The Aggregate Change in Shares and the Level of Stock Prices

William R. Nelson

Federal Reserve Board

January 1999

ABSTRACT

The average change in shares of equity is negatively correlated with estimates of the equity premium calculated using the dividend-ratio model of Campbell and Shiller, as well as with a variant of the model written in terms of the earnings-price ratio. This correlation is consistent with corporations issuing equity when it is a relatively inexpensive source of finance and repurchasing equity when it is a relatively good investment. However, when the retirements of shares resulting from mergers are included, the average change in shares is no longer significantly correlated with the equity premium.

Keywords: Equity issuance, Equity repurchase, Dividend-ratio model, Earnings-price ratio

JEL Classification: G14, G35

---

1 Mail Stop 84, Federal Reserve Board, Washington D.C. 20551, Phone: (202) 452-3579, Fax: (202) 452-2301, Email: wnelson@frb.gov. The analysis and conclusions in this paper are those of the author and do not indicate concurrence by other members of the research staffs, by the Board of Governors, or by the Federal Reserve Banks. William Brainard, Matthew Shapiro, Robert Shiller, William English, Spencer Krane, and Athanasios Orphanides provided helpful comments. Any remaining errors are my own.
The Aggregate Change in Shares and the Level of Stock Prices

Introduction

The predictive power of the issuance or repurchase of shares for future stock returns is well documented (see, for example, Nelson (1999a and b), Loughran and Ritter (1995), and Ikenberry, Lakanishok and Vermaelen (1995)). After firms issue equity their stock tends to do poorly and after firms repurchase equity their stock tends to do well. This pattern of returns suggests firms are more inclined to issue overvalued shares and repurchase undervalued shares, a hypothesis supported in Nelson (1999b) by examining the balance sheet characteristics of firms engaged in equity activity. In that paper, Nelson finds that firms that issue shares tend to appear overvalued by some measures and those that repurchase shares appear undervalued.

The aggregate change in shares may predict total market returns for the same reasons that the change in shares is correlated with stock returns in a cross-section. When the expected returns implied by the valuation of the market are low relative to the yields on other assets (stock prices are high) corporations can access a relatively cheap source of finance by issuing shares. Conversely, when the expected returns are high (stock prices are low) corporations may find their own shares an attractive investment.

One way to evaluate this hypothesis is to see if the change in shares forecasts the stock market. As shown below, the change in shares does, to a limited extent, predict the subsequent five-year premium of the stock market over a riskless rate. Another way is to test if the change in shares is correlated with a measure of the variation in the level of the stock market attributable to changes in the expected equity premium at all future horizons.
I use the Campbell-Shiller (1988, 1989) dividend-ratio model, and also a modification of that model written in terms of earnings, to estimate the expected equity premium. Whereas the Campbell-Shiller model relates the dividend-price ratio to the expected growth in dividends, the modified model presented here relates the earnings-price ratio to the growth of earnings and future payout ratios. The estimates of the expected equity premium generated by the two models are quite similar, and in both cases the change in shares outstanding is negatively correlated with the premium, although the relationship is more precisely estimated by the dividend-ratio model than it is by the modified model.

But the definition of the change in shares matters importantly for this finding. The average change in shares of continuing firms—the average of the variable that is correlated with returns in the cross section—is negatively correlated with the expected equity premium. When the decline in shares resulting from mergers—which likely are motivated by different considerations than the equity repurchases of ongoing operations—are included in the analysis, the average change is not significant. This difference may help explain the situation in the first half of 1998 when the market, by all measures, appeared richly valued, but corporations were, on net, retiring a large amount of equity. Excluding the retirements owing to mergers, corporations were issuing equity.

In what follows, I first derive the estimate of the expected equity premium using the Campbell-Shiller dividend-ratio model and the model rewritten in terms of earnings. After comparing the estimates of the expected future equity premium implied by the two models, I examine the relationship between these estimates and the change in aggregate shares outstanding.
The dividend-ratio model and the earnings-ratio model\footnote{This section and appendix A are based in large part on a manuscript written while a graduate student at Yale in 1993.}

In order to evaluate the relationship between the change in shares and the relative cost of equity finance it is useful to construct a measure of expected future stock returns. The dividend-ratio model developed by Campbell and Shiller (1988, 1989) provides such an estimate. The model decomposes the dividend-price ratio into two pieces, one attributable to expected dividend growth, and a residual which is a measure of expected future stock returns. The model resembles a stochastic version of the Gordon formula which states that when dividends are growing steadily at rate \( g \), and when price equals the present value of dividends discounted at rate \( r \), the dividend price ratio equals \( r - g \).

Since dividends are by definition the product of earnings and the payout ratio (the ratio of dividends to earnings), it is relatively straightforward to rewrite the Gordon and the Campbell-Shiller models to explain the earnings-price ratio in terms of earnings growth and payout ratios.\footnote{Appendix A presents a formula for the earnings-price ratio similar to the Gordon formula for the dividend-price ratio.} One reason to make this transformation is that because earnings are more closely related to economic fundamentals than are dividends and the payout ratio is a choice variable of the firm, the alternative model may be better specified, have more economic content, and could potentially better predict the stock market. For example, Campbell and Shiller (1997) find that the ratio of a ten-year moving average of earnings to lagged stock price does a better job of predicting stock price movements than does the dividend-price ratio, and when estimating the dividend-ratio model, Campbell and Shiller (1988) include a long
moving average of the earnings-price ratio as a component of their VAR, although they do not explicitly model the earnings-price ratio. Another reason to model the earnings-price ratio is that market participants pay closer attention to price-earnings ratios than dividend yields, so a model written in terms of earnings may be of general interest and may provide a framework for future research.

The dividend-ratio model of Campbell and Shiller uses a linearization of the log stock return to express the dividend-price ratio as a function of expected future returns and the expected future growth in dividends. Letting \( r_t \) be the return, \( P_t \) the end-of-period price, and \( D_t \) the dividend, the return is defined as

\[
    r_{t+1} = \log(P_{t+1} + D_{t+1}) - \log P_t. \tag{1}
\]

Campbell and Shiller show that the return can be well approximated by

\[
    r_{t+1} = -\rho \delta_{t+1} + \delta_t + \Delta d_t, \tag{2}
\]

where \( \delta_t \) is the log of the dividend price ratio, \( d_t \) is the log of dividends, \( \rho \) is the sample average of the ratio of price to the sum of price and dividend, and all variables including \( r_t \) are deviations from their mean levels.

Solving equation 2 forward yields

\[
    \delta_t = \sum_{j=1}^{\infty} \rho^{j-1} (r_{t+w} - \Delta d_{t+w}). \tag{3}
\]

an expression which relates the log dividend-price ratio to future returns and growth in
A similar procedure yields an expression for the log earnings-price ratio. Defining \( \varphi_t \) to be the demeaned log of the ratio of dividends to earnings (the payout ratio), \( \lambda_t \) to be earnings, \( \varepsilon_{\text{d}} \) to be the demeaned log of earnings, and \( \varepsilon_t \) to be the demeaned log of the earnings-price ratio, equations 1, 2 and 3 can be rewritten as

\[
\begin{align*}
    r_{t+1} &= \log(P_{t+1} + \lambda_{t+1} \varphi_{t+1}) - \log P_t, \tag{4}
    \\
    r_{t+1} &= -\rho \varepsilon_{t+1} + \varepsilon_t + \Delta \varepsilon_{t+1} + (1-\rho)\varphi_{t+1}, \tag{5}
    \\
    \varepsilon_t &= \sum_{j=1}^{\infty} \rho^{j-1} \left( r_{t+j} - \Delta \varepsilon_{t+j} - (1-\rho)\varphi_{t+j} \right). \tag{6}
\end{align*}
\]

Equation 5 can be derived from equation 2 by appropriate addition and subtraction of the log of earnings. Although equation 6 can be obtained from equation 3 in a similar way, the simplest way to derive equation 6 is to solve equation 5 forward.

Note that in making the transformation from equations 1-3 to equations 4-6, earnings could be replaced with any variable as long as the payout ratio and the earnings-price ratio are appropriately redefined. Furthermore, the earnings-ratio model is based on the same linearization (not just a similar linearization) of the log return used to derive the dividend-ratio model.

I continue to follow Campbell and Shiller by noting that since equation 6 holds ex-
post, it also must hold in expectation. I will refer to the expectation-version of equation 6 as the earnings-ratio model. Letting \( E_t \) denote expectations conditional on information at time \( t \), and utilizing the fact that both the dividend-price ratio and the earnings-price ratio are known at time \( t \), the dividend-ratio model states

\[
\delta_t = E_t \sum_{j=1}^{\infty} \rho^{j-1} \left( r_{t+j} - \Delta d_{t+j} \right), \tag{7}
\]

and the earnings-ratio model states,

\[
\epsilon_t = E_t \sum_{j=1}^{\infty} \rho^{j-1} \left( r_{t+j} - \Delta e_{t+j} - (1-\rho) \varphi_{t+j} \right). \tag{8}
\]

It will be useful to rewrite these models in terms of the expected difference between the market return and the riskless return. Letting \( r^m \) and \( r^b \) refer to the return on the market and on Treasury bills, the sum of the expected future spread of the stock market over the bill rate can be written as

\[
E_t \sum_{j=1}^{\infty} \rho^{j-1} \left( r^m_{t+j} - r^b_{t+j} \right)
= \epsilon_t + E_t \sum_{j=1}^{\infty} \rho^{j-1} \left( \Delta e_{t+j} - r^b_{t+j} + (1-\rho) \varphi_{t+j} \right) \tag{9}
\]

or

\[
= \delta_t + E_t \sum_{j=1}^{\infty} \rho^{j-1} \left( \Delta d_{t+j} - r^b_{t+j} \right).
\]

Forecasts of the right-hand side variables, which I generate below using VARs, allow
for estimation of the sum of expected discounted equity premia. In the discussion below I refer to the expected discounted sum multiplied by \((1-\rho)\) as the equity premium. The equity premium by this measure is the expected weighted average difference in return between the stock market and the riskless rate, with more weight placed on imminent returns than on returns in the distant future. Note that after subtracting the bill rate from the earnings or dividend growth rates all of the variables are real, so no correction for inflation is necessary. While the models are equally valid specified in terms of nominal or real variables, the estimation in terms of real variables avoids the complications that arise from the possibility that inflation, and therefore nominal variables, may be nonstationary. Subtracting the bill rate thus allows for a simpler specification, since it is not necessary to include the rate of inflation in the VAR.\(^4\)

A brief aside on share repurchases and the dividend-ratio model

Given the subject of this paper, it is worth attempting to clarify the relationship between share repurchases, the dividend-price ratio, and the growth rate of dividends. A corporation that adopts a share repurchase program, while keeping constant the amount it transfers to its shareholders through dividends and repurchases combined, will experience a reduction in its dividend-price ratio, but the growth rate of its dividends per share will increase as a result of the repurchases, leaving the dividend-price ratio still well described by the Gordon formula, or the Campbell-Shiller dividend-ratio model.

\(^4\) Similarly, Mankiw, Romer, Shapiro (1991), when testing for the excess variability of stock prices consider the sum of future nominal dividends discounted by the nominal riskless rate plus a constant equity premium, avoiding the measurement errors that arise from deflating by a price index.
To see this, consider a firm that has one infinitely divisible share on which it pays $10 in dividends each period that are discounted at a 10 percent interest rate. Applying the Gordon formula \((D/P = r - g)\), the dividend price ratio will be 10 percent and the price of the firm’s stock will be $100. If the firm decides instead to buy back 5 percent of its shares each year and still pay out $10 a year, the firm will payout $5 to buy back shares and $5 in dividends, and the amount of shares outstanding will shrink 5 percent each year. Since the overall value of the corporation will not change, the price and dividend per share of the stock will grow at 5 percent each year. The dividend-price ratio will fall to 5 percent, but the Gordon formula will still hold. Furthermore, the dividend-price ratio will still be a sufficient statistic for the growth in dividends, consistent with the logic of the dividend-ratio model.

Since the dividend-ratio model remains valid, it is not necessary to take into account net share repurchases when implementing it. However, as noted by Cole, Helwege, and Laster (1996), the growing popularity of share repurchase programs could help account for the low dividend-price ratios prevailing in the late 1990s, although after taking into account share issuance as well as repurchases they estimate the effect to be small. Furthermore, a factor not

---

5 This ignores the different tax treatment of dividends and repurchases. Repurchases are generally more attractive than dividends for tax reasons because the stock owners have the ability to choose when to be assessed the capital gains tax, and because capital gains taxes have generally been lower than income taxes for higher tax brackets. Because of these tax advantages, a change from dividends to repurchases may increase the value of the firm.

6 In fact, it would be incorrect to do so unless the entire analysis was conducted in terms of the sum of all dividends and the total capitalization of the corporations in the sample, as opposed to the per-share dividend and price. Intuitively, the present-value formula should value a stock under all feasible investment strategies, including that of holding the stock forever, ignoring share repurchases or offerings. Alternatively, the present-value formula values the stock for an investor planning to own all the stock of a corporation, buying up offerings and selling back repurchases. The first strategy is best described using the per-share data, the second using the total market data. The analysis here, as elsewhere, is conducted using per-share dividends, earnings, and prices.
noted by Cole et al. is that per-share dividend growth has not increased as would need to be the case if repurchases were driving down dividend-price ratios.

The data

I will now use the dividend-ratio and the earnings-ratio model to test the predictive power of the change in shares for future market returns. To do so, I need a measure of the market return, a measure of the riskless rate, and a measure of the average change in shares. In order to estimate the dividend- and earnings-ratio models I also need data on aggregate corporate earnings, dividends, and stock price per share. All the data are annual and begin in 1926. The data on earnings, dividends, and stock prices are from Shiller (1989), updated by Shiller through the end of 1997. The price series is the January level of the Standard and Poor’s Composite Stock Price Index and the dividends and earnings are the twelve month average of the per-share levels. The data on the Treasury bill rate are from Ibbotson Associates (1997) and are the annual return to rolling over each month the shortest Treasury bill available having a maturity of at least one month.7

The data on changes in shares and the market-weighted average return are taken from the December 1997 CRSP data file. The change is based on the market-weighted average of the monthly change in shares of each issue of common equity—corrected for stock splits and stock dividends—listed on the file. The file covers the NYSE, beginning in 1962, the AMEX and, beginning in 1972, the NASDAQ. The monthly changes are then accumulated to form an annual change. Table 1 presents descriptive statistics of the data used in the VAR and the

---

7 Treasury bill rates for 1997 are from the Federal Reserve Statistical Release H.15.
data are plotted in figure 1.

The decline in shares that occurs when a corporation's equity is retired in its entirety, as when a firm is liquidated, acquired, or goes private, is not included in this measure of the average change in shares because firms that disappear during the month do not have a valid change in shares and are therefore excluded from the overall average. In many cases—liquidation for example—this treatment is correct. However, the appropriateness in the case of mergers is less clear. The elimination of shares in the course of a merger is, on the one hand, a retirement of shares and should be included. However, merger-related retirements are less a statement about management's view of the relative value of their stock (at least the management of the acquired firm) than are standard repurchases, and their inclusion may reduce the forecasting power of the change in shares for expected returns. Furthermore, the predictive power of the change in shares for the cross-section in stock returns documented in Nelson (1999a and b) applies, by necessity, to changes that do not result in the termination of the stock. Given this ambiguity, I examine two measures of the average change in shares: one that includes and one that excludes merger-related retirements.8

The solid line in the top panel of figure 2 plots the average change in shares without the terminal changes and the solid line in the middle panel is the average change including merger-related retirements. The dashed lines are the ratio of net share issuance to the market value of equity for nonfinancial corporations from the Flow of Funds accounts (Federal Reserve Board). All series are four-quarter moving averages. When the mergers are

---

8 I have classified equity issues that are delisted because they are acquired in a merger and those that are delisted because they are acquired in an exchange of stock as merger-related retirements.
included, the average change in shares calculated from the CRSP data tracks the change calculated using the Flow of Funds data quite closely.

One difference between the two series is that the CRSP average uses all corporations and the Flow of Funds data are only for nonfinancial corporations. When the CRSP change is calculated including the merger-related retirements and using only nonfinancial corporations, the bottom panel, the correspondence to the Flow of Funds change is even closer.

Significantly, the change in shares calculated with the merger information and the Flow of Funds change have been trending down in recent years and both are negative at the end of the sample. Excluding the merger-related retirements, the change in shares has been trending up and is near the upper part of its historical range at the end of the sample.

**Regressions of the spread of the stock market over Treasury bills on the change in shares**

Table 2 presents the results of regressions of the subsequent five year stock return less the Treasury bill return on the change in shares. The relative stock return is expressed at an annual rate. The change calculated without the merger-related retirements (as in the top panel of figure 2) significantly forecasts the subsequent premium of the stock market over the return to Treasury bills. A one percentage point increase in the change in shares has historically presaged a 2-½ percentage point decline in the relative annual return to stocks over the following five years. In contrast, the change calculated including the merger-related
retirements does not significantly predict the relative stock returns.\(^9\)

This relationship can also be seen in figure 3 which shows the 5-year lagged change in shares and the 5-year relative return to stocks over bills. The change calculated without merger-related retirements is plotted in the top panel, and the change calculated with merger-related retirements is plotted in the bottom panel.

The difference in the performance of these two measures of the change in shares could help explain the conflict in the late 1990s between the level of the stock market and the net issuance of equity. By all measures, the stock market at that time appeared richly valued, but nonfinancial corporations were retiring a large amount of equity. A factor in the negative net equity issuance was the large volume of shares retired as a result of mergers. These results, and those presented below, suggest merger-related share retirements may have masked the impact of the level of the stock market on corporations’ decisions to issue or repurchase shares.

Regressions of the realized stock returns are of only limited utility in evaluating the effect of the level of the stock market on the change in shares. The five-year horizon is an arbitrary compromise chosen to overcome the difficulty of forecasting short-horizon returns, while not consuming a large portion of the sample. The next section examines the relationship between the change in shares and variation in the level of stock prices attributable to variation in the equity premium.

---

\(^9\) The change in shares by either measure is not logically the most desirable instrument to measure corporations’ reaction to their perception of the equity premium. More appropriate would be the change relative to the change in other corporate assets and liabilities. I have tried using the difference between the change in shares and the change in corporate assets adjusted for inflation, but the resulting series performs significantly worse than the change in shares.
Estimates of the equity premium using the earnings-ratio and the dividend-ratio model

In order to implement the formulas for the equity premium in equation 9, it is necessary to forecast the growth in earnings and the payout ratio, for the earnings-ratio model, and the growth in dividends, for the dividend-ratio model. The estimates of the VARs used to generate these forecasts are shown in table 3. The top panel reports the VAR used to estimate the earnings-ratio model, which includes the log earnings price ratio, the earnings growth rate minus the Treasury bill rate, the log payout ratio, and the change in shares outstanding (excluding merger-related retirements) to calculate the equity premium implied by the earnings-ratio model. The VAR includes one lag of each of the variables. Only one lag is included because chi-squared tests easily accept the exclusion of additional lags.

The lower panel presents the VAR used to estimate the dividend-ratio model. The VAR includes the log dividend-price ratio, the dividend growth rate minus the Treasury bill rate, the earnings growth rate minus the Treasury bill rate, and the change in shares outstanding. The earnings growth rate is included to make the information available to the two VARs as close as possible.

Figure 4 compares the estimates of the equity premium calculated using the two models. The top panel shows the log earnings-price ratio warranted by the sum of expected future earnings growth minus the Treasury bill return and future payout ratios under the assumption that the equity premium equals its sample average, and also the actual log earnings-price ratio. As can be seen in equation 9, the difference between the actual and predicted earnings-price ratio is the sum of the expected future spread of the stock market
over the Treasury bill return. The second panel shows the actual and predicted log dividend-price ratios. The two estimates of the equity premium are shown in the third panel, and the difference between the two estimates of the premium is shown at the bottom. Although the two models are equivalent under certainty, in a stochastic environment the differences between the forecast of dividend growth and the sum of the forecast of earnings growth and the payout ratio could result in different estimates of the equity premium. In fact, in part because the VARs use largely the same information, the estimates are quite similar.

The similarity of the two forecasts of expected returns suggests earnings and payout-ratios add little to the forecast of future dividends. The parameters that multiply the independent variables of the VARs to form the estimates of the equity premium, shown in table 4, reveal the reason for the similarity between the two estimates. In small part, the similarity arises because both models place roughly equal weight on the change in shares. A one percentage point increase in the change in shares corresponds to a 27 basis point decline in the equity premium calculated using the earnings-ratio model and a 20 basis point decline in the equity premium calculated using the dividend-ratio model. The similarity in the estimates of the equity premium owes more importantly, however, to the coefficients on the remaining variables. The contribution of the log earnings-price ratio, the growth in earnings, and the payout ratio to the estimate of the equity premium of the earnings-ratio model can be rewritten to match the contribution of the log dividend-price ratio, the growth in dividends, and the growth in earnings to the estimate of the equity premium of the dividend-ratio model.

---

10 Appendix B describes the calculation of the parameters that define the equity premium, and also defines the other statistics discussed in this section.
plus 0.01 times the growth in dividends. Since the growth in dividends varies only slightly over time, the estimates of the equity premium are nearly identical.

It may also be of interest to recast the coefficient on the change in shares in terms of the level of the stock market. Holding constant the other variables in the VAR, the estimates indicate a one percentage point increase in net share issuance would give rise to an increase of between 5 and 6 percent of the estimate of the difference between the current level of the stock market and what it would be if the equity premium were equal to its average level. That is, the market is likely 5 to 6 percent more overvalued for each percentage point increase in net share issuance, other things equal.

Recall that the definition of the equity premium here is the expectation of the weighted average future spreads of the stock market over Treasury bills. In order to evaluate the plausibility of the estimates of the equity premium, it is instructive to compare them to the empirical realization of the weighted average spread. Figure 5 compares the estimated equity premium from the earnings-ratio model with the realized spread of the stock market over Treasury bills. In the top panel, the realized spread is the discounted sum of the difference between the stock market and Treasury bill return, where each year the sum is calculated through the end of the sample and then scaled to equal the average spread. This method results in noisy estimates of the realized spread near the end of the sample, because the number of net returns entering the sum declines. To keep the scales between the top and bottom panel the same, I cut off several of the more extreme observations near the end of the sample in the top panel. In the bottom panel, the realized spread is calculated each year assuming that the net stock return equals its sample average after the end of the sample. This
technique results in an estimate that is too smooth toward the end of the sample. Both estimates, however, are quite similar toward the beginning of the sample, when the entries in the summation receiving significant weight are the same. During the first half of the sample, the estimated equity premium moves closely with both measures of the realized spread.

Both versions of the realized spread considered above suffer from the fact that the sample is finite. It is also possible to use the stochastic processes estimated by the VARs to calculate the fraction of the variation of the realized spread that is expected variation. The parameters of the VAR and the covariance matrix of the errors together permit an estimate of the variance of the equity premium and the variance of the realized spread; for each model, the ratio of these two variances are shown in the bottom of the two panels of table 4. The estimates of the earnings-ratio model indicate about one half of the variation in the realized spread is expected, and the estimates from the dividend-ratio model suggest three-fourths of the variation in the realized spread is expected. The difference arises from the estimates of the variance of the realized spread indicated by the two models.

Even though the two estimates of the equity premium are similar, the equity premium is more precisely estimated by the dividend-ratio model. The average conditional standard error of the estimate of the premium using the dividend ratio model is 0.21 percent, and the estimate for the earnings-ratio model is 0.37 percent. Evidently, the VAR estimates indicate it is significantly easier to forecast dividend growth than to forecast the sum of earnings growth and payout ratios.
The change in shares and the equity premium

Figure 6 plots the change in shares calculated without the merger-related changes and the estimates of the equity premium calculated using the earnings-ratio and dividend-ratio models. In both cases, there is a clear negative correlation between the change in shares and the equity premium. A regression of the change in shares on the equity premium, shown in table 5, confirms this impression. The top panel reports the results when the change in shares is calculated without the merger-related retirements. The results suggest a one percentage point decline in the equity premium is associated with a one percentage point increase in the change in shares. It requires a 24 percent change in the market to produce a one percentage point change in the equity premium, so the estimates suggest the stock market would have to increase or decrease by one-third to illicit a one standard deviation (1.4 percent) movement in the change in shares.

However, because the change in shares is included in the VAR, the equity premium estimates are themselves a function of the change in shares. Thus, it is not surprising that there is a relationship between the two. Furthermore, the standard errors of the regression coefficients need to be adjusted for the fact that the equity premium used in the regression are estimates, not data. The table also reports estimates of the coefficients and t-statistics from a regression of the change in shares on the equity premium calculated using the VAR estimates. The coefficient is the covariance of the two series divided by the variance of the equity premium, where the covariance and variance are calculated using the VAR parameter estimates, the covariance matrix of the parameters, and the covariance matrix of the data. The estimates calculated from the VAR coefficients are identical to the OLS estimates. The
standard error used to calculate the t-statistic is the quadratic form of the numerical
derivatives of the coefficient with respect to the parameter estimates, and the variance-
covariance matrix of the VAR parameters. The exact calculation of the statistics is described
in detail in Appendix B. Using the correct standard errors, only the coefficient estimated
using the dividend-ratio model is significant at the 5 percent level. The coefficient estimated
using the earnings-ratio model is significant at the 10 percent level.

The bottom panel of table 5 reports the results when the change in shares used in the
VAR and in the regression includes the merger-related retirement of shares. Using the correct
standard errors, shown at the right, the augmented change in shares is not significantly related
to either of the estimates of the equity premium.

The insignificance of the change in shares when merger-related retirements are
included suggests the merger-related retirements are brisk when the equity premium is low
(the market is overvalued) and are slow when the equity premium is high (the market is
undervalued). One simple way to evaluate this hypothesis without repeating the entire
analysis is to regress the percentage of corporations delisted owing to mergers on the
estimates of the equity premium used above. Table 6 presents the results of such a
regression. The dependent variable is the market value of corporations whose stock was
delisted as a result of a merger or exchange as a percent of the market value of all
corporations in the sample at that time. The measure of the equity premium are the estimates
from the earnings-ratio and the dividend-ratio model estimated using the change in shares that
excludes the merger-related retirements. The regression is not corrected for the fact that the
equity premium is an estimate. The results suggest that for each percentage point increase in
the equity premium, the percentage of the market delisted owing to mergers declines by a little less than half a percentage point. That is, the estimates indicate there are more merger-related reductions in shares when the market is overvalued, the opposite of the relationship between non-merger related changes in shares and the level of the market.\textsuperscript{11}

**Conclusion**

The results indicate that the change in shares is negatively correlated with the equity premium, but also suggest the earnings-ratio model is of limited utility. Nevertheless, there are several hypotheses that might be better examined with the earnings-ratio model than the dividend-ratio model: What is the relationship between expected economic activity and expected returns to the stock market? Do firms adjust their payout-ratio in response to variation in the expected returns? Does retention of earnings lead to faster earnings growth? How do market participants value such retentions? Finally, the earnings-price ratio is often of more interest than the dividend price ratio. The earnings-ratio model may be of heuristic value simply for this reason alone.\textsuperscript{12}

The significance of the relationship between the change in shares and the equity premium suggests the level of the stock market influences corporate decisions at least with regard to equity activity. The apparent increase in equity issuance when the expected return

\textsuperscript{11} The literature is somewhat divided on this point. Scherer and Ross note that although there had been general agreement that merger activity and stock prices were positively correlated, the the merger wave of the 1980s appeared to coincide with low, not high, stock prices (see Scherer and Ross (1990), chapter 5).

\textsuperscript{12} For example, Sharpe (1998) uses the earnings-ratio model in conjunction with analysts' earnings forecasts to examine the relationship between expected stock returns and inflation.
on equity is low relative to the Treasury bill rate is reminiscent of the mechanisms relating financial flows and interest rates considered by Brainard and Tobin (1968) "Pitfalls in Financial Model Building." In that paper, the authors introduce the idea that capital investment should be positively related to the ratio of the market value to the replacement cost of capital \( q \). The results reported here indicate a possible first step in the linkage between investment and \( q \)—that corporations issuance and repurchases of equity respond to the level of the stock market—appears to hold true in aggregate.

The analysis in this paper does not address the important second step in the linkage—that corporations adjust their capital investment in response to their issuance or repurchase. Corporations may adjust only other financial variables, not physical capital, in which case the consequences for economic welfare are limited.\(^{13}\)

The significance of the relationship between the change in shares and the equity premium also supports the dividend-ratio model methodology as a mechanism for identifying the component of stock prices attributable to variation in expected returns. Alternative explanations for the apparent variation in the dividend-price ratio in excess of what can be accounted for by changes in expected future dividend growth need now also to address why the unexplained component is negatively correlated with the change in shares.

Finally, the lack of significance when merger-related retirements are included helps explain why during the first half of 1998 nonfinancial corporations retired equity at a $118 billion annual rate even though the market appeared richly valued: Retirements related to

\(^{13}\) As shown in Nelson (1999b), changes in shares owing to changes in capital investment do predict stock returns in the cross section, which suggests capital investment is affected when corporations issue or repurchase stock in response to deviations of their stock price from its fundamental value.
mergers occurred at a $159 billion rate; excluding mergers net equity issuance of nonfinancial corporations was at a $41 billion rate, about the level of issuance excluding mergers that has prevailed during most of the 1990s. Indeed, since the results suggest the pace of merger activity apparently picks up when the equity premium is low, the positive non-merger related net issuance and heavy merger-related retirements is a combination doubly indicative of overvaluation.
Table 1
Descriptive Statistics
Sample: 1926 to 1997

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Earnings-Price Ratio</td>
<td>ε</td>
<td>-2.63</td>
<td>0.34</td>
</tr>
<tr>
<td>Growth in Earnings Less the Treasury Bill Rate</td>
<td>Δe-t^b</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>Log Payout Ratio</td>
<td>φ</td>
<td>-0.57</td>
<td>0.26</td>
</tr>
<tr>
<td>Change in Shares</td>
<td>Δs</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Log Dividend-Price Ratio</td>
<td>δ</td>
<td>-3.20</td>
<td>0.33</td>
</tr>
<tr>
<td>Growth in Dividends Less the Treasury Bill Rate</td>
<td>Δd-t^b</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlations</th>
<th>ε</th>
<th>Δe-t^b</th>
<th>φ</th>
<th>Δs</th>
<th>δ</th>
<th>Δd-t^b</th>
</tr>
</thead>
<tbody>
<tr>
<td>ε</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δe-t^b</td>
<td>0.17</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>φ</td>
<td>-0.39</td>
<td>-0.43</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δs</td>
<td>-0.06</td>
<td>-0.08</td>
<td>-0.08</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>δ</td>
<td>0.71</td>
<td>-0.16</td>
<td>0.37</td>
<td>-0.13</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Δd-t^b</td>
<td>0.24</td>
<td>0.65</td>
<td>-0.32</td>
<td>0.13</td>
<td>-0.01</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: All data are annual and are not in percent. The stock price is the January level of the Standard and Poor Composite Stock Price Index and the dividends and earnings are the twelve-month average of the per-share levels. The data on the Treasury bill rate are from Ibbotson (1997) and are the annual return to rolling over each month the shortest Treasury bill available having a maturity of at least one month. The data on change in shares is the market-weighted average of the change in shares of corporations included listed on the NYSE and AMEX, beginning in 1962, the NASDAQ.

22
Table 2
Regression of Five-Year Market Premium on the Change in Shares
For Various Definitions of the Change in Shares

Dependent Variable: Five-year market-weighted stock return minus the return to rolling over Treasury bills (percent, AR).

<table>
<thead>
<tr>
<th>Definition of Change in Shares</th>
<th>Constant</th>
<th>Change in Shares Lagged 5 Years (Percent)</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merger-related retirements excluded</td>
<td>14.94 (6.73)</td>
<td>-2.48 (-3.23)</td>
<td>0.12</td>
</tr>
<tr>
<td>Merger-related retirements included</td>
<td>11.10 (5.70)</td>
<td>-1.12 (-1.17)</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Note: The data are defined in table 1. The t-statistics—in parentheses—are corrected for the four-year overlap in the relative stock return using the Newey-West correction.
### Table 3
Coefficient (t-Statistic) Estimates from VARs

<table>
<thead>
<tr>
<th>Earnings-Ratio Model</th>
<th>Dependent Variables</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ε&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Δε&lt;sub&gt;t&lt;/sub&gt;-R&lt;sub&gt;ε&lt;/sub&gt;</td>
<td>φ&lt;sub&gt;t&lt;/sub&gt;</td>
<td>ΔS&lt;sub&gt;t&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td>ε&lt;sub&gt;εt-l&lt;/sub&gt;</td>
<td>0.72</td>
<td>-0.22</td>
<td>0.13</td>
<td>-0.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7.88)</td>
<td>(-3.03)</td>
<td>(2.31)</td>
<td>(-0.74)</td>
<td></td>
</tr>
<tr>
<td>Δε&lt;sub&gt;εt-l&lt;/sub&gt;-R&lt;sub&gt;εt-l&lt;/sub&gt;</td>
<td>0.32</td>
<td>0.28</td>
<td>-0.07</td>
<td>-0.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.07)</td>
<td>(2.24)</td>
<td>(-0.77)</td>
<td>(-0.25)</td>
<td></td>
</tr>
<tr>
<td>φ&lt;sub&gt;εt-l&lt;/sub&gt;</td>
<td>0.05</td>
<td>-0.02</td>
<td>0.90</td>
<td>-0.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.35)</td>
<td>(-0.17)</td>
<td>(11.04)</td>
<td>(-1.77)</td>
<td></td>
</tr>
<tr>
<td>ΔS&lt;sub&gt;εt-l&lt;/sub&gt;</td>
<td>2.75</td>
<td>-2.08</td>
<td>1.58</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.53)</td>
<td>(-1.47)</td>
<td>(1.44)</td>
<td>(5.49)</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.56</td>
<td>0.20</td>
<td>0.72</td>
<td>0.35</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dividend-Ratio Model</th>
<th>Dependent Variables</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>δ&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Δd&lt;sub&gt;t&lt;/sub&gt;-R&lt;sub&gt;d&lt;/sub&gt;</td>
<td>Δε&lt;sub&gt;t&lt;/sub&gt;-R&lt;sub&gt;ε&lt;/sub&gt;</td>
<td>ΔS&lt;sub&gt;t&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td>δ&lt;sub&gt;εt-l&lt;/sub&gt;</td>
<td>0.85</td>
<td>-0.10</td>
<td>-0.16</td>
<td>-0.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(11.19)</td>
<td>(-2.28)</td>
<td>(-2.28)</td>
<td>(-1.21)</td>
<td></td>
</tr>
<tr>
<td>Δd&lt;sub&gt;εt-l&lt;/sub&gt;-R&lt;sub&gt;εt-l&lt;/sub&gt;</td>
<td>0.39</td>
<td>-0.01</td>
<td>-0.32</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.48)</td>
<td>(-0.07)</td>
<td>(-1.28)</td>
<td>(0.50)</td>
<td></td>
</tr>
<tr>
<td>Δε&lt;sub&gt;εt-l&lt;/sub&gt;-R&lt;sub&gt;εt-l&lt;/sub&gt;</td>
<td>0.04</td>
<td>0.22</td>
<td>0.30</td>
<td>-0.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.21)</td>
<td>(2.47)</td>
<td>(1.97)</td>
<td>(-0.09)</td>
<td></td>
</tr>
<tr>
<td>ΔS&lt;sub&gt;εt-l&lt;/sub&gt;</td>
<td>3.62</td>
<td>-0.45</td>
<td>-1.82</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.28)</td>
<td>(-0.51)</td>
<td>(-1.22)</td>
<td>(5.24)</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.67</td>
<td>0.23</td>
<td>0.18</td>
<td>0.34</td>
<td></td>
</tr>
</tbody>
</table>

Note: The data are defined in Table 1. The data are annual and the sample is 1927 to 1997. The sample means of the variables have been subtracted.
### Table 4
Statistics on the Estimates of the Equity Premium

#### Earnings-Ratio Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\varepsilon_t$</th>
<th>$\Delta E_{r-p}^t$</th>
<th>$\varphi_t$</th>
<th>$\Delta s_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters of the Equity Premium</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>-0.27</td>
</tr>
<tr>
<td></td>
<td>(2.37)</td>
<td>(0.89)</td>
<td>(1.31)</td>
<td>(2.70)</td>
</tr>
</tbody>
</table>

**Fraction of Variance of Realized Spread Explained by Variation in Equity Premium**: 0.53

**Standard Deviation of Realized Spread (Percent)**: 1.13

**Time-Series Average of Conditional Standard Error of Equity Premium (Percent)**: 0.39

#### Dividend-Ratio Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\delta_t$</th>
<th>$\Delta d_{r-p}^t$</th>
<th>$\Delta e_{r-p}^t$</th>
<th>$\Delta s_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters of the Equity Premium</td>
<td>0.02</td>
<td>-0.01</td>
<td>0.01</td>
<td>-0.20</td>
</tr>
<tr>
<td></td>
<td>(3.85)</td>
<td>(2.20)</td>
<td>(2.44)</td>
<td>(2.63)</td>
</tr>
</tbody>
</table>

**Fraction of Variance of Realized Spread Explained by Variation in Equity Premium**: 0.76

**Standard Deviation of Realized Spread (Percent)**: 1.00

**Time-Series Average of Conditional Standard Error of Equity Premium (Percent)**: 0.21

Note: The data are defined in table 1. The equity premium is the expected discounted future difference between the stock return and the Treasury bill return. The estimate of the premium is a function of the independent variable of the VAR, using the parameters shown. The realized spread is the realized discounted difference between the stock return and the Treasury bill return. The statistics reported are defined in appendix B.
Table 5
Regression Coefficient (t-Statistic) of
Change in Shares on Equity Premium

<table>
<thead>
<tr>
<th>Model Used To Generate Equity Premium</th>
<th>OLS Regression of Change in Shares on Estimated Equity Premium</th>
<th>Coefficient and Standard Error Calculated from VAR Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Change in Shares Calculated Without Merger-Related Retirements</td>
<td></td>
</tr>
<tr>
<td>Earnings-Ratio Model</td>
<td>-1.17</td>
<td>-1.17</td>
</tr>
<tr>
<td></td>
<td>(-6.73)</td>
<td>(-1.65)</td>
</tr>
<tr>
<td>Dividend-Ratio Model</td>
<td>-0.91</td>
<td>-0.91</td>
</tr>
<tr>
<td></td>
<td>(-5.05)</td>
<td>(-2.40)</td>
</tr>
<tr>
<td></td>
<td>Change in Shares Calculated With Merger-Related Retirements</td>
<td></td>
</tr>
<tr>
<td>Earnings-Ratio Model</td>
<td>-1.50</td>
<td>-1.50</td>
</tr>
<tr>
<td></td>
<td>(-4.68)</td>
<td>(-0.72)</td>
</tr>
<tr>
<td>Dividend-Ratio Model</td>
<td>-0.40</td>
<td>-0.40</td>
</tr>
<tr>
<td></td>
<td>(-1.31)</td>
<td>(-0.53)</td>
</tr>
</tbody>
</table>

Note: The data are defined in Table 1. The OLS estimates are the coefficient and t-statistic from a regression of the change in shares outstanding on the estimate of the equity premium, uncorrected for the fact that the equity premium is an estimate based in part on the change in shares. The estimates calculated from the VAR estimates correct for the constructed nature of the equity premium, and are defined in Appendix B.
Table 6
Regression of the Market Value of Firms Delisted Owing to Mergers
As a Percent of Total Market Value on the Equity Premium

<table>
<thead>
<tr>
<th>Model Used To Generate Equity Premium</th>
<th>OLS Coefficient (t-Statistic)</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings-Ratio Model</td>
<td>-0.48 (2.48)</td>
<td>0.08</td>
</tr>
<tr>
<td>Dividend-Ratio Model</td>
<td>-0.46 (2.45)</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Note: The data are defined in table 1. The estimates are the coefficient and t-statistic from a regression of the market value of corporations whose stock was delisted as a result of a merger or exchange as a percentage of the market value of all corporations in the sample at that time on the estimate of the equity premium. The equity premium is an estimate of the expected discounted difference between the return to the market and the return to Treasury bills. The statistics are uncorrected for the fact that the equity premium is an estimate.
Figure 1

Stock Market Data

Log Earnings Price Ratio

Growth in Earnings Minus T-Bill Return

Log Payout Ratio
Figure 1 (cont.)

Stock Market Data

Change in Shares

Log Dividend Price Ratio

Growth in Dividends Minus T-Bill Return
Figure 2

FOF v. CRSP Change in Shares

No Mergers Included in CRSP Change
CRSP, All Corporations

Mergers Included in CRSP Change
CRSP, All Corporations

Mergers Included in CRSP Change
CRSP, Nonfinancial Corps.
Figure 3

Equity Premium(t) and Change in Shares(t-5)

No Mergers

With Mergers
Figure 4

Earnings-Ratio v. Dividend-Ratio Model

Log Earnings-Price Ratio

Log Dividend-Price Ratio

Implied Expected Equity Premium

Difference Between The Expected Premia
Figure 5

Ex-Ante Premium v. Realized Spread

No Post Sample Data

Post Sample Data=Mean
Figure 6
Change in Shares and The Equity Premium

Dividend-Ratio Model

Earnings-Ratio Model
Appendix A: “Gordon” formula for the earnings-price ratio

The Gordon formula states that a share currently paying a dividend of $D_0$ which is expected to grow at rate $g$ should have a price of

$$P_0 = \int_{t=0}^{\infty} e^{(g-r)t} D_0 \, dt = \frac{1}{r-g} D_0; \quad (1)$$

where $r$ is the discount rate appropriate for the stream of payments.

Denote the payout ratio, D/E, by $\Phi$. If the payout ratio is constant, then since $D_t = \Phi E_t$ the growth in dividends and earnings will be equal and the Gordon formula implies

$$\frac{P_0}{E_0} = \frac{1}{r-g} \Phi. \quad (2)$$

Equation (2) shows that of two firms with the same discount rate and the same level and growth rate in earnings, the firm with a higher payout ratio is more valuable. The intuition is straightforward − earnings growth can be achieved in two ways: assets can be accumulated or assets can become more profitable. Of the two, the second is more valuable since it is achieved without additional expense.

In many situations, it is not realistic to assume that the payout ratio would be constant. On the other hand, assuming that the payout ratio grows at an exponential rate (in order to get a closed form solution) is equally unrealistic. A natural specification that allows for a simple solution is that the deviation of the payout ratio from a normal level decays exponentially. Letting $\Phi$ be the normal level, then
\[ \Phi_t = \Phi - e^{-\beta}(\Phi - \Phi_0). \]  

(3)

where \( f \) measures the speed at which the payout ratio returns to its normal level. Letting \( g \) now refer to the growth in earnings and using the above specification for the payout ratio, dividends at time \( t \) are

\[ D_t = \Phi_t E_t = (\Phi - e^{-\beta}(\Phi - \Phi_0)) E_0 e^{gt}. \]  

(4)

Setting the price equal to the present value of the stream of dividends yields

\[ P_0 = \int_{t=0}^{\infty} e^{(g-r)t} \Phi E_0 e^{-r(1-g)(\Phi - \Phi_0)E_0} dt. \]  

(5)

Provided earnings are not growing faster than the discount rate, solving the integral gives

\[ P_0 = \left( \frac{1}{r-g} \Phi - \frac{1}{r+f-g}(\Phi - \Phi_0) \right) E_0. \]  

(6)

Finally, dividing by initial earnings yields an expression for the price-earnings ratio:

\[ \frac{P_0}{E_0} = \frac{1}{r-g} \Phi - \frac{1}{r+f-g}(\Phi - \Phi_0). \]  

(7)

This formula has the same intuition as that in the simpler case where the payout ratio is constant. For a given path of the payout ratio, the higher the growth in earnings, the more valuable the firm. For a given growth in earnings, the higher the payout ratio, the more valuable the firm. In addition, for a payout ratio currently below normal, the faster it rises to

36
normal, the more valuable the path of earnings.

The formula indicates the behavior of the payout ratio can have a profound effect on the price warranted by a given stream of earnings. Consider the following example. If earnings are growing at 5 percent per year, the discount rate is 10 percent and the payout ratio equals the normal payout ratio of 50 percent, the warranted price-earnings ratio is 10. If the payout ratio is currently zero, and approaches the normal rate with a half-life of ten years ($f=6.7\%$) the warranted price-earnings ratio is 5.7. Ignoring the payout ratio results in a price which is nearly twice the correct price.
Appendix B: Derivation of the Statistics

It is convenient at this point to write the VARs in companion form.\textsuperscript{14} For a VAR with \( n \) variables and \( m \) lags, let \( z_t \) be a vector with \( n \times m \) elements where the first \( m \) elements of \( z \) are the current and \( m-1 \) lags of the first variable in the VAR, the next \( m \) elements are the current and \( m-1 \) lags of the second variable, and so forth. With this transformation, a VAR of arbitrary order can be rewritten as a first-order VAR with

\[
z_t = A z_{t-1} + u_t,
\]

(1)

where the matrix \( A \) consists of the coefficients from the VAR for those rows corresponding to the current values of the variables, and of suitably placed 0s and 1s for those rows with lagged variables. Note that the expectation of \( z_{i,t} \) at time \( t \) is \( A^k z_t \). Furthermore, let \( e_x \) be a \( n \times m \) unit vector with a 1 in the row that corresponds to the first entry of variable \( x \).

It will help to define several variables. Let \( q \) be the equity premium: the expected discounted sum of the difference between the stock return and the Treasury bill return, all multiplied by \( (1-\rho) \).

\[
q_t = (1-\rho) E_t \sum_{j=1}^{\infty} \rho^{j-1} \left( r_{t+i}^m - r_{t+i}^b \right)
\]

(2)

The definition of the equity premium (equation 9 in the text) can be written using the companion form notation after solving the convergent sums as

\textsuperscript{14} The notation here closely follows Campbell and Shiller (1989).
\[(Earnings-Ratio) \quad q_t = (1-\rho)(e_{\bar{e}} + e_{\Delta d-e} A(I-\rho A)^{-1})z_t \]
\[(Dividend-Ratio) \quad q_t = (1-\rho)(e_{\bar{e}} + e_{\Delta d-e} A(I-\rho A)^{-1})z_t. \quad (3)\]

Let \( f \) be the vector that when multiplied by the vector \( z \), yields the premium,

\[(Earnings-Ratio) \quad f = (1-\rho)(e_{\bar{e}} + e_{\Delta d-e} A(I-\rho A)^{-1}) \]
\[(Dividend-Ratio) \quad f = (1-\rho)(e_{\bar{e}} + e_{\Delta d-e} A(I-\rho A)^{-1}). \quad (4)\]

Then equation 2 can be rewritten as

\[q_t = f'z_t. \quad (5)\]

The covariance of the change in shares with the equity premium is

\[E q_t \Delta s_t = E f'z_t \Delta s_t. \quad (6)\]

Let \( \theta \) equal the covariance matrix of the data, then, equation 6 simplifies to

\[E q_t \Delta s_t = f'\theta \Delta s_t. \quad (7)\]

The variance of the equity premium is

\[E q_t^2 = f'\theta f. \quad (8)\]

The coefficient estimate of the regression of the change in shares on the equity premium is
the ratio of the covariance of the two (equation 7) to the variance of the premium (equation 8). The parameter, call it $b$, is a highly nonlinear function of the parameters of the VAR (call them $a$). Letting $\Omega$ be the variance-covariance matrix of the parameters, the variance of $b$ is $b_d(a)^T \Omega b_d(a)$. I calculate the derivative of $b$ with respect to $a$ numerically.\textsuperscript{15} Similarly, the variance of the parameters that define the equity premium equal $f_d(a)^T \Omega f_d(a)$, where $f_d(a)$ is the derivative with respect to the parameters of the VAR of the vector that when multiplied by $z$ yields the equity premium. The conditional variance of the estimate of the equity premium resulting from imprecision in the estimation of the VAR parameters is

$$z_t^T f_a(a) \Omega f_a(a) z_t. \quad (9)$$

**Variance of the realized spread of the stock return over the Treasury bill return**

Recall that the definition of the equity premium here is the expectation of the weighted average future spreads of the stock market over Treasury bills. Define the realized spread to be the realization of the weighted average spread; that is, the variable of which the equity premium is the expectation. The variance of the realized spread can be calculated using the estimates of the VARs. It is simpler to derive the variance of the realized spread as estimated by the dividend-ratio model, the result using the earnings-ratio model can then be seen by extension. The variance of the realized spread is the variance of the equity premium plus the variance of the forecast error. The variance of the equity premium is derived above

\textsuperscript{15} Campbell and Shiller (1989) test the restriction that the expected market return equals a constant times the expected growth in consumption. I am following the same technique here to estimate the standard error of $b$ that they follow to estimate the standard error of the constant that multiplies consumption.
in equation 8. The variance of the forecast error is the expected value of the realized less the expected premium, squared:

\[
(1-\rho)^2 E \left\{ \sum_{j=1}^{\infty} \rho^{j-1} \left( r_{t+j} - r_{t+j}^b \right) \right\} \\
- E_i \sum_{j=1}^{\infty} \rho^{j-1} \left( r_{t+j}^m - r_{t+j}^b \right)^2 \\
= (1-\rho)^2 E \left\{ \delta_i \sum_{j=1}^{\infty} \rho^{j-1} \left( \Delta d_{t+j} - r_{t+j}^b \right) \right\} \\
- \delta_i - E_i \sum_{j=1}^{\infty} \rho^{j-1} \left( \Delta d_{t+j} - r_{t+j}^b \right)^2 .
\]  

(10)

Returning to the companion form notation, using the fact that

\[
z_{t+1} = A_{t} z_t + A_{t-1} u_{t-1} + A_{t-2} u_{t-2} + \cdots + A_{t-1} u_{t-1} + u_{t+1},
\]

and dropping everything known at time \( t \), the variance can be written as

\[
\sigma^2_q = (1-\rho)^2 E \left( e_{\Delta d-t}^f \sum_{i=1}^{\infty} \rho^i \left( \sum_{j=1}^{i} A_{t+j} u_{t+j} \right) \right)^2.
\]  

(11)

The summation in equation 12 can be rewritten as

\[
\sigma^2_q = (1-\rho)^2 E \left( e_{\Delta d-t}^f \sum_{i=0}^{\infty} \rho^i \sum_{j=0}^{\infty} (\rho A)^j u_{t+1+j} \right)^2.
\]  

(13)

Given the assumed independence of the \( u \)'s, the expectation of the cross terms will be zero, so I ignore them. Squaring the remaining terms within the parentheses yields:
\[ \sigma_q^2 = (1-\rho)^2 E e^{\Delta_d^b} \left( \sum_{i=0}^{\infty} \rho^{2i} \sum_{j=0}^{\infty} \rho^{2(\cdot)} (\rho A)^j u_{i+1} u_{i+1} (\rho A)^j e_{\Delta_d^b}. \right. \]  

(14)

Taking the expectation and letting \( \Xi \) be the covariance matrix of the residuals yields:

\[ \sigma_q^2 = (1-\rho)^2 e^{\Delta_d^b} \left( \sum_{i=0}^{\infty} \rho^{2i} \sum_{j=0}^{\infty} (\rho A)^j \Xi (\rho A)^j e_{\Delta_d^b}. \right. \]  

(15)

Let

\[ B = \sum_{j=0}^{\infty} (\rho A)^j \Xi (\rho A)^j. \]  

(16)

Then

\[ \sigma_q^2 = (1-\rho)^2 e^{\Delta_d^b} \left( \sum_{i=0}^{\infty} \rho^{2i} B \right) e_{\Delta_d^b}. \]  

(17)

Solving the convergent sum,

\[ \sigma_q^2 = \frac{(1-\rho)^2}{(1-\rho^2)} e^{\Delta_d^b} B e_{\Delta_d^b}. \]  

(18)
Equation 18 is the formula for the variance of the conditional forecast error of the realized spread defined by the dividend-ratio model. The formula for the variance of the conditional forecast error the realized spread defined by the earnings-ratio model is derived by replacing $e_{\Delta q_t}$ with $e_{\Delta q_t} + (1 - \rho) e_{q_t}$. The matrix B does not have a closed form solution. I calculate it by adding a large number of the terms in the summation, which converge quickly; the remaining terms are negligible.
References


