A Guide to FRB/US

A Macroeconomic Model of the United States

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Abstract: FRB/US is a large-scale quarterly econometric model of the U.S. economy, developed to replace the MPS model. Most behavioral equations are based on specifications of optimizing behavior containing explicit expectations of firms, households, and financial markets. Although expectations are explicit, the empirical fits of the structural descriptions of macroeconomic behavior are comparable to those of reduced-form time series models. In most instances, tests do not reject overidentifying restrictions of rational expectations or the hypothesis of serially independent residuals. As modeled, private sector expectations of policy constitute a major transmission channel of monetary policy.

Keywords: Macroeconomic models, private sector learning, rational expectations, vector autoregressions.

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Contents

1 Introduction and Overview

FRB/US is a new quarterly econometric model of the U.S. economy developed to replace the MPS model. The new model has three distinctive features. First, the expectations of private sectors are explicit, and these expectations, especially market perceptions of policy, constitute a major transmission channel of monetary policy. Second, information assumed to be accessed by private sectors can vary in scope and timing, and can include “perfect foresight” or learning from past observations. Third, although expectations are explicit, the empirical fits of the structural descriptions of macroeconomic behavior are comparable to those of reduced-form time series models.

In contrast to the inability of MPS to disentangle forecast and response lags, the structure of FRB/US parses dynamics into contributions of expectations and adjustment frictions. This decomposition enables the new model to provide sharper interpretations of macroeconomic developments and to examine the sensitivity of forecasts and policy scenarios to a range of assumptions about how sectoral expectations are formed. However, the more explicit theoretical structure has been achieved while also incorporating newer statistical techniques aimed at improving the goodness of fit and reliability of empirical estimates.

Despite changes in structure and updated empirical estimates, FRB/US retains several notable characteristics of MPS. One important similarity is the blend of long-run neoclassical conditions for equilibrium with short-run sticky-price disequilibria where monetary policy actions have significant short-run effects on the level of real activity.

FRB/US is a large-scale model, containing some 300 equations and identities. However, the number of stochastic “core” equations or estimated descriptions of the economic behavior of firms, households, and investors is much smaller, around 50 equations. In the current edition of FRB/US (version 1.0), about half of these behavioral equations are based on formal specifications of optimizing behavior containing explicit estimates of the forward expectations of firms and households. Most of the discussion in this guide will focus on this subset of core equations because the format of these structural equations is new. Although the explicit expectations format may be extended to additional core equations in future model editions, most key macroeconomic relationships in version 1.0 of FRB/US now include explicit expectations. These include the estimated structural equations for aggregate consumption, two components of consumer durables, residential construction, investment in producers' durable equipment, inventory investment, labor hours, measures of aggregate price and labor cost, three long-term interest rates, and the value of corporate equity.

1.1 Objectives

In the 25 years since the initial version of the MPS model was completed and brought into operational use at the Board, the practice of macroeconomics has evolved substantially. Notable developments include: greater emphasis on modeling expectations, especially the assumption that expectations are *rational* or consistent with modeled outcomes; more extensive use of dynamic optimization theory to characterize responses of households and firms to shocks; expansion of the types of models in general use to include atheoretic vector autoregressions (VARs) and theoretically-based, general equilibrium models of business cycles; and development of new statistical techniques for estimation of long-run relationships among trending series and for testing facets of equation performance.

Although the design of FRB/US draws on many of these developments, other adaptations and new concepts were required in the construction of the model in order to meet a list of goals. The main objectives that guided the development of FRB/US are:

1. **Use:** (a) The model should be suitable for both forecasting and policy simulations. (b) It should also be able to run simulations of policy and other scenarios under a variety of assumptions about how households, firms and financial markets form expectations, including the extent of available information.

2. **Conceptual design:** (a) Expectations should be explicit. (b) Structural equations for households, firms, and financial markets should be based on economic theories of optimizing behavior.

3. **Statistical implementation:** (a) Estimation of equations in the model should be based on modern time series techniques. (b) Equations should have satisfactory statistical properties, including goodness of historical fit.

4. **Simulation properties:** For shocks that are not unusual in historical perspective, the model's simulated responses should be close to those obtained from atheoretic models that do not impose strong priors, such as VARs. The model should also be able to match established rules of thumb regarding economic relationships under appropriate circumstances.

Some of these objectives are complementary: Explicit representation of expectations, 2a, assists in identifying the contributions of alternative expectations, 1b. However, there are well-known conflicts between the objective for more theoretical rigor, 2b, and the objectives related to forecasting, 1a, and improved statistical properties, 3b. Indeed, major innovations in the design of FRB/US have been motivated to improve the tradeoff between theoretical and empirical properties.

1.2 Implementation

Key equations in FRB/US are based on four fundamental building blocks:

- arbitrage equilibria
- equilibrium planning
- dynamic adjustments
- forecasting (expectations formation)

The way these elements are combined in key equations varies. Equations for financial variables are based on *arbitrage equilibria*. For example, arbitrage equates the rate of return on a bond to a weighted average of expected future values of a short-term interest rate plus a term premium. Because transactions costs are relatively small in financial markets, arbitrage is assumed within each quarter. In contrast, nonfinancial variables are costly to adjust and, thus, move only gradually to eliminate any disequilibria. Equations for nonfinancial variables are based on *equilibrium plans*—values that would be desired in the absence of adjustment frictions—and *dynamic adjustments*. The latter employ a general model of dynamic frictions to estimate the optimal rate at which deviations from equilibria are eliminated. The approach to optimal dynamic adjustments used in FRB/US is a generalization of a costly adjustment approach commonly used in applied macroeconomics.

The main price equation in FRB/US is convenient for illustrating the structure of nonfinancial equations. The equilibrium condition is derived from profit maximization and makes the planned price level a markup over unit factor costs. A timeseries (cointegration) regression is used to estimate the long-run weights on unit labor and energy costs. Once the equilibrium relationship is specified, a generalized frictions model is estimated for price dynamics, including the degree of cyclical variation of the markup. There are several ways to represent price adjustments, as discussed in section 2, but for now it is sufficient to indicate that they depend on expectational terms (future values of the equilibrium price level) and inertial terms (lags of the actual price level).

Equations for financial and nonfinancial variables alike contain expectations of the private sectors. As indicated above, expectations enter financial equations through the definition of *arbitrage equilibria*. For nonfinancial equations, expectations may enter two ways. First, components of the *equilibrium plans* of some equations are present values. For example, the desired level of consumption is derived from the lifecycle model and, thus, depends on the current value of tangible assets and current and expected future values of household income. Second, the *dynamic adjustment* component of each nonfinancial equation depends on expectations because households and firms aim, not at the current equilibrium value, but at the trajectory that the equilibrium is expected to follow.

The last building block, *forecasting*, describes how sectoral expectations are formed. Here, it is useful to distinguish between the assumption made about expectations when FRB/US was estimated, and the various options for specifying expectations when the model is simulated.

For estimation purposes, sectoral expectations were derived from forecasts of small VARs. Although the structure of the VARs varies across equations, the VARs have a similar organization. Each contains a common set of variables—consumption price inflation, output gap, and the federal funds rate—along with one or more sector-specific variables. Because each VAR contains an equation for the federal funds rate, this form of expectations incorporates an average sample view of how monetary policy was conducted historically.

For simulation purposes, several options for expectations are currently available. One is the approach of *VAR expectations* just described. Under a second option, expectations are equal to forecasts from the FRB/US model—an option labeled *full-model expectations*. Both types of expectations can be viewed as reflecting rational behavior, but under different assumptions about the *scope* of information available to individuals. Another dimension of expectations along which FRB/US has flexible capabilities is the *speed* with which individuals learn about changes in the economic environment. The specific application developed so far pertains to how quickly the private sector catches on to changes in the long-run inflation objective of monetary policy—an issue closely related to the topic of policy credibility.

1.3 Characteristics of equations

The economic behavior described by FRB/US can be summarized most easily by focusing on three main sectors:

- **Households** choose equilibrium aggregate consumption based on the lifecycle model, an approach motivated by utility maximization, but are assumed to be quite risk averse and, thus, to discount the future heavily in computing expected income. The dynamic equation for aggregate consumption contains sluggish adjustment of actual consumption toward its equilibrium as well as modest effects of liquidity constraints. Investment in consumer durables and residential construction varies with aggregate consumption as well as with real interest rates and relative prices.

- **Firms** choose investment, inventories, labor hours, and prices based on profit maximization under imperfect competition. Firms also are involved, along with households, in the short-run determination of wages. Adjustment dynamics are estimated to be most rapid for inventories and labor hours and slowest for wages and investment in producers' durable equipment. The speed of adjustment of the aggregate price is intermediate. In addition to its sensitivity to the cost of capital, investment in producers' durable equipment is modestly sensitive to cash flow.

- **Financial markets** set bond rates, stock prices, and the exchange rate by standard arbitrage conditions. Thus, bond yields depend on values of short-term interest rates expected to prevail over the maturity of the bond, and the stock market valuation depends on expected dividends. Term premiums in the bond equations vary countercyclically; the risk premium in the equity market is modeled as a constant. All asset price equations have significant serial correlation in their residuals, which is interpreted as an additional time-varying component of term or risk premiums.

FRB/US also contains "traditional" equations—without explicit expectations—for imports, exports, nonresidential construction, employment, labor force participation, and the relative price of consumption.

With regard to interactions among sectors, an important set of linkages describes the *transmission channels of monetary policy.* As in MPS, key transmission channels operate through medium- and long-term interest rates directly in equations for investment in producers' durable equipment, residential construction, and consumer durables, and indirectly through effects of the value of the stock market on aggregate consumption and effects of the exchange rate on exports and imports. As indicated above, asset prices and bond rates are directly linked to expectations of future federal funds rates which, in turn, are perceived to reflect policy responses to macroeconomic indicators. In addition to forward funds rates, the transmission of monetary policy through sectoral expectations is extended in FRB/US to private sector forecasts of future equilibrium values. For example, under either VAR or full-model expectations, an increase in the current funds rate lowers expected future output and income, thereby restraining current investment and consumption.[1]

1.4 Simulation capabilities and properties

FRB/US currently can be simulated with either VAR or full-model expectations, alternatives that vary the *scope* of information available to the private sector. Although a common view is that economic models have "stark" properties under the assumption of full-model expectations, because of the extensive amount of information provided individuals under this assumption, this

[1]Note that the additional policy transmission channels identified in FRB/US do not necessarily imply larger policy "multipliers" but may reflect only the explicit decomposition of estimated responses among lags due to frictions and lags stemming from the formation of expectations, including learning.

characterization is not generally correct for FRB/US. The model contains significant adjustment frictions that slow responses of nonfinancial variables, even to anticipated events. Also, properties of FRB/US under full-model expectations can be similar to those under VAR expectations, if the shock or change being simulated is not unusual in an historical context. One example is a transitory change in the federal funds rate. Under either VAR or full-model expectations, output moves for a period of time in the opposite direction of the interest rate change, as does inflation, and long-term interest rates change by a fraction of the movement in the funds rate. In this instance, the VAR contains the essential macroeconomic responses so VAR expectations are similar to full-model expectations.

In contrast, unusual shifts can yield quite different outcomes under VAR and full-model expectations—an example is a future change in fiscal policy that is perfectly anticipated under full-model expectations but recognized only as it occurs under VAR expectations. In this instance, macroeconomic variables move in advance of the fiscal change under full-model expectations, but only after the policy change under VAR expectations.

A second dimension of FRB/US simulations that pertains to expectations is the *speed* at which the private sector learns about changes in policy objectives, such as a shift in monetary policy that seeks to reduce the rate of inflation. The private sector's perception of the inflation objective—as distinct from the actual policy objective—is included explicitly in the structure of FRB/US. Thus, consequences of a disinflationary policy can be compared under different assumptions about the credibility of the shift. Under full credibility, perceptions of the inflation objective respond immediately and the output cost of reducing inflation is relatively small. Alternatively, if perceptions of the inflation target adjust more slowly—and only after policy actions to achieve a lower rate of inflation are instigated—the output cost of reducing inflation is higher.

1.5 Organization of remaining sections

Four sections follow. Section 2 presents the specifications of economic behavior in FRB/US, using a bond rate equation to illustrate the structure of the typical financial equation and the aggregate price equation to portray the specification of adjustment dynamics in nonfinancial equations. Section 3 reviews how these approaches have been applied to core behavioral equations. Expectations are explicit in the structure of FRB/US and important to its properties. Section 4 discusses options for expectations that are currently used in simulations of FRB/US; develops the concept of long-run expectations, which are termed expectation *endpoints*; and presents the core equations of the vector autoregression used for VAR-based expectations. Finally, section 5 presents simulations of FRB/US to illustrate its responses to key types of shocks under different assumptions about how expectations are formed.

2 Types of Economic Behavior

This section introduces the types of equations used to describe economic behavior in FRB/US. Discussion of each equation type includes a brief summary of assumptions about economic behavior that motivate the equation format; significant differences, if any, with specifications in alternative macro models; and an example to illustrate how the equation is referenced in later sections. There are only four standard types of equations, each describing a different activity by individuals:

- *Arbitrage equilibria.* This category includes equations for bond yields and the price of equity, based on standard formulations of efficient market pricing. Under the assumption that arbitrage profits are zero, expected returns on assets traded in financial markets are related to an expected baseline rate of return and term or risk premiums.
- *Equilibrium planning.* Variables not determined in financial markets are controlled by individuals in a particular sector, such as consumption by households or fixed capital investment by firms. Each sector selects equilibrium values for its own activities that would be undertaken in the absence of frictions. These equilibrium settings are functions of variables not controlled by individuals within the sector. Examples of predetermined explanatory variables are expected income for household consumption and sales for business capital investment. Equilibrium equations are essentially steady-state relationships and comprise the set of equations most likely to be subject to functional and coefficient restrictions from economic theory.[2]
- *Dynamic adjustments.* Unlike financial market behavior, where asset prices are assumed to reflect equilibrium valuations, activities in remaining sectors are subject to a variety of dynamic frictions. A shift in an equilibrium setting for a variable controlled by a sector will initiate a response to reach the new desired value. However, when responses are constrained, the optimal approach to the revised equilibrium may be spread over many quarters. The format of dynamic adjustment equations in FRB/US is dissimilar to that in other macroeconomic models of rational behavior in that the extent of dynamic frictions is not imposed by priors but determined by statistical testing; consequently, the empirical goodness of fit is comparable to those of atheoretic time series models.
- *Forecasting.* Structural equations in FRB/US require forecasts of explanatory variables. In the case of arbitrage equilibria, principal determinants of financial market yields, such as bond rates, are multiperiod forecasts of the federal funds rate. For variables subject to dynamic

[2]The explicitness of equilibrium equations in FRB/US is useful in empirical checks of theoretical long-run restrictions, using tests for trending variables developed in recent years by time series analysts, vid. Engle and Granger (1987).

frictions, the formulation of optimal adjustment plans requires multiperiod forecasts of the relevant equilibrium values. As indicated later, rational expectations are assumed in estimation where sector expectations are generated by a VAR model of the economy. There are two advantages in assuming rational expectations. First, the unobserved expectations that condition actions of households and firms are replaced by forecasts from an explicit forecast model. Second, the use of explicit expectations permits identification of frictions that impede sectoral dynamic adjustments.[3]

The remainder of this section provides brief discussions of the arbitrage equilibrium equation for the 10-year bond rate, the equilibrium and dynamic adjustment equations for the aggregate price equation, and examples of additional constraints on behavior. Because sectoral expectations require a forecast model of the economy, discussion of this topic is postponed until an overview of the full model is presented.

2.1 Arbitrage equilibria.

The simplest form of economic behavior that is assumed to reflect rational expectations is pricing of financial assets in auction markets. The assumption that auction market forecasts are rational requires that the model's multiperiod forecasts of fundamental variables, such as a representative short-term interest rate, should explain significant movements in the auction prices of long-maturity assets.

Assuming bond valuations reflect rational forecasts of the future path of the federal funds rate, the yield to maturity on a 10-year Treasury bond, $r10$, is a weighted moving average of the funds rates expected over the next 40 quarters,

$$\begin{aligned} r10_t &= \mu_{10} + w_1 r_t + w_2 r^e_{t+1} + w_3 r^e_{t+2} + \ldots + w_{40} r^e_{t+39}, \qquad (1) \\ &= \mu_{10} + 1.0 \text{ leads}_{40}(r^e_{t+i}). \end{aligned}$$

The first term on the right hand side of the equal sign in equation 1 is the term premium for the 10-year bond, μ_{10}. The remaining explanatory variables are forecasts of the federal funds rate in future periods. Current market forecasts based on information available in period t are denoted by the superscript “e”. In this instance, no superscript is attached to the funds rate of the current period because the quarterly bond rate is assumed to incorporate current-quarter information; thus,

[3] Of course, the assumption that expectations of sectors are rational may not be consistent with observed behavior. Rational expectations (RE) impose very tight restrictions on the way that sectoral expectations influence sector responses. In contrast to rejections of RE restrictions in many empirical macroeconomic studies, RE restrictions are generally accepted in FRB/US.

r_t is the realized funds rate in the current period, not a forecast.

The weights, w_i, on expected funds rates over the 40-quarter maturity of the 10-year bond sum to unity. This is indicated in the second line of equation 1 where the relevant weight sum (1.0 in this equation) multiplies a condensed notation for the weighted average of 40-quarter funds rate forecasts, $\text{leads}_{40}(.)$. For a discount bond, the relationship in equation 1 would be a simple moving average with each funds rate forecast receiving the same weight of 1/40. However, in the current example of a 10-year Treasury *coupon* bond, the quarterly weights decline geometrically at a 2% rate consistent with applying an annual discount rate of 8% (approximately the sample mean of the 10-year bond rate) to future coupon payments.

Table 1: 10-Year Government Bond Rate Equation (r10)

equilibrium relationship:	$r10$ = .46 + 1.0 $\text{leads}_{40}(r^e)$ - .79 $\text{leads}_{40}(\tilde{x}^e)$ + .85 $\text{lags}_1(\tilde{\mu}_{10})$.	R^2 .98 SEE .32
properties:	mean response lag to surprise = 0 quarters.	span: 63q1-94q4
definitions:	$r10$ - ten-year government bond rate. r - federal funds rate. $\tilde{x}$ - aggregate output gap. $\tilde{\mu}_{10}$ - term premium residual.	

The estimated equation for the 10-year bond rate in FRB/US is displayed in table 1.[4] The joint assumptions of rational expectations and the absence of frictions on portfolio adjustments leave only the term premium to explain predictable movements in the bond equation residual. Some of the predictable variation in the term premium is explained by a 40-quarter lead of expected deviations of aggregate output from trend output, $\tilde{x}^e_{t+i}$, where the negative coefficient sum, -.79, indicates countercyclical movements in the term premium. Predictable variation in the term premium residual, after accounting for the effect of expected output deviations, is approximated by a one-quarter autoregressive lag, $\text{lags}_1(\tilde{\mu}_{10})$, where the notation for lags parallels that used for leads.

As indicated in table 1, the mean lag response to unanticipated shocks is zero because the bond rate is assumed to incorporate current-quarter news. Additional reported statistics include the

[4]The construction of expectations used in estimating model equations, such as the funds rate forecasts for the 10-year bond rate equation, is discussed in section 4.

proportion of sample variation in the 10-year bond rate that is explained by the estimated equation, R^2 = .98, and the standard deviation of the quarterly equation residual, SEE = .32.

2.2 Rational behavior under frictions.

In contrast to financial markets, where quarterly outcomes are assumed to reflect equilibrium valuations, behavior in most other sectors of FRB/US is constrained by costs of adjustment. This section describes the basic features of behavior in FRB/US when actions planned by sectors are subject to dynamic frictions. For concreteness, the aggregate price equation of FRB/US will illustrate the basic features of rational behavior under frictions.

As indicated earlier, optimal behavior under frictions is described as a two-stage decision process. In the first stage, a sector decides on the equilibrium setting of a variable under its control. The equilibrium setting is the value that would be selected in the absence of frictions and is derived from standard conditions for profit or utility maximization. In the case of the price equation, profit maximization under imperfect competition requires the optimal (log) producer price to be proportional to the (log) marginal cost of production. Because long-run production is Cobb-Douglas in FRB/US, the marginal cost of production can be expressed as a weighted average of unit labor and energy costs. As shown in the first equation of table 2, the long-run elasticities of the aggregate price equilibrium with respect to unit costs of labor and energy are .98 and .02, respectively. To allow for possible cyclical variations in perceived demand elasticities and marginal costs of production, the equilibrium condition also specifies that the optimal price margin may vary with the aggregate unemployment rate. The negative coefficient of the unemployment rate, u, indicates procyclical variation in the equilibrium price, with a decline in the unemployment rate of one percentage point raising the level of the desired price markup by 0.3 percent.

In the second decision stage, after selecting an equilibrium setting, the sector formulates an optimal approach to the equilibrium price. A standard model of optimal price adjustment under frictions is that by Rotemberg (1987), where costs associated with changing prices are proportional to the squared deviation of the current price from last period's price, $(p_t - p_{t-1})^2$. More generally, this quadratic penalty on changing the *level* of sector activity is the standard approximation of frictions used in all areas of applied macroeconomics, in part because it provides a rationale for the familiar partial adjustment model in which a fixed fraction of the distance to the equilibrium is eliminated in each period.[5]

The basic paradigm in FRB/US of adjustment dynamics employs a generalization of this standard adjustment cost specification, which includes not only penalties for changing the level

[5] A recent historical review of dynamic friction specifications in empirical macroeconomics may be found in Brayton and Tinsley (1995).

Table 2: Aggregate Price Equation (p)

equilibrium relationship:	$p^* = .98(w - \rho) + .02p_e - .003u.$	
remarks:	• equilibrium condition includes also effects of farm and import prices.	
dynamic adjustment:	$\Delta p_t = -.10(p_{t-1} - p^*_{t-1})$ + .57lags$_2$(Δp_{t-i}) + .43leads$_\infty$(Δp^{*e}_{t+i}).	R^2 .88 SEE .0025
properties:	mean response lag to surprise = 3.3 quarters.	span: 63q1-94q4
remarks:	• dynamic equation includes an accelerated response to energy price inflation.	
definitions:	p - log price of final sales plus imports less gov't labor and indirect business taxes. w - log compensation per hour (ECI). ρ - log trend labor productivity. p_e - log crude energy price. u - demographically-weighted unemployment rate.	

of an action but also for changing its growth rate or for altering a moving average of recent actions.[6] There are three reasons to consider a more general description of dynamic frictions: First, in contrast to equilibrium relationships, economic theory offers relatively little guidance on the nature of dynamic frictions. Second, the standard prior that firms and households smooth only the levels of macroeconomic aggregates is arbitrary and generally strongly rejected by postwar data. Third, the extended model of frictions developed for FRB/US conveniently turns out to be a restricted version of the familiar vector autoregression (VAR) developed by Sims (1980), enabling FRB/US to take advantage of the data-oriented techniques of modern time series analysis while estimating structural descriptions of rational behavior.

Under the generalized adjustment cost specification, the basic equation in FRB/US describing rational adjustment under frictions contains three sets of regressors: a single regressor consisting of the distance to the equilibrium that remains at the start of the current quarter, $p_{t-1} - p^*_{t-1}$; a second set of regressors consisting of lags of the dependent variable; and a third set of regressors containing expected future changes in the desired equilibrium price. For convenient reference, the dynamic adjustment equation reported in table 2 for the aggregate price level is reproduced here as equation 2:

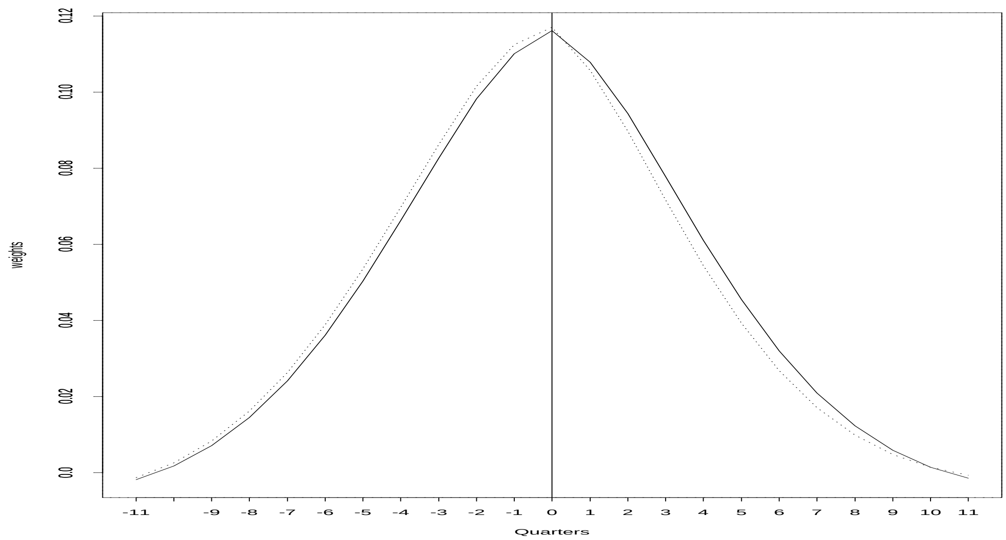
$$\Delta p_t = -.10(p_{t-1} - p^*_{t-1}) + .57\text{lags}_2(\Delta p_{t-i}) + .43\text{leads}_\infty(\Delta p^{*e}_{t+i}). \tag{2}$$

The first term after the equal sign indicates that 10% of the distance to the equilibrium price level is eliminated in each quarter. Additional inertia is indicated in the next set of regressors which includes two lags of the inflation rate, with a weight sum of .57. The coefficients of these lagged terms would be zero if frictions were only associated with costs of changing the level of the price. Thus, an implication of the joint presence of the initial level adjustment term and the lagged inflation terms is that *both* the level and the inflation rate of the aggregate price index are "sticky." The third set of regressors are forecasts of changes in the equilibrium price over the planning horizon of firms. As indicated by the subscript notation for forecasted leads, $\text{leads}_\infty(.)$, the planning horizon is infinite because firms are assumed to be infinitely-lived entities.

In fact, the effective planning horizon of firms is considerably shortened by the rapid rate of decay in weights assigned to distant forecasts of equilibrium price changes. Under rational planning, there are two sources of influence on the relevance of future events. One is the standard rate of return required by investors such as the discount factor applied to future earnings to derive the current market value of equity. In the case of business firms, a fixed quarterly discount factor of .98 is assumed, which is consistent with a postwar annual real rate of return to equity of about

[6]Issues in specifying and estimating more general descriptions of adjustment frictions are discussed in Tinsley (1993) and Kozicki and Tinsley (1995).

Figure 1: Lead and Lag Response Weights of the Aggregate Price Equation



8%. The second and more important source of rapid decay in weights assigned to future periods is the influence of frictions in constraining sectoral adjustments. Intuitively, if frictions are large then the planning horizon must be lengthy because the equilibrium will not be reached quickly. Conversely, if frictions are small, the required planning horizon will be short.

Lead and lag response weights of the estimated price equation are reproduced by the downward-sloping solid lines in figure 1. These weights are obtained by a rearrangement of the dynamic adjustment equation so the price in the current quarter, p_t, is expressed as an infinite distributed lead of future price equilibria, p^*_{t+i}, and an infinite distributed lag of past price equilibria, p^*_{t-i}.[7] Because the aggregate price eventually converges to the equilibrium price, the combined sum of lead and lag weights is unity. Note that the lead and lag response weights resemble a tent with the center pole anchored in the current quarter, designated by the origin. Both the lead and lag response weights are approximately zero within twelve quarters, moving forwards or backwards in time from the current quarter. The lead response weights are slightly more concave downward due to the additional tilt introduced by the discount factor. As reported in table 2, the mean response lag to an unanticipated shock (surprise) is 3.3 quarters; this estimate is the mean lag defined by the left half of the weights in figure 1 that extend over past quarters. By contrast, the response lag to a future event that has been perfectly anticipated (perfect foresight) is the mean lag associated with the full distribution of lead and lag weights and is approximately zero.

[7]See derivations of response weights and additional discussion in von zur Muehlen (1996).

The alternative dotted line in figure 1 indicates the surprisingly modest effect on the response weights of the aggregate price if the quarterly discount factor of the business sector were to be reduced from .98 to .94, consistent with an increase of the annual discount rate from 8% to 24%.[8] Given the sizeable tripling of the discount rate, this exercise demonstrates that frictions on behavior are generally the dominant influence in determining the effective planning horizons of rational firms and households.

Another way to highlight differences in interpreting the dynamic responses of a rational pricing model under frictions is to contrast the FRB/US equation in table 2 with the dynamic structure of a standard reduced-form Phillips curve. If the distinction between actual and equilibrium prices is suppressed except for the contribution of the unemployment rate to the equilibrium, then the FRB/US aggregate price equation can be viewed as a two-sided Phillips curve with current inflation depending on both past inflation and expected future inflation as well as on a cyclical indicator. If it were further assumed that expected future inflation depended only on lagged inflation, then the format of the price equation would resemble that of a simple Phillips curve in which inflation is a function of inflation lags and a cyclical variable. Although the inflation lags appearing in a typical Phillips curve are frequently identified only with inflation expectations, the discussion of rational planning under frictions indicates that reduced-form inflation lags combine the effects of both frictional inertia and expectations. An advantage of the rational adjustment format used in FRB/US is that it provides an empirically tested separation of response lags attributable to frictions on price adjustment from lags associated with forecasting. As implemented in FRB/US, the forecasting lags are conditioned not only on lagged inflation but also on the business sector's perceptions of response lags in other sectors, including those of the central bank.

2.3 Heterogeneous households and firms.

Although equations based on the assumption of identical individuals often provide reasonable estimates of macroeconomic aggregates, in some cases there is no recourse but to acknowledge measurable differences among individuals. This section discusses how the standard equation formats are modified to accommodate heterogeneity among firms and households.

It is often possible to account for heterogeneity if there exists an observable proxy for a category of constrained behavior. For example, in the case of imperfect capital markets, investment spending of some firms may be limited to that which can be financed by retained earnings. Similarly, consumption plans of some individuals may be constrained by current income, rather than wealth, due to an inability to borrow on expected future labor income or an absence of tangible assets for

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[8] As noted in section 3, an annual 25% discount rate is used in the household sector to discount expectations of uncertain future income.

collateralized borrowing. In both examples, the net response can be modeled as a weighted average of the proxy for liquidity-constrained behavior (either retained earnings or current income, in these examples) and the behavior that would be predicted by the standard equation format for rational adjustment under dynamic frictions.

Another way to accommodate heterogeneity is to disaggregate explanatory variables. For example, unlike firms, households have finite lifetimes. Under the assumption of lifecycle averaging of planned consumption, an individual's propensity to spend in each period out of the sum of tangible and intangible wealth is approximately the inverse of the expected remaining lifespan of the individual. Consequently, even for a stable age distribution of households, cyclical or secular changes in the distributions of types of wealth over age cohorts will induce changes in the aggregate propensity to spend from aggregate household wealth. For example, the expected present value of labor income is more important for young households, who have lower propensities to spend; by contrast, the expected present value of distributions from pension funds and social security is more important for retirees who, in the absence of bequest motives, have higher propensities to spend. Effects of heterogeneity in household spending responses are approximated by including the composition of wealth, in addition to total wealth, in equilibrium equations for household consumption.

3 A Bird's Eye View of FRB/US

This section presents the estimated structure of all equations that have been reformulated using specifications required for either of the two primary forms of rational behavior in FRB/US—arbitrage equilibria and dynamic adjustment subject to frictions. As described later in section 4, these equations were estimated under the assumption of rational expectations, implemented by representing sectoral expectations with VAR forecasts. For each equation, the restrictions imposed by rational expectations were tested, as was the hypothesis that the equation residuals were serially independent. Results of these tests, reported in Appendix A, generally support the null hypotheses of rational expectations and serially independent residuals.

Before surveying individual estimated equations, a brief preview of three categories of dynamic adjustment equations may be useful. First, as noted in section 2, the generic format for dynamic adjustment under frictions expresses the first difference of a variable as a function of the gap between the variable and its equilibrium value, lags of first differences of the variable, and leads of expected first differences of its equilibrium. Three dynamic equations that use this standard template are the quarterly adjustments in inventories, price, and wage.

A second category of dynamic adjustment formats is due to heterogeneous behavior, when some households and firms follow the standard adjustment model and others behave according to another criterion. Three equations exhibit heterogeneous behavior: aggregate consumption includes effects of liquidity-constrained households; investment in producers' durable equipment incorporates effects of liquidity-constrained firms; and labor hours include some workers whose hours are costless to adjust. In each case, extra regressors are added to represent the alternative behavior, and coefficient restrictions are imposed so that the proportions of each type of behavior appearing in the aggregate adjustment equation sum to one.

A third dynamic adjustment category is also represented in investment equations. Although investment decisions aim at achieving a desired capital stock, the estimated equations do not include capital stocks among the explanatory variables. This omission is a consequence of a current lack of data on stocks; nevertheless, even if stock data were available, doubts about its quality might argue against its use. Investment equations have been modified to approximately capture effects that arise when the underlying target is a stock. The modifications, which appear in equations for investment in motor vehicles, other durables, residential construction, and producers' durable equipment, involve the inclusion of extra growth rate variables which operate like the output accelerator found in many traditional investment equations.

3.1 Households

Most households base consumption decisions on expected lifetime assets, which equal current property wealth plus the present value of expected after-tax labor and transfer income. Risk aversion causes future income flows to be discounted quite heavily with the consequence that households do not act to fully offset government fiscal actions, as suggested by the Ricardian equivalence hypothesis. Another characteristic of the household sector is that movements in labor force participation are driven by changes in social norms in the long run, represented by time trends, and by the availability of jobs in the short run. The real, after-tax wage has no effect on labor supply. Thus, there are no effects of fiscal policy on the labor force, nor are consumption and labor supply decisions intertwined.

Aggregate consumption (table 3). As in MPS, aggregate consumption is defined as the service flow derived from the stock of consumer durables plus spending on nondurables and services. In the steady state, aggregate consumption depends on total wealth—defined as the after-tax present value of the sum of labor, property, and transfer income—and its composition. Expected future income flows are discounted at a 25 percent annual rate in computing present values due to the aversion of households to the uncertainty of future income. The composition of wealth matters because the lifecycle hypothesis suggests that, absent strong bequest motives, the propensity to spend out of expected lifetime resources at the individual level increases with age. Types of income and assets are not evenly distributed over age groups; thus, consumption aggregated across individuals varies with the composition of total wealth. Based on a linear approximation of the logarithmic equilibrium condition, marginal propensities to consume out of categories of income and tangible wealth are .51 for labor income, 1.05 for transfer income, .39 for property income, .030 for corporate equities, and .075 for other net tangible assets.[9] In addition to total wealth and its composition, desired aggregate consumption also depends positively on the output gap, approximating the effect of countercyclical variation in the perceived riskiness of future income flows.

Table 3: Aggregate Consumption Equation (c)

equilibrium relationship:	$c^* = 1.0v + .62s_{trans} - .15s_{prop} + .52s_{stock} + 1.28s_o + .013\tilde{x}.$
dynamic adjustment:	$\Delta c_t = -.12(c_{t-1} - c^*_{t-1})$ + .17 lags$_1(\Delta c_{t-i})$ + .75 leads$_\infty(\Delta c^{*e}_{t+i})$ + .09Δy_t.
	span: 63q1-95q4 R^2: .54 SEE: .0032 MRL [a]: 7.9 quarters
definitions:	c - log consumption (including service flow of stock of durables). Y - income (labor + transfer + property). y - log Y. V - wealth = leads$_\infty(Y^e)$. v - log V. s_{trans} - transfer wealth / V. s_{prop} - property wealth / V. s_{stock} - value of corp. equity / V. s_o - other net financial and tangible assets / V. $\tilde{x}$ - aggregate output gap.

[a] Mean response lag to a surprise.

The dynamic consumption equation is a weighted average of the behavior of lifecycle and liquidity-constrained households. The share of income associated with the latter group is about 10 percent, based on the estimated coefficient on contemporaneous income growth. This direct effect of income growth is in addition to the contribution of income growth in the VAR forecasting model for expectations of target consumption that was used in estimation of the dynamic consumption equation. Lifecycle consumers adjust spending sluggishly, with a mean response lag to shocks (surprise) of about two years.

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[9]The equilibrium equation contains both the present value of property income and a Flow of Funds estimate of property wealth because the latter controls for differences between the market valuation of property assets and the valuation of property income using a fixed discount rate. For a more detailed description of household consumption and investment, see Reifschneider (1996).

Table 4: Household Investment Equations (c_{dv}, c_{do}, and i_h)

equilibrium relationships:	$c^*_{dv} = 1.0c^* - .46(p_{dv} - p_c) - .41(p_{gas} - p_c) - .03r_{dv}$.
	$c^*_{do} = 1.0c^* - .56(p_{do} - p_c) - .02r_{do} + .004t_{82}$.
	$i^*_h = 1.0c^* - .13r_h - .003t_{47} + .003t_{88}$.
dynamic adjustment:	$\Delta c_{dv,t} = -.30(c_{dv,t-1} - c^*_{dv,t-1})$ - .28 lags$_1(\Delta c_{dv,t-i})$ + 3.22 leads$_\infty(\Delta c^{*e}_{dv,t+i})$ + 7.46 lags$_4(\Delta c^*_{dv,t-i})$.
	span: 63q1-94q4 R^2: .43 SEE: .0054 MRL [a]: 3.3 quarters
	$\Delta c_{do,t} = -.10(c_{do,t-1} - c^*_{do,t-1})$ + .17 lags$_1(\Delta c_{do,t-i})$ + 2.15 leads$_\infty(\Delta c^{*e}_{do,t+i})$ + 1.12 lags$_4(\Delta c^*_{do,t-i})$.
	span: 64q1-95q4 R^2: .34 SEE: .0016 MRL [a]: 7.3 quarters
	$\Delta i_{h,t} = -.09(i_{h,t-1} - i^*_{h,t-1})$ + .38 lags$_1(\Delta i_{h,t-i})$ + 6.10leads$_\infty(\Delta i^{*e}_{h,t+i})$ + 4.15 lags$_4(\Delta i^*_{h,t-i})$.
	span: 63q1-95q4 R^2: .60 SEE: .034 MRL [a]: 5.9 quarters
remark:	• dynamic equation for i_h also includes variables for deposit disintermediation in 1966-7 and for credit controls in 1980.
definitions:	c_{dv} - log consumer expenditures on motor vehicles (constant dollars). c_{do} - log consumer expenditures on other durables (constant dollars). i_h - log investment in residential construction (constant dollars). c^* - log aggregate consumption target (see table 3). p_{dv} and p_{do} - log prices of motor vehicles and other durables. p_c - log price of aggregate consumption. p_{gas} - log retail gasoline price, adjusted for vehicle fuel efficiency. r_{dv}, r_{do} and r_h - costs of capital, motor vehicles, other durables, and housing. t_{82}, t_{47} and t_{88} - quarterly time trends starting 82q1, 47q1 and 88q1.

[a] Mean response lag to a surprise.

Household investment (table 4). The equilibrium or long-run ratios of household investment in motor vehicles, other durables, and housing to aggregate consumption are functions of relative prices, user costs of capital, and time trends (for the equilibrium ratios of other durables and housing). User costs incorporate effects of depreciation and the real rate of interest. The latter is defined as a nominal interest rate—the auto loan rate in each of the durables equations and the fixed-rate mortgage yield in the housing equation—less an expected consumer inflation measure of corresponding maturity, adjusted for the marginal tax rate.

As noted earlier, the dynamic adjustment equation for each component of household investment includes a modification of the standard rational adjustment format to account for the requirement that investment decisions are aimed at achieving a desired stock of capital, in addition to optimal rates of investment. This modification introduces accelerator effects to the investment equations by adding lags and additional leads in the growth rates of desired investment as regressors. As indicated by the large coefficient sums on the growth rates of equilibrium investment in the dynamic adjustment equations of table 4, accelerator effects are most pronounced for two categories of household investment, motor vehicles and housing.

3.2 Firms

Firms produce with a long-run Cobb-Douglas technology, setting equilibrium factor input quantities (labor, capital, and energy) and the price of output to maximize profits. The production function and first-order conditions for profit maximization define the equilibrium equations for investment in producers' durable equipment, labor hours, and the price of output. Energy demand is determined as an exogenous ratio to output rather than by a first-order condition. The firm sector also includes equations for inventories and hourly labor compensation. The structure of the latter is closely related to that of the price equation.

Investment in producers' durable equipment (table 5). The desired capital stock varies proportionally with output and, based on the Cobb-Douglas production structure, inversely with the user cost of capital. In the latter, financing costs are measured with weights of 0.8 on the cost of debt and 0.2 on the cost of equity; the weights were chosen to maximize the fit of the equipment equation. The ratio of target investment to target capital equals the sum of the depreciation rate and the growth rate of output. In the equilibrium equation for investment, which is written in logs, the log of the sum of depreciation and output growth has been linearized so that output growth—the accelerator—appears separately.

Table 5: Business Investment Equations (i_{pd} and k_i)

equilibrium relationships: $i^*_{pd} = 1.0x_b - 1.0r_{pd} + 1.0z_{pd} + 19.5\Delta x_b.$

$k^*_i = 1.0x_b.$

dynamic adjustment: $\Delta i_{pd,t} = -.07(i_{pd,t-2} - i^*_{pd,t-2})$ + .26 lags$_2(\Delta i_{pd,t-i})$ + .47 leads$_\infty(\Delta i^{*e}_{pd,t+i-1})$ + .22 lags$_2(\Delta cf_{t-i})$.

span: 64q1-94q4 R^2: .40 SEE: .0022 MRL[a]: 7.0 quarters

$\Delta k_{i,t} = -.23(k_{i,t-1} - k^*_{i,t-1})$ + .47 lags$_3(\Delta k_{i,t-i})$ + .53 leads$_\infty(\Delta k^{*e}_{i,t+i})$.

span: 62q3-94q4 R^2: .42 SEE: .0065 MRL[a]: 1.3 quarters

remarks:

- dynamic equation for i_{pd} is a weighted average of adjustment model (.78) and cash flow model (.22).
- adjustment model component for i_{pd} includes 1-quarter delivery lag.

definitions:

i_{pd} - log investment in producers' durable equipment (constant dollars).
k_i - log stock of manufacturing and trade inventories (constant dollars).
x_b - log output, business sector (constant dollars).
r_{pd} - log user cost of capital, producer durables.
z_{pd} - log(depreciation rate + mean of Δx_b).
cf - log corporate cash flow (constant dollars).

[a] Mean response lag to a surprise.

The dynamic investment equation augments the generalized adjustment model, which does not fully capture cyclical investment fluctuations, with cash flow. The added variable is motivated by recent empirical literature suggesting that some firms are constrained in their access to capital markets. The resulting equation places 20 percent weight on the growth of cash flow and 80 percent weight on the standard adjustment specification. Considering only the adjustment part of the equation, investment is found to be fairly sluggish, with a mean response lag of almost 2 years. The response includes a one-quarter delivery lag which was found to improve the fit of the equation; the delivery lag causes the timing of the level correction term and the expectations variables to be shifted back one quarter more than in the standard adjustment specification.

Inventory investment (table 5). The model's main inventory equation determines the stock of manufacturing and trade inventories, where the inventory target stock, k^*, is proportional to the output of the business sector, x_b. Relative to the adjustments of most nonfinancial aggregates, firms adjust inventory holdings to revised target levels rapidly with a mean lag response to unanticipated shocks of 1.3 quarters.

Labor input (table 6). In MPS, the equilibrium level of aggregate labor hours was based on the sum of labor requirements across the vintages of capital needed to produce a given level of output. This condition, derived from an assumed putty-clay characteristic of capital, was approximated by making target hours a function of output, total factor productivity (specified as a time trend), and contributions of average capital and energy intensities of existing capacity to the labor-output ratio. Coefficients based on capital and energy intensities were imposed, based on average factor shares, without causing too much deterioration in the equation's goodness of fit. However, with revised NIPA data, imposing the factor-intensity effects in FRB/US led to a substantial deterioration of fit. Thus, the equilibrium condition has been simplified to make target hours a function of output and a pair of time trends capturing the well-known slowdown of productivity growth in the early 1970s.[10]

Labor hours are modeled as heterogeneous with respect to adjustment costs. About one-third of hours is estimated to respond immediately to changes in target hours, with the remainder following the generalized adjustment cost model. Aggregated across both types of labor, the mean response lag to a surprise is very short, less than a quarter.

Prices and Wages (table 6). The steady-state structures of the price and wage equations are derived from the equilibrium Cobb-Douglas assumption that the share of income received by capital is constant in the steady state. If the share of income received by energy producers is stable, then constancy of the capital share implies constancy of the labor share. Price and wage targets also vary procyclically. The two equilibrium conditions jointly determine the real wage and the NAIRU. The latter is constant when defined by a demographically-weighted unemployment rate and a bit less than 6 percent, currently, in terms of the civilian unemployment rate.

The generalized frictions model yields level error correction terms in the dynamic adjustment equations for both price and wage. The level correction term in the price equation is a standard feature of *level* price markup equations and captures variations in price margins. By contrast, a level correction term in wage equations is less common, but consistent with models of wage bargaining.

The dynamics of wages and prices are strongly interrelated because the target wage is largely determined by the price level and, reciprocally, the target price is a function primarily of the wage level. With regard to adjustment speeds, wages are estimated to be more sluggish than prices. The mean response lag in the wage equation (8.7 quarters) is more than twice the mean response lag

[10] A simulation option is available to make labor productivity respond gradually to movements in relative factor prices, similar to the corresponding MPS equation.

Table 6: Aggregate Labor Hours, Wages, and Prices (h, w, and p)

equilibrium relationship:

$$h^* = 1.0x_g - .0069t_{47} + .0042t_{73}.$$

$$w^* = 1.0\rho + 1.02p_g - .02p_e - .01u.$$

$$p^* = .98(w - \rho) + .02p_e - .003u.$$

remark: • equilibrium condition for p also includes effects of farm and import prices.

dynamic adjustment:

$$\Delta h_t = -.15(h_{t-1} - h^*_{t-1}) + .38 \text{ lags}_1(\Delta h_{t-i}) + .41 \text{ leads}_\infty(\Delta h^{*e}_{t+i}).$$
$$+ .31\ \Delta h^*_t - .12 \text{ lags}_1(\Delta h^*_{t-i}).$$

span: 63q1-94q4 R^2: .76 SEE: .0046 MRL [a]: 0.7 quarters

$$\Delta w_t = -.03(w_{t-1} - w^*_{t-1}) + .71 \text{ lags}_3(\Delta w_{t-i}) + .29 \text{ leads}_\infty(\Delta w^{*e}_{t+i}).$$

span: 63q1-94q4 R^2: .82 SEE: .0028 MRL [a]: 8.7 quarters

$$\Delta p_t = -.10(p_{t-1} - p^*_{t-1}) + .57 \text{ lags}_2(\Delta p_{t-i}) + .43 \text{ leads}_\infty(\Delta p^{*e}_{t+i}).$$

span: 63q1-94q4 R^2: .88 SEE: .0025 MRL [a]: 3.3 quarters

remarks:
- dynamic equation for h is a weighted average of standard adjustment model (.69) and immediate response model (.31) .
- dynamic equation for w also includes variables for wage and price controls, employer social insurance contributions, and the minimum wage.
- dynamic equation for p also includes an accelerated response to energy price inflation.

definitions:

h - log hours, nonfarm business sector (employees and self-employed).
w - log compensation per hour (ECI).
p - log price of final sales plus imports less gov' t labor and indirect business taxes.
x_g - log output, nonfarm business sector plus oil imports less housing product (constant dollars).
t_{47} and t_{73}- quarterly time trends starting 47q1 and 73q1.
ρ - log trend labor productivity.
p_g - log price of x_g less indirect business taxes.
p_e - log crude energy price.
u - demographically-weighted unemployment rate.

[a] Mean response lag to a surprise.

(3.3 quarters) in the price equation. Wages are also less responsive to forward expectations, with a coefficient sum of .29 on the expected rate of growth of the equilibrium wage compared with an expectations coefficient sum of .43 in the price equation. The dynamic wage and price equations contain inflation-neutrality restrictions to insure that the equilibrium real wage and NAIRU are independent of the rate of inflation.[11]

Other equations. *Nonresidential construction* is assumed to move with business output and a time trend. In the current version, no effects of interest rates or tax policy are included. *Employment* is determined by a standard error-correction equation in which the employment target depends on aggregate hours and trends in the workweek.

3.3 Financial markets

Equations for three long-term interest rates and the stock market comprise the core of the financial market sector of FRB/US. Unlike nonfinancial behavior, where frictions make it too costly to move immediately to equilibrium values, asset prices are assumed to be in equilibrium continuously.

Long-term bond rates (table 7) are determined according to the expectations theory of the term structure. The theory posits that, up to a term premium, the yield on a long-term bond is given by the expected future path of short-term interests rates. Abstracting from term premia, the yield on a 5-year government bond is a weighted average of expected federal funds rates over the next 20 quarters.[12] The yields on the other two long-term bonds—the 10-year government bond and the Moody AAA corporate bond—are modeled in an analogous way.

Term premia for all three bond equations are estimated to vary negatively with a weighted average of the output gap expected over the maturity of the bond. One can interpret this negative relationship to mean that investors require larger risk premiums when they expect a deterioration in the average performance of the economy over their investment horizon. Consistent with this notion, the sensitivity of the term premium to expected economic conditions increases with the maturity of the bond.

[11]These restrictions set the sum of the coefficients on the lead and lag inflation terms to one. Analogous restrictions were not imposed on other adjustment equations, because they explain the behavior of real variables whose steady-state growth rates are likely to have relatively small variations.

[12]The weights attached to each future value decline about 2 percent per quarter. The implied average duration of the 5-year government bond rate is approximately 4 years. The 10-year government bond and the corporate bond have durations of 7 and 12 years, respectively.

Table 7: Financial Sector Equations (r_5, r_{10}, r_{cb}, and v_s)

5-year gov't bond rate [a]:	$r_{5,t} = .34 + 1.0 \text{ leads}_{20}(r^e_{t+i}) - .62 \text{ leads}_{20}(\tilde{x}^e_{t+i}) + .83 \text{ lag}_1(\tilde{\mu}_{5,t-i})$
	span: 63q1-94q4 R^2: .97 SEE: .47 MRL [b]: 0 quarters
10-year gov't bond rate [a]:	$r_{10,t} = .46 + 1.0 \text{ leads}_{40}(r^e_{t+i}) - .79 \text{ leads}_{40}(\tilde{x}^e_{t+i}) + .85 \text{ lag}_1(\tilde{\mu}_{10,t-i})$
	span: 63q1-94q4 R^2: .99 SEE: .32 MRL [b]: 0 quarters
corporate bond rate [a]:	$r_{cb,t} = 1.21 + 1.0 \text{ leads}_{120}(r^e_{t+i}) - 1.21 \text{ leads}_{120}(\tilde{x}^e_{t+i}) + .87 \text{ lag}_1(\tilde{\mu}_{30,t})$
	span: 63q1-94q4 R^2: .99 SEE: .27 MRL [b]: 0 quarters
stock market wealth:	$v_{s,t} - p_{g,t} = 4.7 + d_t + 50 \text{ leads}_{\infty}(\Delta d^e_{t+i}) - 50(\ (r_{cb,t}/400) - \text{leads}_{120}(\Delta p^e_{c,t+i}) \ )$
	span: 65q1-95q4 R^2: .97 SEE: .20 MRL [b]: 0 quarters
definitions:	r - federal funds rate. $\tilde{x}$ - output gap. $\tilde{\mu}_5$, $\tilde{\mu}_{10}$, and $\tilde{\mu}_{30}$ - term premium residuals for r_5, r_{10}, and r_{cb}. v_s - log stock market wealth (current dollars, flow of funds accounts). d - log national income dividends (constant dollars, deflated by p_g). p_g - log price, business sector output.[c] Δp_c - inflation rate, household consumption price.[c]

[a] For the three bond equations, the reported SEE and R^2 are computed after adjustment for first-order serial correlation of the term-premium residuals.
[b] Mean response lag to a surprise.
[c] Price indexes divided by 100 before taking logarithms.

Stock market wealth (table 7). Similar to the log linearized model in Campbell and Shiller (1989), the real value of the stock market is determined by expectations of the future flow of real dividend payments. Future expected dividends are discounted by the expected opportunity yield on corporate bonds, measured by the current corporate bond rate less expected consumer inflation rates over a 30-year horizon. Both the stream of expected real dividends and the real corporate bond rate are multiplied by a normalization factor due to linearization about the sample average of the real return on equity. As noted in the fourth equation of table 7, on a quarterly basis, the normalization factor is approximately 1/.02 = 50. Additional compensation for equity ownership, apart from the cyclical risk premium already embedded in the corporate bond rate, is captured by freely estimating the intercept of the equity equation. However, the substantial residual serial correlation of this equation suggests the additional risk premium required by households for equity holding is not constant over time.

Mortgage and car loan rates. The equilibrium relationships for the mortgage rate and the loan rate on new cars are based on the 10-year and 5-year government bond rates, respectively. In the short run, movements in the mortgage and new car loan rates reflect partial adjustments to long-run equilibria; the mortgage rate also varies countercyclically.

3.4 Foreign trade and government sectors

In the current model version, these sectors are constructed using unrestricted error correction regressions and not the restricted rational adjustment specifications described in section 2. A brief description follows of the theoretical motivations of the main equations in these sectors.

The equations describing the long-run determinants of real *exports* and *imports* are standard. Real exports depend on foreign GDP and the *real exchange rate*. The latter is defined by open-interest parity arbitrage with an expected long-term real rate of interest and a country risk premium, which is a function of U.S. net foreign indebtedness. Real nonoil imports are a function of domestic GDP and the relative price of imports. Each trade equation contains a time trend and is formulated as an error correction; long-run income elasticities are constrained to unity and long-run price elasticities to minus unity.

The *government sector* of FRB/US is disaggregated into two tiers: federal and state & local. Most variables in this sector are either exogenous or defined through identities. However, tax payments are endogenous, as are transfers to persons, which are estimated to have components that vary countercyclically, and net interest payments, which are functions of stocks of debt outstanding and interest rates.

4 Expectations

Expectations of future events are important determinants of private sector behavior in FRB/US, especially over multiperiod horizons as the initial restraining influences of frictions dissipate. This section discusses the formulation of explicit expectations in FRB/US and options that may be used in hypothetical scenarios as well as those used in estimation.

In estimating the structural descriptions of behavior described in section 3, explicit expectations of the desired equilibria of firms and households were required to identify the frictions that inhibit dynamic adjustments. Although explicit expectations need not be rational, the assumption that sectors formulate rational expectations using a condensed model of the economy was imposed in estimation and not empirically rejected in most instances. *Rational expectations* (RE) denote forecasts of variables that are consistent with expected outcomes of the modeled mechanisms that generate these variables.[13]

Although the assumption of rational expectations has proved to be a useful organizing principle in estimating the model—as was the assumption that financial auction markets are efficient—there are obviously instances when such an assumption is unlikely to provide a good prediction of short-run behavior, as in the aftermath of an unusual event which is not well understood by individuals in any sector, such as the prospect of war or the collapse of a market. Also, prior reasoning cannot indicate the level of detailed information about the general economy that is needed to make rational decisions in a particular sector.

Therefore, several options exist (or are being developed) for FRB/US that vary two dimensions in testing or imposing rational expectations in estimation and simulation. One dimension is the *scope* of rational expectations, where sector forecasts may be consistent for summary aggregates but not necessarily for all forecast components. The other is the *speed* of rational expectations formation where, in the most typical case, private sector perceptions may be consistent with policy goals in the long run but not necessarily over short forecast horizons.

Discussion in this section is organized as follows: The first subsection outlines the main options that specify the scope of information shared by sectors. The second subsection discusses specifications of long-run expectations in FRB/US, including private sector perceptions of long-run policy goals. Finally, a third subsection describes the VAR model used in the estimation of FRB/US to generate sector expectations.

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[13]Note that *rational behavior* by individuals, in the sense of optimal dynamic planning, need not imply rational expectations. As discussed by Sargent (1993), both rational planning and consistent forecasts are required for rational expectations. The first condition of rational behavior is always imposed in FRB/US structural equations, whereas the second condition of consistent perceptions by firms and households is a simulation option. Other options that limit the information or computing ability of individuals are examples of *bounded* or *limited* rationality. Not all bounds on rationality admit rational expectations, even in the long run.

4.1 The scope of sectoral information.

The two main options for the scope of information shared in common by firms, households, and investors are:

Full-model expectations. Under full-model expectations, all sectors use the same forecast model of the economy. The common forecast model is a *closed* version of FRB/US, meaning that that the behavioral responses of all sectors—including policy responses—are specified.[14] The forecast of a variable appearing anywhere in the model will be equal to the forecast generated by the full FRB/US model. The full-model expectations option is the conventional assumption regarding shared information in RE models and provides a useful benchmark of policy effects under complete information and perfect foresight of selected future events.

Because FRB/US is nonlinear, full-model expectations are obtained by iterative, numerical simulations under the assumption that future shocks are either known (perfect foresight) or equal to zero.[15] In discussing the method of solving for full-model expectations, the 10-year bond rate equation is again used as an example. The bond rate requires 40-quarter forecasts of the funds rate and the output gap (trend deviation).

$$r10 = .46 + 1.0\text{leads}_{40}(r^e) - .79\text{leads}_{40}(\tilde{x}^e) + .85\text{lag}_1(\tilde{\mu}_{10}). \tag{3}$$

The full-model expectations solution proceeds by iterative revisions of expectations.[16] Assume that initial "guesses" by investors of 40-quarter forecasts of the funds rates and the output gap are available for each quarter of the forecast horizon. These will define investor forecasts of the bond rate in each forecast period. However, after also solving the full model, where each sector uses its own "guesses" to initialize forecasts of explanatory variables, the forecasts of the funds rate and output generated by the full model will not generally match the initial investor guesses used to construct the bond rate forecasts. Similarly, the initial bond rate guesses used in other equations of the model, such as the guesses by the business sector to forecast the cost of capital for fixed investment, will generally not match the bond rate forecasts produced by equation 3. So, in the

[14] Obviously, individuals in private sectors cannot formulate rational forward plans without a description of expected policies. For purposes of discussion in this section, monetary policy is summarized by an equation describing the responses of the federal funds rate to recent movements in inflation and the trend deviation in output. Other ways to characterize monetary policy are discussed in section 5.

[15] The practice of setting future residuals to zero (thus ignoring nonzero expectations of nonlinear functions of residuals) has become a convention in macroeconomic constructions of full-model expectations, which are often termed *model-consistent* expectations. The bias of expectations generally is small because FRB/US is approximately linear for small shocks and, indeed, for many purposes may be viewed as a large, restricted VAR.

[16] The following is a simplified description of the Fair-Taylor method of solving RE models, which is an iterative Jacobi method of solving systems of dynamic equations.

next iteration, each sector uses the multiperiod forecast of the model to replace its initial guesses. This iterative cycle of updating sectoral expectations by model forecast solutions from the previous iteration and then solving the full model for a new multiperiod forecast sequence continues until sectoral expectations exactly match the full-model forecast. Since FRB/US is approximately linear for small shocks, this iterative process will converge to a unique forecast, if initial guesses are not too distant from the solution.

This cursory description ignores an important issue involving the forecast horizon. Note that the bond rate in the last period of the forecast horizon requires 40 more quarters of funds rates and output gap forecasts. But these variables, in turn, require accompanying bond rate forecasts. This second sequence of more distant bond rates requires 40 additional quarters of funds rates and output gap forecasts, and so on. This infinite recursive sequence is approximated by solving the model for a large number of periods, say 25 years (100 quarters). In practical terms, the impact of distant variables on present actions dissipates rather quickly for most variables. For example, as noted in section 2, the lead weights in the aggregate price equation are approximately zero at the end of a three-year forecast horizon even though the theoretical planning horizon of firms is infinite. Although the effective planning horizon for most variables does not exceed three or four years, notable exceptions are long-maturity instruments such as bonds and equity.

VAR expectations. Under this option, a small vector autoregression (VAR) model of summary macroeconomic aggregates replaces the full FRB/US model as the description of the economy that conditions sectoral forecasts. The same VAR model is used by all sectors, although disaggregated information may be added to provide additional local information within a sector. The VAR model most commonly used for expectations in FRB/US is the *historical* VAR. The historical VAR provides an average-history summary of the dynamic behavior of the economy and was used to generate sectoral expectations in estimation of FRB/US arbitrage and dynamic adjustment equations.[17] Using the same description of policy, general multiplier properties are similar for the historical VAR and the full FRB/US model. This correspondence suggests that the many restrictions in FRB/US required for structural interpretations are not noticeably inconsistent with historical data, reinforcing the results of direct statistical tests of RE restrictions shown in Appendix A. Given the well-behaved and generally white noise residuals of equations in FRB/US under historical VAR expectations, the historical VAR is the standard expectations option for short-term forecast analysis.

If a hypothetical monetary policy is simulated that deviates markedly from the policy responses captured by the historical VAR, it is unlikely that forecasts of summary aggregates from the

[17] Conditions for maximum likelihood properties of estimated friction parameters based on VAR expectations are discussed in Brayton and Tinsley (1995) and Kozicki and Tinsley (1995).

historical VAR model and from FRB/US using historical VAR expectations would agree. In this instance, tests of simulated behavior from the full model would be likely to reject a hypothesis that the historical VAR expectations were rational.[18]

Given the linear structure of VAR models, VAR expectations can be directly obtained by analytical solutions rather than numerical iterations. Thus, the calculation of sectoral VAR expectations, even for infinite forecast horizons, is significantly less computationally demanding than full-model expectations. The matrix manipulations[19] required for VAR expectations typically require only a few seconds on a Sparc10 computer, whereas a standard forecast simulation using full-model expectations can take 25-30 minutes. In the example of the bond rate, equation 3, the funds rate and output gap are explicit variables in the historical VAR. The resultant funds rate and output gap forecasts will be the same for all sectors since all sectors use the same VAR model to generate expectations.

4.2 Long-run expectations.

As noted in previous sections, long-horizon forecasts of the federal funds rate and inflation rate are important determinants of the equity price and bond rates in the current period. Thus, long-horizon expectations constitute a significant policy transmission channel in FRB/US. Short-run policy actions, such as fund rate alterations, can have very different effects depending on how the funds rate movements influence private sector perceptions of long-run policy objectives.

Long-horizon expectations, succinctly called expectation *endpoints* in FRB/US, are generally defined by the steady-state values of equilibrium planning equations. However, the endpoints of the nominal funds rate and consumer inflation rate are explicitly defined in FRB/US, in part to indicate the role of policy in determining these variables but also to better capture historical shifts in private sector perceptions of the endpoints of these variables. These shifting endpoint expectations apply to both full-model expectations and VAR expectations.

A default assumption in many RE models is *symmetric* policy information, where the plans of policymakers are known by all sectors of the economy. By contrast, *asymmetric* policy information is also an option in FRB/US, where private sectors either do not know policy objectives or are sceptical of policy announcements. Explicit endpoint expectations allow the often fuzzy topic

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[18] Another option under development for VAR expectations is a *virtual* VAR, which is a miniaturization of any closed version of FRB/US. If a hypothetical alteration in the full model changes dynamic relationships among summary aggregates included in the VAR, the reduced-form implications of these changes are captured by the virtual VAR. Virtual or "mapped" VARs are iteratively estimated from the outcomes of full model simulations using virtual VAR expectations. This expectations option is very computer-intensive and the subject of ongoing work.

[19] These are simple manipulations of a matrix containing the VAR coefficients. The VAR model is transformed into a first-order autoregressive format. A two-period forecast requires the square of the matrix; a three-period forecast requires the cube of the matrix, and so on.

of policy "credibility" to be precisely framed in FRB/US. If endpoint expectations are based on symmetric policy information, then the inflation endpoint of private sector forecasts will be equal to the long-run inflation goal of a fully credible policy. If policy information is asymmetric then the long-run inflation goal of policy is not known or believed and must be inferred by the private sector from observable indicators, such as past rates of inflation.

Endpoints are defined, usually implicitly, in conventional macroeconomic models as either constants (if variables are detrended) or moving averages (if variables contain a random walk component). The differences in summary statistics and one-period prediction errors between equations estimated with these two endpoint alternatives are often small or insignificant, but profound differences appear in long-horizon forecasts, as shown below. A third category of shifting endpoints, developed for FRB/US, draws on private sector perceptions of long-run expectations. The remainder of this subsection indicates typical effects of alternative endpoints on long-horizon forecasts and reviews the historical measurements used in FRB/US to represent shifting endpoint perceptions of firms, households, and investors.

Constant and moving-average endpoints. To illustrate long-horizon forecast effects of the two endpoints used in conventional forecast models, alternative forecasts of the funds rate are displayed in the two panels of figure 2. Forecasts in the top panel are generated by a four-lag autoregression in the *level* of the funds rate. This model is appropriate if the variable is without a trend (stationary). Note that both forecasts shown in the top panel, one starting from the high level of interest rates in 1980:Q1 and the other starting from the relatively low level in 1986:Q4, converge to and remain at a common forecast. This constant endpoint is approximately the mean of the funds rate in the sample used to estimate the autoregression in the level of the funds rate.

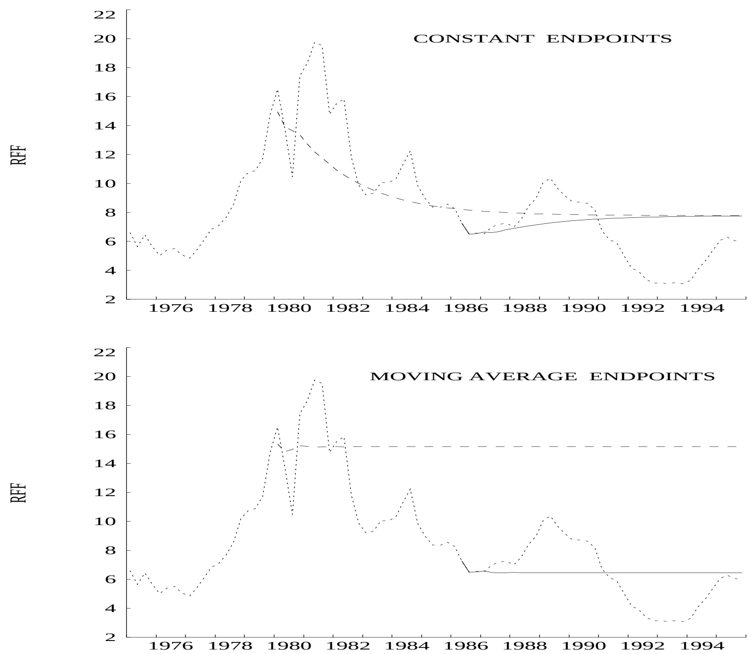
Forecasts in the second panel of figure 2 are generated by a four-lag autoregression in the *first-difference* of the funds rate. This model is often selected if the variable contains a random walk component (making it nonstationary).[20] In this panel, each forecast rapidly converges to and remains at a constant that is near the level of the funds rate at the start of the forecast. This is because, in a first-difference autoregression, the endpoint moves over time and is defined by a weighted moving average of funds rates in the periods immediately prior to the start of the forecast.

A characteristic of the forecasts in both panels of figure 2 is that endpoints are reached rather quickly. Typically, in linear forecasting models of the funds rate, the funds rate endpoint is reached by the fifth or sixth year of the forecast horizon.[21] Consequently, in the case of long-maturity

[20]Many empirical studies in macroeconomics and macrofinance assume postwar nominal interest rates and inflation rates contain random walk components.

[21]Obviously, closure is faster for forecasts that start near the endpoint and slower for initial forecasts that are farther away, such as the forecast in the top panel of figure 2 that begins in 1980:Q1.

Figure 2: Autoregressive funds rate forecasts with alternative endpoints



bonds such as the 10-year bond equation discussed in section 2, the selection of the expected funds rate endpoint will determine much of the variation in the predicted bond rate. As can be inferred from the behavior of the funds rate predictions in figure 2, bond rate predictions from the autoregression in the level of the funds rate *understate* historical bond movements because the long-horizon forecasts of the funds rate converge to the fixed endpoint. On the other hand, bond rate predictions from the autoregression in the first-difference of the funds rate *overstate* historical bond rate variation because the moving-average endpoint is too sensitive to recent levels of the funds rate.

A shifting endpoint for the funds rate. The combination of the rational expectations assumption that bond rates are an average of expected funds rates over the appropriate maturity horizon, as discussed in section 2, and the fact that forward rates in the second five years of a 10-year bond appear to be dominated by the funds rate endpoint suggests that an average of distant forward rates in the observed term structure will provide a direct estimate of investors' time-varying perceptions of the funds rate endpoint. The historical time series in FRB/US of the perceived funds rate endpoint is based on forward rates in the 10-30 year segment of the term structure. For future reference, this expectations endpoint for the funds rate is denoted by the subscript convention, r_∞.

The empirical fit of bond rate equations using this endpoint, r_∞, is substantially better than those using fixed or moving-average endpoints.[22] However, use of this shifting endpoint measurement merely postpones the task of selecting a model that can predict investor perceptions of the nominal interest rate endpoint.

A shifting endpoint for inflation. By the Fisher identity, the expected endpoint of the nominal funds rate is a weighted average of the expected real rate endpoint (determined by the marginal product of capital) and the expected inflation endpoint.[23] Thus, a source of sizeable movements in the nominal interest rate endpoint is the shifting of investor perceptions of the endpoint of expected inflation. Although survey estimates of individual perceptions of the inflation endpoint such as the Philadelphia survey of 10-year inflation expectations are available in recent years, none are available prior to the major shift in policy in late 1979. In order to estimate a longer historical series and to provide a behavioral description of investors' evolving perceptions of endpoints, Kozicki and Tinsley (1996) develop an investor learning model where individuals sequentially test for statistically significant shifts in the endpoint of expected inflation, given a null hypothesis of an unchanged endpoint. Learning is nonlinear with faster responses to signals of a large change in the inflation endpoint than to signals of small changes. However, movements in the aggregate perception of the inflation endpoint are smoothed because the rate of learning varies among individuals in the economy. The inflation endpoint constructed by this learning model is quite similar to the (discontinued) Hoey survey estimates of inflation expected in the second five years of a 10-year horizon and accounts for most of the sample variability of the shifting nominal interest rate endpoint. The Kozicki-Tinsley series of the inflation rate endpoint is spliced with the Philadelphia estimate of expected 10-year inflation to provide the FRB/US historical estimate of the inflation rate endpoint, π_∞, perceived by the private sector; note that this need not be equal to the long-run policy target for inflation.

4.3 The historical VAR.

Under VAR expectations, all sectors share a condensed description of the aggregate economy represented by a three-variable vector autoregression in aggregate output, inflation, and the federal funds rate (where the latter is selected as a summary indicator of monetary policy). The historical

[22] Explicit contrasts of bond rate predictions from funds rate forecast models with alternative endpoints are presented in Kozicki and Tinsley (1996).

[23] Under the assumption that investors arbitrage after-tax real rates, adjusted for differential risk premia, the weights are functions of the tax rate on investor earnings.

VAR is an average-history estimate of reduced form relationships among these three variables over the 32 year sample beginning in 1963.

Estimated equations of the historical VAR are listed in table 8 for the three summary aggregates: the federal funds rate, r; consumption inflation, π; and the trend deviation in aggregate output (the "output gap"), $\tilde{x}$.

Table 8: The Historical VAR

$\Delta r =$	$.03 \text{ lag}_1(\pi - \pi_\infty) + .12 \text{ lag}_1(\tilde{x} - 0) - .05 \text{ lag}_1(r - r_\infty)$	SEE 1.14
	$+ .33 \text{ lag}_3(\Delta\pi) + .22 \text{ lag}_3(\Delta\tilde{x}) - .27 \text{ lag}_3 (\Delta r)$.	R^2 .30
$\Delta\pi =$	$- .17 \text{ lag}_1(\pi - \pi_\infty) + .13 \text{ lag}_1(\tilde{x} - 0) - .01 \text{ lag}_1(r - r_\infty)$	SEE 1.13
	$- .27 \text{ lag}_3(\Delta\pi) - .17 \text{ lag}_3(\Delta\tilde{x}) + .02 \text{ lag}_3 (\Delta r)$.	R^2 .26
$\Delta\tilde{x} =$	$-.02 \text{ lag}_1(\pi - \pi_\infty) - .04 \text{ lag}_1(\tilde{x} - 0) - .21 \text{ lag}_1(r - r_\infty)$	SEE 1.12
	$+ .09 \text{ lag}_3(\Delta\pi) + .19 \text{ lag}_3(\Delta\tilde{x}) + .08 \text{ lag}_3 (\Delta r)$.	R^2 .33
remarks:	• span 1963q1 - 1994q4.	
definitions:	r - federal funds rate. π - inflation rate of personal consumption deflator (chain weights). $\tilde{x}$ - trend deviation of output.	

The equations in table 8 differ from those in conventional VARs due to the presence of explicit endpoints for each of the variables in the historical VAR. As discussed above, each endpoint represents private sector perceptions of the long run outcome for that variable. The format of the equations in the historical VAR enforces this view where, intuitively, all terms on the left-hand and right-hand sides of the VAR equations are zero in the long run. Thus, in the long run, the funds rate will reach the funds rate endpoint, r_∞; the inflation rate will attain the inflation rate endpoint, π_∞; and the output gap will converge to its endpoint, which is zero.[24]

In contrast to analysis of individual equations, the average speed of adjustment of variables toward long run values in a fully interdependent system, such as the historical VAR, is the same for all variables in the system. This is evident in table 8, since each endpoint deviation appears in

[24] Further discussion of endpoint-deviation formulations of VARs may be found in Brayton and Tinsley (1995) and Kozicki and Tinsley (1996).

all equations; thus, if one endpoint deviation is nonzero, then all endpoint deviations generally will be nonzero.

Unlike the structural equations in FRB/US, the equations in VAR models are reduced forms so direct behavioral interpretations are ordinarily not possible. However, under the assumption that the funds rate is the variable most responsive to "news", the funds rate equation may be interpreted as an average-history representation of policy responses to shocks and observed movements of inflation and output. Under this interpretation, the first equation in table 8 indicates that historical policy generally increased the funds rate if either inflation or output were above their endpoints or the funds rate was below its endpoint.[25]

5 Full-System Properties

This section provides an overview of the system properties of FRB/US and demonstrates through a few examples how the model can be used to analyze a rich set of forecast and policy questions, including cases where the public has imperfect knowledge of policy objectives or where monetary or fiscal policy lack credibility. As discussed in previous sections, the dynamic behavior of sectors depends significantly on expectations of households, firms, and financial markets, including anticipations of policy. Assumptions about how expectations are formed, such as the scope of information or the speed of learning, can be tailored in FRB/US to address a wide range of issues.

Unless otherwise indicated, simulations in this section assume that monetary policy responds to economic conditions according to the VAR equation for the federal funds rate in table 8 that reflects the average-sample behavior of historical policy. In figures below, this simulated monetary policy is referenced as the *average historical policy*. To highlight the role of private sector expectations in the transmission of policy effects, simulation results are shown for two cases of expectations formation. Under *VAR expectations*, firms and households use the same estimated VAR to generate forecasts of the future that was used in estimating the FRB/US equations. Under *full-model expectations*, expected values of future variables equal the values forecast by the full FRB/US model.

5.1 System responses to transitory shocks

To provide a brief introduction of the system properties under different assumptions regarding expectation formation, the initial simulations indicate responses of the economy to transitory

[25] Note this interpretation is based on *symmetric* policy information under the implicit assumption that the endpoints of operational policy in the funds rate equation are the same as those perceived by private sectors in the remainder of the model.

shocks in the federal funds rate and in aggregate demand (government spending). Beyond showing representative dynamic responses of key macroeconomic variables in FRB/US, these exercises demonstrate under what conditions VAR expectations differ from full-model expectations and how such differences affect macroeconomic outcomes. The general rule that can be extracted from these (and similar) simulations of transitory shocks is:

Rule of Thumb 1 *For transitory shocks, the dynamic response of most main macroeconomic aggregates differs between VAR expectations and full-model expectations to the extent the shocks applied to the system deviate from average historical experience.*

VAR expectations are based on the historical behavior of macroeconomic aggregates. For shocks that are not unusual in an historical perspective, the summary macroeconomic responses provided by the VAR contain most of the information needed to predict the responses of the full FRB/US model. In contrast, when the simulation experiment strays from the typical pattern of shocks in the economy, differences between VAR and full-model expectations emerge.

Figure 3 shows the responses of inflation, output, the federal funds rate, and the 10-year government bond rate to a one-quarter, 100-basis point positive shock to the VAR equation for the federal funds rate.[26] In this and most of the following figures, the economy's response under full-model expectations is shown as the solid line, while the dashed line represents the results under VAR expectations. Results in all instances are displayed as deviations from a baseline forecast. As the figure shows, the presence of lagged endogenous variables in the funds rate equation amplifies the initial impulse, and the funds rate stays above baseline for about two years. The higher level of the funds rate brings about a decline in aggregate demand and downward pressure on prices. Although not shown in the figure, eventually all variables return to their baseline values as the effects of the shock wear off.

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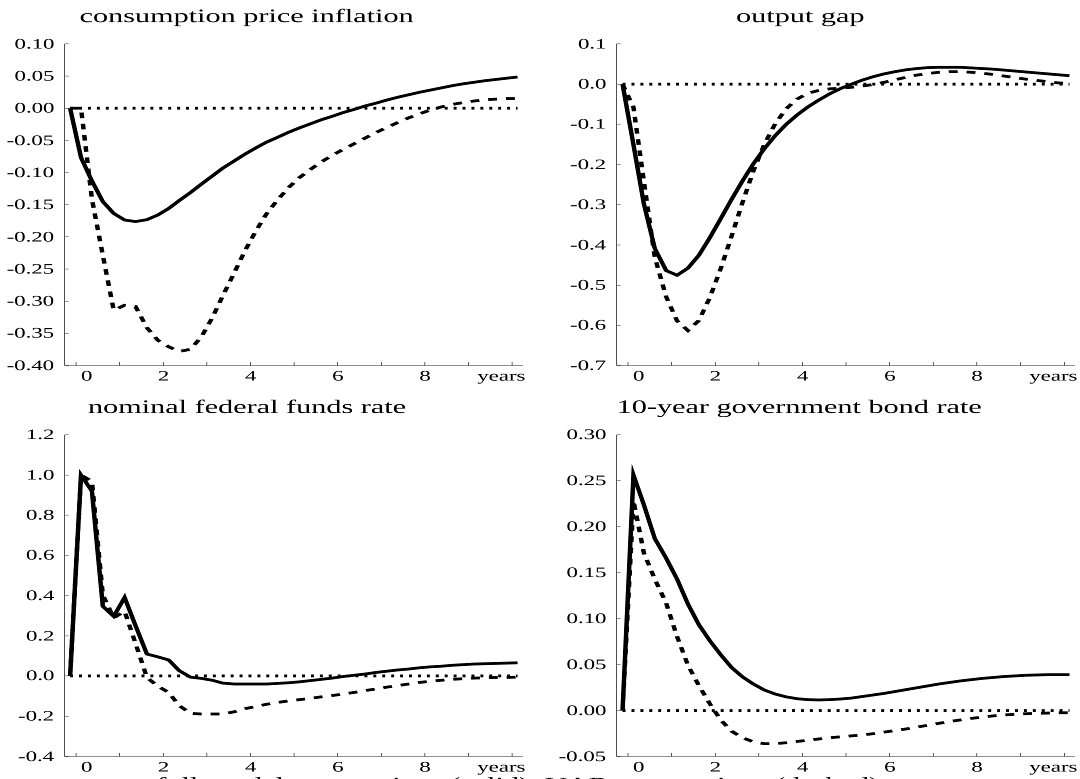
[26] In order to simplify the design of the transitory shocks, each simulation of this type is based on the assumption that the long-run inflation objective of monetary policy is unchanged, as are private perceptions of the policy objective.

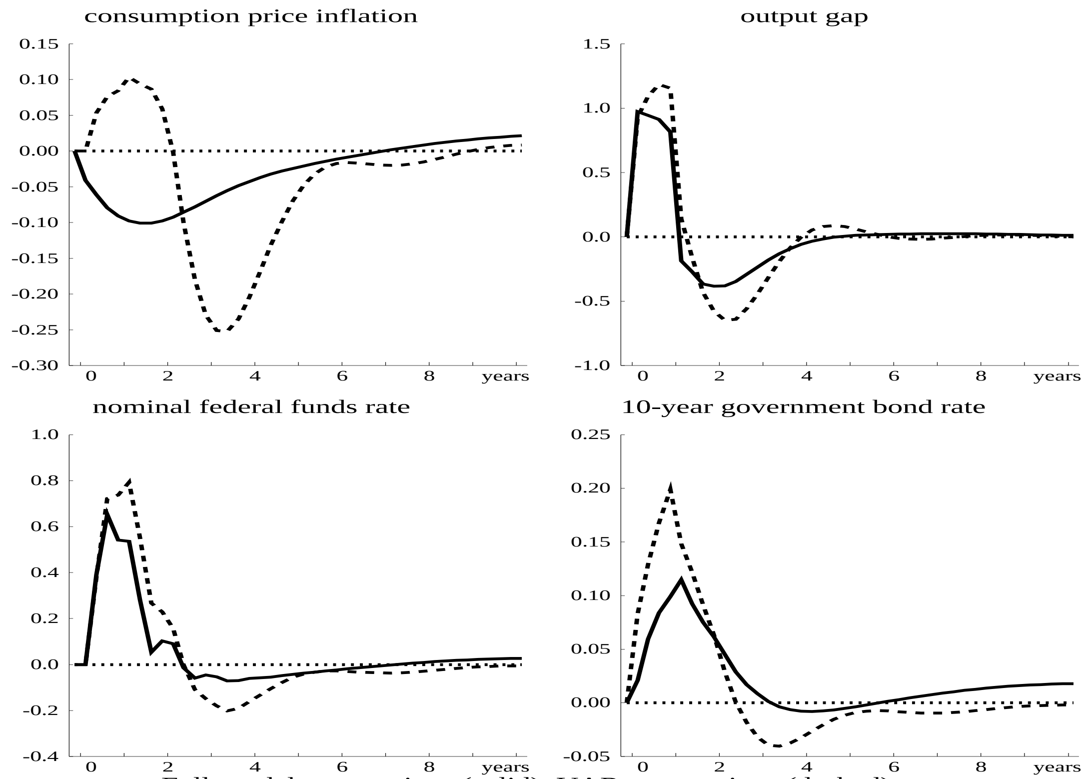
Figure 3
One-quarter, 100-basis-point shock to the federal funds rate
Average Historical Policy
(deviations from baseline, per cent)



full-model expectations (solid); VAR expectations (dashed)

The responses to the interest rate shock under full-model and VAR expectations are quite similar, except for the response of inflation which is somewhat damped under full-model expectations. The relatively minor differences between the dynamic responses under the two types of expectations can be traced to two sources. First, the dynamics of the VAR model, while close, are not identical to the dynamics of the full FRB/US system in this instance, introducing some bias in VAR expectations. To some degree, agents are not using the right model to form expectations of future events. Second, a one-period difference in timing of some responses is due to the assumption under VAR expectations that anticipations generally are formed at the beginning of the quarter and do not depend on contemporaneous information, while full-model expectations implicitly take into consideration observations in the current period.

Figure 4
Four-quarter shock to government spending equal to 1% of GDP
Average Historical Policy
(deviations from baseline, per cent)

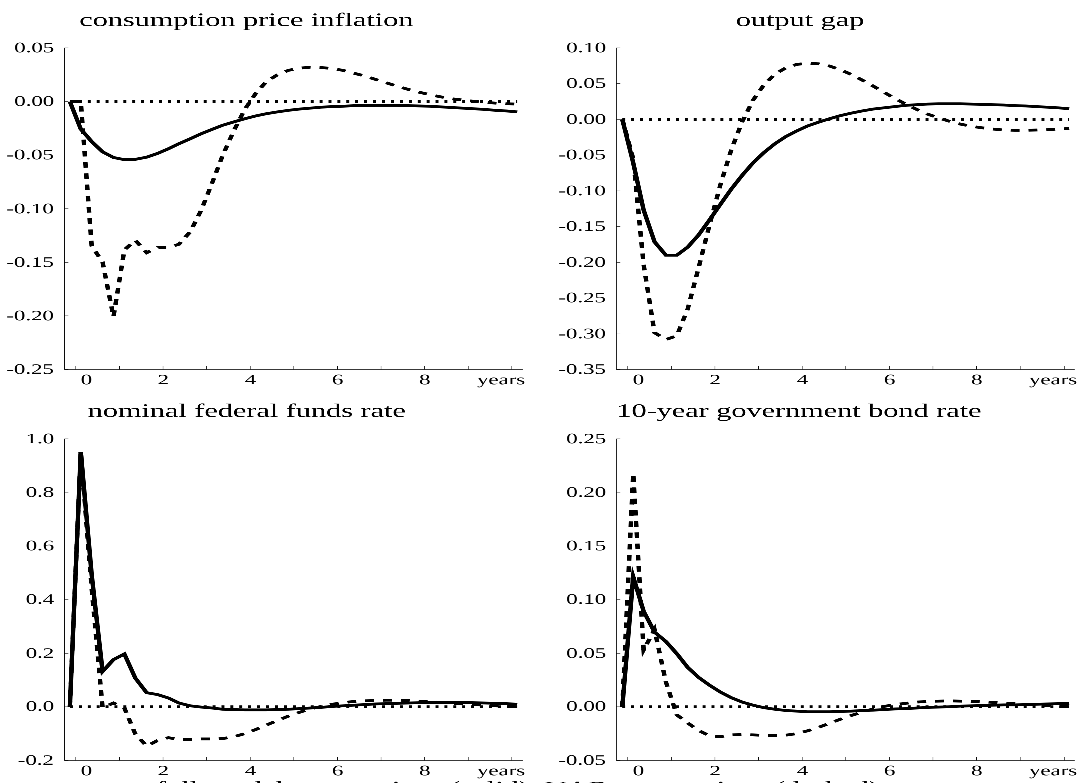


Full-model expectations (solid); VAR expectations (dashed)

Figure 4 shows the response to an increase in government purchases, equal to one percent of GDP, lasting one year. In this experiment, an additional source of expectations bias is introduced into VAR expectations: The hypothesized duration of the demand shock does not correspond to the average historical serial correlation of output. Historically, deviations of output from potential tend to amplify themselves initially and then to die out gradually. Under VAR expectations, firms and households do not "see through" the four-quarter design of the shock and expect output (and inflation) to follow typical historical patterns. The perception of a high level of future activity and inflation drives up prices today. Once the shock ends, expectations of future activity and inflation are revised downward, dampening wage and price inflation. By contrast, under figure 4's example of full-model expectations, firms and households have perfect foresight about this shock and know the spending impulse will only last one year. This leads to a smaller rise in the bond rate and no discernable rise in inflation. In fact, for nearly all of the first eight years, inflation is below baseline. The fall in inflation is due to effects of future low activity and inflation and also to effects of the appreciation of the exchange rate caused by the rise in bond yields. Agents make ex post expectational errors using VAR expectations; but, if the true duration of the shock is not known beforehand, the responses under VAR expectations may be considered reasonable.

The government purchases scenario shown in figure 4 demonstrates how deviations of hypothetical shocks from historical behavior introduce a wedge between VAR expectations and full-model expectations. Figure 5 illustrates another way differences can arise between the two types of expectations—a shift in the actual responsiveness of monetary policy to output and inflation deviations that is not reflected in VAR expectations, at least over the simulation interval. The policy used in this simulation is one estimated over the shorter sample period from 1979 to 1995 (termed the *post-1970's policy*) and is more aggressive in combatting output and inflation deviations than is the average historical policy.

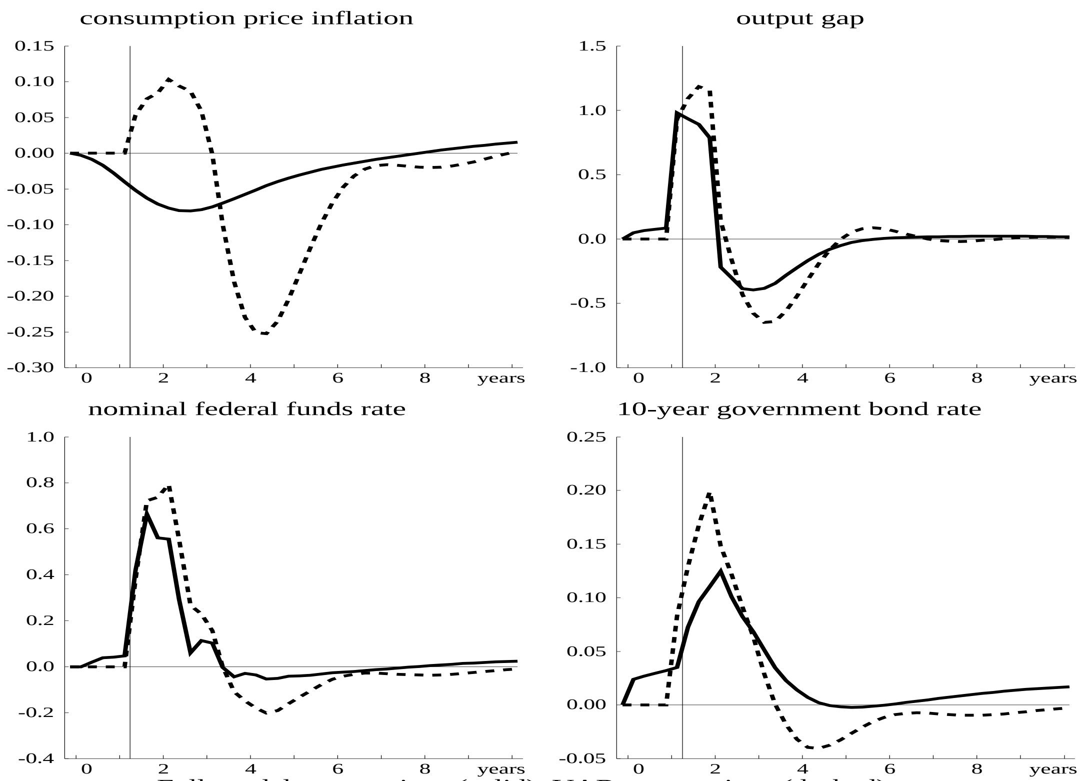
Figure 5
One-quarter, 100-basis-point shock to the federal funds rate
Post-1970s Policy
(deviations from baseline, per cent)



full-model expectations (solid); VAR expectations (dashed)

The simulations reported in figure 5 repeat the interest rate increase shown in figure 3, but with actual monetary policy determined by the average post-1970s policy. In this example, the revised policy is captured by full-model expectations but not by VAR expectations. Differences in responses under VAR expectations from those under full-model expectations are larger than in figure 3, because an additional source of expectations bias has been introduced due to the misperception of monetary policy under VAR expectations, which continue to be based on the average historical characterization of monetary policy. In particular, under VAR expectations the federal funds rate is anticipated to persist at an elevated level far longer than the actual policy entails. This leads to an overly pessimistic view of future output and prices, and an exaggerated response of output and inflation to the shock.

Figure 6
Anticipated future four-quarter shock to government spending
Average Historical Policy
(deviations from baseline, per cent)



Full-model expectations (solid); VAR expectations (dashed)

Finally, implications of perfect foresight are indicated by contrasting the effects of a future shock that is foreseen in one case and unexpected in the other. Figure 6 shows the same four-quarter government purchases shock as in figure 4, with the exception that it occurs one year in the future (at the date designated by the vertical line). For full-model expectations, the shock is assumed to be foreseen in advance whereas under VAR expectations it is unexpected and there is no reaction until the shock occurs. In both cases, monetary policy follows the average historical policy. For VAR expectations, the responses are identical to those in figure 4 aside from a one-year delay. Under full-model expectations, inflation and output rise with the announcement of the future spending increase. The initial rise in inflation is due to labor and product markets that are foreseen to be tighter in the future. Once the fiscal expansion is in full swing, however, inflation begins to fall because of the anticipated weakening of output after the end of the temporary increase in government spending. Also, long bond rates rise upon the announcement of the policy change under full-model expectations, due to the anticipated rise in the federal funds rate.

5.2 System responses to permanent shocks

Thus far, simulations have illustrated the effects of expectation formation on system dynamics for temporary disturbances. Discussion now turns to analysis of a permanent change in monetary policy, where the experiment is a policy that aims to reduce the inflation rate permanently by one percentage point within ten years. Any number of funds rate paths can achieve this objective; in the simulations that follow, the funds rates set by policy are consistent with the planned reduction in inflation but otherwise respond to movements in observed output and inflation using the average post-1970's responses discussed above. We consider two cases of policy credibility. In the first case of "perfect credibility," the private sector fully believes that the announced disinflationary policy will occur as planned. In the second case of "learning," the private sector only slowly adjusts its views about the probability that the full disinflationary program will be carried out. In the latter case, the rate of adjustment in the inflation endpoint is 5% per quarter, so that long-run inflation expectations will have fallen by one-half of one percentage point after 3 1/2 years.[27] These simulations provide the basis for a second rule of thumb:

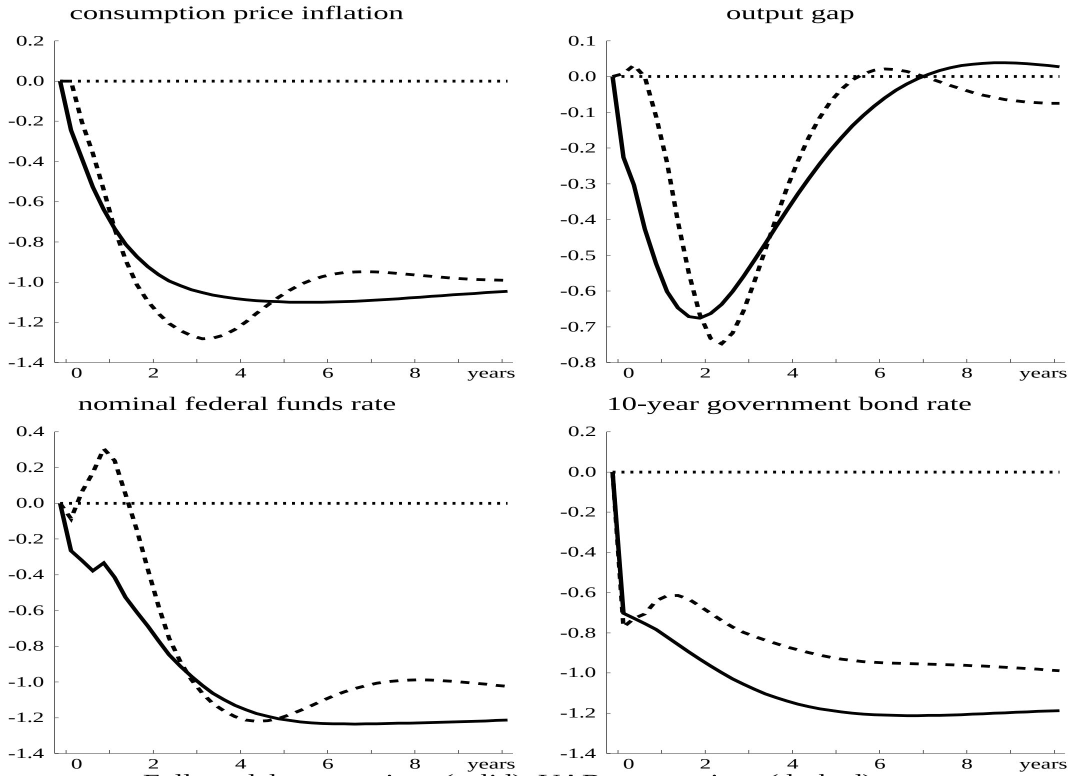
Rule of Thumb 2 *The cost of disinflation, in terms of lost output and employment, is decreasing in the degree of credibility of the policy.*

Figure 7 shows the consequences of a credible policy of disinflation. As with the transitory monetary policy shift of figure 3, there is little difference between the outcomes under VAR and full-model expectations. With the change in the policy taken as known, the information contained in the VAR is sufficient to understand the responses of the full FRB/US model. Sacrifice ratios (cumulative annual increase in the unemployment rate divided by the percentage point decrease in the inflation rate) are also similar. For VAR expectations, the sacrifice ratio is 1.3; for full-model expectations it is 1.7.[28]

[27] This rate of inflation endpoint learning is consistent with the fall in long-run expectations measured by surveys during the disinflation of the 1980's.

[28] Sacrifice ratios are computed at the end of the tenth year of the simulations.

Figure 7
Permanent disinflation of one percentage point
Post-1970s Policy
(deviations from baseline, per cent)



Full-model expectations (solid); VAR expectations (dashed)
Instantaneous recognition of change in target inflation

The assumption of perfect credibility of the disinflationary policy is removed in figure 8.[29] Inflation declines more gradually in this case, as the dampening effects of the credible policy on expected inflation are attenuated in the case of gradual learning. Also, the rapid decline in bond rates in the case of perfect credibility is absent under imperfect credibility. Bond traders, like all agents in the economy, only gradually adjust their views about the long-run objectives of policy. The higher real interest rates generated by this disinflationary policy lead to losses of output that are significantly greater than those under perfect credibility. In terms of the sacrifice ratio, the effect of imperfect credibility is to increase the cost of disinflation from 1.3 to 2.6 for VAR expectations and from 1.7 to 2.3 for full-model expectations.

The credibility of monetary policy is not the only aspect of policy that affects the output and employment cost of disinflation in FRB/US. Also important is the speed at which policy attempts to reduce inflation, with the cost being higher the faster is the desired reduction

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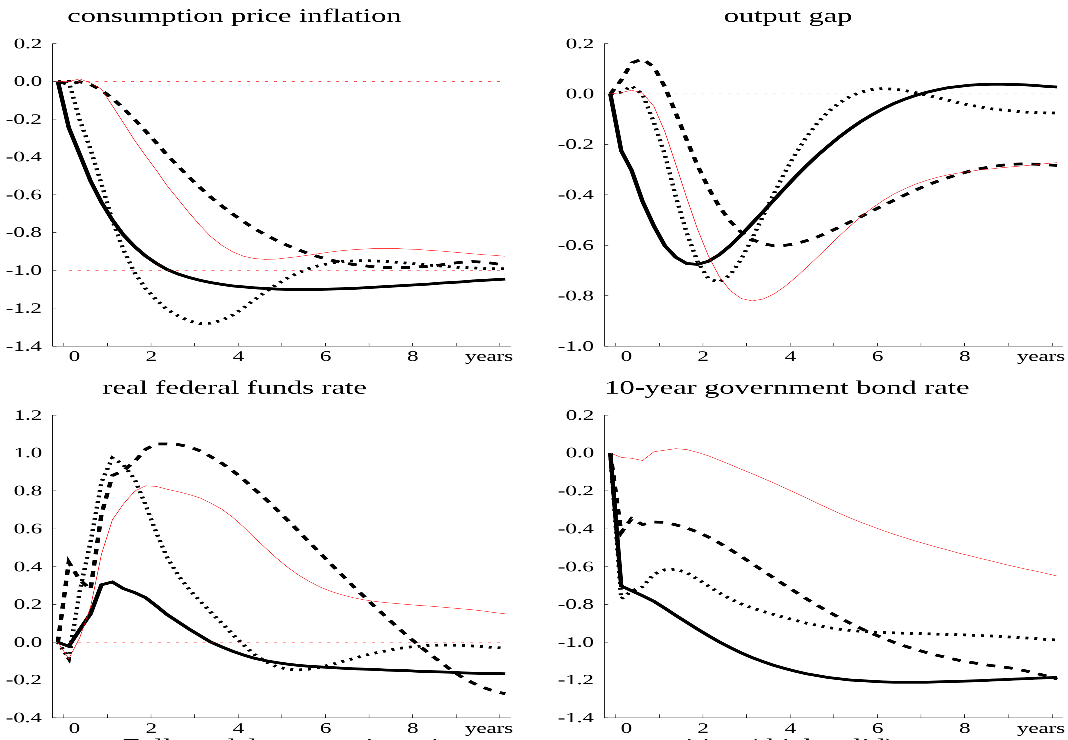
[29]For ease of comparison, the figure also repeats the simulated responses found under full credibility. Unlike the other figures, the federal funds rate is plotted in real terms in figure 8.

Figure 8
Disinflation, with and without learning
Post-1970s Policy
(deviations from baseline, per cent)



Full-model expectations, instantaneous recognition (thick solid)
Full-model expectations, learning, 5% rate (dashed)
VAR expectations, instantaneous recognition (dotted)
VAR expectations, learning, 5% rate (thin solid)

in inflation. In a model where inflation depends only on past observations, the cost of a permanent disinflation is invariant to the speed at which the disinflation occurs. In a model where inflation depends on the expected future values, this invariance disappears. A credible policy to reduce inflation that affects variables in the future will also affect the present. Thus, an effective way of reducing current inflation at little cost in terms of lost output is to “announce” a restrictive future policy that reduces expectations of inflation. The private sector is not being “fooled” because policy must generate a reduction in output below potential at some point to be consistent with the reduction in inflationary expectations. The key is that much of the reduction in inflation is accomplished through lower inflation expectations, as opposed to operating only through reduced aggregate demand. The effect of the speed of disinflation on the sacrifice ratio is illustrated by comparing outcomes for the two “average” descriptions of historical policy. Under VAR expectations with full endpoint credibility, the sacrifice ratio falls from 1.3 to 1.1 if the more gradual average historical policy is substituted for the average post-1970's policy used in figures 7 and 8.

Even though FRB/US is an empirically estimated model with well-behaved statistical properties, the simulations reported in this section demonstrate that the structural design of FRB/US is suitable for analyses aimed at a broad range of macroeconomic policy questions. The model has the flexibility to examine policy issues under different assumptions about policy credibility and the extent of economic information upon which expectations are based.

A Appendix: Testing the Theory

The goodness of fit of the main structural equations in the model is summarized by the proportions of explained variation, R^2, and the standard deviations of equation residuals, SEE, reported in tables 1 through 7 in the main text. This appendix presents two additional empirical tests directed at assessing the adequacy of the theoretical specifications of dynamic adjustments and the assumption of rational expectations.

The first is a test for serial independence of the residuals to determine if the generalized adjustment cost specifications are able to describe dynamic behavior adequately or if significant correlations in the data remain unexplained. The test in the first column of numbers in table A1 indicates the significance of autocorrelations of an equation residual with any of its first twelve lags. The entries in this column are rejection probabilities (p-values) of the null hypothesis of serially independent residuals. A p-value of .05 (.01) or less indicates rejection of the serial independence hypothesis with at least a 95$\%$ (99$\%$) level of confidence. The entries in this column suggest that eight (ten) of the twelve equations examined have white noise residuals.

The second test examines coefficient restrictions imposed by the VAR-based implementation of rational expectations (RE). Using the example of the aggregate price equation, the FRB/US price equation presumes that firms use the VAR to generate predictions of the equilibrium price. These predictions are then weighted by the lead response weights of the price equation (shown earlier in figure 1) to determine the estimated price change in the current period. The potential information in the VAR consists of lagged values of all variables included in the VAR model. Although this information is organized by the VAR to produce minimum mean square errors in predicting the equilibrium price, it may be that firms prefer to organize this information in some other way, such as rule-of-thumb extrapolations. The test in the second column of numbers in table A1 augments the FRB/US dynamic adjustment equation with lagged values of the variables in the sectoral VAR as additional regressors. If the additional VAR regressors are statistically significant then the p-values in the second column will be low, indicating that firms are not using rational expectations (at least as defined by VAR forecasts) in their dynamic adjustment equations. Again, a p-value of .01 or less indicates rejection of RE restrictions with at least a 99$\%$ level of confidence. The entries in the second column suggest that rational expectations restrictions are not rejected for ten of the twelve equations examined.

Table A1: Tests for Serially Independent Residuals and RE Restrictions [a]

equation	serially independent residuals	RE restrictions
aggregate consumption	.24	.30
consumer durables, motor vehicles	.28	.12
other consumer durables	.28	.19
residential investment	.01	.01
producers' durable equipment	.67	.72
inventory investment	.24	$<$ .01
labor hours	.02	.09
aggregate price	.20	.71
wage	.04	.27
5-year Treasury bond rate	.01	.78
10-year Treasury bond rate	.07	.68
corporate bond rate	.37	.91

[a]table entries are rejection probabilities (p-values); a low p-value indicates rejection of a null hypothesis (white noise residuals or RE restrictions).

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