}}, \tag{S11}$$

which jointly determine the dynamics of 11 endogenous variables: C_t, L_t, K_t, I_t, Y_t, R^A_t, N_t, $\bar{\omega}_t$, $\bar{b}_t$, λ_t, ϕ_t.

The standard RBC model is given by equations (S1) to (S5), *plus* the following investment Euler equation,

$$1=\beta E_t\left\{\frac{u'(C_{t+1})}{u'(C_t)}R^A_{t+1}\right\}, \tag{S6'}$$

which jointly determine the path of 6 endogenous variables: C_t, L_t, K_t, I_t, Y_t, R^A_t.